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Local Externalities, Convex Adjustment Costs, and Sunspot Equilibria

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We study the possibility of random changes in the allocation of resources among regions/sectors in the presence of local productive externalities, in an environment without intrinsic uncertainty. We show that "positive" externalities over some range of activity are a necessary condition for existence of such sunspot equilibria. For the two-state sunspot case, we derive sufficient conditions for existence of sunspot equilibria, in terms of the properties of adjustment costs, private technologies, and externalities. *Journal of Economic Literature* Classification Numbers: E32, R13.

1. INTRODUCTION

The presence of productive externalities has been pointed to by a number of authors as a potential source of multiple equilibria. Recent examples of models characterized by both productive externalities and multiple equilibria include [2, 3, 5, 8, 9, 11, 12], among others.

The present paper investigates the potential role of a certain type of productive externalities—which we refer to as "local externalities"—as a source of sunspot fluctuations, i.e., random changes in the allocation of resources in the absence of shocks to preferences, technology, or endowments.[1]

The possibility of sunspot equilibria generated by such productive externalities was analyzed by [11] in the context of an otherwise standard one-sector neoclassical growth model. Furthermore, as argued in [7], other growth models with productive externalities that have been shown

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[1] Papers illustrating the possibility of sunspot equilibria in environments where the first welfare theorem is violated include examples with restricted participation [4], an infinite number of agents [1], monopolistic competition [6], and borrowing constraints [13], among others.

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to be characterized by indeterminacy of perfect foresight paths in a neighborhood of a steady state (e.g., [2, 3]) could also be shown to exhibit stationary sunspot equilibria that remain arbitrarily close to that steady state.

The model we develop has two basic features, shared by other examples found in the literature [8, 9]. First, producers allocate resources across two activities, with the private technology associated with each activity being a function of the *aggregate* level of resources allocated to it. Underlying that assumption is the notion of technological or knowledge spillovers (positive externalities) or congestion effects (negative externalities) that are restricted to a certain "activity"—and thus have a "local" nature— and which can be naturally interpreted as either intra-industry or intra-region spillovers. The second main feature of the model is the presence of adjustment costs, or more precisely, private costs of reallocating resources across activities, i.e., across regions or industries. Under similar assumptions, the papers listed above show how the possiblity of multiple perfect foresight equilibrium paths arises, but, to the extent of our knowledge, no analysis of the potential for sunspot equilibria in that environment can be found in the literature.

The model, presented in Section 2, is deliberately stylized in order to isolate the role of externalities. Given our assumptions, the latter is the only source of potential inefficiency of equilibrium allocations. In particular, we assume a finite number of agents, and complete markets (including sunspot-contingent markets). Because of the presence of externalities, competitive equilibria may be suboptimal, thus making room for sunspot-contingent allocations [4].

In Section 3, we show that the presence of "positive" externalities over some activity range are a necessary condition for existence of finite-state sunspot equilibria. For the two-state sunspot case, an open set of sunspot equilibria is shown to exist if, in addition to the necessary condition above, the marginal adjustment cost schedule is sufficiently "flat." We finish that section by briefly discussing some welfare implications of sunspots in our model.

2. The Model

There is a finite number of identical agents, who act as consumer-producers.[2] Each agent derives utility from the consumption of a single perishable good in two periods, indexed by $t = 1, 2$. We denote his consumption vector by $c(\omega) = [c_1, c_2(\omega)]$, where $\omega \in \Omega$, and Ω is a (finite) set

[2] Instead, we could have assumed the existence of both consumers and firms, with the former selling labor services to the latter in both spot- and state-contingent markets. Our assumption simplifies the notation without affecting any of the results.

of possible realizations of a sunspot variable in the second period. The representative agent maximizes

$$EU(c) \equiv \sum_{\omega \in \Omega} \pi(\omega)\, U(c(\omega)), \tag{1}$$

where U is C^2, strictly increasing, strictly concave, and satisfies $\lim_{c_t \to 0} U_t(\cdot) = +\infty$, for $t = 1, 2$, where $U_t(\cdot) \equiv \partial U(\cdot)/\partial c_t$.

