The Basic New Keynesian Model

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Motivation and Outline

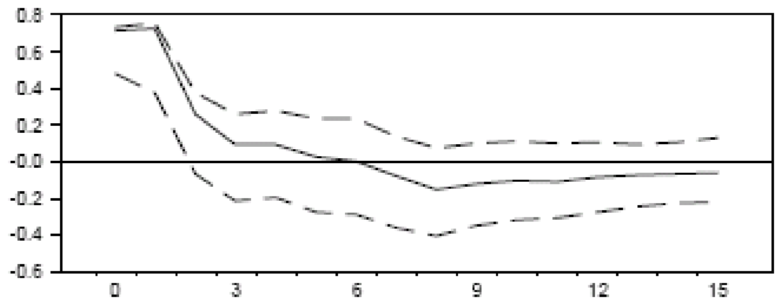
Evidence on Money, Output, and Prices:

- Macro evidence on the effects of monetary policy shocks

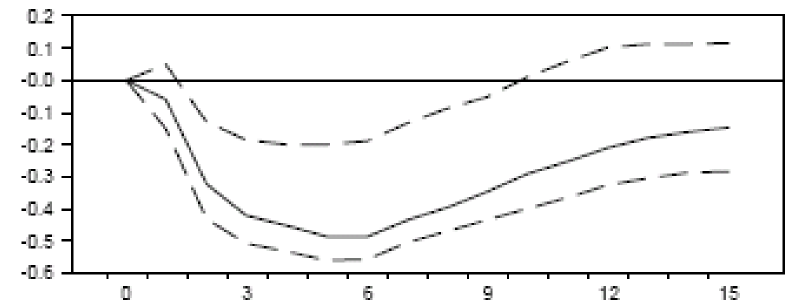
  (i) persistent effects on real variables
  (ii) slow adjustment of aggregate price level
  (iii) liquidity effect

- Micro evidence: significant price and wage rigidities

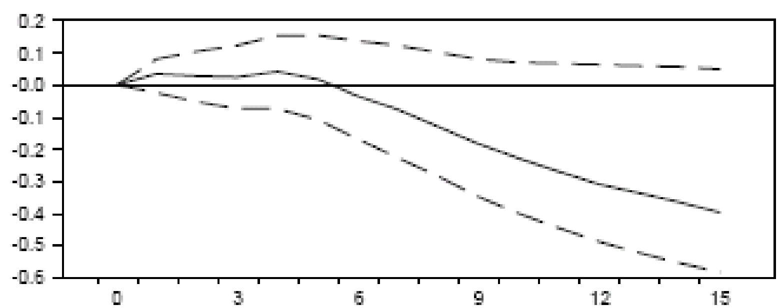
Figure 1. Estimated Dynamic Response to a Monetary Policy Shock



