Rethinking the New Keynesian Model

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Chapter 2
From the Euler Equation to the Consumption Function *

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Abstract

I propose replacing the Euler equation with the consumption function as the building block of aggregate demand in the New Keynesian model. The consumption function makes the policy transmission channels explicit and separates the direct from the general equilibrium effects. It also extends easily to departures from rational expectations and to models with finite horizons. Applications to monetary policy, fiscal policy, animal spirits, overlapping generations and finite-horizon planning are provided to illustrate the usefulness of the proposed approach

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The idea that fluctuations (around trend) in aggregate output are caused by changes in aggregate demand is a defining feature of Keynesian economics, and one fully embraced by the New Keynesian model (NK model, henceforth). Under this view, understanding the forces behind aggregate demand and modelling them adequately should be a central goal of macroeconomics.

In the present chapter I deal with the way the NK framework engages in that goal, focusing on how it models consumption and its role in aggregate fluctuations. I offer a criticism of standard treatments of consumption in NK models, including the textbook formulations that are a staple of graduate macro courses. I argue that, while formally correct, the conventional approach based on the consumption Euler equation fails to provide a rich description of the channels through which consumption affects aggregate demand and output. In order to overcome that limitation, I propose to shift the emphasis from the Euler equation to the consumption function, thus the title of the chapter.

With that purpose in mind, and to keep the analysis as simple as possible, I use a stripped-down NK model as a reference framework. It features a closed economy, no endogenous capital accumulation, and exogenous government purchases. In the present chapter I restrict myself to a version of the model that assumes an infinite-lived representative consumer, though sections 6 and 7 depart from that assumption by analyzing the implications of finite horizons and overlapping generations. Further discussion of the implications of (intragenerational) heterogeneity for some of the issues dealt with in this chapter can be found in chapter 4.

Many of the ideas in this chapter are not new. In particular, the consumption function approach has been adopted in a variety of applications in the literature, sometimes by necessity, but generally without an explicit discussion. A section at the end of the chapter reviews some of that literature. My contribution here lies in putting the consumption function-based analysis center-stage and discussing its merits as explicitly as possible, using a standard framework.

The chapter is organized as follows. Section 1 describes the "traditional" approach to modeling aggregate demand and output in the NK literature, and discusses some of its limitations. Section 2 presents an alternative to that analysis, based on the consumption function. The following sections contain a number of applications that exemplify the usefulness of the proposed approach including the direct vs indirect effects of monetary policy (section 3), the channels of fiscal policy (section 4), animal spirits (section 5), overlapping generations (section 6), and finite horizon planning (section 7). Section 8 contains some notes on the related literature. Section 9 concludes.

1 The Dynamic IS Equation and its Discontents

The dynamic IS equation (DIS, henceforth) constitutes one of the building blocks of the NK model. In its simplest formulation, corresponding to a closed economy without capital accumulation, the DIS combines (i) a goods market clearing condition equating output to the sum of consumption and government purchases and (ii) the intertemporal optimality condition of the representative consumer (i.e. the consumption Euler equation). A standard representation of the DIS equation is given by

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\{\pi_{t+1}\} - \rho\right) + d_t \tag{1}$$

where $y_t \equiv \log Y_t$ denotes (log) aggregate output, i_t is the nominal interest rate, and $\pi_t \equiv p_t - p_{t-1}$ is the rate of inflation between periods $t-1$ and t, with $p_t \equiv \log P_t$ denoting the (log) price level. d_t represents here a generic aggregate demand shifter, to be interpreted below. Throughout, a "hat" symbol denotes deviations of a variable from its steady state value. The purpose of this chapter is threefold: (i) to point to the limitations of (1) for understanding the channels through which different shocks affect aggregate demand, (ii) to provide an alternative, based on the consumption function, and (iii) to show how the latter approach can provide microfoundations for the aggregate demand shock d_t.

1.1 The Dynamic IS Equation: Standard Derivation

Consider an infinite-lived representative consumer whose preferences are given by $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; \Phi_t)$ where C_t denotes consumption, N_t denotes work hours, and Φ_t is an exogenous preference shock. $\beta \equiv \exp\{-\rho\} \in (0,1)$ is the discount factor, with ρ denoting the corresponding discount rate. In what follows period utility is assumed to take the functional form

$$U\left(C_t, N_t; \Phi_t\right) = \begin{cases} \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right)\Phi_t \text{ for } \sigma \neq 1 \\ \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}\right)\Phi_t \quad \text{for } \sigma = 1 \end{cases}$$

Suppose that, among other financial assets, our consumer can buy and sell a nominally riskless one-period discount bond at a price $Q_t \equiv \exp\{-i_t\}$, with i_t denoting the corresponding yield. The intertemporal optimality condition (i.e. also known as the consumption Euler equation) of the representative consumer can be written as

$$Q_t = \beta \mathbb{E}_t\{(C_{t+1}/C_t)^{-\sigma}(\Phi_{t+1}/\Phi_t)(P_t/P_{t+1})\}$$

A log-linear approximation to the above equation around a steady state with constant consumption and $\Phi = 1$ is given by

$$\widehat{c}_t = \mathbb{E}_t\{\widehat{c}_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}_t\{\pi_{t+1}\} - \rho - z_t) \tag{2}$$

where $c_t \equiv \log C_t$ and $z_t \equiv \phi_t - \mathbb{E}_t\{\phi_{t+1}\}$, with $\phi_t \equiv \log \Phi_t$. Note that z_t can be naturally interpreted as a discount rate shock. In what follows I assume that z_t follows a stationary $AR(1)$ process $z_t = \rho_z z_{t-1} + \varepsilon_t^z$, with $\rho_z \in [0,1)$.

In order to derive the dynamic IS equation, I combine the log-linear Euler equation (2) with a goods market clearing condition. In the simple environment considered here with no investment or external sector, the goods market clearing condition can be written as:

$$Y_t = C_t + G_t$$

where G_t denotes government purchases, assumed to be exogenous. In what follows, and with little loss of generality, I assume zero government purchases in the steady state (i.e. $G = 0$, where variables without time subscripts denote steady state values). Accordingly, $C = Y$. Then one can conveniently approximate the above market clearing condition around the steady state as

$$\widehat{y}_t = \widehat{c}_t + g_t \tag{3}$$

where $g_t \equiv G_t/Y$. The dynamic IS equation can then be obtained by combining (3) and (2):

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\{\pi_{t+1}\} - \rho - z_t\right) + g_t - \mathbb{E}_t\{g_{t+1}\} \tag{4}$$

which can equivalently be written in the same form as (1):

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\{\pi_{t+1}\} - \rho\right) + d_t \tag{5}$$

where $d_t \equiv \frac{1}{\sigma} z_t + g_t - \mathbb{E}_t\{g_{t+1}\}$ is the aggregate demand shock.[1]

In the basic NK model the previous equation is combined with an equation describing the evolution of inflation (the so-called New Keynesian Phillips curve; NKPC, henceforth) and a rule for the nominal interest rate. A standard representation of the NKPC, based on the Calvo price-setting model, is given by:[2]

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa \widehat{y}_t + u_t \tag{6}$$

where u_t is an exogenous disturbance that collects the impact of different supply shocks (e.g. changes in technology or distortionary taxes).

A common description of monetary policy is given by the Taylor-type rule:[3]

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t \tag{7}$$

where v_t has the conventional interpretation of an exogenous monetary policy shock.

Equations (5), (6) and (7) constitute the familiar "three-equation model." Formally, they are a system of difference equations which, under certain conditions, determines uniquely the evolution of endogenous variables (y_t, i_t, π_t) as a function of exogenous variables (d_t, u_t, v_t). In addition to the previous three equations, a government budget constraint determines the evolution of debt, given a rule for taxes that guarantees that the transversality condition on that debt is satisfied independently of the trajectory of prices.[4] If, as it is often assumed in simple versions of the model, taxes are lump-sum, the implied path of taxes and debt has no effect on equilibrium output or inflation (this is the well known Ricardian equivalence result).

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[1] Note that I have not discussed how steady state output is determined, for that requires specifying other blocks othe model. In practice all it takes is a system of wage and price equations that must be satisfied in the steady state. For instance, and assuming $y = n$ in the steady state, the (real) wage equation

$$\begin{aligned} \omega &= \mu_w + \sigma c + \varphi n \\ &= \mu_w + (\sigma + \varphi) y \end{aligned}$$

and the price equation

$$\omega = -\mu_p$$

where ω denotes the steady state (log) real wage.(or, equivalently, minus the log price markup), and μ_w and μ_p denote respectively the (exogenous) steady state wage and price markups (in logs). Combining both steady state conditions we obtain

$$y = -\frac{\mu}{\sigma + \varphi}$$

where $\mu \equiv \mu_p + \mu_w$.

[2] See, e.g., Galí (2015, chapter 3) for a derivation.

[3] See Taylor (1993).

[4] The previous environment, characterized by a passive fiscal policy and active monetary policy in the language of Leeper (1991), is the one generally assumed in applications of the NK model.

1.2 Some Unanswered Questions

In the context of the three-equation model above, the DIS equation can be interpreted as determining aggregate demand and, hence, output as a function of the path for the real interest rate $r_t \equiv i_t - \mathbb{E}_t\{\pi_{t+1}\}$ and the exogenous demand shifter, d_t. While this is formally correct, I would like to argue that a focus on the DIS equation as a description of aggregate demand is not fully satisfactory, for its final form and its derivation (as presented above) hide some of the mechanisms at work behind the determination of aggregate demand and cannot handle many applications of interest.

To illustrate my point, it is useful to rewrite (2) and (4) in "level form" through forward iteration:

$$\widehat{c}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}\mathbb{E}_t\{\widehat{r}_{t+k}\} + \frac{1}{\sigma(1-\rho_z)}\ z_t \tag{8}$$

$$\widehat{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}\mathbb{E}_t\{\widehat{r}_{t+k}\} + \frac{1}{\sigma(1-\rho_z)}\ z_t + g_t \tag{9}$$

where $\widehat{r}_t \equiv r_t - \rho$, and where I have imposed the terminal conditions $\lim_{T\to\infty}\mathbb{E}_t\{\widehat{y}_{t+T}\} = \lim_{T\to\infty}\mathbb{E}_t\{\widehat{c}_{t+T}\} = \lim_{T\to\infty}\mathbb{E}_t\{\widehat{g}_{t+T}\} = 0$, requiring eventual convergence to the steady state (at least in expectation).

Equation (9) describes how output is determined as a function of aggregate demand, where the latter is a function of two exogenous variables, z_t and g_t, as well as the path of current and anticipated real interest rates. Such a description of output determination leaves many questions unanswered. Here is a partial list:

- An exogenous change in lump-sum taxes leaves consumption (and output) unchanged, given a constant real interest rate. This is the well known Ricardian equivalence result and is formally captured by the absence of such taxes in (8) and (9). But why shouldn't consumption (and, hence, output) increase when taxes are lowered? The answer to that question is well understood (and discussed explicitly below), but the above formulation is uninformative as to why Ricardian equivalence holds in the present model.

- An increase in government purchases is often argued to have a negative wealth effect on consumption, resulting from higher taxes, current or anticipated. But that channel seems to be absent in the above formulation, since consumption appears to be independent of g_t. Only an intertemporal substitution channel resulting from an eventual rise in the real interest rate in response to the increase in g_t seems to be at work. Where is the wealth effect hiding?

- An anticipated change in r_t of a given size is predicted to have the same effect on consumption and output independently of the horizon at which it is implemented, even though consumers discount the future. This paradoxical result is a version of the so-called forward guidance puzzle. Why does it hold?

- One would expect consumers' waves of optimism or pessimism ("animal spirits") about future output to affect consumers' perceived lifetime wealth and, hence, current consumption and output. A similar outcome would be expected if, for whatever reason, the price of

firms' stocks changes due to the emergence of a bubble (i.e. a deviation from fundamental pricing). But none of those factors seem to be captured by (8) and (9), hence they don't appear to warrant any change in consumption or output, unless they induce a change in the path of real interest rates. How can this be true?

These and other questions cannot be easily answered using the standard three-equation representation of the NK model embedding the DIS equation, as derived above. The reason has to do with the fact that the Euler equation (2) is an *incomplete characterization of the consumption-savings problem* facing the representative consumer. Below I propose an alternative approach to modelling aggregate consumption, one that focuses on the consumption function, while fully consistent with the baseline three-equation model discussed above. The proposed approach provides a satisfactory explanation to the unanswered questions listed above and allows for a better understanding of the channels through which different shocks impact aggregate demand and output.

2 From the Euler Equation to the Consumption Function

Next I derive an approximate log-linear consumption function for the infinite-lived representative consumer. To do so I combine the consumer's intertemporal budget constraint with the intertemporal optimality condition, both log-linearized around a steady state.[5] In the present section I assume an environment with complete markets and no fiscal sector. In Appendix B I show how the analysis presented here carries over to the case of incomplete markets. In a subsequent section below I introduce a fiscal sector and analyze the impact of some fiscal interventions.

