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# Product diversity, endogenous markups, and development traps

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**Abstract**

We analyze the implications of endogenous markups for the dynamics of capital accumulation, in an environment in which the degree of competition increases with economic development. In equilibrium, markups are inversely related to the aggregate capital stock, which makes it possible for the marginal revenue product of capital to be nonmonotonic, even if the marginal product of capital is strictly diminishing. That feature raises the possibility of multiple steady states and, most interestingly, multiple equilibrium paths (converging to different steady states) for given initial conditions. We conclude by discussing some of the predictions of our model and assessing their empirical relevance.

*Key words*: Multiple equilibria; Growth models; Endogenous markups; Convergence

*JEL classification*: L13; O41

## 1. Introduction

Recent research on the mechanics of growth has focused on several departures from a neoclassical environment as possible explanations for some of the observed patterns of per capita income, both across countries and over time.

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Though the assumption of perfect competition is frequently disposed of in that literature (e.g., Romer, 1990), the introduction of market power often plays a peripheral role as a device to sustain nonconvex technologies. In particular, the assumptions on technology and preferences in most of those models imply that optimal markups charged by firms are constant, i.e., unresponsive to changes in demand conditions and/or the number of firms.

In the present paper we analyze the implications for the dynamics of capital accumulation of endogenous markups, in an environment in which the intensity of competition increases with economic development and the associated range of products available. Leaving the details of the model aside for the moment, the main point that we want to convey can be summarized as follows: the presence of market power drives a wedge between the marginal product of capital and the return to investment, which is immediately apparent when looking at the first-order condition for profit maximization of an imperfectly competitive firm:[1]

$$r = MPK\,(1 - 1/\xi) - \delta,$$

where $r$ is the interest rate, $MPK$ is the marginal product of capital, $\xi$ is the price elasticity of demand faced by the firm, and $\delta$ is the depreciation rate. $(1 - 1/\xi)$ is the marginal revenue. The (implicit) optimal markup is given by $\mu = (1 - 1/\xi)^{-1}$. Notice that $MPK(1 - 1/\xi)$ corresponds to the *marginal revenue product of capital* ($MRPK$). The intuition for the presence of $\xi$ in the expression for the interest rate is straightforward: when considering whether to employ an additional unit of capital (at a given rental cost $r + \delta$), any individual firm must take into account the price reduction that is necessary in order to sell the additional output. The size of the price reduction (and thus the wedge between $MRPK$ and $MPK$) will be greater the lower is the perceived price elasticity of demand (i.e., the higher is the optimal markup $\mu$).

Suppose now that the structure of the economy is such that the price elasticity of demand is positively related to the aggregate capital stock (or, equivalently, the size of markups decreases with the capital stock). In that case, $MRPK$ and, as a result, the real interest rate may no longer be decreasing in the capital stock, even in the presence of a diminishing $MPK$. As we show below, that feature raises the possibility of multiple steady states and, most interestingly, multiple equilibrium paths converging to different steady states, for given initial conditions.

In the present paper we describe and analyze an economic environment which endogenously generates the kind of negative relationship between markups and the capital stock discussed above in a way that we find both appealing and

[1] This formulation implicitly assumes a symmetric equilibrium in which the relative price for all goods is one. See below for a formal derivation.

plausible. A key assumption of our model is embedded in the technology used to produce final goods, which implies that the substitutability among intermediate inputs is increasing in the number of those inputs used. That feature is responsible for the negative link between optimal markups and the number of firms in the intermediate goods sector. The argument is closed by showing that, in the presence of overhead costs in the production of intermediate goods, the number of firms in that sector at any point in time is positively related to the level of aggregate demand and output and, consequently, to the aggregate capital stock.

The intuition for the possible positive relationship between the interest rate and the capital stock can be summarized as follows: an increase in the stock of capital raises output, aggregate demand, and the sales and profits of each incumbent firm. The latter effect leads to entry of new firms and, consequently, more competition and lower markups. If the previous effect is strong enough, it might more than offset the decline in the marginal product of capital, thus leading to an increase in the interest rate.

The presence of a nonmonotonic interest rate schedule is then shown to be a potential source of multiple steady states. High-capital, high-income steady states are associated with a larger number of firms, a greater variety of intermediate inputs, and lower markups than low-capital, low-income steady states.

As will become clear below, the results emphasized in the paper hinge on the presence of a negative relationship between the number of firms and equilibrium markups, but not on the particular mechanism chosen to generate such a relationship. Any other choice of technologies, preferences, and market structure that preserved that property would generate similar outcomes.[2]

The existence of multiple steady states will generally (though not always) imply the existence of multiple (off-steady state) equilibrium paths for a range of initial conditions. Needless to say, the emergence of such multiplicity can also be found in other recent papers. Most existing models, however, are characterized by either price-taking behavior or constant markups, and rely on a different mechanism to generate multiple equilibria. More specifically, the examples found in the literature typically embed some form of external increasing returns in production or pecuniary externalities as the basic source of the complementarity between private and aggregate investment that underlies the multiplicity of equilibrium outcomes.[3] Even though our model economy involves

[2] Gali and Zilibotti (1995) develop a model with Cournot competition and free entry, in which the price elasticity of demand at the firm level (evaluated at the symmetric equilibrium) is proportional to the number of firms in the industry.

[3] See, e.g., Benhabib and Farmer (1994), Boldrin (1993), Zilibotti (1993), as well as the surveys by Boldrin and Rustichini (1994) and Benhabib and Galí (1995). Other examples of dynamic general equilibrium models exhibiting multiple equilibria include models of regional/sectoral allocation (e.g., Matsuyama, 1991) and models with pecuniary externalities resulting from technological complementarities (e.g., Ciccone and Matsuyama, 1992).

firms that use a technology subject to increasing returns (because of an overhead capital requirement), it will become clear that the latter is only a device to bound the number of firms in a zero-profit equilibrium, and does not play any role in generating multiple equilibria: in fact, the marginal product of capital is strictly decreasing in our model. In contrast, the possibility of an increasing return to investment (necessary for multiple equilibria) is purely a consequence of the effect of entry on equilibrium markups.[4]

The plan of the paper is as follows. In Section 2 we set up the basic model. Section 3 derives the dynamical system characterizing an equilibrium. Section 4 discusses the existence and possible multiplicity of steady states. Section 5 analyzes the equilibrium dynamics. Section 6 discusses some of the empirical implications of the model and provides some evidence regarding the main underlying mechanism. Section 7 concludes.

