Non-Ricardian Aspects of Fiscal Policy in Chile

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Abstract

This paper examines non-Ricardian effects of government spending shocks in the Chilean economy. We first provide evidence on those effects based on vector autoregressions. We then show that such evidence can be accounted for by a model that features: (i) a sizeable share of non-Ricardian households, i.e. households which do not make use of financial markets and just consume their current labor income; (ii) nominal price and wage rigidities; (iii) an inflation targeting scheme and (iv) a structural balance fiscal rule that reflects the particular Chilean fiscal rule. The model is estimated employing Bayesian techniques. Finally, we use model simulations to demonstrate the countercyclical effects of the Chilean fiscal rule as compared with a zero-deficit rule.

Keywords: Fiscal policy rules, Ricardian agents, Non-Ricardian agents, New Keynesian open economy model, Bayesian estimation.

JEL Codes: H30, E31, F41.

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1 Introduction

In this paper we examine the effects of government spending shocks in the Chilean economy. The study of the effects of those shocks in an emerging market economy is of special interest because of the potential importance in such an economy of non-Ricardian households, i.e. households which do not own any assets nor have any liabilities and just consume their current labor income.[1] The existence of non-Ricardian households has been pointed to as a key ingredient in the transmission mechanism of government spending shocks in some developed economies. There are several factors that may explain non-Ricardian behavior including myopia and lack of access to capital markets. The importance of such behavior is likely to be even greater in less developed economies.

To study the effects of government spending shocks in an economy like the Chilean one is also interesting because of its significant financial and trade openness. The size of the fiscal multiplier generally depend on the response of monetary policy and the degree of flexibility of the exchange rate. In particular, economies with less flexible exchange rate regimes are likely to exhibit larger fiscal multipliers, as the exchange rate regime limits the potential offsetting effects due to the response of the interest rate and the exchange rate to a government spending shock. Recently, Ilzetzki *et al.* (2009) have pointed out that cumulative fiscal multipliers in fixed exchange rate regimes are positive and significant, whereas in flexible exchange rate regimes are basically zero. In the period of study, monetary policy was characterized by the existence of an explicit commitment to an inflation target. In terms of the exchange rate regime, the Chilean economy moved from an exchange rate band in the nineties towards a flexible exchange rate since 2000.

We start our work by presenting empirical evidence on the macroeconomic effects of government spending shocks for the Chilean economy. First, we present evidence based on vector autoregressive (VAR) models that indicates that the fiscal multiplier is positive and large in the Chilean economy. Moreover, the positive consumption multiplier that emerges from this VAR analysis points to the importance of that variable in generating the large GDP multiplier, and suggests the presence of non-Ricardian effects. Secondly, we develop a small open economy model to study the channels through which these shocks are transmitted to the economy. The model features Ricardian and non-Ricardian households along the lines of Galí *et al.* (2007) and Coenen *et al.* (2008). The model is calibrated and estimated for the Chilean economy. For this purpose we model explicitly the fiscal framework under which fiscal policy has been conducted in Chile known as the structural balance rule.

The Chilean fiscal rule ties total government spending to structural revenues. Structural revenues correspond to the sum of cycle-adjusted tax revenues and copper-related fiscal revenues evaluated at what could be considered a long-term copper price. Under this fiscal rule, government spending plus a structural fiscal surplus target must be equal to permanent (structural) revenues. Shocks to GDP (deviations from potential output) and to copper prices that affect transitorily fiscal revenues do not alter the path for government spending (path that is only affected by changes in potential output and the long-term price of copper). For example, the rule implies that if effective copper prices are transitorily above the estimated long-term copper price, the government saves the amount of copper-related fiscal revenues associated to this transitory copper price shock.[2] When officially implemented in 2001, the government announced a structural fiscal surplus target equivalent to one percent of GDP (i.e., structural revenues minus government expenditure equals

[1] See, e.g. , Campbell & Mankiw (1991), Mankiw (2000) and Galí *et al.* (2007).

[2] Potential output and long-term copper price are determined by two committees of experts independent of the government. See Frankel (2011) for a description of the Chilean fiscal rule.

one percent of GDP). We show that the specification of a fiscal policy rule that approximates the Chilean rule leads to consumption and output fiscal multipliers that are positive in the short run, in a way consistent with the evidence.[3]

The structure of the paper is as follows. Section 2 presents VAR evidence on non-Ricardian effects of fiscal policy for the Chilean case. Section 3 introduces a dynamic stochastic general equilibrium model for Chile. The model is calibrated and estimated and results are reported in Section 5. Numerical simulations are presented in Section 6. Finally, Section 7 concludes.

2 Some Evidence of the Effects of Government Spending in Chile

In the present section we provide some evidence on the macroeconomic effects of government spending shocks, using Chilean data for the past two decades. Following much of the literature, we rely on estimated VARs. While the literature has largely focused on the effects of government purchases (often restricted to military ones), we also examine the impact of changes in transfers, since the latter are perceived as an important stabilization tool in Chile and have historically displayed large changes. In both cases, we report impulse response functions (IRFs), as well as estimates of the size of the output and consumption multipliers.

2.1 The Effects of Government Purchases

We first consider a small VAR specification including four variables: government purchases (government consumption plus public investment), GDP (excluding copper and other natural resources), private consumption (of durables and nondurables), and government deficit (excluding copper-related revenues).[4] The first three variables are expressed in logs and normalized by the size of the population. The deficit is normalized by lagged GDP. Data availability restricts the sample to the period 1990Q1-2010Q1. Our VAR includes four lags of all the variables, a constant term and a second order polynomial in time.

Following much of the literature, identification relies on the assumption that government purchases are predetermined relative to the other variables included in the VAR.[5] In other words, we interpret reduced form innovations to government purchases as exogenous shocks to that variable. This is equivalent to ordering government purchases first in a Cholesky factorization of the VAR.

Figure 1 reports the impulse responses to a one standard deviation shock to government purchases, together with the corresponding 95 percent confidence intervals. Note that government purchases increase by nearly close to two percent on impact. In response to that fiscal expansion, both GDP and consumption rise. Both variables display a pattern that is roughly similar over time, with the peak being attained four quarters after the shock in the case of output and three quarters in the case of consumption. Not surprisingly, the deficit increases on impact.

[3] The exercise of implementing a zero deficit rule provides a good benchmark; however, results are not reported. Briefly, a zero-deficit fiscal rule instrumented by transfers leaving public expenditure exogenous (as in Forni *et al.* (2007)) yields positive fiscal multipliers (of consumption and GDP). On the other hand, if the shock is on the government expenditures we find a negative fiscal multiplier for consumption but a positive one for GDP.

[4] We exclude copper and other natural resources activities from GDP because they are mainly affected by supply conditions. This strategy is consistent with the way in which we model GDP in our theoretical model.

[5] See e.g. Blanchard & Perotti (2002), Fatas & Mihov (2001), Galí *et al.* (2007), and Perotti (2008).

Figure 1: Impulse response to government purchases shock (small VAR)

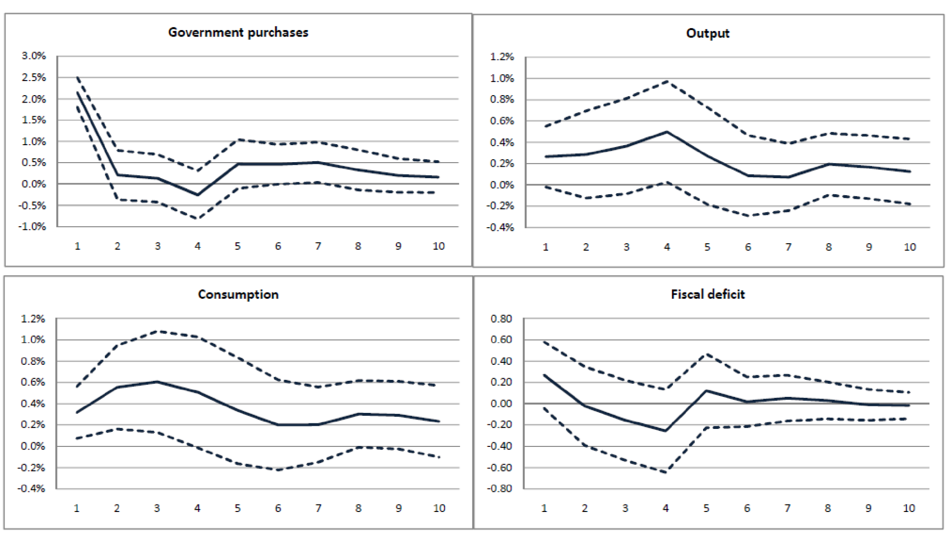


Table 1 reports the corresponding multipliers for both GDP and consumption at different horizons. The *basic multiplier* measures $\frac{dX_{t+k}}{dG_t}$ for $k = 1, 2, 4, 6, 8$, where dG_t is the change in the *level* of government purchases on impact, and dX_{t+k} is the corresponding response in the *level* of GDP (when $X = Y$) or consumption (when $X = C$), k periods after the shock.[6] The GDP multiplier is above one half (0.7) on impact, and reaches a peak value close to 1.3 at a four-quarter horizon, before it goes down. The previous values are similar to those obtained using U.S. data by a variety of authors (see Hall (2009) for a survey of existing results). A look at the consumption multiplier points to the importance of that variable in generating the large GDP multiplier, suggesting the presence of non-Ricardian effects.

In addition to the basic multiplier we also report estimates of the *cumulative multiplier* at different horizons, defined as $(\sum_{j=1}^{k} dX_{t+j})/(\sum_{j=1}^{k} dG_{t+j})$. The latter takes into account not only the size of the initial increase in government purchases, but also its subsequent pattern of adjustment. As shown in Table 1, both the GDP and consumption cumulative multipliers increase in the first year, reflecting the persistence of the GDP and consumption responses in that horizon, beyond that of government purchases themselves.

[6] Using the IRFs for the logs we compute the multiplier as $\frac{dX_{t+k}}{dG_t} = \frac{d \log X_{t+k}}{d \log G_t} \; \frac{X_{t+k}}{G_t}$

Table 1. Effects of government purchases (Small Var)

	Basic		Cumulative	
time/multipliers	dC/dG	dY/dG	dC/dG	dY/dG
t=1	0.585	0.674	0.585	0.674
t=2	1.026	0.727	1.466	1.274
t=4	0.941	1.274	3.528	3.462
t=6	0.372	0.219	3.168	3.062
t=8	0.563	0.496	3.010	2.786

Table 2. Effects of government purchases (Large Var)

	Basic		Cumulative	
time/multipliers	dC/dG	dY/dG	dC/dG	dY/dG
t=1	0.743	1.103	0.743	1.103
t=2	1.300	1.202	2.049	2.313
t=4	1.193	1.429	4.181	4.450
t=6	0.721	1.000	3.888	4.341
t=8	0.639	0.496	3.719	4.079

We explore the robustness of the previous findings to the use of a larger VAR, which includes, in addition to the four variables above, real copper price, total private investment and the real exchange rate. Given the fiscal rule in place, whereby the government is allowed to spend only the fraction of the increase in copper revenues that is consider to be permanent, it is natural to order that price before government purchases, which are now ordered second in the VAR.[7] Figure 2 displays the estimated IRFs to a government purchases shock using the larger VAR. The corresponding multipliers are shown in Table 2 The picture that emerges is, qualitatively and quantitatively, very similar to that obtained using the small VAR. Note that investment also rises in response to the increase in government purchases, suggesting a role for that variable complementary to that of consumption in generating the large GDP multiplier. That amplification effect is likely to be partially offset by the real exchange rate appreciation, which should dampen the growth of aggregate demand. The pattern of the deficit response estimated using the large VAR is also very similar, suggesting again a deficit increase on impact.

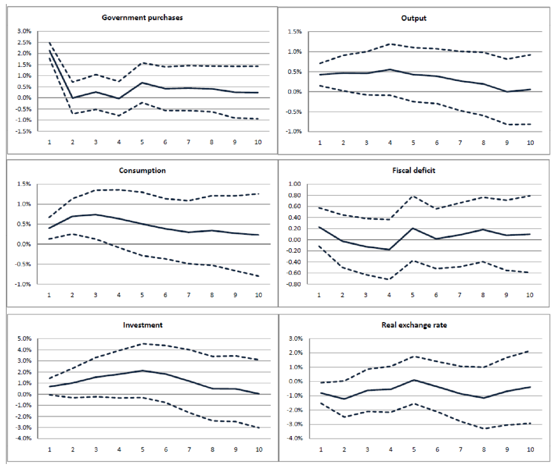
2.2 The Effects of Government Transfers

Next we report estimates of the dynamic effects of government transfers, using an approach analogous to the one in the previous subsection, with total government transfers substituting for government purchases in the two VARs.

Figure 3 reports the impulse responses to a transfer shock. As shown in the first box, the increase in transfers appears to have a similar persistence than the increase in government purchases studied above. The resulting responses of output, consumption and the deficit show a pattern not too different from that obtained for government purchases. Also, the sign of the response of the

[7] The fiscal policy rule in place in Chile establishes that government spending is linked to structural revenues (the permanent component of effective revenues). One component of those structural revenues corresponds to copper related revenues. Structural copper revenues correspond to the revenues that the government would collect if the price of copper was equal to their long run price or permanent price.

Figure 2: Impulse response to government purchases shock (large VAR)



deficit is less clear cut in the case of a shock to transfers. Consideration of the estimated multipliers, shown in Table 3, pointing to similar multipliers for both GDP and consumption.

The evidence based on the large VAR, reported in Figure 4 and Table 4, provides a similar picture. One difference relative to the corresponding findings for purchases is worth pointing out: the real exchange depreciates in response to an increase in transfers.

Table 3. Effects of Government Transfers
(Small VAR)

	Basic		Cumulative	
time/multipliers	dC/dG	dY/dG	dC/dG	dY/dG
t=1	0.447	0.721	0.447	0.721
t=2	1.165	1.107	1.297	1.471
t=4	0.874	1.612	2.375	2.820
t=6	0.090	0.448	1.955	3.162
t=8	0.411	0.486	2.002	2.984

Table 4. Effects of Government Transfers
(Large VAR)

	Basic		Cumulative	
time/multipliers	dC/dG	dY/dG	dC/dG	dY/dG
t=1	0.395	0.877	0.395	0.877
t=2	1.267	1.417	1.338	1.847
t=4	0.683	1.206	2.250	2.758
t=6	0.042	0.720	1.786	3.222
t=8	0.360	0.312	1.784	2.917

Figure 3: Impulse response to government transfers shock (small VAR)

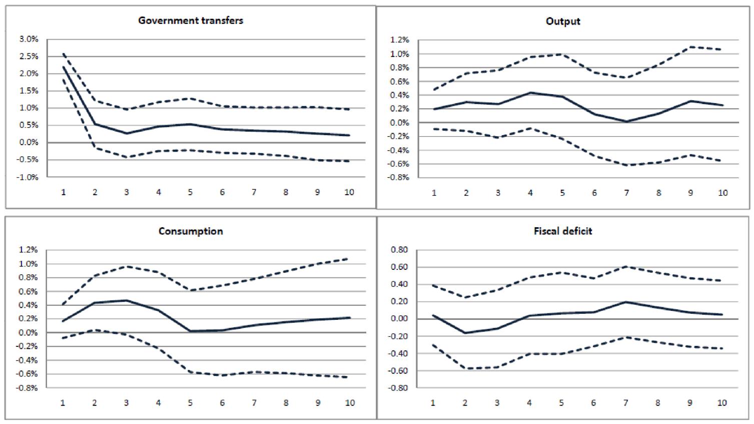
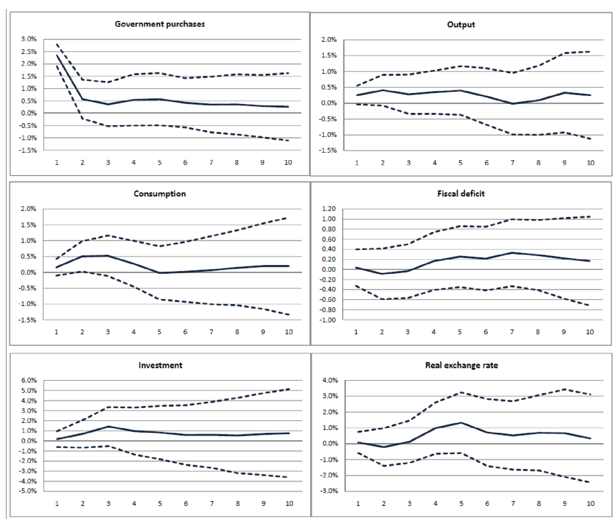


Figure 4: Impulse response to government transfers shock (large VAR)



2.3 Discussion

The evidence presented on the effects of shocks to government purchases and government transfers points towards the existence of positive multiplier effects on GDP. The sign and size of the estimated response of consumption is suggestive of strong non-Ricardian effects, which would account for the size of both the GDP and consumption multipliers. In the next section we develop an open economy New Keynesian model that tries to account for these regularities.

3 A Small Open Economy Model for Chile

This section presents the structure of a DSGE model along the lines of Altig *et al.* (2005), Adjemian *et al.* (2008) and Adolfson *et al.* (2007), but extended to incorporate a role for fiscal policy. We build on the work by Galí *et al.* (2007) and Coenen *et al.* (2008) who develop versions of a New Keynesian model allowing for a fraction of non-Ricardian households, but modified in order to capture particular features of the Chilean economy. Among the latter we have copper income explaining a non-negligible share of government's income, a fiscal rule that seeks to keep government spending closely linked to structural (permanent) fiscal revenues, and an inflation targeting monetary policy regime. A complementary appendix with main model's derivations is available upon request from the authors.

3.1 Consumers

There are two types of consumers: Ricardian (with weight $1-\lambda$) and non-Ricardian (with weight λ), denoted with superscript $j = \{R, N\}$. Ricardian consumers are assumed to have access to financial markets to smooth consumption over time, whereas non-Ricardian ones do not. Implicitly, though, we make an exception to the latter assumption in order to simplify the analysis: we assume full insurance of the risk generated by Calvo wage setting among consumers of a given type (as in Coenen *et al.* (2008)).

Both consumer types are assumed to maximize an objective function of the form $\sum_{t=0}^{\infty} \beta^t U_t^j(h)$ with period utility given by

$$U_t^j(h) = \ln\left(C_t^j(h) - bC_{t-1}^j(h)\right) - \bar{\zeta}\zeta_t \frac{L_t^j(h)^{1+\sigma_L}}{1+\sigma_L}, \tag{1}$$

where $C_t^j(h)$ is a consumption index and $L_t^j(h)$ denotes hours of work. Note that b measures the degree of *internal* habit formation, $\bar{\zeta}$ is a constant, σ_L is the inverse of the Frisch elasticity and ζ_t is a shock to the disutility from work. The latter parameter is assumed to follow an AR(1) process with unconditional mean of one, persistence ρ_ζ and constant variance, σ_ζ^2.[8]

The consumption index takes the form:

$$C_t^j(h) \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^j(h)^{1-\frac{1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^j(h)^{1-\frac{1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where $C_{H,t}^j(h) \equiv \left(\int_0^1 C_{H,t}^j(h,i)^{1-\frac{1}{\epsilon_H}} di\right)^{\frac{\epsilon_H}{\epsilon_H-1}}$ and $C_{F,t}^j(h) \equiv \left(\int_0^1 C_{F,t}^j(h,i)^{1-\frac{1}{\epsilon_F}} di\right)^{\frac{\epsilon_F}{\epsilon_F-1}}$ are CES indexes for domestic and imported consumption goods, respectively, with parameter α determining

[8]Notice that we abuse of notation declaring $C_t^j(h)$ for $j = \{R, N\}$; however, we want to stress that the decision maker is the individual h.

the degree of openness and $\eta > 1$ being the constant elasticity of substitution between domestic and imported goods.

3.1.1 Ricardian Consumers

Ricardian consumers ($h = R$) maximize utility subject to two constraints. First, a flow budget constraint of the form

$$\begin{array}{c} B^R(s^t,h) + S_t B^{R,*}(s^t,h) + (1-\tau_{w,t})\,\mathcal{S}_{WR} W_t^R(h) L_t^R(h) + R_t^k u_t^R(h) K_{t-1}^R(h) \\ -P_t \Phi\left(u_t^R(h)\right) K_{t-1}^R(h) + P_t\left[Tr_t^R(h) - TX_t^R(h)\right] + (1-\tau_{\mathrm{Pr},t})\,\mathsf{Pr}_t^R(h) \leq \\ +\sum_{s^{t+1}|s^t} Q\left(s^{t+1},s^t\right) B^R\left(s^{t+1},h\right) + S_t \mathbb{RP}_t \sum_{s^{t+1}|s^t} Q^*\left(s^{t+1},s^t\right) B^{R,*}\left(s^{t+1},h\right) \\ +\int_0^1 P_{H,t}(i)(C_{H,t}^R(h,i) + I_{H,t}^R(h,i))di + \int_0^1 P_{F,t}(i)(C_{F,t}^R(h,i) + I_{F,t}^R(h,i))di \end{array} \quad (3)$$

The terms on the left hand side represent consumer h's cash inflows, including maturing one-period nominal discount bonds (domestic and foreign), labor income (given by after tax and subsidies wage–$\mathcal{S}_{WR}$ is a subsidy to eliminate monopolistic distortions–times the number of hours worked), income from capital leased to firms net of utilization costs[9], transfers ($Tr_t^R(h)$) net of lump-sum taxes ($TX_t^R(h)$), transfers and profits in the form of net of tax distributed dividends, $(1-\tau_{\mathrm{Pr},t})\mathsf{Pr}_t^R(h)$. Note that S_t is the nominal exchange rate, which measures the number of Chilean pesos (Ch\$) to buy a US dollar (USD). Note also that the utilization rate of physical capital, $u_t^R(h)$, is a choice variable. Following Adolfson *et al.* (2007), the utilization cost function $\Phi(\cdot)$ takes the form:

$$\Phi\left(u_t^R(h)\right) \equiv \frac{\theta}{2}\left(u_t^R(h) - 1 + r^k\right)\left(u_t^R(h) - 1\right) \quad (4)$$

where $\theta > 0$ is a parameter that directly influences the sensitivity of the cost function when $u_t^R(h)$ varies and r^k is the real steady state capital rental rate. Note that capital income simplifies to $R_t^k K_{t-1}^R(h)$ when capital is "fully" utilized ($u_t^R(h) = 1$) because $\Phi(1) = 0$.[10]

The right hand side of (3) includes the various purchases incurred by the Ricardian consumer: consumption, investment, and purchases of (state-contingent) domestic and foreign assets. Note that $\mathbb{RP}_t \equiv \exp\left(-\phi_a\left(\frac{S_t B_{t+1}^*}{P_{t+1}}\right) - \phi_{\Delta S}\left(E_t\left[\frac{S_{t+1}}{S_t}\right] - 1\right) + \phi_t\right)$ is the risk premium function, a factor that adjusts the return at which domestic consumers can borrow or lend to/from the rest of the world. It depends on the country's aggregate net foreign asset position B_t^*, on the expected rate of depreciation $E_t[S_{t+1}/S_t]$, as well as an exogenous risk premium shock ϕ_t.[11] The risk premium function can be viewed as a measure of international asset market incompleteness (asymmetric information, entry costs to build the portfolio, etc.). I_t^R is an investment index given by

$$I_t^R \equiv \left[(1-\alpha)^{\frac{1}{\eta}}\left(I_{H,t}^R\right)^{1-\frac{1}{\eta}} + \alpha^{\frac{1}{\eta}}\left(I_{F,t}^R\right)^{1-\frac{1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \quad (5)$$

[9] In our notation, $K_{t-1}^j(h)$ reflects the agent h's end of period stock of physical capital ready to be used in the productive process in period t.

[10] It follows that $\Phi'(.) = \theta\left[u_t^R(h) - 1\right] + r^k$, which at the steady state $\Phi'(1) = r^k$ and $\Phi''(1) = \theta > 0$.

[11] Note that B_t^* is the sum of the net debt position maintained by Ricardian agents, $(1-\lambda) B_t^{R,*} \equiv \int_\lambda^1 B^{R,*}(s^t,h)dh$, and the government. Besides the usual mechanism stressed by Schmitt-Grohe & Uribe (2001) (i.e., the one that involves deviations from the targeted net foreign position —in this case we assume that it is zero for Chile), we follow Adjemian *et al.* (2008) and Adolfson *et al.* (2009), by adding a second argument which captures the deviation of the expected exchange gross depreciation rate from one. Including the this additional explanatory variable induces a negative correlation between the expected depreciation rate and risk premium, which is a relevant empirical finding (Duarte & Stockman (2005)).

where, in a way analogous to consumption, $I_{H,t}^R \equiv \left(\int_0^1 I_{H,t}^R(j)^{1-\frac{1}{\epsilon_H}} dj\right)^{\frac{\epsilon_H}{\epsilon_H - 1}}$ and $I_{F,t}^R \equiv \left(\int_0^1 I_{F,t}^R(j)^{1-\frac{1}{\epsilon_F}} dj\right)^{\frac{\epsilon_F}{\epsilon_F - 1}}$ represent indexes of domestic and imported investment goods.

A second constraint is given by the law of motion of physical capital:

$$K_t^R(h) = (1-\delta) K_{t-1}^R(h) + \varepsilon_{I,t} I_t^R(h) - \frac{1}{2}\Psi \left(\frac{\varepsilon_{I,t} I_t^R(h)}{K_{t-1}^R(h)} - \delta\right)^2 K_{t-1}^R(h), \tag{6}$$

where δ is the depreciation rate, $\varepsilon_{I,t}$ is an investment-specific technology shock, and $\Psi \geq 0$ is a parameter that scales the quadratic installation costs associated with any positive net investment. The first order conditions (FOC) are presented in the Appendix Section A.1.

3.1.2 Non-Ricardian Consumers

Non-Ricardian consumers ($j = N$) are assumed to have no access to financial markets. Thus, they consume in the same period their wage income and the transfers they receive from the government.[12] Their consumption is thus given by

$$\int_0^1 P_{H,t}(i) C_{H,t}^N(h,i) di + \int_0^1 P_{F,t}(i) C_{F,t}^N(h,i) di = (1-\tau_{w,t}) \mathcal{S}_{WN} W_t^N(h) L_t^N(h) + P_t \left(Tr_t^N(h) - TX_t^N(h)\right) \tag{7}$$

3.1.3 Wage Setting

Wage setting follows closely the formalism in Erceg & Levin (2003), with indexation as in Smets & Wouters (2007). Each consumer is specialized in a differentiated labor service, which is demanded by all firms. The wage elasticity of the demand for each type of labor is constant. Each period, a given consumer can reset optimally the nominal wage for his labor type with probability ϕ_L. Once the new wage is set, the consumer meets fully the demand for its labor type at the quoted wage. Between re-optimization periods we allow the nominal wage to be adjusted mechanically according to the following indexation rule

$$W_t^j(h) = (\Pi_{t-1})^{\xi_L} \left(\bar{\Pi}\right)^{(1-\xi_L)} W_{t-1}^j(h)$$

which makes the rate of change of the individual wage a weighted geometric average of lagged price inflation Π_{t-1} and steady state price inflation $\bar{\Pi}$, with ξ_L representing the weight of the former.

3.2 Firms

There are two types of firms operating in the economy: intermediate goods producers and importers. In addition there are foreign firms, but we do not model their behavior explicitly.

3.2.1 Domestic Producers

We assume a continuum of monopolistically competitive firms, each of which produces a differentiated good. Firm i's production function depends on an exogenous technology, capital and labor:

$$Y_{H,t}(i) = A_{H,t} \left(u_t^R K_{t-1}(i)\right)^\gamma L_t(i)^{1-\gamma} - FC_H, \tag{8}$$

[12] As in Galí *et al.* (2007), we rule out the possibility that non-Ricardian households can smooth consumption through money holdings in contrast with Coenen *et al.* (2008)).

where FC_H is a non-negative fixed cost, measured in terms of output. The labor input bundle $L_t(i)$ is given by the CES function

$$L_t(i) \equiv \left(\lambda^{\frac{1}{\eta_L}} L_t^N(i)^{1-\frac{1}{\eta_L}} + (1-\lambda)^{\frac{1}{\eta_L}} L_t^R(i)^{1-\frac{1}{\eta_L}}\right)^{\frac{\eta_L}{\eta_L-1}}, \tag{9}$$

where η_L is the elasticity of substitution between Ricardian and non-Ricardian labor, and where

$$\begin{aligned} L_t^R(i) &\equiv \left[\left(\frac{1}{1-\lambda}\right)^{\frac{1}{\varepsilon_{LR}}} \int_\lambda^1 L_t^R(i,h)^{1-\frac{1}{\varepsilon_{LR}}} dh\right]^{\frac{\varepsilon_{LR}}{\varepsilon_{LR}-1}}, \\ L_t^N(i) &\equiv \left[\left(\frac{1}{\lambda}\right)^{\frac{1}{\varepsilon_{LN}}} \int_0^\lambda \left(L_t^N(i,h)\right)^{1-\frac{1}{\varepsilon_{LN}}} dh\right]^{\frac{\varepsilon_{LN}}{\varepsilon_{LN}-1}}. \end{aligned}$$

Firms minimize costs subject to (8) and conditional on any given output level. The resulting *real* marginal cost function is (note that we drop the i index since firms have identical costs):

$$MC_{H,t} = \frac{1}{A_{H,t}} \frac{\left(r_t^k\right)^\gamma w_t^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}. \tag{10}$$

Each period, each domestic firm decides how much labor of each type to hire (given the wage $W_t^j(h)$) and how much capital services to rent (given the rental rate R_t^K). In addition, and with probability ϕ_H, any given firm can readjust optimally the price of its good, setting a price $\tilde{P}_{H,t}(i)$. In the absence of reoptimization, the firm's price is adjusted mechanically according to the indexation rule

$$P_{H,t}(i) = (\Pi_{t-1})^{\xi_H} \left(\bar{\Pi}\right)^{(1-\xi_H)} P_{H,t-1}(i)$$

Given its price at any point in time, the firm produces a quantity in order to meet fully the demand for its good.

