## Appendix A



Figure A.1: Pointwise 68 percent real wage responses intervals to monetary shocks, only monetary shocks identified.


Figure A2: Pointwise 68 percent real wage response intervals to monetary shocks, all shocks identified.


Figure A.3: Pointwise 68 percent real wage response intervals to monetary shocks, VAR chosen with BIC.

|  | Flexible price <br> sticky wage model |  |  |  | Sticky price <br> flexible price model |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Standard deviation of <br> monetary shocks 10 <br> times larger | Standard deviation of <br> markup shocks 10 <br> times larger | Basic |  |  |  |  |  |  |
| Horizon | $\mathrm{T}=80$ | $\mathrm{~T}=160$ | $\mathrm{~T}=500$ | $\mathrm{~T}=80$ | $\mathrm{~T}=160$ | $\mathrm{~T}=500$ | $\mathrm{~T}=80$ | $\mathrm{~T}=160$ | $\mathrm{~T}=500$ |
| 0 | 90 | 90 | 90 | 50 | 50 | 49 | 100 | 100 | 100 |
| 1 | 80 | 86 | 88 | 64 | 74 | 77 | 75 | 85 | 94 |
| 2 | 78 | 80 | 86 | 64 | 73 | 81 | 68 | 76 | 83 |
| 3 | 72 | 76 | 83 | 62 | 73 | 80 | 61 | 69 | 78 |
| 4 | 68 | 72 | 81 | 59 | 72 | 83 | 59 | 63 | 70 |

Table A.1: Percentages of correctly signed real wage responses to monetary shocks; median value across 200 Monte Carlo replications. In all panels the VAR has two lags and includes output, real wages, hours, inflation and the nominal rate.

|  | 2 lags |  |  | 4 lags |  |  | 10 lags |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Horizon | $\mathrm{T}=80$ | $\mathrm{~T}=160$ | $\mathrm{~T}=500$ | $\mathrm{~T}=80$ | $\mathrm{~T}=160$ | $\mathrm{~T}=500$ | $\mathrm{~T}=80$ | $\mathrm{~T}=160$ | $\mathrm{~T}=500$ |
| 0 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 1 | 82 | 89 | 97 | 78 | 86 | 95 | 76 | 87 | 96 |
| 2 | 75 | 78 | 90 | 63 | 66 | 85 | 60 | 71 | 83 |
| 3 | 65 | 69 | 84 | 53 | 59 | 70 | 52 | 57 | 72 |
| 4 | 60 | 61 | 76 | 59 | 55 | 63 | 47 | 54 | 59 |

Table A.2: Percentages of correctly signed real wage responses to monetary shocks; median value across 200 Monte Carlo replications. The DGP is the sticky prices, flexible wage model; the VAR includes output, inflation, nominal rate and hours. The correct representation of the $\operatorname{DGP}$ is a $\operatorname{VAR}(2)$.

## Appendix B

The Smets and Wouter's (2003) class of models features nominal frictions (sticky nominal wage and price setting, backward wage and inflation indexation), real frictions (habit formation in consumption, investment adjustment costs, variable capital utilization and fixed costs in production). The class has three blocks and its log-linearized representation (around the steady state) is as follows. The aggregate demand block is:

$$
\begin{equation*}
y_{t}=c_{y} c_{t}+i_{y} i_{t}+g_{y} e_{t}^{g} \tag{24}
\end{equation*}
$$

$$
\begin{equation*}
c_{t}=\frac{h}{1+h} c_{t-1}+\frac{1}{1+h} E_{t} c_{t+1}-\frac{1-h}{(1+h) \sigma_{c}}\left(R_{t}-E_{t} \pi_{t+1}\right)+\frac{1-h}{(1+h) \sigma_{c}}\left(e_{t}^{b}-E_{t} e_{t+1}^{b}\right) \tag{25}
\end{equation*}
$$

$$
\begin{equation*}
i_{t}=\frac{1}{1+\beta} i_{t-1}+\frac{\beta}{1+\beta} E_{t} i_{t+1}+\frac{\phi}{1+\beta} q_{t}-\frac{\beta E_{t} e_{t+1}^{I}-e_{t}^{I}}{1+\beta} \tag{26}
\end{equation*}
$$

$$
\begin{equation*}
q_{t}=\beta(1-\delta) E_{t} q_{t+1}-\left(R_{t}-E_{t} \pi_{t+1}\right)+(1-\beta(1-\delta)) E_{t} r_{t+1} \tag{27}
\end{equation*}
$$