## 2. The model

### 2.1. Final goods

There is a final good, produced by a perfectly competitive representative firm. At each point in time the firm chooses a technology from a set of available constant returns technologies. Each technology is 'defined' by the range of intermediate goods that uses as inputs. Thus, technology $m$ involves the use of intermediate goods in the interval $[0, m]$ and is represented by the production function

$$Y = s(m)\left[m^{-(1-1/\mu(m))}\int_0^m x(z)^{1/\mu(m)}\,\mathrm{d}z\right]^{\mu(m)}, \tag{1}$$

where $Y$ is output and $x(z)$ represents the quantity of intermediate good $z \in [0, m]$ used. $s$: $\boldsymbol{R}_+ \to \boldsymbol{R}_+$ is an increasing function which, as argued below, captures the benefits of input diversity in production. $\mu$: $\boldsymbol{R}_+ \to \boldsymbol{R}_+$ is a continuously differentiable function with the following properties:

$$\mu'(m) < 0, \quad \lim_{m\to 0}\mu(m) = \bar{\mu} \in (1, \infty), \quad \lim_{m\to\infty}\mu(m) = 1,$$

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[4]Galí (1993) develops another example of a model economy with multiple equilibria resulting from endogenous markups. The economic mechanism there is very different from the one in this paper: there is a fixed number of firms, each of which produces a differentiated good which it sells to two different customer types (consumers and other firms) that are characterized by different demand elasticities. As a result, the optimal markup (and the equilibrium interest rate) is related to the share of firms' purchases in aggregate demand, which coincides with the aggregate savings rate. Under appropriate assumptions, an increase in the latter may raise interest rates and create the possibility of multiple equilibria.

The implied elasticity of substitution among intermediate goods is given by $\xi(m) = \mu(m)/\mu(m) - 1$, whose properties follow trivially from our assumptions on $\mu$.

We let interval $[0, M(t)]$ represent the range of intermediate goods (and, thus, of technologies) available in period $t$. Accordingly, the technology choice is subject to the constraint $m \in [0, M(t)]$. Having chosen a technology the representative firm seeks to maximize its profits,

$$\pi_y \equiv \max Y - \int_0^m p(z)\, x(z)\, \mathrm{d}z,$$

subject to (1), and where the price of the final good has been normalized to unity. Letting $E \equiv \int_0^m p(z)\, x(z)$, the solution to that problem yields the set of input demand schedules:

$$x(z) = (p(z)/P)^{-\xi(m)}\,(E/Pm), \quad \text{all } z \in [0, m], \tag{2}$$

where $P \equiv [(1/m) \int_0^m p(z)^{1-\xi(m)} \mathrm{d}z]^{1/(1-\xi(m))}$. Substitution of (2) into (1) allows us to derive the following expression for profits:

$$\pi_y \equiv (s(m)/P - 1)E. \tag{3}$$

By assumption the final goods firm takes $P$ as given.[5] Since $s' > 0$, the firm will always choose $m = M$, i.e., the technology that makes use of the entire range of intermediate inputs available. The existence of an equilibrium will thus require $P = s(m)$, for otherwise the supply of final goods would be either zero or infinite. As a result, profits in the final sector will be zero.

From (3) it should be clear that the choice of a technology that uses the entire range of available inputs will be optimal for an arbitrarily small $s'$. In fact, if $s(\cdot)$ was constant, our firm would be indifferent regarding its technology choice. The 'preference for variety' effect captured by the $s' > 0$ assumption is not the focus of our analysis, but only a device to guarantee that the entire range of available inputs is used at any point in time. Thus, and in order to simplify the algebra and stress the role of changes in the degree of market power instead, we just set $s \equiv 1$ in what follows and assume that the firm always selects technology $M$, i.e., the one that makes use of the entire range of inputs currently available, among the set of equally profitable technologies.[6]

[5] This is a consequence of our assumption of price-taking behavior and the fact that $P$ will be independent of the choice of technology by any individual firm.

[6] It is straightforward to extend the analysis below to the general case $s' > 0$.

### 2.2. Intermediate goods

At any point in time there is a continuum of firms indexed by $z \in [0, M]$ operating in this sector. Each firm produces a differentiated intermediate good that is used by the final goods firm as an input. Each intermediate firm has access to a technology described by the production function

$$x = F(k - v, n), \tag{4}$$

where $x$ is output, $k$ is capital, $v$ is overhead requirement, and $n$ is labor input. Firm index $z$ is omitted to ease the notation. We assume $F$ is twice differentiable, increasing, concave, and homogeneous of degree one in its two arguments. We also assume $\lim_{u \to 0} F(u, u') = 0$, $\lim_{u \to 0} F_1(u, u') = +\infty$, and $\lim_{u \to \infty} F_1(u, u') = 0$, for all $u' \in (0, +\infty)$, and where $F_i$ denotes the partial derivative of $F$ with respect to its $i$th argument.

At each point in time the typical intermediate firm seeks to solve

$$\pi_x = \max\, px - wn - qk,$$

subject to (2) and (4), while taking the rental cost of capital $q$, the wage $w$, and other relevant economy-wide variables ($E$, $P$, and $M$) as given. The first-order conditions for the problem above are

$$p = \mu(M)\,[w/F_2], \tag{5}$$

$$p = \mu(M)\,[q/F_1]. \tag{6}$$

Notice that the terms in brackets in (5) and (6) correspond to the marginal cost, so $\mu$ can be interpreted as the optimal markup.

In equilibrium we must have $p = P = 1$ for all active intermediate firms, given the symmetry of the model. Letting $K$ and $N$ respectively denote the aggregate capital stock and labor supply (where the latter is assumed to be inelastically supplied), it follows from (6) and the fact that $K = Mk$ and $N = Mn$ that

$$q = f'(K - vM)/\mu(M), \tag{7}$$

where $f(u) \equiv F(u, N)$. In addition, output per firm will be given by $x = f(K - vM)/M$. We can thus derive the following expression for an individual firm's profits:

$$\pi_x = \frac{f(K - vM)}{M\,\xi(M)} - \frac{f'(K - vM)}{\mu(M)}\,v \equiv \Pi(K, M), \tag{8}$$

which is defined for $K \geq 0$, and $0 \leq M \leq K/v$.

Notice that, under our assumptions, the profit function $\Pi$ is continuously differentiable on its domain, and $\Pi_1(K, M) > 0$ and $\Pi_2(K, M) < 0$ hold. In words, capital accumulation raises individual firm's profits (given the number of firms), while entry reduces such profits (given the aggregate capital stock). The former effect results from the fact that capital accumulation raises output per firm and reduces overhead costs. The inverse relationship between the number of firms and profits (given $K$) is a consequence of the negative impact of entry on the level of output per firm and markups, as well as the increase in overhead costs.

It is easily checked that $\lim_{M\to 0}\Pi(K,M) = +\infty$ and $\lim_{M\to K/v}\Pi(K,M) = -\infty$. The previous properties, combined with the continuity of $\Pi$, guarantee the existence of a unique $M \in (0, K/v)$ satisfying $\Pi(K,M) = 0$, for any given $K > 0$. That value of $M$ gives the range of intermediate firms that will be active in the free-entry, zero-profit equilibrium, as a function of the aggregate capital stock. Formally,

$$M = m(K), \tag{9}$$

where $m$: $\boldsymbol{R}_+ \to \boldsymbol{R}_+$ is a continuously differentiable function satisfying $\Pi(K, m(K)) = 0$, $0 < m(K) < K/v$, and

$$m'(K) = -\,\Pi_1/\Pi_2 > 0,$$

for all $K > 0$, i.e., the number of firms is increasing in the aggregate stock of capital. In other words, positive (negative) net aggregate savings lead to entry (exit) of firms, thus increasing (decreasing) the variety of intermediate goods available and the intensity of competition.

