Business Cycles with Pricing Cascades*

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Abstract

Business cycles with pronounced inflation can have sectoral origins and often feature a growing share of price-adjusting firms. Rationalizing such phenomena requires enhancing our modeling toolkit. We do that by building a non-linear equilibrium multi-sector framework featuring a general input-output network and *optimal* decisions on the timing and size of price adjustments. The interaction of our ingredients creates equilibrium *cascades*: large movements in aggregates trigger price adjustment decisions on the extensive margin. Following demand shocks, such as monetary interventions, networks *dampen* cascades, thus slowing down price adjustment decisions and giving central banks substantial power to stimulate the real economy with limited inflationary consequences. In contrast, under supply shocks, networks *amplify* cascades, leading to fast increases in the frequency of repricing and large inflationary swings. Applied to Euro Area data, the interaction of networks with cascades allows us to quantitatively match the surges in inflation and repricing frequency in the post-Covid era.

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1 Introduction

The dynamics of aggregate prices and quantities over the business cycle have long been a central theme in economics. Recent events, such as the Covid pandemic and the Russian invasion of Ukraine, have brought renewed attention to the topic, along with new evidence on the cyclical properties of key macro-variables. First, following a prolonged period of stability, we have witnessed the possibility of *large* inflationary swings in advanced economies, marked by persistent double-digit rates of price growth in the US, the UK, and the Euro Area. All the while, the movements in aggregate activity have been milder and transient. Second, we have learned that the inflationary surge has been accompanied by a rising *frequency* at which firms adjust their prices (Cavallo et al., 2024; Montag and Villar, 2025).[1] Third, large shocks hitting specific *sectors* have caused significant consequences for the rest of the economy (Schneider, 2025; Shen et al., 2026). Although informative, such evidence cannot be analyzed using existing theoretical frameworks – however influential – that rely on linearized single-sector setups with a constant frequency of adjustment (Woodford, 2004; Galí, 2015). This discrepancy calls for a new framework for studying aggregate prices and quantities over the business cycle, and this paper develops one.

Our dynamic general equilibrium framework features a combination of three ingredients. First, a multi-sector structure with a fully unrestricted input-output architecture, allowing us to capture empirically realistic production networks. Second, firms making pricing decisions in an optimal state-dependent manner, so that both the extensive and the intensive margins of adjustment are endogenous. Third, a fully non-linear solution strategy, tracing out the response of the economy to arbitrarily large shocks, either aggregate or sector-specific. The *interaction* of our three ingredients delivers a novel theoretical mechanism, namely pricing *cascades*: large movements in aggregates trigger possibly self-reinforcing adjustment decisions at the extensive margin. Crucially, networks can dampen or amplify cascades, depending on the *type of shock* hitting the economy. For demand shocks, such as monetary interventions, networks dampen cascades, leading to muted price responses with a near-constant frequency of adjustment in equilibrium. In contrast, networks amplify cascades following either aggregate or sectoral supply shocks, with strongly non-linear price responses led by the extensive margin. As a result, the mechanism of pricing cascades allows our framework to produce realistic monetary non-neutrality, while simultaneously generating substantial inflationary surges following reasonably-sized and structurally-interpretable shocks. When esti-

[1] Rising frequency of adjustment was also a major feature during the US inflationary surge of the late 1970s-early 1980s (Nakamura et al., 2018), in contrast to the modest role of the extensive margin in the subsequent Great Moderation period (Klenow and Kryvtsov, 2008). Also see Alvarez et al. (2019) for evidence regarding the co-movement between frequency and inflation in Argentina (1988-1997).

mated using Euro Area data, the interaction of networks with cascades allows our model to jointly match the surges in inflation and the repricing frequency in the post-Covid era.

The shock-dependent interaction between production networks and pricing cascades is the key channel that is unique to our model. Under *demand* shocks, networks *dampen* cascades by shrinking the magnitudes of desired price changes, hence making firm-level adjustment decisions less likely relative to the economy that only uses labor in production. In an economy with networks, a demand shock affects both the wage and the price of intermediate inputs. Under countercyclical markups, the price of intermediates moves by less than the wage, attenuating changes in marginal costs. Smaller movements in marginal costs reduce deviations between actual and optimal reset prices, making it less likely that the firm opts for the costly adjustment decision. We formally show that such a reduction in adjustment probability is stronger for firms with a higher total exposure to intermediate inputs, as measured by the *customer centrality* metric. Moreover, heterogeneity in *customer centrality* driven by asymmetries in the input-output structure, in general, matters for aggregates, capturing effects beyond those under roundabout production (Basu, 1995). In the version of our model calibrated to 39 sectors of the Euro Area economy, the aggregate consequences of cascades dampening are substantial. Following a large expansion in money supply (+10%), the economy with fixed menu costs and networks features a rise in the monthly fraction of price-adjusting firms by 0.18, compared to an increase by 0.35 without the linkages. As for monetary non-neutrality, a 1% expansion leads to an impact GDP rise of 0.85% with networks and 0.78% without networks; for the larger shock (+10%), the gap widens: 6% under networks, as opposed to 2.5% without. Hence, cascades dampening leads to substantial additional monetary non-neutrality, which persists even for large shocks and even in the economy with fixed menu costs.

In contrast, networks *amplify* cascades following aggregate or sector-specific *supply shocks*. For example, consider an exogenous contraction in aggregate total factor productivity (TFP). At first, it leads to a one-for-one increase in firm-level marginal costs; on top of that, as long as prices of intermediate inputs also go up in equilibrium, it leads to a further surge in costs. Relative to the economy that only uses labor in production, such a double whammy of marginal cost increases expands deviations between actual and optimal reset prices, making the adjustment decision more likely, especially for firms with a high *customer centrality*. Such cascades amplification is not specific to aggregate supply shocks. To see that, consider a TFP shock to a sector that acts as a supplier of intermediate inputs to the rest of the economy. On top of impacting the shocked sector itself, it also affects the marginal costs, the optimal reset prices and hence the adjustment probability in the downstream industries. We formally derive a novel centrality metric, the *supplier Herfindahl*, which pins down the sensitivity of aggregate adjustment frequency to a sectoral supply shock. The *supplier Herfindahl* increases both in the *average* importance of a sector as a supplier, as well as in the degree to which the sector is a disproportionately important provider of inputs to certain parts of the economy. Quantitatively, following a large aggregate TFP contraction (-10%), the monthly fraction of price-adjusting firms rises by 0.65, as opposed to 0.28 without networks. The latter creates a strong non-linearity in the behavior of aggregate CPI inflation. While a -1% TFP contraction generates 0.59% inflation on impact, a -10% contraction leads to 17% inflation, a surge driven by the sharp increase in the equilibrium adjustment frequency. Beyond aggregate disturbances, we find that large shocks to sectors such as "Chemicals and chemical products", "Food and beverages" and "Crop and animal production" can have a disproportionately large effect on the economy-wide adjustment frequency, and hence generate sharp non-linear inflationary surges.[2]

We conduct a number of empirical validation exercises, showing that our model can capture the observed macro dynamics in the Euro Area, *both* in the low- and high-inflation periods. First, we study the performance of our model under small aggregate and sector-specific shocks, finding that we can reconcile the estimated responses of aggregates to identified monetary and oil shocks in the low-inflation pre-Covid period. Second, we subject our model to the key shocks experienced by the Euro Area economy in the high-inflation (post-)Covid years (2020-2024), and compare the model-implied dynamics of aggregate inflation and adjustment frequency to those observed in the data. In particular, we feed in four series, corresponding to aggregate demand, aggregate labor wedge, as well as the sectoral dynamics of energy and food prices. We find that the model successfully reconciles most of the five percentage point increase in the monthly repricing frequency, as well as the aggregate inflation surge up to 11% at the peak. In contrast, an otherwise identical model with only labor in production, when subjected to the same four shocks, generates at most a one percentage point increase in the repricing frequency, as well as an inflation surge to only 5% at the peak. As for an economy with networks but time-dependent pricing, it generates no change in the adjustment frequency by construction, with a maximum of 7% inflation. These results highlight the quantitative importance of our key theoretical channel – the interaction of networks with pricing cascades – for explaining business cycle dynamics.

Beyond a theory of dampening and amplification, our model presents a theory of *co-existence* of empirically-realistic monetary non-neutrality and inflationary surges in general equilibrium. Crucially, we generate such co-existence while remaining consistent with the microeconomic distribution of price changes *and* under shocks that have

[2]Our baseline results for the propagation of TFP shocks are obtained holding the monetary instrument fixed. To recognize that the endogenous central bank response is important, Appendix D.1 shows that the key results go through quantitatively under an empirically realistic central bank policy rule.

a clear structural interpretation and whose size can be directly disciplined by the data. Generating such co-existence has long been a major challenge in monetary economics. Recent work by Blanco et al. (2024c) and L'Huillier and Phelan (2025) stresses that models which produce reasonable non-neutrality of money and Phillips Curve slope, such as those with time-dependent pricing or stochastic menu costs, struggle to generate double-digit inflation surges without appealing to shocks that are either unreasonably large or lack a clear structural interpretation, such as cost-push or markup shocks. At the same time, models that match inflation and frequency movements with reasonably-sized shocks, such as those with fixed menu costs, imply that money is close to neutral. In this sense, our key theoretical channel, working through the interaction of production networks and pricing cascades, offers a resolution to this long-standing and first-order issue in the literature without deviating from a conventional price-setting setup.

Contribution to the literature Our paper contributes to at least **three** broad strands of the literature. **First**, we add to the vast literature on state-dependent pricing in macroeconomics. Under state-dependent pricing, the probability of a price change is endogenous and affected by idiosyncratic and aggregate shocks, in contrast to time-dependent models such as Taylor (1979) or Calvo (1983). Our main contribution is to the literature on general equilibrium implications of state-dependent pricing, marked by the works of Golosov and Lucas (2007), Gertler and Leahy (2008) and Midrigan (2011) in the context of single-sector models with small aggregate shocks and fixed menu costs.[3] This framework has been further explored analytically by Alvarez et al. (2016, 2022) and Alvarez and Lippi (2022), whose results provide model-based sufficient statistics linking the dynamics of macro aggregates to micro pricing moments. Subsequent work also considers one-sector models with state-dependent pricing subjected to *large* aggregate shocks (Karadi and Reiff, 2019; Cavallo et al., 2024; Blanco et al., 2024a,b,c; Karadi et al., 2024; Gagliardone et al., 2025). Beyond single-sector setups, Carvalho and Kryvtsov (2021) and Caratelli and Halperin (2023) consider multi-sector frameworks with state-dependent pricing, but without an explicit input-output structure. Arguably most related to ours is the work on state-dependent pricing with complementarities, both *micro*, such demand systems with non-constant price elasticities, and *macro*, in the form of symmetric roundabout production structures (Nakamura and Steinsson, 2010; Klenow and Willis, 2016; Mongey, 2021; Alvarez et al., 2023; Nirei and Scheinkman, 2024).

We contribute to this literature by developing the first general equilibrium model, which combines a multi-sector setup with a fully general input-output structure, state-

[3] There are also papers that consider models with menu costs that are random rather than fixed (Dotsey et al., 1999; Nakamura and Steinsson, 2010), or where the price change probability is a *smoothly* increasing function of the gain from adjustment (Caballero and Engel, 2007; Costain and Nakov, 2011).

dependent pricing and arbitrarily large aggregate and sector-specific shocks. Conceptually, we study the interaction of state-dependent pricing with *macro* complementarities, highlighting that the asymmetries in the input-output structure, unique to our setup, matter for the transmission of both aggregate and sector-specific shocks. Our novel channel is also quantitatively relevant, which we show by estimating our model to 39 sectors of the Euro Area economy, fully matching the sector-specific pricing moments as well as the input-output structure.

Second, our paper is related to the growing literature on production networks and their role in connecting micro shocks and frictions with aggregate business cycle fluctuations. The seminal work by Acemoglu et al. (2012) considers a flexible-price efficient setup and shows how production networks can amplify sector- or firm-specific shocks to create aggregate volatility. Subsequent work by Baqaee and Farhi (2020) and Bigio and La'O (2020) provides general first-order aggregation results for microeconomic shocks and distortions in economies with inefficiencies and networks. As for the propagation of large shocks in flexible-price network economies, see Baqaee and Farhi (2019) for results based on second-order approximations, as well as Dew-Becker (2023) for limit cases in a fully non-linear economy. A separate strand of this literature analyzes linearized models with production networks and *time-dependent* pricing, both positively (Pasten et al., 2020; Ghassibe, 2021; Afrouzi and Bhattarai, 2023) and normatively (La'O and Tahbaz-Salehi, 2022; Rubbo, 2023).

We contribute to this literature by showing that the *interaction* of production networks and state-dependent pricing creates a novel source of non-linearity in aggregate business cycles, through pricing *cascades* created by large aggregate or sector-specific shocks. Our results stress that both the source of the shock, either demand- or supply-side, as well as its sectoral origin matter for aggregate fluctuations and the degree of non-linearity. In case of sectoral supply shocks, the position of the industry in the network can matter over an above its equilibrium size, against the network-irrelevance results established in the prior literature (Hulten, 1978).

Third, we contribute to the literature that aims to explain the observed time series of aggregate activity and inflation through the lens of general equilibrium models subjected to structural shocks. The seminal work by Smets and Wouters (2007) estimates a rich dynamic stochastic general equilibrium model with full-information Bayesian techniques, obtaining a decomposition of key macroeconomic aggregates in the United States in terms of aggregate structural shocks. While hugely influential, a key criticism of the approach points to the excessive importance of wage and price markup shocks in decompositions, while such disturbances have ambiguous microfoundations (Chari et al., 2009). Subsequent work by Galí et al. (2012) addresses the criticism by enriching the estimated model with involuntary unemployment. More recent papers consider multi-sector setups with networks, allowing for a role of industry-specific shocks in explaining aggregate fluctuations. In particular, Rubbo (2024) finds that in the US, sectoral shocks account for most of the deflation and subsequent inflation in the immediate aftermath of Covid, while aggregate factors explain most of the price surge in 2021 and beyond. In a multi-country study allowing for international spillovers, Di Giovanni et al. (2023) attribute inflation to 2020 to supply chain bottlenecks, while assigning a major role to both aggregate shocks and energy prices in the subsequent periods.

With our quantitative exercise for the Euro Area in the (post-)Covid era, we contribute by showing that one can explain both the surge in inflation *and* the rise in the aggregate adjustment frequency with four structural shocks: aggregate demand, aggregate labor wedge, as well as sectoral shocks to energy and food prices. Crucially, we find that network amplification of pricing cascades following the sector-specific commodity price shocks is critical for the quantitative fit.

Roadmap The remainder of the paper is structured as follows. Section 2 outlines the optimization problem faced by each type of agent in the economy and the numerical strategy to solve the equilibrium dynamics. Section 3 explains the key model mechanisms in a simplified version of our setup. Section 4 outlines our procedure for estimating the structural parameters of the model to match key sectoral micro-pricing moments for the Euro Area. Section 5 shows our quantitative results for monetary shocks. Section 6 turns to quantitative results for aggregate and sector-specific TFP shocks. Section 7 describes our empirical validation exercises, which assess the ability of our model to explain the aggregate dynamics of inflation and repricing frequency in the Euro Area. Section 8 concludes.

2 Model

We begin by introducing our theoretical model, which presents a combination of three key ingredients. First, it features a number of sectors populated by firms interconnected by production networks, which facilitate trade in intermediate inputs, both within and across sectors. Second, firms make optimal pricing decisions subject to menu costs. Third, we allow for aggregate, sector-specific, and firm-level shocks, and present a numerical strategy that allows us to compute the economy-wide equilibrium dynamic response to an arbitrarily large disturbance of any origin.

2.1 Overview

Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. The economy is populated by three (types of) agents: households, firms, and the government. There is a continuum of identical households, each consuming output and supplying labor. Firms are subdivided into N sectors, indexed by $i \in \{1, 2, ..., N\}$, each sector containing a continuum of monopolistically competitive firms of measure one; we use Φ_i to denote the set of all firms in sector i. The government consists of the central bank, which conducts policy by setting money supply, and the fiscal authority, which collects taxes from firms and rebates them to households in a lump-sum fashion.

2.2 Households

The representative household chooses a sequence of consumption, labor supply, and one-period nominal bond holdings to maximize expected lifetime utility:

$$\max_{\{C_t, L_t, B_{t+1}\}_{t \geqslant 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \tag{1}$$

subject to the period-by-period budget constraint

$$P_t^C C_t + \mathbb{E}_t\{\Lambda_{t,t+1} B_{t+1}\} \leqslant B_t + W_t L_t + \sum_{i=1}^{N} \int_0^1 D_{i,t}(j) dj + T_t, \tag{2}$$

where $0 \leqslant \beta \leqslant 1$ is the discount factor, C_t is consumption, L_t is labor supply, B_t is the level of nominal bond holdings, T_t is the level of lump-sum transfers from the government, $D_{i,t}(j)$ are the dividends received lump-sum from firm j in sector i, $\Pi_t^C = \left(P_t^C / P_{t-1}^C\right)$ is the gross CPI inflation rate, W_t is the nominal wage, and $\Lambda_{t,t+1}$ is the nominal stochastic discount factor of the household.

Total final consumption C_t is given by an aggregator over sector-specific varieties:

$$C_t = \mathcal{D}(C_{1,t}, ..., C_{N,t}) \tag{3}$$

where $\mathcal{D}(\cdot)$ is homogeneous of degree one and non-decreasing in each of the arguments. The household chooses consumption of each of the sector-specific varieties to minimize total expenditure $\sum_i P_{i,t} C_{i,t}$, subject to the aggregator in (3). The minimal cost of assembling such a basket of sectoral varieties aggregating to $C_t = 1$ pins down the consumption price index as $P_t^C = \mathcal{P}^C(P_{1,t}, ..., P_{N,t})$, where $\mathcal{P}^C$ is homogeneous of degree one and non-decreasing in each of the arguments.

Sectoral final consumption $C_{i,t}$ is in turn given by the following aggregator over firm-specific varieties:

$$C_{i,t} = \left\{ \int_0^1 \left[\zeta_{i,t}(j) C_{i,t}(j)\right]^{\frac{\epsilon-1}{\epsilon}} dj \right\}^{\frac{\epsilon}{\epsilon-1}}, \tag{4}$$

where $\epsilon > 1$ is the within-sector elasticity of substitution, $C_{i,t}(j)$ is the final demand for the output of firm $j \in [0,1]$ in sector i at time t, and $\zeta_{i,t}(j)$ is a firm-specific idiosyncratic *quality* process. The quality process follows a random walk in logs:

$$\log \zeta_{i,t}(j) = \log \zeta_{i,t-1}(j) + \sigma_i \varepsilon_{i,t}(j), \tag{5}$$

where $\varepsilon_{i,t}(j)$ is an *i.i.d.* Gaussian innovation with mean zero and standard deviation of one. The final demand for firm j in sector i is: $C_{i,t}(j) = \zeta_{i,t}(j)^{\epsilon-1}\left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\epsilon} C_{i,t}$, whereas the sectoral price index of sector i is given by: $P_{i,t} = \left[\int_0^1 \left(\frac{P_{i,t}(j)}{\zeta_{i,t}(j)}\right)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}$.

The representative household is also subject to a cash-in-advance constraint, which requires that the nominal money holdings are sufficient to cover the aggregate nominal final demand:

$$P_t^C C_t \leqslant M_t. \tag{6}$$

The aggregate money supply process $\{M_t\}_{t\geqslant 0}$ is set by the central bank, and agents treat this process as exogenous. An alternative is to consider a central bank which conducts monetary policy by setting the nominal interest rate according to a Taylor rule; we consider such an extension in Appendix D.1.

We now specify the functional forms for household preferences. First, for household preferences over aggregate consumption and labor supply, we use the log-linear preferences of Golosov and Lucas (2007):

Assumption 1 (Golosov-Lucas preferences)**.** *The utility function over consumption and labor supply is log-linear:* $u(C_t, L_t) = \log C_t - L_t$.

Under such preferences, we obtain the following intra-temporal labor supply condition: $\frac{W_t}{P_t^C} = C_t$. When combined with the cash-in-advance constraint (6), it implies that the nominal wage equals money supply in every period: $W_t = M_t$. In addition, the nominal stochastic discount factor satisfies: $\Lambda_{t,t+1} = \beta \frac{P_t^C C_t}{P_{t+1}^C C_{t+1}} = \beta \frac{M_t}{M_{t+1}}$.

As for the aggregation across final consumption of sectoral varieties, in the baseline model, we assume it takes the Cobb-Douglas form:

Assumption 2 (Consumption aggregation)**.** *The consumption aggregator $\mathcal{D}(\cdot)$ is given by:*

$$\mathcal{D}(C_{1,t}, ..., C_{N,t}) = \iota^C \prod_{i=1}^{N} C_{i,t}^{\varpi_i^C}, \tag{7}$$

where $\iota^C \equiv \prod_{i=1}^N \left(\varpi_i^C\right)^{-\varpi_i^C}$ *is a normalization term and* $\sum_i \varpi_i^C = 1$, $\varpi_i^C \geqslant 0, \forall i$.

Under this assumption, the equilibrium sectoral final consumption shares are constant over time: $\omega_{i,t}^C \equiv \frac{P_{i,t} C_{i,t}}{P_t^C C_t} = \varpi_i^C$. In Appendix D.3 we consider a more general CES aggregator over sectoral consumption varieties.

2.3 Firms: production

The production function of firm j in sector i is given by:

$$Y_{i,t}(j) = \frac{1}{\zeta_{i,t}(j)} \times A_{i,t} \times \mathcal{F}_i\left[L_{i,t}(j), X_{i,1,t}(j), ..., X_{i,N,t}(j)\right], \tag{8}$$

where $\mathcal{F}_i(\cdot)$ is homogeneous of degree one and non-decreasing in inputs; $L_{i,t}(j)$ is the labor used by firm j in sector i at time t, $X_{i,k,t}(j)$ represents intermediate inputs bought by firm j in sector i from sector k at time t. In addition, $A_{i,t}$ is an exogenous sector-specific total factor productivity process, while $\zeta_{i,t}(j)$ is the firm-level idiosyncratic quality process introduced in (5).

The intermediates demand $X_{i,k,t}(j)$ is in turn an aggregator over intermediates bought from each firm in sector k:

$$X_{i,k,t}(j) = \left\{\int_0^1 \left[\zeta_{k,t}(j')X_{i,k,t}(j,j')\right]^{\frac{\epsilon-1}{\epsilon}} dj'\right\}^{\frac{\epsilon}{\epsilon-1}}, \tag{9}$$

where $X_{i,k,t}(j,j')$ are intermediates bought by firm j in sector i from firm j' in sector k, which satisfy the following demand condition in equilibrium: $X_{i,k,t}(j,j') = \zeta_{k,t}(j')^{\epsilon-1}\left(\frac{P_{k,t}(j')}{P_{k,t}}\right)^{-\epsilon} X_{i,k,t}(j)$.

Each firm chooses its labor and intermediate inputs in order to minimize the total cost of production, subject to the production technology in (8). The latter delivers the following marginal cost function for firm j in sector i at time t:

$$MC_{i,t}(j) = \zeta_{i,t}(j) \times \mathcal{Q}_i(W_t, P_{1,t}, ..., P_{N,t}; A_{i,t}) \tag{10}$$

where $\mathcal{Q}_i(\cdot)$ is the common component of the marginal cost index for all firms within a sector, which strictly falls in $A_{i,t}$ and is homogeneous of degree one and non-decreasing in the prices of all inputs.

In our baseline model, we assume that production technology takes a Cobb-Douglas form for all firms in all sectors:

Assumption 3 (Production technology)**.** *The production technology $\mathcal{F}_i(\cdot)$ for a firm j in sector i is given by:*

$$\mathcal{F}_i[L_{i,t}(j), X_{i,1,t}(j), ..., X_{i,N,t}(j)] = \iota_i L_{i,t}(j)^{\overline{\alpha}_i} \prod_{k=1}^{N} X_{i,k,t}(j)^{\overline{\omega}_{ik}}, \tag{11}$$

where $\iota_i \equiv \overline{\alpha}_i^{-\overline{\alpha}_i} \prod_{k=1}^N \overline{\omega}_{ik}^{-\overline{\omega}_{ik}}$ is a normalization term and $\overline{\alpha}_i + \sum_k \overline{\omega}_{ik} = 1$, $\overline{\alpha}_i$, $\overline{\omega}_{ik} \geqslant$

$0, \forall i$.

Under this assumption, the equilibrium labor cost shares and the input-output cost shares are constant over time and the same for all firms within a sector: $\alpha_{i,t} \equiv \frac{W_t L_{i,t}(j)}{MC_{i,t}(j)Y_{i,t}(j)} = \overline{\alpha}_i, \quad \omega_{ik,t} \equiv \frac{P_{k,t}X_{i,k,t}(j)}{MC_{i,t}(j)Y_{i,t}(j)} = \overline{\omega}_{ik}$. As with household preferences, in Appendix D.3 we relax the Cobb-Douglas assumption and consider a more general CES production function.

2.4 Firms: equilibrium size

The goods market clearing condition for firm j in sector i is given by:

$$Y_{i,t}(j) = C_{i,t}(j) + \sum_{k=1}^{N} \int_0^1 X_{k,i,t}(j', j)dj'. \tag{12}$$

Aggregating up to the level of sectors, multiplying both sides by $P_{i,t}$ and dividing by aggregate final nominal demand $P_t^C C_t$, one can express the sectoral sales share (Domar weight) $\lambda_i \equiv \frac{P_{i,t}Y_{i,t}}{P_t^C C_t}$ as:

$$\lambda_{i,t} = \omega_{i,t}^C + \sum_{k=1}^{N} \omega_{ki,t}\lambda_{k,t} \times \frac{\Delta_{k,t}}{\mathcal{M}_{k,t}}, \tag{13}$$

where $\Delta_{k,t}$ is a measure of price dispersion within the sector and $\mathcal{M}_{k,t}$ is a measure of sectoral markup, defined as:

$$\Delta_{k,t} \equiv (P_{k,t}/M_t)^{\epsilon} \int_0^1 \left(\frac{P_{k,t}(j')}{\zeta_{k,t}(j')M_t}\right)^{-\epsilon} dj', \qquad \mathcal{M}_{k,t} \equiv \frac{P_{k,t}}{\mathcal{Q}_{k,t}}. \tag{14}$$

Stacking the equation for sales shares across sectors, we can write it as:

$$\boldsymbol{\lambda}_t = \boldsymbol{\omega}_{C,t} + \tilde{\Omega}_t^T \boldsymbol{\lambda}_t \quad \Longrightarrow \quad \boldsymbol{\lambda}_t = (I - \tilde{\Omega}_t^T)^{-1}\boldsymbol{\omega}_{C,t} \tag{15}$$

where $\tilde{\Omega}_t$ is a $N \times N$ matrix whose $[i, j]$ entry is given by $[\tilde{\Omega}_t]_{i,j} = \omega_{ij,t}\left\{\frac{\Delta_{i,t}}{\mathcal{M}_{i,t}}\right\}$. Having calculated the sectoral sales shares, one obtains the sectoral total output as $Y_{i,t} = \lambda_{i,t} \times M_t/P_{i,t}$ and then the size of an individual firm as $Y_{i,t}(j) = \zeta_{i,t}(j)^{\epsilon-1}\left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\epsilon} Y_{i,t}$.

2.5 Firms: pricing

The nominal profit of firm j in sector i at time t is given by:

$$D_{i,t}(j) = [(1 - \tau_i)P_{i,t}(j) - MC_{i,t}(j)] \times Y_{i,t}(j), \tag{16}$$

where τ_i is an exogenous sector-specific sales tax levied by the government.[4] Denoting by $\tilde{P}_{i,t}(j) \equiv \frac{P_{i,t}(j)}{\zeta_{i,t}(j)M_t}$ the firm's quality-adjusted *real* price and by $\tilde{P}_{i,t} \equiv \frac{P_{i,t}}{M_t}$ the sectoral *real* price index, we can write the firm-level *real* profits $\tilde{D}_{i,t}(j) \equiv \frac{D_{i,t}(j)}{M_t}$ as:

$$\begin{aligned}\tilde{D}_{i,t}(j) &= \left(\frac{P_{i,t}}{M_t}\right)^{\epsilon-1} \times \left[(1-\tau_i)\frac{P_{i,t}(j)}{\zeta_{i,t}(j)M_t} - \frac{\mathcal{Q}_{i,t}}{M_t}\right] \times \left(\frac{P_{i,t}(j)}{\zeta_{i,t}(j)M_t}\right)^{-\epsilon} \times \lambda_{i,t} \\ &= \tilde{D}\left(\tilde{P}_{i,t}(j), \{\tilde{P}_{k,t}, \Delta_{k,t}, A_{k,t}\}_{k=1}^{N}\right). \end{aligned} \tag{17}$$

Note that keeping track of the firm-level real profits requires knowing the firm's real quality-adjusted price, as well as the real sectoral prices, price dispersions, and productivities of all sectors in the economy.

Resetting the *nominal* price $P_{i,t}(j)$ involves the firm paying a sector-specific and possibly time-varying menu cost $\kappa_{i,t}$ measured in units of labor. The optimal reset price maximizes the firm's value, taking into account that this new price may not change for some period of time. In particular, when the nominal price does not change, the log of the quality-adjusted real price $p_{i,t}(j) \equiv \log \tilde{P}_{i,t}(j)$ evolves according to

$$p_{i,t}(j) = p_{i,t-1}(j) + \log\left(\frac{P_{i,t-1}(j)}{\zeta_{i,t}(j)M_t}\right) - \log\left(\frac{P_{i,t-1}(j)}{\zeta_{i,t-1}(j)M_{t-1}}\right) = p_{i,t-1}(j) - \sigma_i\varepsilon_{i,t} - m_t, \tag{18}$$

where $m_t \equiv \Delta \log M_t$.

Without loss of generality, let $\eta_{i,t}(p)$ denote the probability that a firm in sector i with a quality adjusted log relative price p resets its price at t. Consider a firm with a real quality adjusted price p at the end of period t, and let $p_+ \equiv (p - \sigma_i\varepsilon_{i,t+1}(j) - m_{t+1})$. Then this firm's real value at the end of period t is given by the following Bellman equation:

$$\begin{aligned} V_{i,t}(p) \quad &= \quad \tilde{D}_{i,t}(p) + \\ &+ \beta\mathbb{E}_t\left[\{1-\eta_{i,t+1}(p_+)\} V_{i,t+1}(p_+) + \eta_{i,t+1}(p_+)\left(\max_{p'} V_{i,t+1}(p') - \kappa_{i,t+1}\right)\right] \end{aligned} \tag{19}$$

which consists of the current period real profits $\tilde{D}_{i,t}(p)$, as well as the discounted expected continuation value. The latter is computed by taking into account that at time $t+1$ the nominal price does not change with probability $1 - \eta_{i,t+1}(\cdot)$, whereas with probability $\eta_{i,t+1}(\cdot)$ the firm pays the menu cost and optimally resets the nominal price.

Our formulation of the pricing problem covers a wide range of existing models of

[4] The proceeds of these taxes are then rebated to households as a lump-sum transfer $T_t = \sum_{i=1}^{N} \tau_i \int_0^1 P_{i,t}(j)Y_{i,t}(j)dj$.

price setting, corresponding to the different functional forms of $\eta_{i,t}(\cdot)$. In the baseline setup of our model, we consider a specific functional form for the adjustment probability function $\eta_{i,t}(\cdot)$. In particular, following Golosov and Lucas (2007), we assume that a firm adjusts if and only if the value gain from adjustment in a given period exceeds the menu cost:

Assumption 4 (*Ss* pricing)**.** *Consider a firm in sector i with the quality adjusted log relative price p at time t. Then the probability that this firm adjusts its nominal price is given by:*

$$\eta_{i,t}(p) = \mathbf{1}(\mathcal{L}_{i,t}(p) > 0) \tag{20}$$

where $\mathbf{1}(\cdot)$ is the indicator function, and

$$\mathcal{L}_{i,t}(p) = \max_{p'} V_{i,t}(p') - V_{i,t}(p) - \kappa_{i,t} \tag{21}$$

is the gain from adjustment (or loss from inaction), net of the menu cost.

In our quantitative exercises, we focus on the case with fixed sector-specific menu costs ($\kappa_{i,t} = \overline{\kappa}_i$). However, in Appendix D.2 we extend our analysis to sector-specific stochastic menu costs.

Note that although here we specify a problem of *price* setting under nominal rigidities, our setup can automatically handle rigidities in nominal *wage* setting as well by appropriately parameterizing the input-output structure. In particular, consider a setup with a sector called the labor union (LU), such that it only uses labor in production $(\overline{\alpha}_{LU} = 1)$ and moreover it is the only sector purchasing labor directly from households $(\overline{\alpha}_{-LU} = 0)$. Instead, other sectors purchase labor indirectly from the labor union as an intermediate input, such that $\overline{\omega}_{i,LU}$ represents the empirical cost share of labor for sector i. Then any rigidities in the price setting of the labor union sector are isomorphic to nominal wage rigidities. Moreover, the gap between the nominal wage W_t and the price index of the labor union sector $P_{LU,t}$ has a natural interpretation as the aggregate *labor wedge*.[5]

2.6 Equilibrium definition and solution method

In addition to the goods market clearing condition in (12), the equilibrium in our economy is also characterized by clearing of the labor market:

$$L_t = \sum_{i=1}^{N} \int_0^1 L_{i,t}(j)dj + \sum_{i=1}^{N} \kappa_{i,t} \int_0^1 \eta_{i,t}(p_{i,t}(j))dj, \tag{22}$$

[5]More formally, the labor wedge is the gap between the nominal marginal rate of substitution across consumption and labor $P_t^C \times MRS_t^{CL} = W_t$ and the nominal cost of labor faced by firms $P_{LU,t}$.

as well as by clearing in the market for bonds, which are in zero net supply: $B_t = 0$.

