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## Journal of Monetary Economics

journal homepage: www.elsevier.com/locate/jme



# Are central banks' projections meaningful?

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#### ARTICLE INFO

Article history:
Received 26 January 2009
Received in revised form
6 November 2011
Accepted 9 November 2011
Available online 26 November 2011

#### ABSTRACT

Central banks' projections – i.e. forecasts conditional on a given interest rate path – are often criticized on the grounds that their assumptions are inconsistent with the existence of a unique equilibrium in many forward-looking models. The present paper describes three alternative approaches to constructing projections that are not subject to the above criticism, using the New Keynesian model as a reference framework. The three approaches are shown to generate different projections for inflation and output, even though they imply an identical path for the interest rate. The latter result calls into question the meaning and usefulness of such projections.

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## 1. Introduction

The periodic construction and publication of macroeconomic forecasts is a common activity among central banks in advanced industrialized economies. Those forecasts get considerable attention from analysts, market participants and the financial press. Among the central banks that have adopted an explicit inflation-forecast targeting strategy those forecasts play a central role in the internal decision process for, in many cases, policy settings are chosen in order to attain a certain inflation target at a specified horizon. Those forecasts are also viewed as an important element in central banks' communication policy. More generally, and using the words of the ECB, "macroeconomic projections play an important role as a tool for aggregating and organizing existing information on current and future economic developments" (European Central Bank, 2001).

The methods and assumptions behind those forecasts vary considerably across central banks. In particular, practices differ regarding the assumption on the path for the short-term nominal interest rate (henceforth, the interest rate) underlying those forecasts. The relative merits of alternative assumptions on which to condition the published forecasts remains the subject of considerable debate, partly sparked by the recent decision by some central banks to change their practice in that regard.<sup>1</sup>

At the risk of oversimplification, one can distinguish three alternative assumptions underlying central banks' forecasting practices. In some cases the interest rate is assumed to remain constant at its current level over the forecasting horizon, giving rise to so-called *constant interest rate* (*CIR*) forecasts. Other central banks construct their forecasts under the assumption that the interest rate will follow a path consistent with current market expectations, with the latter being approximated with the forward rates implicit in the yield curve prevailing at the time the projection is made.<sup>2</sup> We refer to

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<sup>&</sup>lt;sup>1</sup> See, e.g. the recent announcements by the Riksbank (2007) and the Federal Reserve Board (2007).

<sup>&</sup>lt;sup>2</sup> Of course, it is not easy in practice to disentangle expected rates from liquidity and term premia, but this additional complication is unrelated to the point made in the present paper.

**Table 1**Central banks' forecasting practices.

Reserve Bank of Australia	CIR
Central Bank of Brazil	CIR and ME
European Central Bank	ME
Bank of Japan	ME
Reserve Bank of New Zealand	CBE
Norges Bank	CBE
Riksbank	CBE
Swiss National Bank	CIR
Bank of England	ME
Federal Reserve System	CBE (individual FOMC members)

Note. CIR: constant interest rate; ME: market expectations; CBE: central bank expectations. Source: Bank for International Settlements (2007) and national sources.

those as *market expectations* (*ME*) forecasts. A third practice found among central banks consists in constructing forecasts based on the assumption that the interest rate will follow whatever path the central bank expects it to follow, i.e. a path consistent with the central bank's "own policy rule," independently of whether the latter has been made explicit or not. We refer to those forecasts as *central bank's expectations* (*CBE*) forecasts. Since the latter forecasts are based on the central bank's best assessment of what the interest rate path will be, they can be interpreted as unconditional forecasts. By contrast, CIR or ME forecasts are conditioned on a path of interest rates that does not generally coincide with the central bank's own expectations on that path. As a result, those forecasts are not necessarily the best predictors of future outcomes, as is implicitly reflected in their common labeling as "conditional forecasts" or "projections". For convenience, in the remainder of the paper (as well as in the title) the term "projections" is used to refer to forecasts conditional on a *given* interest rate path.

Each of the abovementioned forecasting procedures has its own advantages and disadvantages, which may explain the observed diversity of practices.<sup>4</sup> That diversity is illustrated in Table 1, which summarizes the assumptions on the interest rate path underlying the forecast practices of several major central banks.

The starting motivation for the present paper is a concern frequently voiced regarding central banks' projections: the inconsistency of their underlying assumptions with the existence of a unique equilibrium in a variety of forward-looking models, including the new vintage of optimizing models commonly used for monetary policy analysis. That argument hinges on the notion that, in order to construct such projections, the path of the interest rate is taken *as given* by the central bank, i.e. it is assumed that the interest rate will vary according to a pre-specified path, independently of how inflation or other endogenous variables may end up evolving. In other words, the model is fed with an exogenous interest rate path as a description of monetary policy, which is known to generate an indeterminate equilibrium (and, hence, indeterminate forecasts) in many dynamic models with forward-looking components.

The present paper seeks to contribute to that debate by clarifying the sense in which macroeconomic projections conditional on a given interest rate path are or are not feasible or well defined. First, it is shown that it is indeed possible to overcome the curse of indeterminacy and to generate *determinate* macroeconomic projections consistent with an arbitrary path for the interest rate. This is done using two well known reference frameworks: the canonical or basic New Keynesian model (e.g., Clarida et al., 1999) and the medium-scale model of Smets and Wouters (2007). As many as three approaches are proposed to construct the desired projections. The first approach, based on the "modest interventions" model of Leeper and Zha (2003), consists of adding unexpected policy shocks to the central bank's rule in order to generate the desired interest rate path.<sup>7</sup> The second and third approaches rely on two alternative interest rate rules designed to generate the desired path of the interest rate as an endogenous equilibrium outcome, while responding systematically to inflation and other macrovariables in a way that guarantees the determinacy of the equilibrium (and, hence, uniqueness of the projections).

