Bubbly Business Cycles

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Introduction

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- Macroeconomics has turned to financial-accelerator models:

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- In recent work: large shocks to net worth
  - theory: interaction of rational bubbles and financial frictions
    - expansionary effects of bubbles
    - bubbles and dynamic inefficiency
  - application: crisis as collapse of bubbles or pyramid schemes in financial markets

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  - theory: interaction of rational bubbles and financial frictions
    - expansionary effects of bubbles
    - bubbles and dynamic inefficiency
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- This paper: research project to
  - develop general model of bubbly business cycles
    - provide a simplified version to develop intuitions
  - evaluate contribution of technology / bubble shocks to recent events

Related literature

- Rational bubbles
  - Samuelson (1958), Tirole (1985)
  - Caballero and Krishnamurthy (2006), Kocherlakota (2008), Farhi and Tirole (2009)
  - Martin and Ventura (2011a, 2011b)
- Financial accelerator
  - Bernanke and Gertler (1989), Kiyotaki Moore (1997)
  - Carlstrom and Fuerst (1997), Bernanke Gertler and Gilchrist (1996), Gertler and Kiyotaki (2010)
- Quantitative OLG
  - Rios-Rull 1996
  - Krueger and Kubler 2004, Glover, Heathcoate, Krueger and Rios-Rull 2011

A model of bubbly business cycles

- OLG: T-period lifetimes, generations of size one
  - each generation composed of workers and entrepreneurs
- Preferences: individual i of generation τ maximizes

$$U_{i,s^t} = E_t \left\{ \sum_{n=t}^{\tau+T} \beta^{n-t} \cdot \frac{c_{i,s^n}^{1-\gamma} - 1}{1-\gamma} \right\},$$

  where $s^t \in S_t$ denotes history of shocks until t

- Individual $i \in I_t$:
  - works in the first $T - T^R$ periods of life
    - income $y_{i,s^t} = \begin{cases} \text{wage} \;\; w_{s^t} & \text{if } i \text{ worker} \\ \text{rents} \;\; z_{i,s^t} & \text{if } i \text{ entrepreneur} \end{cases}$
  - retires in the last T^R periods of life
    - income $y_{i,s^t} = 0$

Optimal savings and portfolios

- Full set of one-period Arrow-Debreu securities
  - $a_{s^t s^{t+1}}$: issued in history s^t, delivers in s^{t+1}
- Flow budget constraint of individual i:

$$c_{i,s^t} = y_{i,s^t} + a_{i,s^{t-1}s^t} - \sum_{s^{t+1}\in S_{t+1}} q_{s^t s^{t+1}} \cdot a_{i,s^t s^{t+1}} \quad \text{and} \quad a_{i,s^{\tau-1}s^\tau} = 0$$

with restriction that $a_{i,s^{\tau+T-1}s^{\tau+T}} \geq 0$

- Optimization implies

$$\pi_{s^t s^{t+1}} \cdot \beta \cdot \left(\frac{c_{i,s^{t+1}}}{c_{i,s^t}}\right)^{-\gamma} = q_{s^t s^{t+1}} \quad \text{and} \quad a_{i,s^{\tau+T-1}s^{\tau+T}} = 0$$

- Note: representative individual within each generation, with c_{τ,s^t} satisfying

$$c_{\tau,s^t} = w_{\tau,s^t} + z_{\tau,s^t} + a_{\tau,s^{t-1}s^t} - \sum_{s^{t+1}\in S_{t+1}} q_{s^t s^{t+1}} \cdot a_{\tau,s^t s^{t+1}} \quad \text{and} \quad a_{\tau,s^{\tau-1}s^\tau} = 0$$

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Firms

- Production undertaken in firms:
  - new: managed by founding entrepreneur
  - old: managed by employees once entrepreneur retires

- All firms produce according to

$$F\left(l_{i,s^t}, k_{i,s^t}\right) = A^Q_{s^t} \cdot l^{1-\alpha}_{i,s^t} \cdot k^{\alpha}_{i,s^t}$$

- Labor markets competitive:

$$w_{s^t} = (1-\alpha) \cdot A^Q_{s^t} \cdot l^{-\alpha} \cdot k^{\alpha}_{s^t}$$

where $k_{s^t} \equiv \sum_{\tau=-\infty}^{t} \int_{i \in I_\tau} k_{i,s^t}$ and $l = 1 - \dfrac{T^R}{T}$.

- Gross output of firm i:

$$F\left(l_{i,s^t}, k_{i,s^t}\right) + p^K_{s^t} \cdot (1-\delta) \cdot k_{i,s^t} - w_{s^t} \cdot l_{i,s^t} = R^K_{s^{t+1}} \cdot k_{i,s^t}$$

where

$$R^K_{s^{t+1}} = \alpha \cdot A^Q_{s^t} \cdot l^{1-\alpha} \cdot k^{\alpha-1}_{s^t} + p^K_{s^t} \cdot (1-\delta)$$

Old vs. new firms

- Investment efficiency:
  - entrepreneurs raise the efficiency of investment
  - firm i's capital stock evolves accorting to:

$$k_{i,s^{t+1}} = \max\left\{A^K_{i,s^t}, \frac{1}{p^K_{s^t}}\right\} \cdot I_{i,s^t}$$

  where I_{i,s^t} is gross investment and

$$A^K_{i,s^t} = \begin{cases} A^K_{s^t} > 1 & \text{if } i \text{ is new} \\ 1 & \text{if } i \text{ is old} \end{cases}$$

- Contracting friction:
  - entrepreneur appropriates share $(1-\phi)$ of gross output
  - entrepreneurial rents

$$z_{i,s^t} = \begin{cases} (1-\phi) \cdot R^K_{s^{t+1}} \cdot k_{i,s^t} & \text{if } i \text{ is new} \\ 0 & \text{if } i \text{ is old} \end{cases}$$

- Assume $\phi \cdot A^K_{s^t} < 1$:
  - in principle, no borrowing by new firms
  - but capital is not the firm's only asset!

Bubbles

- Let V_{i,s^t} denote market value / financing to firm i:

$$V_{i,s^t} = \sum_{s^{t+1} \in S_{t+1}} q_{s^t s^{t+1}} \cdot \left(R^K_{s^{t+1}} \cdot k_{i,s^{t+1}} - z_{i,s^{t+1}} - I_{i,s^{t+1}} + V_{i,s^{t+1}} \right)$$

- Define bubble in firm i as

$$b_{i,s^t} = V_{i,s^t} - I_{i,s^t} \geq 0$$

  difference between market value and gross investment

- In equilibrium:

$$p^K_{s^t} = \sum_{s^{t+1} \in S_{t+1}} q_{s^t s^{t+1}} \cdot R^K_{s^{t+1}}$$

- Old firms: indifferent between investing or not, **no bubble creation**

$$\sum_{s^{t+1} \in S_{t+1}} q_{s^t s^{t+1}} \cdot b_{i,s^{t+1}} = b_{i,s^t} \quad \text{if } i \text{ is old}$$

- New firms: entrepreneurs maximize investment, **possible bubble creation**

$$I_{i,s^t} = \frac{\sum_{s^{t+1} \in S_{t+1}} q_{s^t s^{t+1}} \cdot b_{i,s^{t+1}} - b_{i,s^t}}{1 - \phi \cdot A^K_{s^t}} \quad \text{if } i \text{ is new}$$

