Endogenous Production Networks and Non-Linear Monetary Transmission

Mishel Ghassibe

CREi, UPF, BSE & CEPR *

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Abstract

I develop a tractable dynamic sticky-price model, where input-output linkages are formed endogenously. The model delivers cyclical properties of networks that are consistent with those I estimate using sectoral and firm-level data, conditional on identified real and nominal shocks. A novel source of state dependence in nominal rigidities arises: the strength of complementarities in price setting and monetary non-neutrality increase in the number of suppliers optimally chosen by firms. As a result, the model simultaneously rationalizes the following observed non-linearities in monetary transmission. First, there is *cycle dependence*: the magnitude of real GDP's response to a monetary shock is procyclical. Second, there is *path dependence*: non-neutrality of real GDP is higher following previous periods of loose monetary policy. Third, there is *size dependence*: larger monetary contractions shrink the network and generate a less than proportional decrease in GDP relative to smaller contractions.

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*Centre de Recerca en Economia Internacional (CREi), Ramon Trias Fargas, 25-27, Mercè Rodoreda building, 08005-Barcelona, Spain, mghassibe@crei.cat. The author acknowledges financial support from the JDC2022-049717-I grant funded by MCIN/AEI/ 10.13039/501100011033 and by "European Union NextGenerationEU/PRTR". The author also acknowledges support from the Generalitat de Catalunya, through CERCA and SGR Programme (2021-SGR-01599) and the AEI through the Severo Ochoa Programme for Centres of Excellence in R&D (CEX2019-000915-S).

1 Introduction

Modern firms operate in a complex environment, making a number of decisions as they respond to the evolution of the business cycle. Active supply chain management is one of them: as Figure 1(a) shows, firms in the United States drop almost five suppliers on average in recessions, before regaining them as the economy recovers (FactSet Research Systems Inc., nd).[1] Moreover, Figure 1(b) suggests that such procyclicality in input-output relationships is consistent with aggregate data: the overall cost share of intermediate inputs drops consistently in economic downturns, with an even more pronounced volatility at the sectoral level, as reported in panel (c). Understanding the implications of such active supplier management for the aggregate business cycle, and especially for the conduct of policies aiming to control inflation and economic activity, requires a broadening of our modelling toolkit. I fill this gap by developing a general equilibrium model that studies the endogenous formation of input-output linkages in conjunction with other key decisions made by firms, most notably price setting. The model delivers cyclical properties of production networks that are consistent with those I estimate using sectoral and firm-level data, conditional on both real and nominal shocks. Moreover, it delivers novel insights on the transmission of monetary policy to inflation and economic activity.

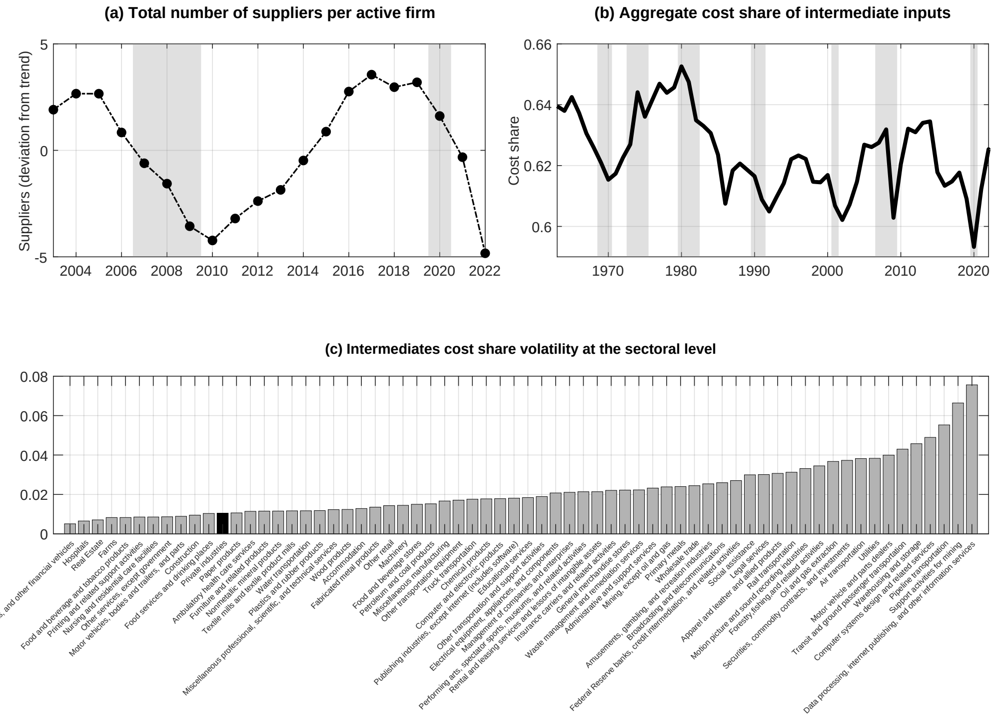
In particular, accounting for the observed network cyclicality allows the model to rationalize multiple observed non-linearities in the effect of central bank interventions on the aggregate economy. This is because the model links the density of the network, given by the number of suppliers each firm optimally chooses to have, to the degree of complementarities in price setting created by the network production structure. Specifically, in states of the world where firms endogenously connect to more suppliers, their unit cost becomes dependent on a larger number of prices set by other firms, raising the cost share of intermediate inputs, which in turn strengthens pricing complementarities and amplifies monetary non-neutrality.[2] Naturally, in states with few linkages across firms the opposite occurs: pricing complementarities weaken and monetary non-neutrality diminishes. This novel mechanism makes the strength of monetary transmission depend on the *phase of the business cycle*, the *past monetary stance* and the *size of the shock* even if the probability of price adjustment is state-independent. Below I explain each of the three non-linearities step-by-step.

First, in high-productivity expansionary states, firms optimally choose to connect to more suppliers, making the real GDP response to monetary shocks larger than in recessions. Monetary non-neutrality is therefore *cycle-dependent*: it is stronger in expansions. The mechanism that generates this effect is as follows. A productivity improvement lowers prices of potential new suppliers relative to the cost of in-house labor, which incentivizes firms to connect to more

[1] FactSet Revere Supply Chain Relationships is a dataset on customer-supplier relationships across a universe of firms in the United States. It contains annual accounts (2003-2022), covering 854,683 unique relationships. When working with that dataset, I extract the cyclical component using an HP-filter with a smoothing parameter of 400 (annual data). However, the results are robust to using alternative detrending procedures, such as fitting a polynomial trend. In Appendix C, Figure C.1 alternatively reports the number of suppliers per firm in per cent (rather than level) deviation from trend, whereas Figure C.2 reports the original data that has not been detrended.

[2] That an increase in the number of suppliers is associated with a higher cost share of intermediates (and *vice versa*) is consistent with the evidence in panels (a) and (b) of Figure 1, which shows that both the number of suppliers and the aggregate cost share of intermediates go down in recessions.

Figure 1: **Intermediate inputs over the business cycle (United States)**



Notes: Panel (a) shows the number of suppliers per active firm in the United States (in deviation from trend) based on the annual FactSet Revere Supply Chain Relationships dataset (2003-2022)(FactSet Research Systems Inc., nd). Panel (b) shows the aggregate cost share of intermediate inputs in the United States (1963-2022), constructed as the ratio of total expenditure on intermediates and total production costs in the United States using annual data published by the US Bureau of Economic Analysis (BEA)(U.S. Bureau of Economic Analysis, 2025a,c). Panel (c) reports the standard deviations of the cyclical components of sector-specific intermediates cost shares in the US, using BEA annual data (U.S. Bureau of Economic Analysis, 2020b). Grey bars indicate NBER recession dates.

suppliers, replacing a part of production that was previously done by labor with intermediate inputs.[3] The resulting increase in the intermediates cost share strengthens pricing complementarities and thus amplifies the non-neutrality of money. On the other hand, in recessions firms optimally decide to disconnect from some suppliers, replacing them with in-house labor, which lowers the cost share of intermediates and makes the economy more money-neutral, as complementarities in price setting weaken. These theoretical findings are consistent with the model-free econometric studies that find a weaker response of GDP to monetary shocks in recessions (Tenreyro and Thwaites, 2016; Alpanda et al., 2021; Jordà et al., 2020).

Second, a monetary easing incentivizes firms to connect to more suppliers, which makes monetary transmission *path-dependent*: following a period of loose monetary policy, any further monetary intervention has a stronger effect on GDP. The mechanism works as follows. Due to rigidities in price setting, a monetary easing makes prices charged by supplier firms cheaper relative to the cost of in-house labor, which incentivizes firms to connect to more suppliers, replacing a part of production that was previously done by labor with intermediate inputs.[4]

[3]This theoretical step relies on the ratio of nominal wage to (input) prices being procyclical, or more specifically that such measure of the real wage rises following a productivity improvement. The seminal work by Basu (1995) shows empirically that, unconditionally, prices of intermediate goods are less procyclical than unit labor costs, suggesting that the relevant real wage is indeed procyclical overall. As for conditional cyclicality, Liu and Phaneuf (2007) and Christiano et al. (2010) provide empirical evidence that real wages rise following technology improvements.

[4]Once again, this step relies on the ratio of nominal wage to (input) prices being procyclical, consistent with the empirical evidence in Basu (1995). Moreover, econometric analysis in Barth and Ramey (2001) and Christiano et al. (2005, 2010) finds that real wages increase following monetary expansions, consistent with my model.

This increases the cost share of intermediate inputs and hence strengthens complementarities in price setting. As a result, any subsequent monetary intervention has a stronger effect on GDP, since it occurs in a state with stronger pricing complementarities. Of course, the converse also holds: any monetary shocks have a weaker effect on real GDP when they are preceded by episodes of tight monetary policy. Such theoretical findings are consistent with the econometric evidence of stronger monetary transmission to GDP under an already loose monetary stance (Alpanda et al., 2021) and additional evidence that finds GDP to be more sensitive to monetary interventions whenever credit is loose (Jordà et al., 2020).

Third, the model produces *size dependence*, as the response of GDP does not change linearly with the size of the monetary shock, even if the probability of price adjustment is state-independent. Larger monetary expansions have a disproportionately *larger* positive effect on real GDP compared to smaller monetary expansions, as the former make the network denser. As a result, achieving a given degree of real GDP expansion requires a (weakly) smaller monetary easing. On the other hand, larger monetary contractions have a disproportionately *smaller* negative effect on real GDP compared to smaller monetary contractions, as the former shrink the network. A consequence of the above is that achieving a given degree of real GDP contraction requires a (weakly) larger monetary tightening. Econometric findings in Alvarez et al. (2017) and Ascari and Haber (2022) suggest that large nominal shocks have a disproportionately smaller effect on real variables, compared to small nominal shocks, which is consistent with my predictions for monetary contractions.

Fourth, I provide novel reduced-form econometric evidence on network cyclicality, conditional on identified productivity and monetary shocks. As a first exercise, I use sectoral data from the US Bureau of Economic Analysis (BEA)(U.S. Bureau of Economic Analysis, 2020b) to construct annual time series of intermediates cost shares for 65 sectors of the US economy. Consistent with the qualitative theoretical prediction, I find that the intermediates cost share rises following a positive productivity shock, and following a monetary easing. Moreover, I find evidence that the effect is asymmetric: expansionary shocks, both to productivity and monetary policy, lead to disproportionately larger magnitudes of responses. The asymmetry holds locally in the sense of being present even for small shocks: for example, while a 1% productivity expansion generates a statistically significant rise in the intermediates cost share, a 1% contraction produces no statistically significant decline. One limitation of using sectoral data is that it does not provide direct evidence that the extensive margin of network adjustment is relevant, while it is central to my theoretical model. I therefore provide additional evidence using data on firm-level linkages in Compustat, as constructed by Atalay et al. (2011, nd), which allows me to study the extensive margin of network adjustment. My findings using firm-level data align with the evidence obtained from sectoral data: productivity and monetary expansions lead to statistically significant increases in the number of linkages. Moreover, the sign asymmetry established with sectoral data continues to hold at the level of firm-level linkages: productivity and monetary expansions lead to larger magnitudes of changes in the number of suppliers, which is true even locally for small shocks.

A quantitative dynamic version of my model, calibrated to 389 sectors of the United States economy, reproduces well the magnitudes of the estimated cost-share responses, including the empirically established asymmetries in network dynamics. There is both a qualitative and a quantitative underpinning to this model performance. At the *qualitative* level, asymmetries arise because endogenous network formation makes firms' supplier-set decisions sharply non-linear in sectoral prices and wages. Each potential supplier relationship is associated with its own productivity term; therefore, adding a supplier is attractive only if the direct cost savings relative to labor are sufficiently large to compensate for the change in productivity it generates. Crucially, since the adoption and severing decisions are threshold-based, expansionary and contractionary shocks of the same size need not produce symmetric responses. The mass of inactive links close to the adoption threshold can differ from the mass of active links close to the severing threshold, and the links on the two margins can differ in their input shares, productivity terms, and price stickiness. Therefore, the precise direction and magnitude of the asymmetry is ultimately a *quantitative* question. The calibrated version of my model disciplines these thresholds by estimating link-specific productivities so that the model's steady state replicates the observed sectoral input-output structure of the US economy. In the calibrated economy, the estimated distribution of link-specific productivities implies that more links lie close to the adoption threshold than to the severing threshold.[5] As a result, expansionary productivity or monetary shocks activate many links that do not exist in steady state, whereas equal-sized contractionary shocks sever fewer existing links. Consequently, expansions generate a larger rise in intermediates cost shares than the corresponding decline produced by contractionary shocks, with the magnitude and direction of the asymmetry consistent with my reduced-form econometric evidence.

While my model emphasizes the extensive margin of network adjustment, fluctuations in the intermediates cost share could also occur purely at the intensive margin, for instance when the production technology takes a constant-elasticity-of-substitution (CES) form in labor and a fixed set of intermediate-input suppliers.[6] However, I show that intensive-margin variation *alone*, in the fixed-network CES benchmarks considered, cannot reproduce the estimated dynamics of the intermediates cost share, especially the observed asymmetries. At the qualitative level this happens since the CES model allows firms to adjust their reliance on intermediates only through a *smooth* intensive margin: they reallocate expenditure across existing inputs according to their prices relative to the composite input price index. Because this adjustment occurs through a differentiable substitution margin, the production block itself gives little scope for

[5] Key to this quantitative result is the interaction between the *sparsity* of the empirical US input-output network and the calibrated distribution of link-specific productivities. In the 2007 BEA table that I target, almost two thirds of potential links are inactive, so the calibration features a large pool of inactive supplier relationships. With a smooth unimodal distribution of link-specific productivities, many of these inactive links lie close to their adoption thresholds. An expansionary shock that lowers supplier prices relative to wages shifts these thresholds downward and can activate many such links, whereas an equal-sized contraction operates on the smaller set of active links and severs only those close enough to their severing thresholds.

[6] Conditional on a specific supplier set, my calibrated endogenous network model features a Cobb-Douglas production technology, which is why the quantitative results attribute all cost share variation to the extensive margin. This functional form restriction is crucial for computational feasibility purposes.

local asymmetries. In fact, the sectoral labor cost share in the CES model is *exactly* log-linear in the wage, sectoral productivity, the sector's own price and its markup. Thus, absent sufficiently asymmetric movements in those equilibrium objects, expansionary and contractionary shocks generate close to mirror-image movements in the intermediates cost share. This contrasts with the endogenous-network model, where local non-linearities are central, since the extensive-margin decision is threshold-based and hence sharply non-linear in prices and wages. At the quantitative level, I consider a fixed-network CES model calibrated to the same 389 sectors of the US economy as the extensive-margin model, and show that the CES model alone cannot reproduce the estimated cost-share dynamics, particularly the asymmetries, under plausible values of the substitution elasticity. Put differently, the quantitative CES network model maps relative price movements into cost share changes through a smooth substitution margin, which is both approximately locally symmetric and insufficiently sensitive to relative price movements to generate the observed magnitude of cost-share responses. The endogenous-network model, by contrast, introduces threshold-based link adoption and severing, which can generate both larger and locally asymmetric cost-share movements.[7]

My proposed mechanism behind non-linear monetary transmission works through cyclical variation in the intermediates cost share, which affects the strength of pricing complementarities. Therefore, since the fixed-network CES model produces endogenous fluctuations in the intermediates cost share, it can also, in principle, generate cycle-, path-, and size-dependence in monetary transmission. However, as highlighted above, because the fixed-network CES model alone cannot quantitatively match the observed cost-share dynamics, it also falls short of producing sizable monetary non-linearities.[8] Widely used estimates of the elasticity of substitution between labor and intermediates lie between 0.5 and 1 (Rotemberg and Woodford, 1996; Atalay, 2017; Oberfield and Raval, 2021); in my quantitative CES network model, values in this range imply a countercyclical intermediates cost share, producing monetary non-neutrality that is *weaker* in expansions and following past episodes of loose policy, contrary to available econometric evidence. Moreover, even under elasticities above 2, as found in some more recent studies (Chan, 2021; Mertens and Schoefer, 2024), the fixed-network CES model produces only mild procyclicality in the intermediates cost share, and hence only a correspondingly weak degree of non-linearity in monetary transmission. Importantly, this does not rule out a role for the intensive margin of input adjustment, nor does it suggest that only the extensive margin matters. Reality most likely reflects a combination of both. Instead, my results suggest that the extensive margin plays a non-trivial role in production-network cyclicality, with important implications

[7] This can be conceptualized by the endogenous-network model having an "effective" substitution elasticity between labor and intermediates that depends on the sign, size and source of the shock. In Appendix E.3, I explicitly compute such effective elasticities in the calibrated model.

[8] Even conditional on producing the same change in the intermediates cost share sector-by-sector, the endogenous-network model and the fixed-network CES model can generate quantitatively different non-linearities in monetary transmission. This is due to the "average input stickiness" channel: for a given intermediates cost share, the supplier composition need not be the same across the two models, and monetary transmission can differ depending on whether spending is allocated towards suppliers with stickier or more flexible prices. However, for the degree of non-neutrality, this is secondary to whether the total intermediates share itself rises or falls with the relevant state. In particular, rising intermediates cost share strengthens (local) non-neutrality regardless of the underlying changes in the composition of intermediates. This is because the overall intermediates price index is always less sensitive to monetary changes than the nominal wage due to countercyclical sectoral markups.

for monetary transmission. My work takes a first step towards incorporating this extensive margin into the study of monetary transmission, leaving room for future work to study its interaction with intensive-margin adjustment.

Contribution to the literature This paper makes a contribution to three strands of the literature. First, it adds to the emerging literature on endogenous production networks in macroeconomics. Seminal studies analytically characterize network formation in environments that are either entirely frictionless or feature frictions with flexible prices (Carvalho and Voigtländer, 2015; Oberfield, 2018; Acemoglu and Azar, 2020a; Taschereau-Dumouchel, 2025). Subsequent studies further advance our understanding of the role of endogenous networks by adding uncertainty (Kopytov et al., 2024), or through detailed quantitative analysis that brings models with endogenous networks to the data (Lim, 2018; Huneeus, 2018). My work contributes to this literature by developing the first dynamic general equilibrium model featuring endogenous network formation under nominal rigidities. I also provide novel reduced-form econometric evidence on network cyclicality in the US, both unconditionally and conditional on identified productivity and monetary shocks, which the model can reproduce.

Second, the paper contributes to the literature on shocks, frictions and macroeconomic policies in multi-sector models with input-output linkages. An important strand of this literature studies the dynamics of monetary transmission under *exogenous* production networks, both analytically (Ghassibe, 2021; Afrouzi and Bhattarai, 2023), quantitatively using detailed calibration (Basu, 1995; Nakamura and Steinsson, 2010; Pasten et al., 2020b), as well as in fully estimated models (Smets et al., 2019; Carvalho et al., 2021). As for optimal monetary policy under fixed exogenous networks, it is studied in important recent papers by La'O and Tahbaz-Salehi (2022), Rubbo (2023) and Qiu et al. (2025). A separate branch of this literature studies, both positively and normatively, fiscal policy in multi-sector models with exogenous networks (Liu, 2019; Bouakez et al., 2023). Another influential strand of this literature studies aggregation properties of any microeconomic frictions and wedges under exogenous networks (Jones, 2011; Grassi, 2017; Baqaee and Farhi, 2020; Bigio and La'O, 2020). I contribute to this literature by providing novel sector-level analytical characterization of monetary transmission under *endogenous* network formation in the static version of my model. I also develop a novel numerical algorithm that solves for the entire path of sectoral prices, quantities and network linkages in response to any MIT shock, nominal or real. My numerical algorithm allows me to quantify the dynamics of transmission of both productivity and monetary changes in a version of my model calibrated to 389 sectors of the US economy.

Third, my work contributes to the literature on non-linearities and state dependence in the transmission of shocks. The theoretical literature on state dependence in monetary transmission includes Santoro et al. (2014), who study asymmetric transmission of monetary policy under loss aversion; McKay and Wieland (2021) who rationalize path dependence in a framework with lumpy durable consumption demand; Alpanda et al. (2021) who explain non-linearities in a model with constraints on household borrowing and refinancing; Eichenbaum et al. (2022) who show how the effect of monetary policy depends on the distribution of savings from refinancing mortgages; Berger et al. (2021) who investigate path-dependence under pre-payable mortgages; Bernstein (2021) who shows that presence of occasionally binding borrowing constraints and household heterogeneity makes responses to monetary transmission stronger in expansions and contractionary shocks more powerful than expansionary ones. There is also an important empirical strand which estimates how the strength of monetary transmission depends on the phase of the business cycle, prior monetary conditions and the size of the shock (Tenreyro and Thwaites, 2016; Jordà et al., 2020; Ascari and Haber, 2022). A separate branch of this literature studies, both theoretically and empirically, the degree of state dependence in the transmission of fiscal shocks (Auerbach and Gorodnichenko, 2012; Ramey and Zubairy, 2018; Demyanyk et al., 2019; Ghassibe and Zanetti, 2022; Jo and Zubairy, 2022). My paper contributes by developing a tractable framework that can simultaneously rationalize *multiple* non-linearities in monetary transmission through a single novel theoretical channel, which is supported by both aggregate and disaggregated data.

The remainder of the paper is structured as follows. Section 2 sets out the general theoretical framework. Section 3 develops an analytically tractable version of the model and presents the key theoretical results. Section 4 develops the numerical algorithm for solving a dynamic forward-looking version of the model and quantifies the key effects. Section 5 provides novel econometric evidence consistent with the key mechanisms of the model. Section 6 concludes.

2 Theoretical framework

I build a dynamic general equilibrium model, which unifies two environments that have so far been treated as separate in the literature. On the one hand, the model features a multi-sector input-output production structure, with firms' pricing decisions subject to sector-specific nominal rigidities. On the other hand, input-output linkages are formed *endogenously* by firms that optimize their production costs, as in Acemoglu and Azar (2020a). As a result, the equilibrium production network responds to real *and* nominal disturbances, both realized and anticipated. Such cyclicality in the production network makes the strength of pricing complementarities, and hence the degree of monetary non-neutrality, depend on the underlying state of the world.

2.1 Model overview

Time is discrete, with outcomes in $t = 0$ exogenously given, and outcomes in $t \geqslant 1$ determined by agents' decisions. There are three types of agents in my model. First, there is a continuum of infinitely lived households. Second, there is a continuum of monopolistically competitive firms, owned by the households, where each firm belongs to one, and only one, of the K sectors; let the set of all firms in sector k be $\Phi_k, \forall k = 1, 2, ..., K$. Third, there is a government, consisting of a central bank that sets the level of money supply in the economy, and a fiscal authority that collects taxes from firms and rebates them to households as a lump-sum transfer.

A crucial feature of my economy is the presence of a production network across sectors, where the input-output linkages are formed endogenously through each firm's choice of its set of suppliers, denoted by $S_k(j) \subseteq \{1, 2, ..., K\}, \forall k, \forall j \in \Phi_k$. For every choice of supplier sectors, there is a given level of productivity pinned down by a predetermined time- and sector-specific mapping $\{\mathcal{A}_{k,t}(S_{k,t})\}_{t=1}^{\infty}$, and $\mathcal{A}_t \equiv [\mathcal{A}_{1,t}(.), \mathcal{A}_{2,t}(.), ..., \mathcal{A}_{K,t}(.)]^T, \forall t$. Importantly, the entire path $\{\mathcal{A}_t\}_{t=1}^{\infty}$ is known to the agents at $t = 0$, and it is expected to remain unchanged forever. For any two mappings $\underline{\mathcal{A}}$ and $\overline{\mathcal{A}}$, the convention is that $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}$ if and only if $\overline{\mathcal{A}}_k(S_k) \geqslant \underline{\mathcal{A}}_k(S_k), \forall S_k, \forall k$.

As for the nominal side of the economy, at $t = 0$ the agents know the initial level of money supply $\mathcal{M}_0$, and expect it to remain unchanged forever. At the beginning of the first period $t = 1$, they discover the future path of the money supply $\{\mathcal{M}_t\}_{t=1}^{\infty}$, and there is no uncertainty from there onwards.

2.2 Firms: production and endogenous choice of suppliers

On the production side, there are K sectors, indexed by $k = 1, 2, ..., K$ with a measure one of firms in each sector; let Φ_k denote the set of all firms in sector k. The production function of firm $j \in \Phi_k$ is given by:

$$Y_{k,t}(j) = \mathcal{F}_k \left[S_{k,t}(j), \mathcal{A}_{k,t}(S_{k,t}(j)), N_{k,t}(j), \{Z_{kr,t}(j)\}_{r \in S_{k,t}(j)} \right] \tag{1}$$

where $S_{k,t}(j) \subseteq \{1, 2, ..., K\}$ is the set of sectors whose firms supply inputs to firm j in sector k at time t, $\mathcal{A}_{k,t}(.)$ is a mapping from the chosen set of suppliers to the associated level of productivity at time t, $N_{k,t}(j)$ is the labor input of firm j in sector k at time t, whereas $Z_{kr,t}(j)$ denotes purchases of intermediate inputs from sector r, which in turn is an aggregator of purchases from all firms in that sector: $Z_{kr,t}(j) \equiv \left(\int_{j' \in \Phi_r} Z_{kr,t}(j, j')^{\frac{\theta-1}{\theta}} dj' \right)^{\frac{\theta}{\theta-1}}$, $\theta > 1$. I impose the following regularity conditions on the production function:

Assumption 1 (Production function)**.** *For every sector $k = 1, 2, ..., K$, the production function satisfies the following conditions: (a) $\mathcal{F}_k$ is strictly quasi-concave, has constant returns to scale in $\left(N_{k,t}(j), \{Z_{kr,t}(j)\}_{r \in S_{k,t}(j)}\right)$, is increasing and continuous in $\mathcal{A}_{k,t}(S_{k,t}(j))$, $N_{k,t}(j)$ and $\{Z_{kr,t}(j)\}_{r \in S_{k,t}(j)}$, and is strictly increasing in $\mathcal{A}_{k,t}(S_{k,t}(j))$ when $N_{k,t}(j) > 0$ and $\{Z_{kr,t}(j)\}_{r \in S_{k,t}(j)} > 0$; (b) labor is an essential factor of production: $\mathcal{F}_k(\cdot, \cdot, 0, \cdot) = 0$; (c) $\mathcal{A}_{k,t}(\varnothing) > 0$.*

Conditional on a particular set of suppliers $S_{k,t}(j)$, each firm's total cost of production at time t is given by $\left[W_t N_{k,t}(j) + \sum_{r \in S_{k,t}(j)} P_{r,t} Z_{kr,t}(j) \right]$, where W_t is the nominal wage and $P_{r,t}$ is the price index of sector r. Taking as given $S_{k,t}(j), W_t$ and $\{P_{r,t}\}_{r \in S_{k,t}(j)}$, each firm chooses labor and intermediate input quantities to minimize the total cost, subject to the production function in (1). The latter delivers the following unit cost function:

$$\mathcal{Q}_{k,t}(j) = \mathcal{Q}_k \left[S_{k,t}(j), \mathcal{A}_{k,t}(S_{k,t}(j)), W_t, \{P_{r,t}\}_{r \in S_{k,t}(j)} \right]. \tag{2}$$

Three properties of the unit cost function should be noted. First, for a given choice of suppliers, the unit cost function is common to all firms within a given sector. Second, given the properties of the production function, $\mathcal{Q}_k$ is decreasing and continuous in $\mathcal{A}_{k,t}(S_{k,t}(j))$ and is increasing and homogenous of degree one in $(W_t, \{P_{r,t}\}_{r \in S_{k,t}(j)})$. Third, as the set of supplier sectors $S_{k,t}(j)$ expands, the unit cost function becomes a function of a larger set of sectoral prices. The latter property is crucial for delivering the state-dependent degree of complementarities in price setting.

Finally, following Acemoglu and Azar (2020a), the set of suppliers is chosen optimally to minimize the unit cost of production in every period:

$$S_{k,t}(j) \in \arg\min_{S_{k,t}(j)} \mathcal{Q}_k \left[S_{k,t}(j), \mathcal{A}_{k,t}(S_{k,t}(j)), W_t, \{P_{r,t}\}_{r \in S_{k,t}(j)} \right]. \tag{3}$$

The above minimization problem highlights the trade-off faced by firms when choosing the optimal set of suppliers: firms would like to purchase inputs from sectors whose combination delivers a high level of productivity, while at the same time avoiding those that charge high prices for their output.[9] Notice that the optimal choice of suppliers is based on aggregate and sectoral variables only. Since all firms within a given sector are ex ante identical, they will be making the same choice of suppliers in equilibrium. To simplify notation, from now on I use $S_{k,t}$ to denote the common choice of suppliers for all firms in sector k at time t. As a result, the unit cost is also common for all firms within a given sector, which from now on I denote by $\mathcal{Q}_{k,t}$.

2.3 Firms: pricing under nominal rigidities

Price stickiness is modeled as a modified finite-horizon version of Calvo (1983). In particular, there exists a finite, deterministic time period $T > 1$, such that in periods $1 \leqslant t \leqslant (T-1)$ firms face a constant and sector-specific probability of price non-adjustment $\alpha_k \in (0,1)$, whereas in periods $t \geqslant T$ firms face no nominal rigidities.[10] The cut-off period $T > 1$ is known by all agents in the economy from $t = 0$ and they expect it to stay fixed forever; naturally, as $T \to \infty$ the price setting problem collapses back to the standard Calvo (1983) pricing.[11]

More precisely, in any period $1 \leqslant t \leqslant (T-1)$ a firm in sector k has probability $(1-\alpha_k)$ of setting its price equal to its optimal value. The optimal reset price at time $1 \leqslant t \leqslant (T-1)$ is chosen to maximize expected future discounted nominal profits:

$$\max_{\hat{P}_{k,t}(j)} \sum_{s=0}^{T-t-1} \alpha_k^s F_{t,t+s} \left[\hat{P}_{k,t}(j) \tilde{Y}_{k,t+s}(j) - (1+\tau_k) \mathcal{Q}_{k,t+s} \tilde{Y}_{k,t+s}(j) \right], \tag{4}$$

where $\tilde{Y}_{k,t+s}(j) = \left[\frac{\hat{P}_{k,t}(j)}{P_{k,t+s}} \right]^{-\theta} Y_{k,t+s}$, $F_{t,t+s}$ is the stochastic discount factor between periods t

[9] In other words, each firm faces a choice across constant-returns-to-scale production technologies and picks the cost-minimizing one.

[10] In Appendix B.2 I consider an extension of my model which also allows for nominal wage rigidity.

[11] I use this modified version of Calvo (1983) pricing for two reasons. First, the special case where $T = 2$ is analytically tractable at the sector-level, as I show in Section 3. Second, given that money is neutral for all $t \geqslant T$, this formulation of pricing allows me to develop a novel numerical algorithm for obtaining a sector-level solution for the forward-looking problem with endogenous network formation, implemented in Section 4.

and $t+s$ and is defined in the next subsection; τ_k is a tax imposed by the government, the revenue from which is rebated to households as a lump-sum transfer. The first-order condition for the optimal reset price for any firm in sector k, $\hat{P}_{k,t}$ is given by:

$$\hat{P}_{k,t}(j) = \hat{P}_{k,t} = (1+\mu_k)\frac{\sum_{s=0}^{T-t-1}\alpha_k^s F_{t,t+s}P_{k,t+s}^{\theta}Y_{k,t+s}\mathcal{Q}_{k,t+s}}{\sum_{s=0}^{T-t-1}\alpha_k^s F_{t,t+s}P_{k,t+s}^{\theta}Y_{k,t+s}}, \quad 1 \leqslant t \leqslant T-1, \quad \forall k \tag{5}$$

where $(1+\mu_k) \equiv (1+\tau_k)\frac{\theta}{\theta-1}$ is the steady-state desired markup. On the other hand, in any period $t \geqslant T$ there are no nominal rigidities, firms maximize contemporaneous profits, and optimally set $P_{k,t} = (1+\mu_k)\mathcal{Q}_{k,t}, t \geqslant T$. Given that the optimal price is identical for all firms within a sector, the sectoral price index can be obtained by aggregation using the ideal sectoral price index $P_{k,t} \equiv \left(\int_{j\in\Phi_k} P_{k,t}(j)^{1-\theta}dj\right)^{\frac{1}{1-\theta}}, \forall k$:

$$P_{k,t} = \begin{cases} \left[\alpha_k P_{k,t-1}^{1-\theta} + (1-\alpha_k)(\hat{P}_{k,t})^{1-\theta}\right]^{\frac{1}{1-\theta}}, & 1 \leqslant t \leqslant (T-1); \\ \\ (1+\mu_k)\mathcal{Q}_{k,t}, & t \geqslant T. \end{cases} \tag{6}$$

Note that while I employ a form of Calvo (1983) pricing for quantitative tractability, the main theoretical channel behind endogenous network formation is not specific to the particular form of nominal rigidity. In particular, as is evident from (3), the key ingredient driving the supplier set decision is the behavior of sectoral relative prices $\{P_{k,t}/W_t\}_{k=1}^K$. As long as those relative prices fall following both productivity expansions and monetary easings, firms would find it optimal to expand their supplier set, subject to regularity conditions (stated later) that are not specific to the form of nominal rigidity. In general, countercyclical relative prices $\{P_{k,t}/W_t\}_{k=1}^K$ are consistent not only with a broad class of time-dependent pricing models (Taylor, 1979; Calvo, 1983), but also with models featuring quadratic costs of price adjustment (Rotemberg, 1982), as well as with pricing subject to either fixed or stochastic menu costs (Golosov and Lucas, 2007; Nakamura and Steinsson, 2010).[12]

[12]While conceptually the same, the quantitative aspects of endogenous network dynamics can be different under alternative pricing models. In particular, under the more quantitatively realistic state-dependent pricing with menu costs, large shocks can in principle drastically increase the fraction of price adjusters, causing relative prices $\{P_{k,t}/W_t\}_{k=1}^K$ to be much more countercyclical under productivity shocks and essentially acyclical under monetary shocks. Naturally, that would make the supplier choice decisions much more sensitive to productivity changes and almost insensitive to monetary interventions. While a full model with endogenous networks and state-dependent pricing is likely computationally intractable, recent work by Ghassibe and Nakov (2025) develops a model with fixed input-output linkages and menu costs that offers some guidance on how large the frequency movements are likely to be. Under the most quantitatively realistic stochastic menu costs, the paper finds that the frequency moves very little for aggregate TFP shocks up to 3% in magnitude and money supply changes up to 15% in absolute value.

2.4 Households

A continuum of infinitely lived households populates the economy and owns all the firms. The representative household makes choices to maximize the lifetime utility

$$\max_{\{C_{t+s},N_{t+s},B_{t+s+1}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} \beta^s \left[\log C_{t+s} - N_{t+s}\right] \tag{7}$$

subject to the budget constraint $P_t^c C_t + [F_{t,t+1}B_{t+1}] \leqslant B_t + W_t N_t + \sum_{k=1}^{K} \int_{j\in\Phi_k} \Pi_{k,t}(j)dj + T_t$, where C_t is aggregate consumption, P_t^c is the consumption price index (defined below), N_t is labor supply, B_{t+1} is the payoff of assets purchased at time t, $F_{t,t+1}$ is the stochastic discount factor between periods t and $t+1$, $\Pi_{k,t}(j)$ denotes nominal profits of firm j in sector k, β is the discount factor for future utility and T_t is the lump-sum transfer from the government.

The composite consumption index C_t is an aggregator for the final consumption of goods produced in the different sectors of my economy: $C_t \equiv u(C_{1,t}, C_{2,t}, ..., C_{K,t})$, where $C_{k,t}$ is in turn an aggregator for the final consumption of goods produced by firms in that sector: $C_{k,t} \equiv \left(\int_{j\in\Phi_k} C_{k,t}(j)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}, \forall k$. I impose the following regularity conditions on the aggregator u:

Assumption 2 (Consumption aggregation)**.** *The consumption aggregator u is continuous, differentiable, increasing, strictly quasi-concave and homogeneous of degree one in $(C_{1,t}, C_{2,t}, ..., C_{K,t})$, and all sectoral consumption goods are normal.*

Households choose their sectoral consumption levels by minimizing the total cost of purchases $\sum_{k=1}^{K} P_{k,t}C_{k,t}$ subject to the consumption aggregator u. The latter also delivers the consumption price index $P_t^c = P_t^c(P_{1,t}, P_{2,t}, ..., P_{K,t})$, as the minimal cost of assembling such a basket.

2.5 Monetary policy

Purchases of final goods are subject to a cash-in-advance constraint, so that $P_t^c C_t = \mathcal{M}_t, \forall t \geqslant 1$. Agents know the initial level of money supply $\mathcal{M}_0$, and at $t = 0$ anticipate it to stay at that level forever. At time $t = 1$ they discover the future path of money supply $\{\mathcal{M}_t\}_{t=1}^{\infty}$ and therefore any $\mathcal{M}_t \neq \mathcal{M}_0$ constitutes a monetary shock at time t to the agents.

2.6 Market clearing and equilibrium

In addition to the optimality conditions, budget constraints and the policy rule above, equilibrium in my economy is characterized by market-clearing conditions in the asset market: $B_t = 0$; the labor market: $N_t = \sum_{k=1}^{K} \int_{j\in\Phi_k} N_{k,t}(j)dj$; and the goods markets: $Y_{k,t}(j) = C_{k,t}(j) + \sum_{r=1}^{K} \int_{j'\in\Phi_r} Z_{rk,t}(j',j)dj', \quad \forall k, \forall j \in \Phi_k$. The equilibrium in my economy can be summarized as follows:

Definition 1 (Equilibrium)**.** *The equilibrium is a collection of prices $\{P_{k,t}(j)|j \in \Phi_k\}_{k=1}^{K}$, wage W_t, allocations $\left\{Y_{k,t}(j), N_{k,t}(j), C_{k,t}(j), \{Z_{kr,t}(j,j')|j' \in \Phi_r\}_{r=1}^{K} | j \in \Phi_k\right\}_{k=1}^{K}$ and supplier choices*

$\{S_{k,t}(j)|j \in \Phi_k\}_{k=1}^{K}$, which, given the exogenous path of productivity mappings $\{\mathcal{A}_t\}_{t=1}^{\infty}$, the exogenous path of money supply $\{\mathcal{M}_t\}_{t=0}^{\infty}$ and the exogenous initial prices $\{P_{k,0}\}_{k=1}^{K}$, satisfy agent optimization and market clearing in every time period $t \geqslant 1$.

3 An analytically tractable version

In this section, I consider an analytically tractable version of the model obtained when nominal rigidities are only present in the first period. This simplification allows me to formally characterize the propagation of monetary shocks to real variables, under different states of productivity and initial levels of money supply. Formal propositions establish that small monetary shocks, which do not affect the shape of the network, have an impact on GDP that is larger whenever the productivity mapping improves or initial money supply rises. Further, large monetary expansions have a more-than-proportional positive effect on GDP relative to small monetary expansions; on the other hand, large monetary contractions have a less-than-proportional negative effect on GDP relative to small monetary contractions.

3.1 Equilibrium in the simplified version

In this section, I focus on the version of my model with $T = 2$, so that nominal rigidities are only present at $t = 1$. I show that this simplification allows me both to formally establish equilibrium existence and uniqueness properties, as well as to analytically characterize the transmission of monetary shocks under different baseline productivity mappings and levels of money supply. The assumption is formally documented below:

Assumption 3 (Horizon of stickiness)**.** *The firms cannot adjust prices fully flexibly until the horizon $T = 2$, so that nominal rigidities are only present at $t = 1$, and prices are fully flexible for all $t \geqslant 2$.*

One implication of the above assumption is that, conditional on a particular choice of suppliers, the optimal reset price at $t = 1$ is given by $\hat{P}_{k,1} = (1 + \mu_k)\mathcal{Q}_{k,1}, \forall k$. The latter delivers a tractable expression for the equilibrium sectoral price index in the first period, namely

$$P_{k,1} = \left[\alpha_k P_{k,0}^{1-\theta} + (1-\alpha_k)\left\{(1+\mu_k)\min_{S_{k,1}} \mathcal{Q}_{k,1}\left[S_{k,1}, \mathcal{A}_{k,1}(S_{k,1}), W_1, \{P_{r,1}\}_{r \in S_{k,1}}\right]\right\}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{8}$$

for $k = 1, 2, ..., K$.

At the same time, prices are flexible after the first period, so that the equilibrium sectoral price in $t \geqslant 2$ is pinned down by $P_{k,t} = (1 + \mu_k)\min_{S_{k,t}} \mathcal{Q}_{k,t}\left[S_{k,t}, \mathcal{A}_{k,t}(S_{k,t}), W_t, \{P_{r,t}\}_{r \in S_{k,t}}\right]$. Moreover, notice that the intratemporal consumption-labor supply condition and the cash-in-advance constraint jointly imply that $W_t = \mathcal{M}_t$, so that the nominal wage equals money supply in every period. This exogeneity of the nominal wage, combined with the fact that the only endogenous component in the pricing equations above is the unit cost function, jointly imply that prices in this simplified setting are pinned down exclusively by the exogenous supply-side factors – the productivity mapping $\mathcal{A}_t$ and desired markups $\{1+\mu_k\}_{k=1}^K$ – in addition to the exogenous level of money supply $\mathcal{M}_t$. It is the latter property that allows me to represent equilibrium sectoral prices as a lattice, which in turn delivers equilibrium existence and uniqueness properties summarized below.

Proposition 1 (Equilibrium)**.** *Suppose Assumptions 1-3 hold. Then, the equilibrium introduced in Definition 1 entails the following properties: (a) it exists; (b) the equilibrium sectoral prices and final sectoral consumptions are unique; (c) the equilibrium supplier choices and remaining sectoral allocations are generically unique.*

Since Assumption 3 implies that money is neutral for $t \geqslant 2$ and given that my interest is in the transmission of monetary shocks to real variables, in the rest of this section I am going to focus exclusively on outcomes at $t=1$. For notational simplicity, I drop time subscripts for variables at $t=1$ for the remainder of this section.

The theoretical results in the remainder of this section are going to be presented in two steps. Before establishing each key finding, I explain it in a stylized setting featuring only two sectors. The simplified two-sector setting is summarized below.

Two-sector setting *Consider the economy with two sectors ($K=2$) and nominal rigidities only present in the first period ($T=2$). For simplicity, firms are not allowed to buy inputs from other firms in their own sector ($k \notin S_k, k=1,2$), and the production function is Cobb-Douglas with the functional form: $Y_k(j) = e(k)\mathcal{A}_k(S_k)N_k(j)^{(1-\omega_{k,-k})}Z_{k,-k}(j)^{\omega_{k,-k}}$, and $e(k) = (1-\omega_{k,-k})^{-(1-\omega_{k,-k})}\omega_{k,-k}^{-\omega_{k,-k}}$. The input-output shares are calibrated such that $\omega_{k,-k}=0.5$ if $-k \in S_k$ and $\omega_{k,-k}=0$ otherwise, for $k=1,2$. Price stickiness is calibrated such that Sector 1 is price flexible ($\alpha_1=0$) and Sector 2 features nominal rigidities ($\alpha_2=0.5$). The productivity mapping is given by $a_k(\varnothing) \equiv \log A_k(\varnothing) = 1, k=1,2$ and $a_k(\{-k\}) \equiv \log A_k(\{-k\}) = \overline{a}, k=1,2$. The exogenous initial prices are set at $P_{k,0}=1, k=1,2$. Finally, assume that $\tau_k = -1/\theta$, so that all market power distortions are optimally removed, and $\theta \to 1^+$, which allows me to obtain closed-form expressions for equilibrium sectoral prices and quantities. In this setting, there are only four possibilities for supplier choices (S_1,S_2), namely $(\varnothing,\varnothing),(\varnothing,\{1\}),(\{2\},\varnothing)$ and $(\{2\},\{1\})$. Letting $m \equiv \log\mathcal{M}$, one can write (log) unit cost for sector k at $t=1$ as $q_k = -a_k(S_k) + (1-\mathbf{1}_{-k\in S_k}\times\omega_{k,-k})\,m + (\mathbf{1}_{-k\in S_k}\times\omega_{k,-k})p_{-k}$. As for the prices, in Sector 1 prices are flexible, so that $p_1=q_1$ in logs. In Sector 2 prices are sticky with probability of adjustment equal to one half ($1-\alpha_2=0.5$), so that under the limit $\theta\to 1^+$ one obtains $P_2 = \mathcal{Q}_2^{0.5}$, or $p_2 = 0.5q_2$ in logs. Combining the expressions for (log) unit costs and prices at $t=1$, one obtains the pairs (q_1,q_2) conditional on the four possible supplier set arrangements:*

	$S_2=\varnothing$	$S_2=\{1\}$
$S_1=\varnothing$	$(m-1, m-1)$	$(m-1, -\overline{a}+m-\frac{1}{2})$
$S_1=\{2\}$	$(-\overline{a}+\frac{3}{4}m-\frac{1}{4}, m-1)$	$(-\frac{10}{7}\overline{a}+\frac{5}{7}m, -\frac{12}{7}\overline{a}+\frac{6}{7}m)$

Another consequence of money neutrality for all $t \geqslant 2$ is that the only change in money supply relevant for real variables at $t = 1$ is that between money supply at $t = 1$ and its baseline level $\mathcal{M}_0$. For ease of presentation of my results in this section, I introduce the following definition of a *monetary shock*:

Definition 2 (Monetary shock)**.** *Let $\varepsilon^m \equiv \log(\mathcal{M}/\mathcal{M}_0)$, where $\mathcal{M}$ is money supply at $t = 1$ and $\mathcal{M}_0$ is the baseline level of money supply, be the monetary shock in my economy.*

In the next subsection, I study properties of my economy's baseline at $t = 1$, which occurs under zero monetary shock ($\varepsilon^m = 0$). The subsequent section studies propagation of a monetary shock $\varepsilon^m \neq 0$ conditional on different baselines.

3.2 Baseline ($\varepsilon^m = 0$)

The baseline is given by the equilibrium evaluated under $\varepsilon^m = 0$. Baseline prices, allocations and supplier choices are pinned down by the productivity mapping $\mathcal{A}$, baseline money supply $\mathcal{M}_0$ and exogenous initial prices $\{P_{k,0}\}_{k=1}^K$. In this subsection, I am going to investigate how, holding the initial prices fixed, changes in the baseline pair $(\mathcal{A}, \mathcal{M}_0)$ affect the equilibrium.[13]

First, I am going to use the two-sector setting introduced earlier to build intuition on how changes in the productivity mapping affect the baseline, *ceteris paribus*.

Example 1 (Productivity mapping and baseline)**.** *In Panel (a) of Figure 2 I use the two-sector setting where, holding baseline money supply fixed at $m_0 = 0$, I consider three different productivity mappings by varying the parameter $\overline{a}$, which represents the productivity associated with using the other sector as a supplier. When $\overline{a} = 0$, neither sector finds it optimal to buy inputs from the other sector, and the equilibrium network is empty. As I increase $\overline{a}$ to 0.65, the sticky-price Sector 2 finds it optimal to purchase inputs from the flexible-price Sector 1, but not vice versa. This is because Sector 2 finds it optimal to lower its unit cost by purchasing inputs from Sector 1, whose price flexibility is associated with lower prices. Finally, as I further increase $\overline{a}$ to 0.8, both sectors find it optimal to connect to each other. This is because the productivity associated with the connection is now sufficiently high to spur the flexible-price Sector 1 to purchase from Sector 2, whose price stickiness prevents it from fully lowering the price. In addition, notice that as I increase the productivity parameter, equilibrium unit costs and hence sectoral prices drop, which lowers the consumption price index and through the cash-in-advance constraint implies that baseline GDP rises.*

I now use the two-sector setting to build intuition on the link between initial money supply and baseline equilibrium:

Example 2 (Money supply and baseline)**.** *In Panel (b) of Figure 2 I use the two-sector setting where, holding productivity mapping fixed at $\overline{a} = 0$, I consider three different initial money supplies by varying the parameter m_0. When $m_0 = 0$, neither sector finds it optimal to buy inputs from the other sector, and the equilibrium network is empty. As I increase m_0 to 4, the flexible-price Sector 1 finds it optimal to purchase inputs from the sticky-price Sector 2, but not*

[13] In Appendix B.1 I consider baselines that differ in the levels of desired markups, parameterized by exogenous changes in the sectoral cost taxes $\{\tau_k\}_{k=1}^K$.

vice versa. This is because the nominal wage rises one-for-one with money supply whereas Sector 2 price rises less than one-for-one, and Sector 1 finds it cheaper to substitute inputs bought from Sector 2 for in-house labor, even though such a connection lowers 1's productivity. Finally, as I further increase m_0 to 8, both sectors find it optimal to connect to each other. This is because the increase in money supply/nominal wage is now so large that even the sticky-price Sector 2 wishes to substitute inputs from Sector 1 for in-house labor, even though such a connection lowers 2's productivity. This happens because Sector 1's price no longer tracks the money supply one-for-one, since it inherits price stickiness from Sector 2. In addition, notice that as I increase money supply, the gap between money supply and equilibrium unit costs and prices, both sectoral and the aggregate consumption index, is growing, which through the cash-in-advance constraint implies that baseline GDP rises.

Therefore, in my simple examples, baseline GDP and the number of suppliers of each sector (weakly) rise, *ceteris paribus*, both as one improves the productivity mapping and as one increases initial money supply. However, the mechanisms through which linkages are added are, in fact, diagonally opposite under the two scenarios. As one improves the productivity mapping, extra sectors that get adopted as suppliers are those that *adjust* their prices downwards by enough to lower the buyer's unit cost for the level of productivity they bring. Conversely, as one increases money supply, sectors that receive additional intermediate customers are those that *do not adjust* their prices upwards enough, so that they are cheaper relative to labor for the level of productivity they bring. Having built the intuition in the two-sector setting, I now formalize the relationship between the baseline pair $(\mathcal{A}, \mathcal{M}_0)$, GDP and supplier choices in equilibrium. As for the effect on GDP, no further assumptions, over and above the ones already made, are required:

Lemma 1 (Baseline GDP)**.** *Suppose Assumptions 1-3 hold. Consider any two baseline pairs $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ such that either $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ or $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$. Then, $C(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \geqslant C(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$.*

The overall mechanism behind the above result is as follows. Holding initial money supply fixed, an improvement in the productivity mapping lowers each unit cost function, which in turn lowers every sectoral price. Firms re-optimize their set of suppliers, which delivers lower new unit costs, since the original set of suppliers remains available. The latter further lowers sectoral prices. The mechanism repeats until a new equilibrium is reached, which features lower sectoral prices and hence a lower consumption price index, which through the cash-in-advance constraint implies a larger GDP. Similarly, holding productivity mapping fixed, larger initial money supply decreases the ratios of sectoral prices to money supply due to sticky prices. Firms re-optimize their set of suppliers, which delivers lower new unit costs, since the original set of suppliers remains available. The latter further lowers the ratios of sectoral prices to money supply. The mechanism repeats until a new equilibrium is reached, which features lower ratios of sectoral prices to money supply and hence a lower ratio of consumption price index to money supply, which through the cash-in-advance constraint implies a larger GDP.