Having specified the optimality and market clearing conditions, we can now formally define the decentralized equilibrium in our economy:

Definition 1 (Equilibrium)**.** *The equilibrium is a collection of prices* $\{P_{i,t}(j)|j \in \Phi_i\}_{i=1}^N$, *allocations* $\left\{Y_{i,t}(j), L_{i,t}(j), C_{i,t}(j), \{X_{i,r,t}(j,j')|j' \in \Phi_r\}_{r=1}^N |j \in \Phi_i\right\}_{i=1}^N$, *wage* W_t *and bond holdings* B_t, *which, given the realizations of firm-level quality* $\{\zeta_{i,t}(j)|j \in \Phi_i\}_{i=1}^N$, *sectoral productivities* $\{A_{i,t}\}_{i=1}^N$ *and money supply* M_t, *satisfy agent optimization and market clearing conditions in every period.*

We compute fully non-linear transitions under the assumption of perfect foresight over aggregate and sectoral shocks, while maintaining uncertainty over the idiosyncratic innovations. Full details of the numerical strategy are given in Appendix C.

3 Pricing cascades and networks: formal results

We now use a simplified version of our model in order to formally introduce the notion of pricing *cascades*: large movements in aggregates that create possibly self-reinforcing price adjustment decisions at the extensive margin. Moreover, we present analytical results regarding the interaction of pricing cascades with networks. In particular, we formally show that networks *dampen* cascades whenever the aggregate cycle is driven by demand shocks, whereas they *amplify* cascades driven by supply shocks. Our analytical results pin down the appropriate network centrality metrics that determine the degree of dampening or amplification. We also present several examples with particular network arrangements in order to solidify the intuition behind our theoretical results.

3.1 Notation

Before presenting the formal results, let us introduce some additional notation to be used in this section. First, for a matrix X, let $X^{(i)}$ denote a vector corresponding to the i-th row of the matrix; similarly, let $X_{(i)}$ denote a vector corresponding to the i-th column of the matrix. Second, for two N-dimensional vectors $\boldsymbol{x}$ and $\boldsymbol{y}$, let the cross-sectional variance and covariances be $Var(\boldsymbol{x}) \equiv \frac{1}{N}\sum_{i=1}^N (x_i - \overline{x})^2$ and $Cov(\boldsymbol{x}, \boldsymbol{y}) \equiv \frac{1}{N}\sum_{i=1}^N (x_i - \overline{x})(y_i - \overline{y})$, where $\overline{x} \equiv \frac{1}{N}\sum_{i=1}^N x_i, \overline{y} \equiv \frac{1}{N}\sum_{i=1}^N y_i$.

3.2 Static economy

In order to obtain the intuition regarding the transmission of large shocks in our model, we consider a simplified setup based on several additional assumptions. First, we assume the economy to be static in the sense that agents fully discount the future:

Assumption 5 (Myopia). *Agents fully discount the future in their objective function:* $\beta = 0$.

This is a simplifying assumption that we do not impose in the full quantitative analysis. However, it helps us make analytic progress by making each firm's value function equal to its contemporaneous profits, implying that the optimal quality-adjusted real reset price for any firm in sector i is given by:

$$\tilde{P}^*_{i,t} = \frac{1}{1-\tau_i}\frac{\epsilon}{\epsilon-1} \times \frac{1}{A_{i,t}}\prod_{k=1}^{N}\tilde{P}_{k,t}^{\varpi_{ik}} = \Gamma_i \times \tilde{\mathcal{Q}}_{i,t} \tag{23}$$

where $\Gamma_i \equiv \frac{1}{1-\tau_i}\frac{\epsilon}{\epsilon-1}$ is the (exogenous) desired markup.

The decision to change prices is based on whether the value gain from adjustment exceeds the menu cost. In the static setup, we can obtain a tractable approximation for the gain from adjustment as a function of the *price gap*, or the difference between the current and the optimal reset price:

Lemma 1 (Adjustment gains). *Suppose Assumptions 1-5 hold. Let* $\tilde{p}_{i,t}(j) \equiv \log \tilde{P}_{i,t}(j) - \log \tilde{P}^*_{i,t}$ *be the price gap for a firm* j *in sector* i *at time* t. *Then the profit gain from price adjustment satisfies:*

$$\tilde{D}^*_{i,t}(j) - \tilde{D}_{i,t}(j) \quad = \quad \frac{1}{2}(\epsilon-1)(1-\tau_i)[\tilde{P}_{i,t}/\tilde{P}^*_{i,t}]^{\epsilon-1}\lambda_{i,t} \times [\tilde{p}_{i,t}(j)]^2 \quad + \quad \mathcal{O}\left(|\tilde{p}_{i,t}(j)|^3\right) \tag{24}$$

where $\tilde{D}^*_{i,t}(j)$ *is profits at the optimal reset price,* $\tilde{P}_{i,t}$ *is the real sectoral price index and* $\lambda_{i,t}$ *is the sectoral sales share (Domar weight).*

To illustrate the interaction between networks and price adjustment decisions, consider the initial period ($t = 0$) in our economy. Unless specified otherwise, for notational convenience we drop the time subscript for all the variables at $t = 0$. If the firm chooses not to adjust its nominal price, then the quality-adjusted real price in the initial period is given by:

$$\log \tilde{P}_i(j) = p_{i,-1}(j) - \sigma_i\varepsilon_i(j) - m \tag{25}$$

where $p_{i,-1}(j)$ is the initial (exogenous) quality-adjusted real price of firm j in sector i, $\varepsilon_i(j)$ is the realization of the firm-level quality shock in period $t = 0$, and $m \equiv \log(M/M_{-1})$ is the realization of money growth at $t = 0$. Given the expression for the optimal reset price in (23), we can write the firm-level price gap in the initial period as:

$$\tilde{p}_i(j) = -\sigma_i\varepsilon_i(j) - m + a_i - \sum_{k=1}^{N}\varpi_{ik}\log\tilde{P}_k + (p_{i,-1}(j) - \gamma_i) \tag{26}$$

where $\gamma_i \equiv \log\Gamma_i = \log\frac{\epsilon}{\epsilon-1}\frac{1}{1-\tau_i}$ and $a_i \equiv \log A_i$.

Without loss of generality, we treat the monetary shock m and productivity shocks $\{a_i\}_{i=1}^N$ as i.i.d. mean zero. In addition, we make the following assumption regarding the initial exogenous quality-adjusted real price:

Assumption 6 (Initial prices)**.** *The initial exogenous quality-adjusted real price of a firm j in sector i is given by:* $\quad p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\tau_i}, \quad \forall i, \forall j \in \Phi_i.$

We also assume a specific form of time variation in the sector-specific menu cost $\kappa_{i,t}$:

Assumption 7 (Sectoral menu costs)**.** *The sector-specific menu cost is given by the process: $\kappa_{i,t} = \overline{\kappa}_i(1-\tau_i)[\tilde{P}_{i,t}/\tilde{P}^*_{i,t}]^{\epsilon-1}\lambda_{i,t}$, where $\overline{\kappa}_i$ is a sector-specific constant.*

Importantly, we do not impose Assumption 6 or 7 in the full quantitative analysis, using them for closed-form results in this section only. In particular, Assumption 7 permits analytic tractability by removing the need to keep track of the width of the inaction region for adjustment decisions, allowing us to exclusively focus on its location instead.

Given the assumption regarding initial quality-adjusted prices, as well as the realizations of aggregate and sectoral variables, the magnitude of the price gap of a specific firm is pinned down by the realization of its idiosyncratic quality innovation $\varepsilon_i(j)$. We can use the approximate profit gain in Lemma 1 to determine the sector-specific *inaction regions*, defining the ranges for idiosyncratic innovations under which the firm will choose not to adjust:

Lemma 2 (Inaction region)**.** *Suppose Assumptions 1-7 hold. Given the realizations of aggregate and sectoral variables, let $\underline{\varepsilon}_i$ and $\overline{\varepsilon}_i$ be thresholds such that a firm in sector i will not adjust the price if it draws an innovation in $[\underline{\varepsilon}_i, \overline{\varepsilon}_i]$. Then,*

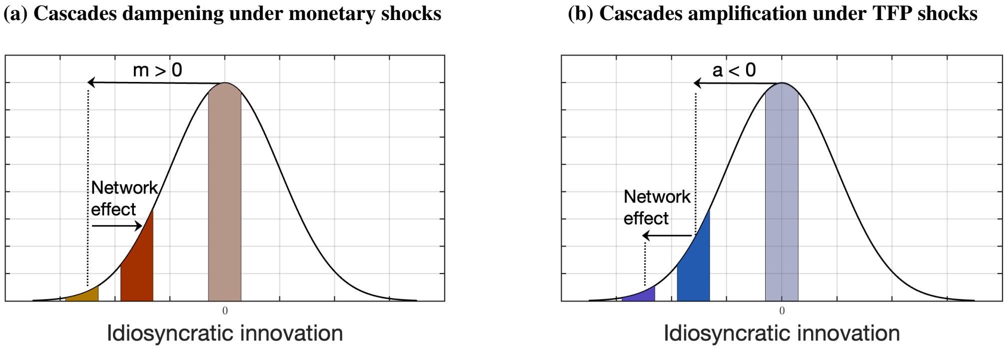
$$\sigma_i[\underline{\varepsilon}_i, \quad \overline{\varepsilon}_i] \quad = \quad -m + a_i - \sum_{k=1}^{N} \overline{\omega}_{ik} \log \tilde{P}_k \quad \pm \quad \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}, \tag{27}$$

where $\quad m \equiv \log(M/M_{-1}) \quad$ *and* $\quad a_i \equiv \log A_i.$

Given the realizations of monetary and productivity shocks, which are independent of the presence of input-output linkages, we can now derive the effect of removing input-output linkages ($\overline{\omega}_{i,k} = 0, \forall i, k$) on the firm-level decision to adjust its price following different types of aggregate/sectoral shocks.

Monetary shocks Consider an increase in the money supply $m > 0$. According to Lemma 2, this increase in money supply, *ceteris paribus*, implies a leftward shift of

Figure 1: **Network effect on the inaction region under monetary and TFP shocks**



Notes: the figure visualizes properties of the firm-level inaction region for price adjustment decisions, derived in Lemma 2. Panel (a) considers a monetary expansion $m > 0$: relative to the economy without input-output linkages ($\varpi_{ij} = 0, \forall i, j$), the presence of production networks attenuates the leftward shift in the inaction region, lowering the probability of price adjustment, *ceteris paribus*. Panel (b) considers an aggregate TFP contraction $a_i = a < 0, \forall i$: relative to the economy without input-output linkages ($\varpi_{ij} = 0, \forall i, j$), the presence of production networks amplifies the leftward shift in the inaction region, increasing the probability of price adjustment, *ceteris paribus*.

the inaction region. In other words, more extreme (negative) realizations of idiosyncratic innovations are needed to prevent adjustment. At the same time, Lemma 2 also implies that as long as the pass-through of the money supply to sectoral prices is incomplete ($\log \tilde{P}_k < 0, \forall k$), the presence of networks attenuates the leftward shift of the inaction region for all firms that have a non-zero cost share of intermediate inputs. As a result, this weakly lowers the probability of price adjustment for any firm, creating *dampening* in price changes. Panel (a) of Figure 1 provides a graphical illustration of this mechanism.

To formalize such interaction of networks and the extensive margin under monetary shocks, the following proposition links the firm-level probability of adjustment to the monetary shock size and input-output characteristics:

Proposition 1. *Suppose Assumptions 1-7 hold. Let ϱ_i be the probability that a firm in sector i decides to adjust its price. Following a monetary shock m, denote by $\Delta\varrho_i(m)$ the change relative to steady-state. Then:*

$$\frac{1}{\chi_i}\Delta\varrho_i(m) \approx \left[\quad m \quad + \quad \overline{\mu} \times \mathcal{C}_i \quad + \quad N \times Cov\left((\overline{\Psi} - I)^{(i)}, \boldsymbol{\mu}\right) \quad \right]^2 \tag{28}$$

where $\chi_i \equiv -\Xi_i''\left(\sqrt{\frac{2\kappa_i}{\epsilon-1}}\right) > 0$ *and* Ξ_i *is CDF of* $\mathcal{N}(0, \sigma_i^2)$, $\mathcal{M}_i$ *is the sectoral markup and* $\overline{\mu} \equiv \frac{1}{N}\sum_{i=1}^{N} \log \mathcal{M}_i$, $\boldsymbol{\mu} \equiv [\log \mathcal{M}_1, ..., \log \mathcal{M}_N]^T$, $\overline{\Psi} \equiv (I - \overline{\Omega})^{-1}$ *is the Leontief inverse matrix, and*

$$\mathcal{C}_i \equiv \sum_{j=1}^{N} \overline{\Psi}_{i,j} - 1 \tag{29}$$

*is the **customer centrality** of sector i.*

It follows that the firm-level probability of adjustment is a square function of the sum of three terms. First, the value of the monetary shock m, whose increases or decreases always lead to a rise in the adjustment probability. Second, the *customer centrality* of the sector to which the firm belongs, multiplied by the average sectoral (log) markup.[6] Third, the covariance between the total exposure to other sectors as a customer and the markups in the supplier sectors.

One can see that as long as the input prices rise less than one-for-one with money supply, which is consistent with countercyclical sectoral markups, networks lead to a reduction in firm-level adjustment probability, *ceteris paribus*. Moreover, this dampening is stronger for firms in sectors with a higher *customer centrality*, implying a higher exposure to intermediates, as well as for those that are more exposed to inputs from sectors with more compressed markups, as captured by the covariance term.

The fact that the individual adjustment probability is approximately a squared function of the sectoral *customer centrality*, *and* that it depends on the relative exposure to sectors with more compressed markups, implies that heterogeneity and asymmetry in the input-output structure generally matter for the behavior of aggregates. As a result, in general, the cascades effects cannot be fully captured by a (symmetric) roundabout production setup (Basu, 1995; Nakamura and Steinsson, 2010) even conditional on the same overall importance of intermediate inputs.

We are now ready to formalize the notion of *cascades dampening*:

Corollary 1 (Cascades dampening)**.** *Suppose Assumptions 1-7 hold. Consider a monetary shock $m > 0$. Then, for any firm in any sector $i : \overline{\alpha}_i \in (0,1)$, production networks lower the probability of adjustment as long as sectoral markups satisfy* $-m < \sum_{j=1}^{N}(\overline{\Psi}_{ij} - \mathbf{1}_{i=j})\mu_j < 0$.

The restriction on the behavior of markups is intuitive: on the one hand, they need to be countercyclical, meaning they turn negative in logs following a monetary expansion; on the other hand, they cannot be too negative, ruling out situations where the nominal sectoral marginal costs fall after the monetary expansion. While Corollary 1 is stated for a monetary expansion $(m > 0)$, the same dampening effect holds for a monetary contraction $(m < 0)$ with the signs of the inequality restrictions on markups reversed $\left(-m > \sum_{j=1}^{N}(\overline{\Psi}_{ij} - \mathbf{1}_{i=j})\mu_j > 0\right)$. In words, following a monetary contraction, cascades dampening requires that markups rise but not by too much, so as to rule out nominal sectoral marginal costs rising.

[6] Recall we set the sectoral taxes $\{\tau_i\}_{i=1}^{N}$ such that all the sectoral price indices are equal to one in steady state. As a result, all the sectoral markups are also equal to one in steady state, or zero in logs. Hence $\mu_i \equiv \log \mathcal{M}_i$ represents the log deviation of sectoral markup from its steady-state value.

Note that while the cascades dampening result above is proved under flexible wages, it extends to an economy with sticky wages modeled through the introduction of an auxiliary labor union sector. Corollary 1′ in Appendix A proves this formally.

Productivity shocks In contrast, consider a sectoral TFP deterioration $a_i < 0$. According to Lemma 2, this productivity change creates a leftward shift of the inaction region for all firms in sector i. Moreover, as long as this productivity decline leads to a rise in price indices of other sectors ($\log \tilde{P}_k > 0, \forall k$). Then Lemma 2 also implies that networks further amplify the leftward shift in the inaction region for all firms in sector i, as long as the cost share of intermediates in that sector is non-zero. In other words, even more extreme (negative) realizations of idiosyncratic innovations are needed to justify non-adjustment. As a result, contrary to the case of monetary shocks, the presence of networks weakly raises the probability of price adjustment for any firm in sector i, thus *amplifying* pricing cascades. Panel (b) of Figure 1 illustrates this mechanism graphically.

We formalize this interaction between the firm-level adjustment probability and supply shocks below, generalizing it to the effect of any combination of sectoral productivity changes:

Proposition 2. *Suppose Assumptions 1-7 hold. Let ϱ_i be the probability that a firm in sector i decides to adjust its price. Following a combination of sectoral productivity shocks $\boldsymbol{a} \equiv \{a_j\}_{j=1}^N$, denote by $\Delta\varrho_i(\boldsymbol{a})$ the change relative to steady-state, then:*

$$\frac{1}{\chi_i}\Delta\varrho_i(\boldsymbol{a}) \quad \approx \quad \left[\sum_{j=1}^{N} \overline{\Psi}_{ij} a_j \quad - \quad \overline{\mu} \times \mathcal{C}_i \quad - \quad N \times Cov\left((\overline{\Psi} - I)^{(i)}, \boldsymbol{\mu}\right)\right]^2 \tag{30}$$

where $\chi_i \equiv -\Xi_i''\left(\sqrt{\frac{2\kappa_i}{\epsilon-1}}\right) > 0$ and Ξ_i is CDF of $\mathcal{N}(0, \sigma_i^2)$, $\mathcal{M}_i$ is the sectoral markup and $\overline{\mu} \equiv \frac{1}{N}\sum_{i=1}^N \log \mathcal{M}_i$, $\boldsymbol{\mu} \equiv [\log \mathcal{M}_1, ..., \log \mathcal{M}_N]^T$, $\overline{\Psi} \equiv (I - \overline{\Omega})^{-1}$ is the Leontief inverse matrix, and $\mathcal{C}_i$ is the customer centrality measure introduced in (29).

It follows that the probability a firm in sector i adjusts is a square function of the sum of three terms. First, the value of each sectoral TFP shock a_j, weighted by the Leontief inverse entry $\overline{\Psi}_{ij}$, measuring the extent to which sector j acts as a supplier to sector i, and hence influences its marginal cost and the reset price. Second, the *customer centrality* of sector i, multiplied by the negative of the average sectoral markup change. Third, the covariance between the total exposure to other sectors as a customer and the markups in the supplier sectors. Therefore, as long as all sectoral prices increase after the TFP contractions, the presence of networks leads to an increase in firm-level adjustment probability, *ceteris paribus*.

A notable special case is that of an aggregate TFP shock $a_j = a, \forall j$:

$$\frac{1}{\chi_i}\Delta\varrho_i(a) \quad \approx \quad \left[a \quad + \quad (a-\overline{\mu})\times\mathcal{C}_i \quad - \quad N\times Cov\left((\overline{\Psi}-I)^{(i)},\boldsymbol{\mu}\right)\right]^2. \quad (31)$$

One can see that following an aggregate TFP contraction $a < 0$, as long as all sectoral prices increase, networks amplify the rise in adjustment frequency, and more so for sectors with a larger *customer centrality*, as well as for those relatively more reliant on intermediate inputs from sectors which pass through more of the aggregate TFP contraction to prices (leading to smaller markup compressions).

Overall, we formalize the notion of *cascades amplification* below:

Corollary 2 (Cascades amplification). *Suppose Assumptions 1-7 hold. Consider a combination of sectoral TFP shocks $a_j \leqslant 0, \forall j$ with at least one strict inequality. Then, for any firm in any sector $i : \overline{\alpha}_i \in (0,1)$, production networks increase the probability of adjustment as long as sectoral markups satisfy* $\sum_{j=1}^{N}(\overline{\Psi}_{ij} - \mathbf{1}_{i=j})(a_j - \mu_j) < 0$.

As before, the restriction on the behavior of markups is intuitive: after the negative TFP shocks, markups can fall, but not to the extent that leads to reductions in nominal sectoral marginal costs.Also, while Corollary 2 is stated for contractionary shocks $(a_j \leqslant 0, \forall j)$, the same amplification result holds for a combination of expansionary TFP shocks $(a_j \geqslant 0, \forall j)$, with the sign of the inequality restriction on markups reversed $\left(\sum_{j=1}^{N}(\overline{\Psi}_{ij} - \mathbf{1}_{i=j})(a_j - \mu_j) > 0\right)$. In words, following expansionary TFP shocks, cascades amplification requires that markups rise, but not to the extent that nominal sectoral marginal costs rise as well.

It follows that a productivity shock to a sector k can, in principle, increase the probability of price adjustment for firms in any other sector i. This is true as long as the price indices of sectors used as suppliers by sector i $(j : \overline{\omega}_{ij} > 0)$ rise following the productivity deterioration. As a result, a sectoral productivity shock can have an effect on average economy-wide probability of adjustment. The proposition below formalizes the sectoral characteristics which pin down such a notion of systemic importance:

Proposition 3. *Suppose Assumptions 1-7 hold. Further, set $\overline{\kappa}_i = \overline{\kappa}, \sigma_i = \sigma, \forall i$ and assume $Cov(\overline{\Psi}_{(i)}, \boldsymbol{\mathcal{C}}) = 0, \forall i$. Let $\varrho \equiv \frac{1}{N}\sum_{i=1}^{N}\varrho_i$ be the average probability of adjustment. Following a TFP shock specific to sector k, a_k, denote by $\Delta\varrho(a_k)$ the change in the average adjustment probability relative to its steady-state value and assume that $Cov\left((\overline{\Psi}-I)^{(i)},\boldsymbol{\mu}\right) = 0, \forall i$, then:*

$$\frac{1}{\chi}\Delta\varrho(a_k) \quad \approx \quad \mathcal{H}_k \times a_k^2 \quad - \quad 2\overline{\mu}\times\overline{\mathcal{C}}\times\mathcal{S}_k\times a_k \quad + \quad \overline{\mu}^2\overline{\mathcal{C}^2} \qquad (32)$$

where $\chi \equiv -\Xi''\left(\sqrt{\frac{2\overline{\kappa}}{\epsilon-1}}\right) > 0$ and Ξ is CDF of $\mathcal{N}(0,\sigma^2)$, $\mathcal{M}_i$ is the sectoral markup *and* $\overline{\mu} \equiv \frac{1}{N}\sum_{i=1}^{N}\log\mathcal{M}_i$, $\boldsymbol{\mu} \equiv [\log\mathcal{M}_1, ..., \log\mathcal{M}_N]^T$, $\mathcal{C}_i$ *is the customer centrality introduced in (29),* $\overline{\mathcal{C}} \equiv \frac{1}{N}\sum_{i=1}^{N}\mathcal{C}_i$, $\overline{\mathcal{C}^2} \equiv \frac{1}{N}\sum_{i=1}^{N}\mathcal{C}_i^2$, $\boldsymbol{\mathcal{C}} \equiv [\mathcal{C}_1, ..., \mathcal{C}_N]^T$, *and*

$$\mathcal{H}_k \quad \equiv \quad \frac{1}{N}\sum_{i=1}^{N}\overline{\Psi}_{i,k}^2, \qquad\qquad \mathcal{S}_k \quad \equiv \quad \frac{1}{N}\sum_{i=1}^{N}\overline{\Psi}_{i,k} \tag{33}$$

are, respectively, the ***supplier Herfindahl*** $(\mathcal{H}_k)$ *and* ***supplier centrality*** $(\mathcal{S}_k)$ *of sector* k*.*

A notable case arises in the high pass-through environment, where $\overline{\mu} \approx 0$, so that:

$$\frac{1}{\chi}\Delta\varrho(a_k) \quad \approx \quad \mathcal{H}_k \times a_k^2. \tag{34}$$

It follows that a key metric that pins down the extent to which a TFP shock to a specific sector can affect the average economy-wide adjustment probability is that sector's *supplier Herfindahl*. In order to understand what makes a sector have a large *supplier Herfindahl*, it is convenient to re-express it as follows:

$$\mathcal{H}_k \quad = \quad \mathcal{S}_k^2 \quad + \quad Var\left(\overline{\Psi}_{(k)}\right). \tag{35}$$

One can see that the *supplier Herfindahl* is given by the sum of the squared *supplier centrality* and the variance of the column of the Leontief inverse corresponding to that sector. The first term, $\mathcal{S}_k^2$, measures the *average* importance of sector k as a direct or indirect supplier to the rest of the economy. The second term, $Var\left(\overline{\Psi}_{(k)}\right)$ measures the dispersion in the degree of direct and indirect exposure of other sectors to inputs bought from sector k. In particular, for a given degree of *average* importance as a supplier, the *supplier Herfindahl* rises for sectors that act as disproportionately important suppliers to some parts of the economy, leading to a high variance term.

3.3 Simple examples

We now solidify the intuition behind the formal results on cascades with the aid of several examples. In particular, we return to the dynamic version of our model, but consider concrete network arrangements in order to highlight the key mechanisms. To facilitate further comparability between monetary and TFP shocks, for the remainder of this subsection we assume that both follow AR(1) in logs with persistence $\rho = 0.9$.[7]

[7]For the simple examples, we also make the more conventional assumption of a fixed sector-specific menu cost, so that $\kappa_{i,t} = \overline{\kappa}_i$.

***Example 1*: roundabout production economy**

First, we consider a one-sector ($N = 1$) roundabout economy, where firms trade intermediate inputs with other firms in the same sector, as in the work of Basu (1995). Figure 2(a) illustrates such an arrangement graphically. Naturally, in the limit where we set the cost share of labor to one ($\overline{\alpha}_1 = 1$), the one-sector economy collapses to that of Golosov and Lucas (2007), where firms only use labor in production.

We use this simple example to illustrate how the presence of the network affects the (impact) response of the aggregate fraction of adjusting firms to monetary and productivity shocks of different sizes. As can be seen in the bottom panel of Figure 2(a), when there are no networks ($\overline{\alpha}_1 = 1$), monetary and productivity shocks are isomorphic in their effect on aggregate frequency. However, as soon as we add the roundabout production structure ($\overline{\alpha}_1 = 0.4$), the aggregate adjustment frequency responds much faster to productivity shocks relative to monetary shocks. This is because under monetary shocks, the network structure shrinks the desired price changes and the price gaps, thus *dampening* cascades, which leads to slower increases in the aggregate fraction of adjusters. In contrast, under TFP shocks, the presence of networks expands movements in desired price changes and hence the price gaps, thus *amplifying* cascades at the firm-level, leading to faster increases in the aggregate fraction of adjusters.

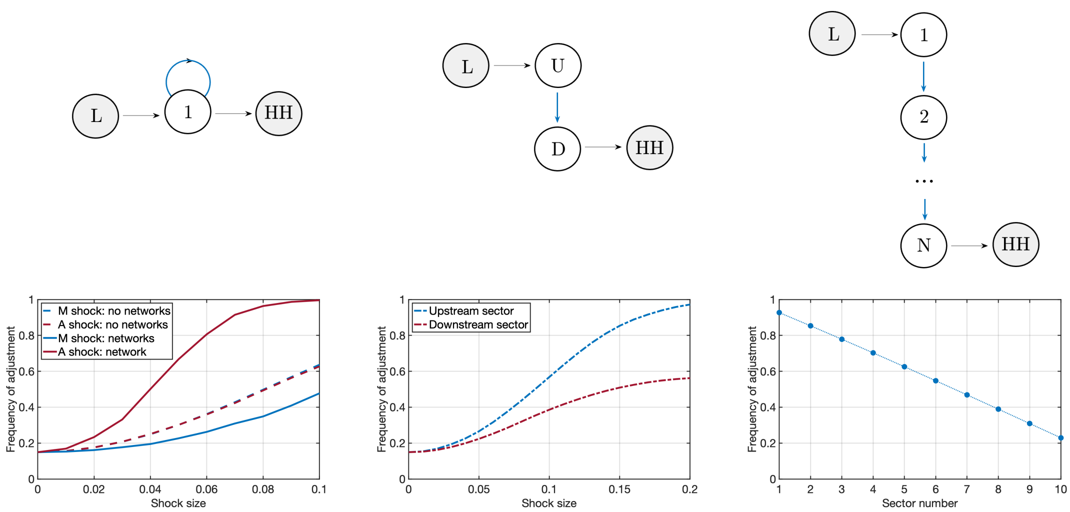
***Example 2*: two-sector vertical chain economy**

For our second example, we consider a two-sector economy ($N = 2$), which illustrates how the position of a sector in the network affects the transmission of sectoral productivity shocks to aggregate frequency. The top panel of Figure 2(b) presents the arrangement graphically: the upstream sector (U) only uses labor in production and supplies its output as an intermediate input to the downstream sector (D). Importantly, the two sectors have the same size in steady-state equilibrium, in the sense of having identical (cost-based) Domar weights. Moreover, their pricing moments are also the same in steady-state, hence their only ex ante difference comes from the position in the network.

We now consider sector-specific productivity shocks of different sizes and record their effect on the aggregate adjustment frequency. In the bottom panel of Figure 2(b), one can see that large shocks to the upstream sector deliver faster increases in the aggregate frequency, relative to equally sized shocks to the downstream sector. This is another way in which networks amplify cascades: shocks to the upstream sector affect the marginal cost, the optimal reset price, and hence the price gaps, of the downstream sector. As a result, shocks to the upstream sector trigger extensive margin price adjustment decisions both in the upstream *and* in the downstream sector. The opposite,

Figure 2: **Three example economies**

(a) Roundabout production economy **(b) Two-sector vertical chain economy** **(c) N-sector vertical chain economy**



Notes: the figure considers three example economies to illustrate the impact response of the aggregate adjustment frequency to aggregate and sector-specific shocks. In all examples, we set $\epsilon = 9$ and calibrate the pricing parameters in each sector to match the steady-state frequency and standard deviation of price changes of 0.15 and 0.10, respectively. Panel (a) considers aggregate TFP and monetary shocks in a one-sector $(N = 1)$ roundabout economy, where each firm's marginal cost is $MC_{1,t}(j) = \zeta_{1,t}(j) \times \frac{1}{A_{1,t}} \times M_t^{\overline{\alpha}_1} P_{1,t}^{1-\overline{\alpha}_1}$, $\overline{\alpha}_1 = 0.4$. Panels (b) and (c) consider sector-specific TFP shocks in vertical chain economies, where the marginal cost of firms in Sector 1 is $MC_{1,t}(j) = \zeta_{1,t}(j)\frac{1}{A_{1,t}}M_t$ and for firms in Sectors $1 < i \leqslant N$ is $MC_{i,t}(j) = \zeta_{i,t}(j)\frac{1}{A_{i,t}}P_{i-1,t}$. Panel (b) sets $N = 2$ and considers sectoral TFP shocks up to 20%, whereas Panel (c) sets $N = 10$ and considers 20% sectoral TFP shocks.

however, is not true: shocks to the downstream sector only affect price gaps in the downstream sector itself and do not affect price adjustment decisions in the upstream sector.

This simple example illustrates an important point: when it comes to the effect of a sector-specific shock on aggregates, the position of the shocked sector in the network can matter over and above its size. In particular, here shocks to the upstream sector have a stronger effect on aggregate adjustment frequency, even though it is as large as the downstream sector in steady state. This runs counter to a number of established network-irrelevance results, where the presence of networks makes no difference over and above its effect on equilibrium size (Hulten, 1978; Baqaee and Farhi, 2020).

Example 3: N-sector vertical chain economy

With our third example, we would like to illustrate how the interaction between the network position and pricing cascades extends beyond the two-sector arrangement. In particular, as depicted in the top panel of Figure 2(c), we consider an N-sector vertical chain economy. In such a setup, Sector 1 is the most upstream sector, which uses labor to produce a good that is supplied as an intermediate to Sector 2, which then supplies intermediates to Sector 3 and so on. Sector N is the least upstream sector, as it sells everything it produces as a final good to households. As before, all the N sectors have the same steady-state pricing moments and are equally big in the sense of having identical (cost-based) Domar weights. The only relevant dimension of heterogeneity is their position in the network.

In the bottom panel of Figure 2(c), we set $N = 10$ and plot the aggregate frequency response to a large (20%) sector-specific productivity shock to each sector. One can see that the shock to the most upstream Sector 1 delivers the largest increase in aggregate frequency. Moreover, the aggregate frequency response falls monotonically as we move down the supply chain. As before, this represents the interaction of networks with pricing cascades: shocks to more upstream sectors affect, directly or indirectly, marginal costs and hence price gaps in a larger number of sectors, thus triggering a bigger increase in aggregate adjustment frequency.

4 Full model with Euro Area data

We now move to the quantitative analysis of our full dynamic model. In this section, we outline the strategy to bring our model to the Euro Area data. In particular, we discipline the structural parameters of the model in order to make it consistent with the Euro Area economy disaggregated into 38 sectors. The household preferences and firms' production function parameters are estimated to match the observed consumption and input-output shares in the World Input-Output Tables. As for the sector-specific menu costs and variances of idiosyncratic shocks, those are estimated to fully match the observed sectoral frequencies and standard deviations of price changes in the PRISMA dataset for the Euro Area.

4.1 Parameterization and Calibration

We discipline the structural parameters of our model to the Euro Area data at a monthly frequency.

For the aggregate parameters, the household discount factor is set to $\beta = 0.96^{1/12}$ as in Golosov and Lucas (2007). The within-sector elasticity of substitution across varieties is $\epsilon = 3$ as in Midrigan (2011).[8] We assume that the aggregate money supply follows a random walk with drift:

$$\log M_t = \overline{\pi} + \log M_{t-1} + \varepsilon_t^M, \tag{36}$$

[8] Setting $\epsilon = 6$ (Blanco et al., 2024b) leaves the main quantitative results virtually unchanged.

where $\overline{\pi}$ is the trend growth rate for money supply, which is also the equilibrium level of trend inflation; ε_t^M is an i.i.d. mean zero money growth innovation. The steady-state money growth rate is $\pi = 2\%$ per year, in line with the inflation target of the European Central Bank (ECB). As for the sectoral total factor productivities, we assume they follow an AR(1) process:

$$\log A_{i,t} = \rho \log A_{i,t-1} + \varepsilon_{i,t}^A, \tag{37}$$

where $\rho \in (0, 1)$ is the persistence parameter and $\varepsilon_{i,t}^A$ is an i.i.d., mean zero, sector-specific productivity innovation. We set the persistence of TFP processes equal to $\rho = 0.9$.

We calibrate our economy to 38 production sectors of the Euro Area economy, following the classification in the World Input-Output Database (WIOD). The final consumption shares $\{\overline{\omega}_i^C\}_{i=1}^N$, the labor cost shares $\{\overline{\alpha}_i\}_{i=1}^N$ and the input-output cost shares $\{\overline{\omega}_{ik}\}_{i,k=1}^N$ are taken from the 2014 input-output tables for the Euro Area based on WIOD.[9] In order to capture the possibility that wages are also sticky, we introduce an auxiliary labor union sector. In particular, we assume that the labor union sector is the only one that directly purchases labor from households and then sells it to the rest of the sectors as an intermediate input. In general, we work with $N = 39$ sectors: 38 production sectors and the auxiliary labor union sector.