Thus, by showing how projections can be constructed using *any* of the above methods, the paper refutes the claim that such forecasts are not feasible due to the indeterminacy underlying the associated equilibrium. The latter finding, however, can hardly provide any consolation to advocates of conditional projections: a simple example illustrates that the three methods will generally *yield different projections for variables other than the interest rate itself*, despite the fact that, by construction, the latter exhibits an identical path across methods. Thus, the problem is not necessarily one of multiplicity of equilibria resulting from an exogenous interest rate path, but one of *multiplicity of rules* that are consistent with the latter path, each having its own implications for other variables. That observation calls into question the

<sup>&</sup>lt;sup>3</sup> Many central banks that publish CBE-type forecasts for inflation and output also report the associated forecasts for the interest rate path (e.g. the RB of New Zealand, Norway's Norges Bank and Sweden's Riksbank). Others do not (the U.S. Federal Reserve).

<sup>&</sup>lt;sup>4</sup> See, e.g., Svensson (2006) for a detailed discussion.

<sup>&</sup>lt;sup>5</sup> See, e.g., Leitemo (2003), Honkapohja and Mitra (2005), and Woodford (2005).

<sup>&</sup>lt;sup>6</sup> See, e.g. Woodford (2003) or Galí (2008) for a detailed discussion.

<sup>&</sup>lt;sup>7</sup> As shown below, the "modest interventions" approach is equivalent to assuming a switch to the usual interest rate rule right after the forecast horizon.

usefulness of projections conditional on a given interest rate path since, in principle, there is no obvious reason to prefer one method over another in order to generate the same path.

That problem does not arise when constructing CBE forecasts, which are based on the rule effectively followed by the central bank when constructing those forecasts. More generally, the problem disappears whenever the projections are conditional on *a rule* (as long as the latter is consistent with a unique equilibrium), and *not* on an *interest rate path* (which may be supported by a multiplicity of rules). It is in that sense that the present paper provides an additional argument against the reporting of central banks' projections conditional on a given interest rate path, while endorsing instead rule-based projections. But once this is accepted, it is not clear why the central bank would want to base its projections on a rule *other than the actual rule it follows* for, among other things, in that case the projections would also correspond to the best unconditional forecasts.

The remainder of the paper is organized as follows. Section 2 describes the reference model used in the subsequent analysis. Section 3 revisits the problem of indeterminacy when the interest rate follows an exogenous path, and shows how this indeterminacy is inherited by the associated projections. Section 4 describes three approaches that overcome that indeterminacy problem. Sections 5 and 6 illustrate, using calibrated versions of the two reference models, how those three approaches lead to significantly different inflation and output projections conditional on an unchanged interest rate, at any finite horizon. Section 7 concludes.

#### 2. A baseline model

The canonical New Keynesian model is used as a reference framework for the analysis of alternative macroprojections.<sup>8</sup> The non-policy block of the model is made up of the following two equations:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\}) \tag{1}$$

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa y_t + u_t \tag{2}$$

where  $y_t$  denotes the output gap,  $i_t$  is the short-term nominal interest rate,  $\pi_t \equiv p_t - p_{t-1}$  is the rate of inflation between t-1 and t (with  $p_t$  denoting the log of the price level), and  $u_t$  is an exogenous cost-push shock which follows an AR(1) process with autoregressive coefficient  $\rho_u \in (0,1]$ . All variables are expressed in deviations from steady state values. Eq. (1) can be obtained by log-linearizing the representative household's Euler equation and a market clearing condition that equates consumption to output. Eq. (2) is a version of the so-called New Keynesian Phillips curve, which can be derived by aggregating the price-setting decisions of monopolistically competitive firms subject to Calvo-type constraints on the frequency of price adjustment, combined with standard assumptions on technology and labor markets. In that context, parameter  $\sigma$  corresponds to the coefficient of relative risk aversion,  $\beta$  is the household's discount factor and  $\kappa$  is a coefficient which is inversely related to the degree of price rigidities. All variables are expressed in terms of deviations from their values in a zero inflation steady state.

A block describing how monetary policy is conducted completes the model. In particular, it is assumed that the central bank follows a simple interest rate rule of the form

$$i_t = \phi \pi_t \tag{3}$$

where  $\phi > 1$ . As is well known, the latter condition guarantees uniqueness of the equilibrium.<sup>10</sup>

The paper's main contribution lies in the presentation of three alternative approaches to constructing forecasts for inflation and output in the above economy, under a (counterfactual) assumption regarding how interest rates evolve over the forecast horizon. But before turning to that analysis, a brief detour is taken to describe the main concern associated with the construction of macroeconomic forecasts conditional on an arbitrary, exogenously given interest rate path.

#### 3. Macroeconomic projections and the indeterminacy problem

For the sake of concreteness, it is assumed that the central bank is interested in the k-period ahead forecasts of inflation and the output gap *conditional* on an arbitrary, exogenous path for the interest rate,  $\{i_t^*\}$ . As mentioned above, both the CIR and ME projections constructed and published by many central banks can be viewed as particular examples of such an exercise since, in both cases, the path of the interest rate is given to the forecaster (i.e. the cental bank), with no room allowed for possible adjustments in response to developments in the economy, at least over the horizon for which the interest rate path is defined.

<sup>&</sup>lt;sup>8</sup> This is the model used in the optimal policy analysis of Clarida et al. (1999). The reader is referred to King and Wolman (1996), Woodford (2003) or Galí (2008) for a detailed description of that model and a derivation of (1) and (2).

<sup>&</sup>lt;sup>9</sup> A constant natural rate of interest is implicitly assumed. Also, constant terms are ignored, to lighten the notation. The analysis below carries over, with suitable modifications, to the case of a time-varying natural rate of interest.

<sup>&</sup>lt;sup>10</sup> As discussed in Svensson and Woodford (2005), if the central bank were to follow instead a "targeting rule," an additional complication would arise as a result of the need to determine the interest rate rule that would be consistent with the central bank's desired allocation but with no other (i.e. the one implying a determinate equilibrium).

