Frost and Fire: A Tale of Two Crises

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Abstract

Financial crises are characterized by depressed asset prices, tight financial constraints, and misallocation of resources. Standard policy responses—such as asset purchases and low interest rates—are generally intended to alleviate these symptoms. This paper distinguishes between two types of crises that appear similar but differ fundamentally in their underlying mechanisms: *fire-sale crises*, where productive firms are forced to sell assets; and *demand-freeze crises*, where productive firms are unable to purchase assets. While both lead to similar observable outcomes, they have contrasting general equilibrium effects and may call for different policy interventions. Notably, conventional policies can be counterproductive in demand-freeze crises, as they may exacerbate financial constraints and further distort resource allocation. Empirical evidence on the pattern of capital reallocation among U.S. firms suggests that demand-freeze crises are, in fact, more common.

Keywords: Financial crises, financial frictions, demand freezes, fire sales, asset purchases, monetary loosening, credit easing, capital reallocation, cleansing effects.

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Some say the world will end in fire,
Some say in ice.
From what I've tasted of desire
I hold with those who favor fire.
But if it had to perish twice,
I think I know enough of hate
To say that for destruction ice
Is also great
And would suffice.

— Robert Frost

1 Introduction

There is substantial evidence linking financial crises to severe misallocation of capital (Caballero and Hammour, 2005; Reis, 2013; Gopinath et al., 2017). A widely accepted narrative emphasizes the role of tightening financial constraints during crises, which hinder the efficient allocation of capital to its most productive uses. This misallocation, in turn, is believed to contribute to declining asset prices, which further exacerbate financial constraints, deepening the downturn in a self-reinforcing cycle (Lorenzoni, 2008).

In response to these dynamics, conventional wisdom typically advocates for credit-easing interventions, such as reductions in interest rates and large-scale asset purchases. These measures are designed to stabilize or raise asset prices, thereby alleviating financial constraints and facilitating a more efficient allocation of capital. As stated by Isabel Schnabel, member of the ECB's Executive Board: *"After the outbreak of COVID-19…asset purchases…reduced systemic stress, prevented broader fire sales and thus averted an excessive tightening of financing conditions that could have threatened the stability of the financial system at large."*[1]

In this paper, we challenge the conventional narrative of financial crises by using a canonical model that links financial distress to capital reallocation dynamics. Our framework, in which financial crises arise due to a tightening of financial constraints, reveals that crises can fall into one of the following categories: *fire-sale crises*, in which productive firms are forced to sell assets under distress; and *demand-freeze crises*, in which productive firms are unable to purchase assets despite potential gains from doing so. Both crisis types align with the prevailing view in that they feature depressed asset prices, binding financial constraints, and a breakdown in the

[1]Speech by Isabel Schnabel, Member of the Executive Board of the ECB, at the 2024 BOJ-IMES Conference on "Price Dynamics and Monetary Policy Challenges: Lessons Learned and Going Forward", 28 May 2024.

efficient reallocation of resources. As a result, both crises reduce aggregate output and welfare.

Despite exhibiting many similar features, fire-sale and demand-freeze crises differ in a critical respect: namely, in the general-equilibrium effects of asset prices on resource allocation. We show that it is useful to think of the effects of a tightening of financial constraints as being composed of two parts. A *direct channel*, which captures the change in output while keeping the price of capital constant; and an indirect *capital-price channel*, which captures the change in output that comes about from the induced change in the price of capital. In fire-sale crises, high productivity entrepreneurs are sellers of capital because they need resources to pay off their debts. Therefore, the decline in the price of capital that takes place during the crisis hurts them—and thus the allocation of capital—by forcing them to liquidate even more capital. In demand-freeze crises, high-productivity entrepreneurs are instead buyers of capital, although they are not able to buy as much capital as they would like to. Thus, the decline in the price of capital that takes place during the crisis benefits them—and thus the allocation of capital—by allowing them to purchase more capital.

Real-world crises likely combine aspects of demand-freeze and fire-sale episodes. To evaluate this, we study an economy calibrated to match firm-level productivity and leverage observed in the U.S. economy. The key finding is that the calibrated economy endogenously produces demand-freeze crises: on average, financially constrained entrepreneurs expand during crises, but less than they would in the absence of financial constraints. As in the data, crises coincide with low capital reallocation precisely because the most productive entrepreneurs—the natural buyers of capital—are constrained. Consequently, the decline in asset prices during crises partly mitigates their severity by facilitating reallocation toward these constrained entrepreneurs.

The distinction between both types of crises has significant implications for the design of policy responses, e.g. interest-rate changes and asset purchases. To see this, it is useful to also decompose the effects of policies into a direct channel, which captures the change in output that would arise if the price of capital were fixed; and an indirect capital-price channel, which comes about from the induced change in the price of capital. In the case of expansionary interest-rate policy, for instance, the direct channel has a positive sign: by relaxing the financial constraint of distressed entrepreneurs, an interest-rate cut enables them to expand their investment thereby boosting output. In the case of asset purchases, the sign of the direct channel is weakly negative, because we assume that the marginal productivity of capital is weakly lower in the public than in the private sector.

Our key object of interest, however, is the capital-price channel of policies. We show that—in line with the conventional wisdom—interest-rate cuts and asset purchases both increase the price of capital during financial crises. Interest-rate cuts do so by raising unconstrained entrepreneurs' valuation of future dividends, whereas asset purchases do so by directly increasing the overall demand for capital. However, this increase in the price of capital has radically different implications for reallocation and output, depending on whether it takes place during a fire-sale or a demand-freeze crisis. During a fire-sale crisis, the capital-price channel strengthens the expansionary effects of policies, because higher asset prices reduce entrepreneurs' need to liquidate capital. In contrast, the capital-price channel weakens the expansionary effect of policies during a demand-freeze crisis, because higher asset prices reduce entrepreneurs' ability to purchase capital. In our calibrated economy, which endogenously gives rise to demand-freeze crises, the indirect channel is in fact dominant: thus, interest-rate cuts and asset purchases during crises actually reduce output!

Thus, our theoretical results suggest that the effects of policy interventions during demand-freeze crises differ substantially from their effects during fire-sale crises. Moreover, although the conventional wisdom is largely based on the narrative of fire-sale crises, a simple calibrated economy suggests that demand-freeze crises may be common. But what about their empirical relevance?

We show that, empirically, a sufficient statistic to detect demand-freeze episodes is a positive covariance between entrepreneurs' marginal product of capital and their net purchases of capital. Conversely, a necessary condition for a fire-sale episode is that this covariance be negative. To assess this in the data, we analyze the cross-sectional relationship between firms' marginal product of capital and their capital investment activity in Compustat data. We find that this covariance is systematically positive, even in times of heightened financial stress as measured through the Gilchrist and Zakrajšek (2012) excess bond premium. Another noteworthy feature of the data is that, consistent with previous evidence (e.g., Eisfeldt and Rampini, 2006), capital reallocation is strongly pro-cyclical. This once again suggests the prevalence of demand-freeze episodes, where reallocation is limited during downturns.

Finally, we extend our model in two key dimensions. First, in the spirit of the recent macro-finance literature, we reinterpret entrepreneurs as a conglomerate of financial intermediaries and firms. This allows us to speak to the role of intermediaries in financial crises through a simple recalibration of the model. In particular, we double the degree of leverage in the calibration, from a ratio of assets that reflects the indebtedness of the non-financial corporate sector, to one providing a more accurate reflection of leverage for the entire private sector in the U.S. economy (e.g., see Gertler and Karadi, 2011). Two key insights emerge: first, the strength of the direct channel increases alongside leverage. Thus, it is now possible for the overall effect of interest-rate cuts to be expansionary during crises. Second, in the high-leverage economy, crises may be of the demand-freeze or fire-sale type depending on their severity. Intuitively, high-leverage can in some instances lead to sales of capital that are significant enough to trigger fire-sale crises. Having said this, even when leverage is increased to take into account the financial sector, most crises in the calibrated economy are of the demand-freeze type.

Second, we extend the model to include financial assets in addition to capital. This allows us to explore the implications of financial asset purchases, which are a common form of intervention in practice. This extension provides a number of interesting insights but does not fundamentally change the analysis. At a general level, financial assets provide a buffer against the misallocation of productive assets: in the event of a crisis, constrained entrepreneurs can sell these assets to maintain their desired level of capital. If the crisis becomes severe enough, however, the price of financial assets falls as they cannot all be absorbed by the private sector at 'fair' value. In this case, a policy of financial-asset purchases raises their price and affects the economy through the two familiar channels. Purchases expand output through the direct channel, i.e., they raise the net worth of constrained entrepreneurs by raising the price of financial assets. But they also affect the economy through the capital-price channel since, by raising the price of financial assets, they reduce their return and thus provide incentives for unconstrained entrepreneurs to expand their holdings of physical capital. We show that the introduction of financial assets does not qualitatively change the behavior of the calibrated economy with high leverage: namely, crises can be of the fire-sale or demand-freeze types depending on their severity.

On the policy front, our model sheds light on a widespread debate on interventions during crises. On the one hand, there is a *benign* view that policies such as asset purchases are necessary to sustain the net worth of market participants thereby mitigating the severity of the crisis. As Ben Bernanke states, *"The purpose of asset purchases is to ease financial conditions...these policies help allocate resources towards productive uses..."*. On the other hand, there is a *distortionary* view that this type of intervention may distort markets and prevent the allocation of resources to their most productive uses. For instance, Ragu Rajan claims that *"Central bank asset purchases can distort market signals and misallocate resources..."*.[2] Through the lens of our theory, both views are valid but apply to different circumstances: whereas the benign view applies to fire-sale crises, the distortionary view applies to demand-freeze crises.

Our paper builds on two strands of literature. First, it is closely related to work examining capital reallocation and its macroeconomic implications under financial frictions (Eisfeldt and Rampini, 2006; Kurlat, 2013; Fuchs et al., 2016; Bigio, 2015; Eisfeldt and Shi, 2018; Asriyan

[2]This view is also reminiscent of the work of Caballero and Hammour (2005), who argued that recessions have a "cleansing" effect by enabling resources to be redirected to the most productive market participants.

et al., 2019; Lanteri and Rampini, 2023; Asriyan et al., 2024). Among these contributions, the closest to ours are Lanteri and Rampini (2023) and Asriyan et al. (2024), who also highlight the general-equilibrium effects of capital reallocation in environments with heterogeneous productivity and financial constraints. Whereas Lanteri and Rampini (2023) focus on the determinants of capital misallocation in steady state, we instead study state-contingent misallocation—namely, how capital is misallocated during financial crises and how this varies with crisis severity. A central distinction in our framework is between fire-sale and demand-freeze crises, depending on whether constrained entrepreneurs are net sellers or net buyers of capital. We show that the same economy can have crises of different types depending on their severity. Moreover, we provide a positive and a normative analysis of policy interventions across both types of crises. We also develop a "sufficient-statistic" approach that allows one to empirically discriminate between the two crisis types using firm-level data, and we find evidence suggesting that demand-freeze episodes may be prevalent.

Second, our normative analysis builds on the literature that studies inefficiencies in the allocation of productive factors arising from financial frictions. A common theme in this work is that individual firms fail to internalize the general-equilibrium effects of their factor demand on prices, giving rise to pecuniary externalities and inefficient allocations. Prominent examples include Caballero and Krishnamurthy (2001), Lorenzoni (2008), Biais and Mariotti (2009), Dávila and Korinek (2017), Itskhoki and Moll (2019), Asriyan et al. (2021), Asriyan et al. (2024), and Buera et al. (2021). Among these, the closest paper to ours is Dávila and Korinek (2017), who provide a general analysis of pecuniary externalities in economies with financial frictions. Our objective, by contrast, is to build a positive theory of financial crises—distinguishing between fire-sale and demand-freeze crises—and to analyze the policy implications of this distinction. Like us, Dávila and Korinek (2017) emphasize that the sign of externalities depends on whether constrained borrowers are net buyers or net sellers of productive assets. Our contribution is to transform this insight into an operational crisis taxonomy, demonstrate its empirical relevance, and show how it alters the interpretation of standard policy tools. In particular, we show why concrete crisis interventions (e.g., interest-rate cuts, asset purchases) may backfire in demand-freeze crises, despite being effective in fire-sale environments.

In this last regard, our work is naturally related to the extensive macroeconomic literature on the financial accelerator, which highlights the central role of asset prices in the propagation of financial crises and business cycles more broadly (e.g., Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997; Krishnamurthy, 2003; Brunnermeier and Sannikov, 2014). We contribute to this literature by showing that the pattern of capital reallocation across firms is a central determinant of general equilibrium outcomes and plays a crucial role in shaping the effectiveness of policy interventions during crises. In particular, we find that the empirical patterns of asset reallocation among U.S. corporations urges caution in interpreting the standard policy prescriptions derived from this literature, which are largely based on the fire-sale narrative of crises. This is also consistent with the recent evidence of Baron et al. (2025), who study banking crises in 46 economies and conclude that losses during these episodes are primarily driven by write-downs of non-performing assets and not by the sale of assets during panics.

A key mechanism underlying our results is the presence of heterogeneity across firms in the equilibrium tightness of their financial constraints—and therefore in their marginal productivity of capital—and the differential responsiveness of these firms to policy changes. In this respect, our work connects to a growing literature empirically documenting firm-level heterogeneity in responses to policy interventions, particularly monetary shocks (e.g., Ottonello and Winberry, 2020; Cloyne et al., 2023; Anderson and Cesa-Bianchi, 2024; Jeenas, 2025).

Finally, our focus on the micro-level details of financial crises as a basis for optimal policy design is shared by recent contributions such as Kurlat (2021) and Robatto (2025). Unlike our framework, these papers model crises as being uniformly driven by fire sales, where the most productive firms become net sellers of assets. Nevertheless, they show that policy responses can still diverge sharply from traditional prescriptions, particularly when fire sales arise from asymmetric information problems à la Akerlof (1970).

2 A Model of Financial Crises

2.1 Environment

We consider an economy that lasts for three periods: $t = 0, 1, 2$. There are two goods: a perishable consumption good and capital. The economy's capital stock is fixed and equal to $\bar{K} > 0$. There are two sets of agents, entrepreneurs and savers, each of unit mass, denoted by I^E and I^S respectively.

Preferences. The preferences of agent $i \in I^E \cup I^S$ are given by:

$$U^i = E_0[c_2^i],$$

where $c_2^i \geq 0$ is agent i's consumption at $t = 2$ and $E_0[\cdot]$ is the expectation operator at $t = 0$.

Endowments. Each entrepreneur is endowed with w units of the consumption good at $t = 0$, while each saver is endowed with W units of it plus $\bar{K}$ units of capital at $t = 0$.

Technology. All agents have access to storage technology with a gross return 1. In addition, entrepreneurs have access to a productive technology that uses capital to produce consumption goods. In particular, if entrepreneur $i \in I^E$ installs $k_0^i \geq 0$ units of capital at $t = 0$, she produces $y_1^i = \rho_1^i \cdot (k_0^i)^\alpha$ units of the consumption good at $t = 1$, where $\alpha \in (0, 1)$. Likewise, if she installs $k_1^i \geq 0$ units of capital at $t = 1$, she produces $y_2^i = \rho_2^i \cdot (k_1^i)^\alpha$ units of the consumption good at $t = 2$. The productivities $\rho_1^i > 0$ and $\rho_2^i > 0$ are idiosyncratic and potentially stochastic.

Markets. In any given period t, entrepreneurs and savers can trade capital in a competitive market at price q_t. To finance investment, entrepreneurs can also issue promises to savers in a competitive credit market. Promises are short-term and (*de jure*) non-contingent: a promise issued at t is a commitment to deliver one unit of the consumption good at $t+1$. We introduce a friction by assuming that an entrepreneur can abscond with a fraction $1 - \theta_t$ of her output at $t = 1, 2$. We focus throughout on the case in which $\theta_1 = 1$ and $\theta_2 = \lambda \leq 1$.

Uncertainty. There is both idiosyncratic and aggregate uncertainty. Idiosyncratic uncertainty refers to the productivities ρ_1^i and ρ_2^i, both of which are realized at $t = 1$.[3] Productivities are drawn independently across entrepreneurs from a common distribution: $(\rho_1^i, \rho_2^i) \sim^{\text{iid}} G$ with support $[\underline{\rho}_1, \bar{\rho}_1] \times [\underline{\rho}_2, \bar{\rho}_2]$. At $t = 1$, there are thus two sources of heterogeneity among entrepreneurs. First, entrepreneurs differ in the *realized* productivity, ρ_1^i, which captures differences in their cash flows or net worth. Second, entrepreneurs differ in the *expected* productivity, ρ_2^i, which captures differences in their investment opportunities going forward. To conserve on notation, we summarize the productivities of entrepreneur i by $s^i = (\rho_1^i, \rho_2^i)$. Aggregate uncertainty instead refers to the value of the parameter λ, which is also realized at $t = 1$, drawn from a distribution F with support $[0, 1]$. We think of λ as capturing credit-market conditions, where high realizations of λ reflect normal times whereas low realizations of λ reflect crisis times.

2.2 Entrepreneurial investment and borrowing

We use b_t^i to denote the total (i.e., net of storage) promises of repayment at t made by entrepreneur $i \in I^E$ at $t - 1$. We assume that entrepreneur i can renegotiate promised repayments ex post, and that she has full bargaining power in renegotiation. Thus, repayments are *de facto* bounded by entrepreneurial collateral. Letting $\widetilde{b}_t^i$ denote the actual repayments made

[3]The assumption that ρ_2^i is realized at $t = 1$ is for simplicity. Our setting is isomorphic to the alternative specification where entrepreneur i receives a signal about ρ_2^i. E.g., if ρ_1^i and ρ_2^i are correlated, then the realization of ρ_1^i is itself a signal about ρ_2^i (see Section 3.3).

by entrepreneur $i \in I^E$ at time t, it follows that:

$$\widetilde{b}_1^i = \min\left\{\rho_1^i \cdot (k_0^i)^\alpha + q_1 \cdot k_0^i,\ b_1^i\right\}, \tag{1}$$

$$\widetilde{b}_2^i = \min\left\{\lambda \cdot \rho_2^i \cdot (k_1^i)^\alpha + q_2 \cdot k_1^i,\ b_2^i\right\}. \tag{2}$$

To streamline the analysis of the entrepreneurs' problem, we will make use of the following two observations. First, let R_t denote the market interest rate, i.e., the return on a non-contingent promise between periods t and $t+1$. Since there is neither consumption nor investment in the aggregate at $t = 0, 1$, storage must necessarily be used and, thus, in equilibrium:

$$R_t = 1 \text{ for } t = 0, 1. \tag{3}$$

Second, as capital is no longer useful after production at $t = 2$, we must have that in equilibrium:

$$q_2 = 0. \tag{4}$$

We are now ready to study entrepreneurial optimization, given asset prices. As is customary, we start with the entrepreneurs' problem at $t = 1$ and then analyze her problem at $t = 0$.

2.2.1 Optimization at $t = 1$

At $t = 1$, entrepreneur $i \in I^E$ has net worth:

$$w_1^i \equiv \rho_1^i \cdot \left(k_0^i\right)^\alpha + q_1(\lambda) \cdot k_0^i - \min\left\{\rho_1^i \cdot (k_0^i)^\alpha + q_1 \cdot k_0^i,\ b_1^i\right\}, \tag{5}$$

where the notation $q_1(\lambda)$ indicates that the price of capital depends on the aggregate state λ, and where we have used the expression for $\widetilde{b}_1^i$ in Equation (1). Entrepreneur $i \in I^E$ then solves:

$$V(w_1^i, s^i, \lambda) \equiv \max_{k_1^i, b_2^i}\ \rho_2^i \cdot (k_1^i)^\alpha + w_1^i - q_1(\lambda) \cdot k_1^i, \tag{6}$$

subject to her budget constraint:

$$q_1(\lambda) \cdot k_1^i \leq w_1^i + b_2^i, \tag{7}$$

and borrowing constraint:

$$b_2^i \leq \lambda \cdot \rho_2^i \cdot \left(k_1^i\right)^\alpha. \tag{8}$$

Note that we have imposed that $\widetilde{b}_2^i = b_2^i$, which is without loss of generality since both ρ_2^i and $q_2 = 0$ are known at the time of borrowing at $t = 1$.