The link between the baseline pair $(\mathcal{A}, \mathcal{M}_0)$ and supplier choices in equilibrium can be

Figure 2: **Equilibrium supplier choices under different baseline conditions**

(a) Equilibrium choices of suppliers under different baseline productivity mappings ($m_0 = 0$)

Recession: $\overline{a} = 0$

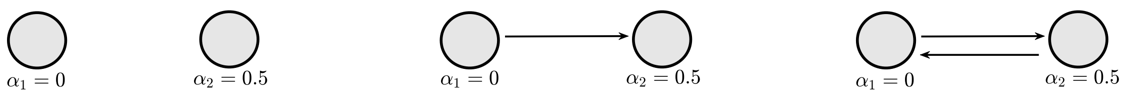
	$\varnothing$	$\{1\}$
$\varnothing$	**(-1,-1)**	$(-1, -\frac{1}{2})$
$\{2\}$	$(-\frac{1}{4}, -1)$	$(0, 0)$

Normal: $\overline{a} = 0.65$

	$\varnothing$	**$\{1\}$**
$\varnothing$	$(-1, -1)$	$\mathbf{(-1, -\frac{23}{20})}$
$\{2\}$	$(-\frac{9}{10}, -1)$	$(-\frac{13}{14}, -\frac{39}{35})$

Expansion: $\overline{a} = 0.80$

	$\varnothing$	**$\{1\}$**
$\varnothing$	$(-1, -1)$	$(-1, -\frac{13}{10})$
$\{2\}$	$(-\frac{21}{20}, -1)$	$\mathbf{(-\frac{8}{7}, -\frac{48}{35})}$



(b) Equilibrium choices of suppliers under different baseline levels of money supply ($\overline{a} = 0$)

Tight money: $m_0 = 0$

	$\varnothing$	$\{1\}$
$\varnothing$	**(-1,-1)**	$(-1, -\frac{1}{2})$
$\{2\}$	$(-\frac{1}{4}, -1)$	$(0, 0)$

Normal money: $m_0 = 4$

	$\varnothing$	$\{1\}$
$\varnothing$	$(3, 3)$	$(3, \frac{7}{2})$
$\{2\}$	$\mathbf{(\frac{11}{4}, 3)}$	$(\frac{20}{7}, \frac{24}{7})$

Loose money: $m_0 = 8$

	$\varnothing$	**$\{1\}$**
$\varnothing$	$(7, 7)$	$(7, \frac{15}{2})$
$\{2\}$	$(\frac{23}{4}, 7)$	$\mathbf{(\frac{40}{7}, \frac{48}{7})}$

Notes: the figure uses the analytically tractable version of my model under $T = 2$, calibrated for $K = 2, \omega_{kk} = 0, \forall k$, $Y_k(j) = e(k)\mathcal{A}_k(S_k)N_k(j)^{(1-\omega_{k,-k})}Z_{k,-k}(j)^{\omega_{k,-k}}$ and $e(k) = (1-\omega_{k,-k})^{-(1-\omega_{k,-k})}\omega_{k,-k}^{-\omega_{k,-k}}$, $a_{k,0} \equiv \log \mathcal{A}_{k,0}, \forall k$ and $m_0 = \log \mathcal{M}_0$, the production technology be given by $a_{1,0}(\varnothing) = 1, \quad a_{1,0}(\{2\}) = \overline{a}, a_{2,0}(\varnothing) = 1, \quad a_{2,0}(\{1\}) = \overline{a}$, $P_{1,0} = P_{2,0} = 1$, the sectoral shares are given by $\omega_{12} = \omega_{c1} = 0.5 = \omega_{21} = \omega_{c2} = 0.5$, and Calvo parameters by $\alpha_1 = 0, \alpha_2 = 0.5$. Finally, assume that $\tau_k = -1/\theta$, and $\theta \to 1^+$.

formalized using the monotone comparative statics theorems of Milgrom and Shannon (1994). Those, however, require imposing further regularity conditions. The first one puts a restriction on the relationship between the unit cost function, the choice of suppliers and the productivity mapping, holding sectoral prices and wage fixed:

Assumption 4. *For all $W, \{P_k\}_{k=1}^K$, the unit cost function $\mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), W, \{P_r\}_{r\in S_k}\right]$ is quasi-submodular in $(S_k, \mathcal{A}_k(S_k))$, for all $k = 1, 2, ..., K$.*

The assumption of quasi-submodularity ensures that, holding wage and prices fixed, an improvement in the productivity mapping does not encourage any sector to shrink its number of suppliers.[14] However, prices also vary across baselines, which brings in the need for a sector regularity condition. The additional restriction is a single-crossing property on the unit cost function, relating joint variations in prices, wages and the set of suppliers:

Assumption 5. *Let $\mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), \{\tilde{P}_r\}_{r\in S_k}\right] \equiv \frac{1}{W}\mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), 1, \{\frac{P_r}{W}\}_{r\in S_k}\right], \forall k$. For all $S_k \subseteq S'_k$ and for all $\{\tilde{P}_r, \tilde{P}'_r\}_{r=1}^K$ such that $\tilde{P}'_r \leqslant \tilde{P}_r, \forall r \neq k$:*

$$\begin{aligned}&\mathcal{Q}_k\left[S'_k, \mathcal{A}_k(S'_k), \{\tilde{P}_r\}_{r\in S'_k}\right] - \mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), \{\tilde{P}_r\}_{r\in S_k}\right] \leqslant 0 \\ \implies \quad &\mathcal{Q}_k\left[S'_k, \mathcal{A}_k(S'_k), \{\tilde{P}'_r\}_{r\in S'_k}\right] - \mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), \{\tilde{P}'_r\}_{r\in S_k}\right] \leqslant 0, \quad \forall k. \end{aligned} \tag{9}$$

The above single-crossing property ensures that a reduction in all sectoral price to wage ratios does not discourage the adoption of a larger number of suppliers by every sector. Such an assumption immediately holds under a range of widely used production functions, most notably Cobb-Douglas with Hicks-neutral technology.[15]

The additional assumptions above allow me to formalize the link between the baseline pair $(\mathcal{A}, \mathcal{M}_0)$ and supplier choices in equilibrium:

Lemma 2 (Baseline supplier choices)**.** *Suppose Assumptions 1-5 hold. Consider any two baseline pairs $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ such that either $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ or $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$. Then, $S_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \supseteq S_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, for all $k = 1, 2, ..., K$.*

The intuition behind the above result is as follows. An improvement in the productivity mapping, *ceteris paribus*, incentivizes firms to connect to more suppliers, as it lowers their unit costs directly through higher productivity and indirectly through lower prices charged by suppliers. Similarly, an increase in initial money supply, *ceteris paribus*, leads to a reduction in sectoral price to wage ratios, implying that it is cost-reducing to substitute intermediates bought

[14] Relaxing this assumption would not necessarily invalidate the possibility that the supplier set expands after a productivity improvement. Instead, it would only break the monotonicity property: the supplier set can still expand locally around certain points, for example where the initial level of technology is low. All in all, the comparative statics for the supplier set in Lemma 2 would become local rather than global in nature. Therefore, the network dynamics following productivity shocks would ultimately be a quantitative question. Empirically, the literature finds that the supplier set expands/supply chain complexity increases with productivity, in line with the quasi-submodularity assumption. At the firm-level, Bernard et al. (2019) find that large and productive Japanese firms have more suppliers than small firms: 10 percent increase in sales is associated with a 3.6 percent increase in the number of suppliers. At a more aggregate level, Bartelme and Gorodnichenko (2015) and more recently Gloria et al. (2024) construct input-output data for a range of economies, finding that wealthier/more productive countries have denser domestic input-output structures.

[15] This is formally shown in Acemoglu and Azar (2020a). See their work for a fuller characterization of families of production functions under which the single-crossing property in Assumption 5 holds.

from other sectors for in-house labor, thus leading to an increase in the number of suppliers. Further, note a key monotonicity property implied by Lemma 2: following either a productivity improvement or a money expansion, firms keep all the existing suppliers and add new ones on top. In other words, the new suppliers are never replacing existing suppliers; instead, they always replace a part of production that was previously done with labor. Similarly, whenever firms drop suppliers in equilibrium, they always replace them with labor, and not by connecting to some other suppliers.

3.3 Propagation of a monetary shock

Having established properties of the baseline, I now consider deviations from the baseline, driven by a non-zero monetary shock ε^m. The mechanics of a monetary shock in terms of its effect on the equilibrium allocations and supplier choices are isomorphic to those for changes in baseline money supply established in the previous subsection. I can therefore immediately formalize comparative statics following a monetary shock.

Lemma 3 (Comparative statics following a monetary shock)**.** *Consider a baseline pair $(\mathcal{A}, \mathcal{M}_0)$, which is perturbed by a monetary shock $\varepsilon^m > 0$. Suppose Assumptions 1-3 hold, then following the monetary shock equilibrium GDP rises relative to its baseline level: $C(\mathcal{A}, \mathcal{M}) > C(\mathcal{A}, \mathcal{M}_0)$. Further, suppose that in addition Assumptions 4 and 5 also hold, then following the monetary shock the set of suppliers for each sector weakly expands: $S_k(\mathcal{A}, \mathcal{M}) \supseteq S_k(\mathcal{A}, \mathcal{M}_0)$, for all $k = 1, 2, ..., K$.*

Note that although the above lemma is stated for an expansionary monetary shock $\varepsilon^m > 0$, it naturally extends to a contractionary shock $\varepsilon^m < 0$, which leads to a reduction in GDP and a weak fall in the number of suppliers for every sector.

From Lemma 3 it follows that a non-zero monetary shock can either change the set of suppliers relative to the baseline, or leave them unchanged. In light of this it is useful to formally distinguish between two such types of monetary shocks, as is done below.

Definition 3 (Small monetary shock)**.** *Define a monetary shock ε^m to be small with respect to the baseline $(\mathcal{A}, \mathcal{M}_0)$ if and only if it leaves the equilibrium supplier choices unchanged for all sectors relative to the baseline: $S_k(\mathcal{A}, \mathcal{M}) = S_k(\mathcal{A}, \mathcal{M}_0)$, $\forall k$. Otherwise, define the monetary shock to be large with respect to the baseline $(\mathcal{A}, \mathcal{M}_0)$.*

Crucially, the above definition helps classify every monetary shock as either small or large *with respect to* a specific baseline pair $(\mathcal{A}, \mathcal{M}_0)$. In principle, as one changes the baseline pair, a shock of a given size can switch from being small to large, or *vice versa*. Moreover, the definition is not necessarily symmetric: for a given baseline, an expansionary shock $\varepsilon^m > 0$ being small does not necessarily imply that a contractionary shock of an equal absolute value is small with respect to the same baseline.

3.3.1 Small monetary shocks

In this section, I study how responses of prices and allocations to a small monetary shock depend on the baseline productivity mapping and initial money supply. As before, I begin by building intuition in the simplified two-sector setting before establishing general results.

First, I use the two-sector setting to consider the same small monetary expansion occurring under different baseline productivity mappings.

Example 3 (Small monetary shocks across productivity mappings)**.** *Consider the two-sector setting with the assumption that consumption aggregation is Cobb-Douglas:* $C = C_1^{\omega_{c1}} C_2^{\omega_{c2}}, \omega_{ck} = 0.5, k = 1, 2$. *Panel (a) of Figure 3 considers three baselines associated with different productivity mappings, as in Example 1. I perturb each of the baselines with the same small monetary expansion, which, by definition, does not change the equilibrium network. As one can see, in the low productivity baseline with the empty network, only the sticky-price Sector 2 responds to the shock; in the medium-productivity baseline the situation is unchanged, as Sector 1 does not buy inputs from Sector 2, and hence inherits no stickiness. However, in high-productivity state, where both sectors buy from each other, both sectors see their final consumption rise. We can see that the magnitude of both sectoral and aggregate final consumption response to a monetary shock is weakly larger in the baselines with higher productivity.*

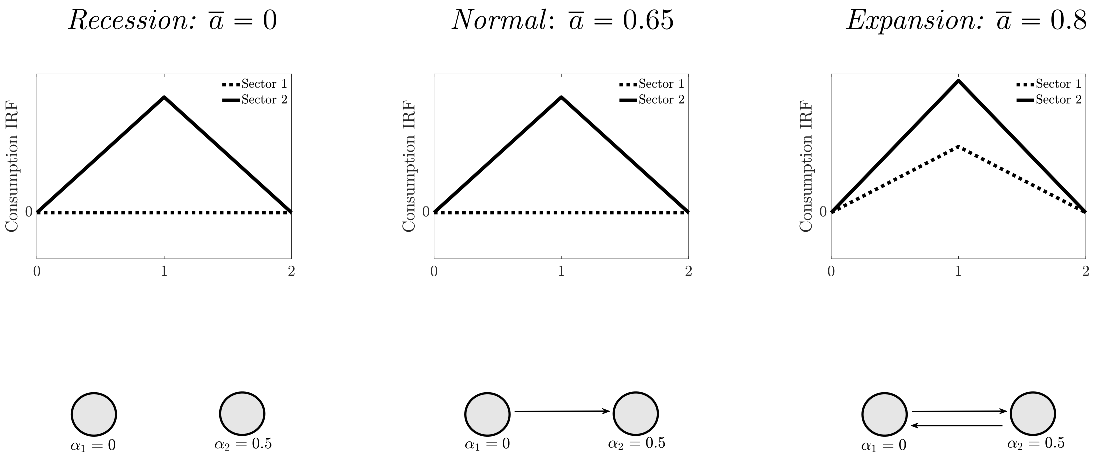
Similarly, I use the two-sector setting to consider the same small monetary expansion occurring under different levels of initial money supply.

Example 4 (Small monetary shocks across initial money supplies)**.** *Consider the two-sector setting with the assumption that consumption aggregation is Cobb-Douglas:* $C = C_1^{\omega_{c1}} C_2^{\omega_{c2}}, \omega_{ck} = 0.5, k = 1, 2$. *Panel (b) of Figure 3 considers three baselines with different levels of initial money supply, as in Example 2. I perturb each of the baselines with the same small monetary expansion, which, by definition, does not change the equilibrium network. One can see that in the tight money baseline only the sticky-price Sector 2 responds to the shock; in the normal money baseline flexible-price Sector 1 buys from Sector 2 and inherits stickiness, hence responding to the shock. Finally, in the loose money baseline both sectors buy from each other and respond by even more than in the normal money state. Overall, one can see that both sectoral and aggregate consumption respond weakly more strongly under the baselines with higher initial money supply.*

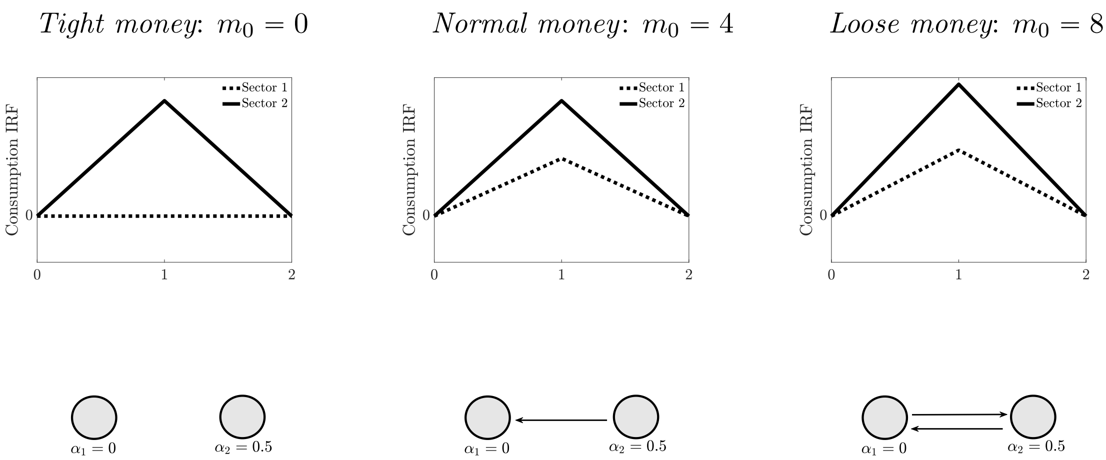
One can see that in the examples above, following a small monetary shock of the same size, consumption responds more strongly in states that feature more linkages across sectors. Key to the mechanism is the behavior of firms' intermediates cost share under different network configurations. In particular, if a firm uses only labor, i.e. the supplier set is empty and the intermediates cost share is zero, then the small monetary expansion generates a one-for-one increase in the nominal wage and also a one-for-one increase in the unit cost. Instead, if the supplier set is non-empty and the intermediates cost share is positive, then the increase in the nominal wage is still one-for-one, but the increase in the unit cost is *less* than one-for-one. This is because the one-for-one increase in the nominal wage is now scaled by the cost share of labor, which is less than one, and the remaining part of $\mathcal{Q}_k$, consisting of intermediates, is moving by strictly *less* than the nominal wage, since prices are sticky. A smaller increase in the unit cost implies a smaller rise in the firm's optimal reset price. The latter, in turn, implies a smaller

Figure 3: **Final aggregate consumption following a small monetary expansion**

(a) IRFs of consumption to a small monetary expansion under recession and expansion ($m_0 = 0$)



(b) IRFs of consumption to a small monetary expansion under tight and loose initial money ($\overline{a} = 0$)



Notes: the figure uses the analytically tractable version of my model under $T = 2$, calibrated for $K = 2, \omega_{kk} = 0, \forall k$, $Y_k(j) = e(k)\mathcal{A}_k(S_k)N_k(j)^{(1-\omega_{k,-k})}Z_{k,-k}(j)^{\omega_{k,-k}}$ and $e(k) = (1-\omega_{k,-k})^{-(1-\omega_{k,-k})}\omega_{k,-k}^{-\omega_{k,-k}}$, $a_{k,0} \equiv \log \mathcal{A}_{k,0}, \forall k$ and $m_0 = \log \mathcal{M}_0$, the production technology be given by $a_{1,0}(\varnothing) = 1, \quad a_{1,0}(\{2\}) = \overline{a}, a_{2,0}(\varnothing) = 1, \quad a_{2,0}(\{1\}) = \overline{a}$, $P_{10} = P_{20} = 1$, the sectoral shares are given by $\omega_{12} = \omega_{c1} = 0.5 = \omega_{21} = \omega_{c2} = 0.5$, and Calvo parameters by $\alpha_1 = 0, \alpha_2 = 0.5$. Finally, assume that $\tau_k = -1/\theta$, and $\theta \to 1^+$.

increase in the overall sectoral price index, since those who get the opportunity to adjust end up increasing their prices by less. Since each sectoral price is rising less, so does the aggregate consumption price index (CPI). Finally, since the CPI is rising by *less*, the cash-in-advance constraint implies that the aggregate consumption (GDP) has to increase by *more*, implying a higher degree of monetary non-neutrality.

Having explained the key mechanism in intuitive terms, I now establish the results formally. Notice that the existence of a small monetary shock around a specific baseline implies that there is a neighborhood within which variations in money supply do not affect the equilibrium set of supplier choices. The latter in turn means that such a neighborhood features no discontinuities created by formation or destruction of linkages, and one can appeal to local properties around the baseline. In order to establish analytically tractable local properties, I make several further assumptions. First, a vital required additional assumption is differentiability of the production function in labor and intermediate inputs:

Assumption 6. *The production function $\mathcal{F}_k$ is differentiable in $\left(N_{k,t}(j), \{Z_{kr,t}(j)\}_{r\in S_{k,t}(j)}\right)$, $\forall k$.*

The second additional assumption concerns the initial sectoral prices, which have so far been assumed to be completely exogenous:

Assumption 7 (Initial sectoral prices)**.** *For every sector $k = 1, 2, ..., K$, the initial sectoral price is given by $P_{k,0} = (1+\mu_k)g(\mathcal{M}_0)\mathcal{Q}_k[S_k, \mathcal{A}_k(S_k), \mathcal{M}_0, \{P_r(\mathcal{A}, \mathcal{M}_0)\}_{r\in S_k}]$, where $(\mathcal{A}, \mathcal{M}_0)$ is the baseline pair, $1 + \mu_k = (1 + \tau_k)\frac{\theta}{\theta-1}$ and $g : \mathbb{R}^+ \to (0, 1)$ is strictly decreasing on the whole domain.*

The role of the g function is to make the initial ($t = 0$) prices change *less than one-for-one* with the baseline level of money supply. The latter, in turn, ensures that the baseline equilibrium set of suppliers changes with $\mathcal{M}_0$, preserving the baseline comparative statics properties established in Lemmas 1 and 2. For example, a move to larger baseline money increases the baseline nominal wage one-for-one; at the same time, the fact that g falls in $\mathcal{M}_0$ ensures that the initial prices rise less than one-for-one. Hence, overall sectoral prices rise by less than the nominal wage, creating an incentive to expand the baseline supplier set.[16]

Armed with the two additional assumptions, I can now begin to formalize the differences in responses to a small monetary shock that arrives under different baselines. As a first step, the lemma below documents baseline-specific local responses of sectoral prices to a monetary shock:

Lemma 4. *Suppose Assumptions 1-7 hold. For any baseline pair $(\mathcal{A}, \mathcal{M}_0)$, let $p_k(\mathcal{A}, \mathcal{M}_0)$ be a first order deviation of $\log P_k(\mathcal{A}, \mathcal{M})$ from $\log P_k(\mathcal{A}, \mathcal{M}_0)$, given $S_k = S_k(\mathcal{A}, \mathcal{M}_0), \forall k$. Consider*

[16] Removing the g function would set the initial prices at the steady-state level. This would not be a problem for changes in $\mathcal{A}$: an improvement in productivity still lowers sectoral prices, while keeping the nominal wages unchanged, which incentivizes the adoption of extra suppliers. At the same time, this *would* be a problem for changes in $\mathcal{M}_0$: the nominal wage, initial prices and the overall sectoral prices would all adjust one-for-one, creating no incentive to change the set of suppliers. Crucially, I do not make this assumption in the fully dynamic forward-looking setting considered in Section 4.

any two baseline pairs $(\underline{\mathcal{A}}, \underline{\mathcal{M}_0})$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ *and a monetary shock* ε^m *which is small with respect to both baselines, then:*

$$\mathbb{p}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathbb{p}(\underline{\mathcal{A}}, \underline{\mathcal{M}_0}) = -\left[\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathcal{L}(\underline{\mathcal{A}}, \underline{\mathcal{M}_0})\right] \mathcal{E}^m \tag{10}$$

where $\mathbb{p} \equiv [p_1, p_2, ..., p_K]^T$, $\mathcal{E}^m \equiv [\varepsilon^m, \varepsilon^m, ..., \varepsilon^m]^T$ *and* $\mathcal{L}$ *is a Leontief inverse given by:*

$$\mathcal{L}(\mathcal{A}, \mathcal{M}_0) \equiv [I - (I - A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)]^{-1} [I - (I - A)\Gamma(\mathcal{M}_0)] \tag{11}$$

where $A \equiv diag(\alpha_1, ..., \alpha_K)$, $\Gamma(\mathcal{M}_0) \equiv diag(\gamma_1(\mathcal{M}_0), ..., \gamma_K(\mathcal{M}_0))$, $\gamma_k \equiv \frac{1}{\alpha_k (g(\mathcal{M}_0))^{1-\theta} + 1 - \alpha_k}$ *and* $[\Omega(\mathcal{A}, \mathcal{M}_0)]_{kr} = \frac{\partial \log \mathcal{Q}_k \left[S_k, \mathcal{A}_k(S_k), W, \{P_{k'}\}_{k' \in S_k}\right]}{\partial \log P_r} |_{\mathcal{M}=\mathcal{M}_0}, \forall k, r.$

The above result details channels through which changes in the baseline affect local properties of price responses to a monetary shock. The key object pinning down the change in local behavior of sectoral prices across baselines is the (negative) difference between a form of Leontief inverse matrices evaluated under those different baselines. The novel Leontief matrix is a product of two terms: $[I - (I - A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)]^{-1}$ and $[I - (I - A)\Gamma(\mathcal{M}_0)]$. The first term shows that if a sector k expands its baseline set of suppliers, adding new non-zero (positive) elements to the matrix $\Omega(\mathcal{A}, \mathcal{M}_0)$, the *direct* influence of such larger reliance on intermediates on local price behavior is going to be scaled by the probability of adjustment in sector k, $1 - \alpha_k$. Indeed, if the sector already has a very low probability of adjustment (α_k close to 1), the additional pricing complementarities coming from the extra suppliers will have a small effect on dampening sector k's price response to a monetary shock. While the product of $(I - A)$ and $\Omega(\mathcal{A}, \mathcal{M}_0)$ captures the direct effect, the inverse in the first term extends the logic to all the additional *direct* and *indirect* exposure to intermediates that a sector gets as the baseline set of suppliers expands.[17] As for the second term, $[I - (I - A)\Gamma(\mathcal{M}_0)]$, it merely scales all the sectoral price responses by the own adjustment probabilities and does not depend on the underlying input-output structure. Notably, in the limit of flexible prices ($\alpha_k = 0, \forall k$), the term $[I - (I - A)\Gamma(\mathcal{M}_0)]$ converges to a matrix of zeros, making the whole Leontief inverse $\mathcal{L}(\mathcal{A}, \mathcal{M}_0)$ a matrix of zeros as well for any baseline. All this indicates is that if one removes nominal rigidities, all sectoral prices adjust one-for-one with the monetary shock, regardless of the underlying network structure, and hence the local behavior of sectoral prices is trivially the same.

Baselines with either a better productivity mapping or a higher money supply, *ceteris paribus*, feature more suppliers for each sector and hence more non-zero elements in the $\Omega(\mathcal{A}, \mathcal{M}_0)$ matrix, whose entries denote cross-elasticities of a sector's unit cost to sectoral prices. The latter affects price responses through two margins. At the extensive margin, having more suppliers leads to a rise in the cost share of intermediates, which strengthens complementarities in price

[17] For example, suppose a supplier of sector k, call it sector r, expands its supplier set to also include sector j. Then the effect of this change on sector k's pricing will depend not only on the cost shares $[\Omega(\mathcal{A}, \mathcal{M}_0)]_{kr}$ and $[\Omega(\mathcal{A}, \mathcal{M}_0)]_{rj}$, but also on the probability of adjustment in sector r. Indeed, if α_r is close to one, the price index in sector r is already barely moving and hence the additional pricing complementarities will not affect either the pricing in sector r itself or the pricing in its customer sector k even if it heavily relies on its inputs.

setting and unambiguously delivers smaller increases in sectoral prices following a small monetary expansion.[18] At the intensive margin, however, the effect of a change in the baseline on the unit cost elasticity to prices of already existing suppliers is ambiguous. Therefore, the effect of a change in the baseline on the responses of sectoral prices to a small monetary shock depends on the relative quantitative importance of the three channels described above.

However, a sufficient condition exists under which the ambiguity described above is resolved. In particular, it occurs in the special case when the intensive margin adjustment described above is absent. The latter happens when, conditional on a supplier relationship existing, the elasticity of a sector's unit cost to that supplier's price is fixed across baselines, which is the case under Cobb-Douglas production function with Hicks-neutral technology:

Assumption 8. *For every sector $k = 1, 2, ..., K$ the production function is Cobb-Douglas with Hicks-neutral technology: $Y_{k,t}(j) = \mathcal{A}_{k,t}(S_{k,t})N_{k,t}(j)^{1-\sum_{r\in S_{k,t}}\omega_{kr}} \prod_{r\in S_{k,t}} Z_{kr,t}(j)^{\omega_{kr}}, \quad \omega_{kr} \geqslant 0$, $\sum_{r=1}^{K}\omega_{kr} < 1$.*

Adding the above assumption ensures that under a baseline with either a better productivity mapping or a higher money supply, the same small monetary expansion delivers smaller increases in all sectoral prices.[19] I formalize this result below:

Proposition 2. *Suppose Assumptions 2-5 and 7-8 hold. Consider any two baseline pairs $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ such that either $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ or $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$. For a monetary shock $\varepsilon^m > 0$ that is small with respect to both baselines, it follows that $p_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \leqslant p_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$ for all $k = 1, 2, ..., K$.*

Importantly, though the above proposition is formulated for a small monetary expansion $\varepsilon^m > 0$, it trivially extends to a small monetary contraction $\varepsilon^m < 0$. In particular, it would follow that under a baseline with either a better productivity mapping or a higher money supply, the same small monetary contraction delivers smaller decreases in all sectoral prices.

I now use properties of price responses established above to study how changes in the baseline affect consumption responses to small monetary shocks. From the cash-in-advance constraint, it follows that the change in aggregate consumption is $[\log C(\mathcal{A}, \mathcal{M}) - \log C(\mathcal{A}, \mathcal{M}_0)] = \varepsilon^m - [\log P^c(\mathcal{A}, \mathcal{M}) - \log P^c(\mathcal{A}, \mathcal{M}_0)]$. Therefore, local properties of GDP around the baseline can be inferred from local properties of the consumption price index. For a monetary shock ε^m that is small with respect to a baseline pair $(\mathcal{A}, \mathcal{M}_0)$ one can write the following local approximation:

$$\log P^c(\mathcal{A}, \mathcal{M}) - \log P^c(\mathcal{A}, \mathcal{M}_0) \approx \sum_{k=1}^{K} \frac{\partial \log P^c(\mathcal{A}, \mathcal{M}_0)}{\partial \log P_k} \left[\log P_k(\mathcal{A}, \mathcal{M}) - \log P_k(\mathcal{A}, \mathcal{M}_0)\right]. \quad (12)$$

[18] Key to this prediction is the specific *pattern* underlying the addition of suppliers in equilibrium: as Lemma 2 indicates, following a productivity or monetary expansion, firms keep all the existing suppliers and add new ones on top. In other words, the new suppliers do not replace existing suppliers; instead, they always replace a part of production that was previously done with labor. This ensures that the cost share of intermediates rises with the addition of extra suppliers, holding the intensive margin of each relationship fixed.

[19] Crucially, imposing Cobb-Douglas is merely *sufficient*, and by no means necessary: it is enough that the intensive margin variation does not make the intermediates cost share fall after productivity/monetary expansions. Notably, the empirical evidence in Section 5 suggests that the intermediates cost share is indeed procyclical.

From Proposition 2 we know local properties of (log-)deviations of sectoral prices as one varies the baseline. However, the variation in elasticities of the consumption price index with respect to sectoral prices as one varies the baseline is ambiguous. In this sense, the local properties of aggregate consumption/GDP around different baselines depend on the relative quantitative properties of movements in sectoral prices and the elasticities that are used to aggregate them.

However, a sufficient condition exists under which the ambiguity described above is resolved. In particular, it occurs in the special case when the elasticities of the consumption price index with respect to sectoral prices remain fixed across all baselines. The latter occurs when the consumption aggregator takes the Cobb-Douglas form, as is detailed below:

Assumption 9. *The consumption aggregator u is Cobb-Douglas:* $C_t = \prod_{k=1}^{K} C_{k,t}^{\omega_{ck}}, \quad \omega_{ck} \geqslant 0, \sum_{k=1}^{K} \omega_{ck} = 1$.

With the additional assumption above, I can now formalize non-linear transmission of a small monetary shock to both sectoral and aggregate consumption, the latter being equivalent to GDP in my economy. First, whenever a small monetary expansion arrives under a baseline with a better productivity mapping, it triggers a consumption increase of a larger magnitude:

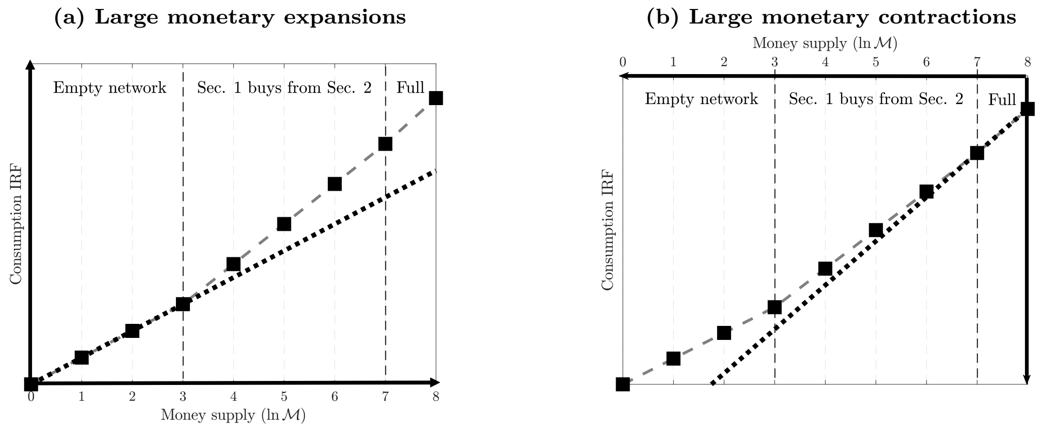
Theorem 1 (Cycle dependence)**.** *Suppose Assumptions 3-5 and 7-9 hold. For any baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, let* $c_k(\mathcal{A}, \mathcal{M}_0) \equiv \log C_k(\mathcal{A}, \mathcal{M}) - \log C_k(\mathcal{A}, \mathcal{M}_0), \forall k$*. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \mathcal{M}_0)$*,* $(\overline{\mathcal{A}}, \mathcal{M}_0)$ *such that* $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}$*. For a monetary shock* $\varepsilon^m > 0$ *which is small with respect to both baselines it follows that* $c_k(\overline{\mathcal{A}}, \mathcal{M}_0) \geqslant c_k(\underline{\mathcal{A}}, \mathcal{M}_0)$ *for all* $k = 1, 2, ..., K$*, and* $c(\overline{\mathcal{A}}, \mathcal{M}_0) \geqslant c(\underline{\mathcal{A}}, \mathcal{M}_0)$*.*

Three points should be noted. First, although the theorem is stated for small monetary expansions, it trivially extends to small monetary contractions. In particular, it follows that if a small monetary contraction arrives in a state with, *ceteris paribus*, higher productivity, the resulting fall in GDP is (weakly) larger. Second, the additional assumption of Cobb-Douglas consumption aggregation implies cycle dependence not only at the level of aggregate GDP, but also at the level of final consumptions of individual sectors. Third, my cycle dependence result provides a theoretical rationale for empirical findings of a procyclical magnitude of the impulse response of GDP to monetary shocks (Tenreyro and Thwaites, 2016, Alpanda et al., 2021, Jordà et al., 2020). In addition, even though optimal monetary policy conduct is beyond the scope of this paper, the result in Theorem 1 has implications for central bank conduct. In particular, in an expansion, generating a local drop in aggregate GDP requires a smaller monetary contraction, all else constant.

In a similar way, I can formalize that whenever a small monetary expansion arrives under a baseline with higher money supply, it triggers a consumption increase of a larger magnitude:

Theorem 2 (Path dependence)**.** *Suppose Assumptions 3-5 and 7-9 hold. For any baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, let* $c_k(\mathcal{A}, \mathcal{M}_0) \equiv \log C_k(\mathcal{A}, \mathcal{M}) - \log C_k(\mathcal{A}, \mathcal{M}_0), \forall k$*. Consider any two baseline pairs* $(\mathcal{A}, \underline{\mathcal{M}}_0)$*,* $(\mathcal{A}, \overline{\mathcal{M}}_0)$ *such that* $\overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$*. For a monetary shock* $\varepsilon^m > 0$ *which is small*

Figure 4: **Aggregate consumption response to large monetary shocks (two sectors)**



Notes: the figure uses the analytically tractable version of the model under $T = 2$, calibrated for $K = 2, \omega_{kk} = 0, \forall k$, $Y_k(j) = e(k)\mathcal{A}_k(S_k)N_k(j)^{(1-\omega_{k,-k})}Z_{k,-k}(j)^{\omega_{k,-k}}$ and $e(k) = (1-\omega_{k,-k})^{-(1-\omega_{k,-k})}\omega_{k,-k}^{-\omega_{k,-k}}$, $a_{k,0} \equiv \log \mathcal{A}_{k,0}, \forall k$ and $m_0 = \log \mathcal{M}_0$, the production technology be given by $a_{1,0}(\varnothing) = 1$, $a_{1,0}(\{2\}) = \overline{a}, a_{2,0}(\varnothing) = 1$, $a_{2,0}(\{1\}) = \overline{a}$, $P_{10} = P_{20} = 1$, the sectoral shares are given by $\omega_{12} = \omega_{c1} = 0.5 = \omega_{21} = \omega_{c2} = 0.5$, and Calvo parameters by $\alpha_1 = 0, \alpha_2 = 0.5$. Finally, assume that $\tau_k = -1/\theta$, and $\theta \to 1^+$. Throughout exercises $\overline{a}_0 = 0$.

with respect to both baselines it follows that $c_k(\mathcal{A}, \overline{\mathcal{M}}_0) \geqslant c_k(\mathcal{A}, \underline{\mathcal{M}}_0)$ *for all* $k = 1, 2, ..., K$, *and* $c(\mathcal{A}, \overline{\mathcal{M}}_0) \geqslant c(\mathcal{A}, \underline{\mathcal{M}}_0)$.

As before, the theorem trivially extends to small monetary contractions: if a small monetary contraction arrives in a state with, *ceteris paribus*, lower money supply, the resulting fall in GDP is (weakly) smaller. My path dependence result provides a theoretical rationale for empirical findings that monetary transmission to GDP is stronger under an already loose monetary stance (Alpanda et al., 2021) and additional evidence that finds that GDP is more sensitive to monetary interventions in states of the world with already loose credit (Jordà et al., 2020). The path dependence result in Theorem 2 also has a clear implication for central bank conduct. In particular, in a loose money state, generating a local drop in aggregate GDP requires a smaller monetary contraction, all else constant.

The intuition behind both theorems is very similar. Baselines with a better productivity mapping or higher initial money supply, *ceteris paribus*, feature more suppliers for every sector in equilibrium. The latter strengthens complementarities in price setting, and hence delivers greater monetary non-neutrality and a consumption response of a larger magnitude.[20]

3.3.2 Large monetary shocks

I now move to studying properties of large monetary shocks which, by definition, are able to change the equilibrium set of suppliers relative to the baseline. As before, I begin with building intuition using the two-sector setting before establishing formal results.

[20] By analogy with small monetary shocks, one could also study the propagation of a small sector-specific productivity shock ζ_k, which does not affect the equilibrium network around the baseline $(\mathcal{A}, \mathcal{M}_0)$. Normalizing $g(\mathcal{M}_0) = 1$, a first-order change in aggregate GDP (c) following the small productivity shock to sector k is $c = \ell_k \times \zeta_k$, where ℓ_k is the k'th element of the vector $\boldsymbol{\ell} \equiv (I - A)\left[I - \Omega(\mathcal{A}, \mathcal{M}_0)^T(I - A)\right]^{-1}\boldsymbol{\omega_c}$, where $A \equiv diag(\alpha_1, ..., \alpha_K)$ is a diagonal matrix of sector-specific Calvo parameters, $\Omega(\mathcal{A}, \mathcal{M}_0)^T$ is the transpose of the matrix of input-output cost shares under the baseline set of suppliers, and $\boldsymbol{\omega_c} \equiv [\omega_{c1}, ..., \omega_{cK}]^T$ is a vector of final consumption shares. This gives a version of Hulten's theorem in my economy.

First, I consider larger monetary expansions in the two-sector setting:

Example 5 (Large monetary expansions). *Panel (a) of Figure 4 considers large monetary shocks in the context of the two-sector setting with an additional assumption on consumption aggregation:* $C = C_1^{\omega_{c1}} C_2^{\omega_{c2}}, \omega_{ck} = 0.5, k = 1, 2$. *Starting with an initial state with* $\overline{a} = m_0 = 0$, *I subject my economy to progressively larger monetary shocks. One can see that shocks smaller than* $\varepsilon^m = 3$ *keep the initially empty network unchanged, and the changes in the aggregate consumption impulse response are exactly proportional to the size of the shock. However, as I consider larger expansions, they turn out to be large enough to expand the network by encouraging the flexible-price Sector 1 to buy from the sticky-price Sector 2, also increasing the slope of the relationship between the monetary shock size and consumption response.*

Similarly, I use the two-sector setting to study large monetary contractions:

Example 6 (Large monetary contractions). *Panel (b) of Figure 4 considers large monetary contractions in the context of the same two-sector setting with an additional assumption on consumption aggregation:* $C = C_1^{\omega_{c1}} C_2^{\omega_{c2}}, \omega_{ck} = 0.5, k = 1, 2$. *I start from a baseline with* $\overline{a} = 0$ *and* $m_0 = 8$. *Initial reductions in money supply leave the baseline full network unchanged and the magnitude of aggregate consumption contraction is exactly proportional to the size of the monetary shock. However, larger contractions break the network, in this case by discouraging Sector 2 from buying from Sector 1, and lower the slope of the relationship between the monetary shock size and consumption response.*

One can see that in Example 5 above, large monetary expansions deliver an increase in GDP that is larger than the one that would occur under fixed networks. In this sense, large monetary expansions deliver more than proportional increases in GDP by expanding the set of suppliers. On the other hand, in Example 6 above, large monetary contractions deliver reductions in GDP that are smaller in magnitude than those under exogenous networks. Therefore, large monetary contractions break supplier relationships and in this way deliver less than proportional decreases in GDP, relative to the outcome under exogenous networks.

In order to formalize the above intuition, I introduce several new concepts. First, I let the exact impulse response of a sectoral price to monetary shock ε^m be given by the following expression: $\tilde{P}_k(\varepsilon^m) \equiv P_k(\mathcal{A}, \mathcal{M})/P_k(\mathcal{A}, \mathcal{M}_0), \forall k$. Second, in order to facilitate comparison with a setting where supplier choices are exogenous, I let $\tilde{P}^e_k(\varepsilon^m; S) \equiv P_k(\mathcal{A}, \mathcal{M}; S)/P_k(\mathcal{A}, \mathcal{M}_0; S), \forall k$ denote exact impulse responses of sectoral prices to monetary shock ε^m in a version of my economy with an *exogenous* production network S. I am now ready to state a proposition which establishes bounds on how the exact impulse responses of sectoral prices grow with the size of the monetary shock:

Proposition 3. *Suppose Assumption 1-3 and 6 hold. For a baseline pair* $(\mathcal{A}, \mathcal{M}_0)$, *and the baseline equilibrium network* S^0, *let* $\varepsilon^m > 0$ *be a small monetary shock and* $E^m > 0$ *be a large monetary shock. Further, let* S^L *be the equilibrium network following the large monetary shock. The ratios of exact impulse responses of sectoral prices satisfy:*

$$\frac{\tilde{P}^e_k(E^m; S^L)}{\tilde{P}^e_k(\varepsilon^m; S^L)} \leqslant \frac{\tilde{P}_k(E^m)}{\tilde{P}_k(\varepsilon^m)} \leqslant \frac{\tilde{P}^e_k(E^m; S^0)}{\tilde{P}^e_k(\varepsilon^m; S^0)}, \tag{13}$$

for all $k = 1, 2, ..., K$, where $\tilde{P}_k(.)$ is the exact impulse response under endogenous networks, whereas $\tilde{P}^e_k(.; S)$ is the exact impulse response under the exogenous network S.

Intuitively, as one moves from a small monetary expansion to a large monetary expansion, the rate at which the exact responses of sectoral prices increase is smaller than what would occur if the network stayed fixed at its baseline level. This is because the opportunity to endogenously select the supplier sectors creates an extra margin along which units costs, and hence prices, can be minimized. This implies that prices increase at most as quickly as under the network fixed at its baseline level. At the same time, this drag on price increases is bounded from below by what would occur if the network was fixed at the level delivered by the large monetary expansion. Crucially, although the above proposition is stated for monetary expansions it trivially extends to monetary contractions. In particular, as one moves from a small monetary contraction to a large monetary contraction, the rate at which the exact magnitudes of prices drops change is larger than what would occur if the network stayed fixed at its baseline level. Once again, this is because the opportunity to optimally select the suppliers creates an extra margin along which unit costs and prices are minimized.

Since the above proposition applies to all sectoral prices, it implies corresponding properties for the aggregate consumption price index. The latter combined with the cash-in-advance in turn delivers properties of aggregate consumption, which also represents GDP in my model:

Theorem 3 (Size dependence)**.** *Suppose Assumption 1-3 and 6 hold. For a baseline pair $(\mathcal{A}, \mathcal{M}_0)$, and the baseline equilibrium network S^0, let $\varepsilon^m > 0$ be a small monetary shock and $E^m > 0$ be a large monetary shock. Further, let S^L be the equilibrium network following the large monetary shock. The ratios of exact impulse responses of GDP satisfy:*

$$\frac{\tilde{C}^e(E^m; S^0)}{\tilde{C}^e(\varepsilon^m; S^0)} \leqslant \frac{\tilde{C}(E^m)}{\tilde{C}(\varepsilon^m)} \leqslant \frac{\tilde{C}^e(E^m; S^L)}{\tilde{C}^e(\varepsilon^m; S^L)}. \tag{14}$$

where $\tilde{C}(.)$ is the exact impulse response under endogenous networks, whereas $\tilde{C}^e(.; S)$ is the exact impulse response under the exogenous network S.

The theorem above formally establishes size-dependence in the response of GDP to monetary shocks in my model. In particular, as one moves from a small monetary expansion to a large monetary expansion, the rate at which the exact impulse response of GDP rises is larger than what would occur if the set of suppliers was fixed at the baseline level. This is due to the fact that, relative to the world where networks are fixed at the baseline level, the opportunity to optimally select suppliers puts a drag on how quickly prices increase following monetary expansions. At the same time, the above theorem also implies that following a small monetary contraction the rate at which the magnitude of exact impulse response of GDP changes is smaller than what would occur if the set of suppliers was fixed at the baseline level.

4 A quantitative dynamic version

After establishing the key properties of my model in a simplified static version, I now quantify the effects in a forward-looking setting calibrated to 389 sectors of the US economy. I develop a novel numerical algorithm for solving a dynamic version of my multi-sector model with sticky prices and endogenous network formation, and use it to determine how changes in baseline productivity and money supply affect equilibrium supplier choices, and how the latter creates non-linearities in the transmission of monetary shocks.

4.1 Forward-looking setting

I now quantify the effects established analytically in the previous section using a dynamic forward-looking version of the model. In particular, in this section I do not make the simplifying assumptions needed to obtain closed-form results, and instead solve the model numerically. Numerical simulations still require functional form assumptions, and I therefore maintain the assumptions of a Cobb-Douglas production function and a Cobb-Douglas consumption aggregator. Moreover, I need to specify the functional form of the productivity mapping, where I augment the mapping of Acemoglu and Azar (2020a) with an aggregate productivity term:

Assumption 10 (Productivity mapping)**.** *For every sector $k = 1, 2, ..., K$ the productivity mapping $\mathcal{A}_{k,t}(S_{k,t})$ takes the following form:*

$$\mathcal{A}_{k,t}(S_{k,t}) = \mathcal{Z}_t e(S_{k,t}) B_{k,0} \prod_{r \in S_{k,t}} B_{kr}, \tag{15}$$

where $e(S_{k,t})$ is a term given by $e(S_{k,t}) \equiv (1 - \sum_{r\in S_{k,t}} \omega_{kr})^{-(1-\sum_{r\in S_{k,t}} \omega_{kr})} \prod_{r \in S_{k,t}} \omega_{kr}^{-\omega_{kr}}$, $\mathcal{Z}_t$ is aggregate productivity, which follows an AR(1) process in logs: $\log \mathcal{Z}_t = \rho_z \log \mathcal{Z}_{t-1} + \zeta_t$, and $\{B_{k,0}\}_k$, $\{B_{kr}\}_{kr}$ are parameters.

Two points should be noted regarding the productivity mapping above. First, it delivers an equilibrium unit cost function given by $\mathcal{Q}_k = \left[\mathcal{Z}_t B_{k,0} \prod_{r\in S_{k,t}} B_{kr}\right]^{-1} W_t \prod_{r\in S_{k,t}} \left(\frac{P_{r,t}}{W_t}\right)^{\omega_{kr}}$, or, writing in logs, $-z_t - b_{k,0} + w_t + \sum_{r\in S_{k,t}} [\omega_{kr}(p_{r,t} - w_t) - b_{kr}]$.[21] The latter implies a tractable rule for whether or not it is optimal to buy inputs from a particular supplier. Namely, given the levels of sectoral prices and the nominal wage, sector k should only buy inputs from sector r if $\omega_{kr}(p_{r,t} - w_t) < b_{kr}$.[22] Second, the entire path of aggregate productivity is known to the

[21] I interpret the value of B_{kr} as reflecting the relative strength of two forces associated with adopting an extra supplier. On one hand, it may lead to positive technological synergies with the firm's existing supply chain, leading to a productivity improvement. On the other hand, it may impose additional costs associated with managing a complex supply chain, leading to a deterioration in productivity. Such trade-offs are widely recognized in the supply chain management literature, see Akın Ateş et al. (2022) for a review. All in all, whenever the effect of technological synergies outweighs the costs of supply chain complexity, then $b_{kr} > 0$, and *vice versa*.

[22] Here the Cobb-Douglas technology assumption is important for computational feasibility. If the production technology were of a more general form such as constant-elasticity-of-substitution (CES), all *relative* sectoral prices (and not just the sectoral price relative to the wage) would matter for link optimality, which would require checking all possible linkage combinations to find the optimal set of suppliers. In other words, conditional on the wage and sectoral prices, it would require checking 2^K conditions on each iteration. For the calibration with $K = 389$ sectors, going from Cobb-Douglas to a more general technology (with intensive margin variation) would imply a move from checking $K^2 = 151,321$ to $2^K = 1.260864 \times 10^{117}$ conditions on every iteration, making the algorithm computationally infeasible.

agents, so that they are aware of $\mathcal{Z}_0$ and $\{\zeta_t\}_{t=1}^{\infty}$ at $t = 0$.[23]

I also need to specify the functional form for the money supply process, which I assume to follow an AR(1) process in log-differences:

Assumption 11 (Money supply). *For a given initial money supply $\mathcal{M}_0$, the money supply in $t \geqslant 1$ takes the following form: $\Delta \log \mathcal{M}_t = \rho_m \Delta \log \mathcal{M}_{t-1} + \varepsilon_t^m, \rho_m \in (0, 1)$.*

The agents are aware of $\mathcal{M}_0$ at $t = 0$ and at the beginning of $t = 1$ discover the entire future path of monetary shocks $\{\varepsilon_t^m\}_{t=1}^{\infty}$, facing no uncertainty beyond that point.[24]

My modified, finite-horizon version of Calvo (1983) pricing, combined with the simple rule for inclusion of suppliers described above, allows me to use backward induction to solve my model at the sector-level. More specifically, I am interested in solving for equilibrium for periods $t < T$, which feature nominal rigidities and hence real variables responding to monetary shocks. For $t \geqslant T$, the problem is static from the firms' pricing perspective, and money is neutral, which can therefore be used as a terminal condition in the backward induction algorithm. Here I outline my method, which numerically solves for sectoral prices, supplier choices and allocations:

Algorithm 1 (Model with sticky prices and endogenous production networks). *Start from a guess for sectoral prices, supplier choices and allocations; let $\mathcal{X}_t^-, \mathcal{X}_t$ and $\mathcal{X}_t^+$ be, respectively, the full set of past, present and future prices, supplier choices and allocations at time t for any $1 \leqslant t \leqslant T-1$. Let $\left\{\{P_{k,t}^0\}_{k=1}^K\right\}_{t=1}^{T-1}$ be sectoral prices from the initial guess. Taking as given exogenous paths for money supply and the productivity mapping, follow the steps below, starting from $t = T - 1$*

(i) Taking as given sectoral supplier choices, as well as past and future variables $\mathcal{X}_t^-, \mathcal{X}_t^+$, solve for prices $\{P_{k,t}\}_{k=1}^K$;

(ii) Given prices $\{P_{k,t}\}_{k=1}^K$, update supplier choices according to the following rule: sector k should only buy inputs from sector r if $\omega_{kr}(p_{r,t} - m_t) < b_{kr}$, where $p_{r,t} \equiv \log P_{r,t}$ and $m_t \equiv \log \mathcal{M}_t$;

(iii) Taking as given $\mathcal{X}_t^-, \mathcal{X}_t^+$, $\{P_{k,t}\}_{k=1}^K$ and $\{S_{k,t}\}_{k=1}^K$, update $\mathcal{X}_t$;

(iv) Repeat (i)-(iii) until convergence within the time period;

(v) If $t > 1$, decrease t by one and go back to (i). Otherwise, compare $\{\{P_{k,t}^0\}_{k=1}^K\}_{t=1}^{T-1}$ with $\{\{P_{k,t}\}_{k=1}^K\}_{t=1}^{T-1}$; if they are equal, stop the algorithm; if they are not equal, set $P_{k,t}^0 = P_{k,t}, \forall k, 1 \leqslant t \leqslant T-1$, set $t = T-1$ and return to (i).

[23]Note that it is trivial to extend the productivity mapping to allow for sector-specific productivity shocks, or even productivity shocks specific to a particular buyer-supplier sectoral pair.