Each agent has an endowment of the consumption good represented by a vector $w = [w_1, w_2] \geqslant 0$. In addition, he has an endowment of a production input (e.g., labor), which we normalize to be equal to one in both periods. The input endowment is allocated to two productive activities indexed by $i = 1, 2$, subject to the constraint

$$\begin{aligned} n_{11} + n_{12} &\leqslant 1, \qquad n_{1i} \geqslant 0, \quad i = 1, 2; \\ n_{21}(\omega) + n_{22}(\omega) &\leqslant 1, \qquad n_{2i}(\omega) \geqslant 0, \quad i = 1, 2, \quad \text{all} \quad \omega \in \Omega, \end{aligned} \tag{2}$$

where n_{ti} denotes the quantify of the input allocated to activity i in period t.

Each activity yields an output of the consumption good in *period* 2, according to the production function

$$y_i(\omega) = \theta(N_{2i}(\omega))\, F(n_{2i}(\omega)), \qquad i = 1, 2, \quad \text{all} \quad \omega \in \Omega, \tag{3}$$

where y_i is the output of activity i. $F(\cdot)$ is a C^2 function defined on the unit interval, and satisfying $F' > 0$, $F'' < 0$, and $\lim_{z \to 0} F'(z) = \infty$. $\theta(\cdot)$ is a technology index whose realization is a function of N_{2i}, the *aggregate* input level allocated to activity i. Without loss of generality we assume N_{2i} is normalized by the number of agents, and let $\theta(\cdot)$ be accordingly defined on the unit interval. We further assume $\theta(\cdot)$ is C^1, and $0 < \theta(z) < \infty$ for all $z \in [0, 1]$. At this stage we do not impose any restrictions on the sign of its derivative.

The probability distribution of N_{2i} $(i = 1, 2)$ is taken as given by each individual agent, who perceives his choice of n_{2i} as having a negligible effect on the aggregate value of that variable. Given the dependence of individual production possibilities on aggregate outcomes, individual decisions have *external effects* which may be the source of inefficient equilibrium allocations. Under our specification, that externality has a *local* nature, in the sense that spillovers take place only *within each activity*, i.e., within a certain economic region or sector, depending on the interpretation.

A final feature of our model concerns adjustment costs. Even though output is not obtained until the second period, production activities are assumed to begin in period 1, for agents have to allocate their input endowment to the two activities in that initial period. Any adjustments in that initial allocation that take place in period two are assumed to be costly, with the size of the adjustment cost being a function $Q(\cdot)$ with argument $d(\omega) \equiv n_{21}(\omega) - n_{11}$. We assume $Q(\cdot)$ is a C^2, strictly convex function defined on $[-1, 1]$ with a minimum at zero (i.e., $Q''(\cdot) > 0$, and $Q'(0) = 0$. It is convenient to introduce a *net* output function, defined as $y(\omega) \equiv y_1(\omega) + y_2(\omega) - Q(d(\omega))$.

Each agent thus maximizes (1), subject to (2) and the budget constraint

$$c_1 - w_1 + \sum_{\omega \in \Omega} p(\omega)[c_2(\omega) - w_2 - y(\omega)] \leqslant 0, \tag{4}$$

where $p(\omega) \geqslant 0$ is the price of the consumption good for delivery in period 2 and state ω. Note that the price for consumption goods delivered (with certainty) in period 1 has been normalized to unity.

3. Competitive Equilibria with Local Externalities

3.a. *Competitive Equilibria*

Our assumptions on $U(\cdot)$, $F(\cdot)$, $Q(\cdot)$, and $\theta(\cdot)$ guarantee the existence of an interior solution to the problem above, with (2) and (4) holding with equality. In order to simplify the notation we define $n_t \equiv n_{t1}$ and $N_t \equiv N_{t1}$, $t = 1, 2$. The first order conditions characterizing that solution are

$$\sum_{\omega \in \Omega} \pi(\omega)\, U_1(c(\omega)) = \lambda, \tag{5}$$

$$\pi(\omega)\, U_2(c(\omega)) = \lambda p(\omega) \qquad \text{all} \quad \omega \in \Omega, \tag{6}$$