Optimization by entrepreneur $i \in I^E$ implies the following demand for capital:

$$k_1^i = k_1\left(w_1^i, s^i, \lambda\right) = \begin{cases} k_1^*\left(s^i, \lambda\right) & \text{if } w_1^i \geq w_1^*\left(s^i, \lambda\right) \\ x \text{ s.t. } q_1\left(\lambda\right) \cdot x = w_1^i + \lambda \cdot \rho_2^i \cdot x^\alpha & \text{otherwise} \end{cases}, \tag{9}$$

where:

$$w_1^*\left(s^i, \lambda\right) \equiv q_1\left(\lambda\right) \cdot k_1^*\left(s^i, \lambda\right) - \lambda \cdot \rho_2^i \cdot \left(k_1^*\left(s^i, \lambda\right)\right)^\alpha \tag{10}$$

denotes the level of net worth that enables entrepreneur $i \in I^E$ to undertake her efficient level of investment, $k_1^*\left(s^i, \lambda\right) \equiv \left(\frac{\alpha \cdot \rho_2^i}{q_1(\lambda)}\right)^{\frac{1}{1-\alpha}}$. The corresponding level of borrowing is then given by:

$$b_2^i = b_2\left(w_1^i, s^i, \lambda\right) = q_1\left(\lambda\right) \cdot k_1\left(w_1^i, s^i, \lambda\right) - w_1^i. \tag{11}$$

Equation (9) says that entrepreneur i's investment is efficient if her net worth exceeds the critical value $w_1^*\left(s^i, \lambda\right)$. When her net worth is below this critical value, entrepreneurial investment is inefficiently low. The lower bound on investment is equal to the amount of capital that the entrepreneur can purchase after debt renegotiation, using only what new borrowing she can obtain. This lower bound is:

$$\underline{k_1}\left(s^i, \lambda\right) \equiv k_1\left(0, s^i, \lambda\right) = \left(\frac{\lambda \cdot \rho_2^i}{q_1\left(\lambda\right)}\right)^{\frac{1}{1-\alpha}}. \tag{12}$$

Given optimal investment and borrowing decisions, it is useful to define the marginal value of wealth for an entrepreneur with net worth w_1^i in state $\left(s^i, \lambda\right)$:

$$\nu^i = \nu(w_1^i, s^i, \lambda) \equiv \begin{cases} 1 & \text{if } w_1^i \geq w_1^*\left(s^i, \lambda\right) \\ \frac{(1-\lambda) \cdot \alpha \cdot \rho_2^i \cdot (k^i)_1^{\alpha-1}}{q_1(\lambda) - \lambda \cdot \alpha \cdot \rho_2^i \cdot (k_1^i)^{\alpha-1}} & \text{otherwise} \end{cases}. \tag{13}$$

If the entrepreneur is unconstrained, Equation (13) says that the marginal value of wealth is one, which is the market interest rate at $t = 1$. If instead the entrepreneur is constrained, Equation (13) says that the marginal value of wealth is greater than one. The reason is that each unit of wealth enables the entrepreneur to purchase an additional $\left(q_1(\lambda) - \lambda \cdot \alpha \cdot \rho_2^i \cdot (k_1^i)^{\alpha-1}\right)^{-1}$ units of capital, each of which raises final consumption by $(1 - \lambda) \cdot \alpha \cdot \rho_2^i \cdot (k_1^i)^{\alpha-1}$ units.

2.2.2 Optimization at $t = 0$

At $t = 0$, entrepreneur $i \in I^E$ chooses her investment k_0^i and debt level b_1^i to solve:

$$\max_{k_0^i, b_1^i} \int V(w_1^i, s^i, \lambda) \cdot dG(\rho_1^i, \rho_2^i) \cdot dF(\lambda), \tag{14}$$

subject to the budget constraint:

$$q_0 \cdot k_0^i = w + \int \min\left\{\rho_1^i \cdot (k_0^i)^\alpha + q_1(\lambda) \cdot k_0^i,\ b_1^i\right\} \cdot dG(\rho_1^i, \rho_2^i) \cdot dF(\lambda). \tag{15}$$

where w_1^i satisfies Equation (5) and where we have replaced $\widetilde{b}_1^i$ in the budget constraint with its definition in Equation (1). The optimal choice of k_0^i is characterized by:

$$\begin{aligned} q_0 = & \int \left(\alpha \cdot \rho_1^i \cdot (k_0^i)^{\alpha-1} + q_1(\lambda)\right) \cdot dG(\rho_1^i, \rho_2^i) \cdot dF(\lambda) \\ & \int_{w_1^i > 0} \frac{\nu(w_1^i, s^i, \lambda) - \xi}{\xi} \cdot \left(\alpha \cdot \rho_1^i \cdot (k_0^i)^{\alpha-1} + q_1(\lambda)\right) \cdot dG(\rho_1^i, \rho_2^i) \cdot dF(\lambda), \end{aligned} \tag{16}$$

where $\xi \equiv \mathbb{E}\left[\nu(w_1^i, s^i, \lambda) \,|w_1^i > 0\right]$ is the marginal value of entrepreneurial wealth at $t = 0$. Given k_0^i, the optimal level of borrowing b_1^i is obtained from Equation (15).

Equation (16) implies that, in choosing borrowing and investment at $t = 0$, the entrepreneur sets the marginal cost of investment, q_0, equal to its expected marginal return plus an adjustment term that captures the hedging value of capital. The sign of this term is determined by the correlation between the return to capital and the marginal value of net worth, ν. When this correlation is positive, capital delivers high payoffs precisely in states in which ν is high, thereby providing a hedge against adverse shocks. In this case, the adjustment term is positive and entrepreneurial investment, k_0, is amplified. When the correlation is negative, capital pays off the most in states where ν is low, reducing its hedging value and dampening investment.

2.3 Properties of equilibrium

We now formally define an equilibrium of the economy and discuss some of its key features.

Definition 1 *A competitive equilibrium consists of a vector of investment and borrowing decisions* $\{k_0^i, k_1^i, b_1^i, b_2^i\}$ *for each entrepreneur* $i \in I^E$ *and a vector of prices* $\{q_0, q_1(\lambda)\}$*, such that entrepreneurs' optimality conditions and budget constraints are satisfied in all states, i.e.,* *Equations* (5), (9)-(11), (15), *and* (16) *hold; and the capital market clears:*

$$k_0^i = \bar{K}, \tag{17}$$

and, for all λ*:*

$$\int k_1\left(w_1^i, s^i, \lambda\right) \cdot dG(\rho_1^i, \rho_2^i) = \bar{K}. \tag{18}$$

Given an equilibrium allocation of capital, aggregate output at $t = 2$ is:

$$Y_2(\lambda) = \int \rho_2^i \cdot k_1\left(w_1^i, s^i, \lambda\right) \cdot dG(\rho_1^i, \rho_2^i). \tag{19}$$

Since individuals only value consumption at $t = 2$, the social welfare in the aggregate state λ can in turn be expressed as:

$$\mathcal{SW}(\lambda) = w + W + \int \rho_1^i \cdot \bar{K}^\alpha \cdot dG(\rho_1^i, \rho_2^i) + Y_2(\lambda). \tag{20}$$

Ultimately, aggregate consumption in this economy is equal to total endowment, $w + W$, plus whatever output is produced through investment. Since all entrepreneurs are ex-ante identical, output at $t = 1$ is mechanically equal to $\int \rho_1^i \cdot \bar{K}^\alpha \cdot dG(\rho_1^i, \rho_2^i)$, while $Y_2(\lambda)$ is determined endogenously and depends on how the capital stock is allocated at $t = 1$. This is our main object of interest.

It is useful at this point to establish a benchmark, which corresponds to the allocation that would obtain in the absence of financial constraints. Capital in this case is allocated efficiently at $t = 1$, which maximizes output at $t = 2$ and capital prices throughout.

Lemma 1 *In the absence of financial constraints, i.e., when* $\lambda = 1$ *with probability one, equilibrium output and prices are given by:*

$$Y_2^\star \equiv \widetilde{\rho}_2 \cdot \bar{K}^\alpha, \tag{21}$$

and:

$$q_1^\star \equiv \alpha \cdot \widetilde{\rho}_2 \cdot \bar{K}^{\alpha-1} \text{ and } q_0^\star \equiv \alpha \cdot \int \rho_1^i \cdot dG(\rho_1^i, \rho_2^i) \cdot \bar{K}^{\alpha-1} + q_1^\star, \tag{22}$$

where:

$$\widetilde{\rho}_2 \equiv \left[\int (\rho_2^i)^{\frac{1}{1-\alpha}} \cdot dG(\rho_1^i, \rho_2^i)\right]^{1-\alpha}. \tag{23}$$

In the absence of financial constraints, aggregate productivity of capital at $t = 2$ is given by $\widetilde{\rho}_2$, which results from allocating the aggregate stock of capital $\bar{K}$ so as to equalize its marginal productivity among entrepreneurs. To analyze the competitive equilibrium with financial constraints and compare it to this benchmark, we make two parametric assumptions that we maintain throughout.

The first assumption guarantees that financial constraints are relevant, i.e., entrepreneurs are not so rich that the competitive equilibrium coincides with the benchmark in Lemma 1:

Assumption 1 *Entrepreneurial endowment w is lower than $\bar{w}$, where:*

$$\bar{w} \equiv \left(\alpha \cdot (\widetilde{\rho}_2)^{\alpha} \cdot (\bar{\rho}_2)^{1-\alpha} + \alpha \cdot \int \rho_1^i \cdot dG(\rho_1^i, \rho_2^i) - \underline{\rho_1} \right) \cdot \bar{K}^{\alpha}. \tag{24}$$

The second assumption guarantees that there are no states of nature in which *all* entrepreneurs renegotiate their debts. Besides capturing the empirically relevant case, this assumption also serves the technical purpose of ensuring that an equilibrium always exists:

Assumption 2 *Entrepreneurial endowment w is greater than $\underline{w}$, where:*

$$\underline{w} \equiv \left(\alpha \cdot \widetilde{\rho}_2 + \alpha \cdot \int \rho_1^i \cdot dG(\rho_1^i, \rho_2^i) - \bar{\rho}_1 \right) \cdot \bar{K}^{\alpha}. \tag{25}$$

Using these two assumptions, the following proposition establishes the existence of competitive equilibrium and compares it to the efficient benchmark in Lemma 1:

Proposition 1 *Suppose Assumptions 1 and 2 hold. Then, a competitive equilibrium exists. Moreover, there exists $\lambda^\star \in (0, 1)$ such that:*

- *If $\lambda \geq \lambda^\star$, all entrepreneurs are unconstrained, $Y_2 = Y_2^\star$ and $q_1 = q_1^\star$.*
- *If $\lambda < \lambda^\star$, some entrepreneurs are constrained, $Y_2 < Y_2^\star$ and $q_1 < q_1^\star$.*

Using Proposition 1, we can partition the set of aggregate states into two subsets, which we respectively refer to as *normal* and *crisis* times. In normal times, $\lambda \geq \lambda^*$, the marginal product of capital is equalized across entrepreneurs, and Y_2 and q_1 are as in the efficient benchmark. In crisis times, instead, $\lambda < \lambda^\star$, financial constraints are tight, the marginal product of capital is not equalized across entrepreneurs, and Y_2 and q_1 are depressed relative to normal times.

3 A tale of two crises

In this section, we characterize the key properties of crisis times. As highlighted in Proposition 1, such times are marked by an inefficient allocation of capital, low output, and depressed asset prices. Although this fits well with common narratives, it is unclear whether low asset prices exacerbate or alleviate the effects of financial crises. We now explore this question, and show that not all crises are alike.

3.1 Fire-sale and demand-freeze crises

We begin by characterizing the behavior of output Y_2 across different crisis states:

Proposition 2 *In equilibrium, the effect of an increase in $\lambda < \lambda^\star$ on output Y_2 is given by:*

$$\frac{dY_2}{d\lambda} = \int \rho_2^i \cdot \alpha \cdot k_1\left(w_1^i, s, \lambda\right)^{\alpha-1} \cdot \frac{dk_1\left(w_1^i, s, \lambda\right)}{d\lambda} \cdot dG\left(\rho_1^i, \rho_2^i\right)$$
$$= \underbrace{\mathbb{E}\left[(\nu^i - 1) \cdot \rho_2^i \cdot (k_1^i)^\alpha \; |\lambda\right]}_{\text{Direct channel}} \underbrace{- \; \mathbb{E}\left[(\nu^i - 1) \cdot \left(k_1^i - k_0^i \cdot \mathbb{1}_{w_1^i > 0}\right) |\lambda\right] \cdot \frac{dq_1}{d\lambda}}_{\text{Capital-price channel}}. \tag{26}$$

Since the capital stock is fixed at $\bar{K}$, changes in λ affect output only through their impact on allocative efficiency, i.e., by reallocating capital across entrepreneurs. Proposition 2 decomposes this effect of λ into what we refer to as the *direct* and *capital-price* channels.

The *direct channel* captures how λ affects capital allocation and output for a *given price of capital*. It reflects the reallocation of capital from unconstrained to constrained entrepreneurs that would occur if the capital price were fixed. This component is always positive because $\nu > 1$ for constrained entrepreneurs.

The *capital-price channel* instead captures the effect of λ on capital reallocation operating through the equilibrium capital price q_1. The sign of this channel is ambiguous: since q_1 is increasing in λ, its sign depends on whether productive entrepreneurs are, on average, buyers or sellers of capital.[4] If relatively productive entrepreneurs (i.e., high-ν) are *sellers* of capital (i.e., $k_1 - k_0 \cdot \mathbb{1}_{w_1 > 0} < 0$), the capital-price channel in Equation (26) is positive and reinforces the direct channel. Intuitively, a higher value of λ raises the price of capital, allowing productive entrepreneurs to service their debts by selling fewer units of capital and thus boosting output. If relatively productive entrepreneurs are instead *buyers* of capital (i.e., $k_1 - k_0 \cdot \mathbb{1}_{w_1 > 0} > 0$), the capital-price channel is negative and works against the direct channel: a higher value of λ raises the price of capital and reduces the amount of capital that productive entrepreneurs can purchase, thereby lowering output. Clearly, entrepreneurs that renegotiate their debts are buyers of capital, since their pre-existing capital stock, k_0, is fully appropriated by creditors.

[4] Throughout, we focus on equilibria in which q_1 is increasing in λ. Although equilibria with the opposite slope may also exist, they are unstable and yield counter-intuitive implications (e.g., higher demand for capital leading to a lower capital price).

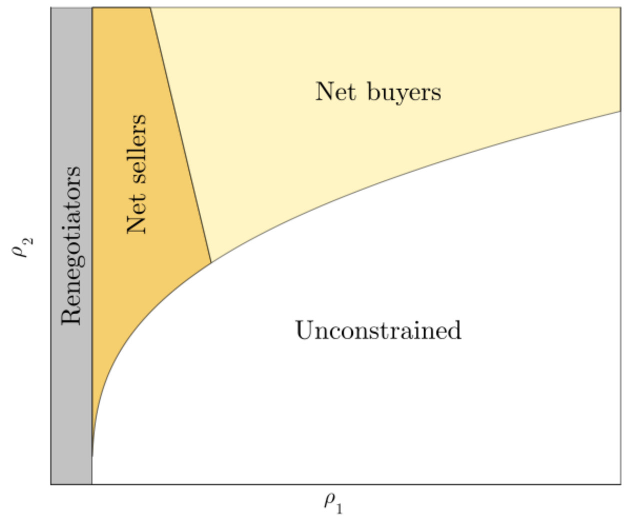

Figure 1: Illustrates the types of entrepreneurs during crises, depending on ρ_1 and ρ_2. Entrepreneur with (ρ_1, ρ_2) in the unshaded (shaded) region is financially unconstrained (constrained), i.e., $\nu = 1$ ($\nu > 1$).

Proposition 2 suggests that general-equilibrium forces play a key role in shaping the effects of financial crises, and that the direction of these forces depends on whether the most productive entrepreneurs are buyers or sellers of capital. Building on this insight, we distinguish between two types of crises:

Definition 2 *Define a financial crisis as a state in which $\lambda < \lambda^\star$, so that financial constraints bind, and output and asset prices are depressed. Moreover, for each crisis state λ, define:*

$$\mathcal{CP}(\lambda) \equiv \mathbb{E}\left[(\nu^i - 1) \cdot (k_1^i - k_0^i \cdot \mathbb{1}_{w_1^i > 0}) \ |\lambda\right]. \tag{27}$$

- *If $\mathcal{CP}(\lambda) < 0$, constrained (i.e., high ν) entrepreneurs are on average sellers of capital and we say there is a* ***fire-sale crisis****.*
- *If $\mathcal{CP}(\lambda) > 0$, constrained (i.e., high ν) entrepreneurs are on average buyers of capital and we say there is a* ***demand-freeze crisis****.*

In what follows, we compare fire-sale and demand-freeze crises and draw out their policy implications. Before doing so, however, Figure 1 characterizes the set of constrained entrepreneurs at $t = 1$ for given values of b_1 and λ. As the figure shows, entrepreneurs can be constrained either because they have low net worth—due to a low realization of ρ_1—or because they have a high demand for capital—due to a high realization of ρ_2. The figure also makes clear that not all constrained entrepreneurs are alike. The shaded gray region depicts entrepreneurs that renegotiate their debts, i.e., those with $\rho_1 \cdot \bar{K}^\alpha + q_1 \cdot \bar{K} < b_1$. The shaded orange region depicts entrepreneurs that repay their debts in full and are net sellers of capital, i.e., those with $k_1 < k_0$. Finally, the shaded yellow region depicts entrepreneurs that repay their debts in full and are net buyers of capital, i.e., those with $k_1 > k_0$.

Figure 1 distinguishes among constrained entrepreneurs according to their behavior—namely, whether they renegotiate their debts or whether they are net sellers or net buyers of capital. It is also useful to distinguish among constrained entrepreneurs according to whether they benefit from, or are hurt by, an increase in the price of capital.

Definition 3 *Consider a constrained entrepreneur i during a crisis, i.e., $\nu^i > 1$:*

- *If $k_1^i - k_0^i \cdot \mathbb{1}_{w_1^i > 0} < 0$, we say entrepreneur i is a fire seller.*
- *If $k_1^i - k_0^i \cdot \mathbb{1}_{w_1^i > 0} > 0$, we say entrepreneur i is a frozen buyer.*

Proposition 2 implies that the direction and strength of the capital-price channel depend on the share of constrained entrepreneurs and on their distribution between fire sellers and frozen buyers, as defined in Definition 3. This distribution is an endogenous object and is difficult to characterize in full generality. For this reason, we use two stylized economies to illustrate the key implications of the theory analytically, before studying these implications numerically in an empirically plausible, calibrated economy.

3.2 Two stylized economies

We introduce two stylized economies, which we use throughout to illustrate the effects of financial crises and show that not all crises are alike. We define a *fire-sale economy* as one in which $\rho_1^i \sim U[0, 2]$ and $\rho_2^i = 1$, for $i \in I^E$. We define a *demand-freeze economy* as one in which $\rho_1^i = 1$ and $\rho_2^i \sim U[0, 2]$, for $i \in I^E$. In both economies, entrepreneurial endowment, w, is assumed to be large enough so as to rule out debt renegotiation in equilibrium.[5] As we shall see, a convenient implication of this setup is that the fire-sale economy suffers only from fire-sale crises, whereas the demand-freeze economy suffers only from demand-freeze crises (Definition 2).