We can now derive the following reduced-form aggregate output function, denoted by $g(K)$, corresponding to a symmetric equilibrium with zero profits:

$$Y = E = m(K)F(k - v, n) = f(K - vm(K)) \equiv g(K). \tag{10}$$

Using (8) it can be easily checked that $0 < vm'(K) < 1$ for all $K > 0$, implying that

$$\partial Y/\partial K = g'(K) = (1 - vm'(K))f' > 0,$$

i.e., aggregate output is increasing in the aggregate capital stock.

### 2.3. Consumers

An infinite-lived representative consumer seeks to maximize the objective function

$$(\sigma/(\sigma - 1)) \int_0^\infty C(t)^{(\sigma-1)/\sigma} \exp(-\rho t)\,\mathrm{d}t, \tag{11}$$

where $\sigma > 0$ is the elasticity of intertemporal substitution and $\rho$ the discount rate.

At each point in time our consumer supplies (inelastically) $N$ units of labor input. In addition, he rents out his capital holdings $K$. Total income is thus given by $wN + qK$, which is fully allocated (in equilibrium) to purchasing the final good. Those purchases are then split between gross investment and consumption, leading to the dynamic budget constraint

$$\dot{K} = wN + rK - C, \tag{12}$$

where $r \equiv q - \delta$ is the (shadow) interest rate and $\delta$ is the rate of capital depreciation. Unless necessary the time dependence of all endogenous variables is not made explicit hereafter in order to ease the notation.

Our consumer is assumed to behave competitively, taking all prices as given. His problem consists in choosing a path for $C$ and $K$ that maximizes (11) subject to (12), the nonnegativity constraints $K \geq 0$ and $C \geq 0$, and an initial condition for the capital stock, given the path of $r$ and $w$. The Euler equation corresponding to that optimal control problem is given by

$$\dot{C}/C = \sigma(r - \rho), \tag{13}$$

with the associated transversality condition

$$\lim_{T \to \infty} K(T)\, C(T)^{-1/\sigma} \exp(-\rho T) = 0. \tag{14}$$

## 3. Equilibrium

In equilibrium, the income perceived by the representative consumer will be equal to the real revenue generated by the intermediate sector, which in turn equals the final good output. Formally,

$$wN + qK = g(K). \tag{15}$$

Using (7) and (9) we can write the equilibrium interest rate as a function of the aggregate capital stock

$$r(K) = f'(K - vm(K))/\mu(m(K)) - \delta. \tag{16}$$

Substituting (15) and (16) into (12) and (13) we obtain the system of differential equations:

$$\dot{K} = g(K) - C - \delta K, \tag{17}$$

$$\dot{C} = \sigma C\,(r(K) - \rho). \tag{18}$$

In order to determine the relevant domain for the above system notice that Inada conditions for $f$ carry over to $g$, i.e., $\lim_{K \to 0} g'(K) = +\infty$, and $\lim_{K \to +\infty} g'(K) = 0$. Those conditions, in turn, guarantee the existence of a maximum sustainable capital stock $\bar{K}$, defined as the smallest possible solution to the equation $g(u) - \delta u = 0$.[7] Given the nonnegativity constraint on consumption and the capital stock, we can restrict our analysis to the set $A = \{(K, C)\colon C \geq 0, 0 \leq K \leq \bar{K}\} \subset \boldsymbol{R}^2$.

We define an equilibrium path for our model economy as a trajectory of the dynamical system (17)–(18), denoted by $(K(t), C(t))$, for all $t \in [0, +\infty)$, satisfying the initial condition $K(0) = K_0$, the transversality condition (14), and such that $(K(t), C(t)) \in A$. Given such an equilibrium path it is straightforward to derive the corresponding paths for aggregate output, the number of firms, the interest rate, and the wage rate using (9), (10), (16), and (15), respectively.

Next we turn to a characterization of equilibria. We start by analyzing the stationary solutions of (17)–(18), and then we turn to more general equilibrium paths.

## 4. Steady state analysis

An interior steady state of our model economy is a vector $(K^*, C^*) \in A$, satisfying

$$r(K^*) = \rho, \tag{19}$$

$$C^* = g(K^*) - \delta K^*. \tag{20}$$

In contrast with the neoclassical model, our assumptions do not guarantee the existence of a stationary equilibrium with positive consumption. Given the continuity of $r(\cdot)$ and the fact that $\lim_{K \to 0} r(K) = +\infty$, the additional condition

$$r(\bar{K}) < 0 \tag{21}$$

is sufficient to guarantee that existence.[8] Hereafter, we assume that (21) is satisfied and concentrate our attention on the possible multiplicity of steady states.

The key to existence of multiple steady states in our model lies in the possibility that the interest rate may be increasing in the capital stock over some range of the latter. In the special case of constant markups ($\mu' = 0$) that possibility is easily ruled out by noticing that variations in $K$ affect $r$ only through their effect on the marginal product of capital, which is strictly decreasing. As a result, $r' < 0$ and (19) will have a single solution.[9] In contrast, when

[7] In general we cannot guarantee the uniqueness of the nonzero solution to $g(u) - \delta u = 0$ for $g$ may not be concave. In order to avoid the uninteresting complication that arises from this fact we just assume $K(0) < \bar{K}$, where $\bar{K} > 0$ denotes the smallest of such solutions.

[8] Though (21) is stronger than necessary for the existence of a steady state, we use it in order to simplify the subsequent analysis of the equilibrium dynamics.

[9] Clearly, a similar result would obtain if $\mu' > 0$.

$\mu' < 0$, the higher demand elasticity brought about by an increase in the capital stock (and the resulting entry of new firms) will tend to increase marginal revenue, thus offsetting decline in the marginal product. Under what conditions will $r$ be increasing? Differentiation of (16) and a straightforward algebraic manipulation imply that $r'(K) > 0$ is equivalent to

$$\varepsilon_\mu \varepsilon_m > \gamma \eta, \tag{22}$$

where $\varepsilon_\mu \equiv -M\mu'/\mu$, $\varepsilon_m \equiv Km'/m$, $\gamma \equiv -(K - vm(K))f''/f'$, and $\eta \equiv (1 - vm')/(1 - vm/K)$. We thus see that a positive relationship between the aggregate capital stock and the interest rate requires at least one of the following: (a) a high elasticity of markups with respect to entry (i.e., a high $\varepsilon_\mu$), (b) a high rate of entry/exit in response to changes in the aggregate capital stock (high $\varepsilon_m$), (c) a slowly diminishing marginal product of capital (low $\gamma$), and (d) a small share of nonoverhead investment in total new investment relative to the share of nonoverhead capital in total capital (low $\eta$). Notice that (a) and (b) imply a high elasticity of markups with respect to the capital stock (i.e., a high $\varepsilon_k \equiv \varepsilon_\mu \varepsilon_m$).