$$\sum_{\omega \in \Omega} p(\omega)\, Q'(d(\omega)) = 0, \tag{7}$$

$$G(n_2(\omega)) = Q'(d(\omega)) \qquad \text{all} \quad \omega \in \Omega, \tag{8}$$

where λ is the shadow value associated with constraint (4) and $G(n_2(\omega)) \equiv \theta(N_2(\omega))\ \ F'(n_2(\omega)) - \theta(1 - N_2(\omega))\ \ F'(1 - n_2(\omega))$. Equations (5) and (6) are standard conditions equating the marginal rate of substitution to relative prices. Equation (7) guarantees that the expected loss of utility resulting from adjustments costs is minimized. Equation (8) equates the *private* marginal gain of shifting input towards activity 1 (in period 2) to the marginal cost of doing so.

Next we to turn to a characterization of competitive equilibria. Given $p(\omega)$ and $N_2(\omega)$ (for all $\omega \in \Omega$), the solution to the representative agent's problem can be easily shown to be unique,[3] and so it must be the same across agents (given our assumption of identical preferences, technology, and endowents). We can thus restrict ourselves to symmetric competitive equilibria, i.e., allocations for which conditions (5)–(8) are satisfied for each consumer, markets clear, and individual and aggregate variables (the latter normalized by the number of agents) take identical values. Letting capital letters denote (per capita) aggregate variables, a competitive equilibrium allocation is formally characterized by the following conditions:

$$C_1 = W_1 \tag{9}$$

$$C_2(\omega) = Y(\omega) + W_2 \qquad \text{all} \quad \omega \in \Omega \tag{10}$$

$$\begin{aligned} Y(\omega) = {} & \theta(N_2(\omega))\, F(N_2(\omega)) + \theta(1 - N_2(\omega))\, F(1 - N_2(\omega)) \\ & - Q(N_2(\omega) - N_1) \qquad \text{all} \quad \omega \in \Omega \end{aligned} \tag{11}$$

$$\sum_{\omega \in \Omega} \pi(\omega)\, U_2(C(\omega))\, G(N_2(\omega)) = 0 \tag{12}$$

$$G(N_2(\omega)) = Q'(N_2(\omega) - N_1) \qquad \text{all} \quad \omega \in \Omega \tag{13}$$

$$\begin{aligned} & 0 \leqslant N_1 \leqslant 1, \qquad 0 \leqslant N_2(\omega) \leqslant 1, \quad 0 < \pi(\omega) < 1, \\ & \text{all} \quad \omega \in \Omega; \quad \sum_{\omega \in \Omega} \pi(\omega) = 1. \end{aligned} \tag{14}$$

3.b. *Perfect Foresight Equilibria*

Consider first the set of competitive *perfect foresight equilibria* (p.f.e.), i.e., solutions to (9)–(14) such that $C_2(\omega) = C_2$, $N_2(\omega) = N_2$, $Y(\omega) = Y$, all $\omega \in \Omega$. Under our assumptions on $U(\cdot)$ and $Q(\cdot)$, (12) and (13) imply that in any p.f.e., $N_1 = N_2 \equiv N$, and $G(N) = 0$. In words, the input allocation among activities is constant over time, no adjustment costs are incurred, and private marginal products are equalized across sectors.[4] Given N, (9) and (10) can then be used to determine C.

Notice that the number of p.f.e. equals the number of solutions to the equation $G(z) \equiv \theta(z)\, F'(z) - \theta(1-z)\, F'(1-z) = 0$, in the unit interval. One such solution is given by $z = 1/2$. The next proposition derives some simple conditions on the "externality function" $\theta(\cdot)$ under which multiple p.f.e. will exist.

[3] This is a consequence of the monotonicity of both the marginal rate of substitution and the (private) marginal product of labor.

PROPOSITION 1 (Necessary and Sufficient Conditions for Multiple p.f.e.). *Let the above assumptions on* $F(\cdot)$ *and* $\theta(\cdot)$ *hold. Then*

$$\theta(z)/\theta(1-z) \leqslant F'(1-z)/F'(z) \qquad \textit{for some} \quad z \in (0, 1/2) \tag{15}$$

is a necessary and sufficient condition for the existence of multiple p.f.e.

