Cities and Public Policy

Urban Economics: Week 10

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The Tiebout Model

Tiebout's (1956) seminal idea

- A reply to Samuelson's case for government intervention
- Private provision of pure public goods is highly inefficient
- If public goods are provided locally, people vote with their feet
- An implicit market for bundles of government services and taxes
- In 1956 "pure theory" in the *JPE* had no equations

Bewley's (1981) seminal model

- Rigorous definition of a Tiebout equilibrium
- Conditions for optimality of local provision of public goods

Tiebout Equilibrium

1. Spatial equilibrium
   - Agents cannot improve their condition by moving
2. Negligible individual impact on government policy
   - Agents take government services and taxes as given
3. Rational policymaking
   - Maximizing the local median voter's welfare or a similar political goal

Optimality conditions

1. As many governments as types of people
   - Every locality has perfectly homogeneous residents
2. Public services rather than non-rival pure public goods
   - No economies of scale: constant cost per person benefiting

Provision of Public Goods

- Measure one of agents with utility

$$u = y - t + \alpha v(g)$$

  - Exogenous pre-tax income y
  - Tax t: no difference between lump-sum and income taxes
  - Public good g with valuation $v(g)$ such that $v' > 0 > v''$

- Provision of public goods in each municipality

$$g = tn^{\gamma}$$

  - A fraction n of the population lives in the municipality
  - γ measures economies or diseconomies of scale
    $\gamma = 1$ is a pure public good and $\gamma = 0$ a public service

- Agents sort into municipalities

Pure Public Goods and Suboptimal Scale

- With pure public goods ($\gamma = 1$) larger communities are more efficient
- There are equilibria with inefficient fragmentation
- Two identical communities with homogeneous agents
  - The symmetric equilibrium always exists
  - Unstable equilibrium: not robust to introducing competitive developers
- Heterogeneous agents: half have $\alpha_i = \bar{\alpha}$ and half $\alpha_i = \underline{\alpha} < \bar{\alpha}$
- Stable equilibrium with two communities and public good provision

$$\bar{g} = v'^{-1}\left(\frac{2}{\bar{\alpha}}\right) > \underline{g} = v'^{-1}\left(\frac{2}{\underline{\alpha}}\right)$$

- Each agent strictly prefers his own community, hence stability

Lack of Efficient Integration

- Suppose everyone lived in one community
- Welfare-maximizing provision of public goods

$$g^* = v'^{-1}\left(\frac{2}{\underline{\alpha} + \bar{\alpha}}\right) > \bar{g}$$

  - Greater public good provision thanks to economies of scale
- Competitive developers can propose $\underline{g}$ to half the population
- The proposal is attractive to all agents with $\alpha_i = \underline{\alpha}$ if and only if

$$\underline{\alpha} v\left(\underline{g}\right) - 2\underline{g} > \underline{\alpha} v\left(g^*\right) - g^*$$

- Satisfied if $\underline{\alpha}$ is low enough: no efficient integrated equilibrium then
- Differentiated taxes by group would be ideal, but they are impossible
  - α_i is unobservable and tax discrimination on observables is illegal

Lack of Efficient Separation

- Should we really be concerned about non-rival local public goods?
  - Schools and roads are rival and excludable
  - Police and fire departments need to grow with city size too
- The opposite problem with public services: insufficient personalization
- Three groups but only two communities
- Two groups must live together: at least one does not get its ideal g
- Pareto inefficiency with the median-voter theorem
  - Two groups get their ideal policy
  - The third has no way of getting them to change
  - Again the impossibility of differentiated taxation

Misallocation

- Another problem arising from having too few municipalities
- Four different groups and as many specific public services
- Binary service provision: $g_i \in \{0, 1\}$ for $i = 1, 2, 3, 4$
- Each person in group i has utility

$$u_i = y - t + v(g_i) + (1 - \mu) v(g_{i+1})$$

  - Close the circle: $g_5 \equiv g_1$

- Inefficient equilibrium:
  - Groups 1 and 3 live together and supply g_1 and g_3
  - Groups 2 and 4 live together and supply g_2 and g_4
- Each agent strictly prefers his own community, hence stability
- Not robust to competitive developers