Equilibrium

Sequence for c_{τ,s^t}, $a_{\tau,s^t s^{t+1}}$, w_{τ,s^t}, z_{τ,s^t}, k_{s^t}, I_{s^t}, b_{s^t}, b^N_{τ,s^t} and $q_{s^t s^{t+1}}$ satisfying:

- Individual optimization (s.t. definitions of w_{τ,s^t}, z_{τ,s^t})

- Aggregate stock and price of capital

$$k_{s^{t+1}} = I_{s^t} + \left(A^K_{s^t} - 1\right) \cdot \sum_{\tau=-\infty}^{t} \frac{\sum_{s^{t+1} \in S_{t+1}} q_{s^t s^{t+1}} \cdot b_{\tau,s^{t+1}} - b_{\tau,s^t}}{1 - \phi \cdot A^K_{s_t}}$$

$$p^K_{s^t} = \sum_{s^{t+1} \in S_{t+1}} q_{s^t s^{t+1}} \cdot R^K_{s^{t+1}} = 1$$

  where we assume some investment by old firms

- Aggregate bubble:

$$\sum_{s^{t+1} \in S_{t+1}} q_{s^t s^{t+1}} \cdot b_{\tau,s^{t+1}} = b_{\tau,s^t} \quad \text{if } \tau \text{ old} \;\; \text{and} \quad \sum_{s^{t+1} \in S_{t+1}} q_{s^t s^{t+1}} \cdot b_{\tau,s^{t+1}} \geq b_{\tau,s^t} \quad \text{if } \tau \text{ new}$$

- Financial markets clear:

$$\sum_{\tau=-\infty}^{t} a_{\tau,s^t s^{t+1}} = R^K_{s^{t+1}} \cdot k_{s^{t+1}} - \sum_{\tau=-\infty}^{t} z_{\tau,s^{t+1}} + b_{s^{t+1}}$$

Quantitative evaluation

- Objectives:
  - Evaluate contribution of technology / bubble shocks to macroeconomic developments of past 25 yrs.

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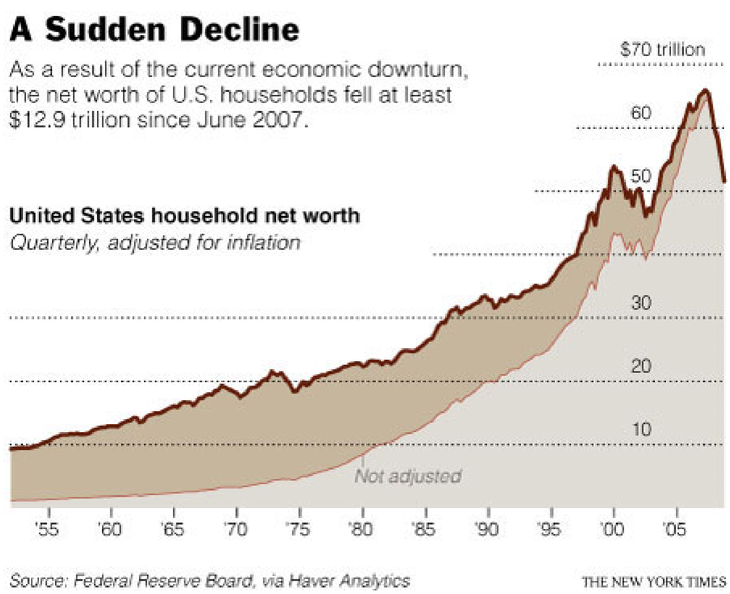
  - Seems worthy of exploring...



- 2000-2002: $5 trillion loss to US household wealth
- 2007-2008: $12.9 trillion loss to US household wealth
- What has changed regarding productive capabilities of the economy? (US GDP $14 trillion in 2009)

Quantitative evaluation

- Objectives:
  - Evaluate contribution of technology / bubble shocks to macroeconomic developments of past 25 yrs.
  - Welfare analysis: quantitative evaluation of costs and benefits of bubbly episodes
- Natural benchmarks to compare with:
  - No bubble: OLG version of RBC model (Rios-Rull '96)
  - Deterministic (constant) bubble: Financial accelerator models (Carlstrom and Fuerst '97, Bernanke, Gertler and Gilchrist '99; Gerltler and Kyotaki '11)
- Not quite there yet...

Quantitative evaluation: challenges

- Dimension of State Space: #Wealth Distribution x #Current Shock Configuration
  - For annual calibration the dimension of state space $\simeq$ 60-70
- Potential solutions:
  - Traditional: linearize around steady state (e.g. Heer and Maussner 07) or quadratic objectives (Rios-Rull '96)
    - In our case: potentially large shocks, not local deviations from steady state
  - Global solution methods based on sparse grids (Krueger and Kubler '04, Glover, Heathcoate, Krueger and Rios-Rull ´11)
    - Good interpolation properties while keeping low the number of evaluation points
- We are close, but: for today, intuition on mechanism

Developing intuitions

- Two simplifications to baseline model:
  - $T = 2$: two-period lifetimes
  - $\beta \to \infty$: all consumption during old age
- Now:
  - workers: w_{s^t} when young
  - entrepreneurs: z_{s^t} when old
- Individual optimization:

$$c_{is^t} = 0 \text{ and } c_{is^{t+1}} = \frac{\left(q_{s^t,s^{t+1}}\right)^{-\frac{1}{\gamma}} \left(\pi_{s^t,s^{t+1}}\right)^{-\frac{1}{\gamma}}}{\sum_{s^{t+1\prime} \in S_{t+1}} \left(q_{s^t,s^{t+1\prime}}\right)^{1-\frac{1}{\gamma}} \left(\pi_{s^t,s^{t+1\prime}}\right)^{-\frac{1}{\gamma}}} \cdot y_{is^t}$$

- Firms: new for one period

$$b_{i,s^t} = \sum_{s^{t+1} \in S_{t+1}} q_{s^t s^{t+1}} \cdot b_{i,s^{t+1}} \text{ if } i \text{ old}$$

$$b^N_{i,s^t} \equiv \sum_{s^{t+1} \in S_{t+1}} q_{s^t s^{t+1}} \cdot b_{i,s^{t+1}} \text{ if } i \text{ new}$$

so that b^N_{i,s^t} denotes bubble creation

Equilibrium

- Aggregate investment by new firms:

$$\frac{1}{1-\phi\cdot A^K_{s^t}}\cdot b^N_{s^t}$$

- Law of motion of aggregate bubble (attractive)

$$\sum_{s^{t+1}\in S_{t+1}} q_{s^t s^{t+1}}\cdot b_{s^{t+1}} = b_{s^t}+b^N_{s^t} \tag{1}$$

- Some investments by old firms in equilibrium (feasibility)

$$(1-\alpha)\cdot A^Q_{s^t}\cdot k^\alpha_{s^t}\cdot l^{1-\alpha} > (1-\delta)\cdot k_{s^t}+b_{s^t}+\frac{1}{1-\phi\cdot A^K_{s^t}}\cdot b^N_{s^t} \tag{2}$$