Proof. See the Appendix

Let $\eta(z) \equiv z\theta'(z)/\theta(z)$ and $\sigma(z) \equiv -zF''(z)/F'(z)$, all $z \in (0, 1)$, denote the elasticity of the technology parameter and the private marginal product, respectively, with respect to the input level. Under our assumptions $\sigma(\cdot) > 0$, all $z \in (0, 1)$. However, $\eta(z)$ can be either positive or negative, depending on whether *positive* or *negative* externalities are effective at a level of activity z. Now we can state the following corollary to Proposition 1.

COROLLARY. (i) *multiple p.f.e. exist only if* $\eta(z) > 0$ *for some* $z \in (0, 1)$;
(ii) *multiple p.f.e. exist if* $\eta(1/2) \geqslant \sigma(1/2)$.

Proof. See the Appendix.

Thus, we see that the presence of positive local externalities ($\eta > 0$) for some range of activity levels is a necessary condition for the existence of p.f.e. other than $N = 1/2$. Whenever such positive externalities are "strong enough" relative to diminishing private marginal product at $N = 1/2$ the existence of multiple p.f.e. is guaranteed. In addition, and as a trivial consequence of the symmetry between activities, one can easily show that the total number of p.f.e. will always be odd.

Notice also that adjustment costs do not enter $G(\cdot)$ and thus play no role in determing the set of p.f.e.. The reason for that "irrelevance" is that on any such equilibria no reallocation of inputs takes place in the second period and thus no adjustment costs are ever incurred.

3.c. *Sunspot Equilibria*

Next we consider the possibility of k-state sunspot equilibria, i.e., allocations (and associated sunspot distributions) satisfying (9)–(14) and such that $N_2(\omega_i) \neq N_2(\omega_j)$, all $\omega_i, \omega_j \in \Omega$, $\omega_i \neq \omega_j$, where $\Omega = \{\omega_1, \omega_2, ..., \omega_k\}$. Each sunspot equilibrium is characterized by beliefs of the following sort about the behavior of the economy in period 2: with probability $\pi(\omega_i)$, sunspot event "ω_i" will take place, and the aggregate level of input allocated to activity one in period two will be given by $N_2(\omega_i)$; this is true for all $\omega_i \in \Omega$. Given those beliefs and the prices prevailing in both spot-and sunspot-contingent markets, each agent chooses the allocation of inputs among activities, as well as the consumption plan, that maximizes his expected utility. In particular, in the first period all agents end up choosing the same level of employment N_1 in activity 1 $(1-N_1$ in activity 2), and consume their own endowment w_1. In period 2, they observe the sunspot realization and the associated value for N_2, and reallocate their input endowment optimally, given their initial allocation N_1 and the adjustment cost schedule they face. Their (identical) input reallocation decision then turns out to be consistent with their initial beliefs on the effects of the sunspot realization on the aggregate allocation among activities.

Clearly, such an equilibrium is fully defined by a vector $[N_1, N_2(\omega_1), ..., N_2(\omega_k), \pi(\omega_1), ..., \pi(\omega_{k-1})]$ satisfying (12)–(14), for, once that vector is known, one can easily determine the equilibrium quantities $C(\omega)$, and $Y(\omega)$, all $\omega \in \Omega$, using conditions (9) and (11). Finally, one can compute the equilibrium-contingent prices $p(\omega)$ by combining (5) and (6) to yield $p(\omega) = \pi(\omega)\, U_2(c(\omega))/\sum_{\omega \in \Omega} \pi(\omega)\, U_1(c(\omega))$, for all $\omega \in \Omega$.

As in many other models found in the literature, the conditions under which sunspot equilibria exist in our model are related to those under which we have multiple p.f.e. In our model, the existence of multiple p.f.e. is a necessary (though not sufficient) condition for existence of sunspot equilibria. This result is formalized in the following proposition and corollary.