Ambiguous Theories of Decentralization

- Decentralization has both advantages and disadvantages
- Public economics and centralization, following Samuelson
  - Decentralization sacrifices economies of scale
  - Centralization internalizes spatial externalities
- The Tiebout argument: decentralization allows preference-matching
  - The argument needs the assumption of policy uniformity
  - Why not centrally coordinated but differentiated policies?
- Political economy
  - Centralization reduces preference-matching via legislative bargaining
  - Decentralization creates competition, providing checks on politicians
    1. Mobility limits rent extraction by local officials
    2. Yardstick competition makes local officials more accountable
  - Ambiguous additional mechanisms in models of political agency

Empirical Analysis

- Given the theoretical ambiguity, we could let the data speak
- Broad-brush picture of U.S. municipalities
  1. Raise income with property taxes
  2. Spend income on schools
     - There are other municipal services and occasionally other taxes
- The empirical literature is overwhelmingly about schools
- The political fractionalization of metropolitan areas varies quite a bit
  - The problem is that it is obviously endogenous
- Hoxby (2000) opened the field up to identification by IV
  - But the Hoxby–Rothstein quarrel represents economics at its worst
  - Politically charged, personally nasty, scientifically difficult to adjudicate
  - IV strategies with many degrees of freedom in specification search

Income Redistribution

- A share $p < 1/2$ of the population are poor and $1 - p$ rich
  - Exogenous gross incomes respectively y_P and $y_R > y_P$
- The government taxes the rich and redistributes to the poor

$$(1 - p)\, t = pr$$

- The poor have utility $u_P = \log(y_P + r)$
- The rich have heterogeneous altruism a and utility

$$u_R = \log(y_R - t) + ap \log(y_P + r)$$

- Altruism a is uniformly distributed on $[\bar{a} - \Delta/2, \bar{a} + \Delta/2]$

Preferences for Redistribution

- The poor prefer expropriation $t = y_R$
- Each of the rich has ideal policy (if positive)

$$r(a) = \frac{a(1-p)y_R - y_P}{1+ap} \text{ with } r'(a) = \frac{(1-p)y_R + py_P}{(1+ap)^2} > 0$$

- At the national level, the median voter is rich and has altruism

$$\hat{a} = \bar{a} + \frac{\Delta}{2}\frac{p}{1-p}$$

- Nation-wide direct democracy yields his preferred redistribution

Local Redistribution and Endogenous Segregation

- People vote on taxes and transfers within their community
  - No desire for outward transfers in equilibrium
- Two equilibria with two locations

1. Two identical copies of the integrated economy
2. Perfect segregation of rich and poor
   - The rich community has $p_1 = 0$ and $t = 0$
   - The poor community has $p_2 = 1$ and $t = y_R$

- No need to assume that altruism is local: only $y_P > \bar{a} y_R$
  - The rich community does not vote to redistribute to the poor one
  - The poor never vote to redistribute to anyone but themselves
- Segregation is always heuristically stable; integration need not be

The Curley Effect

- Far-sighted redistributionists moderate taxes to preserve the tax base
  - Potential for races to the bottom across tax jurisdictions
- But a redistributionist demagogue may want to chase the rich away
- Glaeser and Shleifer (2005): named after Boston mayor James Curley
  - More recently, Coleman Young in Detroit for 24 years
- Complete reversal of the role of foresight
  - A benevolent politician does better when he is forward-looking
  - The cynical demagogue also does better for himself
  - But he does so by doing worse for the voters
- The opposite of the classic "vote with your feet" argument
  - Bad politicians are not disciplined by out-migration
  - On the contrary, they become worse to pursue it

Political Class Warfare

- The demagogue has an intrinsic appeal to the lower-status group
  - Ethnic allegiances when class and ethnicity are aligned
  - Or direct class identity in an ethnically homogeneous polity
- Support from the poor also increases to redistribution
- The rich vote against the demagogue
  - They also have class or ethnic loyalties
  - They oppose redistribution
- But the rich can also flee the demagogue by leaving town
  - The higher redistribution, the more rich people flee
  - Reduced-form assumption of a continuous response

Exploiting Allegiance

- Suppose that the low-status group reliably votes for the demagogue
  - Allegiance is its main motivation
  - Past performance counts less
- His key incentive is to shape the electorate, not to help his supporters