Figure 2 depicts the equilibrium price of capital, q_1, and output, Y_2, as they depend on λ. Panels (a) and (b) show these objects for the fire-sale and demand-freeze economies, respectively. Both economies are consistent with Proposition 1. In each economy, there exists a threshold

[5] In the demand-freeze economy, this is guaranteed by Assumption 2. In the fire-sale economy, the lower bound on entrepreneurial endowment that guarantees this is higher; we provide an explicit expression for it in the proof of Proposition 3 in Appendix A.1.

value $\lambda^\star$ such that, whenever $\lambda \geq \lambda^\star$, financial constraints are slack at $t = 1$, and prices and quantities coincide with those in the efficient benchmark. By contrast, for $\lambda < \lambda^\star$, financial constraints bind and distort the allocation of capital. As a result, both the asset price and output fall relative to the efficient benchmark.

Figure 2 suggests that financial crises in the fire-sale and demand-freeze economies look similar and align with the conventional narrative: they feature low asset prices and depressed output. This similarity, however, masks a crucial difference in the underlying source of the inefficient capital allocation.

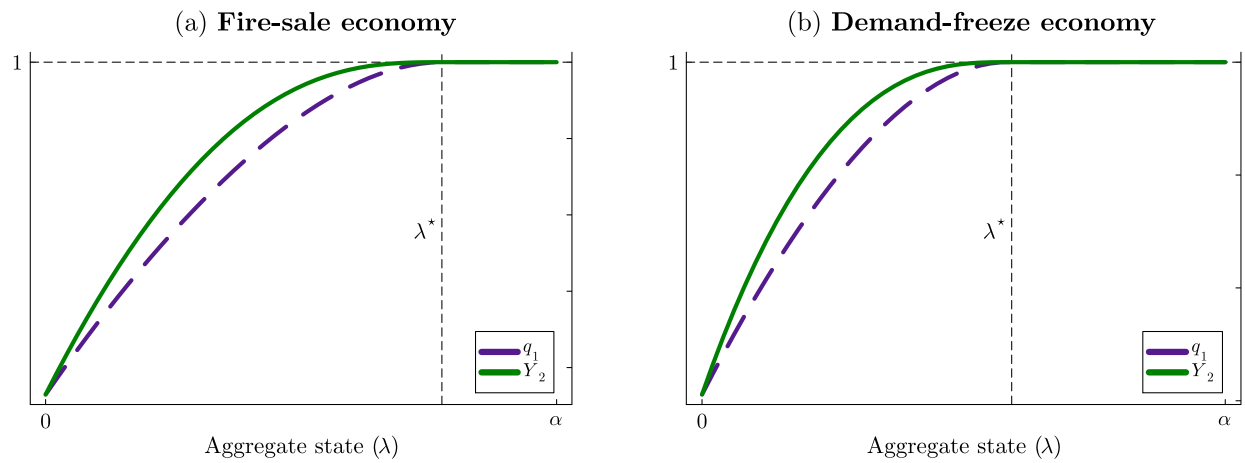

Figure 2: Illustrates output (Y_2) and the price of capital (q_1) across aggregate states (λ), relative to their counterparts, $Y_2^\star$ and $q_1^\star$, in the efficient benchmark (see Lemma 1).

In the fire-sale economy, entrepreneurs at $t = 1$ have the same productivity of investment going forward, ρ_2, but they differ in their net worth, w_1, due to heterogeneity in realized productivity, ρ_1. Thus, during a crisis, it is entrepreneurs with low ρ_1 who are constrained. In the terminology of Definition 3, these entrepreneurs are *fire sellers*: because of their low net worth, they are forced to sell capital to repay their debts.

In the demand-freeze economy, by contrast, entrepreneurs at $t = 1$ have the same net worth, w_1, as they share the same realized productivity, ρ_1, but they differ in the productivity of investment going forward, ρ_2. Therefore, during a crisis, it is entrepreneurs with high ρ_2 who are constrained. In the terminology of Definition 3, these entrepreneurs are *frozen buyers*: due to binding financial constraints, they are unable to purchase as much capital as they would like.

This distinction has crucial implications for the role of capital reallocation during crises, for the sign and strength of the capital-price channel outlined in Proposition 2, and, ultimately, for the effective design of policy.

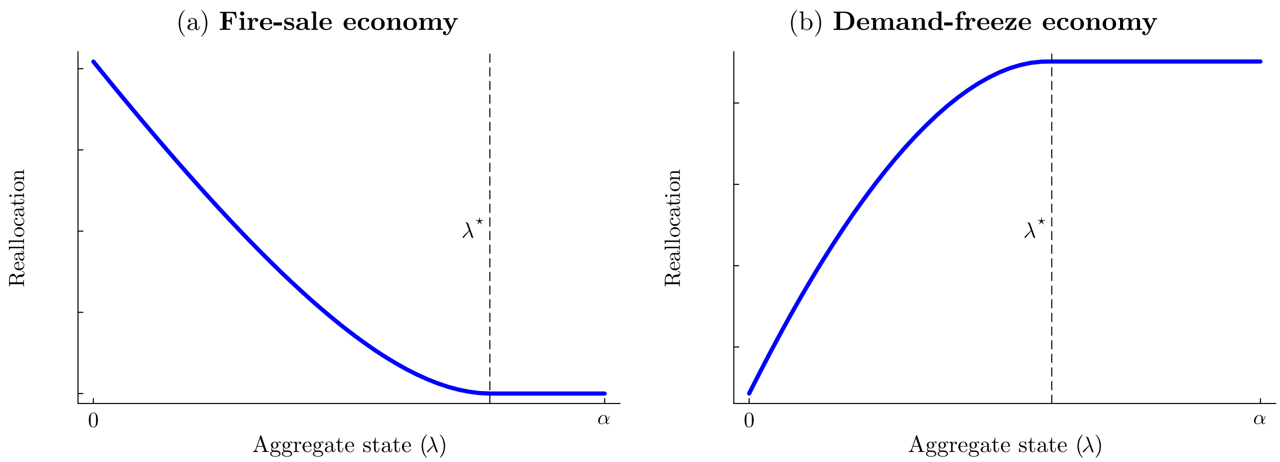

Figure 3: Illustrates the share of capital that is traded, $\int (k_1^i - k_0^i)_+ \cdot dG(\rho_1^i, \rho_2^i)/\bar{K}$, across aggregate states (λ).

Consider first the behavior of capital reallocation during crises. In the fire-sale economy, reallocation is countercyclical because crises are times of expanded trade. During normal times, there is no need to reallocate capital, as all entrepreneurs have the same productivity of investment going forward. In crisis times, however, reallocation is high because entrepreneurs with low net worth are forced to sell part of their capital. Instead, in the demand-freeze economy, capital reallocation is procyclical because crises are times of restricted trade. In normal times, reallocation is high, as entrepreneurs with high ρ_2 can borrow to purchase all the capital they desire. In crisis times, however, reallocation is low because financial constraints limit the amount of capital these entrepreneurs can buy. This distinction between the two economies is illustrated in Figure 3, which depicts capital reallocation in both economies as it depends on λ. We will return to it in Section 6, when we analyze the patterns of reallocation in the data.

Consider next the direction of capital reallocation in the two economies, i.e., who buys and sells capital. In the fire-sale economy, it is entrepreneurs with a high marginal return to investment who sell capital during crises. These entrepreneurs have low net worth and are forced to sell capital to repay their debts. Thus, as illustrated in Panel (a) of Figure 4, the covariance between ν and $k_1 - k_0$ is negative, constrained entrepreneurs are predominantly fire sellers, and—consequently—this economy only gives rise to fire-sale crises (Definition 2).

In the demand-freeze economy, entrepreneurs with a high marginal return to investment are instead buyers of capital during crises. These entrepreneurs wish to expand their capital holdings, but their ability to do so is limited by tight financial constraints. Thus, as illustrated in Panel (b) of Figure 4, the covariance between ν and $k_1 - k_0$ is positive, constrained entrepreneurs are frozen buyers, and—consequently—this economy only gives rise to demand-freeze crises.

Proposition 3 summarizes this key distinction between the two economies.

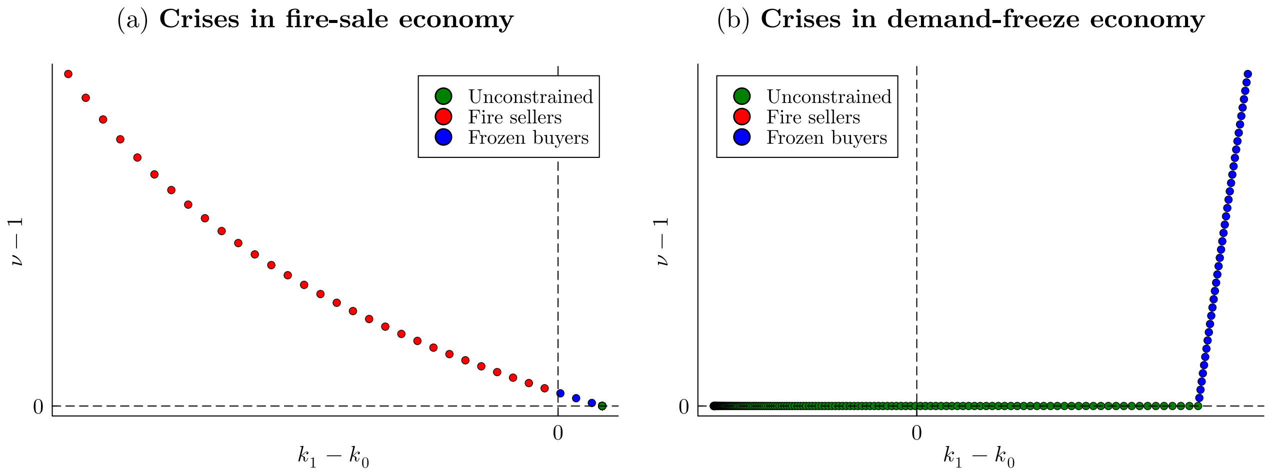

Figure 4: Illustrates the relationship between entrepreneurs' marginal value of net worth (ν) and their net demand for capital ($k_0 - k_1$).

Proposition 3 *In the fire-sale economy, $\mathcal{CP}(\lambda) < 0$ for all crisis states $\lambda < \lambda^{\star}$. Instead, in the demand-freeze economy, $\mathcal{CP}(\lambda) > 0$ for all crisis states $\lambda < \lambda^{\star}$.*

Proposition 3 shows that these two stylized economies provide useful benchmarks to characterize fire-sale and demand-freeze crises. In the fire-sale economy, the fall in the price of capital aggravates the crisis: since high-productivity entrepreneurs are net sellers of capital, low capital prices force them to sell even more capital to repay their debts. In the demand-freeze economy, the fall in the price of capital instead alleviates the crisis: since high-productivity entrepreneurs are net buyers of capital, low capital prices allow them to expand their investment. As we shall see in Section 4, this distinction is crucial for the effective design of policies.

Albeit pedagogical, these stylized economies are special in two respects. First, they are based on simplistic productivity processes in which entrepreneurs differ only in their $t = 1$ or their $t = 2$ productivity. Second, they rely on a parametrization that rules out equilibrium debt renegotiation. We now extend our analysis to a more realistic, calibrated economy.

3.3 Crises in a "calibrated" economy

We assume that entrepreneurial productivity follows an AR(1) process:[6]

$$\log(\rho_1) \sim \mathcal{N}\left(\frac{a}{1-\gamma}, \frac{\sigma_\varepsilon^2}{1-\gamma^2}\right) \quad \text{with } a \text{ chosen so that } E[\rho_1] = 1, \tag{28}$$

$$\log(\rho_2) = a + \gamma \cdot \log(\rho_1) + \varepsilon, \qquad \varepsilon \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right). \tag{29}$$

We set $\gamma = 0.9$ and $\sigma_\varepsilon = 0.1$ and interpret a period as a quarter.[7] The baseline value of γ as a quarterly persistence parameter is common in the firm-dynamics and financial-frictions literature for the U.S. (e.g., Khan and Thomas, 2013; Gilchrist et al., 2014; İmrohoroğlu and Tüzel, 2014). The standard deviation of the innovation, $\sigma_\varepsilon = 0.1$, lies in the middle of the range of values typically used in the firm-dynamics literature.[8]

We set $\alpha = 0.65$, a commonly used value for the degree of decreasing returns in firms' operating profits as a function of capital in the corporate finance literature (e.g., Hennessy and Whited (2007)). The level of entrepreneurial endowment w is chosen to deliver a debt-to-assets ratio of 0.65 at $t = 0$, following the mean ratio of liabilities to total assets for non-financial firms reported by Kalemli-Ozcan et al. (2012). Finally, we assume that aggregate states are distributed according to $F(\lambda) = \lambda^\beta$ and calibrate β to generate a 4% probability of a crisis (e.g., Jordà et al., 2017; Paul, 2020)).

Panel (a) of Figure 5 depicts the price of capital, q_1, and output, Y_2, in the calibrated economy as a function of λ. As in the fire-sale and demand-freeze benchmarks, financial crises are times of low asset prices and low output. Panel (b) of the figure depicts the share of the total capital stock that is reallocated across entrepreneurs. As in the demand-freeze economy of the previous section, the volume of reallocation in the calibrated economy is lower during crises than in normal times.

To shed light on the nature of financial crises in the calibrated economy, Panel (a) of Figure 6 plots a random sample of 20,000 entrepreneurs in (ρ_1, ρ_2)-space for a crisis with $\lambda = 0.1$. The shaded yellow area indicates combinations of (ρ_1, ρ_2) for which entrepreneurs are constrained. Most constrained entrepreneurs are net buyers of capital (blue), with a select few being net sellers (red).[9] In this respect, the calibrated economy echoes the demand-freeze benchmark:

[6] Given that the implied support of the distribution $G(\rho_1, \rho_2)$ is $(0, \infty) \times (0, \infty)$, it is straightforward to verify that Assumptions 1 and 2 are satisfied in the calibrated economy.

[7] In Appendix A.2.1, we extend the analysis to $\gamma = 0.65$ and interpret a period as a year.

[8] For instance, Gilchrist et al. (2014) and İmrohoroğlu and Tüzel (2014) suggest values around $\sigma_\varepsilon \approx 0.15$, while Khan and Thomas (2013) use a value consistent with a quarterly $\sigma_\varepsilon \approx 0.065$.

[9] Entrepreneurs who renegotiate their debt—those with ρ_1 close to zero—are part of the equilibrium but

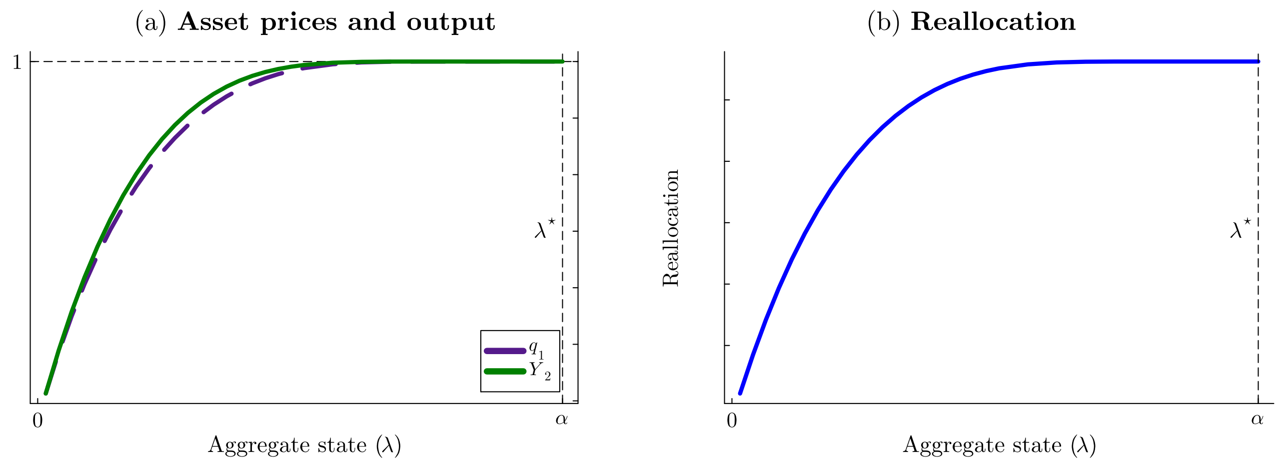

Figure 5: Illustrates the behavior of asset prices, output and capital reallocation across aggregate states (λ) in the calibrated economy.

during crises, most constrained agents are frozen buyers, expanding but less than desired.

Intuitively, this result follows from the interplay of persistent productivity and concave technology. Entrepreneurs with higher net worth (high ρ_1) also tend to have higher future productivity (high ρ_2) and, due to concavity, desired investment rises more than proportionally with ρ_2. This implies that it is the highest-productivity types—those with high (ρ_1, ρ_2)—who are most likely to be financially constrained.

Panel (b) of Figure 6 depicts entrepreneurs' marginal value of net worth, ν, against their capital purchasing position, $k_1 - k_0 \cdot \mathbb{1}_{w_1>0}$, during the crisis. The relation between the two variables is again reminiscent of the demand-freeze economy depicted in Panel (b) of Figure 4. Unconstrained entrepreneurs adjust their capital stock until their marginal value of net worth equals one. Constrained entrepreneurs, who have a high marginal value of net worth, are largely net buyers of capital, but their ability to expand is limited by financial constraints. Unlike the stylized demand-freeze economy, there is now some variation in entrepreneurs' marginal value of wealth for each level of capital purchases: the reason is that, given the calibrated productivity process, entrepreneurs differ in both ρ_1 and ρ_2.

This discussion suggests that, in line with Definition 2, $\mathcal{CP}(\lambda) > 0$ in the calibrated economy when $\lambda = 0.1$. Figure 7 confirms that this extends to other crisis states as well. Panel (a) shows that $\mathcal{CP}(\lambda)$ is positive *for all* $\lambda < \lambda^\star$: namely, in the calibrated economy, all crises are of the demand-freeze type. Panel (b) decomposes $\mathcal{CP}(\lambda)$ into the contributions of net buyers, net sellers, and entrepreneurs who renegotiate their debts.[10] As the figure shows, $\mathcal{CP}(\lambda)$ is driven

do not appear in the simulated sample because the probability of renegotiation is extremely small under the baseline calibration (about 10^{-11}).

[10]More precisely, we conduct this decomposition by partitioning the integral $\mathcal{CP}(\lambda)$ into the corresponding

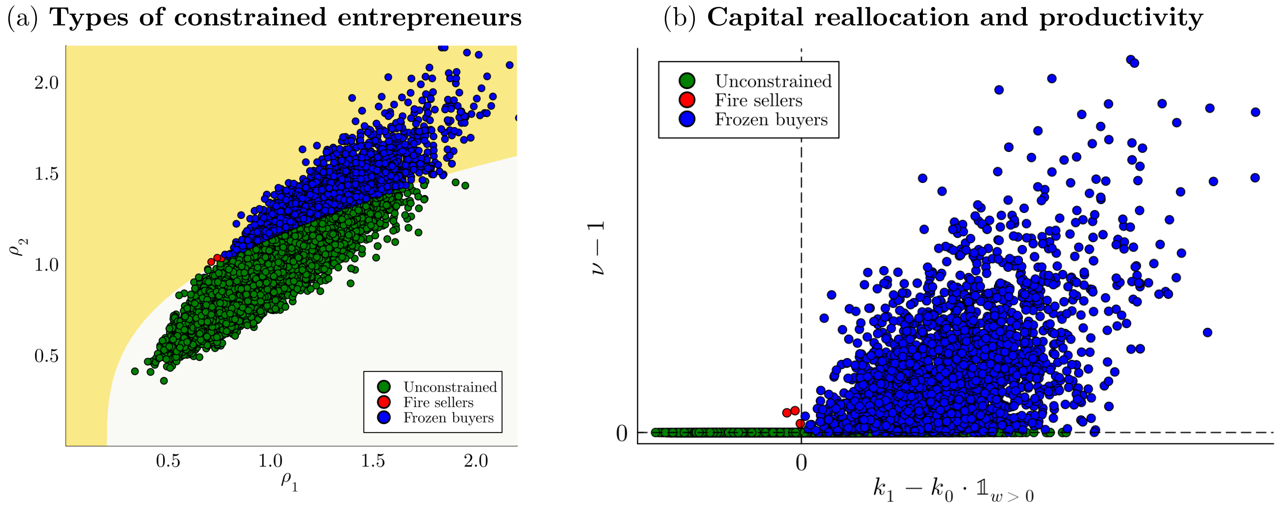

Figure 6: Illustrates the characteristics of financial crises in the calibrated economy.

solely by the prevalence of net buyers: in the calibrated economy, neither debt renegotiation nor fire sellers are common. Interestingly, $\mathcal{CP}(\lambda)$ grows larger as the crisis becomes more severe: this is because low asset prices during severe crises strengthen entrepreneurs' desire to expand.