3.2.2 Importers

There is a continuum of firms which import a good produced overseas at a price $S_t P_{F,t}^*$, "repackage" it and sell it as a differentiated good in the domestic market. Each importer reoptimizes the price of its good with a probability ϕ_F, setting a price $\tilde{P}_{F,t}(i)$, subject to a sequence of demand constraints. In the absence of reoptimization, the price is adjusted according to the indexation rule:

$$P_{F,t}(i) = (\Pi_{t-1})^{\xi_F} \left(\bar{\Pi}\right)^{(1-\xi_F)} P_{F,t-1}(i)$$

Like domestic producers, importers meet the demand for their good at the prevailing price.

3.3 Fiscal Policy

The government purchases goods from both domestic firms and importers. Those purchases are assumed not to have any effect on private utility or productivity. The government allocates its consumption expenditures, given by $\int_0^1 P_{H,t}(i)G_{H,t}(i)di + \int_0^1 P_{F,t}(i)G_{F,t}(i)di$, among the different goods in order to maximize

$$G_t \equiv \left[(1-\alpha_G)^{\frac{1}{\eta}} G_{H,t}^{1-\frac{1}{\eta}} + (\alpha_G)^{\frac{1}{\eta}} G_{F,t}^{1-\frac{1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{11}$$

where $G_{H,t} \equiv \left(\int_0^1 G_{H,t}(i)^{1-\frac{1}{\epsilon_G}} di\right)^{\frac{\epsilon_G}{\epsilon_G-1}}$ and $G_{F,t} \equiv \left(\int_0^1 G_{F,t}(i)^{1-\frac{1}{\epsilon_G}} di\right)^{\frac{\epsilon_G}{\epsilon_G-1}}$. The solution to that problem yields a set of demand functions for each good, which will have to be added to the demand for private consumption and investment purposes. The associated Lagrange multiplier is the 'true' price index $P_{G,t}$:

$$P_{G,t}^{1-\eta} = (1-\alpha_G)P_{H,t}^{1-\eta} + \alpha_G P_{F,t}^{1-\eta}. \tag{12}$$

In addition to purchasing goods, the government taxes consumption, labor income, and profits, it transfers resources to consumers, and issues debt in domestic and foreign goods markets. That activity is summarized in the government budget constraint, which takes the following form:

$$\begin{aligned} P_t Tr_t + g_t P_t Y_t + B_t + S_t B_t^* + (\mathcal{S}_F - 1) P_{F,t} \int_0^1 C_{F,t}(h)dh + (\mathcal{S}_F - 1) P_{F,t} \int_\lambda^1 I_{F,t}^R(h)dh \\ + (\mathcal{S}_F - 1) P_{F,t} G_{F,t} + (\mathcal{S}_{WR} - 1) \int_\lambda^1 W_t^R(h)L_t^R(h)dh + (\mathcal{S}_{WN} - 1) \int_0^\lambda W_t^N(h)L_t^N(h)dh, = \\ \frac{B_{t+1}}{R_t} + \frac{S_t B_{t+1}^*}{R_t^* \mathbb{RP}_t} + \tau_{w,t} \left(\mathcal{S}_{WR} \int_\lambda^1 W_t^R(h)L_t^R(h)dh + \mathcal{S}_{WR} \int_0^\lambda W_t^N(h)L_t^N(h)dh\right) + \tau_{\mathrm{Pr},t} \int_\lambda^1 \mathsf{Pr}_t^R(h)dh \\ + P_t \int_0^1 TX_t(h)dh + P_{cu,t}\kappa X_{cu,t}Y_t + \tau_{cu,t}P_{cu,t}(1-\kappa) X_{cu,t}Y_t + P_{mo,t}X_{mo,t}Y_t \end{aligned} \tag{13}$$

The terms on the left hand side represent different government outlays, including transfers, $Tr_t \equiv \int_0^1 Tr_t(h)dh = \int_\lambda^1 Tr_t^R(h)dh + \int_0^\lambda Tr_t^N(h)dh$, government consumption $P_{G,t}G_t \equiv g_t P_t Y_t$ (where $g_t \equiv \frac{P_{G,t}G_t}{P_t Y_t}$ is the share of government consumption in GDP), repayment of maturing government bonds (both domestic, B_t, and foreign, $S_t B_t^*$), and subsidies on foreign goods expenditures and employment Those outlays are funded through the issuing of new debt (domestic, $\frac{B_{t+1}}{R_t}$, and foreign$\frac{S_t B_{t+1}^*}{R_t^* \mathbb{RP}_t}$), labor income taxes, taxes on profits, lump-sum taxes, and copper-related revenues. The latter are explained briefly next.

Copper production is assumed to be stochastic and exogenous. Consistent with the market structure of copper production in Chile, the state-own company accounts for a share κ of production (all of which accrues to the government as revenue). The remaining share corresponds to foreign companies which are taxed at a rate $\tau_{cu,t}$. We assume that the world copper prices, $P_{cu,t}^*$, are exogenously given, implying a domestic copper price $P_{cu,t} = S_t P_{cu,t}^*$. The share of copper production to GDP, $X_{cu,t}$, follows an exogenous process, described below. In addition, $X_{mo,t}$ represents the output of molybdenum (a byproduct of copper production) as a share of GDP. The molybdenum world price is exogenous and given by $P_{mo,t}^*$. All revenues from molybdenum production accrue to the government.

Following Forni *et al.* (2007), tax rates on wages, benefits and on copper production are allowed to vary according to:

$$\tau_{w,t} = \left(1-\rho_{\tau_w}\right)\tau_w + \rho_{\tau_w}\tau_{w,t-1} + \varepsilon_{\tau_w,t}, \tag{14}$$
$$\tau_{\mathrm{Pr},t} = \left(1-\rho_{\tau_{\mathrm{Pr}}}\right)\tau_{\mathrm{Pr}} + \rho_{\tau_{\mathrm{Pr}}}\tau_{\mathrm{Pr},t-1} + \varepsilon_{\tau_{\mathrm{Pr}},t}, \tag{15}$$
$$\tau_{cu,t} = \left(1-\rho_{\tau_{cu}}\right)\tau_{cu} + \rho_{\tau_{cu}}\tau_{cu,t-1} + \varepsilon_{\tau_{cu},t}, \tag{16}$$

where τ_w, τ_{Pr} and τ_{cu} are long run tax rates, ρ_{τ_w}, $\rho_{\tau_{\mathrm{Pr}}}$, and $\rho_{\tau_{cu}}$ explain the degree of persistency, $\varepsilon_{\tau_w,t}$, $\varepsilon_{\tau_{\mathrm{Pr}},t}$ and $\varepsilon_{\tau_{cu},t}$ are iid shocks with zero means and constant variances.

Fiscal policy in Chile is conducted using a structural balance approach, called the Chilean structural balance fiscal rule.[13] As discussed in the introduction, the Chilean fiscal rule ties government spending to structural/permanent government revenues. Such a rule has been followed explicitly by the Chilean government since 2001 and implicitly since the beginnings of the nineties.[14] We formalize that rule by assuming that total government spending (including interest payments) plus a time varying "surplus target" (*surplus*) must be equal to structural revenues. Structural revenues correspond to the revenues that the government would collect if (i) the price of copper and molybdenum were equal to their long run or "reference" values (denoted by $P_{cu,t}^{ref}$ and $P_{mo,t}^{ref}$ respectively) and (ii) the economy were producing at its steady state level (potential output). The "surplus target"–the difference between government spending and structural revenues– is set by the fiscal authorities. When the fiscal rule was introduced in 2001 the structural surplus target was set at 1% of GDP. The idea was to acknowledge that public debt was at a level higher than what was considered appropriate for a small open economy that faced exogenous credit constraint shocks and given potential future pension liabilities. It is worth noting that even though fiscal policy was not conducted using an explicit rule in the nineties, the "shadow" structural surplus averaged 1% of GDP in that decade. Again, behind this fiscal policy was the goal of reducing government debt to some "long run" (sustainable) level. Motivated by the observed practice, we assume that the structural surplus (*surplus*) is a function of the difference between current government debt and a long term target for government debt ($\overline{B} = B + SB^*$):

$$surplus_t = F\left(\overline{B}_t - \overline{B}\right) + s_t \tag{17}$$

where $F' > 0$. If government debt is higher than its long run target, the structural surplus is positive which reduces government spending given structural revenues. Additionally, we assume that the surplus target depends on an exogenous shock s_t that follows and autoregressive process of order one. In particular, we assume that:

$$s_t = \rho_s s_{t-1} + \varepsilon_{s,t}, \tag{18}$$

where $\varepsilon_{s,t}$ follows an i.i.d. process with mean zero and constant variance $\sigma^2_{\varepsilon_{s,s}}$.

In practice, we assume that $\overline{B} = 0$ (Chile exhibited by the end of the last decade a net creditor position of around 3% of GDP). This formulation allow us to have a well specified fiscal rule (government debt is stationary) while capturing the most relevant aspects of the Chilean fiscal rule. A negative surplus shock (reduction in s) makes room for a rise in total government spending,

[13] Previous papers that have analyzed the effects of the Chilean fiscal rule in DSGE models are Garcia & Restrepo (2007), Medina & Soto (2007) and Kumhof & Laxton (2009)

[14] By "implicitly" we mean that even though there was no explicitly commitment to any fiscal policy rule in that period, fiscal policy outcomes in the nineties resemble the ones that could have been obtained by the implementation of the Chilean fiscal rule of the 2000.

which can be allocated to transfers or consumption. One can show that under this formulation the dynamics of debt are described by:

$$
\begin{aligned}
\overline{B}_{t+1}-\overline{B}_t &= \left(P^{ref}_{cu,t}-P_{cu,t}\right)\kappa X_{cu,t}+\tau_{cu,t}\left(P^{ref}_{cu,t}-P_{cu,t}\right)(1-\kappa)X_{cu,t}+\left(P^{ref}_{mo,t}-P_{mo,t}\right)X_{m,t}\\
&+\tau_{w,t}\left[\mathcal{S}_{WR}\int_\lambda^1 W^R(h)L^R(h)dh+\mathcal{S}_{WR}\int_0^\lambda W^N(h)L^N(h)dh\right]\\
&-\tau_{w,t}\left[\mathcal{S}_{WR}\int_\lambda^1 W^R_t(h)L^R_t(h)dh+\mathcal{S}_{WR}\int_0^\lambda W^N_t(h)L^N_t(h)dh\right]\\
&+\tau_{\Pr,t}\left\{\int_\lambda^1 \mathsf{Pr}^R(h)dh-\int_\lambda^1 \mathsf{Pr}^R_t(h)dh\right\}-surplus_t
\end{aligned}
$$

Clearly, if the current price of copper is above its long term value, we have a fiscal surplus (a reduction in government debt). The same is true for the other determinants of government revenues.

From this particular specification of the Chilean fiscal rule we can derive a more traditional fiscal policy representation for the Bayesian estimation of the structural model, along the lines of our empirical strategy. We assume a specification for government consumption and transfers consistent with the representation of the Chilean fiscal rule just described. In particular, we represent government consumption by the next process

$$g_t=(1-\rho_G)\,g+\rho_G g_{t-1}+\varepsilon_{G,t}, \tag{19}$$

where ρ_G measures the persistence of the process, g is the long run government share, $\frac{P_G G}{PY}$, and $\varepsilon_{G,t}$ is an exogenous a shock with mean zero and constant variance $\sigma^2_{\varepsilon_G}$. Under this specification, shocks to government consumption imply an increase in government debt in the current period and an adjustment in the structural surplus target (*surplus*) from next period. Given our specification, the adjustment in the surplus target translates into an adjustment in government transfers. Consistently, shocks to the surplus target (s) are translated into one-to-one movements in transfers. In particular, a negative shock to the surplus target, increases govenrment transfers. The evolution of transfers mimics the evolution of the surplus target (*surplus*) determined by equations (17) and (18).

3.4 Monetary Policy

We assume that the Central Bank (CB) sets the (gross) nominal interest rate, $R_{rule,t}$ according to a variant of the Taylor rule with partial adjustment, given by

$$R_t = R^{\psi_R}_{t-1}R^{1-\psi_R}_{rule,t}\exp(\varepsilon_{m,t}), \tag{20}$$

$$R_{rule,t} = \left(\frac{\Pi_{A,t}}{\bar{\Pi}_A}\right)^{\psi_\pi}\left(\frac{Y_{r,t}}{\bar{Y}_r}\right)^{\psi_y}, \tag{21}$$

where ψ_R determines the degree of smoothing, and $\varepsilon_{m,t}$ is an exogenous i.i.d. monetary policy shock. The target values are steady state GDP without the copper sector, $\bar{Y}_r$, and inflation, $\bar{\Pi}_A$, assumed to be 1 for simplicity.[15] According with the Taylor principle, the reaction parameter

[15] This is without loss of generality, since during the 2000s the inflation rate in Chile fluctuated quite closely around the 3% inflation target. In the empirical implementation we substract this target.

to annualized inflation deviations ψ_π should be larger than one, where $\Pi_{A,t} \equiv \Pi_t^4$, while ψ_y for quarterly data should be around 0.5/4.

We have also studied an extension of the rule above that allows for a systematic interest rate response to nominal exchange rate variations. That extension could be useful to accommodate the policy regime from 1986:1 to 2001:2, as documented by Medina & Soto (2007). In the analysis that follows we ignore this term since in this paper we focus on the sample period 2001:3-2010:1.

4 Equilibrium and Aggregation

We first state clearing conditions in the markets for domestic inputs. Thus, for labor services of household h the market clearing condition is given by

$$L_t(h) = \int_0^1 L_t(h,i)di,$$

where $L_t(h,i)$ is firm i's demand for labor services from household h. A similar condition must hold for all $h \in [0,1]$.

Given that only Ricardian households engage in capital accumulation, the market clearing condition in the market for that input is given by

$$K_t = (1-\lambda) K_t^R$$

where $(1-\lambda) K_t^R = \int_\lambda^1 K_t^R(h)dh$. Similarly, for other asset holdings we have

$$\begin{aligned} B_t &= (1-\lambda) B_t^R \\ B_t^* &= (1-\lambda) B_t^{R,*} - B_t^{G,*} \end{aligned}$$

(notice that $B_t^{G,*}$ is the amount of liabilities so with the negative sign converts to net holdings). In the same manner, aggregate real variables such as consumption and investment are:

$$\begin{aligned} C_t &= \lambda C_t^N + (1-\lambda) C_t^R, \\ I_t &= (1-\lambda) I_t^R, \end{aligned}$$

where C_t^R and C_t^N come from aggregators similar to (2) and $(1-\lambda) I_t^R = \int_\lambda^1 I_t^R(h)dh$.

Market clearing in the home produced goods implies that supply given by the aggregated version of equation (8) equals demand:

$$Y_{H,t} = \Delta_{H,t} \left[T_{H,t}^{-\eta} (1-\alpha)(C_t + I_t) + T_{GH,t}^{-\eta} (1-\alpha_G) G_t \right] + (\alpha_C^* + \alpha_I^*) \left(\frac{T_{H,t}}{RER_t} \right)^{-\eta} Y_t^*. \quad (22)$$

After some little algebra we can derive the following expression for aggregate output, Y_t, and aggregate output without copper, $Y_{r,t}$:[16]

$$Y_t = \frac{(C_t + I_t)\left[1 - \Delta_{F,t}\alpha (T_t T_{H,t})^{1-\eta}\right] + \Phi\left(u_t^R\right) K_{t-1}}{1 - RER_t\left(p_{cu,t}^* X_{cu,t}^{share} + p_{mo,t}^* X_{mo,t}^{share}\right) - \left[1 - \Delta_{F,t}\alpha_G (T_t T_{GH,t})^{1-\eta}\right] g_t}, \quad (23)$$

$$Y_{r,t} = \frac{(C_t + I_t)\left[1 - \Delta_{F,t}\alpha (T_t T_{H,t})^{1-\eta}\right] + \Phi\left(u_t^R\right) K_{t-1}}{1 - \left[1 - \Delta_{F,t}\alpha_G (T_t T_{GH,t})^{1-\eta}\right] g_t}. \quad (24)$$

[16] For details see the derivation in Section A.2.

Notice that the central bank targets $Y_{r,t}$ instead of Y_t. From equation (23) we can isolate the consumption and investment levels as follows:

$$C_t+I_t = \frac{Y_t\left\{1 - RER_t\left(p^*_{cu,t}X^{share}_{cu,t} + p^*_{mo,t}X^{share}_{mo,t}\right) - \left[1-\Delta_{F,t}\alpha_G\left(T_tT_{GH,t}\right)^{1-\eta}\right]g_t\right\} - \Phi\left(u^R_t\right)K_{t-1}}{\left(1-\Delta_{F,t}\alpha\left(T_tT_{H,t}\right)^{1-\eta}\right)}. \tag{25}$$

The evolution of net foreig assets under incomplete international asset markets is:[17]

$$\frac{S_{t-1}B^*_t}{P_{t-1}}\frac{S_t}{S_{t-1}}\frac{1}{\Pi_t} + NX_t = \frac{1}{R^*_t\mathbb{RP}_t\left(\cdot,\cdot,\cdot\right)}\frac{S_tB^*_{t+1}}{P_t}, \tag{26}$$

where we employed the following net exports definition:

$$\begin{aligned} NX_t \equiv\ & RER_t\left(p^*_{cu,t}\kappa X^{share}_{cu,t} + p^*_{mo,t}X^{share}_{mo,t}\right)Y_t \\ & +\Delta_{H,t}\left[\frac{T^{1-\eta}_{H,t}}{MC_{H,t}}\left(1-\alpha\right)\left(C_t+I_t\right) + \frac{T^{1-\eta}_{GH,t}}{MC_{H,t}}\left(1-\alpha_G\right)g_tY_t\right] \\ & +\frac{T_{H,t}}{MC_{H,t}}\left(\alpha^*_C+\alpha^*_I\right)\frac{T^{-\eta}_{H,t}}{RER^{-\eta}_t}Y^*_t - T_HFC_H - \Delta_{H,t}\left(C_t+I_t\right) - g_tY_t - \Phi\left(u^R_t\right)K_{t-1}, \end{aligned} \tag{27}$$

where we take into account that $C_t + I_t$, come from Eq. (25).

The model has seventeen exogenous driving forces are collected in the following vector:

$$\mathbf{v}_t = \left(v_{m,t}, \zeta_t, RER_{F,t}, \Pi^*_t, Y^*_t, A_{H,t}, x^{share}_{cu,t}, x^{share}_{mo,t}, R^*_t, \phi_{,t}, \varepsilon_{I,t}, \tau_{w,t}, \tau_{\mathrm{Pr},t}, \tau_{cu,t}, p^*_{cu,t}, p^*_{mo,t}\right)$$

which is assumed to follow the process

$$\underset{(17\times1)}{\mathbf{v}_t} = \underset{(17\times17)}{\boldsymbol{\rho}}\underset{(17\times1)}{\mathbf{v}_{t-1}} + \underset{(17\times1)}{\boldsymbol{\varepsilon}_t},$$

where $\boldsymbol{\rho}$ is a diagonal matrix containing the corresponding autoregressive coefficients, and $\{\boldsymbol{\varepsilon}_t\}$ is the vector of exogenous serially uncorrelated shocks with zero mean and diagonal variance-covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$.

5 Calibration and Estimation

We estimate the model above using Bayesian methods. First, we define the measurement equation which links the observed variables with the model's solution or law of motion.[18] Then, the Kalman filter is employed to evaluate the posterior density (which is proportional to the product of the likelihood and the assumed prior densities).[19]

To be consistent with the assumptions involving technology in the model, we get rid of the trend of non-stationary variables by filtering the data with a (deterministic) quadratic trend (in accordance with our VAR estimation). Moreover, we lower the observed inflation rate by the

[17] For further details on the derivation refer to the appendix Section A.3.
[18] Calculations are performed with the set of routines included in DYNARE, Juillard (2005)
[19] For details on these aspects see Fornero (2010).

target, 3 percent. Similarly, for the interest rate we subtract a neutral interest rate of 5 percent (the inflation target plus an assumed steady state real rate of 2 percent).

We restrict estimation to the sample period 2001Q3-2010Q1, a period characterized by a well defined monetary policy based on an inflation target and a flexible exchange rate.

We calibrate a subset of parameters. These are, $\beta = 0.9878$ which is consistent with a neutral annual interest rate of 5 %. Import shares $\alpha = \alpha_G = 0.3$ approximate the import/GDP ratio. The settings $\alpha_C^* = \alpha_I^* = 0.0004$ are consistent with the share of Chilean GDP to world GDP (0.35%). The elasticities of substitution among varieties of intermediate and final imported goods are $\varepsilon_H = \varepsilon_F = 11$, consistent with markups $\mu_H = \mu_F = \mathcal{S}_F = 1.1$. Further, the elasticities of substitution among varieties of labor types are $\varepsilon_{LR} = \varepsilon_{LN} = 9$ which imply markups $\mu_{WR} = \mathcal{S}_{WR} = \mu_{WN} = \mathcal{S}_{WN} = 1.125$. In addition, $\bar{\zeta} = 7.5$ as in Adolfson *et al.* (2007), the annual depreciation rate is assumed to be 10% ($\delta = 0.025$), and some steady state ratios and relative prices are $X_{cu}^{share} = 0.044$, $X_{mo}^{share} = 0.01$, $g = 0.094$, $A_H = 1$, $\tau_w = 0.2$, $\tau_{\mathrm{Pr}} = 0.17$ and $T = T_H = T_{GH} = 1$. We also left calibrated the Calvo price and wage probabilities because of lack of identification under usual priors. Furthermore, the habit formation parameter affects the steady state due to the assumption of internal habit formation; therefore we calibrate it to 0.8. For exogenous processes of copper and molybdenum shares which are not identified, $\rho_{x_{cu}}$ and $\rho_{x_{mo}}$, we assume an autoregressive coefficient of 0.1.[20] Last but not least, the elasticity η is calibrated to 2.

The crucial parameter λ is left calibrated to 0.50 due to lack of identification. Data from the Household Financial Survey (EFH) implemented by the Central Bank of Chile in 2007 suggest a λ value of 0.29. This value is computed by adding the fraction of households that requested a financial credit and were rejected one or more times, the fraction that did not apply to any financial credit because they expected to be rejected and the fraction that considered themselves unable to afford the credit payments. All things considered, we calibrate λ to a conservative 0.5 since the data from the EFH corresponds to a period in which credit expanded rapidly towards first time credit holders.[21]

The prior densities are quite standard. We choose a gamma density for the friction parameter of investment Ψ with prior mean 50 and SD equal to 20. The prior mean for the elasticity of the $\mathbb{RP}$ respect to the asset position is 0.04 with prior SD of one tenth of the mean with Beta distribution. Similar density type is chosen for persistence parameters)such as $\psi_R and the \rho$'s) with mean 0.5 and variance 0.2. The priors for Taylor rule parameters are quite standard, see Smets & Wouters (2003). For variances of standard errors and measurement errors we assume inverted gamma distributions with 20 and 1 degrees of freedom, depending on whether the errors refer variables or on shares (which vary less), respectively.

The set of observed variables includes 11 time series which are gathered in the vector oZ_t=($oY_{r,t}$, oY_t^*, oC_t, oI_t, $o\Pi_t$, $o\Pi_t^*$, oR_t, oR_t^*, ow_t, $oRER_t$, og_t)'. Since the current model version does not have a balance growth path, the data has been filtered up employing a linear quadratic trend or if the resulting detrended time series is not stationary we applied the Hoddrick Prescott filter, then we scale variables with the SS values. In addition, we allow for measurement errors which are included in the vector meZ_t =(me$Y_{r,t}$, meY_t^*, meC_t, meI_t, meΠ_t, meΠ_t^*, meR_t, meR_t^*, mew_t, meRER_t, meg_t)'. In the case of interest rates and inflation, which are not filtered, we substract the

[20] We tried also a VAR(1) for foreign variables as it is usually done in the literature; however, off diagonal elements of the persistency matrix turned out to be not statiscatically different from zero. Thus, we specify AR(1) processes for R^*, Π^* and Y^*.

[21] Defining credit-constrained households as those who cannot access to low-cost credit and hence end up using high-cost credit (credit cards), Ruiz-Tagle (2009) finds that at least 41% of Chilean households were credit constrained in 2004.

neutral interest rates and inflation targets (foreign inflation in demeaned). Measurement errors are assumed to be i.i.d.

Table 5. Estimation results Chilean fiscal rule

Parameters	Prior density	Prior mean	Prior SD	Post. mean	0.05	0.95
Ψ	Γ	50	20	64.3307	37.3497	91.4607
ϕ_a	β	0.04	0.004	0.0393	0.0326	0.0465
θ	N	1	0.25	0.9359	0.5269	1.4169
ψ_R	β	0.5	0.15	0.8441	0.6771	0.9445
ψ_π	N	1.5	0.15	1.249	0.9751	1.5452
ψ_{yr}	β	0.125	0.05	0.1729	0.067	0.2745
ρ_ζ	β	0.5	0.2	0.7033	0.338	0.9501
ρ_{RER_F}	β	0.5	0.2	0.9338	0.8781	0.974
ρ_{ϕ_a}	β	0.5	0.2	0.5098	0.1845	0.8135
ρ_{π^*}	β	0.5	0.2	0.4853	0.3284	0.636
ρ_{y^*}	β	0.5	0.2	0.4913	0.1717	0.8071
ρ_{A_H}	β	0.5	0.2	0.7555	0.4927	0.9325
ρ_G	β	0.5	0.2	0.7138	0.5341	0.8921
ρ_{R^*}	β	0.5	0.2	0.4861	0.2121	0.7808
ρ_{ε_I}	β	0.5	0.2	0.5875	0.2482	0.8941
$\rho_{v_{tr}}$	β	0.5	0.2	0.5565	0.2293	0.8551

SD of shocks	Prior density	Prior mean	g.l.	Post. mean	0.05	0.95
v_m	Γ^{-1}	0.01	20	0.0038	0.002	0.0052
ε_ζ	Γ^{-1}	0.01	20	0.0424	0.0029	0.0689
ε_{RER_F}	Γ^{-1}	0.01	20	0.0032	0.002	0.0043
ε_{π^*}	Γ^{-1}	0.037	20	0.014	0.0111	0.0169
ε_{A_H}	Γ^{-1}	0.01	20	0.0054	0.0036	0.0074
ε_{ϕ_a}	Γ^{-1}	0.01	20	0.0044	0.0023	0.0064
ε_I	Γ^{-1}	0.01	20	0.0122	0.0027	0.026
ε_G	Γ^{-1}	0.001	1	0.0038	0.0026	0.0052
ε_{tr}	Γ^{-1}	0.01	20	0.0061	0.0025	0.0098
ε_s	Γ^{-1}	0.01	1	0.0085	0.0026	0.015

SD meas. errors	Prior density	Prior mean	g.l.	Post. mean	0.05	0.95
me_{YR}	Γ^{-1}	0.001	1	0.001	0.0003	0.0016
me_C	Γ^{-1}	0.001	1	0.0007	0.0003	0.0011
me_I	Γ^{-1}	0.001	1	0.0711	0.0558	0.0847
me_π	Γ^{-1}	0.001	1	0.0037	0.0002	0.0193
me_R	Γ^{-1}	0.001	1	0.0006	0.0002	0.0009
me_W	Γ^{-1}	0.001	1	0.0256	0.0182	0.033
me_{RER}	Γ^{-1}	0.001	1	0.0468	0.0352	0.0592
me_{Y^*}	Γ^{-1}	0.001	1	0.0007	0.0003	0.0012
me_{π^*}	Γ^{-1}	0.001	1	0.0006	0.0002	0.0011
me_{R^*}	Γ^{-1}	0.001	1	0.0007	0.0003	0.0012
me_g	Γ^{-1}	0.001	1	0.0021	0.0009	0.0037

6 Simulations

In this section we present impulse response functions (IRF) to various shocks under the structural balance fiscal rule introduced above. The analysis focuses on the implied size of the consumption and output fiscal multipliers. In Figure 5 we present the dynamic response of the economy for a government spending (consumption) shock, ε_G, equal to 1% of GDP. Note that the impact on output and consumption is positive. Government expenditure increases following (19). Since transfers only respond gradually to offset the increase in spending, through changes in the surplus target, the shock is more expansionary and stimulates consumption and output. This is a critical difference to the case in which the government follows a balance budget rule. Under this formulation, the transfers will have to adjust to fully offset the increase in government consumption. This impulse response is consistent with the VAR evidence reported in a previous section.

Figure 6 displays the IRFs to a positive shock to the total factor productivity. As a result of that shock marginal costs decrease, nominal wages tend to increase but since they are sticky cannot react immediately; however, real wages go up due to deflationary pressures caused by the shock. Also there would be an appreciation of the real exchange rate that would mitigate the expansion of exports. Consumption of Ricardian agents reacts positively, whereas for non-Ricardian agents consumption remains negative during 2 quarters. The higher consumption of Ricardian agents under the Chilean fiscal rule can be associated to the fact that under this specification of fiscal policy, agents understand that the government is going to save, so they consume more.

Figure 5: A positive shock to g_t of size 1%.