- Law of motion of capital stock:

$$k_{s^{t+1}} = (1-\alpha)\cdot A^Q_{s^t}\cdot k^\alpha_{s^t}\cdot l^{1-\alpha} - b_{s^t}+\frac{A^K_{s^t}-1}{1-\phi A^K_{s^t}}\cdot b^N_{s^t} \tag{3}$$

  - crowding-out effect: b_{s^t}
  - reallocation effect: $b^N_{s^t}$

- Competitive equilibrium: sequence of k_{s^t}, b_{s^t} and $b^N_{s^t}$ satisfying Equations (1)-(3)

Bubbly episodes

- Interpretation: investor sentiment shocks $v_{s^t} \in \{F, B\}$
- Economy oscillates between:
  - Fundamental state: $b_{s^t} = 0$
  - Bubbly episodes: $b_{s^t} > 0$
- For analytical convenience: focus on particular class of examples
  - Constant probability of beginning /end
    - $\Pr\left(v_{s^{t+1}} = B \left| v_{s^t} = F\right.\right) = q$ and $\Pr\left(v_{s^{t+1}} = B \left| v_{s^t} = F\right.\right) = p$
  - Constant rate of bubble creation
    - during bubbly episode: $b^N_{s^t} = n \cdot b_{s^t}$
  - Full depreciation

Bubbly episodes (II): recursive representation

- Define $x_{s^t} \equiv \dfrac{b_{s^t}}{(1-\alpha)\cdot A^Q_{s^t}\cdot l^{1-\alpha}\cdot k^{\alpha}_{i,s^t}}$

- Equilibrium: sequence of x_{s^t} satisfying

$$\frac{\sum_{s^{t+1}\in S_{t+1}} \pi_{s^t s^{t+1}}\cdot\left(\frac{\alpha}{1-\alpha}+x_{s^{t+1}}\right)^{-\gamma}}{\sum_{s^{t+1\prime}\in S_{t+1}} \pi_{s^t s^{t+1\prime}}\cdot\left(\frac{\alpha}{1-\alpha}+x_{s^{t+1\prime}}\right)^{-\gamma}}\cdot\frac{x_{s^{t+1}}}{x_{s^t}}=\frac{\frac{\alpha}{1-\alpha}\cdot(1+n)}{1+\left(\frac{A^K_{s^t}-1}{1-\phi A^K_{s^t}}\cdot n-1\right)\cdot x_{s^t}},$$

and

$$x_{s^t}\leq\frac{1-\phi\cdot A^K_{s^t}}{1-\phi\cdot A^K_{s^t}+n}.$$

- Intuition: bubble must be attractive and feasible

Bubbly episodes (III)

- Law of motion of capital stock:

$$k_{s^{t+1}} = \left[1 + \left(\frac{A^K_{s^t} - 1}{1 - \phi A^K_{s^t}} \cdot n - 1\right) \cdot x_{s^t}\right] \cdot (1-\alpha) \cdot A^Q_{s^t} \cdot k^{\alpha}_{s^t} \cdot l^{1-\alpha}$$

- Two benchmark episodes:
  - **Conventional bubbles** (Samuelson-Tirole)

$$\frac{A^K_{s^t} - 1}{1 - \phi A^K_{s^t}} \cdot n < 1$$

    * Contractionary (raise the interest rate and crowd out k)
    * Do not require financial frictions
    * Require dynamic inefficiency
  - **Non-conventional bubbles** (Martin-Ventura 2011)

$$\frac{A^K_{s^t} - 1}{1 - \phi A^K_{s^t}} \cdot n > 1$$

    * Expansionary (lower interest rate and crowd in k)
    * Require financial frictions
    * *Do not require dynamic inefficiency.*

Example 1: deterministic economy

- No technology shocks: $A^K_{s^t} = \overline{A^K}$ and $A^Q_{s^t} = \overline{A^Q}$

- Bubbly episode that never ends: $p = 0$

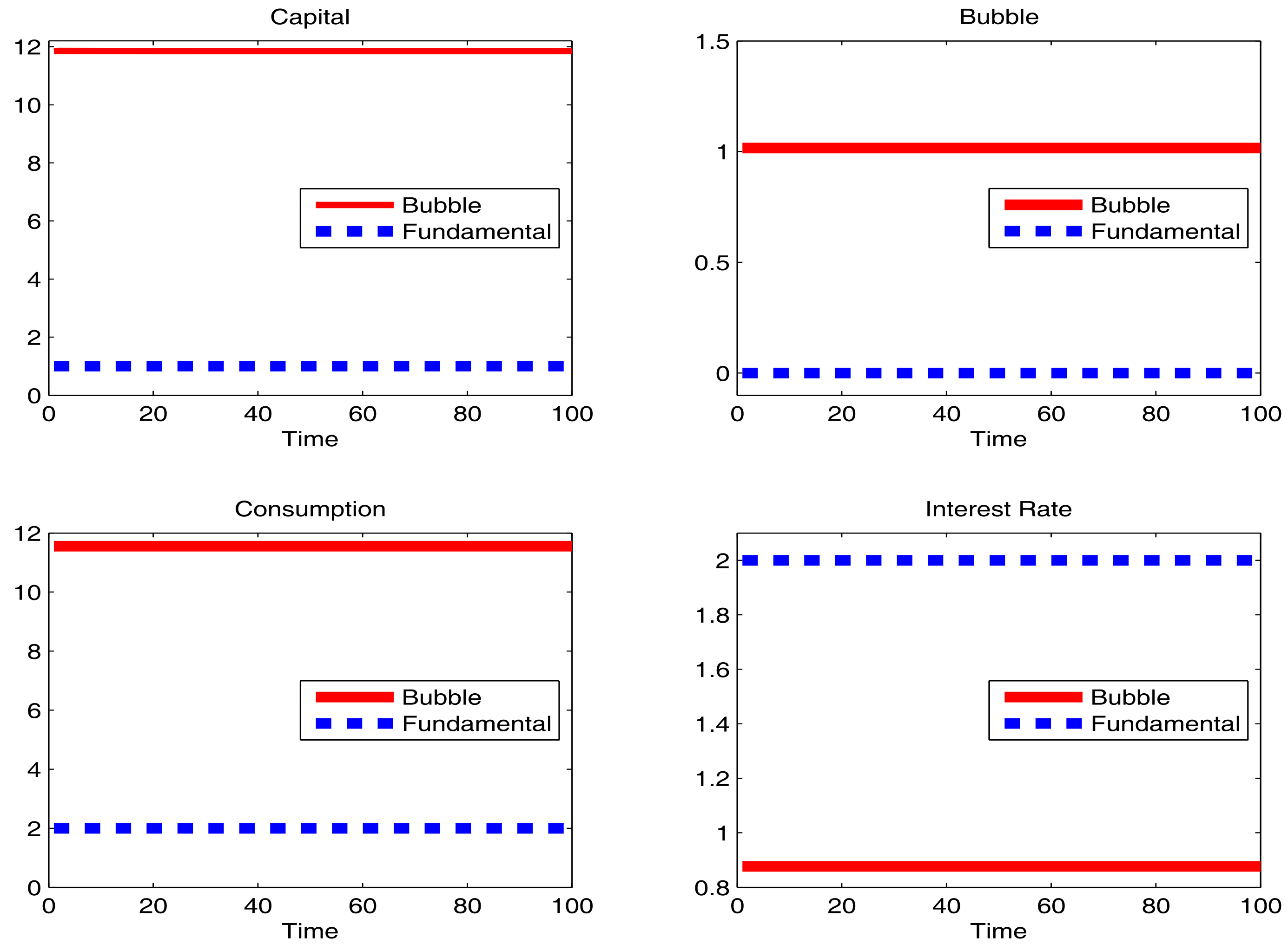
- With bubbly episode
  - High investor sentiment sustain bubble / bubble creation
  - Helps overcome contracting friction
    - higher borrowing by new firms
    - higher efficient investment
  - In example: $x_{s^t} \approx 12\%$ sustains six-fold increase in k and c