Combining the assumed interest rate path with Eqs. (1) and (2) yields the system of difference equations

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \mathbf{A}_0 \begin{bmatrix} E_t \{ \pi_{t+1} \} \\ E_t \{ y_{t+1} \} \end{bmatrix} + \mathbf{B}_0 \begin{bmatrix} u_t \\ i_t^* \end{bmatrix}$$

$$\tag{4}$$

where

$$\mathbf{A}_{0} = \begin{bmatrix} \beta + \frac{\kappa}{\sigma} & \kappa \\ \frac{1}{\sigma} & 1 \end{bmatrix}; \quad \mathbf{B}_{0} = \begin{bmatrix} 1 & -\frac{\kappa}{\sigma} \\ 0 & -\frac{1}{\sigma} \end{bmatrix}$$
 (5)

Letting  $\mathbf{x}_t = [\pi_t, y_t]'$  and  $\mathbf{z}_t \equiv [u_t, i_t^*]'$  one can rewrite the previous dynamical system in a more compact way as

$$\mathbf{x}_t = \mathbf{A}_0 E_t \{ \mathbf{x}_{t+1} \} + \mathbf{B}_0 \mathbf{z}_t \tag{6}$$

Iterating forward on (6) the following expression for k-horizon projections as of time t can be obtained

$$E_t\{\mathbf{x}_{t+k}\} = \mathbf{A}_0^{-k}(\mathbf{x}_t - \mathbf{f}_t) \tag{7}$$

where  $\mathbf{f}_t \equiv \mathbf{B}_0 \mathbf{z}_t + \mathbf{A}_0 \mathbf{B}_0 E_t \{\mathbf{z}_{t+1}\} + \cdots + \mathbf{A}_0^{k-1} \mathbf{B}_0 E_t \{\mathbf{z}_{t+k-1}\}$ . Thus, in order for the projections at any k-horizon to be well defined, the current values of  $\mathbf{x}_t$  (and, thus, of inflation and the output gap) must be determined uniquely, which in turn requires that the solution to (6) exist and be unique. A necessary and sufficient condition for that to be the case is that the two eigenvalues of  $\mathbf{A}_0$  lie within the unit circle, given that  $\mathbf{x}_t$  consists of two non-predetermined variables. <sup>11</sup> But as is well known, and formally re-stated in the following lemma, that condition is *not* satisfied under the exogenous interest rate path regime assumed here. <sup>12</sup>

**Lemma.** Let  $\lambda_1$  and  $\lambda_2$  denote the eigenvalues of  $\mathbf{A}_0$ , where  $\lambda_1 \leq \lambda_2$ . Both  $\lambda_1$  and  $\lambda_2$  are real and satisfy  $0 < \lambda_1 < 1 < \lambda_2$ .

**Proof.** From the properties of matrices and their eigenvalues,  $tr(\mathbf{A}_0) = 1 + \beta + (\kappa/\sigma) = \lambda_1 + \lambda_2$  and  $det(\mathbf{A}_0) = \beta = \lambda_1 \lambda_2$ . Solving the implied system of equations one gets  $\lambda_1 = (1 + \beta + (\kappa/\sigma) - \sqrt{(1 + \beta + (\kappa/\sigma))^2 - 4\beta})/2$  and  $\lambda_2 = (1 + \beta + (\kappa/\sigma) + \sqrt{(1 + \beta + (\kappa/\sigma))^2 - 4\beta})/2$ . Note that

$$\Delta = \left(1 + \beta + \frac{\kappa}{\sigma}\right)^2 - 4\beta > (1 + \beta)^2 - 4\beta = (1 - \beta)^2 \tag{8}$$

from which it follows that both eigenvalues are real. Also, (8) implies  $\lambda_2 > (1 + \beta + (\kappa/\sigma) + \sqrt{(1-\beta)^2})/2 > 1$ .

Note also that  $\sqrt{(1+\beta+(\kappa/\sigma))^2-4\beta} < 1+\beta+(\kappa/\sigma)$ , implying  $\lambda_1 > 0$ . Finally,  $\lambda_1 < 1$  follows from the fact that  $\lim_{(\kappa/\sigma)\to 0} \lambda_1 = \beta < 1$ , and  $\partial \lambda_1/\partial (\kappa/\sigma) < 0$ .

The multiplicity of non-explosive solutions to (6) associated with the exogenous interest rate path implies that the equilibrium value for  $\mathbf{x}_t$  is not uniquely determined. In other words there exist a continuum of values for  $\mathbf{x}_t$  that are consistent with a rational expectations equilibrium. It follows from (7), that there is also a continuum of k-horizon projections for the output gap and inflation,  $E_t\{y_{t+k}\}$  and  $E_t\{\pi_{t+k}\}$ , consistent with the assumed interest rate path.

## 4. Macroeconomic projections without the curse of indeterminacy

Three alternative approaches are presented next that allow one to construct inflation and output gap projections that are not subject to "the curse of indeterminacy" described above. A feature common to the three approaches is their reliance on some rule that turns the interest rate into an endogenous variable, while guaranteeing that the desired interest rate path  $\{i_t^*\}$  is realized *in equilibrium*. In all cases it is assumed that the interest rate along that path converges to its value consistent with a zero inflation steady state, at least in expectation. Given that variables are expressed in terms of deviations from steady state, this implies that  $\lim_{i\to\infty} E_t\{i_{t+i}^*\} = 0$ .

## 4.1. Interest rate rule I

The first approach proposed in order to generate unique, well defined projections assumes that the central bank adopts the interest rate rule

$$i_t = \phi \pi_t + \nu_t \tag{9}$$

<sup>11</sup> See, e.g. Blanchard and Kahn (1980). Throughout we restrict the analysis to equilibria that remain in a neighborhood of the zero inflation steady

state.

12 The argument involves a straightforward variation of the analysis in Bullard and Mitra (2002). See also Galí (2008, Chapter 4).

where  $\phi > 1$  and  $v_t$  is perceived to be an exogenous, zero mean *i.i.d.* monetary policy shock. Notice that the latter shock is appended to the "usual" rule (3), in order to capture the deviations ("modest interventions," using Leeper and Zha's terminology) required in order to keep the nominal interest rate on the desired path  $\{i_t^*\}$ .<sup>13</sup>

It is easy to check that there exists a unique stationary solution to the system made up of (1), (2), and (8), given the assumption of an inflation coefficient greater than one. <sup>14</sup> The form of that solution can be guessed to be

$$y_t = au_t + cv_t; \quad \pi_t = bu_t + dv_t \tag{10}$$

Using the method of undetermined coefficients it is straightforward to determine the values of the four coefficients, which are given by  $a = -(\phi - \rho_u)\Lambda_u$ ,  $b = \sigma(1 - \rho_u)\Lambda_u$ ,  $c = -1/(\sigma + \kappa\phi)$ , and  $d = -\kappa/(\sigma + \kappa\phi)$ , where  $\Lambda_u \equiv 1/(\sigma(1 - \rho_u)(1 - \beta\rho_u) + \kappa(\phi - \rho_u))$ .