Unlike in Section 3.2 above, we do not allow for time variation in sectoral menu costs. Instead, we consider the more conventional fixed menu cost setup (Golosov and Lucas, 2007), allowing the menu costs to vary in the cross-section only:

Assumption 7′ (Fixed menu costs). *The sector-specific menu cost follows the following process: $\kappa_{i,t} = \overline{\kappa}_i$, where $\overline{\kappa}_i$ is a sector-specific constant.*

This leaves us with two parameters per sector to estimate: the menu cost $\overline{\kappa}_i$ and the standard deviation of firm-level shocks σ_i. Below we outline the estimation strategy, both for the 38 production sectors and for the additional labor union sector.

Production sectors In line with evidence on petrol prices in Gautier et al. (2023), we assume that the sectors "Coke and Petroleum Products" and "Mining and Quarrying" have fully flexible prices (zero menu costs) at monthly frequency. For the remaining 36 production sectors, we estimate the parameters $\{\overline{\kappa}_i\}_i$ and $\{\sigma_i\}_i$ to match the observed sector-specific monthly frequency and standard deviation of (non-zero) price changes

[9] We make use of the EMuSe Calibration Toolkit developed by Hinterlang et al. (2023), which constructs the Euro Area input-output table by combining accounts of individual countries in the WIOD. Since our model does not feature capital, we measure labor cost shares by dividing labor compensation by the sum of labor compensation and intermediate inputs expenditure.

(excluding sales) in steady state. We use the data from Gautier et al. (2024), who provide pricing moments at the level of products for Euro Area economies. Note that while the pricing moments data come at the level of consumption products (COICOP codes), the production sectors and the input-output structure in our model are disciplined at the level of NACE sectoral definitions. In order to construct measures of pricing moments in a classification consistent with our sectoral accounts, we use the concordance between NACE sectors and COICOP product codes developed by Kouvavas et al. (2021), which is summarized in Table B.5.[10] Figure B.4 shows that under the estimated parameters, we exactly match the frequency and standard deviation of price changes in every production sector in steady state.

Labor union sector We calibrate the price setting parameters in the labor union sector to exactly match the monthly frequency and standard deviation of nominal wage changes reported in Costain et al. (2022), who base their moments on data from the International Wage Flexibility Project (IWFP, Dickens et al., 2007). In particular, we target a frequency of nominal wage changes of $1/12$ (once a year) and a standard deviation of 6.52%.

Note that the wage moments reported by Costain et al. (2022)/IWFP are based on data that pools across 16 countries, some of which are not part of the Euro Area.[11] While we are not aware of a comprehensive quantitative study of (micro) wage-setting specifically for the Euro Area, we address this coverage issue in two ways. First, we have verified that our monthly wage adjustment frequency target $(1/12)$ is consistent with the qualitative survey results of the European Wage Dynamics Network (Walque et al., 2010), where the majority of firms report resetting wages once a year. Second, as a robustness check, Section 5.3 presents results under an alternative calibration that sets the pricing parameters in the labor union sector to exactly match the frequency and standard deviation of nominal wage changes in France (Le Bihan et al., 2012).[12] To the best of our knowledge, this is the only comprehensive quantitative study of wage microdata for any Euro Area economy. The French data shows an average monthly frequency of wage changes equal to 0.1634, with a standard deviation of 4.58%, implying a slightly higher degree of wage flexibility than under Costain et al. (2022).

All in all, in steady state, the estimated menu cost parameters imply that the total

[10]More specifically, we employ the following procedure. First, for each Euro Area economy, we construct sector-specific frequencies and standard deviations of price changes as the median across products corresponding to that sector. Second, in order to obtain Euro Area-level frequency and standard deviation of price changes we take the average across countries for each sector.

[11]The International Wage Flexibility Project (IWFP) analyzes harmonized microdata on wage changes for continuing workers, compiling 31 datasets covering 16 countries: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Sweden, Switzerland, the United Kingdom, and the United States.

[12]In addition, Section 5.3 also considers the benchmark of fully flexible nominal wages.

amount of labor that goes towards paying the cost of price and wage adjustment is around 1% of aggregate employment. As Figure B.5 shows, there is some variation across sectors, with the maximum of around 4% for sectors such as "Textiles, clothes, leather" and "Insurance and pension".

Auxiliary economies We also parameterize three auxiliary economies, for the purpose of benchmarking them against our baseline setup. First, we estimate the firm-level pricing parameters in a counterfactual economy without input-output linkages, for the same set of sector-specific frequencies and standard deviations of price and wage changes in steady state. Such an economy features no linkages across the 38 production sectors ($\overline{\omega}_{ij} = 0, \forall i, \forall j \neq LU$), which are only linked to the labor union sector instead ($\overline{\omega}_{i,LU} = 1, \forall i \neq LU$). Second, to assess the specific contribution of asymmetries in the input-output structure, we study an otherwise identical economy with a fully symmetric roundabout production structure, such that $\overline{\omega}_{i,LU} = \overline{\alpha}$ and $\overline{\omega}_{i,j} = (1-\overline{\alpha})/38, \quad \forall i, j \neq LU$, where $\overline{\alpha} \equiv \frac{1}{38}\sum_i \overline{\alpha}_i = 0.33$. In such a symmetric roundabout setup, all production sectors have an identical cost share of labor (equal to the empirical economy-wide average), and hence an equal overall cost share of intermediate inputs. Moreover, the purchases of intermediate inputs are shared equally across all the 38 production sectors. Third, we consider an economy with the empirically observed input-output linkages, but featuring time-dependent price-setting as in Calvo (1983). The latter setup corresponds to having constant sector-specific pricing hazards ($\eta_i(p) = \overline{\eta}_i, \forall i$) and zero menu costs ($\overline{\kappa}_i = 0, \forall i$). We therefore estimate sector-specific constant hazards and standard deviations of idiosyncratic shocks to match the same sectoral frequencies and standard deviations of price and wage changes as in the steady state of our baseline setup. Importantly, all three auxiliary economies feature nominal wage rigidity, since the pricing parameters in the labor union sector are still calibrated to match the frequency and standard deviation of wage changes.

4.2 Sectoral characteristics

In order to better understand the cross-sectional properties of the sectors we consider in our quantitative setup, we evaluate the relevant centrality measures introduced in Section 3. First, recall that in Propositions 1 and 2, the effect of networks on the probability of adjustment in a given sector depends on the *customer centrality* measure $\mathcal{C}_i$. Intuitively, the customer centrality measure captures the total reliance of a sector on intermediate inputs, both direct and indirect. Naturally, if a sector only uses labor purchased from the household, its customer centrality measure collapses to zero. Table B.1 in Appendix B reports the customer centrality measure for each of the 38 production sectors. The two sectors with the largest customer centrality are "Coke and petroleum products" (4.35) and "Chemicals and chemical products" (4.25), followed by "Paper and paper products" (3.97) and "Food and beverages" (3.94), while the smallest customer centrality is in "Education" (1.55).

Second, we quantify the *Supplier Herfindahl* $\mathcal{H}_i$ and the *Supplier Centrality* $\mathcal{S}_i$ measures introduced in (33). Recall that in Proposition 3, the *Supplier Herfindahl* of a sector plays a key role in determining its ability to influence the average economy-wide probability of adjustment. In turn, according to (35), $\mathcal{H}_i$ is the sum of $\mathcal{S}_i^2$ and the cross-sectional variance of the i'th column of the Leontief inverse. The *Supplier Centrality* measure captures the *average* importance of a sector as a seller, either directly or indirectly, of intermediate inputs. As is reported in Table B.1, the distribution of supplier centrality features a heavy right tail, with three sectors having a disproportionately larger measure than the rest: those are "Administration and support" (0.23), "Legal, accounting, management" (0.20) and "Chemicals and chemical products" (0.19). Those sectors are also the ones with the largest *Supplier Herfindahl*, though in a different order: "Chemicals and chemical products" (0.12), "Administration and support" (0.08) and "Legal, accounting, management" (0.08). The difference comes from the fact that while "Chemicals and chemical products" has a smaller supplier centrality than the other two, it is a disproportionately important supplier to some sectors, which increases the variance term and drives up the *Supplier Herfindahl*. See Table B.3 for a decomposition of each sector's *Supplier Herfindahl* into the (squared) *Supplier Centrality* component, as well as the variance component.

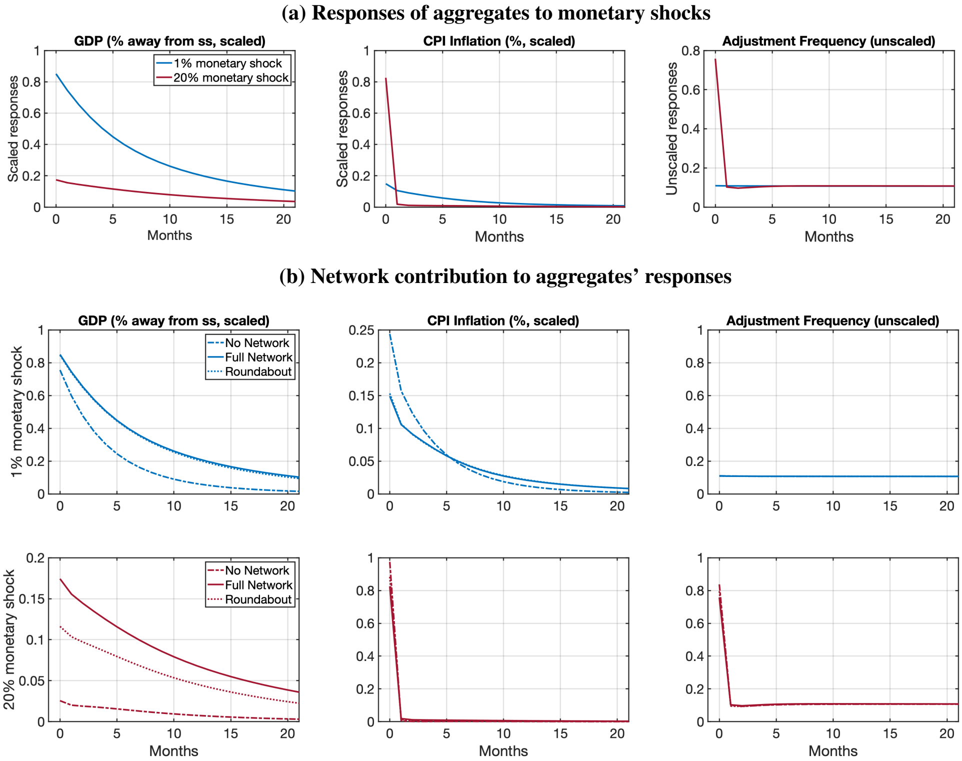
5 Quantitative results: monetary shocks

For our first set of quantitative results, we present the general equilibrium dynamics of our economy following monetary shocks of different sizes. We show that the aggregate repricing frequency response to large monetary shocks is substantially attenuated by the presence of networks, so that the effect of cascades dampening is quantitatively sizable. As a result, the economy with networks features much stronger monetary non-neutrality, which manifests in a substantial flattening of the fully non-linear Phillips Curve.

5.1 Aggregate dynamics

Figure 3(a) shows the scaled (per % shock) responses of aggregate CPI inflation, aggregate GDP, as well as the unscaled monthly fraction of adjusting firms following one-time monetary shocks of two different magnitudes: $\varepsilon_0^M = 1\%$ and $\varepsilon_0^M = 20\%$.

Figure 3: **Aggregate responses to monetary shocks**



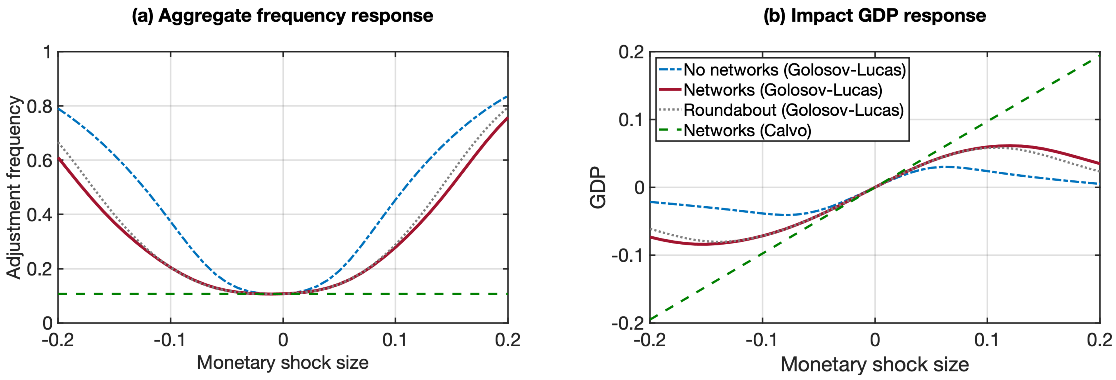
Notes: Panel (a) shows the responses of aggregate GDP, CPI inflation and aggregate monthly adjustment frequency following 1% and 20% one-time monetary shocks in the baseline economy with fixed menu costs and networks; Panel (b) additionally shows the corresponding responses in the otherwise identical economies with a symmetric roundabout production network and without networks, respectively.

Two key features are apparent. First, the scaled response of inflation increases in the size of the monetary shock, which represents a strong *size effect*. As can be seen in the frequency panel, this happens as the monthly fraction of adjusting firms increases rapidly with larger shocks, reaching over 0.75 for the 20% monetary shock.

Second, as shown in Figure 3(b), the contribution of production networks to the magnitudes of responses differs markedly between shocks of different sizes. For the small 1% shock, networks dampen the response of inflation and, as a result, amplify the response of aggregate GDP. This is the effect of production networks known from the prior literature, which employs linearized models with time-dependent pricing: input-output linkages create pricing complementarities, dampening inflation and amplifying the GDP response. At the same time, for the large 20% shock, the amplification of the aggregate GDP response due to networks is much greater. Importantly, this is because the 20% monetary shock delivers a markedly smaller increase in the monthly repricing frequency relative to the economy without networks, which is the cascades *dampening* effect introduced earlier.[13] Figure 3(b) also reports the responses in an otherwise

[13]In Figure E.1 we construct, for each size of the monetary shock, the difference between the output response with and without networks, as a fraction of the former. One can see that for small monetary

Figure 4: **Impact responses to monetary shocks**



Notes: Panels (a) and (b) show the impact responses of, respectively, the aggregate adjustment frequency and aggregate real GDP following monetary shocks of different sizes in the baseline economy with fixed menu costs and networks, as well as in the otherwise identical economies without networks, with Calvo (1983) pricing and with symmetric roundabout production.

identical economy with a symmetric roundabout production structure. One can see that for the small 1% shock it behaves practically identically to the baseline economy with the full empirically observed input-output structure. At the same time, following the large 20% monetary expansion, the symmetric roundabout economy features a GDP rise that is more than a third smaller than under the baseline economy, while also producing a larger inflation and frequency response. It follows that asymmetries in the input-output structure are quantitatively important for aggregate dynamics under large monetary shocks, while being less relevant under smaller shocks.[14]

In Figure 4(a), we further investigate the interaction between networks and the response of repricing frequency by looking at a wide range of shock sizes and signs. One can see that networks consistently dampen the response of the aggregate monthly repricing frequency to monetary shocks of all sizes that we consider. Moreover, as shocks get larger, network asymmetries make a quantitatively sizable contribution. For example, following a 20% monetary expansion, networks overall deliver a frequency dampening from 0.84 to 0.76, with almost half of the effect accounted for by the network asymmetries.[15]

Figure 4(b) reports the impact response of aggregate real GDP to monetary shocks of different sizes, showing that cascades dampening by networks substantially amplifies

shocks, the contribution is in the neighborhood of 10-20%. Such magnitudes are consistent with prior estimates of network contributions in linearized time-dependent setups (Ghassibe, 2021). As the size of the shock increases, however, the contribution of the network increases dramatically, reaching almost 80% for a 15% monetary expansion.

[14]This is consistent with the modest net importance of network asymmetries for aggregate dynamics following monetary shocks in linearized economies with time-dependent pricing (Pasten et al., 2020).

[15]See Figure E.3 for the aggregate inflation response under monetary shocks of different sizes. Moreover, in Appendix D.4 we present a decomposition which quantifies the contribution of frequency (extensive margin) to aggregate inflation responses, confirming that for large shocks most of the network effect on inflation indeed works through the extensive margin channel.

the degree of monetary non-neutrality. In the baseline economy with networks, the aggregate real GDP response is hump-shaped in the size of monetary shocks and is maximized following a 12% monetary expansion, delivering an increase of almost 6%. At the same time, the equivalent economy without networks has its real GDP response maximized following a 5% monetary shock, corresponding to a smaller increase of just over 3%. Moreover, as the monetary shocks get larger, network asymmetries contribute increasingly to the GDP response, accounting for almost one third of the effect under the 20% shock. Figure 4(b) also shows the responses in the alternative setup with time-dependent Calvo (1983) pricing, where even under the fully non-linear solution, the GDP response changes essentially linearly in the size of the shock.

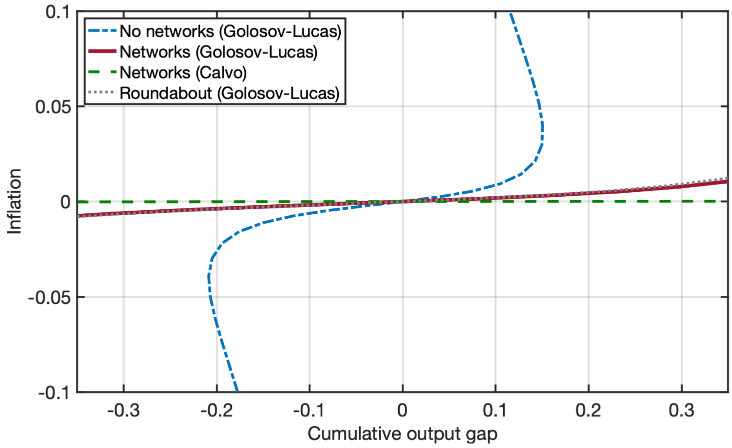
5.2 Phillips Curves

Figure 5 illustrates the trade-off between GDP response and inflation under monetary interventions of different sizes. In particular, the figure traces out a non-linear "Phillips curve" in the cumulative output gap–CPI inflation space, under different model configurations.[16] In the baseline economy with networks and menu costs, the local slope for a small 1% shock is 0.0183, whereas for the otherwise identical economy without networks it is more than three times steeper at 0.0624. As shocks become larger, the slope flattening generated by networks strengthens: for a 10% shock, the baseline economy has a slope of 0.0715, eight times smaller than the value of 0.5784 under no networks. That the degree of flattening is stronger for the larger monetary shock is a consequence of cascades dampening, which slows down the rise in adjustment frequency under networks. Note that the economy with networks and Calvo (1983) pricing produces a much smaller slope of around 0.0006 for both shock sizes.

In Figure E.2(a) (Appendix) we show that the slope flattening result is robust to alternative measures capturing the inflation/GDP trade-off. In particular, we consider the impact slope given by the ratio of inflation and GDP responses on impact, as well as the 3-month slope (Holden, 2025) given by the ratio of cumulative CPI and average GDP response in the 3 months following the monetary shock. Figure E.2(b) also compares the model-based slopes to those implied by the estimated dynamics of inflation and economic activity in response to identified monetary shocks of Miranda-Agrippino and

[16] To be precise, we plot the impact response of aggregate CPI inflation against the cumulative output gap, measured as the infinite sum of real GDP responses, discounted by the household's $\beta = 0.96^{1/12}$. It is motivated by the form of the textbook New Keynesian Phillips Curve (Galí, 2015): $\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t = \kappa \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t y_{t+s}$. Under the perfect foresight monetary shocks, the ratio of impact inflation response to the cumulative (discounted) gap can therefore be interpreted as the Phillips Curve slope for the particular shock size. Naturally, this is not a structural object in our model and there exist alternative slope measures, which we consider in Figure E.2 in the Appendix.

Figure 5: **Fully non-linear "Phillips Curves"**



Notes: the figure shows the fully non-linear "Phillips Curves", obtained by tracing out the impact response of CPI inflation and the cumulative real GDP response ("output gap") following monetary shocks of different sizes. The "Phillips Curves" are constructed for three economies: the baseline economy with fixed menu costs and networks, as well as the otherwise identical economies without networks, with Calvo (1983) pricing and with symmetric roundabout production.

Ricco (2021).[17] Taking the 3-month slope as a specific example, the Miranda-Agrippino and Ricco (2021) responses produce a value of 0.45. Our baseline model produces a 3-month slope of 0.74, which is much closer to the data than the slope in the menu cost economy without networks (1.36), or in the economy with networks and Calvo (1983) pricing (0.10).

5.3 Role of nominal wage rigidity

While our baseline results discipline the pricing parameters in the labor union sector to the frequency and standard deviation of nominal wage changes from Costain et al. (2022), in Figure E.5 we consider alternative degrees of wage flexibility. In particular, we study the propagation of monetary shocks under the higher frequency of wage adjustment found in the French microdata (Le Bihan et al., 2012), as well as under fully flexible wages. Figure E.5(a) shows the responses following a 1% monetary shock: relative to the baseline degree of wage rigidity, the slightly more flexible wages of Le Bihan et al. (2012) produce a stronger (scaled) response of inflation (0.18% vs. 0.16%), with an even stronger response of 0.22% under fully flexible wages. Naturally, the latter also

[17]We consider the responses in Figure 3 of Miranda-Agrippino and Ricco (2021), using the industrial production response as proxy for the real GDP dynamics. To maintain a tight mapping between the data and the model, we measure the implied dynamics of nominal GDP by the sum of the estimated impulse responses of CPI and industrial production. We then feed in that path of nominal GDP as $\{M_t\}_{t\geqslant 0}$ into the model, record the equilibrium dynamics of inflation and real GDP, and compute the slope measures. A caveat is that the Miranda-Agrippino and Ricco (2021) analysis is done for the United States, whereas our model is calibrated for the Euro Area. However, we believe the estimated responses are still informative about the relevant *order of magnitude* of the slope conditional on monetary shocks, which is unlikely to be substantially different in the United States and in the Euro Area.

implies a slightly weaker response of GDP as wages get more flexible. At the same time, Figure E.5(b) shows that under the larger 10% monetary shock, the responses of inflation and GDP are virtually identical for all the levels of wage rigidity considered. Key to this result is the behavior of the fraction of adjusters in the labor union sector. Under small monetary shocks, the frequency of wage adjustment stays at the steady-state level, implying stronger aggregate nominal wage responses, and hence a more pronounced aggregate inflation, in economies with more flexible wages in steady state. At the same time, as confirmed in Figure E.6, as the monetary shock reaches 10% in absolute value, the monthly fraction of adjusters rises to virtually one regardless of the steady-state frequency of wage adjustment, delivering identical dynamics of the nominal wage and other macro aggregates.[18]

5.4 Disaggregated dynamics

We also study sector-level behavior following a large monetary shock. In Figure E.8 we report changes in sectoral monthly adjustment frequencies, as well as scaled impact responses of sectoral price indices, following a 10% monetary shock, showing that the cascades dampening effect holds at the level of each sector. Further, Figure E.10(a) confirms that there is an approximately linear relationship between the (square root of) sectoral frequency changes following a 10% monetary shock and the sectoral customer centrality measure introduced in (29), in line with the formal result in Proposition 1.[19]

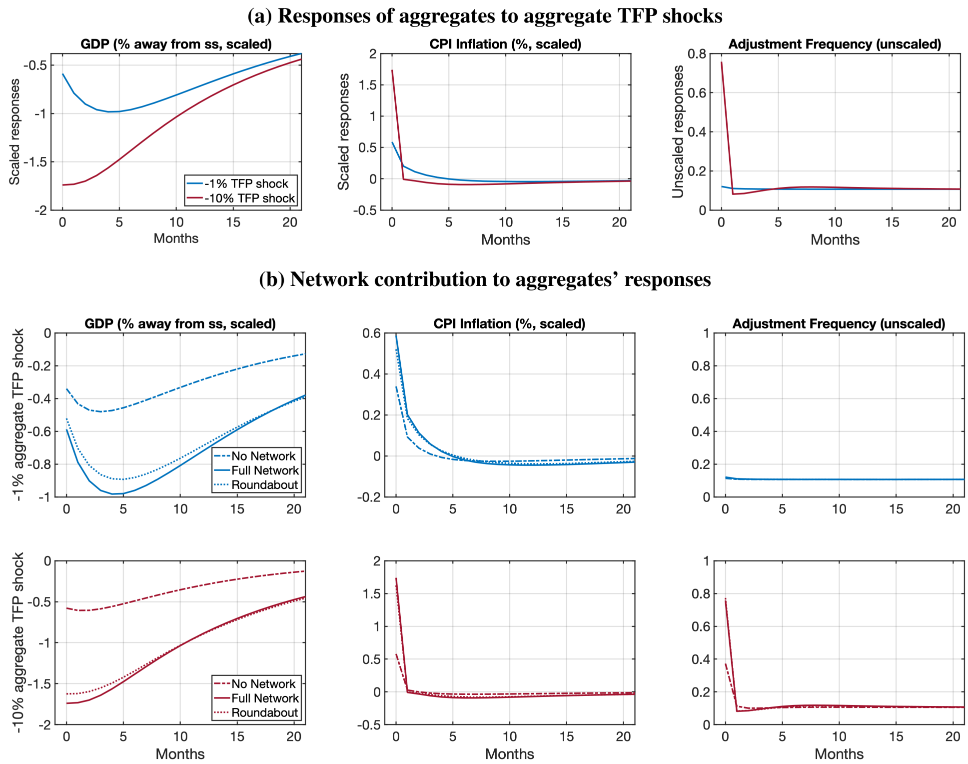
6 Quantitative results: TFP shocks

For our second set of quantitative results, we turn to the general equilibrium dynamics following aggregate and sector-specific total factor productivity (TFP) shocks. First, we show that following large aggregate TFP shocks, the economy with networks features a much stronger response of the repricing frequency, implying that the cascades amplification channel is indeed quantitatively important. Second, our numerical results confirm that TFP shocks to specific sectors, particularly those with a large *Supplier Herfindahl*, can lead to sizable movements in the aggregate repricing frequency,

[18]In Figure E.7 we also study the strength of the cascades dampening effect under the three different steady-state levels of wage rigidity. In particular, for each wage rigidity calibration, we report the impact responses of aggregate adjustment frequency and GDP in economies with and without networks. The results imply that the degree of dampening is virtually unaffected by the steady-state degree of wage rigidity. This is consistent with the steady-state level of wage rigidity having no effect on aggregate dynamics following large monetary shocks .

[19]Formally, we regress the square root of the change in sectoral adjustment frequency on the measure of customer centrality, further controlling for the covariance between that sector's row of $(\overline{\Psi}-I)$ and the vector of sectoral markups, own markup change, the steady-state frequency of adjustment and the sectoral standard deviation of price changes. The regression specification is based on the result in Proposition 1.

Figure 6: **Aggregate responses to aggregate TFP shocks**



Notes: Panel (a) shows the responses of aggregate real GDP, CPI inflation and aggregate monthly adjustment frequency following -1% and -10% one-time aggregate TFP shocks in the baseline economy with fixed menu costs and networks; Panel (b) additionally shows the corresponding responses in the otherwise identical economies with a symmetric roundabout production network and without networks, respectively.

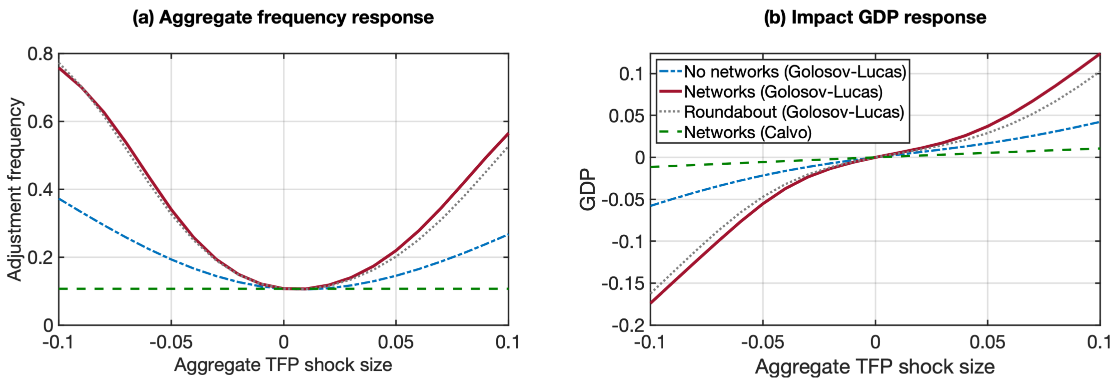
generating non-linearities in aggregate inflation. The latter highlights the importance of modelling the empirical asymmetries in the input-output structure for studying the propagation of large sectoral shocks.

6.1 Aggregate TFP shocks

6.1.1 Aggregate dynamics

In Figure 6(a), we report scaled (per % shock) responses of aggregate GDP and CPI inflation, as well as the unscaled responses of the aggregate monthly fraction of adjusters following two negative aggregate TFP shocks: $\varepsilon_0^A = -1\%$ and $\varepsilon_0^A = -10\%$. Just as with monetary shocks in the previous section, there is substantial size dependence: for the -1% shock the impact scaled response of GDP is -0.6%, whereas it is -1.7% for the -10% shock. At the same time, the scaled response of CPI inflation increases in the magnitude of the aggregate TFP shock, implying that the aggregate price changes rise more than proportionally with the size of the TFP innovation. Quantitatively, the -1% shock generates a scaled impact response of CPI inflation of 0.6%, whereas the -10% shock corresponds to a normalized response of almost 1.7% on impact. Key to

Figure 7: **Impact responses to aggregate TFP shocks**



Notes: Panels (a) and (b) show the impact responses of, respectively, the aggregate adjustment frequency and aggregate real GDP following aggregate TFP shocks of different sizes in the baseline economy with fixed menu costs and networks, as well as in the otherwise identical economies without networks, with Calvo (1983) pricing and with symmetric roundabout production.

the observed size dependence is the endogenous response of the monthly fraction of adjusters: for the -1% shock it remains unchanged, whereas the larger -10% shock brings the fraction of adjusters to almost 80%.

In order to understand the contribution of networks to the observed size dependence, in Figure 6(b) we document the responses to the same aggregate TFP shocks in an otherwise identical economy without networks. For the -1% shock, networks amplify the response of aggregate GDP by a factor of two, while the scaled response of CPI inflation is nearly 0.3% without networks versus 0.6% under networks. Importantly, for the larger -10% shock, the network amplification of both aggregate GDP and CPI inflation is stronger. Regarding the response of inflation, this is the opposite of our findings under monetary shocks, where the network delivers stronger *dampening* of the inflation response for larger innovations. To understand the difference, it is instructive to look at the response of the adjustment frequency. One can see that for the -10% shock, the monthly fraction of adjusters increases substantially more in the economy with networks. This is the exact opposite of what happens under monetary shocks, where networks dampen the response of the adjustment frequency. Instead, under TFP shocks, networks make the response of the adjustment frequency stronger, representing cascades amplification. Figure 6(b) also shows the response in the otherwise identical economy with a symmetric roundabout production structure. One can see that, for both shock sizes, the responses of GDP and inflation are smaller in magnitude than in the baseline economy with the full network structure, suggesting that network asymmetries matter for aggregate dynamics.

In Figure 7(a), we further investigate the interaction between the aggregate monthly repricing frequency and the size of the aggregate TFP shock. Once again, contrary to the findings under monetary shocks, networks consistently and substantially amplify the response of the aggregate fraction of adjusters to aggregate TFP shocks. For example, following a -10% aggregate TFP shock, the economy with networks features a rise in the fraction of adjusters to 75%, while without networks the aggregate adjustment frequency rises to just below 40%. In this sense, the cascades amplification effect is quantitatively sizable.[20]

The cascades amplification channel is also important for generating non-linearity in aggregate GDP and inflation response. In Figure 7(b), we plot the impact response of GDP to aggregate TFP shocks of different signs and sizes. The GDP response rises much faster in the economy with networks relative to the no-network benchmark, being almost three and a half times higher after a -10% shock. We also report the real GDP responses in an economy with networks, but with time-dependent (Calvo, 1983) pricing, where the responses are much smaller and change essentially linearly with shock size. Note that by the cash-in-advance constraint (6), under TFP shocks the impact response of inflation is just a mirror image of the impact GDP response, implying that the cascades amplification channel generates substantial inflation non-linearity as well.

6.1.2 Disaggregated dynamics

We also study sector-level behavior following a large aggregate TFP shock. In particular, Figure E.9 reports the impact responses of sector-specific monthly fractions of adjusters and scaled sectoral price indices to a -10% aggregate TFP shock, confirming that the cascades amplification effect holds at the level of individual sectors. Moreover, Figure E.11(a) shows that there is an approximately linear relationship between the (square root of the) change in the fraction of adjusters in a given sector and its customer centrality, in line with Proposition 2.

6.2 Sectoral TFP shocks

For our next set of results, we study the transmission of sector-specific TFP shocks. Our particular interest is in how large shocks originating in specific parts of the economy can affect macroeconomic aggregates.

In Figure 8, we subject each production sector of our economy to a large negative TFP shock (one sector at a time) and report the impact response of the aggregate monthly fraction of adjusters. Panel (a) shows the results under a $\varepsilon_{i,0}^{A} = -20\%$ shock. First, for the majority of sectors, the effect of their own shock on aggregate frequency is relatively modest. However, there are notable exceptions: individual shocks to "Food

[20] Appendix D.4 presents an inflation decomposition, showing that under large aggregate TFP shocks networks strengthen the inflation response mainly through the cascades amplification channel of the adjustment frequency change (extensive margin channel).

and beverages", "Crop and animal production", and "Chemicals and chemical products" generate an aggregate frequency increase of over 0.04. Second, for all sectors, networks enhance the aggregate frequency response to the sector-specific TFP shock, representing the cascades amplification effect. Moreover, the amplification is especially strong for the three aforementioned sectors that are particularly important for aggregate frequency. Third, in the otherwise identical economy with a symmetric roundabout production structure, the response of aggregate frequency is either the same or noticeably smaller for the vast majority of sectoral shocks, with the difference particularly stark for the three sectors that can deliver a sizable aggregate frequency effect in the baseline model. In this sense, accounting for the empirical asymmetries in the input-output structure is crucial for accurately capturing the ability of specific shocks to affect aggregate adjustment frequency.