In sum, once the economy is calibrated to an empirically disciplined productivity process and allows for equilibrium debt renegotiation, financial crises primarily take the form of demand freezes. During crises, asset prices and output fall, capital reallocation is muted, and constrained entrepreneurs—those with a high marginal value of net worth—are net buyers of capital. Consequently, in the calibrated economy the *capital-price channel* identified in Proposition 2 dampens, rather than amplifies, the effects of crises: namely, lower asset prices facilitate the reallocation of capital toward constrained entrepreneurs. As we now show, this has first-order implications for policy design.

4 Policy implications

We now turn to policy design for managing financial crises. Our focus is on credit-easing measures—reductions in policy rates and large-scale asset purchases—which, according to conventional wisdom, help mitigate the economic costs of crises.

net buyers', net sellers', and renegotiators' terms.

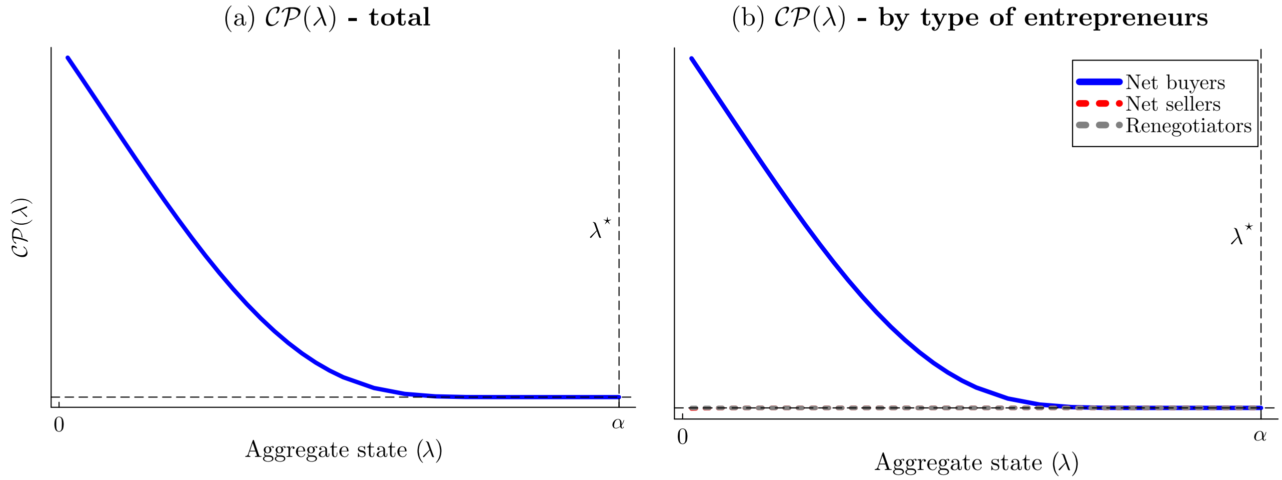

Figure 7: Illustrates $\mathcal{CP}(\lambda)$ and its decomposition across aggregate states (λ) in the calibrated economy.

4.1 Crisis interventions

We focus throughout on the ex-post effect of policies, i.e., on the effects of crisis interventions at $t = 1$. We interpret our analysis as applying to policies implemented in a particular state $\lambda < \lambda^\star$, which do not affect entrepreneurial choices at $t = 0$ because any such state has zero probability of occurring from an ex-ante perspective.

We model interest-rate policies as taxes or subsidies to the use of storage at $t = 1$. Letting τ denote the tax rate imposed on storage at $t = 1$, it follows that $R_1 = 1 - \tau$ in equilibrium. Thus, we henceforth use R_1 directly to capture the policy. To model asset purchases, we assume that the government can purchase K^g units of capital at $t = 1$ and use them to produce output at $t = 2$ according to a public technology:

$$Y_2^g = \rho^g \cdot K^g. \tag{30}$$

We assume the costs/revenues generated by either policy are transferred lump-sum to savers.[11]

In the presence of crisis interventions, aggregate output can be expressed as:

$$Y_2(\lambda, R_1, K^g) = \int \rho_2^i \cdot k_1\left(w^i, s^i, \lambda\right)^\alpha \cdot dG(\rho_1^i, \rho_2^i) + \rho^g \cdot K^g, \tag{31}$$

where:

$$k_1\left(w^i, s^i, \lambda\right) \equiv \min\left\{\left(\frac{\alpha \cdot \rho_2^i}{q_1(\lambda, R_1, K^g) \cdot R_1}\right)^{\frac{1}{1-\alpha}}, x\right\} \tag{32}$$

[11] Although purchases of real assets by governments are uncommon in practice, they turn out to be useful for understanding the more commonly used financial-asset purchases, which we consider in Section 5.

for $x \geq 0$ that satisfies:

$$q_1\left(\lambda, R_1, K^g\right) \cdot x = w_1^i + \frac{\lambda \cdot \rho_2^i \cdot x^\alpha}{R_1}. \tag{33}$$

Finally, the marginal value of net worth for entrepreneur i is given by:

$$\nu(w_1^i, s^i, \lambda) \equiv \frac{(1-\lambda) \cdot \alpha \cdot \rho_2^i \cdot k_1\left(w^i, s^i, \lambda\right)^{\alpha-1}}{q_1(\lambda, R_1, K^g) \cdot R_1 - \lambda \cdot \alpha \cdot \rho_2^i \cdot k_1\left(w^i, s^i, \lambda\right)^{\alpha-1}} \cdot R_1. \tag{34}$$

The notation $Y_2(\lambda, R_1, K^g)$ and $q_1(\lambda, R_1, K^g)$ makes explicit that output and asset prices now also depend on policies R_1 and K^g. Equations (32)–(34), in turn, generalize the capital demand and the marginal value of net worth of Equations (9) and (13) to the case where the interest rate may not equal the technological return to storage, i.e., $R_1 \neq 1$, and where the capital market-clearing condition is modified to:

$$\int k_1\left(w^i, s^i, \lambda\right) \cdot dG(\rho_1^i, \rho_2^i) + K^g = \bar{K}. \tag{35}$$

Equations (31)–(35) can be used to characterize the effects of policy interventions on output and welfare, where the latter is as before given by Equation (20).

It is immediate that a decline in R_1 boosts the demand for capital of all entrepreneurs and, hence, raises the equilibrium price of capital q_1. To analyze the overall effect of the policy on output, it is again useful to decompose it into a direct and an indirect capital-price channel.

Proposition 4 *Consider a competitive equilibrium in a crisis state $\lambda < \lambda^\star$ absent policy intervention. The effect of a small increase in the interest rate on output Y_2 is given by:*

$$\frac{dY_2}{dR_1} = \underbrace{-\ \mathbb{E}\left[\left(\nu^i - 1\right) \cdot \lambda \cdot \rho_2^i \cdot \left(k_1^i\right)^\alpha \ |\lambda\right]}_{\text{Direct channel}} \ \underbrace{-\ \mathcal{CP}(\lambda) \cdot \frac{dq_1}{dR_1}}_{\text{Capital-price channel}}, \tag{36}$$

where $\mathcal{CP}(\lambda)$ is as in Definition 2.

The *direct channel* in Equation (36) captures the effect of the interest rate on the allocation of capital and output for a *given price of capital*. It is unambiguously negative: a lower rate of interest relaxes financial constraints, thereby promoting reallocation from unconstrained to constrained entrepreneurs (who have the higher marginal value of net worth).

The *capital-price channel* in Equation (36) captures instead the effect of the interest rate on capital reallocation that operates through the equilibrium price of capital q_1. Its sign hinges on the nature of the crisis at hand. In a fire-sale crisis, $\mathcal{CP}(\lambda) < 0$ and – since $dq_1/dR_1 < 0$ – the capital-price channel reinforces the expansionary effect of an interest-rate cut. By raising q_1, a lower rate reduces the need for productive entrepreneurs to liquidate capital to repay debts. In a demand-freeze crisis, $\mathcal{CP}(\lambda) > 0$ and the capital-price channel instead attenuates the expansion from a rate cut. By raising q_1, a lower rate limits the ability of productive entrepreneurs to purchase capital.

The effects of asset purchases can likewise be expressed as the sum of two components.

Proposition 5 *Consider a competitive equilibrium in a crisis state $\lambda < \lambda^\star$ absent policy intervention. The effect of a small increase in asset purchases on output Y_2 is given by:*

$$\frac{dY_2}{dK^g} = \underbrace{\rho^g - q_1}_{\text{Direct channel}} \quad \underbrace{- \mathcal{CP}(\lambda) \cdot \frac{dq_1}{dK^g}}_{\text{Capital-price channel}}, \tag{37}$$

where $\mathcal{CP}(\lambda)$ is as in Definition 2.

The *direct channel* in Equation (37) captures the effect of asset purchases on output for a *given price of capital*. Since asset purchases reallocate capital toward the public technology, the sign of the direct component depends on the marginal productivity of capital in the public sector, ρ^g, relative to its marginal productivity in the private sector, q_1.

The *capital-price channel* in Equation (37) captures instead the effect of asset purchases that operates through the equilibrium price of capital q_1. Once again, its sign hinges on the nature of the crisis at hand. In a fire-sale crisis, $\mathcal{CP}(\lambda) < 0$, so the capital-price channel is expansionary. By raising q_1, asset purchases reduce the need for productive entrepreneurs to liquidate capital. In a demand-freeze crisis, $\mathcal{CP}(\lambda) > 0$, so the capital-price channel is contractionary. By raising q_1, asset purchases limit the ability of productive entrepreneurs to purchase capital.

In sum, both interest-rate cuts and asset purchases raise the equilibrium price of capital q_1. The resulting price increase can mitigate a fire-sale crisis (by easing deleveraging) but worsen a demand-freeze crisis (by pricing out constrained buyers). As we show next, the capital-price channel can be large and even dominate the direct channel. We begin by analyzing both policies in the stylized fire-sale and demand-freeze economies of Section 3.2, and then turn to the calibrated benchmark of Section 3.3.

4.2 Policy in two stylized economies

We return to the fire-sale economy, in which entrepreneurs differ only in ρ_1, and the demand-freeze economy, in which they differ only in ρ_2. As in Section 3.2, entrepreneurial net worth is assumed to be sufficiently high to rule out debt renegotiation in equilibrium. We also assume that the marginal productivity of capital in the public sector, ρ^g, is equal to its marginal productivity in the private sector under no intervention.

Figure 8 illustrates the effects of interest-rate policy and asset purchases in the fire-sale economy during a crisis with $\lambda = 0.1$. In this setting, the capital-price channel *reinforces* policy stimulus because $\mathcal{CP}(\lambda) < 0$ by Proposition 3. Accordingly, Panel (a) shows that rate cuts raise both the capital price q_1 and output Y_2. Panel (b) reports the effects of asset purchases. Initially, purchases raise output through the capital-price channel. Eventually, however, their effect can turn contractionary because of the direct channel: as purchases increase and more units of capital are transferred to the public sector, the productivity wedge between the private and public sectors, $q_1 - \rho^g$, widens.

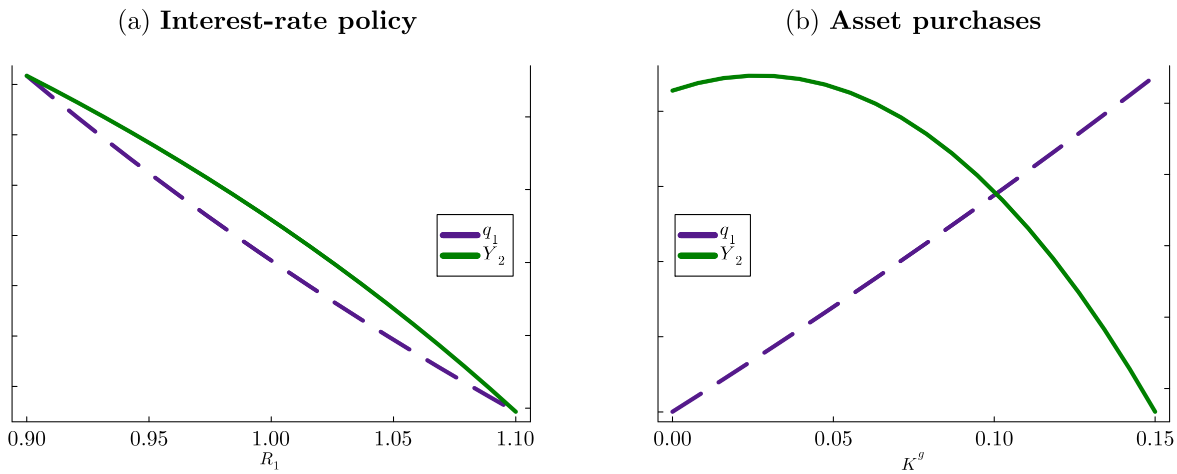

Figure 8: Illustrates Y_2 and q_1 as a function of R_1 and K^g in a fire-sale economy in crisis state $\lambda = 0.1$. Panel (b) assumes that government productivity, ρ^g, equals the marginal productivity of unconstrained entrepreneurs absent intervention, $q_1(\lambda, 1, 0)$.

Figure 9 illustrates instead the effects of interest-rate policy and asset purchases in the demand-freeze economy during a crisis with $\lambda = 0.1$. In this environment, the capital-price channel *weakens* policy stimulus because $\mathcal{CP}(\lambda) > 0$ by Proposition 3.

Panel (a) shows that, in the case of interest-rate policy, the capital-price channel is in fact dominant. As a result, interest-rate cuts *reduce* output—even though they raise the capital price q_1! The higher price of capital tightens the constraints of productive entrepreneurs who are net buyers, thereby outweighing the direct expansionary effect of the lower rate. Panel (b) displays the effects of asset purchases, which are also contractionary in the demand-freeze economy. This is unsurprising: the direct channel is weakly negative under our assumptions (the public sector is weakly less productive than the private sector), and the capital-price channel

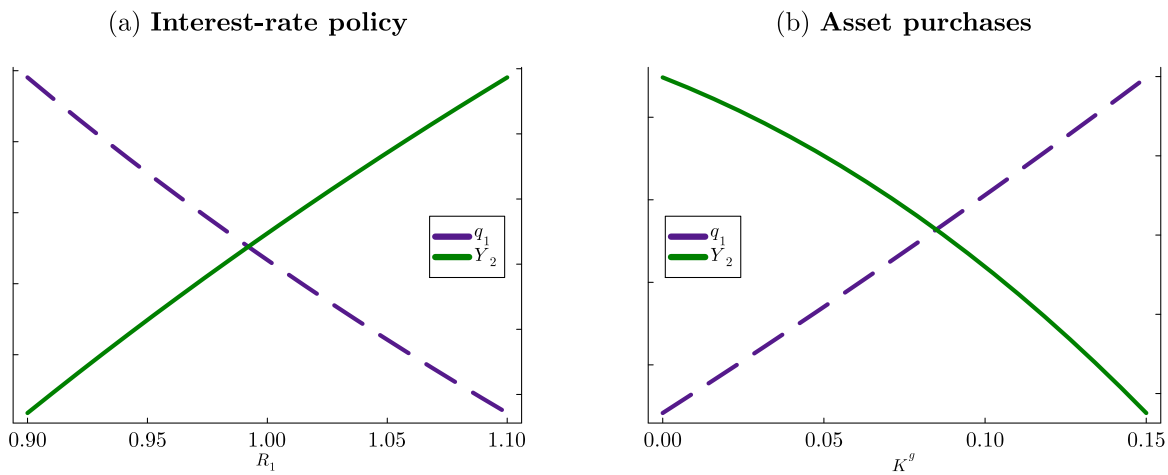

Figure 9: Illustrates Y_2 and q_1 as a function of R_1 and K^g in a demand-freeze economy in crisis state $\lambda = 0.1$. Panel (b) assumes that government productivity, ρ^g, equals the marginal productivity of unconstrained entrepreneurs absent intervention, $q_1(\lambda, 1, 0)$.

is always negative in a demand-freeze crisis, as higher capital prices further restrict the ability of constrained entrepreneurs to acquire capital.

Although the fire-sale and demand-freeze economies share key crisis features—depressed asset prices and heightened misallocation—the policy implications differ markedly. In the fire-sale economy, instruments that raise the capital price q_1—policy-rate cuts and asset purchases—operate through both the direct and the capital-price channels to increase output (with asset purchases turning contractionary only when the public–private productivity wedge becomes sufficiently large). By contrast, in the demand-freeze economy, the capital-price channel dominates: higher q_1 impedes reallocation toward constrained buyers and more than offsets the direct effect, so rate cuts reduce output and asset purchases are contractionary.

4.3 Policy in the calibrated economy

We now turn to the effects of policy in the calibrated economy. Panels (a) and (b) in Figure 10 respectively illustrate the effects of interest-rate policy and asset purchases under the calibration of Section 3.3. As before, the figure depicts the effect of policies when $\lambda = 0.1$ and assumes that the marginal productivity of capital in the public sector, ρ^g, is equal to its productivity in the private sector in the absence of policy.

We already established that, due to the combination of persistent productivity and a concave technology, crises in the calibrated economy are of the demand-freeze type. Panel (a) shows

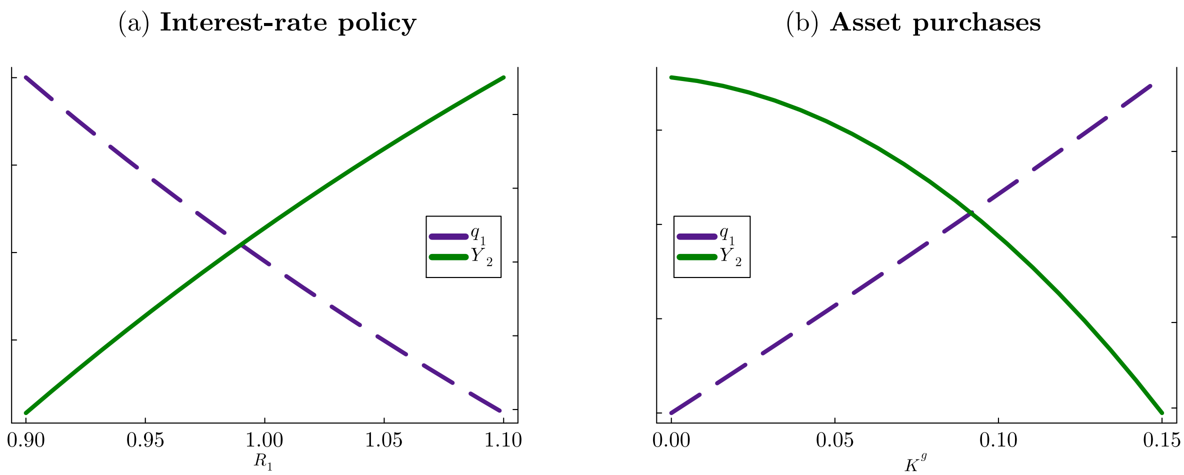

Figure 10: Illustrates Y_2 and q_1 as a function of R_1 and K^g in crisis state $\lambda = 0.1$ in the calibrated economy. Panel (b) assumes that government productivity, ρ^g, equals the marginal productivity of unconstrained entrepreneurs absent intervention, $q_1(\lambda, 1, 0)$.

that the capital-price channel of interest-rate cuts is not only positive but actually dominates the direct channel. As a result, interest-rate cuts are contractionary in the calibrated economy. Panel (b) shows that asset purchases are likewise contractionary. The direct channel becomes increasingly negative as the marginal productivity of capital in the private sector rises above ρ^g, and the capital-price channel is negative throughout because—given the demand-freeze nature of the crisis—higher asset prices hinder the reallocation of capital toward constrained entrepreneurs.