- Expansion and dynamic inefficiency
  - Existence requires dynamically inefficient chain of investments
  - In fundamental equilibrium: savings > capital income

$$\begin{aligned} (1-\alpha) \cdot \overline{A^Q} \cdot l^{1-\alpha} \cdot k^{\alpha}_{s^{t+1}} &> \alpha \cdot \overline{A^Q} \cdot l^{1-\alpha} \cdot k^{\alpha}_{s^{t+1}} \\ 0.5 &> \alpha \end{aligned}$$

  - If not satisfied, bubbly episode must generate dynamic inefficiency: expansionary!

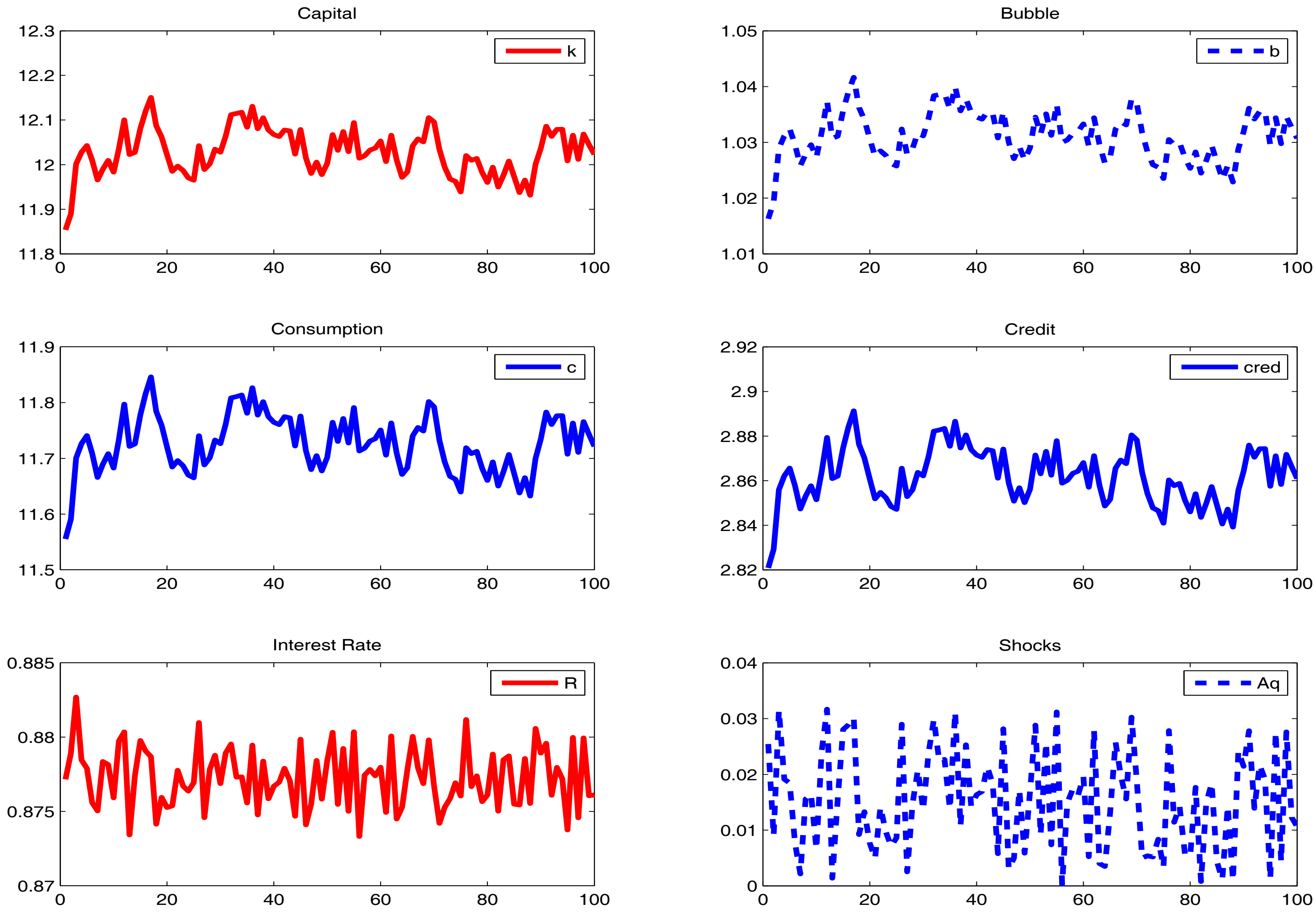
Example 1: deterministic economy



Example 2: stochastic economy with deterministic bubble

- Technology shocks: $A^Q_{s^t} \in \left[A^Q_L, A^Q_H\right]$ and $A^K_{s^t} \in \left[A^K_L, A^K_H\right]$
- Bubbly episode that never ends: $p = 0$
- Fundamental shocks have the usual effects
  - High values of $A^Q_{s^t}$
    - Raise output, consumption, capital accumulation
    - Lower interest rate: raise borrowing and investment by new firms
  - High values of $A^K_{s^t}$
    - Raise output and consumption with a lag
    - Raise borrowing and investment by new firms
- Interaction with bubble
  - Shocks to $A^Q_{s^t}$: proportional effect on output and bubble (x_{s^t} unaffected)
  - Shocks to $A^K_{s^t}$: lower interest rate and growth rate of bubble
  - Bubble amplifies effects of technology shocks ($\uparrow$ volatility)
    - aggregate effects proportional to intermediation
    - intermediation proportional to aggregate bubble creation