The kind of nonmonotonicity of $r(K)$ discussed above is a necessary (but not sufficient) condition for the existence steady states. A stronger condition, which is both necessary and sufficient, is the existence of a capital stock $K^{**} \in (0, \bar{K})$ satisfying

$$r(K^{**}) = \rho, \qquad r'(K^{**}) > 0.$$

The previous condition follows immediately from the continuity of $r(K)$, combined with the Inada condition at zero and (21). Notice also that under those assumptions the number of equilibria must be (generically) odd. Fig. 1

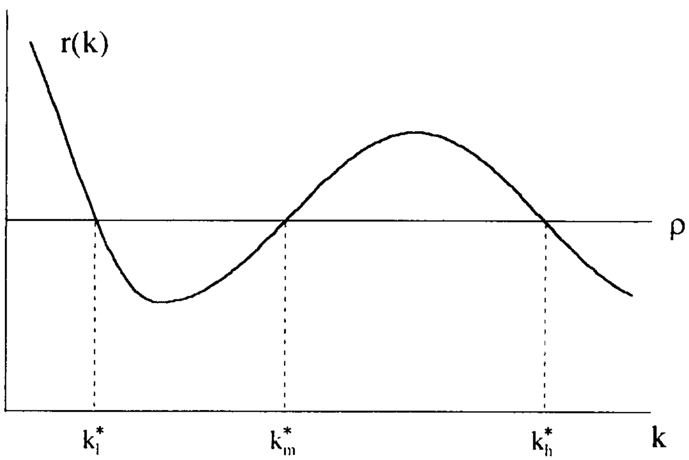


Fig. 1

illustrates the previous point by displaying an $r(K)$ schedule yielding three steady states.

We end this section by illustrating the possibility of multiple steady states in the model analyzed in this paper with a numerical example that is roughly consistent with some of the evidence discussed below. The production function is Cobb–Douglas, given by $F(k - v, n) = A(k - v)^{\alpha} n^{1-\alpha}$. The elasticity of substitution among inputs is linear and given by $\xi(m) = \phi + \varepsilon m$. Though rather restrictive, the previous two assumptions allow us to derive a closed-form expression for the number of firms consistent with a zero-profit equilibrium as a function of the aggregate capital stock:

$$m(K) = (\psi/2)\left(\sqrt{1 + 4\alpha\varepsilon K/v(1 + \alpha(\phi - 1))^2} - 1\right), \tag{23}$$

where $\psi = (1 + \alpha(\phi - 1))/\alpha\varepsilon$. The equilibrium interest rate in that case is given by

$$r(K) = A\alpha(K - vm(K))^{-(1-\alpha)}(1 - 1/\varepsilon m(K)) - \delta. \tag{24}$$

We choose parameter values so that the model roughly matches some features of the data discussed below.[10] The settings are $\alpha = 0.8$, $A = 0.397$, $v = 0.15$, $\phi = 1.27$, $\varepsilon = 0.05$, $\delta = 0.1$, and $\rho = 0.04$. Under the previous assumptions equation $r(K) = \rho$ has three solutions, corresponding to three interior steady states: $K_l^* = 0.35$, $K_m^* = 2.22$, and $K_h^* = 11.1$. The associated steady-state income levels are, respectively, $Y_l^* = 0.05$, $Y_m^* = 0.34$, and $Y_h^* = 1.78$. The implied steady-state markups are given by $\mu_l^* = 4.47$, $\mu_m^* = 3.62$, and $\mu_h^* = 2.21$.[11]

## 5. Equilibrium dynamics

Fig. 2 displays the phase diagram corresponding to the dynamical system (17)–(18) for a version of the model characterized by a single steady state.[12] Trajectories below (above) the $\dot{K} = 0$ schedule correspond to an increasing (decreasing) aggregate capital stock, as represented by the horizontal arrows.

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[10]The existence of multiple equilibria in our example is, admittedly, rather fragile: a relatively small change in the discount rate or the depreciation rate (for instance) could easily restore uniqueness. We view this as an unpleasant feature of the parametrization chosen (Cobb–Douglas technology, plus linear elasticity of substitution) in order to derive an interest rate mapping in closed form, not a general property of the model.

[11]Notice that these are value-added-based markups, and are thus higher than the corresponding markups on *total* marginal costs, which would be scaled down by the share of materials (see, e.g., Domowitz, Hubbard, and Petersen, 1988).

[12]The $\dot{K} = 0$ schedule is represented under the assumption that $g(K)$ is strictly concave in $(0, \bar{K})$.

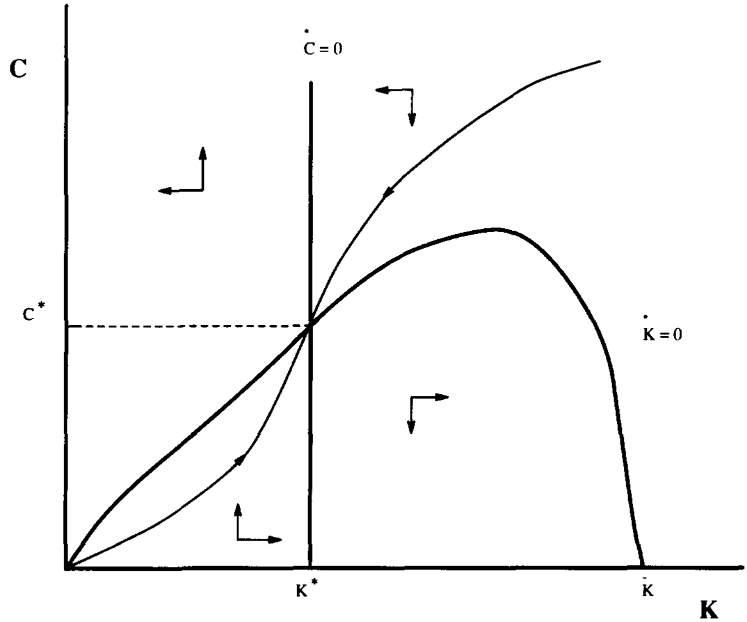


Fig. 2

The $\dot{C} = 0$ schedule is given, in addition to the $K$ axis, by a vertical line at $K^*$, the unique solution to (19). As shown in the Appendix, the unique steady state is always a saddle in this case. The equilibrium dynamics are qualitatively identical to those of the neoclassical model with perfect competition and constant returns (Cass, 1965; Koopmans, 1965): given an initial capital stock $K_0 \in (0, \bar{K})$, there is a unique equilibrium path, which converges monotonically to the steady state. That path belongs to the stable manifold of the steady state. Any trajectory above that manifold can be shown to deplete the stock of capital in finite time, forcing consumption to jump to zero, thus violating the consumer's Euler equation. Similarly, trajectories below the stable manifold cannot qualify as equilibria for they eventually violate the transversality condition.[13]

In the presence of multiple steady states the characterization of the equilibrium dynamics is more involved, since there are a number of cases that need to be considered even if we restrict ourselves to the case of three steady states, as we do below. We begin our discussion by stating some general results regarding the

[13] Given (21) and the fact that any such trajectory would converge to $(\bar{K}, 0)$, it must be the case that $\lim_{T \to \infty}(\dot{K}/K - (1/\sigma)\,(\dot{C}/C) - \rho) = -r(\bar{K}) > 0$ holds along such a trajectory, which violates the transversality condition.

*local* stability properties of the different steady states, leaving its formal derivation for the Appendix. Let $(K_1^*, C_1^*), (K_2^*, C_2^*), \ldots$ denote different interior steady states, with $K_i^* > K_j^*$ whenever $i > j$.

