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Price smoothing policies*

A welfare analysis

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In post-WWII experience U.S. monetary authorities have attempted to eliminate seasonal fluctuations in prices and nominal interest rates. Developments in financial markets and recently discovered empirical regularities regarding the seasonal cycle seem to make these activities questionable. Using a money-in-the-utility-function model this paper analyzes the welfare properties of price and interest rate smoothing policies and the sense in which the distinction between seasonal and cyclical fluctuations is relevant. It is shown that smoothing policies are welfare improving, but not optimal, and that the origin of the shocks, not the persistence of the fluctuations, is relevant in formulating policies.

1. Introduction

For much of the post-WWII experience it has been common practice for the U.S. monetary authorities to react to seasonal variations in the demand for money and credit in such a way as to keep the price level and nominal interest rates relatively free of seasonal fluctuations. Pursuing these seasonal activities was motivated in the earlier part of the century by the perceived connection between seasonal movements in interest rates and banking panics [see Kemmerer (1910), Friedman and Schwartz (1963), Miron (1986), and Canova (1991), among others]. However, after the FDIC began insuring commercial bank deposits in 1934, there seems little reason to have continued

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this practice since the apparent link between seasonals and banking panics was severed.

Some justifications for continuing these seasonal activities exist in the current literature. For example, Poole and Lieberman (1972) claim that the elimination of seasonality in nominal interest rates arising from seasonal fluctuations in money demand (as opposed to seasonal movements in aggregate demand) achieves an efficient allocation of resources. Goodfriend (1987) argues that central banks regard price instability as costly because of a Lucas (1972) aggregate supply effect. He reassesses the idea that central banks care about interest rate instability because of potential threats to the financial system. In Barro (1989) nominal interest rate smoothing is desirable for the same reasons cited by Goodfriend.

Earlier work by Friedman and Schwartz (1963, p. 295) also suggested that a price smoothing policy, following seasonal movements in the money demand to output ratio, is 'perhaps desirable' since there are seasons when the community desires smaller real balances relative to income at any given interest rate. However, more recently Friedman (1982b, p. 406) contends:

> Even if the seasonal in the demand function were known, it is by no means obvious that there is any social gain from the Fed's offsetting it. After all, the markets are well equipped to handle regular seasonals.

In discussing the formulation of price smoothing policies, Friedman and Schwartz also stressed the distinction between seasonal and cyclical fluctuations in the money demand to output ratio. They claim that the proper price smoothing response depends on which of the two cycles is present in the economy (p. 295). Barsky and Miron (1989) implicitly question this distinction by demonstrating the existence of several statistical similarities between what they call the 'seasonal' and the 'business' cycle. In particular their analysis reveals that several real variables (output in particular) possess similar features at seasonal and business cycle frequencies. Therefore Friedman and Schwartz's distinction seems appropriate only if cyclical shocks affect money demand differently than seasonal shocks.

In this paper I analyze the welfare implications of price and interest rate smoothing policies and examine whether the distinction between seasonal and cyclical shocks matters in the formulation of smoothing policies. The exercises are conducted in the context of a money-in-the-utility-function (MIUF) model similar to the one employed by LeRoy (1984a, b) and Danthine and Donaldson (1986). While these studies emphasized the effect of certain shocks on commodity and asset prices, this paper concentrates primarily on the welfare properties of various government portfolio choices in response to certain shocks. Although other attempts to assess the welfare implications of alternative government portfolios exist in the literature [e.g., Sargent and

Wallace (1982), Wallace (1988)], the analysis so far has been restricted to deterministic environments.

The model considered includes money and one additional asset which dominates money in rate of return. This additional asset is included because it is widely recognized that alternative government portfolios may be neutral if no rate of return dominance exists [see Wallace (1981), Chamley and Polemarchakis (1985), Sargent and Smith (1987), and Gottardi (1988)]. Also, contrary to both Barro and Goodfriend who describe simple choices for money supply paths, I examine a class of open market price smoothing policies. This approach has the advantage of more closely resembling the policies employed by the Fed in the post-WWII era. The policies considered have real effects because they influence either the level of real cash balances or their rate of return.[1]

The way money is incorporated in a general equilibrium model is still a controversial issue in monetary theory [see Danthine and Donaldson (1986) for a discussion and references]. Since the results may depend on the framework of analysis used, the paper also discusses the robustness of the welfare conclusions and of the seasonal/cyclical distinction using two alternative specifications (a cash-in-advance and an overlapping generations model).

The paper demonstrates that in a MIUF model smoothing policies can be welfare improving, but they are not optimal [see also Friedman (1969, pp. 46–48)]. Since at an optimum the return of money must fluctuate with the state of the economy, it is not clear why smoothing policies have received more attention in practice than policies attempting to drive the economy toward optimality. As far as the distinction between seasonal and cyclical fluctuations is concerned, it is shown that the design of welfare improving policies may depend on the origin of the shocks, but not on the structure of their transition matrix (i.e., the persistence of the fluctuations). Finally, the paper shows that these conclusions are robust both to alternative ways of introducing money in the economy and to the life span of the agents in the economy, but that they may be altered in an environment where agents are heterogeneous.

The rest of the paper is organized as follows: The next section presents the MIUF model, computes the equilibria for a set of benchmark policies, and examines the response of the price level and of the interest rate to two types of shocks. Section 3 discusses the relevance of the distinction between seasonal and cyclical fluctuation. Section 4 considers the welfare properties of a class of smoothing policies. Section 5 examines the robustness of the results to alternative ways of introducing money in the economy. Section 6 contains the conclusions.

[1]Although it has been argued that the ability of monetary policy to affect real variables through these channels may be quantitatively unimportant, alternative model specifications with more interesting real results are lacking [see Barro (1989) for a discussion of this issues].

2. The model

There are two agents in the economy: a representative consumer who derives utility from consumption and real cash balances and saves using money or government bonds and a government that finances current deficits with an inflation tax and bond issues.