$\Rightarrow$ Inefficient and self-defeating redistribution

  - The demagogue cares little about transferring from the rich to the poor
  - He really only wants to drive the rich out of town
- The demagogue's own supporters may be harmed
  - The two groups can be complements in production
  - Simple loss from taxing beyond the maximum of the Laffer curve

Local Public Finance

Two cities or regions

- Identical endowment of land $1/2$ per region
- Endogenous population N and $1-N$ respectively
- Homogeneous exogenous labor income y
- Exogenous tax revenue requirements $T_1 \neq T_2$
- Poll taxes (or income taxes) t_c
- Property taxes on land τ_c

Land Consumption

- Optimal housing consumption $p_c = v'(L_c)$
- Land consumption surplus

$$V_c = v(L_c) - v'(L_c) L_c$$

- Land markets clear

$$L_1 = \frac{1}{2N} \text{ and } L_2 = \frac{1}{2(1-N)}$$

- Housing consumption surplus differential

$$\begin{aligned} \Delta_V(N) \equiv & \; v\left(\frac{1}{2N}\right) - v'\left(\frac{1}{2N}\right)\frac{1}{2N} \\ & - \left[v\left(\frac{1}{2(1-N)}\right) - v'\left(\frac{1}{2(1-N)}\right)\frac{1}{2(1-N)}\right] \end{aligned}$$

The First Best

- An individual i owns land $\bar{L}_1^i$ and $\bar{L}_2^i$.
- His utility in city c is

$$U_c^i = y + (p_1 - \tau_1)\,\bar{L}_1^i + (p_2 - \tau_2)\,\bar{L}_2^i - t_c + \max_L \{v(L) - p_c L\}$$

  - Land consumption L at equilibrium price p_c
  - Increasing, concave utility of land $v(L)$

- Aggregate welfare

$$y - T_1 - T_2 + Nv\left(\frac{1}{2N}\right) + (1-N)\,v\left(\frac{1}{2(1-N)}\right)$$

- Social optimum

$$N^* = \frac{1}{2} \Rightarrow \Delta_V(N^*) = 0$$

The Optimal Tax Base

- Spatial equilibrium

$$\Delta_V(N) = t_1 - t_2$$

1. Cities levy only income taxes $t_1 = T_1/N$ and $t_2 = T_2/(1-N)$
   - ★ The first best cannot be achieved for any $T_1 = T_2$
   - ★ We would expect the most taxed city to be smaller
   - ★ There could be multiple equilibria, but all are inefficient
2. Cities levy only property taxes on land $\tau_c = 2T_c$
   - ★ The first best is achieved