Combining (1) with the above solution one can obtain the following expression for t+k output gap and inflation:

$$y_{t+k} = E_{t+k}\{y_{t+k+1}\} + \frac{1}{\sigma}E_{t+k}\{\pi_{t+k+1}\} - \frac{1}{\sigma}i_{t+k}^* = -(\phi - 1)\Lambda_u \rho_u u_{t+k} - \frac{1}{\sigma}i_{t+k}^*$$
(11)

and

$$\pi_{t+k} = \beta E_{t+k} \{ \pi_{t+k+1} \} + \kappa y_{t+k} + u_{t+k} = (1 - \rho_u)(\sigma + \kappa \phi) \Lambda_u u_{t+k} - \frac{\kappa}{\sigma} i_{t+k}^*$$
(12)

Thus, the central bank's k-horizon forecasts are given by

$$E_{t}\{y_{t+k}\} = -(\phi - 1)\Lambda_{u}\rho_{u}^{k+1}u_{t} - \frac{1}{\sigma}i_{t+k}^{*}$$
(13)

$$E_t\{\pi_{t+k}\} = (1 - \rho_u)(\sigma + \kappa \phi) \Lambda_u \rho_u^k u_t - \frac{\kappa}{\sigma} i_{t+k}^*$$

$$\tag{14}$$

Note that, ex-post, the central bank imposes a sequence of realizations for  $v_t$  that guarantee that the desired interest rate path  $\{i_t^*\}$  is attained. Formally, this requires

$$v_t = t^* - \phi \pi_t \tag{15}$$

for all t. Combining the previous condition with the expression for equilibrium inflation above we obtain an expression for  $v_t$  in closed form

$$v_t = \left(1 + \frac{\kappa \phi}{\sigma}\right) i_t^* - \phi(\sigma + \kappa \phi) (1 - \rho_u) \Lambda_u u_t = \left(1 + \frac{\kappa \phi}{\sigma}\right) (i_t^* - i_t^r)$$
(16)

where  $i_t^r \equiv \phi \sigma (1 - \rho_u) \Lambda_u u_t$  is the interest rate that would prevail under the baseline or "usual" rule. Thus, it is clear that in general  $\{v_t\}$  will not satisfy the *i.i.d.* assumption ex-post, thus violating the rationality of expectations. This should be recognized by agents if the "intervention" were to last long enough.<sup>15</sup>

The second and third methods described below are not subject to the previous shortcoming, being fully consistent with the assumption of rational expectations. Before turning to them, a brief detour is taken to present an equivalence result that may be of independent interest.

## 4.1.1. Modest interventions vs. switching rules: an equivalence result

If the interest rate path on which projections must be conditioned is assumed to revert back after the forecast horizon to a level determined by the usual rule, the problem of indeterminacy can be shown to go away, allowing the central bank to construct well defined conditional projections.

To see this assume that the central bank sets the interest rate according to the rule

$$i_{t+j} = i_{t+j}^*$$
 (17)

for j = 0, 1, 2, ..., k and

$$i_{t+i} = \phi \pi_{t+i} \tag{18}$$

for j = k+1, k+2,..., i.e. it sets the interest rate at the level determined by the given exogenous path up to the desired forecast horizon, and switches to its regular interest rate rule after that.<sup>16</sup> Such an approach is used by Laséen and Svensson (2008), as a way to simulate arbitrary time-varying interest rate rules in the context of the Riksbank's estimated DSGE model (Ramses).

Note that from time t+k+1 onward the equilibrium dynamics are described by (1)–(3), which are associated with a unique equilibrium. One can solve for that equilibrium using the method of undetermined coefficients, after guessing that

<sup>&</sup>lt;sup>13</sup> See also the application in Smets and Wouters (2005).

<sup>&</sup>lt;sup>14</sup> See, e.g., Bullard and Mitra (2002).

<sup>&</sup>lt;sup>15</sup> This is acknowledged by Leeper and Zha (2003) who argue that the deviations from the rule should be small enough not to induce a change in agents' expectations about the regime in place.

<sup>&</sup>lt;sup>16</sup> In that sense, the switching rule approach is more restrictive than the modest interventions approach (or the second and third approaches described below), since it can only generate finite horizon projections conditional on a *finite* horizon interest rate path.

both the output gap and inflation will be proportional to the cost-push shock. This yields the following expressions:

$$y_{t+k+1} = au_{t+k+1}; \quad \pi_{t+k+1} = bu_{t+k+1}$$
 (19)

where a and b are given by the same expressions as above.

Combining the previous result with (1), (2) and the fact that  $i_{t+k} = i_{t+k}^*$ , we can write the equilibrium conditions corresponding to period t+k as

$$y_{t+k} = E_t \{ y_{t+k+1} \} + \frac{1}{\sigma} E_t \{ \pi_{t+k+1} \} - \frac{1}{\sigma} i_{t+k}^* = -(\phi - 1) \rho_u \Lambda_u u_{t+k} - \frac{1}{\sigma} i_{t+k}^*$$
 (20)

$$\pi_{t+k} = \beta E_t \{ \pi_{t+k+1} \} + \kappa y_{t+k} + u_{t+k} = (1 - \rho_u)(\sigma + \kappa \phi) \Lambda_u u_{t+k} - \frac{\kappa}{\sigma} i_{t+k}^*$$
 (21)

which are the expressions identical to (9) and (10) above. The corresponding projections as of time t are thus uniquely determined and correspond to those generated by the "modest interventions" method.