Figure 11 extends these results beyond the illustrative case of $\lambda = 0.1$. Starting from the non-intervention equilibrium, Panel (a) shows the marginal effect of the interest rate on output across values of λ. The derivative is positive for all $\lambda < \lambda^\star$, implying that—in the calibrated economy—an interest-rate cut is contractionary in *all* crisis states. Moreover, the contractionary impact is strongest in the most severe crises. The reason is that the direct channel (depicted by the dashed black line) vanishes as $\lambda \to 0$, when entrepreneurs cannot borrow at all (see Equation (36)), whereas the capital-price channel (depicted by the dashed teal line) grows stronger as λ becomes small.

Panel (b) shows the marginal effect of asset purchases on output across λ: asset purchases are likewise contractionary throughout, and increasingly so as the crisis intensifies. Note that, given our assumption that ρ^g is equal to the marginal productivity of the private sector when $K^g = 0$, the capital-price channel captures the full effect of asset purchases on output.

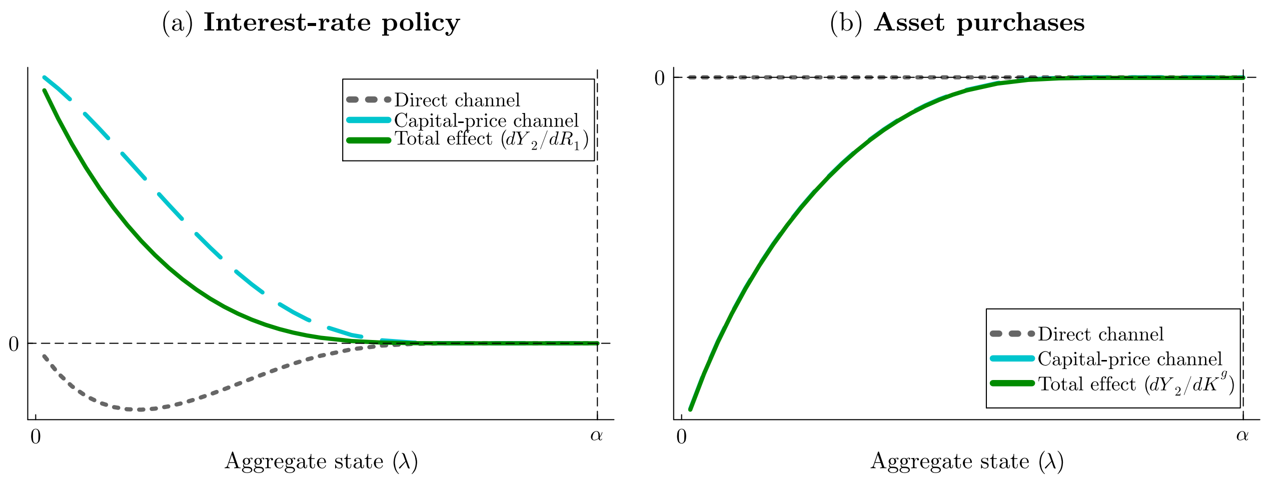

Figure 11: Illustrates effect of policy on output in the calibrated economy, across aggregate states (λ). Panel (b) assumes that government productivity, ρ^g, equals the marginal productivity of unconstrained entrepreneurs absent intervention, $q_1(\lambda, 1, 0)$.

4.4 Discussion

The analysis so far shows that, while financial crises share common features—low asset prices, binding financial constraints, and capital misallocation—the mechanisms behind them differ markedly across fire-sale and demand-freeze environments. In fire-sale crises, productive entrepreneurs are net sellers of capital, so low prices exacerbate deleveraging. In demand-freeze crises, constrained entrepreneurs are net buyers, so low prices facilitate reallocation toward more productive agents. The sign of the capital-price channel captures this distinction.

When the model is calibrated to empirically plausible productivity dynamics, crises consistently take the form of demand freezes. Constrained entrepreneurs are predominantly net buyers, reallocation is muted in crises, and the capital-price channel is positive for all $\lambda < \lambda^\star$. Hence, lower asset prices ease rather than aggravate the crisis.

These findings have direct implications for policy. Although, in principle, the net effect of both channels may operate in either direction, in the calibrated economy the capital-price channel dominates. Higher capital prices impede reallocation toward constrained buyers, so interest-rate cuts become contractionary and asset purchases uniformly so—even though the direct effects of these policies may be expansionary or only weakly negative.

Our framework abstracts from other channels that may justify the use of interest-rate cuts or asset purchases—such as aggregate-demand management,[12]—but our results indicate that the capital-price channel can substantially weaken their effectiveness. Hence, even when other

[12] See, for instance, Caballero and Simsek (2020) for how interest-rate policy can be used for aggregate-demand management during crises.

channels are operative, the presence of the capital-price channel urges caution in the deployment of such policies.

The main takeaway is that diagnosing crises is essential. Fire-sale and demand-freeze crises may look similar at a superficial level, but their propagation mechanisms—and therefore the effects of policy—differ sharply. As we show next, this distinction remains central once we extend the framework to include the financial sector and consider financial-asset purchases.

5 The role of leverage and financial assets

This section extends the model along two dimensions. First, following recent work in macro-finance, we reinterpret entrepreneurs as conglomerates of financial intermediaries and firms. This reinterpretation permits us to analyze the role of intermediaries in financial crises through a simple recalibration of the model. Second, we augment the environment to include financial assets in addition to capital, which enables us to study the implications of financial-asset purchases—a common form of intervention in practice. Throughout, we refer to these conglomerates as banker-entrepreneurs.

5.1 A model of banker-entrepreneurs

A convenient way to reinterpret our model is to view entrepreneurs as conglomerates of financial intermediaries and firms. Following much of the recent macro-finance literature, we assume that the economy is populated by banker-entrepreneurs who borrow from savers and have direct access to the production technology (e.g., Gertler and Karadi, 2011; Amador and Bianchi, 2024). All other aspects of the model remain unchanged.

Under this reinterpretation, all conceptual results continue to hold. The only modification arises in the calibration. Previously, we calibrated a ratio of debt to assets of 0.65, implying a leverage (i.e., assets to equity) ratio of $1/(1 - 0.65) \approx 2.86$, to represent the non-financial corporate sector. We now follow the simple approach of Gertler and Karadi (2011) and double this target leverage ratio (to 5.7) to capture the private sector as a whole. Higher leverage increases the sector's vulnerability to adverse shocks and leads to deeper crises.

Figure 12 plots equilibrium variables as functions of the aggregate state λ. Panel (a) shows that, as λ falls below $\lambda^\star$, asset prices and output decline gradually at first and then collapse. The mechanism is that banker-entrepreneurs, being more highly leveraged, face tight financial constraints during crises. Initially, these constraints reduce capital reallocation, much as in the baseline calibration. However, once crises become sufficiently severe to induce large-scale

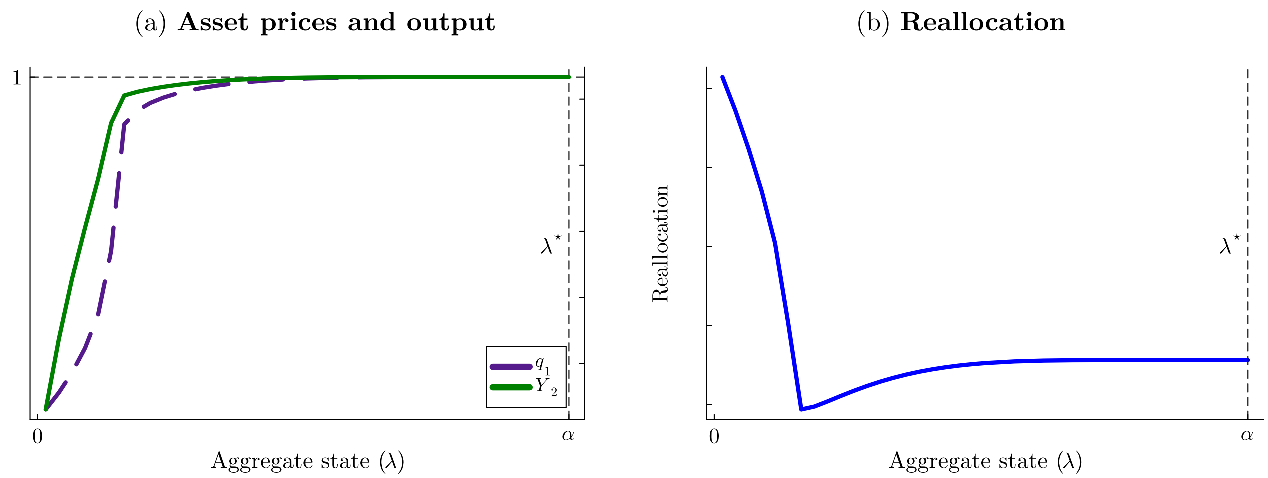

Figure 12: Illustrates asset prices, output and capital reallocatiacross aggregates states (λ) in the economy with banker-entrepreneurs.

capital sales caused by widespread renegotiations, reallocation rises sharply.

Figure 13 provides a closer look at the dynamics of a severe crisis, focusing on the case $\lambda = 0.1$. Panel (a) shows that, relative to the baseline calibration in Figure 6, binding constraints become much more widespread when leverage is increased. As a result, a sizable fraction of banker-entrepreneurs turn into fire sellers. Panel (b) depicts the relation between the marginal value of net worth and capital reallocation. Unlike in the baseline, this relation is now non-monotonic: among fire sellers it is negative, since the most constrained sellers reduce their capital holdings the most; whereas among net buyers it is positive, since the most constrained buyers expand their holdings the most.

How do the policy implications change in this environment? First, given the higher leverage of banker-entrepreneurs, the direct channel of interest-rate policy becomes substantially stronger than under the baseline calibration. In fact, Panel (a) in Figure 14 shows that the direct channel now largely dominates during crises, especially when crises are severe. More crucially, Figure 14 shows that the sign of $\mathcal{CP}(\lambda)$ now changes with the severity of the crisis. Initially, $\mathcal{CP}(\lambda) > 0$ and crises are of the demand-freeze type. As λ falls and crises become more severe, though, there is a region where $\mathcal{CP}(\lambda)$ turns negative and the crisis turns into a fire-sale episode. This happens because, due to their high leverage, many banker-entrepreneurs are forced to liquidate substantial shares of their capital in order to repay their debts. As λ falls even further and the crisis intensifies, though, $\mathcal{CP}(\lambda)$ becomes again positive and the crisis turns once more a demand-freeze episode. The reason is that, in such a severe crisis, an increasing

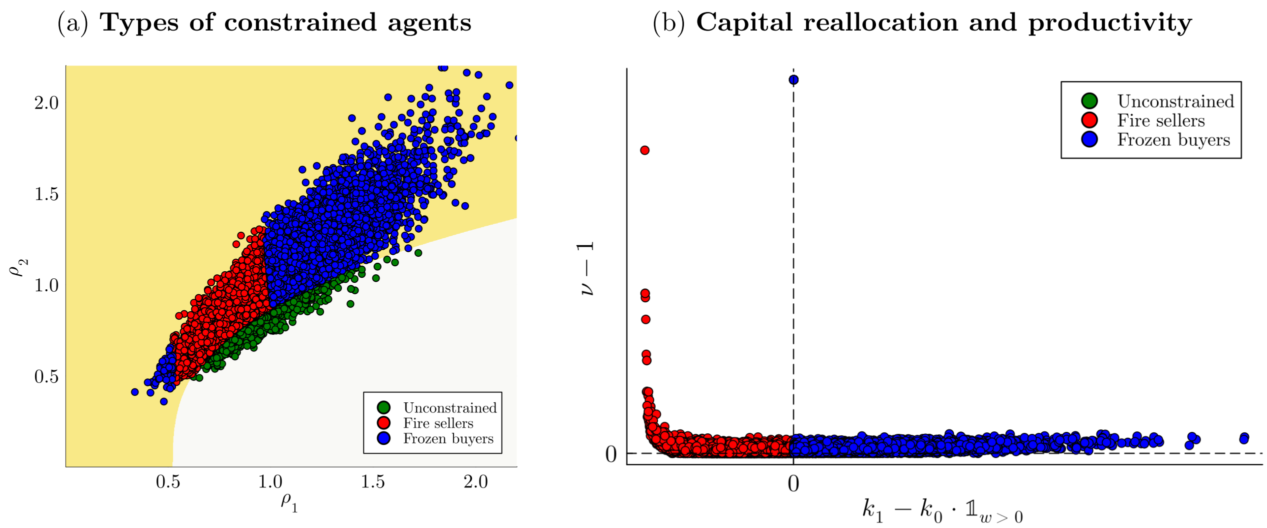

Figure 13: Illustrates the characteristics of financial crises in the calibrated economy with banker-entrepreneurs.

share of banker-entrepreneurs are unable to fully repay their debts and need to renegotiate them, thereby behaving as frozen buyers (Definition 3).

In a similar vein, Panel (b) shows that the effect of asset purchases on the economy now also depends on the severity of the crisis. Due to the high leverage, mild and severe crises are of the demand-freeze type, whereas intermediate ones are of the fire-sale type.

This analysis of the highly-leveraged economy extends our previous analysis, and it paints a more nuanced picture of crises. Depending on their severity, crises can now be of the fire-sale or demand-freeze type. Since high leverage strengthens the direct channel, interest-rate cuts are now expansionary even during demand-freeze crises, although less than they would be in the absence of the capital-price channel. Moreover, when comparing across all crises states, demand-freeze crises still appear to be more common than fire-sale crises. The reason is that, once crises are severe enough to trigger widespread debt renegotiation, the net worth of these banker-entrepreneurs is wiped out and they *de facto* become frozen buyers.

5.2 Financial-asset purchases

We now extend the model of the previous section to include financial assets. Specifically, suppose that at $t = 0$ there is a unit supply of a Lucas tree that yields a dividend of δ units of the consumption good at both $t = 1$ and $t = 2$. We interpret this tree as representing financial claims (e.g., mortgages) that are held by banker-entrepreneurs and are technologically distinct from productive capital. Banker-entrepreneurs are assumed to operate these assets

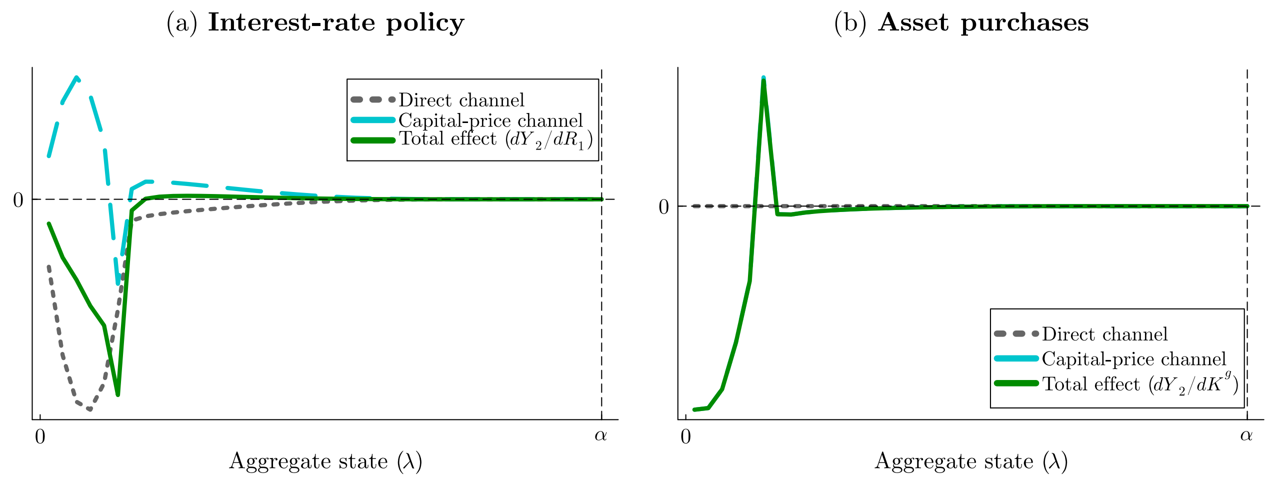

Figure 14: Illustrates the direct and the capital-price channels of policy across aggregate states (λ).

more efficiently than savers: if a saver holds the tree between t and $t+1$, it yields only $\beta \cdot \delta$ units of fruit, where $\beta < 1$. Throughout, we focus on values of β that are sufficiently low to guarantee that savers never hold the tree in equilibrium. Finally, we maintain the assumption that banker-entrepreneurs can abscond with a fraction $1 - \lambda$ of their output at $t = 2$, which now includes the dividends from the tree.

Letting z_t^i denote banker-entrepreneur i's holdings of the tree at time t and $p_1(\lambda)$ denote the price of trees in state λ, her net worth at $t = 1$ is now given by :

$$w_1^i = \rho_1^i \cdot (k_0^i)^\alpha + q_1(\lambda) \cdot k_0^i + [\delta + p_1(\lambda)] \cdot z_0^i - \min\{\rho_1^i \cdot (k_0^i)^\alpha + q_1(\lambda) \cdot k_0^i + [\delta + p_1(\lambda)] \cdot z_0^i,\ b_1^i\}. \quad (38)$$

At this stage, banker-entrepreneur i chooses capital, tree holdings, and borrowing to solve:

$$V(w_1^i, s^i, \lambda) = \max_{k_1^i, z_1^i, b_2^i} \left\{\rho_2^i \cdot (k_1^i)^\alpha + \delta \cdot z_1^i - b_2^i\right\} \quad (39)$$

subject to:

$$q_1(\lambda) \cdot k_1^i + p_1(\lambda) \cdot z_1^i = w_1^i + b_2^i, \quad (40)$$

$$b_2^i \leq \lambda \cdot [\rho_2^i \cdot (k_1^i)^\alpha + \delta \cdot z_1^i], \quad (41)$$

$$z_1^i \geq 0. \quad (42)$$

Banker-entrepreneurs' optimal investment k_1^i is characterized as in Equations (9) and (10), except that the unconstrained level of capital now satisfies:

$$k_1^*\left(s^i,\lambda\right) \equiv \left(\frac{\alpha\cdot\rho_2^i}{q_1\left(\lambda\right)\cdot\frac{\delta}{p_1}}\right)^{\frac{1}{1-\alpha}}, \tag{43}$$

as the opportunity cost of capital investment now depends on the return on trees, δ/p_1.

In addition to the capital market–clearing condition (see Equation (18)), equilibrium must also satisfy:

$$\int w_1^i\cdot di \geq q_1(\lambda)\cdot\bar{K} + p_1(\lambda) - \lambda\cdot\left(\int \rho_2^i\cdot(k_1^i)^\alpha\cdot dG(\rho_1^i,\rho_2^i)+\delta\right), \tag{44}$$

which ensures that banker-entrepreneurs have, as a group, sufficient resources to purchase the entire stock of capital and the unit supply of trees. The resulting equilibrium features two types of crises, depending on whether the tree price is equal to or below its "fair" value δ.

In the first type of crisis, the aggregate net worth of banker-entrepreneurs is large enough to absorb the supply of trees at the fair price $p_1=\delta$. In this case, Equation (43) implies that the desired level of capital investment is unchanged relative to the baseline model. The only difference is that each banker-entrepreneur benefits from additional net worth—equal to the present value of the tree dividends $2\cdot\delta$—which helps support capital investment.

In the second type of crisis, the aggregate net worth of banker-entrepreneurs is insufficient to purchase all the trees at the fair value δ. Thus, p_1 is depressed below δ and its equilibrium value is the one that satisfies Equation (44) with equality. Because the rate of return on trees exceeds one in this case, unconstrained banker-entrepreneurs reallocate resources toward tree holdings to equate returns across assets. As a consequence, for a given price of capital $q_1(\lambda)$, the desired level of capital investment decreases (see Equation (43)). Intuitively, once the price of trees is depressed below δ, unconstrained banker-entrepreneurs shift their spending away from capital towards trees, potentially freeing-up capital to be used by constrained banker-entrepreneurs.