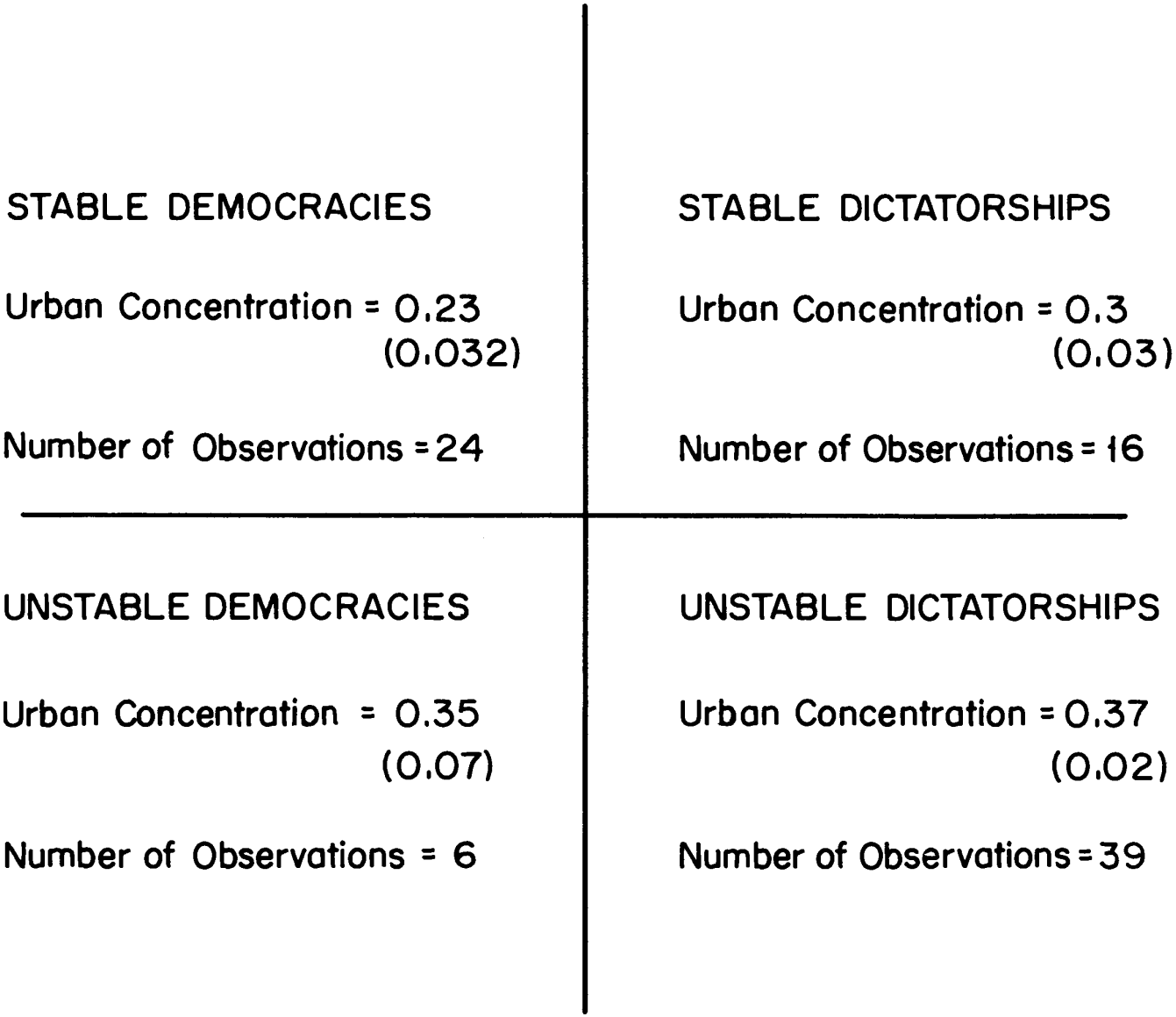
$\Rightarrow$ Efficiency of taxing the immobile factor

- More broadly, spatial elasticity vs. supply/demand elasticity

Primate Cities

- Some countries are dominated by their capital city
  - Montevideo: 59% of the population of Uruguay
  - Santiago: 48% of the population of Chile
  - Dublin: 39% of the population of Ireland
  - Athens: 35% of the population of Greece
  - Copenhagen: 35% of the population of Denmark
  - Buenos Aires: 32% of the population of Argentina
- Unsurprisingly, these tend to be smaller countries
  - Evidence of an efficient scale for big cities
- Economic geography plays a role
  - Share of agriculture, transportation network
- Politics are the main driving factor (Ades and Glaeser 1995)

Politics and Urban Concentration



Redistributing Income to the Capital

- Two locations
  1. The capital city, with population N
  2. Everywhere else, with population $1 - N$
- Two ways for the citizens to oust the incumbent ruler
  1. An election in which everyone votes
  2. A revolt by the capital's inhabitants
- Two policy instruments
  1. Transfer to each resident of the capital t_C
  2. Transfer to every other citizen t_H
     - Government budget constraint

$$Nt_C + (1 - N)\, t_H = T$$

Transfers and Political Support

1. Transfers to all regions generate electoral support
   - Citizen i opposes the ruler with probability $f(t_i) : f' < 0$
   - The incumbent is replaced in an election with probability

$$(1-\delta) E\left(Nf(t_C) + (1-N) f(t_H)\right) \text{ with } E' > 0$$

   - More extreme: elections decided by the rural median voter
2. Transfers to the capital reduce the probability of revolt
   - Citizen i takes part in the revolt with probability $g(t_C) : g' < 0$
   - The incumbent is replaced after a revolt with probability

$$\delta R\left(Ng(t_C)\right) \text{ with } R' > 0$$

- Dictatorship vs. democracy: exogenous parameter $\delta \in [0,1]$

Bread and Circuses

- Take the population of the capital N as given
- Favoritism towards the capital measured by $\Delta \equiv t_C - t_H$
- The ruler is solely concerned with remaining in power

$$\min \left\{ (1-\delta) E \left(N f \left(t_C \right) + (1-N) f \left(t_H \right) \right) + \delta R \left(N g \left(t_C \right) \right) \right\}$$

subject to

$$t_C = T + (1-N)\Delta \text{ and } t_H = T - N\Delta$$

- First-order condition

$$(1-\delta) E' \left[f' \left(t_C \right) - f' \left(t_H \right) \right] = \delta R' g' \left(t_C \right)$$