[24]The quantitative model considers central bank policy set as an exogenous money growth path rather than as an endogenous policy rule. An endogenous central bank policy rule would *qualitatively* preserve the network cyclicality and non-linear monetary transmission properties as long as the sectoral price-to-wage ratios remain procyclical. At the same time, a quantitative assessment of this channel would require a substantial extension to my numerical algorithm. I believe this is an exciting and policy-relevant agenda, which I leave for future work at this stage.

Notice that step (i) involves solving for sectoral prices and allocations, taking the production network *as given*. This allows me to approximate the solution in step (i) by considering a log-linear approximation around the deterministic steady state, holding the choice of suppliers fixed. The supplier choices are then computed in step (ii) using those approximate solutions for sectoral prices. This decoupling of the "smooth" optimal pricing problem and the "non-smooth" supplier choice problem allows me to substantially speed up the algorithm, while preserving the non-linearity generated by the extensive margin of production network formation.[25]

4.2 Calibration

I calibrate my model at an annual frequency, for $K = 389$ sectors of the United States economy. Below I summarize my calibration strategy.

Aggregate parameters I set $\beta = 0.99$ to target an annualized real interest rate of 1% in steady state. The within-sector elasticity of substitution is set at $\theta = 6$ (Galí, 2015). The persistence parameters of productivity and money supply growth are set at $\rho_z = 0.86$ and $\rho_m = 0.80$, respectively.[26] As for the threshold beyond which there are no nominal rigidities, I choose $T = 50$, so that after fifty years all firms can adjust their prices with certainty. Finally, I normalize $\mathcal{Z}_0 = 1$.

Sectoral parameters Sector-specific Calvo parameters $\{\alpha_k\}_{k=1}^K$ are set as one minus the sector-specific frequency of price adjustment from Pasten et al. (2020a,b). The steady-state markups in my economy are $\{(1+\tau_k)\frac{\theta}{\theta-1}\}_{k=1}^K$, and I set sectoral tax rates $\{\tau_k\}_{k=1}^K$ to match the sectoral markups from De Loecker et al. (2020) in steady state. The observed input-output shares $\{\omega_{kr}\}_{kr}$, as well as the final consumption shares $\{\omega_{ck}\}_k$, are calibrated using the 2007 BEA Detail-level Input-Output accounts.[27] The input-output cost shares for linkages not observed in the 2007 accounts are imputed following Acemoglu and Azar (2020a) by setting each unobserved share equal to 0.95 times the labor cost share of the customer sector (k) times the relative outdegree of the supplier sector (r): $\omega_{kr} = 0.95 \times \left(1 - \sum_{r' \in S_k} \omega_{kr'}\right) \times \frac{\sum_{k':r \in S_{k'}} \omega_{k'r}}{\sum_{k',r':r' \in S_{k'}} \omega_{k'r'}}$. In other words, I impute the cost share of inputs going from sector r to sector k as 0.95 times the labor cost share in the customer sector k, multiplied by the observed outdegree of the supplier sector r, normalized by the total outdegree of all the sectors in the economy. This ensures that all the imputed cost shares are proportional to the observed outdegree of the supplier sector and that the row sums of the full input–output matrix (including the linkages absent in 2007) are less than one so that labor is an essential input as required by Assumption 1. Finally, the productivity parameters $\{B_{k,0}\}_k$ and $\{B_{kr}\}_{kr}$ are estimated to ensure the steady-state equilibrium of my economy under $\mathcal{M}_t = \mathcal{Z}_t = 1, \forall t$ simultaneously matches the observed

[25] See Poirier (2026) for formal convergence properties of the type of algorithm that I employ.

[26] The persistence parameter for the exogenous productivity process is set as the annual counterpart to the quarterly persistence estimate by Smets and Wouters (2007). As for the money growth persistence parameter, it is set as the annual counterpart to the estimated monthly M2 growth persistence in the US.

[27] The sectoral input-output table, final consumptions and markups are taken directly from the replication package for Acemoglu and Azar (2020a,b). The underlying input-output accounts are based on U.S. Bureau of Economic Analysis (2020a,d).

input-output linkages and real GDP in 2007. The full details of the algorithm used to estimate $\{B_{k,0}\}_k$ and $\{B_{kr}\}_{kr}$ are given in Appendix D.1.[28]

4.3 Simulation results

4.3.1 Baseline economies

Variations in the baseline of my economy are driven by changes in the sequence $\{\zeta_t\}_{t=1}^{\infty}$, which pins down the path of aggregate productivity, as well as changes in the money supply sequence $\{\mathcal{M}_t\}_{t=1}^{\infty}$ discovered at $t = 1$. Crucially, I keep $\mathcal{M}_0 = 1$ across all baselines, which is also used to pin down the common initial prices for all the baselines.[29]

I consider two sets of variations in the baseline. First, I hold $\zeta_t = 0, \forall t \geqslant 2$, and consider values of $\zeta_1 = \{-3\%, -2\%, -1\%, 0, 1\%, 2\%, 3\%\}$. In this way, I look at baselines with both low and high productivity paths, all of which eventually converge to the initial value of $\mathcal{Z}_0 = 1$; the case of $\zeta_1 = 0$ where aggregate productivity remains fixed at the initial value is set as a benchmark against which the other baselines are compared. Second, I hold $\zeta_t = 0, \forall t \geqslant 1$ and consider different baseline paths of money supply $\{\mathcal{M}_t\}_{t=1}^{\infty}$ discovered at $t = 1$. In particular, I assume that in the unmodelled past before $t = 0$ the money supply starts from $\mathcal{M}_{-\infty} = 1$ and follows the process in Assumption 11, experiencing a one-time, non-repeating shock $\varepsilon^m_{-\infty}$, creating a permanent change in the level of money supply by $t = 1$, at which it then stays forever. I consider values of $\varepsilon^m_{-\infty} = \{-6\%, -4\%, -2\%, 0, 2\%, 4\%, 6\%\}$, where the case of $\varepsilon^m_{-\infty} = 0$ is treated as a benchmark against which all the other baselines are compared.[30]

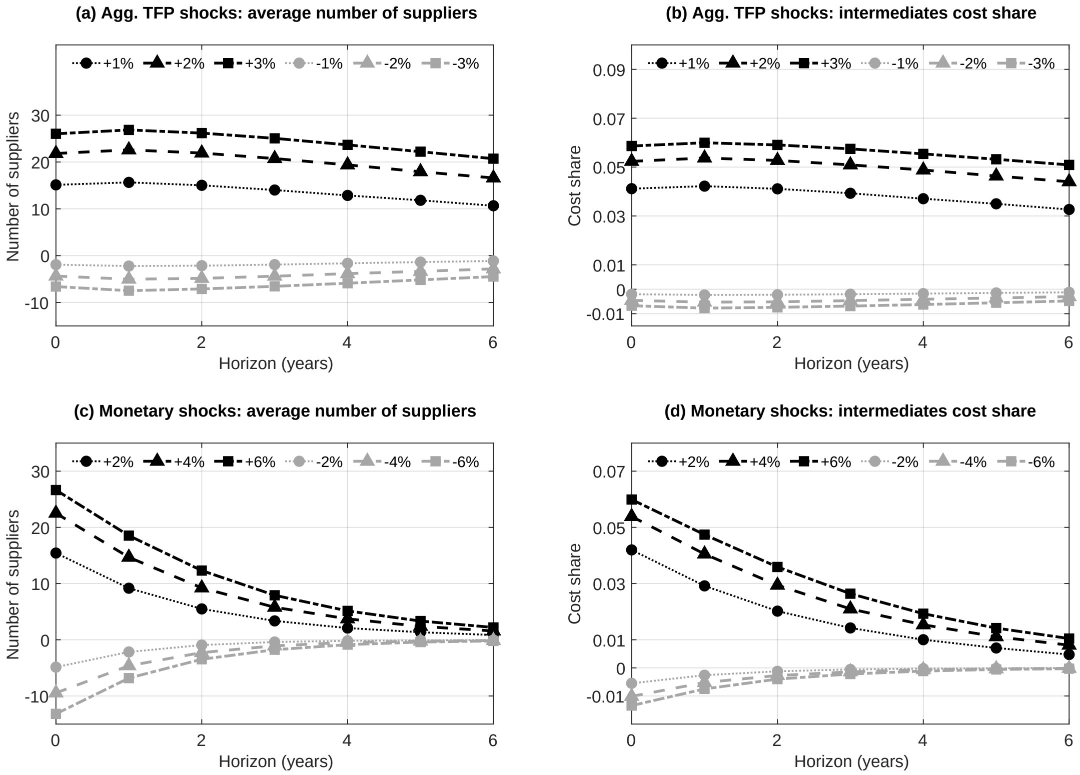
Panels (a) and (b) of Figure 5 consider baselines with different aggregate productivity paths and their associated paths of average number of suppliers and average intermediates cost share, which measures the average share of total costs that goes to suppliers. Relative to the fixed-productivity baseline ($\zeta_1 = 0$), letting $\zeta_1 = 1\%$ is associated with a long-lived increase in the average number of suppliers by around 15 per sector, which translates into an increase in average intermediates cost share by around 0.04. Similarly, setting $\zeta_1 = 3\%$, which delivers a baseline with an even better aggregate productivity, increases the average number of suppliers per sector by around 27, or an approximately 0.06 increase in average intermediates cost share. Importantly, the results suggest that the effect of changing the baseline productivity path is not symmetric. Specifically, a low-productivity baseline with $\zeta_1 = -1\%$ decreases the number of suppliers only by approximately 2 per sector, or a small decrease of 0.002 in average intermediates cost share. Under an even worse productivity baseline with $\zeta_1 = -3\%$, the average number

[28] Given that the productivity parameters are pinned down by inequality restrictions, the values of $\{B_{kr}\}_{kr}$ are likely set- rather than point-identified. I have experimented with applying the estimation algorithm to a variety of initial guesses for the parameters: the quantitative properties of the resulting impulse responses are robust to different initial guesses, even though the precise productivity parameter estimates are slightly different.

[29] In particular, initial prices are set as a fixed markup over the steady-state unit cost: $P_{k,0} = (1+\mu_k) \times \left[B_{k,0} \prod_{r \in S_{k,0}} B_{kr}\right]^{-1} \times \mathcal{M}_0^{1-\sum_{r\in S_{k,0}} \omega_{kr}} \prod_{r\in S_{k,0}} P_{r,0}^{\omega_{kr}}$.

[30] For example, suppose we start at $\mathcal{M}_{-\infty} = 1$, and experience a one-time shock $\varepsilon^m_{-\infty} = 2\%$. Then, $\log \mathcal{M}_1 = \sum_{t=1}^{\infty} \Delta \log \mathcal{M}_{1-t} + \log \mathcal{M}_{-\infty} = \varepsilon^m_{-\infty} \sum_{t=1}^{\infty} \rho_m^{t-1} = \frac{\varepsilon^m_{-\infty}}{1-\rho_m} = \frac{0.02}{1-0.8} = 0.1$. All in all, the $\varepsilon^m_{-\infty} = 2\%$ shock corresponds to a one-time permanent jump of $\log \mathcal{M}_t, t \geqslant 1$, by 0.1, or a 10% permanent increase in the baseline level of money supply that is discovered at $t = 1$. Under $\varepsilon^m_{-\infty} = 0$, the level of money supply simply stays at $\mathcal{M}_0 = 1$, just as the agents expected at $t = 0$.

Figure 5: **Numbers of suppliers and intermediates cost shares**



Notes: Panels (a) and (b) show the average number of suppliers and average intermediates cost shares across baselines where $\mathcal{M}_t = 1, \forall t \geqslant 0$ and $\zeta_t = 0, \forall t \geqslant 2$, and consider values of $\zeta_1 = \{-3\%, -2\%, -1\%, 1\%, 2\%, 3\%\}$, relative to $\zeta_1 = 0$; Panels (c) and (d) show the average number of suppliers and average intermediates cost shares across baselines where $\mathcal{M}_0 = 1$, $\zeta_t = 0, \forall t \geqslant 1$ and values of $\varepsilon^m_{-\infty} = \{-6\%, -4\%, -2\%, 2\%, 4\%, 6\%\}$, relative to the case where $\varepsilon^m_{-\infty} = 0$.

of suppliers falls by around 8 per sector, equivalent to a 0.01 reduction in average intermediates cost share.[31]

As for results under baselines with different initial levels of money supply, those are reported in Panels (c) and (d) of Figure 5. Relative to the benchmark case ($\varepsilon^m_{-\infty} = 0$), a looser money supply baseline under $\varepsilon^m_{-\infty} = 2\%$ features an initial rise in the average number of suppliers by around 16, or an increase in the average intermediates cost share by around 0.045. A baseline with an even looser initial money supply ($\varepsilon^m_{-\infty} = 6\%$) is associated with an initial increase in the average number of suppliers by around 28, equivalent to a 0.06 increase in average intermediates cost share. Just like with variations in baseline aggregate productivity, the effect of variations in initial money supply is not symmetric. In particular, a baseline with tighter money supply ($\varepsilon^m_{-\infty} = -2\%$) has an initial drop in the average number of suppliers by 5, or a 0.005 reduction in average intermediates cost share. An even tighter initial money supply ($\varepsilon^m_{-\infty} = -6\%$) features a drop of 15 in the average number of suppliers, or a 0.012 reduction in the average intermediates cost share.

An important feature of the quantitative results above is the asymmetry in the response of

[31] Note that the network dynamics are very persistent. For baselines with different levels of productivity, this persistence has two sources. First, there is the exogenous persistence, coming from the autoregressive process assumed for $\mathcal{Z}_t$. Second, there is the endogenous persistence coming from price stickiness: since changes in (relative) prices are staggered, so are the optimal supplier choices. To separate the two sources, Figure D.2 reports the network dynamics under transitory productivity changes ($\rho_z = 0$). The amount of persistence in this case remains substantial, suggesting that price stickiness is an important source of endogenous persistence of the network.

the production network to expansionary and contractionary shocks. Key to this quantitative result is the empirical *sparsity* of the network that I target in steady-state: in the 2007 BEA table almost two thirds of potential links are missing. The link-specific productivities used to rationalize this observed pattern are drawn from a Gaussian distribution. In the baseline calibration, this combination of empirical sparsity and estimated productivities places many inactive links close to their adoption thresholds. As a result, expansionary shocks move a larger mass of economically relevant inactive links across their adoption thresholds than equal-sized contractionary shocks move active links across their severing thresholds. More formally, note that a firm in sector k is indifferent between linking to a sector r if and only if $\omega_{kr}(p_{r,t}-m_t) = b_{kr}$. Hence, define the relative price threshold $\varsigma_{kr,t} \equiv \left[\frac{b_{kr}}{\omega_{kr}} - (p_{r,t} - m_t)\right]$. For an active link, $\varsigma_{kr,t} > 0$ measures the increase in the relative price of supplier r required to sever the link; for an inactive link, $-\varsigma_{kr,t} > 0$ measures the relative-price decline required to activate it. Figure D.1 in the Appendix plots the distribution of such thresholds in steady state. One can see that over 2700 linkages that *do not exist* in steady state require a relative price drop of 1% or less in order to be activated. At the same time, fewer than 400 linkages that *do exist* in steady state would be severed following relative price increases of 1% or less. Hence, for a given size of an expansionary shock that lowers relative prices, more linkages are activated. A factor that makes the thresholds larger in absolute value for linkages that already exist is that the productivity parameters for connections that are optimally formed in steady state $\{b_{kr}|r \in S_k\}_{k=1}^{K}$ are on average larger than for those connections that are not formed $\{b_{kr}|r \notin S_k\}_{k=1}^{K}$.

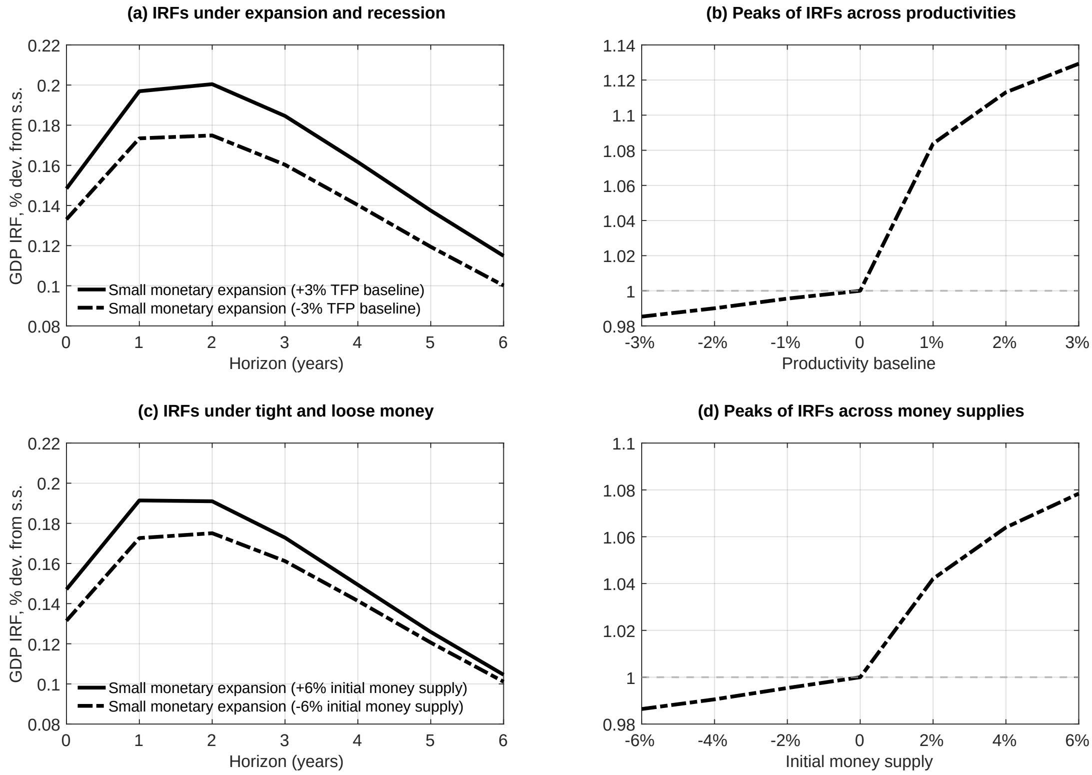
4.3.2 Small monetary shocks

I now perturb each of the baselines considered in the previous subsection with the same monetary shock, which is small in the sense that it leaves the equilibrium set of suppliers unchanged relative to the baseline in every period. The aim is to study how variations in the baseline affect the magnitude of monetary transmission to GDP.

Figure 6(a) shows IRFs of GDP to the same small monetary expansion under baselines with low ($\zeta_1 = -3\%$) and high ($\zeta_1 = 3\%$) aggregate productivity. One can see that under high productivity the response of GDP is persistently higher across horizons, despite the fact that the size of the shock is the same across the two baselines. This is because, as seen in Figure 5(a), under the high productivity baseline the average number of suppliers is higher, which strengthens complementarities in price setting and amplifies monetary non-neutrality. Moreover, Figure 5(a) also shows that under the high productivity baseline the increase in the average number of suppliers is persistent, which is what creates persistently higher non-neutrality of money, and explains why the gap between the two IRFs holds across horizons.

In order to further quantify such interplay between baseline productivity and monetary non-neutrality, Figure 6(b) plots peaks of IRFs of GDP to the same small monetary expansion across baselines with different aggregate productivities. For ease of interpretation, the peak under the neutral productivity baseline ($\zeta_1 = 0$) is normalized to one. One can see that peak

Figure 6: **Impulse responses of GDP to a small monetary expansion across baselines**

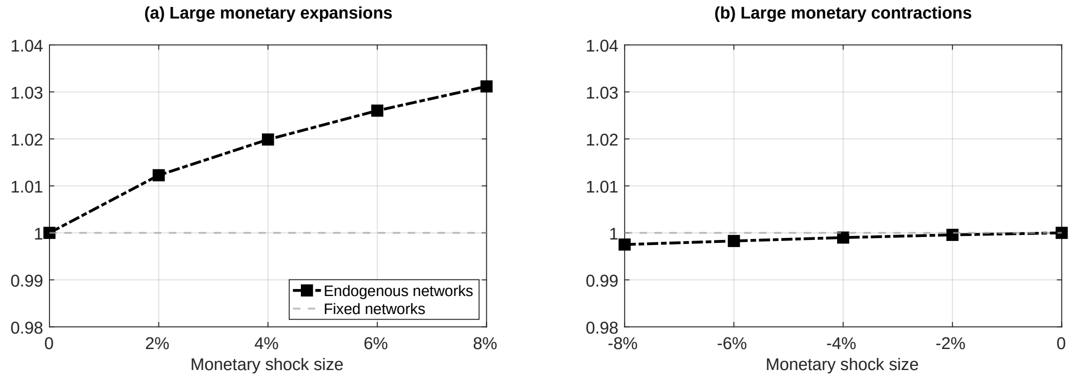


Notes: Panel (a) shows IRFs of GDP to the same small monetary expansion ($\varepsilon_1^m = 0.000001\%$), scaled per 1% of the shock, around two baselines: ($\mathcal{M}_0 = 1, \zeta_1 = -3\%$) and ($\mathcal{M}_0 = 1, \zeta_1 = 3\%$); Panel (b) shows peaks of IRFs to the same small monetary expansion around several baselines, namely those with $\mathcal{M}_0 = 1$ and $\zeta_1 = \{-3\%, -2\%, -1\%, 0, 1\%, 2\%, 3\%\}$, where that peak for $\zeta_1 = 0$ is normalized to one. Panel (c) shows the scaled IRFs of GDP to the same small monetary expansion around two baselines, with ($\zeta_t = 0, \forall t \geqslant 1, \varepsilon_{-\infty}^m = \{-6\%\}$) and ($\zeta_t = 0, \forall t \geqslant 1, \varepsilon_{-\infty}^m = \{6\%\}$); Panel (d) shows peaks of IRFs to the same small monetary expansion around several baselines, namely those with $\zeta_t = 0, \forall t \geqslant 1$ and $\varepsilon_{-\infty}^m = \{-6\%, -4\%, -2\%, 0, 2\%, 4\%, 6\%\}$, where that peak for $\varepsilon_{-\infty}^m = 0$ is normalized to one.

response of GDP under the highest productivity baseline is almost one-sixth larger than under the lowest productivity baseline. However, the rate at which monetary non-neutrality varies with baseline productivity is highly non-linear. In particular, the change in non-neutrality between the highest productivity baseline ($\zeta_1 = 3\%$) and the benchmark case ($\zeta_1 = 0$) is almost seven times larger than the change between the benchmark case and the lowest productivity baseline ($\zeta_1 = -3\%$). In this sense, in my quantitative model monetary policy becomes powerful in expansions relative to normal times, but its effectiveness does not wane by nearly as much in recessions. The latter is explained by the asymmetry in the relationship between the average number of suppliers and baseline productivity, as documented in Figure 5(a).

I now repeat the above exercise for baselines that differ in their initial money supply. Figure 6(c) shows IRFs of GDP to the same small monetary expansion under baselines with tight ($\varepsilon_{-\infty}^m = -6\%$) and loose ($\varepsilon_{-\infty}^m = 6\%$) initial money supply. One can see that under loose initial money supply the response of GDP is higher across horizons. As before, this is because under the loose initial money supply baseline the average number of suppliers is higher, which strengthens complementarities in price setting and amplifies monetary non-neutrality. Notice that according to Figure 5(c), the change in average number of suppliers triggered by a change in initial money supply is much shorter lived than that caused by changes in baseline productivity.

Figure 7: **IRFs of GDP to large monetary shocks relative to fixed networks**



Notes: Panel (a) shows peaks of IRFs to large monetary expansions around the baseline with $\zeta_t = 0, \forall t \geqslant 1$ and $\mathcal{M}_0 = 1$, for shock values $\varepsilon_1^m = \{0, 2\%, 4\%, 6\%, 8\%\}$, where that peak for $\varepsilon_1^m = 0$ is normalized to one; Panel (b) shows peaks of IRFs to large monetary contractions around the baseline with $\zeta_t = 0, \forall t \geqslant 1$ and $\mathcal{M}_0 = 1$, for shock values $\varepsilon_1^m = \{-8\%, -6\%, -4\%, -2\%, 0\}$, where that peak for $\varepsilon_1^m = 0$ is normalized to one.

As a result, additional monetary non-neutrality created by high initial money supply is shorter lived and, as can be seen in Figure 6(c), IRFs of GDP under different baseline money supplies converge to each other faster than in Figure 6(a).

In Figure 6(d) I plot peaks of GDP responses to the same small monetary expansion across baselines with different initial money supply. One can see that peak response of GDP under the baseline with the loosest money supply ($\varepsilon_{-\infty}^m = 6\%$) is almost one-tenth larger than under the baseline with the tightest money supply ($\varepsilon_{-\infty}^m = -6\%$). As in Figure 6(b) the relationship between peak GDP and changes in the baseline is highly non-linear: the drop in potency of monetary policy under tight initial money supply is much smaller than the rise in potency under loose initial money. And just as before, this is explained by the asymmetry in the relationship between the average number of suppliers and baseline money supply, as documented in Figure 5(c).

4.3.3 Large monetary shocks

Having studied properties of small monetary shocks, I now turn to monetary shocks that are large enough to affect equilibrium supplier choices. Recall that in the previous section I formally established that large monetary expansions expand the set of suppliers, which leads to a larger GDP response relative to the case where networks are fixed. Conversely, large monetary contractions reduce the number of linkages and thus deliver a smaller contraction in GDP relative to what would happen under exogenous networks.

In Figure 7 I report results where I fix the baseline at $\mathcal{M}_0 = \mathcal{Z}_t = 1, \forall t$, and subject my economy to one-time large monetary expansions $\varepsilon_1^m = \{2\%, 4\%, 6\%, 8\%\}$ and large monetary contractions $\varepsilon_1^m = \{-8\%, -6\%, -4\%, -2\%\}$. For each shock size, I normalize GDP IRFs with the response of GDP that would occur if the same shock hit an otherwise identical economy with fixed exogenous input-output linkages.

Panel (a) reports normalized peaks of GDP IRFs to large monetary expansions. Relative to an economy with fixed networks, the peak response of GDP to a 2% monetary expansion is larger by just over 1 percent, whereas for an 8% shock the response is 3 percent larger. Consistently with my formal results in the previous section, large monetary expansions deliver a more-than-proportional response of GDP, despite fully time-dependent probabilities of price adjustment.

Results for large monetary contractions are reported in Panel (b). Relative to an economy with fixed exogenous production networks, an 8% monetary contraction delivers a drop in GDP that is 0.3 percent smaller at the trough, with the gap shrinking down to just over 0.1 percent for a 4% contraction. Overall, the effects are consistent with formal results from the previous section, though their quantitative relevance is much smaller than under large monetary expansions. This is a consequence of the asymmetric sensitivity of equilibrium supplier choices to changes in money supply that was documented in Panels (c) and (d) of Figure 5, where reductions in money supply do not eliminate as many linkages as are added under an increase in money supply of the same magnitude.

5 Empirical evidence

Having established and quantified my theoretical results, I now turn to empirical assessment of the key mechanisms. In this section I use both sectoral and firm-level data to estimate network cyclicality conditional on identified technology and monetary shocks. I show that the empirical results are consistent with the key theoretical predictions of my model.

5.1 Evidence using sector-level data

5.1.1 Data

In this subsection I use sector-level data on intermediates cost share in order to perform an econometric evaluation of the predictions of my model regarding business cycle fluctuations in the shape of the production network. In particular, I use US data on sector-level employee compensation and expenditure on intermediate inputs, published by the Bureau of Economic Analysis (BEA)(U.S. Bureau of Economic Analysis, 2025a,c), to construct the share of total costs going to intermediate inputs:

$$\delta_{k,t} = \frac{\text{Intermediate inputs costs}_{k,t}}{\text{Compensation of Employees}_{k,t} + \text{Intermediate inputs costs}_{k,t}}. \tag{16}$$

I construct such intermediates cost shares for 65 three-digit sectors of the US economy at an annual frequency between 1987-2017.[32] In the case of a Cobb-Douglas production function, this measure maps directly into the model-based measure of sector-specific intermediates cost share in a particular time period, given by $\sum_{r \in S_{k,t}} \omega_{kr}, \forall k$.

[32] The granular ("Detail"-level) 6-digit BEA input-output accounts are only published every five years, with tables for 2007, 2012 and 2017 currently available under a consistent sectoral classification. Therefore, while I use the 2007 accounts to calibrate the steady-state of my model to 389 sectors of the US economy, unfortunately I cannot conduct a time series analysis of the evolution of linkages, or of the intermediates cost shares, using that dataset.

5.1.2 Econometric strategy

Once those measures have been constructed, I use the local projection approach of Jordà (2005) in order to estimate horizon-specific impulse responses of sectoral intermediates cost share to productivity and monetary shocks:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta_H s_t + \boldsymbol{\gamma}'_H \boldsymbol{x}_{k,t-1} + \varepsilon_{k,t+H}, \tag{17}$$

for $H = 0, 1, ..., \overline{H}$, where $\delta_{k,t}$ is the intermediates cost share of sector k, s_t is an identified exogenous shock, $\boldsymbol{x}_{k,t-1}$ is a vector of control variables and $\alpha_{k,H}$ represents horizon-specific sectoral fixed effects.[33] The estimated value of β_H gives the horizon-specific response of the intermediates cost share to an identified exogenous shock. According to my theory, the intermediates cost share should, *ceteris paribus*, rise following expansionary productivity and monetary shocks, and *vice versa*.

A feature of my theory is that changes in productivity and monetary conditions can affect the equilibrium set of suppliers in a non-linear fashion. For example, Section 4 documents that improvements in productivity and money supply affect the intermediates cost share much more strongly than contractionary shocks of the same magnitude. In order to test for the presence of such non-linearities in the data, I consider the following non-linear local projection specification:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta_H^{linear} s_t + \beta_H^{sign} s_t \times \mathbf{1}\{s_t > 0\} + \beta_H^{size} s_t \times |s_t| + \boldsymbol{\gamma}'_H \boldsymbol{x}_{k,t-1} + \varepsilon_{k,t+H}, \tag{18}$$

for $H = 0, 1, ..., \overline{H}$, where it follows that $\beta_H^{sign} > 0$ indicates that positively-valued shocks make the impulse response more positive, whereas $\beta_H^{size} > 0$ indicates that shocks which are large in magnitude make the response scale up more than proportionally.

In the baseline results presented in this subsection, I use identified aggregate TFP shocks from Fernald (2014, 2020) and identified annual monetary shocks from Romer and Romer (2004) that have been extended until 2007 by Wieland and Yang (2020).[34]

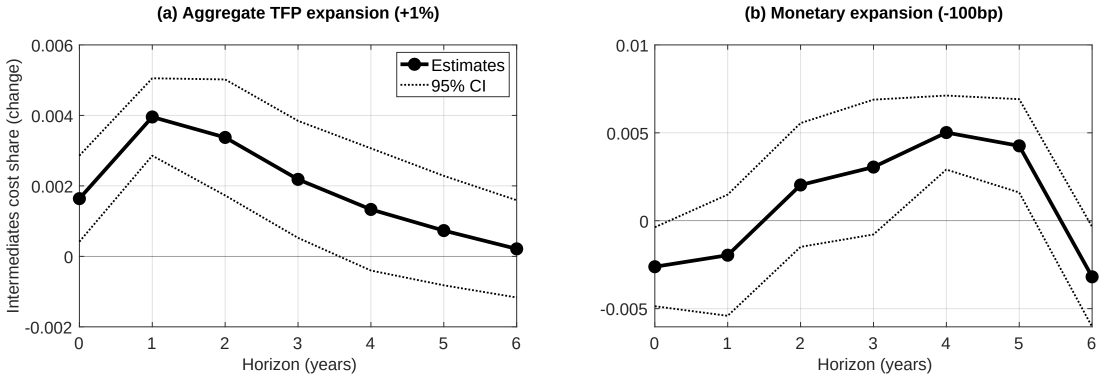
5.1.3 Estimation results

Figure 8 shows the estimation results using the linear local projection specification (17). Panel (a) shows that following a positive 1% shock to aggregate TFP, the intermediates cost share responds positively on impact, reaching a peak increase of 0.004 after one year, and then gradually declines back to zero; such positive response is consistent with my theory. As for monetary

[33] The set of controls is specified in the descriptions to figures presenting each of the estimation results. The regressions can use aggregate US macro time series, namely real GDP, nominal GDP, CPI, the effective federal funds rate, and the WTI oil price, which are, respectively, FRED series GDPC1, GDP, CPIAUCSL, DFF, and WTISPLC, (U.S. Bureau of Economic Analysis, 2020c, 2025b; U.S. Bureau of Labor Statistics, 2020; Board of Governors of the Federal Reserve System (US), 2020; Federal Reserve Bank of St. Louis, 2025).

[34] Note that the aggregate TFP shocks that I use in local projections in Section 5 are not the raw TFP series of Fernald (2014), but rather an innovation I extract from an AR(1) regression using that series. In other words, starting from the raw annual TFP index in levels ($\{TFP_t\}_t$), I run the regression $\log TFP_t = \alpha + \rho \log TFP_{t-1} + \beta \times t + \varepsilon_t^{TFP}$, where t is a time trend, and then use the fitted residuals $\{\hat{\varepsilon}_t^{TFP}\}_t$ as shocks in local projections. As for the monetary shocks, I start from the monthly series of Romer and Romer (2004), extended to 2007 by Wieland and Yang (2020), and turn them into an annual series by summing up the shocks within each calendar year. The updated monetary shocks can be accessed at Wieland (2021).

Figure 8: **Linear IRFs of intermediates cost share to TFP and monetary shocks**



Notes: Panel (a) shows estimated IRFs of the intermediates cost share to a productivity shock based on Fernald (2014), using the specification in (17); the set of controls includes one lead of productivity shock and one lag of intermediates cost share, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows the estimated IRFs of intermediates cost share to a monetary shock based on Romer and Romer (2004), using the specification in (17); the set of controls includes one lead of monetary shock and one lag of intermediates cost share, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence intervals.

expansions, Panel (b) shows that a surprise one-time 100 bp easing leads to a slight decline in the intermediates cost share on impact, after which it gradually increases, reaching a peak of 0.005 increase after four years; the response at longer horizons is consistent with theoretical predictions of the model.

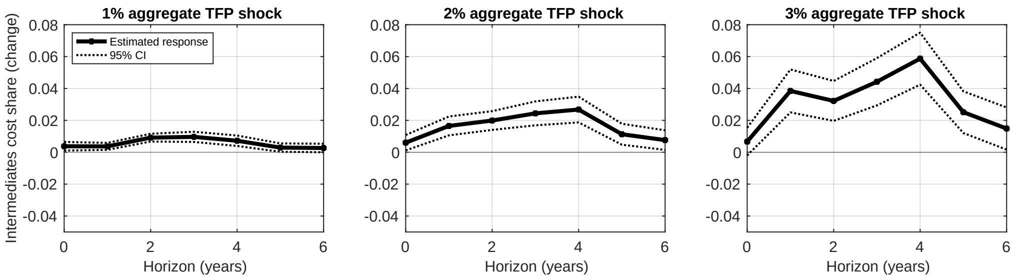
Figure 9 reports the estimation results under the non-linear local projections in (18). Panel (a) shows the responses of the intermediates cost share to aggregate TFP shocks of different signs and sizes. Following a 1% productivity expansion, the intermediates cost share rises by around 0.01 at the peak, whereas a 3% productivity expansion delivers a maximum increase in the intermediates cost share by around 0.06 at the peak. Importantly, these differences imply that the responses estimated allowing for non-linearities are not equal to linear IRFs scaled by the size of the shock, suggesting an important role for size effects. As for the negative TFP shocks, a 1% productivity contraction does not generate a drop in intermediates cost share, whereas a 3% contraction generates a modest drop in intermediates cost share of 0.02 at the trough. The latter suggests the presence of sign effects: productivity contractions do not produce drops in intermediates cost share of the same magnitude as the rises in cost share produced by productivity expansions.

In Panel (b) of Figure 9 I turn to the effect of monetary shocks of different signs and sizes on the intermediates cost share. A 50 bp monetary easing raises intermediates cost share by around 0.005 at the peak, whereas a 150 bp easing produces a rise of close to 0.02. Once again, size effects are present. As for monetary contractions, a 50 bp tightening produces a drop close to 0.005, while the larger 150 bp tightening delivers a drop of around 0.01 at the trough. Just like in the case of productivity changes, sign effects are detected: monetary contractions produce drops in the intermediates cost share that are smaller in magnitude than the increases produced by easings of the same size.

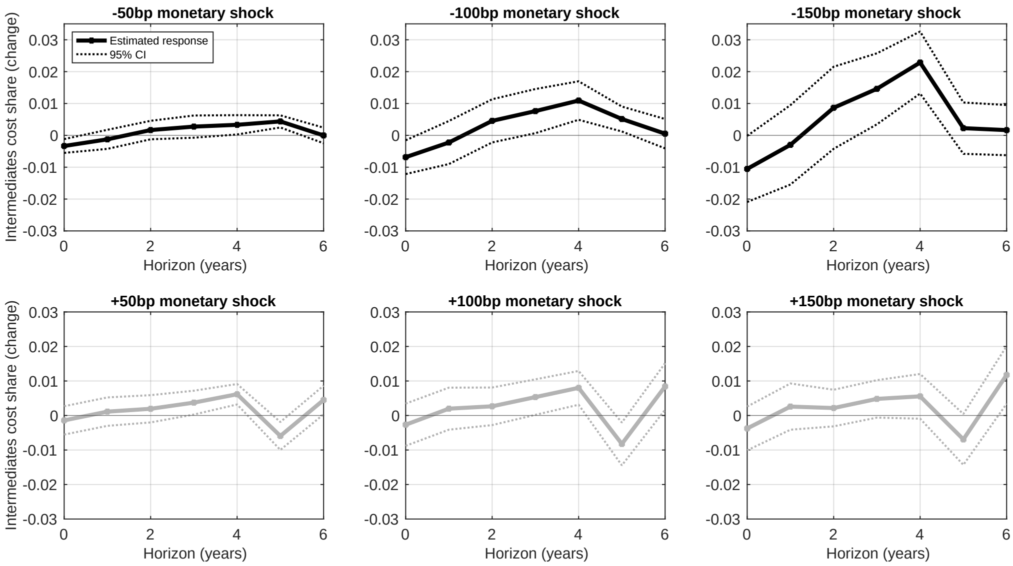
Note that for both productivity and monetary shocks, one sees important non-linearities in the response of the intermediates cost share. In particular, it follows that the responses to

Figure 9: **Non-linear IRFs of intermediates cost shares**

(a) Aggregate TFP shocks

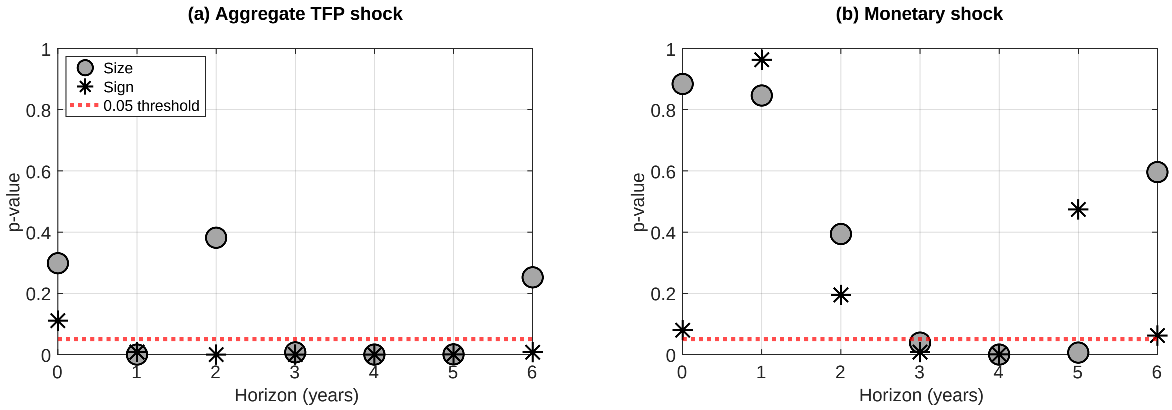


(b) Monetary shocks



Notes: Panel (a) shows the estimated non-linear IRFs of intermediates cost share to productivity shocks based on Fernald (2014), using the specification in (18); the set of controls includes one lead of productivity shock and one lag of intermediates cost share, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows the estimated non-linear IRFs of the intermediates cost share to monetary shocks based on Romer and Romer (2004), using the specification in (18); the set of controls includes one lead of monetary shock and one lag of intermediates cost share, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence intervals.

Figure 10: p-values for tests of significance of non-linear effects (sectoral data)



Notes: Panels (a) and (b) show the p-values for the test that the horizon-specific coefficients on non-linear sign and size effects from specification (18) ($\beta_H^{sign}, \beta_H^{size}$) are significantly different from zero, for Fernald (2014) productivity and Romer and Romer (2004) monetary shocks, respectively.

expansionary shocks are larger in magnitude than the effects of equally large contractionary shocks (sign effect); and that the responses to larger shocks are not the same as linearly scaled responses to smaller shocks (size effect). I now formally test whether the coefficients in (18) that capture the non-linearities ($\beta_H^{sign}, \beta_H^{size}$) are significantly different from zero. Figure 10 reports the p-values associated with the tests. For the aggregate TFP shocks, the sign effect is formally significant at all horizons beyond the impact response; the size effect is significant at 1-,3-,4- and 5-year horizons. For monetary shocks, the sign effect is significant at 3- and 4-year horizons, whereas the size effect is significant at 3-,4- and 5-year horizons.

5.1.4 Estimated vs. model-based responses

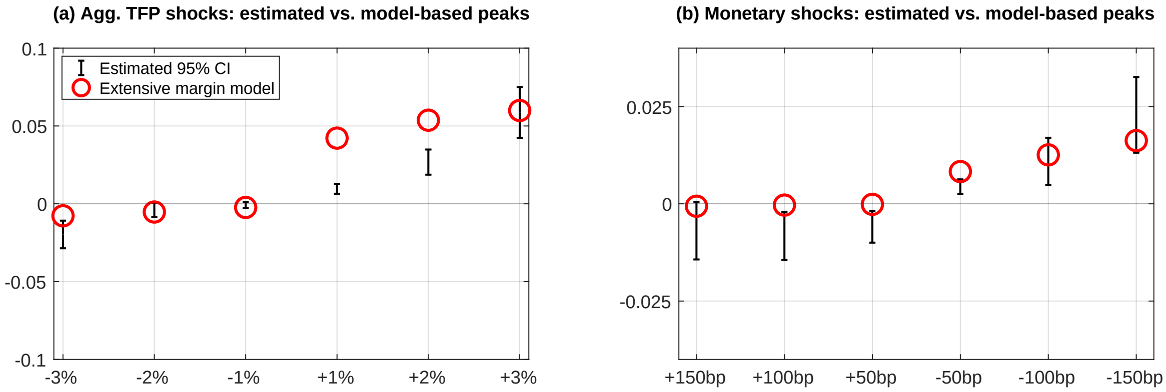
I now compare the estimated responses of the intermediates cost share with their equivalents generated by the quantitative dynamic model in Section 4.

Panel (a) of Figure 11 reports the peaks (troughs) of the intermediates cost share response to expansionary (contractionary) aggregate TFP shocks, both model-based and estimated (with 95% confidence intervals). Two features stand out. First, the model matches the magnitudes of responses for most shock sizes, with the exception of +1% and +2% shocks, where it over-predicts the response. Second, the asymmetry between expansionary and contractionary shocks, which is economically and statistically significant in the data, is matched by the model.

In Panel (b) of Figure 11 I turn to monetary shocks. Recall that the model features shocks to money supply, as opposed to interest rate shocks used in estimation. I therefore use the econometric procedure outlined in Appendix F.1 to translate interest rate shocks into an implied path of money supply growth, which I then feed into the model. One can see that the model-based peaks (troughs) lie within the confidence intervals of their estimated equivalents, with the fit particularly good for monetary expansions.

While Figure 11 focuses on the peaks (troughs) of the responses, Figure D.3 in the Appendix compares entire estimated and model-based IRFs.

Figure 11: **Intermediates cost share: estimated vs. model-based responses**



Notes: Panel (a) shows the peaks (troughs) of estimated responses of the intermediates cost share to expansionary (contractionary) productivity shocks of Fernald (2014) vs. the corresponding responses under the quantitative model in Section 4. Panel (b) shows the peaks (troughs) of estimated responses of the intermediates cost share to expansionary (contractionary) monetary shocks of Romer and Romer (2004) vs. the corresponding responses under the quantitative model in Section 4. See Appendix F.1 for the procedure that translates the Romer and Romer (2004) shocks into the money growth path that is then fed into the quantitative model.

5.1.5 Discussion: margin of adjustment

While my model successfully matches most features of the estimated responses, it does not immediately rule out alternative theories. Indeed, movements in the intermediates cost share could be driven by either the extensive or the intensive margin, while my quantitative model operates solely through the former. I now show that the estimated dynamics of the intermediates cost share are difficult to rationalize with a model that relies *exclusively* on the intensive margin of network adjustment. In particular, here I summarize the exercises in Appendix E, which considers economies with a fixed exogenous set of suppliers and a constant elasticity of substitution (CES) production, showing that it struggles to match the estimated responses.

CES roundabout model I first consider a *fully non-linear* one-sector ($K = 1$) roundabout economy (Basu, 1995) with the technology $Y_t(j) = \mathcal{Z}_t \left[(1-\bar{\delta})^{\frac{1}{\eta}} N_t^{\frac{\eta-1}{\eta}}(j) + \bar{\delta}^{\frac{1}{\eta}} Z_t^{\frac{\eta-1}{\eta}}(j) \right]^{\frac{\eta}{\eta-1}}$, where $\eta > 0$ is the elasticity of substitution across labor and intermediates. The rest of the model is unchanged. The aggregate cost share of intermediates is $\delta_t = \bar{\delta} \times \frac{(P_t/W_t)^{1-\eta}}{(1-\bar{\delta})+\bar{\delta}(P_t/W_t)^{1-\eta}}$. While Appendix E describes the full calibration, here I focus on the elasticity η. Under $\eta = 0.78$ (Oberfield and Raval, 2021), a widely established value in the literature, the model misses the data along three important dimensions (Figure E.1): (i) it produces *countercyclical* δ_t; (ii) movements in δ_t are essentially symmetric across recessions and expansions; (iii) the size of fluctuations in δ_t is an order of magnitude lower than in the data. I then estimate my own value of η by matching model-based and empirical responses of δ_t, obtaining $\eta^* = 2.27$.[35] As Figure E.3 shows, under the estimated value the model succeeds in producing procyclical δ_t, but the other two issues remain: the responses are essentially symmetric (no sign effect) and the magnitudes of responses are still substantially lower than in the data for expansionary shocks. All in all, I conclude that the CES roundabout model, even if directly disciplined to be as close

[35] The elasticity of 2.27 is also within the range of estimates in some more recent papers, such as Chan (2021) and Mertens and Schoefer (2024).

as possible to the data, does not reproduce its salient features.

CES network model I also consider a *fully non-linear* K-sector economy with an exogenous supplier set and the technology: $Y_{k,t}(j) = \mathcal{Z}_t \left[\left(1 - \sum_{r=1}^{K} \omega_{kr}\right)^{\frac{1}{\eta}} N_{k,t}^{\frac{\eta-1}{\eta}}(j) + \sum_{r=1}^{K} \omega_{kr}^{\frac{1}{\eta}} Z_{kr,t}^{\frac{\eta-1}{\eta}}(j) \right]^{\frac{\eta}{\eta-1}}$. I calibrate the model to $K = 389$ sectors, with the same strategy as in the main text. Under $\eta = 0.78$ the CES network model has the same issues as the roundabout setting: δ_t is countercyclical, the responses are symmetric and much smaller than in the data (Figure E.4). Setting $\eta = 2.27$ helps produce procyclical δ_t, but the issues with symmetry and magnitudes remain.[36] All in all, I similarly conclude that the CES network model does not reproduce the salient features of the cost share responses. I also quantify the non-linearity in monetary transmission generated by the CES network model. Under $\eta = 2.27$, where δ_t is procyclical, the model produces cycle-, path- and size-dependence as the extensive margin model, but the strength of non-linearity is substantially weaker (Figures E.7, E.9).

Importantly, the above results do not rule out a role for the intensive margin altogether, nor do they imply that only the extensive margin matters. Instead, they suggest that the extensive margin likely plays a non-trivial role, and I test this formally in the next subsection.

5.2 Evidence using firm-level data

5.2.1 Data

A unique prediction of my model is that the adjustment in the relative reliance on intermediate inputs can happen at the extensive margin. In this subsection, I use firm-level data on the number of suppliers of US publicly listed firms in order to perform an econometric evaluation of the *extensive margin* of network adjustment over the business cycle. In particular, I use the dataset constructed by Atalay et al. (2011, nd) based on US Compustat data, which contains a time series for the firm-level *indegree*, which is the number of suppliers the firm has that are also included in the dataset.[37]

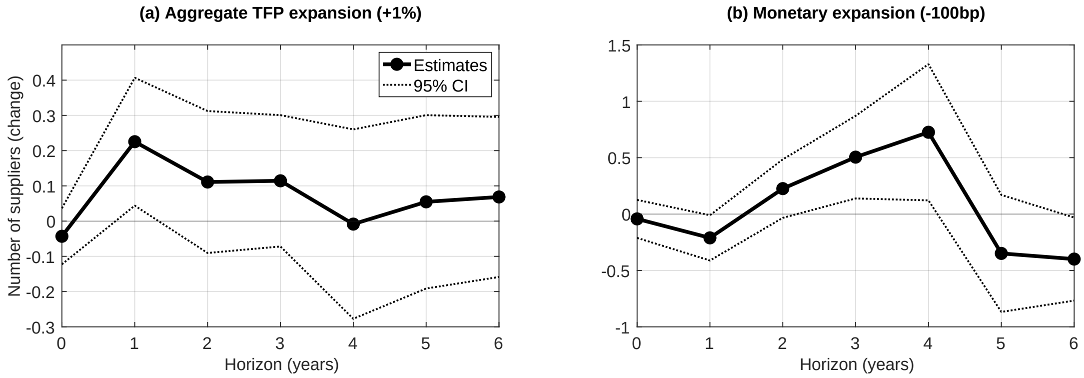
5.2.2 Econometric strategy

Using the indegree measure, I once again follow the local projection approach of Jordà (2005) in order to estimate horizon-specific impulse responses of a firm's number of suppliers to pro-

[36] Note that the sectoral cost share of labor $(1-\delta_{k,t})$ admits an *exact* log-linear form: $\log(1-\delta_{k,t}) = \log\left(1 - \sum_r \omega_{kr}\right) + (1-\eta)(m_t - p_{k,t} - z_t + \mu_{k,t})$, where $z_t \equiv \log \mathcal{Z}_t$ and $\mu_{k,t} \equiv \log(P_{k,t}/\mathcal{Q}_{k,t})$ is the sectoral markup. For the shock magnitudes and values of η that I consider, the sectoral price and markup dynamics is approximately linear, which in turn contributes the approximate linearity of the sectoral cost shares of labor and intermediates as well.

[37] Note that the FactSet Revere dataset, which I use in Figure 1(a), has a clear advantage in terms of the breadth of coverage relative to the Compustat-based data in Atalay et al. (2011, nd). In particular, the Atalay et al. (2011, nd) dataset only covers publicly listed firms, and only formally captures major input-output relationships, namely those where customers contribute to more than 10% of a supplier's revenue. Instead, FactSet Revere covers both listed and non-listed firms and does not restrict its coverage to major relationships only. At the same time, the annual FactSet Revere data starts only in 2003, while the Atalay et al. (2011, nd) data starts in 1976 and thus has a clear advantage in terms of its time dimension. Given that identified monetary shocks series ends in 2007 (due to the subsequent zero lower bound period) I use the Atalay et al. (2011, nd) dataset for regression analysis in this section.

Figure 12: **Linear IRFs of the number of suppliers to TFP and monetary shocks**



Notes: Panel (a) shows the estimated IRFs of the number of suppliers to a productivity shock based on Fernald (2014), using the specification in (19); the set of controls includes four lags of the number of suppliers, one lag of productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows estimated IRFs of the number of suppliers to a monetary shock based on Romer and Romer (2004), using the specification in (19); the set of controls includes one lead of monetary shock, four lags of the number of suppliers, one lag of monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence intervals.

ductivity and monetary shocks:

$$indeg_{j,t+H} = \alpha_{j,H} + \beta_H s_t + \boldsymbol{\gamma}'_H \boldsymbol{x}_{j,t-1} + \varepsilon_{j,t+H}, \tag{19}$$

for $H = 0, 1, ..., \overline{H}$, where $indeg_{j,t}$ is the indegree (number of suppliers) of firm j in year t, s_t is an identified exogenous shock, $\boldsymbol{x}_{j,t-1}$ is a vector of control variables and $\alpha_{j,H}$ represents horizon-specific firm fixed effects. The estimated value of β_H gives the horizon-specific response of the number of suppliers to an identified exogenous shock; according to my theory, the number of suppliers should, *ceteris paribus*, rise following expansionary productivity and monetary shocks, and *vice versa*.

