Nonlinear Monetary Policy Tradeoffs

Davide Debortoli[1,*], Mario Forni[2], Luca Gambetti[3], and Luca Sala[4]

Abstract: We measure the inflation-unemployment tradeoff associated with monetary easing and tightening, during booms and recessions, using a novel nonlinear Proxy-SVAR approach. We find evidence of significant nonlinearities for the U.S. economy (1973:M1 - 2019:M6): stimulating economic activity during recessions is associated with minimal costs in terms of inflation, and reducing inflation during booms delivers small costs in terms of unemployment. These results can be rationalized by a simple model with downward nominal wage rigidities, which is also used to assess the validity of our empirical approach.

Keywords: monetary policy, inflation-unemployment tradeoff, structural VAR models, Proxy-SVAR.

Classification: C32, E32

[*]**Correspondence address:** davide.debortoli@upf.edu

The authors gratefully acknowledge the financial support from the Spanish Ministry of Economy and Competitiveness through the Severo Ochoa Programme for Centres of Excellence in R&D (Barcelona School of Economics CEX2024-001476-S) and through grants PID2024-160049NB-I00 (Debortoli) and PGC2018-094364-B-I00 (Gambetti), from the FAR 2017 Department of Economics "Marco Biagi" (Forni) and from the Italian Ministry of Research and University, PRIN 2017 grant J44I20000180001 (Forni, Gambetti and Sala).

1 Introduction

The presence of an inflation-unemployment tradeoff —or for short the "monetary policy tradeoff"— is at the heart of monetary policymaking. How much inflation is needed to stimulate economic activity? What are the costs of reducing inflation in terms of unemployment? These long-lasting questions became particularly relevant in recent times, as the US and European economy faced the deepest postwar crisis in 2008-09, and more recently the highest surge in inflation since the 1970's.

The magnitude of the monetary policy tradeoff is traditionally measured as the (inverse) slope of a Phillips curve, i.e. a relationship linking inflation and unemployment. The corresponding estimates are usually obtained within *linear* settings, where it is implicitly assumed that the inflation-unemployment tradeoff is constant, and independent from the sign of the monetary intervention, the underling economic conditions, or other factors. However, there are several reasons to question the validity of that assumption. On the one hand, at least since the Great Depression (Keynes, 1936, Chapter 21) it has been argued that monetary tightening is more powerful than monetary easing, due to their potentially different effects on prices, wages, credit conditions, etc.[1] On the other hand, since the late 1980's the inflation rate appears to be largely insensitive to movements in the unemployment rate —as if the Phillips curve had flattened, or disappeared. In this context, a constant inflation-unemployment tradeoff would have the following implications: (i) central banks could stimulate economic activity with minimal costs in terms of inflation; and (ii) reducing inflation would be associated with a very large increase in the unemployment rate. Both implications are clearly extreme, and of doubtful relevance for policymaking purposes.

[1]Some examples in that regard are models with asymmetric price adjustments (e.g., Ball and Mankiw (1994)), or occasionally binding financial constraints (e.g. Bernanke, 1993, and De Long and Summers, 1998).

In this paper, we provide new evidence on the nature of the monetary policy tradeoff for the US economy, and study in particular whether the size of the tradeoff depends on the sign of the monetary intervention (easing vs tightening) and the state of the economy (booms vs recessions).

The contribution of the paper is twofold. First, from a methodological viewpoint, building on the work of Mertens and Ravn (2013), Stock and Watson (2018) and Plagborg-Møller and Wolf (2021) we extend the Proxy-VAR-IV approach to a nonlinear context. The economy is described by a Vector Moving Average (VMA) augmented with *nonlinear* functions of the monetary policy shock. These nonlinear functions give rise to a nonlinear dynamic transmission of monetary policy. The model admits a simple VARX representation where the shock and its nonlinear functions constitute the exogenous variables. Even though the exogenous shock is not observed, under relatively mild assumptions —i.e. the existence of a valid instrument for the shock and the existence of a linear monetary policy rule— it can be estimated as the fitted value of the regression of the instrument on the residuals of a (misspecified) *linear* VAR, where the nonlinear functions of the shock are neglected. The monetary tradeoff is then calculated as the ratio of the (cumulative) impulse responses of inflation and unemployment (or viceversa), *conditional* on identified monetary shocks. In our application, we distinguish between the effects of positive and negative monetary shocks, during booms and recessions. However, the methodology can be applied to study several kinds of non linearities, such as interactions between shocks, and both symmetric and asymmetric "size" effects.[2]