The representative consumer is assumed to enter period t with financial wealth $\mathcal{A}_t$ (which includes the payoffs from last period's financial holdings) and to receive labor income $W_t N_t$, both expressed in terms of period t's consumption good. Such resources are used to purchase consumption goods (in an amount C_t) and a financial portfolio yielding a stochastic one-period ahead payoff $\mathcal{A}_{t+1}$. The market price of that portfolio is given by $\mathbb{E}_t\{\Lambda_{t,t+1}\mathcal{A}_{t+1}\}$, where $\Lambda_{t,t+1}$ is the stochastic discount factor, a familiar representation that follows from the assumption of a complete set of Arrow securities, as shown in Appendix A.[6]

The consumer's period budget constraint can thus be written as:

$$C_t + \mathbb{E}_t\{\Lambda_{t,t+1}\mathcal{A}_{t+1}\} = \mathcal{A}_t + W_t N_t \tag{10}$$

[5] A similar approach to deriving an approximate consumption function was originally proposed by Campbell and Mankiw (1989) as a benchmark for the empirical analysis of deviations from the permanent income hypothesis. The title of this chapter corresponds indeed to the title of one of the sections in their paper. The Campbell-Mankiw approach adopted here generalizes the "exact" consumption function originally used in Flavin (1981), derived under the assumption of a constant interest rate and a quadratic utility.

[6] I use $\Lambda_{t,t+1}$ as shortcut notation for $\Lambda(s^t, s^{t+1}) \equiv Q^A(s^t, s^{t+1})/\xi(s^{t+1}|s^t)$ where $Q^A(s^t, s^{t+1})$ is the price at history node s^t of an Arrow security paying one unit of consumption if s^{t+1} is realized (and nothing otherwise), with $\xi(s^{t+1}|s^t)$ denoting the probability of s^{t+1} conditional on s^t.

Iterating (10) forward and imposing the transversality condition $\lim_{T\to\infty}\mathbb{E}_t\{\Lambda_{t,T}\mathcal{A}_{t+T}\}=0$ one obtains the consumer's intertemporal budget constraint (C-IBC, henceforth):

$$\sum_{k=0}^{\infty}\mathbb{E}_t\{\Lambda_{t,t+k}C_{t+k}\}=\mathcal{A}_t+\sum_{k=0}^{\infty}\mathbb{E}_t\{\Lambda_{t,t+k}W_{t+k}N_{t+k}\} \tag{11}$$

where $\Lambda_{t,t+k}$ is defined recursively as $\Lambda_{t,t+k}=\Lambda_{t,t+k-1}\Lambda_{t+k-1,t+k}$ for $k=1,2,...$ with $\Lambda_{t,t}=1$. As shown in Appendix A, utility maximization requires

$$\Lambda_{t,t+k}=\beta^k U_{c,t+k}/U_{c,t}$$

for all t and $t+k$, and which must hold state by state. Under the assumed functional form for utility I have $U_{c,t}=C_t^{-\sigma}\Phi_t$, implying

$$\Lambda_{t,t+k}=\beta^k(C_{t+k}/C_t)^{-\sigma}(\Phi_{t+k}/\Phi_t)$$

which can be written in log-deviations from steady state as:

$$\widehat{\lambda}_{t,t+k}=\sigma(\widehat{c}_t-\widehat{c}_{t+k})+\phi_{t+k}-\phi_t \tag{12}$$

where $\widehat{\lambda}_{t,t+k}\equiv\log(\Lambda_{t,t+k}/\beta^k)=\widehat{\lambda}_{t,t+1}+...+\widehat{\lambda}_{t+k-1,t+k}$ for $k=1,2,3,...$

On the other hand the price of a one-period nominally riskless discount bond is given by

$$Q_t\equiv\exp\{-i_t\}=\mathbb{E}_t\{\Lambda_{t,t+1}(P_t/P_{t+1})\}$$

which can be rewritten in log-deviations from steady state as:

$$\mathbb{E}_t\{\widehat{\lambda}_{t,t+1}\}=-\widehat{r}_t$$

Accordingly I can write

$$\mathbb{E}_t\{\widehat{\lambda}_{t,t+k}\}=\sum_{j=0}^{k-1}\mathbb{E}_t\{\widehat{\lambda}_{t+j,t+1+j}\}=-\sum_{j=0}^{k-1}\mathbb{E}_t\{\widehat{r}_{t+j}\} \tag{13}$$

for $k=1,2,3,...$

Taking conditional expectations on both sides of (12), and using (13) to substitute for $\mathbb{E}_t\{\widehat{\lambda}_{t,t+k}\}$ one can rewrite the consumer's intertemporal optimality condition at different horizons as:

$$\mathbb{E}_t\{\widehat{c}_{t+k}\}=\widehat{c}_t+\frac{1}{\sigma}\sum_{j=0}^{k-1}\mathbb{E}_t\{\widehat{r}_{t+j}\}-\frac{1}{\sigma}\sum_{j=0}^{k-1}\mathbb{E}_t\{z_{t+j}\} \tag{14}$$

for $k=1,2,3,...$The second and third terms on the right hand side capture the intertemporal substitution of consumption, driven by (i) the relative price of consumption at different dates and (ii) exogenous discount rate shocks, respectively.

Next I derive a log-linear approximation to the C-IBC (11). First, note that under the assumption of fundamental valuation of stocks I have

$$Q_t^S=\sum_{k=0}^{\infty}\mathbb{E}_t\{\Lambda_{t,t+k}D_{t+k}\}$$

where $D_t \equiv Y_t - W_t N_t$ denotes firms' aggregate profits in period t (which are distributed as dividends) and where Y_t denotes aggregate output (or, equivalently, aggregate income). One can thus rewrite the C-IBC as:

$$\sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k}(C_{t+k} - Y_{t+k})\} = F_t$$

where $F_t \equiv \mathcal{A}_t - Q_t^S$ denotes the value of individual financial wealth in excess of firms' equity value.

As discussed explicitly below, goods market equilibrium requires that $C_t = Y_t$ for all t. Similarly, equilibrium in financial markets requires that the financial wealth $\mathcal{A}_t$ held by the representative consumer must equate the value of firms' shares, Q_t^S, the only asset available in positive net supply.[7] Neither condition is, however, a constraint of the individual consumer's problem, which explains why it is not imposed in the C-IBC. On the other hand, those conditions inform the properties of the steady state around which the C-IBC is approximated, and which is assumed to satisfy $Y = C$ and $\mathcal{A} = Q^S$. Thus, log-linearizing the C-IBC around that steady state yields:

$$\sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{c}_{t+k}\} = \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{y}_{t+k}\} + \widehat{f}_t \tag{15}$$

where $\widehat{f}_t \equiv (\mathcal{A}_t - Q_t^S)/Y$.

Combining (15) with optimality condition (14), rearranging terms and simplifying, yields the following log-linearized consumption function:

$$\widehat{c}_t = (1-\beta)\left(\widehat{f}_t + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{y}_{t+k}\}\right) - \frac{\beta}{\sigma}\sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\} + \frac{\beta}{\sigma(1-\beta\rho_z)} z_t \tag{16}$$

Several remarks are in order. Firstly, and as shown in Appendix B, an identical approximate consumption function can be derived for the case of incomplete markets under the assumption of a sequential market for one-period bonds. Secondly, equation (16) shows the different determinants of *individual* consumption *in partial equilibrium*, i.e. conditional on a given individual financial wealth and taking as given current and expected future income and interest rates. To understand the role of the different determinants consider first the case of a constant real rate and no preference shocks ($\widehat{r}_t = 0$, and $z_t = 0$ for all t). In that case, (14) implies that $\widehat{c}_t = \mathbb{E}_t\{\widehat{c}_{t+k}\}$ for $k = 1, 2, 3, ..$ and all t, i.e. the consumer seeks to maintain his consumption constant over time *in expectation*. As implied by (16), in that case consumption is given by

$$\widehat{c}_t = (1-\beta)[\widehat{f}_t + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{y}_{t+k}\}] \equiv \widehat{y}_t^P \tag{17}$$

The previous expression corresponds to the unique level of consumption that satisfies the requirement of a constant expected consumption path, while exhausting the budget constraint. This is precisely the meaning of $\widehat{y}_t^P$, a variable often referred to as *permanent income*.[8] Relative

[7] This assumption is relaxed in section 4, where government debt is introduced.

[8] See, e.g., Flavin (1981), Campbell (1987) and Campbell and Mankiw (1989).

to this benchmark of full smoothing of consumption, changes in current or anticipated interest rates or and/or in the preference shifter, captured respectively by the second and third terms on the right hand side of (16), bring about anticipated changes in consumption, so that its trajectory satisfies (14) at all times.

Thirdly, the (contemporaneous) marginal propensity to consume, $\partial\widehat{c}_t/\partial\widehat{y}_t$, associated with the above consumption function is given by $1-\beta$, which is small for any plausible calibration of the discount factor. Note that $1-\beta$ is also the slope of the linear function mapping current income to current consumption in the traditional Keynesian cross diagram, with the remaining terms in (16) playing the role of shifters of that mapping.

Finally, note that (16) implies that the impact of an anticipated interest rate change on consumption is subject to discounting, i.e. it is decreasing with the horizon of the anticipated change. To gain intuition for that property consider the impact of a (temporary) increase in the interest rate at horizon T ($\widehat{r}_{t+T} > 0$) on individual consumption, starting from a steady state ($\widehat{f}_t = z_t = 0$) and conditional on a constant income path ($\widehat{y}_{t+k} = 0$ for $k = 0, 1, 2, ..$). The anticipated increase in the real interest rate will lower (log) consumption by β^{T+1}/σ (times the increase in $\widehat{r}_{t+T}$), and stay constant at that lower level until period $t+T$. The resulting savings will lead to a gradual increase in financial wealth $\widehat{f}_{t+k}$, for $k = 1, 2, ..T$. In period $t+T$ the higher real rate is realized and (log) consumption jumps up in period $t+T+1$ in according to the Euler equation, stabilizing at a plateau $(1-\beta^{T+1})/\sigma$ above the initial path, supported by the higher financial wealth. For any given initial reduction of consumption, the further is the horizon k of the anticipated rate increase, the greater will be the accumulation of financial wealth and the higher the eventual level at which consumption will plateau. Accordingly, a smaller initial decline will be needed to satisfy both the C-IBC and the intertemporal optimality condition. Figure 1 illustrates this point, by showing the trajectories of consumption in response to an anticipated increase in the real rate at two distinct horizons, $T = 8$ and $T = 40$, under the assumption that $\beta = 0.995$ and $\sigma = 1$. Nevertheless, and as shown below, the discounting implied by the partial equilibrium logic just described is exactly offset in general equilibrium by the income response, leading to an undiscounted relation that has been the focus of a literature on the forward guidance puzzle.

Note that one can alternatively write the consumption function (16) as:

$$\widehat{c}_t = (1-\beta)(\widehat{f}_t + \widehat{y}_t) - \frac{\beta}{\sigma}\widehat{r}_t + \widehat{c}_t^F + \widehat{c}_t^Z$$

where

$$\widehat{c}_t^F \equiv (1-\beta)\sum_{k=1}^{\infty}\beta^k \mathbb{E}_t\{\widehat{y}_{t+k}\} - \frac{\beta}{\sigma}\sum_{k=1}^{\infty}\beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\} \tag{18}$$

is the forward-looking component of consumption based on expectations of future income and interest rates, while

$$\widehat{c}_t^Z \equiv \frac{\beta}{\sigma(1-\beta\rho_z)}z_t \tag{19}$$

is the exogenous (or "autonomous") component of consumption, which is proportional to the preference shock. The previous representation highlights the "static" dependence of consumption on contemporaneous variables ($\widehat{f}_t$, $\widehat{y}_t$, $\widehat{r}_t$), by contrast to the terms involving expectations or exogenous terms.

2.1 Equilibrium

The above analysis of consumption has been a partial equilibrium one, i.e. with the representative consumer taking as given income and interest rates, and ignoring market clearing considerations. Next I relax the assumption of an exogenous path for income, and show how output is determined by the equilibrium in the goods market, while still taking as given the path of the real interest rate.

For simplicity, I start by considering the case in which consumption is the only component of aggregate demand. In that case goods market clearing implies $\widehat{y}_t = \widehat{c}_t$ for all t. On the other hand, asset market clearing requires $\widehat{f}_t = 0$ for all t, since financial wealth, both individual and aggregate, must be allocated in equilibrium to the only financial asset in positive net supply, i.e. firms' equity.[9] Accordingly, I can write:

$$\begin{aligned} \widehat{y}_t &= (1-\beta)\widehat{y}_t - \frac{\beta}{\sigma}\widehat{r}_t + \widehat{c}^F_t + \widehat{c}^Z_t \\ &= -\frac{1}{\sigma}\widehat{r}_t + \frac{1}{\beta}\left(\widehat{c}^F_t + \widehat{c}^Z_t\right) \end{aligned} \tag{20}$$

Note that $\frac{1}{\beta}$ can be interpreted as a *static aggregate demand multiplier*, which corresponds to the $\frac{1}{1-mpc}$ term familiar from introductory textbooks, with the marginal propensity to consume given in this case by $mpc = 1-\beta$. The resulting static multiplier in this baseline model will be above but close to one for any plausible value of β. Equation (20) can thus be interpreted as a *static IS equation*, representing the level of output consistent with equilibrium in the goods market as a function of the current real interest rate, with $\widehat{c}^F_t + \widehat{c}^Z_t$ playing the role of a shifter of the IS curve.