Figure 15 summarizes the discussion. The economy is calibrated so that, as in Section 5.1, banker-entrepreneurs have a leverage ratio of 5.7 and trees represent 30% of their assets.[13] Panel (a) of the figure displays equilibrium prices of capital and trees, as well as output, for different crisis states λ. For relatively mild crises, the presence of trees mitigates the downturn: constrained banker-entrepreneurs sell trees in order to sustain investment in capital. This

[13] This approximate target of 30% follows from mapping the $t=0$ value of physical capital ($q_0\cdot\bar{K}$) in our model to the value of total nonfinancial assets held by the U.S. nonfinancial business sector ($\approx$ \$49 trillion), and the value of the Lucas tree (p_0) to that of the total value of loans owed by the U.S. household (and nonprofit organizations) sector ($\approx$ \$20 trillion), based on the U.S. Federal Reserve's Flow of Funds in 2025Q2.

mechanism is visible in Panel (b), where the reallocation of trees rises as λ declines, while the reallocation of capital remains initially almost unchanged. However, once the crisis becomes sufficiently severe, the aggregate net worth of banker-entrepreneurs is no longer sufficient to purchase the outstanding stock of trees at their fair value. Beyond this point, further reductions in λ depress the price of trees, and the price of capital and output as well. In these deep crises, tighter financial constraints trigger greater reallocation of both capital and trees. Trees in particular command high returns and therefore become especially attractive to unconstrained entrepreneurs, who redirect their investment from capital to trees.

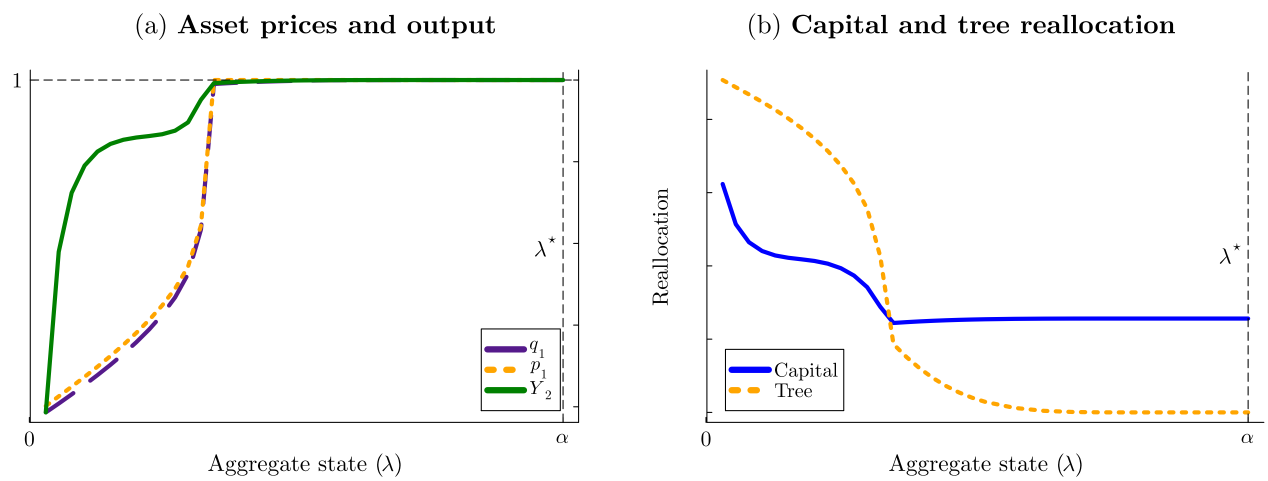

Figure 15: Illustrates the effect of crises in the calibrated economy with financial assets.

Thus, the introduction of financial assets delivers a natural insight: their reallocation across banker-entrepreneurs can—so long as crises are not too severe—prevent the inefficient reallocation of real assets. This observation may suggest that supporting tree prices—for instance, through financial-asset purchases—should always be desirable because it helps avert inefficient movements of capital. As we show next, however, this intuition can be misleading.

To analyze the effects of financial-asset purchases, consider a government that buys Z^g units of trees from the private sector. As before, we assume that trees yield the same dividend in the government's hands as in those of banker-entrepreneurs. We therefore adopt the benign benchmark in which government tree purchases have no direct resource cost.[14]

Given this, the effect of a marginal increase in government tree purchases on aggregate output

[14]An alternative specification is that the government is no better than savers at operating trees. Our results below, however, show that even under the benign benchmark tree purchases may be undesirable.

can be written as:

$$\frac{dY_2}{dZ^g} = \underbrace{\mathbb{E}\left[\frac{\alpha \cdot \rho_2^i \cdot (k_1^i)^{\alpha-1} - q_1 \cdot \frac{\delta}{p_1}}{q_1 - \lambda \cdot \alpha \cdot \rho_2^i \cdot (k_1^i)^{\alpha-1}} \cdot \mathbb{1}_{w_1^i>0} | \lambda\right] \cdot \frac{dp_1}{dZ^g}}_{\text{Direct channel}}$$
$$\underbrace{- \mathbb{E}\left[\frac{\alpha \cdot \rho_2^i \cdot (k_1^i)^{\alpha-1} - q_1 \cdot \frac{\delta}{p_1}}{q_1 - \lambda \cdot \alpha \cdot \rho_2^i \cdot (k_1^i)^{\alpha-1}} \cdot (k_1^i - k_0^i \cdot \mathbb{1}_{w_1^i>0}) \, |\lambda\right] \cdot \frac{dq_1}{dZ^g}}_{\text{Capital-price channel}} . \tag{45}$$

Equation (45) decomposes the effect of tree purchases on output. As before, the first term captures the effect of the policy for a *given price of capital*. This effect is always weakly positive: tree purchases raise their equilibrium price and thus the net worth of banker-entrepreneurs, enabling those who are constrained and not renegotiating to increase their investment in capital. The second term instead captures the capital-price channel, which is negative (positive) whenever the crisis is of the demand-freeze (fire-sale) type. By raising the equilibrium price of trees p_1, tree purchases raise the demand for capital of all banker-entrepreneurs. Constrained banker-entrepreneurs *can* invest more in capital because their net worth increases in p_1, whereas unconstrained banker-entrepreneurs *want* to invest more in capital because the return on trees declines in p_1. Therefore, tree purchases raise the equilibrium price of capital, and this price effect reduces (increases) output whenever the crisis is of the demand-freeze (fire-sale) type.

Panel (a) in Figure 16 depicts the effect of tree purchases across different values of λ for the highly-leveraged banker-entrepreneur economy, decomposing it into the direct and the capital-price channels. As in the analysis of Section 5.1, the type of crises that the economy experiences varies with λ. For relatively mild crises, $p_1 = \delta$ and tree purchases have no effect on asset prices or output. The picture changes once crises are severe enough to depress p_1 below δ. Initially, crises are of the fire-sale type, and both the direct and the capital-price channels have a positive effect on output. In this region, the higher price of trees raises the net worth of banker-entrepreneurs and the resulting increase in the price of capital reduces their need to liquidate capital in order to repay debts. As λ falls and crises become more severe, though, the capital-price channel turns negative and eventually becomes dominant. As in Section 5.1, the reason is that an increasing share of banker-entrepreneurs renegotiate their debts when crises are sufficiently severe, and—as we have already explained—they behave as frozen buyers.

Panel (b) in Figure 16 depicts the effects of financial-asset purchases for the specific crisis in which $\lambda = 0.1$. In this case, the crisis is of the demand-freeze type and the indirect capital-price channel is dominant. Thus, aggregate output decreases with asset purchases, Z^g, despite the fact that the prices of all assets, p_1 and q_1, are monotonically increasing in Z^g.

(a) **Marginal effect of tree purchases on output** (b) **Tree purchases, output, and prices** $(\lambda = 0.1)$

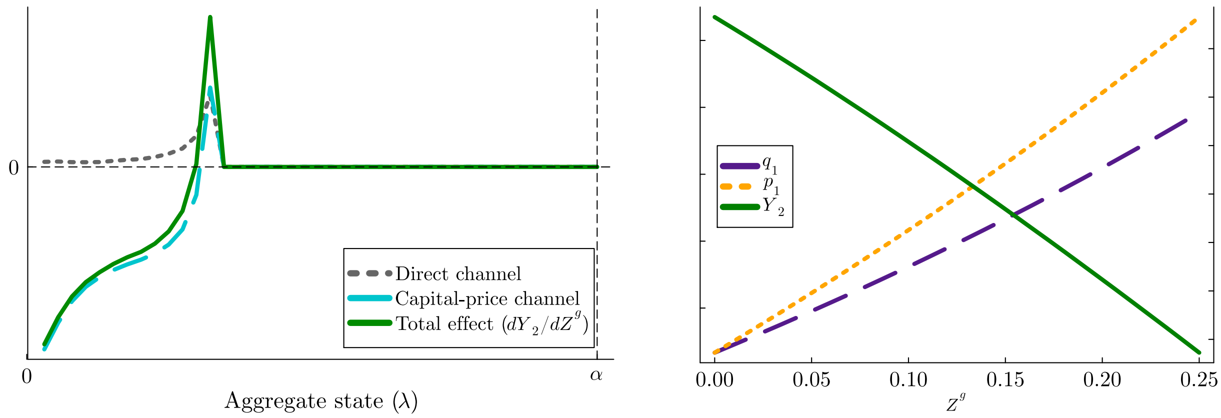

Figure 16: Illustrates the effect of financial-asset purchases in the calibrated banker-entrepreneur economy.

6 Reallocation in the data

Our analysis underscores the importance of distinguishing between fire-sale and demand-freeze episodes. Real-world crises likely combine aspects of both. In what follows, we use the predictions of our model to motivate a reduced-form analysis of the patterns of capital reallocation over the business cycle, both in the aggregate and across firms, in U.S. data.

To do so, we express the expectation term $\mathcal{CP}(\lambda)$ in the capital-price channel as follows:

$$\begin{aligned}\mathcal{CP}(\lambda) &= \mathbb{E}\left[(\nu^i - 1) \cdot (k_1^i - k_0^i \cdot \mathbb{1}_{w_1^i > 0}) \mid \lambda\right] \\ &= \mathbb{COV}\left(\nu^i, k_1^i - k_0^i \mid \lambda\right) + \mathbb{E}\left[(\nu^i - 1) \cdot k_0^i \cdot \mathbb{1}_{w_1^i = 0} \mid \lambda\right] \\ &\geq \mathbb{COV}\left(\nu^i, k_1^i - k_0^i \mid \lambda\right), \end{aligned} \tag{46}$$

where the last inequality follows because the product of $\nu - 1$ and k_0 is weakly positive.

Equation (46) says that the expectations term in the capital-price channel is bounded below by the covariance between ν and net capital purchases, $k_1 - k_0$. This has important conceptual and empirical implications. Conceptually, it yields a sufficient statistic for detecting a demand-freeze crisis: if $\mathbb{COV}(\nu^i, k_1^i - k_0^i \mid \lambda) > 0$, the crisis must be of the demand-freeze variety.[15]

[15] It also yields a necessary statistic for detecting a fire-sale crisis. If a crisis is of the fire-sale variety, the covariance must be negative.

Empirically, this covariance is directly measurable using firm-level data by relating the marginal product of capital to net investment.[16] In what follows, we implement this test using Compustat.

6.1 Data

As the firm-level dataset we use the quarterly Compustat universe of publicly listed U.S. incorporated firms, during the period 1983Q4–2019Q4.[17] Our central measure of firm i's overall (gross) capital accumulation ($I_{i,t}$) is its reported *Capital Expenditures* in quarter t (denoted as $Capx_{i,t}$), net of *Sale of Property Plant, and Equipment* ($Sale_{i,t}$). When constructing investment rates, we take its ratio relative to the book value of tangible capital stock $k_{i,t-1}$, in place at the end of quarter $t-1$, measured as the Compustat item *Property, Plant, and Equipment (Net)*.[18] To more directly study the reallocation of *existing* capital, we also consider only focusing on the proxies for the sale of existing capital with $Sale_{i,t}$, and the purchases of existing capital with the *Acquisitions* item ($Acq_{i,t}$). This follows conventional approaches in the literature on capital reallocation (e.g., Eisfeldt and Rampini, 2006). As $Capx_{i,t}$ does not include the capital obtained through acquiring other firms, we also consider an acquisitions-augmented gross investment measure $I^a_{i,t} \equiv Capx_{i,t} - Sale_{i,t} + Acq_{i,t}$.

As the proxy for firm i's (log) marginal revenue product of capital in quarter t ($\text{MRPK}_{i,t}$), we employ its (log) average revenue product of capital ($\text{ARPK}_{i,t}$). The justification for this is that all our panel regressions include detailed (SIC2) industry fixed effects, so as long as $\text{ARPK}_{i,t}$ is proportional to $\text{MRPK}_{i,t}$ conditional on industry, the SIC2 $\times$ time fixed effects absorb industry-specific capital shares, and the remaining within-industry variation in $\log(\text{ARPK}_{i,t})$ is identical to that of $\log(\text{MRPK}_{i,t})$.[19]

Our estimated panel regressions also include a variety of other firm-level observables as controls. As the measure of a firm's leverage we employ its total debt divided by its total assets, both measured at book values. We measure firms' balance sheet liquidity (or, *liquidity ratio* for short) as the ratio of cash and short-term investments to total assets. As the measure

[16] We make use of the observation that the marginal value of funds, ν^i, of entrepreneur i monotonically increases in that entrepreneur's marginal productivity of capital, $\text{MRPK}_i = \alpha \cdot \rho^i_2 \cdot (k^i_1)^{\alpha-1}$ (Equation (13)).

[17] Our analysis is limited to publicly listed U.S. firms. To the extent that financial constraints and reallocation dynamics among private firms differ systematically, this may influence the inference about the aggregate economy. However, since publicly listed firms account for a disproportionate share of aggregate capital, the patterns we document remain informative about the bulk of U.S. capital reallocation activity.

[18] See Appendix B.1 for further detail on sample selection and variable construction.

[19] Under more general conditions than those of our model—such as imperfect competition with heterogeneous markups, or production functions where the physical marginal product is not proportional to the physical average product within industries—this proxy is imperfect. Nonetheless, we follow the approach conventional in much of the misallocation literature (e.g., Midrigan and Xu, 2014; David and Venkateswaran, 2019) and use $\text{ARPK}_{i,t}$ as our productivity proxy.

of firms' size we employ (log) total book assets. We measure Tobin's q as the market-to-book ratio of total assets. We discuss further details on the sample selection and the construction of other variables used in Appendix B.1.

6.2 The cyclicality of capital reallocation in the aggregate

We first provide evidence that capital reallocation activity is procyclical, as already found in existing work, e.g., by Eisfeldt and Rampini (2006). To do so, we construct aggregate turnover measures based on acquisitions and sales to proxy for capital reallocation. While our data is quarterly, we construct these measures over an annual horizon in order to eliminate any effects of seasonality over quarters. Moreover, micro-level acquisitions and capital sales can be lumpy and erratic, making it potentially difficult to precisely detect systematic responses in investment over shorter horizons. Also, for comparability with Eisfeldt and Rampini (2006) who use the annual Compustat database, we compute a generic measure of turnover over a given year $[t, t+3]$ as the ratio of the sum of all observed firms' acquisitions and/or capital sales during the interval, relative to the sum of their total assets (or tangible capital) in place at the end of quarter $t-1$.

Table 1 reports the correlations between the time series of various aggregate turnover measures during the year implied by quarters $[t, t+3]$, and either real GDP growth between $t-1$ and $t+3$ or—to proxy for financial crises—the Gilchrist and Zakrajšek (2012) excess bond premium (EBP) measured in quarter t. The correlations indicate that capital turnover activity is procyclical in the aggregate—higher whenever real GDP growth is high or when the EBP is low. The correlations are statistically significantly different from zero for a majority of cases. Through the lens of our model, this procyclicality of reallocation activity is the first stylized fact that hints towards the U.S. economy being more likely prone to demand-freeze rather than fire-sale crises.

6.3 Capital investment and reallocation in the cross-section

We now study how capital accumulation and sales behavior varies in the *cross-section* of firms, and whether these cross-sectional relations change during times of financial turmoil, as measured by the EBP. To do so, we estimate simple panel regressions of the following specification type:

$$y_{i,t} = \alpha_i + \delta_{s,t} + \beta \log(\text{ARPK}_{i,t+1}) + \gamma \log(\text{ARPK}_{i,t+1}) \times s_t^{EBP} + \zeta X_{i,t-1} + u_{i,t} \qquad (47)$$

Table 1: Cyclicality of aggregate reallocation measures in Compustat

	Correlation	
Turnover rate	Real GDP growth	EBP
$\frac{\text{Acq.+Sale}}{\text{Assets}_{t-1}}$	0.2643***	-0.2668***
$\frac{\text{Acq.+Sale}}{\text{PP\&E}_{t-1}}$	0.0749	-0.2650***
$\frac{\text{Sale}}{\text{Assets}_{t-1}}$	0.3332***	-0.0397
$\frac{\text{Sale}}{\text{PP\&E}_{t-1}}$	0.2987***	-0.0658
$\frac{\text{Acq.}}{\text{Assets}_{t-1}}$	0.1967**	-0.2817***
$\frac{\text{Acq.}}{\text{PP\&E}_{t-1}}$	0.0323	-0.2604***

Notes: Correlation coefficients between the time series of aggregate turnover measures during the year implied by quarters $[t, t+3]$, and real GDP growth between $t-1$ and $t+3$ and the Gilchrist and Zakrajšek (2012) excess bond premium (EBP) measured in quarter t. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

where $y_{i,t}$ is generic notation for the capital accumulation or sale activity for firm i during quarter t. α_i and $\delta_{s,t}$ denote firm and time×industry (SIC2) fixed effects, respectively. $\text{ARPK}_{i,t+1}$ is measured as the ratio of firm i's revenues in $t+1$ to the capital stock in place at the end of t. Note that this is the relevant timing of interest implied by the indirect component in our model, i.e., whether the covariance of investment activity in t positively or negatively related to the marginal productivity of capital in $t+1$.

$X_{i,t}$ is a vector of firm-level controls, measured as of quarter t. In our baseline, $X_{i,t}$ includes the firm's leverage, liquidity ratio, Tobin's q and log total assets. s_t^{EBP} denotes an indicator of financial turmoil in quarter t, as calculated based on EBP_t. We consider both a continuous specification, with $s_t^{EBP} = EBP_t$, and dummy-specifications, with s_t^{EBP} being an indicator variable of whether EBP_t is above a certain percentile of the observed $\{EBP_t\}$ series during the sample period. $u_{i,t}$ are regression error terms. We drop extreme observations of firm-level variables to control for outliers (see Appendix B.1.1) and consider standard errors clustered two-way at the time×industry (SIC3) and firm levels.

In our benchmark specification, $y_{i,t} = 100 \times I_{i,t}/k_{i,t-1}$, meaning i's gross investment rate (net of capital sale) in quarter t. Table 2 presents the corresponding estimates of β and γ. The first column corresponds to a specification in which $s_t^{EBP} = 1$ whenever EBP_t is above the median observed EBP value in our sample. The positive coefficient of $\hat{\beta}$ says that at times of low EBP, meaning "normal times" (when $s_t^{EBP} = 0$), more productive firms accumulate relatively more capital. Quantitatively, a 10% higher $ARPK_{i,t+1}$ predicts a 0.086 pp higher investment rate.