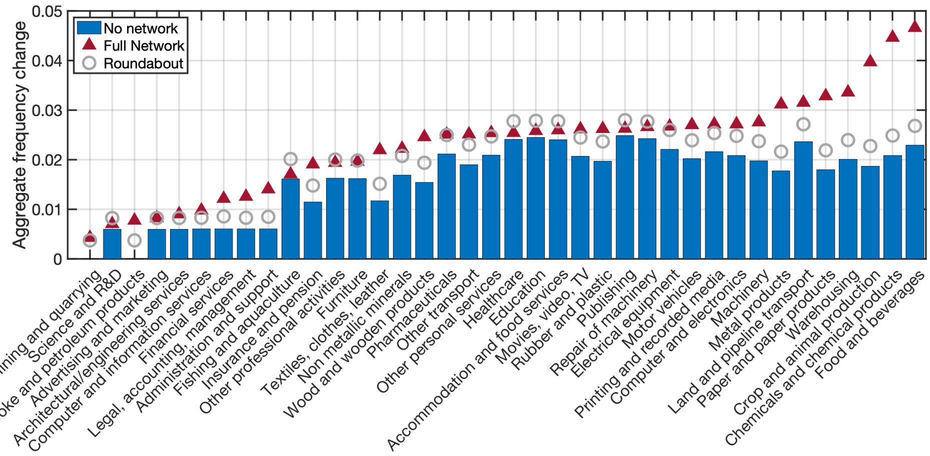
In Panel (b) of Figure 8 we repeat the exercise for even larger $\varepsilon^A_{i,0} = -50\%$ sectoral shocks. In the baseline model with networks and menu costs, most sectors still generate a relatively modest aggregate monthly frequency surge. However, under the larger -50% shock several sectors generate an even more disproportionate aggregate effect than before: shocks to "Administration and support" and "Chemicals and chemical products" produce an almost 0.15 surge in the aggregate monthly fraction of adjusters. Crucially, such large aggregate effects of sector-specific shocks are missed almost entirely in both the no-network benchmark *and* in the symmetric roundabout economy. Ignoring the asymmetries in the input-output linkages would predict that -50% shocks to "Administration and support" and "Chemicals and chemical products" generate merely 0.04 increases in aggregate frequency, as opposed to almost 0.15 under the empirically observed network. Once again, this stresses that network asymmetries are of paramount quantitative importance when it comes to the transmission of large sectoral shocks to aggregate adjustment frequency.

An important pattern emerges: large shocks to certain sectors have the capacity to disproportionately affect the aggregate adjustment frequency. Moreover, this disproportionate importance stems from network amplification. In order to shed light on how network characteristics affect the systemic importance of a sector for aggregate frequency, in Figure 9(a) we plot an estimated linear relationship between the aggregate frequency changes following -20% shocks and the sectoral *supplier Herfindahls* $\mathcal{H}_i$, introduced in (33).[21] The relationship is clearly positive: a 0.02 increase in *supplier Herfindahl* is associated with a 0.01 additional increase in aggregate frequency. This is consistent with the formal result in Proposition 3. Moreover, such a key role of the *supplier Herfindahl* sheds light on why the symmetric roundabout economy fails to pro-

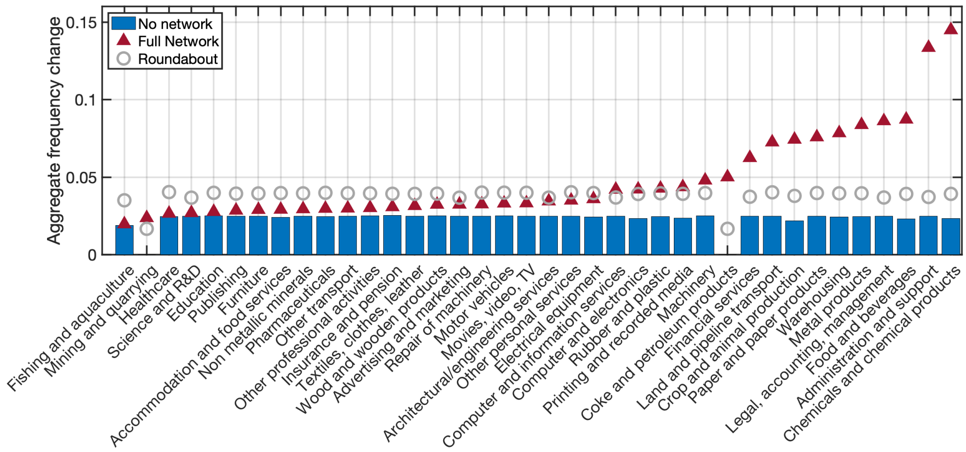
[21]Formally, we regress the change in aggregate frequency on $\mathcal{H}_i$, further controlling for $\mathcal{S}_i$, as well as the sectoral steady-state adjustment frequency and standard deviation of price changes.

Figure 8: **Aggregate monthly frequency responses to sectoral TFP shocks**

(a) Responses of aggregate frequency to -20% sectoral TFP shocks



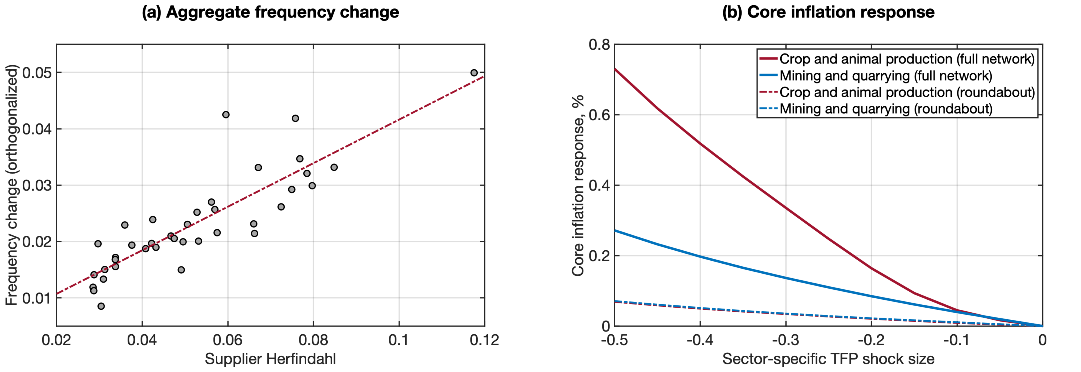
(b) Responses of aggregate frequency to -50% sectoral TFP shocks



Notes: the figure shows the impact responses of aggregate monthly adjustment frequency to large sector-specific TFP shocks in the baseline economy with menu costs and networks, as well as in the otherwise identical economies without networks and with a symmetric roundabout production. Panel (a) considers -20% sector-specific TFP shocks, whereas Panel (b) considers larger -50% sector-specific TFP shocks.

duce sizable increases in aggregate frequency under sectoral shocks. Figure B.3(b) plots the values of sectoral *supplier Herfindahl* in the baseline economy with empirically observed input-output linkages, as well as in the counterfactual symmetric roundabout economy. One can see that in the symmetric roundabout economy the *supplier Herfindahl* is the same for all sectors and, for most sectors, drastically smaller than in the baseline economy with asymmetries. This, in turn, is caused by the quantitative importance of the $Var\left(\overline{\Psi}_{(k)}\right)$ term for the *supplier Herfindahl* in the baseline economy (see Figure B.3(a)), representing the effect of being a disproportionately important supplier to some part of the economy, which is largely missing in the symmetric roundabout setup.

Figure 9: **Sectoral TFP shocks, aggregate responses and the *supplier Herfindahl***



Notes: Panel (a) shows the fitted linear relationship between the sectoral *supplier Herfindahl* and the response of aggregate monthly adjustment frequency to a -20% TFP shock to that sector (further controlling for the sectoral *supplier centrality*, as well as the steady-state adjustment frequency and standard deviation of price changes). Panel (b) shows the impact effect of TFP shocks to “Mining and quarrying” and “Crop and animal production” sectors on annulized core inflation (excludes the two shocked commodity sectors) in the baseline economy with menu costs and networks, as well as in an otherwise identical economy with a symmetric roundabout production.

The cascades amplification channel, which strengthens the effect of sectoral shocks on aggregate frequency, has further important implications for non-linearities in aggregate inflation. In order to see that, in Figure 9(b) we consider how progressively larger shocks to the two commodity sectors, “Mining and quarrying” and “Crop and animal production”, affect aggregate *core* inflation, which excludes movements in the commodity prices themselves. First, one can see that as the negative TFP shocks to the “Crop and animal production” sector become larger in magnitude, core inflation responds more than proportionally on impact, thus rising in a fast non-linear fashion. This is because, as documented in Figure 8, shocks to “Crop and animal production” create sizable increases in the aggregate adjustment frequency. Second, far less non-linearity in core inflation response occurs under shocks to “Mining and quarrying”. This is because shocks to the latter sector do not induce substantial movements in aggregate adjustment frequency. Another key finding is that the magnitude of the core inflation response, as well as the degree of non-linearity, becomes drastically weaker in the counterfactual symmetric roundabout economy. As one can see, shocks to the two commodity sectors have essentially the same effect on core inflation (since the input-output structure is symmetric), with the inflation response changing approximately linearly in shock size due to the largely missing frequency response, as documented in Figure 8.

7 Empirical validation

We now assess the ability of our model to capture the empirically observed macro dynamics in the Euro Area. First, we study the performance of our model under small aggregate and sector-specific shocks. Our results show that our quantitative model can reconcile the observed responses of Euro Area macro aggregates to identified monetary and oil shocks in the low-inflation pre-Covid period. Second, turning to large shocks, we assess whether the interaction between networks and pricing cascades is important for quantitatively explaining the macro dynamics in the Euro Area in the (post-)Covid era. To do that, we feed four structurally interpretable shock series into our model, corresponding to the widely perceived major drivers of the post-Covid business cycle: money supply, energy price movements, food price movements, and labor market conditions. We show that when subjected to those four series, our model successfully captures the rise in the aggregate repricing frequency *and* the surge in consumer price inflation in the Euro Area.

7.1 Small shocks

In our first set of empirical validation exercises, we assess the ability of our model to reproduce the observed macro dynamics under small aggregate and sector-specific shocks. To do that, we estimate linear IRFs of Euro Area aggregates to identified monetary and oil shocks and then compare them to model equivalents.

7.1.1 Estimated impulse responses

We use the local projection approach of Jordà (2005) in order to estimate horizon-specific impulse responses of aggregate variables to identified shocks:

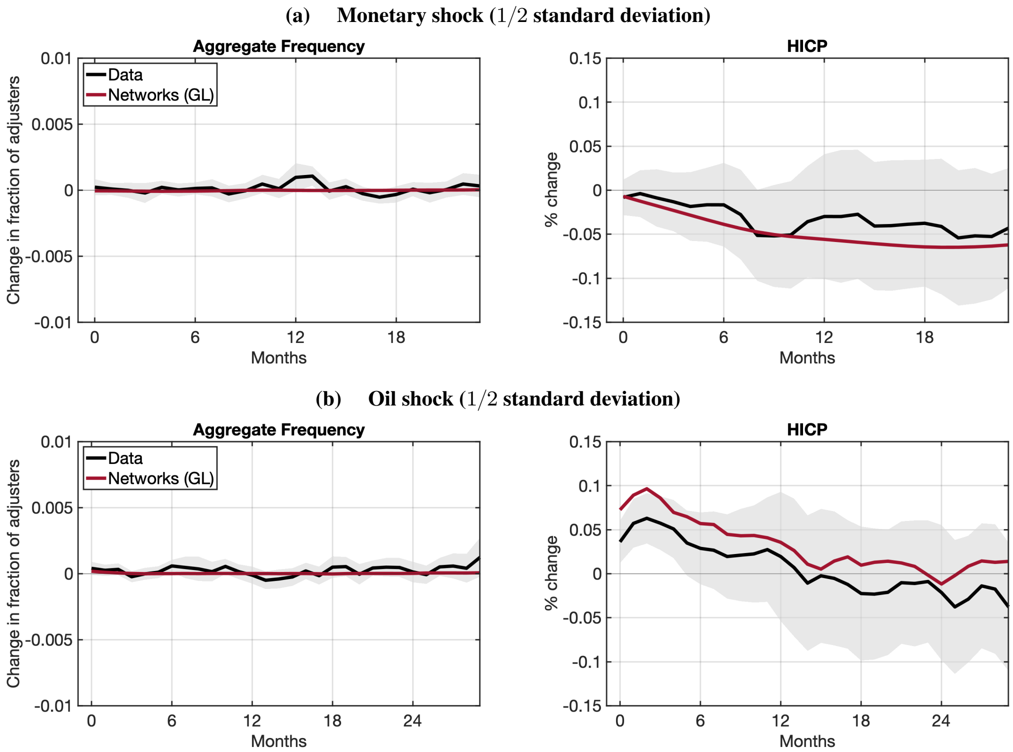
$$y_{t+H} = \alpha_H + \beta_H s_t + \boldsymbol{\gamma}'_H \boldsymbol{x}_{t-1} + \varepsilon_{t+H}, \tag{38}$$

for $H = 0, 1, ..., \overline{H}$, where y_t is the dependent variable of interest, s_t is an identified exogenous shock, $\boldsymbol{x}_{t-1}$ is a vector of control variables and α_H represents horizon-specific fixed effects.[22] The estimated value of β_H gives the horizon-specific response of the variable of interest to an identified exogenous shock. Our dependent variables of interest include Euro Area HICP (in logs), nominal and real aggregate GDP (in logs), the (real) IMF Energy Price Index (in logs), the risk-free nominal interest rate (in basis points), as well as the aggregate monthly fraction of price adjusting firms (in levels).[23]

[22] The set of controls is specified in the descriptions to figures presenting each of the estimation results.

[23] To maintain a tight link with our theoretical model, we only consider the aggregate final demand (private and government expenditure) portion of GDP, which we linearly interpolate (in logs) to transform quarterly into monthly data. The raw IMF Energy Price Index is reported in USD, which is why we first adjust it by the USD/EUR exchange rate and then deflate it by the Euro Area nominal GDP to get a real energy price index. Following Jarociński and Karadi (2020), we measure the nominal risk-free rate by the sovereign German 1-year yield. The time series on the fraction of adjusting firms (excluding sales) comes from Gautier et al. (2025), seasonally adjusted by removing monthly dummies.

Figure 10: **Responses to small shocks: model vs. data (Euro Area)**



Notes: Panel (a) shows the estimated IRFs of the aggregate monthly adjustment frequency and aggregate HICP to a contractionary monetary shock of Jarociński and Karadi (2020), as well as the corresponding responses in the baseline model. For aggregate frequency, the set of controls includes 12 lags of aggregate frequency and HICP; for HICP, the set of controls includes 12 lags of HICP, industrial production, risk-free nominal interest rate, real IMF energy and food price indices and the Euro Stoxx 50 index. Panel (b) shows the estimated IRFs of the aggregate adjustment frequency and aggregate HICP to a contractionary oil shock of Känzig (2021), as well as the corresponding responses in the baseline model. For aggregate frequency, the set of controls includes 12 lags of the oil shock, aggregate frequency and HICP; for HICP, the set of controls includes 12 lags of the oil shock, HICP, industrial production, real IMF energy and food price indices and the risk-free nominal interest rate. Shaded areas represent 95% confidence intervals.

We use the identified Euro Area monetary shocks of Jarociński and Karadi (2020), as well as the identified oil shocks of Känzig (2021).[24] Our estimation sample spans monthly data between January of 1999 and December of 2019. By focusing on the pre-Covid period, we explicitly study macro dynamics when aggregate fluctuations were relatively moderate and shocks were small. Indeed, the in-sample standard deviation of our monetary shock measure is merely 3.6 basis points, whereas a one-standard deviation oil shock corresponds to an impact jump in the real energy price index of just under 5%.

7.1.2 Model fit

Monetary shock Panel (a) of Figure 10 shows the estimated responses of the aggregate monthly fraction of adjusters and HICP to a $1/2$ standard deviation contractionary

[24] We use the "poor man's" version of Jarociński and Karadi (2020) shocks, whereas the Känzig (2021) shocks are identified from a SVAR estimated on the pre-Covid sample.

monetary shock. One can see that the adjustment frequency response is flat and tightly estimated around zero, whereas the HICP experiences a trough of around -0.05% after 7 months.[25] In order to assess the ability of our baseline theoretical model to reproduce such dynamics, we feed the estimated path of nominal GDP following the monetary shock (reported in Figure F.12(a)) as $\{M_t\}_{t\geqslant 0}$ into our model.[26] As Figure 10(a) shows, the baseline model with menu costs and networks successfully reproduces the flat response of the adjustment frequency, as well as matches the drop in HICP.

Oil shock Panel (b) of Figure 10 reports the dynamics of the aggregate monthly adjustment frequency and HICP following a $1/2$ standard deviation contractionary oil shock. As with monetary shocks, the frequency response is flat around zero, whereas the HICP instead experiences an increase of just over 0.05% at the peak. To obtain theoretical counterparts to the responses, we feed TFP shocks to the "Mining and Quarrying" sector of our model to exactly match the estimated response of the energy price index (reported in Figure F.12(b)) in general equilibrium. In order to incorporate the possibility of a monetary policy response to the oil shock, we simultaneously feed in the estimated nominal GDP response (also reported in Figure F.12(b)) as the money supply path.[27] Figure 10(b) shows that the model successfully matches the flat response of the aggregate fraction of adjusters, while also generating an HICP increase that lies within the estimated confidence interval.

7.1.3 Extensions and robustness checks

In addition to the small shock exercises above, we perform several extensions and robustness checks, summarized here.

Alternative models In Figure F.13 we compare the estimated responses to small monetary and oil shocks to those generated by alternative theoretical models. One can see that relative to the baseline setup with menu costs and networks, the model with networks and Calvo (1983) pricing severely underpredicts the drop in HICP following the

[25] Since Gautier et al. (2025) use HICP product weights to construct their aggregate frequency time series, for the rest of this section we also compute model-based aggregate frequency by applying sectoral final consumption weights to sector-specific frequency movements.

[26] We use the nominal GDP, as opposed to money supply, as the empirical counterpart of M_t, since empirical measures of money supply (such as M0 or M2) are heavily influenced by velocity fluctuations, while our model assumes velocity is constant due to the cash-in-advance constraint $P_tC_t = M_t$, in line with most of the theoretical literature. Note that the model-based responses in Figure 10(a) assume that entire path of $\{M_t\}_{t\geqslant 0}$ is known at $t = 0$; see Section 7.1.3 for alternative results when the agents discover the path of M_t period-by-period instead.

[27] One can see that the nominal GDP is estimated to fall, which through the lens of our model indicates a monetary tightening in response to the oil price increase.

small monetary contraction, while simultaneously notably overpredicting the persistence in the HICP response following the oil shock. At the same time, the model with menu costs but without networks overshoots the HICP drop under the monetary contraction, while also generating a counterfactual deflation following the exogenous oil price surge.

In Figure F.14 we run a formal comparison across the three models by computing a measure of mean squared distance between the empirical and the theoretical responses.[28] One can see that for both monetary and oil shocks, the distance to the data is minimized under the baseline model with menu costs and networks. Moreover, this holds true for both HICP and real GDP responses.

Unanticipated shocks The model-based responses in Figure 10 are produced under the assumption that at $t = 0$ agents observe the entire path of monetary and TFP shocks that we feed in. In Figure F.15 we relax the perfect foresight assumption and present model-based responses when the agents discover the path of money supply and the price index of "Mining and Quarrying" period-by-period instead.[29] The results confirm the ability of our baseline model to reproduce the estimated dynamics of adjustment frequency and the HICP following small unanticipated monetary and oil shocks as well.

Sectoral responses While in Figure 10 we focus on the dynamics of aggregate HICP, in Figures F.16 and F.17 we additionally report the estimated HICP responses to monetary and oil shocks at the level of four major sectoral groups, as well as the corresponding responses in our baseline model. In particular, we consider the price level dynamics of Energy, Food, Services, as well as Non-Energy Industrial Goods (labeled simply as "Goods").[30] Overall, one can see that our baseline model matches the sectoral price responses reasonably well, with the fit being better at the shorter horizon and when the shocks are fed into the model in the unanticipated (period-by-period) fashion.

[28] We compute the distance between data and model as $\frac{1}{\overline{H}+1}\sum_{H=0}^{\overline{H}}\left[(IRF_H^{data} - IRF_H^{model})/SE_H^{data}\right]^2$, where IRF_H is the data/model impulse response at horizon H and SE_H^{data} is the estimated standard error of the horizon-H empirical response, and we set $\overline{H} = 36$ (months).

[29] In each period, agents learn the contemporaneous value of the exogenous process, and form model-based expectations about its future evolution. For money supply, they make forecasts based on the random walk in (36), whereas the real energy price index is expected to revert back to steady state as an AR(1) process in logs with persistence 0.96. To implement this exercise, we run a new perfect foresight transition for each time period, using each period's solution as initial conditions for the next period.

[30] See Table B.1 for a mapping between the 38 production sectors used in our quantitative setup and the four broad sectoral groups. Note that the empirical response for Energy uses the "HICP: Liquid fuels and fuels and lubricants for personal transport equipment" series, rather than the broader "HICP: Energy" series, since the former maps more closely to the energy sectors in our theoretical model.

7.2 Large shocks

We now assess whether the interaction between networks and pricing cascades is important for quantitatively explaining macroeconomic dynamics in the Euro Area in the (post-)Covid era, particularly the sizable surges in inflation and the adjustment frequency. To do that, we feed four structurally interpretable shock series into our model, corresponding to the widely perceived major drivers of business cycles: money supply, energy price movements, food price movements, and labor market conditions.

7.2.1 Four exogenous shock series

In our exercise, we consider four exogenous monthly shock series, spanning the period between January 2019 and June 2024.

First, we feed in the Euro Area nominal GDP in order to approximate the aggregate money supply series $\{M_t\}_{2019:1}^{2024:6}$. We treat this series as an amalgamation of monetary and fiscal stance in the (post-)Covid era, capturing the overall aggregate demand conditions. Second, we are fitting an exogenous TFP process in the labor union sector $\{A_t^{LU}\}_{2019:1}^{2024:6}$ to make sure the nominal cost of labor faced by firms exactly matches the observed Euro Area nominal hourly earnings series in equilibrium. Equivalently, this amounts to fitting an exogenous process for the aggregate labor wedge. We believe this is important, since the labor market in our model is much too parsimonious to reconcile the observed wage dynamics, which is in turn crucial for price setting.[31]

Third and fourth, we fit exogenous TFP processes in the "Mining and quarrying" and "Crop and animal production" sectors, $\{A_t^{\text{ENERGY}}\}_{2019:1}^{2024:6}$ and $\{A_t^{\text{FOOD}}\}_{2019:1}^{2024:6}$, in order to exactly match the real IMF Energy Price Index and the IMF Food Price Index as the respective sectoral price indices in equilibrium.[32] In this way, we subject the model to empirically-realistic commodity price shocks, which represent a supply-side influence on aggregate inflation. Since the global commodity prices are largely orthogonal to the Euro Area economic conditions, we believe it is plausible to assume those are driven purely by exogenous sector-specific shocks.

7.2.2 Explaining the surge in frequency and inflation

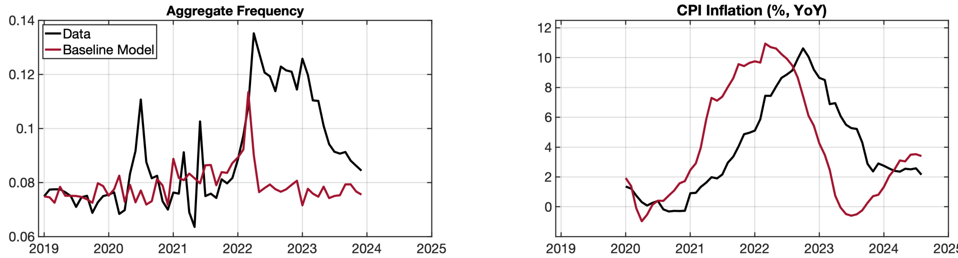
Figure 11(a) shows the observed changes in aggregate monthly adjustment frequency and aggregate inflation in the Euro Area, as well as the variation generated by the four

[31] A more realistic labor market setup would feature search-and-matching frictions with, for example, a bargaining process for the wage.

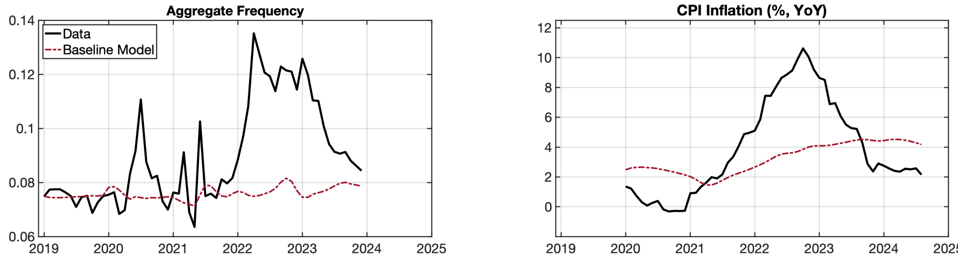
[32] The IMF Energy and Food Price Indices track the respective price movements in US dollars. As before, in order to transform them into model-consistent Euro Area *real* sectoral price indices, we apply two transformations. First, we adjust them by movements in the US dollar/Euro nominal exchange rate. Second, we deflate the exchange rate adjusted series by Euro Area nominal GDP.

Figure 11: **Explaining the observed surge in frequency and inflation (Euro Area)**

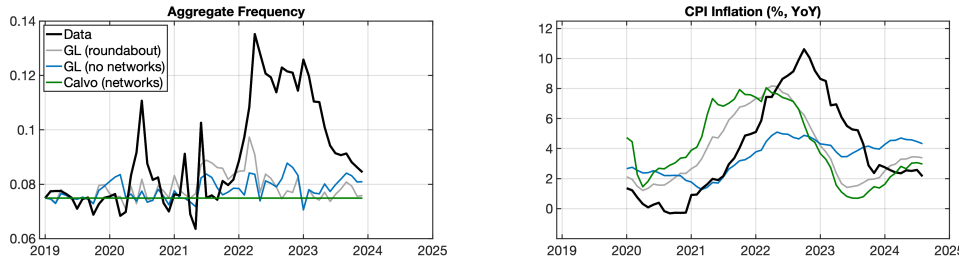
(a) Baseline model: all shocks



(b) Baseline model: no commodity shocks



(c) Alternative models: all shocks



Notes: the figure shows the observed and model-implied changes in the Euro Area aggregate adjustment frequency and year-on-year CPI inflation. Panel (a) considers the baseline model with menu costs and networks, subjected to all four shocks; Panel (b) considers the baseline model, which is subjected to the aggregate demand and labor wedge shocks, but not the energy and food price shocks; Panel (c) considers the models with menu costs and no networks, the model with networks and Calvo (1983) pricing, as well as the model with menu costs and symmetric roundabout production, subjected to all four shocks.

shocks in our baseline non-linear model with menu costs and production networks. In Panel (a), one can see that the baseline model successfully reproduces most of the surge in the aggregate adjustment frequency, as observed in the Euro Area microdata. In addition, the baseline model can also generate the magnitude of the observed increase in aggregate CPI inflation.

In order to discern the relative contribution of supply (commodity) shocks, in Figure 11(b) we compare the actual Euro Area data with the model-implied variation generated exclusively by the aggregate demand and the aggregate labor wedge shocks. It follows that the model produces essentially no surge in the adjustment frequency, whereas the peak inflation response is just below 5%, as opposed to almost 11% in the data. Since it is the supply-side commodity shocks that generate pricing cascades that get amplified by networks, one can see that the key mechanism specific to our model is quantitatively important for matching the observed pricing dynamics.

To highlight that our mechanism requires an *interaction* of large shocks, networks and state-dependent pricing, in Figure 11(c) we consider alternative modeling setups, which omit our model ingredients one-by-one. In particular, an otherwise identical model without networks, when subjected to the same four shock series, produces no major surge in adjustment frequency and less than half of observed inflation at the peak. At the same time, a model with networks and time-dependent Calvo (1983) pricing generates zero variation in frequency by construction, while generating only 7% inflation at the peak. As for the model with menu costs and a symmetric roundabout production structure it also produces around 7% inflation at the peak, while producing less than half of the frequency surge generated by the baseline model with the full network structure, suggesting that input-output asymmetries are quantitatively important.

7.2.3 Extensions and robustness checks

We perform several extensions and robustness checks under large shocks, which are summarized here.

Unanticipated commodity shocks In Figure 11 the model-based responses are produced under the assumption of perfect foresight, so that in January of 2019 the agents know the entire future path of the four shocks considered. However, especially for movements in commodity prices, a more realistic setup is where the agents discover the values period-by-period instead. To assess the implications of such unanticipated shocks, in Figure F.19 we assume that the agents learn the values of food and energy prices only at the beginning of each period.[33] Two important results emerge under such a setup with unanticipated shocks. First, the baseline model with menu costs and networks produces an even larger surge in aggregate monthly adjustment frequency, reaching 0.13 at the peak, matching the entire frequency surge observed in the data. Second, the model-generated inflation still matches the 11% peak observed in the data well.

Comparison with the US While our empirical validation exercises focus on the post-Covid experience in the Euro Area (EA), the surge in adjustment frequency also occurred in other advanced economies, most notably in the United States (US). As one

[33] At the beginning of each period, agents learn the contemporaneous values of TFP shocks in the "Crop and animal production" and "Mining and quarrying" sectors and expect them to revert back to steady state as an AR(1) process with persistence of 0.99. At the same time, we maintain the assumption of perfect foresight over the money growth process and the labor union sector price. Effectively, we run a new perfect foresight transition for each time period, using each period's solution as initial conditions for the next period.

can see in Figure F.20, the US exhibited a near-constant monthly fraction of price adjusters in the run-up to Covid, only to witness a frequency surge by nearly 15 percentage points in early 2022, almost triple the rise observed in the EA (Montag and Villar, 2025). While a full quantitative analysis of the US post-Covid price dynamics is beyond the scope of our paper, we believe the key theoretical principles delivered by our framework can shed light on at least some causes of the US-EA differences. In particular, recall that in (34) we established that the response of the average economy-wide probability of adjustment to a large sectoral TFP shock is approximately proportional to the *supplier Herfindahl* of the shocked sector. Figure B.3(c) compares the values of the *supplier Herfindahl* for our 38 production sectors between the US and the EA. One can see that, relative to the EA, the US *supplier Herfindahl* is notably larger for "Mining and quarrying" (0.11 vs. 0.05) and "Crop and animal production" (0.08 vs. 0.06). For "Mining and Quarrying" in particular, this implies that, all else fixed, an equally large surge in energy prices should produce a more than twice larger rise in the adjustment frequency in the US. In our view, this provides one reason for the larger surge in the monthly fraction of the adjusters in the US, while certainly leaving room for other factors, such as differentially generous fiscal transfers and different consumption baskets relative to the Euro Area.

As a further empirical test of the link between the *supplier Herfindahl* and adjustment frequency dynamics, Figure F.21 reports the estimated responses of the monthly adjustment frequency in the EA and the US to a large (4 standard deviations) oil shock of Känzig (2021).[34] Following such a large surge in energy prices, the monthly fraction of adjusters in the EA is estimated to rise by 0.07 at the peak, compared to 0.13 in the US. The response in the US is almost twice as large, consistent with the *supplier Herfindahl* of the "Mining and Quarrying" sector being approximately twice as large in the US.

8 Conclusions

Recent business cycle episodes have shed light on novel aspects of macro fluctuations, such as inflationary swings driven by large sectoral shocks, as well as the crucial role of the extensive margin of price adjustment. Rationalizing such evidence requires broadening our modeling toolkit, which we do by developing a theoretical framework featuring an economy with production networks, state-dependent pricing and large shocks.

[34] In order to estimate the response to a large shock, we make use of the non-linear local projection approach (Ascari and Haber, 2022): $freq_{t+H} = \alpha_H + \beta_H^{lin} s_t + \beta_H^{sign} s_t^2 + \beta_H^{size} s_t^3 + \boldsymbol{\gamma}'_H \boldsymbol{x}_{t-1} + \varepsilon_{t+H}$, where $freq_{t+H}$ is the aggregate monthly fraction of adjusters H months after the shock and s_t is the identified oil shock. The presence of quadratic and cubic shocks tractably captures the possible non-linearities in the sign and size of the shock, respectively.

The *interaction* of our three ingredients creates a novel theoretical channel, namely pricing *cascades*: large movements in aggregates trigger price adjustment decisions at the extensive margin. Beyond its conceptual novelty, we show that the interaction of networks with pricing cascades is quantitatively important for rationalizing the Euro Area inflationary experience in the (post-)Covid era.

Key to our theoretical mechanism is the differential interaction of networks and pricing cascades, depending on the type of shock driving the business cycle. In particular, under demand shocks, such as central bank interventions, networks *dampen* cascades, slowing down the movements of aggregate adjustment frequency and inflation, and strengthening monetary non-neutrality. On the other hand, networks *amplify* cascades following aggregate or sector-specific supply shocks, leading to a strong inflationary spiral led by the rising fraction of adjusting firms. Quantitatively, we find the network amplification of cascades set off by large movements in energy and food prices to be a crucial contributor to the surges in Euro Area inflation and adjustment frequency between 2020 and 2024.