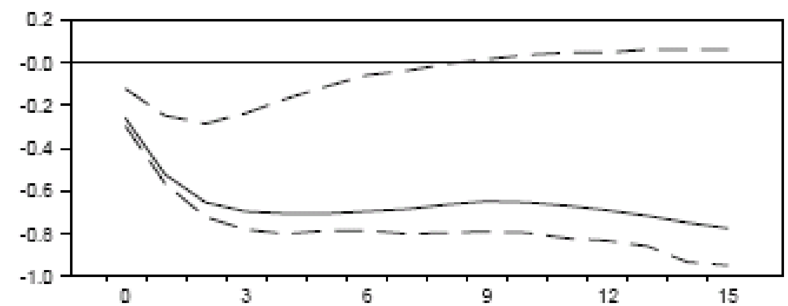
Federal funds rate



GDP



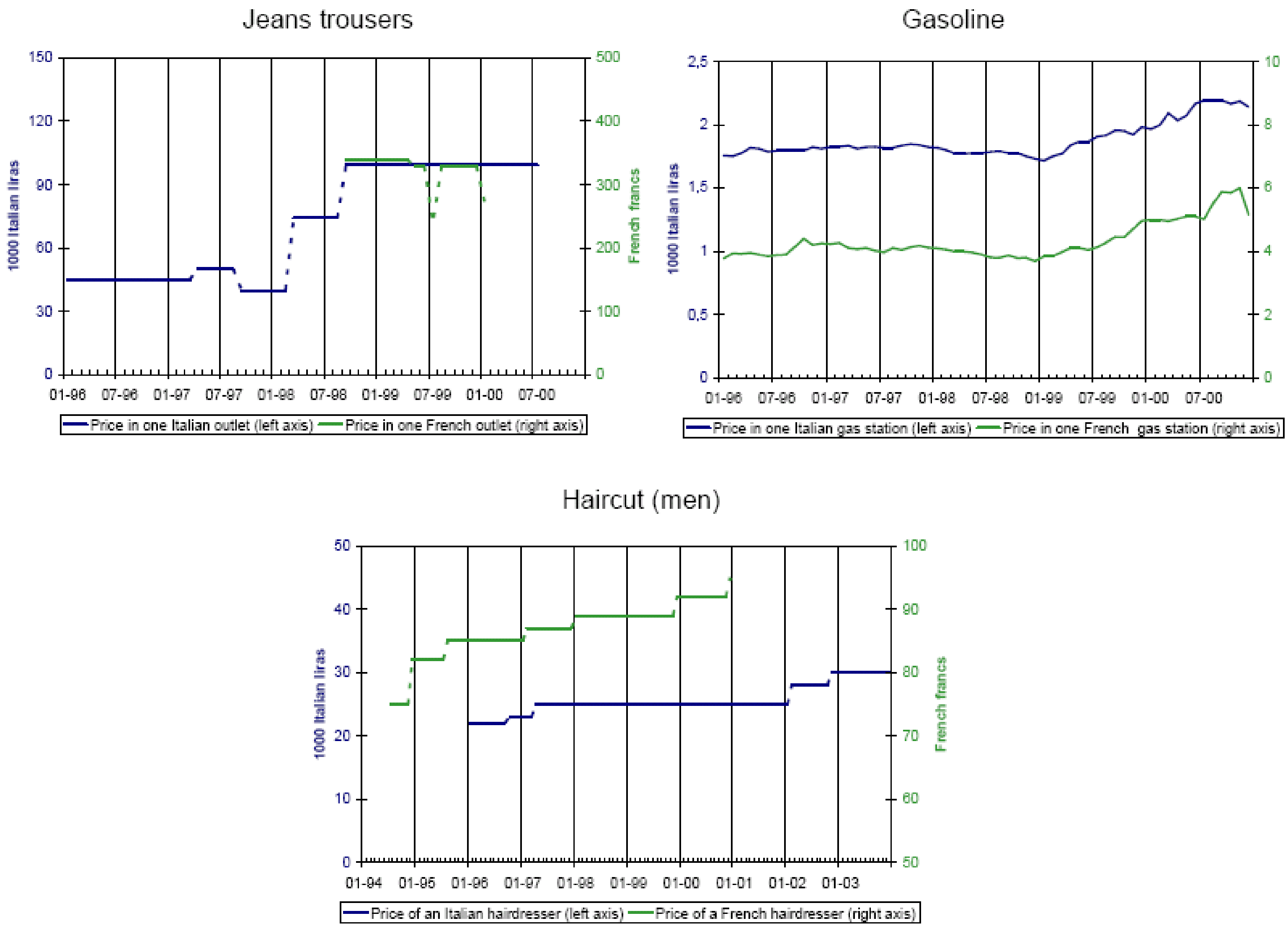
GDP deflator



M2

Source: Christiano, Eichenbaum and Evans (1999)

Figure 1 - Examples of individual price trajectories (French and Italian CPI data)



Note : Actual examples of trajectories, extracted from the French and Italian CPI databases. The databases are described in Baudry *et al*. (2004) and Veronese *et al*. (2005). Prices are in levels, denominated in French Francs and Italian Lira respectively. The dotted lines indicate events of price changes.

Source: Dhyne et al. (JEP, 2006)

TABLE 1. Measures of price stickiness in the euro area and the US (% per month unless otherwise stated).

	Statistics	Euro area	US
CPI*	Frequency	15.1	24.8
	Average duration (*months*)	13.0	6.7
	Median duration (*months*)	10.6	4.6
PPI†	Frequency	20.0	n.a
Surveys‡	Frequency	15.9	20.8
	Average duration (*months*)	10.8	8.3
NKPC§	Average durations (*months*)	13.5–19.2	7.2–8.4
Internet prices¶	Frequency	79.2	64.3

Source: Álvarez et al. (JEEA, 2006)

Motivation and Outline

Evidence on Money, Output, and Prices:

- Macro evidence on the effects of monetary policy shocks

  (i) persistent effects on real variables
  (ii) slow adjustment of aggregate price level
  (iii) liquidity effect

- Micro evidence: significant price and wage rigidities

  $\Rightarrow$ in conflict with the predictions of classical monetary models

A Baseline Model with Nominal Rigidities

- monopolistic competition
- sticky prices (staggered price setting)
- competitive labor markets, closed economy, no capital accumulation

Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t; Z_t\right)$$

where

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$$

subject to

$$\int_0^1 P_t(i)C_t(i)di + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

for $t = 0, 1, 2, \ldots$ plus solvency constraint.

Example:

$$\lim_{T\to\infty} E_t \left\{\frac{B_T}{P_T}\right\} \geq 0$$

Optimality conditions

1. Optimal allocation of expenditures

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$$

implying

$$\int_0^1 P_t(i)C_t(i)di = P_tC_t$$

where