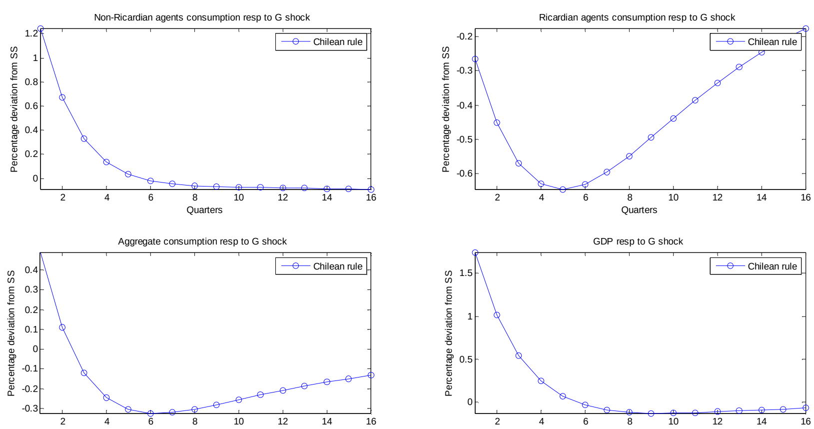


Figure 6: A positive productivity shock of 1%

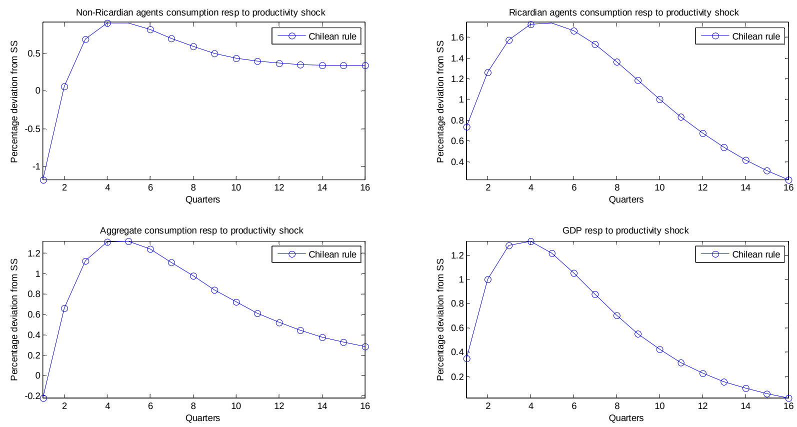


Figure 7 illustrates a shock in the copper-to-GDP share of 1 percentage point. The multiplier of GDP is positive. Consumption of Ricardian agents increases. A fraction of this increase is explained by the fact that under the Chilean fiscal rule the government is saving the temporary increase in revenues, which is compatible with larger consumption levels for Ricardian agents. The response of non-Ricardian agents' consumption is interesting to analyze. Under a balanced budget rule, all the temporary increase in revenues would be transferred to the public, leading to a large increase in consumption of non-Ricardian households in the short run (as opposed to Ricardian agents, who smooth consumption and hence save much of the transfer). By contrast, the Chilean rule would fix the expenditure to a constant, thereby government savings would increase.

Figure 7: A positive shock to the copper-to-GDP share of 1%

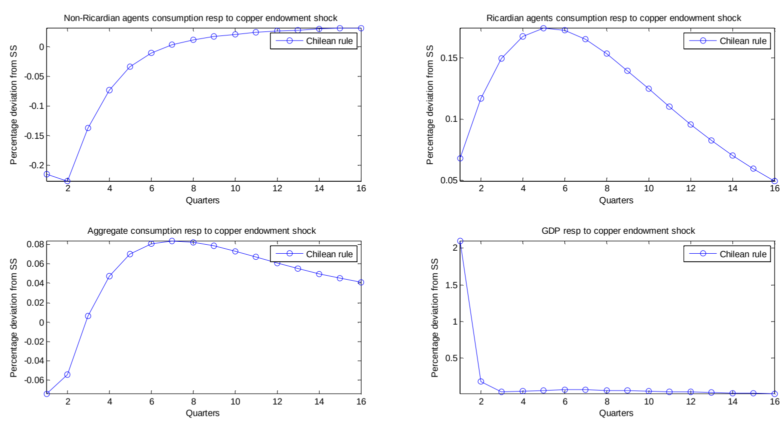


Figure 8 considers a shock to transfers of 1 percent. Note that the estimated persistence of the AR(1) process for the transfers process is 0.56. Ricardian consumers save the temporary increase in transfers, whereas non-Ricardian agents consume all. The positive response of consumption by non-Ricardian agents leads to an aggregate consumption multiplier that is positive for about one year. GDP increases as well, the response path suggests a larger multiplier than consumption.

Figure 8: A positive transfers shock of 1%.

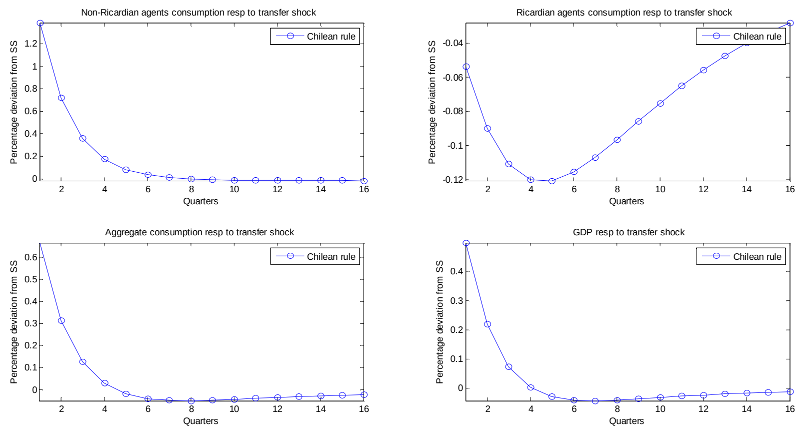
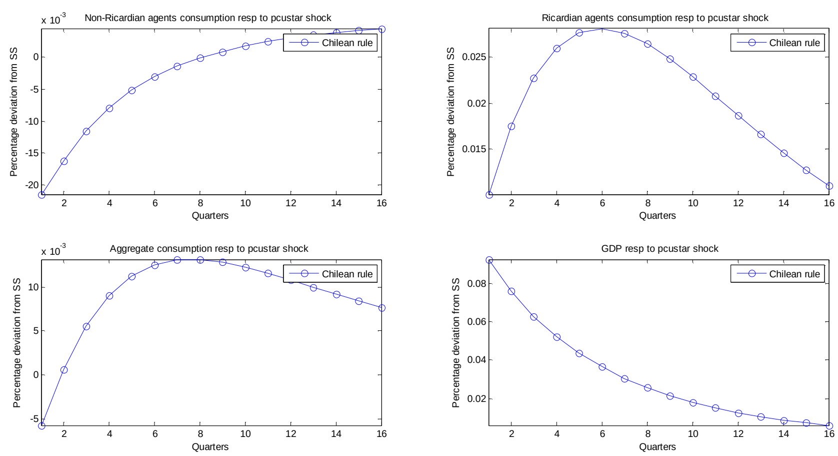


Figure 9 reports a positive shock of 1 percent in the price of copper relative to the foreign price index. The results are qualitatively similar to those observed in Figure 5. The GDP multiplier is positive as well as Ricardian consumption. Non-Ricardian consumption decreases under the Chilean rule. The reason being that the government saves saves for a while by buying public debt.[22]

Figure 9: A positive shock to the copper price of 1%.

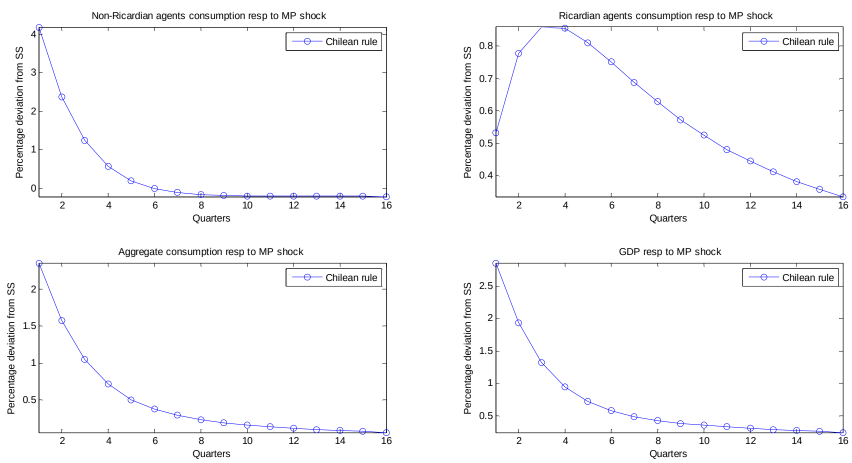


In Figure 10 we report responses to an expansive monetary policy under estimated parameters. The drop of interest rates cause a hump-shaped consumption pattern for Ricardian agents, while for non-Ricardians responses are monotonic. Overall aggregate consumption and GDP expand as it would be expected in any New Keynesian model like ours. Non-Ricardian consumption expands due to increases in wages and tax revenues (that are distributed through transfers, which turn out to be mitigated by the Chilean fiscal rule). Of course, the drop in interest rates makes it less

[22] The GDP multiplier remains also positive in the case of a zero-deficit rule, results not shown. Non-Ricardian consumption increases under a zero deficit rule because the government distributes higher transfers.

attractive to invest in domestic fixed income assets in comparison with foreign assets, leading to a depreciation of the domestic currency.

Figure 10: An expansive monetary policy: shock to interest rate instrument of 1%



7 Conclusions

This paper presents VAR evidence of fiscal multipliers that are large and robust for Chile. The evidence we present indicates that aggregate real consumption and real GDP expand significantly when transfers and /or government expenditure go up. Results from small VARs (four variables) suggest that basic multipliers of consumption peak at the second quarter with values larger than one, while the peak is slightly delayed and higher in magnitude when considering output multipliers. Accumulated multipliers grow steadily and peak between 4 and 6 quarters and then the expansionary effect come to halt and start to fall at a lower level. Values range from 2.4 to 3.5 for consumption and 3.2 to 3.5 for output. Large VARs take explicitly into account the fact that Chile is a small open economy in the specification by including three additional variables (copper price as exogenous, total private investment and the RER) produce consumption and output responses that are more expansive to government purchases shocks. The large VAR with transfers shocks exhibits fiscal multipliers similar to the ones obtained from the small VAR.

We confront this evidence with the prediction of a DSGE model for the Chilean economy. The model features two household types: Ricardian and non-Ricardian. The former solve a typical dynamic programming problem, whereas non-Ricardian households consume labor income and transfers within the period. We assume a standard specification for monetary policy but allow for two fiscal policies rules: one that imposes a balanced budget, while the other approximates the Chilean fiscal policy rule characterized by expenditure responding to structural revenues.

The results indicate that when a balanced budget rule is instrumented by transfers (leaving public expenditure exogenous) a public transfer shock yields positive fiscal multipliers of consumption and output. On the other hand, if government purchases are shocked instead, the balanced budget rule causes a negative fiscal multiplier for consumption but a positive one for GDP. Interestingly, the implementation of a fiscal policy rule that approximates the Chilean fiscal rule in the model leads to the finding that both the consumption and output fiscal multipliers are positive in the short run.

References

Adjemian, S., Darracq-Paries, M., & Smets, F. 2008. *A quantitative perspective on optimal monetary policy cooperation between the US and the Euro Area*. Working Paper Series 884. European Central Bank.

Adolfson, M., Laseén, S., Lindé, J., & Villani, M. 2007. Bayesian estimation of an open economy DSGE model with incomplete pass-through. *Journal of International Economics*, **72**(2), 481–511.

Adolfson, M., Laseén, S., Lindé, J., & Svensson, L. 2009 (Aug). *Monetary Policy Trade-Offs in an Estimated Open-Economy DSGE Model*. Working Paper Series 232. Sveriges Riksbank (Central Bank of Sweden).

Altig, D., Christiano, L., Eichenbaum, M., & Linde, J. 2005 (January). *Firm-Specific Capital, Nominal Rigidities and the Business Cycle*. Working Paper 11034. National Bureau of Economic Research.

Blanchard, O., & Perotti, R. 2002. An Empirical Characterization Of The Dynamic Effects Of Changes In Government Spending And Taxes On Output. *The Quarterly Journal of Economics*, **117**(4), 1329–1368.

Campbell, J., & Mankiw, N. G. 1991. *The Response of Consumption to Income*. NBER Reprints 1645. National Bureau of Economic Research, Inc.

Coenen, G., McAdam, P., & Straub, R. 2008. Tax reform and labour-market performance in the euro area: A simulation-based analysis using the New Area-Wide Model. *Journal of Economic Dynamics and Control*, **32**(8), 2543–2583.

Duarte, M., & Stockman, A. 2005. Rational speculation and exchange rates. *Journal of Monetary Economics*, **52**(1), 3–29.

Erceg, C., & Levin, A. 2003. Imperfect credibility and inflation persistence. *Journal of Monetary Economics*, **50**(4), 915–944.

Fatas, A., & Mihov, I. 2001 (Apr). *The Effects of Fiscal Policy on Consumption and Employment: Theory and Evidence*. CEPR Discussion Papers 2760. C.E.P.R. Discussion Papers.

Fornero, J. 2010 (March). *Ricardian Equivalence Proposition in a NK DSGE Model for two Large Economies: The EU and the US*. Working Papers Central Bank of Chile 563. Central Bank of Chile.

Forni, Lorenzo, Monteforte, Libero, & Sessa, Luca. 2007 (November). *The general equilibrium effects of fiscal policy: estimates for the euro area*. Temi di discussione (Economic working papers) 652. Bank of Italy, Economic Research Department.

Frankel, J. 2011. *A Solution to Fiscal Procyclicality: the Structural Budget Institutions Pioneered by Chile*. Working Papers 604. Central Bank of Chile.

Galí, J., López-Salido, J.D., & Vallés, J. 2007. Understanding the Effects of Government Spending on Consumption. *Journal of the European Economic Association*, **5**(1), 227–270.

Garcia, C., & Restrepo, J. 2007 (Jul). *The Case for a Countercyclical Rule-based Fiscal Regime.* ILADES-Georgetown University Working Papers inv183. Ilades-Georgetown University, School of Economics and Bussines.

Hall, R.E. 2009. *By how much does GDP rise if the Government buys more output?* NBER Working Papers 15496. National Bureau of Economic Research, Inc.

Ilzetzki, E. O., Mendoza, E., & Vegh, C. 2009. *How big (small?) are fiscal multipliers?* Tech. rept. mimeo.

Juillard, M. 2005. *DYNARE Manual.* Tech. rept. CEPREMAP. Mimeo.

Kumhof, M., & Laxton, D. 2009 (April). *Simple, Implementable Fiscal Policy Rules.* IMF Working Papers 09/76. International Monetary Fund.

Mankiw, N. Gregory. 2000. The Savers-Spenders Theory of Fiscal Policy. *American Economic Review*, **90**(2), 120–125.

Medina, J.P., & Soto, C. 2007 (December). *Copper Price, Fiscal Policy and Business Cycle in Chile.* Working Papers Central Bank of Chile 458. Central Bank of Chile.

Perotti, R. 2008. In Search of the Transmission Mechanism of Fiscal Policy. *Pages 169–226 of: NBER Macroeconomics Annual 2007, Volume 22.* National Bureau of Economic Research, Inc.

Ruiz-Tagle, J. 2009. *Households credit constraints and the cost of credit.* Tech. rept. Mimeo.

Schmitt-Grohe, S., & Uribe, M. 2001. Stabilization Policy and the Costs of Dollarization. *Journal of Money, Credit and Banking*, **33**(2), 482–509.

Smets, F., & Wouters, R. 2003. An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, **1**(5), 1123–1175.

Smets, F., & Wouters, R. 2007. Shocks and Frictions in the US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, **97**(3), 586–606.

Woodford, M. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press.

Appendix to "Non-Ricardian Aspects of Fiscal Policy in Chile" by Céspedes, Fornero and Gali (2011)

A Derivation of particular equations of the model

A.1 First order conditions: the Ricardian consumer

The Lagrangian summarizes the (constrained) intertemporal problem that the Ricardian consumer faces:

$$E_t\left[\begin{array}{c}\sum_{a=0}^{\infty}\beta^{t+a}\left[\ln\left(C^R_{t+a}(h)-bC^R_{t+a-1}(h)\right)-\bar{\zeta}\zeta_{t+a}\frac{L^R_{t+a}(h)^{1+\sigma_L}}{1+\sigma_L}\right]\\ +\sum_{a=0}^{\infty}\beta^{t+a}\tilde{\Lambda}^R_{t+a}\left[\begin{array}{c}B^R\left(s^{t+a},h\right)+S_{t+a}B^{R,*}(s^{t+a},h)+\left(1-\tau_{w,t+a}\right)\mathcal{S}_{WR}W^R_{t+a}(h)L^R_{t+a}(h)\\ +P_{t+a}\left[Tr^R_{t+a}(h)-TX^R_{t+a}(h)\right]-\sum_{s^{t+a+1}|s^{t+a}}Q\left(s^{t+a+1},s^{t+a}\right)B^R\left(s^{t+a+1},h\right)\\ -S_{t+a}\sum_{s^{t+a+1}|s^{t+a}}Q^*\left(s^{t+a+1},s^{t+a}\right)B^{R,*}\left(s^{t+a+1},h\right)\\ +\left(1-\tau_{\mathrm{Pr},t+a}\right)\mathsf{Pr}^R_{t+a}(h)+R^k_{t+a}u^R_{t+a}(h)K^R_{t+a-1}(h)-P_{t+a}\Phi\left(u^R_{t+a}(h)\right)K^R_{t+a-1}(h)\\ -P_{t+a}\left[C^R_{t+a}(h)+I^R_{t+a}(h)\right]\end{array}\right]\\ +\sum_{a=0}^{\infty}\beta^{t+a}\Xi_{t+a}\left[\begin{array}{c}K^R_{t+a}(h)-(1-\delta)K^R_{t+a-1}(h)\\ -\varepsilon_{I,t+a}I^R_{t+a}(h)+\frac{1}{2}\Psi\left(\frac{\varepsilon_{I,t+a}I^R_{t+a}(h)}{K^R_{t+a-1}(h)}-\delta\right)^2K^R_{t+a-1}(h)\end{array}\right]\end{array}\right],$$

where $\tilde{\Lambda}_t$ and $\Xi_t\equiv\tilde{\Lambda}_tP_tq_t$ (q_t is the Tobin's q) are Lagrange multipliers relative to the CBC and law of motion of capital, $\sum_{s^{t+a+1}|s^{t+a}}Q\left(s^{t+a+1},s^{t+a}\right)$ and $\sum_{s^{t+a+1}|s^{t+a}}Q^*\left(s^{t+a+1},s^{t+a}\right)$ are relevant discount factors for home and foreign assets for consumer $j=R$.[23] Perfect risk sharing implies that:

$$\sum_{s^{t+a+1}|s^{t+a}}Q(s^{t+a+1},s^{t+a}) = R^{-1}_{t+a},\quad\text{and}\tag{28}$$

$$\sum_{s^{t+a+1}|s^{t+a}}Q^*\left(s^{t+a+1},s^{t+a}\right) = \frac{\left(R^*_{t+a}\right)^{-1}}{\mathbb{RP}_{t+a}\left(\cdot,\cdot,\cdot\right)},\tag{29}$$

where $\mathbb{RP}_{t+a}\left(\cdot,\cdot,\cdot\right)$ is defined in footnote 11.

The Ricardian consumer chooses consumption allocations, home and foreign asset holdings, investment, capital, the utilization rate and labor hours (derived in a separated section). The resulting FOCs can be summarized in the following.

First, the FOC w.r.t. consumption reads as:

$$\Lambda^R_t=\left[C^R_t(h)-bC^R_{t-1}(h)\right]^{-1}-\beta bE_t\left[C^R_{t+1}(h)-bC^R_t(h)\right]^{-1},\tag{30}$$

where $\Lambda^R_t\equiv\tilde{\Lambda}^R_tP_t$ is the (real) Lagrange multiplier that equalizes the marginal utility w.r.t. consumption.

[23] This notation is more explicit than in Woodford (2003) Ch. 3: s^{t+h} stands for the history of states of the world that have taken place untill $t+h$, thus $Q\left(s^{t+h+1},s^{t+h}\right)$ indicates the value of the discount factor in a particular state at $t+h+1$ (among all possible states).

Second, FOCs w.r.t. home and foreign asset holdings yield the following Euler equations:

$$\frac{1}{R_t} = \beta E_t \left[\frac{\Lambda_{t+1}^R}{\Lambda_t^R \Pi_{t+1}} \right], \tag{31}$$

$$\frac{1}{R_t^* \mathbb{RP}_t(\cdot,\cdot,\cdot)} = \beta E_t \left[\frac{\Lambda_{t+1}^R}{\Lambda_t^R} \frac{S_{t+1}}{S_t} \Pi_{t+1}^{-1} \right], \tag{32}$$

respectively, where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ stands for gross aggregate inflation.

Combining (31) with (32) allow us to derive the uncovered interest rate parity (UIP) reads as:

$$\frac{E_t\left[\Lambda_{t+1}^R S_t / S_{t+1} \Pi_{t+1}^{-1}\right]}{E_t\left[\Lambda_{t+1}^R \Pi_{t+1}^{-1}\right]} = \frac{R_t}{R_t^* \mathbb{RP}_t(\cdot,\cdot,\cdot)}.$$

Third, FOCs w.r.t. investment, capital and the utilization rate are:

$$\Lambda_t^R = \varepsilon_{I,t} \left[\Xi_t - \Xi_t \Psi \left(\frac{\varepsilon_{I,t} I_t^R(h)}{K_{t-1}^R(h)} - \delta \right) \right], \tag{33}$$

$$\Xi_t = \beta E_t \left\{ \begin{array}{c} \frac{\Lambda_{t+1}^R}{P_{t+1}} \left[R_{t+1}^k u_{t+1}^R(h) - P_{t+1} \Phi\left(u_{t+1}^R(h)\right) \right] \\ +\Xi_{t+1} \left[(1-\delta) - \frac{1}{2} \Psi \left(\frac{\varepsilon_{I,t+1} I_{t+1}^R(h)}{K_t^R(h)} - \delta \right)^2 + \Psi \left(\frac{\varepsilon_{I,t+1} I_{t+1}^R(h)}{K_t^R(h)} - \delta \right) \frac{\varepsilon_{I,t+1} I_{t+1}^R(h)}{K_t^R(h)} \right] \end{array} \right\}, \tag{34}$$

$$R_t^k = P_t \Phi'\left(u_t^R(h)\right). \tag{35}$$

A.2 Derivation of the GDP identity

In this subsection we derive the gross domestic product (GDP) identity Eq. (23) in the main text. Begin with GDP definition: the sum of consumption, investment, government spending and net exports (minus resources lost due to adjustment of capital utilization):

$$\begin{aligned} Y_t &= C_t + I_t + \frac{P_{G,t} G_t}{P_t} + RER_t \left(p_{cu,t}^* X_{cu,t}^{share} + p_{mo,t}^* X_{mo,t}^{share} \right) Y_t \\ &\quad - \Delta_{F,t} \left[T_t T_{H,t} \left(C_{F,t} + I_{F,t}^R \right) + \frac{P_{F,t}}{P_t} \frac{P_{G,t}}{P_{G,t}} G_{F,t} \right] + \Phi\left(u_t^R\right) K_{t-1}, \\ Y_t &= C_t + I_t + \frac{P_{G,t} G_t}{P_t Y_t} Y_t + RER_t \left(p_{cu,t}^* X_{cu,t}^{share} + p_{mo,t}^* X_{mo,t}^{share} \right) Y_t \\ &\quad - \Delta_{F,t} \left[T_t^{1-\eta} T_{H,t}^{1-\eta} \alpha \left(C_t + I_t \right) + \frac{P_{F,t}}{P_t} \frac{P_{G,t}}{P_{G,t}} \underbrace{\alpha_G \left(T T_{GH,t} \right)^{-\eta} G_t}_{} \frac{Y_t}{Y_t} \right] + \Phi\left(u_t^R\right) K_{t-1}, \end{aligned}$$

notice that $g_t \equiv \frac{P_{G,t} G_t}{P_t Y_t}$, $\frac{P_{F,t}}{P_t} \frac{P_{G,t}}{P_{G,t}} \alpha_G \left(T_t T_{GH,t} \right)^{-\eta} G_t \frac{Y_t}{Y_t} = \frac{P_{F,t}}{P_{G,t}} \alpha_G \left(T_t T_{GH,t} \right)^{-\eta} \frac{P_{G,t} G_t}{P_t Y_t} Y_t$:

$$Y_t = C_t + I_t + g_t Y_t + RER_t \left(p_{cu,t}^* X_{cu,t}^{share} + p_{mo,t}^* X_{mo,t}^{share} \right) - \Delta_{F,t} T_t^{1-\eta} \left[T_{H,t}^{1-\eta} \alpha \left(C_t + I_t \right) + \alpha_G T_{GH,t}^{1-\eta} g_t Y_t \right] + \Phi\left(u_t^R\right) K_{t-1},$$

$$Y_t\left[1-RER_t\left(p^*_{cu,t}X^{share}_{cu,t}+p^*_{mo,t}X^{share}_{mo,t}\right)-g_t+\Delta_{F,t}\alpha_G\left(T_tT_{GH,t}\right)^{1-\eta}g_t\right]$$
$$=\left[1-\Delta_{F,t}\alpha\left(T_tT_{H,t}\right)^{1-\eta}\right](C_t+I_t)+\Phi\left(u^R_t\right)K_{t-1},$$

and reorganizing we get Eq. (23).

A.3 Derivation of the net foreign asset (NFA) position

This section derives the NFA under incomplete international asset markets.

First, notice that domestic nominal aggregated benefits that accrue to Ricardian households are:

$$\begin{aligned}\mathsf{Pr}_t &= P_t\mathfrak{B}_t=P_t\left(\mathfrak{B}_{H,t}+\mathfrak{B}_{F,t}\right),\\ &= P_t\left(\frac{T_{H,t}}{MC_{H,t}}Y_{H,t}-w^R_t(1-\lambda)L^R_t-w^N_t\lambda L^N_t-r^k_tu^R_tK_{t-1}-T_{H,t}FC_H\right)\\ &\quad +P_t\left[(\mu_F-1)\frac{S_tP^*_{F,t}}{P_t}\left(C_{F,t}+I_{F,t}+G_{F,t}\right)\right],\end{aligned}\tag{36}$$

where $\mathfrak{B}_{H,t}\equiv(1-\lambda)\mathfrak{B}^R_{H,t}=\int_\lambda^1\mathfrak{B}^R_{H,t}(h)dh$ and $\mathfrak{B}_{F,t}\equiv(1-\lambda)\mathfrak{B}^R_{F,t}=\int_\lambda^1\mathfrak{B}^R_{F,t}(h)dh$ are real benefits of domestic intermediate producers and final goods importers, respectively. Notice that Eq. (36) assumes complete home bias in stocks property holdings. Once the subsidy to the importer, $\mathcal{S}_F$, is considered the price turns out to be $\frac{\mu_F}{\mathcal{S}_F}P_{F,t}=P_{F,t}$ (recall that $P_{F,t}\equiv\mu_FS_tP^*_{F,t}$) and the LOOP is restored if and only if $\mathcal{S}_F=\mu_F$.[24] In other words, thanks to the subsidy the consumer effectively pays a price which is identical to the marginal cost (as in a perfectly competitive environment). Furthermore, recall that $\frac{S_tP^*_{F,t}}{P_t}=\frac{S_tP^*_{F,t}}{P_{F,t}}\frac{P_{F,t}}{P_t}=RER_{F,t}T_{H,t}T_t$. Aggregate profits from private copper firms are:

$$\mathsf{Pr}_{cu,t}=P_tRER_tp^*_{cu,t}\left(1-\tau_{cu,t}\right)(1-\kappa)X^{share}_{cu,t}Y_t.\tag{37}$$

Second, for convenience rewrite the CBCs for Ricardian and non-Ricardian households, Eqs. (3) and (7), respectively:[25]

$$B^R\left(s^t,h\right)+S_tB^{R,*}_t(h)+\left(1-\tau_{w,t}\right)\mathcal{S}_{WR}W^R_tL^R_t+P_t\left[Tr^R_t(h)-TX^R_t(h)\right]$$
$$-\sum\nolimits_{s^{t+1}|s^t}Q\left(s^{t+1},s^t,h\right)B^R\left(s^{t+1},h\right)-S_t\sum\nolimits_{s^{t+1}|s^t}Q^*\left(s^{t+1},s^t,h\right)B^{R,*}_{t+1}(h)+\left(1-\tau_{\mathrm{Pr},t}\right)\mathsf{Pr}^R_t(h)$$
$$+R^k_tu^R_t(h)K^R_{t-1}(h)-P_t\Phi\left(u^R_t(h)\right)K^R_{t-1}(h)-P_t\left[C^R_t(h)+I^R_t(h)\right]=0,$$
$$\left(1-\tau_{w,t}\right)\mathcal{S}_WW^N_t(h)L^N_t(h)+P_t\left[Tr^N_t(h)-TX^N_t(h)\right]-P_tC^N_t(h)=0.$$

[24] Notice that $\frac{T_H}{MC_H}=\mu_H$ at the steady state.

[25] Recall that due to the demand aggregation (i.e., integration across goods) $C^R_t(h)+I^R_t(h)=\Delta_{H,t}T_{H,t}\left[C^R_{H,t}(h)+I^R_{H,t}(h)\right]+\Delta_{F,t}T_{H,t}T_t\left[C^R_{F,t}(h)+I^R_{F,t}(h)\right]$ and $C^N_t(h)=\Delta_{H,t}T_{H,t}C^N_{H,t}(h)+\Delta_{F,t}T_{H,t}T_tC^N_{F,t}(h)$, for any household h.