*Proposition 1. All steady states with an* odd *index are saddles.*

*Proposition 2. Let* $(K_e^*, C_e^*)$ *be a steady state with an* even *index. If* $g'(K_e^*) - \delta > 0$, *then* $(K_e^*, C_e^*)$ *is either an unstable node or an unstable focus. If* $g'(K_e^*) - \delta < 0$, *then* $(K_e^*, C_e^*)$ *is a stable node or a stable focus.*

Unfortunately, the size of $g'(K_e^*)$ depends in a complicated way on $\mu, f$, and the values taken by all the exogenous parameters. We can derive the following expression:

$$g'(K_e^*)/\delta = \mu(1 - vm')(1 + \rho/\delta),$$

where $\mu$ and $m'$ are evaluted at $K_e^*$. Thus we see that, *ceteris paribus*, $(K_e^*, C_e^*)$ will be unstable whenever the discount rate or the markup are sufficiently high, and/or the depreciation rate is low.

Figs. 3 and 4 show two alternative phase diagrams describing equilibrium dynamics consistent with (17)–(18) when the economy has three steady states, denoted by $(K_l^*, C_l^*)$, $(K_m^*, C_m^*)$, and $(K_h^*, C_h^*)$, and henceforth referred to as $\boldsymbol{L}$, $\boldsymbol{M}$, and $\boldsymbol{H}$, respectively. The $\dot{K} = 0$ schedule is shown to have a shape similar to that in Fig. 2. Again, trajectories below (above) that schedule correspond to an increasing (decreasing) aggregate capital stock. The $\dot{C} = 0$ schedule is now given, in addition to the $K$ axis, by the three vertical lines at $K_l^*$, $K_m^*$, and $K_h^*$, which partition the phase diagram into four regions. Given (18) and the properties of $r(K)$ (see Fig. 1), consumption can be shown to be increasing in the first and third regions (starting from the left) and decreasing in the second and fourth (as represented by the arrows).

Next we focus on the phase portrait in Fig. 3, which corresponds to an economy in which $\boldsymbol{M}$ is unstable. The instability of $\boldsymbol{M}$ follows from the fact that $g'(K_m^*) - \delta > 0$ (i.e., $\boldsymbol{M}$ lies on the upward-sloping segment of the $\dot{K} = 0$ schedule). The solid lines with arrows represent trajectories that satisfy the equilibrium conditions. As the figure makes clear, the set of equilibria depends on the size of the initial capital stock relative to the benchmarks $K_1$ and $K_2$, defined by the projections onto the $K$ axis of the left-most point of $\boldsymbol{H}$'s stable manifold and the right-most point of $\boldsymbol{L}$'s stable manifold, respectively. Economies for which $K(0) \in (0, K_1)$ have a unique equilibrium, which converges monotonically to the low steady state $\boldsymbol{L}$. Those economies can be thought of as being stuck in a 'poverty trap'. If $K(0) \in (K_2, \bar{K})$, the equilibrium is also unique, but now it converges to the high steady state $\boldsymbol{H}$. Finally (and most interestingly), if $K(0) \in (K_1, K_2)$, there are multiple trajectories that satisfy all the equilibrium

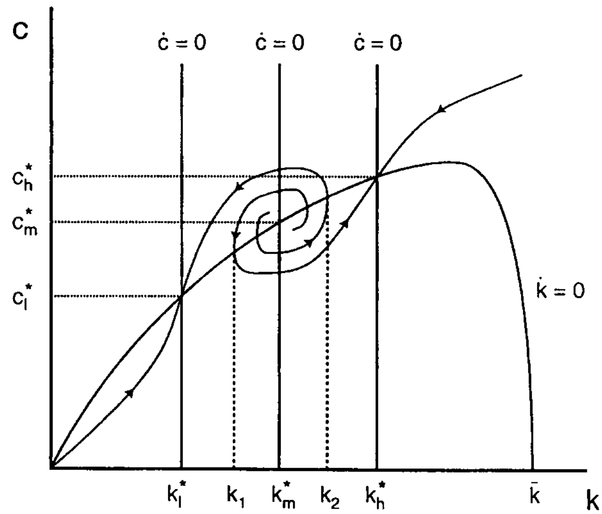


Fig. 3

conditions.[14] Whether the economy converges to the high or the low steady state depends on which of those equilibrium paths is actually 'selected'. That, in turn, depends on how agents coordinate their initial expectations on the future path of the economy, expectations which will be self-fulfilling given the perfect-foresight nature of our model. In other words, the initial conditions (defined by the initial capital stock) are no longer sufficient to pin down the outcome that will be observed: the latter is a function of agents' expectations as well. As a result that outcome can differ for economies with identical fundamentals (including initial conditions).

The range of initial conditions for which multiple equilibrium paths exist depends on the extent of the 'overlap' of the stable manifolds associated with ***L*** and ***H***, represented by the interval $(K_1, K_2)$. The extent of that overlap region depends in a complicated way on parameter values. In particular, for sufficiently low values of $\sigma$ the eigenvalues associated with the linearization of (17)–(18) will be real (and positive), in which case we cannot rule the vanishing of the overlap region and the implied emergence of *determinate* equilibrium dynamics with a 'threshold': if $K(0) < K_m^*$, there is a (unique) equilibrium path converging to ***L***; if $K(0) > K_m^*$, the (also unique) equilibrium converges to ***H*** instead. In other words, the steady state to which an economy converges is determined by the

[14] In fact, if ***M*** is a focus (i.e., if the associated eigenvalues are complex) the number of equilibria becomes infinite (though countable) as $K(0) \to K_m^*$.

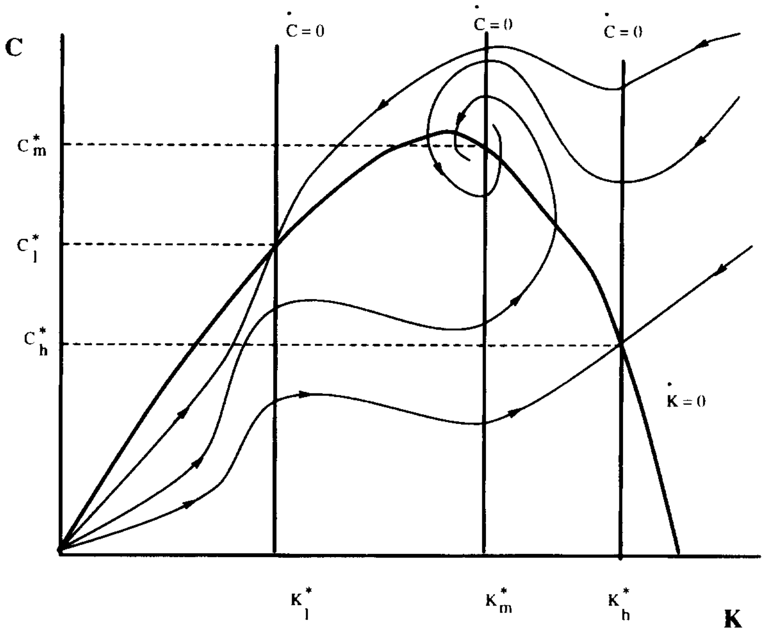


Fig. 4

initial capital stock in that case. A completely different situation may arise if the left branch of ***H***'s stable manifold has its origin at (0, 0) instead of ***M***: in that case the poverty trap associated with low capital stock levels disappears: no matter how low $K(0)$ is, there is always an equilibrium path which converges to ***H*** (in addition to other possible equilibria).[15]