One should note, however, that the previous equivalence result hinges critically on  $E_t\{y_{t+k+1}\}$  and  $E_t\{\pi_{t+k+1}\}$  corresponding in both cases to the expected value conditional on the baseline rule, and hence on being history-independent. The previous observation implies that the equivalence result presented above will generally not carry over to a more general setting with endogenous state variables (e.g., the Smets–Wouters (2007) model used below).

## 4.2. Interest rate rule II

Consider next an interest rate rule of the form

$$i_t = i_t^* - \gamma i_{t-1}^* + \gamma (\pi_t + \sigma \Delta y_t) \tag{22}$$

where  $\gamma$  is a constant coefficient satisfying  $\gamma > 1.^{17}$  Combining (22) with (1) we obtain the difference equation

$$i_t - i_t^* = \frac{1}{\nu} E_t \{ i_{t+1} - i_{t+1}^* \}$$
 (23)

Note that under the assumption that  $\gamma > 1$  the only non-explosive solution to (23) is  $i_t = i_t^*$  for all t. In other words, by following rule (22) the central bank can support any desired interest rate path  $\{i_t^*\}$ .

The equilibrium dynamics under rule (22) are described by (2) and

$$y_{t} = E_{t}\{y_{t+1}\} - \frac{1}{\sigma}(i_{t}^{*} - \gamma i_{t-1}^{*} + \gamma(\pi_{t} + \sigma \Delta y_{t}) - E_{t}\{\pi_{t+1}\})$$
(24)

where the latter equation can be obtained by using (22) to eliminate the interest rate in (1). Equivalently, and more compactly, we can write (2) and (3) as

$$\begin{bmatrix} \pi_t \\ y_t \\ y_{t-1} \end{bmatrix} = \mathbf{A}_1 \begin{bmatrix} E_t \{ \pi_{t+1} \} \\ E_t \{ y_{t+1} \} \\ y_t \end{bmatrix} + \mathbf{B}_1 \begin{bmatrix} u_t \\ i_t^* - \gamma i_{t-1}^* \end{bmatrix}$$
 (25)

where

$$\mathbf{A}_{1} \equiv \begin{bmatrix} \beta & 0 & \kappa \\ 0 & 0 & 1 \\ \frac{\beta \gamma - 1}{\sigma \gamma} & -\frac{1}{\gamma} & 1 + \frac{1}{\gamma} + \frac{\kappa}{\sigma} \end{bmatrix}; \quad \mathbf{B}_{1} \equiv \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ \frac{1}{\sigma} & \frac{1}{\sigma \gamma} \end{bmatrix}$$

$$(26)$$

Note that the system (25) involves one predetermined and two non-predetermined variables. Thus, it has a unique non-explosive solution if and only if two eigenvalues of  $\mathbf{A}_1$  lie inside, and one outside, the unit circle. The following proposition establishes a necessary and sufficient condition for that property to obtain.

**Proposition 1.** A necessary and sufficient condition for (25) to have a unique non-explosive solution is given by  $\gamma > 1$ .

**Proof.** The characteristic polynomial of  $A_1$  is given by

$$p_{A_1}(z) = z^3 - \left(1 + \beta + \frac{\kappa}{\sigma} + \frac{1}{\gamma}\right)z^2 + \left(\frac{1}{\gamma}\left(1 + \beta + \frac{\kappa}{\sigma}\right) + \beta\right)z - \frac{\beta}{\gamma} = (z - \lambda_1)(z - \lambda_2)(z - \gamma^{-1})$$

$$\tag{27}$$

where  $\lambda_1 \equiv (1+\beta+(\kappa/\sigma)-\sqrt{(1+\beta+(\kappa/\sigma))^2-4\beta})/2$  and  $\lambda_2 \equiv (1+\beta+(\kappa/\sigma)+\sqrt{(1+\beta+(\kappa/\sigma))^2-4\beta})/2$ . Using the same logic as in the proof of the Lemma above we conclude that both  $\lambda_1$  and  $\lambda_2$  are real, and satisfy the inequality  $0 < \lambda_1 < 1 < \lambda_2$ . Thus, and as long as  $\gamma > 1$ , two eigenvalues of **A** lie inside and one outside, the unit circle.

<sup>&</sup>lt;sup>17</sup> The present rule generalizes the one considered in Galí (2003, 2009) to the case of an arbitrary interest rate path.

More generally, one can show that an interest rate rule of the form

$$i_t = \gamma \pi_t + \varphi \Delta y_t + v_t \tag{28}$$

generates a unique equilibrium, where  $\gamma > 1$  and  $\{\nu_t\}$  is an arbitrary exogenous process. By setting  $\varphi \equiv \sigma \gamma$  and  $\nu_t = i_t^* - \gamma i_{t-1}^*$  one can guarantee that such a unique equilibrium is associated with the desired interest rate path  $\{i_t^*\}$ .

Letting  $\mathbf{x}_t = [\pi_t, y_t, y_{t-1}]'$  and  $\mathbf{z}_t = [u_t, t_t^* - \gamma t_{t-1}^*]'$ , we can compute projections  $E_t \{\pi_{t+k}\}$  and  $E_t \{y_{t+k}\}$  using

$$E_t\{\mathbf{x}_{t+k}\} = \mathbf{A}_1^{-k}(\mathbf{x}_t - \mathbf{g}_t) \tag{29}$$

where  $\mathbf{g}_t \equiv \mathbf{B}_1 \mathbf{z}_t + \mathbf{A}_1 \mathbf{B}_1 E_t \{\mathbf{z}_{t+1}\} + \cdots + \mathbf{A}_1^{k-1} \mathbf{B}_1 E_t \{\mathbf{z}_{t+k-1}\}$ , and where  $\mathbf{x}_t$  is the unique solution to (25), which can be obtained using standard formulae (e.g., Blanchard and Kahn, 1980).

It should be noted here that the role of parameter  $\gamma$  is restricted to guaranteeing that the target path for the interest rate is attained, which is the case for *any* value of  $\gamma$  larger than one. The particular choice of  $\gamma$  has no influence, however, on the resulting equilibrium path of inflation and the output gap and, hence, on the corresponding projections. To see this, note that under rule (22) the linear combination  $\pi_t + \sigma \Delta y_t$  will be equal to  $i_{t-1}^*$ , for all t. The latter condition can in turn be combined with inflation Eq. (2) to obtain a difference equation for the output gap, which can be solved independently of  $\gamma$ .