There is one perishable good in the economy and agents receive a random endowment x_t of it each period. There are two nominal assets in the model: money M_t and a one-period bond with face value B_t. The price of money in terms of consumption is p_t and the price of a default-free bond paying one unit of currency at maturity is S_t. Then, the gross nominal interest rate on risk-free bonds is $1/S_t$, which is a known quantity at t.

The government is composed of two distinct authorities. A fiscal authority that exogenously produces a net-of-interest deficit $D_t = G_t - T_t$ and a monetary authority that takes D_t as given and finances it subject to the constraint:

$$D_t = m_t^s - m_{t-1}^s \pi_{t-1} + b_t^s - b_{t-1}^s r_{t-1}. \tag{1}$$

Here $m_t^s = M_{t+1}^s p_t$ and $b_t^s = B_{t+1}^s p_t S_t$, $\pi_{t-1} = p_t / p_{t-1}$ is the gross real return on currency, and $r_{t-1} = \pi_{t-1}/S_{t-1}$ is the gross real return on bonds. At time 1 the finance constraint is given by

$$D_1 = m_1^s + b_1^s - p_1 * (M_0^s + B_0^s). \tag{2}$$

The representative consumer maximizes

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, m_t, \varepsilon_t), \tag{3}$$

where ε_t is a preference shock and $0 \leq \beta \leq 1$ is a discount factor. The constraint she faces is

$$c_t + m_t + b_t \leq x_t + m_{t-1}\pi_{t-1} + b_{t-1} r_{t-1}, \tag{4}$$
$$c_t \geq 0, \quad m_t \geq 0, \quad \forall t.$$

The time t choice variables for the consumer are c_t, m_t, b_t. Let $W_t = m_{t-1}\pi_{t-1} + b_{t-1} r_{t-1}$ be the financial wealth available to agents at the beginning of time t and let $\xi_t = (x_t, \varepsilon_t)$ be the realization of a two-state Markov chain, with $\infty > x_h \geq x_l > 0$ and $\infty > \varepsilon_h \geq \varepsilon_l > 0$ and time-invariant transition function:

$$P(\xi_t = \xi_i \mid \xi_{t-1} = \xi_j) = \Psi(i, j). \tag{5}$$

$\Psi(i,j)$ is assumed to induce a unique time-invariant measure on ξ_t which describes the long-run (average) behavior of the process and has the following properties:

Assumption 1. $1 \geq \Psi(i,j) \geq 0$, $\forall i,j$; $\sum_j \Psi(i,j) = 1$.

The individual decision problem can be cast into a dynamic programming framework by defining Z as the maximum expected utility obtainable by a consumer who begins the period with W_t when the states of the economy are $(\xi_t, D_t, M^s_{t+1}, B^s_{t+1})$ and the prices are p_t and S_t. Such a function, if it exists, satisfies

$$Z(W, \xi, D, M^s, B^s) = \max_{(c,m,b)} U(c, m, \varepsilon) + \beta \mathrm{E} Z(W', \xi', D', M^{s\prime}, B^{s\prime}). \qquad (6)$$

To insure the existence of a solution to the consumer's problem we impose the following conditions:

Assumption 2. There exist a $\hat{m} < \infty$ such that $U(c_t, \hat{m}, \varepsilon_t) \geq U(c_t, m_t, \varepsilon_t)$, $\forall m_t$, and $U_2(c_t, \hat{m}, \varepsilon_t) = 0$.

Assumption 3. For all c_t, $m_t \in [0, \hat{m}]$ and for fixed ε_t, $U(.,.,.)$ is differentiable, increasing in c and m, and strictly concave.

Assumption 4. $V(c_t, m_t, \varepsilon_t) = U_2(c_t, m_t, \varepsilon_t)/U_1(c_t, m_t, \varepsilon_t) \to \infty$ as $m_t \to 0$ for fixed c_t and ε_t, and $V(c_t, m_t, \varepsilon_t) \to 0$ as $c_t \to 0$ for fixed m_t and ε_t.

Assumption 2 is innocuous. None of the results are altered if we let $\hat{m} \to \infty$. It is nevertheless useful since it facilitates the comparison with a cash-in-advance setup (see section 5) where there exists a natural upper bound to the value of real cash balances [see Wallace (1988)] and with previous work on optimal monetary growth [see, e.g., Friedman (1969) and Abel (1987)]. Assumptions 3 and 4 are standard [see, e.g., Danthine and Donaldson (1986) or Wallace (1988)] and insure that the consumer's indifference curves are well-behaved. Under these assumptions the necessary and sufficient conditions for the consumer problem can be summarized by the following expression for the intratemporal marginal rate of substitution between c_t and m_t:

$$V(c_t, m_t, \varepsilon_t) = 1 - \frac{\beta \mathrm{E}_t U_1(c_{t+1}, m_{t+1}, \varepsilon_{t+1}) \pi_t}{U_1(c_t, m_t, \varepsilon_t)} = 1 - S_t. \qquad (7)$$

Note that (7) embeds the notion of rate of return dominance for money. The marginal utility of holding real bonds, expressed in terms of expected marginal utility of consumption, exceeds the return on money [see Townsend (1987)]. This anomaly arises because, in a monetary equilibrium and from the point of view of agents, the return on money must be augmented by the indirect yield obtained by loosening the liquidity constraint before it is compared with other assets. In the environment considered here there are potential profits to be made by appropriately intermediating between the two assets. Therefore, in general, different government portfolios will not be neutral.