The next step in the analysis of the goods market equilibrium consists in recognizing that future output entering the expression for $\widehat{c}^F_t$ is not exogenous, but it is instead determined in a way similar to (20). This requires combining (20) with (18), which yields after some algebra:[10]

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\widehat{r}_t + \frac{1}{\sigma}z_t \tag{21}$$

which is the standard (recursive) representation of the *dynamic IS equation (DIS)*.

Note that by contrast with (16), the DIS equation implies no discounting of future interest rates. To see this, ignore preference shocks, iterate (21) forward and assume $\lim_{T\to\infty}\mathbb{E}_t\{\widehat{y}_{t+T}\} = 0$, to obtain

$$\widehat{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}\mathbb{E}_t\{\widehat{r}_{t+k}\} \tag{22}$$

[9] Note that that $\widehat{f}_t = 0$, the condition for equilibrium in asset markets, follows from equilibrium in goods markets and the C-IBC (15), a manifestation of Walras' law.

[10] As a preliminary step note that $\widehat{c}^F_t$ and $\widehat{c}^Z_t$ admit the recursive representation:

$$\widehat{c}^F_t = \beta\mathbb{E}_t\{\widehat{c}^F_{t+1}\} + (1-\beta)\beta\mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{\beta^2}{\sigma}\mathbb{E}_t\{\widehat{r}_{t+1}\}$$

$$\widehat{c}^Z_t = \beta\mathbb{E}_t\{\widehat{c}^Z_{t+1}\} + \frac{\beta}{\sigma}z_t$$

Multiplying both sides of (20) by the operator $(1-\beta L_t^{-1})$ and simplifying yields equation (21) in the text.

Note that, in contrast with consumption function (16) above, which takes current and future income as given, (21) and (22) already embed the impact on aggregate consumption of the income changes (current and anticipated) that will be induced by any shift in expectations regarding future interest rates. In particular, the further the horizon of an expected transitory interest rate hike, the longer the decline in income that will precede it, and the larger the negative impact on consumption, in a way which exactly offsets the discounting that arises in partial equilibrium, when income is taken as given and only the direct effects of the anticipated interest rate hike are accounted for.

The DIS equation (21) is one of the blocks of the three-equation New Keynesian model, and as complete a description of the goods market as it is required in order to solve for the equilibrium of that model. The consumption function (16) can be (and is generally) sidestepped in that process. Yet, its use, even if only implicitly, yields interesting insights that would otherwise remain buried in the DIS. The next two sections illustrate this point by analyzing (i) the direct vs indirect effects of monetary policy shocks, and (ii) the channels of fiscal policy.

3 Direct vs Indirect Effects of Monetary Policy Shocks

Starting from a steady state, I model an exogenous tightening of monetary policy in period t as a policy intervention that induces a persistent increase in the real interest rate described by:

$$\widehat{r}_{t+k} = \rho_r^k$$

for $k = 0, 1, 2, ...$and where $\rho_r \in [0, 1)$ measures the persistence of the tighter policy. We can use consumption function (16) to determine the *direct* or partial equilibrium effect (i.e. conditional on the path of output) of that monetary policy intervention on consumption:

$$\begin{aligned} \frac{\partial \widehat{c}_t}{\partial \widehat{r}_t} &= -\frac{\beta}{\sigma} \sum_{k=0}^{\infty} (\beta \rho_r)^k \\ &= -\frac{\beta}{\sigma(1-\beta\rho_r)} \end{aligned}$$

The *total* effect takes into account the induced income changes. It can be determined by substituting for $\widehat{r}_{t+k}$ in (22) to obtain

$$\frac{d\widehat{c}_t}{d\widehat{r}_t} = -\frac{1}{\sigma(1-\rho_r)}$$

Note that, as expected, the size of both the direct and the total effect is increasing in the persistence of the interest rate increase, ρ_r. There is an important difference between the two, however: as $\rho_r \to 1$ the impact is bounded for the direct effect (due to discounting), but unbounded for the total effect (due to the lack of discounting).

More generally, the impact general equilibrium (GE) multiplier, i.e. the ratio of the total effect to the direct effect is given by:

$$\Xi_r = 1 + \frac{1-\beta}{\beta(1-\rho_r)}$$

which is increasing in ρ_r. The relationship is highly nonlinear though, with the GE multiplier remaining close to one, except for values of ρ_r close to unity.[11] That nonlinearity is captured clearly by Figure 2, which displays the GE multiplier as a function of ρ_r, under an assumed discount factor setting of $\beta = 0.995$ and $\sigma = 1$.

4 The Channels of Fiscal Policy in the New Keynesian Model

Next I introduce a fiscal block in the model above, and examine its interactions with private consumption. The government period budget constraint is given by:

$$G_t + B_t^G = \mathbb{E}_t\{\Lambda_{t,t+1} B_{t+1}^G\} + T_t \tag{23}$$

with lump-sum taxes T_t, government purchases G_t and government debt B_t^G, all in real terms. Iterating (23) forward, and imposing the transversality condition $\lim_{T\to\infty} \mathbb{E}_t\{\Lambda_{t,T} B_{t+T}^G\} = 0$ yields the government's intertemporal budget constraint (G-IBC, henceforth):[12]

$$B_t^G = \sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k}(T_{t+k} - G_{t+k})\}$$

Note that in the steady state:

$$(1-\beta)B^G = T - G$$

Log-linearization of the G-IBC around that steady state yields:

$$\begin{aligned} \widehat{b}_t^G &= \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{t}_{t+k} - \widehat{g}_{t+k}\} + (1-\beta) b^G \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t\{\widehat{\lambda}_{t,t+k}\} \\ &= \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{t}_{t+k} - \widehat{g}_{t+k}\} - b^G \beta \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\} \end{aligned} \tag{24}$$

where $t_t \equiv T_t/Y$, $g_t \equiv G_t/Y$, and $b_t^G \equiv B_t^G/Y$. The last term on the right hand side captures the income effect of interest rate changes. In particular, under the assumption that $B^G > 0$ (and, hence, $T - G > 0$) an increase in current or future interest rates reduces the resources available to the government, thus requiring higher taxes and/or lower government spending (current or future).

The consumer's period budget constraint is now given by

[11] As stressed in Kaplan, Moll and Violante (2018) this is a distinctive property of the representative agent model relative to models in which a non-negligible fraction of consumers and hand-to-mouth. See chapter 4 for a discussion.

[12] Note that the transversality condition follows from the consumer's transversality condition $\lim_{T\to\infty} \mathbb{E}_t\{\Lambda_{t,T} \mathcal{A}_{t+T}\} = 0$, combined with the fact that in equilibrium $\mathcal{A}_t = Q_t^S + B_t^G$ and $Q_t^S \geq 0$ together with the assumption of $B_t^G \geq 0$.

$$C_t + \mathbb{E}_t\{\Lambda_{t,t+1}\mathcal{A}_{t+1}\} = \mathcal{A}_t + W_t N_t - T_t$$

with the corresponding C-IBC:

$$\begin{aligned}\sum_{k=0}^{\infty}\mathbb{E}_t\{\Lambda_{t,t+k}C_{t+k}\} &= \mathcal{A}_t + \sum_{k=0}^{\infty}\mathbb{E}_t\{\Lambda_{t,t+k}(W_{t+k}N_{t+k} - T_{t+k})\} \\ &= F_t + \sum_{k=0}^{\infty}\mathbb{E}_t\{\Lambda_{t,t+k}(Y_{t+k} - T_{t+k})\}\end{aligned}$$

where, as above, I have defined $F_t \equiv \mathcal{A}_t - Q_t^S$ as the excess value of financial holdings $\mathcal{A}_t$ over the value of firms $Q_t^S = \sum_{k=0}^{\infty}\mathbb{E}_t\{\Lambda_{t,t+k}D_{t+k}\}$.

Note that in a steady state equilibrium $F = B^G$ and hence $C = Y - T + (1-\beta)B^G$. Log-linearizing the C-IBC around that steady state yields:

$$\begin{aligned}0 &= \widehat{f}_t + \sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{y}_{t+k} - \widehat{t}_{t+k} - \Gamma\widehat{c}_{t+k}\} - (1-\beta)b^G\sum_{k=1}^{\infty}\beta^k\mathbb{E}_t\{\widehat{\lambda}_{t,t+k}\} \\ &= \widehat{f}_t + \sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{y}_{t+k} - \widehat{t}_{t+k} - \Gamma\widehat{c}_{t+k}\} + b^G\beta\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{r}_{t+k}\} \qquad (25)\end{aligned}$$

where $\Gamma \equiv C/Y \leq 1$. In order to simplify the algebra and with little loss of generality I assume $G = 0$ in what follows, implying $\Gamma = 1$.

Combining (25) with intertemporal optimality condition (14) (and ignoring preference shocks, for simplicity) and rearranging terms yields the consumption function:

$$\widehat{c}_t = (1-\beta)\left(\widehat{f}_t + \sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{y}_{t+k} - \widehat{t}_{t+k}\}\right) + \left[(1-\beta)b^G - \frac{1}{\sigma}\right]\beta\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{r}_{t+k}\} \qquad (26)$$

Note that, in addition to the intertemporal substitution effect discussed in the previous section, current or anticipated changes in the interest rate also have an income effect, to the extent that $b^G > 0$. In that case, and for any given path of taxes, an increase in the interest rate raises the resources available to the consumer and leads to a rise in consumption.

Imposing the G-IBC (24) in the consumption function (26), as well as the equilibrium condition $\widehat{f}_t = \widehat{b}_t^G$, one can rewrite (26) as:

$$\widehat{c}_t = (1-\beta)\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{y}_{t+k} - \widehat{g}_{t+k}\} - \frac{\beta}{\sigma}\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{r}_{t+k}\} \qquad (27)$$

The *partial equilibrium* effect of government purchases on consumption is given by $-(1-\beta)\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{g}_{t+k}\}$. That term captures the negative impact on wealth, working through higher taxes, current or anticipated. Note also that the impact of an anticipated change in government purchases is decreasing with the horizon. As discussed below, part of the negative impact on consumption is offset by the induced expansion of output.

On the other hand the path of government debt and taxes $\{\widehat{b}_t^G, \widehat{t}_t\}$ does not affect consumption. This is the well known *Ricardian equivalence* property. The previous analysis makes the logic behind that property fully transparent: fiscal policy affects consumption through its impact on $\widehat{b}_t^g - \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{t}_{t+k}\} + b^G \beta \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\}$, a term which the G-IBC equates to $-\sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{g}_{t+k}\}$. Thus, given the path of government purchases, the *timing* of the taxes needed to finance them is irrelevant (and so is the associated path of government deficits and debt). In addition one can see that the income effect of interest rate changes on consumption, proportional to $b^G \beta \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\}$ vanishes, with only the intertemporal substitution effect, $-\frac{1}{\sigma} \beta \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\}$ left. The reason is that any increase in the consumer's resources resulting from higher interest rates is exactly matched by a commensurate loss of resources by the government, which requires an increase in current and/or future taxes that has an exact offsetting effect.[13]

The previous analysis took the path of income as given. Next I bring its endogeneity into the picture through the goods market clearing condition $Y_t = C_t + G_t$ for all t. Rewriting the latter condition in terms of log-deviations from steady state and combining it with (27) yields:

$$\begin{aligned} \widehat{y}_t &= \widehat{c}_t + \widehat{g}_t \\ &= (1-\beta) \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{y}_{t+k}\} - \frac{\beta}{\sigma} \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\} + (\widehat{g}_t - \widehat{g}_t^P) \end{aligned} \tag{28}$$

where $\widehat{g}_t^P \equiv (1-\beta) \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{g}_{t+k}\}$ can be interpreted as the *permanent* component of government purchases, equal to the annuity associated with the present value of taxes needed to finance $\{\widehat{g}_t\}$. Note that $\widehat{g}_t^P$ captures the impact of government purchases on consumption, resulting from the tax financing needs, and for any given path of income and interest rates. Equation (28) makes clear that only the "temporary" component of government purchases $\widehat{g}_t - \widehat{g}_t^P$ will have an impact on aggregate demand and output. Thus, an increase in government purchases such that $\widehat{g}_t = \widehat{g}_t^P$ would be exactly offset by a change in consumption of the opposite sign resulting from the reduction in the consumer's lifetime resources due to current and anticipated higher taxes, leaving aggregate demand unchanged.

Writing (28) in recursive form yields

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma} \widehat{r}_t + (\widehat{g}_t - \mathbb{E}_t\{\widehat{g}_{t+1}\}) \tag{29}$$

which is the standard DIS equation in the presence of government purchases. In the particular case of an exogenous $AR(1)$ process for $\widehat{g}_t$:

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma} \widehat{r}_t + (1-\rho_g) \widehat{g}_t \tag{30}$$

More generally, iterating (29) forward, and assuming $\lim_{T \to \infty} \mathbb{E}_t\{\widehat{g}_{t+T}\} = 0$:

[13] The previous result is conditional on the path of the real interest rate. If the experiment is defined instead by a path for the nominal rate, with nominal government debt in positive net supply, the fiscal response determines the path of inflation and hence that of the real rate, and thus becomes a key determinant of the effects of monetary policy, even under Ricardian equivalence. See Caramp and Silva (2023).

$$\widehat{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}\mathbb{E}_t\{\widehat{r}_{t+k}\} + \widehat{g}_t \tag{31}$$

By embedding the induced effects on output, the DIS fails to capture the mechanisms through which changes in government purchases end up having an effect on aggregate output. The next subsection catalogs these channels explicitly, making use of the consumption function above.