#### 4.3. Interest rate rule III

Consider finally an interest rate rule of the form

$$i_{t} = i_{t}^{*} - \gamma i_{t-1}^{*} + \gamma (\pi_{t} + r_{t-1})$$
(30)

where  $r_t \equiv i_t - E_t \{\pi_{t+1}\}$  is the (ex-ante) real interest rate and  $\gamma > 1$ . Combining (30) with the definition of the real interest rate, yields again a difference equation of the form

$$i_t - i_t^* = \frac{1}{\nu} E_t \{ i_{t+1} - i_{t+1}^* \} \tag{31}$$

whose only stationary solution is  $i_t = i_t^*$  for all t given our assumption that  $\gamma > 1$ .

The equilibrium dynamics under rule (30) are described by three equations: inflation Eq. (2), the dynamic IS Eq. (1) and the interest rate rule (30), with the latter two rewritten in terms of the real interest rate, that is,

$$y_t = E_t \{ y_{t+1} \} - \frac{1}{\sigma} r_t \tag{32}$$

$$r_t + E_t\{\pi_{t+1}\} = i_t^* - \gamma i_{t-1}^* + \gamma (\pi_t + r_{t-1}) \tag{33}$$

The previous equilibrium conditions can be written in compact form as

$$\begin{bmatrix} \pi_t \\ y_t \\ r_{t-1} \end{bmatrix} = \mathbf{A}_2 \begin{bmatrix} E_t \{ \pi_{t+1} \} \\ E_t \{ y_{t+1} \} \\ r_t \end{bmatrix} + \mathbf{B}_2 \begin{bmatrix} u_t \\ i_t^* - \gamma i_{t-1}^* \end{bmatrix}$$
(34)

$$\mathbf{A}_{2} \equiv \begin{bmatrix} \beta & \kappa & -\frac{\kappa}{\sigma} \\ 0 & 1 & -\frac{1}{\sigma} \\ \frac{1}{\gamma} - \beta & -\kappa & \frac{1}{\gamma} + \frac{\kappa}{\sigma} \end{bmatrix}; \quad \mathbf{B}_{2} \equiv \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ -1 & -\frac{1}{\gamma} \end{bmatrix}$$

$$(35)$$

Once again, the system of difference Eq. (34) involves two non-predetermined and one predetermined variables. Thus, it has a unique non-explosive solution if and only if two eigenvalues of  $A_2$  lie inside, and one outside, the unit circle. The following proposition establishes that the condition  $\gamma > 1$  is both necessary and sufficient condition for uniqueness to obtain.

**Proposition 2.** A necessary and sufficient condition for (34) to have a unique non-explosive solution is given by  $\gamma > 1$ .

**Proof.** The characteristic polynomial of  $A_2$  is given by

$$p_{A_2}(z) = z^3 - \left(1 + \beta + \frac{\kappa}{\sigma} + \frac{1}{\gamma}\right)z^2 + \left(\frac{1}{\gamma}\left(1 + \beta + \frac{\kappa}{\sigma}\right) + \beta\right)z - \frac{\beta}{\gamma}$$
(36)

which is identical to that of  $A_1$ . Hence, the same condition for uniqueness of the equilibrium applies.  $\Box$ 

Letting now  $\mathbf{x}_t = [\pi_t, y_t, r_{t-1}]'$  and  $\mathbf{z}_t = [u_t, i_t^* - \gamma i_{t-1}^*]'$ , we can compute conditional forecasts  $E_t \{\pi_{t+k}\}$  and  $E_t \{y_{t+k}\}$  using

$$E_t\{\mathbf{x}_{t+k}\} = \mathbf{A}_2^{-k}(\mathbf{x}_t - \mathbf{g}_t) \tag{37}$$

where  $\mathbf{g}_t = \mathbf{B}_2 \mathbf{z}_t + \mathbf{A}_2 \mathbf{B}_2 E_t \{\mathbf{z}_{t+1}\} + \dots + \mathbf{A}_2^{k-1} \mathbf{B}_2 E_t \{\mathbf{z}_{t+k-1}\},$  and where  $\mathbf{x}_t$  is the unique solution to (34).

Note finally that under rule (30) the linear combination  $\pi_t + r_{t-1}$  is (ex-post) equal to  $i_{t-1}^*$ , for all t. That condition can be combined with (2) and (16) to obtain a system of three difference equations with three endogenous variables ( $\pi_t$ ,  $y_t$ , and  $r_t$ ),

which is independent of  $\gamma$ . Hence, and as it was the case for interest rate rule II, the specific choice of  $\gamma$  has no influence on the resulting projections.

### 5. Multiple determinate projections: an illustration using the basic New Keynesian model

The previous section has described three alternative approaches to the construction of projections conditional on a given path for the nominal interest rate which are not subject to the problem of indeterminacy. Such a multiplicity of approaches raises a natural question: are the inflation and output gap projections generated by the different approaches identical, if they are conditioned on the same interest rate path? The answer to that question is negative, as the numerical examples shown below make clear. Furthermore, and most importantly, the differences across projections obtained using different rules are quantitatively large.

For the purposes of the present section it is assumed that the economy is described by the canonical New Keynesian model used above. In the next section a similar exercise is carried out using the estimated DSGE model of Smets and Wouters (2007), a more realistic framework and, hence, one for which quantitative predictions can be taken more seriously.

Consider the case of a central bank which, as of time t, wants to produce conditional forecasts of inflation and the output gap for period t+k. As above, the non-policy block of the economy is described by Eqs. (1) and (2). For simplicity, the economy is assumed to be at its steady state position in period t-1, i.e.  $y_{t-1} = \pi_{t-1} = i_{t-1} = 0$ . A cost-push shock of unit size is assumed to hit the economy in period t, vanishing over time in proportion to  $\rho_k^k$ , for  $k = 1, 2, 3, \ldots$ 

What are the model-based projections for inflation and the output gap in period t+k conditional on the central bank keeping the interest rate unchanged? Those projections are computed next under each of the three approaches discussed

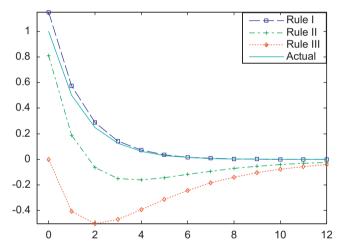


Fig. 1. CIR projections in the basic NK model: inflation.