2.1. Equilibrium

An equilibrium for the economy is defined as follows:

Definition. A competitive equilibrium is a collection of bounded sequences for consumer choice variables (c_t, m_t, b_t), prices $(p_t > 0,\ S_t > 0)$, and government portfolio choices (M^s_{t+1}, B^s_{t+1}), such that

- (c_t, m_t, b_t) solve (6).
- (M^s_{t+1}, B^s_{t+1}) satisfy (1).
- $x_t = G_t + c_t$.
- $m_t = M^s_{t+1} p_t$ and $b_t = B^s_{t+1} S_t p_t$.
- $r_t > 1,\ \forall t$, and $S_t = \pi_t / r_t$.

To compute the equilibrium for this economy we need to impose more structure on the activities of the government. The following assumption describes the conduct of the fiscal authorities and a benchmark policy employed by the monetary authority in managing the government portfolio.

Assumption 5. $0 \leq D_t = D \leq \delta$, $\forall t$ and for an appropriately chosen δ; $M^s_{t+1} = (1 + \Upsilon) M^s_t$, $\Upsilon > (\leq) 0$; $M^s_0 + B^s_0 > 0$; and $\theta_t = b_t / m_t = \theta$, $\forall t$.

Assumption 5 allows us to concentrate entirely on the management of the government portfolio (the fiscal deficit is constant) and restricts the activities of the monetary authority to be time- and state-independent. The money supply is allowed to grow or shrink as long as the composition of the government portfolio, denoted by θ, is unchanged over time. The condition $M^s_0 + B^s_0 > 0$

insures that p_1 is positive. Note that, given M_0^s and B_0^s, policy choices are fully described by the parameter θ. One obvious choice that satisfies all of these conditions is $M_{t+1}^s = M_0^s$ and $\theta = 0$.

Let $\mu = m_l/m_h$ and $\lambda = U_1(x_l - D, m_l, \varepsilon_l)/U_1(x_h - D, m_h, \varepsilon_h)$. The conditional returns on money are given by

$$\mathrm{E}(\pi | t = h) = \frac{\Psi(h,h) + \Psi(h,l)\mu}{1 + \Upsilon},$$
$$\mathrm{E}(\pi | t = l) = \frac{\Psi(l,l) + \Psi(l,h)\mu^{-1}}{1 + \Upsilon}. \tag{8}$$

Using (8), the feasibility condition, the restrictions appearing in Assumption 4, and $B_{t+1}^s = 0$, $\forall t$, (7) can be written as

$$V(x_l - D, m_l, \varepsilon_l) = 1 - \frac{\beta(\Psi(l,h)/\mu\lambda + \Psi(l,l))}{1 + \Upsilon}, \tag{9}$$

$$V(x_h - D, m_h, \varepsilon_h) = 1 - \frac{\beta(\Psi(h,l)\mu\lambda + \Psi(h,h))}{1 + \Upsilon}. \tag{10}$$

The ratio of (9) to (10) implicitly defines a function $\phi(\mu)$. The equilibrium for the economy is given by a fixed point to $\mu = \phi(\mu)$.

Proposition 1. There exists a unique fixed point to the equation $\mu = \phi(\mu)$. It is given by $\mu^* = \lambda^{-1}$.

(The proofs of all propositions are in the appendix.) The next proposition isolates the effect of each of the two shocks on the price level and on the nominal interest rate.

Proposition 2. a) If $U_{13} > (\leq) 0$ and $x_h = x_l$, then $\mu^* > (\leq) 1$ and $S_h < (\geq) S_l$. b) If $\varepsilon_h = \varepsilon_l$, then $\mu^* < 1$ and $S_h > S_l$.

An interpretation of part a) of this proposition is the following: if a high ε implies stronger consumption desires for a given endowment and a high (low) ε is realized, the representative consumer wishes to consume more (less) currently and cares less (more) about the future. In order to consume more (less) she attempts to liquidate (acquire) real balances. If the government follows state- and time-independent policies, the attempt to liquidate (acquire) real balances will push the price level and the nominal interest rate up (down).[2] The restriction $U_{13} > 0$ is satisfied for example for $U(c_t, m_t, \varepsilon_t) = c_t^{\alpha_1} m_t^{\alpha_2} \varepsilon^{\alpha_3}$ with $\alpha_1 + \alpha_2 + \alpha_3 = 1$. Note that preferences disturbances are modelled here as 'real' shocks. Note also that 'real' preference disturbances which have no effect on the marginal utility of consumption leave the price level and the nominal interest rates unchanged.

An interpretation of part b) of proposition appears in both LeRoy (1984b) and Danthine and Donaldson (1986), and it is only briefly summarized here. An unexpected high endowment requires that the level of real money balances adjust to keep the service of money constant and the goods market in equilibrium. Since the price of money in terms of consumption adjusts procyclically to match movements in the marginal utility of consumption, the price level and the nominal interest rate will be low (high) when the endowment is high (low).

From Proposition 2 it is immediate to note that the responses of the price level and the nominal interest rate are independent of the serial correlation properties of the shocks. This is a well-known result for endowment shocks, but some explanations are required for the case of preference shocks. The serial correlation properties of real preference shocks do not matter here because they induce variations over time in the marginal utility of consumption. As in the case of endowment shocks, agents will have negative (positive) excess demand for real balances when the marginal utility of consumption is low (high) since changes in the marginal utility of consumption change the opportunity cost of holding money. Therefore preference disturbances of this type require adjustments both in the goods and in the money market and cannot be classified as 'sunk' costs in the terminology of Leroy (1984a).

One could also consider preference shocks that affect only the desire to hold *nominal* cash balances relative to income.[3] In this case adjustments will occur only in the money market. That is to say, for a *given* level of the real interest rate, the price level and the nominal interest rate will adjust to keep supply and demand for money in equilibrium. In this situation preference shocks will have features similar to money supply shocks. Since they do not affect the marginal utility of consumption, they are 'sunk' costs and the serial correlation properties of the disturbances will matter for the equilibrium level of p_t and S_t. This type of shock is not considered in this paper because it implies some form of money illusion (there are times when identical amounts of nominal cash

[2] It can be shown that when a intertemporal preference shock (a random β) is used in place of an intratemporal shock (a random ε) with $U_{13} > 0$, the results of Propositions 1 and 2 still hold. This is trivially the case when shocks are i.i.d., but it also holds when serial dependence is allowed. See Canova (1988) for this exercise.