- Dictators fear urban revolts, so they favor the capital

$$\delta > 0 \Leftrightarrow t_C > t_H$$

All Roads Lead to Rome

- People flock towards transfers: $N(\Delta) : N' > 0$
- Suppose the ruler can commit to transfer levels t_C, t_H
- His first-order condition becomes

$$\begin{aligned} & N(1-N)\left\{(1-\delta)E'\left[f'(t_C)-f'(t_H)\right]+\delta R'g'(t_C)\right\} \\ & +\left\{(1-\delta)E'\left[f(t_C)-f(t_H)\right]+\delta R'g(t_C)\right\}N' \\ & -\Delta\left\{(1-\delta)E'\left[Nf'(t_C)+(1-N)f'(t_H)\right]+\delta NR'g'(t_C)\right\} \end{aligned}$$

1. Generate support preferentially in the capital to avoid revolt
2. Avoid attracting people to the capital where they can revolt
3. Avoid attracting people to the capital where they must be bought off

$\Rightarrow$ The ruler wishes to commit to lower favoritism for the capital

Dictatorship and Primate Cities

- Does dictatorship lead to a larger capital?
- The ruler wishes to commit to favorable transfers if and only if

$$-\frac{g'(T)}{g(T)} > \frac{N'(0)}{N(0)[1-N(0)]}$$

  - The effect on the probability of revolt is high
  - The effect on the number of potential revolters is low
- Is there a commitment device?
  - Transfers can be revised after citizens have chosen location
  - Ex post, when revising transfers $N' \approx 0$
  - Then the dictator always chooses $\delta > 0$
  - The city grows large in anticipation of lack of commitment
- On the other hand, the could be barriers to internal migration

Economics and Urban Primacy

Dependent variable: log of average population in main city (1970–1985)	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	1.136	2.014	1.156	0.651	0.808	0.297
	(0.878)	(0.934)	(0.942)	(1.109)	(1.082)	(1.063)
Capital city dummy	0.424	0.465	0.374	0.336	0.283	0.408
	(0.204)	(0.196)	(0.181)	(0.200)	(0.180)	(0.188)
Log of average nonurbanized population	0.595	0.553	0.583	0.640	0.623	0.641
	(0.068)	(0.066)	(0.063)	(0.073)	(0.072)	(0.071)
Log of average urbanized population outside the main city	0.059	0.066	0.063	0.058	0.054	0.045
	(0.050)	(0.045)	(0.042)	(0.042)	(0.040)	(0.038)
Log of land area	0.167	0.155	0.115	0.109	0.113	0.120
	(0.051)	(0.049)	(0.049)	(0.054)	(0.053)	(0.055)
Log of average real GDP per capita	0.034	0.058	0.165	0.193	0.149	0.166
	(0.129)	(0.131)	(0.127)	(0.146)	(0.149)	(0.148)
Average share of the labor force outside of agriculture	2.656	2.556	2.704	2.623	2.782	3.071
	(0.554)	(0.567)	(0.549)	(0.547)	(0.518)	(0.516)
Share of trade in GDP		−0.609	−0.676	−0.463	−0.404	−0.519
		(0.225)	(0.204)	(0.228)	(0.240)	(0.244)

Politics and Urban Primacy

Dictatorship dummy based on Gastil's index of political rights		0.444	0.324	0.442	0.705	
		(0.154)	(0.156)	(0.148)	(0.181)	
Africa dummy			0.160	0.127	0.172	
			(0.263)	(0.260)	(0.257)	
Latin America dummy			0.390	0.342	0.295	
			(0.159)	(0.158)	(0.162)	
New democracy				0.428		
				(0.177)		
Revolution and coups					2.372	
					(0.772)	
Dictatorship dummy × revolution and coups					−2.705	
					(0.803)	
Number of observations	85	85	85	85	85	85
Adjusted R^2	0.81	0.81	0.82	0.83	0.83	0.84

Note. All variables are averages of their 1970, 1975, 1980, and 1985 observations. The 1985 observation is missing for the Share of labor outside of agriculture. The Dictatorship dummy takes a value of one for countries with an average Gastil index larger than three. White-corrected standard errors in parentheses.