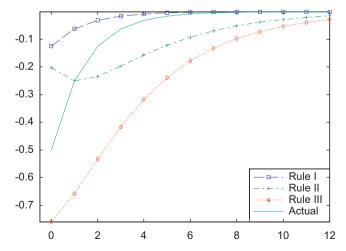


Fig. 2. CIR projections in the basic NK model: output.

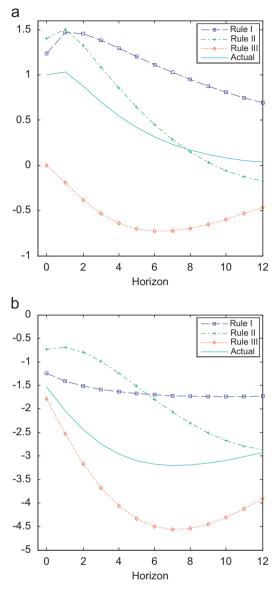


Fig. 3. CIR projections in the Smets-Wouters model: productivity shocks: (a) inflation and (b) output.

above for a calibrated version of the baseline model. For the purposes of this exercise, the following parameter values are assumed, which are similar to those often used in the literature:  $\beta=0.99$ ,  $\sigma=1$ ,  $\kappa=0.1$ ,  $\phi=1.5$  and  $\rho_u=0.5$ . The size of the shock is normalized so that the response of (annualized) inflation on impact under the "true" rule (3) is of one percentage point. It should be clear that the main finding of this section is a qualitative one, and does not hinge on the details of the calibration.

Figs. 1 and 2 display the inflation and output gap projections at horizons up to 12 quarters associated with the three rules described in the previous section. The figures also display the (unconditional) forecast associated with the "true" rule (3). Note that, even though the three rules support an unchanged interest rate through the forecast horizon (and beyond, in the case of rules II and III), their associated projections for inflation and output are very different. The differences among them involve both the size of the projected changes in inflation and the output gap as a result of the assumed cost-push shock, but also in the patterns of those responses and, in one case, even the sign of those responses. One can also see that none of them tracks the unconditional forecast well, but this was to be expected since the latter implies a different interest rate path. Interestingly, the constant interest rate projections differ even in terms of the sign of their deviation from the unconditional forecasts.

<sup>&</sup>lt;sup>18</sup> As argued above, the specific choice of  $\gamma$  has no influence on the projections, as long as it is larger than one, which it is assumed here.

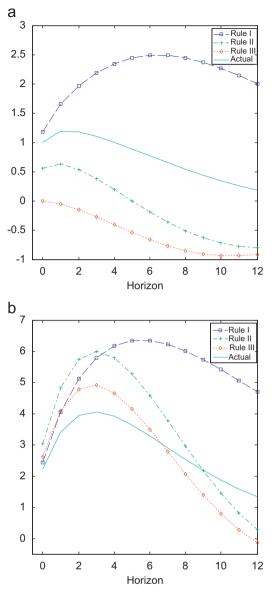


Fig. 4. CIR projections in the Smets-Wouters model: investment shocks: (a) inflation and (b) output.

Why do the three rules considered above generate different projections despite being associated with an identical nominal interest rate path? Put simply, the intuition behind that result is that any given nominal interest rate path is consistent with different paths of the real rate and expected inflation. In the New Keynesian model, the presence of sticky prices makes it possible for the central bank to influence real variables, including the real interest rate. The three rules considered achieve the same nominal rate path through different combinations of real rates and expected inflation. Not surprisingly, each of those combinations is associated with different paths for the output gap and, as a result, for inflation as well. This leads, in turn, to different projections.<sup>19</sup>

Most importantly, the previous finding clearly calls into question the usefulness of projections conditional on a given interest rate path, since there is no obvious reason to prefer one method over another in order to generate such projections. Put differently, the information required in order to compute well defined macroeconomic projections goes beyond the specification of the interest rate path and the horizon of the projections. A complete description would need to

<sup>&</sup>lt;sup>19</sup> In the case of ME-based projections, one might argue that there should be a preference for the rule that does a better job at matching private sector expectations of other macrovariables, in addition to the nominal interest rate. But, any model-based private sector forecasts would have to be conditioned on some interest rate rule. A well defined approach would thus consist in computing projections conditional on the policy rule assumed by the private sector. But then it is not clear that those forecasts should be attached much value if the latter rule differs from the true one.

include the nature and specification of the *policy rule* that will be followed in order to support the desired interest rate path.

## 6. Multiple determinate projections: an illustration using the Smets-Wouters model

The present section reports the findings of an exercise similar to the one described above, but using a more realistic version of the New Keynesian model, namely, the estimated DSGE model of Smets and Wouters (2007). Relative to the basic New Keynesian model above, the Smets–Wouters model incorporates a number of features, including endogenous capital accumulation (subject to adjustment costs), habit formation in consumption, variable capital utilization, staggered wage and price setting with partial indexation, and as many as seven different structural shocks. That model, as well as the related models in Smets and Wouters (2003) and Christiano et al. (2005), can be viewed as the backbone of the estimated DSGE models developed at central banks in recent years and used for monetary policy analysis and forecasting. The reader is referred to the original Smets and Wouters (2007) paper (and its companion technical appendix) for details.