$$P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon}di\right)^{\frac{1}{1-\epsilon}}$$

2. Other optimality conditions

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta E_t\left\{\frac{U_{c,t+1}}{U_{c,t}}\frac{P_t}{P_{t+1}}\right\}$$

Specification of utility:

$$U(C_t, N_t; Z_t) = \begin{cases} \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right) Z_t & \text{for } \sigma \neq 1 \\ \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}\right) Z_t & \text{for } \sigma = 1 \end{cases}$$

where

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

Optimality conditions:

$$w_t - p_t = \sigma c_t + \varphi n_t$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1 - \rho_z) z_t$$

where $i_t \equiv -\log Q_t$ and $\rho \equiv -\log \beta$

Ad-hoc money demand:

$$m_t - p_t = c_t - \eta i_t$$

Firms

- Continuum of firms, indexed by $i \in [0,1]$
- Each firm produces a differentiated good
- Identical technology

  $$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

  where

  $$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

- Probability of resetting price in any given period: $1-\theta$, independent across firms (Calvo (1983)).
- $\theta \in [0,1]$: index of price stickiness
- Implied average price duration $\frac{1}{1-\theta}$

Aggregate Price Dynamics

$$P_t = \left[\theta(P_{t-1})^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

Dividing by P_{t-1} :

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}$$

Log-linearization around zero inflation steady state

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \tag{1}$$

or, equivalently

$$p_t = \theta p_{t-1} + (1-\theta)p_t^*$$

Optimal Price Setting

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} (1/P_{t+k}) \left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to:

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \tag{2}$$

for $k = 0, 1, 2, ...$where $\Lambda_{t,t+k} \equiv \beta^k U_{c,t+k}/U_{c,t}$

Optimality condition:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} (1/P_{t+k}) \left(P_t^* - \mathcal{M} \Psi_{t+k|t} \right) \right\} = 0$$

where $\Psi_{t+k|t} \equiv \mathcal{C}'_{t+k}(Y_{t+k|t})$ and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon - 1}$.