Integrating first over goods and then over agents, we obtain:

$$(1-\lambda) B_t^R + (1-\lambda) S_t B_t^{R,*} + (1-\tau_{w,t}) \mathcal{S}_{WR} D_{W,t}^R W_t^R L_t^R + P_t (1-\lambda) Tr_t^R$$

$$-P_t (1-\lambda) TX_t^R - \tfrac{(1-\lambda) B_{t+1}^R}{R_t} - \frac{(1-\lambda) S_t B_{t+1}^{R,*}}{R_t^* \exp\left(-\phi_a \left(\frac{S_t B_{t+1}^*}{P_t}\right) - \phi_{\Delta S}\left(E_t\left[\frac{S_{t+1}}{S_t}\right]-1\right)+\phi_t\right)} + (1-\tau_{\mathrm{Pr},t}) \mathsf{Pr}_t$$

$$+R_t^k u_t^R K_{t-1} - P_t \Phi\left(u_t^R\right) K_{t-1} - P_t (1-\lambda)\left[\Delta_{H,t} T_{H,t}\left(C_{H,t}^R + I_{H,t}^R\right) + \Delta_{F,t} T_{H,t} T_t \left(C_{F,t}^R + I_{F,t}^R\right)\right] = 0,$$

$$(1-\tau_{w,t}) \mathcal{S}_{WN} W_t^N L_t^N + P_t \lambda \left(Tr_t^N - TX_t^N\right) - P_t \lambda \left[\Delta_{H,t} T_{H,t} C_{H,t}^N + \Delta_{F,t} T_{H,t} T_t C_{F,t}^N\right] = 0,$$

where we replaced C_t^R and I_t^R by their equivalents taking into account demands' structure. To simplify the algebra we assume that $\Delta_{H,t} = \Delta_{F,t}$ and combine both restrictions to obtain:

$$\begin{aligned}
&(1-\lambda) B_t^R + (1-\lambda) S_t B_t^{R,*} + (1-\tau_{w,t})\left[\mathcal{S}_{WR} D_{W,t}^R W_t^R L_t^R + \mathcal{S}_{WN} D_{W,t}^N W_t^N L_t^N\right] \\
&\quad + P_t Tr_t - P_t TX_t - \tfrac{(1-\lambda) B_{t+1}^R}{R_t} - \tfrac{(1-\lambda) S_t B_{t+1}^{R,*}}{R_t^* \mathbb{RP}_t(\cdot,\cdot,\cdot)} + (1-\tau_{\mathrm{Pr},t}) \mathsf{Pr}_t \\
&\quad + R_t^k u_t^R K_{t-1} - P_t \Phi\left(u_t^R\right) K_{t-1} - \Delta_{H,t} P_t C_t - \Delta_{H,t} P_t (1-\lambda) I_t^R = 0.
\end{aligned} \tag{38}$$

Third, taking into account the GBC (13) and assuming that period-to-period outflows are equal to sources of income, we can calculate lump-sum transfers consistent with a zero-deficit rule:

$$\begin{aligned}
P_t Tr_t \quad = \quad & -g_t P_t Y_t + \left(\frac{B_{t+1}}{R_t} - B_t\right) + \left(\frac{S_t B_{t+1}^{G,*}}{R_t^* \mathbb{RP}_t(\cdot,\cdot,\cdot)} - S_t B_t^{G,*}\right) \\
& + S_t P_t^* p_{cu,t}^* \left[\kappa + \tau_{cu,t}(1-\kappa)\right] X_{cu,t}^{share} Y_t + S_t P_t^* p_{mo,t}^* X_{mo,t}^{share} Y_t \\
& + \tau_{w,t}\left[\mathcal{S}_{WR} D_{W,t}^R W_t^R L_t^R + \mathcal{S}_{WN} D_{W,t}^N W_t^N L_t^N\right] \\
& + \tau_{\mathrm{Pr},t} \mathsf{Pr}_t + P_t TX_t - (\mathcal{S}_F - 1)\Delta_{F,t} P_F \left(C_{F,t} + I_{F,t} + G_{F,t}\right) \\
& - (\mathcal{S}_{WR} - 1) D_{W,t}^R W_t^R L_t^R - (\mathcal{S}_{WN} - 1) D_{W,t}^N W_t^N L_t^N.
\end{aligned}$$

Combine transfers from the previous equation with Eq. (38) and cancel out common terms:[26]

$$\begin{gathered}
(1-\lambda) B_t^R + (1-\lambda) S_t B_t^{R,*} + (1-\tau_{w,t})\left[\mathcal{S}_{WR} D_{W,t}^R W_t^R L_t^R + \mathcal{S}_{WN} D_{W,t}^N W_t^N L_t^N\right] \\
+ \left(\begin{array}{c}
-g_t P_t Y_t + \left(\frac{B_{t+1}}{R_t} - B_t\right) + \left(\frac{S_t B_{t+1}^{G,*}}{R_t^* \mathbb{RP}_t(\cdot,\cdot,\cdot)} - S_t B_t^{G,*}\right) \\
+ S_t P_t^* p_{cu,t}^* \left[\kappa + \tau_{cu,t}(1-\kappa)\right] X_{cu,t}^{share} Y_t + S_t P_t^* p_{mo,t}^* X_{mo,t}^{share} Y_t \\
+ \tau_{w,t}\left[\mathcal{S}_{WR} D_{W,t}^R W_t^R L_t^R + \mathcal{S}_{WN} D_{W,t}^N W_t^N L_t^N\right] \\
+ \tau_{\mathrm{Pr},t} \mathsf{Pr}_t + P_t TX_t - (\mathcal{S}_F - 1)\Delta_{F,t} P_F \left(C_{F,t} + I_{F,t} + G_{F,t}\right) \\
- (\mathcal{S}_{WR} - 1) D_{W,t}^R W_t^R L_t^R - (\mathcal{S}_{WN} - 1) D_{W,t}^N W_t^N L_t^N.
\end{array}\right) \\
- P_t TX_t - \tfrac{(1-\lambda) B_{t+1}^R}{R_t} - \tfrac{(1-\lambda) S_t B_{t+1}^{R,*}}{R_t^* \mathbb{RP}_t(\cdot,\cdot,\cdot)} + (1-\tau_{\mathrm{Pr},t}) \mathsf{Pr}_t \\
+ R_t^k u_t^R K_{t-1} - P_t \Phi\left(u_t^R\right) K_{t-1} - \Delta_{H,t} P_t C_t - \Delta_{H,t} P_t (1-\lambda) I_t^R = 0.
\end{gathered}$$

[26] Notice that $P_t\left[(\mu_F - 1)\frac{S_t P_{F,t}^*}{P_t}\left(C_{F,t} + I_{F,t} + G_{F,t}\right)\right] - (\mathcal{S}_F - 1) P_F \left(C_{F,t} + I_{F,t} + G_{F,t}\right) = 0$, since $\mu_F - 1 - \mathcal{S}_F + 1 = \mu_F - \mathcal{S}_F = 0$.

$$
\begin{gathered}
S_t\left[(1-\lambda)B_t^{R,*}-B_t^{G,*}\right]-g_tP_tY_t\\
+\tfrac{S_t\left[B_{t+1}^{G,*}-(1-\lambda)B_{t+1}^{R,*}\right]}{R_t^*\mathbb{RP}_t(\cdot,\cdot,\cdot)}+S_tP_t^*p_{cu,t}^*\left[\kappa+\tau_{cu,t}(1-\kappa)\right]X_{cu,t}^{share}Y_t+S_tP_t^*p_{mo,t}^*X_{mo,t}^{share}Y_t\\
-(\mathcal{S}_F-1)\Delta_{F,t}P_F(C_{F,t}+I_{F,t}+G_{F,t})+W_t^RL_t^R+W_t^NL_t^N.\\
+\mathsf{Pr}_t+R_t^ku_t^RK_{t-1}-P_t\Phi\left(u_t^R\right)K_{t-1}-\Delta_{H,t}P_t(C_t+I_t)=0.
\end{gathered}
$$

and taking into account the equation for benefits, Eq. (36):

$$
\begin{gathered}
S_t\left\{(1-\lambda)B_t^{R,*}-B_t^{G,*}-\tfrac{(1-\lambda)B_{t+1}^{R,*}-B_{t+1}^{G,*}}{R_t^*\mathbb{RP}_t(\cdot,\cdot,\cdot)}\right\}-g_tP_tY_t\\
+S_tP_t^*p_{cu,t}^*\left[\kappa+\tau_{cu,t}(1-\kappa)\right]X_{cu,t}^{share}Y_t+S_tP_t^*p_{mo,t}^*X_{mo,t}^{share}Y_t\\
+P_t\left(\tfrac{T_{H,t}}{MC_{H,t}}Y_{H,t}-T_{H,t}FC_H\right)-P_t\Phi\left(u_t^R\right)K_{t-1}-\Delta_{H,t}P_t(C_t+I_t)=0,
\end{gathered}
$$

recall that $B_t^*=(1-\lambda)B_t^{R,*}-B_t^{G,*}$ where it is understood that $B_t^{R,*}$ are net holdings of private agents, while $-B_t^{G,*}$ are net holdings of the government, which explains the negative sign; so replacing:

$$
\begin{gathered}
S_tB_t^*+S_tP_t^*\left\{p_{cu,t}^*\left[\kappa+\tau_{cu,t}(1-\kappa)\right]X_{cu,t}^{share}+p_{mo,t}^*X_{mo,t}^{share}\right\}P_tY_t+P_t\left(\tfrac{T_{H,t}}{MC_{H,t}}Y_{H,t}-T_{H,t}FC_H\right)\\
-\tfrac{S_tB_{t+1}^*}{R_t^*\mathbb{RP}_t(\cdot,\cdot,\cdot)}-\Delta_{H,t}P_t(C_t+I_t)-P_t\left(g_tY_t+\Phi\left(u_t^R\right)K_{t-1}\right)=0.
\end{gathered}
$$

Replacing $Y_{H,t}$ by its equal from the equilibrium conditions (22), the previous equation in *real* terms becomes:

$$
\begin{gathered}
\frac{S_tB_t^*}{P_t}\frac{S_{t-1}}{S_{t-1}}\frac{P_{t-1}}{P_{t-1}}+RER_t\left\{\left[\kappa+\tau_{cu,t}(1-\kappa)\right]p_{cu,t}^*X_{cu,t}^{share}+p_{mo,t}^*X_{mo,t}^{share}\right\}Y_t\\
+\frac{T_{H,t}}{MC_{H,t}}\left\{\Delta_{H,t}\left[(1-\alpha)\underbrace{T_{H,t}^{-\eta}(C_t+I_t)}+(1-\alpha_G)T_{GH,t}^{-\eta}G_t\frac{P_{G,t}}{P_{G,t}}\frac{Y_t}{Y_t}\right]+(\alpha_C^*+\alpha_I^*)\frac{T_{H,t}^{-\eta}}{RER_t^{-\eta}}Y_t^*\right\}\\
-T_{H,t}FC_H-\Delta_{H,t}\underbrace{(C_t+I_t)}-g_tY_t-\Phi\left(u_t^R\right)K_{t-1}=\frac{S_tB_{t+1}^*}{R_t^*\mathbb{RP}_t(\cdot,\cdot,\cdot)}\frac{1}{P_t}
\end{gathered}
\tag{39}
$$

where $T_{H,t}G_t\frac{P_{G,t}}{P_{G,t}}\frac{Y_t}{Y_t}=\frac{P_{H,t}}{P_{G,t}}\frac{P_{G,t}G_t}{P_tY_t}Y_t=T_{GH,t}g_tY_t$. We can rewritte the previous equation as following:

$$
\frac{S_{t-1}B_t^*}{P_{t-1}}\frac{S_t}{S_{t-1}}\frac{1}{\Pi_t}+NX_t-RER_tp_{cu,t}^*(1-\tau_{cu,t})(1-\kappa)X_{cu,t}^{share}Y_t=\frac{1}{R_t^*\mathbb{RP}_t(\cdot,\cdot,\cdot)}\frac{S_tB_{t+1}^*}{P_t},\tag{40}
$$

where we employed the following definition for net exports (strickly speaking, we should add Eq. (37) and deduce this amount to get the true measure of net exports including copper and molybdenum):

$$
\begin{aligned}
NX_t\equiv\;& RER_t\left(p_{cu,t}^*X_{cu,t}^{share}+p_{mo,t}^*X_{mo,t}^{share}\right)Y_t+\Delta_{H,t}\left[\frac{T_{H,t}^{1-\eta}}{MC_{H,t}}(1-\alpha)\underbrace{(C_t+I_t)}+\frac{T_{GH,t}^{1-\eta}}{MC_{H,t}}(1-\alpha_G)g_tY_t\right]\\
&+\frac{T_{H,t}}{MC_{H,t}}(\alpha_C^*+\alpha_I^*)\frac{T_{H,t}^{-\eta}}{RER_t^{-\eta}}Y_t^*-T_HFC_H-\Delta_{H,t}\underbrace{(C_t+I_t)}-g_tY_t-\Phi\left(u_t^R\right)K_{t-1},
\end{aligned}
\tag{41}
$$

where Eq. (37) provides the clue to gauge the net rents balance of the balance of payments, i.e. benefits of foreign mining companies:

$$NR_t \equiv -RER_t p^*_{cu,t} \left(1-\tau_{cu,t}\right)\left(1-\kappa\right) X^{share}_{cu,t} Y_t. \tag{42}$$

Thus, it is required in equilibrium that $NX + NR = 0$ to avoid debt accumulation. In the case of Chile, it is the case that long run data supports a ratio NX-to-GDP of 2% while the rents balance is a deficit of the approximately the same magnitude. Besides, recall that terms signaled with $\underbrace{}$, in (39) come from Equation (25).

Alternatively, we may rewrite Eq. (26) in terms of Y_t:

$$\underbrace{\frac{S_{t-1}}{P_{t-1}}\frac{B^*_t}{Y_t}}_{\mathring{B}^*_t}\frac{S_t}{S_{t-1}}\frac{1}{\Pi_t} + \underbrace{\frac{NX_t}{Y_t}}_{\mathring{N}X_t} + \underbrace{\frac{NR_t}{Y_t}}_{\mathring{N}R_t} = \frac{1}{R^*_t \mathbb{RP}_t\left(\cdot,\cdot,\cdot\right)}\underbrace{\frac{Y_{t+1}}{Y_t}}_{=1}\underbrace{\frac{S_t}{P_t}\frac{B^*_{t+1}}{Y_{t+1}}}_{\mathring{B}^*_{t+1}}, \tag{43}$$

and

$$\begin{aligned}
\frac{NX_t}{Y_t} \quad &\equiv \quad RER_t\left(p^*_{cu,t}X^{share}_{cu,t} + p^*_{mo,t}X^{share}_{mo,t}\right)Y_t + \Delta_{H,t}\left[\frac{T^{1-\eta}_{H,t}}{MC_{H,t}}\left(1-\alpha\right)\frac{\left(C_t+I_t\right)}{Y_t} + \frac{T^{-\eta}_{GH,t}}{MC_{H,t}}\left(1-\alpha_G\right)g_t\right] \\
&\quad + \frac{T_{H,t}}{MC_{H,t}}\left(\alpha^*_C + \alpha^*_I\right)\frac{T^{-\eta}_{H,t}}{RER^{-\eta}_t}\frac{Y^*_t}{Y_t} - \frac{T_{H,t}FC_H}{Y_t} - \frac{\Delta_{H,t}\left(C_t+I_t\right)}{Y_t} - g_t - \Phi\left(u^R_t\right)\frac{K_{t-1}}{Y_t}, \qquad (44) \\
\frac{NR_t}{Y_t} \quad &\equiv \quad -RER_t p^*_{cu,t}\left(1-\tau_{cu,t}\right)\left(1-\kappa\right)X^{share}_{cu,t}. \qquad (45)
\end{aligned}$$

Either we can employ (26) and (27) or (43) and (44).

A.4 Derivation of T_{GH}

Begin with the definition of $T_{GH,t}$:

$$T_{GH,t} \equiv \frac{P_{H,t}}{P_{G,t}},$$

where $P_{G,t}$ comes from Eq. (12). Besides, observe that $T_{GH,t}$ relates statically with $T_t \equiv \frac{P_{F,t}}{P_{H,t}}$:

$$\begin{aligned}
T_{GH,t} \quad &= \quad \frac{P_t}{P_t}\frac{P_{H,t}}{P_{G,t}} = \frac{P_{H,t}}{P_t}\frac{P_t}{P_{G,t}} = T_{H,t}\left[\frac{\left(1-\alpha\right)P^{1-\eta}_{H,t} + \alpha P^{1-\eta}_{F,t}}{\left(1-\alpha_G\right)P^{1-\eta}_{H,t} + \alpha_G P^{1-\eta}_{F,t}}\right]^{\frac{1}{1-\eta}}, \\
T^{1-\eta}_{GH,t} \quad &= \quad T^{1-\eta}_{H,t}\frac{\left(1-\alpha\right)P^{1-\eta}_{H,t} + \alpha P^{1-\eta}_{F,t}}{\left(1-\alpha_G\right)P^{1-\eta}_{H,t} + \alpha_G P^{1-\eta}_{F,t}}\frac{P^{1-\eta}_{H,t}}{P^{1-\eta}_{H,t}} = T^{1-\eta}_{H,t}\frac{1-\alpha+\alpha T^{1-\eta}_t}{1-\alpha_G+\alpha_G T^{1-\eta}_t}.
\end{aligned}$$

Thus,

$$T_{GH,t} = T_{H,t}\left[\frac{1-\alpha+\alpha T^{1-\eta}_t}{1-\alpha_G+\alpha_G T^{1-\eta}_t}\right]^{\frac{1}{1-\eta}}. \tag{46}$$

B Steady State (complete asset markets)

$$\begin{aligned} i &= i^* = \frac{\Pi}{\beta}, \\ \Pi &= \Pi_H = \Pi_F = \Pi^* = 1. \end{aligned}$$

For home producers it follows from optimality conditions that at the SS $T_H \equiv \frac{P_H}{P} = \mu_H MC_H$, which implies that the *real* marginal cost is ($MC_H = \frac{MC_H^{nom}}{P}$):

$$MC_H = \frac{T_H}{\mu_H}, \tag{47}$$

and similarly for home importers:

$$P_F = \frac{\mu_F}{\mathcal{S}_F} MC_F^{nom} = \frac{\mu_F}{\mathcal{S}_F} SP_F^*,$$

which yields the marginal cost in terms of imported good prices:

$$1 = \frac{\mu_F}{\mathcal{S}_F} \frac{SP_F^*}{P_F} \Rightarrow RER_F = \frac{\mathcal{S}_F}{\mu_F}, \tag{48}$$

where we employed the definition $RER_F \equiv \frac{SP_F^*}{P_F}$. In terms of the GDP deflator:

$$\begin{aligned} \frac{P_H}{P_H}\frac{P_F}{P} &= \frac{\mu_F}{\mathcal{S}_F} \underbrace{\frac{SP_F^*}{P}\frac{P_H}{P_H}\frac{P_F}{P_F}}_{\text{represents } MC_F}, \\ \frac{P_H}{P}\frac{P_F}{P_H} &= \frac{\mu_F}{\mathcal{S}_F}\frac{SP_F^*}{P_F}\frac{P_H}{P}\frac{P_F}{P_H}, \\ T_H T &= \underbrace{\frac{\mu_F}{\mathcal{S}_F}\frac{SP_F^*}{P_F} T_H T}_{=MC_F}, \\ T_H T \frac{\mathcal{S}_F}{\mu_F} &= MC_F. \end{aligned}$$

so it follows that the real marginal cost of the imported good is:

$$MC_F = T_H T \frac{\mathcal{S}_F}{\mu_F} = RER_F T_H T. \tag{49}$$

Next, we seek to pin down SS inputs' prices. First, consider the FOC w.r.t. capital (34) at the SS (the assumption is that $u = 1$, $\Phi(1) = 0$, $\varepsilon_I = 1$):

$$\Xi = \beta \left\{ \Lambda \frac{R^k}{P} + \Xi \left[(1-\delta) - \frac{1}{2}\Psi \left(\frac{I^R}{K^R} - \delta \right)^2 + \Psi \left(\frac{I^R}{K^R} - \delta \right) \frac{I^R}{K^R} \right] \right\}$$

taking into account (33) at the SS, $\Lambda^R = \Xi - \Xi\Psi\left(\frac{I^R}{K^R} - \delta\right)$, to express the previous equation in terms of one multiplier:

$$
\begin{aligned}
\Xi &= \beta\left\{\left[\Xi - \Xi\Psi\left(\frac{I^R}{K^R} - \delta\right)\right] r^k + \Xi\left[(1-\delta) - \frac{1}{2}\Psi\left(\frac{I^R}{K^R} - \delta\right)^2 + \Psi\left(\frac{I^R}{K^R} - \delta\right)\frac{I^R}{K^R}\right]\right\}, \\
1 &= \beta\left\{\left[1 - \Psi\left(\frac{I^R}{K^R} - \delta\right)\right] r^k + (1-\delta) - \frac{1}{2}\Psi\left(\frac{I^R}{K^R} - \delta\right)^2 + \Psi\left(\frac{I^R}{K^R} - \delta\right)\frac{I^R}{K^R}\right\},
\end{aligned}
$$

and finally at the SS materialized investment level is identical to the desired level, just to replace the capital that is depreciated, $\frac{I^R}{K^R} = \delta$ (this result comes from the law of motion of capital (6) at the SS, $I^R = \delta K^R$). Thus,

$$
\begin{aligned}
1 &= \beta\left[r^k + (1-\delta)\right], \\
r^k &= \frac{1}{\beta} - (1-\delta). \qquad (50)
\end{aligned}
$$

Second, to obtain w, recall that from Eq. (10) the *real* marginal cost at the SS is:

$$
MC_H = \frac{1}{A_H}\frac{\left(r^k\right)^\gamma w^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}
$$

which equalized to (47), i.e. $\frac{T_H}{\mu_H} = \frac{1}{A_H}\frac{\left(r^k\right)^\gamma w^{1-\gamma}}{\gamma^\gamma(1-\gamma)^{1-\gamma}}$, leads to:

$$
w = \left[A_H \gamma^\gamma (1-\gamma)^{1-\gamma} \frac{T_H}{\mu_H\left(\frac{1}{\beta} - (1-\delta)\right)^\gamma}\right]^{\frac{1}{1-\gamma}}, \qquad (51)
$$

where r^k comes from (50).

From the production function optimality condition (marginal rate of transformation is equal to relative input price) we get:

$$
\frac{1-\gamma}{\gamma} = \frac{wL}{r^k K} \Rightarrow \frac{K}{L} = \frac{\gamma}{(1-\gamma)}\frac{w}{r^k}. \qquad (52)
$$

Total nominal domestic profits are Pr=Pr_H+Pr_F. Home traders' nominal profits are $\mathsf{Pr}_H = P\mathfrak{B}_H = \frac{P_H Y_H}{MC_H} - WL - R^k K - P_H FC_H$ and real profits (here real means in terms of the consumption bundle C) are given by:

$$
\mathfrak{B}_H = \frac{T_H Y_H}{MC_H} - wL - r^k K - T_H FC_H. \qquad (53)
$$

Under perfect competition and constant returns to scale, the no entry condition guarantees that real benefits are zero at the steady state ($\mathfrak{B}_{H,t} = 0$). Thus, the Euler theorem states that the value of the production equals the value added from inputs, or f(inputs)=$\sum$(price inputs*inputs' quantities):

$$
T_H Y_H = wL + r^k K. \qquad (54)
$$

We rewrite Eq. (53) taking into account Eq. (47) and Eq. (54) we may find out the value of FC_H such that $\mathfrak{B}_{H,t}=0$ holds:

$$\mathfrak{B}_H=\mu_H Y_H - T_H Y_H - T_H FC_H = 0 \Rightarrow FC_H = \frac{(\mu_H - T_H)}{T_H} Y_H, \tag{55}$$

and taking into account that Y_H includes FC_H, it is straightforward that:

$$\begin{aligned} FC_H &= \frac{(\mu_H - T_H)}{T_H}\left(A_H K^{\gamma} L^{1-\gamma} - FC_H\right), \\ \left(1+\frac{\mu_H - T_H}{T_H}\right) FC_H &= \frac{(\mu_H - T_H)}{T_H} A_H \left(\frac{K}{L}\right)^{\gamma} L, \\ FC_H &= \frac{(\mu_H - T_H)}{\mu_H} A_H \left(\frac{K}{L}\right)^{\gamma} L, \end{aligned} \tag{56}$$

where $\frac{K}{L}$is given by Eq. (52). Thus, to check that benefits $\mathfrak{B}_H$ are zero just substitute FC_H into Eq. (55):

$$\mathfrak{B}_H = (\mu_H - T_H) Y_H - T_H \left[\frac{(\mu_H - T_H)}{\mu_H} A_H \left(\frac{K}{L}\right)^{\gamma} L\right],$$

and taking into account that $Y_H = A_H \left(\frac{K}{L}\right)^{\gamma} L - \frac{(\mu_H - T_H)}{\mu_H} A_H \left(\frac{K}{L}\right)^{\gamma} L = \frac{T_H}{\mu_H} A_H \left(\frac{K}{L}\right)^{\gamma} L$:

$$\mathfrak{B}_H = (\mu_H - T_H)\left[\frac{T_H}{\mu_H} A_H \left(\frac{K}{L}\right)^{\gamma} L\right] - T_H \frac{(\mu_H - T_H)}{\mu_H} A_H \left(\frac{K}{L}\right)^{\gamma} L = 0. \tag{57}$$

Similarly, real benefits for home importers are:

$$\mathfrak{B}_F = \frac{P_F}{P}\left(C_F + I_F + G_F\right) - \underbrace{RER_F T_H T}_{=MC_F}\left(C_F + I_F + G_F\right),$$

Multiply the first term on the RHS by $\frac{SP_F^*}{SP_F^*}\frac{P_F}{P_F}$, and arrange properly (recall $RER_F \equiv \frac{SP_F^*}{P_F} = \frac{\mathcal{S}_F}{\mu_F}$ and $\frac{P_F}{P} \equiv T_H T$):

$$\begin{aligned} \mathfrak{B}_F &= \underbrace{\frac{P_F}{SP_F^*}}_{=\mu_F/\mathcal{S}_F}\underbrace{\frac{SP_F^*}{P_F}}_{=RER_F}\underbrace{\frac{P_F}{P}}_{=T_H T}\left(C_F + I_F + G_F\right) - RER_F T_H T\left(C_F + I_F + G_F\right), \\ &= \left(\frac{\mu_F}{\mathcal{S}_F} - 1\right) RER_F T_H T\left(C_F + I_F + G_F\right) = 0. \end{aligned} \tag{58}$$

The marginal utility of consumption from equation (30) at the SS, both for $\{R,N\}$ are:

$$\Lambda^j = \frac{(1-\beta b)}{C^j (1-b)}. \tag{59}$$

As the real wage is divided by the markup in the SS, the wage equation evaluated at the SS simplifies to:

$$-\bar{\zeta}\left(L^j\right)^{\sigma_L} + \frac{\Lambda^j}{\mu_{Wj}} \mathcal{S}_{Wj} w^j = 0,$$

which implies that:

$$L^j = \left(\frac{\Lambda^j}{\bar{\zeta} \mu_{Wj}} \mathcal{S}_{Wj} w^j \right)^{1/\sigma_L},$$

and taking into account the real wage at the SS, Eq. (51):

$$L^j = \left\{ \frac{\Lambda^j}{\bar{\zeta} \mu_{Wj}} \mathcal{S}_{Wj} \left[A_H \gamma^\gamma (1-\gamma)^{1-\gamma} \frac{T_H}{\mu_H \left(\frac{1}{\beta} - (1-\delta) \right)^\gamma} \right]^{\frac{1}{1-\gamma}} \right\}^{1/\sigma_L}.$$

The market clearance condition for home produced goods, Eq. (22), at the SS can be written as (recall that $\Delta_H = \Delta_F = u^R = 1$):

$$A_H K^\gamma L^{1-\gamma} - FC_H = \underbrace{(1-\alpha) T_H^{-\eta} (C+I) + (1-\alpha_G) T_{GH}^{-\eta} G}_{\text{home absorption}} + \underbrace{(\alpha_C^* + \alpha_I^*) \left(\frac{T_{H,t}}{RER_t} \right)^{-\eta} Y^*}_{\text{exports}}. \quad (60)$$

We assume that at the SS the nominal trade balance is zero, i.e., the value of *total* exports (LHS) equals the value of imports (RHS):

$$P_H (C_H^* + I_H^* + G_H^*) + P \cdot RER \left(p_{cu}^* X_{cu}^{share} + p_{mo}^* X_{mo}^{share} \right) Y = P_F (C_F + I_F + G_F), \text{ (nominal)}$$

or in *real* terms, i.e., w.r.t. bundle C good price deflator, P:

$$\underbrace{T_H (\alpha_C^* + \alpha_I^*) \left(\frac{T_H}{RER} \right)^{-\eta} Y^*}_{\text{intermediate Xs (Q)}} + \underbrace{RER \left(p_{cu}^* X_{cu}^{share} + p_{mo}^* X_{mo}^{share} \right) Y}_{\text{copper \& molybdenum Xs (Q)}} = T_H T \underbrace{\left[\alpha (T_H T)^{-\eta} (C+I) + \alpha_G (T_{GH} T)^{-\eta} G \right]}_{\text{intermediate Ms (Q)}}, \quad (61)$$

The intermediate export level that is consistent with the zero trade balance (expressed in terms of the home intermediate good) is:

$$(\alpha_C^* + \alpha_I^*) \left(\frac{T_H}{RER} \right)^{-\eta} Y^* = T \left[\alpha (T_H T)^{-\eta} (C+I) + \alpha_G (T_{GH} T)^{-\eta} G \right] - \frac{1}{T_H} \left[RER \left(p_{cu}^* X_{cu}^{share} + p_{mo}^* X_{mo}^{share} \right) Y \right]. \quad (62)$$