Example 2: stochastic economy with deterministic bubble



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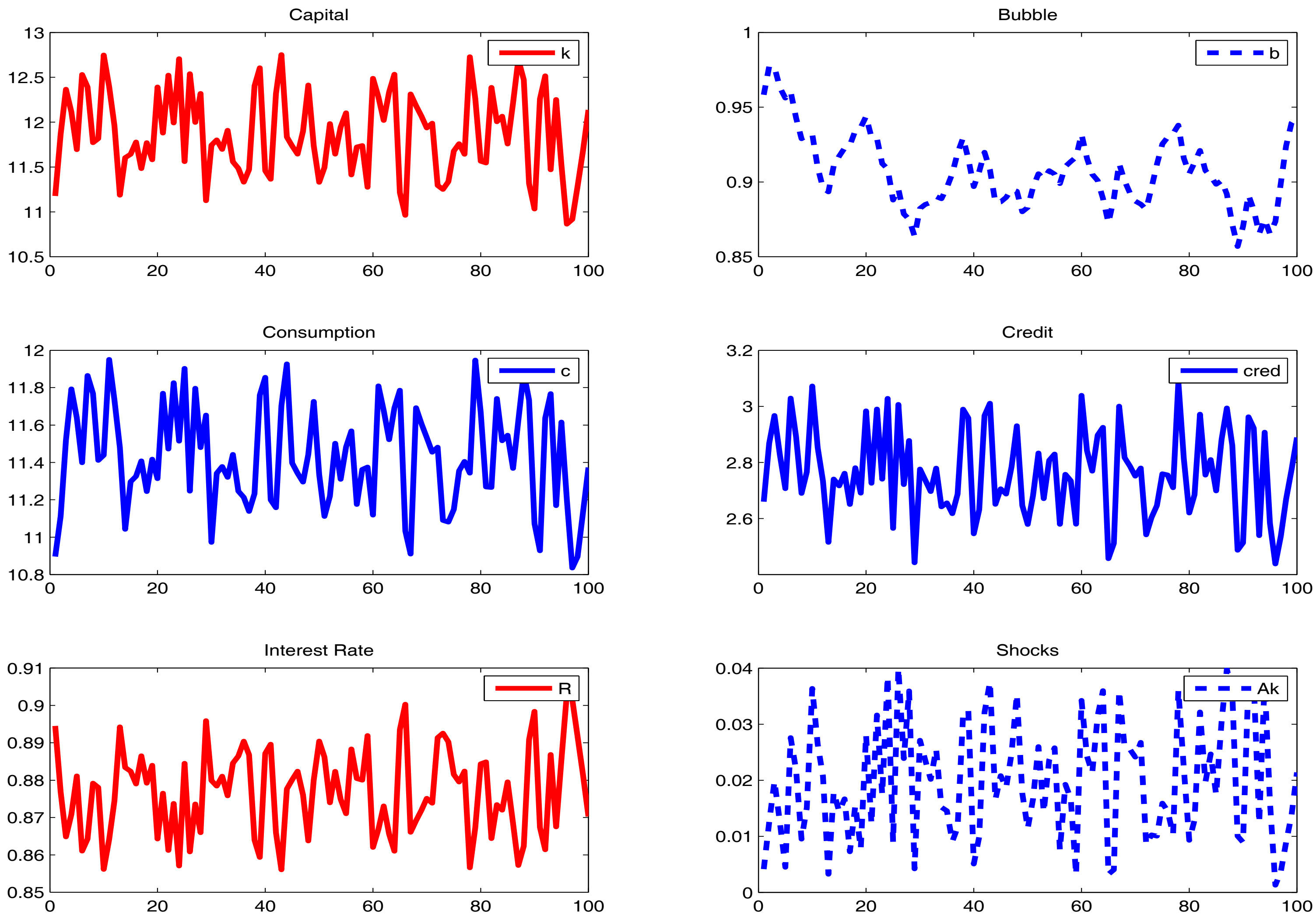

Example 3: bubbly business cycles

- No technology shocks: $A^K_{s^t} = \overline{A^K}$ and $A^Q_{s^t} = \overline{A^Q}$
- Stochastic bubbly episodes: $p > 0$, $q > 0$
  - shocks to x_{s^t} and to $x^N_{s^t}$
- Huge effects of investor sentiment shocks
  - Bubbly episodes of approx. 20 periods
  - Bubble peaks at approx. 8% of wages
  - Increase of capital stock, consumption, efficient investment: $> 500\%$
  - When episode ends: increases disappear in two periods
- Main insight
  - Large equilibrium effects of investor sentiment shocks
  - Despite rationality and risk aversion
    - ∗ risk aversion increases the effects