As discussed earlier, my model predicts a non-linear relationship between changes in productivity and monetary conditions, and the equilibrium set of suppliers. In order to test for such non-linear adjustment at the extensive margin, I once again extend the previous specification to include additional terms:

$$indeg_{j,t+H} = \alpha_{j,H} + \beta_H^{linear} s_t + \beta_H^{sign} s_t \times \mathbf{1}\{s_t > 0\} + \beta_H^{size} s_t \times |s_t| + \boldsymbol{\gamma}'_H \boldsymbol{x}_{j,t-1} + \varepsilon_{j,t+H}, \tag{20}$$

for $H = 0, 1, ..., \overline{H}$, where it follows that $\beta_H^{sign} > 0$ indicates that positively-valued shocks make the impulse response more positive, whereas $\beta_H^{size} > 0$ indicates that larger shocks make the response more positive.

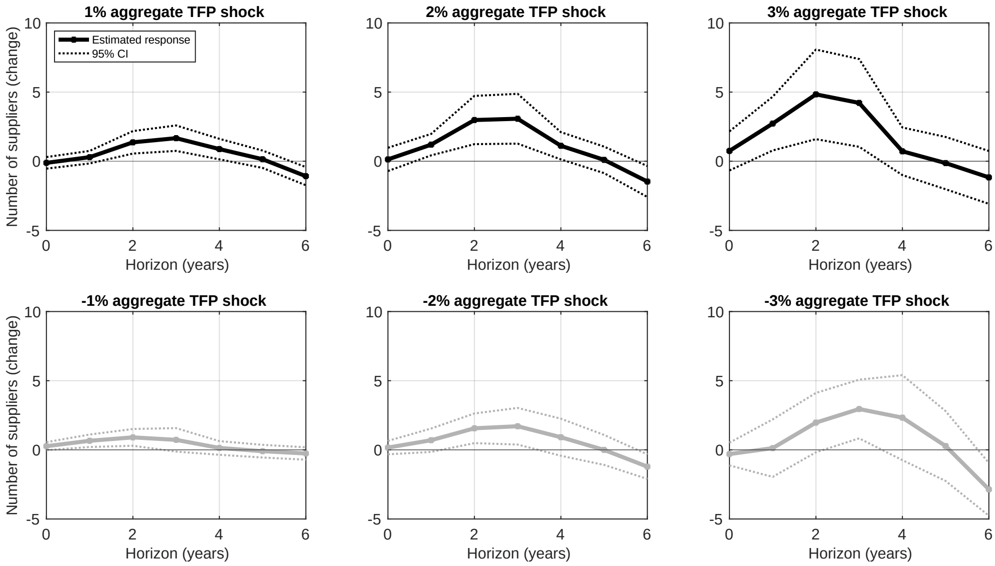
In the baseline results presented in this subsection, I once again use identified annual productivity shocks from Fernald (2014, 2020) and identified annual monetary shocks from Romer and Romer (2004) that have been extended by Wieland and Yang (2020).

5.2.3 Estimation results

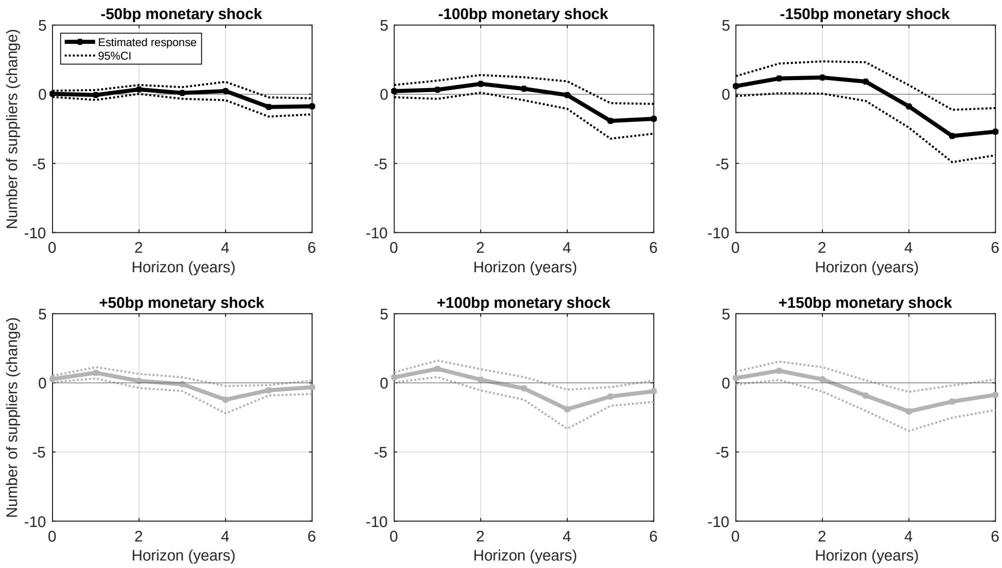
Figure 12 shows estimation results using linear local projections in (19). As one can see, following a positive 1% shock to aggregate TFP, the (average) number of suppliers responds

Figure 13: **Non-linear IRFs of number of suppliers**

(a) Aggregate TFP shocks

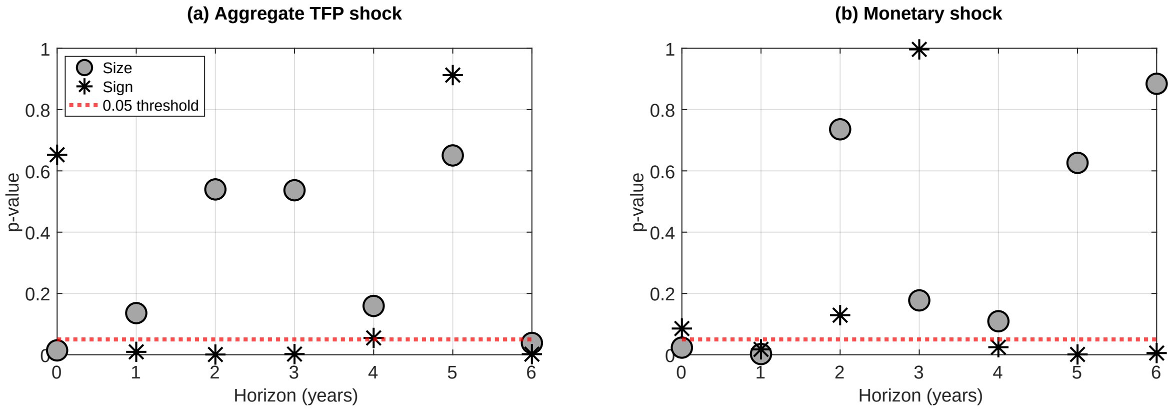


(b) Monetary shocks



Notes: Panel (a) shows the estimated non-linear IRFs of the number of suppliers to productivity shocks based on Fernald (2014), using the specification in (20); the set of controls includes one lead of productivity shock, four lags of the number of suppliers, one lag of productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows estimated non-linear IRFs of the number of suppliers to monetary shocks based on Romer and Romer (2004), using the specification in (20); the set of controls includes one lead of monetary shock, four lags of the number of suppliers, one lag of monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence intervals.

Figure 14: p-values for tests of significance of non-linear effects (firm-level data)



Notes: Panels (a) and (b) show p-values for the test that the horizon-specific coefficients on non-linear sign and size effects from specification (20) ($\beta_H^{sign}, \beta_H^{size}$) are significantly different from zero, for Fernald (2014) productivity and Romer and Romer (2004) monetary shocks, respectively.

positively on impact, reaching a peak increase of around 0.20 after one year, and then gradually declines back to zero; such positive response is consistent with my theory. As for a monetary expansion, a surprise one-time 100 bp easing first leads to a small, statistically insignificant decline, after which it gradually increases, reaching a peak of 0.85 increase after four years; once again, the response at longer horizons is both statistically significant and consistent with theoretical predictions.

Figure 13 reports the results obtained using the non-linear local projections in (20). Panel (a) shows the responses of the (average) number of suppliers to aggregate TFP shocks of different signs and sizes. Following a 1% productivity expansion, the number of suppliers rises by almost 2 at the peak, whereas a 3% productivity expansion increases the number of suppliers per firm by approximately 5. Turning to aggregate TFP contractions, one can see that they are unable to deliver drops in the number of suppliers for the first five years after the shock, with some minor statistically significant reductions detected at the 6-year horizon. The latter suggests that sign effects, previously found in the sector-level estimation and in the quantitative version of my model, are similarly a feature of empirical behavior of the firm-level number of suppliers.

In Panel (b) of Figure 13 I present the effect of monetary shocks of different signs and sizes on the (average) number of suppliers. A 50 bp monetary easing raises the number of suppliers by almost 1 at the peak, whereas a 150 bp easing produces a rise of close to 2. As for monetary contractions, a 50 bp tightening produces a drop close to 1, while the larger 150 bp tightening delivers a drop of around 2 at the trough.

As with sector-level estimation, I formally test whether the coefficients in (20) that capture the non-linearities ($\beta_H^{sign}, \beta_H^{size}$) are significantly different from zero. Figure 14 reports the p-values associated with the tests. For the aggregate TFP shocks, the sign effect is formally significant at 1-,2-,3-,4- and 6-year horizons; the size effect is significant on impact and at the 6-year horizon. For monetary shocks, the sign effect is significant at the 1-,4-,5- and 6-year horizons, whereas the size effect is significant on impact and at the 1-year horizon.

5.3 Further results and robustness checks

In Appendix F, I conduct a number of further econometric exercises and robustness checks, which I summarize here.

Estimation in broad sectoral groups In Appendix F.2.2 I re-estimate the effect of identified shocks on the intermediates cost share, allowing for non-linearity as in specification (18), at the level of six broad sectoral groups: "Non-Manufacturing Goods", "Durable Manufacturing Goods", "Non-Durable Manufacturing Goods", "Trade, Transportation and Warehousing", "Information, FIRE and Professional Services" and "Education, Healthcare and Other Services". The results suggest that largest sensitivities of the intermediates cost share to changes in productivity are detected in "Trade, Transportation and Warehousing" and "Durable Manufacturing Goods", whereas very little responsiveness is found in "Information, FIRE and Professional Services" and "Education, Healthcare and Other Services". As for the effect of monetary shocks, by far the largest response is found in "Non-Manufacturing Goods", whereas the smallest effect is found in "Education, Healthcare and Other Services".

Role of intermediates inventories A potential concern with the evidence on the cyclicality of the sectoral intermediates cost shares is that it may reflect timing patterns in the use of *inventories* of intermediate inputs. For example, in a recession, firms may stop purchasing new intermediates and decide to use up their intermediates inventories instead. Combined with continued payments to labor, this would mechanically decrease the measured intermediates cost share even without the endogenous network mechanism. To address this issue, in Appendix F.2.3 I extend the specification in (17) to control for whether or not a given sector relies heavily on inventories of intermediates, and check whether the cost share procyclicality result continues to hold among the sectors that *do not* strongly accumulate intermediates inventories.[38] The results in Figures F.21 and F.22 suggest that the intermediates cost share procyclicality, conditional on either productivity or monetary shocks, remains statistically significant among the sectors that do not rely heavily on inventories of intermediates. While the high-inventory sectors do show stronger responses to the identified shocks, the difference with the low-inventory sectors is not statistically significant.

Alternative non-linear specification While in specifications (18) and (20) I capture possible non-linearities using interactions with a sign dummy and with the absolute value of the shock, I also consider an alternative non-linear specification. In particular, I add quadratic and cubic shocks, which similarly capture possible sign and size effects. Figure F.3 reports the estimated response of the intermediates cost share using this alternative specification, formally introduced in (F.2). Reassuringly, the results remain virtually unchanged relative to the non-

[38] Sectors in the low-inventory category are those in the broad categories of "Trade, Transportation and Warehousing", "Information, FIRE and Professional Services" and "Education, Healthcare and Other Services", whereas the high-inventory sectors are those in "Non-Manufacturing Goods", "Durable Manufacturing Goods" and "Non-Durable Manufacturing Goods".

linear specification in (18), both in terms of magnitudes of effects, as well as when it comes to sign and size effects and their statistical significance (Figure F.11). Similarly, Figure F.17 reports the estimation results for the indegree using the alternative non-linear specification, formally captured in (F.4). One can see that relative to the specification (20), results remain virtually unaffected, both in terms of magnitudes of effects, as well as when it comes to sign effects and their statistical significance (Figure F.19).

Estimation with oil shocks While the baseline econometric estimation uses an aggregate supply shock, namely the TFP shock of Fernald (2014), in Appendix F.4 I obtain additional empirical results for a sector-specific supply shock, namely the oil news shock of Känzig (2021b,a). For the effect of oil price shocks on the intermediates cost share, Panel (a) of Figure F.10 reports the estimation results using the non-linear local projection specification (18). One can see that oil price increases lead to statistically significant drops in the intermediate cost share, and *vice versa* for oil price decreases. I also study the response of the firm-level indegree (number of suppliers) to oil price shocks. In particular, Panel (a) of Figure F.18 reports estimation results using the non-linear local projection specification in (20). One can see that oil price increases lead to a statistically significant drop in the number of suppliers, and *vice versa* for oil price decreases.

Role of extensive margin variation In Appendix F.3.5 I use the Atalay et al. (2011) dataset to investigate the relationship between the firm-level intermediates cost share and the number of suppliers. In particular, I estimate the association between the intermediates cost share and the number of suppliers, as well as measure the fraction of (within) variation in the intermediates cost share that is accounted for by the extensive margin of the network. Since the dataset formally only captures the major input-output linkages, which are the *least* likely to be the marginal ones to be dropped and regained over the business cycle, the estimated fraction of variation explained by the extensive margin is likely a conservative *lower bound*. Therefore, I also repeat the estimation on sub-samples of firms with more extensive reporting of suppliers. Two major findings emerge from the analysis, reported in Table F.24. First, there is a statistically significant and robust association between the intermediates cost share and the number of suppliers, with a 1% increase in the number of suppliers being associated with around 1.2% increase in the intermediates cost share. Second, the full sample implies that approximately 20% of within (cyclical) variation in the intermediates cost share is explained by the extensive margin; focusing on sub-samples with richer coverage implies a much larger role for the extensive margin, with as much as 40-50% of the intermediates cost share variation explained by the number of suppliers.

Pattern of supplier adoption and termination The network variation results in Lemmas 2 and 3 imply a very specific *pattern* underlying the addition of suppliers: following a productivity or monetary expansion, firms keep all the existing suppliers and add new ones on top.

Similarly, whenever firms drop suppliers, they replace them with labor, and not by connecting to some other suppliers. While the evidence in Section 5.2 points to an overall procyclicality in the number of suppliers per firm, in Appendix F.3.4 I provide additional results that the suppliers are gained and lost in a manner that is consistent with my theory. To do that, I use the Atalay et al. (2011) dataset to measure, for each firm, the number of suppliers gained and lost in every year relative to the previous year. The results in Figures F.23 and F.24 find no evidence that overall indegree increases driven by TFP or monetary expansions are accompanied by rises in the number of suppliers lost, in line with my theoretical predictions. Similarly, I find that overall drops in the number of suppliers driven by contractionary TFP or monetary shocks are driven by a statistically significant rise in the number of suppliers lost and no statistically significant change in the number of suppliers gained. Once again, such evidence is consistent with the theoretical predictions of my model.

Response of the outdegree In Appendix F.3 I also study the response of the firm-level *outdegree*, which is the number of *customer* firms that the firm has. Figure F.13 reports the estimation results using the linear specification (19): one can see that both an aggregate TFP expansion and a monetary easing produce a statistically significant increase in the average number of customers. An additional prediction of my model is that under productivity shocks, firms with a higher probability of price adjustment see larger fluctuations in their outdegree and *vice versa* for monetary shocks. I formally test this prediction using the specification (F.6), which controls for the average duration of price spells of a given firm.[39] Figure F.14 shows that following an aggregate TFP expansion only the firms with low price duration see an increase in their number of customers immediately after the shock. By contrast, following a monetary easing, it is only the firms with the high price duration that see their number of customers increase in the immediate aftermath of the shock.

6 Conclusion

I develop a dynamic multi-sector general equilibrium model with sticky prices, in which input-output linkages are formed endogenously through firms' optimizing decisions. I provide novel empirical evidence on the cyclical behavior of production networks and show that the model can replicate the observed variation, conditional on either productivity or monetary shocks driving the cycle. In particular, in the data the reliance on intermediates rises following productivity improvements and monetary easings. In the model, technological improvements also increase the reliance on intermediates, as expanding the set of suppliers lowers firms' unit costs directly through higher productivity and indirectly through lower prices charged by suppliers. Similarly,

[39] In order to measure the average duration of price spells for a firm j (d_j), I first match each firm to a 6-digit sector based on the BEA input-output accounts. I then use the 6-digit measures of adjustment frequency (Pasten et al., 2020a,b) to assign a probability of adjustment $(1-\alpha_j)$ to each firm, and finally measure the duration of price spells as $d_j = -1/\log \alpha_j$. Naturally, the mapping from sector-level frequencies to firm-level durations is necessarily noisy. However, I aim to partly alleviate this concern by splitting firms by whether their duration exceeds a specific threshold (e.g., 15 months) rather than fully exploiting the intensive margin variation in the price duration that is prone to measurement error.

monetary easings also expand the production network in the model, as they make prices charged by supplier firms cheaper relative to the cost of in-house labor.

The model delivers a novel mechanism for state-dependence in monetary transmission, which is generated by cyclical variation in the strength of complementarities in price setting. This mechanism makes the strength of monetary transmission depend on the *phase of the business cycle*, *past monetary stance* and the *size of the shock* even if the probability of price adjustment is state-independent. In particular, in periods of high productivity and loose monetary policy, firms optimally decide to connect to more suppliers, which strengthens pricing complementarities and amplifies monetary non-neutrality. Conversely, episodes of low productivity and tight monetary policy lead to sparser networks and weaker complementarities in price setting, which diminishes monetary non-neutrality. At the same time, larger monetary expansions have a disproportionately *larger* positive effect on GDP compared to smaller monetary expansions, as the former expand the production network. Larger monetary contractions instead have a disproportionately *smaller* negative effect on GDP, as they shrink the network. The non-linearities in the transmission of monetary shocks in the theoretical model are consistent with the econometric evidence (Tenreyro and Thwaites, 2016; Jordà et al., 2020; Alpanda et al., 2021; Ascari and Haber, 2022).

The analysis offers multiple avenues for future research. First, the optimal choice of suppliers can be studied in an open-economy setting, where firms minimize their production costs by deciding whether to purchase intermediate inputs from domestic producers or to import them instead. Such a mechanism would pin down the optimal degree of participation in global value chains, with important implications for both monetary and trade policies. Second, the analysis has focused on the positive aspects of monetary transmission under endogenous production networks, while future work could study the optimal conduct of monetary policy.

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Supplemental Appendix

Endogenous production networks and non-linear monetary transmission

(Mishel Ghassibe, *CREi, UPF, BSE & CEPR*)

A Detailed proofs

Proof of Proposition 1. (a) Existence;

In periods $t \geqslant 2$ prices are flexible, and firms' problem is static and collapses to that studied by Acemoglu and Azar (2020). Hence, it suffices to show existence at $t = 1$, where prices are sticky. For convenience, let $\overline{P}_k \equiv \frac{P_k}{\mathcal{M}}$ and $\overline{\mathcal{Q}}_k \equiv \frac{\mathcal{Q}_k}{\mathcal{M}}$ be ratios of sectoral prices and unit costs to money supply.

As a first step, I am going to establish existence of equilibrium prices and supplier choices. Subsequently, I am going to show how existence of equilibrium prices and supplier choices implies existence of equilibrium quantities.

Rewrite equation for sectoral price aggregation, using the fact that every sectoral production function exhibits constant returns to scale:

$$P_k^{1-\theta} = \alpha_k P_{k,0}^{1-\theta} + (1-\alpha_k)\left[\mathcal{M}(1+\mu_k)\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k})\right]^{1-\theta}$$

$$\overline{P}_k = \left[\alpha_k \left[\frac{P_{k,0}}{\mathcal{M}}\right]^{1-\theta} + (1-\alpha_k)\left[(1+\mu_k)\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k})\right]^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{A.1}$$

or $\overline{P}_k = f_k[\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k}); \mathcal{M}]$, where $f_k' > 0$, $f_k'' < 0 \forall k$, and $f_k(.; \overline{\mathcal{M}}) < f_k(.; \underline{\mathcal{M}}), \forall k : \alpha_k > 0$ for any $\overline{\mathcal{M}} > \underline{\mathcal{M}}$. Define the following function:

$$\kappa(\{\overline{P}_k\}_{k=1}^K) =$$
$$= \left[f_1[\min_{S_1} \overline{\mathcal{Q}}_1(S_1, \mathcal{A}_1(S_1), \{\overline{P}_r\}_{r\in S_1}); \mathcal{M}], ..., f_K[\min_{S_K} \overline{\mathcal{Q}}_K(S_K, \mathcal{A}_K(S_K), \{\overline{P}_r\}_{r\in S_K}); \mathcal{M}]\right]^T \tag{A.2}$$

One can show that $\kappa(.)$ has a fixed point, and that its fixed point corresponds to equilibrium ratios of sectoral prices to money supply. This follows from the fact that the set $\mathbb{L} = \{\overline{P}_k \geqslant 0, \forall k : \overline{P}_k = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k}); \mathcal{M}]\}$ is a complete lattice with respect to operations

$$\mathbb{P} \bigwedge \mathbb{Q} = [\min(P_1, Q_1), ..., \min(P_K, Q_K)]^T$$

and

$$\mathbb{P} \bigvee \mathbb{Q} = [\max(P_1, Q_1), ..., \max(P_K, Q_K)]^T.$$

I now formally establish the latter point.

First, a producer in any sector k can produce using labor only, incurring a unit cost of $\overline{\mathcal{Q}}_k(\varnothing, \mathcal{A}_k(\varnothing), \varnothing)$, which delivers the price to money supply ratio that is given by the expression $\overline{P}_k' \equiv f_k[\overline{\mathcal{Q}}_k(\varnothing, \mathcal{A}_k(\varnothing), \varnothing); \mathcal{M}], \forall k$, which does not depend on sectoral prices, and so $\kappa(\{\overline{P}_k\}_{k=1}^K) \leqslant [\overline{P}_1', ..., \overline{P}_K']^T$ for all $\{\overline{P}_k\}_{k=1}^K$. Since labor is an essential input by assumption, it follows that at zero prices the unit cost is positive for any choice of suppliers: $\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), 0_{|S_k|\times 1}) > 0, \forall S_k, k$.

Let $\overline{P}_k'' \equiv \kappa_k(0_{|S_k|\times 1}) = f_k[\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), 0_{|S_k|\times 1}); \mathcal{M}], \forall k$. Since the unit cost function is increasing in sectoral prices and $f'_k > 0, \forall k$, it follows that $\kappa(\{\overline{P}_k\}_{k=1}^K) \geqslant \kappa(0_{|S_k|\times 1}) = [\overline{P}''_1, ..., \overline{P}''_K]^T$ for all $\{\overline{P}_k\}_{k=1}^K$. Hence, $\mathbb{O} \equiv \times_{k=1}^K [\overline{P}''_k, \overline{P}'_k]$ is a complete lattice and κ maps $\mathbb{O}$ to $\mathbb{O}$.

Second, for any $\{\overline{P}_k\}_{k=1}^K$ and $\{\overline{\overline{P}}_k\}_{k=1}^K$ such that $\overline{P}_k \leqslant \overline{\overline{P}}_k \forall k$ we have

$$\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k}) \leqslant \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{\overline{P}}_r\}_{r\in S_k}), \forall S_k, \quad \forall k$$

and hence

$$\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k}) \leqslant \min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{\overline{P}}_r\}_{r\in S_k}), \quad \forall k$$

which further implies

$$f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k})] \leqslant f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{\overline{P}}_r\}_{r\in S_k})], \quad \forall k$$

and

$$\kappa(\{\overline{P}_k\}_{k=1}^K) \leqslant \kappa(\overline{\{\overline{P}_k\}}_{k=1}^K).$$

Hence, κ is an order-preserving function which maps a complete lattice $\mathbb{O}$ onto itself. By Knaster-Tarski theorem, it follows that κ has a minimal fixed point, which yields equilibrium ratios of sectoral prices to money supply $\{\overline{P}_k\}_{k=1}^K$ and supplier choices $\{S_k\}_{k=1}^K$. Trivially, equilibrium sectoral prices are then recovered as $P_k = \mathcal{M} \times \overline{P}_k, \forall k$. As for equilibrium firm-level prices within a particular sector, those are given by $P_{k,0}$ for non-adjusters at $t = 1$ and by $(1 + \mu_k)\mathcal{Q}_k(S_k, \mathcal{A}_k(S_k), \mathcal{M}, \{P_r\}_{r\in S_k})$ for adjusters at $t = 1$.

Having established existence of equilibrium prices and supplier choices, I now show that their existence allows me to recover equilibrium allocations. First, the cash-in-advance constraint allows me to recover aggregate final consumption $C_t = \mathcal{M}_t/P_t^c$, where $P_t^c = P_t^c[P_{1,t}, ..., P_{k,t}]^T$ is the consumption price index. Given aggregate final consumption, sectoral consumptions $C_{k,t} = C_{k,t}(\{P_{k,t}\}_{k=1}^K, C_t)$ are pinned down uniquely from properties of the aggregator $u(.)$. As for firm-level consumptions, those are pinned down as $C_{k,t}(j) = \left(\frac{P_{k,t}(j)}{P_{k,t}}\right)^{-\theta} C_{k,t}$. Let $z_{kr,t} \equiv \frac{Z_{kr,t}(j)}{Y_{k,t}(j)}$ and $n_{k,t} \equiv \frac{N_{k,t}(j)}{Y_{k,t}(j)}$ be firm-level ratios of factor demand to output; given assumed properties of the production function, those are common to all firms within a particular sector, and independent of output. From goods market clearing condition:

$$C_{k,t}(j) + \sum_{r=1}^K \int_{j'\in\Phi_r} Z_{rk,t}(j', j)dj' = Y_{k,t}(j), \quad \forall k, \forall j \in \Phi_k \tag{A.3}$$

$$C_{k,t} + \sum_{r=1}^K z_{rk,t} \int_{j'\in\Phi_r} \left(\frac{P_{r,t}(j')}{P_{r,t}}\right)^{-\theta} Y_{r,t}dj' = Y_{k,t}, \quad \forall k, \forall j \in \Phi_k \tag{A.4}$$

$$C_{k,t} + \sum_{r=1}^K z_{rk,t}\Delta_{r,t}Y_{r,t} = Y_{k,t}, \quad \forall k, \forall j \in \Phi_k \tag{A.5}$$

where $\Delta_{r,t} = \int_{j\in\Phi_r} \left(\frac{P_{r,t}(j)}{P_{r,t}}\right)^{-\theta} dj$ is price dispersion in sector r. The expression above allows me to recover sectoral outputs:

$$[Y_{1,t}, ..., Y_{k,t}]^T = (I - \Omega_t')^{-1}[C_{1,t}, ..., C_{k,t}]^T, \tag{A.6}$$

where Ω_t is a matrix whose entries are given by $[\Omega_t]_{kr} = z_{rk,t}\Delta_{r,t}$. Given sectoral outputs, firm-level outputs can be recovered as $Y_{k,t}(j) = \left(\frac{P_{k,t}(j)}{P_{k,t}}\right)^{-\theta} Y_{k,t}$, further allowing to pin down factor demands: $Z_{kr,t}(j) = z_{kr,t}Y_{k,t}(j)$, $N_{k,t}(j) = n_{k,t}Y_{k,t}(j)$.

(b) Uniqueness of sectoral prices and final consumptions;

As before, uniqueness of sectoral prices under no nominal rigidities ($t \geqslant 2$) is shown in Acemoglu and Azar (2020). Here I only need to establish uniqueness of equilibrium prices under nominal rigidities in $t = 1$.

Let $\{\overline{P}_k\}_{k=1}^K$ be the minimal element of complete lattice $\mathbb{L}$ introduced in (a); suppose $\{\overline{P}_k^*\}_{k=1}^K$ is another set of equilibrium ratios of sectoral prices to money supply; then it must be that $\overline{P}_k^* \geqslant \overline{P}_k, \forall k$ with a strict inequality for at least one sector. Below I show by contradiction that the latter implies $\{\overline{P}_k\}_{k=1}^K$ is the unique equilibrium set of ratios of sectoral prices to money supply.

Given assumed properties of the production function, it follows that $\overline{\mathcal{Q}}_k$ is concave in $\{\overline{P}_k\}_{k=1}^K$. Moreover, pointwise minimum of a finite collection of concave functions is also concave, and it hence follows that $\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k})$ is also concave. Recall that $f_k' > 0, f_k'' < 0, \forall k$ so that f_k is increasing and concave, and hence

$$\kappa_k(\{\overline{P}_k\}_{k=1}^K) = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k})]$$

is also concave for all $k = 1, ..., K$.

Let $\nu \in (0,1)$ be such that $\nu\overline{P}_k^* \leqslant \overline{P}_k, \forall k$ with at least one $r = 1, 2, ..., K$ such that $\nu\overline{P}_r^* = \overline{P}_r$. Then:

$$\kappa_r(\{\overline{P}_r\}_{r=1}^K) - \overline{P}_r \geqslant \kappa_r(\nu\{\overline{P}_r^*\}_{r=1}^K) - \nu\overline{P}_r^* \geqslant \kappa_r(0)(1-\nu) + \underbrace{\kappa_r(\{\overline{P}_r^*\}_{r=1}^K)\nu - \nu\overline{P}_r^*}_{=0} \geqslant (1-\nu)\kappa_r(0) > 0.$$

The final inequality implies $\kappa_r(\{\overline{P}_r\}_{r=1}^K) > \overline{P}_r$, which contradicts that $\{\overline{P}_k\}_{k=1}^K$ is a fixed point. Hence, $\{\overline{P}_k\}_{k=1}^K$ is the unique fixed point, and equilibrium sectoral prices are unique.

Uniqueness of sectoral prices implies uniqueness of the aggregate consumption price index, and through the cash-in-advance constraint, it follows that equilibrium aggregate final consumption is also unique. Finally, properties of the consumption aggregator u imply that given unique sectoral prices and aggregate consumption, sectoral consumptions are also unique.

(c) Generic uniqueness of supplier choices and remaining quantities;

Generic uniqueness of sectoral supplier choices follows directly from the proof in Acemoglu and Azar (2020), since the possibility of multiplicity in equilibrium networks is unrelated to the degree of nominal rigidities.

From (A.6) one can see that equilibrium sectoral outputs are function of the amount of inputs sectors are buying from each other. Since supplier choices are generically unique, so are sectoral outputs in equilibrium. Finally, since factor demands are pinned down, among other things, by sectoral outputs, sectoral labor demands and desired purchases of intermediate inputs are also generically unique.

□

Proof of Lemma 1. (a) $\overline{\mathcal{A}} > \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0 = \mathcal{M}_0$;

Let $\{\overline{P}_k^0\}_{k=1}^K$ be equilibrium ratios of sectoral prices to money supply under the baseline pair $(\underline{\mathcal{A}}, \mathcal{M}_0)$. Naturally, they satisfy the following fixed point condition:

$$\overline{P}_k^0 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \underline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}); \mathcal{M}_0], \quad \forall k. \tag{A.7}$$

Suppose the productivity mapping changes to $\overline{\mathcal{A}} > \underline{\mathcal{A}}$, while baseline money supply remains unchanged. Define $\{\overline{P}_k^1\}_{k=1}^K$ such that they satisfy:

$$\overline{P}_k^1 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}); \mathcal{M}_0], \quad \forall k \tag{A.8}$$

Since $\overline{\mathcal{Q}}_k$ is decreasing in $\mathcal{A}_k$ and $f_k' > 0$, it follows that:

$$\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}) < \min_{S_k} \overline{\mathcal{Q}}_k(S_k, \underline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}), \quad \forall k \tag{A.9}$$

$$\Rightarrow f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}); \mathcal{M}_0] < f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \underline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}); \mathcal{M}_0], \quad \forall k \tag{A.10}$$

$$\Rightarrow \overline{P}_k^1 < \overline{P}_k^0, \quad \forall k. \tag{A.11}$$

As before, define $\kappa_k(\{\overline{P}_k\}_{k=1}^K) = f_k[\min_{S_k} \overline{\mathcal{Q}_k}(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_k\}_{k=1}^K; \mathcal{M}_0], \forall k$. The new equilibrium ratios of sectoral prices to money supply are the minimal fixed point of $\kappa(.)$. For $\tau \geqslant 1$, define $\overline{P}_k^\tau = \kappa_k(\{\overline{P}^{\tau-1}\}_{k=1}^K)$. Since $\kappa(.)$ is increasing, it follows that:

$$\overline{P}_k^1 < \overline{P}_k^0, \forall k \Rightarrow \kappa(\{\overline{P}^1\}_{k=1}^K) < \kappa(\{\overline{P}^0\}_{k=1}^K) \tag{A.12}$$

$$\Rightarrow \overline{P}_k^2 < \overline{P}_k^1, \forall k \Rightarrow \kappa(\{\overline{P}^2\}_{k=1}^K) < \kappa(\{\overline{P}^1\}_{k=1}^K) \tag{A.13}$$

$$\Rightarrow \overline{P}_k^3 < \overline{P}_k^2, \forall k \Rightarrow ... \Rightarrow \overline{P}_k^t < \overline{P}_k^{t-1}, \forall k \tag{A.14}$$

$$\Rightarrow \lim_{t \to \infty} \overline{P}_k^t < \overline{P}_k^0, \forall k. \tag{A.15}$$

Since $\kappa(.)$ is continuous, $\lim_{t \to \infty} \overline{P}_k^t$ is a fixed point of $\kappa(.)$. Further, since the new equilibrium is a minimal fixed point, it must be that $\overline{P}_k(\overline{\mathcal{A}}, \mathcal{M}_0) \leqslant \lim_{t\to\infty} \overline{P}_k^t < \overline{P}_k^0 = \overline{P}_k(\underline{\mathcal{A}}, \mathcal{M}_0), \forall k$.

From the definition of the aggregate consumption price index it can be deduced that

$$\overline{P}^c(\overline{\mathcal{A}}, \mathcal{M}_0) = P^c(\frac{P_1(\overline{\mathcal{A}}, \mathcal{M}_0)}{\mathcal{M}_0}, ..., \frac{P_K(\overline{\mathcal{A}}, \mathcal{M}_0)}{\mathcal{M}_0}) =$$
$$= P^c(\overline{P}_1(\overline{\mathcal{A}}, \mathcal{M}_0), ..., \overline{P}_K(\overline{\mathcal{A}}, \mathcal{M}_0)) < P^c(\overline{P}_1(\underline{\mathcal{A}}, \mathcal{M}_0), ..., \overline{P}_K(\underline{\mathcal{A}}, \mathcal{M}_0)) = \overline{P}^c(\underline{\mathcal{A}}, \mathcal{M}_0).$$

From the cash-in-advance constraint, $C(\mathcal{A}, \mathcal{M}_0) = 1/P^c(\mathcal{A}, \mathcal{M}_0)$, and hence $C(\overline{\mathcal{A}}, \mathcal{M}_0) > C(\underline{\mathcal{A}}, \mathcal{M}_0)$.

(b) $\overline{\mathcal{A}} = \underline{\mathcal{A}} = \mathcal{A}, \overline{\mathcal{M}}_0 > \underline{\mathcal{M}}_0$;

Similarly, let $\{\overline{P}_k^0\}_{k=1}^K$ be equilibrium ratios of sectoral prices to money supply under the baseline pair $(\mathcal{A}, \underline{\mathcal{M}}_0)$. Suppose $\underline{\mathcal{M}}_0$ rises to $\overline{\mathcal{M}}_0 > \underline{\mathcal{M}}_0$, and define $\{\overline{P}_k^1\}_{k=1}^K$ such that they satisfy:

$$\overline{P}_k^1 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \overline{\mathcal{M}}_0], \quad \forall k \tag{A.16}$$

Since $f_k[\quad . \quad ; \overline{\mathcal{M}}_0] < f_k[\quad . \quad ; \underline{\mathcal{M}}_0], \forall k$, it follows that:

$$f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \overline{\mathcal{M}}_0] < f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \underline{\mathcal{M}}_0], \quad \forall k \tag{A.17}$$
$$\Rightarrow \overline{P}_k^1 < \overline{P}_k^0, \quad \forall k. \tag{A.18}$$

The rest of the proof follows from (a), where it is similarly shown that $\overline{P}_k(\mathcal{A}, \overline{\mathcal{M}}_0) < \overline{P}_k(\mathcal{A}, \underline{\mathcal{M}}_0), \forall k$, and hence $\overline{P}^c(\mathcal{A}, \overline{\mathcal{M}}_0) < \overline{P}^c(\mathcal{A}, \underline{\mathcal{M}}_0)$ and $C(\mathcal{A}, \overline{\mathcal{M}}_0) > C(\mathcal{A}, \underline{\mathcal{M}}_0)$. □

Proof of Lemma 2. (a) $\overline{\mathcal{A}} > \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0 = \mathcal{M}_0$; Let $S_k^0 = S_k(\underline{\mathcal{A}}, \mathcal{M}_0), \forall k$, be the original set of supplier choices in equilibrium, which satisfy:

$$S_k^0 \in \arg\min_{S_k} \mathcal{Q}_k(S_k, \underline{\mathcal{A}}_k(S_k), \mathcal{M}_0, \{P_r(\underline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k}) =$$
$$= \arg\min_{S_k} \mathcal{M}_0 \overline{\mathcal{Q}}_k(S_k, \underline{\mathcal{A}}_k(S_k), \{\overline{P}_r(\underline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k})$$
$$= \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \underline{\mathcal{A}}_k(S_k), \{\overline{P}_r(\underline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k}), \quad \forall k.$$

Suppose $\underline{\mathcal{A}}$ improves to $\overline{\mathcal{A}} > \underline{\mathcal{A}}$, and define supplier choices S^1 such that it is true that $S_k^1 \in \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r(\underline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k}), \forall k$. From Theorem 4 of Milgrom and Shannon (1994) it follows that $S_k^1 \supseteq S_k^0, \forall k$.

From the proof of Lemma 1 it is known that $\overline{P}_k(\overline{\mathcal{A}}, \mathcal{M}_0) < \overline{P}_k(\underline{\mathcal{A}}, \mathcal{M}_0), \forall k$, and hence

$$\arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r(\overline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k}) \supseteq \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r(\underline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k})$$
$$\Rightarrow S_k(\overline{\mathcal{A}}, \mathcal{M}_0) \supseteq S_k^1 \supseteq S_k(\underline{\mathcal{A}}, \mathcal{M}_0), \quad \forall k. \tag{A.19}$$

(b) $\overline{\mathcal{A}} = \underline{\mathcal{A}} = \mathcal{A}, \overline{\mathcal{M}}_0 > \underline{\mathcal{M}}_0$;

Let $S^0 = S(\mathcal{A}, \underline{\mathcal{M}}_0)$ be the original equilibrium network, which satisfies

$$S_k^0 \in \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r(\mathcal{A}, \underline{\mathcal{M}}_0)\}_{r \in S_k}), \quad \forall k. \tag{A.20}$$

Suppose $\underline{\mathcal{M}}_0$ changes to $\overline{\mathcal{M}}_0 > \underline{\mathcal{M}}_0$. From the proof of Lemma 1 it is known that $\overline{P}_k(\mathcal{A}, \overline{\mathcal{M}}_0) < \overline{P}_k(\mathcal{A}, \underline{\mathcal{M}}_0), \forall k$, and hence by Theorem 4 of Milgrom and Shannon (1994):

$$\arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r(\mathcal{A}, \overline{\mathcal{M}}_0)\}_{r \in S_k}) \supseteq \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r(\mathcal{A}, \underline{\mathcal{M}}_0)\}_{r \in S_k}),$$

$$\Rightarrow S_k(\mathcal{A}, \overline{\mathcal{M}}_0) \supseteq S_k(\mathcal{A}, \underline{\mathcal{M}}_0), \quad \forall k. \tag{A.21}$$

□

Proof of Lemma 3. Proof follows directly from Lemmas 1 and 2, where variation in money supply is driven by the monetary shock, as opposed to changes in baseline money supply, which is instead kept fixed. □

Proof of Lemma 4. First, notice that under the assumptions used in this lemma, if a shock $\varepsilon^m > 0$ is small with respect to a baseline pair $(\mathcal{A}, \mathcal{M}_0)$, then any other monetary shock $\overline{\varepsilon}^m \in (0, \varepsilon^m)$ is also small with respect to the baseline pair $(\mathcal{A}, \mathcal{M}_0)$. The proof of that follows by contradiction. Suppose there exists $\overline{\varepsilon}^m \in (0, \varepsilon^m)$ such that $S_k(\mathcal{A}, \mathcal{M}_0 \exp(\overline{\varepsilon}^m)) \supset S_k(\mathcal{A}, \mathcal{M}_0)$ for at least one k. Then for those k it follows $S_k(\mathcal{A}, \mathcal{M}_0 \exp(\overline{\varepsilon}^m)) \supset S_k(\mathcal{A}, \mathcal{M}_0 \exp(\varepsilon^m)) = S_k(\mathcal{A}, \mathcal{M}_0)$ where $\varepsilon^m > \overline{\varepsilon}^m$, which implies that for those sectors the set of suppliers shrinks as money supply increases, which contradicts Theorem 4 of Milgrom and Shannon (1994). Similarly, suppose there exists $\overline{\varepsilon}^m \in (0, \varepsilon^m)$ such that $S_k(\mathcal{A}, \mathcal{M}_0 \exp(\overline{\varepsilon}^m)) \subset S_k(\mathcal{A}, \mathcal{M}_0)$ for at least one k. Then for those k it follows $S_k(\mathcal{A}, \mathcal{M}_0 \exp(\overline{\varepsilon}^m)) \subset S_k(\mathcal{A}, \mathcal{M}_0)$ where $\varepsilon^m > 0$, which once again implies that for those sectors the set of suppliers shrinks as money supply increases, which contradicts Theorem 4 of Milgrom and Shannon (1994).

Hence, starting from the baseline $(\mathcal{A}, \mathcal{M}_0)$, the solution to my model for any monetary shock $\overline{\varepsilon}^m \in (0, \varepsilon^m]$ coincides with the solution to a restricted version of my model where the set of suppliers is kept fixed at $S(\mathcal{A}, \mathcal{M}_0)$. In the rest of this proof I consider such restricted version of my model for an unrestricted value of monetary shock, and ultimately evaluate solution to that model under $\overline{\varepsilon}^m \in (0, \varepsilon^m]$. Write sectoral price aggregation in such restricted model:

$$P_k^{1-\theta} = \alpha_k P_{k,0}^{1-\theta} + (1 - \alpha_k) \left[(1 + \mu_k) \mathcal{Q}_k(S_k, \mathcal{A}_k(S_k), \mathcal{M}, \{P_r\}_{r \in S_k}\right]^{1-\theta} \tag{A.22}$$

For a given baseline $(\mathcal{A}, \mathcal{M}_0)$ and model parameters, equilibrium prices are pinned down by the value of the monetary shock ε^m. The model with a fixed network features no discontinuities and since the unit cost function is differentiable by assumption, one can write a first order Taylor approximation around $\varepsilon^m = 0$:

$$\log P_k(\varepsilon^m) = \log P_k(0) + \frac{(1-\alpha_k)\left[(1+\mu_k)\mathcal{Q}_k(0)\right]^{1-\theta}}{\alpha_k P_{k,0}^{1-\theta} + (1-\alpha_k)\left[(1+\mu_k)\mathcal{Q}_k(0)\right]^{1-\theta}}\left[\log \mathcal{Q}_k(\varepsilon^m) - \log \mathcal{Q}_k(0)\right] \quad \text{(A.23)}$$

By assumption, $P_{k,0} = (1+\mu_k)g(\mathcal{M}_0)\mathcal{Q}_k(0)$:

$$\log P_k(\varepsilon^m) = \log P_k(0) + \frac{(1-\alpha_k)}{\alpha_k g(\mathcal{M}_0)^{1-\theta} + (1-\alpha_k)}\left[\log \mathcal{Q}_k(\varepsilon^m) - \log \mathcal{Q}_k(0)\right] \quad \text{(A.24)}$$

$$= \log P_k(0) + (1-\alpha_k)\gamma_k(\mathcal{M}_0)\left[\varepsilon^m + \log \overline{\mathcal{Q}}(\varepsilon^m) - \log \overline{\mathcal{Q}}(0)\right]. \quad \text{(A.25)}$$

where $\gamma_k(\mathcal{M}_0) \equiv \frac{1}{\alpha_k g(\mathcal{M}_0)^{1-\theta} + 1 - \alpha_k}, \forall k$. Further,

$$\log \overline{P}_k(\varepsilon^m) - \log \overline{P}_k(0) + \varepsilon^m = (1-\alpha_k)\gamma_k(\mathcal{M}_0)\varepsilon^m + \\ + (1-\alpha_k)\gamma_k(\mathcal{M}_0)\sum_{r\in S_k}\frac{\partial \log \mathcal{Q}_k(0)}{\partial \log P_r}\left[\log \overline{P}_r(\varepsilon^m) - \log \overline{P}_r(0)\right]. \quad \text{(A.26)}$$

Letting $\overline{p}_k \equiv [\log \overline{P}_k(\varepsilon^m) - \log \overline{P}_k(0)]$ and $\overline{\mathbb{p}} = [\overline{p}_1, \overline{p}_2, ..., \overline{p}_K]^T$, one can re-write the above conveniently in matrix form:

$$\overline{\mathbb{p}} = [(I-A)\Gamma(\mathcal{M}_0) - I]\mathcal{E}^m + (I-A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)\overline{\mathbb{p}} \quad \text{(A.27)}$$

$$\overline{\mathbb{p}} = -[I - (I-A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)]^{-1}[I - (I-A)\Gamma(\mathcal{M}_0)]\mathcal{E}^m \quad \text{(A.28)}$$

where $A = diag(\alpha_1, ..., \alpha_K)$, $\Gamma(\mathcal{M}_0) = diag(\gamma_1(\mathcal{M}_0), ..., \gamma_K(\mathcal{M}_0)), \mathcal{E}^m = [\varepsilon^m, \varepsilon^m, ..., \varepsilon^m]^T$ and $[\Omega(\mathcal{A}, \mathcal{M}_0)]_{kr} = \frac{\partial \log \mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), W, \{P_{k'}\}_{k'\in S_k}\right]}{\partial \log P_r}|_{\mathcal{M}=\mathcal{M}_0}, \forall k, r$.

Return to the original version of my model, where network formation is endogenous. For a monetary shock $\varepsilon^m > 0$ that is small with respect to two baselines $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$, the expression in (A.28) holds locally around both baselines. Further, for those two baselines $\overline{p}_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \overline{p}_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = p_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \varepsilon^m - p_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}) + \varepsilon^m = p_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - p_k(\underline{\mathcal{A}}, \underline{\mathcal{M}})$, and so $\overline{\mathbb{p}}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \overline{\mathbb{p}}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = \mathbb{p}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathbb{p}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$. Combining this with (A.28) delivers the final result:

$$\mathbb{p}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathbb{p}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = -\left[\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathcal{L}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)\right]\mathcal{E}^m \quad \text{(A.29)}$$

where $\mathcal{L}(\mathcal{A}, \mathcal{M}_0) = [I - (I-A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)]^{-1}[I - (I-A)\Gamma(\mathcal{M}_0)]$. □

Proof of Proposition 2. Let $D \equiv \Omega(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \Omega(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, then one can rewrite the difference between the Leontief Inverses as follows:

$$\begin{aligned}\mathcal{L}(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)-\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)=&(I-A)\Gamma(\underline{\mathcal{M}}_0)\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)\mathcal{L}(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)\\&-(I-A)\Gamma(\overline{\mathcal{M}}_0)\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)\\&-(I-A)\Gamma(\overline{\mathcal{M}}_0)D\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)\\&+(I-A)(\Gamma(\overline{\mathcal{M}}_0)-\Gamma(\underline{\mathcal{M}}_0)).\end{aligned}\tag{A.30}$$

Let $\Xi\equiv diag\left(\frac{\gamma_1(\overline{\mathcal{M}}_0)}{\gamma_1(\underline{\mathcal{M}}_0)},...,\frac{\gamma_K(\overline{\mathcal{M}}_0)}{\gamma_K(\underline{\mathcal{M}}_0)}\right)$, then the above can be re-written as:

$$\begin{aligned}&\mathcal{L}(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)-\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)=-[I-(I-A)\Gamma(\underline{\mathcal{M}}_0)\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)]^{-1}\times\\&\quad\times[(I-A)\Gamma(\overline{\mathcal{M}}_0)D\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)+(I-A)(I-\Xi)\Gamma(\underline{\mathcal{M}}_0)(I-\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0))].\end{aligned}\tag{A.31}$$

From the definition of $\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)$:

$$\begin{aligned}\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)&=[I-(I-A)\Gamma(\overline{\mathcal{M}})\Omega(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)]^{-1}[I-(I-A)\Gamma(\overline{\mathcal{M}}_0)]\\&=I+[I-(I-A)\Gamma(\overline{\mathcal{M}_0})\Omega(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)]^{-1}(I-A)\Gamma(\overline{\mathcal{M}}_0)(\Omega(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)-I)\\&\Rightarrow I-\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)=[I-\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)]+\\&+\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)[I-(I-A)\Gamma(\overline{\mathcal{M}}_0)\Omega(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)]^{-1}(I-A)\Gamma(\overline{\mathcal{M}}_0)(I-\Omega(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)).\end{aligned}\tag{A.32}$$

Suppose we have any two baselines pairs $(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)$ such that either $\overline{\mathcal{A}}\geqslant\underline{\mathcal{A}},\overline{\mathcal{M}}_0=\underline{\mathcal{M}}_0$ or $\overline{\mathcal{A}}=\underline{\mathcal{A}},\overline{\mathcal{M}}_0\geqslant\underline{\mathcal{M}}_0$. Under Cobb-Douglas production function entries of the Ω matrix in the Leontief Inverse are given by shares in the production function, so that $[\Omega]_{kr}=\omega_{kr}$ if sector k buys inputs from sector r and $[\Omega]_{kr}=0$ otherwise. As a result, $D=\Omega(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)-\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)$ is a $K\times K$ matrix with non-negative entries, since from Lemma 2 the $(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)$ baseline will have (weakly) more suppliers for every sector. Further, from the properties of γ_k, it follows that all entries of the Ξ matrix are positive and less than or equal to one. Finally, since labor is an essential input, it follows that sum of rows of $(I-\Omega(\mathcal{A},\mathcal{M}))$ for any baseline pair are positive and less than or equal to one.

Combining the above observations with the expressions in (A.31) and (A.32), it follows that for any two baselines pairs $(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)$ such that either $\overline{\mathcal{A}}\geqslant\underline{\mathcal{A}},\overline{\mathcal{M}}_0=\underline{\mathcal{M}}_0$ or $\overline{\mathcal{A}}=\underline{\mathcal{A}},\overline{\mathcal{M}}_0\geqslant\underline{\mathcal{M}}_0$, and a monetary shock $\varepsilon^m>0$ that is small with respect to both baselines, it holds that:

$$\mathbb{p}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)-\mathbb{p}(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)=-\left[\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)-\mathcal{L}(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)\right]\mathcal{E}^m\leqslant 0_{K\times 1}.\tag{A.33}$$

or $p_k(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)\leqslant p_k(\underline{\mathcal{A}},\underline{\mathcal{M}}_0),\forall k$ in scalar form. □

Proof of Theorem 1. Under Cobb-Douglas aggregation of sectoral consumption it follows that sectoral consumption demand takes the following form:

$$C_k = \omega_{ck}\left[\frac{P_k}{P^c}\right]^{-1} C = \omega_{ck}\frac{P^c C}{P_k} = \omega_{ck}\frac{\mathcal{M}}{P_k}. \tag{A.34}$$

From the above it follows that $\log C_k = \log \omega_{ck} + \log \mathcal{M} - \log P_k$ and evaluating the latter under baseline and following the shock yields:

$$\begin{aligned} \log C_k(\mathcal{A},\mathcal{M}) - \log C_k(\mathcal{A},\mathcal{M}_0) &= \log \mathcal{M} - \log P_k(\mathcal{A},\mathcal{M}) - \log \mathcal{M}_0 + \log P_k(\mathcal{A},\mathcal{M}_0) \\ c_k(\mathcal{A},\mathcal{M}_0) &= \varepsilon^m - p_k(\mathcal{A},\mathcal{M}_0). \end{aligned} \tag{A.35}$$

For the two baselines $(\underline{\mathcal{A}},\mathcal{M}_0)$, $(\overline{\mathcal{A}},\mathcal{M}_0)$ such that $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}$, and a monetary shock $\varepsilon^m > 0$ that is small with respect to both baselines it follows:

$$c_k(\overline{\mathcal{A}},\mathcal{M}_0) - c_k(\underline{\mathcal{A}},\mathcal{M}_0) = -\left[p_k(\overline{\mathcal{A}},\mathcal{M}_0) - p_k(\underline{\mathcal{A}},\mathcal{M}_0)\right] \geqslant 0, \quad \forall k \tag{A.36}$$

where the last part follows from Proposition 2.