Fig. 4 displays yet another possible kind of equilibrium dynamics, corresponding to an economy in which $g'(K_m^*) - \delta < 0$ (i.e., ***M*** lies on the downward-sloping segment of $\dot{K} = 0$). ***M*** is stable in this case. The extent of the multiplicity of equilibria is even greater now: given any initial capital stock $K(0) \in (0, \bar{K})$ there exists a *continuum* of trajectories consistent with equilibrium. Two of those trajectories correspond to the stable manifolds of ***L*** and ***H***, respectively, and converge toward those steady states. In addition there is an open set of equilibrium paths [each of which can be indexed by the initial choice of consumption $C(0)$] which corresponds to the area between the two previous

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[15]The symmetric situation, corresponding to the case in which the right branch of ***L***'s stable manifold originates on the $K = \bar{K}$ locus, leads instead to the possibility of convergence to the low steady state ***L***, for any initial capital stock $K(0)$.

stable manifolds. All of those paths converge to $M$. Thus, in that case, the possibility of nonconvergence in income levels extends to all economies, regardless of initial conditions.[16]

The possibility of multiple equilibrium paths in our model results from the *complementarity* among individual investment decisions whenever the interest rate is increasing in the aggregate capital stock over some range of the latter. In that case, the expectation that the economy as a whole will follow a path characterized by high investment, high entry rates, and low markups raises the anticipated *private* return to investment, thus inducing individual savings decisions that are consistent, *in the aggregate*, with the initial expectations (given our perfect foresight assumption).

Needless to say, many other authors have developed models that exhibit, in their reduced form, similar complementarities which are the source of multiple equilibria. From that point of view the model is formally related to some recent work that aims at examining the implications of external increasing returns and/or pecuniary externalities in the context of dynamic models with capital accumulation and which often finds in those features a potential source of multiple equilibria.[17] Yet, the model developed above points to a completely different possible source of multiplicity: the interaction between the extent of competition (related to the number of firms and the range of available products), the size of demand elasticities and markups, and the private return to investment.

## 6. Empirical issues

The model developed and analyzed above has a number of interesting empirical implications. Some of the predictions are also shared by other dynamic models with multiple equilibria, while others are specific to the class of models with endogenous markups. We discuss some of these in turn.[18]

[16] A Hopf bifurcation occurs as $g'(K_m^*) - \delta$ switches sign and, as a result, the eigenvalues of the linearized system around $M$ cross the imaginary axis. In that case a periodic orbit will emerge for a range of parameter values, as the stability of the middle steady state changes. We view that possibility as not particularly relevant from an empirical point of view. Given that, and in order to save space, we omit a discussion of the possible dynamics associated with the presence of a periodic orbit.

[17] See references in Section 1.

[18] A more detailed discussion of the empirical relevance of a number of growth models with multiple equilibria can be found in Benhabib and Gali (1995).

### 6.1. Multiple equilibria in growth models: Some empirical implications

In the presence of multiple equilibria of the sort generated by the model above (and other models with similar reduced-form dynamics) the income levels of different economies may fail to converge, even asymptotically. Such a property seems consistent with the lack of evidence of per capita income convergence across the sample of countries in the Summers–Heston data set (Romer, 1989; Barro, 1991). In our model the possibility of no convergence does not rely on the presence of differences in fundamentals: two economies may diverge even if they have identical preferences and technologies. This stands in contrast with the well-known predictions of the neoclassical growth model.

Lack of income convergence across economies with identical fundamentals but *different initial conditions* (i.e., initial stocks of accumulated factors) is a prediction of a variety of models with determinate equilibria, including endogenous growth models (e.g., Lucas, 1988) as well as models with 'threshold' dynamics (e.g., Azariadis and Drazen, 1990).[19] The class of models with multiple equilibria discussed here go beyond that possibility: no convergence may obtain even for economies with *both identical fundamentals and identical initial conditions*. That property offers a potential explanation for 'economic miracles', i.e., sudden, seemingly unpredictable fast growth episodes that lead to a growing income gap between economies that were very similar at some point, as illustrated by the examples of South Korea and the Philippines discussed in Lucas (1990). Furthermore, the existence of multiple equilibrium paths of the sort implied by our model (see, e.g., Fig. 3) allows for the possibility of 'catching up' followed by 'overtaking' between countries that are initially endowed with different capital stock levels (but identical fundamentals). That possibility, which cannot be accounted for by either endogenous growth models or threshold models, appears to be consistent with the evidence of substantial 'reshuffling' over time in the distribution of relative income across countries provided by Quah (1993).[20]

The existence of a multiplicity of steady states and equilibrium paths can also be reconciled with three additional pieces of evidence. First, it is consistent with the existence of 'convergence clubs', i.e., income convergence among a limited set of countries. The existence of such 'clubs', empirically identified by a number of authors (Baumol et al., 1989), is clearly at odds with the predictions of endogenous growth models. In contrast, the model developed above implies that,

[19] Endogenous growth models predict no convergence even for economies with 'similar' initial conditions. Threshold models (with bounded growth) predict convergence across economies whose initial conditions are sufficiently close.

[20] Benhabib and Gali (1995) show that the basic evidence provided by Quah is robust to controlling for possible differences in technology levels across countries. Clearly, that evidence could be reconciled with determinate equilibrium models once we allowed for large enough random shocks.

asymptotically, economies are sorted out by the steady state to which they converge. *Conditional on selecting paths that lie on the same stable manifold*, two economies with (possibly) very different initial conditions (but otherwise identical) will eventually see their income levels converge.[21] Second, the model implies that interest rates will be asymptotically equalized across economies, even in the absence of international capital mobility. In contrast with the neoclassical model, such equalization of interest rates may coexist with very different levels of capital and income, corresponding to different steady states. That implication of the model allows us to reconcile the absence of large differences in interest rates across countries (and the seemingly related failure of capital to flow from rich to poor countries) with the existence of potentially large (and persistent) income gaps among the same countries.[22]

### *6.2. Economic development and markups: Some evidence*

Finally, we turn to some evidence pertaining to a more specific prediction of our model, namely, the presence of a negative relationship between the size of markups and the level of development of an economy, as measured by (per capita) income or capital stock levels. As discussed above, the existence of such a link between development and markups is a necessary (though not sufficient) condition for our model to yield multiple equilibria. Assessing the empirical relevance of that relationship faces an unavoidable difficulty: the need to construct measures of markups. In order to do so we use the methodology proposed by Hall (1988), which exploits an immediate implication of one of the first-order conditions for profit maximization, namely, condition (5), evaluated at the symmetric equilibrium (i.e., at $p = 1$). A straightforward manipulation yields

$$\mu = (F_2 N/Y)/(wN/Y) \equiv \alpha_n/s_n, \tag{25}$$

where $\alpha_n$ is the elasticity of output with respect to employment and $s_n$ is the labor income share. We want to stress the fact that (25) will hold as long as firms maximize profits given the wage, even in the presence of unspecified distortions in labor markets (e.g., wage-setting by unions, minimum wages, etc.). Even

[21] Notice that in our model the selection of an equilibrium path lying on the same stable manifold corresponds to a choice of a savings rate. In other words, observed saving rates may act as a 'proxy' for the steady state to which economies will end up approaching. That observation allows us to reconcile our model with the evidence of convergence conditional on savings rates found in Mankiw, Romer, and Weil (1992).