PROPOSITION 2 [Necessary Condition for Existence of Sunspot Equilibria]. *Let the above assumptions on $F(\cdot)$, $\theta(\cdot)$, and $Q(\cdot)$ hold. Then* (15) *is a necessary condition for the existence of finite-state sunspot equilibria.*

Proof. See the Appendix

The following corollary follows trivially from Propositions 1 and 2.

COROLLARY. *The existence of multiple p.f.e. is a necessary condition for finite-state sunspot equilibria to exist.*

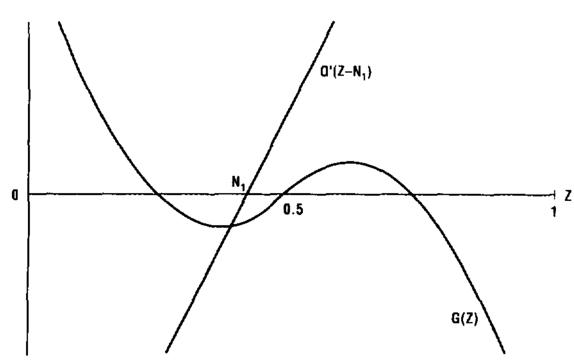

FIG. 1. The case of multiple perfect foresight equilibria but no sunspot equilibria.

The converse of the previous corollary is not true: multiplicity of p.f.e. is not sufficient to guarantee the existence of sunspot equilibria. This is illustrated in Fig. 1, representing an economy for which $G(z)$ vanishes at three different z values, but does not intersect $Q'(z-N_1)$ more than once, for any $0 \leqslant N_1 \leqslant 1$.

A complete characterization of the conditions for existence of sunspot equilibria, expressed in terms of the properties of utility, production, and adjustment cost functions is not attempted here. Instead, we focus on the class of *two-state* sunspot equilibria, for which we derive a *sufficient* condition for existence.

LEMMA. *Let the above assumptions on* $F(\cdot)$, $\theta(\cdot)$, *and* $Q(\cdot)$ *hold. Let* $\Omega = \{\alpha, \beta\}$ *be the set of relevant sunspot events, with associated probabilities* $\pi(\alpha) = \pi$ *and* $\pi(\beta) = 1-\pi$, $0 < \pi < 1$. *Let* $L(z) \equiv \theta(z) F'(z)$, $z \in [0, 1]$, *denote the* private *marginal product of the input in a symmetric equilibrium. Then there exists a two-state sunspot equilibrium, characterized by* $N_1 = 1/2$, *if the following condition holds*:

$$0 < Q''(0) < 4L(1/2)(\eta(1/2) - \sigma(1/2)). \tag{16}$$

Proof. See the Appendix.

Given the existence of a sunspot equilibrium, and because of the symmetry of the model, there is a trivial dimension of multiplicity: if $[N_1, N_2(\alpha), N_2(\beta), \pi]$ defines a sunspot equilibrium, so does $[1-N_1, 1-N_2(\alpha), 1-N_2(\beta), \pi]$. However, and as the next proposition makes precise, the set of sunspot equilibria is much larger.

PROPOSITION 3. *Assume that the conditions of the Lemma hold, including condition* (16). *Then there exists an open set of two-state sunspot equilibria.*

Proof. See the Appendix

Note that the assumption $\eta(1/2) - \sigma(1/2) > 0$, though sufficient for multiple p.f.e., is not enough to guarantee the existence of sunspot equilibria. A sufficient condition for the latter involves the adjustment cost function; specifically, (16) shows that a two-state sunspot equilibrium will exist as long as $\eta(1/2) - \sigma(1/2)$ and/or $L(1/2)$ are large enough relative to the "slope" of the marginal adjustment cost schedule evaluated at zero, i.e., at a point where no adjustment takes place. In other words, our sufficient condition will be satisfied if adjustment costs do not increase much with the size of the adjustment, or if, in the symmetric equilibrium, the (shadow) wage rate is high, and positive externalities are strong relative to the rate of decrease in private marginal products.