The construction of inflation and output projections under alternative rules using the Smets-Wouters model requires two main changes relative to the analysis above. First, and given the presence of endogenous state variables, the exogenous monetary policy shocks  $\{v_t\}$  that must be fed into rule I ("modest interventions") in order to keep the nominal

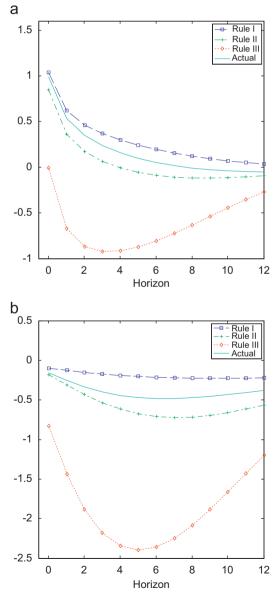


Fig. 5. CIR projections in the Smets-Wouters model: price markup shocks: (a) inflation and (b) output.

interest rate along the desired path (a constant one, in the present exercise) are computed using an iterative procedure. Secondly, rule II has to be modified in a way consistent with the specification of the Euler equation in the Smets–Wouters model, which is given by

$$x_{t} = E_{t}\{x_{t+1}\} - \frac{1}{\tilde{\sigma}}(i_{t} - E_{t}\{\pi_{t+1}\})$$
(38)

where  $x_t \equiv (1-\chi)c_t - \chi c_{t-1} - \psi n_t$ , with  $c_t$  and  $n_t$  denoting, respectively, (log) consumption and (log) hours, and where  $\chi$  and  $\psi$  are themselves functions of exogenous parameters. Note that (38) differs from (1) due to three factors: (i) consumption and output no longer coincide due to the presence of capital accumulation, (ii) habit formation is allowed for  $(\chi \ge 0, \tilde{\sigma} \ge \sigma)$ , and (iii) the utility function is (potentially) nonseparable in consumption and hours  $(\psi \ge 0)$ . Accordingly, the interest rate rule corresponding to (22), and which combined with (38) will support an arbitrary exogenous interest rate path  $\{i_t^*\}$  is now given by

$$i_t = i_t^* - \gamma i_{t-1}^* + \gamma (\pi_t + \tilde{\sigma} \Delta x_t) \tag{39}$$

Finally, note that rule III does not require any modification and can be implemented "as is" in the Smets–Wouters model. Figs. 3–6 display the inflation and output projections over a twelve-quarter horizon implied by rules I through III under the assumption of a constant interest rate path  $(i_{t+j}^* = 0, j = 0, 1, 2, ...)$ , and in response to four different structural shocks: a neutral

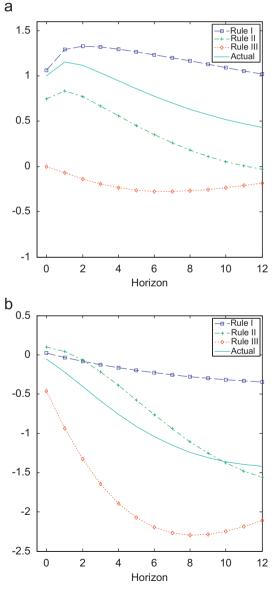


Fig. 6. CIR projections in the Smets-Wouters model: wage markup shocks: (a) inflation and (b) output.

technology shock ("productivity"), an investment-specific technology shock ("investment"), a price markup shock, and a wage markup shock. These four shocks account for the bulk of the forecast error variance decomposition of inflation and output at a ten-quarter horizon in Smets and Wouters (2007). In each case, the inflation and output forecasts consistent with the "true" interest rate rule are also displayed, though now the latter rule corresponds to the estimated interest rate rule in Smets and Wouters (2007), which is substantially richer than (3) above.<sup>20</sup> Again, the size of each shock is normalized so that the response of (annualized) inflation on impact under the true rule is of one percentage point. For the purposes of the present exercise the model is calibrated according to the baseline calibration/estimation in Smets and Wouters (2007).

The projections based on the Smets-Wouters model reported in Figs. 3–6 confirm the main finding obtained in the context of the basic New Keynesian model, as discussed in the previous section: even though the three rules considered support the same path for the nominal interest rate (a flat path in this case), their implied projections for both inflation and output are generally very different, with the gaps among them being economically significant and often of a size of one-hundred or more basis points. Of course, those projections also display important differences relative to those generated by the actual rule, though this should come as no surprise since the latter implies a different path for the nominal interest rate. As can be seen in Fig. 5, rules I and II, when conditioned on price markup shocks, possibly provide the only example of quantitatively similar – though far from identical – projections (which in turn happen to be similar to those generated by the actual rule). Yet, even in that case, rule III involves very different projections, with no reason to prefer of discard the latter relative to those implied by rules I and II, as argued above.

#### 7. Concluding remarks

There appears to be a growing tendency among central banks to construct and report macroeconomic projections consistent with their own views about the future evolution of nominal interest rates. Many economists have welcome that development on different grounds, including its likely benefits from the viewpoint of transparency and ease of communication of the monetary authority's decisions and overall strategy to the public.

The present paper has provided an additional argument for the adoption of projections based on the central bank's own interest rate forecasts. In particular, it has argued that the alternative, i.e. conditioning projections on an exogenously given interest rate path, rests on shaky theoretical grounds. The latter assessment does not follow from the often heard argument that such projections will typically be indeterminate in forward-looking models. On the contrary, as many as three different approaches have been presented above to constructing determinate projections conditional on an arbitrary interest rate path. Instead, the main shortcoming of those projections lies precisely in the multiplicity of methods (each associated with a different rule) that are available to generate them, together with the fact that the different methods generally yield divergent projections for variables other than the interest rate itself. That observation calls into question the usefulness of projections conditional on a given interest rate path since, in principle, there is no obvious reason to prefer one method over another.

#### **Acknowledgments**

I am grateful for comments and suggestions to the editor Marty Eichenbaum, two anonymous referees, Marc Giannoni, Frank Smets, Lars Svensson, Raf Wouters and participants at the CREI Faculty Lunch, SNB Monetary Policy Conference, and SED Meetings (Cambridge, MA). Davide Debortoli and Tomaz Cajner provided excellent research assistance. I am grateful to the Government of Catalonia and the Ministerio de Ciencia e Innovación for financial support.

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<sup>&</sup>lt;sup>20</sup> In particular, the nominal interest rate is allowed to respond, in addition to inflation, to its own lag, as well as the level and first-difference of the output gap.

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