[3] This would be the case for example if $U(c_t, m_t, \varepsilon_t) = U_1(c_t) + U_2(M_t \varepsilon_t)/p_t)$. Note also the difference between this setup, where preference shocks affect nominal balances, and the case $U_{13} = 0$ of part a) of Proposition 2, where preference shocks affect the desire to hold real balances.

holdings yield a higher level of utility) on the part of agents which is difficult to justify in an optimizing framework with full information.

3. Seasonal and cyclical shocks

Friedman and Schwartz (1963) contend that it is important to distinguish between cyclical and seasonal fluctuations for policymaking. Barsky and Miron (1989) implicitly question this separation by showing substantial similarities between what they call the 'seasonal' and the 'business' cycle for several US variables.

In the present context we can distinguish cyclical and seasonal shocks in two ways: either in terms of their origin (preference or endowment shocks) or in terms of the mean duration of the fluctuations (the persistence of the shocks). In the former case a reasonable distinction would be to identify seasonal shocks with preference shocks and cyclical shocks with endowment shocks. In the latter case, a natural terminology would be to call 'seasonal' those fluctuations with a short period of oscillation (low persistence) and 'cyclical' those fluctuations with a longer period of oscillation (high persistence). Note that when the period of oscillation goes to zero the fluctuations become deterministic. That is, when $\Psi_{l,l} = \Psi_{h,h} = 0$, the economy experiences a two-period deterministic cycle. Therefore, this second distinction allows us to analyze another basic difference of seasonal and cyclical variations, namely that the former are usually thought to be deterministic while the latter are not.

Given this taxonomy, the results of Proposition 2 may reconcile both the Friedman and Schwartz and Barsky and Miron positions.

If different cycles are identified with different sources of shocks (which is probably what Friedman and Schwartz had in mind), Proposition 2 indicates that, when $U_{13} > 0$, the response of the price level and of the nominal interest rate do depend on the source of the fluctuation. In this case the distinction between seasonal and cyclical shocks becomes vital for stabilization purposes but not for the reasons suggested by Friedman and Schwartz (p. 295).

However, when $U_{13} < 0$, the model predicts that preference shocks will produce the same responses in the price level and in the nominal interest rate as endowment shocks. In this case, the 'seasonal' and the 'business' cycle in nominal variables will look alike.

If, on the other hand, the seasonal and the business cycle are identified with different persistence in the shocks (which is more closely related to the statistical concept employed by Barsky and Miron in their empirical analysis and, probably, to the idea behind Friedman's quote in the introduction), then the responses of the price level and of the nominal interest rate to the two shocks will be identical. Proposition 2 shows that in equilibrium the behavior of the price level and of the nominal interest rate is independent of the persistence of shocks (the entries of the Ψ matrix). In other words, two economies that are subject to the same source of shock but differ in their persistence are indistinguishable as far as price and interest rate movements are concerned. This holds in the case when both economies are subject to perturbations that are of stochastic nature, but also when one economy is subject to stochastic variations while the other is subject to variations that are of deterministic nature.

Hence, if $U_{13} > 0$ and the 'seasonal' cycle and the 'business' cycle are generated by different shocks, the model supports the Friedman and Schwartz claims. However, if the 'seasonal' cycle and the 'business' cycle originate from the same type of shock or originate from two different types of shocks but $U_{13} < 0$, then the actual data will exhibit the empirical regularities discovered by Barsky and Miron.

4. Government intermediation

In this section I design government policies that smooth out fluctuations in the price level and in the nominal interest rate. I first analyze the situation when either endowment shocks or preference shocks affect the economy, but not both. Later in the section I consider the case when both shocks operate jointly in the economy.

Since from Proposition 2, movements in the price level and in the nominal interest rate for the case of preference shocks with $U_{13} < 0$ are similar to the case of endowment shocks, the qualitative features of smoothing policies for these two situations will be identical. Therefore, the analysis of this section considers only the case of endowment shocks or preference shocks which positively affect the marginal utility of consumption.[4] The policies of this section are stated in terms of θ.

Assumption 6. $$\theta_i \in \left[\max\left(\frac{D - m_i + m_j r_j}{m_i - m_j r_j}, -1 \right); \frac{D - \bar{m}_i}{\bar{m}_i - \bar{m}_j r_j} \right], \quad j \neq i, \quad \forall j.$$

Assumption 6 poses limits on the size of the government's intermediation activities. The upper limit is derived by noting that since m is decreasing in θ, the condition $r_t > 1$, $\forall t$, imposes restrictions on the size of the financiable deficit. Similarly, the lower limit follows from noting that if θ becomes 'too' negative, the equilibrium may not exist either because total return on money exceeds the return on bonds or because the sum of the monetary base and the face value of the government debt becomes negative.

[4] In the case of preference shocks when $U_{13} = 0$, no price or interest rate movements obtain and no government action is required.

First, I analyze the welfare properties of policies which stabilize the price level. Proposition 2 suggests a simple way to design such policies. Regardless of the source of the shocks and for a given β, the fixed point $\mu = \lambda^{-1}$ is in fact a relationship between p and M^s in various states and price level and money supply movements are inversely related. Therefore a price smoothing policy calls for negative (positive) comovements in the money supply when preferences (endowments) are shocked.

Proposition 3. There exist (θ_h, θ_l) policies that make π^* constant. These policies are welfare improving.