Consider deviation of aggregate consumption from baseline:

$$c(\overline{\mathcal{A}},\mathcal{M}_0) - c(\underline{\mathcal{A}},\mathcal{M}_0) = \sum_{k=1}^{K} \omega_{ck}\left[c_k(\overline{\mathcal{A}},\mathcal{M}_0) - c_k(\underline{\mathcal{A}},\mathcal{M}_0)\right] \geqslant 0. \tag{A.37}$$

□

Proof of Theorem 2. Recall from the proof of Theorem 1 it is true that $c_k(\mathcal{A},\mathcal{M}_0) = \varepsilon^m - p_k(\mathcal{A},\mathcal{M}_0), \forall k$. For the two baselines $(\mathcal{A},\underline{\mathcal{M}}_0)$, $(\mathcal{A},\overline{\mathcal{M}}_0)$ such that $\overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$, and a monetary shock $\varepsilon^m > 0$ that is small with respect to both baselines it follows:

$$c_k(\mathcal{A},\overline{\mathcal{M}}_0) - c_k(\mathcal{A},\underline{\mathcal{M}}_0) = -\left[p_k(\mathcal{A},\overline{\mathcal{M}}_0) - p_k(\mathcal{A},\underline{\mathcal{M}}_0)\right] \geqslant 0 \tag{A.38}$$

where the last part follows from Proposition 2.

Consider deviation of aggregate consumption from baseline:

$$c(\mathcal{A},\overline{\mathcal{M}}_0) - c(\mathcal{A},\underline{\mathcal{M}}_0) = \sum_{k=1}^{K} \omega_{ck}\left[c_k(\mathcal{A},\overline{\mathcal{M}}_0) - c_k(\mathcal{A},\underline{\mathcal{M}}_0)\right] \geqslant 0. \tag{A.39}$$

□

Proof of Proposition 3. Let $\{\overline{P}_k^0\}_{k=1}^K$ be equilibrium ratios of sectoral prices to money supply under the initial level of money supply $\mathcal{M}_0$. As shown in the proof of Lemma 1, they satisfy:

$$\overline{P}_k^0 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \mathcal{M}_0], \quad \forall k \tag{A.40}$$

where f_k is an increasing function. Further, we know that the equilibrium network under the initial level of money supply $\mathcal{M}_0$ is S^0.

Suppose that following a large monetary shock E^m the level of money supply rises to $\mathcal{M}^L > \mathcal{M}_0$. Define $\{\overline{P}_k^1\}_{k=1}^K$ such that they satisfy:

$$\overline{P}_k^1 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \mathcal{M}^L], \quad \forall k \tag{A.41}$$

Let $\{\overline{P}_k^1(S^0)\}_{k=1}^K$ be the equivalent of $\{\overline{P}_k^1\}_{k=1}^K$ in an economy with an exogenous production network, given by S^0, then it follows that:

$$\begin{aligned}\overline{P}_k^1(S^0) = f_k[\overline{\mathcal{Q}}_k(S_k^0, \mathcal{A}_k(S_k^0), \{\overline{P}_r^0\}_{r\in S_k^0}); \mathcal{M}^L] \geqslant & f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \mathcal{M}^L] \\ = & \overline{P}_k^1, \quad \forall k \end{aligned}\tag{A.42}$$

given that the economy with endogenous networks has an extra layer of minimization.

As in the proof of Lemma 1, we can define $\{\overline{P}_k^2\}_{k=1}^K$ and $\{\overline{P}_k^2(S^0)\}_{k=1}^K$ as the outcome of the next stage of fixed point iteration (with endogenous and exogenous network, respectively), and it follows that:

$$\begin{aligned}\overline{P}_k^2(S^0) = f_k[\overline{\mathcal{Q}}_k(S_k^0, \mathcal{A}_k(S_k^0), \{\overline{P}_r^1(S^0)\}_{r\in S_k^0}); \mathcal{M}^L] \geqslant & f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^1\}_{r\in S_k}); \mathcal{M}^L] \\ = & \overline{P}_k^2, \quad \forall k \end{aligned}\tag{A.43}$$

where the inequality follows from two sources: first, $\overline{P}_k^1(S^0) \geqslant \overline{P}_k^1, \forall k$; second, the economy with endogenous networks has an extra layer of minimization. By induction, it then follows that for any stage $t \geqslant 1$ of the fixed point iteration:

$$\begin{aligned}\overline{P}_k^t(S^0) = f_k[\overline{\mathcal{Q}}_k(S_k^0, \mathcal{A}_k(S_k^0), \{\overline{P}_r^{t-1}(S^0)\}_{r\in S_k^0}); \mathcal{M}^L] \geqslant & f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^{t-1}\}_{r\in S_k}); \mathcal{M}^L] \\ = & \overline{P}_k^t, \quad \forall k \end{aligned}$$

$$\Rightarrow \lim_{t\to\infty} \overline{P}_k^t(S^0) \geqslant \lim_{t\to\infty} \overline{P}_k^t, \quad \forall k$$

$$\Rightarrow \overline{P}_k(\mathcal{M}^L; S^0) \geqslant \overline{P}_k(\mathcal{M}^L), \quad \forall k \tag{A.44}$$

$$\Rightarrow P_k(\mathcal{M}^L; S^0)/\mathcal{M}^L \geqslant P_k(\mathcal{M}^L)/\mathcal{M}^L, \quad \forall k$$

$$\Rightarrow P_k(\mathcal{M}^L; S^0) \geqslant P_k(\mathcal{M}^L), \quad \forall k \tag{A.45}$$

where $P_k(\mathcal{M}^L)$ is the equilibrium price index of sector k under money supply $\mathcal{M}^L$ and endogenous network, whereas $P_k(\mathcal{M}^L; S^0)$ is the equilibrium price index of sector k under money supply $\mathcal{M}^L$ and exogenous network S^0. We can further see that:

$$\begin{aligned}
P_k(\mathcal{M}^L) &\leqslant P_k(\mathcal{M}^L; S^0), \quad \forall k \\
\Rightarrow \frac{P_k(\mathcal{M}^L)}{P_k(\mathcal{M}_0; S^0)} &\leqslant \frac{P_k(\mathcal{M}^L; S^0)}{P_k(\mathcal{M}_0; S^0)}, \quad \forall k \\
\Rightarrow \tilde{P}_k(E^m) &\leqslant \tilde{P}^e_k(E^m; S^0), \quad \forall k \\
\Rightarrow \frac{\tilde{P}_k(E^m)}{\tilde{P}_k(\varepsilon^m)} &\leqslant \frac{\tilde{P}^e_k(E^m; S^0)}{\tilde{P}^e_k(\varepsilon^m; S^0)}, \quad \forall k
\end{aligned} \tag{A.46}$$

where we know that $\tilde{P}_k(\varepsilon^m) = \tilde{P}^e_k(\varepsilon^m; S^0)$ since, by definition, the small monetary shock ε^m leaves the equilibrium network unchanged at S^0. This establishes the first inequality in the proposition.

In order to prove the second inequality, return to our definition of $\{\overline{P}^0_k\}_{k=1}^K$, which gives the equilibrium ratios of sectoral prices to money supply under the initial level of money supply $\mathcal{M}_0$. Suppose that following a small monetary shock ε^m the level of money supply rises to $\mathcal{M}^S > \mathcal{M}_0$. Define $\{\overline{P}^1_k\}_{k=1}^K$ such that they satisfy:

$$\overline{P}^1_k = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}^0_r\}_{r\in S_k}); \mathcal{M}^S], \quad \forall k \tag{A.47}$$

Recall that S^L is the equilibrium network under money supply $\mathcal{M}^L$. Let $\{\overline{P}^1_k(S^L)\}_{k=1}^K$ be the equivalent of $\{\overline{P}^1_k\}_{k=1}^K$ in an economy with an exogenous production network, given by S^L, then:

$$\begin{aligned}
\overline{P}^1_k(S^L) = f_k[\overline{\mathcal{Q}}_k(S^L_k, \mathcal{A}_k(S^L_k), \{\overline{P}^0_r\}_{r\in S^L_k}); \mathcal{M}^S] &\geqslant f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}^0_r\}_{r\in S_k}); \mathcal{M}^S] \\
&= \overline{P}^1_k, \quad \forall k
\end{aligned} \tag{A.48}$$

given that the economy with endogenous networks has an extra layer of minimization.

As before, by induction, for any stage $t \geqslant 1$ of the fixed point iteration:

$$\begin{aligned}
\overline{P}^t_k(S^L) = f_k[\overline{\mathcal{Q}}_k(S^L_k, \mathcal{A}_k(S^L_k), \{\overline{P}^{t-1}_r(S^L)\}_{r\in S^L_k}); \mathcal{M}^S] &\geqslant f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}^{t-1}_r\}_{r\in S_k}); \mathcal{M}^S] \\
&= \overline{P}^t_k, \quad \forall k
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \lim_{t\to\infty} \overline{P}^t_k(S^L) &\geqslant \lim_{t\to\infty} \overline{P}^t_k, \quad \forall k \\
\Rightarrow \overline{P}_k(\mathcal{M}^S; S^L) &\geqslant \overline{P}_k(\mathcal{M}^S), \quad \forall k
\end{aligned} \tag{A.49}$$

$$\begin{aligned}
\Rightarrow P_k(\mathcal{M}^S; S^L)/\mathcal{M}^S &\geqslant P_k(\mathcal{M}^S)/\mathcal{M}^S, \quad \forall k \\
\Rightarrow P_k(\mathcal{M}^S; S^L) &\geqslant P_k(\mathcal{M}^S), \quad \forall k
\end{aligned} \tag{A.50}$$

where $P_k(\mathcal{M}^S)$ is the equilibrium price index of sector k under money supply $\mathcal{M}^S$ and endogenous network, whereas $P_k(\mathcal{M}^S; S^L)$ is the equilibrium price index of sector k under money supply $\mathcal{M}^S$ and exogenous network S^L. Further, we know that since ε^m is a small monetary shock, it does not change the equilibrium network relative to S^0. Therefore, $P_k(\mathcal{M}^S) = P_k(\mathcal{M}^S; S^0), \forall k$, where $P_k(\mathcal{M}^S; S^0)$ is the the equilibrium price index of sector k under money supply $\mathcal{M}^S$ and exogenous network S^0. We can therefore see that:

$$\begin{aligned}
P_k(\mathcal{M}^S; S^0) = P_k(\mathcal{M}^S) &\leqslant P_k(\mathcal{M}^S; S^L), \quad \forall k \\
\Rightarrow \frac{1}{P_k(\mathcal{M}^S; S^0)} &\geqslant \frac{1}{P_k(\mathcal{M}^S; S^L)}, \quad \forall k \\
\Rightarrow \frac{P_k(\mathcal{M}^L; S^L)}{P_k(\mathcal{M}^S; S^0)} &\geqslant \frac{P_k(\mathcal{M}^L; S^L)}{P_k(\mathcal{M}^S; S^L)}, \quad \forall k \\
\Rightarrow \frac{P_k(\mathcal{M}^L; S^L)/P_k(\mathcal{M}^0; S^0)}{P_k(\mathcal{M}^S; S^0)/P_k(\mathcal{M}^0; S^0)} &\geqslant \frac{P_k(\mathcal{M}^L; S^L)/P_k(\mathcal{M}^0; S^L)}{P_k(\mathcal{M}^S; S^L)/P_k(\mathcal{M}^0; S^L)}, \quad \forall k \\
\Rightarrow \frac{\tilde{P}_k(E^m)}{\tilde{P}_k(\varepsilon^m)} &\geqslant \frac{\tilde{P}^e_k(E^m; S^L)}{\tilde{P}^e_k(\varepsilon^m; S^L)}, \quad \forall k \qquad \text{(A.51)}
\end{aligned}$$

which gives us the second inequality in the proposition.

Combining (A.46) and (A.51):

$$\frac{\tilde{P}^e_k(E^m; S^L)}{\tilde{P}^e_k(\varepsilon^m; S^L)} \leqslant \frac{\tilde{P}_k(E^m)}{\tilde{P}_k(\varepsilon^m)} \leqslant \frac{\tilde{P}^e(E^m; S^0)}{\tilde{P}^e_k(\varepsilon^m; S^0)}, \qquad \text{(A.52)}$$

for all $k = 1, 2, ..., K$. □

Proof of Theorem 3. Recall that $\overline{P}_k(\mathcal{M}^L) \leqslant \overline{P}_k(\mathcal{M}^L; S^0) \forall k$. Hence,

$$\begin{aligned}
\frac{P^c(\mathcal{M}^L)}{\mathcal{M}^L} = P^c(\overline{P}_1(\mathcal{M}^L), ..., \overline{P}_K(\mathcal{M}^L)) &\leqslant P^c(\overline{P}_1(\mathcal{M}^L; S^0), ..., \overline{P}_K(\mathcal{M}^L; S^0)) \\
&= \frac{P^c(\mathcal{M}^L; S^0)}{\mathcal{M}^L}. \qquad \text{(A.53)}
\end{aligned}$$

Combined with the cash-in-advance constraint:

$$\begin{aligned}
\frac{C(\mathcal{M}^L)}{C(\mathcal{M}^L; S^0)} &= \frac{P^c(\mathcal{M}^L; S^0)}{P^c(\mathcal{M}^L)} \geqslant 1 \\
\Rightarrow C(\mathcal{M}^L) &\geqslant C(\mathcal{M}^L; S^0) \\
\Rightarrow \frac{C(\mathcal{M}^L)}{C(\mathcal{M}^S; S^0)} &\geqslant \frac{C(\mathcal{M}^L; S^0)}{C(\mathcal{M}^S; S^0)} \\
\Rightarrow \frac{C(\mathcal{M}^L)/C(\mathcal{M}_0; S^0)}{C(\mathcal{M}^S; S^0)/C(\mathcal{M}_0; S^0)} &\geqslant \frac{C(\mathcal{M}^L; S^0)/C(\mathcal{M}_0; S^0)}{C(\mathcal{M}^S; S^0)/C(\mathcal{M}_0; S^0)} \\
\Rightarrow \frac{\tilde{C}(E^m)}{\tilde{C}(\varepsilon^m)} &\geqslant \frac{\tilde{C}^e(E^m; S^0)}{\tilde{C}^e(\varepsilon^m; S^0)}. \qquad \text{(A.54)}
\end{aligned}$$

Recall that $\overline{P}_k(\mathcal{M}^S) = \overline{P}_k(\mathcal{M}^S; S^0) \leqslant \overline{P}_k(\mathcal{M}^S; S^L) \forall k$. Hence,

$$\begin{aligned}\frac{P^c(\mathcal{M}^S; S^0)}{\mathcal{M}^S} &= P^c(\overline{P}_1(\mathcal{M}^S; S^0), ..., \overline{P}_K(\mathcal{M}^S; S^0)) \leqslant P^c(\overline{P}_1(\mathcal{M}^S; S^L), ..., \overline{P}_K(\mathcal{M}^S; S^L)) \\ &= \frac{P^c(\mathcal{M}^S; S^L)}{\mathcal{M}^S}. \end{aligned} \tag{A.55}$$

Combined with the cash-in-advance constraint:

$$\begin{aligned} \frac{C(\mathcal{M}^S; S^0)}{C(\mathcal{M}^S; S^L)} &= \frac{P^c(\mathcal{M}^S; S^L)}{P^c(\mathcal{M}^S; S^0)} \geqslant 1 \\ \Rightarrow \frac{1}{C(\mathcal{M}^S; S^0)} &\leqslant \frac{1}{C(\mathcal{M}^S; S^L)} \\ \Rightarrow \frac{C(S^L, \mathcal{M}^L)}{C(S^0, \mathcal{M}^s)} &\leqslant \frac{C(S^L, \mathcal{M}^L)}{C(S^L, \mathcal{M}^s)} \\ \Rightarrow \frac{C(\mathcal{M}^L; S^L)/C(\mathcal{M}_0; S^0)}{C(\mathcal{M}^S; S^0)/C(\mathcal{M}_0; S^0)} &\leqslant \frac{C(\mathcal{M}^L; S^L)/C(\mathcal{M}_0; S^L)}{C(\mathcal{M}^S; S^L)/C(\mathcal{M}_0; S^L)} \\ \Rightarrow \frac{\tilde{C}(E^m)}{\tilde{C}(\varepsilon^m)} &\leqslant \frac{\tilde{C}^e(E^m; S^L)}{\tilde{C}^e(\varepsilon^m; S^L)}. \end{aligned} \tag{A.56}$$

Combining (A.54) and (A.56):

$$\frac{\tilde{C}^e(E^m; S^0)}{\tilde{C}^e(\varepsilon^m; S^0)} \leqslant \frac{\tilde{C}(E^m)}{\tilde{C}(\varepsilon^m)} \leqslant \frac{\tilde{C}^e(E^m; S^L)}{\tilde{C}^e(\varepsilon^m; S^L)}. \tag{A.57}$$

□

B Proofs of additional results

B.1 Business cycles driven by exogenous markup shocks

The main text considers baselines that vary in their productivity mappings $\mathcal{A}$ and initial levels of money supply $\mathcal{M}_0$. That said, my model also permits that baselines differ in the levels of exogenous desired markups $\{1 + \mu_k\}_{k=1}^K$, which I discuss in this section. In particular, given that $1 + \mu_k = (1 + \tau_k)\frac{\theta}{\theta - 1}$, shifts in the sectoral tax rates $\{\tau_k\}_{k=1}^K$ represent exogenous changes in baseline desired sectoral markups.

In terms of the effect on real GDP and supplier choices, exogenous decreases in baseline markups are isomorphic to exogenous improvements in the productivity mapping. In particular, *ceteris paribus*, an exogenous decrease in each sectoral desired markup lowers each sectoral price. Firms re-optimize their set of suppliers, which delivers lower new unit costs, since the original set of suppliers remains available. The latter further lowers sectoral prices. The mechanism repeats until a new equilibrium is reached, which features lower sectoral prices and hence a lower consumption price index, which implies a larger GDP. As for the effect on equilibrium supplier choices, an exogenous decrease in each sectoral desired markup incentivizes firms to connect to more suppliers, as lower desired markups decrease the prices charged by potential suppliers, while leaving the nominal wage unchanged. Below I establish these results formally.

Lemma B1 (Baseline GDP)**.** *Suppose Assumptions 1-3 hold. Consider two otherwise identical baselines, which differ only in the exogenous sectoral tax rates given by* $\{\underline{\tau}_k\}_{k=1}^K$ *and* $\{\overline{\tau}_k\}_{k=1}^K$*, respectively. Suppose that* $\underline{\tau}_k \leqslant \overline{\tau}_k, \forall k$*, so that one of the baselines has (weakly) larger tax rates in every sector, and hence (weakly) larger desired markups. Then,* $C\left(\{\underline{\tau}_k\}_{k=1}^K\right) \geqslant C\left(\{\overline{\tau}_k\}_{k=1}^K\right)$*.*

Proof. Let $\{\overline{P}_k^0\}_{k=1}^K$ be equilibrium ratios of sectoral prices to money supply under the baseline sectoral tax rates $\{\overline{\tau}_k\}_{k=1}^K$. Naturally, they satisfy the following fixed point condition:

$$\overline{P}_k^0 = \left[\alpha_k \left[\frac{P_{k,0}}{\mathcal{M}}\right]^{1-\theta} + (1-\alpha_k)\left[(1+\overline{\tau}_k)\frac{\theta}{\theta-1}\min_{S_k}\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k})\right]^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{B.1}$$

or

$$\overline{P}_k^0 = f_k[\min_{S_k}\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \overline{\tau}_k], \quad \forall k \tag{B.2}$$

where $f_k[\quad . \quad ; \overline{\tau}_k] \geqslant f_k[\quad . \quad ; \underline{\tau}_k]$, for any $\overline{\tau}_k \geqslant \underline{\tau}_k$, $\forall k$.

Suppose each $\overline{\tau}_k$ falls to $\underline{\tau}_k \leqslant \overline{\tau}_k, \forall k$, and define $\{\overline{P}_k^1\}_{k=1}^K$ such that they satisfy:

$$\overline{P}_k^1 = f_k[\min_{S_k}\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \underline{\tau}_k], \quad \forall k \tag{B.3}$$

Since $f_k[\quad . \quad ; \underline{\tau}_k] \leqslant f_k[\quad . \quad ; \overline{\tau}_k], \forall k$, it follows that:

$$f_k[\min_{S_k}\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \underline{\tau}_k] \leqslant f_k[\min_{S_k}\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \overline{\tau}_k], \quad \forall k \tag{B.4}$$

$$\Rightarrow \overline{P}_k^1 \leqslant \overline{P}_k^0, \quad \forall k. \tag{B.5}$$

The rest of the proof follows directly from the proof of part (a) of Lemma 1, where from fixed point iteration it follows that $\overline{P}_k(\{\overline{\tau}_k\}_{k=1}^K) \geqslant \overline{P}_k(\{\underline{\tau}_k\}_{k=1}^K), \forall k$, and hence $\overline{P}^c(\{\overline{\tau}_k\}_{k=1}^K) \geqslant \overline{P}^c(\{\underline{\tau}_k\}_{k=1}^K)$ and $C(\{\underline{\tau}_k\}_{k=1}^K) \geqslant C(\{\overline{\tau}_k\}_{k=1}^K)$. □

Lemma B2 (Baseline supplier choices)**.** *Suppose Assumptions 1-5 hold. Consider two otherwise identical baselines, which differ only in the exogenous sectoral tax rates given by* $\{\underline{\tau}_k\}_{k=1}^K$ *and* $\{\overline{\tau}_k\}_{k=1}^K$*, respectively. Suppose that* $\underline{\tau}_k \leqslant \overline{\tau}_k, \forall k$*, so that one of the baselines has (weakly) larger tax rates in every sector, and hence (weakly) larger desired markups. Then,* $S_k\left(\{\underline{\tau}_k\}_{k=1}^K\right) \supseteq S_k\left(\{\overline{\tau}_k\}_{k=1}^K\right)$*, for all* $k = 1, 2, ..., K$*.*

Proof. Let $S^0 = S(\{\overline{\tau}_k\}_{k=1}^K)$ be the equilibrium network under the sectoral tax rates $\{\overline{\tau}_k\}_{k=1}^K$. Naturally, it satisfies:

$$S_k^0 \in \arg\min_{S_k}\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r(\{\overline{\tau}_k\}_{k=1}^K)\}_{r\in S_k}), \quad \forall k. \tag{B.6}$$

Suppose each $\overline{\tau}_k$ falls to $\underline{\tau}_k \leqslant \overline{\tau}_k, \forall k$. From the proof of Lemma B1 it is known that $\overline{P}_k(\{\overline{\tau}_k\}_{k=1}^K) \geqslant \overline{P}_k(\{\underline{\tau}_k\}_{k=1}^K), \forall k$, and hence by Theorem 4 of Milgrom and Shannon (1994):

$$\arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r(\{\underline{\tau}_k\}_{k=1}^K)\}_{r\in S_k}) \supseteq \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r(\{\overline{\tau}_k\}_{k=1}^K)\}_{r\in S_k}),$$
$$\Rightarrow S_k(\{\underline{\tau}_k\}_{k=1}^K) \supseteq S_k(\{\overline{\tau}_k\}_{k=1}^K), \quad \forall k. \tag{B.7}$$

□

B.2 Nominal wage rigidity

The key results of the analysis in the main text are established in a setting where price setting is subject to nominal rigidities, while the nominal wage is fully flexible. However, my framework can be easily extended to allow for nominal rigidities in both price and wage setting, while leaving the key results qualitatively unchanged.

In particular, consider a version of my model with $K+1$ sectors, where the additional sector is populated by firms which act as labor unions.[40] More specifically, they buy labor directly from households at rate W_t and sell those labor services to firms in the remaining K sectors of the economy. Moreover, labor union firms do not use any inputs other than labor and do not sell any output to households. Nominal rigidities in the price setting of firms in the labor union sector are then isomorphic to aggregate nominal wage rigidity. In particular, suppose each labor union firm faces an exogenous probability of price adjustment $(1-\alpha_u) \in (0,1)$. Then under $T = 2$, the price index of the labor union sector at $t = 1$ is given by $P_u = \left[\alpha_u P_{u,0}^{1-\theta} + (1-\alpha_u)W^{1-\theta}\right]^{\frac{1}{1-\theta}}$.[41] The combination of the cash-in-advance constraint and the consumption-labor supply condition still implies that $W = \mathcal{M}$, meaning that the price index of the labor union sector is pinned down exclusively by the exogenous money supply $\mathcal{M}$. However, an exogenous increase/decrease in $\mathcal{M}$ implies an increase/decrease in P_u that is less than one-for-one, implying a degree of nominal rigidity in the cost of purchasing labor services.

Firms in the remaining K sectors of the economy cannot purchase labor directly from households, instead buying it from the labor union sector. Then the unit cost function for any firm in a non-union sector k is given by $\mathcal{Q}_{k,t}\left[S_{k,t}, \mathcal{A}_{k,t}(S_{k,t}), P_{ut}, \{P_{r,t}\}_{r\in S_{k,t}}\right]$, and the optimal choice of supplier sectors $S_{k,t}$ reduces to deciding whether to use more labor at the price P_{ut}, or instead buy from a non-union sector r at $P_{r,t}$. Then, the comparative statics following changes in the productivity mapping, exogenous desired markups and money supply are qualitatively unchanged relative to the results under flexible wages. In particular, either an improvement in the productivity mapping or a fall in desired markups of non-union sectors leaves the nominal price of labor P_u unchanged, while lowering the prices of the non-union sectors. The latter leads to a larger real GDP and encourages the adoption of more suppliers, as they are cheaper relative

[40] Such an approach to introducing nominal wage rigidity has been used in multi-sector models with exogenous production networks, notably Rubbo (2023).

[41] I normalize the sectoral productivity of all firms in the union sector to one in all periods: $\mathcal{A}_{u,t} = 1, \forall t$. Further, I assume that the tax rate in the union sector is set to exactly offset any distortions coming from market power: $(1+\tau_u)\frac{\theta}{\theta-1} = 1$. Those assumptions are without loss of generality, the comparative statics of real GDP and supplier choices remain qualitatively unchanged when those assumptions are relaxed.

to labor, just as in the flexible-wage case. An increase in money supply delivers a less than one-for-one increase in the nominal price of labor P_u, and (weakly) an *even smaller* increase in the prices of the non-union sectors. Intuitively, any nominal rigidities in the price of labor become part of the unit cost function, and any nominal rigidities in price setting emerge over and above the rigidity in the unit cost function. Put simply, wage stickiness makes sectoral prices *even stickier* in terms of the magnitude of response to the monetary expansion. As a result, prices in the non-union sectors rise by less than the nominal price of labor, which in turn rises by less than the money supply. The latter implies an increase in real GDP, as well as an expanded set of suppliers, as they are now cheaper relative to labor. Below I establish these results formally for changes in money supply under nominal wage rigidity. Note that the equivalence is trivial for changes in the productivity mapping or changes in the desired markups of non-union sectors, since those do not affect the price of labor P_u.

Lemma B3 (Baseline GDP)**.** *Suppose Assumptions 1-3 hold. Consider the extended version of the model with $K+1$ sectors, where the additional sector contains a continuum of labor unions, whose price setting is subject to a lottery with an adjustment probability $(1-\alpha_u) \in (0,1)$. Further, consider any two otherwise identical baselines which differ only in the level of money supply given by $\underline{\mathcal{M}}_0$ and $\overline{\mathcal{M}}_0$, respectively, where $\underline{\mathcal{M}}_0 \leqslant \overline{\mathcal{M}}_0$. Then, $C(\overline{\mathcal{M}}_0) \geqslant C(\underline{\mathcal{M}}_0)$.*

Proof. The price index of the union sector is given by

$$P_u = \left[\alpha_u P_{u,0}^{1-\theta} + (1-\alpha_u) W^{1-\theta}\right]^{\frac{1}{1-\theta}} = \left[\alpha_u P_{u,0}^{1-\theta} + (1-\alpha_u)\mathcal{M}^{1-\theta}\right]^{\frac{1}{1-\theta}} = P_u(\mathcal{M}) \tag{B.8}$$

where $P_u(\mathcal{M})$ strictly rises in $\mathcal{M}$, whereas $\frac{P_u(\mathcal{M})}{\mathcal{M}}$ strictly falls in $\mathcal{M}$.

For any non-union sector $k = 1, 2, ..., K$, define $\check{P}_k \equiv \frac{P_k}{P_u}$ to be the ratio of the sectoral price index to the price index of the labor union sector. Let $\{\check{P}_k^0\}_{k=1}^K$ be equilibrium ratios of sectoral prices to labor union price index under the baseline money supply $\underline{\mathcal{M}}_0$. Naturally, they satisfy the following fixed point condition:

$$\check{P}_k^0 = \left[\alpha_k \left[\frac{P_{k,0}}{P_u(\underline{\mathcal{M}}_0)}\right]^{1-\theta} + (1-\alpha_k)\left[(1+\mu_k)\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r^0\}_{r\in S_k})\right]^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{B.9}$$

or

$$\check{P}_k^0 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r^0\}_{r\in S_k}); P_u(\underline{\mathcal{M}}_0)], \quad \forall k \tag{B.10}$$

where $f_k[\quad . \quad ; \overline{P_u}] \leqslant f_k[\quad . \quad ; \underline{P_u}], \forall k$ for any $\overline{P_u} \geqslant \underline{P_u}$.

Suppose the baseline level of money supply rises from $\underline{\mathcal{M}}_0$ to $\overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$, and define $\{\check{P}_k^1\}_{k=1}^K$ such that they satisfy:

$$\check{P}_k^1 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r^0\}_{r\in S_k}); P_u(\overline{\mathcal{M}}_0)], \quad \forall k \tag{B.11}$$

Since $P_u(\overline{\mathcal{M}}_0) \geqslant P_u(\underline{\mathcal{M}}_0)$, then $f_k[\quad . \quad ; P_u(\overline{\mathcal{M}}_0)] \leqslant f_k[\quad . \quad ; P_u(\underline{\mathcal{M}}_0)], \forall k$, it follows that:

$$f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r^0\}_{r\in S_k}); P_u(\overline{\mathcal{M}}_0)] \leqslant f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r^0\}_{r\in S_k}); P_u(\underline{\mathcal{M}}_0)] \tag{B.12}$$

$$\Rightarrow \check{P}_k^1 \leqslant \check{P}_k^0 = \check{P}_k(\underline{\mathcal{M}}_0), \quad \forall k. \tag{B.13}$$

In the same way as in proof of part (a) of Lemma 1, from fixed point iteration it then follows that $\check{P}_k(\overline{\mathcal{M}}_0) \leqslant \check{P}_k(\underline{\mathcal{M}}_0), \forall k$. From the cash-in-advance constraint:

$$\begin{aligned} C(\overline{\mathcal{M}}_0) = \left(\frac{P^c(P_1, ..., P_K)}{\overline{\mathcal{M}}_0}\right)^{-1} &= \left(P^c\left(\frac{P_1}{\overline{\mathcal{M}}_0}, ..., \frac{P_K}{\overline{\mathcal{M}}_0}\right)\right)^{-1} \\ &= \left(\frac{P_u(\overline{\mathcal{M}}_0)}{\overline{\mathcal{M}}_0} P^c(\check{P}_1(\overline{\mathcal{M}}_0), ..., \check{P}_K(\overline{\mathcal{M}}_0))\right)^{-1}. \end{aligned} \tag{B.14}$$

Since $\check{P}_k(\overline{\mathcal{M}}_0) \leqslant \check{P}_k(\underline{\mathcal{M}}_0), \forall k$ and $\frac{P_u(\overline{\mathcal{M}}_0)}{\overline{\mathcal{M}}_0} \leqslant \frac{P_u(\underline{\mathcal{M}}_0)}{\underline{\mathcal{M}}_0}$, it follows that

$$C(\overline{\mathcal{M}}_0) \geqslant C(\underline{\mathcal{M}}_0). \tag{B.15}$$

□

Lemma B4 (Baseline supplier choices)**.** *Suppose Assumptions 1-5 hold. Consider the extended version of the model with $K+1$ sectors, where the additional sector contains a continuum of labor unions, whose price setting is subject to a lottery with an adjustment probability $(1-\alpha_u) \in (0,1)$. Further, consider any two otherwise identical baselines which differ only in the level of money supply given by $\underline{\mathcal{M}}_0$ and $\overline{\mathcal{M}}_0$, respectively, where $\underline{\mathcal{M}}_0 \leqslant \overline{\mathcal{M}}_0$. Then, $S_k(\overline{\mathcal{M}}_0) \supseteq S_k(\underline{\mathcal{M}}_0)$, for all the non-union sectors $k = 1, 2, ..., K$.*

Proof. Let $S^0 = S(\underline{\mathcal{M}}_0)$ be the equilibrium network under the baseline money supply $\underline{\mathcal{M}}_0$. Naturally, for every non-union sector $k = 1, 2, .., K$ it satisfies:

$$\begin{aligned} S_k^0 \in \arg\min_{S_k} \mathcal{Q}_k(S_k, \mathcal{A}_k(S_k), P_u(\underline{\mathcal{M}}_0), &\{P_r(\underline{\mathcal{M}}_0)\}_{r\in S_k}) \\ &= \arg\min_{S_k} P_u(\underline{\mathcal{M}}_0)\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r(\underline{\mathcal{M}}_0)\}_{r\in S_k}) \\ &= \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r(\underline{\mathcal{M}}_0)\}_{r\in S_k}), \quad \forall k. \end{aligned} \tag{B.16}$$

Suppose the baseline level of money supply rises from $\underline{\mathcal{M}}_0$ to $\overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$. From the proof of Lemma B3 it is known that $\check{P}_k(\overline{\mathcal{M}}_0) \leqslant \check{P}_k(\underline{\mathcal{M}}_0), \forall k$, and hence by Theorem 4 of Milgrom and Shannon (1994):

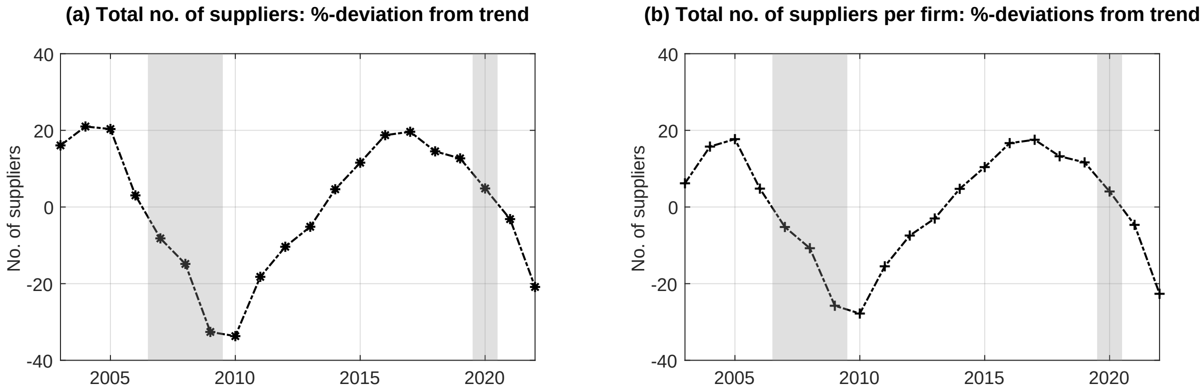
$$\arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r(\overline{\mathcal{M}}_0)\}_{r\in S_k}) \supseteq \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r(\underline{\mathcal{M}}_0)\}_{r\in S_k}), \quad \forall k \tag{B.17}$$

$$\Rightarrow S_k(\overline{\mathcal{M}}_0) \supseteq S_k(\underline{\mathcal{M}}_0), \quad \forall k. \tag{B.18}$$

□

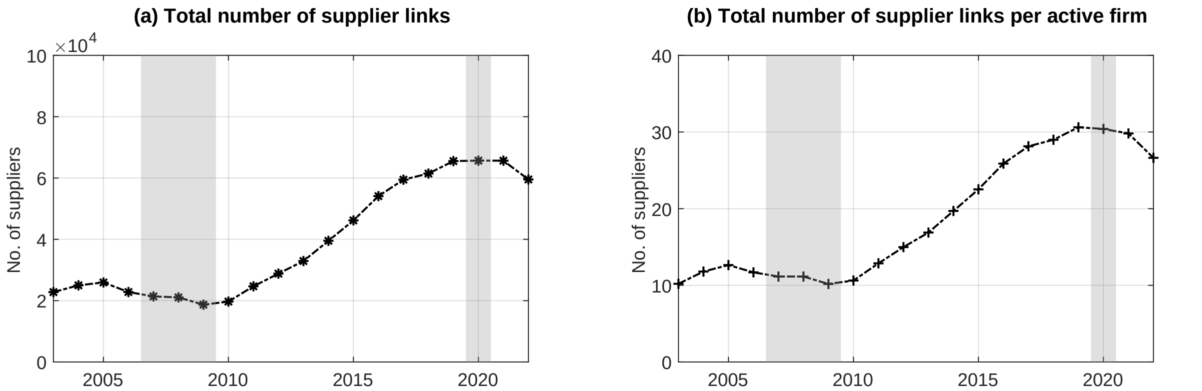
C Additional data figures

Figure C.1: **Suppliers over the business cycle: %-deviations from trend**



Notes: Panels (a) and (b) show, respectively, the total number of supplier relationships and the number of suppliers per active firm and in the United States (in %-deviation from trend) based on the annual FactSet Revere Supply Chain Relationships dataset (2003-2022); the series are detrended using an HP filter with smoothing parameter 400. Grey bars indicate NBER recession dates.

Figure C.2: **Suppliers over the business cycles: non-detrended data**



Notes: Panels (a) and (b) show, respectively, the total number of supplier relationships and the number of suppliers per active firm and in the United States based on the annual FactSet Revere Supply Chain Relationships dataset (2003-2022). The series have not been detrended. Grey bars indicate NBER recession dates.

D Quantitative dynamic model: additional results

D.1 Steady-state parameter estimation

Here I provide more details on how the productivity parameters $B \equiv [\{B_{k,0}\}_k, \{B_{kr}\}_{kr}]$, or $\{b_{k,0}\}_k$ and $\{b_{kr}\}_{kr}$ in logs, are estimated. The overall strategy is to estimate those parameters such that an observed set of supplier choices $S = \{S_k\}_{k=1}^K$, which is in turn based on an observed input-output matrix, emerges as the equilibrium supplier choice in the deterministic steady-state of my model.

For convenience, note that only relative sectoral prices $\overline{P}_k \equiv P_k/W = P_k/M$ are pinned down in steady-state. Moreover, for a given set of productivity parameters B and supplier choices S, the steady-state relative prices can be computed analytically as:

$$\overline{\boldsymbol{p}}(B,S) = [I - \Omega(S)]^{-1} \times [-\log \boldsymbol{\alpha}(B,S) + \log(1+\boldsymbol{\mu})] \tag{D.1}$$

where $\overline{\boldsymbol{p}} \equiv [\log \overline{P}_1, ..., \log \overline{P}_K]^T, \boldsymbol{\mu} \equiv [\mu_1, ..., \mu_K]^T$ and $\boldsymbol{\alpha}(B,S) \equiv [\alpha_1(B,S), ..., \alpha_K(B,S)]^T$, where $\alpha_k(B,S) \equiv B_{k,0} \prod_{r \in S_k} B_{kr}$.

Recall that a link between sectors k and r exists if and only if $\omega_{kr}\overline{p}_r < b_{kr}$. The algorithm below uses this optimality principle to pin down a set of productivity parameters B for a given set of supplier choices S and the associated input-output cost shares given by the matrix $\Omega : \omega_{kr} = [\Omega]_{kr}$.

Algorithm D.1. *For a given set of supplier choices $S = \{S_k\}_{k=1}^K$ and a scaling parameter $m > 0$, take an initial guess for productivity parameters B^0 and compute initial relative prices $\overline{\boldsymbol{p}}^0 = \overline{\boldsymbol{p}}(B^0, S)$. Then follow the steps below for all pairs of sectors $\{k, r\}_{k,r=1}^K$:*

(i) If $r \in S_k$ and $b_{kr}^0 \leqslant \omega_{kr}\overline{p}_r^0$, then update $b_{kr}^1 = \omega_{kr}\overline{p}_r^0 + (\omega_{kr}\overline{p}_r^0 - \varrho_{kr})$ where ϱ_{kr} is a draw from truncated $\mathcal{N}\left(\frac{m}{K}, \frac{1}{K^2}\right)$ distribution with support over the interval $(-\infty, \omega_{kr}\overline{p}_r^0]$;

(ii) If $r \notin S_k$ and $b_{kr}^0 > \omega_{kr}\overline{p}_r^0$, then update $b_{kr}^1 = \omega_{kr}\overline{p}_r^0 - (\omega_{kr}\overline{p}_r^0 - \varrho_{kr})$ where ϱ_{kr} is a draw from truncated $\mathcal{N}\left(\frac{m}{K}, \frac{1}{K^2}\right)$ distribution with support over the interval $(-\infty, \omega_{kr}\overline{p}_r^0]$;

(iii) If $r \in S_k$ and $b_{kr}^0 > \omega_{kr}\overline{p}_r^0$ or if $r \notin S_k$ and $b_{kr}^0 \leqslant \omega_{kr}\overline{p}_r^0$, then update $b_{kr}^1 = b_{kr}^0$;

(iv) Recompute the relative sectoral prices with the updated productivity parameters: $\overline{\boldsymbol{p}}^1 = \overline{\boldsymbol{p}}(B^1, S)$. If $\overline{\boldsymbol{p}}^1 = \overline{\boldsymbol{p}}^0$, then quit the algorithm; otherwise, set $\overline{\boldsymbol{p}}^0 = \overline{\boldsymbol{p}}^1$ and $B^0 = B^1$ and return to (i).

The initial guesses for the parameters are set as follows. The $\{b_{k,0}\}$ parameters are set to independent draws from $\mathcal{N}(m, 1)$. As for the initial guesses of $\{b_{k,r}\}_{k,r=1}^K$, those are taken as independent draws from $\mathcal{N}\left(-\frac{m}{K}, \frac{1}{K^2}\right)$.[42]

[42]Initializing $\{b_{k,0}\}$ and $\{b_{k,r}\}_{k,r=1}^K$ at common values of m and $-m/K$, respectively, produces a calibrated productivity distribution that leaves the dynamic responses virtually unchanged. I have also considered alternative variances of the updating distribution. In particular, instead of using draws from truncated $\mathcal{N}\left(\frac{m}{K}, \frac{1}{K^2}\right)$, I have considered variances of $(0.8/K)^2$ and $(1.2/K)^2$. Reassuringly, the resulting calibrations produce dynamic responses that are very close to the baseline.

Applying the algorithm to actual data requires setting numerical values to several objects. First, I take the observed set of suppliers S from the BEA Detail level input-output accounts for 2007: a given supplier exists if and only if the input-output cost share is positive in the accounts. Second, I set the scaling parameter m to make sure the steady-state of my model matches the level of real GDP in 2007. Third, the matrix of input-output cost shares Ω is set as follows. For the linkages that exist in 2007, the cost shares ω_{kr} is set to the observed cost share. For the linkages that do not exist in 2007, I follow Acemoglu and Azar (2020) in imputing the cost shares as $\omega_{kr} = 0.95 \times \left(1 - \sum_{r' \in S_k} \omega_{kr'}\right) \times \frac{\sum_{k': r \in S_{k'}} \omega_{k'r}}{\sum_{k', r': r' \in S_{k'}} \omega_{k'r'}}$. This ensures that all the imputed cost shares are proportional to the observed outdegree of the supplier sector and that the row sums of the full input–output matrix (including the linkages absent in 2007) are less than one so that labor is an essential input as required by Assumption 1.

D.2 Relative price thresholds

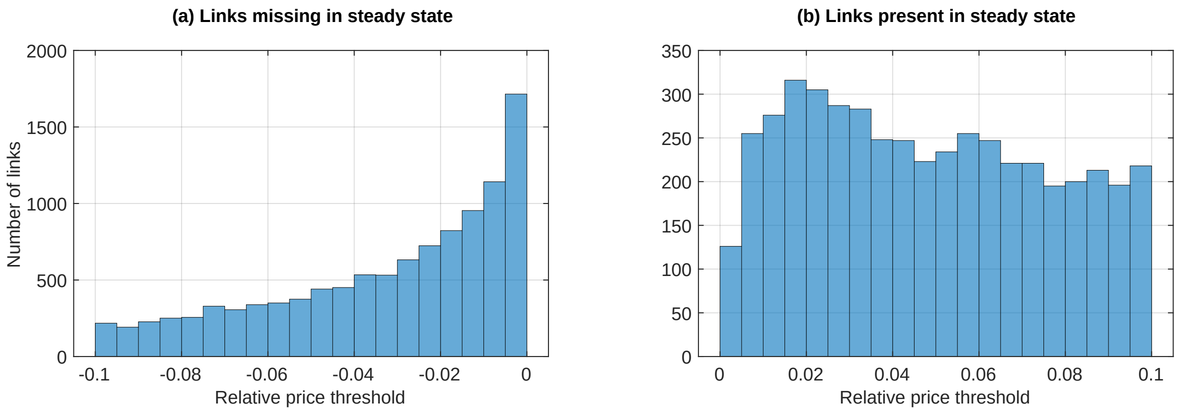
Note that a firm in sector k is indifferent between linking to sector r if and only if $\omega_{kr}\overline{p}_r = b_{kr}$. Therefore, if the (k, r) relationship currently exists ($\omega_{kr}\overline{p}_r < b_{kr}$), then the relative price in sector r needs to increase by $\varsigma_{kr} \equiv \left(\frac{b_{kr}}{\omega_{kr}} - \overline{p}_r\right)$ in order to bring a firm in sector k to the point of indifference. Similarly, it would have to drop by the same amount if there relationship between k and r did not exist at $\overline{p}_r$. I call $\{\varsigma_{kr}\}_{kr}$ the *relative price thresholds*, as they indicate the amount a relative price needs to go up for an existing relationship to sever, or go down for a currently non-existing relationship to form.

Figure D.1 plots the distributions of relative price thresholds in the steady state of my model, focusing on thresholds up to 10% in absolute value. Panel (a) shows that over 2700 linkages that do not exist in steady state require a relative price drop of 1% or less in order to be activated. At the same time, fewer than 400 linkages that do exist in steady state would be severed following relative price increases of 1% or less. This is the key reason behind the asymmetry in network dynamics following expansionary and contractionary shocks. Shocks that lead to drops in relative prices, such as expansionary productivity and monetary shocks, activate many more linkages than the number of relationships severed following equally-sized contractionary shocks, which deliver increases in relative prices.

D.3 Endogenous network persistence

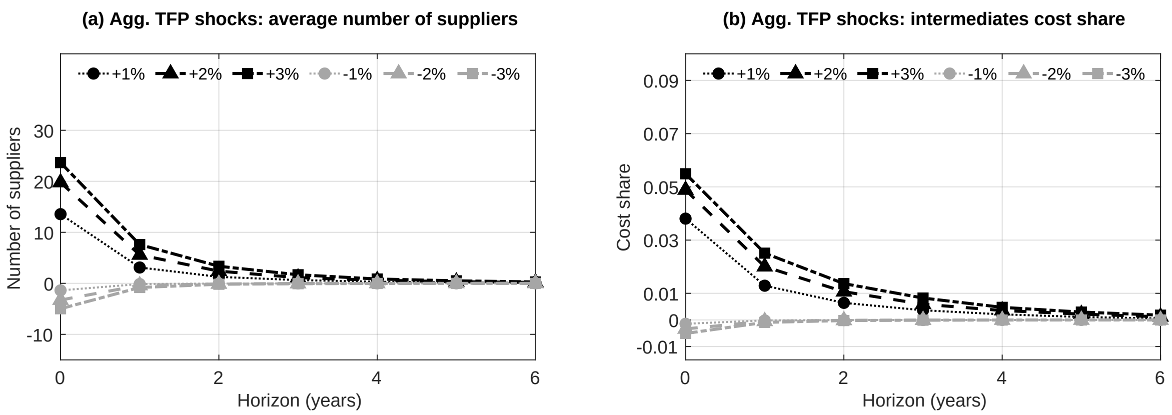
In Figure D.2 I plot the responses of the average number of suppliers and the average intermediates cost share following purely transitory ($\rho_z = 0$) aggregate TFP shocks of different sizes. One can see that despite the transitory nature of the shocks, the equilibrium network responses are persistent. This is because the presence of nominal rigidities makes (relative) price changes persistent even following transitory shocks. In this sense, the combination of endogenous supplier choice and sticky prices delivers endogenous persistence in equilibrium production networks.

Figure D.1: **Histograms of steady-state relative price thresholds (389 sectors)**



Notes: Panel (a) reports the steady-state relative price thresholds for input-output linkages that are missing in the steady state of the quantitative model. Panel (b) reports the steady-state relative price thresholds for input-output linkages that are instead present in the steady state of the quantitative model.

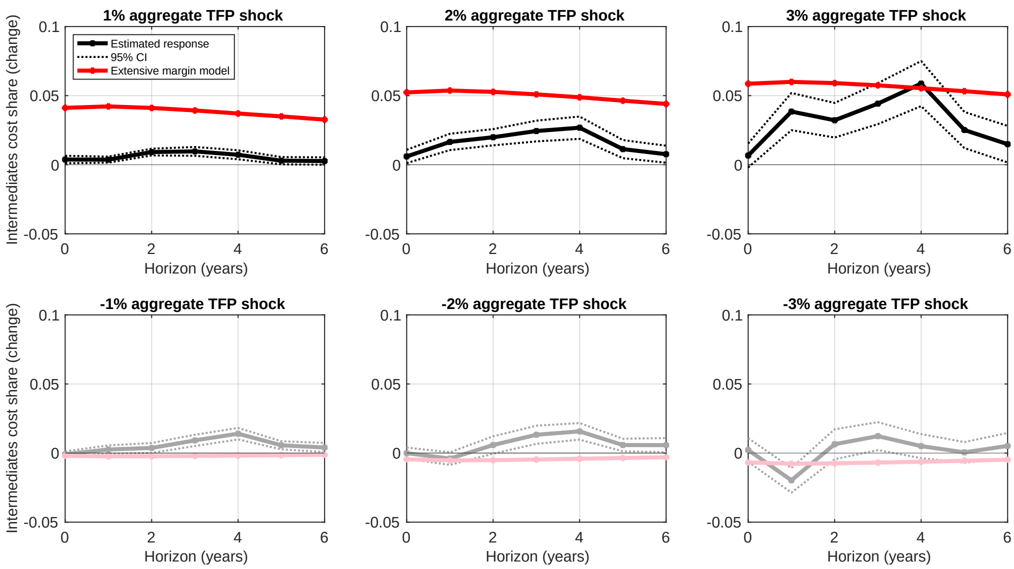
Figure D.2: **Baseline networks driven by transitory productivity changes**($\rho_z = 0$)



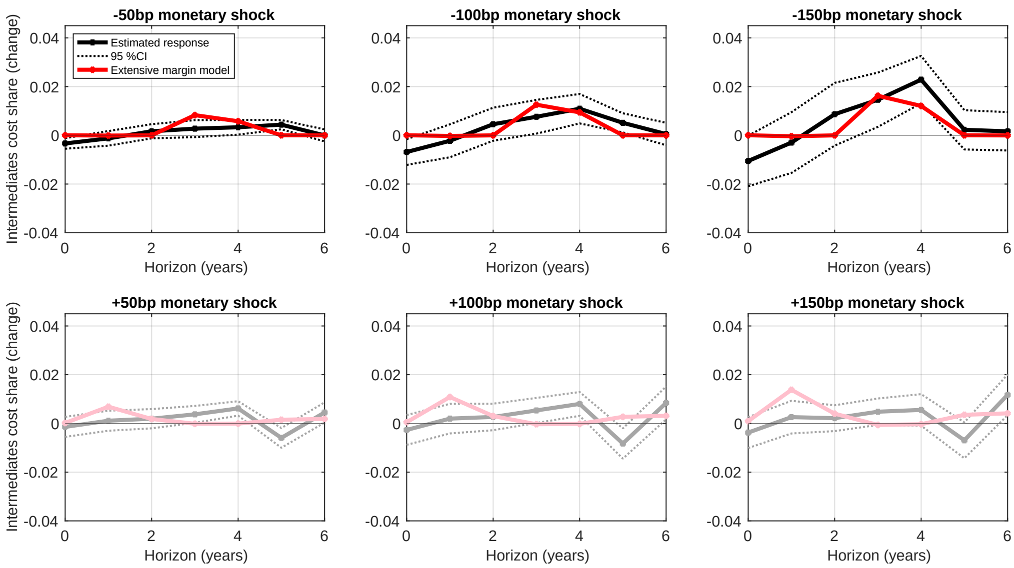
Notes: Panels (a) and (b) show the average number of suppliers and average intermediates cost shares across baselines where $\mathcal{M}_0 = 1$ and $\zeta_t = 0, \forall t \geqslant 2$, and consider values of $\zeta_1 = \{-3\%, -2\%, -1\%, 1\%, 2\%, 3\%\}$ and $\rho_z = 0$ relative to $\zeta_1 = 0$.