[22] The 'interest rate puzzle' was initially raised by King and Rebelo (1993) in the context of the standard neoclassical growth model. Lucas (1990) offers an alternative explanation based on differences in human capital levels and externalities.

though $\alpha_n$ is not directly observable, some data pertaining to $s_n$ is available: Table 1 displays labor income share measures for a subset of countries in the Summers–Heston sample, for years 1965 and 1985. The reported shares have been computed as the ratio of compensation of employees to GDP, using international data from the U.N. publication *National Accounts Statistics: Main Aggregates and Detailed Tables*. A quick glance at Table 1 points to the presence of substantial differences across countries in those shares. Our model predicts that such differences should be related to differences in the level of economic development. More precisely, there should be a negative relationship between the level of markups and the income or capital stock levels. We assess the empirical relevance of that prediction by estimating the following equation:

$$\log(1/s_n^i) = \beta_0 + \beta_1 \log(Z^i) + u^i, \tag{26}$$

where $i$ is a country index and $Z^i$ denotes either GNP ($Y^i$) or the capital stock ($K^i$) for country $i$. We estimate (26) for two years (1965 and 1985), alternatively using population size or employment to normalize $Y^i$ and $K^i$. GNP, population, and employment data are taken from the Summers–Heston (1991) data set. We use the capital stock measures constructed by Benhabib and Spiegel (1994). Our model predicts that $\beta_1$ should be negative in (26). Alternative OLS estimates of (26) obtained are displayed in Table 2 (standard errors are reported in parentheses).[23]

Notice that all $\beta_1$ estimates are negative and significantly different from zero at conventional confidence levels. Furthermore, the point estimates across specifications appear to be remarkably close, lying between $-0.06$ and $-0.12$. Those estimates have the interpretation of markup elasticities with respect to the capital stock or GNP and imply that richer economies are also, on average, more competitive. The average (across four equations) estimate of the elasticity of the markup with respect to $K$ is $-0.08$, while the estimated elasticity with respect to $Y$ is slightly below $-0.09$.

Given our constructed measure of markups, the significant negative relationship between markups and levels of development detected in the data is a reflection of the fact that, on average, rich countries tend to have higher labor income shares. Needless to say, that empirical observation has a number of possible theoretical explanations in addition to the presence of an inverse relationship between markup and capital (or income) levels. Possible alternative explanations include departures from the assumption of a Cobb–Douglas technology (e.g., CES with an elasticity of substitution less than one), employment decisions

[23] Notice that OLS is a consistent estimator for the previous coefficients under the assumption of a common elasticity parameter $\alpha_n^i$ across countries (as a Cobb–Douglas technology with identical 'shares' would imply) or, more generally, if the cross-country distribution of $\alpha_n^i$ is uncorrelated with the corresponding income or capital distribution.

Table 1
Labor income shares

| # | Country | $s_n$ (1965) | $s_n$ (1985) |
|---|---|---|---|
| 1 | Algeria | | 35.3 |
| 2 | Angola | | |
| 3 | Benin | | 19.8 |
| 4 | Botswana | 31.0 | 28.9 |
| 5 | Burkina Fasso | | 25.3 |
| 6 | Burundi | | 19 |
| 7 | Cameroon | | 26.9 |
| 8 | Cape Verde | | |
| 9 | Central African R. | | |
| 10 | Chad | | |
| 11 | Comoros | | |
| 12 | Congo | | 27.2 |
| 13 | Egypt | 40.7 | |
| 14 | Ethiopia | | |
| 15 | Gabon | | 29 |
| 16 | Gambia | | |
| 17 | Ghana | | |
| 18 | Guinea | | |
| 19 | Guinea Bissau | | 31.3 |
| 20 | Ivory Coast | | 35.1 |
| 21 | Kenya | 39.7 | 35.6 |
| 22 | Lesotho | | |
| 23 | Liberia | | |
| 24 | Madagascar | 32.9 | |
| 25 | Malawi | 23.9 | 20.2 |
| 26 | Mali | | 24.7 |
| 27 | Mauritania | | |
| 28 | Mauritius | 50.3 | 39.5 |
| 29 | Morocco | | |
| 30 | Mozambique | | |
| 31 | Niger | 12.7 | 17.6 |
| 32 | Nigeria | | 21.9 |
| 33 | Rwanda | | 23.7 |
| 34 | Senegal | | |
| 35 | Seychelles | | 37.1 |
| 36 | Sierra Leone | | 18 |
| 37 | Somalia | | |
| 38 | S. Africa | 53.1 | 53 |
| 39 | Sudan | | 34.3 |
| 40 | Swaziland | 42.2 | 46.6 |
| 41 | Tanzania | 29.0 | 14.1 |
| 42 | Togo | | 28 |
| 43 | Tunisia | | |
| 44 | Uganda | | |
| 45 | Zaire | 29.7 | |
| 46 | Zambia | 37.5 | 39.8 |

Table 1 (continued)

| # | Country | $s_n$ (1965) | $s_n$ (1985) |
|---|---|---|---|
| 47 | Zimbabwe | 53.2 | 52.4 |
| 48 | Afghanistan | | |
| 49 | Bahrain | | 42.6 |
| 50 | Bangladesh | | |
| 51 | Myanmar | | 39.2 |
| 52 | China | | |
| 53 | Honk Kong | | 50.4 |
| 54 | India | | |
| 55 | Iran | | |
| 56 | Iraq | 22.3 | 31.5 |
| 57 | Israel | 49.1 | 46.7 |
| 58 | Japan | 44.4 | 54.2 |
| 59 | Jordan | 61.0 | 41.3 |
| 60 | Korea | 26.6 | 39.5 |
| 61 | Kuwait | 14.4 | 31.1 |
| 62 | Malaysia | 37.9 | 33.4 |
| 63 | Nepal | | 55.1 |
| 64 | Oman | | 28.7 |
| 65 | Pakistan | | |
| 66 | Philippines | | 30.2 |
| 67 | Saudi Arabia | | 42.4 |
| 68 | Singapore | | |
| 69 | Sri Lanka | 43.3 | 44.8 |
| 70 | Syria | 30.1 | |
| 71 | Taiwan | | |
| 72 | Thailand | 23.6 | 25.8 |
| 73 | U. Arab Emirates | | 25.1 |
| 74 | Yemen | | |
| 75 | Austria | 47.8 | 53.1 |
| 76 | Belgium | 48.2 | 56.1 |
| 77 | Cyprus | | |
| 78 | Denmark | 49.7 | 53.7 |
| 79 | Finland | 49.2 | 54.9 |
| 80 | France | 46.8 | 54.9 |
| 81 | W. Germany | 49.5 | 56 |
| 82 | Greece | 30.5 | 41.4 |
| 83 | Hungary | | 45.2 |
| 84 | Iceland | | 47.1 |
| 85 | Ireland | 48.4 | 53.9 |
| 86 | Italy | 45.5 | 46.1 |
| 87 | Luxembourg | 52.7 | 59.6 |
| 88 | Malta | 48.5 | 46.8 |
| 89 | Netherlands | 52.6 | 51.8 |
| 90 | Norway | 49.8 | 47.9 |
| 91 | Poland | | |
| 92 | Portugal | | 47.3 |
| 93 | Spain | 46.7 | 45.7 |