Flexible price case $(\theta = 0)$:

$$P_t^* = \mathcal{M} \Psi_{t|t}$$

Zero inflation steady state

$$\Lambda_{t,t+k} = \beta^k \ ; \ P_t^*/P_{t-1} = P_t/P_{t+k} = 1 \ \Rightarrow \ Y_{t+k|t} = Y \ ; \ \Psi_{t+k|t} = \Psi_t \ ; \ P_t = \mathcal{M}\Psi_t$$

Linearized optimal price setting condition:

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t\{\psi_{t+k|t}\}$$

where $\psi_{t+k|t} \equiv \log \Psi_{t+k|t}$ and $\mu \equiv \log \mathcal{M}$

Particular Case: $\alpha = 0$ (constant returns)

$$\Longrightarrow \psi_{t+k|t} = \psi_{t+k}$$

Recursive form:

$$p_t^* = \beta\theta E_t\{p_{t+1}^*\} + (1-\beta\theta)p_t - (1-\beta\theta)\widehat{\mu}_t$$

where $\mu_t \equiv p_t - \psi_t$ and $\widehat{\mu}_t \equiv \mu_t - \mu$

Combined with price dynamics equation yields:

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda\widehat{\mu}_t$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

General case: $\alpha \in [0,1)$

$$\begin{aligned}\psi_{t+k|t} &= w_{t+k} - mpn_{t+k|t} \\ &= w_{t+k} - (a_{t+k} - \alpha n_{t+k|t} + \log(1-\alpha))\end{aligned}$$

$$\psi_{t+k} \equiv w_{t+k} - (a_{t+k} - \alpha n_{t+k} + \log(1-\alpha))$$

$$\begin{aligned}\psi_{t+k|t} &= \psi_{t+k} + \alpha(n_{t+k|t} - n_{t+k}) \\ &= \psi_{t+k} + \frac{\alpha}{1-\alpha}(y_{t+k|t} - y_{t+k}) \\ &= \psi_{t+k} - \frac{\alpha\epsilon}{1-\alpha}(p_t^* - p_{t+k})\end{aligned}$$

Optimal price setting equation:

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{p_{t+k} - \Theta\widehat{\mu}_{t+k}\right\}$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon} \in (0,1]$.

Recursive form:

$$p_t^* = \beta\theta E_t\{p_{t+1}^*\} + (1-\beta\theta)p_t - (1-\beta\theta)\Theta\widehat{\mu}_t$$

Combined with price dynamics equation yields:

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda\widehat{\mu}_t$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\,\Theta$$

Equilibrium

Goods markets clearing

$$Y_t(i) = C_t(i)$$

for all $i \in [0,1]$ and all t.

Letting $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$:

$$Y_t = C_t$$

Combined with Euler equation:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1-\rho_z)z_t$$

Labor market clearing:

$$\begin{aligned} N_t &= \int_0^1 N_t(i)di \\ &= \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} di \\ &= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di \end{aligned}$$

Up to a first order approximation:

$$n_t = \frac{1}{1-\alpha}(y_t - a_t)$$

Average price markup and output

$$\begin{aligned}\mu_t &\equiv p_t - \psi_t \\ &= -(w_t - p_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\ &= -(\sigma y_t + \varphi n_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\ &= -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha)\end{aligned}$$

Under flexible prices:

$$\mu = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha)$$

implying

$$y_t^n = \psi_{ya} a_t + \psi_y$$

where $\psi_y \equiv -\frac{(1-\alpha)(\mu - \log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha} > 0$ and $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$. Thus,

$$\widehat{\mu}_t = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(y_t - y_t^n)$$

New Keynesian Phillips Curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

where $\widetilde{y}_t \equiv y_t - y_t^n$ and $\kappa \equiv \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$.

The Non-Policy Block of the Basic New Keynesian Model

New Keynesian Phillips Curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

Dynamic IS equation

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n)$$

where r_t^n is the *natural rate of interest*, given by

$$r_t^n = \rho - \sigma(1-\rho_a)\psi_{ya} a_t + (1-\rho_z) z_t$$

Missing block: description of monetary policy (determination of i_t).

Equilibrium under a Simple Interest Rate Rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t$$

where $\widehat{y}_t \equiv y_t - y$ and

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

Equivalently:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widetilde{y}_t + \phi_y \widehat{y}_t^n + v_t$$

where $\widehat{y}_t^n \equiv y_t^n - y$.

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T \; u_t$$

where

$$\begin{aligned} u_t &\equiv \widehat{r}_t^n - \phi_y \widehat{y}_t^n - v_t \\ &= -\psi_{ya}(\phi_y + \sigma(1-\rho_a))a_t + (1-\rho_z)z_t - v_t \end{aligned}$$

and

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma+\phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

with $\Omega \equiv \frac{1}{\sigma+\phi_y+\kappa\phi_\pi}$.

Uniqueness condition (Bullard and Mitra):

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$$

Exercise: analytical solution (method of undetermined coefficients).