Given the presence of externalities, competitive equilibria in our model will generally be suboptimal, even in the perfect foresight case. The presence of sunspots adds, however, two sources of inefficiency which are absent from p.f.e. First, the sunspot realization in period 2 leads to a *costly* reallocation of resources across activities. Second, the uncertainty regarding period 2 consumption (which depends on the sunspot realization) tends to reduce the consumer's expected utility. Yet, we cannot rule out the possibility that the allocation associated with a given sunspot equilibrium dominates a given p.f.e. allocation from a welfare viewpoint (as long as the latter is suboptimal). This would be the case if sunspots led to greater levels of (gross) output in period 2 (relative to a given p.f.e.), which more than offset the negative effects of adjustment costs and uncertainty. If, on the other hand, p.f.e. dominated sunspot equilibria, it would be straightforward for a policymaker to increase the representative agent's utility by setting up a simple tax scheme that raised the "slope" of the marginal cost schedule sufficiently to effectively rule out sunspot equilibria (as in Fig. 1).

Appendix

Proof of Proposition 1. Our assumptions on $F(\cdot)$ and $\theta(\cdot)$ guarantee that $G(z)$ is continuous on $(0, 1)$, with $G(0) > 0$, $G(1) < 0$, and $G(1/2) = 0$. If $G(z)$ is to vanish for some $z \in (0, 1/2)$ (and thus, by symmetry of $G(\cdot)$, for $1 - z$ as well) it must be the case that $G(z) \leqslant 0$ for some $z \in (0, 1/2)$. Conversely, if $G(z) \leqslant 0$ for some $z \in (0, 1/2)$ there exists some $z' \in (0, z)$ such that $G(z') = 0$. However, given $\theta(\cdot) > 0$ and $F'(\cdot) > 0$, $G(z) \leqslant 0$ is equivalent to the condition in the proposition.

Proof of the Corollary to Proposition 1. Given our assumption of strict concavity of $F(\cdot)$, we have $F'(1-z)/F'(z) < 1$ for all $z \in (0, 1/2)$. Thus, the inequality in proposition 1 requires that $\theta(z) < \theta(1-z)$ for some $z \in (0, 1/2)$, and (i) follows from the definition of $\eta(\cdot)$. Given continuity of $G(\cdot)$, a sufficient condition for $G(z) \leqslant 0$ to hold for some $z \in (0, 1/2)$ (and, hence, for multiple p.f.e.) is given by $G'(1/2) \geqslant 0$; (ii) then follows from the definition of $\eta(\cdot)$, $\sigma(\cdot)$, and $G(\cdot)$.

Proof of Proposition 2. Suppose not, i.e., suppose $[N_1, N_2(\omega_1), ..., N_2(\omega_k)]$ defines a k-state sunspot equilibrium, while $G(z) > 0$ for all $z \in (0, 1/2)$. Assume first that $N_1 \leqslant 1/2$. Given the strict monotonicity of $Q'(\cdot)$ and the fact that $Q'(0) = 0$, it must be the case that $Q'(z - N_1) = G(z)$ holds only for $z \leqslant 1/2$, so that $G(N_2(\omega)) > 0$ all $\omega \in \Omega$, given (13). However, if that is the case, (12) fails to hold, and $[N_1, N_2(\omega_1), ..., N_2(\omega_k)]$ cannot define a sunspot equilibrium. A symmetric argument can be applied to the case of $N_1 \geqslant 1/2$.

Proof of the Lemma. Note that, for a given N_1 value, we just need to show the existence of $0 < N_2(\alpha) < 1$, $0 < N_2(\beta) < 1$, and $0 < \pi < 1$, such that $G(N_2(\alpha)) = Q'(N_2(\alpha) - N_1)$, $G(N_2(\beta)) = Q'(N_2(\beta) - N_1)$, and $\pi U_2(C(\alpha)) G(N_2(\alpha)) + (1-\pi) U_2(C(\beta)) G(N_2(\beta)) = 0$.