The negative estimate of $\hat{\gamma}$ says that in times of financial turmoil, this positive cross-sectional relation between productivity and investment weakens. However, since $|\hat{\gamma}| < |\hat{\beta}|$, the relation between productivity and investment *remains* positive even when EBP is high. Moreover, as we increase the cutoff for "high EBP", and thus focus on the most severe 10% observations of EBP, this weakening of reallocation towards high-productivity firms becomes more pronounced. But again, the cross-sectional relationship between productivity and investment remains positive. The final column of Table 2 provides the estimates for the continuous specification which imply that even at the highest observed value of EBP_t in the sample (3.47 in 2008Q4), the imputed cross-sectional relationship would not change sign.

Table 2: Panel regressions for investment rates in "normal" and "crisis" times

	High EBP_t cutoff:			
Dep. var.: $100 \times I_{i,t}/k_{i,t-1}$	p50	p75	p90	Cont.
$\log(ARPK_{i,t+1})$	0.8632***	0.8389***	0.8235***	0.8157***
	(12.76)	(13.22)	(13.42)	(13.55)
$\log(ARPK_{i,t+1}) \times High\ EBP_t$	-0.1265**	-0.1383**	-0.1705**	
	(-2.2)	(-2.16)	(-2.21)	
$\log(ARPK_{i,t+1}) \times EBP_t$				-0.1418***
				(-3.46)
Observations	351,801	351,801	351,801	351,801
Controls	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
Time-Industry FE	✓	✓	✓	✓

Notes: Regression coefficient estimates for β and γ in specification (47), with t-statistics in parentheses. Standard errors clustered two-way at the time$\times$industry (SIC3) and firm levels. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

Table 3 in Appendix B.2 shows that the same results hold for the acquisitions-augmented investment rates. Moreover, Tables 4–6 in Appendix B.2 show that analogous takeaways apply when we focus separately on reported capital expenditures, or the acquisition of existing firms or likelihood of selling physical capital.[20] More productive firms spend more on capital expenditures, engage more actively in acquisitions and are less likely to sell existing capital in normal times. In crisis times, the relations of capital expenditures and acquisitions with productivity weaken quantitatively, but there is no evidence that they change sign. The relation of sales with productivity in turn either does not change or becomes slightly stronger depending on the

[20]When considering sale of capital as a standalone variable, we focus on the extensive margin indicator of whether a firm is selling capital or not, due to the high prevalence of zeros and missing observations in this variable in Compustat.

cutoff percentile of EBP that is used to define crises.

Thus, also in the cross-section of firms, we uncover patterns of capital reallocation that are more compatible with demand-freeze crises: on average, capital reallocation goes in the "right" direction, but simply less so in times of financial crises.

7 Conclusions

This paper has argued that not all financial crises are alike, even when they display the same symptoms of depressed asset prices, tight financial constraints, and impaired reallocation. By distinguishing between fire-sale crises—where productive agents are forced sellers—and demand-freeze crises—where productive agents are constrained buyers—we show that asset prices play a fundamentally different role across crisis types. In fire-sale crises, low prices amplify misallocation by forcing productive agents to contract even further, while in demand-freeze crises they partially offset financial constraints by enabling productive agents to expand even further. This distinction is captured in a simple but powerful sufficient statistic: the covariance between firms' marginal productivity and their net capital purchases. When this covariance is positive, crises are of the demand-freeze type, and higher asset prices impede, rather than facilitate, efficient reallocation. Our calibrated model and firm-level evidence both point decisively in this direction: U.S. downturns are characterized by muted reallocation, productive firms expanding less than desired, and capital flowing in the "right" direction albeit too weakly—hallmarks of demand-freeze episodes

These findings have direct implications for crisis management. Policies designed to raise asset prices, such as interest-rate cuts and asset purchases, are often justified by a fire-sale narrative in which higher prices relieve distress and restore efficiency. In demand-freeze crises, however, the same interventions have adverse side effects—and may even backfire—by pricing constrained, productive buyers out of the market, worsening misallocation and reducing output through general-equilibrium effects. Thus, our analysis suggests that effective crisis management requires identifying the underlying reallocation mechanism before intervening. Asset price support is not universally stabilizing, and its allocative consequences depend critically on who is on which side of the market. Recognizing this distinction is essential for designing interventions that mitigate, rather than inadvertently deepen, the long-run economic costs of financial crises.

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A Model Appendix

A.1 Proofs and Derivations

Proof of Lemma 1. Absent financial constraints, entrepreneurial investment at $t = 1$ is efficient and given by:

$$k_1^i = k_1^\star(s^i, 1) = \left(\frac{\alpha \cdot \rho_2^i}{q_1}\right)^{\frac{1}{1-\alpha}}.$$

Capital market clearing at $t = 1$ then implies:

$$\int k_1^\star(s^i, 1)\, di = \bar{K},$$

which yields the expression for $q_1 = q_1^\star$ in Equation (22).

Analogously, the expression for $q_0 = q_0^\star$ in Equation (22) follows from the optimality condition for investment in Equation (16). Absent financial constraints, $\nu(w_1^i, s^i, 1) = 1$ for all $(w_1^i, s^i, 1)$, so the Euler equation reduces to equating the marginal cost of capital to its expected marginal product. Imposing capital market clearing, $k_0^i = \bar{K}$ for all i, delivers $q_0^\star$.

Finally, aggregate output at $t = 2$ is obtained by aggregating individual outputs:

$$y_2^i = \rho_2^i \cdot (k_1^i)^\alpha = \rho_2^i \cdot \left(\frac{\alpha \cdot \rho_2^i}{q_1^\star}\right)^{\frac{\alpha}{1-\alpha}},$$

which, combined with $q_1^\star$ in Equation (22), yields $Y_2 = Y_2^\star$ as given in Equation (21). ■

Proof of Proposition 1. We first establish equilibrium existence.

In any equilibrium, capital prices must satisfy $q_0 \leq q_0^\star$ and $q_1(\lambda) \leq q_1^\star$ for all λ, where $(q_0^\star, q_1^\star)$ are the efficient prices characterized in Lemma 1. We thus restrict attention to prices in the compact set $[0, q_0^\star] \times [0, q_1^\star]$.

Consider first the capital market at $t = 1$ in aggregate state λ. For any $(q_0, q_1(\lambda))$ in this set, aggregate capital demand at $t = 1$ is given by:

$$D_1(q_0, q_1(\lambda)) \equiv \int k_1^i\, di,$$

where the dependence on q_0 arises through entrepreneurs' net worth. This function has the following properties: (i) it is continuous in $(q_0, q_1(\lambda))$, since individual capital demands are continuous; (ii) $D_1(q_0, q_1^\star) \leq \bar{K}$, as $q_1^\star$ corresponds to the unconstrained efficient allocation; (iii) $D_1(q_0, q_1(\lambda))$ exceeds $\bar{K}$ for $q_1(\lambda)$ close to zero. While the latter is immediate when $\lambda > 0$, it also holds for $\lambda = 0$ by Assumption 2, which guarantees a positive mass of entrepreneurs with strictly positive net worth and hence unbounded capital demand as $q_1(\lambda) \to 0$. By continuity, there therefore exists a price $q_1(\lambda) \in (0, q_1^\star]$ such that $D_1(q_0, q_1(\lambda)) = \bar{K}$.

Next consider the capital market at $t = 0$. Aggregate capital demand is given by:

$$D_0(q_0, \{q_1(\lambda)\}) \equiv \int k_0^i di,$$

where individual demands follow from the optimality condition in Equation (16), combined with Equation (13) evaluated at the optimal k_1^i given by Equation (9). This demand satisfies analogous properties: (i) $D_0(q_0, \{q_1(\lambda)\})$ is continuous in $(q_0, \{q_1(\lambda)\})$; (ii) $D_0(q_0^\star, \{q_1(\lambda)\}) \leq \bar{K}$; (iii) $D_0(q_0, \{q_1(\lambda)\})$ exceeds $\bar{K}$ for q_0 close to zero. Hence, for any schedule $\{q_1(\lambda)\}$ clearing the capital market at $t = 1$, there exists a price $q_0 \in (0, q_0^\star]$ such that $D_0(q_0, \{q_1(\lambda)\}) = \bar{K}$.

Therefore, there exists a vector of prices $(q_0, \{q_1(\lambda)\})$ such that the capital markets clear at both dates and all aggregate states λ, establishing the existence of a competitive equilibrium.

We now establish the existence of a threshold $\lambda^\star$ that partitions aggregate states.

Fix an aggregate state λ at $t = 1$. In equilibrium, all entrepreneurs are unconstrained if and only if the entrepreneur with the lowest cash flow $\rho_1^i = \underline{\rho}_1$ and the highest investment opportunity $\rho_2^i = \bar{\rho}_2$ is unconstrained. Given an equilibrium price q_0 at $t = 0$, this entrepreneur is unconstrained if and only if her net worth plus the maximum amount of credit she can obtain suffices to finance her efficient level of investment:

$$\max\{0,\ \underline{\rho}_1 \cdot \bar{K}^\alpha + w - q_0 \cdot \bar{K}\} \ \geq\ q_1^\star \cdot \left(\frac{\alpha \cdot \bar{\rho}_2}{q_1^\star}\right)^{\frac{1}{1-\alpha}} \ -\ \lambda \cdot \bar{\rho}_2 \cdot \left(\frac{\alpha \cdot \bar{\rho}_2}{q_1^\star}\right)^{\frac{\alpha}{1-\alpha}},$$

where $q_1^\star$ is the efficient price characterized in Lemma 1.

The threshold $\lambda^\star$ is then the value of λ at which the above inequality holds with equality. Clearly, $\lambda^\star < 1$ since the right hand size equals zero at $\lambda = \alpha$. That $\lambda^\star > 0$ in turn follows by Assumption 1, which ensures that this inequality is violated when $\lambda = 0$ and q_0 equals the efficient price $q_0^\star$, which is an upper bound on the date 0 market clearing price. Finally, it is straightforward to show that the output $Y_2(\lambda)$ and the asset price $q_1(\lambda)$ are depressed below their efficient counterpart when $\lambda < \lambda^\star$, since for all such λ capital at $t = 1$ is misallocated and, thus, the aggregate capital demand is depressed. ■

Proof of Proposition 2. Differentiation of aggregate output $Y_2(\lambda)$ with respect to λ yields:

$$\begin{aligned}\frac{dY_2}{d\lambda} &= \int \rho_2^i \cdot \alpha \cdot k_1\left(w_1^i, s^i, \lambda\right)^{\alpha-1} \cdot \frac{dk_1\left(w_1^i, s^i, \lambda\right)}{d\lambda} \cdot dG\left(\rho_1^i, \rho_2^i\right) \\ &= \int \left(\rho_2^i \cdot \alpha \cdot k_1\left(w_1^i, s^i, \lambda\right)^{\alpha-1} - q_1(\lambda)\right) \cdot \frac{dk_1\left(w_1^i, s^i, \lambda\right)}{d\lambda} \cdot dG\left(\rho_1^i, \rho_2^i\right), \end{aligned} \tag{48}$$

where, to write the second line, we have used the fact that capital market clearing implies:

$$\int \frac{dk_1\left(w_1^i, s^i, \lambda\right)}{d\lambda} \cdot dG\left(\rho_1^i, \rho_2^i\right) = 0.$$

From Equation (9), we have that for constrained entrepreneurs:

$$\frac{dk_1\left(w_1^i, s^i, \lambda\right)}{d\lambda} = \frac{\rho_2^i \cdot k_1\left(w_1^i, s^i, \lambda\right)^{\alpha}}{q_1(\lambda) - \lambda \cdot \alpha \cdot k_1\left(w_1^i, s, \lambda\right)^{\alpha-1}} + \frac{k_0^i \cdot \mathbb{1}_{w_1^i > 0} - k_1\left(w_1^i, s^i, \lambda\right)}{q_1(\lambda) - \lambda \cdot \alpha \cdot k_1\left(w_1^i, s^i, \lambda\right)^{\alpha-1}} \cdot \frac{dq_1(\lambda)}{d\lambda}, \tag{49}$$

where $\mathbb{1}_{w_1^i > 0}$ is an indicator function that takes value one if $w_1^i > 0$ (i.e., if the entrepreneur has not defaulted) and zero otherwise. We obtain the stated result by combining Equation (49) with Equation (48), and using the definition of ν_1^i in Equation (13). ■

Proof of Proposition 3. Assume that in equilibrium no entrepreneur renegotiates her debt obligations.

Fire-sale economy. Consider the fire-sale economy in a crisis state $\lambda < \lambda^\star$. Since for all $i \in I^E$, $\rho_2^i = 1$, it follows that: (i) k_1^i is weakly increasing in ρ_1^i, and strictly so if $i \in I^E$ is financially constrained; and (ii) ν_1^i is weakly decreasing in ρ_1^i, and strictly so if $i \in I^E$ is financially constrained. Since $k_0^i = \bar{K}$ for all $i \in I^E$, it follows that $\mathcal{CP}(\lambda) < 0$.

Demand-freeze economy. Consider the demand-freeze economy in a crisis state $\lambda < \lambda^\star$. Since for all $i \in I^E$, $\rho_1^i = 1$, it follows that: (i) k_1^i is weakly increasing in ρ_2^i, and strictly so if either $i \in I^E$ is financially unconstrained or is financially constrained and $\lambda > 0$; and (ii) ν_1^i is weakly increasing in ρ_2^i, and strictly so if $i \in I^E$ is financially constrained. Since $k_0^i = \bar{K}$ for all $i \in I^E$, it follows that $\mathcal{CP}(\lambda) > 0$.

No renegotiation. That no entrepreneur renegotiates her debts in the demand-freeze economy is ensured by Assumption 2, since in this economy entrepreneurs have the same net worth; thus, either all renegotiate or none.

For the fire-sale economy, it must be that the poorest entrepreneur, i.e., one with $\rho_1^i = 0$, does not renegotiate. Moreover, it suffices to rule out renegotiation by this entrepreneur in the worst crisis state $\lambda = 0$. Hence, such an entrepreneur does renegotiate whenever

$$b_1 \leq q_1(0) \cdot \bar{K}, \tag{50}$$

that is, if her promised repayments are fully collateralized by the value of capital in the worst crisis state. Since, all else equal, $q_1(0)$ is lower when b_1 is higher, it suffices to check this inequality at the highest value of b_1, i.e., if normal times are expected w.p. 1 ex ante:

$$\begin{aligned} b_1^{\max} &= q_0^{fb} \cdot \bar{K} - w \\ &= 2 \cdot \alpha \cdot \bar{K}^{\alpha} - w. \end{aligned} \tag{51}$$

Therefore, the condition on entrepreneurial endowment becomes:

$$w \geq 2 \cdot \alpha \cdot \bar{K}^{\alpha} - q \cdot \bar{K}, \tag{52}$$

where q is the market-clearing price in the crisis state $\lambda = 0$ when entrepreneurial endowment w satisfies (52) at equality, which is given by the maximal solution to the market clearing condition:

$$\begin{aligned} \frac{1}{2} \cdot \int_0^{\frac{q \cdot \left(\frac{\alpha}{q}\right)^{\frac{1}{1-\alpha}}}{\bar{K}^{\alpha}}} \frac{\rho \cdot \bar{K}^{\alpha}}{q} \cdot d\rho + \left(1 - \frac{1}{2} \cdot \frac{q \cdot \left(\frac{\alpha}{q}\right)^{\frac{1}{1-\alpha}}}{\bar{K}^{\alpha}}\right) \cdot \left(\frac{\alpha}{q}\right)^{\frac{1}{1-\alpha}} &= \bar{K} \\ &\iff \\ \left(1 - \frac{1}{4} \cdot \frac{q \cdot \left(\frac{\alpha}{q}\right)^{\frac{1}{1-\alpha}}}{\bar{K}^{\alpha}}\right) \cdot \left(\frac{\alpha}{q}\right)^{\frac{1}{1-\alpha}} &= \bar{K}. \end{aligned} \tag{53}$$

This completes the proof. ■

Proof of Proposition 4. We analyze the effect of changes in the interest R_1 in the absence of government purchases (i.e., $K^g = 0$). Differentiation of aggregate output with respect to R_1 yields:

$$\begin{aligned} \frac{dY_2}{dR_1} &= \int \rho_2^i \cdot \alpha \cdot k_1\left(w_1^i, s^i, \lambda\right)^{\alpha-1} \cdot \frac{dk_1\left(w_1^i, s^i, \lambda\right)}{dR_1} \cdot dG\left(\rho_1^i, \rho_2^i\right) \\ &= \int \left(\rho_2^i \cdot \alpha \cdot k_1\left(w_1^i, s^i, \lambda\right)^{\alpha-1} - q_1(\lambda, R_1, 0) \cdot R_1\right) \cdot \frac{dk_1\left(w_1^i, s^i, \lambda\right)}{dR_1} \cdot dG\left(\rho_1^i, \rho_2^i\right), \end{aligned} \tag{54}$$

where we have used the fact that capital market clearing implies:

$$\int \frac{dk_1\left(w_1^i, s^i, \lambda\right)}{dR_1} \cdot dG\left(\rho_1^i, \rho_2^i\right) = 0.$$

From Equation (9), we have that for constrained entrepreneurs:

$$\frac{dk_1\left(w_1^i, s^i, \lambda\right)}{dR_1} = -\frac{\frac{\lambda \cdot \rho_2^i \cdot k_1\left(w_1^i, s, \lambda\right)^{\alpha}}{R_1^2}}{q_1(\lambda, R_1, 0) - \frac{\lambda \cdot \alpha \cdot k_1\left(w_1^i, s, \lambda\right)^{\alpha-1}}{R_1}} + \frac{k_0^i \cdot \mathbb{1}_{w_1^i > 0} - k_1\left(w_1^i, s, \lambda\right)}{q_1(\lambda, R_1, 0) - \frac{\lambda \cdot \alpha \cdot k_1\left(w_1^i, s, \lambda\right)^{\alpha-1}}{R_1}} \cdot \frac{dq_1}{dR_1} \tag{55}$$

where $\mathbb{1}_{w_1^i > 0}$ is an indicator function that takes value one if $w_1^i > 0$ (i.e., if the entrepreneur has not defaulted) and zero otherwise. We obtain the stated result by combining Equations (54) and (55), evaluated at $R_1 = 1$, and using the definition of ν_1^i in Equation (13). ■

Proof of Proposition 5. We analyze the effect of changes in public purchase of capital K^g in the absence of interest-rate policies (i.e., $R_1 = 1$). Differentiation of aggregate output with respect to K^g yields:

$$\begin{aligned}
\frac{dY_2}{dK^g} &= \rho^g + \int \rho_2^i \cdot \alpha \cdot k_1\left(w_1^i, s^i, \lambda\right)^{\alpha-1} \cdot \frac{dk_1\left(w_1^i, s^i, \lambda\right)}{dK^g} \cdot dG\left(\rho_1^i, \rho_2^i\right) \\
&= \rho^g - q_1(\lambda, 1, K^g) + \int \left(\rho_2^i \cdot \alpha \cdot k_1\left(w_1^i, s^i, \lambda\right)^{\alpha-1} - q_1(\lambda, 1, K^g)\right) \cdot \frac{dk_1\left(w_1^i, s^i, \lambda\right)}{dK^g} \cdot dG\left(\rho_1^i, \rho_2^i\right),
\end{aligned} \tag{56}$$

where we have used the fact that capital market clearing implies:

$$\int \frac{dk_1\left(w_1^i, s^i, \lambda\right)}{dK^g} \cdot dG\left(\rho_1^i, \rho_2^i\right) = -1.$$

Since K^g affects individual capital holdings only through its effect on the capital price, we can rewrite Equation (56) as:

$$\begin{aligned}
\frac{dY_2}{dK^g} &= \rho^g - q_1(\lambda, 1, K^g) \\
&\quad + \int \left(\rho_2^i \cdot \alpha \cdot k_1\left(w_1^i, s^i, \lambda\right)^{\alpha-1} - q_1(\lambda, 1, K^g)\right) \cdot \frac{dk_1\left(w_1^i, s^i, \lambda\right)}{dq_1} \cdot \frac{dq_1(\lambda, 1, K^g)}{dK^g} \cdot dG\left(\rho_1^i, \rho_2^i\right)
\end{aligned} \tag{57}$$