Externalities and Suboptimal Outcomes

- An economy with two locations: the city and the hinterland
  - The city hosts fraction N of the population
  - Congestion externalities lead to urban utility $V(N)$
  - The hinterland yields reservation utility $\bar{U}$
- Decentralized equilibrium $\hat{N}$ such that

$$V(\hat{N}) - \bar{U} = 0$$

- Social optimum

$$N^* = \arg\max \{NV(N) + (1 - N)\bar{U}\}$$

such that

$$V(N^*) + N^* V'(N^*) - \bar{U} = 0$$

Restricting City Size

- Generically, the decentralized equilibrium is suboptimal
  - Agglomeration economies $V'(N) > 0$
  - Agglomeration diseconomies $V'(N) < 0$
- We may believe that $V'(N) > 0$ on most of its domain
- But if the decentralized equilibrium is stable

$$V'(\hat{N}) < 0$$

- Then the city is too large

$$V(\hat{N}) + \hat{N}V'(\hat{N}) - \bar{U} < 0$$

- Optimal policy should always be reducing the size of the city

Fine-Tuning City Size?

- With two cities the free market yields $\hat{N}_1$ such that

$$V_1\left(\hat{N}_1\right) = V_2\left(1 - \hat{N}_1\right)$$

- The social optimum is N_1^* such that

$$V_1\left(N_1^*\right) + N_1^* V_1'\left(N_1^*\right) = V_2\left(1 - N_1^*\right) + \left(1 - N_1^*\right) V_2'\left(1 - N_1^*\right)$$

- Equilibrium stability requires that

$$V_1'\left(\hat{N}_1\right) + V_2'\left(1 - \hat{N}_1\right) < 0$$

$\Rightarrow$ No general result without knowing V_1 and V_2

Helping People, Not Places

1. Helping poor places inefficiently dislocates economic activity
   - From more productive places to less productive places
2. Place-based aid fails to transfer money to the poor
   - In equilibrium it flows to property owners
   - Really poor people do not own immobile factors
3. Subsidies to poor areas induce concentrated poverty
   - The poor are harmed rather than helped

- Clear to regional economists, highly controversial with the public
  - New Orleans, hurricane Katrina, and Glaeser (2005)
  - NPR: Glaeser vs. the furious caller 17:45–22:45
    - ★ More Glaeser at 6:00–9:00, 32:00–34:00, 37:30–37:40

Technology

- Production function for location i

$$Y_i = A_i K_i^{\alpha} L_i^{\beta} \bar{Z}_i^{1-\alpha-\beta}$$

  - Productivity A_i and fixed capital $\bar{Z}_i$
  - Mobile capital K_i with exogenous price p_K

- Labor demand

$$w_i = \beta \left(\frac{\alpha}{p_K} \right)^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} \left(\frac{\bar{Z}_i}{L_i} \right)^{\frac{1-\alpha-\beta}{1-\alpha}}$$

- Fixed requirement of one unit of housing per person
- Construction costs r_C

Asymmetric Shock

- Region 2 is relatively more productive than region 1
- Region 1 has a given housing stock H_1 and rents $r_1 < r_C$
  - Unprofitable to build in the depressed region
- Region 2 builds to $1 - H_1$ and has rents r_C
- Spatial equilibrium for workers

$$w_1 - r_1 = w_2 - r_C$$

- Labor-market equilibrium

$$w_1 = \omega A_1^{\frac{1}{1-\alpha}} \left(\frac{\bar{Z}_1}{H_1} \right)^{\frac{1-\alpha-\beta}{1-\alpha}} < w_2 = \omega A_2^{\frac{1}{1-\alpha}} \left(\frac{\bar{Z}_2}{1 - H_1} \right)^{\frac{1-\alpha-\beta}{1-\alpha}}$$