Figure D.3: **Non-linear IRFs of intermediates cost shares: model vs. data**

(a) Aggregate TFP shocks

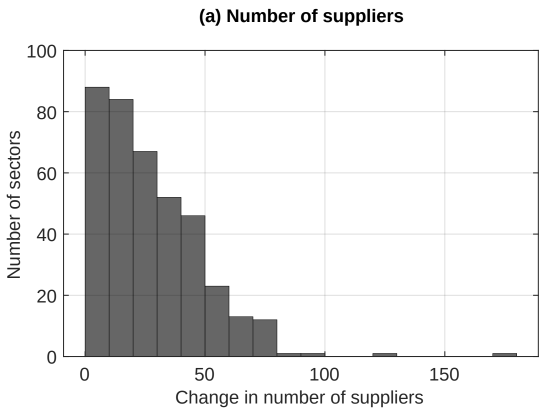


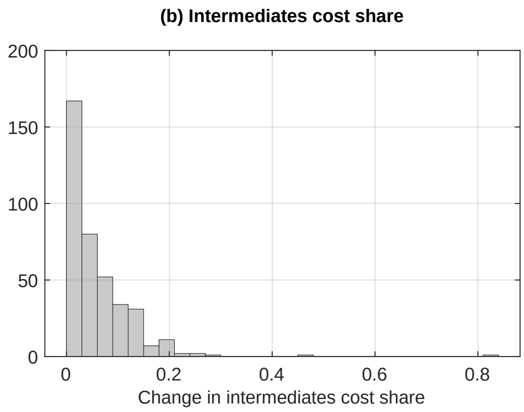
(b) Monetary shocks



Notes: Panel (a) shows the IRFs of the average intermediates cost share in the baseline quantitative endogenous network model to aggregate TFP shocks, as well as the corresponding estimated non-linear IRFs to productivity shocks based on Fernald (2014), using the specification in (18). Panel (b) shows the IRFs of the average intermediates cost share in the baseline quantitative endogenous network model to monetary shocks and the corresponding estimated non-linear IRFs to monetary shocks based on Romer and Romer (2004), using the specification in (18). See Appendix F.1 for the procedure that translates the Romer and Romer (2004) shocks into the money growth path that is then fed into the quantitative model. Dotted lines represent 95% confidence intervals.

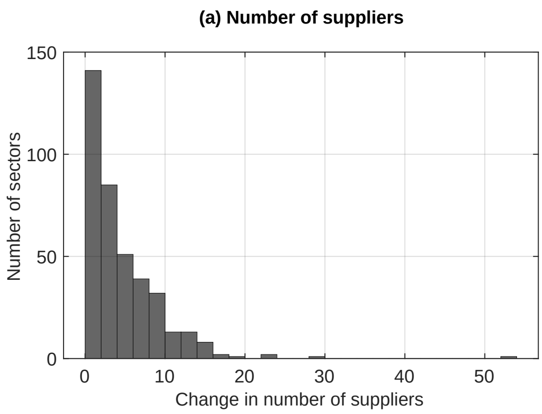
Figure D.4: **Sectoral peak responses to +3% aggregate TFP shock**

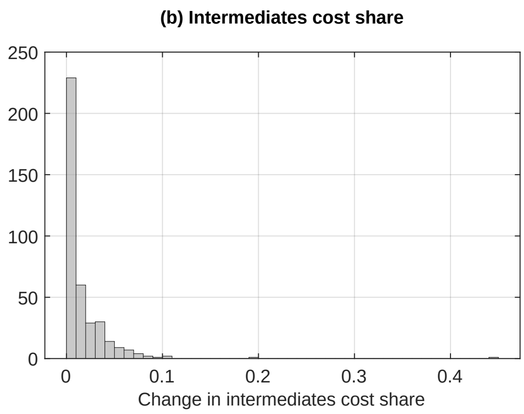




Notes: Panels (a) and (b) show the histogram of peak sector-specific responses of the number of suppliers and intermediates cost shares for a baseline with $\mathcal{M}_0 = 1$, $\zeta_t = 0, \forall t \geqslant 2$, and $\zeta_1 = +3\%$ relative to $\zeta_1 = 0$.

Figure D.5: **Sectoral peak responses to -150 bp monetary shock**





Notes: Panels (a) and (b) show the histogram of peak sector-specific responses of the number of suppliers and intermediates cost shares under $\mathcal{M}_0 = 1$ and $\zeta_t = 0, \forall t \geqslant 1$ following a monetary shock corresponding to 150 bp interest rate cut. See Appendix F.1 for the procedure that translates the Romer and Romer (2004) shocks into the money growth path that is then fed into the quantitative model.

E CES models

The model in the main text emphasizes variation in the intermediates cost share arising from supplier choices at the extensive margin. In this section I assess whether the observed cost share variation could instead be plausibly captured purely by intensive-margin variation. To do that, I consider modelling setups with a fixed exogenous set of suppliers, but with a constant-elasticity of substitution (CES) production technology, which can create endogenous variation in the intermediates cost share at the intensive margin.

E.1 CES roundabout model

I first consider a one-sector ($K = 1$) roundabout economy in the spirit of Basu (1995). In particular, each firm in the production sector has access to the following technology:

$$Y_t(j) = \mathcal{Z}_t \left[(1-\bar{\delta})^{\frac{1}{\eta}} N_t^{\frac{\eta-1}{\eta}}(j) + \bar{\delta}^{\frac{1}{\eta}} Z_t^{\frac{\eta-1}{\eta}}(j) \right]^{\frac{\eta}{\eta-1}}, \tag{E.1}$$

where $\eta > 0$ is the elasticity of substitution across labor and intermediates. When $\eta \in (0,1)$, labor and intermediates are complements; when $\eta > 1$, they are substitutes instead. As $\eta \to 1$, the production function becomes Cobb-Douglas, where the production inputs are neither complements nor substitutes.

With the exception of the production technology, the rest of the modelling setup remains unchanged relative to the main text. In particular, the production sector is still populated by a continuum of firms with the substitution elasticity across them given by $\theta > 1$. Pricing is still subject to Calvo (1983) nominal rigidities, with the probability of adjustment $(1-\alpha)$. Household preferences also remain unchanged, and the cash-in-advance constraint stays in place. The exogenous processes for aggregate TFP and money supply growth are still AR(1) with persistence ρ_z and ρ_m, respectively.[43]

In equilibrium, the aggregate cost share of intermediate inputs is given by:

$$\delta_t \equiv \frac{P_t Z_t(j)}{\mathcal{Q}_t(j) Y_t(j)} = \bar{\delta} \times \frac{(P_t/W_t)^{1-\eta}}{(1-\bar{\delta}) + \bar{\delta}(P_t/W_t)^{1-\eta}}. \tag{E.2}$$

Note that under Cobb-Douglas production ($\eta = 1$), the cost share is fixed at $\bar{\delta}$. Whenever labor and intermediates are complements ($\eta \in (0,1)$), δ_t co-moves with the relative price P_t/W_t. Instead, whenever the production inputs are substitutes ($\eta > 1$), the intermediates cost share counter-moves with P_t/W_t. It follows that the value of η is crucial for the cyclical properties of the intermediates cost share.

Calibration As in the main text, I calibrate the model to the US at annual frequencies. I set $\beta = 0.99$ to target annualized real rate of 1% in steady state. The within-sector elasticity

[43] An additional small change relative to the main text is that I set the steady-state tax to $1+\tau = \frac{\theta-1}{\theta}$, which eliminates the inefficient markup, ensuring all (relative) prices are equal to one in the deterministic steady state.

stays at $\theta = 6$. The steady-state intermediates cost share is $\overline{\delta} = 0.62$. The Calvo parameter is set to $\alpha = 0.22$. The persistence parameters of productivity and money supply growth are set at $\rho_z = 0.86$ and $\rho_m = 0.80$, respectively.

As for the value of the substitution elasticity η, I consider two calibration strategies. First, I consider values of η established in widely-cited studies, which tend to find an elasticity between 0.5 and one (Rotemberg and Woodford, 1996; Atalay, 2017; Oberfield and Raval, 2021). For concreteness, I take $\eta = 0.78$ from Oberfield and Raval (2021). Second, I obtain my own estimate of η by matching the model-implied responses of δ_t following productivity and monetary shocks, to those obtained using non-linear local projections (18) in Section 5.1.[44]

Elasticity from prior literature Figure E.1 reports the fully non-linear perfect foresight transitions of δ_t in the CES roundabout model under $\eta = 0.78$ following aggregate TFP and monetary (interest rate) shocks of different sizes. In addition, the Figure also reports the empirical responses of the intermediates cost share based on the local projection specification in (18). I use (scaled) responses of nominal GDP (NGDP) growth to identified interest rate shocks (Figure F.1) in order to obtain a sequence of money growth rate shocks $\{\varepsilon_t^m\}_t$ that generates the same path of NGDP/money supply growth as an exogenous change in the nominal interest rate. For each size of interest rate shock, I then feed the corresponding sequence of money growth rate shocks into the model.

Three aspects of the model-based responses put it at major odds with the data. First, unlike in the estimated responses, the model-based intermediates cost share δ_t is countercyclical: it falls after expansionary productivity and monetary shocks, and *vice versa*. This is a consequence of inputs being complements under $\eta = 0.78$: expansionary shocks make labor relatively more expensive than intermediates, lowering the cost share of the latter. Second, the model, even though solved fully-non-linearly, produces essentially no asymmetry in the responses of δ_t. By contrast, the estimated responses have a particularly pronounced (and statistically significant) asymmetry based on the sign of the shock, with expansionary shocks producing much bigger responses. Third, the magnitudes of fluctuations in the intermediates cost share produced by the model are markedly lower than in the estimated responses. For example, following the large +3% aggregate TFP shock, the model generates a reduction in intermediates cost share by only 0.003 at the trough, as opposed to an increase by 0.06 in the data.

Overall, it follows that the CES roundabout model, calibrated to $\eta = 0.78$, does not reproduce the salient cyclical properties of the intermediates cost share observed in the data.

Own elasticity estimate I now follow a different strategy for pinning down the elasticity parameter η: instead of taking an available estimate from the literature, I estimate it by matching the model-based and empirical impulse-response functions. More specifically, I search for a value of η that minimizes the squared normalized deviations between peaks of IRFs based on the CES roundabout model and those estimated with the non-linear local projection in (18) across

[44] I also set the sectoral tax to target a price to wage ratio of one in steady state.

different shock types and sizes. More formally, the estimated elasticity parameter satisfies:

$$\eta^* \in \arg\min_{\eta} \left[\sum_{\zeta} \left\{ \frac{\delta(\zeta;\eta) - \hat{\delta}(\zeta)}{\hat{\sigma}(\zeta)} \right\}^2 \quad + \quad \sum_{\varepsilon^m} \left\{ \frac{\delta(\varepsilon^m;\eta) - \hat{\delta}(\varepsilon^m)}{\hat{\sigma}(\varepsilon^m)} \right\}^2 \right], \tag{E.3}$$

where $\delta(.;\eta)$ is the peak(trough) of the model-based IRF following an expansionary(contractionary) shock, $\hat{\delta}$ is the corresponding estimated peak(trough) and $\hat{\sigma}$ is the standard error of the relevant estimated peak(trough). For estimation, I consider one-time aggregate TFP shocks of sizes $\zeta \in \{-3\%, -2\%, -1\%, 1\%, 2\%, 3\%\}$, as well as monetary (interest rate) shocks of sizes $\varepsilon^m \in \{-150\text{bp}, -100\text{bp}, -50\text{bp}, 50\text{bp}, 100\text{bp}, 150\text{bp}\}$. As before, I translate interest rate shocks into money growth shocks using the estimated relationship between NGDP growth and Romer and Romer (2004) shocks in Figure F.1.

Figure E.2 presents the estimation results. Panel (i) shows that the overall squared distance from the data is minimized at $\eta^* = 2.27$. While higher than the elasticities found in older established studies (as discussed, those tend to be close to one from below), my elasticity is within the range of estimates in some more recent papers, such as Chan (2021) and Mertens and Schoefer (2024). In Panel (ii) I report the peaks/troughs of IRFs in data, as well as in the CES roundabout model under $\eta = 2.27$. One can see while the CES roundabout model can in general successfully match the IRF troughs following contractionary TFP and monetary shocks, it systematically underpredicts the peak of δ_t response to expansionary shocks. This problem is particularly severe for expansionary monetary shocks: following a -150 bp easing, the estimated peak rise in intermediates cost share is almost 0.02, as opposed to only 0.002 generated by the CES model. For reference, I also report the peaks/troughs generated by the extensive margin network model studied in the main text. Notably, while the extensive margin model is not explicitly disciplined to match any of the cost share dynamics following shocks, it successfully produces the required asymmetry in the responses, and generates cost share fluctuations that are generally quantitatively consistent with the data, especially for monetary shocks.

In Figure E.3 I report full IRFs of the intermediates cost share generated by the CES roundabout model with $\eta = 2.27$, as well as the corresponding empirical responses based on local projections in (18). One can see that, relative to the results under $\eta = 0.78$, the model successfully generates a procyclical intermediates cost share, since the production inputs are now substitutes. However, the two other issues remain in place: the model-based responses are still essentially symmetric across expansionary and contractionary shocks, and the magnitudes of fluctuations are generally smaller than in the data, especially for monetary shocks.

I have also conducted an additional exercise, where I estimate the elasticity parameter to match the IRFs following either only aggregate TFP shocks, or only monetary shocks. For the estimation procedure that only targets responses to aggregate TFP shocks, I obtain $\eta^* = 2.17$. This is not very different from the baseline estimate of 2.27, and does not majorly affect the model fit. At the same time, targeting only responses to monetary shocks delivers $\eta^* = 11.33$. While this drastically improves the model fit for monetary shocks, it comes at the expense of substantially overpredicting the response of δ_t to aggregate TFP shocks. For example, for $\eta = 11.33$ the CES roundabout model predicts that a 3% aggregate TFP expansion leads to an increase of 0.15 in the intermediates cost share, against just 0.06 in the data.

All in all, I find that even if the elasticity of substitution in the CES roundabout model is explicitly estimated to bring the δ_t responses as close as possible to the data, does not, in general, reproduce the estimated behavior of the intermediates cost share. Moreover, when it comes to generating asymmetries in the responses, and especially for monetary shocks, it is outperformed by the baseline extensive margin network model. Another issue with the CES roundabout model is that is struggles to simultaneously explain the cost share dynamics following TFP and monetary shocks, as signified by drastically different elasticities required to fit the two types of shocks separately. At the same time, the baseline extensive margin model does not have this issue.

E.2 CES network model

I now turn to a K-sector economy with fixed exogenous networks at the extensive margin, and with CES production functions. In particular, each firm in a sector k has access to the following technology:

$$Y_{k,t}(j) = \mathcal{Z}_t \left[\left(1 - \sum_{r=1}^{K} \omega_{kr}\right)^{\frac{1}{\eta}} N_{k,t}^{\frac{\eta-1}{\eta}}(j) + \sum_{r=1}^{K} \omega_{kr}^{\frac{1}{\eta}} Z_{kr,t}^{\frac{\eta-1}{\eta}}(j) \right]^{\frac{\eta}{\eta-1}}. \tag{E.4}$$

As before, $\eta > 0$ is the elasticity of substitution across production inputs. Once again, with the exception of the production technology, the rest of the modelling setup remains unchanged relative to the main text.[45]

The equilibrium cost share of intermediate inputs for any firm in sector k is given by:

$$\delta_{k,t} \equiv \frac{\sum_{r=1}^{K} P_{r,t} Z_{kr,t}(j)}{\mathcal{Q}_{k,t}(j) Y_{k,t}(j)} = \sum_{r=1}^{K} \omega_{kr} \times \frac{(P_{r,t}/W_t)^{1-\eta}}{\left(1 - \sum_{r=1}^{K} \omega_{kr}\right) + \sum_{r=1}^{K} \omega_{kr} (P_{r,t}/W_t)^{1-\eta}}. \tag{E.5}$$

One can see, once again, that the value of substitution elasticity is a key determinant of the cyclicality of the intermediates cost share.

Calibration As in the main text, I calibrate the model to $K = 389$ sectors of the US economy, at annual frequencies. I set $\beta = 0.99$ to target annualized real rate of 1% in steady state. The within-sector elasticity of substitution stays at $\theta = 6$. The persistence parameters of productivity and money supply growth are set at $\rho_z = 0.86$ and $\rho_m = 0.80$, respectively.

I calibrate the input-output cost shares $\{\omega_{kr}\}_{kr}$ and the final consumption shares $\{\omega_{ck}\}_k$ to match the corresponding observed shares in the 2007 BEA Detail-level input-output accounts. Sector-specific Calvo parameters $\{\alpha_k\}_{k=1}^{K}$ are set as one minus the sector-specific frequency of price adjustment from Pasten et al. (2020a,b).

[45] As in the roundabout case, I set the steady-state taxes to $1 + \tau_k = \frac{\theta-1}{\theta}, \forall k$, which ensures all (relative) sectoral prices are equal to one in the deterministic steady state.

As in the CES roundabout case, I consider two calibration strategies for the substitution elasticity η. First, I consider $\eta = 0.78$, corresponding to the value estimated by Oberfield and Raval (2021). Second, since the 389-sector model is too large to permit explicit estimation of η by IRF matching, I consider $\eta = 2.27$ as the estimated elasticity from the CES roundabout economy. [46]

Elasticity from prior literature In Figure E.4, I set $\eta = 0.78$ and report fully non-linear perfect foresight dynamics of the average intermediates cost share $\left(\delta_t \equiv \frac{1}{K}\sum_{k=1}^{K}\delta_{k,t}\right)$ under baselines with different levels of productivity and money supply. My aim is to compare the dynamics of the intermediates cost share with that generated by the extensive margin network model in the main text, as reported in Panels (b) and (d) of Figure 5. Three major differences relative to the extensive margin network model arise. First, the δ_t dynamics are countercyclical in the CES network model: the intermediates cost share is larger in contractionary baselines. As in the CES roundabout case, this is due to the fact that under $\eta = 0.78$ labor and intermediates are complements. Second, in the network CES economy the δ_t responses show essentially no asymmetry in the shock sign, whereas in the extensive margin model expansionary shocks deliver much larger changes in the intermediates cost share. Third, the magnitudes of fluctuations in the intermediates cost share are much smaller in the network CES model: following the large +3% increase in productivity, δ_t changes by just over 0.002, as opposed to the increase of nearly 0.06 in the extensive margin model.

In Figure E.6, I compare IRFs following small monetary expansions around baselines with different levels of productivity and money supply. One can see that the (peaks of) IRFs are marginally larger around baselines with lower productivity and smaller money supply. This is in contrast, both qualitatively and quantitatively, to the results under the extensive margin model, which generates larger IRFs around expansionary baselines, with a much stronger degree of non-linearity, as shown by the gray lines in Panels (b) and (d) of Figure E.6. Similarly, Figure E.8 shows that for large monetary shocks, the CES network model predicts size dependence that is opposite to that under the extensive margin model. In particular, one can see that as one doubles the size of the monetary expansion, the response of GDP less than doubles, and *vice versa*. One can also see that in addition to predicting the opposite pattern of size dependence, the extensive margin model also produces much stronger non-linearity following large monetary shocks.

Own elasticity estimate I now set the elasticity at the value explicitly estimated with the CES roundabout model ($\eta = 2.27$), and repeat the exercises for the dynamics of the intermediates cost share and the degree of non-linearity in monetary transmission.

Figure E.5 reports the dynamics of the average intermediates cost share δ_t under different productivity money supply baselines. First, note that the dynamics are now procyclical: δ_t is larger under expansionary baselines, just as in the extensive margin network model. This

[46] I also set sectoral taxes to target sectoral price to wage ratios of one in steady state.

is because labor and intermediate inputs are now substitutes, and labor is relatively more expensive under expansionary baselines. Second, while procyclical, the responses of δ_t remain symmetric in the shock sign, in contrast to the extensive margin model. Quantitatively, under recessionary baselines, the CES network model produces drops in δ_t that are similar to those in the extensive margin network model. In contrast, the CES network model produces much more modest increases in δ_t under expansionary baselines: for example, following a 3% increase in aggregate TFP, the intermediates cost share rises by just under 0.02 at the peak, as opposed to almost 0.06 in the extensive margin model.

Turning to IRFs following small monetary shocks around different baselines, Figure E.7 shows that under $\eta = 2.27$ the responses are stronger around expansionary baselines, just as in the extensive margin network model. However, while qualitatively similar, the quantitative properties of such non-linearities are different. On the one hand, the drop in the degree of monetary non-neutrality observed in contractionary baselines is similar in the CES and the extensive margin network models. On the other hand, the extensive margin model predicts a much stronger increase in monetary non-neutrality around expansionary baselines. In particular, around the high (+3%) productivity baseline, the extensive margin predicts an increase in monetary non-neutrality that is 6 times larger than in the CES model. Similarly, for the high money supply baseline ($\varepsilon^m_{-\infty} = 6\%$), the additional efficacy of monetary policy is twice as large in the extensive margin model. Finally, Figure E.9 shows that under $\eta = 2.27$ the network CES model produces the same qualitative pattern of size dependence as the extensive margin model. However, the quantitative strength of such non-linearity is different. While the drop in the degree of monetary non-neutrality under large monetary contractions is similar in magnitudes across the two models, the additional non-linear GDP response under large monetary expansions is much more sizable quantitatively in the extensive margin model.

All in all, the CES network model under $\eta = 2.27$ shows that intensive-margin adjustment can generate procyclical intermediates cost shares and non-linear monetary transmission that are qualitatively in line with the extensive-margin model. Quantitatively, however, it does not generate the same increase in δ_t in expansionary states and therefore produces a much smaller degree of non-linearity in monetary transmission.

E.3 Relationship to existing estimates of the substitution elasticity η

To clarify the relationship between my model and existing estimates of the substitution elasticity between labor and intermediates based on CES production (e.g. Atalay, 2017), let me first outline the structure underlying the econometric strategy of such papers. This comparison also highlights the different mechanisms driving intermediates cost-share cyclicality in my model and in fixed-linkage CES environments.

Recall that in the CES roundabout model, the aggregate cost share of intermediate inputs is $\delta_t = \bar{\delta} \times \frac{(P_t/W_t)^{1-\eta}}{(1-\bar{\delta})+\bar{\delta}(P_t/W_t)^{1-\eta}} = \bar{\delta} \times \frac{(P_t)^{1-\eta}}{(1-\bar{\delta})W_t^{1-\eta}+\bar{\delta}P_t^{1-\eta}} = \bar{\delta} \times \left[\frac{P_t}{P_t^{in}}\right]^{1-\eta}$, where $P_t^{in} \equiv \left[(1-\bar{\delta})W_t^{1-\eta} + \bar{\delta}P_t^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is the overall input price index, combining the wage W_t and the price index of intermediate inputs P_t. Taking logs from both sides, it follows that:

$$\log \delta_t \quad = \quad \log \overline{\delta} \quad + \quad (1-\eta)\log(P_t/P_t^{in}). \tag{E.6}$$

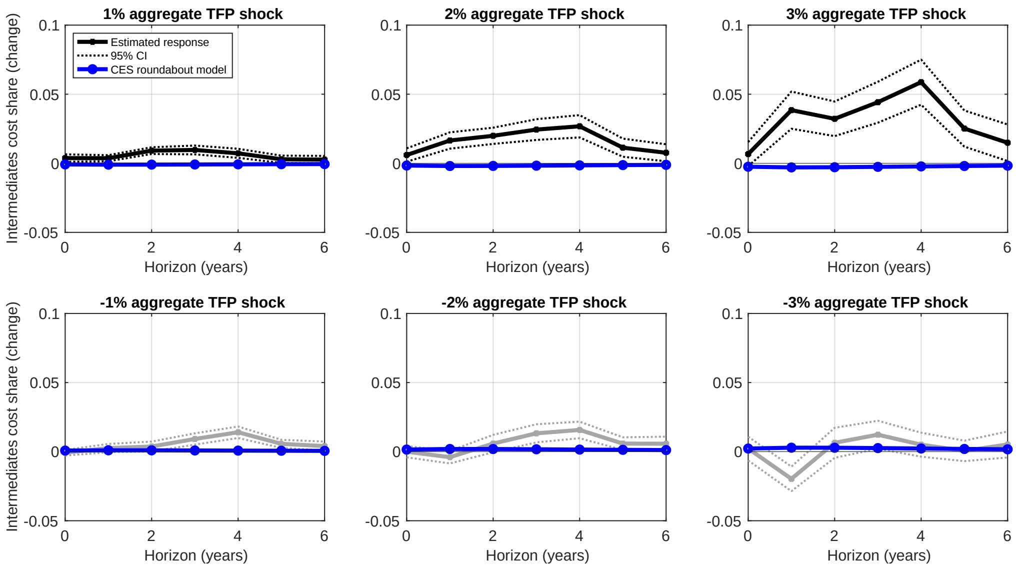
One can see that whenever labor and intermediates are complements ($\eta \in (0,1)$), then δ_t is countercyclical: a rise in productivity or money supply makes intermediates relatively cheaper than labor (falling P_t/P_t^{in}), implying a drop in δ_t. Conversely, whenever labor and intermediates are substitutes ($\eta > 1$), then δ_t is procyclical. The Atalay-style regression is directly based on (E.6), which is estimated either with OLS or by using instruments to create an exogenous variation in (P_t/P_t^{in}). Under the preferred specification with instrumental variables, Atalay (2017) estimates $\hat{\eta} = 0.88$, with the 99% confidence interval [-0.02, 1.78].

Let me now show what happens if one applies the methodology of Atalay (2017) to estimate elasticities using data generated by my model.[47] My model is too complex to simulate synthetic time series generated by random sequences of shocks. Therefore, I use time series produced by perfect foresight transitions following one-time unanticipated changes in productivity or money supply, as in Section 4.3.1. The key finding is that the estimated *effective* elasticity depends heavily on the sign, size and the source of the shock that produces the time series used in estimation. Under aggregate TFP shocks, a +1% shock implies an effective elasticity of 7.52, whereas a larger +3% shock produces an effective elasticity estimate of 4.04. As for TFP contractions, a -1% shock gives an effective elasticity of 1.57, while a -3% contraction implies a value of 1.68. Turning to variation driven by money supply changes, I similarly estimate the effective elasticity under time series produced by different values of the $\varepsilon^m_{-\infty}$ shock. Under a 2% expansion, the effective elasticity estimate is 7.33, while a larger 6% shock delivers a value of 4.11. Under monetary contractions, the effective elasticity estimates are smaller: 1.92 for a -2% shock and 1.79 for a -6% shock. These findings show that when the data-generating process features endogenous extensive-margin network adjustment, an Atalay-style regression does not recover a stable structural elasticity. Instead, it recovers a reduced-form effective elasticity that varies with the sign, size and source of the shock, reflecting the threshold-based adoption and severing of supplier links.

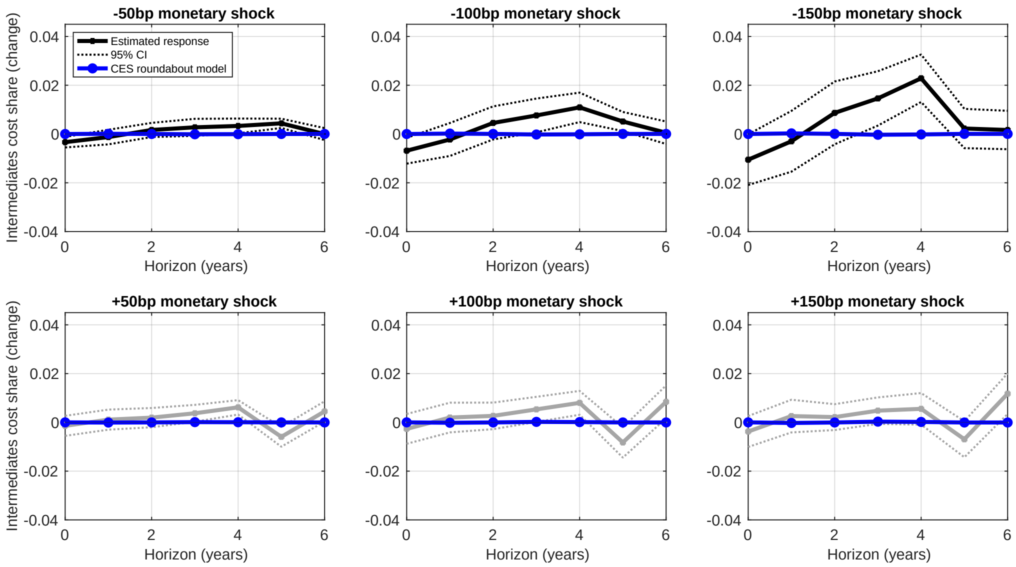
[47] I use a version of the specification (E.6) extended to a multi-sector setup: $\log \frac{\delta_{k,t}}{1-\delta_{k,t}} = \alpha_k + (1-\eta)\sum_r \frac{\omega_{kr}}{(1-\sum_{r'}\omega_{kr'})}\log(P_{r,t}/W_t) + \varepsilon_{k,t}$, which I estimate with OLS allowing for sectoral fixed effects. Note that, consistent with Atalay (2017), $\{\omega_{kr}\}_{kr}$ are cost shares associated with linkages that are observed in the data. I interpret the estimated $\hat{\eta}$ as an effective elasticity rather than as a primitive structural parameter.

Figure E.1: **Non-linear IRFs of intermediates cost shares: CES roundabout model vs. data** ($\eta = 0.78$)

(a) Aggregate TFP shocks



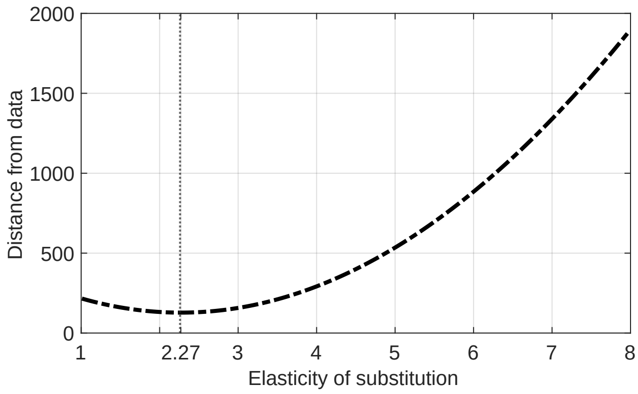
(b) Monetary shocks



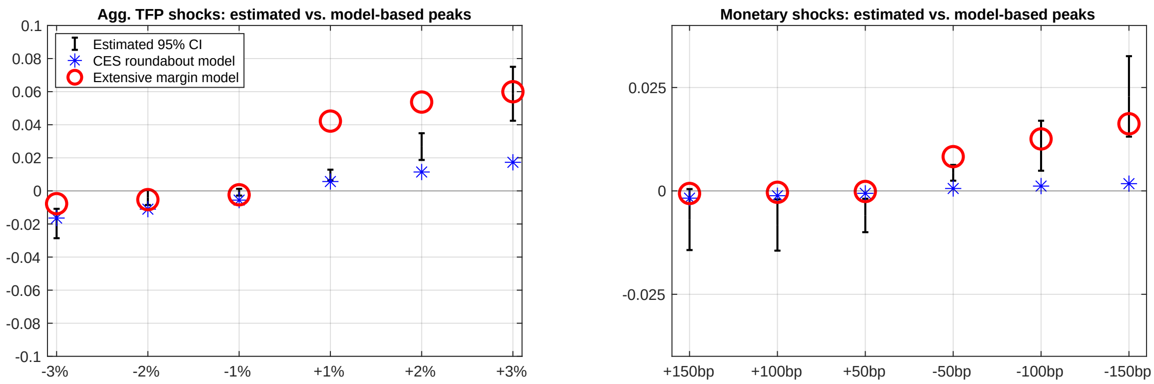
Notes: Panel (a) shows the IRFs of the aggregate intermediates cost share in the CES roundabout model under $\eta = 0.78$ to aggregate TFP shocks, as well as the corresponding estimated non-linear IRFs to productivity shocks based on Fernald (2014), using the specification in (18). Panel (b) shows the IRFs of the average intermediates cost share in the CES roundabout model under $\eta = 0.78$ to monetary shocks and the corresponding estimated non-linear IRFs to monetary shocks based on Romer and Romer (2004), using the specification in (18). See Appendix F.1 for the procedure that translates the Romer and Romer (2004) shocks into the money growth path that is then fed into the CES roundabout model. Dotted lines represent 95% confidence intervals.

Figure E.2: **Estimation of the elasticity of substitution η**

(i) Estimated elasticity of substitution

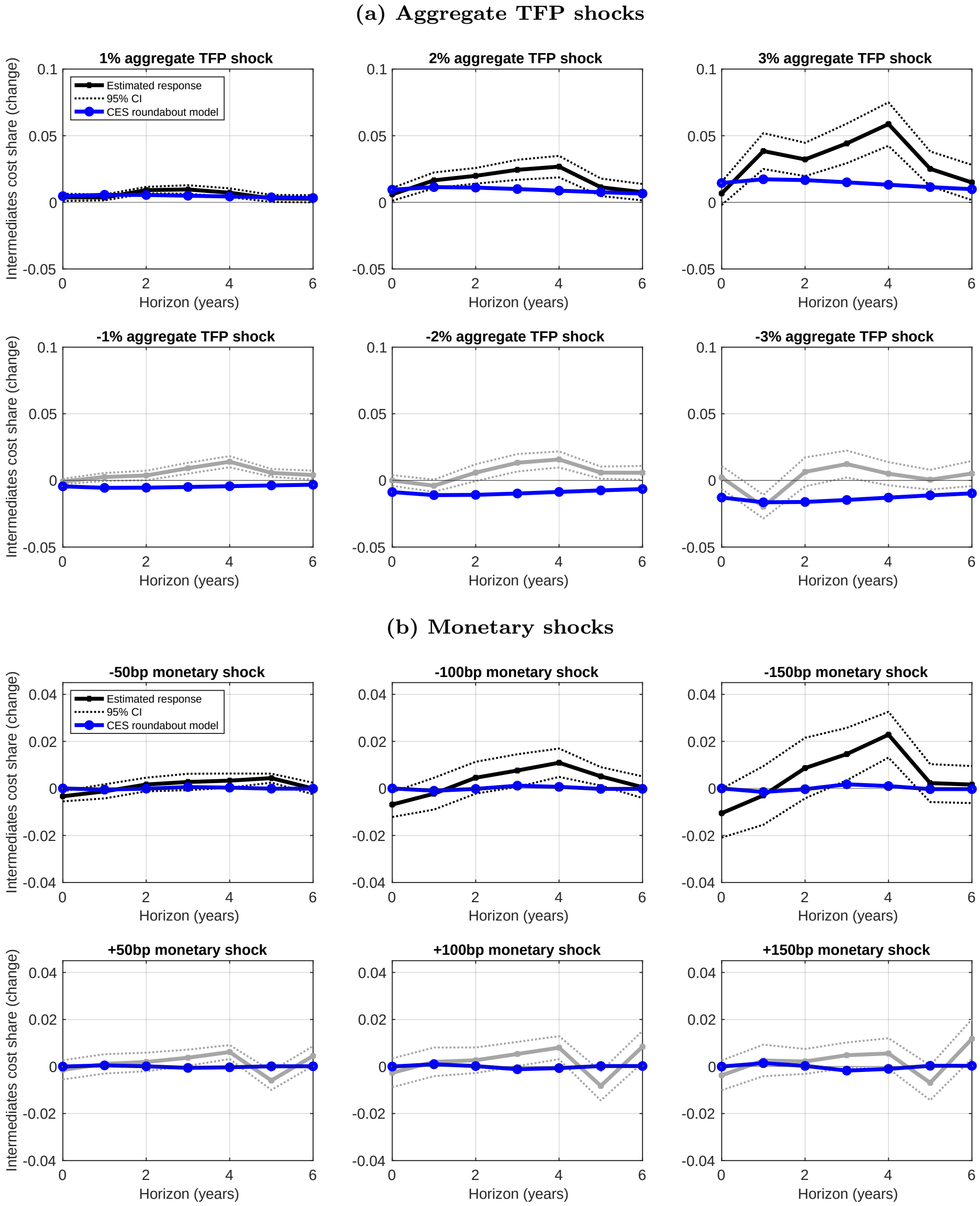


(ii) Peaks of IRFs of the intermediates cost share: models vs. data



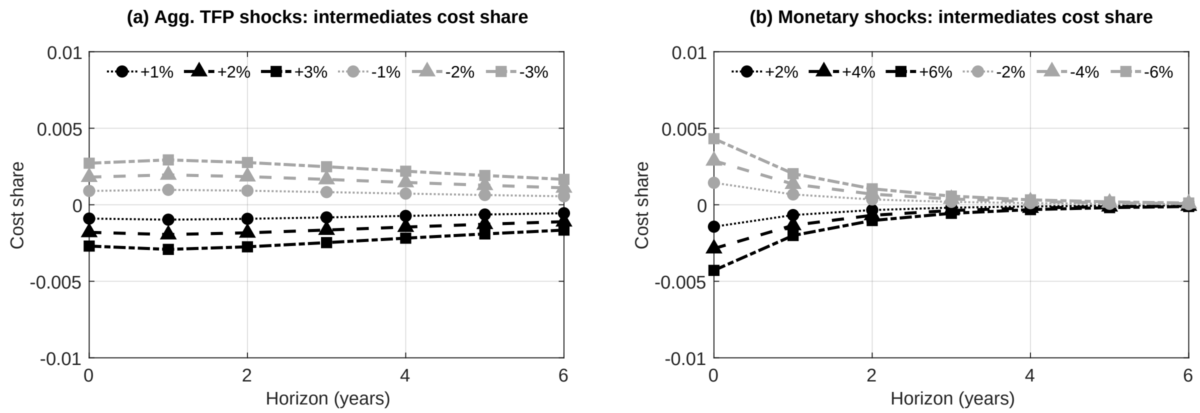
Notes: Panel (i) shows the results of the estimation procedure for η, described in (E.3): the distance to the data is minimized at $\eta^* = 2.27$. Panel (ii)(a) shows the peaks (troughs) of estimated responses of the intermediates cost share to expansionary (contractionary) productivity shocks of Fernald (2014) vs. the corresponding responses under the baseline endogenous network model in Section 4, as well as in the CES roundabout model under $\eta = 2.27$; Panel (ii)(b) shows the peaks (troughs) of estimated responses of the intermediates cost share to expansionary (contractionary) monetary shocks of Romer and Romer (2004) vs. the corresponding responses under the baseline endogenous network model in Section 4, as well as in the CES roundabout model under $\eta = 2.27$. See Appendix F.1 for the procedure that translates the Romer and Romer (2004) shocks into the money growth path that is then fed into the models.

Figure E.3: **Non-linear IRFs of intermediates cost shares: CES roundabout model vs. data** ($\eta = 2.27$)



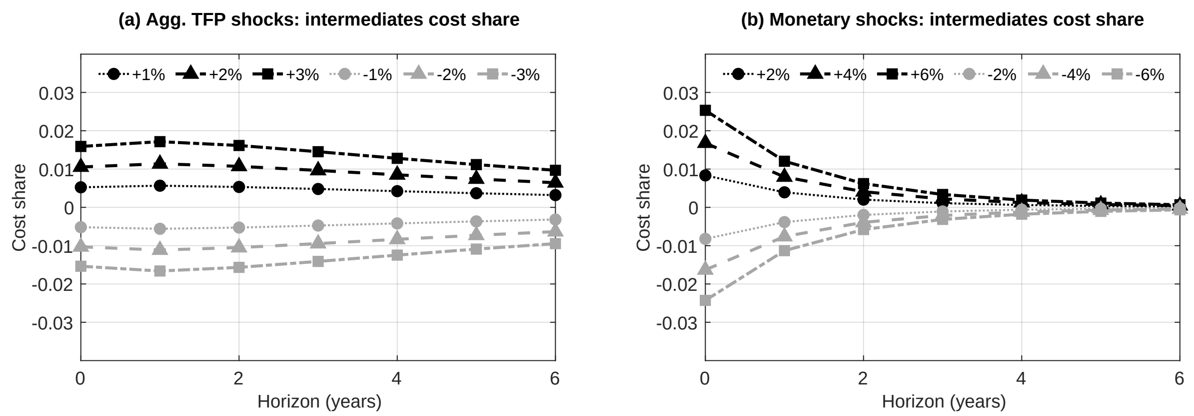
Notes: Panel (a) shows the IRFs of the aggregate intermediates cost share in the CES roundabout model under $\eta = 2.27$ to aggregate TFP shocks, as well as the corresponding estimated non-linear IRFs to productivity shocks based on Fernald (2014), using the specification in (18). Panel (b) shows the IRFs of the average intermediates cost share in the CES roundabout model under $\eta = 2.27$ to monetary shocks and the corresponding estimated non-linear IRFs to monetary shocks based on Romer and Romer (2004), using the specification in (18). See Appendix F.1 for the procedure that translates the Romer and Romer (2004) shocks into the money growth path that is then fed into the CES roundabout model. Dotted lines represent 95% confidence intervals.

Figure E.4: **Intermediates cost shares: CES network model** ($\eta = 0.78$)



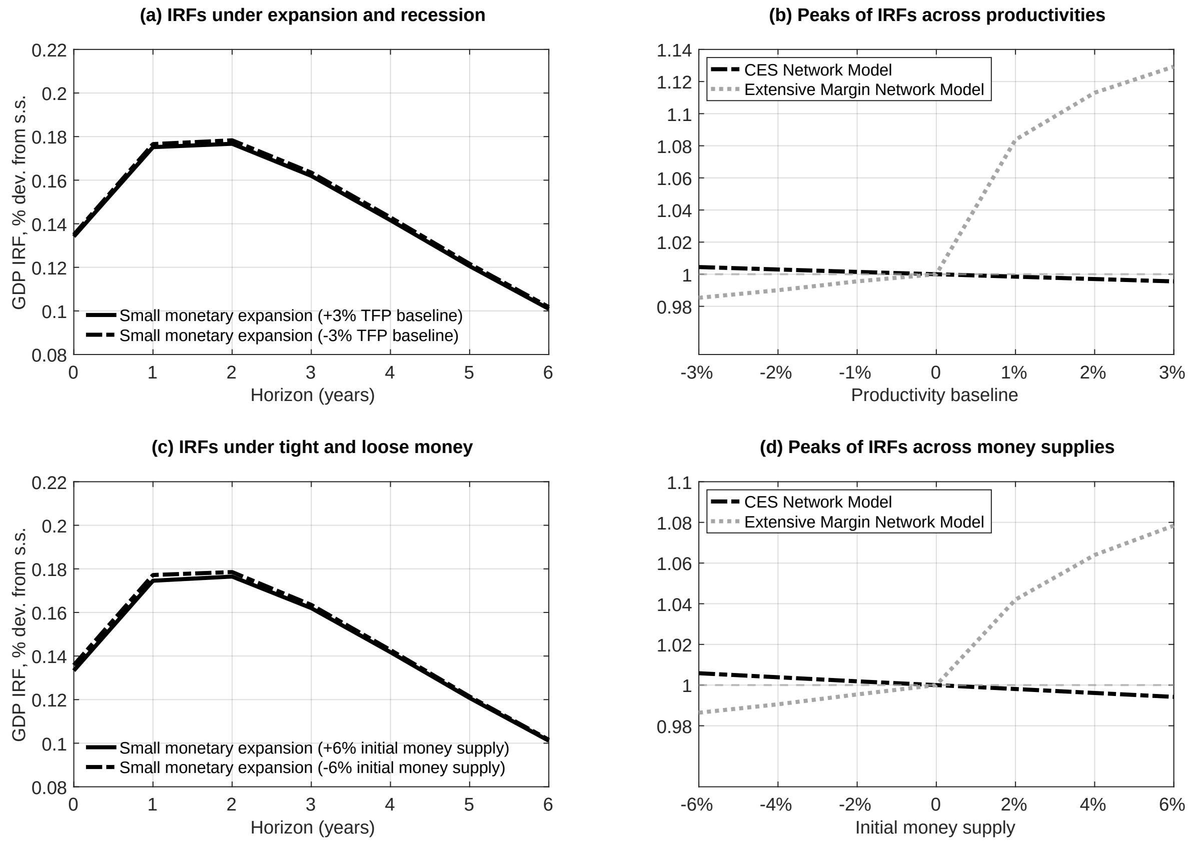
Notes: Panel (a) shows the average intermediates cost shares across baselines where $\mathcal{M}_t = 1, \forall t \geqslant 0$ and $\zeta_t = 0, \forall t \geqslant 2$, and consider values of $\zeta_1 = \{-3\%, -2\%, -1\%, 1\%, 2\%, 3\%\}$, relative to $\zeta_1 = 0$ in the CES network model ($\eta = 0.78$); Panel (b) shows the average intermediates cost shares across baselines where $\mathcal{M}_0 = 1$, $\zeta_t = 0, \forall t \geqslant 1$ and values of $\varepsilon^m_{-\infty} = \{-6\%, -4\%, -2\%, 2\%, 4\%, 6\%\}$, relative to the case where $\varepsilon^m_{-\infty} = 0$ in the CES network model ($\eta = 0.78$).

Figure E.5: **Intermediates cost shares: CES network model** ($\eta = 2.27$)



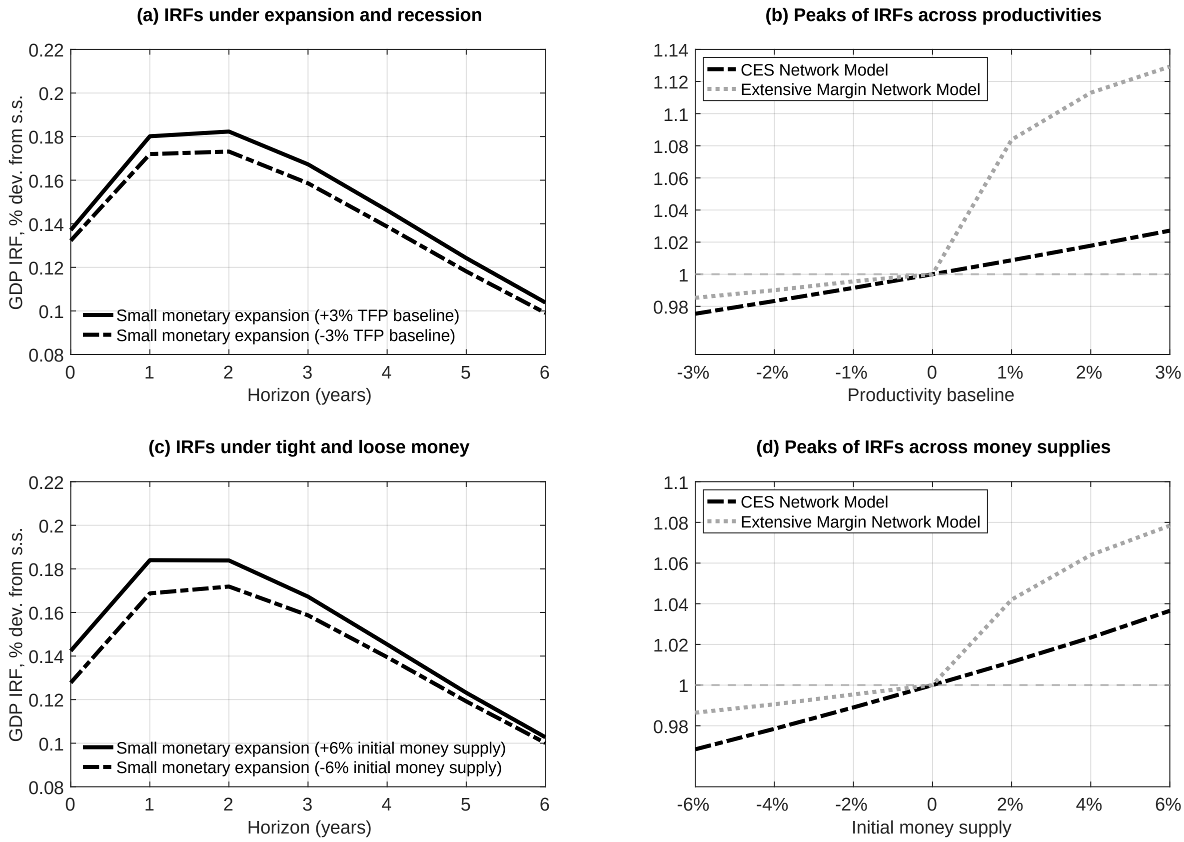
Notes: Panel (a) shows the average intermediates cost shares across baselines where $\mathcal{M}_t = 1, \forall t \geqslant 0$ and $\zeta_t = 0, \forall t \geqslant 2$, and consider values of $\zeta_1 = \{-3\%, -2\%, -1\%, 1\%, 2\%, 3\%\}$, relative to $\zeta_1 = 0$ in the CES network model with ($\eta = 2.27$); Panel (b) shows the average intermediates cost shares across baselines where $\mathcal{M}_0 = 1$, $\zeta_t = 0, \forall t \geqslant 1$ and values of $\varepsilon^m_{-\infty} = \{-6\%, -4\%, -2\%, 2\%, 4\%, 6\%\}$, relative to the case where $\varepsilon^m_{-\infty} = 0$ in the CES network model ($\eta = 2.27$).

Figure E.6: **GDP IRFs to a small monetary expansion: CES network model** ($\eta = 0.78$)



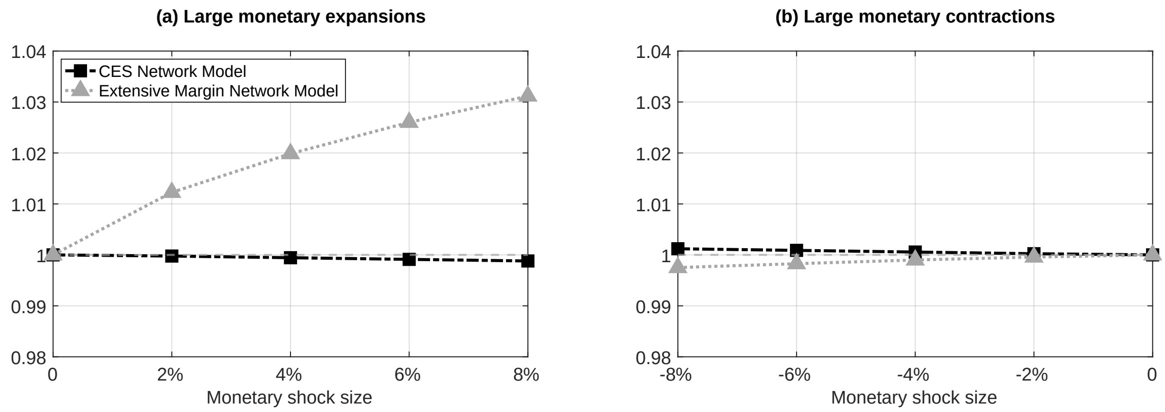
Notes: Panel (a) shows IRFs of GDP to the same small monetary expansion ($\varepsilon_1^m = 0.000001\%$), scaled per 1% of the shock, around two baselines: ($\mathcal{M}_0 = 1, \zeta_1 = -3\%$) and ($\mathcal{M}_0 = 1, \zeta_1 = 3\%$) in the CES network model ($\eta = 0.78$); Panel (b) shows peaks of IRFs to the same small monetary expansion around several baselines, namely those with $\mathcal{M}_0 = 1$ and $\zeta_1 = \{-3\%, -2\%, -1\%, 0, 1\%, 2\%, 3\%\}$, where that peak for $\zeta_1 = 0$ is normalized to one, in both the CES network model ($\eta = 0.78$) and the baseline endogenous network model. Panel (c) shows the scaled IRFs of GDP to the same small monetary expansion around two baselines, with ($\zeta_t = 0, \forall t \geqslant 1, \varepsilon_{-\infty}^m = \{-6\%\}$) and ($\zeta_t = 0, \forall t \geqslant 1, \varepsilon_{-\infty}^m = \{6\%\}$) in the CES network model ($\eta = 0.78$); Panel (d) shows peaks of IRFs to the same small monetary expansion around several baselines, namely those with $\zeta_t = 0, \forall t \geqslant 1$ and $\varepsilon_{-\infty}^m = \{-6\%, -4\%, -2\%, 0, 2\%, 4\%, 6\%\}$, where that peak for $\varepsilon_{-\infty}^m = 0$ is normalized to one, in both the CES network model ($\eta = 0.78$) and the baseline endogenous network model.