Equilibrium under an Exogenous Money Growth Process

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

Money demand

$$l_t \equiv m_t - p_t = \widetilde{y}_t - \eta i_t + y_t^n$$

Substituting into dynamic IS equation

$$(1 + \sigma\eta)\, \widetilde{y}_t = \sigma\eta E_t\{\widetilde{y}_{t+1}\} + \widehat{l}_t + \eta E_t\{\pi_{t+1}\} + \eta \widehat{r}_t^n - \widehat{y}_t^n$$

Identity:

$$\widehat{l}_{t-1} = \widehat{l}_t + \pi_t - \Delta m_t$$

Equilibrium dynamics:

$$\mathbf{A_{M,0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ \widehat{l}_{t-1} \end{bmatrix} = \mathbf{A_{M,1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \widehat{l}_t \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \widehat{r}_t^n \\ \widehat{y}_t^n \\ \Delta m_t \end{bmatrix} \tag{3}$$

where

$$\mathbf{A_{M,0}} \equiv \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad \mathbf{A_{M,1}} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B_M} \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Uniqueness condition:

$\mathbf{A_M} \equiv \mathbf{A_{M,0}^{-1}} \mathbf{A_{M,1}}$ has two eigenvalues inside and one outside the unit circle.

Calibration

Households: $\sigma = 1$; $\varphi = 5$; $\beta = 0.99$; $\epsilon = 9$; $\eta = 4$; $\rho_z = 0.5$

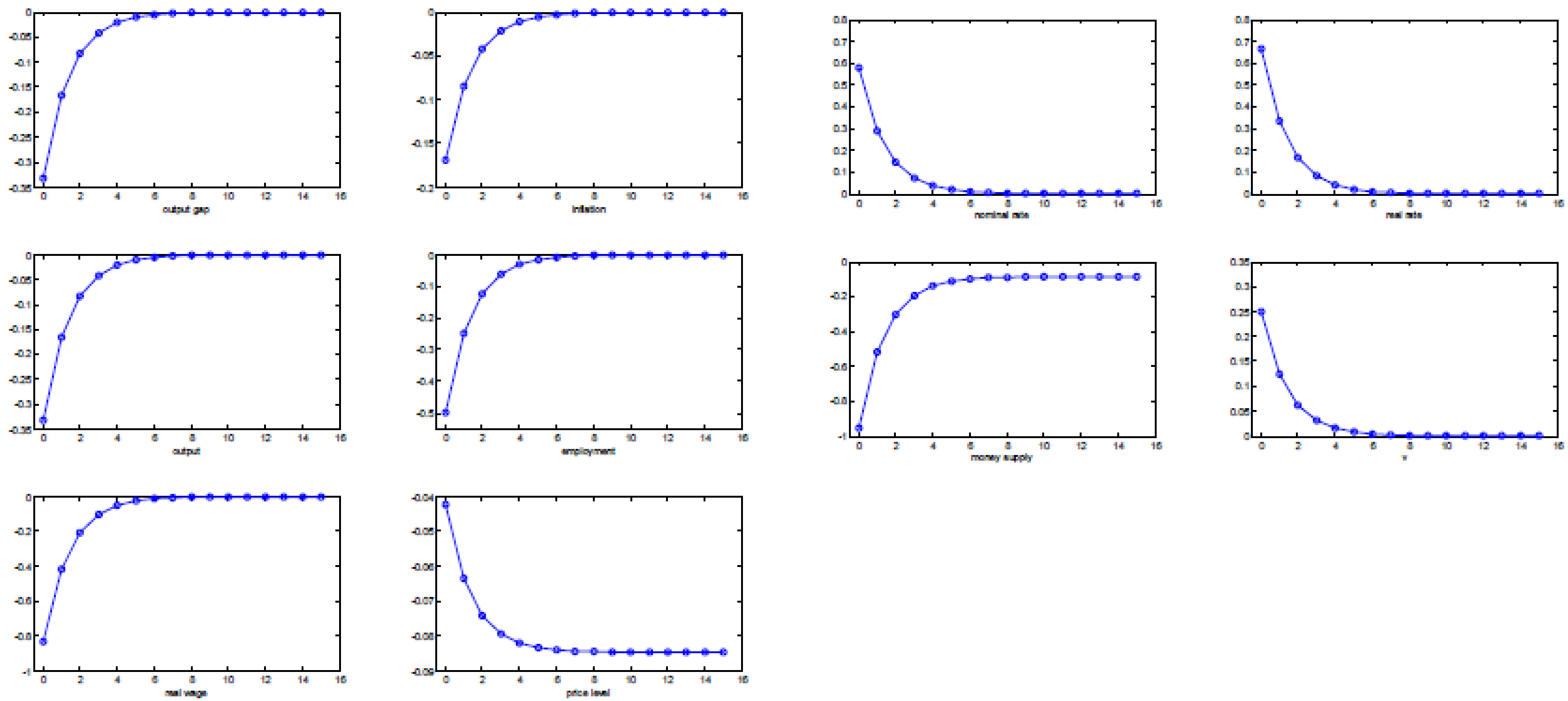
Firms: $\alpha = 1/4$; $\theta = 3/4$; $\rho_a = 0.9$