We seek to introduce the information of the zero trade balance into the SS solution, so we replace Eq. (62) into the equilibrium condition of home (intermediate) produced goods, i.e. into (60): $Y_H = C_H + I_H + G_H + C_H^* + I_H^*$ (in real terms w.r.t. P_H). Further, we substitute the fixed cost FC_H, from Eq. (56). The equilibrium condition becomes:

$$\begin{aligned} A_H \left(\frac{K}{L} \right)^\gamma L - \frac{(\mu_H - T_H)}{\mu_H} A_H \left(\frac{K}{L} \right)^\gamma L &= (1-\alpha) T_H^{-\eta} (C+I) + (1-\alpha_G) T_{GH}^{-\eta} G \\ &+ T^{1-\eta} \left[\alpha T_H^{-\eta} (C+I) + \alpha_G T_{GH}^{-\eta} G \right] - \frac{RER \left(p_{cu}^* X_{cu}^{share} + p_{mo}^* X_{mo}^{share} \right) Y}{T_H}, \end{aligned}$$

$$\left(1-\frac{\mu_H - T_H}{\mu_H}\right) A_H \left(\frac{K}{L}\right)^{\gamma} L \quad = \quad \left(1-\alpha+\alpha T^{1-\eta}\right) T_H^{-\eta}\left(C+I\right)$$
$$+\left(1-\alpha_G+\alpha_G T^{1-\eta}\right) T_{GH}^{-\eta} G - \frac{RER\left(p_{cu}^{*} X_{cu}^{share} + p_{mo}^{*} X_{mo}^{share}\right) Y}{T_H},$$

notice that in the LHS $1-\frac{\mu_H - T_H}{\mu_H} = \frac{\mu_H - \mu_H + T_H}{\mu_H} = \frac{T_H}{\mu_H}$ and in the RHS $\frac{P_G}{P} G = \frac{P_G G}{PY} Y = gY \Rightarrow G = \frac{P}{P_G} gY \Leftrightarrow G = \left[\frac{(1-\alpha)+\alpha T^{1-\eta}}{(1-\alpha_G)+\alpha_G T^{1-\eta}}\right]^{\frac{1}{1-\eta}} gY$, so replacing yields:

$$\frac{T_H}{\mu_H} A_H \left(\frac{K}{L}\right)^{\gamma} L \quad = \quad \left(1-\alpha+\alpha T^{1-\eta}\right) T_H^{-\eta}\left(C+I\right)$$
$$+\left[\left(1-\alpha_G+\alpha_G T^{1-\eta}\right) T_{GH}^{-\eta}\left[\frac{(1-\alpha)+\alpha T^{1-\eta}}{(1-\alpha_G)+\alpha_G T^{1-\eta}}\right]^{\frac{1}{1-\eta}} g - \frac{RER\left(p_{cu}^{*} X_{cu}^{share} + p_{mo}^{*} X_{mo}^{share}\right)}{T_H}\right] \underbrace{Y}$$

and recall that the term $\underbrace{}$ is the real GDP:

$$\begin{aligned}
Y \quad &= \quad C + I + \frac{P_G}{P} G + RER\left(p_{cu}^{*} X_{cu}^{share} + p_{mo}^{*} X_{mo}^{share}\right) Y - T_H T\left(C_F + I_F + G_F\right) + 0K, \\
&= \quad C + I + \left[\frac{(1-\alpha_G)+\alpha_G T^{1-\eta}}{(1-\alpha)+\alpha T^{1-\eta}}\right]^{\frac{1}{1-\eta}} G \\
&\quad\; + RER\left(p_{cu}^{*} X_{cu}^{share} + p_{mo}^{*} X_{mo}^{share}\right) Y - T_H T\left[\alpha\left(T_H T\right)^{-\eta}\left(C+I\right) + \alpha_G\left(T_{GH} T\right)^{-\eta} G\right], \\
&= \quad \left[1-\alpha\left(T_H T\right)^{1-\eta}\right]\left(C+I\right) \\
&\quad\; + \left[1-\alpha_G T_{GH}^{-\eta} T^{1-\eta} T_H \left[\frac{(1-\alpha)+\alpha T^{1-\eta}}{(1-\alpha_G)+\alpha_G T^{1-\eta}}\right]^{\frac{1}{1-\eta}}\right] gY + RER\left(p_{cu}^{*} X_{cu}^{share} + p_{mo}^{*} X_{mo}^{share}\right) Y,
\end{aligned}$$

but since $T_H\left[\frac{(1-\alpha)+\alpha T^{1-\eta}}{(1-\alpha_G)+\alpha_G T^{1-\eta}}\right]^{\frac{1}{1-\eta}} = \frac{P_H}{P}\frac{P}{P_G} = \frac{P_H}{P_G} = T_{GH}$, it simplifies to:

$$\begin{aligned}
Y \quad &= \quad \left[1-\alpha\left(T_H T\right)^{1-\eta}\right]\left(C+I\right) + \left\{\left[1-\alpha_G\left(T_{GH} T\right)^{1-\eta}\right] g + RER\left(p_{cu}^{*} X_{cu}^{share} + p_{mo}^{*} X_{mo}^{share}\right)\right\} Y, \\
&= \quad F_1\left(C+I\right), \qquad (63)
\end{aligned}$$

where we defined the constant:

$$F_1 \equiv \frac{\left[1-\alpha\left(T_H T\right)^{1-\eta}\right]}{1-\left\{\left[1-\alpha_G\left(T_{GH} T\right)^{1-\eta}\right] g + RER\left(p_{cu}^{*} X_{cu}^{share} + p_{mo}^{*} X_{mo}^{share}\right)\right\}}. \qquad (64)$$

Taking into account the latter relationship we get:

$$\frac{T_H}{\mu_H} A_H \left(\frac{K}{L}\right)^{\gamma} L = \left\{\begin{array}{c} \left(1-\alpha+\alpha T^{1-\eta}\right) T_H^{1-\eta} \\ +\left[\left(1-\alpha_G+\alpha_G T^{1-\eta}\right) T_{GH}^{-\eta}\left[\frac{(1-\alpha)+\alpha T^{1-\eta}}{(1-\alpha_G)+\alpha_G T^{1-\eta}}\right]^{\frac{1}{1-\eta}} g - \frac{RER\left(p_{cu}^{*} X_{cu}^{share} + p_{mo}^{*} X_{mo}^{share}\right)}{T_H}\right] F_1 \end{array}\right\} \left(C + \underbrace{I}_{=\delta}\right. \qquad (65)$$

or in a more compact form:

$$\left[\frac{T_H}{\mu_H}A_H\left(\frac{K}{L}\right)^{\gamma}-\digamma_2\delta\frac{K}{L}\right]L=\digamma_2 C,$$

where

$$\digamma_2\equiv\left\{\begin{array}{c}\left(1-\alpha+\alpha T^{1-\eta}\right)T_H^{1-\eta}\\+\left[\left(1-\alpha_G+\alpha_G T^{1-\eta}\right)T_{GH}^{-\eta}\left[\frac{(1-\alpha)+\alpha T^{1-\eta}}{(1-\alpha_G)+\alpha_G T^{1-\eta}}\right]^{\frac{1}{1-\eta}}g-\frac{RER\left(p_{cu}^{*}X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right)}{T_H}\right]\digamma_1\end{array}\right\}. \tag{66}$$

First, begin with non-Ricardian consumers' SS, whose consumption is deduced from the aggregated version of Eq. (7):

$$\lambda C^N=(1-\tau_w)\,\mathcal{S}_{WN}\int_0^{\lambda}w^N(h)L^N(h)dh+\lambda Tr^{N,net},$$

where $\int_0^{\lambda}w^N(h)L^N(h)dh=\int_0^{\lambda}w^N(h)\lambda^{-1}\left(\frac{w^N(h)}{w^N}\right)^{-\varepsilon_{LN}}L^N dh=w^N L^N\underbrace{\int_0^{\lambda}\lambda^{-1}\left(\frac{w^N(h)}{w^N}\right)^{1-\varepsilon_{LN}}dh}_{=1}$,

so:

$$\lambda C^N=(1-\tau_w)\,\mathcal{S}_{WN}w^N L^N+\lambda Tr^{N,net},$$

where we define $Tr^{N,net}\equiv Tr^N-TX^N$ as the amount of net of lump-sum taxes transfers received from the government. We assume that $Tr^{N,net}=Tr^{R,net}=Tr^{net}$ and it comes from the real aggregated GBC Eq. (13) evaluated at the SS:[27]

$$\begin{aligned}Tr^{net} &= \left(\frac{\Pi}{R}-1\right)b+\left(\frac{\Pi}{R^*}-1\right)RERb^{G,*}+\tau_w\left[\mathcal{S}_{WR}w^R L^R+\mathcal{S}_{WN}w^N L^N\right]+\tau_{\mathrm{Pr}}\,(1-\lambda)\,\mathsf{Pr}^R\\&\quad+RER\left(p_{cu}^{*}\kappa X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right)Y+\tau_{cu}RERp_{cu}^{*}\,(1-\kappa)\,X_{cu}^{share}Y\\&\quad-\left(\mathcal{S}_{WR}-1\right)w^R L^R-\left(\mathcal{S}_{WN}-1\right)w^N L^N-gY,\end{aligned} \tag{67}$$

where $R=\frac{\Pi}{\beta}$, $b=(1-\lambda)\frac{B^R}{P}$, $b^{G,*}=\frac{B^{G,*}}{P}$ and $\mathsf{Pr}^R=0$ due to results from Eqs. (53) and (58) (the latter result is due to the fact that we purposely set FC_H so that Eq. (57) holds, while for $\mathfrak{B}_F=0$ to be true, subsidies given to importers should not be taken into account again, i.e. positive benefits from importers vanish with subsidies $-\left(\mathcal{S}_F-1\right)\frac{P_F}{P}\left(C_F+I_F+G_F\right)$ which explains why we omit them). Thus, C^N can be written as:

$$\lambda C^N=(1-\tau_w)\,\mathcal{S}_{WN}w^N L^N+\lambda\left[\begin{array}{c}\left(\frac{\Pi}{R}-1\right)b+\left(\frac{\Pi}{R^*}-1\right)RERb^{G,*}+\tau_w\left[\mathcal{S}_{WR}w^R L^R+\mathcal{S}_{WN}w^N L^N\right]\\+RER\left(p_{cu}^{*}\kappa X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right)Y+\tau_{cu}RERp_{cu}^{*}\,(1-\kappa)\,X_{cu}^{share}Y\\-\left(\mathcal{S}_{WR}-1\right)w^R L^R-\left(\mathcal{S}_{WN}-1\right)w^N L^N-gY\end{array}\right],$$

[27] Notice that we employ similar relationships when evaluating the integral at the SS, i.e., $\tau_w\mathcal{S}_{WR}\int_{\lambda}^{1}w^R(h)L^R(h)dh$ $=\tau_w\mathcal{S}_{WR}\int_{\lambda}^{1}w^R(h)\,(1-\lambda)^{-1}\left(\frac{w^R(h)}{w^R}\right)^{-\varepsilon_{LR}}L^R dh=w^R L^R\underbrace{\int_{\lambda}^{1}(1-\lambda)^{-1}\left(\frac{w^R(h)}{w^R}\right)^{1-\varepsilon_{LR}}dh}_{=1}$.

Grouping L^N and L^R yield:

$$\begin{aligned}
\lambda C^N &= \left[(1-\tau_w)\mathcal{S}_{WN} - \lambda(\mathcal{S}_{WN}-1) + \lambda\tau_w\mathcal{S}_{WN}\right] w^N L^N + \lambda\left(\frac{\Pi}{R}-1\right)b + \lambda\left(\frac{\Pi}{R^*}-1\right) RER b^{G,*} \\
&\quad + \lambda RER\left(p^*_{cu}\kappa X^{share}_{cu} + p^*_{mo}X^{share}_{mo}\right)Y + \lambda\tau_{cu}RERp^*_{cu}(1-\kappa)X^{share}_{cu}Y \\
&\quad + \lambda\left[\tau_w\mathcal{S}_{WR} - (\mathcal{S}_{WR}-1)\right] w^R L^R - \lambda g Y, \\
&= \left[(1-\tau_w)\mathcal{S}_{WN} - \lambda(\mathcal{S}_{WN}-1) + \lambda\tau_w\mathcal{S}_{WN}\right] w^N L^N + \lambda\left[\tau_w\mathcal{S}_{WR} - (\mathcal{S}_{WR}-1)\right] w^R L^R \\
&\quad + \lambda\left[RER\left(p^*_{cu}\kappa X^{share}_{cu} + p^*_{mo}X^{share}_{mo}\right) + \tau_{cu}RERp^*_{cu}(1-\kappa)X^{share}_{cu} - g + \left(\frac{\Pi}{R}-1\right)\frac{b}{Y} + \left(\frac{\Pi}{R^*}-1\right)RER\frac{b^{G,*}}{Y}\right]
\end{aligned}$$

substitute Y by its equal from Eq. (63):

$$\begin{aligned}
\lambda C^N &= \left[(1-\tau_w)\mathcal{S}_{WN} - \lambda(\mathcal{S}_{WN}-1) + \lambda\tau_w\mathcal{S}_{WN}\right] w^N L^N + \lambda\left[\tau_w\mathcal{S}_{WR} - (\mathcal{S}_{WR}-1)\right] w^R L^R \\
&\quad + \lambda\left[\begin{array}{c} RER\left(p^*_{cu}\kappa X^{share}_{cu} + p^*_{mo}X^{share}_{mo}\right) + \tau_{cu}RERp^*_{cu}(1-\kappa)X^{share}_{cu} \\ -g + \left(\frac{\Pi}{R}-1\right)\frac{b}{Y} + \left(\frac{\Pi}{R^*}-1\right)RER\frac{b^{G,*}}{Y} \end{array}\right] \digamma_1 (C+I).
\end{aligned}$$

Further, substitute I by $\delta\frac{K}{L}L$, where L comes from the labor aggregator Eq. (9):

$$\begin{aligned}
\lambda C^N &= \left[(1-\tau_w)\mathcal{S}_{WN} - \lambda(\mathcal{S}_{WN}-1) + \lambda\tau_w\mathcal{S}_{WN}\right] w^N L^N + \lambda\left[\tau_w\mathcal{S}_{WR} - (\mathcal{S}_{WR}-1)\right] w^R L^R \\
&\quad + \lambda\left[\begin{array}{c} RER\left(p^*_{cu}\kappa X^{share}_{cu} + p^*_{mo}X^{share}_{mo}\right) + \tau_{cu}RERp^*_{cu}(1-\kappa)X^{share}_{cu} \\ -g + \left(\frac{\Pi}{R}-1\right)\frac{b}{Y} + \left(\frac{\Pi}{R^*}-1\right)RER\frac{b^{G,*}}{Y} \end{array}\right] \digamma_1 \left(C+\delta\frac{K}{L}L\right)
\end{aligned}$$

define the constant

$$\digamma_3 \equiv \lambda\left[RER\left[p^*_{cu}\kappa X^{share}_{cu} + p^*_{mo}X^{share}_{mo} + \tau_{cu}p^*_{cu}(1-\kappa)X^{share}_{cu}\right] - g + \left(\frac{\Pi}{R}-1\right)\frac{b}{Y} + \left(\frac{\Pi}{R^*}-1\right)RER\frac{b^{G,*}}{Y}\right]\digamma_1,$$

where $\digamma_1$ is defined in Eq. (64) and reorganize:

$$\begin{aligned}
\lambda C^N = &\left[(1-\tau_w)\mathcal{S}_{WN} - \lambda(\mathcal{S}_{WN}-1) + \lambda\tau_w\mathcal{S}_{WN}\right] w^N L^N \\
&+ \lambda\left[\tau_w\mathcal{S}_{WR} - (\mathcal{S}_{WR}-1)\right] w^R L^R + \digamma_3\left(C+\delta\frac{K}{L}L\right),
\end{aligned}$$

define the constants $\digamma_4 \equiv \lambda$, $\digamma_5 \equiv (1-\tau_w)\mathcal{S}_{WN} - \lambda(\mathcal{S}_{WN}-1) + \lambda\tau_w\mathcal{S}_{WN}$, $\digamma_6 \equiv \lambda\left[\tau_w\mathcal{S}_{WR} - (\mathcal{S}_{WR}-1)\right]$ and $\digamma_7 \equiv \digamma_3$ and rewrite:

$$\digamma_4 C^N = \digamma_5 w^N L^N + \digamma_6 w^R L^R + \digamma_7\left(C+\delta\frac{K}{L}L\right). \tag{68}$$

Plugging C^N from (68) into the aggregation condition, (recall that $\digamma_4 \equiv \lambda$) $C = \lambda C^N + (1-\lambda)C^R$, yields:

$$\begin{aligned}
C &= \digamma_5 w^N L^N + \digamma_6 w^R L^R + \digamma_7 C + \digamma_7\delta\frac{K}{L}L + (1-\lambda)C^R, \\
C &= \frac{\digamma_5}{1-\digamma_7}w^N L^N + \frac{\digamma_6}{1-\digamma_7}w^R L^R + \frac{\digamma_7\delta\frac{K}{L}}{1-\digamma_7}L + \frac{(1-\lambda)}{1-\digamma_7}C^R. \qquad (69)
\end{aligned}$$

Next, continue with Ricardian consumers' SS. Begin with the aggregated real version of the CBC (3) evaluated at the SS:[28]

$$\begin{gathered}(1-\lambda) C^R = (1-\lambda) b^R + RER (1-\lambda) b^{R,*} + (1-\tau_w) \mathcal{S}_{WR} w^R L^R \\ + (1-\lambda) Tr^{R,net} - \tfrac{\Pi}{R} (1-\lambda) b^R \\ -\tfrac{\Pi}{R^*} RER (1-\lambda) b^{R,*} + (1-\tau_{\mathrm{Pr}}) (1-\lambda) \mathsf{Pr}^R + r^k (1-\lambda) K^R - (1-\lambda) I^R\end{gathered}$$

recall that in equilibrium $b^{R,*} = (1-\lambda) \frac{B^{R,*}}{P}$, $K = (1-\lambda) K^R$, $I = (1-\lambda) I^R$, $\mathsf{Pr}= (1-\lambda)\mathsf{Pr}^R = (1-\lambda) 0 = 0$:

$$\begin{gathered}(1-\lambda) C^R = \left(1-\tfrac{\Pi}{R}\right) b + \left(1-\tfrac{\Pi}{R^*}\right) RER b^{R,*} + (1-\tau_w) \mathcal{S}_{WR} w^R L^R \\ + (1-\lambda) Tr^{R,net} + r^k K - I\end{gathered}$$

where bellow we hint on how the domestic debt level and the NFA behave in the SS. The latter should be consistent with the calibration of the NX (see bellow). We replace $Tr^{R,net} = Tr^{net}$, take into account that Tr^{net} comes from Eq. (67), recall that $I = \delta \frac{K}{L} L$ and considera debt ratios to GDP:

$$\begin{aligned}(1-\lambda) C^R &= \left(1-\frac{\Pi}{R}\right) \frac{b}{Y} Y + \left(1-\frac{\Pi}{R^*}\right) RER \frac{b^{R,*}}{Y} Y + (1-\tau_w) \mathcal{S}_{WR} w^R L^R \\ &+ (1-\lambda) \left\{ \begin{array}{c} \left(\frac{\Pi}{R}-1\right) \frac{b}{Y} Y + \left(\frac{\Pi}{R^*}-1\right) RER \frac{b^{G,*}}{Y} + \tau_w \left[\mathcal{S}_{WR} w^R L^R + \mathcal{S}_{WN} w^N L^N\right] \\ + RER \left(p^*_{cu} \kappa X^{share}_{cu} + p^*_{mo} X^{share}_{mo} + \tau_{cu} p^*_{cu} (1-\kappa) X^{share}_{cu}\right) Y \\ - (\mathcal{S}_{WR}-1) w^R L^R - (\mathcal{S}_{WN}-1) w^N L^N - gY \end{array} \right\} \\ &+ \left(r^k-\delta\right) \frac{K}{L} L,\end{aligned}$$

substitute Y by its equal from Eq. (63):

$$\begin{aligned}(1-\lambda) C^R &= \left[\begin{array}{c} \left(1-\frac{\Pi}{R}\right) \lambda \frac{b}{Y} + \left(1-\frac{\Pi}{R^*}\right) RER \frac{b^{R,*}}{Y} + (1-\lambda) \left(\frac{\Pi}{R^*}-1\right) RER \frac{b^{G,*}}{Y} \\ + (1-\lambda) \left(RER \left(p^*_{cu} \kappa X^{share}_{cu} + p^*_{mo} X^{share}_{mo} + \tau_{cu} p^*_{cu} (1-\kappa) X^{share}_{cu}\right) - g\right) \end{array} \right] \underbrace{F_1 \left(C + \delta \frac{K}{L} L\right)}_{=Y} \\ &+ \left[(1-\tau_w) \mathcal{S}_{WR} + (1-\lambda) \tau_w \mathcal{S}_{WR} - (1-\lambda)(\mathcal{S}_{WR}-1)\right] w^R L^R \\ &+ \left[\tau_w \mathcal{S}_{WN} - (\mathcal{S}_{WN}-1)\right] (1-\lambda) w^N L^N + \left(r^k-\delta\right) \frac{K}{L} L\end{aligned}$$

Define,

$$F_8 \equiv \left\{ \begin{array}{c} \left(1-\frac{\Pi}{R}\right) \lambda \frac{b}{Y} + \left(1-\frac{\Pi}{R^*}\right) RER \frac{b^{R,*}}{Y} + (1-\lambda) \left(\frac{\Pi}{R^*}-1\right) RER \frac{b^{G,*}}{Y} \\ + (1-\lambda) \left[RER \left(p^*_{cu} \kappa X^{share}_{cu} + p^*_{mo} X^{share}_{mo} + \tau_{cu} p^*_{cu} (1-\kappa) X^{share}_{cu}\right) - g\right] \end{array} \right\} F_1,$$

simplify: $(1-\tau_w) \mathcal{S}_{WR} + (1-\lambda) \tau_w \mathcal{S}_{WR} - (1-\lambda)(\mathcal{S}_{WR}-1)$ to $((1-\tau_w) \lambda \mathcal{S}_{WR} + 1 - \lambda)$ and re-

[28] See footnote 27.

organize:

$$(1-\lambda)\,C^R = F_8 C$$
$$+\left[(1-\tau_w)\,\lambda\mathcal{S}_{WR}+1-\lambda\right]w^R L^R$$
$$+\left[\tau_w\mathcal{S}_{WN}-(\mathcal{S}_{WN}-1)\right]w^N(1-\lambda)\,L^N$$
$$+\left[F_8+\frac{r^k-\delta}{\delta}\right]\delta\frac{K}{L}L,$$

define further constants: $F_9 \equiv F_8$, $F_{10} \equiv \left[(1-\tau_w)\,\lambda\mathcal{S}_{WR}+1-\lambda\right]$, $F_{11} \equiv \left[\tau_w\mathcal{S}_{WN}-(\mathcal{S}_{WN}-1)\right]$ and $F_{12} \equiv \left[F_8+\frac{r^k-\delta}{\delta}\right]\delta\frac{K}{L}$, so we finally get:

$$C^R = \frac{F_9}{1-\lambda}C+\frac{F_{10}}{1-\lambda}w^R L^R+F_{11}w^N L^N+\frac{F_{12}}{1-\lambda}L. \tag{70}$$

Combining (69) with (70) yields:

$C = \frac{F_5}{(1-F_7)}w^N L^N+\frac{F_6}{(1-F_7)}w^R L^R+\frac{F_7\delta\frac{K}{L}}{(1-F_7)}L+\frac{(1-\lambda)}{(1-F_7)}C^R$ and $C^R = \frac{F_9}{1-\lambda}C+\frac{F_{10}}{1-\lambda}w^R L^R+F_{11}w^N L^N+\frac{F_{12}}{1-\lambda}L$.

$$C^R = \frac{F_9}{1-\lambda}\left(\frac{F_5}{1-F_7}w^N L^N+\frac{F_6}{1-F_7}w^R L^R+\frac{F_7\delta\frac{K}{L}}{1-F_7}L+\frac{(1-\lambda)}{1-F_7}C^R\right)+\frac{F_{10}}{1-\lambda}w^R L^R+F_{11}w^N L^N+\frac{F_{12}}{1-\lambda}L$$

$$\left[1-\frac{F_9}{1-F_7}\right]C^R = \frac{F_9}{1-\lambda}\left(\frac{F_5}{1-F_7}w^N L^N+\frac{F_6}{1-F_7}w^R L^R+\frac{F_7\delta\frac{K}{L}}{1-F_7}L\right)+\frac{F_{10}}{1-\lambda}w^R L^R+F_{11}L^N+\frac{F_{12}}{1-\lambda}L$$

$$\left[\frac{1-F_7-F_9}{1-F_7}\right]C^R = \frac{1}{1-\lambda}\left(\frac{F_9F_6}{1-F_7}+F_{10}\right)w^R L^R+\frac{1}{1-\lambda}\left(\frac{F_9F_5}{1-F_7}+(1-\lambda)\,F_{11}\right)w^N L^N+\frac{1}{1-\lambda}\left(\frac{F_9F_7\delta\frac{K}{L}}{(1-F_7)}+F_{12}\right)$$

$$C^R = \left[\frac{1-F_7}{1-F_7-F_9}\right]\frac{1}{1-\lambda}\left[\left(\frac{F_9F_6}{1-F_7}+F_{10}\right)w^R L^R+\left(\frac{F_9F_5}{1-F_7}+(1-\lambda)\,F_{11}\right)w^N L^N+\left(\frac{F_9F_7\delta\frac{K}{L}}{1-F_7}+F_{12}\right)L\right]. \tag{71}$$

now plug (71) into Eq. (69):

$C = \frac{F_5}{(1-F_7)}w^N L^N+\frac{F_6}{(1-F_7)}w^R L^R+\frac{F_7\delta\frac{K}{L}}{(1-F_7)}L$
$+\frac{(1-\lambda)}{(1-F_7)}\left\{\left[\frac{1-F_7}{1-F_7-F_9}\right]\frac{1}{1-\lambda}\left[\left(\frac{F_9F_6}{1-F_7}+F_{10}\right)w^R L^R+\left(\frac{F_9F_5}{1-F_7}+(1-\lambda)\,F_{11}\right)w^N L^N+\left(\frac{F_9F_7\delta\frac{K}{L}}{1-F_7}+F_{12}\right)L\right]\right\}$

grouping:

$C = \left\{\frac{F_5}{(1-F_7)}+\frac{(1-\lambda)}{(1-F_7)}\left[\frac{1-F_7}{1-F_7-F_9}\right]\frac{1}{1-\lambda}\left(\frac{F_9F_5}{1-F_7}+(1-\lambda)\,F_{11}\right)\right\}w^N L^N$
$+\left\{\frac{F_6}{(1-F_7)}+\frac{(1-\lambda)}{(1-F_7)}\left[\frac{1-F_7}{1-F_7-F_9}\right]\frac{1}{1-\lambda}\left(\frac{F_9F_6}{1-F_7}+F_{10}\right)\right\}w^R L^R$
$+\left\{\frac{F_7\delta\frac{K}{L}}{(1-F_7)}+\frac{(1-\lambda)}{(1-F_7)}\left[\frac{1-F_7}{1-F_7-F_9}\right]\frac{1}{1-\lambda}\left(\frac{F_9F_7\delta\frac{K}{L}}{1-F_7}+F_{12}\right)\right\}L$

symplifying:

$$C = \left\{F_5 + \left[\tfrac{1-F_7}{1-F_7-F_9}\right]\left(\tfrac{F_9 F_5}{1-F_7} + (1-\lambda) F_{11}\right)\right\}(1-F_7)^{-1} w^N L^N$$
$$+ \left\{F_6 + \left[\tfrac{1-F_7}{1-F_7-F_9}\right]\left(\tfrac{F_9 F_6}{1-F_7} + F_{10}\right)\right\}(1-F_7)^{-1} w^R L^R$$
$$+ \left\{F_7 \delta \tfrac{K}{L} + \left[\tfrac{1-F_7}{1-F_7-F_9}\right]\left(\tfrac{F_9 F_7 \delta \frac{K}{L}}{1-F_7} + F_{12}\right)\right\}(1-F_7)^{-1} L$$

now plug the latter equation into $\lambda C^N = F_5 w^N L^N + F_6 w^R L^R + F_7 \left(C + \delta \frac{K}{L} L\right)$:

$$\lambda C^N = F_5 w^N L^N + F_6 w^R L^R + F_7 \left\{ \begin{array}{c} \left\{F_5 + \left[\tfrac{1-F_7}{1-F_7-F_9}\right]\left(\tfrac{F_9 F_5}{1-F_7} + (1-\lambda) F_{11}\right)\right\}(1-F_7)^{-1} w^N L^N \\ + \left\{F_6 + \left[\tfrac{1-F_7}{1-F_7-F_9}\right]\left(\tfrac{F_9 F_6}{1-F_7} + F_{10}\right)\right\}(1-F_7)^{-1} w^R L^R \\ + \left\{F_7 \delta \tfrac{K}{L} + \left[\tfrac{1-F_7}{1-F_7-F_9}\right]\left(\tfrac{F_9 F_7 \delta \frac{K}{L}}{1-F_7} + F_{12}\right)\right\}(1-F_7)^{-1} L \end{array} \right\} + F_7 \delta \frac{K}{L} L. \tag{72}$$

and grouping:

$$\begin{aligned} \lambda C^N \quad = \quad & \left\{ \underbrace{F_6 + \frac{F_7 F_6}{1-F_7}} + \frac{F_7 \left[\frac{1-F_7}{1-F_7-F_9}\right]\left(\frac{F_9 F_6}{1-F_7} + F_{10}\right)}{1-F_7} \right\} w^R L^R \\ & + \left\{ \underbrace{F_5 + \frac{F_7 F_5}{1-F_7}} + \frac{F_7 \left[\frac{1-F_7}{1-F_7-F_9}\right]\left(\frac{F_9 F_5}{1-F_7} + (1-\lambda) F_{11}\right)}{1-F_7} \right\} w^N L^N \\ & + F_7 \left\{ \underbrace{\delta \frac{K}{L} + \frac{F_7 \delta \frac{K}{L}}{1-F_7}}_{\frac{\delta \frac{K}{L} - F_7 \delta \frac{K}{L} + F_7 \delta \frac{K}{L}}{1-F_7}} + \frac{\left[\frac{1-F_7}{1-F_7-F_9}\right]\left(\frac{F_9 F_7 \delta \frac{K}{L}}{1-F_7} + F_{12}\right)}{1-F_7} \right\} L. \end{aligned}$$

the terms signaled with $\underbrace{\quad}$ reduce, so we get:

$$\begin{aligned} \lambda C^N \quad = \quad & \frac{1}{1-F_7} \left\{ F_6 + F_7 \left[\frac{1-F_7}{1-F_7-F_9}\right]\left(\frac{F_9 F_6}{1-F_7} + F_{10}\right)\right\} w^R L^R \\ & + \frac{1}{1-F_7} \left\{ F_5 + F_7 \left[\frac{1-F_7}{1-F_7-F_9}\right]\left(\frac{F_9 F_5}{1-F_7} + (1-\lambda) F_{11}\right)\right\} w^N L^N \\ & + \frac{F_7}{1-F_7} \left\{ \delta \frac{K}{L} + \left[\frac{1-F_7}{1-F_7-F_9}\right]\left(\frac{F_9 F_7 \delta \frac{K}{L}}{1-F_7} + F_{12}\right)\right\} L. \end{aligned} \tag{73}$$