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Supplemental Appendix

Business Cycles with Pricing Cascades

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A Detailed proofs

Proof of Lemma 1. We want to find a second-order approximation of the firm-level profit function $\tilde{D}_{i,t}(j)$ in the log quality-adjusted real price of that firm $\log \tilde{P}_{i,t}(j)$ near the optimum $\log \tilde{P}^*_{i,t}$. By definition of the optimal reset price, $\frac{\partial \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)}|_{\tilde{P}_{i,t}(j)=\tilde{P}^*_{i,t}} = 0$. As for the second derivative, one can show that:

$$\frac{\partial^2 \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)^2} = \left[(1-\tau_i)e^{(1-\epsilon)\log \tilde{P}_{i,t}(j)}(1-\epsilon)^2 - \epsilon^2 \tilde{\mathcal{Q}}_{i,t}e^{-\epsilon \log \tilde{P}_{i,t}(j)}\right] \times \tilde{P}^{\epsilon}_{i,t}Y_{i,t}. \tag{A.1}$$

Evaluating the second derivative at $\log \tilde{P}^*_{i,t}$, and after some algebra, one obtains:

$$\frac{\partial^2 \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)^2}|_{\tilde{P}_{i,t}(j)=\tilde{P}^*_{i,t}(j)} = -(\epsilon-1)(1-\tau_i)\left[\tilde{P}_{i,t}/\tilde{P}^*_{i,t}\right]^{\epsilon-1}\lambda_{i,t}. \tag{A.2}$$

Therefore, one can write the second-order approximation as:

$$\tilde{D}_{i,t}(j) = \tilde{D}^*_{i,t}(j) + \frac{1}{2}\frac{\partial^2 \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)^2}|_{\tilde{P}_{i,t}(j)=\tilde{P}^*_{i,t}(j)} \times [\tilde{p}_{i,t}(j)]^2 + \mathcal{O}\left(|\tilde{p}_{i,t}(j)|^3\right), \tag{A.3}$$

where $\tilde{p}_{i,t}(j) \equiv [\log \tilde{P}_{i,t}(j) - \log \tilde{P}^*_{i,t}]$ is the firm-level price gap. Inserting the expression for the second derivative, one obtains:

$$\tilde{D}^*_{i,t}(j) - \tilde{D}_{i,t}(j) = \frac{1}{2}(\epsilon-1)(1-\tau_i)\left[\tilde{P}_{i,t}/\tilde{P}^*_{i,t}\right]^{\epsilon-1}\lambda_{i,t}\times[\tilde{p}_{i,t}(j)]^2 + \mathcal{O}\left(|\tilde{p}_{i,t}(j)|^3\right). \tag{A.4}$$

$\square$

Proof of Lemma 2. Focusing on period $t = 0$, a firm adjusts its price if the profit gain from adjustment exceeds the menu cost:

$$\tilde{D}_{i,0}(j)^* - \tilde{D}_{i,0}(j) \geqslant \kappa_{i,0} \tag{A.5}$$

Using the approximation for the profit gain in Lemma 1, as well as the menu cost form in Assumption 7, one can further rewrite the adjustment condition as:

$$\frac{1}{2}(\epsilon-1)(1-\tau_i)\left[\tilde{P}_{i,0}/\tilde{P}^*_{i,0}\right]^{\epsilon-1}\lambda_{i,0}\times[\tilde{p}_{i,0}(j)]^2 \geqslant \overline{\kappa}_i(1-\tau_i)[\tilde{P}_{i,0}/\tilde{P}^*_{i,0}]^{\epsilon-1}\lambda_{i,0}, \tag{A.6}$$

$$\implies \quad [\tilde{p}_{i,0}(j)]^2 \geqslant \frac{2\overline{\kappa}_i}{\epsilon-1}. \tag{A.7}$$

Using the expression for the price gap in (26), as well as the assumption $p_{i,-1}(j) =$

$\log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\tau_i}$, the adjustment condition becomes:

$$\left| -\sigma_i \varepsilon_{i,0}(j) - m_0 + a_{i,0} - \sum_{k=1}^{N} \overline{\omega}_{ik} \log \tilde{P}_{k,0} \right| \geqslant \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}. \tag{A.8}$$

Therefore, the inaction region is given by:

$$[\sigma_i \underline{\varepsilon}_{i,0}, \quad \sigma_i \overline{\varepsilon}_{i,0}] \quad = \quad -m_0 + a_{i,0} - \sum_{k=1}^{N} \overline{\omega}_{ik} \log \tilde{P}_{k,0} \quad \pm \quad \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}, \tag{A.9}$$

where $\quad m_0 \equiv \log(M_0/M_{-1})$ and $\quad a_{i,0} \equiv \log A_{i,0}$. □

Proof of Proposition 1. Consider a monetary shock m, setting all productivity shocks to zero in logs: $a_i = 0, \forall i$. The probability that a firm in sector i draws an idiosyncratic shock outside of the inaction region, and hence chooses to adjust its price, is given by:

$$\varrho_i(m) = 1 - \left[\Xi_i \left(x_i(m) + \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}} \right) - \Xi_i \left(x_i(m) - \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}} \right) \right], \tag{A.10}$$

where Ξ_i is the CDF of $\mathcal{N}(0, \sigma_i^2)$ $x_i(m) \equiv -m - \sum_{j=1}^{N} \overline{\omega}_{ij} \log \tilde{P}_j(m)$.

Consider a second-order approximation of ϱ_i in $x_i(m)$ near steady state $(x_i(m = 0) = 0)$:

$$\varrho_i(x_i(m)) \quad \approx \quad \varrho_i(0) \quad + \quad \varrho_i'(0) x_i \quad + \quad \frac{1}{2}\varrho''(0) x_i^2. \tag{A.11}$$

Given that $\varrho_i'(0) = 0$ and $\varrho_i''(0) = -2\Xi_i'' \left(\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}} \right) > 0$, it follows that the deviation of adjustment probability from steady state $\Delta\varrho_i(m) \equiv \varrho_i(x_i(m)) - \varrho_i(0)$ is:

$$\Delta\varrho_i(m) \quad \approx \quad -\Xi_i'' \left(\sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}} \right) x_i^2(m). \tag{A.12}$$

From the definition of sectoral markup $\mu_i \equiv \log \mathcal{M}_i = \log(P_i/\mathcal{Q}_i)$:

$$\log P_i \quad = \quad \mu_i + \overline{\alpha}_i \log M + \sum_{j=1}^{N} \overline{\omega}_{ij} \log P_j - a_i. \tag{A.13}$$

Stacking across sectors and inverting, it follows that in the absence of sectoral productivity shocks, the real sectoral prices satisfy:

$$\sum_{j=1}^{N} \overline{\omega}_{ij} \log \tilde{P}_j = \sum_{j=1}^{N} (\overline{\Psi}_{ij} - \mathbf{1}_{i=j})\mu_j = N \left[\frac{1}{N} \sum_{j=1}^{N} (\overline{\Psi}_{ij} - \mathbf{1}_{i=j})\mu_j \right] \tag{A.14}$$

$$= N \left[\frac{1}{N} \sum_{j=1}^{N} (\overline{\Psi}_{ij} - \mathbf{1}_{i=j}) \frac{1}{N} \sum_{j=1}^{N} \mu_j + Cov\left((\overline{\Psi} - I)^{(i)}, \boldsymbol{\mu} \right) \right] \tag{A.15}$$

$$= \overline{\mu} \times \mathcal{C}_i + N \times Cov\left((\overline{\Psi} - I)^{(i)}, \boldsymbol{\mu} \right). \tag{A.16}$$

Combining (A.12) and (A.16), it follows that:

$$\frac{1}{\chi_i} \Delta \varrho_i(m) \approx \left[m + \overline{\mu} \times \mathcal{C}_i + N \times Cov\left((\overline{\Psi} - I)^{(i)}, \boldsymbol{\mu} \right) \right]^2, \tag{A.17}$$

where $\chi_i \equiv -\Xi_i'' \left(\sqrt{\frac{2\kappa_i}{\epsilon - 1}} \right) > 0.$ □

Proof of Corollary 1. Consider a monetary shock $m > 0$. The restriction on markups, combined with (A.14) delivers that:

$$-m < \sum_{j=1}^{N} (\overline{\Psi}_{ij} - \mathbf{1}_{i=j})\mu_j < 0 \Leftrightarrow \qquad -m < \sum_{j=1}^{N} \overline{\omega}_{ij} \log \tilde{P}_j < 0$$

$$\Leftrightarrow \qquad 0 < m + \sum_{j=1}^{N} \overline{\omega}_{ij} \log \tilde{P}_j < m.$$

As a result:

$$\chi_i \times \left(m + \sum_{j=1}^{N} \overline{\omega}_{ij} \log \tilde{P}_j \right)^2 < \chi_i \times m^2. \tag{A.18}$$

The LHS above is the probability of adjustment in the baseline economy with networks, whereas the RHS is the probability of adjustment in the otherwise identical economy without networks ($\overline{\omega}_{ij} = 0, \forall i, j$). Hence, networks reduce the probability of adjustment following the monetary shock $m > 0$ for any firm that uses intermediate inputs. □

Proof of Proposition 2. Consider a collection of sectoral productivity shocks $\boldsymbol{a} \equiv \{a_j\}_{j=1}^{N}$, setting the monetary shock to zero in logs: $m = 0$. The probability that a firm in sector i draws an idiosyncratic shock outside of the inaction region, and hence chooses to adjust its price, is given by:

$$\varrho_i(\boldsymbol{a}) = 1 - \left[\Xi_i \left(x_i(\boldsymbol{a}) + \sqrt{\frac{2\kappa_i}{\epsilon - 1}} \right) - \Xi_i \left(x_i(\boldsymbol{a}) - \sqrt{\frac{2\kappa_i}{\epsilon - 1}} \right) \right], \tag{A.19}$$

where Ξ_i is the CDF of $\mathcal{N}(0, \sigma_i^2)$ $x_i(\boldsymbol{a}) \equiv a_i - \sum_{j=1}^{N} \overline{\omega}_{ij} \log \tilde{P}_j(\boldsymbol{a})$.

Consider a second-order approximation of ϱ_i in $x_i(m)$ near steady state ($x_i(\boldsymbol{a} = \mathbf{0}) = 0$):

$$\varrho_i(x_i(\boldsymbol{a})) \approx \varrho_i(\mathbf{0}) + \varrho_i'(\mathbf{0})x_i + \frac{1}{2}\varrho''(\mathbf{0})x_i^2. \tag{A.20}$$

Given that $\varrho_i'(\mathbf{0}) = 0$ and $\varrho_i''(\mathbf{0}) = -2\Xi_i''\left(\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right) > 0$, it follows that the deviation of adjustment probability from steady state $\Delta\varrho_i(\boldsymbol{a}) \equiv \varrho_i(x_i(\boldsymbol{a})) - \varrho_i(\mathbf{0})$ is:

$$\Delta\varrho_i(\boldsymbol{a}) \approx -\Xi_i''\left(\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right) x_i^2(\boldsymbol{a}). \tag{A.21}$$

From the definition of sectoral markup $\mu_i \equiv \log \mathcal{M}_i = \log(P_i/\mathcal{Q}_i)$:

$$\log P_i = \mu_i + \overline{\alpha}_i \log M + \sum_{j=1}^{N} \overline{\omega}_{ij} \log P_j - a_i. \tag{A.22}$$

Stacking across sectors and inverting, it follows that the real sectoral prices satisfy:

$$\sum_{j=1}^{N} \overline{\omega}_{ij} \log \tilde{P}_j = \sum_{j=1}^{N} (\overline{\Psi}_{ij} - \mathbf{1}_{i=j})(\mu_j - a_j) = N\left[\frac{1}{N}\sum_{j=1}^{N} (\overline{\Psi}_{ij} - \mathbf{1}_{i=j})(\mu_j - a_j)\right] \tag{A.23}$$

$$= N\left[\frac{1}{N}\sum_{j=1}^{N}(\overline{\Psi}_{ij} - \mathbf{1}_{i=j})\frac{1}{N}\sum_{j=1}^{N}\mu_j + Cov\left((\overline{\Psi} - I)^{(i)}, \boldsymbol{\mu}\right)\right] - \sum_{j=1}^{N}(\overline{\Psi}_{ij} - \mathbf{1}_{i=j})a_j \tag{A.24}$$

$$= \overline{\mu} \times \mathcal{C}_i + N \times Cov\left((\overline{\Psi} - I)^{(i)}, \boldsymbol{\mu}\right) - \sum_{j=1}^{N}(\overline{\Psi}_{ij} - \mathbf{1}_{i=j})a_j. \tag{A.25}$$

Combining (A.21) and (A.25), it follows that:

$$\frac{1}{\chi_i}\Delta\varrho_i(\boldsymbol{a}) \approx \left[\sum_{j=1}^{N} \overline{\Psi}_{ij} a_j - \overline{\mu} \times \mathcal{C}_i - N \times Cov\left((\overline{\Psi} - I)^{(i)}, \boldsymbol{\mu}\right)\right]^2, \tag{A.26}$$

where $\chi_i \equiv -\Xi_i''\left(\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right) > 0$. □

Proof of Corollary 2. Consider a collection of sectoral productivity shocks $a_j \leqslant 0$, with at least one strict inequality. The restriction on markups, combined with (A.23) delivers that:

$$
\begin{aligned}
\sum_{j=1}^{N}(\overline{\Psi}_{ij}-\mathbf{1}_{i=j})(a_j-\mu_j)<0 \Leftrightarrow & \qquad \sum_{j=1}^{N}\overline{\omega}_{ij}\log\tilde{P}_j>0 \\
\Leftrightarrow & \qquad a_i-\sum_{j=1}^{N}\overline{\omega}_{ij}\log\tilde{P}_j<a_i\leqslant 0, \forall i
\end{aligned}
$$

As a result:

$$
\chi_i\times\left(a_i-\sum_{j=1}^{N}\overline{\omega}_{ij}\log\tilde{P}_j\right)^2>\chi_i\times a_i^2. \tag{A.27}
$$

The LHS above is the probability of adjustment in the baseline economy with networks, whereas the RHS is the probability of adjustment in the otherwise identical economy without networks ($\overline{\omega}_{ij}=0,\forall i,j$). Hence, networks increase the probability of adjustment following the collection of productivity shocks $a_j\leqslant 0$, with at least one strict inequality. □

Proof of Proposition 3. Consider a productivity shock to sector k, setting all other sectoral shocks to zero in logs ($a_j=0,\forall j\neq k$). Then by Proposition 2, the change in adjustment probability in sector i is given by:

$$
\frac{1}{\chi}\Delta\varrho_i(a_k) \quad\approx\quad \left[\ \overline{\Psi}_{ik}a_k-\quad\overline{\mu}(a_k)\times\mathcal{C}_i\quad-\quad N\times Cov\left((\overline{\Psi}-I)^{(i)},\boldsymbol{\mu}(a_k)\right)\ \right]^2. \tag{A.28}
$$

Given the assumption that $Cov\left((\overline{\Psi}-I)^{(i)},\boldsymbol{\mu}(a_k)\right)=0,\forall i$, it follows that:

$$
\frac{1}{\chi}\Delta\varrho_i(a_k) \quad\approx\quad \overline{\Psi}_{ik}^2a_k^2-2\overline{\Psi}_{ik}a_k\overline{\mu}(a_k)\mathcal{C}_i+\overline{\mu}(a_k)^2\mathcal{C}_i^2 \tag{A.29}
$$

$$
\frac{1}{N}\sum_{i=1}^{N}\frac{1}{\chi}\Delta\varrho_i(a_k) \quad\approx\quad a_k^2\frac{1}{N}\sum_{i=1}^{N}\overline{\Psi}_{ik}^2-2a_k\overline{\mu}(a_k)\frac{1}{N}\sum_{i=1}^{N}\overline{\Psi}_{ik}\mathcal{C}_i+\overline{\mu}(a_k)^2\frac{1}{N}\sum_{i=1}^{N}\mathcal{C}_i^2 \tag{A.30}
$$

$$
\frac{1}{N}\sum_{i=1}^{N}\frac{1}{\chi}\Delta\varrho_i(a_k)\approx\mathcal{H}_ka_k^2-2a_k\overline{\mu}(a_k)\left(\frac{1}{N}\sum_{i=1}^{N}\overline{\Psi}_{ik}\frac{1}{N}\sum_{i=1}^{N}\mathcal{C}_i+Cov(\overline{\Psi}_{(k)},\boldsymbol{\mathcal{C}})\right)+\overline{\mu}(a_k)^2\overline{\mathcal{C}^2}. \tag{A.31}
$$

Given the assumption that $Cov(\overline{\Psi}_{(k)},\boldsymbol{\mathcal{C}})=0,\forall k$, one obtains the following expression for the change in the average probability of adjustment:

$$
\frac{1}{\chi}\Delta\varrho(a_k) \quad\approx\quad \mathcal{H}_k\times a_k^2\quad-\quad 2\overline{\mu}(a_k)\times\overline{\mathcal{C}}\times\mathcal{S}_k\times a_k\quad+\quad\overline{\mu}(a_k)^2\overline{\mathcal{C}^2}. \tag{A.32}
$$

□

Corollary 1′ (Cascades dampening under sticky wages). *Suppose Assumptions 1-7 hold. Consider a monetary shock $m > 0$ in an economy where one of the sectors is a labor union (LU). Then, production networks lower the probability of adjustment for any firm in a production sector $i : \overline{\omega}_{i,LU} \in (0,1)$ as long as sectoral markups satisfy $-(m + \mu_{LU}) < \sum_{j \neq LU} (\overline{\Psi}_{ij}^{-LU} - \mathbf{1}_{i=j})\mu_j < 0$, where $\overline{\Psi}^{-LU} \equiv \left(I - \overline{\Omega}^{-LU}\right)^{-1}$ is the Leontief inverse for the input-output matrix that excludes the rows and columns of the labor union sector.*

Proof. The real sectoral price index of any non-labor union sector $(i \neq LU)$ can be written as:

$$\begin{aligned} \log \tilde{P}_i &= \mu_i + \overline{\omega}_{i,LU} \log \tilde{P}_{LU} + \sum_{j \neq LU} \overline{\omega}_{ij} \log \tilde{P}_j \\ &= \mu_i + \log \tilde{P}_{LU} + \sum_{j=1}^{N} \overline{\omega}_{ij} (\log \tilde{P}_j - \log \tilde{P}_{LU}). \end{aligned} \tag{A.33}$$

Stacking $(\log \tilde{P}_i - \log \tilde{P}_{LU})$ across sectors and inverting:

$$(\log \tilde{P}_i - \log \tilde{P}_{LU}) = \sum_{j \neq LU} \overline{\Psi}_{ij}^{-LU} \mu_j, \quad \forall i \neq LU. \tag{A.34}$$

The inequality restrictions in the Corollary imply that:

$$\sum_{j \neq LU} \overline{\omega}_{ij} (\log \tilde{P}_j - \log \tilde{P}_{LU}) < 0, \tag{A.35}$$

and

$$m + \log \tilde{P}_{LU} + \sum_{j \neq LU} \overline{\omega}_{ij} (\log \tilde{P}_j - \log \tilde{P}_{LU}) > 0. \tag{A.36}$$

As a result:

$$\chi_i \times \left(m + \log \tilde{P}_{LU} + \sum_{j \neq LU} \overline{\omega}_{ij} (\log \tilde{P}_j - \log \tilde{P}_{LU}) \right)^2 < \chi_i \times \left(m + \log \tilde{P}_{LU} \right)^2, \quad \forall i \neq LU. \tag{A.37}$$

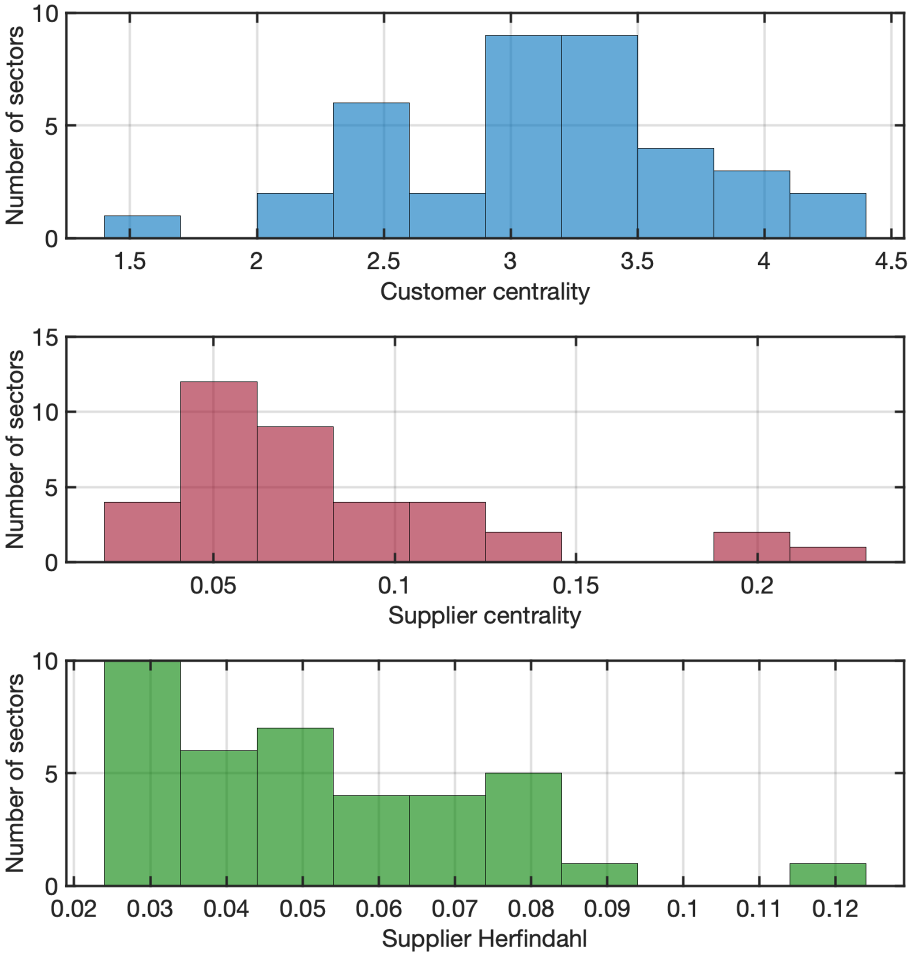
The LHS above is the probability of adjustment for a firm in production sector i in the baseline economy with networks and sticky wages due to a labor union, whereas the RHS is the probability of adjustment in the otherwise identical economy without linkages to other production sectors $(\overline{\omega}_{ij} = 0, \forall j \neq LU)$. Hence, networks reduce the probability of adjustment following the monetary shock $m > 0$ for any production firm that uses intermediate inputs. Note that the real price index of the labor union $\log \tilde{P}_{LU} = \mu_{LU}$ is the same in both economies since the labor union does not purchase inputs from any production sector and the wages paid to workers are exogenous and pinned down by money supply. □

B Additional calibration details (Euro Area)

Table B.1: **Cross-sectional sectoral characteristics**

Sector Name	*Sectoral Group*	*Consumption Share*	*Customer Centrality*	*Supplier Centrality*	*Supplier Herfindahl*
Crop and animal production	Food	0.0290	3.4452	0.0823	0.0595
Fishing and aquaculture	Food	0.0039	3.2103	0.0291	0.0297
Mining and quarrying	Energy	0.0051	3.1919	0.0928	0.0491
Food and beverages	Food	0.1430	3.9448	0.1112	0.0758
Textiles, clothes, leather	Goods	0.0403	3.6296	0.0653	0.0749
Wood and wooden products	Goods	0.0044	3.6121	0.0607	0.0660
Paper and paper products	Goods	0.0079	3.9664	0.0955	0.0785
Printing and recorded media	Goods	0.0035	3.3331	0.0544	0.0359
Coke and petroleum products	Energy	0.0398	4.3485	0.1061	0.0562
Chemicals and chemical products	Energy	0.0162	4.2462	0.1890	0.1175
Pharmaceuticals	Goods	0.0140	3.4977	0.0457	0.0408
Rubber and plastic	Goods	0.0088	3.7245	0.0791	0.0425
Non metallic minerals	Goods	0.0066	3.3615	0.0523	0.0422
Metal products	Goods	0.0081	3.1201	0.1118	0.0671
Computer and electronics	Goods	0.0175	3.2884	0.0773	0.0570
Electrical equipment	Goods	0.0115	3.3162	0.0633	0.0467
Machinery	Goods	0.0066	3.3086	0.0837	0.0506
Motor vehicles	Goods	0.0514	3.8439	0.0670	0.0662
Other transport	Goods	0.0057	3.6416	0.0481	0.0532
Furniture	Goods	0.0224	3.1077	0.0426	0.0338
Repair of machinery	Services	0.0030	2.9322	0.0568	0.0338
Land and pipeline transport	Services	0.0398	2.9952	0.1234	0.0528
Warehousing	Services	0.0125	3.1653	0.1291	0.0768
Accommodation and food services	Services	0.1475	2.9424	0.0527	0.0285
Publishing	Services	0.0138	2.9781	0.0462	0.0304
Movies, video, TV	Services	0.0131	2.9446	0.0600	0.0495
Computer and information services	Services	0.0069	2.4193	0.0922	0.0475
Financial services	Services	0.0391	2.6784	0.1442	0.0724
Insurance and pension	Services	0.0502	3.2978	0.0648	0.0575
Legal, accounting, management	Services	0.0079	2.2538	0.1976	0.0797
Architectural/engineering services	Services	0.0037	2.3319	0.0793	0.0432
Science and R&D	Services	0.0020	2.4415	0.0333	0.0287
Advertising and marketing	Services	0.0020	2.8706	0.0574	0.0337
Other professional activities	Services	0.0069	2.3978	0.0517	0.0313
Administration and support	Services	0.0293	2.4434	0.2261	0.0848
Education	Services	0.0261	1.5508	0.0400	0.0287
Healthcare	Services	0.0743	2.0342	0.0340	0.0309
Other personal services	Services	0.0765	2.3334	0.0631	0.0376

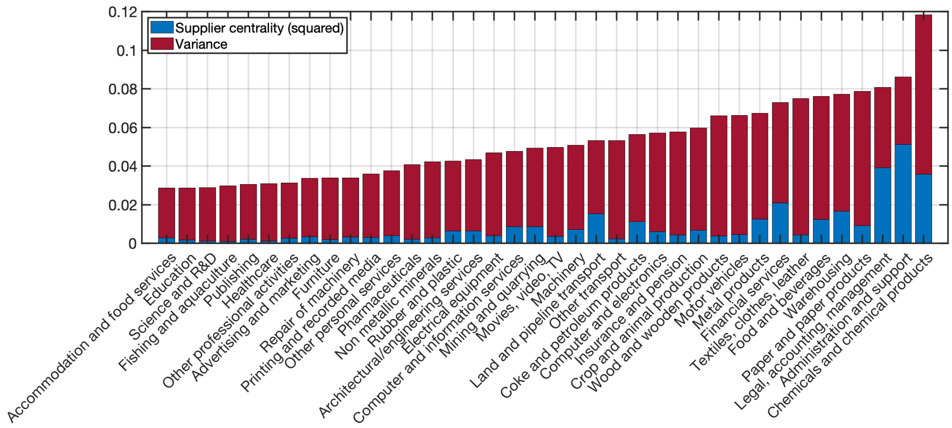
Figure B.2: **Distributions of centrality measures (Euro Area, 38 sectors)**



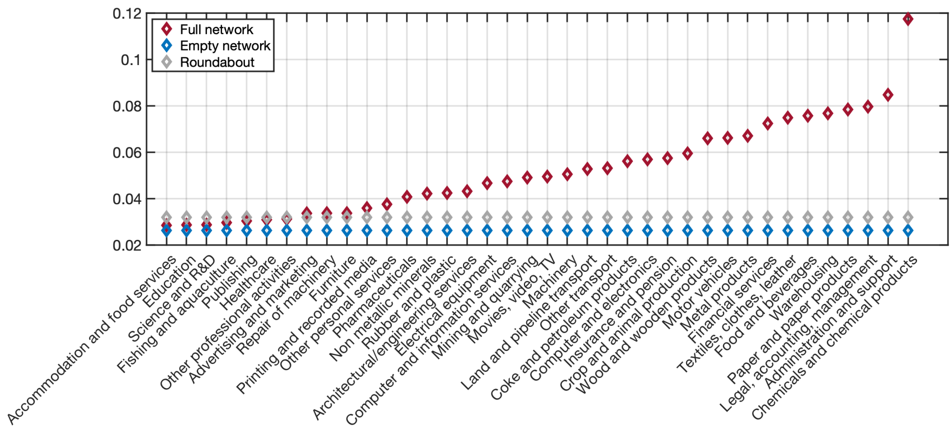
Notes: the panels show the distributions of customer centrality $\mathcal{C}$, supplier centrality $\mathcal{S}$ and the supplier Herfindahl $\mathcal{H}$ measures across the 38 production sectors in the Euro Area.

Figure B.3: **Values of sectoral Supplier Herfindahls $\mathcal{H}_i$**

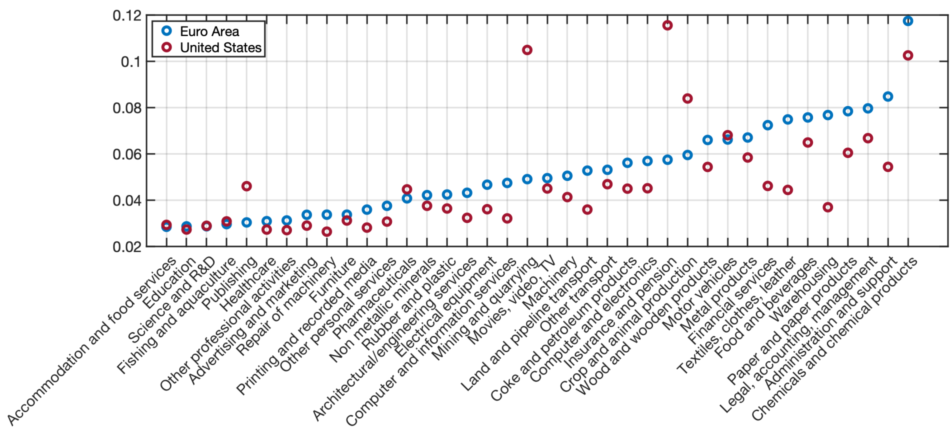
(a) Decomposition of sectoral Supplier Herfindahls (Euro Area)



(b) Sectoral Supplier Herfindahls in alternative economies (Euro Area)

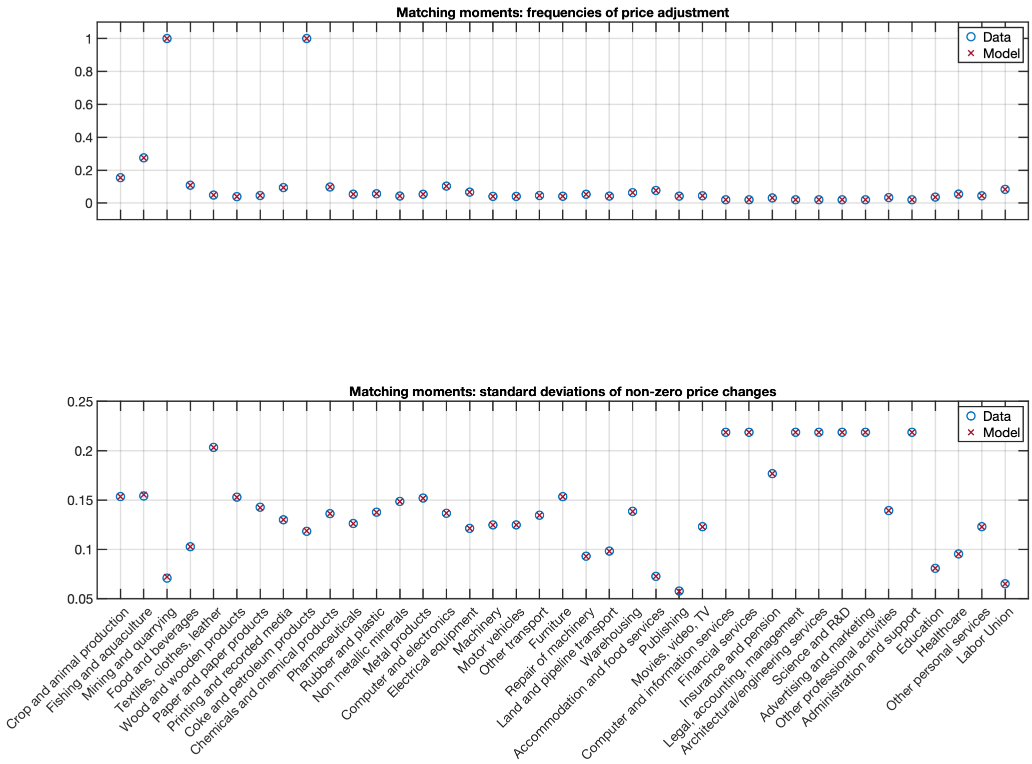


(c) Sectoral Supplier Herfindahls in Euro Area vs. United States



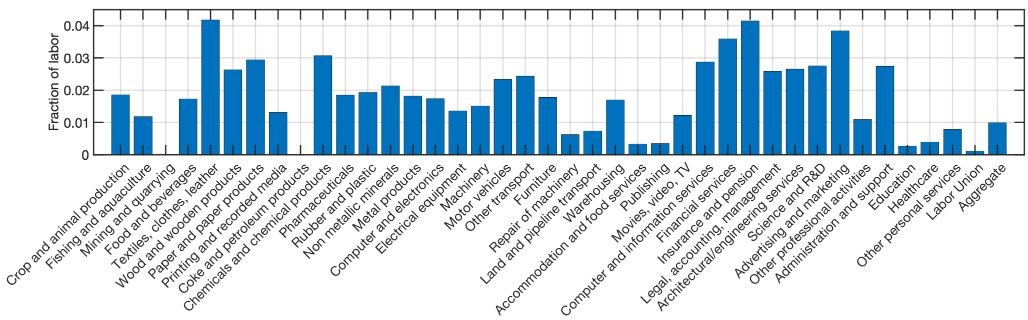
Notes: Panel (a) shows the decomposition of the supplier Herfindahl $\mathcal{H}_i$ into the square of supplier centrality $\mathcal{S}_i^2$ and the variance of the column of the Leontief inverse $Var(\overline{\Psi}_i)$. Panel (b) compares the supplier Herfindahl in the baseline economy with the full network structure to the economy with a symmetric roundabout production, as well as the no-network economy. Panel (c) compares the observed supplier Herfindahls in the Euro Area and the United States.

Figure B.4: **Matching pricing moments for each sector**



Notes: the figure shows the sector-specific frequencies and sizes of price adjustment, as well as the corresponding model-based steady-state values under our estimated values of sectoral menu costs and standard deviations of idiosyncratic quality innovations.

Figure B.5: **Menu cost payment as a fraction of total labor (steady state)**



Notes: the figure shows the fraction of total labor used in every sector that is devoted to menu cost payment in steady state.