Policy rules: $\phi_\pi = 1.5$, $\phi_y = 0.125$; $\rho_v = \rho_m = 0.5$

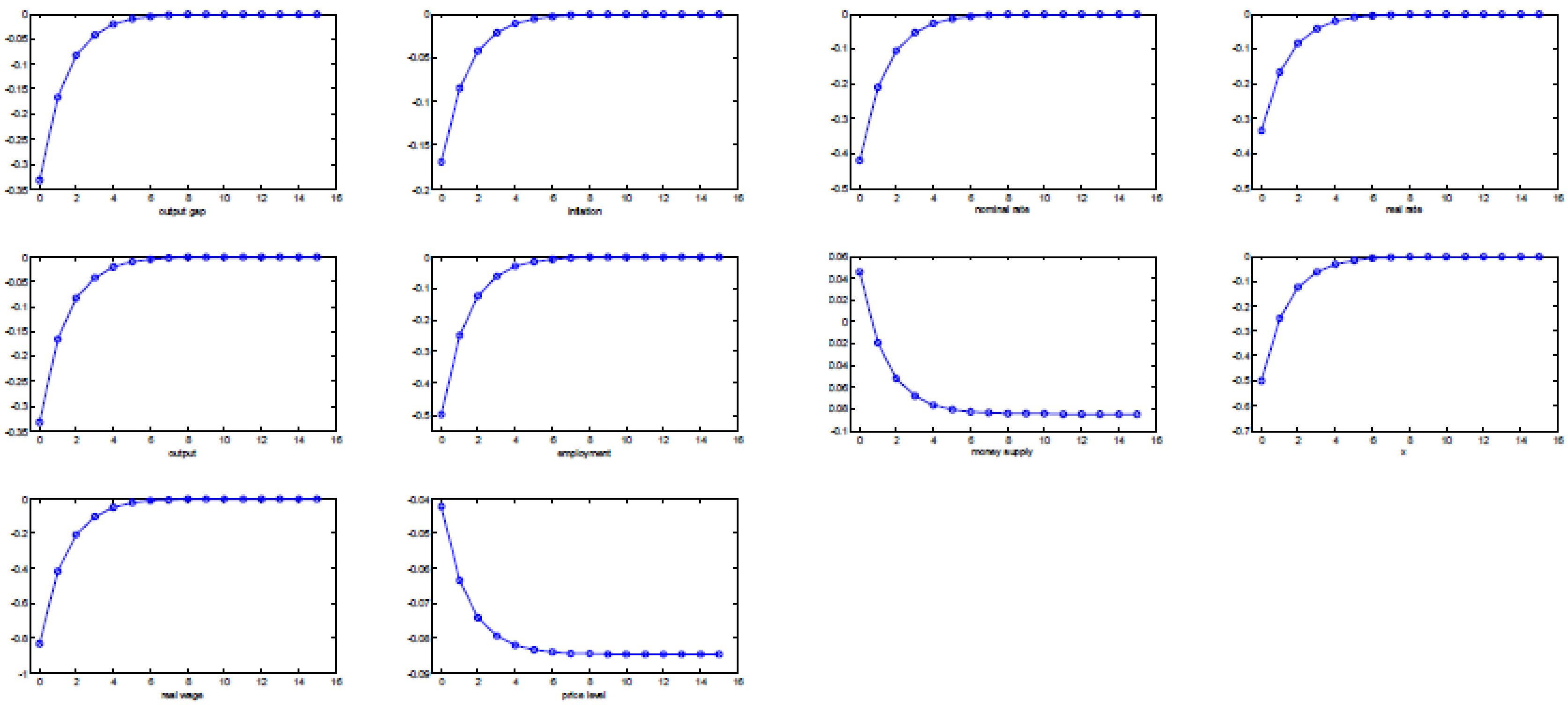
Dynamic Responses to Exogenous Shocks

- Monetary policy, discount rate, technology
- Interest rate rule vs. money growth rule

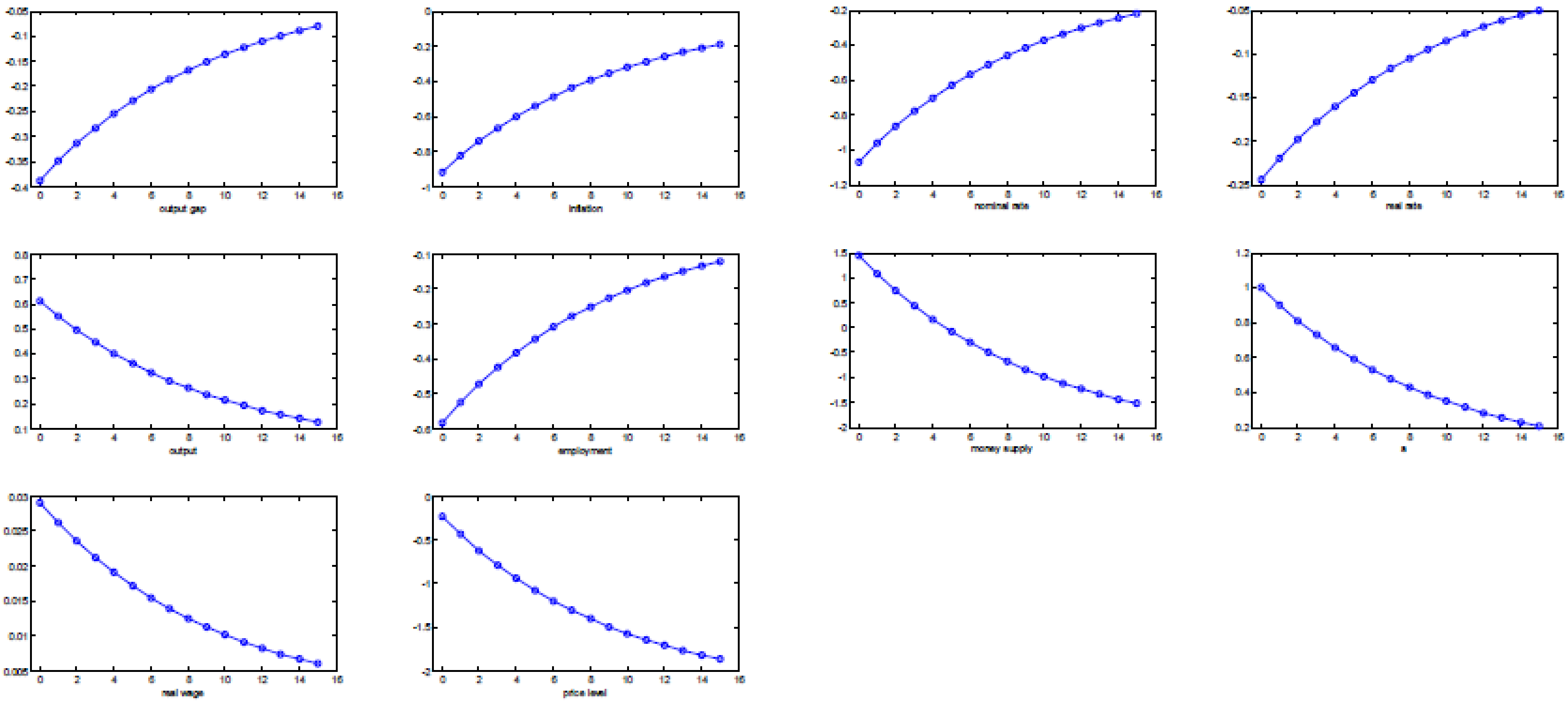
Dynamic responses to a monetary policy shock: Interest rate rule