Here we show that such a two-state sunspot equilibrium exists for $N_1 = 1/2$. Recall that, under our assumptions on $F(\cdot)$, $G(\cdot)$ is continuously differentiable, with $\lim_{z\to 0} G(z) > 0$ and $\lim_{z\to 1} G(z) < 0$. Futhermore, $Q'(z - 1/2)$ is strictly increasing in z and continuous, with $Q'(0) = 0$. Suppose that $G'(1/2) > Q''(0)$ holds. Then it must be the case that $Q'(z - 1/2)$ intersects $G(z)$ for some $0 < z < 1/2$ *and* for some $1/2 < z < 1$. Let $0 < N_2^*(\alpha) < 1/2$ and $1/2 < N_2^*(\beta) < 1$ respectively denote those intersection points (i.e., solutions of (12)). Under our assumptions it is clear that $G(N_2^*(\alpha)) < 0$ and $G(N_2^*(\beta)) > 0$. Letting $C^*(\alpha)$ and $C^*(\beta)$ be the corresponding consumption vectors defined by (9)–(10), it follows that $\pi^* = U_2(C^*(\beta)) G(N_2^*(\beta))/[U_2(C^*(\beta)) G(N_2^*(\beta)) - U_2(C^*(\alpha)) G(N_2^*(\alpha))]$ satisfies $0 < \pi^* < 1$, and guarantees that (13) holds. Thus, there exist $[N_2^*(\alpha), N_2^*(\beta), \pi^*]$ satisfying (12)–(14). But it is straightforward to show, using the definition of $G(\cdot)$, that $G'(1/2) = 4L(1/2)\,(\eta(1/2) - \sigma(1/2))$, and so the condition in the proposition guarantees the existence of a sunspot equilibrium with $N_1 = 1/2$.

Proof of Proposition 3. The existence of an open set $S \subset \mathbb{R}^4$ containing $x^* \equiv [1/2, N_2^*(\alpha), N_2^*(\beta), \pi^*]$, and such that every $x \equiv [N_1, N_2(\alpha), N_2(\beta), \pi] \in S$ satisfies (12)–(13) (thus defining a sunspot equilibrium) follows from a straightforward application of the implicit function theorem. To see this, define a vector-valued function $\mathbf{H}: \mathbb{R}^4 \to \mathbb{R}^3$, with components $H_1(x) \equiv G(N_2(\alpha)) - Q'(N_2(\alpha) - N_1)$, $H_2(x) \equiv G(N_2(\beta)) - Q'(N_2(\beta) - N_1)$, and $H_3(x) \equiv \pi U_2(C(\alpha)) G(N_2(\alpha)) + (1-\pi) U_2(C(\beta)) G(N_2(\beta))$, and where $C(\omega)$ is defined in terms of x by (9)–(10). A sunspot equilibrium is thus defined by a vector x satisfying $\mathbf{H}(x) = \mathbf{0}$, as well as (14). Let $\mathbf{DH}(x) \equiv [\mathbf{D}_n\mathbf{H}(x), \mathbf{D}_\pi\mathbf{H}(x)]$ denote the derivative of $\mathbf{H}(\cdot)$ at x, with $\mathbf{D}_n\mathbf{H}(x)$ and $\mathbf{D}_\pi\mathbf{H}(x)$ denoting the derivatives of $\mathbf{H}(\cdot)$ with respect to $n \equiv [N_1, N_2(\alpha), N_2(\beta)]$ and π, respectively. One can easily show that $\mathbf{D}_n\mathbf{H}(x^*)$ is (generically) nonsingular. Hence, we can use the implicit function theorem [10, p. 224] to show the existence of open sets $S \subset \mathbb{R}^4$ and $P \subset \mathbb{R}$, with $x^* \in S$, and $\pi^* \in P$, and a mapping $v: P \to \mathbb{R}^3$ such that $\mathbf{H}(v(\pi), \pi) = 0$, with $(v(\pi), \pi) \in S$, for all $\pi \in P$, and with derivative given by $v'(\pi) = -\mathbf{D}_n\mathbf{H}(x^*)^{-1}\,\mathbf{D}_\pi\mathbf{H}(x^*)$. Let B be an open ball around x^* and such that condition (14) is satisfied for any $x \in B$ (i.e., B must be contained by the nonnegative unit 4-cell). It follows that $S \cap B$ is a nonempty open set containing allocations that satisfy (12)–(14), thus defining an open set of sunspot equilibria.

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