4.1 Government Spending Shocks: Transmission Channels

Starting from steady state, consider an unexpected persistent increase in government purchases given by:

$$\widehat{g}_{t+k} = \rho_g^k \widehat{g}_t \tag{32}$$

where $\rho_g \in [0, 1)$. Under the assumption of a constant real interest rate, the dynamic impact of the fiscal intervention on output, as determined by (31) is given by

$$\widehat{y}_{t+k} = \rho_g^k \widehat{g}_t$$

thus implying a unit multiplier at all horizons, i.e. $\frac{d\widehat{y}_{t+k}}{d\widehat{g}_{t+k}} = 1$ for $k = 0, 1, 2, 3...$[14].

Such an analysis, based on the DIS equation, does not uncover the channels through which the change in $\widehat{g}_t$ is eventually transmitted to output. A better insight can be attained by using consumption function (27). Consider the effect of $\widehat{g}_t$ on $\widehat{y}_t$, i.e. the impact effect. Three different channels can be shown to be at work in determining that impact. First, there is the direct impact on aggregate demand, given by $\widehat{g}_t$ itself. Secondly, the increase in consumption induced by the higher income, current and anticipated, and conditional on taxes remaining unchanged, provides an additional boost to aggregate demand and output. That induced effect is given by

$$(1-\beta)\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{y}_{t+k}\} = \frac{1-\beta}{1-\beta\rho_g}\widehat{g}_t$$

which is increasing in the persistence of the fiscal expansion, measured by ρ_g. Finally, there is a negative effect on consumption due to the anticipation of higher taxes, current or future. That effect is given by:

$$\begin{aligned} -(1-\beta)\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{t}_{t+k}\} &= -(1-\beta)\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{g}_{t+k}\} \\ &= -\frac{1-\beta}{1-\beta\rho_g}\widehat{g}_t \end{aligned}$$

Note that in the present model with an infinite-lived representative consumer the negative effect on consumption resulting from higher taxes exactly offsets the positive effect resulting from the higher induced income, thus making the total effect equal to the direct effect. This is not, however, a general result: the exact offsetting of the income and tax channels no longer

[14] See, e.g., Woodford (2011).

obtains once one departs from the baseline model analyzed here, as in the overlapping generations model introduced in section 6 of the present chapter or the model with heterogenous consumers in chapter 4.

Finally, note that a fourth channel emerges associated with current and anticipated changes in the real interest rate in response to the fiscal shock, with the impact effect given by

$$-\frac{1}{\sigma}\sum_{k=0}^{\infty}\mathbb{E}_t\{\widehat{r}_{t+k}\} \tag{33}$$

In that case the final size of the spending multiplier, and its dynamic profile will depend on the sign, size and pattern of the interest rate adjustment. That adjustment will generally depend on the monetary policy rule in place. Building on the analysis of the previous section, note that the impact of the interest rate adjustment measured by (33) has itself two components. First, a direct effect, associated with the intertemporal substitution of consumption resulting from the interest rate adjustment, which can be read off the consumption function (27) and is given by

$$-\frac{\beta}{\sigma}\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{r}_{t+k}\} \tag{34}$$

The second component of the consumption response results from the change in current and future income triggered by the interest rate adjustment itself, and is given by:

$$\begin{aligned}(1-\beta)\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{y}_{t+k}\} &= -\frac{1-\beta}{\sigma}\sum_{k=0}^{\infty}\beta^k\sum_{h=k}^{\infty}\mathbb{E}_t\{\widehat{r}_{t+h}\}\\ &= -\frac{1}{\sigma}\sum_{k=0}^{\infty}(1-\beta^{k+1})\mathbb{E}_t\{\widehat{r}_{t+k}\}\end{aligned}$$

which can be checked to correspond to the difference between the total effect (33) and the direct effect (34) of the interest rate adjustment.

The discussion above illustrates how the use of the consumption function allows one to identify the different channels through which a change in government spending affect consumption and, hence, output (which feeds back into consumption). By contrast, the conventional approach focusing exclusively on the consumption Euler equation

$$\begin{aligned}\widehat{c}_t &= \mathbb{E}_t\{\widehat{c}_{t+1}\}-\frac{1}{\sigma}\widehat{r}_t\\ &= -\frac{1}{\sigma}\sum_{k=0}^{\infty}\mathbb{E}_t\{\widehat{r}_{t+k}\}\end{aligned}$$

fails to reveal the existence of general equilibrium effects resulting from higher income as well as the negative wealth effects associated with higher taxes. Instead it conveys the impression that only intertemporal substitution effects are at work, while mismeasuring the latter since their true measure is given by $-\frac{\beta}{\sigma}\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{r}_{t+k}\}$, as shown above.

5 Animal Spirits

Next I depart from the baseline model of consumption developed above by allowing for deviations from rationality when forming expectations about future variables. The goal is to show how those expectation biases or *animal spirits* can be introduced in the model, how they enter the equations describing the equilibrium and how they can be a source of fluctuations by themselves. For simplicity, in this section I shut down the fiscal sector and ignore sources of fluctuations other than *animal spirits*, as formalized below.

I use the operator $\mathbb{E}_t^*\{\cdot\}$ to denote subjective expectations. The latter are assumed to satisfy the law of iterated expectations (i.e. $\mathbb{E}_t^*\{\mathbb{E}_{t+k}^*\{X_{t+T}\}\} = \mathbb{E}_t^*\{X_{t+T}\}$). Accordingly, the C-IBC can now be written as

$$\sum_{k=0}^{\infty} \mathbb{E}_t^*\{\Lambda_{t,t+k} C_{t+k}\} = F_t + \sum_{k=0}^{\infty} \mathbb{E}_t^*\{\Lambda_{t,t+k} Y_{t+k}\}$$

where. as above, $F_t \equiv \mathcal{A}_t - Q_t^S$ but with the value of stocks now given by $Q_t^S \equiv \sum_{k=0}^{\infty} \mathbb{E}_t^*\{\Lambda_{t,t+k} D_{t+k}\}$, which may capture overvaluation or undervaluation of stocks relative to their "true" fundamental value due to waves of irrational optimism or pessimism regarding future dividends.

Following the same steps as above, with subjective expectations substituting for rational expectations at every step, one can now derive the log-linearized individual consumption function:

$$\widehat{c}_t = (1-\beta)\left(\widehat{f}_t + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t^*\{\widehat{y}_{t+k}\}\right) - \frac{\beta}{\sigma} \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t^*\{\widehat{r}_{t+k}\}$$

Equivalently, one can write

$$\widehat{c}_t = (1-\beta)\left(\widehat{f}_t + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{y}_{t+k}\}\right) - \frac{\beta}{\sigma} \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\} + s_t \tag{35}$$

where

$$s_t \equiv (1-\beta) \sum_{k=1}^{\infty} \beta^k [\mathbb{E}_t^*\{\widehat{y}_{t+k}\} - \mathbb{E}_t\{\widehat{y}_{t+k}\}] - \frac{\beta}{\sigma} \sum_{k=1}^{\infty} \beta^k [\mathbb{E}_t^*\{\widehat{r}_{t+k}\} - \mathbb{E}_t\{\widehat{r}_{t+k}\}]$$

I interpret fluctuations in s_t as capturing *irrational* waves of optimism or pessimism regarding future income and interest rates. For concreteness I assume

$$\mathbb{E}_t^*\{\widehat{y}_{t+k}\} - \mathbb{E}_t\{\widehat{y}_{t+k}\} = \gamma^{k-1}\zeta_t$$

$$\mathbb{E}_t^*\{\widehat{r}_{t+k}\} - \mathbb{E}_t\{\widehat{r}_{t+k}\} = -\varpi\gamma^{k-1}\zeta_t$$

for $k = 1, 2, 3, ...$where ζ_t represents an exogenous expectational bias shock ("animal spirits"), which is assumed to follow an $AR(1)$ process with autoregressive coefficient $\rho_\zeta \in [0, 1)$. Note that at any point in time the expectational bias decreases with the horizon, with the rate of decline being inversely related to parameter $\gamma \in [0, 1)$. Finally, $\varpi > 0$ is just a scaling parameter

with the sign normalized so that positive values of ζ_t are associated with an "optimistic" bias (high output, low interest rates).[15] Thus one can write

$$s_t = \beta\chi\zeta_t$$

where $\chi \equiv \frac{\sigma(1-\beta)+\beta\varpi}{\sigma(1-\beta\gamma)} > 0$. Substituting the previous expression into (35), and combining the resulting consumption function with the goods market and asset market clearing conditions (i.e., $\widehat{c}_t = \widehat{y}_t$ and $\widehat{f}_t = 0$, for all t) yields the augmented dynamic IS equation

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\widehat{r}_t + \chi(1-\beta\rho_\zeta)\zeta_t$$

which corresponds to (1) with $d_t \equiv \chi(1-\beta\rho_\zeta)\zeta_t$. Hence one can see how the use of the consumption function, which explicitly incorporates expectations of future income and interest rates, makes it possible to introduce shocks to animal spirits capturing irrational waves of optimism or pessimism (relative to the rational benchmark), that can be embedded into the dynamic IS equation, thus providing microfoundations for the disturbance appended to that equation. Such derivation would not have been possible if one had relied exclusively on the consumer's Euler equation.

6 Overlapping Generations

The Euler equation-based approach to deriving the DIS in the baseline NK model hinges critically on the assumption of an infinite-lived representative consumer. The introduction of heterogeneity, by contrast, requires the aggregation of individual consumption functions in order to determine aggregate consumption. This is illustrated in the present section through the analysis of a model economy with overlapping generations of finite-lived individuals. That framework captures a particular type of heterogeneity: an intergenerational one.[16] The objective of the analysis is to understand how the introduction of finite horizons affects the behavior of aggregate demand and output, as reflected in the DIS, and the response of the economy to fiscal interventions.

The analysis follows closely that in Galí (2021), and is based on the "perpetual youth" model of Yaari (1965) and Blanchard (1985). Henceforth I refer to this model as the OLG-NK model. The size of the population is constant and normalized to one. Each individual has a constant probability γ of surviving into the following period, independently of his age. Each period a cohort of size $1-\gamma$ is born. Thus, the size in period $t \geq s$ of the cohort born in period s is given by $(1-\gamma)\gamma^{t-s}$. Individuals consume, supply labor and manage their own firms, which they set up when they are born. A firm shuts down and stops generating dividends when its founder dies and/or retires.

[15] The model can be trivially genralized to allow for independent biases expectations of the two variables:

$$\mathbb{E}^*_t\{\widehat{y}_{t+k}\} - \mathbb{E}_t\{\widehat{y}_{t+k}\} = \gamma^k_y\zeta^y_t$$

$$\mathbb{E}^*_t\{\widehat{r}_{t+k}\} - \mathbb{E}_t\{\widehat{r}_{t+k}\} = -\gamma^k_r\zeta^r_t$$

[16] Chapter 4 analyzes an economy with heterogeneous infinite-lived consumers.

At any point in time, two types of individuals coexist in the economy, "active" and "inactive." Active individuals supply labor and manage their own firms, which they set up when they are born. Each active individual faces a constant probability $1-\upsilon$ of becoming "inactive", i.e. of permanently losing his job and quitting his entrepreneurial activities, due to retirement or skill obsolescence. The previous assumptions imply that the size of the active population (and, hence, the measure of firms) at any point in time is constant and given by $\alpha \equiv (1-\gamma)/(1-\upsilon\gamma) \in (0,1]$. Henceforth I assume $\beta < \upsilon^{\sigma}$, which guarantees a positive real interest rate in the steady state.[17]

All active individuals get the same wage W_t and work the same number of hours, N_t/α, which are determined by firms and taken as given by the individual.

Let $\mathcal{A}_{t|s}$ denote financial wealth at the start of period t for a member of cohort $s \leq t$. For individuals other than those born in the current period $\mathcal{A}_{t|s} = \mathcal{Z}_{t|s}/\gamma$, where $\mathcal{Z}_{t|s}$ denotes the payoff at t generated by the financial portfolio purchased in period $t-1$ and the term $1/\gamma$ captures the additional return on wealth resulting from an annuity contract. As in Blanchard (1985), that contract has the holder receive each period from a (perfectly competitive) insurance firm an annuity payment proportional to his financial wealth, in exchange for transferring that wealth to the insurance firm upon death.

Complete markets in state-contingent securities are assumed. The market value in period t of a portfolio of securities generating a random payoff $\mathcal{Z}_{t+1}$ is given by $\mathbb{E}_t\{\Lambda_{t,t+1}\mathcal{Z}_{t+1}\}$, where $\Lambda_{t,t+1}$ is the usual stochastic discount factor. Only individuals who are alive can trade in securities markets. As in the baseline model, the intertemporal optimality condition is given by

$$\Lambda_{t,t+k} = \beta^k \left(\frac{C_{t+k|s}}{C_{t|s}}\right)^{-\sigma} \tag{36}$$

for $k = 1, 2, 3, ...$ where $C_{t|s}$ denotes period t consumption for a member of cohort $s \leq t$, and where (36) holds for each state in $t+k$ in which the individual remains alive.[18] Note that an implication of complete markets is that consumption is the same for all (alive) members of a given cohort, independently of whether they are active or inactive.