Figure E.7: **GDP IRFs to a small monetary expansion: CES network model** ($\eta = 2.27$)



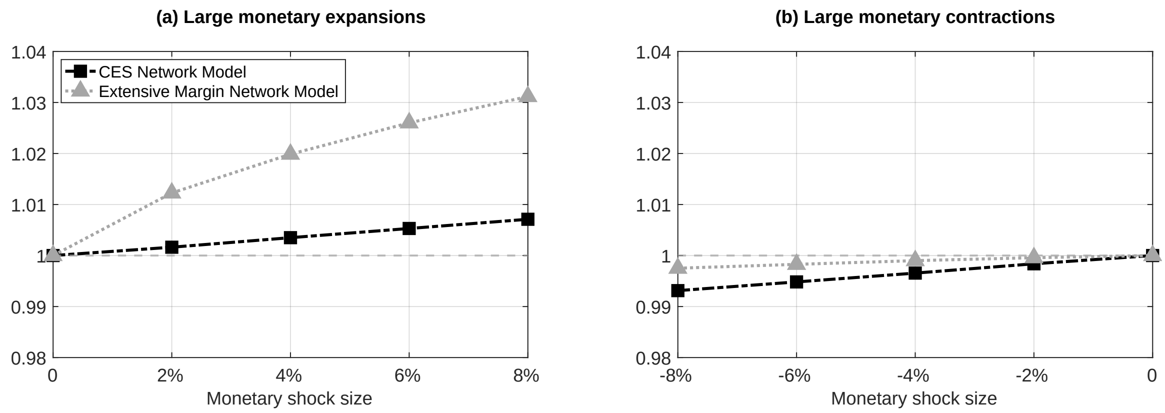
Notes: Panel (a) shows IRFs of GDP to the same small monetary expansion ($\varepsilon_1^m = 0.000001\%$), scaled per 1% of the shock, around two baselines: ($\mathcal{M}_0 = 1, \zeta_1 = -3\%$) and ($\mathcal{M}_0 = 1, \zeta_1 = 3\%$) in the CES network model ($\eta = 2.27$); Panel (b) shows peaks of IRFs to the same small monetary expansion around several baselines, namely those with $\mathcal{M}_0 = 1$ and $\zeta_1 = \{-3\%, -2\%, -1\%, 0, 1\%, 2\%, 3\%\}$, where that peak for $\zeta_1 = 0$ is normalized to one, in both the CES network model ($\eta = 2.27$) and the baseline endogenous network model. Panel (c) shows the scaled IRFs of GDP to the same small monetary expansion around two baselines, with ($\zeta_t = 0, \forall t \geqslant 1, \varepsilon_{-\infty}^m = \{-6\%\}$) and ($\zeta_t = 0, \forall t \geqslant 1, \varepsilon_{-\infty}^m = \{6\%\}$) in the CES network model ($\eta = 2.27$); Panel (d) shows peaks of IRFs to the same small monetary expansion around several baselines, namely those with $\zeta_t = 0, \forall t \geqslant 1$ and $\varepsilon_{-\infty}^m = \{-6\%, -4\%, -2\%, 0, 2\%, 4\%, 6\%\}$, where that peak for $\varepsilon_{-\infty}^m = 0$ is normalized to one, in both the CES network model ($\eta = 2.27$) and the baseline endogenous network model.

Figure E.8: **GDP IRFs to large monetary shocks: CES network model** ($\eta = 0.78$)



Notes: Panel (a) shows peaks of IRFs to large monetary expansions around the baseline with $\zeta_t = 0, \forall t \geqslant 1$ and $\mathcal{M}_0 = 1$, for shock values $\varepsilon_1^m = \{0, 2\%, 4\%, 6\%, 8\%\}$, where that peak for $\varepsilon_1^m = 0$ is normalized to one, in both the CES network model ($\eta = 0.78$) and the baseline endogenous network model; Panel (b) shows peaks of IRFs to large monetary contractions around the baseline with $\zeta_t = 0, \forall t \geqslant 1$ and $\mathcal{M}_0 = 1$, for shock values $\varepsilon_1^m = \{-8\%, -6\%, -4\%, -2\%, 0\}$, where that peak for $\varepsilon_1^m = 0$ is normalized to one, in both the CES network model ($\eta = 0.78$) and the baseline endogenous network model.

Figure E.9: **GDP IRFs to large monetary shocks: CES network model** ($\eta = 2.27$)



Notes: Panel (a) shows peaks of IRFs to large monetary expansions around the baseline with $\zeta_t = 0, \forall t \geqslant 1$ and $\mathcal{M}_0 = 1$, for shock values $\varepsilon_1^m = \{0, 2\%, 4\%, 6\%, 8\%\}$, where that peak for $\varepsilon_1^m = 0$ is normalized to one, in both the CES network model ($\eta = 2.27$) and the baseline endogenous network model; Panel (b) shows peaks of IRFs to large monetary contractions around the baseline with $\zeta_t = 0, \forall t \geqslant 1$ and $\mathcal{M}_0 = 1$, for shock values $\varepsilon_1^m = \{-8\%, -6\%, -4\%, -2\%, 0\}$, where that peak for $\varepsilon_1^m = 0$ is normalized to one, in both the CES network model ($\eta = 2.27$) and the baseline endogenous network model.

F Additional econometric exercises

F.1 Interest rate shocks and the model

In the model, monetary policy shocks represent innovations to the rate of money supply growth, while the econometric results are obtained under nominal interest rate shocks. In order to provide a tighter link between the model-based and the econometric results, I conduct a procedure that translates interest rate shocks into their model-based equivalents. To do that, I estimate the effect of the identified interest rate shocks on the rate of nominal GDP growth. I use nominal GDP, as opposed to a measure of money supply such as M0 or M2, since the latter incorporate fluctuations in money velocity, which the model assumes to be constant ($P_t^C C_t = M_t$). Therefore, nominal GDP is the most direct empirical counterpart of the notion of money supply in my model.

I run the following local projections with annual US data (1969-2007):

$$\Delta ngdp_{t+H} = \alpha_H + \beta_H s_t + \boldsymbol{\gamma}'_H \boldsymbol{x}_{t-1} + \varepsilon_{t+H}, \tag{F.1}$$

for $H = 0, 1, ..., \overline{H}$, where $\Delta ngdp_t$ is the growth rate of nominal GDP, s_t is the identified interest rate shock of Romer and Romer (2004) and $\boldsymbol{x}_{t-1}$ is the set of controls including one lead and lag of the shock, as well as single lags of (log) real GDP, nominal GDP growth, the Federal funds rate, as well as a time trend. Figure F.1 presents the results: following a -100 bp interest rate shock, the nominal GDP growth starts to rise gradually, with the peak increase of almost +1% achieved after three years. Therefore, in order to mimic a -100 bp interest rate shock in the model, I can feed in a sequence of money growth rate shocks that exactly replicate the estimated response of nominal GDP growth. Naturally, by linearly rescaling the estimates, I can also mimic an interest rate shock of any sign or size.

F.2 Further exercises using sector-level data

F.2.1 Alternative non-linear estimation using sectoral data

Here I consider an alternative approach to modeling non-linearity in my local projections setting. In particular, I re-estimate (18), where instead of adding interactions with a sign dummy and absolute value of the shock, I add quadratic and cubic shocks, which similarly capture possible sign and size effects:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta_H^{lin} s_t + \beta_H^{sign} s_t^2 + \beta_H^{size} s_t^3 + \boldsymbol{\gamma}'_H \boldsymbol{x}_{k,t-1} + \varepsilon_{k,t+H}, \tag{F.2}$$

for $H = 0, 1, ..., \overline{H}$, where it follows that $\beta_H^{sign} > 0$ indicates that positively-valued shocks make the impulse response more positive, whereas $\beta_H^{size} > 0$ indicates that larger shocks make the response more positive. Figure F.3 reports estimation results using this alternative specification. Reassuringly, results remain virtually unchanged relative to the non-linear specification in (18), both in terms of magnitudes of effects, as well as when it comes to sign and size effects and their statistical significance, as can be seen from the p-values reported in Figure F.11.

F.2.2 Estimation in broad sectoral groups

In Figures F.4-F.9 I re-estimate the effect of identified shocks on the intermediates cost share, allowing for non-linearity as in specification (18), at the level of six broad sectoral groups: "Non-Manufacturing Goods", "Durable Manufacturing Goods", "Non-Durable Manufacturing Goods", "Trade, Transportation and Warehousing", "Information, FIRE and Professional Services" and "Education, Healthcare and Other Services". This allows me to determine if the observed network cyclicality properties are driven by specific parts of the US economy.

The results suggest that largest sensitivities of the intermediates cost share to changes in productivity are detected in "Trade, Transportation and Warehousing" and "Durable Manufacturing Goods", whereas very little responsiveness is found in "Information, FIRE and Professional Services" and "Education, Healthcare and Other Services". As for the effect of monetary shocks, by far the largest response is found in "Non-Manufacturing Goods", whereas the smallest effect is found in "Education, Healthcare and Other Services".

F.2.3 Role of intermediates inventories

A potential concern with the evidence on the cyclicality of the sectoral intermediates cost shares is that it may reflect timing patterns in the use of *inventories* of intermediate inputs. For example, in a recession, firms may stop purchasing new intermediates and decide to use up their intermediates inventories instead. Combined with continued payments to labor, this would mechanically decrease the measured intermediates cost share even without the endogenous network mechanism. To address this issue, I extend the specification in (17) to control for whether or not a given sector relies heavily on inventories of intermediates:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta_H s_t + \beta_H^{inv} s_t \times LI_k + \boldsymbol{\gamma}'_H \boldsymbol{x}_{k,t-1} + \varepsilon_{k,t+H}, \tag{F.3}$$

where LI_k is a dummy variable equal to one if sector k does not rely heavily on intermediates inventories. Sectors in the low-inventory category are those in the broad categories of "Trade, Transportation and Warehousing", "Information, FIRE and Professional Services" and "Education, Healthcare and Other Services", whereas the high-inventory sectors are those in "Non-Manufacturing Goods", "Durable Manufacturing Goods" and "Non-Durable Manufacturing Goods". Figure F.21 reports the responses of the intermediates cost share to a 1% aggregate TFP shock. One can see that the low-inventory sectors exhibit a statistically significant rise in the intermediates cost share. While the cost share rise is more pronounced among the high-inventory sectors, the difference is not statistically significant, as shown in panel (c). Similarly, Figure F.22 presents the results for a -100 bp monetary easing, showing that the low-inventory sectors exhibit a statistically significant rise in the cost share of intermediates. As before, the increase is more pronounced among the high-inventory sectors, though panel (c) shows that the difference is not statistically significant. All in all, the results suggest that the intermediates cost share procyclicality remains statistically significant even among sectors that do not rely heavily on inventories of intermediates.

F.3 Further exercises using firm-level data

F.3.1 Alternative non-linear specification

Here I once again consider the alternative approach to modeling non-linearity in my local projections setting, this time applied to the firm-level data on the number of suppliers.

$$indeg_{j,t+H} = \alpha_{j,H} + \beta_H^{linear} s_t + \beta_H^{sign} s_t^2 + \beta_H^{size} s_t^3 + \boldsymbol{\gamma}'_H \boldsymbol{x}_{j,t-1} + \varepsilon_{j,t+H}, \tag{F.4}$$

for $H = 0, 1, ..., \overline{H}$. Figure F.17 reports the estimation results. One can see that, relative to the specification (20), results remain virtually unchanged, both in terms of magnitudes of effects, as well as when it comes to sign effects and their statistical significance, as can be seen from the p-values reported in Figure F.19.

F.3.2 Response of the outdegree

I also estimate the response of the firm-level *outdegree*, which is the number of customer firms that a firm has. In particular, I first consider the following linear specification:

$$outdeg_{j,t+H} = \alpha_{j,H} + \beta_H s_t + \boldsymbol{\gamma}'_H \boldsymbol{x}_{j,t-1} + \varepsilon_{j,t+H}. \tag{F.5}$$

for $H = 0, 1, ..., \overline{H}$, where $outdeg_{j,t}$ is the number of customer firms that a firm j has at time t and s_t is an identified shock. The estimation results are reported in Figure F.13. One can see that both an aggregate TFP expansion and a monetary easing produce a statistically significant increase in the average number of customers.

F.3.3 Outdegree and the role of price rigidity

My model also makes an additional prediction regarding the behavior of a firm's outdegree (number of customers), which depends on the nature of the shock hitting the economy, as well as the probability that the firm gets to adjust its price. First, under productivity shocks, firms with a higher probability of price adjustment see larger fluctuations in their outdegree. In particular, following a productivity expansion, firms that are more likely to be adopted as additional suppliers are, *ceteris paribus*, those that get the opportunity to lower their prices. At the same time, following a productivity contraction, firms that are likely to be dropped as suppliers are also the ones that get the chance to adjust their prices upwards in light of rising costs. Second, for monetary shocks, the prediction is the opposite: it is the firms with a *lower* adjustment probability that experience larger fluctuations in their outdegree. Indeed, following a monetary easing, firms that are more likely to be adopted as new suppliers are the ones that *do not* get to increase their prices, thus remaining cheaper than labor. On the other hand, after a monetary contraction, firms that do not get the chance to cut their prices are more likely to be dropped as suppliers, since they are likely more expensive than labor whose cost is falling. I formally test these predictions of my model using the following specification:

$$outdeg_{j,t+H} = \alpha_{j,H} + \beta_H^{linear} s_t + \beta_H^{dur} s_t \times \mathbf{1}\{d_j > \overline{d}\} + \boldsymbol{\gamma}'_H \boldsymbol{x}_{j,t-1} + \varepsilon_{j,t+H}, \tag{F.6}$$

for $H = 0, 1, ..., \overline{H}$, where $outdeg_{j,t}$ is the number of customer firms that a firm j has at time t, s_t is an identified shock and d_j is the average duration of price spells of firm j, which provides an economically interpretable measure of the frequency with which that firm adjusts its prices. Hence, $\mathbf{1}\{d_j > \overline{d}\}$ is a dummy for whether the firm has prices that are particularly "sticky", in the sense that the average duration of price spells is above some threshold $\overline{d}$. In order to measure d_j, I first match each firm to a 6-digit sector based on the Bureau of Economic Analysis (BEA) input-output accounts. I then use the 6-digit measures of adjustment frequency (Pasten et al., 2020a,b) to assign a probability of adjustment $(1 - \alpha_j)$ to each firm, and finally measure the duration of price spells as $d_j = -1/\log \alpha_j$. For productivity shocks (where $s_t > 0$ is expansionary), my model predicts that $\beta_H^{linear} \geqslant 0$ and $\beta_H^{dur} \leqslant 0$. At the same time, for monetary shocks (where $s_t > 0$ is contractionary), my model predicts that $\beta_H^{linear} \leqslant 0$ and $\beta_H^{dur} \leqslant 0$.

In Figure F.14 I show the results of estimating (F.6) under $\overline{d} = 15$ months. In other words, I treat firms with the average duration of price spells of less than 15 months as relatively "price flexible", whereas those with the average duration above 15 months are treated as relatively "price sticky". In Panels (a) and (b) one can see that following a 1% aggregate TFP expansion, the outdegree rises on impact and for the further 5 years for the "price flexible" firms, whereas for the "price sticky" group there is no statistically significant increase until the second year. Panel (c) shows that the difference in impact responses is statistically significant. Moreover, it is consistent with my theory, which predicts that early on only the relatively price flexible firms attract new customers, as they are the ones that are able to lower their prices in light of the higher productivity. Panels (d) and (e) show the results for a -100 bp monetary easing: it follows that the relatively "price sticky" firms see a larger increase in their outdegree, relative to the "price flexible" category. Panel (f) shows that this difference is indeed statistically significant. Once again, this is consistent with the predictions of my theory: following a monetary expansion, it is the firms that *do not* get to reset their prices upwards that should attract new customers, since those firms' prices remain significantly below the rising cost of labor. While the results above are presented for a price duration threshold of 15 months, in Figures F.15 and F.16 I show that the findings are robust to using thresholds of $\overline{d} = 16.5$ months and $\overline{d} = 13.5$ months, respectively.

F.3.4 The pattern of lost and gained suppliers

The network variation results in Lemmas 2 and 3 imply a very specific *pattern* underlying the addition of suppliers: following a productivity or monetary expansion, firms always keep all the existing suppliers and add new ones on top. In other words, the new suppliers do not replace existing suppliers; instead, they always replace a part of production that was previously done with labor. Similarly, whenever firms drop suppliers in equilibrium, they replace them with labor, and not by connecting to some other suppliers. While the evidence in Section 5.2 points to an overall procyclicality in the number of suppliers per firm, here I provide additional results that the suppliers are gained and lost in a manner that is consistent with Lemma 2. To do that, I use the Compustat-based input-output data of Atalay et al. (2011) to measure, for each firm j, the number of suppliers gained ($gain_{j,t}$) and lost ($loss_{j,t}$) in every year t relative to the previous year. Naturally, the difference between gained and lost suppliers equals the overall change in the number of suppliers between years t and $t-1$: $(gain_{j,t} - loss_{j,t}) = \Delta indeg_{j,t}$. I then estimate the following specification:

$$y_{j,t+H} = \alpha^{y}_{j,H} + \beta^{y}_{H} s_t + \beta^{y,sign}_{H} s_t \times \mathbf{1}\{s_t > 0\} + \boldsymbol{\gamma}'^{y}_{H} \boldsymbol{x}_{j,t-1} + \varepsilon_{j,t+H}, \tag{F.7}$$

where $y \in \{gain, loss, \Delta indeg\}$. The specification allows one to separately estimate the response of the number of suppliers gained, lost, as well as the overall change in the indegree, further controlling for whether the shock s_t is positive or negative, as captured by the coefficient $\beta^{y,sign}_{H}$. Recall that Section 5.2 finds that the overall number of suppliers is procyclical. Therefore, for an expansionary TFP/monetary shock, the evidence that would go *against* the pattern in my model (Lemma 2) is if number of lost suppliers systematically rises. In particular, that would imply that firms increase their indegree by dropping some existing suppliers and more than compensating for the loss with new ones. Similarly, for a contractionary shock, the evidence that would go *against* my model is if the number of gained suppliers systematically rises. In particular, that would imply that firms decrease their indegree by gaining some new suppliers, while also dropping an even larger number of existing ones.

Figure F.23 shows the estimated responses for expansionary and contractionary aggregate TFP shocks. In panel (a) one can see that following a +1% shock, the overall change in the number of suppliers ($\Delta indeg_{j,t}$) initially shows a statistically significant increase, before turning negative at longer horizons. This pattern is consistent with a temporary increase in the indegree *level* ($indeg_{j,t}$) as documented in Figure 12. Crucially, panels (b) and (c) find that at the horizons where there is a positive change in the overall number of suppliers, the number of suppliers gained also rises, while the number of suppliers lost falls; at the same time, as the response of ($\Delta indeg_{j,t}$) turns negative, the gains fall and the losses rise. Overall, consistent with my model, I find no evidence that indegree increases driven by a TFP expansion are accompanied by rises in the number of suppliers lost. As for TFP contractions, panels (d)-(f) show that they lead to a (delayed) drop in the number of suppliers, which is driven by a statistically significant rise in the number of suppliers lost and no statistically significant change in the number of suppliers gained. Once again, such evidence is consistent with the theoretical predictions for exogenous TFP contractions.

Figure F.24 repeats the exercises for monetary shocks. Panels (a)-(c) show the results for a monetary expansion, documenting an impact positive effect on the overall change in the number of suppliers ($\Delta indeg_{j,t}$), which is reversed at the 5-year horizon. The initial positive change in the number of suppliers is driven by a drop in the number of suppliers lost, whereas the eventual reversal is driven by a drop in the number of suppliers gained. Crucially, consistent with my model, there is no evidence that rises in the overall number of suppliers are accompanied by a rise in the number of lost suppliers. As for a monetary contraction, panels (d)-(f) show that they lead to an overall drop in the number of suppliers by the 4-year horizon, which is driven by a statistically significant rise in the number of lost suppliers and no statistically significant change in the number of gained suppliers. Once again, this is consistent with my theoretical predictions.

Overall, the estimation exercises suggest that the pattern of lost and gained suppliers in response to TFP and monetary shocks is consistent with the theoretical predictions of my model.

F.3.5 The role of extensive margin variation

In addition to reporting the number of suppliers ($indeg_{j,t}$), the Compustat-based input-output data of Atalay et al. (2011) also reports the total value spent by each firm on intermediate inputs bought from the self-reported suppliers in the dataset ($inval_{j,t}$), as well as the firm-level total cost of goods sold ($cogs_{j,t}$). While the dataset has limitations, since it only covers publicly listed firms and only formally captures major input-output relationships, one can nonetheless use it to investigate the relationship between the in-sample cost share of intermediate inputs $\delta_{j,t} \equiv inval_{j,t}/cogs_{j,t}$ and the number of suppliers ($indeg_{j,t}$). To do that, I consider the following specification:

$$\log \delta_{j,t} = \alpha_j + \beta \log indeg_{j,t} + \varepsilon_{j,t}. \tag{F.8}$$

The specification can be used to estimate the association between the intermediates cost share and the number of suppliers (captured by β), as well as to assess the fraction of variation in the intermediates cost share that is explained by the extensive margin of the network. In particular, to measure the importance of the extensive margin, I compute the partial R^2 of $\log indeg_{j,t}$, which corresponds to the fraction of the intermediates cost share explained by the number of suppliers, once $\log \delta_{j,t}$ has been residualized against the fixed effects. In case where one only considers firm-level fixed effects (and no further controls), such partial R^2 measure corresponds to the fraction of *within*-firm (cyclical) variation explained by the extensive margin. For the baseline results, I estimate (F.8) on the sample of all firms that report at least one supplier in a given year. Importantly, the limited coverage of the Compustat-based dataset implies that the estimated fraction of variation explained by the extensive margin is likely a conservative *lower* *bound.* First, the dataset formally only captures the major input-output linkages, which are the *least* likely to be the marginal ones to be dropped and regained over the business cycle. Second, the amount of within-firm (cyclical) variation in the dataset is generally limited: the median firm has only 9 time series observations (i.e., there are 9 years where it reports suppliers in the dataset). As detailed below, beyond the baseline sample, I consider several additional exercises that aim to address the coverage issues.

Table F.24 reports the estimation results. In column (1) one can see that the specification with firm-level fixed effects only implies a highly statistically significant association of around 1.24 between $\log \delta_{j,t}$ and $\log indeg_{j,t}$: a 1% increase in the number of suppliers is associated with a 1.24% increase in the cost share of intermediates. Moreover, the partial R^2 of $\log indeg_{j,t}$ is around 0.22, implying that 22% of within-firm (cyclical) variation in the intermediates cost share is explained by the extensive margin. In columns (2) and (3) I further saturate the specification with year and year-sector (2-digit SIC) fixed effects, respectively. Importantly, the estimated association coefficient remains stable and statistically significant across the specifications with the estimated partial R^2 of $\log indeg_{j,t}$ also stable near 0.20. All in all, estimation on the baseline sample robustly shows a statistically significant positive association between the intermediates cost share and the number of suppliers, with around 20% of (residualized) variation in the intermediates cost share attributed to the extensive margin of the network.

As argued above, the coverage issues in the Atalay et al. (2011) dataset imply that the estimated fraction of variation explained by the extensive margin is likely a conservative lower bound. To address this issue, I re-estimate the specification (F.8) on sub-samples with superior coverage. First, I consider sub-samples of firms with a richer time dimension. In column (4) I focus on the sample of firms with at least 15 time series observations: one can see that the estimated association coefficient remains significant and virtually unchanged relative to the baseline sample, whereas the partial R^2 of $\log indeg_{j,t}$ is higher at 0.32. Column (5) shows that further restricting the sample to firms with at least 25 years of time series observations once again leaves the association coefficient stable and implies an even higher partial R^2 of 0.44. All in all, it follows that focusing on firms with more within (time series) variation leaves the association coefficient between the intermediates cost share and the number of suppliers virtually unchanged relative to the baseline sample, while implying a much higher fraction of variation explained by the extensive margin, over 40% for the firms with the richest coverage. In fact, in panel (a) of Figure F.25 I find that the estimated partial R^2 of $\log indeg_{j,t}$ rises monotonically in the (minimal) time series dimension of the sample considered.

As a second way to address the coverage issue, I consider sub-samples of firms that have a higher *median* number of suppliers across the years in which they are present in the sample. In other words, I focus on firms that, over the years they feature in the dataset, have more suppliers reported among other Compustat firms. In this way I aim to capture firms that report both key and marginal suppliers, with the latter being more likely to be dropped and adopted over the business cycle. In column (6) I focus on the sample of firms with the median number of suppliers (over the years) of at least 3: one can see that the estimated association coefficient remains significant and virtually unchanged relative to the baseline sample, whereas the partial R^2 of $\log indeg_{j,t}$ is higher at 0.40. Column (7) shows that further restricting the sample to firms with a median of at least 5 suppliers once again leaves the association coefficient stable and implies an even higher partial R^2 of 0.51. It follows that having a more extensive reporting of suppliers in the sample leaves the association coefficient between the intermediates cost share and the number of suppliers virtually unchanged relative to the baseline sample, while implying a much higher fraction of variation explained by the extensive margin, over 50% for the firms with the richest coverage. Moreover, in panel (b) of Figure F.25 I find that the estimated partial R^2 of $\log indeg_{j,t}$ rises monotonically in the (minimal) median number of suppliers.

Overall, two major findings emerge from the analysis using the Atalay et al. (2011) input-output data. First, there is a statistically significant and robust association between the intermediates cost share and the number of suppliers, with a 1% increase in the number of suppliers being associated with around 1.2% increase in the intermediates cost share. Second, the full sample implies that approximately 20% of within (cyclical) variation in the intermediates cost share is explained by the extensive margin, which is likely a conservative lower bound due to coverage issues; focusing on sub-samples with richer coverage implies a much larger role for the extensive margin, with as much as 40-50% of the intermediates cost share variation explained by the number of suppliers.

F.4 Responses to oil price shocks

While the baseline econometric estimation uses an aggregate supply shock, namely the TFP shock of Fernald (2014), I also obtain additional empirical results for a sector-specific supply shock, namely the oil news shock of Känzig (2021).[48]

First, in order to translate the units of the oil news shock into movements in (real) oil prices, I run the following local projection with annual US data (1975-2017):

$$p^{oil}_{t+H} = \alpha_H + \beta_H s_t + \boldsymbol{\gamma}'_H \boldsymbol{x}_{t-1} + \varepsilon_{t+H}, \tag{F.9}$$

for $H = 0, 1, ..., \overline{H}$, where p^{oil}_{t+H} is (log of) real oil price, obtained by deflating the WTI spot price by US CPI, s_t is the oil news shock and $\boldsymbol{x}_{t-1}$ is a vector of controls consisting of one lead and lag of the shock, as well as two lags of log real oil price, log real GDP, the Federal funds rate and log US CPI. Figure F.2 shows the results: a positive unit shock generates a just over 5% increase in the real oil price on impact, with the rise being statistically significant up to 2 years out. I use this estimation in order to scale the size of the shock to obtain, say, 10% or 15% increase in real oil price on impact.

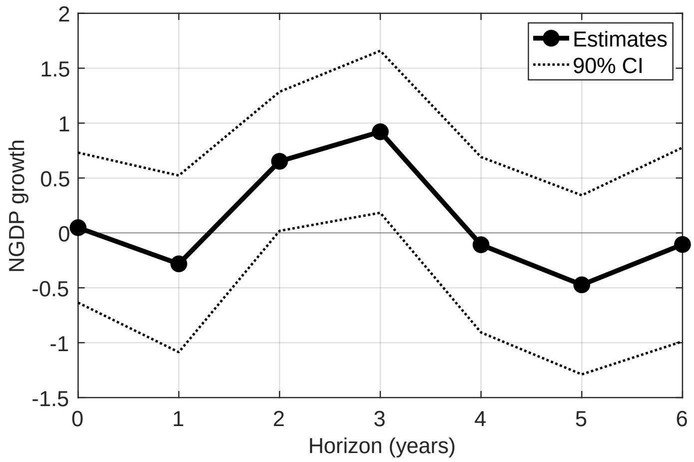
I first study the effect of oil price shocks on the intermediates cost share. Panel (a) of Figure F.10 reports the estimation results using the non-linear local projection specification (18). One

[48] I start with the monthly oil news shock series from Känzig (2021), and then add up the shocks within each calendar year in order to obtain an annual series.

can see that oil price increases lead to statistically significant drops in the intermediate cost share: for instance, following a +10% increase the share drops by 0.01 four years after the shock. At the same time, oil price decreases lead to increases in the intermediates cost share, though milder in magnitude. Figure F.12 finds that the sign and size asymmetries are statistically significant both on impact and at further horizons after the shock. Panel (b) of Figure F.10 shows that the results are robust to using the alternative non-linear local projection specification in (F.2).

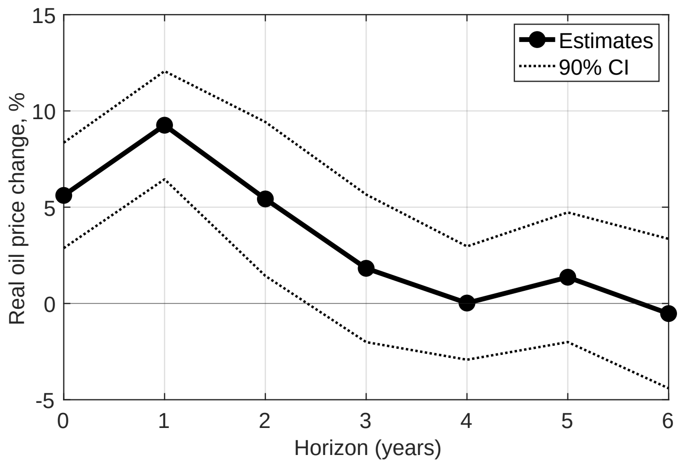
I also study the response of the firm-level indegree (number of suppliers) to oil price shocks. In particular, Panel (a) of Figure F.18 reports estimation results using the non-linear local projection specification in (20). One can see that oil price increases lead to a statistically significant drop in the number of suppliers: for example, a +10% oil price increase leads to a reduction by almost two suppliers per firm four years after the shock. As for oil price decreases, they lead to statistically significant increases in the number of suppliers per firm. Panel (b) of Figure F.18 shows that the results are robust to using the alternative non-linear local projection specification in (F.4). Further, Figure F.20 reports the significance of sign and size effects across the horizons. All in all, the dynamics of the number of suppliers per firm is consistent with the predictions of the model in the main text, and is also qualitatively in line with the responses of the intermediates cost share.

Figure F.1: **Response of nominal GDP growth to a monetary easing (-100 bp)**



Notes: the figure shows the estimated IRF of nominal GDP growth to a -100 bp monetary shock based on Romer and Romer (2004), using the specification in (F.1); the set of controls includes one lead and lag of the shock, as well as single lags of (log) real GDP, nominal GDP growth, the Federal funds rate, as well as a time trend. Dotted lines represent 90% confidence intervals.

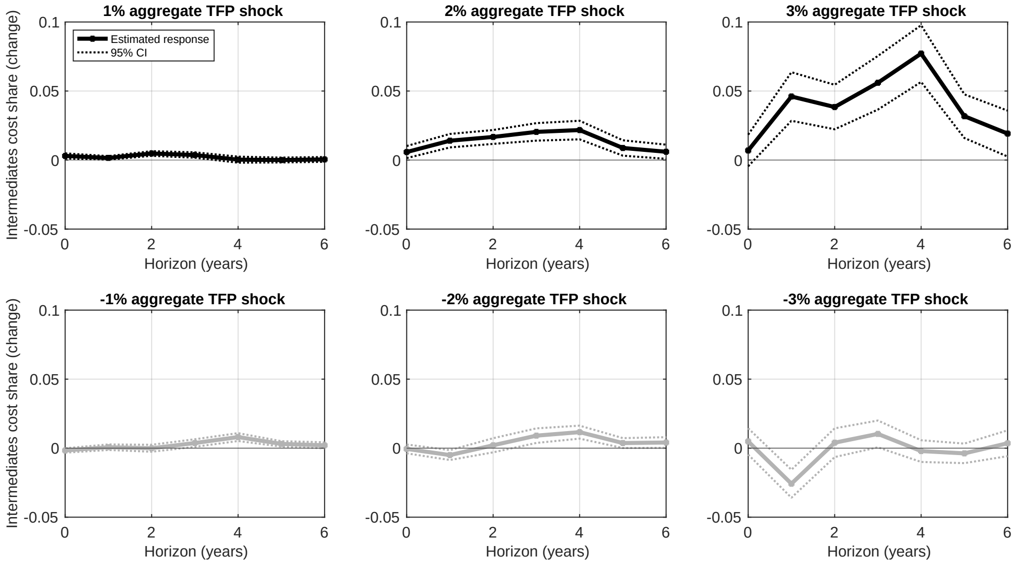
Figure F.2: **Response of real oil price to 1 unit oil news shock**



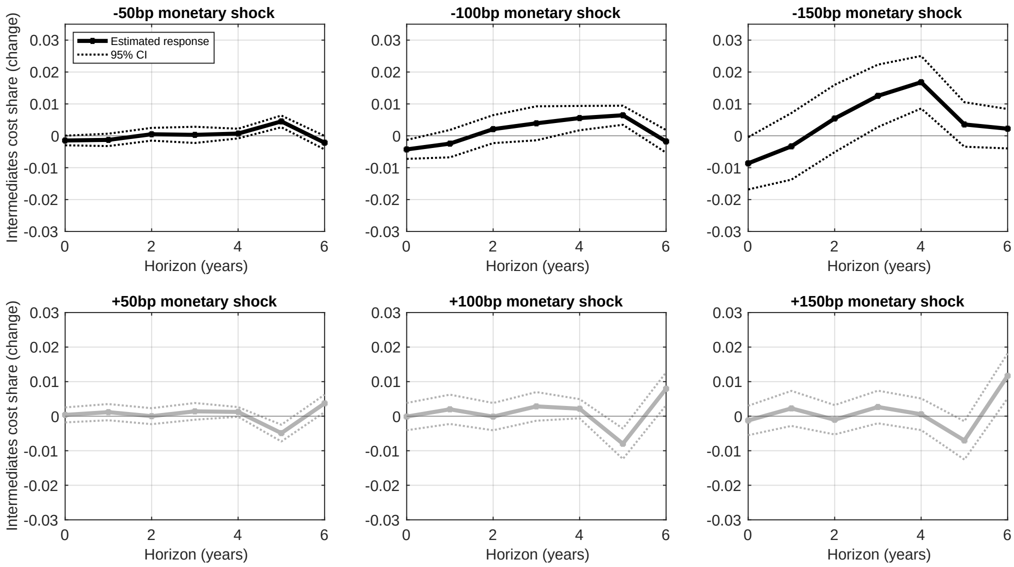
Notes: the figure shows the estimated IRF of real oil price to a 1 unit of oil news shock based on Känzig (2021), using the specification in (F.9); the set of controls includes one lead and lag of the shock, as well as two lags of log real oil price, log real GDP, the Federal funds rate and log US CPI. Dotted lines represent 90% confidence intervals.

Figure F.3: **Non-linear IRFs of intermediates cost shares**

(a) Aggregate TFP shocks



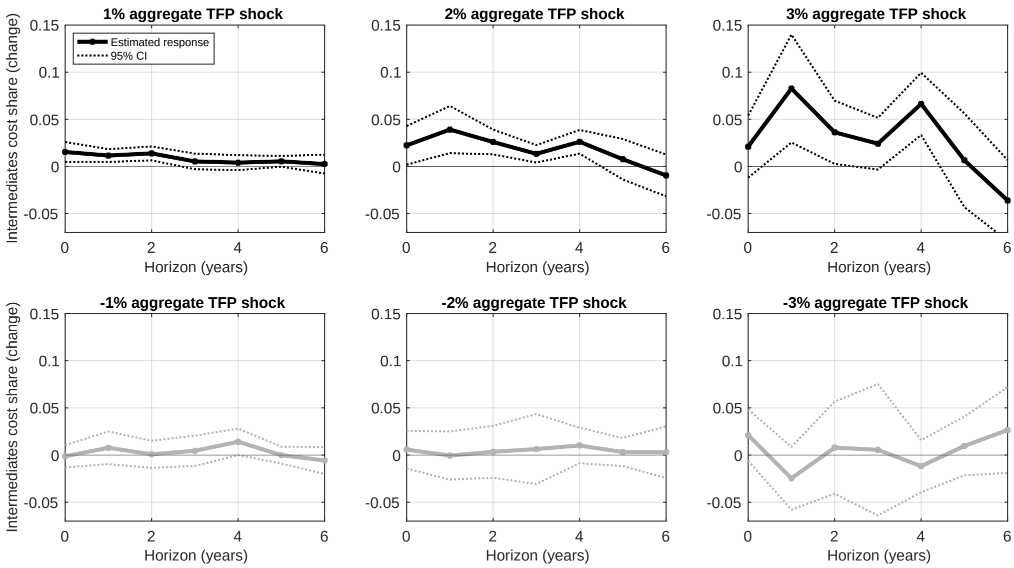
(b) Monetary shocks



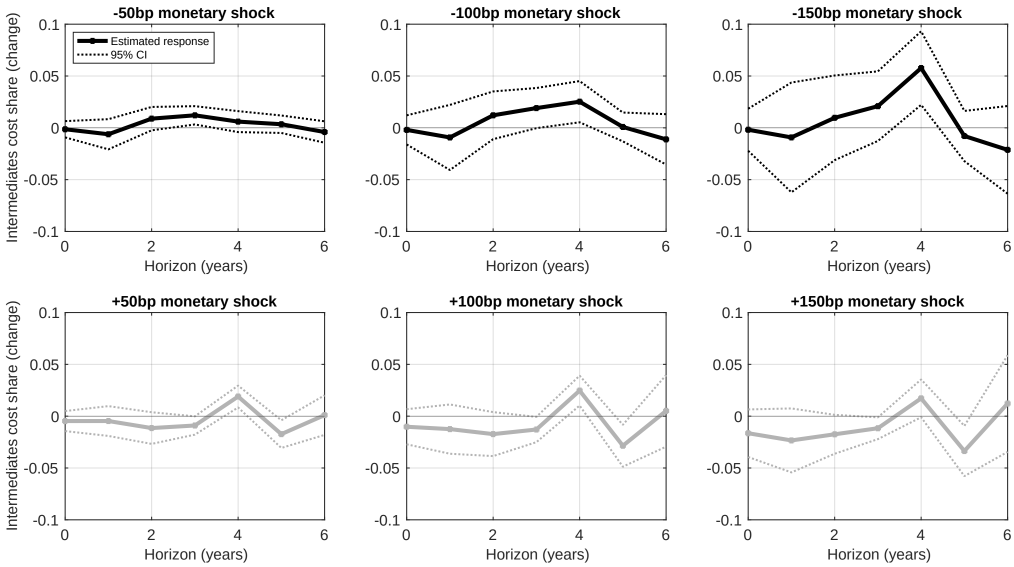
Notes: Panel (a) shows the estimated non-linear IRFs of intermediates cost share to productivity shocks based on Fernald (2014), using the specification in (F.2); the set of controls includes one lead of productivity shock and one lag of intermediates cost share, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows the estimated non-linear IRFs of the intermediates cost share to monetary shocks based on Romer and Romer (2004), using the specification in (F.2); the set of controls includes one lead of monetary shock and one lag of intermediates cost share, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence intervals.

Figure F.4: **Non-linear IRFs of intermediates cost shares: Non-Manufacturing Goods**

(a) Aggregate TFP shocks



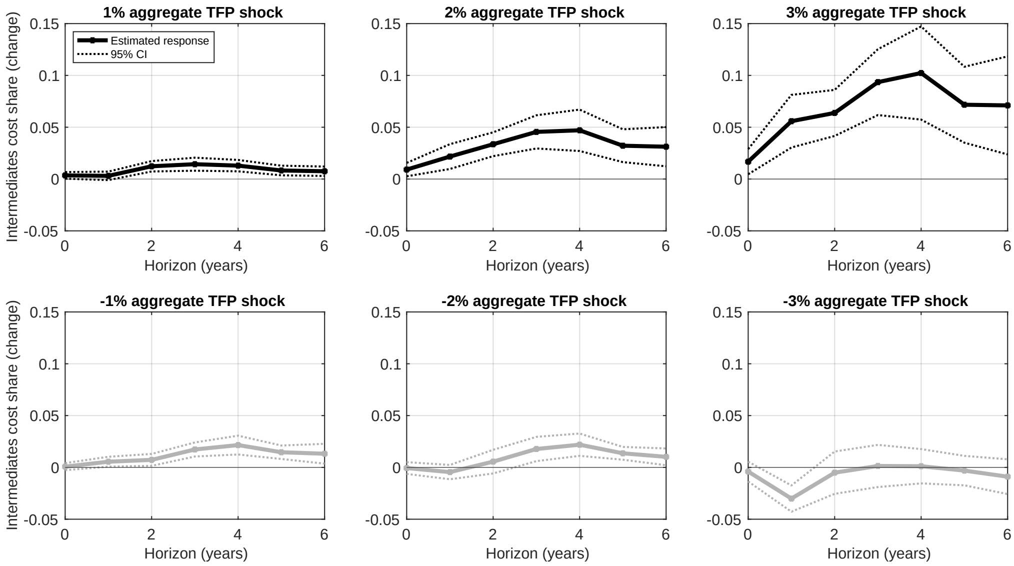
(b) Monetary shocks



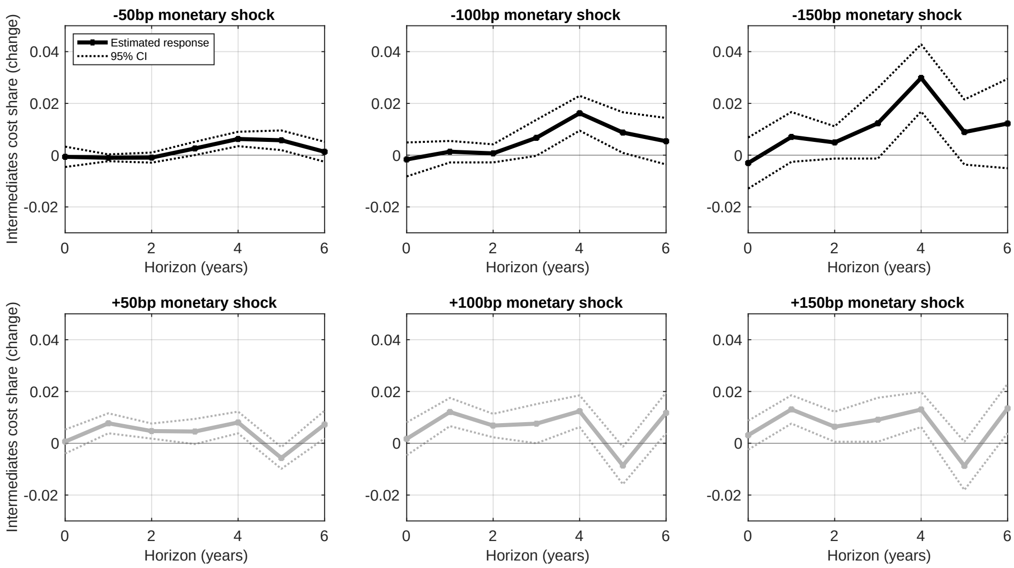
Notes: Panel (a) shows the estimated non-linear IRFs of intermediates cost share to productivity shocks based on Fernald (2014), using the specification in (18); the set of controls includes one lead of productivity shock and one lag of intermediates cost share, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows the estimated non-linear IRFs of the intermediates cost share to monetary shocks based on Romer and Romer (2004), using the specification in (18); the set of controls includes one lead of monetary shock and one lag of intermediates cost share, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence intervals.

Figure F.5: **Non-linear IRFs of intermediates cost shares: Durable Manufacturing Goods**

(a) Aggregate TFP shocks



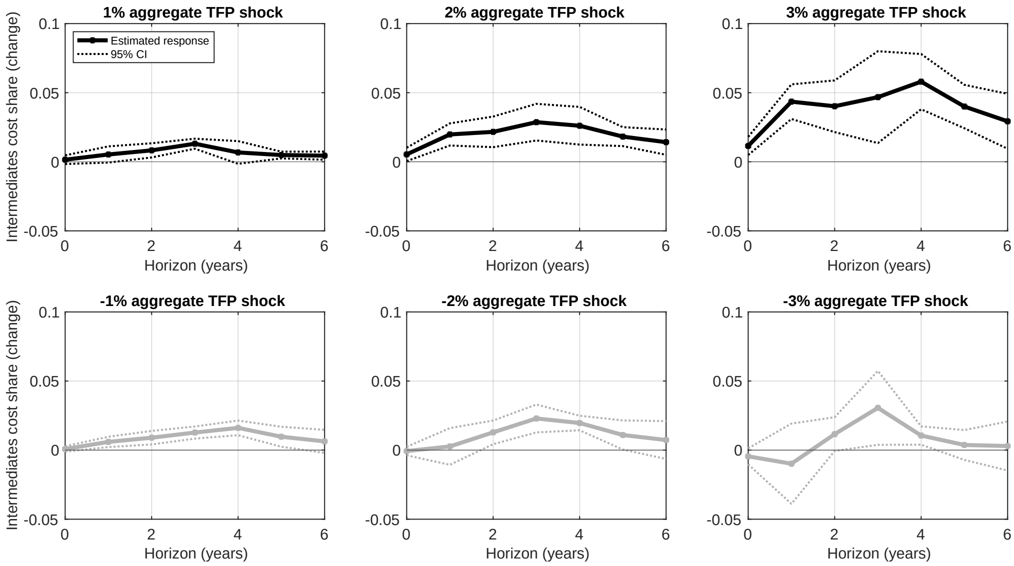
(b) Monetary shocks



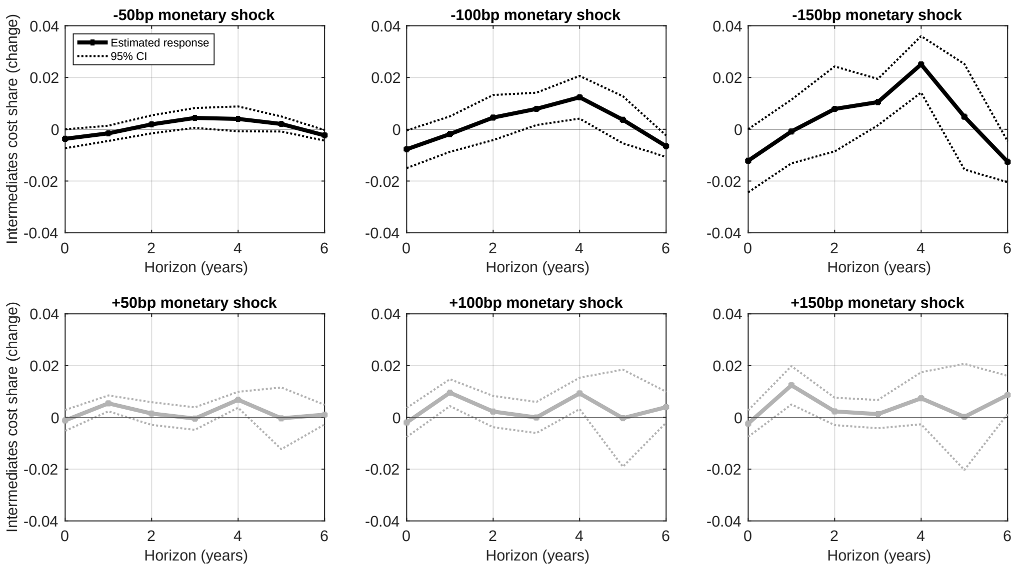
Notes: Panel (a) shows the estimated non-linear IRFs of intermediates cost share to productivity shocks based on Fernald (2014), using the specification in (18); the set of controls includes one lead of productivity shock and one lag of intermediates cost share, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows the estimated non-linear IRFs of the intermediates cost share to monetary shocks based on Romer and Romer (2004), using the specification in (18); the set of controls includes one lead of monetary shock and one lag of intermediates cost share, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence intervals.

Figure F.6: **Non-linear IRFs of intermediates cost shares: Non-Durable Manufacturing Goods**

(a) Aggregate TFP shocks



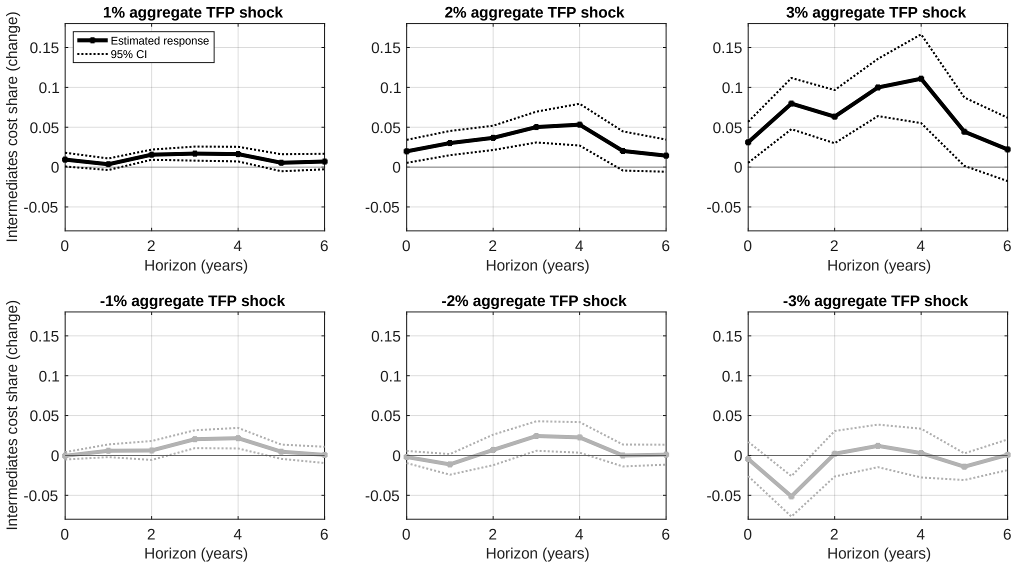
(b) Monetary shocks



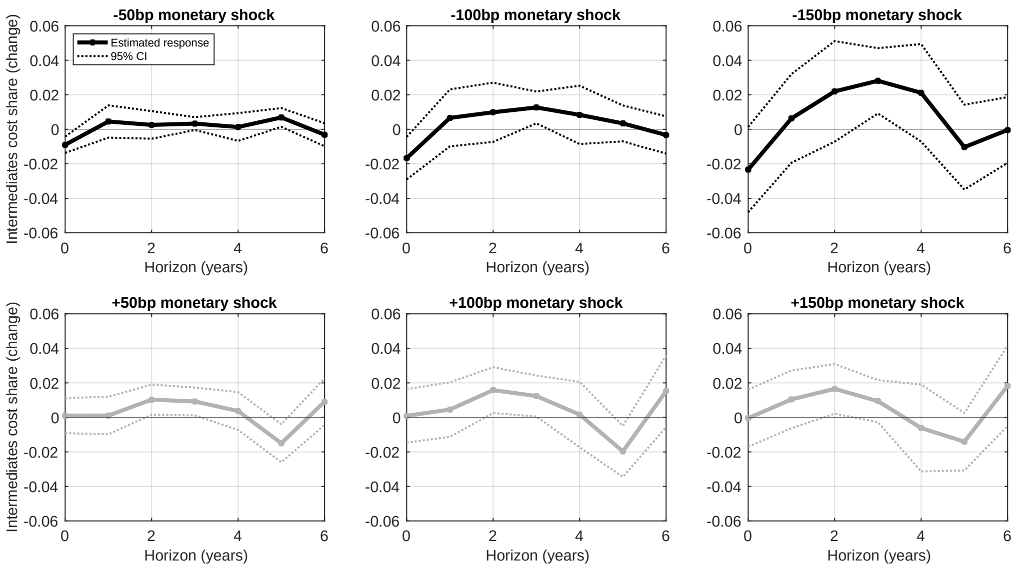
Notes: Panel (a) shows the estimated non-linear IRFs of intermediates cost share to productivity shocks based on Fernald (2014), using the specification in (18); the set of controls includes one lead of productivity shock and one lag of intermediates cost share, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows the estimated non-linear IRFs of the intermediates cost share to monetary shocks based on Romer and Romer (2004), using the specification in (18); the set of controls includes one lead of monetary shock and one lag of intermediates cost share, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence intervals.