If we have had only one labor supply as in Forni *et al.* (2007), the labor supply for the Ricardian agent coupled with $C^R = f(L)$ from the previous expression gives a solution for L and C. In our case, we would have three unknowns but just two equations ($C^R = f(L^R, L^N)$ and $C^R = f(L^R)$ from labor supply). Therefore, we are compelled to find the solution for a system of equations comprising the variables: C^N, C^R, L^N, L^R, L, Λ^N and Λ^R and the following equations:

$$\left\{\begin{array}{l}
(1-F_7)\,\lambda C^N = \left\{F_6 + F_7\left[\frac{1-F_7}{1-F_7-F_9}\right]\left(\frac{F_9F_6}{1-F_7}+F_{10}\right)\right\} w^R L^R \\
+\left\{F_5 + F_7\left[\frac{1-F_7}{1-F_7-F_9}\right]\left(\frac{F_9F_5}{1-F_7}+(1-\lambda)\,F_{11}\right)\right\} w^N L^N \\
+F_7\left\{\delta\frac{K}{L}+\left[\frac{1-F_7}{1-F_7-F_9}\right]\left(\frac{F_9F_7\delta\frac{K}{L}}{1-F_7}+F_{12}\right)\right\}L, \quad \text{Eq. (73)} \\
C^R = \left[\frac{1-F_7}{1-F_7-F_9}\right]\frac{1}{1-\lambda}\left[\left(\frac{F_9F_6}{1-F_7}+F_{10}\right)w^R L^R + \left(\frac{F_9F_5}{1-F_7}+(1-\lambda)\,F_{11}\right)w^N L^N + \left(\frac{F_9F_7\delta\frac{K}{L}}{1-F_7}+F_{12}\right)L\right], \text{Eq. (71)} \\
L = \left[\lambda^{1/\eta_L}\left(L^N\right)^{1-\frac{1}{\eta_L}} + (1-\lambda)^{1/\eta_L}\left(L^R\right)^{1-\frac{1}{\eta_L}}\right]^{\frac{\eta_L}{(\eta_L-1)}}, \quad \text{aggreg. labor demand} \\
L^N = \left\{\frac{\Lambda^N}{\zeta\mu_{WN}}\mathcal{S}_{WN}w^N\right\}^{1/\sigma_L}, \text{labor supply N} \\
L^R = \left\{\frac{\Lambda^R}{\zeta\mu_{WR}}\mathcal{S}_{WR}w^R\right\}^{1/\sigma_L}, \text{labor supply R} \\
\Lambda^N = (1-\beta b)\left[C^N(1-b)\right]^{-1}, U_c^N \\
\Lambda^R = (1-\beta b)\left[C^R(1-b)\right]^{-1}, U_c^R
\end{array}\right.$$

Notice that $\left[\frac{T_H}{\mu_H}A_H\left(\frac{K}{L}\right)^{\gamma} - F_2\delta\frac{K}{L}\right]L = F_2\left[\lambda C^N + (1-\lambda)\,C^R\right]$, is redundant since we employed both CBCs.

Next, with the solution of L evaluate the domestic production function at the SS value of L yields:

$$\begin{aligned}
Y_H &= A_H\left(\frac{K}{L}\right)^{\gamma}L - \frac{(\mu_H - T_H)}{\mu_H}A_H\left(\frac{K}{L}\right)^{\gamma}L \\
&= \frac{T_H}{\mu_H}A_H\left(\frac{K}{L}\right)^{\gamma}L, \qquad (74)
\end{aligned}$$

and from (56) FC_H turns out to be:

$$FC_H = \frac{(\mu_H - T_H)}{\mu_H}A_H\left(\frac{K}{L}\right)^{\gamma}L. \qquad (75)$$

Next, employ the relationship (62) implying balanced trade in order to isolate Y^*:

$$(\alpha_C^* + \alpha_I^*)\left(\frac{T_H}{RER}\right)^{-\eta}Y^* = T\left[\alpha\,(T_H T)^{-\eta}(C+I) + \alpha_G\,(T_{GH}T)^{-\eta}G\right] - \frac{1}{T_H}RER\left(p_{cu}^*X_{cu}^{share} + p_{mo}^*X_{mo}^{share}\right)Y,$$

$$\begin{aligned}
(\alpha_C^* + \alpha_I^*)\left(\frac{T_H}{RER}\right)^{-\eta}Y^* &= T\alpha\,(T_H T)^{-\eta}(C+I) \\
&+\left[T\,(T_{GH}T)^{-\eta}\alpha_G\left[\frac{(1-\alpha)+\alpha T^{1-\eta}}{(1-\alpha_G)+\alpha_G T^{1-\eta}}\right]^{\frac{1}{1-\eta}}g - \frac{1}{T_H}RER\left(p_{cu}^*X_{cu}^{share} + p_{mo}^*X_{mo}^{share}\right)\right]\underbrace{Y,}_{\text{Eq. (63)}}
\end{aligned}$$

$$\begin{aligned}
(\alpha_C^* + \alpha_I^*)\left(\frac{T_H}{RER}\right)^{-\eta}Y^* &= T\,(T_H T)^{-\eta}\alpha\,(C+I) \\
&+\left[T\,(T_H T)^{-\eta}\alpha_G\left[\frac{(1-\alpha)+\alpha T^{1-\eta}}{(1-\alpha_G)+\alpha_G T^{1-\eta}}\right]^{\frac{1}{1-\eta}}g - \frac{RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)}{T_H}\right]F_1\,(C+I)
\end{aligned}$$

$$(\alpha_C^* + \alpha_I^*)\left(\frac{T_H}{RER}\right)^{-\eta}Y^* = \left\{\alpha T^{1-\eta}T_H^{-\eta} + \left[\begin{array}{c} T\,(T_H T)^{-\eta}\alpha_G\left[\frac{(1-\alpha)+\alpha T^{1-\eta}}{(1-\alpha_G)+\alpha_G T^{1-\eta}}\right]^{\frac{1}{1-\eta}}g \\ -\frac{RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)}{T_H}\end{array}\right]F_1\right\}(C+I),$$

isolating Y^* and assuming that RER equals 1:

$$Y^* = \frac{\left(C+\delta\frac{K}{L}L\right)}{(\alpha_C^* + \alpha_I^*)\,T_H^{-\eta}}\left\{\alpha T^{1-\eta}T_H^{-\eta} + \left[\begin{array}{c} T\,(T_H T)^{-\eta}\alpha_G\left[\frac{(1-\alpha)+\alpha T^{1-\eta}}{(1-\alpha_G)+\alpha_G T^{1-\eta}}\right]^{\frac{1}{1-\eta}}g \\ -\frac{RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)}{T_H}\end{array}\right]F_1\right\},$$

$$Y^{*}=\frac{\left(C+\delta\frac{K}{L}L\right)}{\left(\alpha_{C}^{*}+\alpha_{I}^{*}\right)}F_{13}, \tag{76}$$

where C and $I=\delta\frac{K}{L}L$ were calculated above and we defined:

$$F_{13}=\alpha T^{1-\eta}+\left[T^{1-\eta}\alpha_{G}\left[\frac{(1-\alpha)+\alpha T^{1-\eta}}{(1-\alpha_{G})+\alpha_{G}T^{1-\eta}}\right]^{\frac{1}{1-\eta}}g-T_{H}^{\eta-1}RER\left(p_{cu}^{*}X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right)\right]F_{1}.$$

Further SS substitutions are straightforward to calculate given previous relationships.

C Steady State (incomplete asset markets)

Recall that the wedge in interest rates is one at the SS, i.e.,

$$\exp\left(-\phi_{a}\left(0\right)-\phi_{\Delta S}(0)+\ln(1)\right)=\exp(0)=1.$$

This means that the relationships described in the previous section still hold.

Consider the real net exports, equation (27), evaluated at the steady state:

$$\begin{aligned}NX &\equiv \left(RER\left(p_{cu}^{*}X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right)+\frac{(1-\alpha_{G})\,T_{GH}^{1-\eta}g}{MC_{H}}\right)Y \\ &\quad +\frac{1}{MC_{H}}\left\{T_{H}^{1-\eta}\left(1-\alpha\right)\left(C_{t}+I_{t}\right)+\frac{\left(\alpha_{C}^{*}+\alpha_{I}^{*}\right)T_{H}^{1-\eta}}{RER^{-\eta}}Y^{*}\right\}-T_{H}FC_{H}-(C+I)-gY,\end{aligned}$$

Since (47), then:

$$\begin{aligned}NX &\equiv \left\{RER\left(p_{cu}^{*}X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right)+\frac{(1-\alpha_{G})\,T_{GH}^{1-\eta}\mu_{H}g}{T_{H}}\right\}Y \\ &\quad +\mu_{H}\left\{\underbrace{T_{H}^{-\eta}\left(1-\alpha\right)\left(C_{t}+I_{t}\right)}_{T_{H}(C_{H}+I_{H})}+\underbrace{\frac{\left(\alpha_{C}^{*}+\alpha_{I}^{*}\right)T_{H}^{-\eta}}{RER^{-\eta}}Y^{*}}_{T_{H}\left(C_{H}^{*}+I_{H}^{*}\right)}\right\}-T_{H}FC_{H}-\underbrace{(C+I)}_{T_{H}(C_{H}+I_{H})+T_{H}T(C_{F}+I_{F})}-gY,\end{aligned}$$

dividing by Y:

$$\begin{aligned}NX &\equiv RER\left(p_{cu}^{*}X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right)+\mu_{H}\left\{T_{H}^{-\eta}\left(1-\alpha\right)\frac{(C+I)}{Y}+\left(1-\alpha_{G}\right)T_{GH}^{1-\eta}\frac{g}{T_{H}}+\frac{\left(\alpha_{C}^{*}+\alpha_{I}^{*}\right)T_{H}^{-\eta}}{RER^{-\eta}}\frac{Y^{*}}{Y}\right\} \\ &\quad -\frac{T_{H}FC_{H}}{Y}-\frac{(C+I)}{Y}-gY,\end{aligned}$$

summing and substracting in the RHS by $T_{H}\left[\left(1-\alpha\right)T_{H}^{-\eta}\frac{(C_{t}+I_{t})}{Y}+\left(1-\alpha_{G}\right)T_{GH}^{1-\eta}\frac{g}{T_{H}}+\frac{(\alpha_{C}^{*}+\alpha_{I}^{*})T_{H}^{-\eta}}{RER^{-\eta}}Y^{*}\frac{1}{Y}\right]$

to cancel out the fix cost:

$$\begin{aligned}\frac{NX}{Y} \quad &\equiv \quad RER\left(p^*_{cu}X^{share}_{cu} + p^*_{mo}X^{share}_{mo}\right) \\ &\underbrace{+\left(\mu_H - T_H\right)\left[T_H^{-\eta}\left(1-\alpha\right)\frac{(C+I)}{Y} + \left(1-\alpha_G\right)T_{GH}^{1-\eta}\frac{g}{T_H} + \frac{\left(\alpha^*_C + \alpha^*_I\right)T_H^{-\eta}}{RER^{-\eta}}\frac{Y^*}{Y}\right] - \frac{T_H FC_H}{Y}}_{=0} \\ &+T_H\left[T_H^{-\eta}\left(1-\alpha\right)\frac{(C+I)}{Y} + \left(1-\alpha_G\right)T_{GH}^{1-\eta}\frac{g}{T_H} + \frac{\left(\alpha^*_C + \alpha^*_I\right)T_H^{-\eta}}{RER^{-\eta}}\frac{Y^*}{Y}\right] - \frac{(C+I)}{Y} - gY,\end{aligned}$$

$$\begin{aligned}\frac{NX}{Y} \quad &\equiv \quad RER\left(p^*_{cu}X^{share}_{cu} + p^*_{mo}X^{share}_{mo}\right) + \left[\frac{\left(1-\alpha_G\right)T_{GH}^{1-\eta}}{T_H} - 1\right]g \\ &+T_H\underbrace{\left(\alpha^*_C + \alpha^*_I\right)\frac{T_H^{-\eta}}{RER^{-\eta}}Y^*}_{C^*_H + I^*_H}\frac{1}{Y} + \left[\left(1-\alpha\right)T_H^{1-\eta} - 1\right]\frac{(C+I)}{Y},\end{aligned}$$

At the steady state, $T_H = T = RER = 1$, so the net exports simplify to:

$$\frac{NX}{Y} \equiv \underbrace{RER\left(p^*_{cu}X^{share}_{cu} + p^*_{mo}X^{share}_{mo}\right) + \left(\alpha^*_C + \alpha^*_I\right)\frac{Y^*}{Y}}_{=\text{Xs}} - \underbrace{\alpha_G g - \alpha\frac{(C+I)}{Y}}_{=\text{Ms}}.$$

Now, consider Y^* from Eq. (76). Assuming $T_H = T = RER = 1$ implies that $F_{13} = \alpha + \left(\alpha_G g - RER\left(p^*_{cu}X^{share}_{cu} + p^*_{mo}X^{share}_{mo}\right)\right)F_1$ and $F_1 = \frac{(1-\alpha)}{1-[(1-\alpha_G)g + RER(p^*_{cu}X^{share}_{cu} + p^*_{mo}X^{share}_{mo})]}$, so Eq. (76) becomes:

$$Y^* = \frac{\left(C + \delta\frac{K}{L}L\right)}{\left(\alpha^*_C + \alpha^*_I\right)}\left[\alpha + \frac{\left[\alpha_G g - RER\left(p^*_{cu}X^{share}_{cu} + p^*_{mo}X^{share}_{mo}\right)\right]\left(1-\alpha\right)}{1 - \left[\left(1-\alpha_G\right)g + RER\left(p^*_{cu}X^{share}_{cu} + p^*_{mo}X^{share}_{mo}\right)\right]}\right], \tag{77}$$

$$
\begin{aligned}
Y^* &= \frac{(C+I)}{(\alpha_C^*+\alpha_I^*)}\left\{\frac{\begin{pmatrix}\alpha-\alpha\left[(1-\alpha_G)\,g+RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\right]\\ +\left[\alpha_G g-RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\right](1-\alpha)\end{pmatrix}}{1-\left[(1-\alpha_G)\,g+RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\right]}\right\},\\
&= \frac{(C+I)}{(\alpha_C^*+\alpha_I^*)}\left\{\frac{\begin{pmatrix}\alpha-\left[\alpha\,(1-\alpha_G)\,g+\alpha RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\right]+\alpha_G g\\ -RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)-\alpha\left[\alpha_G g-RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\right]\end{pmatrix}}{1-\left[(1-\alpha_G)\,g+RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\right]}\right\},\\
&= \frac{(C+I)}{(\alpha_C^*+\alpha_I^*)}\left\{\frac{\begin{pmatrix}\alpha-\alpha g+\alpha\alpha_G g-\alpha RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\\ +\alpha_G g-RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\\ -\alpha\alpha_G g+\alpha RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\end{pmatrix}}{1-\left[(1-\alpha_G)\,g+RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\right]}\right\},\\
&= \frac{(C+I)}{(\alpha_C^*+\alpha_I^*)}\left\{\frac{\alpha-\alpha g+\alpha_G g-RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)}{1-\left[(1-\alpha_G)\,g+RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\right]}\right\},\\
&= \frac{(C+I)}{(\alpha_C^*+\alpha_I^*)}\left\{\frac{\alpha+(\alpha_G-\alpha)\,g-RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)}{1-\left[(1-\alpha_G)\,g+RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\right]}\right\},
\end{aligned}
$$

and $Y=\frac{1-\alpha}{1-\{(1-\alpha_G)g+RER(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share})\}}(C+I)$, from (63), so we get:

$$
\begin{aligned}
\frac{NX}{Y} &= \underbrace{RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)+(\alpha_C^*+\alpha_I^*)\frac{Y^*}{Y}}_{=\text{Xs}}-\underbrace{\alpha_G g-\alpha\frac{(C+I)}{Y}}_{=\text{Ms}},\\
&= RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)+(\alpha_C^*+\alpha_I^*)\frac{\frac{(C+I)}{(\alpha_C^*+\alpha_I^*)}\left\{\frac{\alpha+(\alpha_G-\alpha)g-RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)}{1-[(1-\alpha_G)g+RER(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share})]}\right\}}{\frac{1-\alpha}{1-[(1-\alpha_G)g+RER(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share})]}(C+I)}\\
&\quad -\alpha_G g-\alpha\frac{(C+I)}{\frac{1-\alpha}{1-\{(1-\alpha_G)g+RER(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share})\}}(C+I)},\\
&= RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\\
&\quad +\frac{\left\{1-\left[(1-\alpha_G)\,g+RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\right]\right\}}{(1-\alpha)}\left\{\frac{\alpha+(\alpha_G-\alpha)\,g-RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)}{1-\left[(1-\alpha_G)\,g+RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\right]}\right\}\\
&\quad -\alpha_G g-\frac{\alpha-\alpha\left[(1-\alpha_G)\,g+RER\left(p_{cu}^*X_{cu}^{share}+p_{mo}^*X_{mo}^{share}\right)\right]}{1-\alpha},
\end{aligned}
$$

$$
\begin{aligned}
\frac{NX}{Y} &= RER\left(p_{cu}^{*}X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right)+\frac{\alpha+\left(\alpha_G-\alpha\right)g-RER\left(p_{cu}^{*}X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right)}{1-\alpha} \\
&\quad -\alpha_G g-\frac{\alpha-\alpha\left[\left(1-\alpha_G\right)g+RER\left(p_{cu}^{*}X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right)\right]}{1-\alpha}, \\
&= \frac{\begin{pmatrix} \left(1-\alpha\right)RER\left(p_{cu}^{*}X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right)+\alpha+\left(\alpha_G-\alpha\right)g-RER\left(p_{cu}^{*}X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right) \\ -\alpha_G g\left(1-\alpha\right)-\alpha+\alpha\left\{\left(1-\alpha_G\right)g+RER\left(p_{cu}^{*}X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right)\right\} \end{pmatrix}}{1-\alpha}, \\
&= \frac{\begin{pmatrix} RER\left(p_{cu}^{*}X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right)-\alpha RER\left(p_{cu}^{*}X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right)+\alpha+\alpha_G g-\alpha g \\ -RER\left(p_{cu}^{*}X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right)-\alpha_G g\left(1-\alpha\right)-\alpha+\alpha g-\alpha\alpha_G g+\alpha RER\left(p_{cu}^{*}X_{cu}^{share}+p_{mo}^{*}X_{mo}^{share}\right) \end{pmatrix}}{1-\alpha}, \\
&= \frac{0}{1-\alpha}=0.
\end{aligned}
$$

D Calvo wage and price setting

D.1 Wage Equation

D.1.1 Normal model

First, we derive the wage equation for Ricardian households. Restrictions are the relevant labor demand faced by them is a slightly modified version of the labor demand that results from the firm's problem and the CBC Eq. (3). We write the Lagrangian in real terms as follows (only terms that matter are displayed):

$$
\mathrm{E}_t\left[\begin{array}{c} \sum_{a=0}^{\infty}\left(\phi_L\beta\right)^a\left[...-\bar{\zeta}_L\frac{\zeta_{L,t+a}}{1+\sigma_L}\left[\left(1-\lambda\right)^{-1}\left(\frac{\tilde{W}_t^R(h)}{P_t}\right)^{-\varepsilon_{LR}}\left[\frac{P_t}{P_{t+a}}\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\left(\bar{\Pi}\right)^{a(1-\xi_L)}\right]^{-\varepsilon_{LR}}\left(\frac{W_{t+a}^R}{P_{t+a}}\right)^{\varepsilon_{LR}}L_{t+a}^R\right]^{1+\sigma_L}\right. \\ +\sum_{a=0}^{\infty}\left(\phi_L\beta\right)^a\Lambda_{t+a}^R\left(...+\mathcal{S}_{WR}\frac{\tilde{W}_t^R(h)}{P_t}\left(1-\lambda\right)^{-1}\left(\frac{\tilde{W}_t^R(h)}{P_t}\right)^{-\varepsilon_{LR}}\left[\frac{P_t}{P_{t+a}}\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\left(\bar{\Pi}\right)^{a(1-\xi_L)}\right]^{1-\varepsilon_{LR}}\left(\frac{W_{t+a}^R}{P_{t+a}}\right)^{\varepsilon_{LR}}L_{t+a}^R\right. \end{array}\right.
$$

We differentiate it w.r.t. $\frac{\tilde{W}_t^R(h)}{P_t}$, so that we get the following FOC:

$$
\mathrm{E}_t\left[\begin{array}{c} \sum_{a=0}^{\infty}\left(\phi_L\beta\right)^a\bar{\zeta}_L\left(-\zeta_{L,t+a}\right)\left[\left(1-\lambda\right)^{-1}\left(\frac{\tilde{W}_t^R(h)}{P_t}\right)^{-\varepsilon_{LR}}\left[\frac{P_t}{P_{t+a}}\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\left(\bar{\Pi}\right)^{a(1-\xi_L)}\right]^{-\varepsilon_{LR}}\left(\frac{W_{t+a}^R}{P_{t+a}}\right)^{\varepsilon_{LR}}L_{t+a}^R\right]^{\sigma_L}\times \\ \left(-\varepsilon_{LR}\right)\left(1-\lambda\right)^{-1}\left(\frac{\tilde{W}_t^R(h)}{P_t}\right)^{\varepsilon_{LR}-1}\left[\frac{P_t}{P_{t+a}}\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\left(\bar{\Pi}\right)^{a(1-\xi_L)}\right]^{-\varepsilon_{LR}}\left(\frac{W_{t+a}^R}{P_{t+a}}\right)^{\varepsilon_{LR}}L_{t+a}^R \end{array}\right]+
$$

$$
\mathrm{E}_t\left[\sum_{a=0}^{\infty}\left(\phi_L\beta\right)^a\Lambda_{t+a}^R\left(1-\varepsilon_{LR}\right)\mathcal{S}_{WR}\left(1-\lambda\right)^{-1}\left(\frac{\tilde{W}_t^R(h)}{P_t}\right)^{-\varepsilon_{LR}}\left[\frac{P_t}{P_{t+a}}\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\left(\bar{\Pi}\right)^{a(1-\xi_L)}\right]^{1-\varepsilon_{LR}}\left(\frac{W_{t+a}^R}{P_{t+a}}\right)^{\varepsilon_{LR}}L_{t+a}^R\right]= 0,
$$

taking invariant elements outside the summation:

$$\bar{\zeta}_L \varepsilon_{LR} \lambda^{-(1+\sigma_L)} \left(\frac{\tilde{W}_t^R(h)}{P_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)-1} \mathrm{E}_t \left[\begin{array}{c} \sum_{a=0}^{\infty} (\phi_L\beta)^a \zeta_{L,t+a} \left[\frac{P_t}{P_{t+a}} \left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L} (\bar{\Pi})^{a(1-\xi_L)} \right]^{-\varepsilon_{LR}(1+\sigma_L)} \\ \times \left(w_{t+a}^R\right)^{\varepsilon_{LR}(1+\sigma_L)} \left(L_{t+a}^R\right)^{1+\sigma_L} \end{array} \right] +$$

$$(1-\varepsilon_{LR}) \mathcal{S}_{WR} (1-\lambda)^{-1} \left(\frac{\tilde{W}_t^R(h)}{P_t}\right)^{-\varepsilon_{LR}} \mathrm{E}_t \left[\begin{array}{c} \sum_{a=0}^{\infty} (\phi_L\beta)^a \Lambda_{t+a}^R \left[\frac{P_t}{P_{t+a}} \left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L} (\bar{\Pi})^{a(1-\xi_L)} \right]^{1-\varepsilon_{LR}} \\ \times \left(w_{t+a}^R\right)^{\varepsilon_{LR}} L_{t+a}^R \end{array} \right] =$$

$$0,$$

$$\bar{\zeta}_L \varepsilon_{LR} \lambda^{-(1+\sigma_L)} \left(\frac{\tilde{W}_t^R(h)}{P_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)-1} \mathrm{E}_t \left[\begin{array}{c} \sum_{a=0}^{\infty} (\phi_L\beta)^a \zeta_{L,t+a} \left[\frac{P_t}{P_{t+a}} \left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L} (\bar{\Pi})^{a(1-\xi_L)} \right]^{-\varepsilon_{LR}(1+\sigma_L)} \\ \times \left(w_{t+a}^R\right)^{\varepsilon_{LR}(1+\sigma_L)} \left(L_{t+a}^R\right)^{1+\sigma_L} \end{array} \right] =$$

$$(\varepsilon_{LR}-1) \mathcal{S}_{WR} (1-\lambda)^{-1} \left(\frac{\tilde{W}_t^R(h)}{P_t}\right)^{-\varepsilon_{LR}} \mathrm{E}_t \left[\begin{array}{c} \sum_{a=0}^{\infty} (\phi_L\beta)^a \Lambda_{t+a}^R \left[\frac{P_t}{P_{t+a}} \left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L} (\bar{\Pi})^{a(1-\xi_L)} \right]^{1-\varepsilon_{LR}} \\ \times \left(w_{t+a}^R\right)^{\varepsilon_{LR}} L_{t+a}^R \end{array} \right],$$

$$\frac{\bar{\zeta}_L \varepsilon_{LR} (1-\lambda)^{-\sigma_L}}{(\varepsilon_{LR}-1)\mathcal{S}_{WR}} \frac{\mathrm{E}_t \left[\sum_{a=0}^{\infty} (\phi_L\beta)^a \zeta_{L,t+a} \left[\frac{P_t}{P_{t+a}} \left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L} (\bar{\Pi})^{a(1-\xi_L)} \right]^{-\varepsilon_{LR}(1+\sigma_L)} \left(w_{t+a}^R\right)^{\varepsilon_{LR}(1+\sigma_L)} \left(L_{t+a}^R\right)^{1+\sigma_L} \right]}{\mathrm{E}_t \left[\sum_{a=0}^{\infty} (\phi_L\beta)^a \Lambda_{t+a}^R \left[\frac{P_t}{P_{t+a}} \left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L} (\bar{\Pi})^{a(1-\xi_L)} \right]^{1-\varepsilon_{LR}} \left(w_{t+a}^R\right)^{\varepsilon_{LR}} L_{t+a}^R \right]} =$$

$$\left(\frac{\tilde{W}_t^R(h)}{P_t}\right)^{-\varepsilon_{LR}} \left(\frac{\tilde{W}_t^R(h)}{P_t}\right)^{\varepsilon_{LR}(1+\sigma_L)+1}$$

$$\left(\frac{\tilde{W}_t^R(h)}{P_t}\right)^{1+\varepsilon_{LR}\sigma_L} =$$

$$= \frac{\bar{\zeta}_L \varepsilon_{LR}}{(\varepsilon_{LR}-1) \mathcal{S}_{WR} (1-\lambda)^{\sigma_L}} \frac{\mathrm{E}_t \left[\sum_{a=0}^{\infty} (\phi_L\beta)^a \zeta_{L,t+a} \left[\frac{P_t}{P_{t+a}} \left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L} (\bar{\Pi})^{a(1-\xi_L)} \right]^{-\varepsilon_{LR}(1+\sigma_L)} \left(w_{t+a}^R\right)^{\varepsilon_{LR}(1+\sigma_L)} \left(L_{t+a}^R\right)^{1+\sigma_L} \right]}{\mathrm{E}_t \left[\sum_{a=0}^{\infty} (\phi_L\beta)^a \Lambda_{t+a} \left[\frac{P_t}{P_{t+a}} \left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L} (\bar{\Pi})^{a(1-\xi_L)} \right]^{1-\varepsilon_{LR}} \left(w_{t+a}^R\right)^{\varepsilon_{LR}} L_{t+a}^R \right]}$$

$$\mathcal{H}_{1,t}^{R,w} = \zeta_t \left(w_t^R\right)^{\varepsilon_{LR}(1+\sigma_L)} \left(L_t^R\right)^{1+\sigma_L} + \phi_L\beta E_t \left\{ \left[\frac{\Pi_{t+1}}{(\Pi_t)^{\xi_L} \bar{\Pi}^{(1-\xi_L)}} \right]^{\varepsilon_{LR}(1+\sigma_L)} \underbrace{\zeta_{L,t+1} w_{t+1}^{\varepsilon_{LR}(1+\sigma_L)} L_{t+1}^{1+\sigma_L} + \ldots}_{\mathcal{H}_{1,t+1}^{R,w}} \right\},$$

$$\mathcal{H}_{1,t}^{R,w} = \zeta_t \left(w_t^R\right)^{\varepsilon_{LR}(1+\sigma_L)} \left(L_t^R\right)^{1+\sigma_L} + \phi_L\beta E_t \left\{ \left[\frac{\Pi_{t+1}}{(\Pi_t)^{\xi_L} \bar{\Pi}^{(1-\xi_L)}} \right]^{\varepsilon_{LR}(1+\sigma_L)} \mathcal{H}_{1,t+1}^{R,w} \right\},$$

$$\mathcal{H}_{2,t}^{R,w}=\left(w_t^R\right)^{\varepsilon_{LR}}\Lambda_t^R L_t^R+\phi_L\beta E_t\left\{\left[\frac{\Pi_{t+1}}{(\Pi_t)^{\xi_L}\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LR}-1\mathcal{H}R,w}\mathcal{H}_{2,t+1}^{R,w}\right\},$$

rearranging yields,

$$\begin{aligned}\left(\frac{\tilde{W}_t^R(h)}{P_t}\right)^{1+\varepsilon_{LR}\sigma_L} &= \frac{\varepsilon_{LR}}{(\varepsilon_{LR}-1)\,\mathcal{S}_{WR}}\frac{\bar{\zeta}_L}{(1-\lambda)^{\sigma_L}}\frac{\mathcal{H}_{1,t}^{R,w}}{\mathcal{H}_{2,t}^{R,w}},\\ \frac{\tilde{W}_t^R(h)}{P_t} &= \left(\frac{\mu_{WR}}{\mathcal{S}_{WR}}\frac{\bar{\zeta}_L}{(1-\lambda)^{\sigma_L}}\frac{\mathcal{H}_{1,t}^{R,w}}{\mathcal{H}_{2,t}^{R,w}}\right)^{\frac{1}{1+\varepsilon_{LR}\sigma_L}}. \qquad (78)\end{aligned}$$

where $\mu_{WR}\equiv\frac{\varepsilon_{LR}}{(\varepsilon_{LR}-1)}$ is the markup associated.