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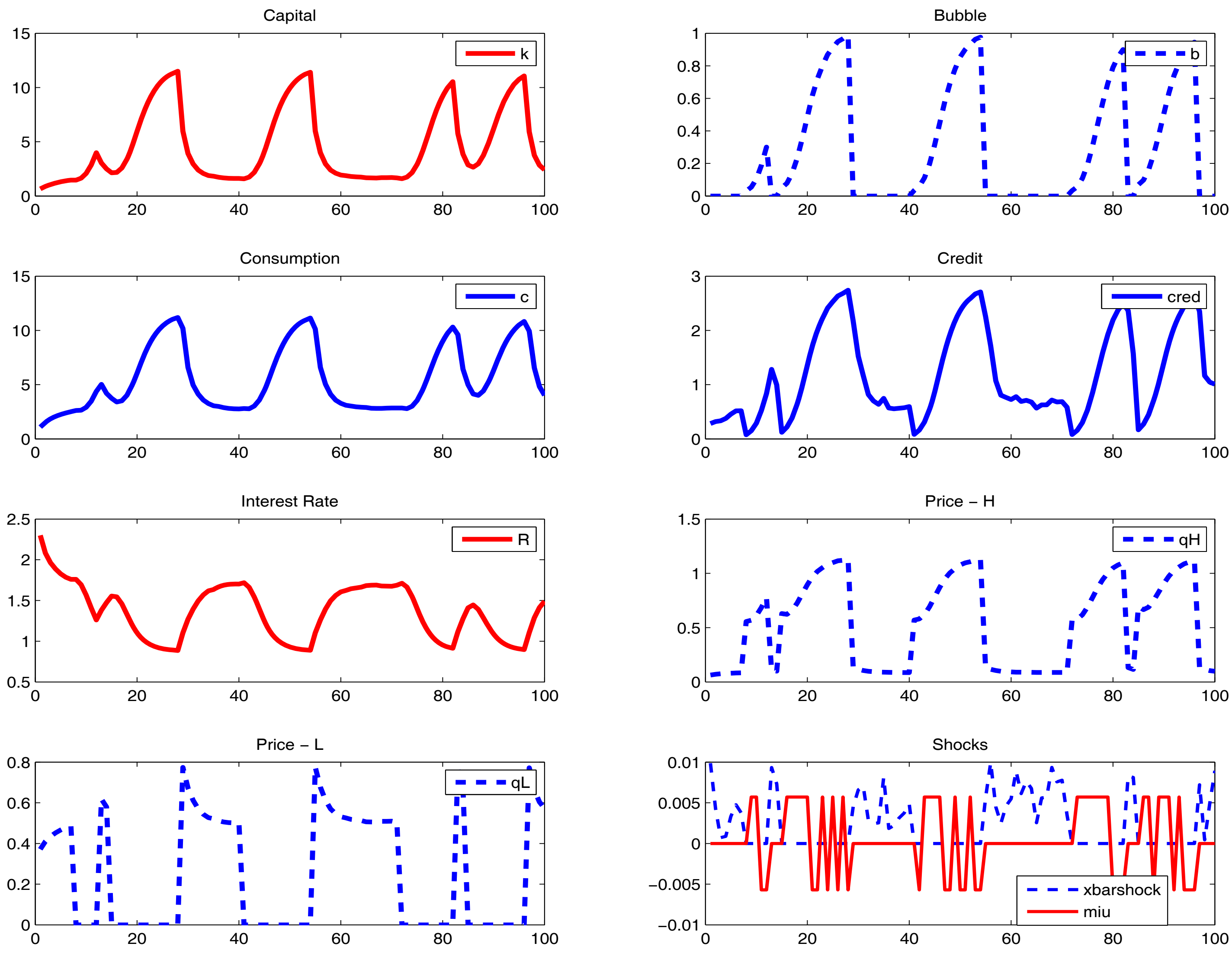
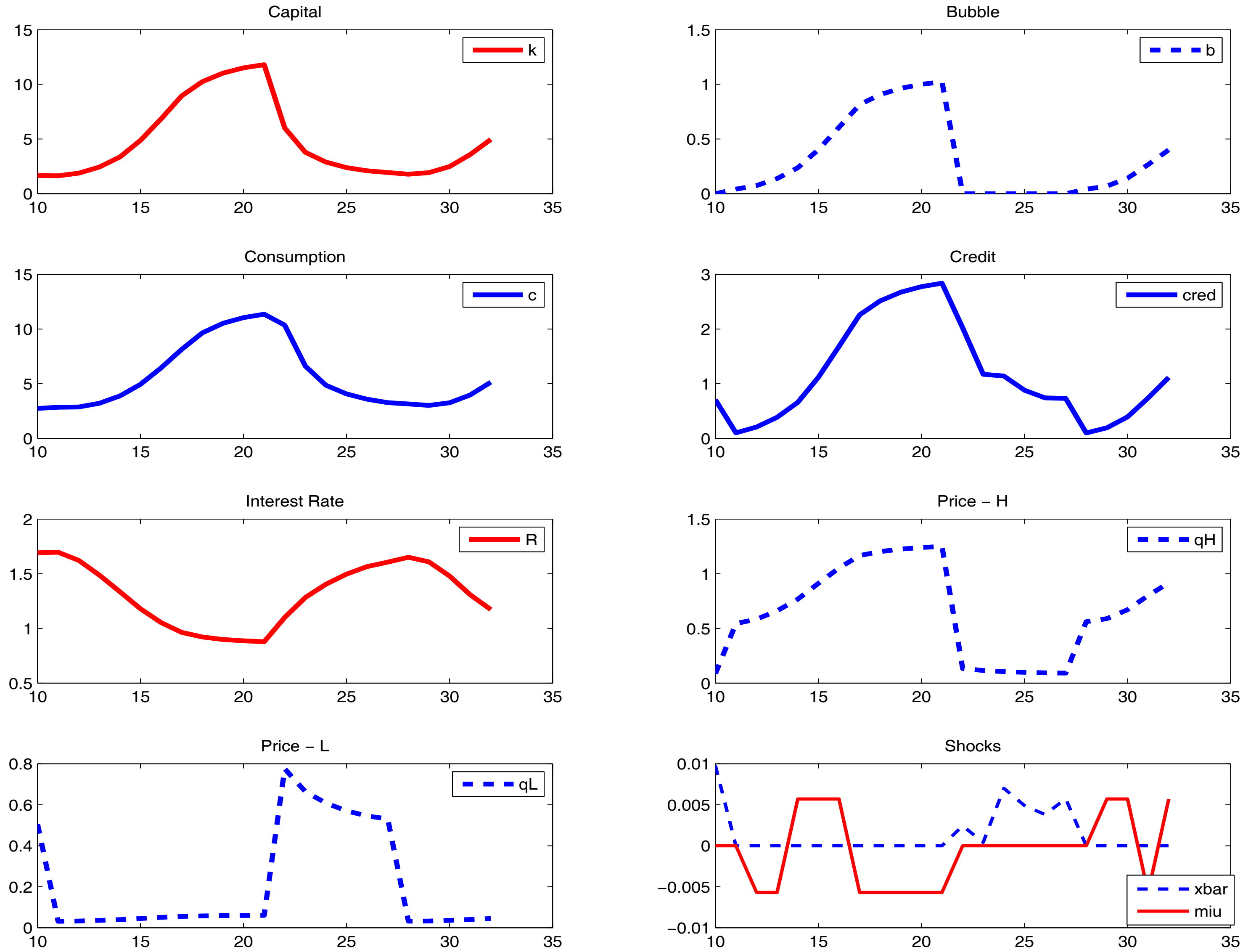
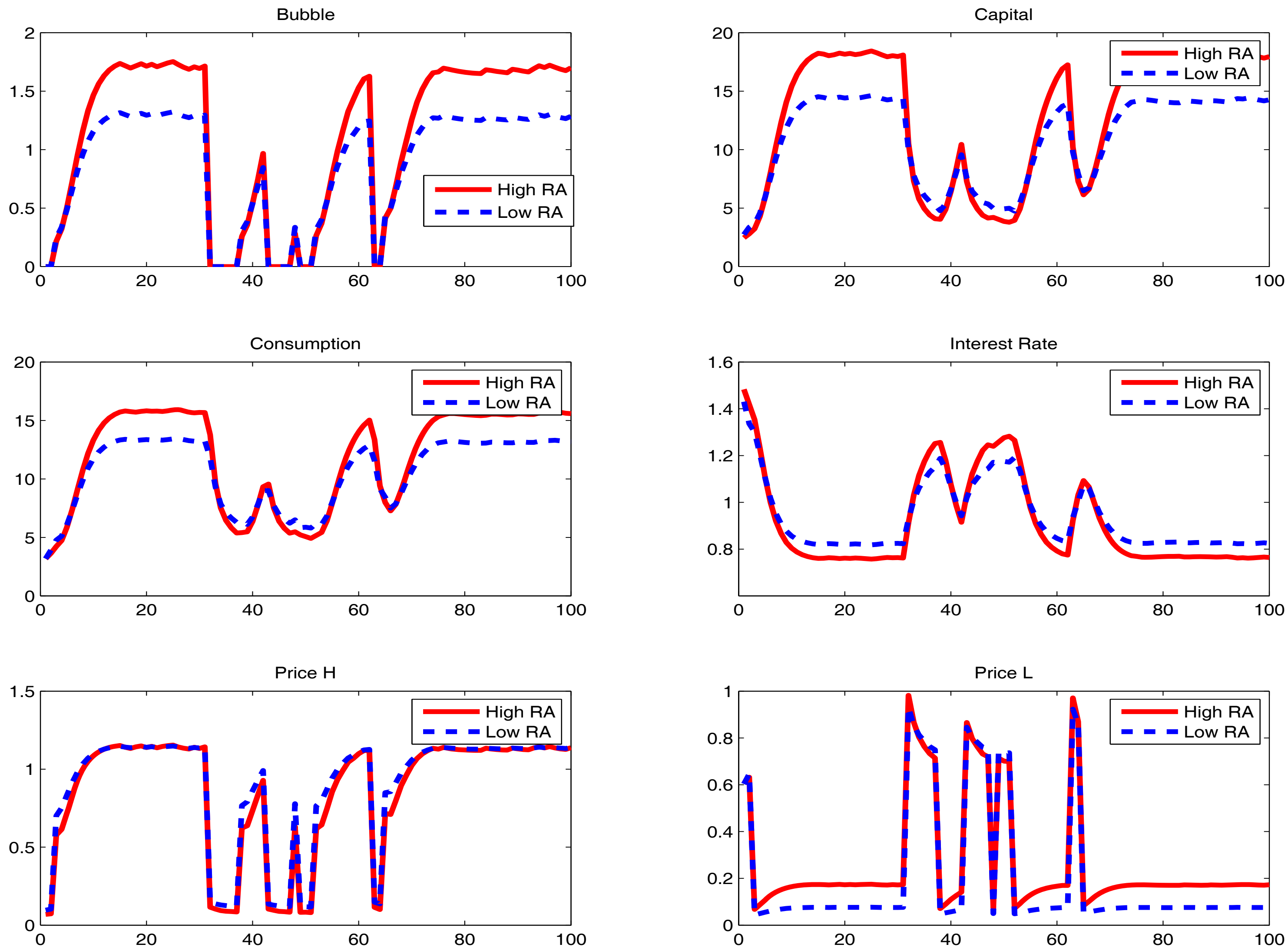