We can write the period budget constraint for an active individual from cohort s as:

$$C_{t|s} + \gamma\mathbb{E}_t\{\Lambda_{t,t+1}\mathcal{A}^a_{t+1|s}\} = \mathcal{A}^a_{t|s} + W_t N_{t|s} - T^a_{t|s} \tag{37}$$

where $T^a_{t|s}$ denotes a lump-sum tax.

The corresponding budget constraint for an inactive individual is given by:

$$C_{t|s} + \gamma\mathbb{E}_t\{\Lambda_{t,t+1}\mathcal{A}^r_{t+1|s}\} = \mathcal{A}^r_{t|s} - T^r_{t|s} \tag{38}$$

In what follows, and with little loss of generality, I assume uniform employment among active individuals (i.e. $N_{t|s} = \frac{1}{\alpha}N_t$) as well as uniform taxation restricted to the same group (i.e. $T^a_{t|s} = \frac{1}{\alpha}T_t$ and $T^r_{t|s} = 0$ for all $s \leq t$ and all t). Iterating (37) and (38) forward and imposing the transversality condition $\lim_{T\to\infty} \gamma^T \mathbb{E}_t\{\Lambda_{t,t+T}\mathcal{A}_{t+T|s}\} = 0$ yields the corresponding

[17] The assumption of a positive real interest rate rules out the possibility of rational bubbles as well as a permanent rollover of government debt, which for simplicity we ignore in our analysis.

[18] For simplicity I am ignoring preference shocks in this section.

individual C-IBCs:

$$\sum_{k=0}^{\infty}\gamma^k\mathbb{E}_t\{\Lambda_{t,t+k}C_{t+k|s}\}=\mathcal{A}^a_{t|s}+\frac{1}{\alpha}\sum_{k=0}^{\infty}(\gamma\upsilon)^k\mathbb{E}_t\{\Lambda_{t,t+k}(W_{t+k}N_{t+k}-T_{t+k})\}$$

$$\sum_{k=0}^{\infty}\gamma^k\mathbb{E}_t\{\Lambda_{t,t+k}C_{t+k|s}\}=\mathcal{A}^r_{t|s}$$

As shown in the Appendix, the previous C-IBCs can be combined with the intertemporal optimality condition (36) and aggregated across consumers to yield the aggregate consumption function:

$$\begin{aligned}C_t\sum_{k=0}^{\infty}(\beta\gamma)^k\mathbb{E}_t\{(\Lambda_{t,t+k}/\beta^k)^{1-\frac{1}{\sigma}}\} &= \mathcal{A}_t+\sum_{k=0}^{\infty}(\gamma\upsilon)^k\mathbb{E}_t\{\Lambda_{t,t+k}(W_{t+k}N_{t+k}-T_{t+k})\}\\ &= F_t+\sum_{k=0}^{\infty}(\gamma\upsilon)^k\mathbb{E}_t\{\Lambda_{t,t+k}(Y_{t+k}-T_{t+k})\} \qquad (39)\end{aligned}$$

where $F_t\equiv\mathcal{A}_t-Q^S_t$, the excess of aggregate wealth $\mathcal{A}_t$ relative to the aggregate value of (currently alive) firms' equity $Q^S_t=\sum_{k=0}^{\infty}(\gamma\upsilon)^k\mathbb{E}_t\{\Lambda_{t,t+k}D_{t+k}\}$.[19]

On the other hand, the government budget constraint in period t is given by

$$B^G_t=T_t+\mathbb{E}_t\{\Lambda_{t,t+1}B^G_{t+1}\} \qquad (40)$$

where for simplicity $G_t=0$ for all t is assumed in order to focus on tax and debt policies. Under the assumption that $\lim_{T\to\infty}\mathbb{E}_t\{\Lambda_{t,t+T}B^G_{t+T}\}=0$ one can write the implied G-IBC as:[20]

$$B^G_t=\sum_{k=0}^{\infty}\mathbb{E}_t\{\Lambda_{t,t+k}T_{t+k}\} \qquad (41)$$

For future reference, the log-linearized version around a steady state of (40) and (41) is given by:

$$\widehat{b}^G_t=\Lambda\mathbb{E}_t\{\widehat{b}^G_{t+1}\}+\widehat{t}_t-\Lambda b^G\widehat{r}_t \qquad (42)$$

$$\widehat{b}^G_t=\sum_{k=0}^{\infty}\Lambda^k\mathbb{E}_t\{\widehat{t}_{t+k}-\Lambda b^G\widehat{r}_{t+k}\} \qquad (43)$$

Next I analyze some of the implications of the OLG-NK model for the transmission of monetary and fiscal policy. The next section studies a baseline version of the model without a fiscal sector and derives a surprising equivalence result.

[19] In Galí (2021) I analyze the case in which a rational bubble $Q^B_t>0$ may emerge in equilibrium, absorbing part of aggregate wealth, i.e. $\mathcal{A}_t=Q^S_t+Q^B_t+B^G_t$. By implying a positive real interest rate in the steady state, assumption $\beta<\upsilon^\sigma$ made above rules out that possibility. See Galí (2021) for a detailed discussion.

[20] In contrast with the representative agent model, that condition doesn't follow from the transversality condition of consumers. It will hold, however, if the government follows a tax rule that guarantees a constant steady state debt ratio and the steady state real interest rate is positive, as assumed here.

6.1 A Baseline OLG-NK Model

In this subsection I assume $T_t = B_t^G = 0$ for all t. In this case $F_t = 0$ in equilibrium, and one can write down the aggregate consumption function as:

$$C_t \sum_{k=0}^{\infty} (\beta\gamma)^k \mathbb{E}_t\{(\Lambda_{t,t+k}/\beta^k)^{1-\frac{1}{\sigma}}\} = \sum_{k=0}^{\infty} (\gamma\upsilon)^k \mathbb{E}_t\{\Lambda_{t,t+k} Y_{t+k}\} \tag{44}$$

As shown in Appendix C, the previous equation can be evaluated at a steady state with $C = Y$ and $\Lambda_{t,t+k} = \Lambda^k$ in order to determine the implied steady state discount factor $\Lambda \equiv \exp\{-r\}$, which is given by:

$$\Lambda = \frac{\beta}{\upsilon^\sigma}$$

thus implying $r = \rho + \sigma \log \upsilon$, which is assumed to be positive.[21] Note that, given Y, a higher incidence of retirement (i.e. a lower υ) reduces expected lifetime income and consumption, thus requiring a lower interest rate in order to restore equilibrium in the goods market.

Log-linearization of (44) about the steady state yields the approximate aggregate consumption function (see Appendix C for details):

$$\widehat{c}_t = (1 - \beta\gamma\upsilon^{1-\sigma}) \sum_{k=0}^{\infty} (\beta\gamma\upsilon^{1-\sigma})^k \mathbb{E}_t\{\widehat{y}_{t+k}\} - \frac{\beta\gamma\upsilon^{1-\sigma}}{\sigma} \sum_{k=0}^{\infty} (\beta\gamma\upsilon^{1-\sigma})^k \mathbb{E}_t\{\widehat{r}_{t+k}\} \tag{45}$$

Note that the resulting consumption function has the same form as in the baseline model with an infinite-lived representative consumer, but with the effective discount factor β now being replaced with $\beta\gamma\upsilon^{1-\sigma}$. A shorter expected horizon, captured by a lower γ, thus implies stronger discounting of future income and interest rates, but a higher marginal propensity to consume. The effect of a higher retirement hazard, which corresponds to a lower υ, has an ambiguous effect on the degree of discounting. This is due to the presence of two forces working in opposite directions. On the one hand a lower υ leads active individuals to put less weight on future income and interest rates, and hence more discounting. This is the partial equilibrium effect, which holds conditional on the steady state real interest rate. On the other hand, a lower υ tends to lower the interest rate, since it raises desired savings for any level of income, the more so the larger is σ, the curvature of the utility function. This is the general equilibrium effect. If $\sigma < 1$, the partial equilibrium, "horizon" effect dominates, with a lower υ implying greater discounting of future income and higher marginal propensity to consume. If $\sigma > 1$, the converse is true, with the general equilibrium, interest rate effect dominating. In the log utility case ($\sigma = 1$), both effects exactly cancel each other and υ drops out of the aggregate consumption function.

The consumption function (45) can be conveniently rewritten in recursive form as:

$$\widehat{c}_t = \beta\gamma\upsilon^{1-\sigma} \mathbb{E}_t\{\widehat{c}_{t+1}\} + (1 - \beta\gamma\upsilon^{1-\sigma})\widehat{y}_t - \frac{\beta\gamma\upsilon^{1-\sigma}}{\sigma}\widehat{r}_t$$

Combined with the goods market clearing condition, $\widehat{c}_t = \widehat{y}_t$, the previous relation implies the recursive DIS equation:

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\widehat{r}_t$$

[21] In Galí (2021) I study the consequences of a negative stady state real rate, including the possibility of rational bubbles.

which is independent of γ and υ and hence identical to the one in a representative consumer economy. Intuitively, the effect of shorter horizons in enhancing the discounting of future income and interest rates is exactly offset by their positive impact on the marginal propensity to consume. In particular, the total effect of a monetary policy intervention is invariant to the values of γ and υ, though its decomposition between direct and indirect effects is not, with the size of the direct effect increasing in $\beta\gamma\upsilon^{1-\sigma}$, as made clear by (45).

The previous equivalence property is, however, not a robust feature of the OLG-NK model: it breaks down in the presence of a fiscal sector when government debt is in positive net supply in the steady state, as shown in the next subsection.

6.2 Fiscal Policy and the Breakdown of Ricardian Equivalence

Next I reintroduce an infinite-lived fiscal authority with period and intertemporal budget constraints given by (40) and (41). For simplicity I have assumed $G_t = 0$ for all t and focus instead on the effects of debt and tax policies and how those effects are shaped by the greater discounting associated with finite lives and retirement. For the sake of tractability and with little loss of generality given the purpose of this subsection, I restrict the analysis to the case of log utility, i.e. $\sigma = 1$. Derivations details can be found in Appendix C.

Under the assumptions above, aggregate consumption function (39) simplifies to:

$$C_t = (1-\beta\gamma)\left[B_t^G + \sum_{k=0}^{\infty}(\gamma\upsilon)^k \mathbb{E}_t\{\Lambda_{t,t+k}(Y_{t+k} - T_{t+k})\}\right] \tag{46}$$

where I have already imposed the equilibrium condition $F_t = B_t^G$.

Evaluating the previous expression at a steady state with $C = Y$ and $(1-\Lambda)B^G = T$ allows one to solve for $\Lambda \equiv \exp\{-r\}$, the steady state discount factor:

$$\Lambda = \left(\frac{\beta}{\upsilon}\right)\left(\frac{\gamma\upsilon}{\gamma\upsilon + (1-\beta\gamma)(1-\gamma\upsilon)b^G}\right) \leq \frac{\beta}{\upsilon}$$

Thus, and as long as $\gamma\upsilon < 1$, higher government debt in the steady state is associated with a higher real interest rate (corresponding to a lower discount factor). The infinite-lived consumer model is a limiting case corresponding to $\gamma\upsilon = 1$, in which case $\Lambda = \beta$ implying a steady state real rate of ρ invariant to the debt ratio, a manifestation of Ricardian equivalence.[22]

To examine the impact of tax policy consider the log-linearized version of (46) around the steady state:

$$\widehat{c}_t = (1-\beta\gamma)\left[\widehat{b}_t^G + \sum_{k=0}^{\infty}(\Lambda\gamma\upsilon)^k\left(\mathbb{E}_t\{\widehat{y}_{t+k}\} - \mathbb{E}_t\{\widehat{t}_{t+k}\} - \frac{[1-(1-\Lambda)b^G]\Lambda\gamma\upsilon}{1-\Lambda\gamma\upsilon}\mathbb{E}_t\{\widehat{r}_{t+k}\}\right)\right] \tag{47}$$

The previous expression allows us to determine the *direct* or partial equilibrium effect of a change in taxes on consumption, conditional on a given path of income and real interest rates. Starting from a steady state, consider a short term tax change $\widehat{t}_t = \Delta$, followed by

[22] As discussed above, throughout I assume $\beta < \upsilon \leq 1$ in order to guarantee a positive real interest rate in the steady state.

a gradual reversal given by $\widehat{t}_{t+k} = -\frac{1-\Lambda\phi}{\Lambda\phi}\phi^k\Delta$ for $k = 1, 2, ...$where $0 \leq \phi < 1$, and with a higher ϕ capturing a more gradual consolidation. Appendix C shows that this tax policy intervention satisfies the government's transversality condition. Using (47) the *direct* effect on current consumption is easily determined to be given by:

$$\widehat{c}_t = -\frac{(1-\beta\gamma)(1-\gamma\upsilon)}{1-\beta\gamma\Theta\phi}\Delta \tag{48}$$

where $\Theta \equiv \frac{\Lambda\upsilon}{\beta} = \frac{\gamma\upsilon}{\gamma\upsilon+(1-\beta\gamma)(1-\gamma\upsilon)b^G} \in (0, 1]$. Thus, and to the extent that $\gamma\upsilon < 1$, a tax reduction ($\Delta < 0$) financed by higher debt and followed by a sequence of tax increases aimed at attaining fiscal consolidation, stimulates current consumption. In other words, Ricardian equivalence no longer holds in this environment. That direct effect is greater (i) the shorter is consumers' horizon of higher taxes (i.e. the lower $\gamma\upsilon$) and (ii) the more gradual is the consolidation (i.e., the higher ϕ) since the impact of future tax changes on current consumption is smaller the further into the future is their implementation.