Intuitively, the policies of Proposition 3 require the government to lend when x_t is low or ε_t is high, so that the money stock is expanded to finance the fiscal deficit and the lending activities. When x_t is low (high), real money balances are depressed (high). By increasing (decreasing) the money stock when the marginal utility of real money balances is high (low), the monetary authority can effectively contain fluctuations in instantaneous utility. Similarly, when ε is high (low), real money balances are depressed (high) and the price level moves procyclically with preference shocks. Therefore, in order to hold the price level free of fluctuations, the monetary authority must undertake open market operations which induce positive comovements between the money stock and the shock. The welfare improvement comes from the fact that it is possible to choose a $\pi^* \geq \lambda\pi_h^* + (1-\lambda)\pi_l^*$ which is preferred by agents, where $0 < \lambda < 1$ is a constant and where π_h^* and π_l^* are the equilibrium real returns on the currency when $\theta_h = \theta_l = 0$.

Next, I examine the welfare properties of policies that stabilize the level of real money balances.

Proposition 4. There exist (θ_h, θ_l) policies that make $\mu^* = 1$. These policies are welfare improving.

As in Proposition 3, the stabilization of the level of real money balances requires the monetary authority to lend when the endowment is low or when the preference shock is high. Similarly, the welfare improvement comes from the fact that it is possible to choose a $m^* \geq \lambda m_h^* + (1-\lambda)m_l^*$ which is preferred by agents, where $0 < \lambda < 1$ is a constant and where m_l^* and m_h^* are the equilibrium levels of real money balances when $\theta_h = \theta_l = 0$.

The policies of Propositions 3 and 4 will not, in general, stabilize the nominal interest rate. This is because even though the return on the currency is kept free of fluctuations, the real rate of interest $U_{1i}/\beta U_{1j}$ is state-dependent for $i \neq j$ and fluctuates with the shocks. To put it in another way, since both shocks affect the goods and the money market, one instrument (the θ) is insufficient to stabilize both the price level and the nominal interest rate at the same time.

The next proposition analyzes the welfare implications of policies that smooth interest rate fluctuations.

Proposition 5. There exist (θ_h, θ_l) policies that stabilize the nominal interest rate. The policies are welfare improving.

As in Proposition 3, the stabilization of the nominal interest rate requires open market operations that expand the supply of currency when x_t is low (or ε_t is high). Therefore, by lending in those states when the nominal interest rate is higher than average, the monetary authority can eliminate undesired fluctuations in the nominal interest rate.

The policies described in Propositions 3–5 are not unique in the sense that they do not pin down the equilibrium value of π^*, m^*, or S^*. In other words, there is a continuum of (θ_h, θ_l) pairs, indexed by the magnitude of π^*, m^*, or S^* chosen, that are welfare improving. This indeterminacy problem arises because, in general, the monetary authority can make a profit by continuously intermediating across states and the propositions do not specify how to dispose of it. In the framework we consider there are two ways of disposing of the profits: one is to assume that they are either redistributed to the agents in a lump sum fashion or returned to the fiscal authorities. This solution violates the assumption of exogeneity of fiscal policy. Alternatively, one could impose the long-run condition that profits from intermediation are zero on average. In this case the policies of Propositions 3–5 are unique, in the sense that there is only one triplet, $(\pi^*, \theta_h, \theta_l)$, $(\mu^*, \theta_h, \theta_l)$, or $(S^*, \theta_h, \theta_l)$, that simultaneously satisfies all equilibrium conditions.[5] Also, the exogeneity of fiscal policy is maintained.

Although Propositions 3–5 are intuitively very similar, they differ in several respects. First, the policies of Proposition 4 do not eliminate fluctuations in the price level. However, for $\mu^* = 1$, the conditional return on the currency is stabilized [see eq. (8)]. Therefore, the policies of Proposition 4 make the expected price level constant. On the other hand, the policies of Proposition 5 induce fluctuations in the price level since they constrain the expected return on currency to match fluctuations in the real side of the economy [see Sims (1983)].

Second, the size of the welfare gains varies with the policy goal. An exact calculation of these gains is impossible to obtain here since Propositions 3–5 do not pin down a unique equilibrium value for π_t, m_t, or S_t. However, it is possible to get an idea of qualitative ordering of the gains if, e.g., we consider deterministically fluctuating preference and endowment disturbances and chose to stabilize π, m, and S at π_h, m_h, and S_h in the case of endowment shocks or at π_l, m_l, and S_l in the case of preference shocks. In this case, stabilizing real money balances is at

[5] An alternative conditions which pins down the equilibrium level of the triplets is that the average, net of interest value of government lending is zero.

least as good as stabilizing the price level, while stabilizing the price level is at least as good as stabilizing the nominal interest rate. This is because nominal interest rate smoothing induces price fluctuations and therefore fluctuations in real money balances that are of larger size than without intervention, while the stabilization of the price level reduces fluctuations in real money balances. Since risk-averse agents prefer less fluctuations in real balances, the results obtains. Hence, unless the nominal interest rate at which to smooth is appropriately selected, the policy may induce welfare costs, while with the policies of Propositions 3 and 4 gains are insured, no matter what the price level is or the expected price level chosen. If one takes a conservative attitude regarding the welfare consequences of the policy and recognizes that there are no *a priori* reasons to expect the government to know precisely at what level to stabilize the nominal interest rate, one may prefer to stabilize the price level as opposed to the nominal interest rate. Note that desirability of price smoothing policies is a result of the fact that the real interest rate fluctuates in response to both types of shocks. In an economy where shocks do not affect the real interest rate, for example when there are shocks to *nominal* cash balances, the above conclusion need not hold.

Third, the policies of Propositions 3–5 differ in terms of the size of the market activities of the government unless the money demand function is linear in each state of the world. For example, if $U(c_t, m_t, \varepsilon_t) = c_t^{\alpha_1} m_t^{\alpha_2} \varepsilon_t^{\alpha_3}$, the size of the fluctuations in the government's portfolio is smaller when the monetary authority tries to smooth real money balances. However, since the costs associated with managing a large government portfolio are not explicitly modelled in this paper, the relative size of government activities does not matter for the welfare of the economy.