Equation (24) is the aggregate resource constraint. Total output, $y_{t}$, is absorbed by consumption, $c_{t}$, investment, $i_{t}$, and exogenous government spending, $e_{t}^{g}$. Equation (25) is a dynamic IS curve: $e_{t}^{b}$ is a preference shock, $\sigma_{c}$ the coefficient of relative risk aversion and $h$ the coefficient of external habit formation. The dynamics of investment are in equation (26); $\phi$ represents the elasticity of the costs of adjusting investments, $q_{t}$ the value of existing capital, $e_{t}^{I}$ a shock to the investment's adjustment cost function and $\beta$ the discount factor. Equation (27) characterizes Tobin's q: the current value of the capital stock positively depends on its expected future value and its expected return, and negatively on the ex-ante real interest rate. The aggregate supply block is:

$$
\begin{equation*}
y_{t}=\omega\left(\alpha k_{t-1}+\alpha \psi r_{t}+(1-\alpha) n_{t}+e_{t}^{z}\right) \tag{28}
\end{equation*}
$$

$$
\begin{gather*}
k_{t}=(1-\delta) k_{t-1}+\delta i_{t} \\
\pi_{t}=\frac{\beta}{1+\beta \mu_{p}} E_{t} \pi_{t+1}+\frac{\mu_{p}}{1+\beta \mu_{p}} \pi_{t-1}+\kappa_{p} m c_{t} \\
w_{t}=\frac{\beta}{1+\beta} E_{t} w_{t+1}+\frac{1}{1+\beta} w_{t-1}+\frac{\beta}{1+\beta} E_{t} \pi_{t+1}-\frac{1+\beta \mu_{w}}{1+\beta} \pi_{t}+\frac{\mu_{w}}{1+\beta} \pi_{t-1}-\kappa_{w} \mu_{t}^{W}  \tag{31}\\
n_{t}=-w_{t}+(1+\psi) r_{t}^{k}+k_{t-1} \tag{32}
\end{gather*}
$$

Equation (28) is the aggregate production function. In equilibrium $\psi r_{t}$ equals the capital utilization rate and $e_{t}^{z}$ is a total factor productivity (TFP) shock. Fixed costs of production are given by $\omega-1$ and $\alpha$ is the capital share. The law of motion of capital accumulation is in equation (29). Equation (30) links inflation to marginal costs, $m c_{t}=\alpha r_{t}^{k}+(1-\alpha) w_{t}-e_{t}^{z}+e_{t}^{\mu_{p}}$ and $e_{t}^{\mu_{p}}$ is a markup shock. The parameter $\kappa_{p}=\frac{1}{1+\beta \mu_{p}} \frac{\left(1-\beta \zeta_{p}\right)\left(1-\zeta_{p}\right)}{\zeta_{p}}$, is the slope of the Phillips curve and depends on $\zeta_{p}$, the probability that firms face for not being able to change prices in the Calvo setting. The parameter $\mu_{p}$ determines the degree of price indexation. Equation (31) links the real wage to expected and past wages, to inflation and to the marginal rate of substitution between consumption and leisure, $\mu_{t}^{W}=w_{t}-\sigma_{l} n_{t}-\frac{\sigma_{c}}{1-h}\left(c_{t}-h c_{t-1}\right)-e_{t}^{\mu_{w}}$, where $\sigma_{l}$ is the inverse of the elasticity of hours to the real wage, $e_{t}^{\mu_{w}}$ a labor supply shock and $\kappa_{w}=\frac{1}{1+\beta} \frac{\left(1-\beta \zeta_{w}\right)\left(1-\zeta_{w}\right)}{\left(1+\frac{\left(1+\varepsilon^{w}\right)^{\prime} \sigma^{\prime}}{\varepsilon}\right) \zeta_{w}}$. Equation (32) follows from the equalization of marginal costs. The monetary rule is

$$
\begin{equation*}
R_{t}=\rho_{R} R_{t-1}+\left(1-\rho_{R}\right)\left(\gamma_{\pi} \pi_{t}+\gamma_{y} y_{t}\right)+e_{t}^{R} \tag{33}
\end{equation*}
$$

where $\varepsilon_{t}^{R}$ is a monetary policy shock.
Equations (24) to (33) define a system of 10 equations in ten unknowns, $\left(\pi_{t}, y_{t}, c_{t}, i_{t}, q_{t}, l_{t}, w_{t}\right.$, $\left.k_{t}, r_{t}, R_{t}\right)$. The model features seven exogenous disturbances: TFP, $e_{t}^{z}$, investment-specific, $e_{t}^{I}$, preference, $e_{t}^{b}$, government spending, $e_{t}^{g}$, monetary policy, $e_{t}^{R}$, price markup $e_{t}^{\mu_{p}}$ and labor supply, $e_{t}^{\mu_{w}}$ shocks. The vector of disturbances $S_{t}=\left[e_{t}^{z}, e_{t}^{I}, e_{t}^{b}, e_{t}^{g}, e_{t}^{R}, e_{t}^{\mu_{p}}, e_{t}^{\mu_{w}}\right]^{\prime}$, satisfies:

$$
\begin{equation*}
\log \left(S_{t}\right)=(I-\varrho) \log (\bar{S})+\varrho \log \left(S_{t-1}\right)+V_{t} \tag{34}
\end{equation*}
$$

where $V \sim \operatorname{iid}\left(0^{\prime}, \Sigma_{v}\right), \varrho$ is diagonal with roots less than one in absolute value and $\bar{S}=E(S)$.
In table B. 1 we present the intervals used to compute robust restrictions presented in table 5 together with the parameters for the DGP used in section 4.4.