Figure F.7: **Non-linear IRFs of intermediates cost shares: Trade, Transportation and Warehousing**

(a) Aggregate TFP shocks



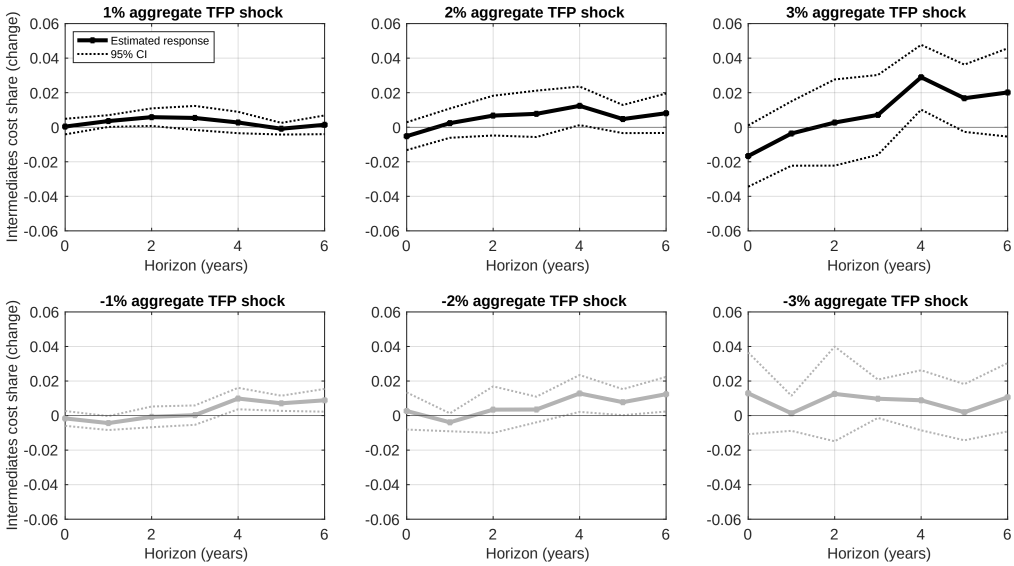
(b) Monetary shocks



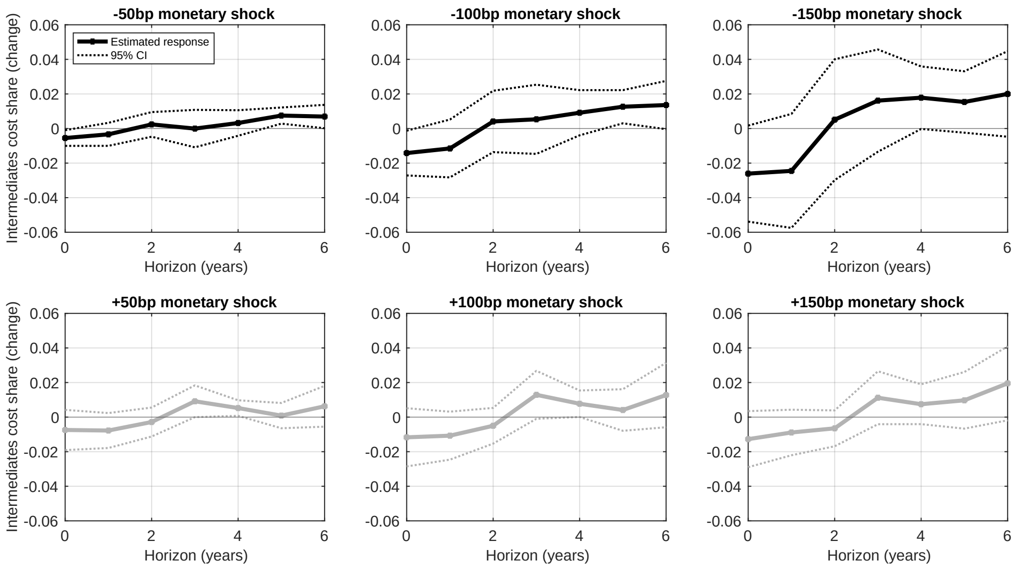
Notes: Panel (a) shows the estimated non-linear IRFs of intermediates cost share to productivity shocks based on Fernald (2014), using the specification in (18); the set of controls includes one lead of productivity shock and one lag of intermediates cost share, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows the estimated non-linear IRFs of the intermediates cost share to monetary shocks based on Romer and Romer (2004), using the specification in (18); the set of controls includes one lead of monetary shock and one lag of intermediates cost share, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence intervals.

Figure F.8: **Non-linear IRFs of intermediates cost shares: Information, FIRE and Professional Services**

(a) Aggregate TFP shocks



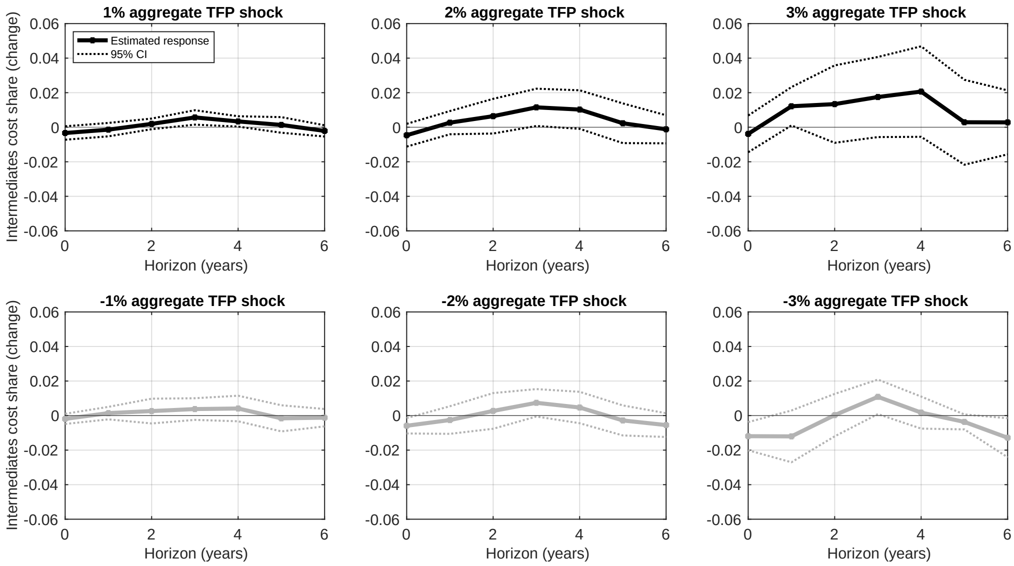
(b) Monetary shocks



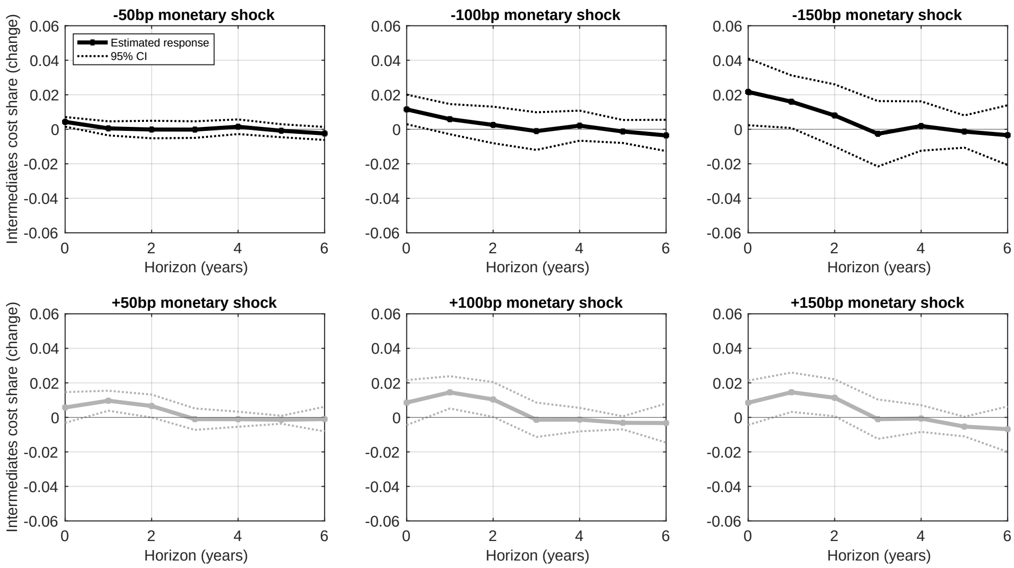
Notes: Panel (a) shows the estimated non-linear IRFs of intermediates cost share to productivity shocks based on Fernald (2014), using the specification in (18); the set of controls includes one lead of productivity shock and one lag of intermediates cost share, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows the estimated non-linear IRFs of the intermediates cost share to monetary shocks based on Romer and Romer (2004), using the specification in (18); the set of controls includes one lead of monetary shock and one lag of intermediates cost share, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence intervals.

Figure F.9: **Non-linear IRFs of intermediates cost shares: Education, Healthcare and Other Services**

(a) Aggregate TFP shocks



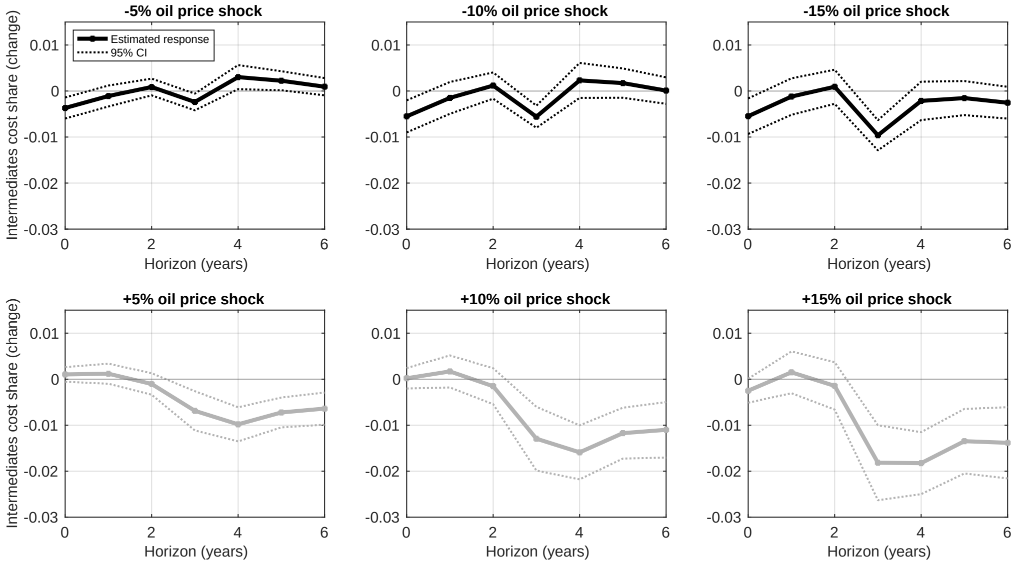
(b) Monetary shocks



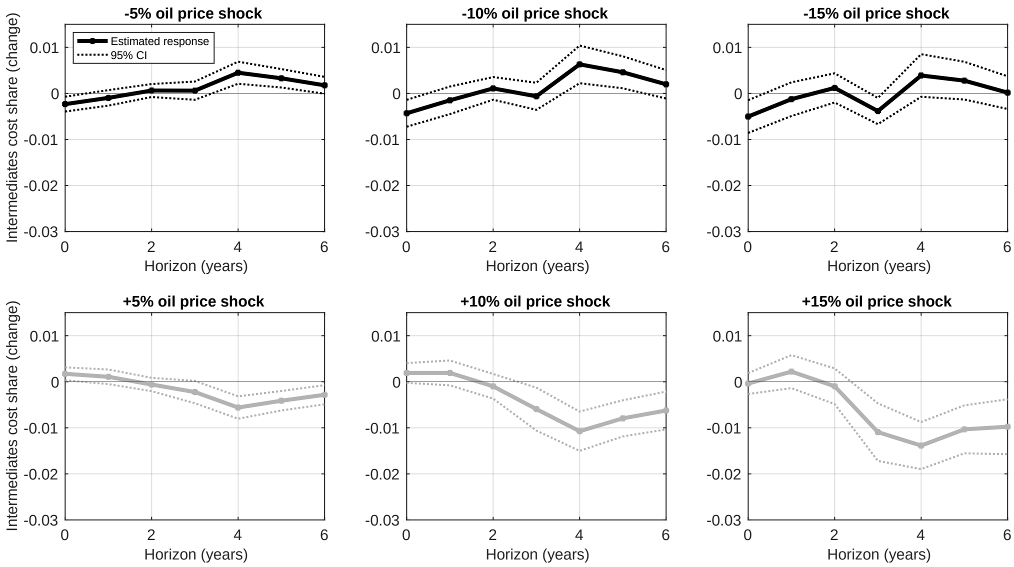
Notes: Panel (a) shows the estimated non-linear IRFs of intermediates cost share to productivity shocks based on Fernald (2014), using the specification in (18); the set of controls includes one lead of productivity shock and one lag of intermediates cost share, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows the estimated non-linear IRFs of the intermediates cost share to monetary shocks based on Romer and Romer (2004), using the specification in (18); the set of controls includes one lead of monetary shock and one lag of intermediates cost share, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence intervals.

Figure F.10: **Non-linear IRFs of intermediates cost shares**

(a) Oil shocks: specification (18)

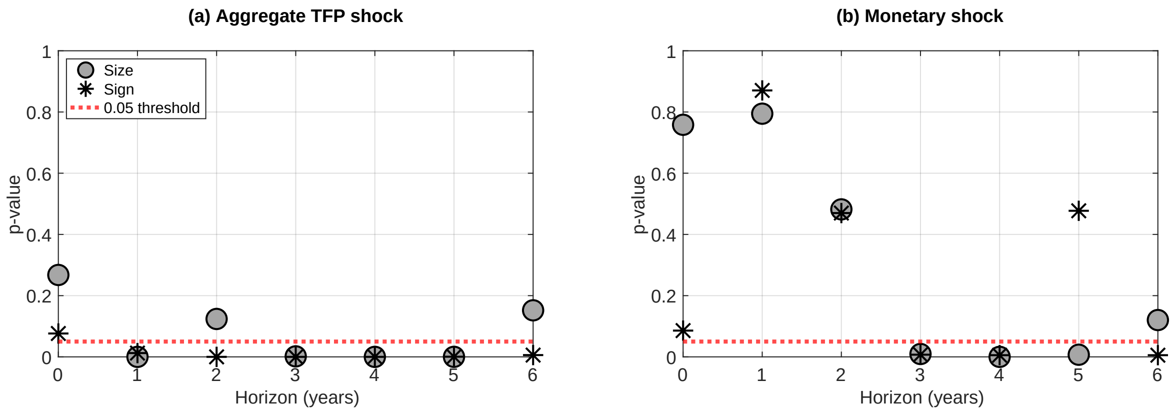


(b) Oil shocks: specification (F.2)



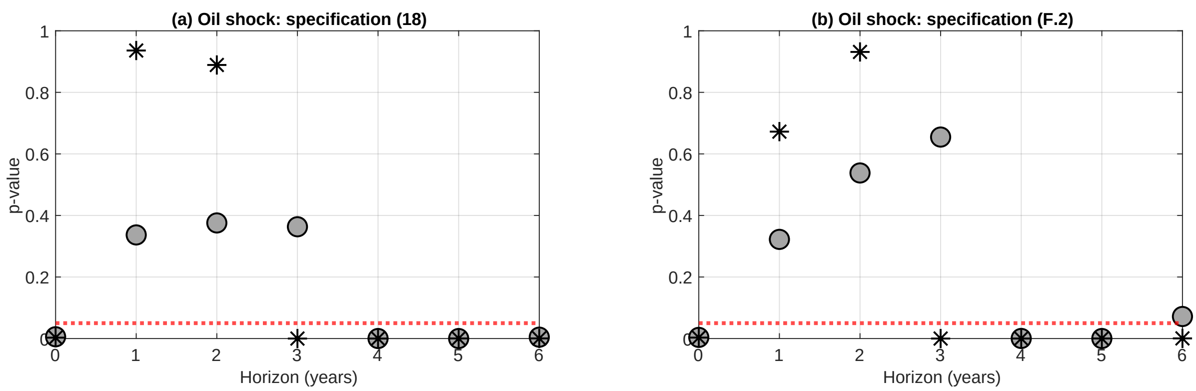
Notes: Panel (a) shows the estimated non-linear IRFs of intermediates cost share to oil shocks based on Känzig (2021), using the specification in (18); the set of controls includes one lag of the intermediates cost share, one lead and lag of the shock, as well as two lags of log real oil price, log real GDP, the Federal funds rate and log US CPI. Panel (b) shows the estimated non-linear IRFs of intermediates cost share to oil shocks based on Känzig (2021), using the specification in (F.2); the set of controls includes one lag of the intermediates cost share, one lead and lag of the shock, as well as two lags of log real oil price, log real GDP, the Federal funds rate and log US CPI.

Figure F.11: p**-values for tests of significance of non-linear effects (sectoral data)**



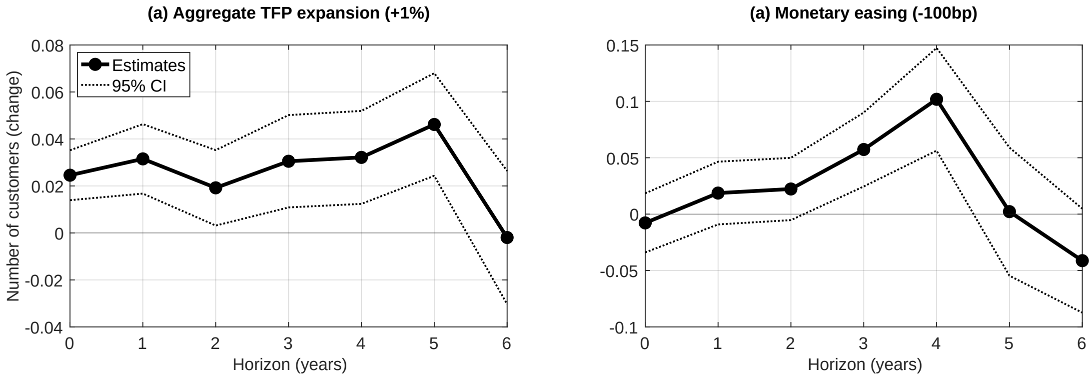
Notes: Panels (a) and (b) show the p-values for the test that the horizon-specific coefficients on non-linear sign and size effects from specification (F.2) ($\beta_H^{sign}, \beta_H^{size}$) are significantly different from zero, for Fernald (2014) productivity and Romer and Romer (2004) monetary shocks, respectively.

Figure F.12: p**-values for tests of significance of non-linear effects (sectoral data)**



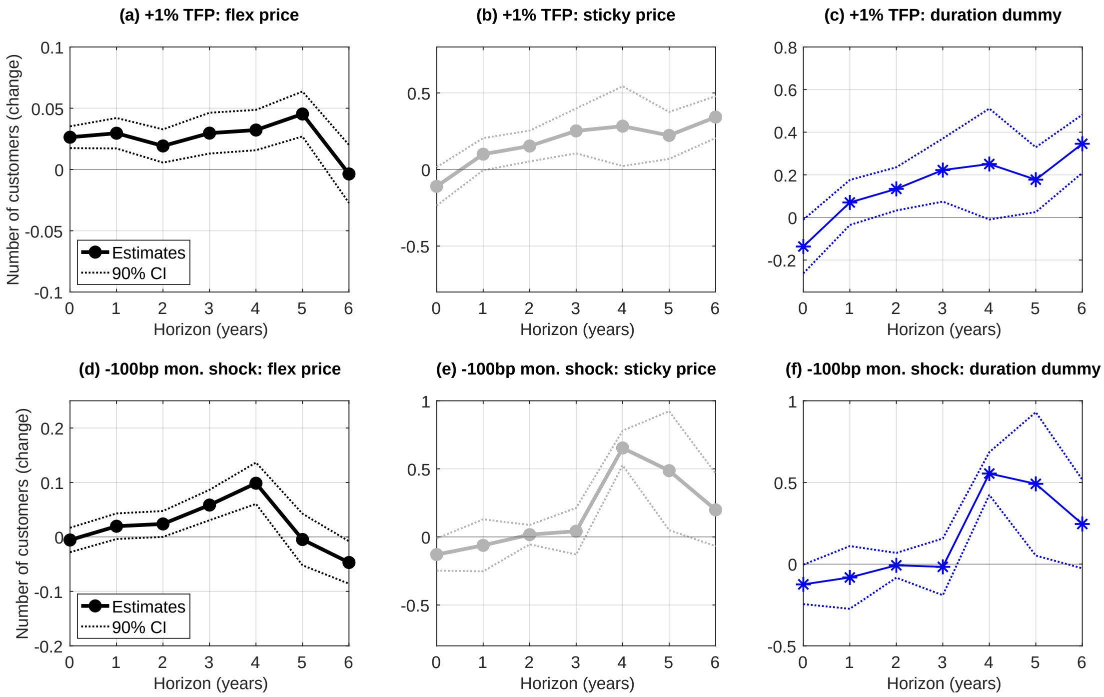
Notes: Panels (a) and (b) show the p-values for the test that under the oil shocks based on Känzig (2021), the horizon-specific coefficients on non-linear sign and size effects ($\beta_H^{sign}, \beta_H^{size}$) are significantly different from zero, for specifications (18) and (F.2) respectively.

Figure F.13: **Linear IRFs of number of customers to TFP and monetary shocks**



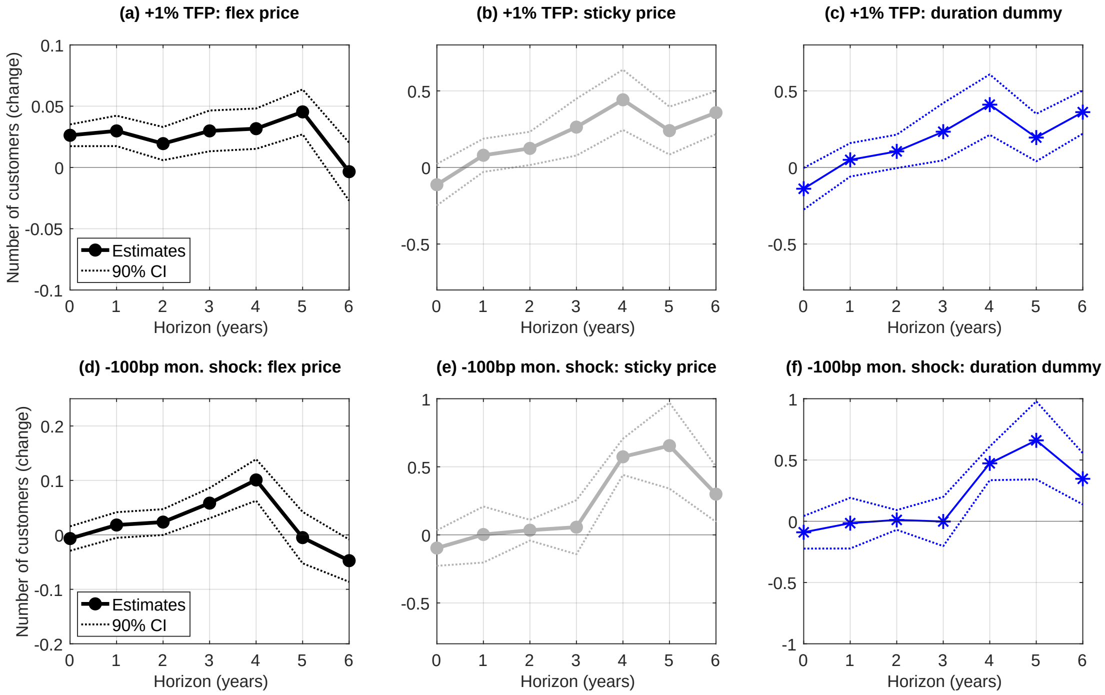
Notes: Panel (a) shows the estimated IRFs of the number of customers to a productivity shock based on Fernald (2014), using the specification in (19) with the number of customers (outdegree) as the dependent variable; the set of controls includes four lags of the number of customers, one lag of productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows estimated IRFs of the number of customers to a monetary shock based on Romer and Romer (2004), using the specification in (19); the set of controls includes one lead of monetary shock, four lags of the number of customers, one lag of monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence intervals.

Figure F.14: **IRFs of number of customers: role of price duration ($\bar{d} = 15$ months)**



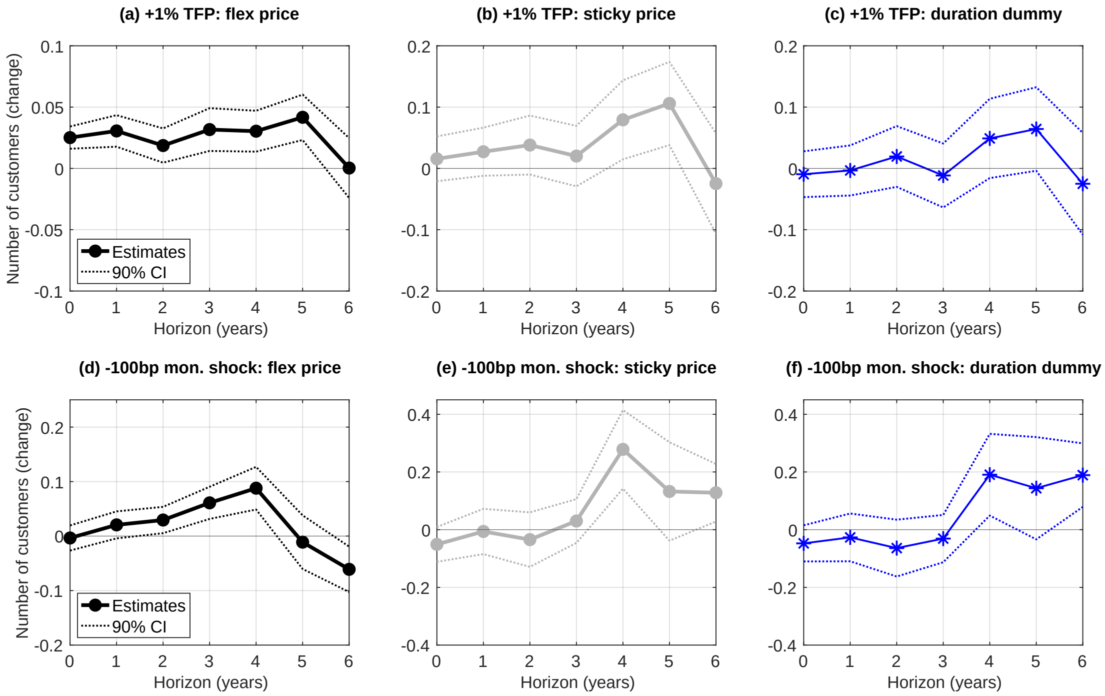
Notes: Panels (a)-(c) show the estimated IRFs of the number of customers to a productivity shock based on Fernald (2014), using the specification in (F.6). Panels (d)-(f) show estimated IRFs of the number of customers to a monetary shock based on Romer and Romer (2004), using the specification in (F.6).

Figure F.15: **IRFs of number of customers: role of price duration ($\bar{d} = 16.5$ months)**



Notes: Panels (a)-(c) show the estimated IRFs of the number of customers to a productivity shock based on Fernald (2014), using the specification in (F.6). Panels (d)-(f) show estimated IRFs of the number of customers to a monetary shock based on Romer and Romer (2004), using the specification in (F.6).

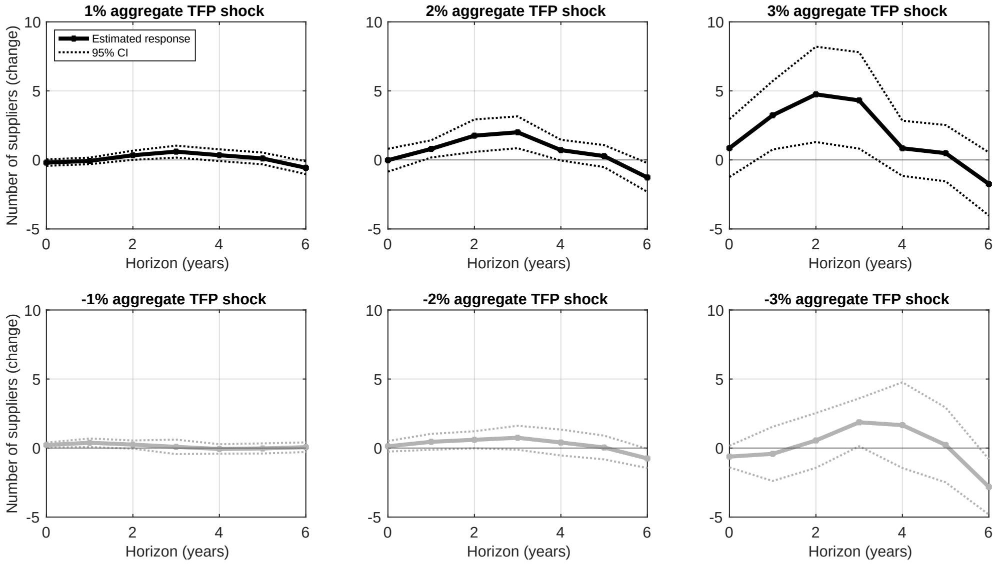
Figure F.16: **IRFs of number of customers: role of price duration ($\bar{d} = 13.5$ months)**



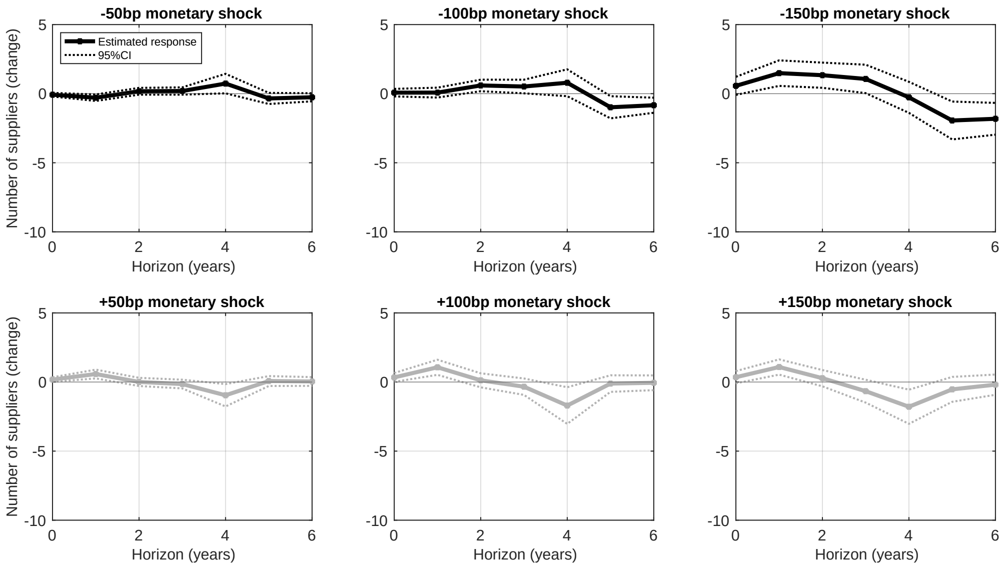
Notes: Panels (a)-(c) show the estimated IRFs of the number of customers to a productivity shock based on Fernald (2014), using the specification in (F.6). Panels (d)-(f) show estimated IRFs of the number of customers to a monetary shock based on Romer and Romer (2004), using the specification in (F.6).

Figure F.17: **Non-linear IRFs of number of suppliers**

(a) Aggregate TFP shocks



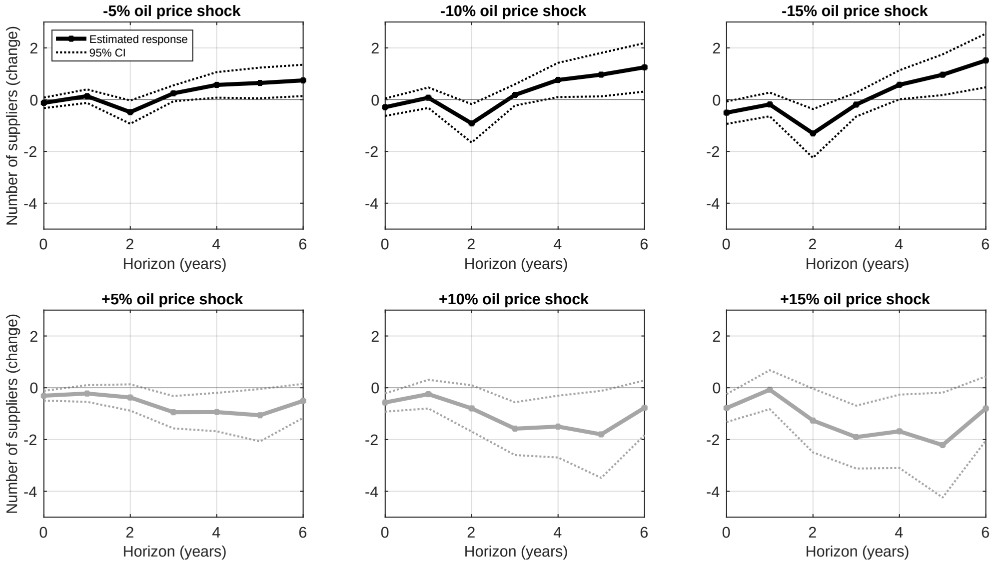
(b) Monetary shocks



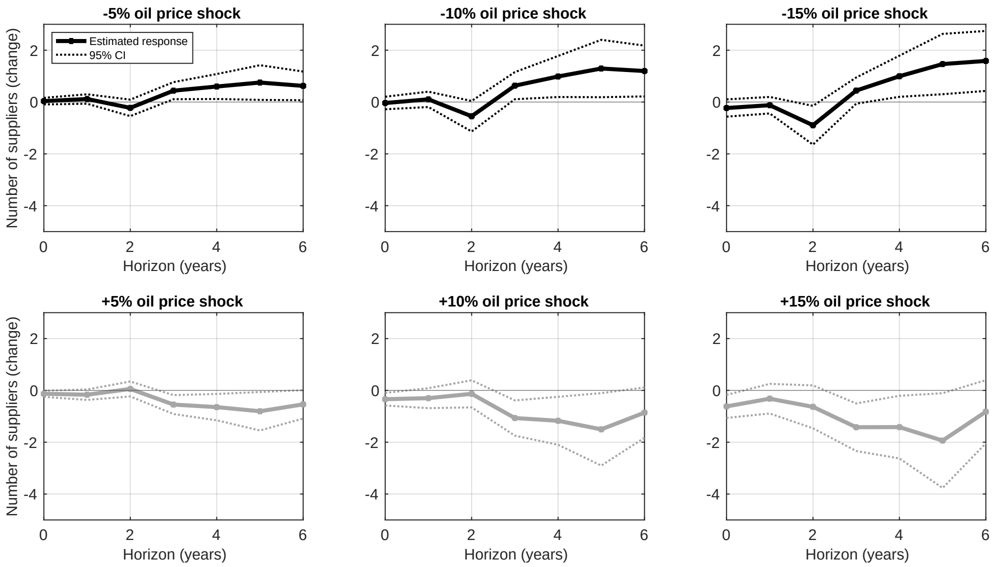
Notes: Panel (a) shows the estimated non-linear IRFs of the number of suppliers to productivity shocks based on Fernald (2014), using the specification in (F.4); the set of controls includes one lead of productivity shock, four lags of the number of suppliers, one lag of productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows estimated non-linear IRFs of the number of suppliers to monetary shocks based on Romer and Romer (2004), using the specification in (F.4); the set of controls includes one lead of monetary shock, four lags of the number of suppliers, one lag of monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence intervals.

Figure F.18: **Non-linear IRFs of number of suppliers**

(a) Oil shocks: specification (20)

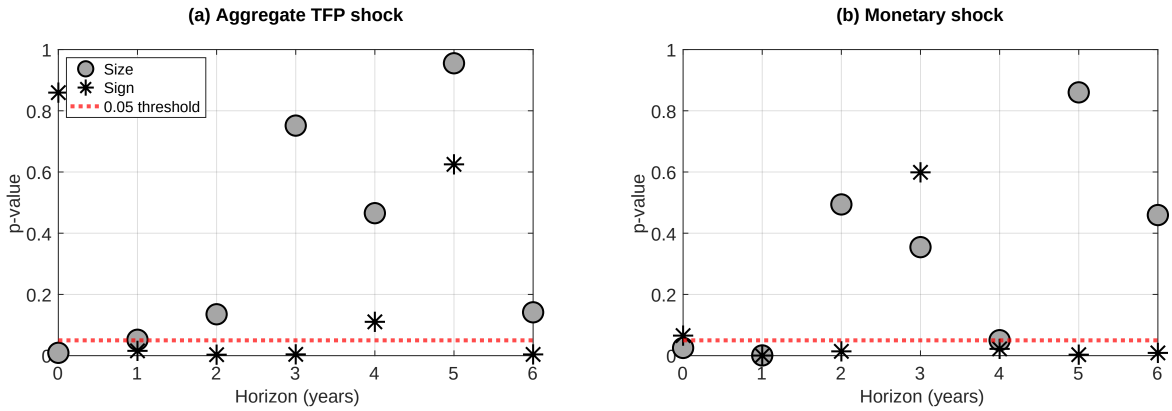


(b) Oil shocks: specification (F.4)



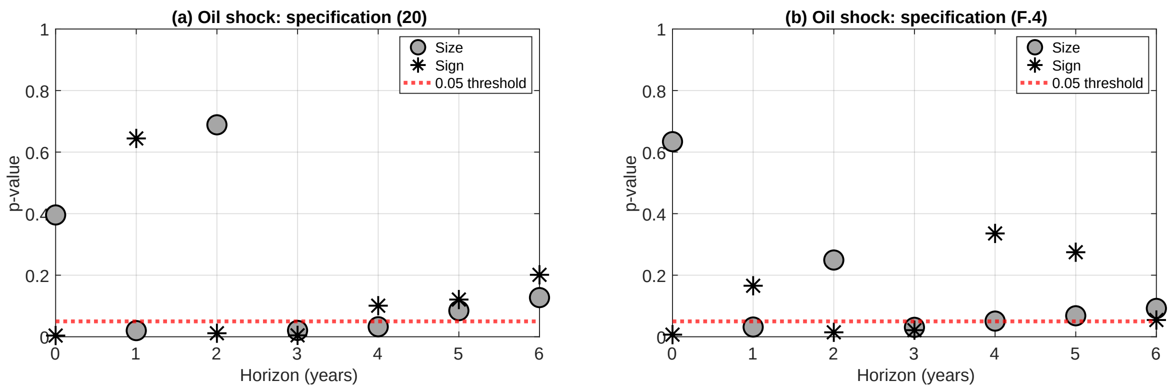
Notes: Panel (a) shows the estimated non-linear IRFs of the number of suppliers to oil shocks based on Känzig (2021), using the specification in (20); the set of controls includes one lead and lag of the shock, four lags of the number of suppliers, as well as single lags of log real oil price, log real GDP, the Federal funds rate, as well as a time trend. Panel (b) shows the estimated non-linear IRFs of the number of suppliers to oil shocks based on Känzig (2021), using the specification in (F.4); the set of controls includes one lead and lag of the shock, four lags of the number of suppliers, as well as single lags of log real oil price, log real GDP, the Federal funds rate, as well as a time trend.

Figure F.19: p**-values for tests of significance of non-linear effects (firm-level data)**



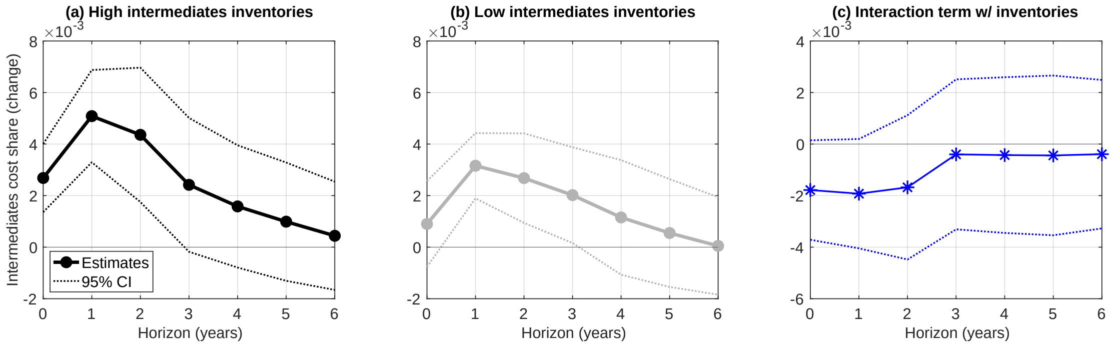
Notes: Panels (a) and (b) show the p-values for the test that the horizon-specific coefficients on non-linear sign and size effects from specification (F.4) $(\beta_H^{sign}, \beta_H^{size})$ are significantly different from zero, for Fernald (2014) productivity and Romer and Romer (2004) monetary shocks, respectively.

Figure F.20: p**-values for tests of significance of non-linear effects (firm-level data)**



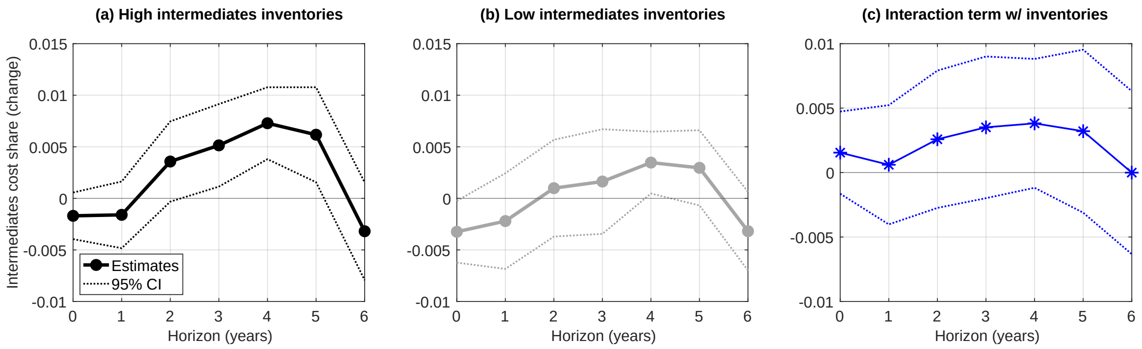
Notes: Panels (a) and (b) show the p-values for the test that under the oil shocks based on Känzig (2021), the horizon-specific coefficients on non-linear sign and size effects $(\beta_H^{sign}, \beta_H^{size})$ are significantly different from zero, for specifications (20) and (F.4) respectively.

Figure F.21: **Intermediates cost share and agg. TFP shocks: the role of inventories**



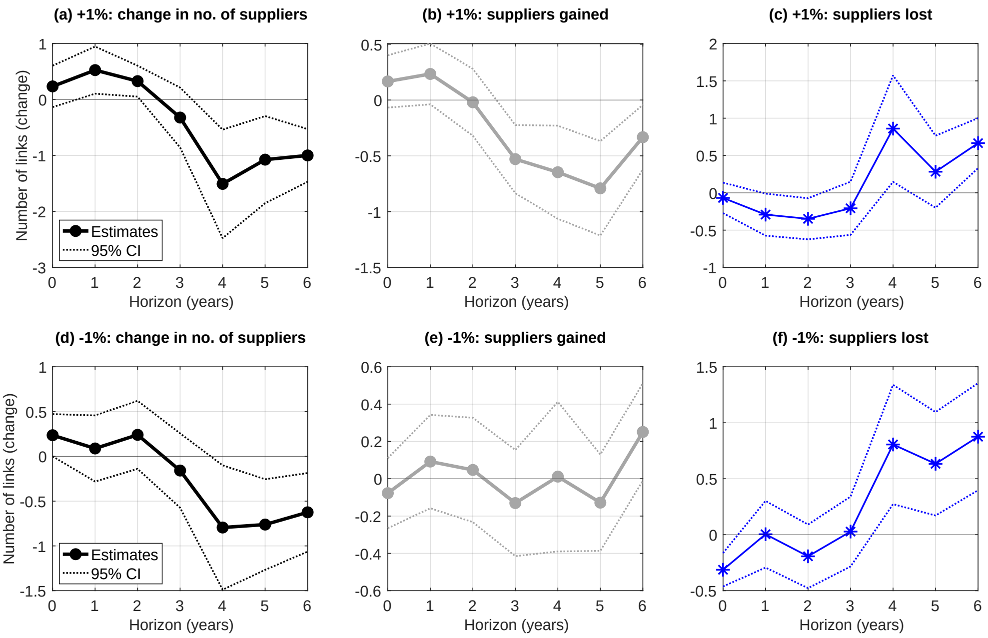
Notes: the figure shows the estimated IRFs of the intermediates cost share to a productivity shock based on Fernald (2014), using the specification in (F.3); the set of controls includes one lead of productivity shock and one lag of intermediates cost share, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend.

Figure F.22: **Intermediates cost share and monetary shocks: the role of inventories**



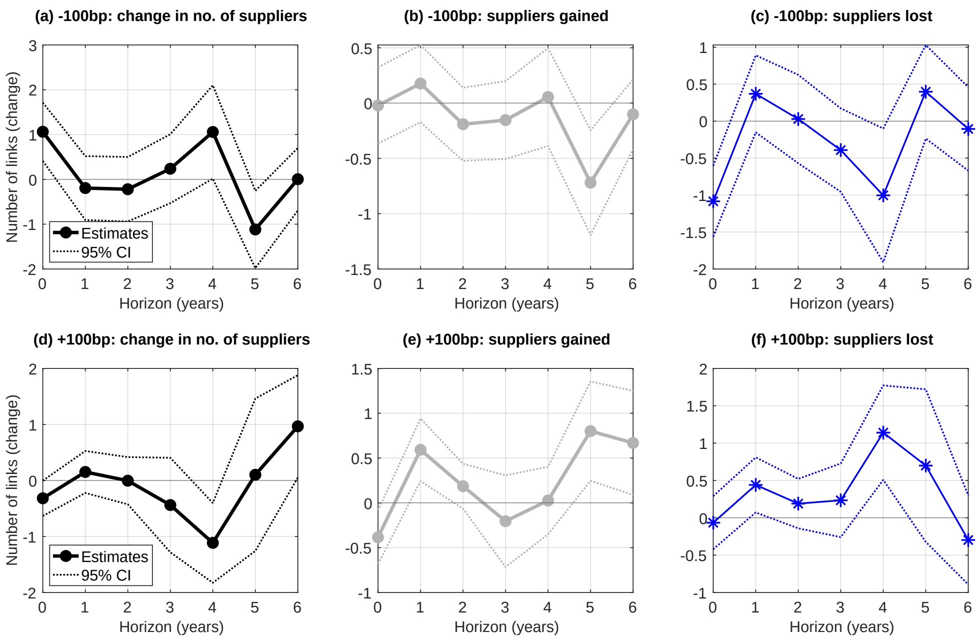
Notes: the figure shows the estimated IRFs of intermediates cost share to a monetary shock based on Romer and Romer (2004), using the specification in (F.3); the set of controls includes one lead of monetary shock and one lag of intermediates cost share, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence intervals.

Figure F.23: **Aggregate TFP shocks: separate effects on suppliers gained and lost**



Notes: the figure shows the estimated IRFs of the overall change in the number of suppliers, the number of suppliers gained and the number of suppliers lost to a +1% and a -1% productivity shock based on Fernald (2014), using the specification in (F.7).

Figure F.24: **Monetary shocks: separate effects on suppliers gained and lost**



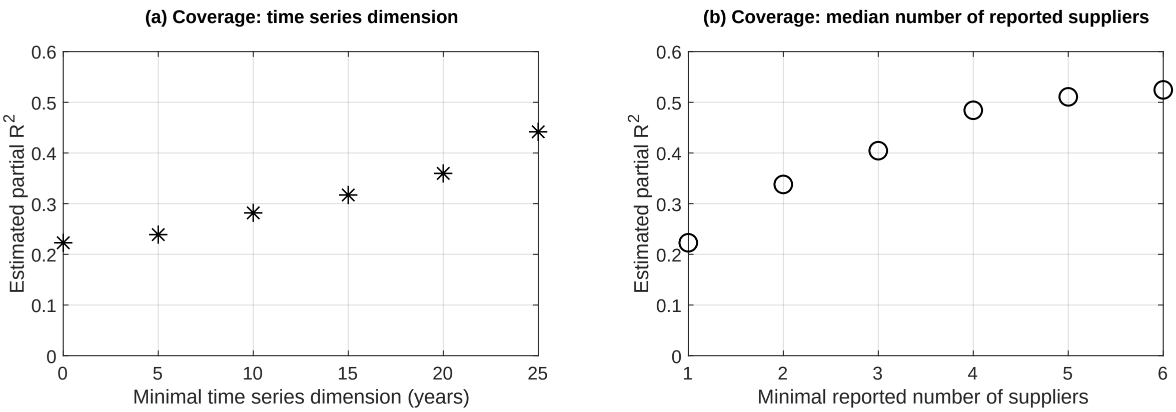
Notes: the figure shows the estimated IRFs of the overall change in the number of suppliers, the number of suppliers gained and the number of suppliers lost to a -100 bp and a +100 bp monetary shock based on Romer and Romer (2004), using the specification in (F.7).

Table F.24: **Intermediates cost share and the number of suppliers (Compustat)**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Dependent variable: $\log \delta_{j,t}$							
Sample	Full	Full	Full	$T_j \geqslant 15$	$T_j \geqslant 25$	$\overline{indeg}_j \geqslant 3$	$\overline{indeg}_j \geqslant 5$
$\log indeg_{j,t}$	1.24***	1.18***	1.20***	1.24***	1.22***	1.25***	1.27***
	(0.05)	(0.05)	(0.05)	(0.07)	(0.09)	(0.07)	(0.09)
Firm fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year fixed effects	No	Yes	No	No	No	No	No
Year × SIC2 fixed effects	No	No	Yes	No	No	No	No
Partial R^2: $\log indeg_{j,t}$	0.22	0.19	0.20	0.32	0.44	0.40	0.51
No. of unique firms	2,098	2,098	2,098	148	43	199	97
No. of firm-year obs.	10,390	10,390	10,390	3,233	1,217	2,854	1,636

Notes: the table shows the estimated linear regression of the firm-level (log) intermediates cost share $\log \delta_{j,t}$ on the firm-level (log) number of suppliers $\log indeg_{j,t}$, as well as fixed effects, using the Compustat-based dataset of Atalay et al. (2011). The full sample spans annual data between 1976 and 2008 and includes firms that report at least one supplier in a given year. In the restricted sample, T_j indicates the number of years in which a firm j features in the sample, whereas $\overline{indeg}_j$ is firm j's in-sample median number of suppliers over the years it is present in the full sample. Robust standard errors are reported in parentheses, with *** indicating that the p-value is less than 0.01.

Figure F.25: **Within-variation in the intermediates cost share due to indegree**



Notes: the figure shows that partial R^2 of $\log indeg_{i,j}$ in the regression (F.8), estimated on subsamples with more extensive coverage. Panel (a) defines the subsample by the minimal time series dimension of each firm; panel (b) defines the subsample by the minimal median number of reported suppliers across the years each firm features in the dataset.

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