Finally, there may also be an institutional reason to prefer price smoothing policies to interest smoothing policies. If the adjustment of goods markets to price changes is slower than the adjustment of financial markets to fluctuations in interest rates, one may prefer the policies of Propositions 3 and 4 to reduce the transition period to a new equilibrium. However, since these adjustment costs are not explicitly considered in the paper, this difference is not crucial to rank policies.

Mankiw and Miron (1990) show that in a simple classical model where the real rate is predetermined with respect to monetary variables, the welfare gains obtained by stabilizing nominal interest rate seasonals are quantitatively negligible. Although it is intuitively plausible that the welfare gains induced by stabilization policies are not large because real balances constitute only a small fraction of agents' wealth, there are several aspects of their numerical calculations which are debatable. First, they measure the increase in welfare in terms of the consumer surplus under a (log) linear money demand function. If agents are highly risk-averse, a (log) linear money demand function may be a poor approximation to (7) [see, e.g., Dotsey and Mao (1992)] and the size of the gains is underestimated. Second, in their money demand function the real interest rate is not seasonal. As is argued in section 2, this type of money demand function emerges from the framework used in this paper only when preference shocks are modelled as 'nominal' money demand shocks. Since these shocks have no impact on the marginal rate of substitution between t and $t+1$ consumption, it is not surprising to find that the welfare costs of these seasonal shocks are small. In addition, because both the endowment and the 'real' preference shocks considered here affect the real interest rate, the range of applicability of their calculations appears to be very narrow. Put in another way, whether the real interest rate is seasonal or not is crucial in deciding whether the triangle under the demand curve for money is small or not. Since the empirical evidence on the behavior of the real interest rate in industrialized countries is slim, more work needs to be done before a measure of the gains of stabilization can be provided. Third, Proposition 5 indicates that there are many levels at which the interest rate can be stabilized if appropriate open market operations are conducted. Mankiw and Miron choose the interest rate that is the average level between states but there is no apparent reason for doing so. For example, one could choose to stabilize the nominal interest rate at a value close to producing the satiation level of real balances. With a Cobb–Douglas utility function, this scenario is bound to induce welfare gains that are larger than those reported by Mankiw and Miron. Therefore, their numerical calculations lack robustness and depend on the stabilization rule employed.

Next, I examine whether the policies of Propositions 3–5 are optimal. It is easy to show that if the monetary authority is free from budget requirements (i.e., $D_t = 0, \forall t$), the value of θ_t which maximizes the discounted present value of the utility of the representative consumer is one which delivers the satiation level of real balances in each state of the economy (i.e., $U_{mt} = 0, \forall t$). Such a policy simply requires enough lending in each state of the world so as to drive the return on money to the state-dependent interest rate.

If, on the other hand, the monetary authority must finance a deficit and has two independent instruments (e.g., it can manipulate M_t and B_t independently of each other), the features of the optimal policy are similar in spirit to the one proposed by Abel (1987): select M_t so as to provide the satiation level of real balances and use B_t to satisfy the government budget constraint.[6] Also in this case, and consistent with Friedman's (1969) prescription, optimality is achieved by setting the real rate of return on money equal to the state-dependent real rate of interest.

If we restrict the monetary authority to use only one instrument to both finance the deficit and to maximize the discounted utility of the representative

[6]Abel's model differs from ours in three respects: it is a finitely-lived agent model, it includes capital accumulation, and it does not restrict fiscal policy to be constant.

consumer subject to the resource constraint, the optimal policy will be of the 'second-best' type as in Weiss (1980). Maximization of (3), subject to (1), the optimal household decision (7) and the resource constraint lead to the following optimality conditions:

$$\frac{\beta \mathrm{E}_t U_{2t+1} m_{t+1}}{U_{2t} m_t} = \frac{1}{r_t}, \tag{11}$$

where $r_t = U_{1t}/\mathrm{E}_t U_{1t+1}$. Intuitively, optimality requires a choice of θ_t which sets the average marginal rate of substitution of real balances over two subsequent periods equal to the inverse of the state-dependent real rate of interest (the ratio of discounted marginal utility of consumption). Note that the weights here are given by the t and $t+1$ levels of real money balances.

Additional insights can be gained by rewriting (11) in the following two alternative forms:

$$\frac{\beta \mathrm{E}_t U_{2t+1} \pi_t}{U_{2t}} = \frac{1}{r_t(1+\Upsilon)}, \tag{12}$$

$$\frac{\beta \mathrm{E}_t U_{2t+1} S_t}{U_{2t}} = \frac{1}{r_t^2(1+\Upsilon)}. \tag{13}$$

Note that in (12) the weighting factor is the expected rate of return on real balances.

From eqs. (11)–(13) it is clear that the stabilization of the price level, of real money balances, or of the interest rate will not achieve optimality. Such policies will simply set the weights in these expressions equal to 1 and will not, in general, provide a return on real balances which is the same as the real rate of interest (or functions of it). Therefore, the provision of an optimal level of government intermediation in this model is inconsistent with stabilization goals. Hence, it is legitimate to wonder why the monetary authority has actively pursued policies that eliminate interest rate fluctuations instead of driving the economy toward the constrained optimum. One possible answer to this question may be that setting the weighted expected return on real balances as close as possible to the state-dependent real rate may require large amounts of lending in both states of the world. If lending large amounts is unfeasible, either because it violates the conditions of Assumption 6 or because it violates the long-run zero-profit condition, the stabilization of the price level or of real balances can be seen as a second-best choice which may reduce the 'distance' of the equilibrium level of real money balances from the constrained optimum.