Table B.5: **NACE to COICOP Mapping**

Sector name (NACE code)	*Product name (COICOP code)*
Crop and animal production (01)	Rice (011110); Flours and other cereals (011120); Beef and veal (011210); Pork (011220); Lamb and goat (011230); Poultry (011240); Other meat (011250); Fresh or chilled fruit (011610); Fresh or chilled vegetables other than potatoes and other tubers (ND) (011710); Sugar (011810); Salt, spices and culinary herbs (ND) (011920); Cigarettes (022010); Cigars (022020); Other tobacco products (022030); Plants and flowers (093320)
Fishing and aquaculture (03)	Fresh or chilled fish (011310); Fresh or chilled seafood (011330)
Food and beverages (10 & 11)	Rice (011110); Flours and other cereals (ND) (011120); Bread (011130); Other bakery products (011140); Pasta products and couscous (ND) (011160); Breakfast cereals (ND) (011170); Other cereal products (ND) (011180); Beef and veal (011210); Pork (ND) (011220); Poultry (011240); Dried, salted or smoked meat (011270); Other meat preparations (ND) (011280); Fresh or chilled fish (011310); Frozen fish (011320); Fresh or chilled seafood (ND) (011330); Frozen seafood (011340); Dried, smoked or salted fish and seafood (011350); Fresh whole milk (011410); Fresh low fat milk (011420); Preserved milk (011430); Yoghurt (011440); Cheese and curd (011450); Other milk products (011460); Butter (011510); Margarine and other vegetable fats (011520); Olive oil (011530); Other edible oils (011540); Other edible animal fats (011550); Frozen vegetables other than potatoes and other tubers (011720); Dried vegetables, other preserved or processed vegetables (011730); Potatoes (011740); Crisps (ND) (011750); Other tubers and products of tuber vegetables (ND) (011760); Sugar (011810); Chocolate (011830); Confectionery products (011840); Edible ices and ice cream (011850); Artificial sugar substitute (011860); Sauces, condiments (011910); Salt, spices and culinary herbs (011920); Baby food (011930); Ready-made meals (011940); Other food products n.e.c. (011990); Coffee (012110); Tea (012120); Cocoa and powdered chocolate (012130); Mineral or spring waters (012210); Soft drinks (012220); Fruit and vegetable juices (012230); Spirits and liqueurs (021110); Alcoholic soft drinks (021120); Wine from grapes (021210); Wine from other fruits (021220); Fortified wines (021230); Wine-based drinks (021240); Lager beer (021310); Other alcoholic beer (021320); Low and non-alcoholic beer (021330); Beer-based drinks (021340)
Textiles, clothes, leather (13, 14 & 15)	Clothing materials (031100); Garments for men (031210); Garments for women (031220); Garments for infants (0 to 2 years) and children (3 to 13 years) (031230); Other articles of clothing (031310); Clothing accessories (031320); Footwear for men (032110); Footwear for women (032120); Footwear for infants and children (032130); Carpets and rugs (051210); Furnishing fabrics and curtains (052010); Bed linen (052020); Table linen and bathroom linen (052030); Other household textiles (052090); Other personal effects n.e.c. (123290)
Wood and wooden products (16)	Household furniture (051110); Garden furniture (051120); Other furniture and furnishings (051190); Other floor coverings (051220); Non-motorised small tools (055210)
Paper and paper products (17)	Paper products (095410)
Printing and recorded media (18)	Equipment for the reception, recording and reproduction of sound (091110); Equipment for the reception, recording and reproduction of sound and vision (091120); Portable sound and vision devices (091130); Other equipment for the reception, recording and reproduction of sound and picture (091190); Cameras (091210); Accessories for photographic and cinematographic equipment (091220); Optical instruments (091230); Newspapers (095210); Magazines and periodicals (095220); Miscellaneous printed matter (095300)

Continued on next page

Table B.5: **NACE to COICOP Mapping**

Sector name (NACE code)	*Product name (COICOP code)*
Chemical and chemical products (20)	Cleaning and maintenance products (056110); Articles for personal hygiene and wellness, esoteric products and beauty products (121320)
Pharmaceuticals (21)	Pharmaceutical products (061100); Pregnancy tests and mechanical contraceptive devices (061210); Other medical products n.e.c. (061290
Rubber and plastic (22)	Non-electric kitchen utensils and articles (054030); Tyres (072110)
Non metallic minerals (23)	Glassware, crystal-ware, ceramic ware and chinaware (054010)
Metal products (24 & 25)	Heaters, air conditioners (053140); Cutlery, flatware and silverware (054020); Non-electric kitchen utensils and articles (054030); Miscellaneous small tool accessories (055220)
Computer and electronics (26)	Other medical products n.e.c. (061290); Fixed telephone equipment (082010); Mobile telephone equipment (082020); Other equipment of telephone and telefax equipment (082030); Equipment for the reception, recording and reproduction of sound (091110); Equipment for the reception, recording and reproduction of sound and vision (091120); Portable sound and vision devices (091130); Other equipment for the reception, recording and reproduction of sound and picture (091190); Cameras (091210); Accessories for photographic and cinematographic equipment (091220); Optical instruments (091230); Personal computers (091310); Accessories for information processing equipment (091320); Calculators and other information processing equipment (091340); Clocks and watches (123120)
Electrical equipment (27)	Lighting equipment (051130); Refrigerators, freezers and fridge-freezers (053110); Clothes washing machines, clothes drying machines and dish washing machines (053120); Cookers (053130); Heaters, air conditioners (053140); Cleaning equipment (053150); Other major household appliances (053190); Food processing appliances (053210); Coffee machines, tea makers and similar appliances (053220); Irons (053230); Toasters and grills (053240); Other small electric household appliances (053290); Motorised major tools and equipment (055110); Fixed telephone equipment (082010)
Machinery (28)	Motorised major tools and equipment (055110)
Motor vehicles (29)	Motorised major tools and equipment (055110); New motor cars (071110); Motor cycles (071200)
Other transport (30)	Motor cycles (071200); Bicycles (071300); Animal drawn vehicles (071400); Camper vans, caravans and trailers (092110); Aeroplanes, microlight aircraft, gliders, hang-gliders and hot-air balloons (D) (092120); Boats, outboard motors and fitting out of boats (D) (092130)
Furniture (31 & 32)	Household furniture (051110); Garden furniture (051120); Other furniture and furnishings (051190)
Repair of machinery (33)	Repair of household appliances (053300); Repair, leasing and rental of major tools and equipment (055120); Repair of non-motorised small tools and miscellaneous accessories (055230); Repair of therapeutic appliances and equipment (061330); Maintenance and repair of personal transport equipment (072300); Repair of audiovisual, photographic and information processing equipment (091500); Maintenance and repair of other major durables for recreation and culture (092300); Repair of equipment for sport, camping and open-air recreation (093230); Repair of electric appliances for personal care (121220); Repair of other personal effects (123230)
Land and pipeline transport (49, 50 & 51)	Passenger transport by train (073110); Passenger transport by underground and tram (073120); Passenger transport by bus and coach (073210); Passenger transport by taxi and hired car with driver (073220); Removal and storage services (S) (073620); Other purchased transport services n.e.c. (073690)
Warehousing (52)	Removal and storage services (S) (073620); Other purchased transport services n.e.c. (073690)

Continued on next page

Table B.5: **NACE to COICOP Mapping**

Sector name (NACE code)	*Product name (COICOP code)*
Accommodation and food services (55 & 56)	Restaurants, cafés and dancing establishments (111110); Fast food and take away food services (111120); Canteens (111200); Hotels, motels, inns and similar accommodation services (112010); Holiday centres, camping sites, youth hostels and similar accommodation services (112020); Accommodation services of other establishments (112030)
Publishing (58)	Software (091330); Fiction books (095110); Educational text books (095120); Other non-fiction books (095130); Binding services and E-book downloads (095140); Newspapers (095210); Magazines and periodicals (095220); Miscellaneous printed matter (095300)
Movies, video, TV (59)	Cinemas, theatres, concerts (094210); Other cultural services (094290)
Computer and information services (60-63)	Wired telephone services (S) (083010); Wireless telephone services (S) (083020); Internet access provision services (083030); Bundled telecommunication services (S) (083040); Other information transmission services (S) (083050); Software (091330); Television and radio licence fees, subscriptions (094230); Other fees and services (127040)
Financial services (64)	Fees and service charges of brokers, investment counsellors (126220); Legal services and accountancy (127020); Other fees and services (127040)
Insurance and pension (65 & 66)	Insurance connected with the dwelling (125200); Private insurance connected with health (125320); Motor vehicle insurance (125410); Travel insurance (125420); Other insurance (125500); Fees and service charges of brokers, investment counsellors (126220); Legal services and accountancy (127020); Other fees and services (127040)
Legal, accounting, management (69&70)	Fees and service charges of brokers, investment counsellors (126220); Legal services and accountancy (127020); Other fees and services (127040)
Architectural/engineering services (71)	Other services related to dwelling (044490); Other fees and services (127040)
Science and R&D (72)	Other fees and services (127040)
Advertising and marketing (73)	Other fees and services (127040)
Other professional activities (74)	Other fees and services (127040)
Administration and support (82&84)	Administrative fees (127010); Other fees and services (127040)
Education (85)	Pre-primary education (ISCED-97 level 0) (101010); Primary education (ISCED-97 level 1) (101020); Secondary Education (102000); Secondary Education (102001); Post-Secondary Non-Tertiary Education (103000); Tertiary Education (104000); Education Not Definable By Level (105000); Education Not Definable By Level (105001); Education Not Definable By Level (105002); Education Not Definable By Level (105003)
Healthcare (86)	General practice (062110); Specialist practice (062120); Dental services (062200); Services of medical analysis laboratories and X-ray centres (062310); Thermal-baths, corrective-gymnastic therapy, ambulance services and hire of therapeutic equipment (062320); Other paramedical services (062390); Hospital services (063000)
Other personal services (96)	Cleaning of clothing (031410); Recreational and sporting services — Attendance (094110); Recreational and sporting services — Participation (094120); Hairdressing for men and children (121110); Hairdressing for women (121120); Personal grooming treatments (121130); Funeral services (127030)

C Details of the numerical algorithm

C.1 Steady state computation on a grid

For each sector, we solve the stationary Bellman equation and price distribution on an evenly spaced grid of log prices $\mathbf{\Gamma}$ with step size Δp, $p_j \in \left[\underline{p}, \underline{p}+\Delta p, ..., \overline{p}\right]$, $j = 1, .., J$ grid points, so that $V_j = V(p_j)$. The expectation $\mathbb{E}\left[V(p - \sigma\varepsilon_{t+1} - \pi)|p = p_j\right]$ is calculated as $\boldsymbol{TV}$ where we define transition matrix

$$\boldsymbol{T} = \begin{bmatrix} \mathcal{T}_{1,1} & \mathcal{T}_{1,2} & \cdots & \mathcal{T}_{1,J} \\ \mathcal{T}_{2,1} & \mathcal{T}_{2,2} & \cdots & \mathcal{T}_{2,J} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{T}_{J,1} & \mathcal{T}_{J,2} & \cdots & \mathcal{T}_{J,J} \end{bmatrix}.$$

with elements

$$\mathcal{T}_{j,k} = \Psi\left(\frac{p_{k+1/2} - (p_j - \pi)}{\sigma}\right) - \Psi\left(\frac{p_{k-1/2} - (p_j - \pi)}{\sigma}\right),$$

and where $p_{k-1/2} \equiv (p_{k-1} + p_k)/2$, $p_{k+1/2} \equiv (p_k + p_{k+1})/2$ and $\Psi(\cdot)$ is the standard normal cumulative distribution function.

We also define the vectors

$$\boldsymbol{\phi} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_J \end{bmatrix}, \ \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_J \end{bmatrix}, \ \boldsymbol{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_J \end{bmatrix}, \ \boldsymbol{D} = \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_J \end{bmatrix}$$

The Bellman equation in matrix notation is then given by

$$\boldsymbol{V} = \boldsymbol{D} + \beta\left[\boldsymbol{T}\left((1-\eta)\cdot\boldsymbol{V}\right) + \boldsymbol{T}\left(\eta\cdot(\boldsymbol{\phi}'\boldsymbol{V} - \kappa w)\right)\right]$$

where $\cdot$ denotes element-by-element multiplication. Vector $\boldsymbol{\phi}$ distributes unit probability mass to grid points adjacent to p^* according to the logit formula

$$\boldsymbol{\phi} = \frac{\exp(\chi\boldsymbol{V})}{\sum_{\Gamma}\exp\left(\chi\boldsymbol{V}\right)}$$

with precision $\chi = 50000$. Note that $\boldsymbol{\phi}'\boldsymbol{V}$ performs smooth maximization as in eq.(19).

To solve the problem for N sectors with input-output linkages, we use the following algorithm. Start with a guess for the vector of steady-state price dispersions Δ_k and sectoral taxes τ_k, then:[35]

1. Given $\pi = \overline{\pi}$, compute the transition matrix $\boldsymbol{T}$
2. Using $\frac{W}{M} \equiv w = 1$, compute[36] $\omega_{ik} = \varpi_{ik} \times \frac{(P_k/M)^{1-\theta_i}}{\overline{\alpha}_i + \sum_{k'=1}^{N} \varpi_{ik'}(P_{k'}/M)^{1-\theta_i}} = \varpi_{ik}$
3. λ is given by eq.(13) and η by eq.(20)
4. With that, construct the profit matrix $\boldsymbol{D}$ as in eq.(17)
5. Iterate backward on the value function $\boldsymbol{V}$ above to convergence
6. To compute the distribution, iterate forward on

$$\boldsymbol{g} = (1-\eta) \cdot (\boldsymbol{T}'\boldsymbol{g}) + \boldsymbol{\phi}\eta' (\boldsymbol{T}'\boldsymbol{g}) \,. \tag{C.1}$$

until convergence of $\boldsymbol{g}$.

7. Given the distribution, compute the residual vectors $resid_1$ and $resid_2$ as in

$$resid_1 = \Delta_k - (P_k/M)^{\epsilon} \int_0^1 \left(\frac{P_k(j')}{\zeta_k(j')M} \right)^{-\epsilon} dj', \tag{C.2}$$

$$resid_2 = (P_k/M)^{1-\epsilon} - \int_0^1 \left(\frac{P_k(j')}{\zeta_k(j')M} \right)^{1-\epsilon} dj' \tag{C.3}$$

8. Search for a vector of sectoral price dispersions and taxes such that $resid \rightarrow 10^{-12}$.

C.2 Solving for impulse-responses in sequence space

We compute fully non-linearly the responses to an MIT shock in the space of sequences, iterating backward in time on the value function and forward in time on the law of motion of the distribution, under the assumption of perfect foresight. The steps are similar to those for computing the steady state; only this time we keep track of the sequences over time. We start by guessing sequences for time t from 0 to $T = 500$ months, for sectoral prices and price dispersions (our starting guess simply equals the steady-state value for these variables). The key assumption is that all stationary variables must return to steady state by period T. Given this initial guess, we compute the price of the final good and consumption over time using their definitions. Given that, we calculate λ_t as in eq.(13). We compute the profits D_t as in eq.(17). Iterating backward in time

[35]We start with the guess $\Delta_k = 1$ and $\tau_k = -1/\epsilon$

[36]We are searching for taxes τ_k such that the steady state equilibrium is symmetric in sectoral prices, $P_k/M = 1$

from $t = T$ to $t = 0$, we solve for the value function as in eq.(19). Given the value function, we can compute the gain from adjustment $\mathcal{L}_t$ and the adjustment hazard η_t. Once the backward iteration on the value function reaches period 0, we start from the steady-state distribution and iterate forward in time on the law of motion of the price distribution from period 1 until period T. Given the distribution, we can compute real sectoral price indices as $P_{i,t}/M_t = \left[\int_0^1 \left(\frac{P_{i,t}(j)}{\zeta_{i,t}(j)M_t}\right)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}$, and the sectoral price dispersions by $\Delta_{k,t} \equiv (P_{k,t}/M_t)^{\epsilon} \int_0^1 \left(\frac{P_{k,t}(j')}{\zeta_{k,t}(j')M_t}\right)^{-\epsilon} dj'$. This provides us with an updated guess, with which we repeat the previous steps until the change in the sequences (of sectoral prices and price dispersions) falls below a convergence tolerance.

D Quantitative model: extensions and robustness checks

D.1 Endogenous monetary policy

In our baseline results, the central bank conducts policy by setting an exogenous path of money supply. We now consider an extension that adds realism to the monetary policy conduct. In particular, we use the cashless limit setup of Woodford (2004) and Galí (2015), where the central bank conducts policy by setting the level of the nominal interest rate, which also responds endogenously to movements in macroeconomic aggregates.

The representative household chooses a sequence of consumption, labor supply and one-period nominal bond holdings to maximize expected lifetime utility:

$$\max_{\{C_t, L_t, B_t\}_{t \geqslant 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \tag{D.1}$$

subject to the period-by-period budget constraint

$$P_t^C C_t + B_t \leqslant (1 + i_{t-1}) B_{t-1} + W_t L_t + \sum_{i=1}^{N} \int_0^1 D_{i,t}(j) dj + T_t, \tag{D.2}$$

where C_t is consumption, L_t is labor supply, B_t is the level of nominal bond holdings, T_t is the level of lump-sum transfers from the government, $D_{i,t}(j)$ are the dividends received lump-sum from firm j in sector i at time t, $\Pi_t^C = \left(P_t^C / P_{t-1}^C\right)$ is the gross CPI inflation rate, W_t is the nominal wage and i_t is the nominal interest rate set by the central bank.

The nominal interest rate follows the following Taylor-type rule:

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + (1 - \rho_i) \left[\phi_\pi \hat{\pi}_t^C + \phi_c c_t\right] + \varepsilon_t^i, \tag{D.3}$$

where $\hat{\imath}_t \equiv \log \frac{1+i_t}{\overline{\Pi}/\beta}$ is the log-deviation of the nominal interest rate from its steady-state value, $\hat{\pi}_t^C \equiv \log(\Pi_t^C / \overline{\Pi})$ is deviation of CPI inflation from target and $c_t \equiv \log C_t / \overline{C}$ is aggregate GDP in deviation from steady state. In the rule, $\rho_i \in [0, 1)$ determines the degree of policy persistence, $\phi_\pi > 0$ and $\phi_c > 0$ pin down how aggressively the central bank responds to deviations of inflation and GDP from their steady-state values, and ε_t^i is the monetary policy shock.

We assume the following form of households' preferences:

$$u(C_t, L_t) = \frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \frac{L_t^{1+\varphi}}{1 + \varphi}. \tag{D.4}$$

Note that the Golosov-Lucas log-linear preferences which we use in the main text arise as a special case when $\sigma \to 1$ and $\varphi = 0$.

Given the presence of possibly non-zero steady-state inflation and the non-stationarity of the quality processes, we appropriately normalize our variables. Unlike in the main text, where we normalize by money supply, in the current cashless setting, we instead normalize by the aggregate CPI price level P_{t-1}^C. In particular, we let $\tilde{P}_{i,t}(j) \equiv \frac{P_{i,t}(j)}{\zeta_{i,t}(j)P_{t-1}^C}$ be the quality-adjusted real price, $\tilde{P}_{i,t} \equiv \frac{P_{i,t}}{P_{t-1}^C}$ be the real sectoral price, and $\tilde{W}_t \equiv \frac{W_t}{P_{t-1}^C}$ be the real wage. Then the equilibrium conditions for the aggregate real variables are given by:

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left[\frac{1+i_t}{\Pi_{t+1}^C} C_{t+1}^{-\sigma} \right] \tag{D.5}$$

$$C_t^{\sigma} L_t^{\varphi} = \tilde{W}_t / \Pi_t^C \tag{D.6}$$

$$L_t = \Pi_t^C \frac{C_t}{\tilde{W}_t} \left[1 - \sum_{i=1}^{N} \lambda_{i,t} \left(1 - \frac{\Delta_{i,t}}{\mathcal{M}_{i,t}} \right) \right] + \sum_{i=1}^{N} \kappa_{i,t} \int_0^1 \eta_{i,t}(j) dj. \tag{D.7}$$

where $\lambda_{i,t}$ is the sectoral Domar weight (sales) share, $\Delta_{i,t}$ is the within-sector dispersion of real prices, and $\mathcal{M}_{i,t}$ is the sectoral markup, which are given by:

$$\lambda_{i,t} = \omega_{i,t}^C + \sum_{k=1}^{N} \omega_{k,i,t} \lambda_{k,t} \frac{\Delta_{k,t}}{\mathcal{M}_{k,t}}, \qquad \Delta_{i,t} \equiv \tilde{P}_{i,t}^{\epsilon} \int_0^1 \tilde{P}_{i,t}(j)^{-\epsilon} dj, \qquad \mathcal{M}_{i,t} \equiv \frac{\tilde{P}_{i,t}}{\tilde{\mathcal{Q}}_{i,t}}. \tag{D.8}$$

The real sectoral price indices and marginal costs in turn satisfy:

$$\tilde{P}_{i,t}^{1-\epsilon} = \int_0^1 \tilde{P}_{i,t}(j)^{1-\epsilon} dj, \quad \tilde{\mathcal{Q}}_{i,t} = \mathcal{Q}_i \left[\tilde{W}_t, \tilde{P}_{1,t}, ..., \tilde{P}_{N,t}; A_{i,t} \right], \quad \Pi_t^C = \mathcal{P}^C \left[\tilde{P}_{1,t}, ..., \tilde{P}_{N,t} \right]. \tag{D.9}$$

If the nominal price is not adjusted, then the quality-adjusted real price evolves according to:

$$p_{i,t}(j) = p_{i,t-1}(j) - \sigma_i \varepsilon_{i,t}(j) - \pi_{t-1}^C, \tag{D.10}$$

where $\pi_{t-1}^C \equiv \log \Pi_{t-1}^C$.

The per-period real profits of a firm are given by:

$$\tilde{D}_{i,t}(j) = \tilde{P}_{i,t}^{\epsilon-1} \left[(1-\tau_{i,t}) \tilde{P}_{i,t}(j) - \tilde{\mathcal{Q}}_{i,t} \right] \tilde{P}_{i,t}(j)^{-\epsilon} \times \lambda_{i,t} \times C_t \times \Pi_t^C. \tag{D.11}$$

Finally, consider a firm with real quality-adjusted price p at the end of period t, and let $p_+ \equiv (p - \sigma_i \varepsilon_{i,t+1}(j) - \pi_t^C)$, where $\pi_t^C \equiv \log \Pi_t^C$. Then this firm's real value at the end of period t is given by the following Bellman equation:

$$\begin{aligned} V_{i,t}(p) &= \tilde{D}_{i,t}(p) + \\ &+ \beta \mathbb{E}_t \left[\{1 - \eta_{i,t+1}(p_+)\} \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{\Pi_t^C}{\Pi_{t+1}^C} V_{i,t+1}(p_+) \right] + \\ &+ \beta \mathbb{E}_t \left[\eta_{i,t+1}(p_+) \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{\Pi_t^C}{\Pi_{t+1}^C} \left(\max_{p'} V_{i,t+1}(p') - \kappa_{i,t+1} \tilde{W}_{t+1} \right) \right]. \end{aligned}$$

To facilitate closer comparability with the baseline results in the cash economy, we set $\sigma = 1$ and $\varphi = 0$. As for the Taylor rule, under our monthly calibration, we set $\rho_i = 0.9$, $\phi_\pi = 1.5$ and $\phi_c = 0.5/12$.

In Figure D.1 we report impulse responses of the aggregate adjustment frequency, GDP, inflation, and the nominal interest rate to an annualized one-time monetary policy shock of -500 basis points. It follows that in the baseline economy with networks, the aggregate frequency rises up to only 0.11, as opposed to 0.13 without networks. Therefore, cascades dampening carries through even under endogenous monetary policy. Moreover, one can also see that the cumulative GDP response is larger in the economy with networks, so that cascades dampening contributes towards additional monetary non-neutrality.

As for supply shocks, in Figure D.2 we report the responses of aggregates to a one-time transitory ($\rho = 0$) aggregate TFP shock of -5%. We find that the response of frequency is stronger in the economy with networks, so that cascades amplification also holds under endogenous monetary policy. One can also see that the response of aggregate CPI inflation is larger under networks, marking the contribution of cascades amplification.

D.2 Stochastic menu costs

In our baseline results, we work under the assumption that nominal price re-setting is subject to a fixed sector-specific menu cost as in Golosov and Lucas (2007). In order to illustrate that our channel of interaction between networks and pricing cascades is not limited to the fixed menu cost setup, as an extension, we consider a stochastic menu cost setup instead.[37] More specifically, we use the CalvoPlus setup of Nakamura and Steinsson (2010), which assumes that each period a randomly selected fraction of firms within each sector draws a menu cost of zero, whereas the complementary fraction is

[37]We study the random menu cost setup in the context of the cash economy. However, we can also feasibly study random menu costs in the cashless limit.

still subject to the fixed menu cost.

Formally, the CalvoPlus setup corresponds to the following functional form of the probability of adjustment function $\eta_{i.t}(.)$:

Assumption 4′ (CalvoPlus pricing). *Consider a firm in sector i with the quality adjusted log relative price p at time t. Then the probability that this firm adjusts its nominal price is given by:*

$$\eta_{i,t}(p) = \ell_i + (1 - \ell_i) \times \mathbf{1}(L_{i,t}(p) > 0) \tag{D.12}$$

where ℓ_i is the sectoral probability of drawing a zero menu cost, $\mathbf{1}(\cdot)$ is the indicator function, and

$$L_{i,t}(p) = \max_{p'} V_{i,t}(p') - V_{i,t}(p) - \overline{\kappa}_i \tag{D.13}$$

is the gain from adjustment (or loss from inaction), net of the menu cost.

Crucially, as the non-zero menu cost tends to infinity ($\overline{\kappa}_i \to \infty$), the pricing problem collapses to the time-dependent model of Calvo (1983), as only the randomly selected fraction ℓ_i in each sector gets to adjust. At the same time, setting the probability of drawing a zero menu cost to zero ($\ell_i = 0$) collapses the pricing problem in that sector to the fixed menu cost setup of Golosov and Lucas (2007).

In order to quantitatively discipline the probabilities of free adjustment, we estimate them so that, in steady state around 75% of all price adjustments are free in each sector, following Nakamura and Steinsson (2010) and Blanco et al. (2024c). As before, the non-zero menu costs and standard deviation of idiosyncratic shocks are estimated to jointly match the sector-specific frequencies and standard deviations of price changes in the Euro Area.

In Figure D.3 we study the responses of aggregate repricing frequency and GDP to monetary shocks of different sizes under CalvoPlus pricing. In panel (a) one can see that the response of aggregate repricing frequency, both with and without networks, is dampened relative to otherwise identical economies with fixed menu costs. This is because the presence of free adjustment opportunities implies that much larger shocks are needed for firms to get pushed out of their inaction region. At the same time, just like in the economy with fixed menu costs, the economy with networks features smaller frequency movements, which is the effect of dampening pricing cascades. As for the GDP responses in panel (b), the economy with networks and random menu costs features much stronger non-neutrality than an otherwise identical economy without networks.

As for the propagation of supply shocks, in Figure D.4 we report the responses of aggregate repricing frequency and CPI inflation to aggregate TFP shocks of different sizes. Panel (a) shows that, as with monetary shocks, the introduction of random menu costs dampens the responses of frequency to aggregate TFP shocks, both with and without networks. At the same time, one can see that conditional on CalvoPlus pricing, the economy with networks features stronger movements in aggregate frequency, implying that networks amplify cascades, just as in the economy with fixed menu costs. In Figure D.5 we show the responses of aggregate frequency to sectoral TFP shocks, confirming the cascades amplification effect under random menu costs.

D.3 Alternative elasticity of substitution across sectors

In our baseline analysis, we use Cobb-Douglas aggregation across sectors, as well as a Cobb-Douglas production technology. In this subsection, we relax this assumption and consider more general constant elasticity of substitution (CES) aggregation across sectoral consumptions, as well as across productive inputs.

First, we consider the following CES final consumption aggregator:

Assumption 2′ (CES consumption aggregation). *The consumption aggregator $\mathcal{D}(\cdot)$ is given by:*

$$\mathcal{D}(C_{1,t},...,C_{N,t}) = \left(\sum_{i=1}^{N} \varpi_i^{C\,\frac{1}{\theta_c}} C_{i,t}^{\frac{\theta_c - 1}{\theta_c}} \right)^{\frac{\theta_c}{\theta_c - 1}}, \tag{D.14}$$

where $\theta_c > 0$ is the elasticity of substitution and $\sum_i \varpi_i^C = 1$, $\varpi_i^C \geqslant 0, \forall i$.

Under this assumption, the equilibrium final consumption shares are given by:

$$\omega_{i,t}^C = \varpi_i^C \times \frac{\tilde{P}_{i,t}^{1-\theta_c}}{\sum_{k=1}^{N} \varpi_k^C \tilde{P}_{k,t}^{1-\theta_c}} \tag{D.15}$$

which is constant in the special case when the sectoral consumption aggregator is Cobb-Douglas ($\theta_c = 1$). It follows that the final consumption shares are time-varying and depend on relative movements in (real) sectoral price indices. Whenever final sectoral varieties are complements, $\theta_c \in (0,1)$, a relative increase in a sectoral price index leads to a rise in that sector's final consumption share, and *vice versa* whenever the varieties are substitutes, $\theta_c > 1$.

Similarly, we also assume the following CES production technology:

Assumption 3′ (CES production technology). *The production technology $\mathcal{F}_i(\cdot)$ for a firm j in sector i is given by:*

$$\mathcal{F}_i[L_{i,t}(j), X_{i,1,t}(j), ..., X_{i,N,t}(j)] = \left(\overline{\alpha}_i^{\frac{1}{\theta_i}} L_{i,t}^{\frac{\theta_i - 1}{\theta_i}}(j) + \sum_{k=1}^{N} \varpi_{ik}^{\frac{1}{\theta_i}} X_{i,k,t}^{\frac{\theta_i - 1}{\theta_i}}(j) \right)^{\frac{\theta_i}{\theta_i - 1}}, \tag{D.16}$$

where $\theta_i > 0$ is the elasticity of substitution and $\overline{\alpha}_i + \sum_k \varpi_{ik} = 1$, $\overline{\alpha}_i, \varpi_{ik} \geqslant 0, \forall i$.

Such a production technology delivers the following equilibrium cost shares of labor and intermediate inputs:

$$\alpha_{i,t} = \overline{\alpha}_i \times \frac{1}{\overline{\alpha}_i + \sum_{k'=1}^{N} \overline{\omega}_{ik'} \tilde{P}_{k',t}^{1-\theta_i}}, \qquad \omega_{ik,t} = \overline{\omega}_{ik} \times \frac{\tilde{P}_{k,t}^{1-\theta_i}}{\overline{\alpha}_i + \sum_{k'=1}^{N} \overline{\omega}_{ik'} \tilde{P}_{k',t}^{1-\theta_i}} \tag{D.17}$$

which are constant in the special case when the production function is Cobb-Douglas ($\theta_i = 1$). As with consumption aggregation, time variation in the input cost shares is pinned down by relative movements in (real) input prices. As before, whenever inputs are complements, a relative increase in the price of an input leads to an increase in the cost share of that input, and *vice versa* whenever inputs are substitutes.

We now revisit our key quantitative exercises in an economy with fixed menu costs and CES aggregation. We calibrate $\theta_c = \theta_i = 0.001, \forall i$, to consider an economy where goods are almost perfect complements, capturing the potential difficulty of substituting both consumption and production varieties. This may represent the supply chain disruptions that we observed during and after the Covid pandemic across the globe.

In Figure D.6, we study the propagation of monetary shocks in our economy with CES aggregation. First, once can see that, just like under Cobb-Douglas, networks dampen the response of frequency to monetary shocks. In other words, our key mechanism of interaction of networks and the extensive margin continues to hold under CES aggregation. Quantitatively, conditional on the presence of networks, moving from Cobb-Douglas to CES with $\theta_c = \theta_i = 0.001, \forall i$ delivers slightly larger frequency movements during expansions and slightly smaller frequency movements during monetary contractions. This is because under complements, sectors with rising prices see their input and consumption shares rise, thus creating a pro-inflation asymmetry.

As for supply disturbances, in Figure D.7 we study the propagation of aggregate TFP shocks. Just as in the economy with Cobb-Douglas, networks amplify the response of aggregate repricing frequency to aggregate TFP shocks. Therefore, our key mechanism that networks amplify pricing cascades continues to hold under CES aggregation. In Figure D.8 we show the responses of aggregate frequency to sectoral TFP shocks, confirming the cascades amplification effect under CES aggregation.

D.4 Inflation decomposition

We also quantify the contribution of cascades dampening and amplification to the response of aggregate inflation in Figure E.3. To do that, we follow Costain and Nakov (2011) and Blanco et al. (2024b) in making use of the following inflation decomposition:

$$\Delta\pi = \Delta\pi^{\text{Calvo}} + \Delta\pi^{\text{Extensive}} + \Delta\pi^{\text{Selection}}, \tag{D.18}$$

where $\Delta\pi$ is the deviation of (net) aggregate CPI inflation rate from its steady-state value, $\Delta\pi^{\text{Calvo}}$ is the inflation component attributed purely to the change in the intensive margin of price changes (holding fixed the fraction and composition of adjusters), $\Delta\pi^{\text{Extensive}}$ is the component driven exclusively by the change in the fraction of adjusters and $\Delta\pi^{\text{Selection}}$ stands for the component driven by changes in the composition of adjusters.[38] Crucially, the decomposition holds both in the economy with and without networks, allowing us to compute the contribution of each of the three components to the difference in inflation response driven by the input-output linkages. Figure E.4 shows the resulting decomposition for monetary and aggregate TFP shocks: the difference is explained mainly by the selection effect for smaller shocks, and by the extensive margin component for large shocks. Therefore, for large shocks, most of the network contribution to the dampening (monetary shocks) and amplification (aggregate TFP shocks) of the inflation response works through the extensive margin effect, representing the role of cascades.

D.5 Disaggregated dynamics

In Figure E.8 we report changes in sectoral adjustment frequencies, as well as scaled impact responses of sectoral price indices, following a monetary shock of $\varepsilon_0^M = 10\%$. A few important patterns emerge. First, all sectoral frequencies increase, with the natural exception of the fully price-flexible sectors. The largest rise in the fraction of adjusters is in "Education" (+0.65), whereas the smallest (non-zero) increase is in "Financial services" (+0.08). Second, removing production networks leads to a larger increase in adjustment frequency in every sector, so that cascades dampening also holds at the sector-by-sector level. Third, cascades dampening translates into smaller increases in sectoral price indices in the economy with networks.