Regional Subsidies

- A small subsidy s for hiring in region 1

$$w_1 = \omega A_1^{\frac{1}{1-\alpha}} \left(\frac{\bar{Z}_1}{H_1} \right)^{\frac{1-\alpha-\beta}{1-\alpha}} + s$$

- Financed by a tax $t = sH_1 / (1 - H_1)$ on hiring in region 2

$$w_2 = \omega A_2^{\frac{1}{1-\alpha}} \left(\frac{\bar{Z}_2}{1 - H_1} \right)^{\frac{1-\alpha-\beta}{1-\alpha}} - s \frac{H_1}{1 - H_1}$$

- Nominal wages rise one to one: a perfectly effective scheme ...
- But house prices in region 1 rise by

$$\Delta r_1 = \frac{s}{1 - H_1}$$

- The subsidy is small when $r_1 + \Delta r_1 < r_C$

Winners and Losers

- Renters in both regions suffer the same real wage decline

$$\Delta (w - r) = -s \frac{H_1}{1 - H_1}$$

  - Nominal wage decline in region 2
  - Nominal wage increase but greater rent increase in region 1

- Any worker who owns a house in region 1 gains

$$\Delta w_1 = s$$

  - Nominal wage increase in region 1
  - Nominal wage decline but asset appreciation in region 2

- A landlord who did not work anywhere would gain even more, Δr_1

Misallocation and Aggregate Losses

- A large subsidy S such that both regions build new houses
- Region 1 has population $h > H_1$ and house prices $p_C > p_1$
- Spatial equilibrium is

$$w_1 = \omega A_1^{\frac{1}{1-\alpha}} \left(\frac{\bar{Z}_1}{h} \right)^{\frac{1-\alpha-\beta}{1-\alpha}} + S = \omega A_2^{\frac{1}{1-\alpha}} \left(\frac{\bar{Z}_2}{1-h} \right)^{\frac{1-\alpha-\beta}{1-\alpha}} - \frac{Sh}{1-h} = w_2$$

- Aggregate output is

$$Y(h) = \frac{\omega}{\beta} \left[A_1^{\frac{1}{1-\alpha}} h^{\frac{\beta}{1-\alpha}} \bar{Z}_1^{\frac{1-\alpha-\beta}{1-\alpha}} + A_2^{\frac{1}{1-\alpha}} (1-h)^{\frac{\beta}{1-\alpha}} \bar{Z}_2^{\frac{1-\alpha-\beta}{1-\alpha}} \right] < Y(H_1)$$

- The free-market allocation is efficient

$$Y'(H_1) = \frac{1}{1-\alpha} \left[w_1(H_1) - w_2(H_1) \right] < 0 \Rightarrow Y(h) < Y(H_1)$$

Human Capital Spillovers

- Simpler production function: $Y_i = A_i L_i$
- L_i now measure human capital
  - Half the population has human capital $E > 1$ per capita
  - The other half has unit human capital per capita
- A fraction q_i of region-i residents has high human capital

$$q_1 H_1 + q_2 (1 - H_1) = \frac{1}{2}$$

- Local spillovers on children's human capital
  - Probability $p(q_i)$ of having a high-human capital child
  - Valued V by all parents

Contraction After a Negative Shock

- The depressed region is small $H_1 < 1/2$

$\Rightarrow$ All its residents have low human capital

$$q_1 = 0 \text{ and } q_2 = \frac{1}{2(1 - H_1)}$$

1. Housing in the depressed region has value $r_1 > 0$

$$A_2 - r_C + Vp\left(\frac{1}{2(1 - H_1)}\right) = A_1 - r_1 + Vp(0)$$

2. Housing in the depressed region is worthless

$$A_2 - r_C + Vp\left(\frac{1}{2(1 - \bar{h})}\right) = A_1 + Vp(0)$$

   - Its population falls to $\bar{h} < H_1$

Keeping the Poor Away

- Let housing in region 1 be free
- Aggregate welfare maximization

$$\max_h \left\{ (1-h) \left[A_2 - r_C + Vp\left(\frac{1}{2(1-h)} \right) \right] + h\left[A_1 + Vp(0) \right] \right\}$$