| Parameter | Description | Support | DGP |
| :--- | :--- | :---: | :---: |
| $\sigma_{c}$ | Risk aversion coefficient | $[1,6]$ | 2 |
| $\sigma_{l}$ | Inverse Frish labor supply elasticity | $[0.5,4.0]$ | 1.9 |
| $h$ | Consumption habit | $[0.1,0.8]$ | 0.7 |
| $\omega$ | Fixed cost | $[1.0,1.80]$ | 1.2 |
| $\phi$ | Adjustment cost parameter | $[0.0001,0.02]$ | 0.018 |
| $\delta$ | Capital depreciation rate | $[0.015,0.03]$ | 0.025 |
| $\alpha$ | Capital share | $[0.1,0.05]$ | 0.3 |
| $\sigma^{2}$ | Capacity utilization elasticity | 0.5 |  |
| $\psi$ | Degree of price stickiness | $[0.4,0.9]$ | 0.7 |
| $\zeta_{p}$ | Price indexation | $[0.4,0.9]$ | 0.2 |
| $\mu_{p}$ | Degree of wage stickiness | 0.8 |  |
| $\zeta_{w}$ | Wage indexation | $[0.2,0.8]$ | 0.5 |
| $\mu_{w}$ | Steady state markup in labor market | $[0.1,1.8]$ | 1.0 |
| $\varepsilon^{w}$ | Share of government consumption | $[0.10,0.25]$ | 0.2 |
| $g_{y}$ | Lagged interest rate coefficient | $[0.2,0.95]$ | 0.74 |
| $\rho_{R}$ | Inflation coefficient on interest rate rule | $[1.1,3.0]$ | 1.18 |
| $\gamma_{\pi}$ | Output coefficient on interest rate rule | $[0.0,1.0]$ | 0.0 |
| $\gamma_{y}$ | Persistence of shocks $i=z, b, I, \mu_{p}, \mu_{w}$ | $[0,0.9]$ | 0.8 |
| $\boldsymbol{\varrho}_{i}$ | Discount factor | 0.99 | 0.99 |
| $\beta$ | Steady state inflation | 1.016 | 1.016 |
| $\pi^{s}$ | Standard deviation expenditure shock |  | 0.1 |
| $s_{g}$ | Standard deviation preference shock |  | 0.066 |
| $s_{b}$ | Standard deviation technology shock |  | 0.0064 |
| $s_{z}$ | Standard deviation investment shock |  | 0.557 |
| $s_{i}$ | Standard deviation price markup shock |  | 0.221 |
| $s_{p}$ | Standard deviation wage markup shock |  | 0.135 |
| $s_{w}$ | Standard deviation monetary shock | 0.0026 |  |
| $s_{m}$ |  |  |  |

Table B.1: Supports for the structural parameters and parameters of the DGP, Smets and Wouters model.

## Appendix C

In this appendix we show that failute to impose the uniqueness condition in identification could lead to large biases. For this purpose, we generate density estimates of the unconstrained $(4,4)$ element of the matrix

$$
D=\left[\begin{array}{cccc}
-1 & 1 & 1 & 1 \\
1 & -1 & 1 & 1 \\
1 & 1 & -1 & 1 \\
1 & 1 & 1 & -1
\end{array}\right]
$$

in a static four variable VAR, $y=D e$, where $e$ has diagonal variance with elements $[1,1,1,2]$, identifying the last shock only using restrictions on the $(j, 4)>0, j=1,2,3$ elements of the matrix (scheme 1), identifying the last shock only using the same restrictions and the restriction that the other three shocks can not generate a similar pattern of responses (scheme $2)$ and identifying all the shocks using the restrictions on the $(j, i), j=, 1,2,3 ; i=1, \ldots, 4$ elements of the matrix.


Figure C.1: Density of the response under different identification schemes. Scheme 1 sign restrictions, one shock; Scheme 2 sign plus uniqueness restrictions, on shock; Scheme 3 sign restrictions all shocks. Vertical bar: true value.

Figure C. 1 shows that the distribution of responses in scheme 1 (dotted line) and in scheme 2 (solid line) looks very different: 30 percent of the mass of the estimated distribution
is above zero in scheme 1 and only 9 percent is above zero when the additional uniqueness restrictions are imposed; the median of the distribution is a better estimator of the true value in scheme 2. Thus, while not a substitute for identifying all the shocks, which can be seen gives very precise information about the sign and the magnitude of the unrestricted element, imposing the uniqueness condition may help to sharpen inference when only a subset of the shocks is identified.