Dynamic responses to a discount rate shock: Interest rate rule



Dynamic responses to a technology shock: Interest rate rule



Estimated Effects of Technology Shocks

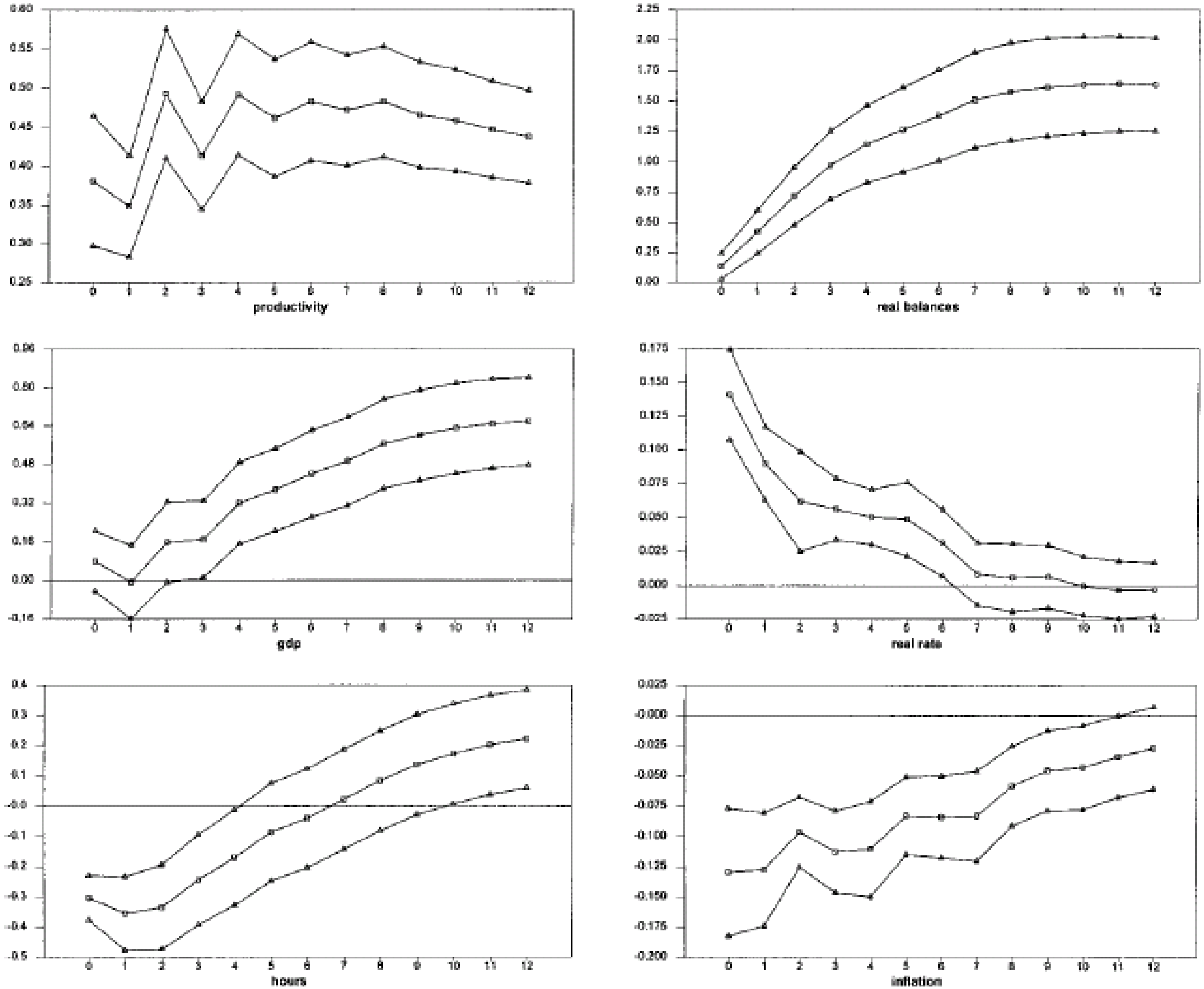
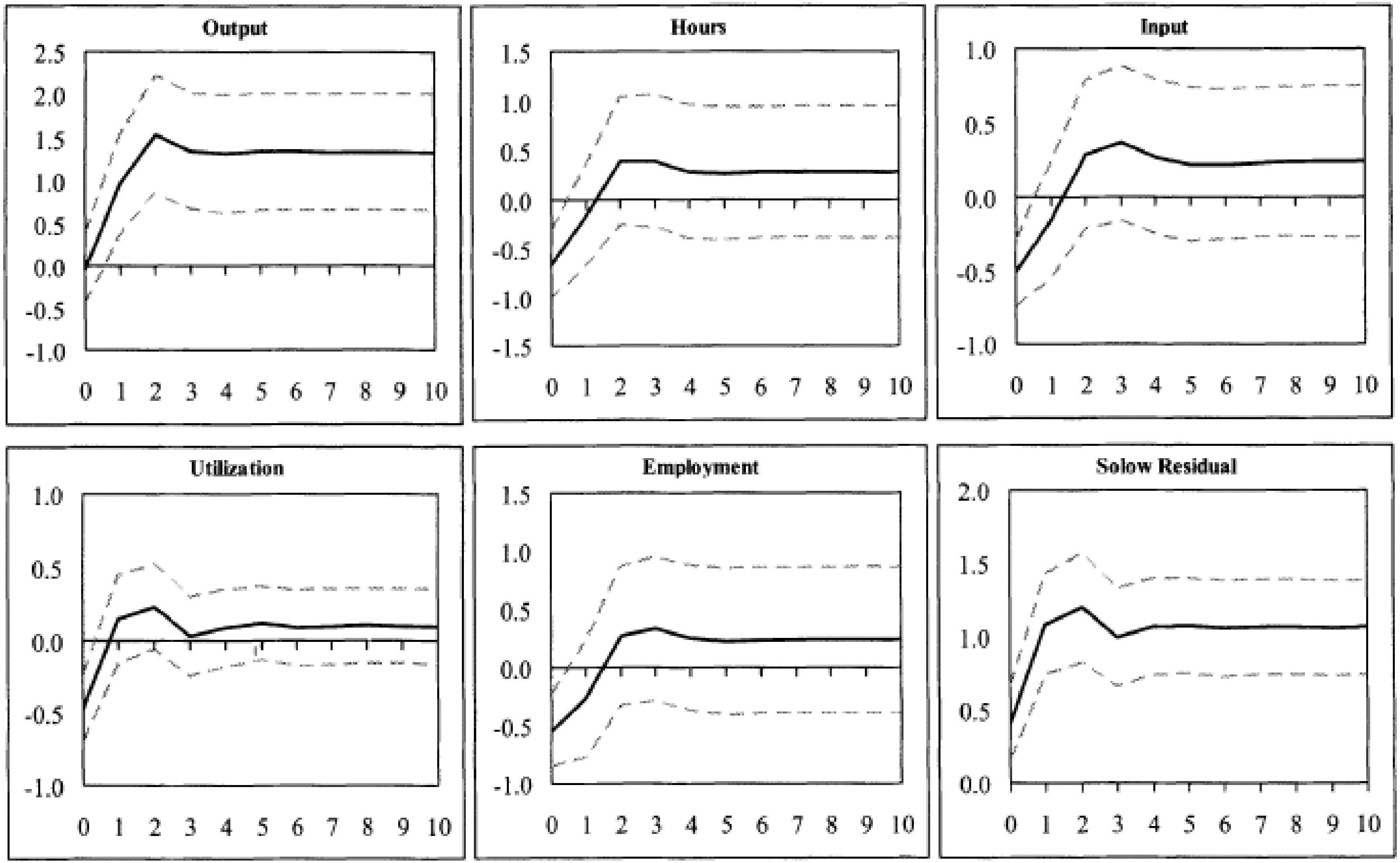