The aggregate wage dynamics are given by:

$$\left(w_t^R\right)^{1-\varepsilon_{LR}}=(1-\phi_L)\left(\frac{\mu_{WR}}{\mathcal{S}_{WR}}\frac{\bar{\zeta}_L}{(1-\lambda)^{\sigma_L}}\frac{\mathcal{H}_{1,t}^{w}}{\mathcal{H}_{2,t}^{w}}\right)^{\frac{1-\varepsilon_{LR}}{1+\varepsilon_{LR}\sigma_L}}+\phi_L\left(w_{t-1}^R\right)^{1-\varepsilon_{LR}}\left[\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\left(\bar{\Pi}\right)^{(1-\xi_L)}}\right]^{\varepsilon_{LR}-1}. \qquad (79)$$

The wage dispersion is defined as:

$$D_{W,t}^R\equiv\int_\lambda^1(1-\lambda)^{-(1+\sigma_L)}\left(\frac{W_t^R(h)}{W_t^R}\right)^{-\varepsilon_{LR}(1+\sigma_L)}dh,$$

$D_{W,t}^R\,(1-\lambda)^{-(1+\sigma_L)}\int_0^\lambda\left(\frac{W_t^R(h)/P_t}{W_t^R/P_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)}dh$, which under Calvo wages is equivalent to:

$$D_{W,t}^R=(1-\lambda)^{-(1+\sigma_L)}\left(\frac{W_t^R}{P_t}\right)^{\varepsilon_{LR}(1+\sigma_L)}\times$$
$$\left[\begin{array}{l}(1-\phi_L)\left(\frac{\tilde{W}_t^R}{P_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)}+(1-\phi_L)\,\phi_L\left(\frac{\tilde{W}_{t-1}^R}{P_{t-1}}\right)^{-\varepsilon_{LR}(1+\sigma_L)}\left[\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\left(\bar{\Pi}\right)^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}\\ +(1-\phi_L)\,\phi_L\left(\frac{\tilde{W}_{t-2}^R}{P_{t-2}}\right)^{-\varepsilon_{LR}(1+\sigma_L)}\left[\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\left(\bar{\Pi}\right)^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}\left[\frac{\Pi_{t-1}}{(\Pi_{t-2})^{\xi_L}\left(\bar{\Pi}\right)^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}+...\end{array}\right],$$

so the indexing term can be written as:

$$\begin{aligned}\Theta_{t,t-1}^R &\equiv \left[\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\left(\bar{\Pi}\right)^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)},\\ \Theta_{t,t-2}^R &\equiv \left[\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\left(\bar{\Pi}\right)^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}\left[\frac{\Pi_{t-1}}{(\Pi_{t-2})^{\xi_L}\left(\bar{\Pi}\right)^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)},\\ &...\\ \Theta_{t,t-j}^R &\equiv \prod_{j=0}^{\infty}\left[\frac{\Pi_{t-j+1}}{(\Pi_{t-j})^{\xi_L}\left(\bar{\Pi}\right)^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}\end{aligned}$$

$$D_{W,t}^{R}=(1-\lambda)^{-(1+\sigma_L)}\left(\frac{W_t^R}{P_t}\right)^{\varepsilon_{LR}(1+\sigma_L)}\left[\sum_{a=0}^{\infty}(1-\phi_L)\,\phi_L^a\left(\frac{\tilde{W}_{t-a}}{P_{t-a}}\right)^{-\varepsilon_{LR}(1+\sigma_L)}\Theta_{t,t-a}^{R}\right],\tag{80}$$

$$\begin{aligned}D_{W,t}^{R} &= (1-\lambda)^{-(1+\sigma_L)}\left(\frac{W_t^R}{P_t}\right)^{\varepsilon_{LR}(1+\sigma_L)}(1-\phi_L)\left(\frac{\tilde{W}_t^R}{P_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)}\\ &\quad+(1-\lambda)^{-(1+\sigma_L)}\left(\frac{W_t^R}{P_t}\right)^{\varepsilon_{LR}(1+\sigma_L)}(1-\phi_L)\sum_{a=1}^{\infty}\phi_L^a\left(\frac{\tilde{W}_{t-a}^R}{P_{t-a}}\right)^{-\varepsilon_{LR}(1+\sigma_L)}\Theta_{t,t-a}^{R}.\end{aligned}$$

Multiplying and dividing by $\left(\frac{W_{t-1}^R}{P_{t-1}}\right)^{\varepsilon_{LR}(1+\sigma_L)}$ in the infinite summation in the RHS yields:

$$\begin{aligned}D_{W,t}^{R} &= (1-\lambda)^{-(1+\sigma_L)}\left(\frac{W_t^R}{P_t}\right)^{\varepsilon_{LR}(1+\sigma_L)}(1-\phi_L)\left(\frac{\tilde{W}_t^R}{P_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)}\\ &\quad+(1-\lambda)^{-(1+\sigma_L)}\left(\frac{W_t^R}{P_t}\right)^{\varepsilon_{LR}(1+\sigma_L)}\frac{\left(\frac{W_{t-1}^R}{P_{t-1}}\right)^{\varepsilon_{LR}(1+\sigma_L)}}{\left(\frac{W_{t-1}^R}{P_{t-1}}\right)^{\varepsilon_{LR}(1+\sigma_L)}}\sum_{a=0}^{\infty}(1-\phi_L)\,\phi_L^{a+1}\left(\frac{\tilde{W}_{t-a-1}^R}{P_{t-a-1}}\right)^{-\varepsilon_{LR}(1+\sigma_L)}\Theta_{t,t-a}^{R},\end{aligned}$$

$$\begin{aligned}&= (1-\lambda)^{-(1+\sigma_L)}\left(\frac{W_t^R}{P_t}\right)^{\varepsilon_{LR}(1+\sigma_L)}(1-\phi_L)\left(\frac{\tilde{W}_t^R}{P_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)}+\phi_L\left[\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\left(\bar{\Pi}\right)^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}\\ &\quad\times\frac{\left(\frac{W_t^R}{P_t}\right)^{\varepsilon_{LR}(1+\sigma_L)}}{\left(\frac{W_{t-1}^R}{P_{t-1}}\right)^{\varepsilon_{LR}(1+\sigma_L)}}\underbrace{(1-\lambda)^{-(1+\sigma_L)}\left(\frac{W_{t-1}}{P_{t-1}}\right)^{\varepsilon_{LR}(1+\sigma_L)}\left[\sum_{a=0}^{\infty}(1-\phi_L)\,\phi_L^a\left(\frac{\tilde{W}_{t-a-1}}{P_{t-a-1}}\right)^{-\varepsilon_{LR}(1+\sigma_L)}\Theta_{t-1,t-a-1}^{R}\right]}_{D_{W,t-1}^{R}\text{ by Eq. (80)}},\end{aligned}$$

$$\begin{aligned}D_{W,t}^{R} &= (1-\lambda)^{-(1+\sigma_L)}\left(\frac{W_t^R}{P_t}\right)^{\varepsilon_{LR}(1+\sigma_L)}(1-\phi_L)\left(\frac{\tilde{W}_t^R}{P_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)}+\phi_L\left[\frac{\frac{W_t}{P_t}}{\frac{W_{t-1}}{P_{t-1}}}\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\left(\bar{\Pi}\right)^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}D_{W,}^{R}\\ &= (1-\lambda)^{-(1+\sigma_L)}\left(\frac{W_t^R}{P_t}\right)^{\varepsilon_{LR}(1+\sigma_L)}(1-\phi_L)\left(\frac{\tilde{W}_t^R}{P_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)}+\phi_L\left[\frac{\Pi_{W,t}}{(\Pi_{t-1})^{\xi_L}\left(\bar{\Pi}\right)^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}D_{W,t-1}^{R}.\end{aligned}$$

Second, we derive the wage equation for non-Ricardian households. Operating restrictions are the labor demand faced by Non-Ricardian agents (that results from the firm's problem) and the CBC Eq. (7). We write the Lagrangian in real terms as follows (only terms that matter are displayed):

$$
\mathrm{E}_t\left[\begin{array}{c}\sum_{a=0}^{\infty}(\phi_L\beta)^a\left[...-\bar{\zeta}_L\frac{\zeta_{L,t+a}}{1+\sigma_L}\left[\lambda^{-1}\left(\frac{\tilde{W}_t^N(h)}{P_t}\right)^{-\varepsilon_{LN}}\left[\frac{P_t}{P_{t+a}}\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\left(\bar{\Pi}\right)^{a(1-\xi_L)}\right]^{-\varepsilon_{LR}}\left(\frac{W_{t+a}^N}{P_{t+a}}\right)^{\varepsilon_{LN}}L_{t+a}^N\right]^{1+\sigma_L}\right.\\ +\sum_{a=0}^{\infty}(\phi_L\beta)^a\Lambda_{t+a}^N\left(...+\mathcal{S}_{WN}\frac{\tilde{W}_t^N(h)}{P_t}\lambda^{-1}\left(\frac{\tilde{W}_t^N(h)}{P_t}\right)^{-\varepsilon_{LN}}\left[\frac{P_t}{P_{t+a}}\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\left(\bar{\Pi}\right)^{a(1-\xi_L)}\right]^{1-\varepsilon_{LR}}\left(\frac{W_{t+a}^N}{P_{t+a}}\right)^{\varepsilon_{LN}}L_{t+a}^N+...\right)\end{array}\right]
$$

Proceeding in a similar way as before, we get the FOCs and after some manipulations we obtain:

$$
\begin{aligned}
\left(\frac{\tilde{W}_t^N(h)}{P_t}\right)^{1+\varepsilon_{LN}\sigma_L} &= \frac{\bar{\zeta}_L\varepsilon_{LN}}{(\varepsilon_{LN}-1)\,\mathcal{S}_{WN}\lambda^{\sigma_L}}\frac{\dashv_{1,t}^{N,w}}{\dashv_{2,t}^{N,w}},\\
\frac{\tilde{W}_t^N(h)}{P_t} &= \left(\frac{\mu_{WN}}{\mathcal{S}_{WN}}\frac{\bar{\zeta}_L}{\lambda^{\sigma_L}}\frac{\mathcal{H}_{1,t}^{N,w}}{\mathcal{H}_{2,t}^{N,w}}\right)^{\frac{1}{1+\varepsilon_{LN}\sigma_L}},
\end{aligned}\tag{81}
$$

where $\mu_{WN}\equiv\frac{\varepsilon_{LN}}{(\varepsilon_{LN}-1)}$ is the markup associated and

$$
\mathcal{H}_{1,t}^{N,w}=\zeta_t\left(w_t^N\right)^{\varepsilon_{LN}(1+\sigma_L)}\left(L_t^N\right)^{1+\sigma_L}+\phi_L\beta E_t\left\{\left[\frac{\Pi_{t+1}}{(\Pi_t)^{\xi_L}\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LN}(1+\sigma_L)}\mathcal{H}_{1,t+1}^{N,w}\right\},
$$

$$
\mathcal{H}_{2,t}^{N,w}=\left(w_t^N\right)^{\varepsilon_{LN}}\Lambda_t^N L_t^N+\phi_L\beta E_t\left\{\left[\frac{\Pi_{t+1}}{(\Pi_t)^{\xi_L}\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LN}-1}\mathcal{H}_{2,t+1}^{N,w}\right\}.
$$

The aggregate wage dynamics are given by:

$$
\left(w_t^N\right)^{1-\varepsilon_{LN}}=(1-\phi_L)\left(\frac{\mu_{WN}}{\mathcal{S}_{WN}}\frac{\bar{\zeta}_L}{\lambda^{\sigma_L}}\frac{\mathcal{H}_{1,t}^{w}}{\mathcal{H}_{2,t}^{w}}\right)^{\frac{1-\varepsilon_{LN}}{1+\varepsilon_{LN}\sigma_L}}+\phi_L\left(w_{t-1}^N\right)^{1-\varepsilon_{LN}}\left[\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LN}-1},\tag{82}
$$

and wage dispersion is:

$$
D_{W,t}^N=\lambda^{-(1+\sigma_L)}\left(\frac{W_t^N}{P_t}\right)^{\varepsilon_{LN}(1+\sigma_L)}(1-\phi_L)\left(\frac{\tilde{W}_t^N}{P_t}\right)^{-\varepsilon_{LN}(1+\sigma_L)}+\phi_L\left[\frac{\Pi_{W,t}}{(\Pi_{t-1})^{\xi_L}\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LN}(1+\sigma_L)}D_{W,t-1}^N.
$$

D.1.2 Wage inflation model

First, we obtain the wage equation for Ricardian households. Beginning with Equation (78), multiply both members of by $\left(\frac{P_t}{W_t^R}\right)^{1+\varepsilon_{LR}\sigma_L}$, so the LHS becomes $\left(\frac{\tilde{W}_t^R(h)}{P_t}\right)^{1+\varepsilon_{LR}\sigma_L}\left(\frac{P_t}{W_t^R}\right)^{1+\varepsilon_{LR}\sigma_L}$ $=\left(\frac{\tilde{W}_t^R(h)}{W_t^R}\right)^{1+\varepsilon_{LR}\sigma_L}\left(\frac{P_t}{P_t}\right)^{1+\varepsilon_{LR}\sigma_L}=\left(\frac{\tilde{W}_t^R(h)}{W_t^R}\right)^{1+\varepsilon_{LR}\sigma_L}=\left(\frac{\tilde{W}_t^R(h)}{W_t^R}\right)^{1+\varepsilon_{LR}\sigma_L}\equiv\ddot{W}_t^R(h)^{1+\varepsilon_{LR}\sigma_L}$. On the other hand, the RHS of (78) becomes:

$$
\frac{\mu_{WR}}{\mathcal{S}_{WR}}\frac{\bar{\zeta}_L}{(1-\lambda)^{\sigma_L}}\frac{\left(\frac{P_t}{W_t^R}\right)^{\varepsilon_{LR}(\sigma_L+1)}}{\left(\frac{P_t}{W_t^R}\right)^{\varepsilon_{LR}-1}}\frac{\mathrm{E}_t\left[\sum_{a=0}^{\infty}(\phi_L\beta)^a\zeta_{L,t+a}\left[\frac{P_t}{P_{t+a}}\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\left(\bar{\Pi}\right)^{a(1-\xi_L)}\right]^{-\varepsilon_{LR}(1+\sigma_L)}\left(\frac{W_{t+a}^R}{P_{t+a}}\right)^{\varepsilon_{LR}(1+\sigma_L)}\left(L_{t+a}^R\right)^{1+\sigma_L}\right]}{\mathrm{E}_t\left[\sum_{a=0}^{\infty}(\phi_L\beta)^a\Lambda_{t+a}\left[\frac{P_t}{P_{t+a}}\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\left(\bar{\Pi}\right)^{a(1-\xi_L)}\right]^{1-\varepsilon_{LR}}\left(\frac{W_{t+a}^R}{P_{t+a}}\right)^{\varepsilon_{LR}}L_{t+a}^R\right]},
$$

$$\frac{\mu_{WR}}{\mathcal{S}_{WR}}\frac{\bar{\zeta}_L}{(1-\lambda)^{\sigma_L}}\frac{\mathrm{E}_t\left[\sum_{a=0}^{\infty}(\phi_L\beta)^a\zeta_{L,t+a}\left[\underbrace{\frac{P_t}{P_{t+a}}}\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\left(\bar{\Pi}\right)^{a\left(1-\xi_L\right)}\right]^{-\varepsilon_{LR}\left(1+\sigma_L\right)}\underbrace{\left(\frac{P_t}{P_{t+a}}\right)^{\varepsilon_{LR}(1+\sigma_L)}}\left(\frac{W_{t+a}^R}{W_t}\right)^{\varepsilon_{LR}\left(1+\sigma_L\right)}\left(L_{t+a}^R\right)^{1+\sigma_L}\right]}{\mathrm{E}_t\left[\sum_{a=0}^{\infty}(\phi_L\beta)^a\Lambda_{t+a}\left[\underbrace{\frac{P_t}{P_{t+a}}}\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\left(\bar{\Pi}\right)^{a\left(1-\xi_L\right)}\right]^{1-\varepsilon_{LR}}\underbrace{\left(\frac{P_t}{W_t}\right)^{\varepsilon_{LR}-1}\left(\frac{W_{t+a}^R}{P_{t+a}}\right)^{\varepsilon_{LR}-1}}\frac{W_{t+a}^R}{P_{t+a}}L_{t+a}^R\right]},$$

$$\frac{\mu_{WR}}{\mathcal{S}_{WR}}\frac{\bar{\zeta}_L}{(1-\lambda)^{\sigma_L}}\frac{\mathrm{E}_t\left[\sum_{a=0}^{\infty}(\phi_L\beta)^a\zeta_{L,t+a}\left[\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\left(\bar{\Pi}\right)^{a\left(1-\xi_L\right)}\right]^{-\varepsilon_{LR}\left(1+\sigma_L\right)}\left(\frac{W_{t+a}^R}{W_t}\right)^{\varepsilon_{LR}\left(1+\sigma_L\right)}\left(L_{t+a}^R\right)^{1+\sigma_L}\right]}{\mathrm{E}_t\left[\sum_{a=0}^{\infty}(\phi_L\beta)^a\Lambda_{t+a}\left[\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\left(\bar{\Pi}\right)^{a\left(1-\xi_L\right)}\right]^{1-\varepsilon_{LR}}\left(\frac{W_{t+a}^R}{W_t}\right)^{\varepsilon_{LR}-1}\frac{W_{t+a}^R}{P_{t+a}}L_{t+a}^R\right]},$$

$$\frac{\mu_{WR}}{\mathcal{S}_{WR}}\frac{\bar{\zeta}_L}{(1-\lambda)^{\sigma_L}}\frac{\mathrm{E}_t\left[\sum_{a=0}^{\infty}\left(\phi_L\beta\right)^a\zeta_{L,t+a}\left[\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\left(\bar{\Pi}\right)^{a(1-\xi_L)}\right]^{-\varepsilon_{LR}(1+\sigma_L)}\Pi_{W,t+a}^{\varepsilon_{LR}(1+\sigma_L)}\left(L_{t+a}^R\right)^{1+\sigma_L}\right]}{\mathrm{E}_t\left[\sum_{a=0}^{\infty}\left(\phi_L\beta\right)^a\Lambda_{t+a}\left[\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\left(\bar{\Pi}\right)^{a(1-\xi_L)}\right]^{1-\varepsilon_{LR}}\Pi_{W,t+a}^{\varepsilon_{LR}-1}\frac{W_{t+a}^R}{P_{t+a}}L_{t+a}^R\right]},$$

which has the following recursive representation (applying symmetry):

$$\left(\ddot{W}_t^R\right)^{1+\varepsilon_{LR}\sigma_L}=\frac{\mu_{WR}}{\mathcal{S}_{WR}}\frac{\bar{\zeta}_L}{(1-\lambda)^{\sigma_L}}\frac{\mathcal{H}_{1,t}^{R,w}}{\mathcal{H}_{2,t}^{R,w}}$$

where:

$$\mathcal{H}_{1,t}^{R,w}=\zeta_{L,t}\left(L_t^R\right)^{1+\sigma_L}+\phi_L\beta\mathrm{E}_t\left[\zeta_{L,t+1}\left[\frac{\Pi_{W,t+1}^R}{\left(\frac{P_t}{P_{t-1}}\right)^{\xi_L}\bar{\Pi}^{\left(1-\xi_L\right)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}\left(L_{t+1}^R\right)^{1+\sigma_L}\right]$$

$$+\mathrm{E}_t\left[\sum_{a=2}^{\infty}\left(\phi_L\beta\right)^a\zeta_{L,t+a}\left[\frac{\Pi_{W,t+a}^R}{\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\bar{\Pi}^{a\left(1-\xi_L\right)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}\left(L_{t+a}^R\right)^{1+\sigma_L}\right]$$

updating one period we get:

$$\mathcal{H}_{1,t+1}^{R,w}=\zeta_{L,t+1}\left(L_{t+1}^R\right)^{1+\sigma_L}+\mathrm{E}_t\left[\sum_{a=1}^{\infty}\left(\phi_L\beta\right)^a\zeta_{L,t+a}\left[\frac{\Pi_{W,t+a}^R}{\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\bar{\Pi}^{a\left(1-\xi_L\right)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}\left(L_{t+a}^R\right)^{1+\sigma_L}\right],$$

Thus:

$$\mathcal{H}_{1,t}^{R,w}=\zeta_{L,t}\left(L_t^R\right)^{1+\sigma_L}+\phi_L\beta\mathrm{E}_t\left[\left[\frac{\Pi_{W,t+1}^R}{\Pi_t^{\xi_L}\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}\mathcal{H}_{1,t+1}^{R,w}\right].$$

Similarly, for $\mathcal{H}_{2,t}^{R,w}$:

$$\mathcal{H}_{2,t}^{R,w}=\Lambda_t\frac{W_t^R}{P_t}L_t^R+\phi_L\beta\mathrm{E}_t\left[\left[\frac{\Pi_{W,t+1}^R}{\Pi_t^{\xi_L}\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LR}-1}\mathcal{H}_{2,t+1}^{R,w}\right].$$

The aggregate wage dynamics are given by:

$$
\begin{aligned}
1 &= (1-\phi_L)\left(\frac{\mu_{WR}}{\mathcal{S}_{WR}}\frac{\bar{\zeta}_L}{(1-\lambda)^{\sigma_L}}\frac{\mathcal{H}^{R,w}_{1,t}}{\mathcal{H}^{R,w}_{2,t}}\right)^{\frac{1-\varepsilon_{LR}}{1+\varepsilon_{LR}\sigma_L}} + \phi_L \frac{\left(w^R_{t-1}\right)^{1-\varepsilon_{LR}}}{\left(w^R_t\right)^{1-\varepsilon_{LR}}}\left[\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\,\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LR}-1}, \\
1 &= (1-\phi_L)\left(\frac{\mu_{WR}}{\mathcal{S}_{WR}}\frac{\bar{\zeta}_L}{(1-\lambda)^{\sigma_L}}\frac{\mathcal{H}^{R,w}_{1,t}}{\mathcal{H}^{R,w}_{2,t}}\right)^{\frac{1-\varepsilon_{LR}}{1+\varepsilon_{LR}\sigma_L}} + \phi_L \left[\frac{\Pi^R_{W,t}}{\Pi_t}\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\,\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LR}-1}, \\
1 &= (1-\phi_L)\left(\frac{\mu_{WR}}{\mathcal{S}_{WR}}\frac{\bar{\zeta}_L}{(1-\lambda)^{\sigma_L}}\frac{\mathcal{H}^{R,w}_{1,t}}{\mathcal{H}^{R,w}_{2,t}}\right)^{\frac{1-\varepsilon_{LR}}{1+\varepsilon_{LR}\sigma_L}} + \phi_L \left[\frac{\Pi^R_{W,t}}{(\Pi_{t-1})^{\xi_L}\,\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LR}-1}.
\end{aligned}
$$

The wage dispersion is defined as: $D^R_{W,t} \equiv \int_\lambda^1 (1-\lambda)^{-(1+\sigma_L)}\left(\frac{W^R_t(h)}{W^R_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)} dh$, which is equal to:

$$
D^R_{W,t} = \left[\begin{array}{c}
(1-\phi_L)\left(\frac{\tilde{W}^R_t}{W^R_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)} + (1-\phi_L)\,\phi_L\left(\frac{\tilde{W}^R_{t-1}}{W^R_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)}\left[\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)} \\
+(1-\phi_L)\,\phi_L\left(\frac{\tilde{W}^R_{t-2}}{W^R_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)}\left[\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\left(\bar{\Pi}\right)^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}\left[\frac{\Pi_{t-1}}{(\Pi_{t-2})^{\xi_L}\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)} + \ldots
\end{array}\right],
$$

so the indexing term can be written as:

$$
\begin{aligned}
\Theta^R_{t,t-1} &\equiv \left[\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\,\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}, \\
\Theta^R_{t,t-2} &\equiv \left[\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\,\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}\left[\frac{\Pi_{t-1}}{(\Pi_{t-2})^{\xi_L}\,\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}, \\
&\ldots \\
\Theta^R_{t,t-j} &\equiv \prod_{j=0}^{\infty}\left[\frac{\Pi_{t-j+1}}{(\Pi_{t-j})^{\xi_L}\,\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)}
\end{aligned}
$$

$$
D^R_{W,t} = \sum_{a=0}^{\infty}(1-\phi_L)\,\phi_L^a\left(\frac{\tilde{W}^R_{t-a}}{W^R_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)}\Theta^R_{t,t-a},
$$

$$
D^R_{W,t} = (1-\phi_L)\left(\frac{\tilde{W}^R_t}{W^R_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)} + (1-\phi_L)\sum_{a=1}^{\infty}\phi_L^a\left(\frac{\tilde{W}^R_{t-a}}{W^R_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)}\Theta^R_{t,t-a}.
$$

Multiplying and dividing the previous expression by $\left(\frac{W^R_t}{W^R_{t-a}}\right)^{-\varepsilon_{LR}(1+\sigma_L)}$ into the infinite summation in the RHS yields:

$$
D^R_{W,t} = (1-\phi_L)\left(\frac{\tilde{W}^R_t}{W^R_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)} + \frac{\left(\frac{W^R_t}{W^R_{t-a}}\right)^{-\varepsilon_{LR}(1+\sigma_L)}}{\left(\frac{W^R_t}{W^R_{t-a}}\right)^{-\varepsilon_{LR}(1+\sigma_L)}}\sum_{a=0}^{\infty}(1-\phi_L)\,\phi_L^{a+1}\left(\frac{\tilde{W}^R_{t-a-1}}{W^R_t}\right)^{-\varepsilon_{LR}(1+\sigma_L)}\Theta^R_{t,t-a},
$$

$$
\begin{aligned}
D_{W,t}^{R} &= (1-\phi_L)\left(\frac{\tilde{W}_t^R}{W_t^R}\right)^{-\varepsilon_{LR}(1+\sigma_L)} + \phi_L\left[\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\left(\bar{\Pi}\right)^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)} \\
&\quad\times\left(\frac{W_t^R}{W_{t-a}^R}\right)^{-\varepsilon_{LR}(1+\sigma_L)} \underbrace{\sum_{a=0}^{\infty}(1-\phi_L)\,\phi_L^a\left(\frac{\tilde{W}_{t-a-1}^R}{W_{t-a-1}^R}\right)^{-\varepsilon_{LR}(1+\sigma_L)}\Theta_{t-1,t-a-1}^R,}_{D_{W,t-1}^R}
\end{aligned}
$$

$$
D_{W,t}^{R} = (1-\phi_L)\left(\frac{\tilde{W}_t^R}{W_t^R}\right)^{-\varepsilon_{LR}(1+\sigma_L)} + \phi_L\left[\frac{\Pi_t\Pi_{W,t}^R}{(\Pi_{t-1})^{\xi_L}\left(\bar{\Pi}\right)^{(1-\xi_L)}}\right]^{\varepsilon_{LR}(1+\sigma_L)} D_{W,t-1}^R.
$$