To better understand the role production networks play in shaping the frequency responses, Figure E.10(a) plots an estimated linear relationship between the (square root of) sectoral frequency changes following a 10% monetary shock and the sectoral customer centrality measure, which we introduced in (29).[39] As can be seen, *ceteris*

[38] Formally, inflation in sector i in the absence of the monetary shock is $\pi_i = \int \tilde{p}\eta_i(\tilde{p})\mathrm{d}g_i(\tilde{p})$, where $\tilde{p}$ is the desired log price change, $\eta_i(\tilde{p})$ is the adjustment hazard, and $g_i(\tilde{p})$ is the ergodic distribution of desired price changes across firms in the sector. The monetary shock changes the desired price changes of all firms in the sector to $\tilde{p} + \delta$, where $\delta = \underline{p}^* - p^* + \Delta m$ and where $\underline{p}^*$ is the log reset price in the first period after the money shock and p^* is the log reset price in the absence of the shock. The money shock changes the inflation rate to $\underline{\pi}_i = \int(\tilde{p} + \delta)\underline{\eta_i}(\tilde{p})\mathrm{d}g_i(\tilde{p})$ where $\underline{\eta_i}(\tilde{p})$ is the new adjustment hazard after the shock. It follows that $\Delta\pi_i \equiv \underline{\pi}_i - \pi_i = \delta \int \eta_i(\tilde{p})\mathrm{d}g_i(\tilde{p}) \quad + \quad \delta \int(\underline{\eta_i}(\tilde{p}) - \eta_i(\tilde{p}))\mathrm{d}g_i(\tilde{p}) \quad + \int \tilde{p}(\underline{\eta_i}(\tilde{p}) - \eta_i(\tilde{p}))\mathrm{d}g_i(\tilde{p})$. Multiplying both sides by the final consumption share $\overline{\omega}_i^C$ and summing across sectors, one obtains the decomposition for aggregate CPI inflation in (D.18).

[39] Formally, we regress the square root of the change in sectoral adjustment frequency on the measure of customer centrality, further controlling for the covariance between that sector's row of $(\overline{\Psi} - I)$ and the vector of sectoral markups, own markup change, the steady-state frequency of adjustment and the sectoral

paribus, there is an approximately linear negative association between the square root of the frequency change and customer centrality, consistent with the formal result in Proposition 1. This is because, all else constant, cascades dampening is stronger for sectors that are more exposed to intermediate inputs.

Similarly, we also investigate the association between customer centrality and the degree of size dependence in sectoral price responses to small vs. large monetary shocks. We measure size dependence in sectoral price indices by the difference in (square roots of) *scaled* impact responses to 10% and 0.1% monetary shocks. In Figure E.10(b) we plot an estimated linear relationship between the measure of size dependence and the sectoral customer centrality.[40] As one would expect, larger customer centrality is associated with a smaller degree of size dependence, which in turn is driven by the smaller movement in the fraction of adjusters following the large monetary shock.

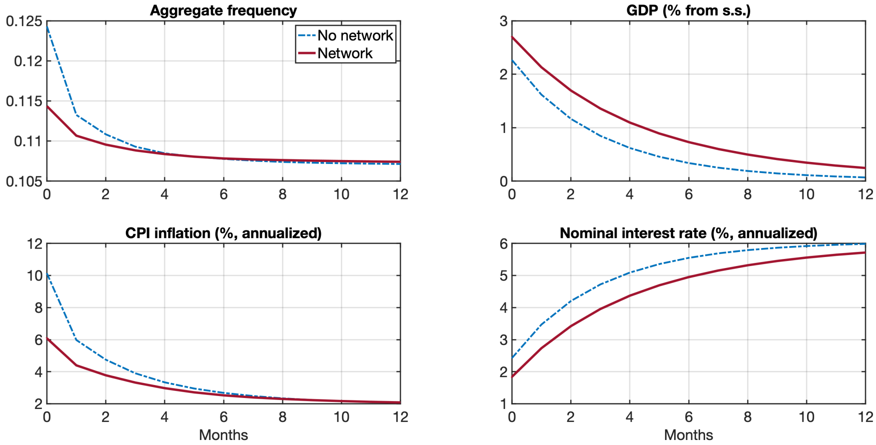
We now analyze the responses of individual sectors to an aggregate TFP shock. In particular, Figure E.9 reports the impact responses of sector-specific fractions of adjusters and scaled sectoral price indices to a large aggregate TFP shock ($\varepsilon_0^A = -10\%$). Some key results are of note. First, with the exception of the two fully flexible sectors, frequency of adjustment rises in all the sectors. The largest rise is in "Publishing" (+0.95), whereas the smallest positive change is in "Legal, accounting, management" (+0.18). Second, the cascades amplification effect holds sector-by-sector: in an otherwise identical economy without networks, frequency increases by less in every sector. Third, the cascades amplification leads to larger sectoral price increases in every sector.

In order to pin down the role played by network characteristics in shaping sectoral frequency responses, Figure E.11(a) plots an estimated linear relationship between the (square root of the) change in the fraction of adjusters in a given sector and its customer centrality, introduced in (29), following a -5% aggregate TFP shock. One can see that, *ceteris paribus*, there is an approximately linear positive relationship between the square root of the frequency change and the customer centrality, consistent with the result in Proposition 2 for an aggregate TFP shock. This is the cascades amplification mechanism in action: higher customer centrality means the sector has a larger total exposure to intermediate inputs; as a result, an aggregate TFP shock leads to a larger desired price change, making the adjustment decision more likely.

standard deviation of price changes. The regression specification is based on the result in Proposition 1.

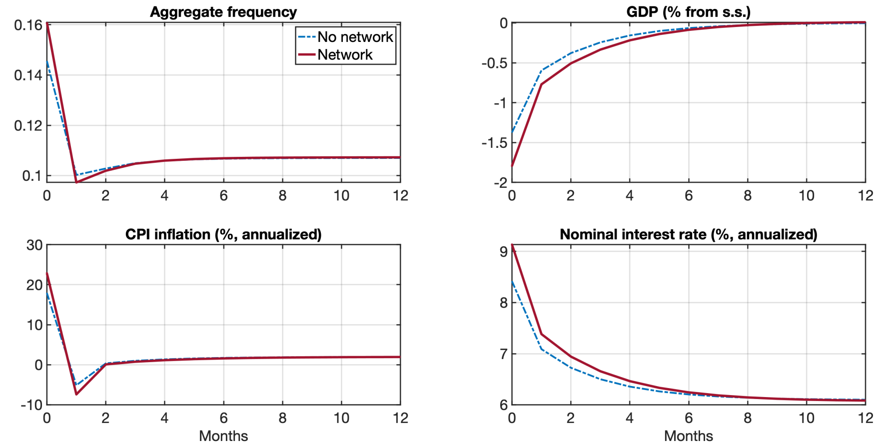
[40] As before, we regress the sectoral measure of size dependence on the measure of customer centrality, further controlling for the covariance between that sector's row of $(\overline{\Psi} - I)$ and the vector of sectoral markups, own markup change, the steady-state frequency of adjustment and the sectoral standard deviation of price changes.

Figure D.1: **Aggregate responses to a monetary shock under a Taylor rule**



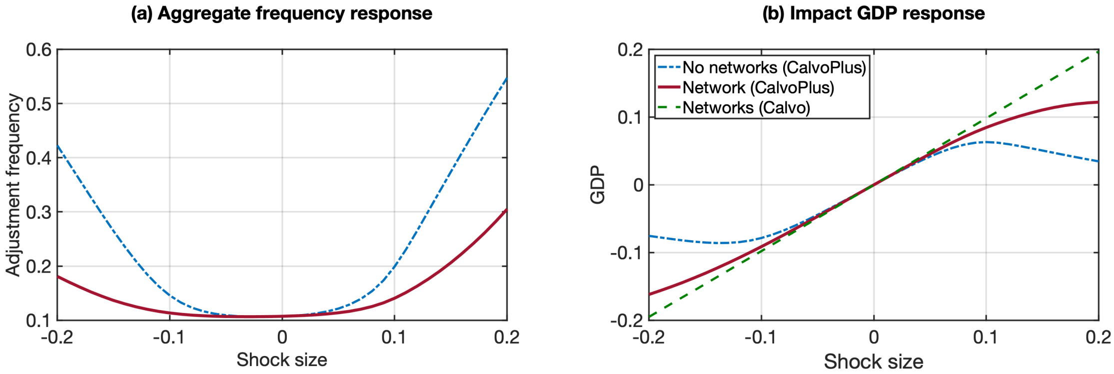
Notes: the figure shows the responses of aggregate adjustment frequency, GDP, CPI inflation and the nominal interest rate to a one-time -500 basis points (annualized) shock to the Taylor rule ($\varepsilon_0^i = -0.05/12$).

Figure D.2: **Aggregate responses to an aggregate TFP shock under a Taylor rule**



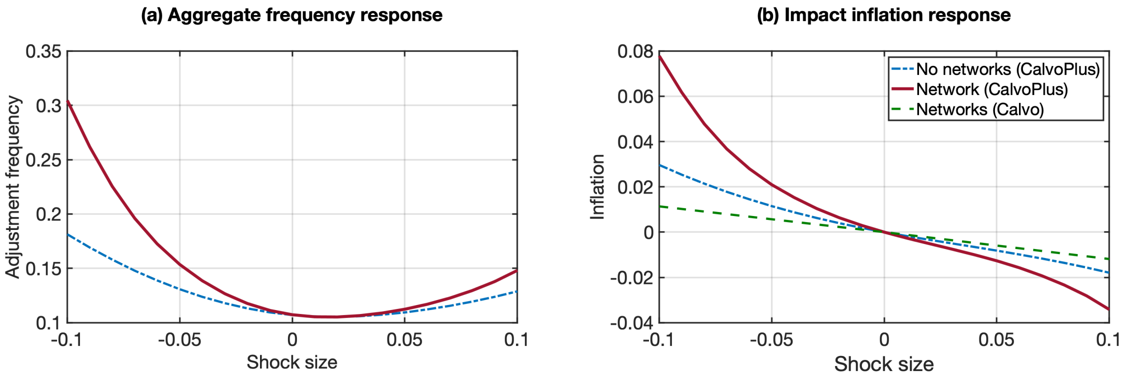
Notes: the figure shows the responses of aggregate adjustment frequency, GDP, CPI inflation and the nominal interest rate to a one-time transitory ($\rho = 0$) aggregate TFP shock of -5%.

Figure D.3: **Frequency and GDP responses to monetary shocks: CalvoPlus**



Notes: Panels (a) and (b) show the impact responses of, respectively, the aggregate adjustment frequency and aggregate real GDP following monetary shocks of different sizes in the economy with CalvoPlus pricing and networks, as well as in the otherwise identical economies with CalvoPlus pricing and without networks and with Calvo (1983) pricing and networks.

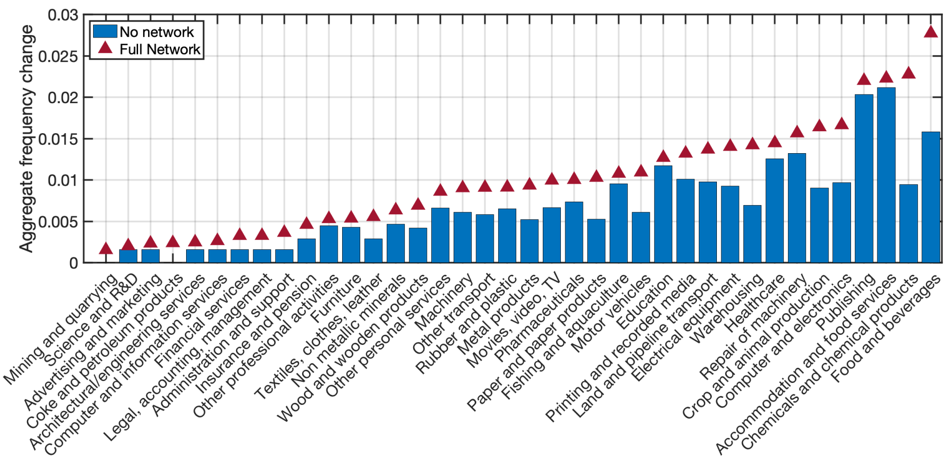
Figure D.4: **Frequency and inflation responses to agg. TFP shocks: CalvoPlus**



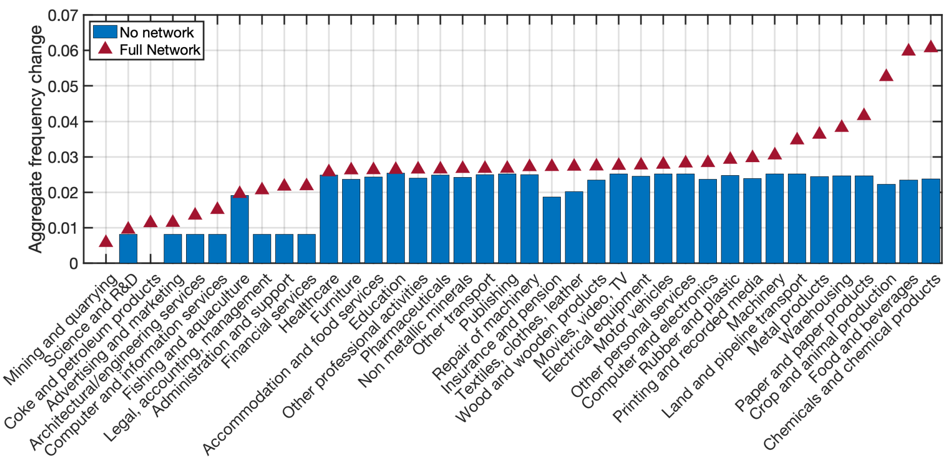
Notes: Panels (a) and (b) show the impact responses of, respectively, the aggregate adjustment frequency and aggregate CPI inflation following aggregate TFP shocks of different sizes in the economy with CalvoPlus pricing and networks, as well as in the otherwise identical economies with CalvoPlus pricing and without networks and with Calvo (1983) pricing and networks.

Figure D.5: **Aggregate frequency responses to sectoral TFP shocks: CalvoPlus**

(a) Responses of aggregate frequency to -20% sectoral TFP shocks

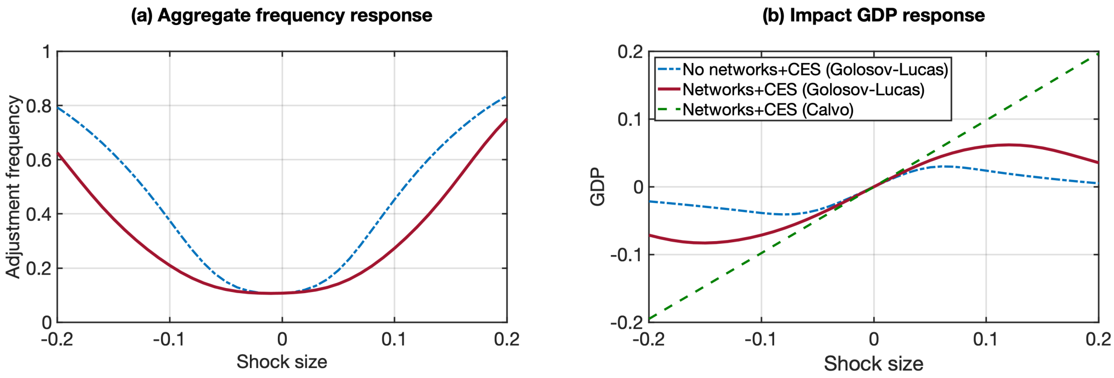


(b) Responses of aggregate frequency to -50% sectoral TFP shocks



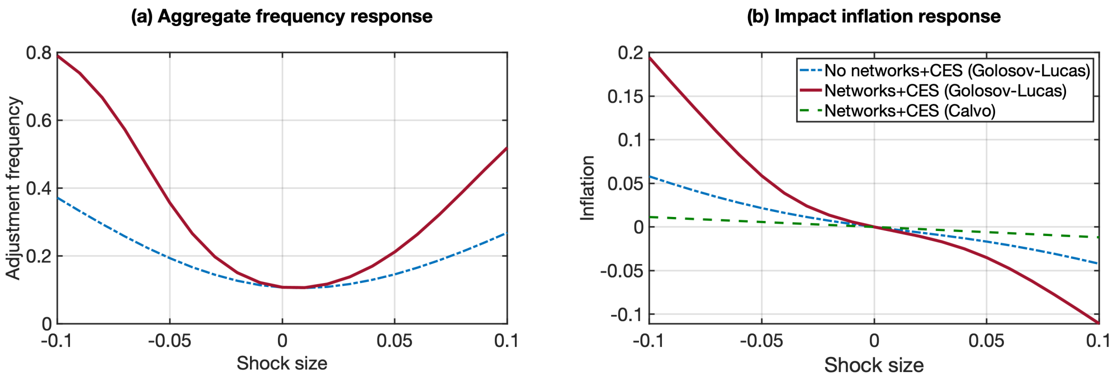
Notes: the figure shows the impact responses of aggregate adjustment frequency to large sector-specific TFP shocks in the economy with CalvoPlus pricing and networks, as well as in the otherwise identical economy without networks. Panel (a) considers -20% sector-specific TFP shocks, whereas Panel (b) considers larger -50% sector-specific TFP shocks.

Figure D.6: **Frequency and GDP responses to monetary shocks: CES aggregation**



Notes: Panels (a) and (b) show the impact responses of, respectively, the aggregate adjustment frequency and aggregate real GDP following monetary shocks of different sizes in the economy with CES aggregation, fixed menu costs and networks, as well as in the otherwise identical economies with without networks and with Calvo (1983) pricing and networks.

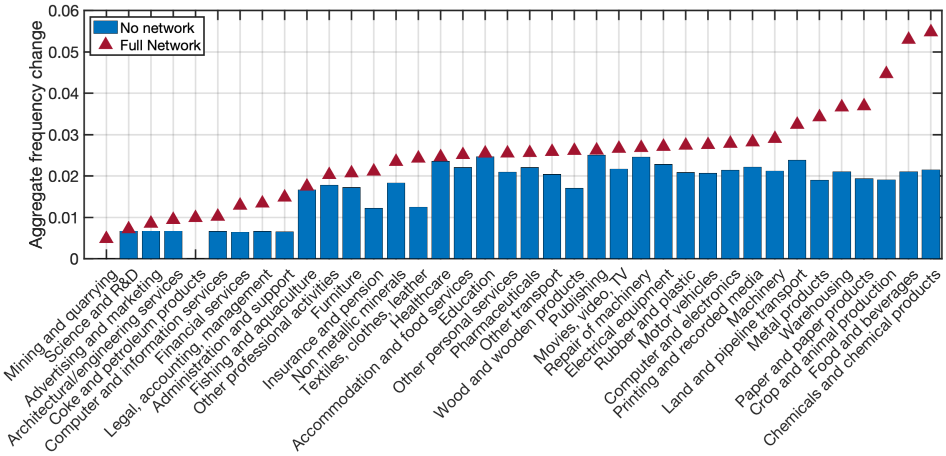
Figure D.7: **Frequency and inflation responses to agg. TFP shocks: CES aggregation**



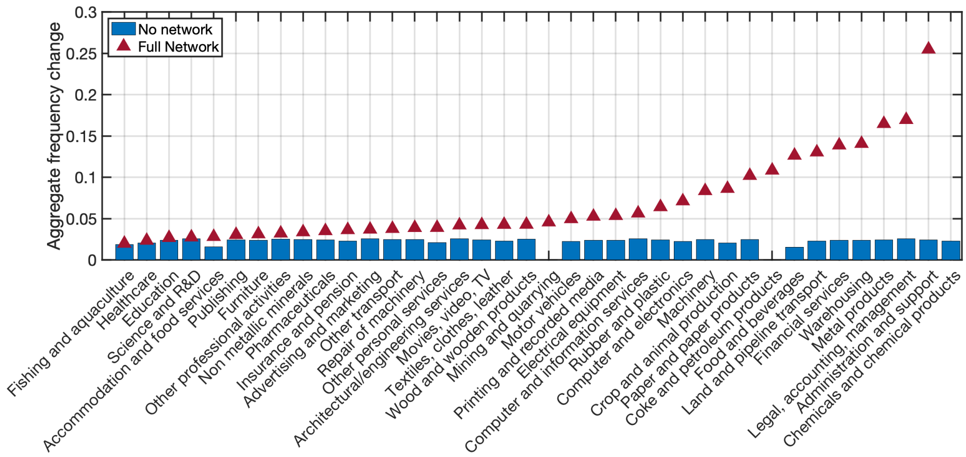
Notes: Panels (a) and (b) show the impact responses of, respectively, the aggregate adjustment frequency and aggregate CPI inflation following aggregate TFP shocks of different sizes in the economy with CES aggregation, fixed menu costs and networks, as well as in the otherwise identical economies with without networks and with Calvo (1983) pricing and networks.

Figure D.8: **Aggregate frequency responses to sectoral TFP shocks: CES agg.**

(a) Responses of aggregate frequency to -20% sectoral TFP shocks



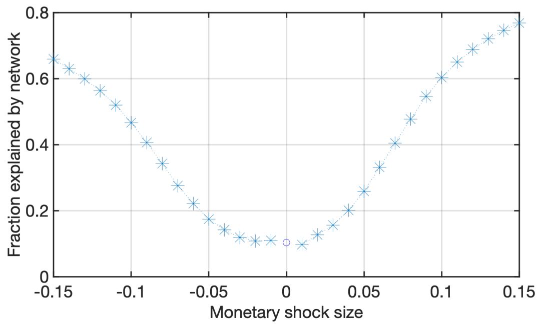
(b) Responses of aggregate frequency to -50% sectoral TFP shocks



Notes: the figure shows the impact responses of aggregate adjustment frequency to large sector-specific TFP shocks in the economy with CES aggregation, fixed menu costs and networks, as well as in the otherwise identical economy without networks. Panel (a) considers -20% sector-specific TFP shocks, whereas Panel (b) considers larger -50% sector-specific TFP shocks.

E Quantitative model: additional figures

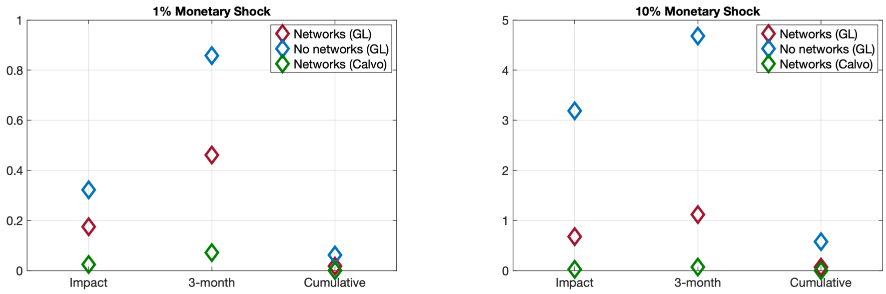
Figure E.1: **Network amplification of GDP responses to monetary shocks**



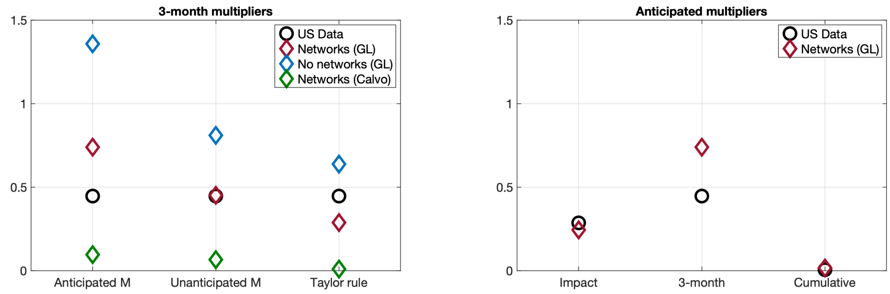
Notes: the figure shows the contribution of networks to the impact responses of GDP to monetary shocks of different sizes under fixed menu costs.

Figure E.2: **Phillips multipliers across horizons and models**

(a) Phillips multipliers following one-time monetary shocks

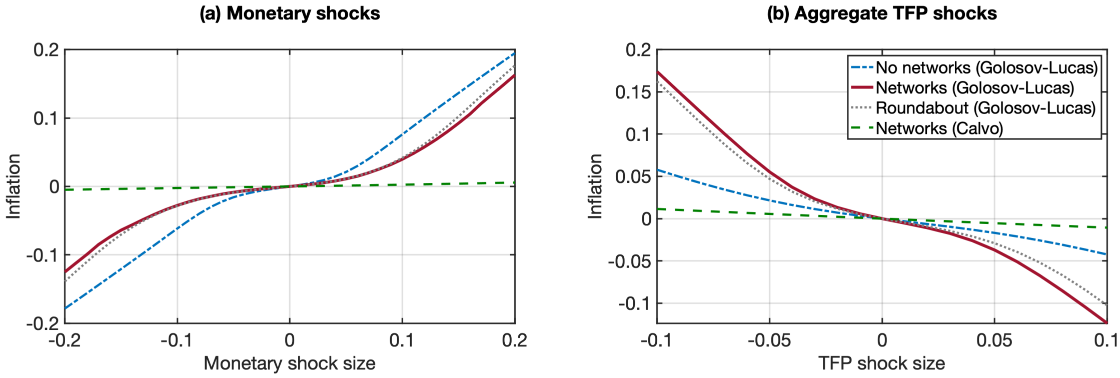


(b) Phillips multipliers based on Miranda-Agrippino and Ricco (2021) responses



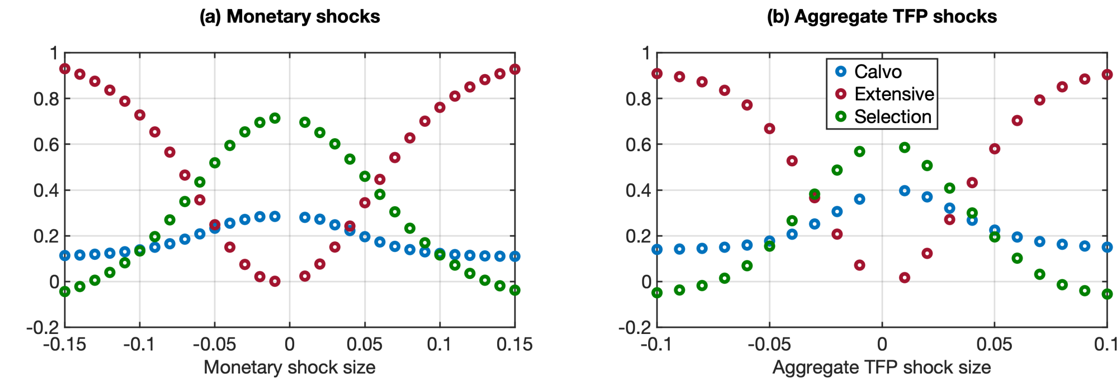
Notes: Panel (a) shows the impact, 3-month and cumulative Phillips multipliers following one-time 1% and 10% monetary shocks in the baseline model with networks and menu costs, as well as in the otherwise identical economies without networks and with networks and Calvo (1983) pricing. Panel (b) shows Phillips multiplier under the money process implied by the responses in Figure 3 of Miranda-Agrippino and Ricco (2021).

Figure E.3: **Impact CPI inflation responses in the baseline model**



Notes: Panels (a) and (b) show the impact responses of CPI inflation following, respectively, monetary and aggregate TFP shocks of different sizes in the baseline economy with fixed menu costs and networks, as well as in the otherwise identical economies without networks and with networks and Calvo (1983) pricing

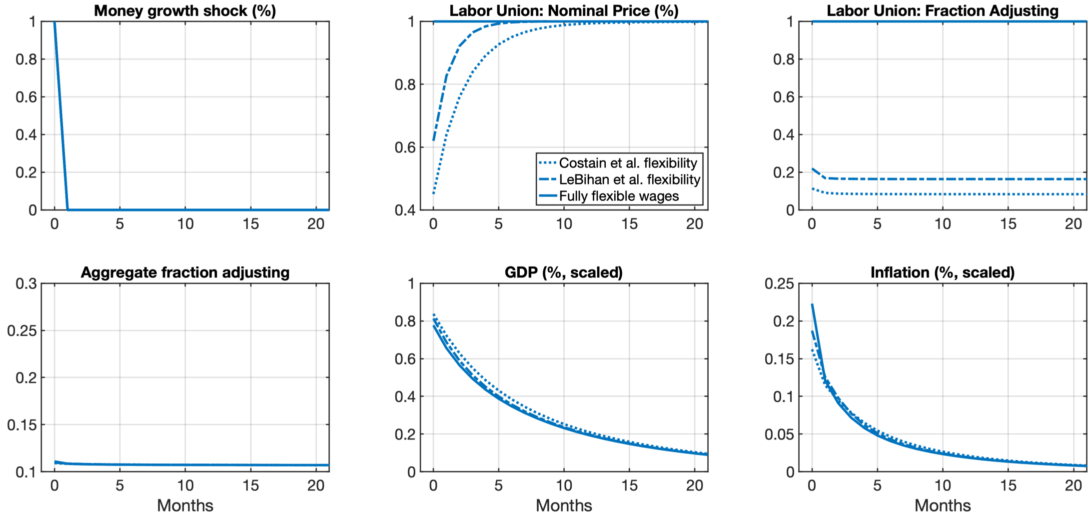
Figure E.4: **Extensive margin contribution to the network effect**



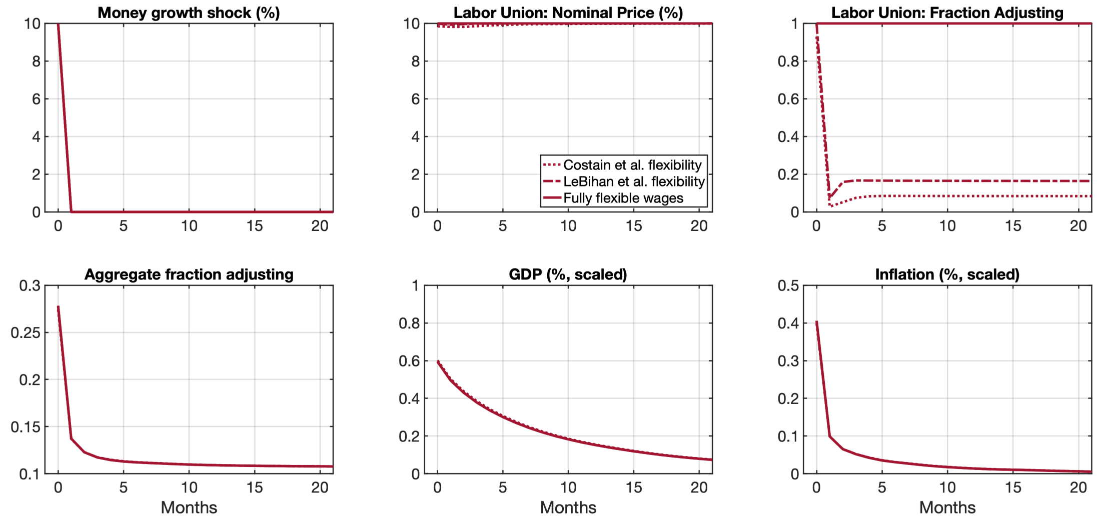
Notes: Panels (a) and (b) use the decomposition in (D.18) to show the proportional contributions of the Calvo, Extensive and Selection components to the effect of networks on the impact response of CPI inflation to monetary and aggregate TFP shocks, respectively.

Figure E.5: **Aggregate responses to monetary shocks: role of wage flexibility**

(a) Responses of aggregates to a 1% monetary shock

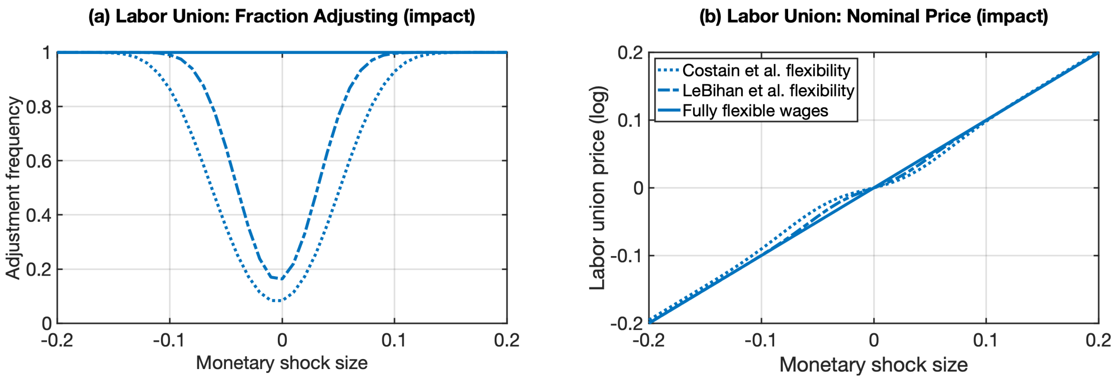


(b) Responses of aggregates to a 10% monetary shock



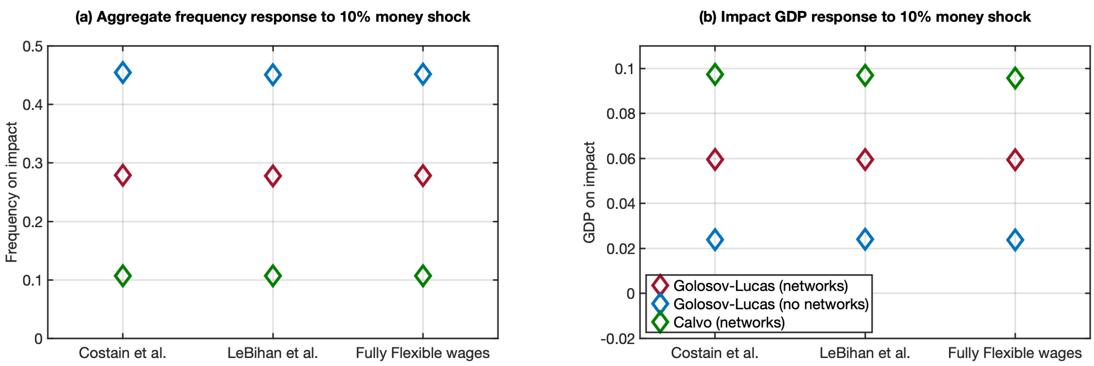
Notes: the figure shows responses to 1% and 10% monetary shocks in the economy with menu costs and networks under different levels of nominal wage flexibility: the baseline level of Costain et al. (2022), the more flexible level of Le Bihan et al. (2012), as well as under fully flexible nominal wages.