FIGURE 4. ESTIMATED IMPULSE RESPONSES FROM A FIVE-VARIABLE MODEL: U.S. DATA, FIRST-DIFFERENCED HOURS (POINT ESTIMATES AND ±2 STANDARD ERROR CONFIDENCE INTERVALS)

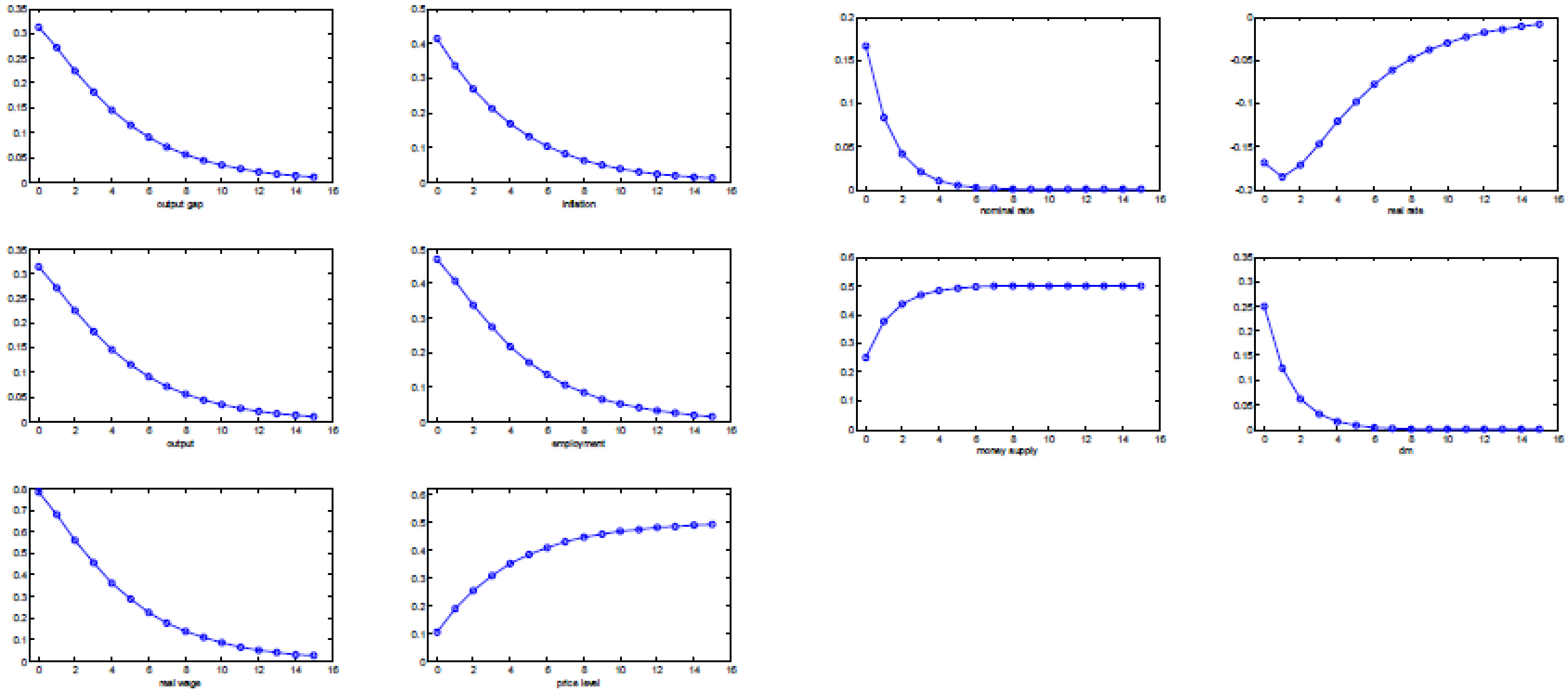
Source: Galí (1999)

Estimated Effects of Technology Shocks

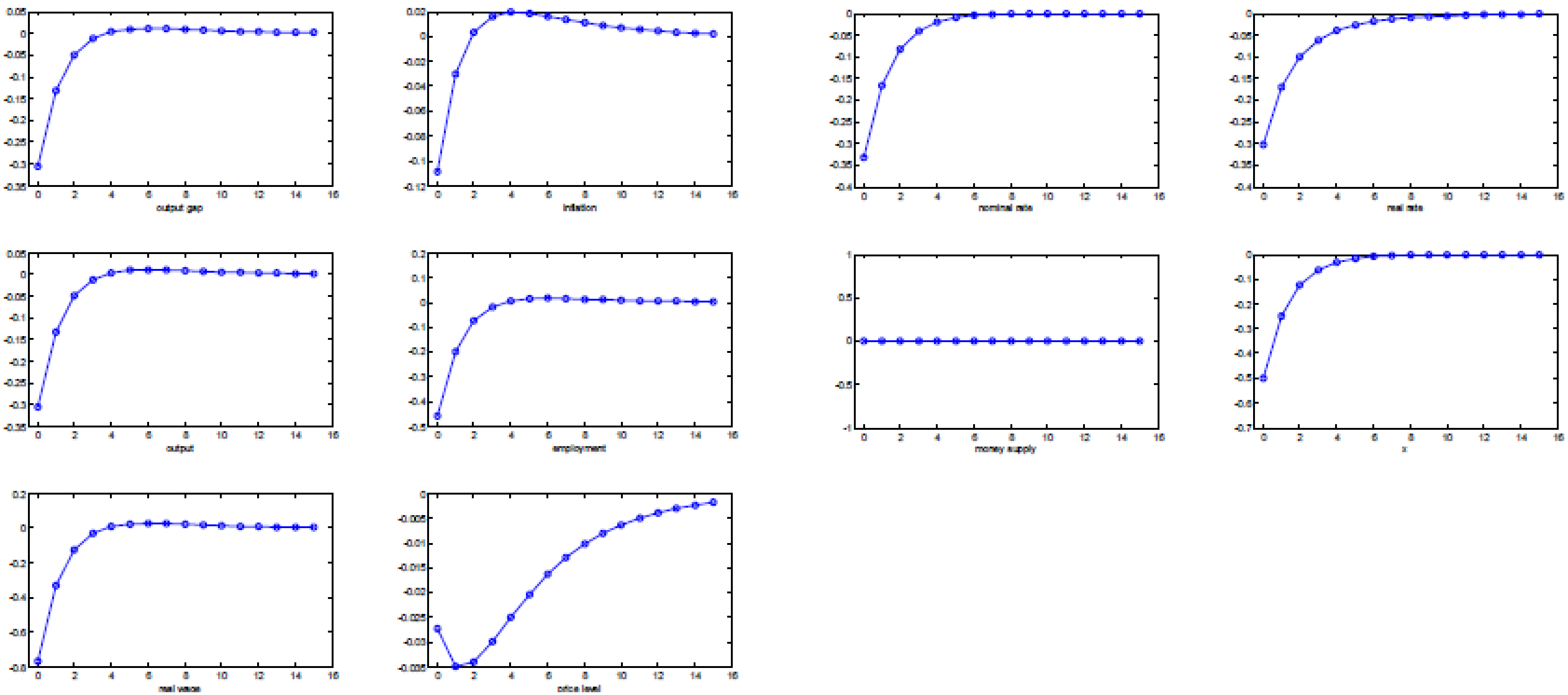


Source: Basu, Fernald and Kimball (2006)

Dynamic responses to a monetary policy shock: Money growth rule



Dynamic responses to a discount rate shock: Money growth rule



Dynamic responses to a technology shock: Money growth rule