Next I show how to determine the total effect of a tax intervention on consumption and output. For simplicity, I assume that the interest rate remains unchanged (i.e. $\widehat{r}_{t+k} = 0$ for $k = 0, 1, 2, ...$). Imposing the goods market equilibrium condition $\widehat{y}_t = \widehat{c}_t$ for all t in (47) and rewriting the resulting expression in recursive form (after some rearranging of terms):

$$\widehat{y}_t = \Theta\mathbb{E}_t\{\widehat{y}_{t+1} - (1-\beta\gamma)\widehat{b}^G_{t+1}\} + \frac{1-\beta\gamma}{\beta\gamma}(\widehat{b}^G_t - \widehat{t}_t)$$

Using (42) to substitute for $\widehat{b}^G_t - \widehat{t}_t$ implies

$$\widehat{y}_t = \Theta\mathbb{E}_t\{\widehat{y}_{t+1}\} + \frac{(1-\beta\gamma)(1-\gamma\upsilon)\Lambda}{\beta\gamma}\mathbb{E}_t\{\widehat{b}^G_{t+1}\}$$

which can be iterated forward to yield

$$\widehat{y}_t = \frac{(1-\beta\gamma)(1-\gamma\upsilon)\Lambda}{\beta\gamma}\sum_{k=0}^{\infty}\Theta^k\mathbb{E}_t\{\widehat{b}^G_{t+1+k}\} \tag{49}$$

where I have assumed $\lim_{T\to\infty}\Theta^T\mathbb{E}_t\{\widehat{b}^G_{t+T}\} = 0$. Equation (49) makes clear that, to the extent that $\gamma\upsilon < 1$, any temporary tax reduction that causes a persistent buildup of government debt will have an expansionary effect on output (given a constant real interest rate). This will be true as long as debt remains above its steady state value, even if taxes are eventually raised.

As shown in Appendix C, the specific tax intervention introduced above implies a path of government debt given by:

$$\widehat{b}^G_{t+1+k} = -\frac{\Delta}{\Lambda}\phi^k$$

Accordingly, substituting the previous expression into (49) and simplifying:

$$\widehat{y}_t = -\frac{(1-\beta\gamma)(1-\gamma\upsilon)}{\beta\gamma(1-\Theta\phi)}\Delta$$

Note that the tax multiplier is increasing nonlinearly with ϕ, the delay in the subsequent tax reversal. Furthermore, the larger is b^G the smaller is Θ, and hence the smaller the multiplier, since a larger fiscal consolidation is required down the road, due to the higher interest rate.[23]

Now it is possible to determine the "general equilibrium multiplier" of tax policies, i.e. the ratio of the total effect to the direct effect on output of the assumed tax intervention. That multiplier is given by[24]

$$\Xi_f = 1 + \frac{1-\beta\gamma}{\beta\gamma(1-\Theta\phi)}$$

which is increasing in ϕ and Θ, i.e. the multiplier effect that result from the induced changes in output are larger the more gradual the subsequent fiscal consolidation (and the lower the steady state real interest rate.

6.3 Finite Horizons, Government Debt and the Forward Guidance Puzzle

Next I examine the consequences of a positive steady state debt ratio $b^G > 0$ on the transmission of monetary policy, with the latter modeled as a path for the real interest rate. As in the previous subsection I maintain the assumption of $\sigma = 1$. It is important to note that, away from the zero government debt steady state, changes in interest rates have an (first order) impact on the government budget constraint (as captured in (43)), and how taxes and/or debt are adjusted in response will impinge on economic outcomes. In what follows, and for concreteness, I assume that taxes are adjusted each period in order to exactly offset any variation in interest payments thus leaving government debt unchanged. Formally this requires setting $\widehat{t}_t = \Lambda b^G \widehat{r}_t$ for all t. Imposing the previous condition, as well as $\widehat{b}^G_t = 0$, on consumption function (47) yields:

$$\widehat{c}_t = (1-\beta\gamma)\left[\sum_{k=0}^{\infty}(\Lambda\gamma\upsilon)^k\left(\mathbb{E}_t\{\widehat{y}_{t+k}\} - \frac{\Lambda[\gamma\upsilon+(1-\gamma\upsilon)b^G]}{1-\Lambda\gamma\upsilon}\mathbb{E}_t\{\widehat{r}_{t+k}\}\right)\right]$$

Rewriting the previous equation in recursive form and imposing the goods market equilibrium condition $\widehat{y}_t = \widehat{c}_t$ yields, after some algebra, the difference equation

$$\widehat{y}_t = \Theta\mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{(1-\beta\gamma)\Theta[\gamma\upsilon+(1-\gamma\upsilon)b^G]}{\gamma\upsilon(1-\beta\gamma\Theta)}\widehat{r}_t \tag{50}$$

which can be thought of as a modified DIS equation with discounting. When $\gamma\upsilon < 1$ and $b^G > 0$ then $\Theta < 1$, so that (50) can be solved forward to obtain:

$$\widehat{y}_t = -\frac{(1-\beta\gamma)\Theta[\gamma\upsilon+(1-\gamma\upsilon)b^G]}{\gamma\upsilon(1-\beta\gamma\Theta)}\sum_{k=0}^{\infty}\Theta^k\mathbb{E}_t\{\widehat{r}_{t+k}\}$$

[23] In the particular case of zero steady state debt, $\Lambda = \beta/\upsilon$, and the previous expression simplifies to:

$$\widehat{y}_t = -\frac{(1-\beta\gamma)(1-\gamma\upsilon)}{\beta\gamma(1-\phi)}\Delta$$

[24] Note that this is valid only if $\gamma\upsilon < 1$ since both the direct and total effects are zero when $\gamma\upsilon = 1$.

Note that in this case the coefficients associated with future interest rates decrease with the horizon k, thus overturning the forward guidance puzzle. The previous analysis makes clear that the apparent resolution of that puzzle in Del Negro et al. (2023) is not a result of finite horizons *alone*, but a consequence of the coexistence of finite horizons *and* a positive net supply of government debt.[25]

7 Finite-Horizon Planning

Next I consider a modification of the baseline model often referred to as *finite-horizon planning*.[26] It assumes that while consumers are infinite-lived, they are capable of forming expectations of future variables only up to T periods ahead. The simplified version presented here assumes that consumers expect all the variables they take as given to return to their steady state values after T periods. Thus, a consumer with horizon T has a (log-linearized) consumption function of the form:[27]

$$\widehat{c}_t^T = (1-\beta)\left(\sum_{k=0}^{T}\beta^k\mathbb{E}_t\{\widehat{y}_{t+k}\}\right) - \frac{\beta}{\sigma}\sum_{k=0}^{T}\beta^k\mathbb{E}_t\{\widehat{r}_{t+k}\} \tag{51}$$

The distribution of planning horizons across the population is assumed to be geometric, with a fraction $(1-\gamma)\gamma^T$ of consumers displaying a horizon T, for $T = 0, 1, 2, ...$with $\gamma \in [0, 1]$. The aggregation of individual consumption functions implies

$$\begin{aligned}\widehat{c}_t &= (1-\gamma)\sum_{T=0}^{\infty}\gamma^T\widehat{c}_t^T \\ &= (1-\beta)\left(\sum_{k=0}^{\infty}(\beta\gamma)^k\mathbb{E}_t\{\widehat{y}_{t+k}\}\right) - \frac{\beta}{\sigma}\sum_{k=0}^{\infty}(\beta\gamma)^k\mathbb{E}_t\{\widehat{r}_{t+k}\}\end{aligned}$$

Combined with the goods market clearing condition, $\widehat{y}_t = \widehat{c}_t$, yields

$$\begin{aligned}\widehat{y}_t &= (1-\beta)\left(\sum_{k=0}^{\infty}(\beta\gamma)^k\mathbb{E}_t\{\widehat{y}_{t+k}\}\right) - \frac{\beta}{\sigma}\sum_{k=0}^{\infty}(\beta\gamma)^k\mathbb{E}_t\{\widehat{r}_{t+k}\} \\ &= \beta\gamma\mathbb{E}_t\{\widehat{y}_{t+1}\} + (1-\beta)\widehat{y}_t - \frac{\beta}{\sigma}\widehat{r}_t\end{aligned}$$

Rearranging terms the following recursive DIS equation can be derived:

$$\widehat{y}_t = \gamma\mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\widehat{r}_t$$

[25]Note that in the particular case of $b^G = 0$ (and hence $\Theta = 1$) the DIS simplifies to:

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \widehat{r}_t$$

which is identical to the standard recursive representation of the DIS in the model with an infinite-lived consumer, characterized by the absence of discounting.

[26]See Woodford (2019) and Dupraz and Marx (2025).

[27]Again I am ignoring preference shocks, for simplicity.

Note that the DIS equation above implies genuine discounting relative to the baseline model and the variations thereof considered in previous sections.[28] The reason for this is that the additional discounting of variations in future income and interest rates implied by the assumption of finite-horizon planning is not offset by a higher marginal propensity to consume, which remains unchanged at $1 - \beta$.[29]

8 Notes on the Literature

The consumption Euler equation has long been a key block of dynamic macro models with a utility maximizing representative consumer, including those in the real business cycle (e.g. Prescott (1986)) as well as in the New Keynesian tradition (e.g. Woodford (2003), Galí (2015)). Efforts to account for the behavior of consumption both in the cross-section and time series dimension had traditionally focused on the consumption function (e.g. Friedman (1957), Modigliani and Brumberg (1954)) until the seminal work by Hall (1978) shifted the attention to empirical tests of the Euler equation. Yet, much of the subsequent empirical work on aggregate consumption that followed had as a theoretical benchmark a log-linearized consumption function similar to the one used in this chapter (e.g. Flavin (1981), Campbell (1987)).

The importance of adopting a consumption function approach under departures from the benchmark of rational expectations becomes apparent in some of the macro applications with adaptive learning (e.g. Preston (2005), Eusepi and Preston (2011)), finite-horizon planning (Woodford (2019)), lack of common knowledge (Angeletos and Lian (2018)), level-k thinking and reflective equilibrium (García-Schmidt and Woodford (2019), Farhi and Werning (2019)), among others.

The presence of heterogeneity generally requires the adoption of the consumption function approach, due to the need to derive an aggregate consumption function through the aggregation of individual ones. Applications include models of overlapping generations with finite horizons (e.g., Del Negro et al. (2023), Galí (2021), Angeletos et al. (2024)), as well as models with infinite-lived heterogeneous households in the HANK tradition (e.g., Kaplan et al. (2018), Auclert et al. (2024)).

9 Summary and Conclusions

The present chapter has sought to point out the limitations of restricting the characterization of consumption to the Euler equation, a common approach in the analysis of the NK model. I propose instead to take the consumption function as a starting point. A tractable representation of the latter can be obtained by combining the Euler equation with the intertemporal budget constraint, after log-linearizing both. The resulting consumption function can then be combined with a goods market clearing condition in order to derive the dynamic IS equation, which already

[28] With exception of the overlapping generations model with non-zero steady state government debt. The behavioral NK model in Gabaix (2020) generates a similar discounting.

[29] A key assumption behind this property (and one embedded in the derivation of (51)) is that consumers with a k-horizon planning do not necessarily anticipate their individual consumption to be back at its steady state value after k periods. This seems a natural assumption since consumption is an endogenous choice variable, in contrast to income and interest rates, which are perceived as exogenous.

embeds general equilibrium effects. A number of issues have been discussed to illustrate the benefits of the proposed approach.

- Direct vs indirect effects of exogenous interest rate changes
- Irrelevance of budget deficits, conditional on the path of government spending (Ricardian equivalence), and negative wealth effects of government purchases on consumption.
- Introduction of irrational waves of optimism and pessimism, and analysis of their impact on economic activity
- Derivation of an aggregate consumption function in models with overlapping generations of finite-lived consumers, which makes transparent its differential implications relative to the baseline model with an infinite-lived representative consumer

By contrast, none of the previous issues can be addressed transparently under the standard approach to deriving the equilibrium conditions of the NK model, as found for instance in Galí (2015).

From a more practical perspective, the analysis of the present chapter has shown that, through a suitable definition, variable z_t in dynamic IS equation (21) can accommodate a variety of aggregate demand shocks, including government spending, animal spirits, or discount rate shocks. In the next chapter I maintain the assumption that (21) determines aggregate demand without taking a stance on the nature of the exogenous forces ultimately driving fluctuations in z_t, which I refer generically to as the aggregate demand shock.