- Social optimum $h^* < \bar{h}$ such that

$$A_2 - r_C + Vp\left(\frac{1}{2(1-h^*)} \right) = A_1 + Vp(0) + \frac{V}{2(1-h^*)} p'\left(\frac{1}{2(1-h^*)} \right)$$

- Subsidy s to live in region 1

$$A_2 - r_C + Vp\left(\frac{1}{2(1-h)} \right) - s\frac{h}{1-h} = A_1 + Vp(0) + s$$

- Statically optimal subsidy

$$s^* = \frac{V}{2} p'\left(\frac{1}{2(1-h^*)} \right) > 0$$

Hurting the Poor

- The poor impose a negative externality on the richer region only
  - The marginal poor resident dilutes q_2 and thus lowers $p(q_2)$
  - There is no such effect in the poor region with $q_1 = 0$
- A statically beneficial subsidy may perpetrate poverty
  - This can lower long-run social welfare (Bénabou 1996)
- Share of low human capital parents with high human capital children

$$p_L(h) = \left(\frac{1}{2} - h\right) p\left(\frac{1}{2(1-h)}\right) + hp(0)$$

  - Those who move to region 1 lose educational opportunities
  - Do those who remain in region 2 gain more?
  - Not necessarily, because some of the gains accrue to the educated
- E.g., the poor always lose if $p(0) = 0$ and $qp'/p < 2$

Indexed Transfer Payments

- Should transfer payments be indexed to local price levels?
- Not with a utilitarian welfare function (Kaplow 1996)
  - Or pretty much any welfare function other than maximin
- Government budget constraint

$$Nt_1 + (1 - N)\, t_2 = T$$

- Objective function

$$\max \left\{ NV\left(t_1 / p_1\right) + (1 - N)\, V\left(t_2 / p_2\right) \right\}$$

- Optimal transfers

$$\frac{1}{p_1} V'\left(\frac{t_1^*}{p_1}\right) = \frac{1}{p_2} V\left(\frac{t_2^*}{p_2}\right)$$

No Spatial Mobility

- Smaller real transfers to residents of more expensive regions
  - Economize on nominal transfers that provide less bang for the buck
- What about nominal transfers?

1. Concavity of V pushes towards egalitarianism
2. The price effect pushes towards differentiation

- These are really an income and a substitution effect
- With an isoelastic objective function
  - Log utility has t_i^* independent of p_i
  - More linear than log makes t_i^* decrease with p_i
  - More concave than log makes t_i^* increase with p_i

Spatial Equilibrium

- Utility $U_i = u\left(t_i / p_i, A_i\right)$ with amenities A_i
- Congestion $p_1\left(N\right)$ such that $p_1' > 0$
- No congestion in region 2, so $p_2 = 1$
- Spatial equilibrium defines $N\left(t_1\right)$ such that

$$u\left(\frac{t_1}{p_1\left(N\right)}, A_1\right) = u\left(\frac{T - Nt_1}{1 - N}, A_2\right)$$

- By the implicit function theorem

$$N'\left(t_1\right) = \left[u_y^1 \frac{1}{p_1\left(N\right)} + u_y^2 \frac{N}{1 - N}\right] \left[u_y^1 \frac{t_1 p_1'\left(N\right)}{\left[p_1\left(N\right)\right]^2} + u_y^2 \frac{T - t_1}{\left(1 - N\right)^2}\right]^{-1} > 0$$

Spatial Mobility

- Welfare maximization

$$\max \left\{ N\left(t_1\right) V\left(U_1\right) + \left[1 - N\left(t_1\right)\right] V\left(U_2\right) \right\}$$

- Optimal transfers (η denotes elasticity)

$$N\left(t_1\right) \frac{dU_1}{dt_1} + \left[1 - N\left(t_1\right)\right] \frac{dU_1}{dt_1}$$

$$\frac{u_y^1}{p_1} \left(1 - \eta_N^p \eta_t^N\right) = u_y^2 \left(1 - \frac{t_2 - t_1}{t_1} \eta_t^N\right)$$

1. Savings from moving people away from congestion in region 1
2. Savings from moving people away from high transfers (t_2 or t_1)
3. Amenities can affect marginal utilities u_y^1 and u_y^2