Example 3: a closer look at an episode



Example 3: role of risk aversion



Example 4: types of bubble shocks

- No technology shocks: $A^K_{s^t} = \overline{A^K}$ and $A^Q_{s^t} = \overline{A^Q}$

- Bubbly episode that never ends: $p = 0$

  – shocks to x_{s^t} and to $x^N_{s^t}$

- Shocks to existing bubble x_{s^t}

  – Contractionary

  – Crowding-out of capital

  – Decrease in consumption and intermediation

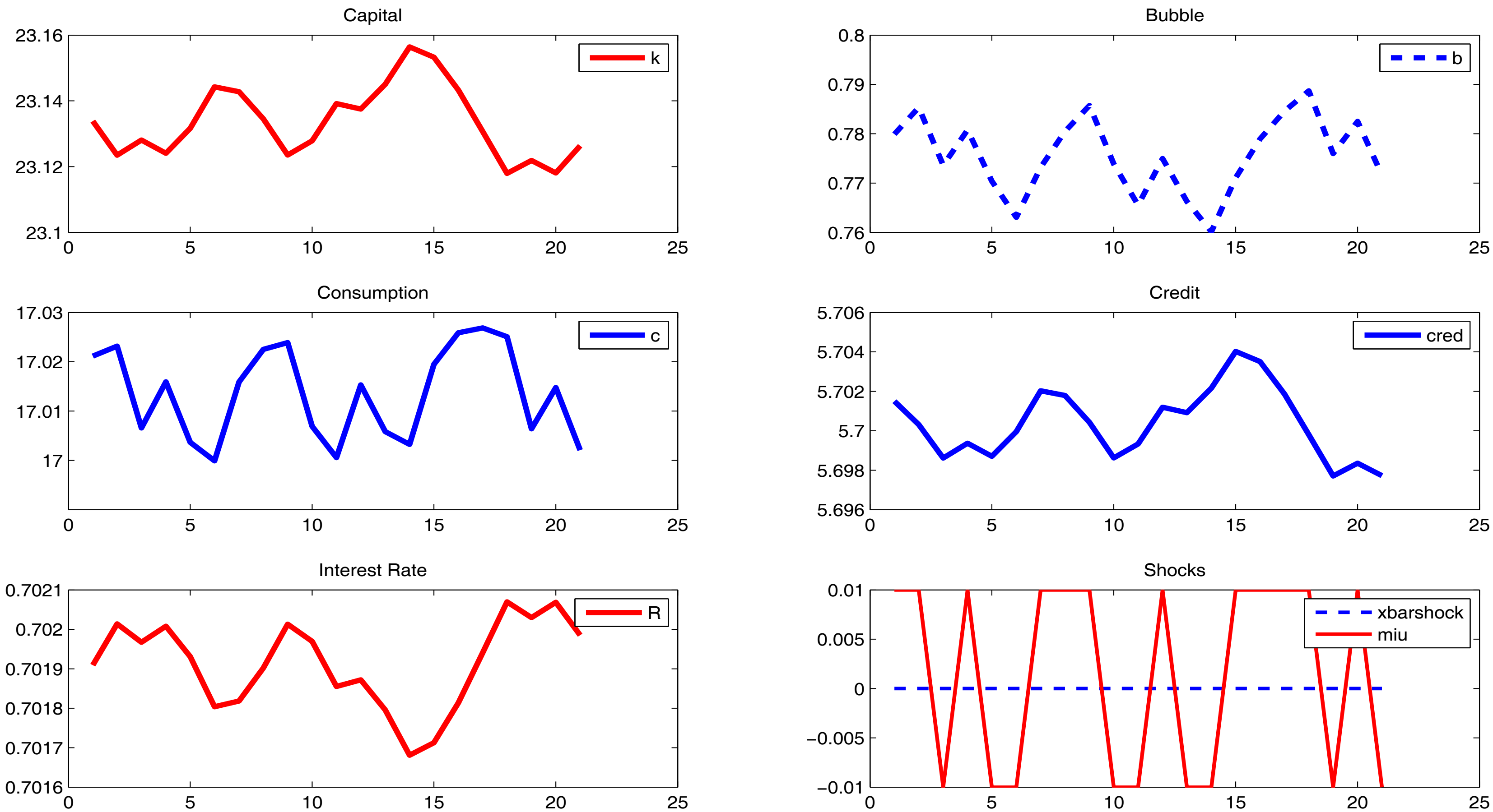
- Shocks to bubble creation $x^N_{s^t}$

  – Expansionary

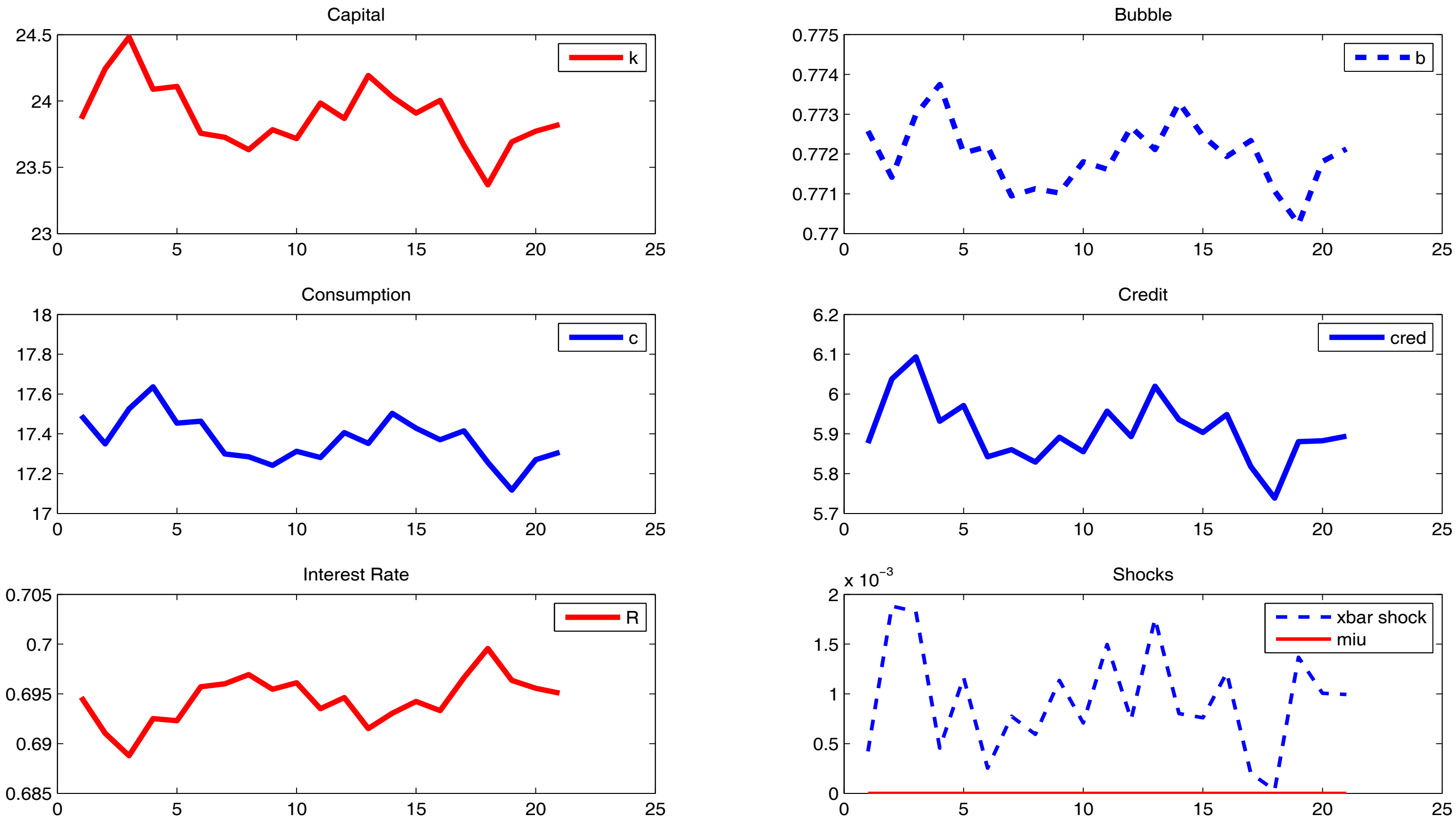
  – Reallocation of resources towards efficient investment

  – Increase in consumption

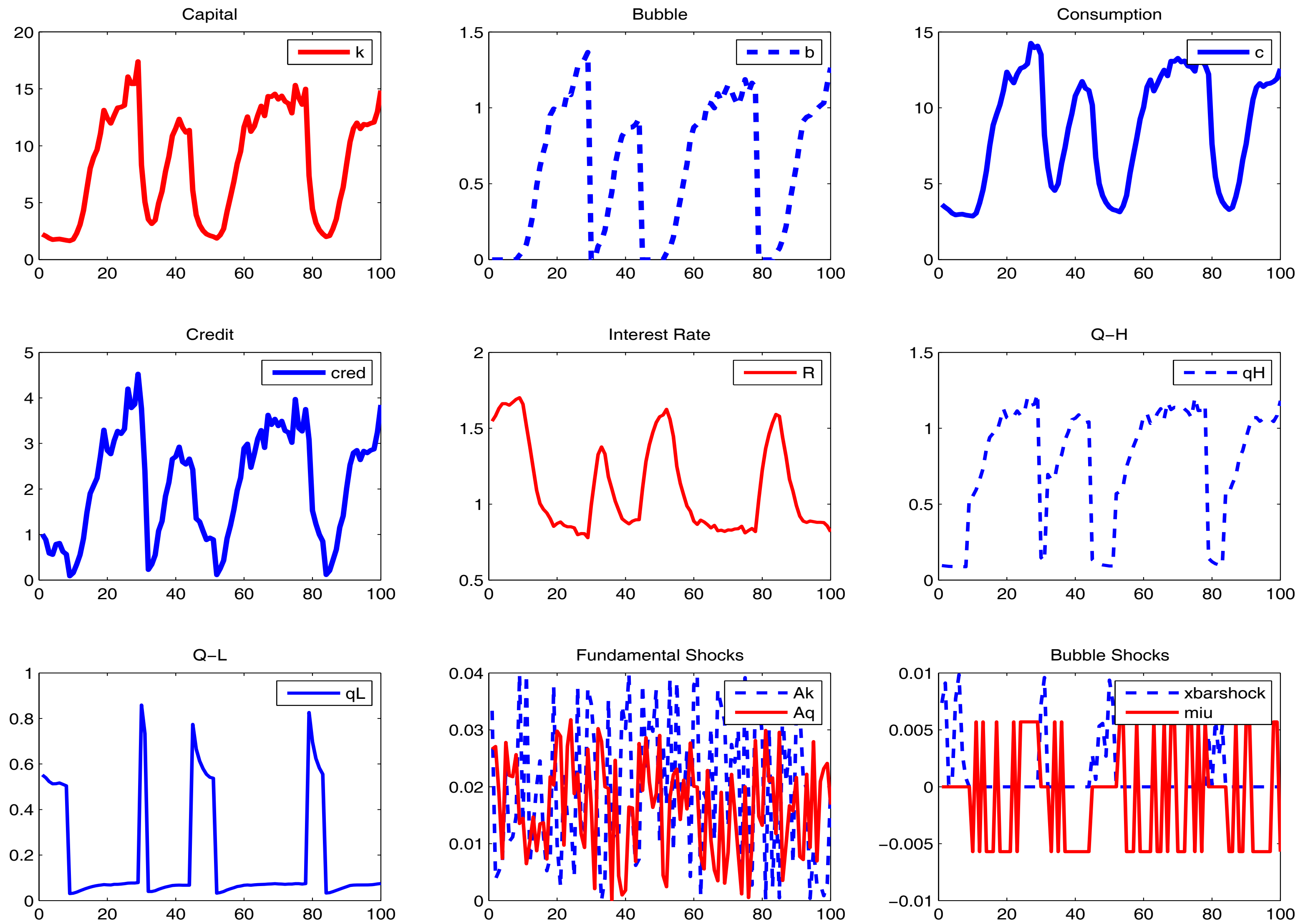
Example 4: shocks to x_{s^t}



Example 4: shocks to $x^N_{s^t}$



Example 5: the full economy



Conclusions

- This paper: research project to
  - develop general model of bubbly business cycles
    - provide a simplified version to develop intuitions
  - evaluate contribution of technology / bubble shocks to recent events (PENDING)
- Main message: rationality consistent with large macroeconomic effects of investor sentiment shocks

Parametrization

Table 1: Parameter values for figures

Parameter	Description	Value	Shock
α	Capital Share	$2/3$	-
ε	Measure of entrepreneurs	0	-
$1-\phi$	Entrepreneurial rent	0.75	-
γ	Risk aversion coefficient	2	$\gamma' = 8$
A^Q	Total factor productivity	3	$[-0.005\%, 0.005\%]$
A^k	Investment efficiency	3.77	$[-0.005\%, 0.005\%]$
$\overline{x}$	Initial bubble	0.02	
n	Growth Rate of Bubble	0.14	
μ	Shocks to existing bubbles	$--$	$\pm\ 0.005$
q	Probability of bubble episode starting	0.15	-
p	Probability of bubble bursting	0.5	-