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APPENDIX

Appendix A. Derivation of the C-IBC under Complete Markets

In the presence of complete sequential markets, the period budget constraint in history node s^t can be written as

$$C(s^t) + \sum_{s^{t+1}|s^t} Q^A(s^t, s^{t+1})\mathcal{A}(s^{t+1}) = \mathcal{A}(s^t) + W(s^t)N(s^t)$$

where $Q^A(s^t, s^{t+1})$ is the price at s^t of an Arrow security paying one unit of the consumption bundle in s^{t+1} (and zero otherwise). Equivalently one can write

$$C(s^t) + \mathbb{E}\{\Lambda(s^t, s^{t+1})\mathcal{A}(s^{t+1})|s^t\} = \mathcal{A}(s^t) + W(s^t)N(s^t)$$

where $\Lambda(s^t, s^{t+1}) \equiv \frac{Q^A(s^t, s^{t+1})}{\xi(s^{t+1}|s^t)}$ and $\xi(s^{t+1}|s^t)$ the probability of s^{t+1} conditional on being in node s^t.

Using simplified conventional notation that omits explicit state-dependence one can rewrite the above period budget constraint as follows:

$$C_t + \mathbb{E}_t\{\Lambda_{t,t+1}\mathcal{A}_{t+1}\} = \mathcal{A}_t + W_t N_t$$

which corresponds to (10) in the main text. Iterating the previous difference equation forward, letting $\Lambda_{t,t+k} = \Lambda_{t,t+1}\Lambda_{t+1,t+2}...\Lambda_{t+k-1,t+k}$ for $k = 1, 2, 3...$and imposing the transversality condition $\lim_{T\to\infty} \mathbb{E}_t\{\Lambda_{t,T}\mathcal{A}_{t+T}\} = 0$ yields the C-IBC in the text.

The consumer's intertemporal optimality condition is given by

$$Q^A(s^t, s^{t+1})U_c(s^t) = \xi(s^{t+1}|s^t)\beta U_c(s^{t+1})$$

which must hold state by state. Equivalently, and using the more convenient notation:

$$\Lambda_{t,t+1} = \beta U_{c,t+1}/U_{c,t}$$

It follows that

$$\Lambda_{t,t+k} = \beta^k U_{c,t+k}/U_{c,t}$$

Under the assumed functional form for utility:

$$\Lambda_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\sigma}(\Phi_{t+k}/\Phi_t)$$

which can be rewritten in terms of log-deviations from steady state as follows:

$$\widehat{\lambda}_{t,t+k} = \sigma(\widehat{c}_t - \widehat{c}_{t+k}) + \phi_{t+k} - \phi_t$$

where $\widehat{\lambda}_{t,t+k} \equiv \log(\Lambda_{t,t+k}/\beta^k)$. Note that all the conditions above hold state by state, for all s^t and s^{t+k}.

Taking conditional expectations yields:

$$\mathbb{E}_t\{\widehat{\lambda}_{t,t+k}\} = \sigma(\widehat{c}_t - \mathbb{E}_t\{\widehat{c}_{t+k}\}) + \mathbb{E}_t\{\phi_{t+k}\} - \phi_t$$

and the analysis follows as in the text.

Appendix B. Derivation of the C-IBC under Incomplete Markets

In the absence of complete markets, one can always write the following equation describing the evolution of wealth:

$$\mathcal{A}_{t+1} = \mathcal{R}_{t,t+1}(\mathcal{A}_t + W_t N_t - C_t)$$

where $\mathcal{R}_{t,t+1}$ is the gross real return on the financial portfolio held by the consumer between t and $t+1$. Dividing both sides by $\mathcal{R}_{t,t+1}$, taking conditional expectations and iterating forward one can derive:

$$\sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k} C_{t+k}\} = \mathcal{A}_t + \sum_{k=0}^{\infty} \mathbb{E}_t\{*_{t,t+k} W_{t+k} N_{t+k}\} \tag{B.1}$$

where now $*_{t,t+k} \equiv \mathcal{R}_{t,t+1}^{-1}\mathcal{R}_{t+1,t+2}^{-1}...\mathcal{R}_{t+k-1,t+k}^{-1}$ and where the transversality condition $\lim_{T\to\infty} \mathbb{E}_t\{\Lambda_{t,T}\mathcal{A}_{t+T}\} = 0$ has been assumed.

The consumer's optimality condition is given by

$$U_{c,t} = \beta \mathbb{E}_t\{\mathcal{R}_{t,t+1} U_{c,t+1}\}$$

Iterating forward and letting $\mathcal{R}_{t,t+k} \equiv \mathcal{R}_{t,t+1}\mathcal{R}_{t+1,t+2}...\mathcal{R}_{t+k-1,t+k}$ one can write:

$$\begin{aligned} U_{c,t} &= \beta^k \mathbb{E}_t\{\mathcal{R}_{t,t+k} U_{c,t+k}\} \\ &= \beta^k \mathbb{E}_t\{*_{t,t+k}^{-1} U_{c,t+k}\} \end{aligned}$$

Under the assumed functional form for utility:

$$1 = \beta^k \mathbb{E}_t\{\Lambda_{t,t+k}^{-1}(C_{t+k}/C_t)^{-\sigma}(\Phi_{t+k}/\Phi_t)\} \tag{B.2}$$

In log-deviations from steady state,

$$\mathbb{E}_t\{\widehat{\lambda}_{t,t+k}\} = \sigma(\widehat{c}_t - \mathbb{E}_t\{\widehat{c}_{t+k}\}) + \mathbb{E}_t\{\phi_{t+k}\} - \phi_t \tag{B.3}$$

which is formally identical to the equation derived in the complete markets case.

On the other hand, the price of a one-period nominal discount bond, $Q_t \equiv \exp\{-i_t\}$, must satisfy the optimality condition:

$$\begin{aligned} Q_t &= \beta \mathbb{E}_t\{(U_{c,t+1}/U_{c,t})(P_t/P_{t+1})\} \\ &= \beta \mathbb{E}_t\{(C_{t+1}/C_t)^{-\sigma}(\Phi_{t+1}/\Phi_t)(P_t/P_{t+1})\} \end{aligned}$$

or, in log-deviations from steady state

$$0 = \widehat{r}_t - \sigma(\widehat{c}_t - \mathbb{E}_t\{\widehat{c}_{t+1}\}) + \mathbb{E}_t\{\phi_{t+1}\} - \phi_t$$

where $r_t \equiv i_t - \mathbb{E}_t\{\pi_{t+1}\}$ and $\widehat{r}_t \equiv r_t - \rho$.

Rearranging terms and iterating forward one obtains:

$$0 = \sum_{j=0}^{k-1} \mathbb{E}_t\{\widehat{r}_{t+j}\} - \sigma(\widehat{c}_t - \mathbb{E}_t\{\widehat{c}_{t+k}\}) + \mathbb{E}_t\{\phi_{t+k}\} - \phi_t$$

or, equivalently:

$$\mathbb{E}_t\{\widehat{c}_{t+k}\} = \widehat{c}_t + \frac{1}{\sigma}\sum_{j=0}^{k-1}\mathbb{E}_t\{\widehat{r}_{t+j}\} - \frac{1}{\sigma}\sum_{j=0}^{k-1}\mathbb{E}_t\{z_{t+j}\} \tag{B.4}$$

for $k = 1, 2, 3, ...$where $z_t \equiv \phi_t - \mathbb{E}_t\{\phi_{t+1}\}$.

The value of firms' equity, Q_t^S, must satisfy the optimality condition

$$\begin{aligned} Q_t^S &= D_t + \beta\mathbb{E}_t\{(U_{c,t+1}/U_{c,t})Q_{t+1}^S\} \\ &= D_t + \beta\mathbb{E}_t\{(C_{t+1}/C_t)^{-\sigma}(\Phi_{t+1}/\Phi_t)Q_{t+1}^S\} \end{aligned}$$

Iterating forward and assuming a transversality condition $\lim_{T\to\infty}\beta^T\mathbb{E}_t\{(C_{t+T}/C_t)^{-\sigma}(\Phi_{t+T}/\Phi_t)Q_{t+T}^S\} = 0$ yields

$$Q_t^S = \sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{(C_{t+k}/C_t)^{-\sigma}(\Phi_{t+k}/\Phi_t)D_{t+k}\}$$

One can rewrite the intertemporal budget constraint (B.1) as:

$$\sum_{k=0}^{\infty}\mathbb{E}_t\{\Lambda_{t,t+k}(C_{t+k} - Y_{t+k})\} = F_t + H_t$$

where $F_t \equiv \mathcal{A}_t - Q_t^S$ and $H_t \equiv \sum_{k=0}^{\infty}\mathbb{E}_t\{[\beta^k(C_{t+k}/C_t)^{-\sigma}(\Phi_{t+k}/\Phi_t) - *_{t,t+k}]D_{t+k}\}$.

Log-linearizing the previous equation around a steady state with $C = Y$ and $F = H = 0$ yields:

$$\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{c}_{t+k}\} = \widehat{f}_t + \sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{y}_{t+k}\}$$

where $\widehat{f}_t \equiv (\mathcal{A}_t - Q_t^S)/Y$ and where I have used the fact that up to a first order $H_t \simeq 0$, as implied by (B.2). Combining the previous approximate intertemporal budget constraint with the optimality condition (B.4) yields the same log-linearized consumption function (16) as in the main text.

Appendix C. Overlapping Generations: Derivations

Consider an individual born in period s who is active in period $t \geq s$. His intertemporal budget constraint is given by

$$\begin{aligned} \sum_{k=0}^{\infty}\gamma^k\mathbb{E}_t\{\Lambda_{t,t+k}C_{t+k|s}\} &= \mathcal{A}_{t|s}^a + \sum_{k=0}^{\infty}\gamma^k\mathbb{E}_t\{\Lambda_{t,t+k}(W_{t+k}N_{t+k|s} - T_{t+k|s})\} \\ &= \mathcal{A}_{t|s}^a + \frac{1}{\alpha}\sum_{k=0}^{\infty}(\gamma\upsilon)^k\mathbb{E}_t\{\Lambda_{t,t+k}(W_{t+k}N_{t+k} - T_{t+k})\} \end{aligned}$$

where $N_{t+k|s} = \frac{1}{\alpha}N_{t+k}$ and $T_{t+k|s} = \frac{1}{\alpha}T_{t+k}$ if our individual remains active in period $t+k$ and $N_{t+k|s} = T_{t+k|s} = 0$ if inactive in period $t+k$. The two events will occur with probabilities υ^k and $1-\upsilon^k$, respectively, which accounts for the second equality. The notation makes clear that $C_{t|s}^a = C_{t|s}^r = C_{t|s}$ due to complete markets.

On the other hand, the intertemporal budget constraint of an individual who is inactive in period t is given by:

$$\sum_{k=0}^{\infty} \gamma^k \mathbb{E}_t\{\Lambda_{t,t+k} C_{t+k|s}\} = \mathcal{A}^r_{t|s}$$

Aggregating the above C-IBCs across consumers, one can obtain the aggregate C-IBC:

$$\sum_{k=0}^{\infty} \gamma^k \mathbb{E}_t\{\Lambda_{t,t+k} C_{t+k}\} = \mathcal{A}_t + \sum_{k=0}^{\infty} (\gamma\upsilon)^k \mathbb{E}_t\{\Lambda_{t,t+k}(W_{t+k}N_{t+k} - T_{t+k})\}$$

Combining each of the two constraints above with the intertemporal optimality condition, $\Lambda_{t,t+k} = \beta^k (C_{t+k|s}/C_{t|s})^{-\sigma}$ yields:

$$C_{t|s} \sum_{k=0}^{\infty} (\beta\gamma)^k \mathbb{E}_t\{(\Lambda_{t,t+k}/\beta^k)^{1-\frac{1}{\sigma}}\} = \mathcal{A}^a_{t|s} + \frac{1}{\alpha} \sum_{k=0}^{\infty} (\gamma\upsilon)^k \mathbb{E}_t\{\Lambda_{t,t+k}(W_{t+k}N_{t+k} - T_{t+k})\}$$

$$C_{t|s} \sum_{k=0}^{\infty} (\beta\gamma)^k \mathbb{E}_t\{(\Lambda_{t,t+k}/\beta^k)^{1-\frac{1}{\sigma}}\} = \mathcal{A}^r_{t|s}$$

Aggregating across all individuals alive in period t yields the following aggregate consumption function:

$$\begin{aligned} C_t \sum_{k=0}^{\infty} (\beta\gamma)^k \mathbb{E}_t\{(\Lambda_{t,t+k}/\beta^k)^{1-\frac{1}{\sigma}}\} &= \mathcal{A}_t + \sum_{k=0}^{\infty} (\gamma\upsilon)^k \mathbb{E}_t\{\Lambda_{t,t+k}(W_{t+k}N_{t+k} - T_{t+k})\} \\ &= F_t + \sum_{k=0}^{\infty} (\gamma\upsilon)^k \mathbb{E}_t\{\Lambda_{t,t+k}(Y_{t+k} - T_{t+k})\} \end{aligned}$$

where $F_t \equiv \mathcal{A}_t - Q^S_t$, the excess of aggregate wealth $\mathcal{A}_t$ relative to the aggregate value of (currently alive) firms' equity $Q^S_t = \sum_{k=0}^{\infty} (\gamma\upsilon)^k \mathbb{E}_t\{\Lambda_{t,t+k} D_{t+k}\}$.