Second, we obtain similar relationships for non-Ricardian households. Beginning with Equation (81), multiply both members of by $\left(\frac{P_t}{W_t^N}\right)^{1+\varepsilon_{LN}\sigma_L}$, so the LHS becomes $\left(\frac{\tilde{W}_t^N(h)}{P_t}\right)^{1+\varepsilon_{LN}\sigma_L}\left(\frac{P_t}{W_t^N}\right)^{1+\varepsilon_{LN}\sigma_L}$ $=\left(\frac{\tilde{W}_t^N(h)}{W_t^N}\right)^{1+\varepsilon_{LN}\sigma_L}\left(\frac{P_t}{P_t}\right)^{1+\varepsilon_{LN}\sigma_L} = \left(\frac{\tilde{W}_t^N(h)}{W_t^N}\right)^{1+\varepsilon_{LN}\sigma_L} = \left(\frac{\tilde{W}_t^N(h)}{W_t^N}\right)^{1+\varepsilon_{LN}\sigma_L} \equiv \ddot{W}_t^N(h)^{1+\varepsilon_{LN}\sigma_L}$. On the other hand, the RHS of (81) becomes:

$$
\frac{\mu_{WN}}{\mathcal{S}_{WN}}\frac{\bar{\zeta}_L}{\lambda^{\sigma_L}}\frac{\left(\frac{P_t}{W_t^N}\right)^{\varepsilon_{LN}(\sigma_L+1)}}{\left(\frac{P_t}{W_t^N}\right)^{\varepsilon_{LN}-1}}\frac{\mathrm{E}_t\left[\sum_{a=0}^{\infty}(\phi_L\beta)^a\zeta_{L,t+a}\left[\frac{P_t}{P_{t+a}}\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\bar{\Pi}^{a(1-\xi_L)}\right]^{-\varepsilon_{LN}(1+\sigma_L)}\left(\frac{W_{t+a}^N}{P_{t+a}}\right)^{\varepsilon_{LN}(1+\sigma_L)}\left(L_{t+a}^N\right)^{1+\sigma_L}\right]}{\mathrm{E}_t\left[\sum_{a=0}^{\infty}(\phi_L\beta)^a\Lambda_{t+a}\left[\frac{P_t}{P_{t+a}}\left(\frac{P_{t-1+a}}{P_{t-1}}\right)^{\xi_L}\bar{\Pi}^{a(1-\xi_L)}\right]^{1-\varepsilon_{LN}}\left(\frac{W_{t+a}^N}{P_{t+a}}\right)^{\varepsilon_{LN}}L_{t+a}^N\right]},
$$

and after analogous manipulations we obtain:

$$
\left(\ddot{W}_t^N\right)^{1+\varepsilon_{LN}\sigma_L} = \frac{\mu_{WN}}{\mathcal{S}_{WN}}\frac{\bar{\zeta}_L}{\lambda^{\sigma_L}}\frac{\mathcal{H}_{1,t}^{N,w}}{\mathcal{H}_{2,t}^{N,w}},
$$

where $\mathcal{H}_{1,t}^{N,w}$ and $\mathcal{H}_{2,t}^{N,w}$ are:

$$
\mathcal{H}_{1,t}^{N,w} = \zeta_{L,t}\left(L_t^N\right)^{1+\sigma_L} + \phi_L\beta\mathrm{E}_t\left[\left[\frac{\Pi_{W,t+1}^N}{\Pi_t^{\xi_L}\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LN}(1+\sigma_L)}\mathcal{H}_{1,t+1}^{N,w}\right].
$$

$$
\mathcal{H}_{2,t}^{N,w} = \Lambda_t\frac{W_t^N}{P_t}L_t^N + \phi_L\beta\mathrm{E}_t\left[\left[\frac{\Pi_{W,t+1}^N}{\Pi_t^{\xi_L}\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LN}-1}\mathcal{H}_{2,t+1}^{N,w}\right].
$$

The aggregate wage dynamics for non-Ricardian households are given by:

$$
1 = (1-\phi_L)\left(\frac{\mu_{WN}}{\mathcal{S}_{WN}}\frac{\bar{\zeta}_L}{\lambda^{\sigma_L}}\frac{\mathcal{H}_{1,t}^{N,w}}{\mathcal{H}_{2,t}^{N,w}}\right)^{\frac{1-\varepsilon_{LN}}{1+\varepsilon_{LN}\sigma_L}} + \phi_L\left[\frac{\Pi_{W,t}^N}{(\Pi_{t-1})^{\xi_L}\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LN}-1}.
$$

The wage dispersion is defined as: $D_{W,t}^N \equiv \int_0^\lambda \lambda^{-(1+\sigma_L)}\left(\frac{W_t^N(h)}{W_t^N}\right)^{-\varepsilon_{LN}(1+\sigma_L)}dh$, which under Calvo wage setting is equal to:

$$D_{W,t}^N = \left[\begin{array}{c} (1-\phi_L)\left(\frac{\tilde{W}_t^N}{W_t^N}\right)^{-\varepsilon_{LN}(1+\sigma_L)} + (1-\phi_L)\,\phi_L\left(\frac{\tilde{W}_{t-1}^N}{W_t^N}\right)^{-\varepsilon_{LN}(1+\sigma_L)} \left[\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LN}(1+\sigma_L)} \\ +(1-\phi_L)\,\phi_L\left(\frac{\tilde{W}_{t-2}^N}{W_t^N}\right)^{-\varepsilon_{LN}(1+\sigma_L)} \left[\frac{\Pi_t}{(\Pi_{t-1})^{\xi_L}\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LN}(1+\sigma_L)} \left[\frac{\Pi_{t-1}}{(\Pi_{t-2})^{\xi_L}\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LN}(1+\sigma_L)} + ... \end{array} \right],$$

after some manipulations we get:

$$D_{W,t}^N = (1-\phi_L)\left(\frac{\tilde{W}_t^N}{W_t^N}\right)^{-\varepsilon_{LN}(1+\sigma_L)} + \phi_L\left[\frac{\Pi_t\Pi_{W,t}^N}{(\Pi_{t-1})^{\xi_L}\,\bar{\Pi}^{(1-\xi_L)}}\right]^{\varepsilon_{LN}(1+\sigma_L)} D_{W,t-1}^N.$$

D.2 Home intermediate producers' price setting

If $\phi_H \to 0$, the relative price $T_{H,t}$ is a markup over the $MC_{H,t}$:

$$T_H = \frac{P_H}{P} = \mu_H MC_H.$$

The domestic intermediate producers' price results from the following problem:

$$\max_{\tilde{P}_{H,t}(i)} E_t\left\{\sum_{h=0}^{\infty}(\phi_H\beta)^h\,\Omega_{t,t+h}\left[\begin{array}{c}\tilde{P}_{H,t}(i)\mathbb{Y}_{H,t+h}(i)\left[\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}(\bar{\Pi})^{h(1-\xi_H)}\right] \\ -P_{t+h}MC_{H,t+h}\mathbb{Y}_{H,t+h}(i)\end{array}\right]\right\},$$

$$\begin{aligned} s.t.\quad MC_{H,t} &= \frac{1}{A_{H,t}}\frac{\left(r_t^k\right)^{\gamma} w_t^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}, \\ \mathbb{Y}_{H,t+h}(i) &= \left(\frac{\tilde{P}_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon_H}\left(\frac{P_{H,t}}{P_{H,t+h}}\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}(\bar{\Pi})^{h(1-\xi_H)}\right)^{-\varepsilon_H}\mathbb{Y}_{H,t+h}. \end{aligned}$$

where $\Omega_{t,t+h} \equiv \frac{\Lambda_{t+h}}{\Lambda_t}\frac{P_t}{P_{t+h}}$ and $\mathbb{Y}_{H,t+h}$ is the domestic demand of home intermediates (do not confuse with production) defined as $\mathbb{Y}_{H,t+h} \equiv C_{H,t+h} + I_{H,t+h} + G_{H,t+h}$. Write the corresponding *nominal* Lagrangean as:

$$\pounds = E_t\left\{\begin{array}{l}\sum_{h=0}^{\infty}(\phi_H\beta)^h\,\Omega_{t,t+h}\tilde{P}_{H,t}(i)\left(\frac{\tilde{P}_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon_H}\left(\frac{P_{H,t}}{P_{H,t+h}}\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}\bar{\Pi}^{h(1-\xi_H)}\right)^{-\varepsilon_H}\mathbb{Y}_{H,t+h}\left[\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}\bar{\Pi}^{h(1-\xi_H)}\right] \\ \qquad -\sum_{h=0}^{\infty}(\phi_H\beta)^h\,\Omega_{t,t+h}P_{t+h}MC_{H,t+h}\left(\frac{\tilde{P}_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon_H}\left(\frac{P_{H,t}}{P_{H,t+h}}\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}\bar{\Pi}^{h(1-\xi_H)}\right)^{-\varepsilon_H}\mathbb{Y}_{H,t+h}\end{array}\right.$$

The resulting FOC w.r.t. $\tilde{P}_{H,t}(i)$ is:

$$\begin{aligned} &E_t\left\{\sum_{h=0}^{\infty}(\phi_H\beta)^h\,\Omega_{t,t+h}(1-\varepsilon_H)\left(\frac{\tilde{P}_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon_H}\left(\frac{P_{H,t}}{P_{H,t+h}}\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}\bar{\Pi}^{h(1-\xi_H)}\right)^{-\varepsilon_H}\mathbb{Y}_{H,t+h}\left[\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}\bar{\Pi}^{h(1-\xi_H)}\right]\right\} \\ &-E_t\left\{\sum_{h=0}^{\infty}(\phi_H\beta)^h\,\Omega_{t,t+h}P_{t+h}MC_{H,t+h}(-\varepsilon_H)\left(\tilde{P}_{H,t}(i)\right)^{-\varepsilon_H-1}P_{H,t}^{\varepsilon_H}\left(\frac{P_{H,t}}{P_{H,t+h}}\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}\bar{\Pi}^{h(1-\xi_H)}\right)^{-\varepsilon_H}\mathbb{Y}_{H,t+h}\right\} = 0, \end{aligned}$$

$$\left(\varepsilon_H-1\right)\tilde{P}_{H,t}^{-\varepsilon_H}(i)E_t\left\{\sum_{h=0}^{\infty}\left(\phi_H\beta\right)^h\Omega_{t,t+h}P_{H,t}^{\varepsilon_H}\left(\frac{P_{H,t}}{P_{H,t+h}}\right)^{-\varepsilon_H}\left(\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}\left(\bar{\Pi}\right)^{h(1-\xi_H)}\right)^{1-\varepsilon_H}\mathbb{Y}_{H,t+h}\right\}$$

$$=\varepsilon_H\left(\tilde{P}_{H,t}(i)\right)^{-\varepsilon_H-1}E_t\left\{\sum_{h=0}^{\infty}\left(\phi_H\beta\right)^h\Omega_{t,t+h}P_{t+h}MC_{H,t+h}P_{H,t}^{\varepsilon_H}\left(\frac{P_{H,t}}{P_{H,t+h}}\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}\left(\bar{\Pi}\right)^{h(1-\xi_H)}\right)^{-\varepsilon_H}\mathbb{Y}_{H,t+h}\right\},$$

replacing $\Omega_{t,t+h}\equiv\frac{\Lambda_{t+h}}{\Lambda_t}\frac{P_t}{P_{t+h}}$, and taking into account the markup definition $\mu_H\equiv\frac{\varepsilon_H}{(\varepsilon_H-1)}$:

$$\tilde{P}_{H,t}(i)=\mu_H\frac{\frac{P_t}{\Lambda_t}E_t\left\{\sum_{h=0}^{\infty}(\phi_H\beta)^h\Lambda_{t+h}\frac{1}{P_{t+h}}P_{t+h}MC_{H,t+h}P_{H,t+h}^{\varepsilon_H}\left(\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}(\bar{\Pi})^{h(1-\xi_H)}\right)^{-\varepsilon_H}\mathbb{Y}_{H,t+h}\right\}}{\frac{P_t}{\Lambda_t}E_t\left\{\sum_{h=0}^{\infty}(\phi_H\beta)^h\Lambda_{t+h}\frac{1}{P_{t+h}}\left(\frac{1}{P_{H,t+h}}\right)^{-\varepsilon_H}\left(\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}(\bar{\Pi})^{h(1-\xi_H)}\right)^{1-\varepsilon_H}\mathbb{Y}_{H,t+h}\right\}},$$

$$\tilde{P}_{H,t}(i)=\mu_H\frac{E_t\left\{\sum_{h=0}^{\infty}(\phi_H\beta)^h\Lambda_{t+h}MC_{H,t+h}P_{H,t+h}^{\varepsilon_H}\left(\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}(\bar{\Pi})^{h(1-\xi_H)}\right)^{-\varepsilon_H}\mathbb{Y}_{H,t+h}\right\}}{E_t\left\{\sum_{h=0}^{\infty}(\phi_H\beta)^h\Lambda_{t+h}\frac{1}{P_{t+h}}P_{H,t+h}^{\varepsilon_H}\left(\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}(\bar{\Pi})^{h(1-\xi_H)}\right)^{1-\varepsilon_H}\mathbb{Y}_{H,t+h}\right\}},$$

dividing both sides by $P_{H,t}$ yields:

$$\frac{\tilde{P}_{H,t}(i)}{P_{H,t}}=\mu_H\frac{E_t\left\{\sum_{h=0}^{\infty}(\phi_H\beta)^h\Lambda_{t+h}MC_{H,t+h}P_{H,t+h}^{\varepsilon_H}\left(\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}(\bar{\Pi})^{h(1-\xi_H)}\right)^{-\varepsilon_H}\mathbb{Y}_{H,t+h}\right\}}{P_{H,t}^{1-\varepsilon_H}P_{H,t}^{\varepsilon_H}E_t\left\{\sum_{h=0}^{\infty}(\phi_H\beta)^h\Lambda_{t+h}\frac{1}{P_{t+h}}P_{H,t+h}^{\varepsilon_H}\left(\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}(\bar{\Pi})^{h(1-\xi_H)}\right)^{1-\varepsilon_H}\mathbb{Y}_{H,t+h}\right\}},$$

$$\frac{\tilde{P}_{H,t}(i)}{P_{H,t}}=\mu_H\frac{E_t\left\{\sum_{h=0}^{\infty}(\phi_H\beta)^h\Lambda_{t+h}MC_{H,t+h}\left(\frac{P_{H,t+h}}{P_{H,t}}\right)^{\varepsilon_H}\left(\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}(\bar{\Pi})^{h(1-\xi_H)}\right)^{-\varepsilon_H}\mathbb{Y}_{H,t+h}\right\}}{E_t\left\{\sum_{h=0}^{\infty}(\phi_H\beta)^h\Lambda_{t+h}\frac{P_{H,t}}{P_{t+h}}\frac{P_{H,t+h}}{P_{H,t+h}}\left(\frac{P_{H,t+h}}{P_{H,t}}\right)^{\varepsilon_H}\left(\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}(\bar{\Pi})^{h(1-\xi_H)}\right)^{1-\varepsilon_H}\mathbb{Y}_{H,t+h}\right\}},$$

$$\frac{\tilde{P}_{H,t}(i)}{P_{H,t}}=\mu_H\frac{E_t\left\{\sum_{h=0}^{\infty}\left(\phi_H\beta\right)^h\Lambda_{t+h}MC_{H,t+h}\left(\frac{P_{H,t+h}}{P_{H,t}}\right)^{\varepsilon_H}\left(\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}\left(\bar{\Pi}\right)^{h(1-\xi_H)}\right)^{-\varepsilon_H}\mathbb{Y}_{H,t+h}\right\}}{E_t\left\{\sum_{h=0}^{\infty}\left(\phi_H\beta\right)^h\Lambda_{t+h}T_{H,t+h}\left(\frac{P_{H,t+h}}{P_{H,t}}\right)^{\varepsilon_H-1}\left(\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}\left(\bar{\Pi}\right)^{h(1-\xi_H)}\right)^{1-\varepsilon_H}\mathbb{Y}_{H,t+h}\right\}}$$

$$\mathcal{Z}_{H1,t+1}=\Lambda_tMC_{H,t}\mathbb{Y}_{H,t}+\sum_{h=1}^{\infty}\left(\phi_H\beta\right)^h\Lambda_{t+h}MC_{H,t+h}\left(\frac{P_{H,t+h}}{P_{H,t}}\right)^{\varepsilon_H}\left(\left(\frac{P_{H,t-1+h}}{P_{H,t-1}}\right)^{\xi_H}\left(\bar{\Pi}\right)^{h(1-\xi_H)}\right)^{-\varepsilon_H}\mathbb{Y}_{H,t+h},$$

$$\mathcal{Z}_{H1,t+1}=\Lambda_{t+1}MC_{H,t+1}^{-\varepsilon_H}\mathbb{Y}_{H,t+1}+...,$$

it follows that:

$$\mathcal{Z}_{H1,t}=\Lambda_tMC_{H,t}\left(C_{H,t}+I_{H,t}+G_{H,t}\right)+\phi_H\beta E_t\left\{\left(\frac{P_{H,t+1}}{P_{H,t}}\right)^{\varepsilon_H}\left(\left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{\xi_H}\left(\bar{\Pi}\right)^{(1-\xi_H)}\right)^{-\varepsilon_H}\mathcal{Z}_{H1,t+1}\right\},$$

$$\mathcal{Z}_{H2,t} = \Lambda_t T_{H,t} \left(C_{H,t} + I_{H,t} + G_{H,t}\right) + \phi_H \beta E_t \left\{ \left(\frac{P_{H,t+1}}{P_{H,t}}\right)^{\varepsilon_H - 1} \left(\left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{\xi_H} \left(\bar{\Pi}\right)^{(1-\xi_H)} \right)^{1-\varepsilon_H} \mathcal{Z}_{H2,t+1} \right\}.$$

Thus, we obtain:

$$\frac{\tilde{P}_{H,t}(i)}{P_{H,t}} = \mu_H \left(\frac{\mathcal{Z}_{H1,t}}{\mathcal{Z}_{H2,t}}\right),$$

so that the price dynamics are given by:

$$1 = (1-\phi_H) \left(\mu_H \left(\frac{\mathcal{Z}_{H1,t}}{\mathcal{Z}_{H2,t}}\right)\right)^{1-\varepsilon_H} + \phi_H \left[\frac{\Pi_t}{\left(\Pi_{H,t-1}\right)^{\xi_H} \left(\bar{\Pi}\right)^{1-\xi_H}}\right]^{\varepsilon_H - 1}.$$

D.3 Home importers Calvo pricing

The domestic importer solves the following problem:

$$\max_{\tilde{P}_{F,t}(i)} E_t \left\{ \begin{array}{c} \sum_{h=0}^{\infty} \left(\phi_F \beta\right)^h \Omega_{t,t+h} \mathcal{S}_F \tilde{P}_{F,t}(i) \mathbb{Y}_{F,t+h}(i) \left[\left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F} \left(\bar{\Pi}\right)^{h(1-\xi_F)} \right] \\ -S_{t+h} P^*_{F,t+h}(i) \mathbb{Y}^*_{F,t+h}(i) \end{array} \right\},$$

$$s.t. \quad : \quad \mathbb{Y}_{F,t+h}(i) = \left(\frac{\mathcal{S}_F \tilde{P}_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon_F} \left(\frac{P_{F,t}}{P_{F,t+h}} \left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F} \left(\bar{\Pi}\right)^{h(1-\xi_F)}\right)^{-\varepsilon_F} \mathbb{Y}_{F,t+h},$$

where we employ the definitions of $RER_{F,t} \equiv \frac{S_t P^*_{F,t}}{P_{F,t}}$, and the fact that $\mathbb{Y}_{F,t+h} \equiv C_{F,t+h} + I_{F,t+h} + G_{F,t+h}$. The FOC w.r.t. $\tilde{P}_{F,t}(i)$:

$$\begin{aligned} & E_t \left\{ \sum_{h=0}^{\infty} \left(\phi_F \beta\right)^h \Omega_{t,t+h} \left(1-\varepsilon_F\right) \left(\frac{\mathcal{S}_F \tilde{P}_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon_F} \left(\frac{P_{F,t}}{P_{F,t+h}} \left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F} \left(\bar{\Pi}\right)^{h(1-\xi_F)}\right)^{-\varepsilon_F} \mathbb{Y}_{F,t+h} \left[\left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F} \left(\bar{\Pi}\right)^{h(1-\xi_F)} \right] \right\} \\ & -E_t \left\{ \sum_{h=0}^{\infty} \left(\phi_F \beta\right)^h \Omega_{t,t+h} S_{t+h} P^*_{F,t+h} \left(-\varepsilon_F\right) \left(\mathcal{S}_F \tilde{P}_{F,t}(i)\right)^{-\varepsilon_F - 1} P_{F,t}^{\varepsilon_F} \left(\frac{P_{F,t}}{P_{F,t+h}} \left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F} \left(\bar{\Pi}\right)^{h(1-\xi_F)}\right)^{-\varepsilon_F} \mathbb{Y}_{F,t+h} \right\} = 0 \end{aligned}$$

$$\begin{aligned} & \left(\varepsilon_F - 1\right) \left(\mathcal{S}_F \tilde{P}_{F,t}(i)\right)^{-\varepsilon_F} E_t \left\{ \sum_{h=0}^{\infty} \left(\phi_F \beta\right)^h \Omega_{t,t+h} P_{F,t}^{\varepsilon_F} \left(\frac{P_{F,t}}{P_{F,t+h}}\right)^{-\varepsilon_F} \left(\left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F} \left(\bar{\Pi}\right)^{h(1-\xi_F)} \right)^{1-\varepsilon_F} \mathbb{Y}_{F,t+h} \right\} \\ & = \varepsilon_F \left(\mathcal{S}_F \tilde{P}_{F,t}(i)\right)^{-\varepsilon_F - 1} E_t \left\{ \sum_{h=0}^{\infty} \left(\phi_F \beta\right)^h \Omega_{t,t+h} S_{t+h} P^*_{F,t+h} P_{F,t+h}^{\varepsilon_F} \left(\left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F} \left(\bar{\Pi}\right)^{h(1-\xi_F)} \right)^{-\varepsilon_F} \mathbb{Y}_{F,t+h} \right\}, \end{aligned}$$

$$\begin{aligned} & \left(\varepsilon_F - 1\right) \left(\mathcal{S}_F \tilde{P}_{F,t}\right)^{-\varepsilon_F} (i) E_t \left\{ \sum_{h=0}^{\infty} \left(\phi_F \beta\right)^h \Omega_{t,t+h} P_{F,t+h}^{\varepsilon_F} \left(\left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F} \left(\bar{\Pi}\right)^{h(1-\xi_F)} \right)^{-\varepsilon_F} \mathbb{Y}_{F,t+h} \right\} \\ & = \varepsilon_F \left(\mathcal{S}_F \tilde{P}_{F,t}(i)\right)^{-\varepsilon_F - 1} E_t \left\{ \sum_{h=0}^{\infty} \left(\phi_F \beta\right)^h \Omega_{t,t+h} S_{t+h} P^*_{F,t+h} P_{F,t}^{\varepsilon_F} \left(\frac{P_{F,t}}{P_{F,t+h}} \left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F} \left(\bar{\Pi}\right)^{h(1-\xi_F)}\right)^{1-\varepsilon_F} \mathbb{Y}_{F,t+h} \right\}, \end{aligned}$$

replacing $\Omega_{t,t+h} \equiv \frac{\Lambda_{t+h}}{\Lambda_t} \frac{P_t}{P_{t+h}}$, and taking into account the markup definition $\mu_F \equiv \frac{\varepsilon_F}{(\varepsilon_F - 1)}$:

$$\tilde{P}_{F,t}(i)=\frac{\mu_F}{\mathcal{S}_F}\frac{\frac{P_t}{\Lambda_t}E_t\left\{\sum_{h=0}^{\infty}(\phi_F\beta)^h\Lambda_{t+h}\frac{1}{P_{t+h}}S_{t+h}P^*_{F,t+h}P_{F,t+h}^{\varepsilon_F}\left(\left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F}(\bar{\Pi})^{h(1-\xi_F)}\right)^{-\varepsilon_F}\mathbb{Y}_{F,t+h}\right\}}{\frac{P_t}{\Lambda_t}E_t\left\{\sum_{h=0}^{\infty}(\phi_F\beta)^h\Lambda_{t+h}\frac{1}{P_{t+h}}\left(\frac{1}{P_{F,t+h}}\right)^{-\varepsilon_H}\left(\left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F}(\bar{\Pi})^{h(1-\xi_F)}\right)^{1-\varepsilon_F}\mathbb{Y}_{F,t+h}\right\}},$$

$$\tilde{P}_{F,t}(i)=\frac{\mu_F}{\mathcal{S}_F}\frac{E_t\left\{\sum_{h=0}^{\infty}(\phi_F\beta)^h\Lambda_{t+h}\frac{P_{F,t+h}}{P_{t+h}}\frac{S_{t+h}P^*_{F,t+h}}{P_{F,t+h}}P_{F,t+h}^{\varepsilon_F}\left(\left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F}(\bar{\Pi})^{h(1-\xi_F)}\right)^{-\varepsilon_F}\mathbb{Y}_{F,t+h}\right\}}{E_t\left\{\sum_{h=0}^{\infty}(\phi_F\beta)^h\Lambda_{t+h}\frac{P_{F,t+h}}{P_{t+h}}P_{F,t+h}^{\varepsilon_H-1}\left(\left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F}(\bar{\Pi})^{h(1-\xi_F)}\right)^{1-\varepsilon_F}\mathbb{Y}_{F,t+h}\right\}},$$

$$\tilde{P}_{F,t}(i)=\frac{\mu_F}{\mathcal{S}_F}\frac{E_t\left\{\sum_{h=0}^{\infty}(\phi_F\beta)^h\Lambda_{t+h}T_{H,t+h}T_{t+h}RER_{F,t+h}P_{F,t+h}^{\varepsilon_F}\left(\left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F}(\bar{\Pi})^{h(1-\xi_F)}\right)^{-\varepsilon_F}\mathbb{Y}_{F,t+h}\right\}}{E_t\left\{\sum_{h=0}^{\infty}(\phi_F\beta)^h\Lambda_{t+h}T_{H,t+h}T_{t+h}P_{F,t+h}^{\varepsilon_H-1}\left(\left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F}(\bar{\Pi})^{h(1-\xi_F)}\right)^{1-\varepsilon_F}\mathbb{Y}_{F,t+h}\right\}},$$

dividing both sides by $P_{F,t}$ yields:

$$\frac{\tilde{P}_{F,t}(i)}{P_{F,t}}=\frac{\mu_F}{\mathcal{S}_F}\frac{E_t\left\{\sum_{h=0}^{\infty}(\phi_F\beta)^h\Lambda_{t+h}T_{H,t+h}T_{t+h}RER_{F,t+h}P_{F,t+h}^{\varepsilon_F}\left(\left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F}(\bar{\Pi})^{h(1-\xi_F)}\right)^{-\varepsilon_F}\mathbb{Y}_{F,t+h}\right\}}{P_{F,t}^{1-\varepsilon_F}P_{F,t}^{\varepsilon_F}E_t\left\{\sum_{h=0}^{\infty}(\phi_F\beta)^h\Lambda_{t+h}T_{H,t+h}T_{t+h}P_{F,t+h}^{\varepsilon_H-1}\left(\left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F}(\bar{\Pi})^{h(1-\xi_F)}\right)^{1-\varepsilon_F}\mathbb{Y}_{F,t+h}\right\}},$$

$$\frac{\tilde{P}_{F,t}(i)}{P_{F,t}}=\frac{\mu_F}{\mathcal{S}_F}\frac{E_t\left\{\sum_{h=0}^{\infty}(\phi_F\beta)^h\Lambda_{t+h}T_{H,t+h}T_{t+h}RER_{F,t+h}\left(\frac{P_{F,t+h}}{P_{F,t}}\right)^{\varepsilon_H}\left(\left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F}(\bar{\Pi})^{h(1-\xi_F)}\right)^{-\varepsilon_F}\mathbb{Y}_{F,t+h}\right\}}{\left\{\sum_{h=0}^{\infty}(\phi_F\beta)^h\Lambda_{t+h}T_{H,t+h}T_{t+h}\left(\frac{P_{F,t+h}}{P_{F,t}}\right)^{\varepsilon_H-1}\left(\left(\frac{P_{F,t-1+h}}{P_{F,t-1}}\right)^{\xi_F}(\bar{\Pi})^{h(1-\xi_F)}\right)^{1-\varepsilon_F}\mathbb{Y}_{F,t+h}\right\}},$$

Solving the FOC, yields:

$$\frac{\tilde{P}_{F,t}(i)}{P_{F,t}}=\frac{\mu_F}{\mathcal{S}_F}\left(\frac{\mathcal{Z}_{F1,t}}{\mathcal{Z}_{F2,t}}\right),$$

where:

$$\begin{aligned}\mathcal{Z}_{F1,t} &= \Lambda_tT_{H,t}T_tRER_{F,t}\left(C_{F,t}+I_{F,t}+G_{F,t}\right)+\phi_F\beta E_t\left\{\left[\frac{\Pi_{F,t+1}}{(\Pi_{F,t})^{\xi_F}(\bar{\Pi})^{1-\xi_F}}\right]^{\varepsilon_F}\mathcal{Z}_{F1,t+1}\right\},\\ \mathcal{Z}_{F2,t} &= \Lambda_tT_{H,t}T_t\left(C_{F,t}+I_{F,t}+G_{F,t}\right)+\phi_F\beta E_t\left\{\left[\frac{\Pi_{F,t+1}}{(\Pi_{F,t})^{\xi_F}(\bar{\Pi})^{1-\xi_F}}\right]^{\varepsilon_F-1}\mathcal{Z}_{F2,t+1}\right\}.\end{aligned}$$

so that the price dynamics are given by (recall the normalization w.r.t. $P_{F,t}$):

$$1=(1-\phi_F)\left(\mu_F\left(\frac{\mathcal{Z}_{F1,t}}{\mathcal{Z}_{F2,t}}\right)\right)^{1-\varepsilon_F}+\phi_F\left[\frac{\Pi_t}{(\Pi_{F,t-1})^{\xi_F}(\bar{\Pi})^{1-\xi_F}}\right]^{\varepsilon_F-1}.$$