Figure E.6: **Labor union response to monetary shocks: role of wage flexibility**



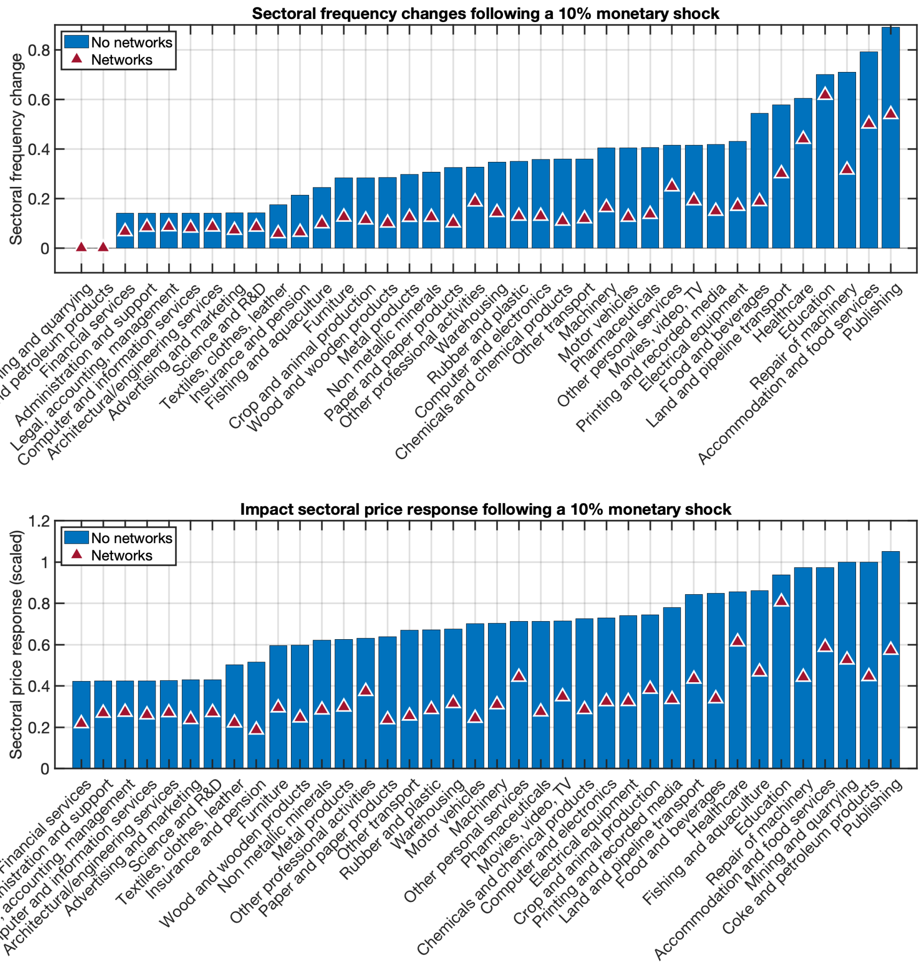
Notes: the figure shows the impact responses of the fraction of adjusters in the labor union sector, as well as its nominal price level, in response to monetary shocks of different sizes and under different levels of nominal wage flexibility: the baseline level of Costain et al. (2022), the more flexible level of Le Bihan et al. (2012), as well as under fully flexible nominal wages.

Figure E.7: **Cascades dampening under monetary shocks: role of wage flexibility**



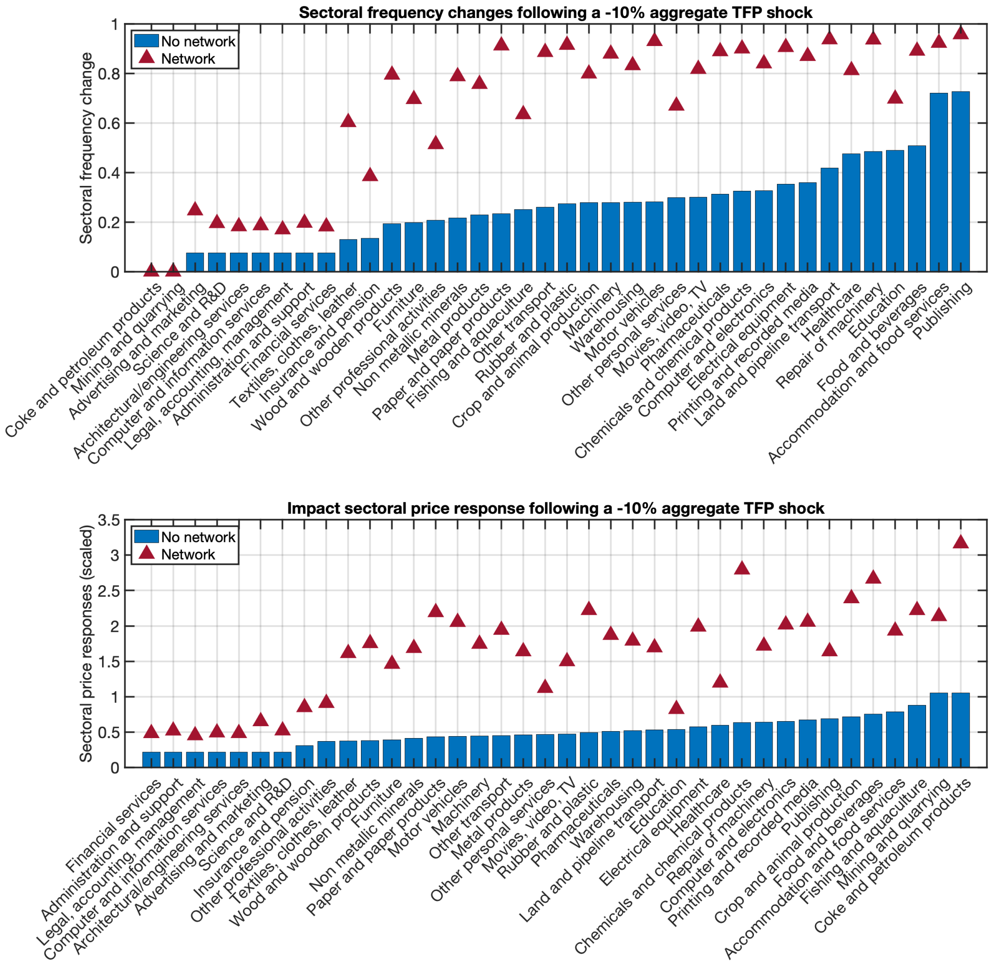
Notes: Panels (a) and (b) show the the impact responses of, respectively, aggregate adjustment frequency and aggregate real GDP following a 10% monetary shock in the baseline economy with fixed menu costs and networks, as well as in the otherwise identical economy without networks and the economy with networks and Calvo (1983) pricing.

Figure E.8: **Sectoral responses to a 10% monetary shock**



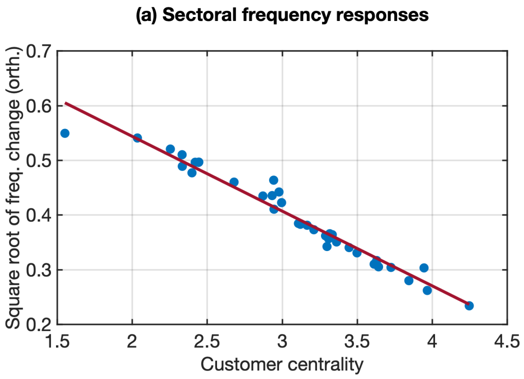
Notes: the top Panel shows the impact responses of sector-specific adjustment frequency to a one-time 10% monetary shock; the bottom Panel similarly shows the scaled impact responses of sectoral price indices one-time 10% monetary shock.

Figure E.9: **Sectoral responses to a aggregate -10% TFP shock**



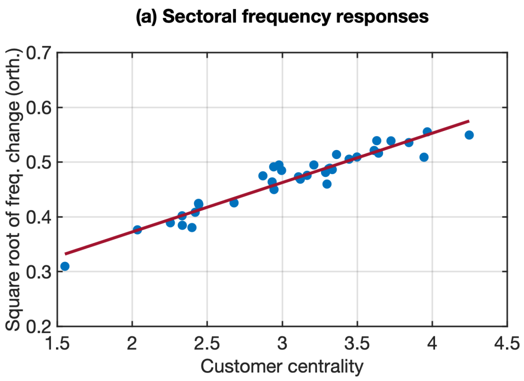
Notes: the top Panel shows the impact responses of sector-specific adjustment frequency to a one-time -10% aggregate TFP shock; the bottom Panel similarly shows the scaled impact responses of sectoral price indices one-time -10% aggregate TFP shock.

Figure E.10: **Sectoral responses to a monetary shock vs. Customer centrality**



Notes: Panels (a) and (b) plot fitted linear relationships between the sectoral Customer centrality and, respectively, the responses of sectoral frequencies and the degree of size dependence in sectoral prices following a 10% monetary shock. The vertical axes show the fitted response to Customer centrality, to which we add the fitted response to the control variables ($Cov\left((\overline{\Psi} - I)^{(i)}, \boldsymbol{\mu}\right)$, own markup change, frequency, size of of price changes) evaluated at their sample means. The regression sample excludes the labor union sector, as well as the two sectors with fully flexible prices.

Figure E.11: **Sectoral responses to an aggregate TFP shock vs. Customer centrality**

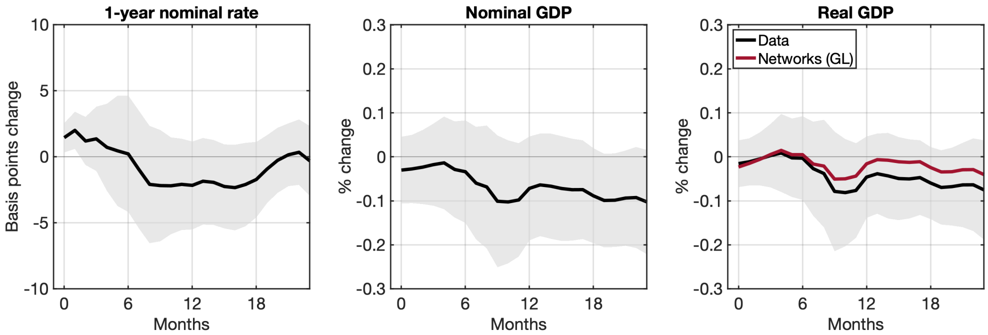


Notes: Panels (a) and (b) plot fitted linear relationships between the sectoral Customer centrality and, respectively, the responses of sectoral frequencies and the degree of size dependence in sectoral prices following a -5% aggregate TFP shock. The vertical axes show the fitted response to Customer centrality, to which we add the fitted response to the control variables ($Cov\left((\overline{\Psi} - I)^{(i)}, \boldsymbol{\mu}\right)$, own markup change, frequency, size of of price changes) evaluated at their sample means. The regression sample excludes the labor union sector, as well as the two sectors with fully flexible prices.

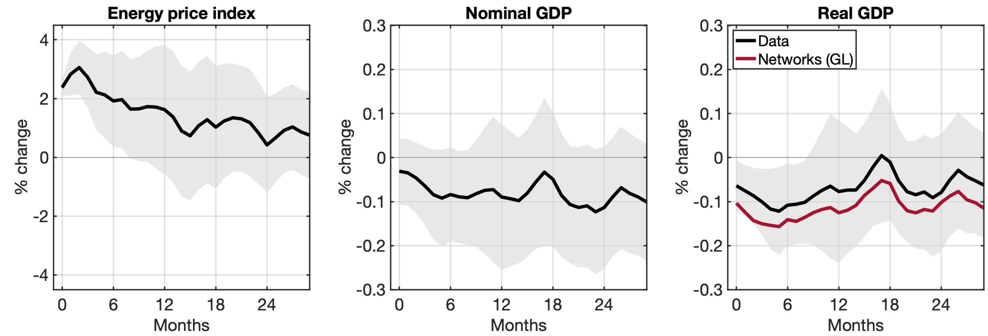
F Empirical validation: additional figures

Figure F.12: **Additional responses to small shocks (Euro Area)**

(a) Monetary shock ($1/2$ **standard deviation)**



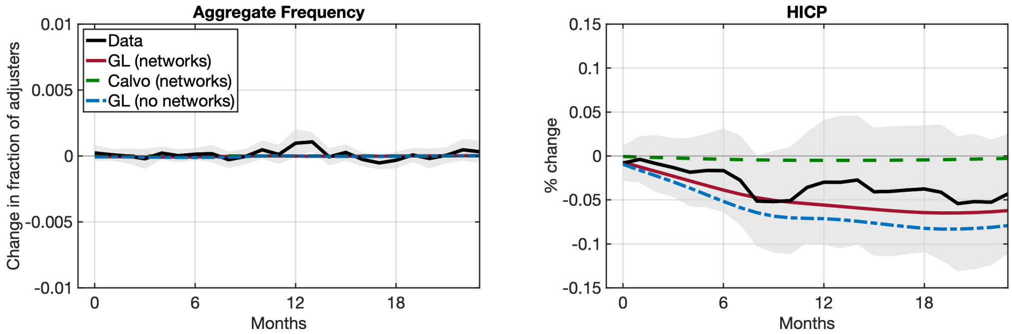
(b) Oil shock ($1/2$ **standard deviation)**



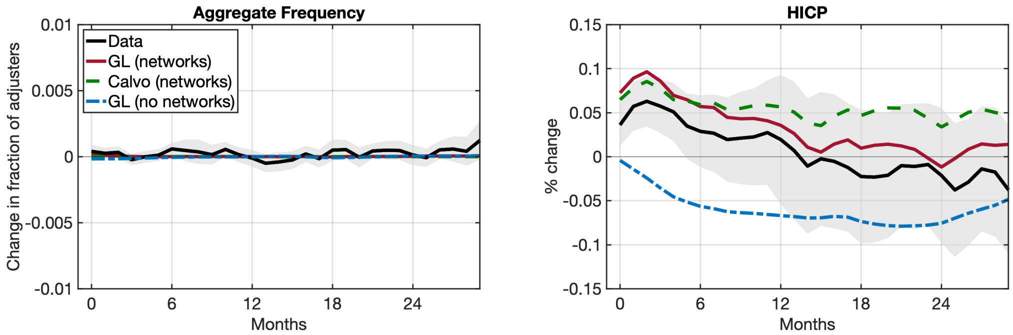
Notes: Panel (a) shows the estimated IRFs of the 1-year nominal interest rate, nominal and real GDP to a contractionary monetary shock of Jarociński and Karadi (2020), as well as the real GDP response in the baseline model. For each of the three dependent variables of interest, the set of controls includes 12 lags of HICP, industrial production, risk-free nominal interest rate, real IMF energy and food price indices and the Euro Stoxx 50 index. Panel (b) shows the estimated IRFs of the real energy price index, nominal and real GDP to a contractionary oil shock of Känzig (2021), as well as the real GDP response in the baseline model. For each of the three dependent variables of interest, the set of controls includes 12 lags of the oil shock, HICP, industrial production, real IMF energy and food price indices and the risk-free nominal interest rate. Shaded areas represent 95% confidence intervals.

Figure F.13: **Responses to small shocks: models vs. data (Euro Area)**

(a) Monetary shock (1/2 **standard deviation)**



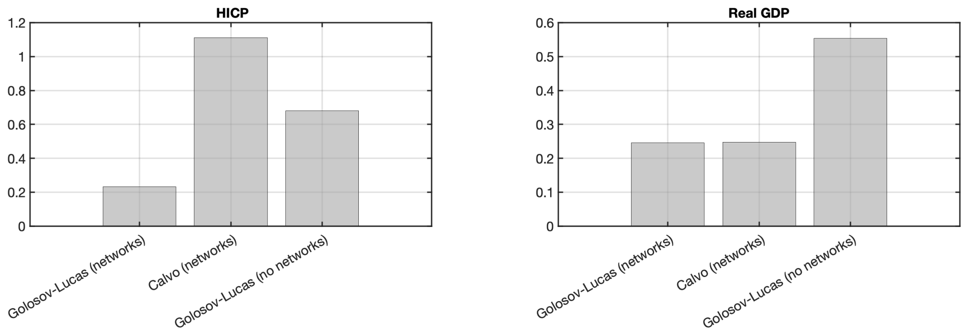
(b) Oil shock (1/2 **standard deviation)**



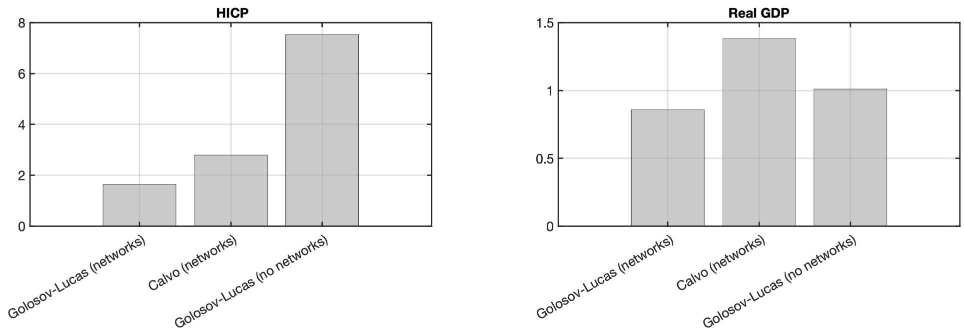
Notes: Panel (a) shows the estimated IRFs of the aggregate adjustment frequency and aggregate HICP to a contractionary monetary shock of Jarociński and Karadi (2020), as well as the corresponding responses in the baseline and alternative models. For aggregate frequency, the set of controls includes 12 lags of aggregate frequency and HICP; for HICP, the set of controls includes 12 lags of HICP, industrial production, risk-free nominal interest rate, real IMF energy and food price indices and the Euro Stoxx 50 index. Panel (b) shows the estimated IRFs of the aggregate adjustment frequency and aggregate HICP to a contractionary oil shock of Känzig (2021), as well as the corresponding responses in the baseline and alternative models. For aggregate frequency, the set of controls includes 12 lags of the oil shock, aggregate frequency and HICP; for HICP, the set of controls includes 12 lags of the oil shock, HICP, industrial production, real IMF energy and food price indices and the risk-free nominal interest rate. Shaded areas represent 95% confidence intervals.

Figure F.14: **Small shocks: mean squared distance between models and the data**

(a) Monetary shock ($1/2$ standard deviation)



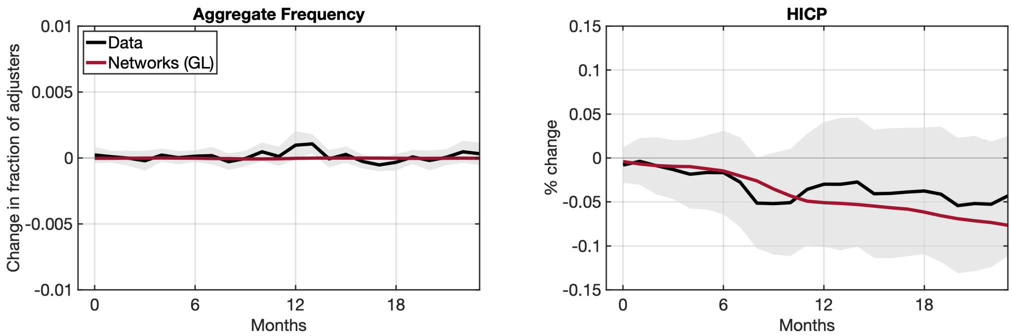
(b) Oil shock ($1/2$ standard deviation)



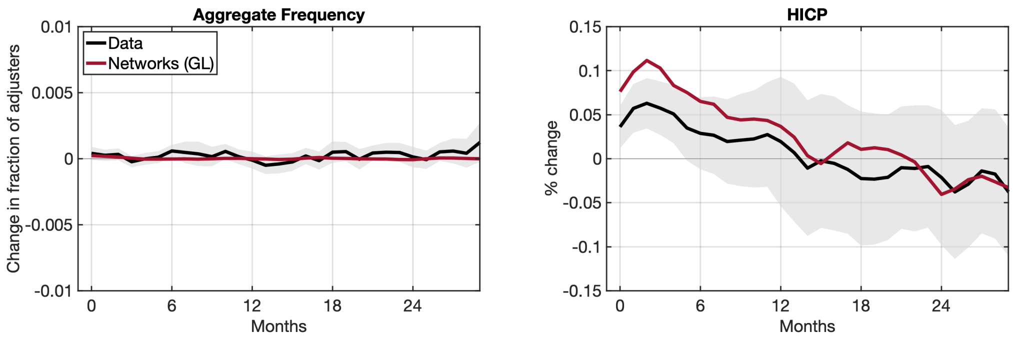
Notes: the figure reports the mean squared distance between the model-based and estimated responses of HICP and real GDP to small monetary and oil shocks, based on Figures F.12 and F.13. The distance between data and model is $\frac{1}{\overline{H}+1}\sum_{H=0}^{\overline{H}}\left[(IRF_H^{data} - IRF_H^{model})/SE_H^{data}\right]^2$, where IRF_H is the data/model impulse response at horizon H and SE_H^{data} is the estimated standard error of the horizon-H empirical response, and we set $\overline{H} = 36$ (months).

Figure F.15: **Responses to small unanticipated shocks: model vs. data (Euro Area)**

(a) Monetary shock (1/2 standard deviation)

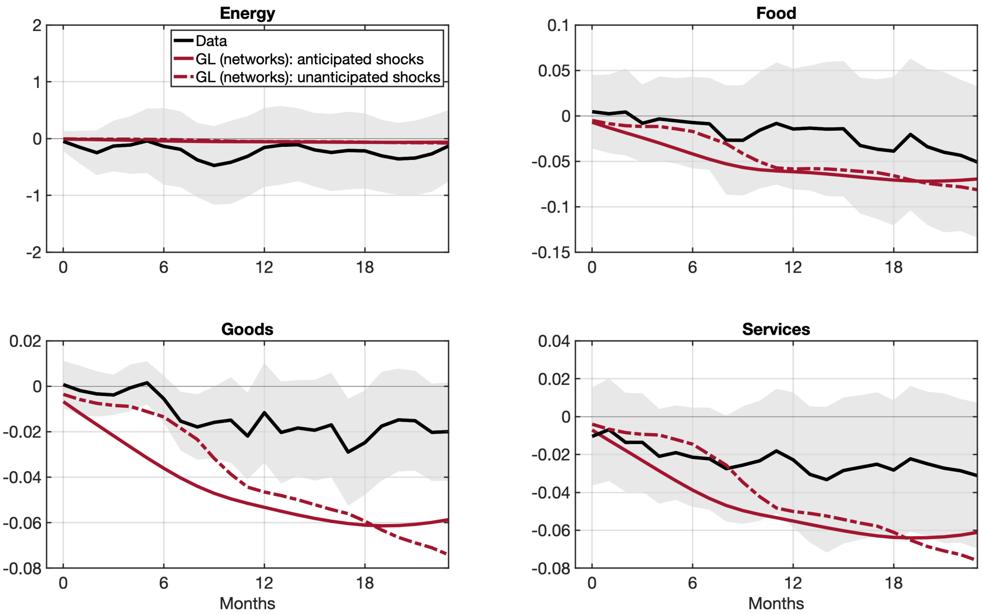


(b) Oil shock (1/2 standard deviation)



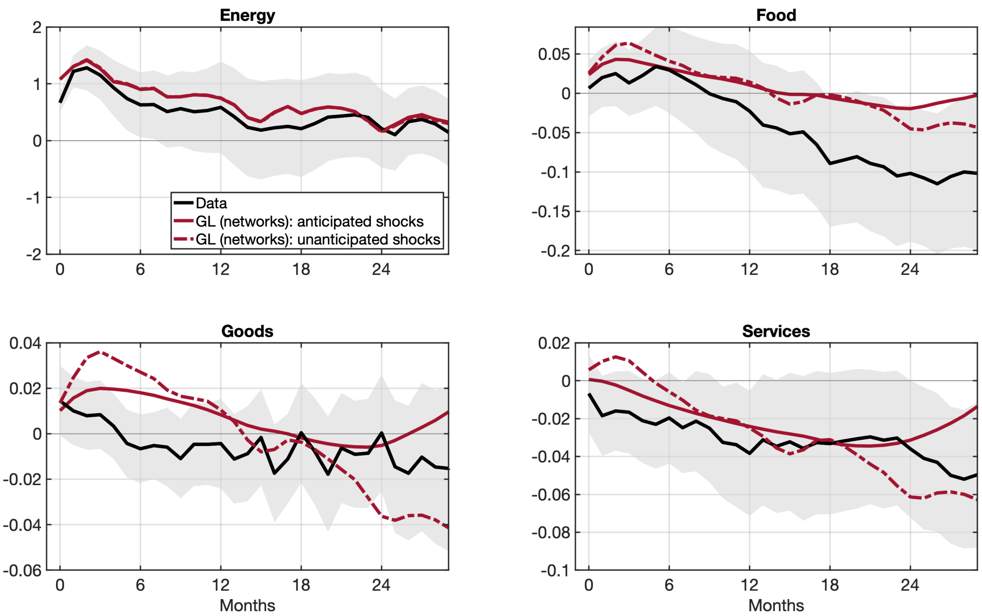
Notes: Panel (a) shows the estimated IRFs of the aggregate adjustment frequency and aggregate HICP to a contractionary monetary shock of Jarociński and Karadi (2020), as well as the corresponding responses in the baseline model under the assumption that the agents discover the money supply path period-by-period. For aggregate frequency, the set of controls includes 12 lags of aggregate frequency and HICP; for HICP, the set of controls includes 12 lags of HICP, industrial production, risk-free nominal interest rate, real IMF energy and food price indices and the Euro Stoxx 50 index. Panel (b) shows the estimated IRFs of the aggregate adjustment frequency and aggregate HICP to a contractionary oil shock of Känzig (2021), as well as the corresponding responses in the baseline model under the assumption that the agents discover the path of the real energy price index and money supply period-by-period. For aggregate frequency, the set of controls includes 12 lags of the oil shock, aggregate frequency and HICP; for HICP, the set of controls includes 12 lags of the oil shock, HICP, industrial production, real IMF energy and food price indices and the risk-free nominal interest rate. Shaded areas represent 95% confidence intervals.

Figure F.16: **Sectoral price responses to a monetary shock** $(1/2$ **standard deviation)**



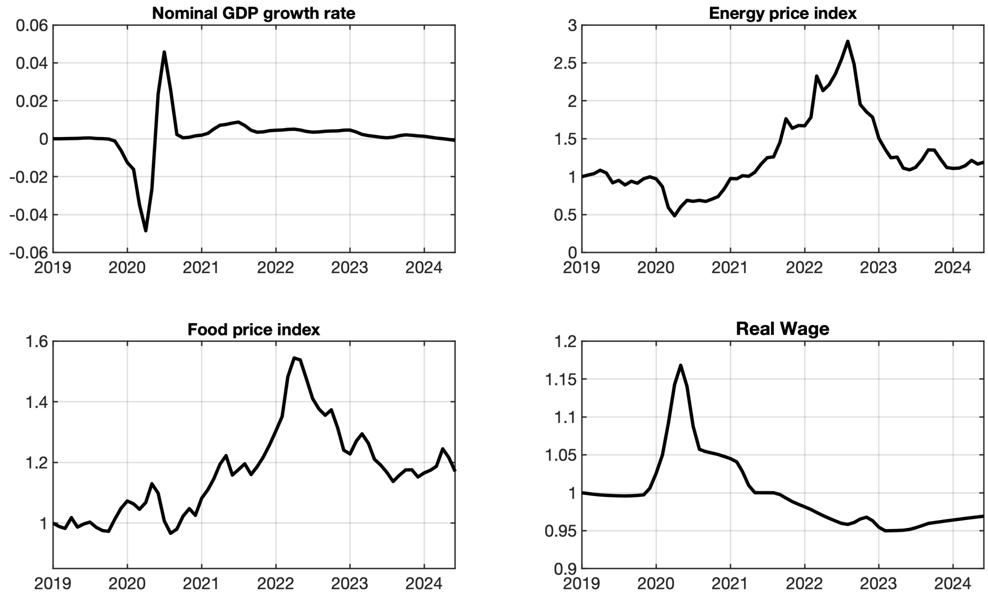
Notes: the figure shows the estimated IRFs of the sectoral HICPs to a contractionary monetary shock of Jarociński and Karadi (2020), as well as the corresponding responses in the baseline model with and without assuming perfect foresight. For each sectoral HICP, the set of controls includes 12 lags of aggregate HICP, industrial production, risk-free nominal interest rate, real IMF energy and food price indices and the Euro Stoxx 50 index. Shaded areas represent 95% confidence intervals.

Figure F.17: **Sectoral price responses to an oil shock** $(1/2$ **standard deviation)**



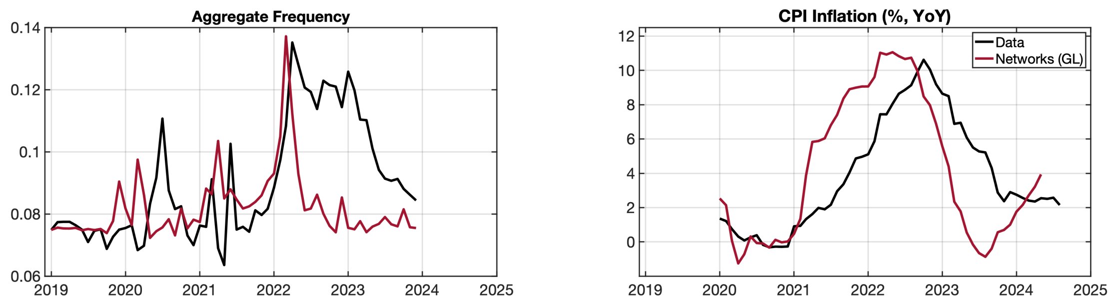
Notes: the figure shows the estimated IRFs of sectoral HICPs to a contractionary oil shock of Känzig (2021), as well as the corresponding responses in the baseline model with and without assuming perfect foresight. For each sectoral HICP, the set of controls includes 12 lags of the oil shock, HICP, industrial production, real IMF energy and food price indices and the risk-free nominal interest rate. Shaded areas represent 95% confidence intervals.

Figure F.18: **Large shock exercise: targeted processes (Euro Area)**



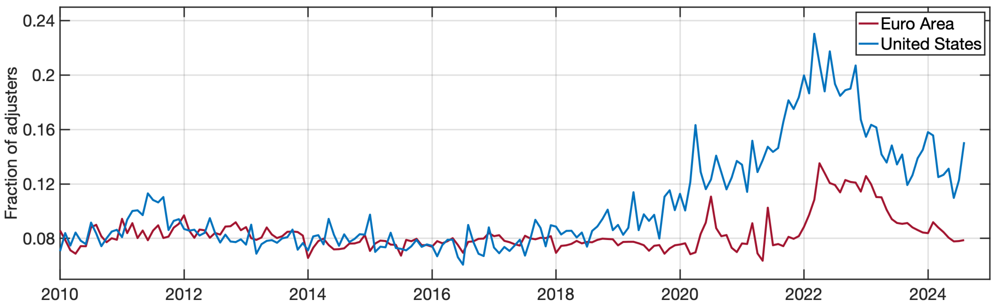
Notes: the figure shows the time series of four processes that we target in our large shock exercises in Section 7.2: nominal GDP growth rate, the real energy price index, the real food price index, as well as the real wage measured by average hourly earnings deflated by nominal GDP.

Figure F.19: **Explaining the observed surge: unanticipated commodity shocks**



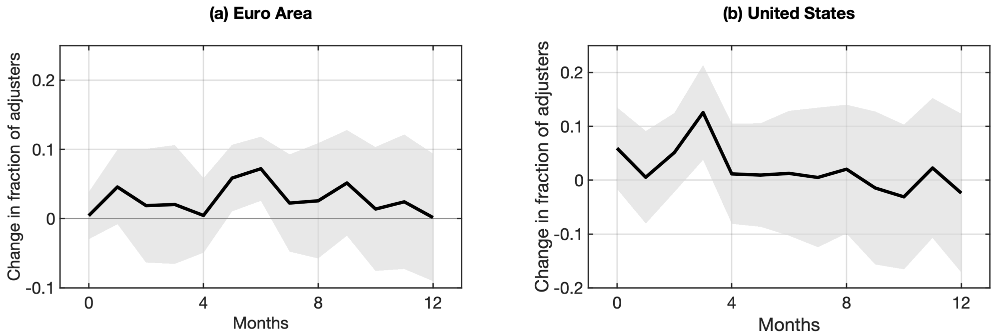
Notes: the figure shows the observed and model-implied changes in the Euro Area aggregate adjustment frequency and year-on-year CPI inflation in the baseline model with menu costs and networks, under the assumption that the values of energy and food prices are discovered period-by-period.

Figure F.20: **Monthly fraction of adjusters: Euro Area vs. United States**



Notes: the figure shows the time series of the monthly fraction of price adjusters based on micro CPI data for the Euro Area (Gautier et al., 2025) and the United States (Montag and Villar, 2025). Both series are based on data that excludes sales and have been seasonally adjusted by removing month dummies.

Figure F.21: **Response of adjustment frequency to a large oil shock (4 s.d.)**



Notes: Panels (a) and (b) show the estimated IRFs of the aggregate adjustment frequency in the Euro Area and the United States to a 4 standard deviation contractionary oil shock of Känzig (2021). The estimation uses the non-linear local projection specification: $freq_{t+H} = \alpha_H + \beta_H^{lin} s_t + \beta_H^{sign} s_t^2 + \beta_H^{size} s_t^3 + \gamma_H' \boldsymbol{x}_{t-1} + \varepsilon_{t+H}$, where $freq_{t+H}$ is the aggregate monthly fraction of adjusters H months after the shock and s_t is the identified oil shock. For aggregate adjustment frequency, the set of controls includes 12 lags of the oil shock, aggregate frequency and HICP. Shaded areas represent 95% confidence intervals.