Table 1 (continued)

| # | Country | $s_n$ (1965) | $s_n$ (1985) |
|---|---|---|---|
| 94 | Sweden | 55.5 | 58.3 |
| 95 | Switzerland | 54.7 | 62 |
| 96 | Turkey | | 19.8 |
| 97 | United Kingdom | 60.5 | 55.3 |
| 98 | Yugoslavia | | |
| 99 | Bahamas | | 42.4 |
| 100 | Barbados | 51.7 | |
| 101 | Canada | 51.9 | 54.2 |
| 102 | Costa Rica | 43.0 | 46.6 |
| 103 | Dominica | | |
| 104 | Dominican Republic | | |
| 105 | El Salvador | | |
| 106 | Grenada | | |
| 107 | Guatemala | | |
| 108 | Haiti | | |
| 109 | Honduras | 43.8 | 45.4 |
| 110 | Jamaica | 47.1 | 43.6 |
| 111 | Mexico | 32.6 | 28.6 |
| 112 | Nicaragua | | |
| 113 | Panama | 64.9 | 49.6 |
| 114 | St. Lucia | | |
| 115 | Trinidad & Tobago | | 60.9 |
| 116 | United States | 57.1 | 59.6 |
| 117 | St. Vincent | | |
| 118 | Argentina | | |
| 119 | Bolivia | 27.0 | 32.3 |
| 120 | Brazil | 29.8 | |
| 121 | Chile | 40.8 | 33 |
| 122 | Colombia | 36.6 | 40.6 |
| 123 | Ecuador | 41.8 | 20.9 |
| 124 | Guyana | 47.7 | |
| 125 | Paraguay | 35.7 | 31 |
| 126 | Peru | 40.2 | 27.4 |
| 127 | Suriname | | 64.6 |
| 128 | Uruguay | 46.9 | 38.1 |
| 129 | Venezuela | 42.9 | 35.1 |
| 130 | Australia | 52.2 | 50.9 |
| 131 | Fiji | 49.8 | 45.2 |
| 132 | Indonesia | | |
| 133 | New Zealand | 50.1 | 49.8 |
| 134 | Papua New Guinea | | 39.3 |
| 135 | Solomon Is. | | 44.4 |
| 136 | Tonga | | 44.2 |
| 137 | Vanuatu | | 42.8 |
| 138 | Western Samoa | | |

*Source*: Author's calculations using data from Table 3.1 of *National Accounts statistics: Main Aggregates and Detailed Tables* of the United Nations.

Table 2
Markup regressions

| Dependent variable | $\hat{\beta}_0$ | $\hat{\beta}_1$ |
|---|---|---|
| $K$ (1965, per capita) | 0.99<br>(0.06) | − 0.06<br>(0.03) |
| $K$ (1965, per worker) | 1.06<br>(0.09) | − 0.07<br>(0.03) |
| $K$ (1985, per capita) | 1.11<br>(0.05) | − 0.09<br>(0.02) |
| $K$ (1985, per worker) | 1.23<br>(0.07) | − 0.10<br>(0.02) |
| $Y$ (1965, per capita) | 1.44<br>(0.32) | − 0.07<br>(0.04) |
| $Y$ (1965, per worker) | 1.50<br>(0.34) | − 0.07<br>(0.03) |
| $Y$ (1985, per capita) | 1.87<br>(0.28) | − 0.11<br>(0.03) |
| $Y$ (1985, per worker) | 2.11<br>(0.31) | − 0.12<br>(0.03) |

by firms which are not based on profit maximization conditional on the wage (e.g., efficient contracts), inappropriate accounting of self-employment, etc. Sorting out the relative importance of those alternative explanations is a challenging task that falls well beyond the scope of this paper.

## 7. Concluding remarks

We have analyzed the implications of endogenous markups for the dynamics of capital accumulation, in an environment in which the size of markups is related to the number of firms supplying differentiated inputs to a final goods sector. Even though the technology available to firms is characterized by a diminishing marginal product of capital (and no productive externalities are present), the equilibrium interest rate may increase as the economy accumulates capital (at least over some range), for positive net investment is associated with entry, increased competition, and, ceteris paribus, a higher marginal revenue. That nonmonotonicity of interest rates may in turn generate multiple steady states as well as multiple equilibrium paths (for given initial conditions).

A key property of the model is the existence of an inverse relationship between the size of markups and the level of output and the capital stock. We have provided some evidence that points to the presence of such a relationship in the data. We find some other properties of the model quite appealing from an empirical point of view, since they provide a potential explanation for the lack of unconditional convergence, the presence of convergence clubs and poverty traps, the emergence of economic miracles, and the absence of capital flows from rich to poor countries. The interpretation of those phenomena given by our model does not rely on the existence of differences in fundamentals (technology, preferences, policies), but on the role of initial conditions and expectations in determining the fate of an economy in the presence of multiple equilibria.

## Appendix: Local stability analysis

Linearization of (17) and (18) around a steady state $(K^*, C^*)$ yields the dynamical system

$$\begin{bmatrix} \dot{K} \\ \dot{C} \end{bmatrix} = \begin{bmatrix} g' - \delta & -1 \\ \sigma C^* r' & 0 \end{bmatrix} \begin{bmatrix} K - K^* \\ C - C^* \end{bmatrix},$$

where both $g'$ and $r'$ are evaluated at $K^*$. The corresponding eigenvalues are

$$\lambda = \tfrac{1}{2}[(g' - \delta) \pm \sqrt{(g' - \delta)^2 - 4\sigma C^* r'}].$$

We can distinguish among the following (generic) cases:

*Case 1:* $r' < 0$. Under our assumptions, this condition holds for any *odd* steady state. In this case, both eigenvalues are real and have opposite signs, so the steady state is a saddle.

*Case 2:* $r' > 0$. This condition holds for any *even* steady state. In this case the real part of both eigenvalues will have the same sign. The following generic configurations are possible:

(a) $g' - \delta > 0$ and $(g' - \delta)^2 - 4\sigma C^* r' > 0$. Both eigenvalues are real and positive. The steady state is an unstable node.
(b) $g' - \delta > 0$ and $(g' - \delta)^2 - 4\sigma C^* r' < 0$. The eigenvalues form a complex conjugate pair, with a positive real part. The steady state is an unstable focus.
(c) $g' - \delta < 0$ and $(g' - \delta)^2 - 4\sigma C^* r' > 0$. Both eigenvalues are real and negative. The steady state is a stable node.
(d) $g' - \delta < 0$ and $(g' - \delta)^2 - 4\sigma C^* r' < 0$. The eigenvalues form a complex conjugate pair, with a negative real part. The steady state is a stable focus.

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