From Equation (9), we have that for constrained entrepreneurs:

$$\frac{dk_1\left(w_1^i, s^i, \lambda\right)}{dq_1} = \frac{k_0 \cdot \mathbb{1}_{w_1^i > 0} - k_1\left(w_1^i, s^i, \lambda\right)}{q_1(\lambda, 1, K^g) - \lambda \cdot \alpha \cdot k_1\left(w_1^i, s^i, \lambda\right)^{\alpha-1}} \tag{58}$$

where $\mathbb{1}_{w_1^i > 0}$ is an indicator function that takes value one if $w_1^i > 0$ (i.e., if the entrepreneur has not defaulted) and zero otherwise. We obtain the stated result by combining Equations (58) and (57), evaluated at $K^g = 0$, and using the definition of ν_1^i in Equation (13). ■

Derivation of Equation (45)**.** Here, we derive the effect of changes in public purchases of trees, Z^g, in the absence of interest-rate or capital-purchase policies (i.e., $R_1 = 1$ and $K^g = 0$). Let us denote endogenous variables as a function of the aggregate state and policies, i.e., (λ, R_1, K^g, Z^g). The economy can be in one of two regimes. In the first regime, $p_1 = \delta$. Entrepreneurs as a group have enough resources to purchase trees at its "fair" price, so that public purchases of trees have no effect on the price of trees and – thus – on the allocation of capital. In the second regime, $p_1 < \delta$. In this case, entrepreneurs as a group do not have enough resources to purchase trees at price δ, so that the tree price is depressed, $p_1 < \delta$, and public purchases of trees raise p_1 and – thus – increase the net worth of entrepreneurs and affect the allocation of capital. In this case, differentiation of aggregate output with respect to Z^g yields:

$$\begin{aligned}\frac{dY_2}{dZ^g} &= \int \rho_2^i \cdot \alpha \cdot k_1\left(w_1^i, s^i, \lambda\right)^{\alpha-1} \cdot \frac{dk_1\left(w_1^i, s^i, \lambda\right)}{dZ^g} \cdot dG\left(\rho_1^i, \rho_2^i\right) \\ &= \int \left(\rho_2^i \cdot \alpha \cdot k_1\left(w_1^i, s^i, \lambda\right)^{\alpha-1} - q_1 \cdot \frac{\delta}{p_1}\right) \cdot \frac{dk_1\left(w_1^i, s^i, \lambda\right)}{dZ^g} \cdot dG\left(\rho_1^i, \rho_2^i\right),\end{aligned} \tag{59}$$

where we have once again used the fact that capital market clearing implies:

$$\int \frac{dk_1\left(w_1^i, s^i, \lambda\right)}{dZ^g} \cdot dG\left(\rho_1^i, \rho_2^i\right) = 0.$$

From Equations (40) and (41), we have that for "constrained" entrepreneurs (i.e., for i such that $\rho_2^i \cdot \alpha \cdot k_1\left(w_1^i, s^i, \lambda\right)^{\alpha-1} > q_1 \cdot \frac{\delta}{p_1}$):

$$\frac{dk_1\left(w_1^i, s^i, \lambda\right)}{dp_1} = \frac{\mathbb{1}_{w_1^i > 0}}{q_1 - \lambda \cdot \alpha \cdot k_1\left(w_1^i, s^i, \lambda\right)^{\alpha-1}}, \tag{60}$$

$$\frac{dk_1\left(w_1^i, s^i, \lambda\right)}{dq_1} = -\frac{k_1\left(w_1^i, s^i, \lambda\right) - k_0^i \cdot \mathbb{1}_{w_1^i > 0}}{q_1 - \lambda \cdot \alpha \cdot k_1\left(w_1^i, s^i, \lambda\right)^{\alpha-1}}. \tag{61}$$

Combining these expressions with Equation (59) and using the fact that $\frac{dk_1^i}{dZ^g} = \frac{dk_1^i}{dp_1} \cdot \frac{dp_1}{dZ^g} + \frac{dk_1^i}{dq_1} \cdot \frac{dq_1}{dZ^g}$, we obtain Equation (45) in the main text.

A.2 Calibration robustness analysis

A.2.1 Annual calibration of baseline economy

In the baseline "calibrated" economy, we interpret one model period as a quarter. This implies that the calibrated persistence of γ in the entrepreneurial productivity process is high, and the standard deviation of the innovation σ_ε is correspondingly low. In this Appendix, we show that all the main results from this "calibrated" economy are robust to considering an annual calibration frequency instead.

Namely, we keep all other calibration targets unchanged, but solve the baseline model with the alternative parameterization of the persistence parameter $\gamma = 0.65$ ($\approx 0.9^4$). To ensure that this recalibration does not fundamentally change the level nor variation of productivity embedded in the distributions of $\log(\rho_t), t = 1, 2$, and it only affects the strength of the positive relationship between ρ_1 and ρ_2, we also adjust the parameters a and σ_ε to keep the unconditional means and variances of $\log(\rho_t)$ equal to their values in the baseline calibration.[21]

Figures 17, 18, 19, and 20 below correspondingly replicate Figures 5, 6, 7, and 11 from the main text for the annual recalibration. Other than fire sellers now being slightly more prominent in the population of entrepreneurs in a crisis, all the main takeaways from the baseline calibrated economy survive. This includes the findings that all crises remain demand-freezes, and that the capital-price effect dominates the direct effect of interest-rate policy for all values of λ.

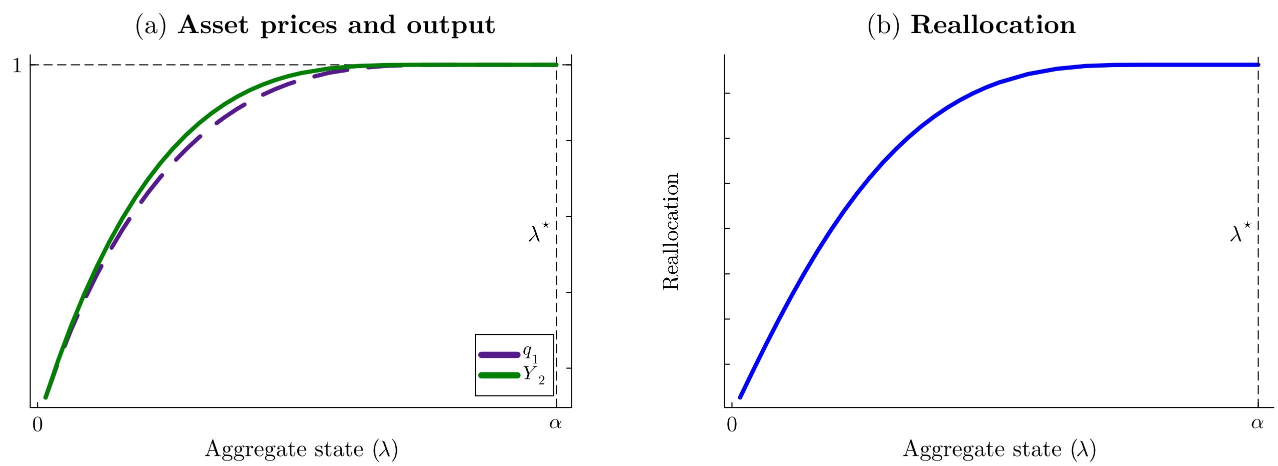

Figure 17: Behavior of asset prices, output and capital reallocation across aggregate states (λ) in annual baseline calibration.

[21]Note that this assumption also ensures that the first-best levels of output in the model are unaffected by this recalibration.

(a) **Types of constrained entrepreneurs** (b) **Capital reallocation and productivity**

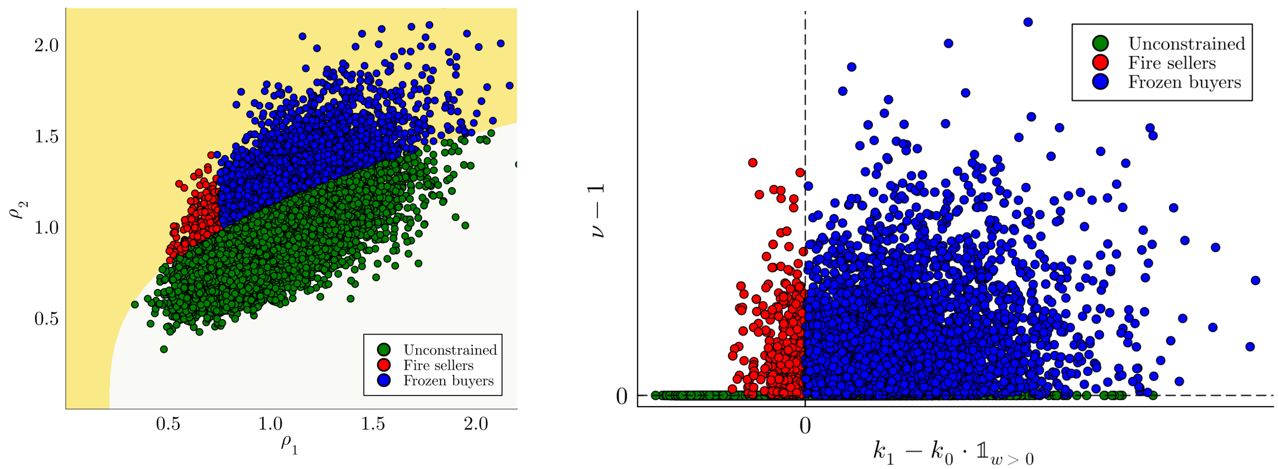

Figure 18: Characteristics of financial crises in annual baseline calibration.

(a) $\mathcal{CP}(\lambda)$ **- total** (b) $\mathcal{CP}(\lambda)$ **- by type of entrepreneurs**

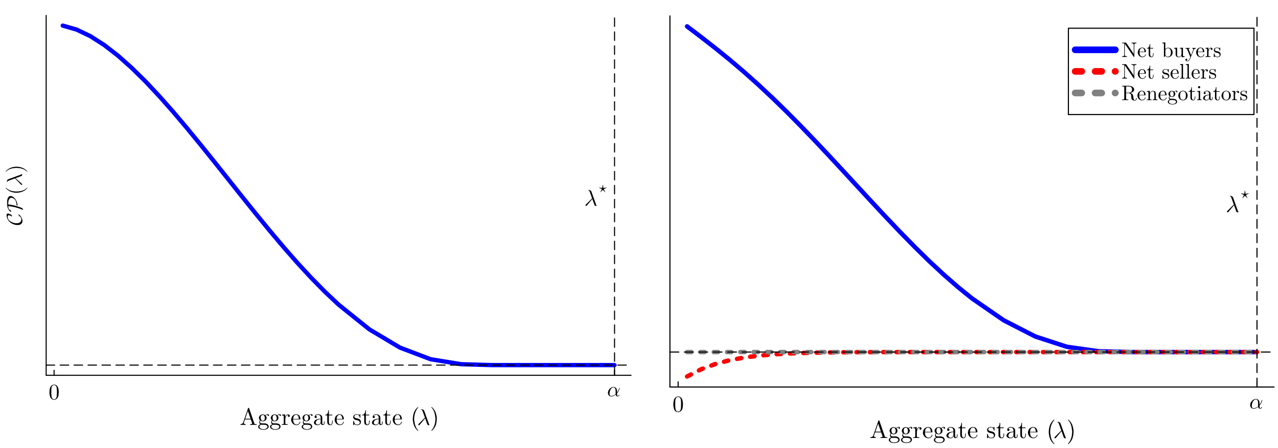

Figure 19: $\mathcal{CP}(\lambda)$ and its decomposition across aggregate states (λ) in annual baseline calibration.

B Data Appendix

B.1 Compustat data for firm-level analysis

In this Appendix we outline the steps taken to construct the variables and select the sample of Compustat firm-quarters that we use in the empirical analysis of Section 6.

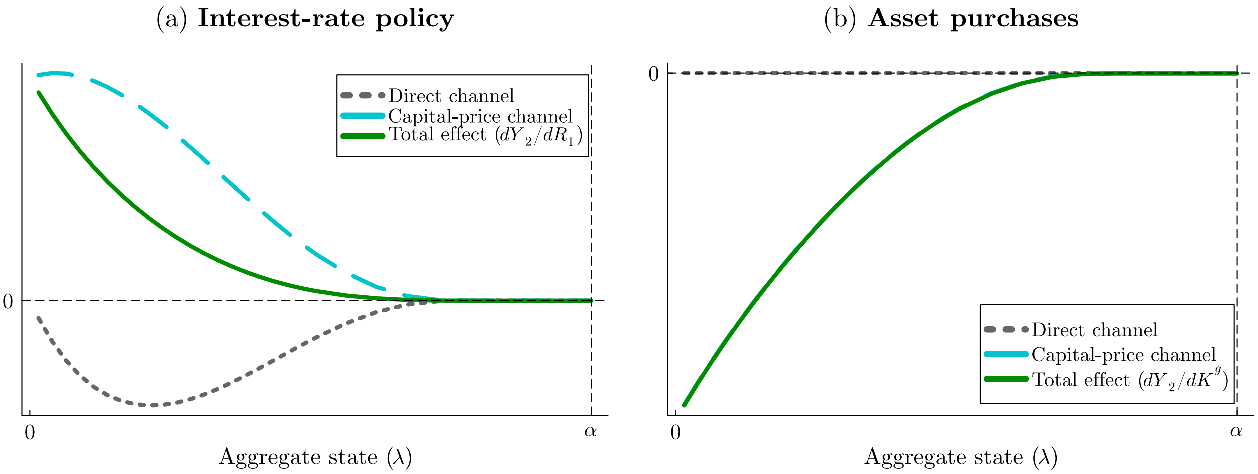

Figure 20: Effect of policy on output in the annually calibrated economy, across aggregate states (λ).

B.1.1 Sample selection

We follow conventional sample selection criteria from the literature. We exclude all firms that are not incorporated in the United States and that are in the financial industry (SIC code between 6000 and 6999), utilities (SIC between 4900 and 4999), or quasi-governmental sectors (SIC code between 9000 and 9999), or for which no SIC industry code is available. In addition, we drop all firm-quarters for which measurements of *Total assets* (Compustat data item 44, $ATQ_{i,t}$), *Property, Plant and Equipment (Net)* (item 42, $PPENTQ_{i,t}$), *Sales* (item 2, $SALEQ_{i,t}$) are negative.

In addition to these sample selection criteria, when estimating the panel regressions specifications such as (47), we drop observations of specific firm-level variables in given firm-quarters identified as outliers which might significantly affect the estimates. We drop the individual firm-quarter observations of $ARPK_{i,t}$ above and below the 1st and 99th percentiles within each SIC1-quarter-cell. For the controls in $X_{i,t-1}$, we drop the individual firm-quarter observations of the liquidity ratio, leverage and Tobin's q for which the observation is above the 99th percentile of the corresponding variable's quarter t cross-section. As for the outcome variables in the panel regression specifications, i.e., investment rates or acquisitions-to-assets rates, we drop the observations below the 1st and above the 99th percentile. We do this separately within each quarter t.

B.1.2 Construction of variables and summary statistics

We construct the key variables employed in Section 6 as follows.

1. As the measure of firms' *fixed capital stocks* in place at the end of quarter t, we employ the Compustat $PPENTQ_{i,t}$, and denote it as $k_{i,t}$.

2. We measure *gross investment* for firm i in quarter t as the quarterly *Capital Expenditures* ($Capx_{i,t}$) constructed based on the Compustat reported *Year-to-date Capital Expenditures* (item 90, $CAPXY_{i,t}$).

3. As the measure of sales of existing capital we use the quarterly sale of property, plant, and equipment, inferred from the Compustat Year-to-Date variable *Sale of Property, Plant, and Equipment* (item 83, $SPPEY_{i,t}$).

4. As the measure of purchases of existing capital (and other productive assets) we use the quarterly acquisitions, inferred from the Compustat Year-to-Date variable *Acquisitions* (item 94, $AQCY_{i,t}$).

5. As the measure of firm *Size*, we employ *Total Assets* $ATQ_{i,t}$.

6. We define *Leverage* as total debt divided by $ATQ_{i,t}$, with total debt computed as the sum of *Debt in Current Liabilities* (item 45, $DLCQ_{i,t}$) and *Total Long-Term Debt* (item 51, $DLTTQ_{i,t}$).

7. We measure the *Liquidity ratio* as *Cash and Short-Term Investments* (item 38, $CHEQ_{i,t}$) divided by $ATQ_{i,t}$.

8. We construct Tobin's q, as the book value of total assets ($ATQ_{i,t}$), plus the difference between the market value of common equity ($PRCCQ_{i,t} \times CSHOQ_{i,t}$) and the book value *Common Equity – Total* (item 59, $CEQQ_{i,t}$), minus *Deferred Taxes* ($TXDBQ_{i,t}$), all scaled by $ATQ_{i,t}$. We infer missing firm-quarters for $TXDBQ_{i,t}$ to be zero.

B.2 Additional empirical results

Table 3: Panel regressions for acquisitions-augmented investment rates in "normal" and "crisis" times

	High EBP_t cutoff:			
Dep. var.: $100 \times I^a_{i,t}/k_{i,t-1}$	p50	p75	p90	Cont.
$\log(ARPK_{i,t+1})$	2.1219***	1.9746***	1.9487***	1.9123***
	(13.01)	(12.93)	(13.12)	(13.21)
$\log(ARPK_{i,t+1}) \times High\ EBP_t$	-0.5524***	-0.4496***	-0.7426***	
	(-4.09)	(-2.9)	(-4.04)	
$\log(ARPK_{i,t+1}) \times EBP_t$				-0.5913***
				(-5.73)
Observations	342,714	342,714	342,714	342,714
Controls	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
Time-Industry FE	✓	✓	✓	✓

Table 4: Panel regressions for gross investment rates in "normal" and "crisis" times

	High EBP_t cutoff:			
Dep. var.: $100 \times Capx_{i,t}/k_{i,t-1}$	p50	p75	p90	Cont.
$\log(ARPK_{i,t+1})$	0.8804***	0.8617***	0.8418***	0.8351***
	(15.54)	(16.3)	(16.46)	(16.58)
$\log(ARPK_{i,t+1}) \times High\ EBP_t$	-0.1212**	-0.1514***	-0.1721***	
	(-2.54)	(-2.94)	(-2.62)	
$\log(ARPK_{i,t+1}) \times EBP_t$				-0.1408***
				(-4.19)
Observations	445,381	445,381	445,381	445,381
Controls	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
Time-Industry FE	✓	✓	✓	✓

Table 5: Panel regressions for acquisition rates in "normal" and "crisis" times

	High EBP_t cutoff:			
Dep. var.: $100 \times Acq_{i,t}/k_{i,t-1}$	p50	p75	p90	Cont.
$\log(ARPK_{i,t+1})$	0.8217***	0.7629***	0.7395***	0.725***
	(12.47)	(12.54)	(12.98)	(13)
$\log(ARPK_{i,t+1}) \times High\ EBP_t$	-0.2467***	-0.2311***	-0.2983***	
	(-4.23)	(-3.23)	(-3.54)	
$\log(ARPK_{i,t+1}) \times EBP_t$				-0.235***
				(-4.78)
Observations	424,659	424,659	424,659	424,659
Controls	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
Time-Industry FE	✓	✓	✓	✓

Table 6: Panel regressions for capital sale indicator in "normal" and "crisis" times

	High EBP_t cutoff:			
Dep. var.: $\mathbf{1}[Sale_{i,t} > 0]$	p50	p75	p90	Cont.
$\log(ARPK_{i,t+1})$	-0.0084***	-0.0095***	-0.0102***	-0.0099***
	(-5.14)	(-6.06)	(-6.56)	(-6.4)
$\log(ARPK_{i,t+1}) \times High\ EBP_t$	-0.0031**	-0.0015	0.0018	
	(-2.55)	(-1.08)	(1.04)	
$\log(ARPK_{i,t+1}) \times EBP_t$				-0.0008
				(-0.79)
Observations	359,757	359,757	359,757	359,757
Controls	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
Time-Industry FE	✓	✓	✓	✓