Finally, one should note that the policies of Propositions 3–5 impose stringent conditions on the actions of the monetary authority. They require constant monitoring in order to quickly adjust the composition of the portfolio with the state of the economy, knowledge of the sources of shocks and of the position of consumer optimal decision rule in each state in the (m, π) or (m, S) plane. Since the monetary authority may not be able to identify which of the two shocks is driving the economy, I next describe a policy which stabilizes the price level but does not require an exact knowledge of the sources of shocks.

Proposition 6. Assume that the monetary authority observes fluctuations in real money balances but does not know the origin of the shocks. Let the transition probability of the shocks be known to the monetary authority. Then there exist $\theta(m)$ price smoothing policies that are welfare improving.

The intuition behind Propositions 3 and 6 is similar. The monetary authority can smooth out fluctuations in the price level and improve welfare by lending when real money balances are depressed. The major difference between the two propositions is that in Proposition 6 the policies are conditional on the level of real balances, while in Proposition 3 price smoothing policies were designed to be contingent on the state of the economy. As in Proposition 3, the policy is not unique. Finally, welfare improves since the policy reduces fluctuations in m_t and risk-averse agents prefer smaller to larger fluctuation in m_t.

5. A robustness analysis

It is widely known that conclusions regarding the effects on the price level and on the nominal interest rate of shocks to the primitives of the economy may depend on the way money is introduced in the model [for example, LeRoy (1984a) and LeRoy and Raymon (1987)]. Since the way money is best incorporated in a general equilibrium model is still controversial, it is worthwhile to examine whether the conclusions derived in the previous sections survive when alternative ways of introducing money in the economy are considered.

Canova (1988) shows that the conclusions do survive in a cash-in-advance (CIA) productive economy populated by a representative infinitely-lived agent, where money has only a transaction function, output is produced with labor, and leisure enters as an argument of the utility function. Intuitively, the responses of the price level and of the nominal interest rate to various shocks are similar in the MIUF and in the above CIA model because, with leisure in the utility function and in an equilibrium with a binding CIA constraint, leisure choices directly determine the money demand function of the agents in the economy. Therefore the marginal rate of substitution between consumption and money balances in the CIA model has the same qualitative features as eq. (7) [see also Feenstra (1986)].[7] In addition, as in the MIUF model, the serial correlation properties of the shocks do not matter for the equilibrium level of prices and interest rates. The existence of price (or nominal interest rate) smoothing policies which are welfare improving in this CIA model is then a direct extension of the conclusions that Wallace (1988) derived in a somewhat similar but deterministic environment.[8]

In an overlapping generations (OG) model where agents of each generation are identical and live two periods, the situation is more complicated. First, the existence of an equilibrium can be shown only under a set of restrictive assumptions [see, e.g., Wallace (1980) or Lin (1989)]. Second, even for stationary environments, the equilibrium need not be unique [see, e.g., Leroy-Raymon (1987)].

Under restrictive assumptions which insure existence and uniqueness of the equilibrium, Canova (1988) shows that the price level and the nominal interest rate respond as in Proposition 2b, when endowments are shocked, and as in Proposition 2a, if preference shocks affect agents' utility in both periods of their life. When money is held for both transaction and storage purposes, the optimal decision rule for the agents' problem has the same format as eq. (7). Therefore, for a given initial endowment of money and bonds in the hands of the old at time zero, the equilibrium is still characterized by a fixed point to the equation $\mu = \phi(\mu)$ and the price level and the nominal interest rate will adjust to equilibrate fluctuations in the supply and demand for goods. When money is held for storage purposes only and the money supply grows at a rate which is independent of the state of the economy, a similar result applies. Independently of the sources of shocks, the price level adjusts to maintain the goods market in equilibrium and the arbitrage condition between the return on real cash balances and on real bonds. These adjustments have qualitative features which are similar to those specified in Proposition 2.

In these OG models the transition probabilities of the shocks do not matter for the equilibrium responses of the price level and the interest rates. Once again, this result obtains because both preference and endowment shocks affect the intertemporal marginal rate of substitution between consumption in the two periods of life.

[7]Feenstra shows the general equivalence between CIA and MIUF models and conjectures that $U_{12} > 0$ is a sufficient condition for obtaining a zero elasticity of substitution between real balances and consumption. The use of leisure in the utility function in the CIA model insures that this condition is satisfied. Also, the upper bound to the marginal utility of real balances in the MIUF model makes the money demand function qualitatively similar to the money demand function in the CIA model, where there exists natural upper bound for real cash balances (given by the endowment). However, despite these similarities, the conditions required for the existence of the equilibrium in the two cases are different. For details see Canova (1988).

[8]Wallace shows that in a deterministic CIA model there is a role for government intermediation and that the welfare implications of the policy are identical to those obtained in a deterministic MIUF models.

The welfare implications of government intermediation depend on the specific framework of analysis used. For example, if money has only a storage function, government intermediation may smooth price (or nominal interest rate) fluctuations but alternative portfolio decisions may have no welfare effects [see, e.g., Wallace (1981) or Chamley and Polemarchakis (1985)]. If money is held for both transaction and storage purposes, the equilibria obtained under alternative government policies are, in general, noncomparable. This is because the stabilization of the price level (or of the nominal interest rate) is not necessarily preferred by old agents at time zero who live in a high endowment or low preference state.

The presence of heterogeneity within generations alters some of these conclusions. For example, in a stochastic version of the model employed by Sargent and Wallace (1982), it is easy to show that the serial correlation properties of the shocks for the equilibrium level of prices and nominal interest rates [see Canova (1988)]. In addition, if the government acts competitively in the open market, price and nominal interest rate smoothing policies are not necessarily welfare improving since alternative government portfolios may induce different intertemporal opportunities for agents of the same generation.[9]

6. Conclusions

This paper attempts to answer the following two questions: why do monetary authorities choose to smooth seasonals in the price and/or in nominal interest rates? Is it important for policy purposes to distinguish between seasonal and cyclical fluctuations in the price level and in the nominal interest rate? Using a welfare criterion it is shown that for a wide variety of monetary economies there are smoothing policies which improve welfare. Such polices require the exchange of assets in the open market, with the government lending when real balances are depressed, the return on the currency is low, or the nominal interest rate is high. These policies, however, are not optimal and can be justified only if the absorption of the amount of lending required to achieve optimality is unfeasible at the current interest rate.