On the other hand, the government budget constraint in period t is given by

$$T_t + \mathbb{E}_t\{\Lambda_{t,t+1} B^G_{t+1}\} = B^G_t$$

where I have assumed for simplicity $G_t = 0$ for all t in order to focus on tax and debt policies. Under the assumption that $\lim_{T\to\infty} \mathbb{E}_t\{\Lambda_{t,t+T} B^G_{t+T}\} = 0$ one can write the implied G-IBC as:[30]

$$B^G_t = \sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k} T_{t+k}\}$$

Next I consider separately the cases without and with a fiscal sector.

[30] Note that in this case the consumer's TVC does not guarantee that the assumed government TVC is satisfied.

a) The Case with no Fiscal Sector ($B_t^G = T_t = 0$ all t)

In this case $F_t = 0$ in equilibrium, and one can write down the aggregate consumption function as:

$$C_t \sum_{k=0}^{\infty} (\beta\gamma)^k \mathbb{E}_t\{(\Lambda_{t,t+k}/\beta^k)^{1-\frac{1}{\sigma}}\} = \sum_{k=0}^{\infty} (\gamma\upsilon)^k \mathbb{E}_t\{\Lambda_{t,t+k} Y_{t+k}\} \tag{C.1}$$

Evaluating the previous equation at a steady state with $C = Y$ and $\Lambda_{t,t+k} = \Lambda^k$ yields

$$\begin{aligned} \sum_{k=0}^{\infty} (\beta\gamma)^k (\Lambda/\beta)^{(1-\frac{1}{\sigma})k} &= \sum_{k=0}^{\infty} (\Lambda\gamma\upsilon)^k \\ \frac{1}{1 - \Lambda^{1-\frac{1}{\sigma}} \beta^{\frac{1}{\sigma}} \gamma} &= \frac{1}{1 - \Lambda\gamma\upsilon} \end{aligned}$$

thus implying:

$$\Lambda = \frac{\beta}{\upsilon^\sigma}$$

We can now rewrite (C.1) as follows:

$$C_t \sum_{k=0}^{\infty} (\Lambda\gamma\upsilon)^k \mathbb{E}_t\{(\Lambda_{t,t+k}/\Lambda^k)^{1-\frac{1}{\sigma}}\} = \sum_{k=0}^{\infty} (\Lambda\gamma\upsilon)^k \mathbb{E}_t\{(\Lambda_{t,t+k}/\Lambda^k) Y_{t+k}\}$$

which can be log-linearized to yield:

$$\frac{1}{1 - \Lambda\gamma\upsilon} \widehat{c}_t + \left(1 - \frac{1}{\sigma}\right) \sum_{k=1}^{\infty} (\Lambda\gamma\upsilon)^k \mathbb{E}_t\{\widehat{\lambda}_{t,t+k}\} = \sum_{k=1}^{\infty} (\Lambda\gamma\upsilon)^k \mathbb{E}_t\{\widehat{\lambda}_{t,t+k}\} + \sum_{k=0}^{\infty} (\Lambda\gamma\upsilon)^k \mathbb{E}_t\{\widehat{y}_{t+k}\}$$

or, equivalently, after some algebraic manipulation using the fact that $\mathbb{E}_t\{\widehat{\lambda}_{t,t+k}\} = -\sum_{h=0}^{k-1} \mathbb{E}_t\{\widehat{r}_{t+h}\}$

$$\begin{aligned} \widehat{c}_t &= (1 - \Lambda\gamma\upsilon) \sum_{k=0}^{\infty} (\Lambda\gamma\upsilon)^k \mathbb{E}_t\{\widehat{y}_{t+k}\} - \frac{\Lambda\gamma\upsilon}{\sigma} \sum_{k=0}^{\infty} (\Lambda\gamma\upsilon)^k \mathbb{E}_t\{\widehat{r}_{t+k}\} \\ &= \Lambda\gamma\upsilon \mathbb{E}_t\{\widehat{c}_{t+1}\} + (1 - \Lambda\gamma\upsilon)\widehat{y}_t - \frac{\Lambda\gamma\upsilon}{\sigma} \widehat{r}_t \end{aligned}$$

Combined with the goods market clearing condition, $\widehat{c}_t = \widehat{y}_t$, the previous relation implies the recursive DIS equation:

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma} \widehat{r}_t$$

which has exactly the same form as in the infinite-lived representative agent model.

b) The Case with a Fiscal Sector

Under the assumption that $\sigma = 1$, consumption function (C.1) can be written as:

$$C_t = (1 - \beta\gamma) \left[B_t^G + \sum_{k=0}^{\infty} (\gamma\upsilon)^k \mathbb{E}_t\{\Lambda_{t,t+k}(Y_{t+k} - T_{t+k})\} \right] \tag{C.2}$$

where I have imposed the equilibrium condition $F_t = B_t^G$.

Evaluating the previous expression at a steady state with $C = Y$ and $(1-\Lambda)B^G = T$ and letting $b^G \equiv B^G/Y$:

$$1 = (1-\beta\gamma)\left[b^G + \sum_{k=0}^{\infty}(\Lambda\gamma\upsilon)^k[1-(1-\Lambda)b^G]\}\right]$$

which allows one to solve for $\Lambda \equiv \exp\{-r\}$, the steady state discount factor:

$$\begin{aligned}\Lambda &= \frac{\beta\gamma}{\gamma\upsilon + (1-\beta\gamma)(1-\gamma\upsilon)b^G} \\ &= \left(\frac{\beta}{\upsilon}\right)\left(\frac{\gamma\upsilon}{\gamma\upsilon + (1-\beta\gamma)(1-\gamma\upsilon)b^G}\right) \leq \frac{\beta}{\upsilon}\end{aligned}$$

Log-linearization of (C.2) around the steady state yields:

$$\begin{aligned}\widehat{c}_t &= (1-\beta\gamma)\left[\widehat{b}_t^G + \sum_{k=0}^{\infty}(\Lambda\gamma\upsilon)^k\left(\mathbb{E}_t\{\widehat{y}_{t+k}\} - \mathbb{E}_t\{\widehat{t}_{t+k}\} + [1-(1-\Lambda)b^G]\mathbb{E}_t\{\widehat{\lambda}_{t,t+k}\}\right)\right] \\ &= (1-\beta\gamma)\left[\widehat{b}_t^G + \sum_{k=0}^{\infty}(\Lambda\gamma\upsilon)^k\left(\mathbb{E}_t\{\widehat{y}_{t+k}\} - \mathbb{E}_t\{\widehat{t}_{t+k}\} - \frac{[1-(1-\Lambda)b^G]\Lambda\gamma\upsilon}{1-\Lambda\gamma\upsilon}\mathbb{E}_t\{\widehat{r}_{t+k}\}\right)\right]\end{aligned}$$

On the other hand, log-linearization of the G-IBC yields:

$$\begin{aligned}\widehat{b}_t^G &= \sum_{k=0}^{\infty}\Lambda^k[\mathbb{E}_t\{\widehat{t}_{t+k}\} + (1-\Lambda)b^G\mathbb{E}_t\{\widehat{\lambda}_{t,t+k}\}] \\ &= \sum_{k=0}^{\infty}\Lambda^k[\mathbb{E}_t\{\widehat{t}_{t+k}\} - \Lambda b^G\mathbb{E}_t\{\widehat{r}_{t+k}\}]\end{aligned}$$

Note that, starting from a steady state with $\widehat{b}_t^G = 0$, (42) implies a path of government debt given by:

$$\mathbb{E}_t\{\widehat{b}_{t+1+k}^G\} = -\Lambda^{-(k+1)}\sum_{h=0}^{k}\Lambda^h[\mathbb{E}_t\{\widehat{t}_{t+h}\} - \Lambda b^G\mathbb{E}_t\{\widehat{r}_{t+h}\}]$$

Thus, the specific tax intervention assumed in section 6.2 implies

$$\widehat{b}_{t+1}^G = -\frac{1}{\Lambda}\Delta$$

and for $k = 1, 2, ...$

$$\begin{aligned}\widehat{b}_{t+1+k}^G &= -\frac{1}{\Lambda^{k+1}}\sum_{h=0}^{k}\Lambda^h\widehat{t}_{t+h} \\ &= -\frac{\Delta}{\Lambda^{k+1}}\left(1 - \frac{1-\Lambda\phi}{\Lambda\phi}\sum_{h=1}^{k}(\Lambda\phi)^h\right) \\ &= -\frac{\Delta}{\Lambda}\phi^k\end{aligned}$$

which satisfies the government transversality conditions $\lim_{T\to\infty} \Theta^T \mathbb{E}_t\{\widehat{b}^G_{t+T}\} = 0$ and $\lim_{T\to\infty} \Lambda^T \mathbb{E}_t\{\widehat{b}^G_{t+T}\} = 0$.

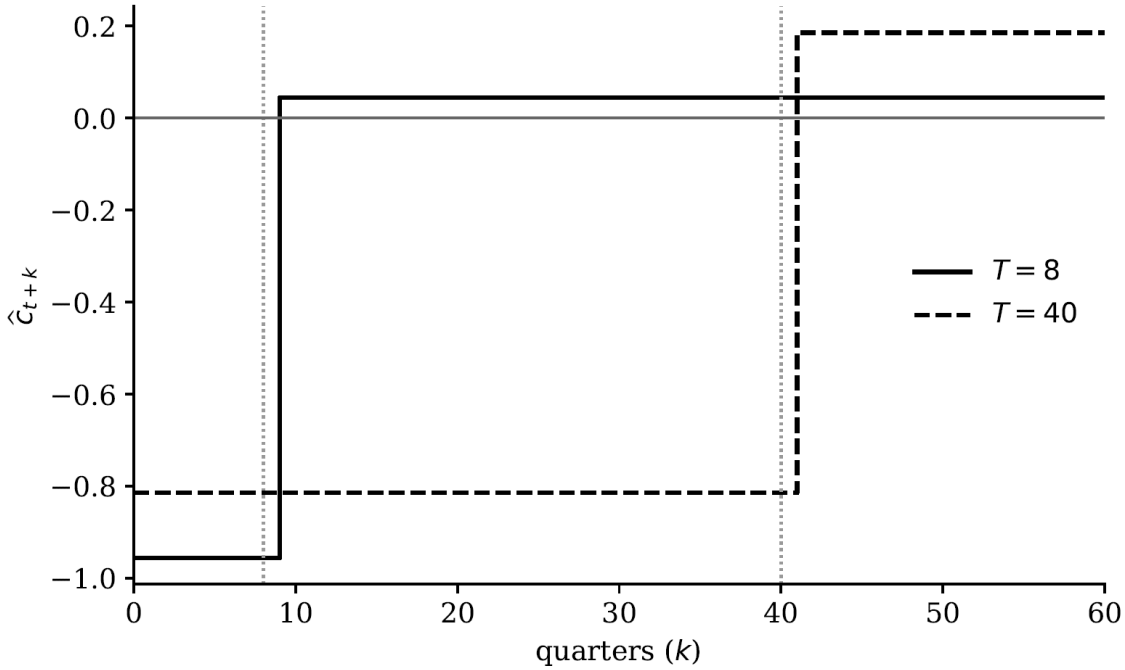


Figure 1

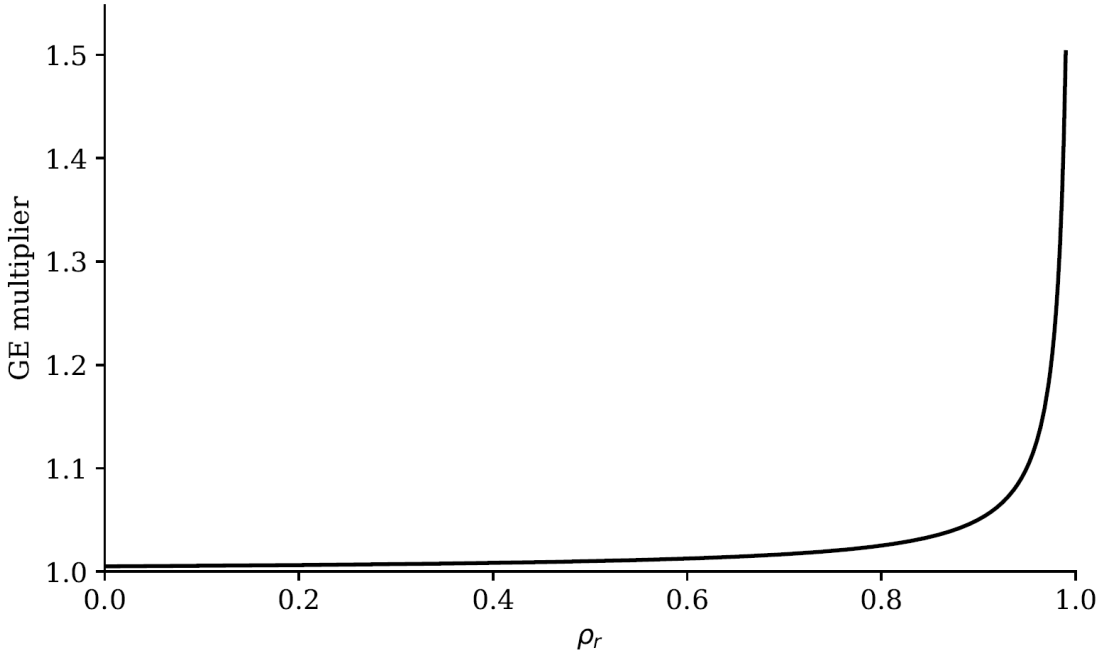


Figure 2