The paper also shows that the design of smoothing policies which are welfare improving is independent of the persistence of the shocks. Therefore, if one believes that seasonal and cyclical disturbances are just aggregate shocks with a different Markov structure, the seasonal/cyclical distinction is not critical

[9]This result differs from the one obtained by Sargent and Wallace. If the monetary authority rather than providing discount loans, uses only open market operations, then stabilization of the price level may induce fluctuations in the real interest rate. This changes the opportunity cost of consuming today vs. tomorrow for at least one class of agents.

for stabilization policies. A distinction which may be more useful for policy purposes is in the source of the shocks.

Finally, we argued that these results are robust to the way money is introduced in the model and hold both in infinitely-lived and finitely-lived economies. The presence of heterogeneity within generations, however, may alter these conclusions. In such an environment the persistence of the shocks may be important in determining the behavior of the price level and of the nominal interest rate. Also, policies designed to smooth the price level may generate consumption allocations that are noncomparable with those achieved in a situation where the monetary authority follows state-independent policies.

Appendix

Proof of Proposition 1. The proof is similar to LeRoy (1984b). The domain of $\phi(\mu)$ is the closed interval $[\mu_1, \mu_2]$ and $\phi(\mu)$ is continuous and monotonically decreasing in μ. Therefore a solution exists and it is unique. To verify that $\mu = \lambda^{-1}$ is an equilibrium, it is sufficient to note that when $\mu = \lambda^{-1}$, $V_h = V_l$.

Proof of Proposition 2a. We show that for $U_{13} > 0$ if $\mu^* = 1$, then $\phi(\mu^*) > 1$. If $\mu^* = 1$, $m_h = m_l = m$, so $U_1(x - D, m, \varepsilon_h) > U_1(x - D, m, \varepsilon_l)$. Since μ^* is a fixed point, $V_h = V_l$. Therefore $1 \leq U_{1h}/U_{1l} = U_{2h}/U_{2l}$ and $\phi(\mu^*) > 1$. The proof for $U_{13} \leq 0$ is identical and omitted. To examine the behavior of $1/S_t$ note that in equilibrium we have

$$S_t = 1 - V(x - D, m_t, \varepsilon_t) = \frac{1}{(1 + \Upsilon_t)(1 + \theta) - 1}\left(\frac{D}{m_t r_t} + \theta\right). \tag{A.1}$$

For $\theta = 0$ and for $r_t > 1, \forall t$, it is immediate to see that if $\mu^* > (\leq) 1, m_h^* < (\geq) m_l^*$ and $S_h < (\geq) S_l$.

Proof of Proposition 2b. Omitted. See LeRoy (1984b).

Proof of Proposition 3. Since (1) is continuous in m^s, b^s, there exists a pair (θ_h, θ_l) for all $i = h, l$ satisfying (A.1) and

$$\pi_h = \pi_l = \pi^*,$$

given r_i. The pair is given by

$$\theta_i = \frac{\pi^* \Upsilon m_i - D}{m_i(r_i - \pi^*(1 + \Upsilon))}. \tag{A.2}$$

Note that for fixed $(r_i, Y, \pi^* \leq r_i/(1 + Y))$, $|\theta_i|$ is increasing in m_i. Therefore, in the case of endowment shocks, $0 \geq \theta_h > \theta_l$, and in the case of preference shocks, $0 \geq \theta_l > \theta_h$. Finally, to show that the policy is welfare improving it is sufficient to pick $\pi^* \in [\lambda\pi_h^* + (1 - \lambda)\pi_l^*; \min_i(r_i/(1 + Y))]$, where π_i^* is the solution to the problem when $\theta = 0$, $\forall i$, and $0 < \lambda = 1$ is a constant.

Proof of Proposition 4. The proof is identical to the one of Proposition 3. There exists a pair (θ_h, θ_l) satisfying (A.1) and

$$m_h = m_l = m^*.$$

The pair is given by

$$\theta_i = \frac{\pi_i Y m^* - D}{m^*(r_i - \pi_i(1 + Y))}. \tag{A.3}$$

Since $|\theta_i|$ is increasing in π_i for each m, $0 \geq \theta_h > \theta_l$ in the case of an endowment shock and $0 \geq \theta_l > \theta_h$ in the case of preference shocks. To show that the policy is welfare improving it is sufficient to notice that, for any $m^* \in [\bar{m}, \hat{m}]$, U is increasing in m in all states and that $\bar{m} = \lambda m_l^* + (1 - \lambda)m_h^*$ is preferred by agents for any $0 < \lambda < 1$.

Proof of Proposition 5. The proof is identical to the one of Proposition 3, where in place of $\pi_l = \pi_h = \pi^*$ we set $S_l = S_h = S^*$. The pair is given by

$$\theta_i = \frac{S^* Y m_i - D/r_i}{m_i[1 - S^*(1 + Y)]}. \tag{A.4}$$

Since $|\theta_i|$ is increasing in π_i for each m and for fixed r_i and it is increasing in r_i for each m and for fixed π_i, $0 \geq \theta_h > \theta_l$ in the case of an endowment shock and $0 \geq \theta_l > \theta_h$ in the case of preference shocks. Welfare improves by choosing $S^* \in [0; \lambda(S_l^*) + (1 - \lambda)(S_h^*)]$, where S_i^* is the equilibrium value of S_i when $\theta = 0$, $\forall i$.

Proof of Proposition 6. The proof is identical to the one of Proposition 3 (except that the policy is now indexed by m instead of i) and it is omitted.

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