Second, from an economic viewpoint, we find that the inflation-unemployment tradeoff varies substantially, depending on the sign of the monetary intervention and the state of the economy,

[2]Forni et al. (2024) adapts the procedure to study the nonlinear effects of financial shocks. Forni et al. (2025) uses the method to study the nonlinear effects of oil supply shocks. Brianti et al. (2024) applies the method within a factor model to study the asymmetric effects of demand shocks. Franconi (2024) uses the method to study whether monetary policy is less effective in periods of geopolitical stress. In these papers, the authorship of the method is clearly attributed to the present work.

thus calling into question typical predictions associated with linear —and possibly nearly flat— Phillips curves. In particular, the inflation costs of stimulating economic activity are found to be small (and insignificant) during recessions. At the same time, the employment costs of lowering inflation are found to be moderate, especially during economic expansions —e.g. we find that reducing inflation by 1 percentage point requires an increase in the unemployment rate of roughly 0.5 percentage points. In other words, our results suggest that central banks can engage into disinflationary policies without necessarily incurring very large unemployment costs.

For policymaking purposes, our results suggest that both monetary easing in recessions and tightening in expansions are associated with relatively favourable tradeoffs. The tradeoff worsens dramatically for other types of policies (e.g. tightening in recessions). In this respect, our analysis provides a different perspective relative to previous results on the nonlinear effects of monetary policy shocks. For instance, the recent works of Tenreyro and Thwaites (2016) and Barnichon and Matthes (2018) suggest that monetary policy is not very effective at stimulating economic activity, especially during recessions —as if the central bank was "pushing on a string". Yet, we also show that a monetary easing during recessions has a moderate effect on prices, so that the central bank faces a relatively favourable inflation-unemployment tradeoff. Thus, monetary policy can be a useful tool, even during recessions: it is possible to achieve a favourable balance between inflation and unemployment, as long as the central bank's interventions are sufficiently aggressive to achieve the desired economic stimulus.

We also show that a sign- and state-dependent monetary policy tradeoff can arise in a simple model economy with labour market frictions, in the form of downward nominal wage rigidities.[3] The model gives rise to inflation and output dynamics that are qualitatively very similar to their

[3]Similar models can be found in Kim and Ruge-Murcia (2009), Benigno and Ricci (2011) and Schmitt-Grohé and Uribe (2016), and Benigno and Eggertson (2023), among others.

empirical counterparts. We then apply our empirical approach to artificial data generated by the model, and show that the resulting estimates capture entirely the nonlinearities of the underlying economy.

The remainder of the paper is organized as follows. Section 2 contains a brief review of the related literature. Section 3 formally defines our measure of the monetary policy tradeoff. Section 4 discusses the econometric approach. Section 5 presents the empirical evidence, Section 6 presents a model with downward nominal wage rigidities, and Section 7 concludes.

2 Related literature

This paper contributes to the vast empirical literature about the inflation-unemployment tradeoff, starting from the original evidence of Phillips (1958) and Samuelson and Solow (1960), and followed by the empirical works on the New-Keynesian Phillips curve of Roberts (1995), Fuhrer and Moore (1995), Galí and Gertler (1999) and Sbordone (2002), among others.[4] More specifically, our paper is related to the recent body of that literature proposing novel approaches to identify the empirical relationship between measures of inflation and economic activity. A number of authors (e.g. McLeay and Tenreyro, 2020, Beraja, Hurst and Ospina, 2019, and Hazell et al., 2022) exploit variations at the regional level to overcome the simultaneity problem of distinguishing between demand and supply shocks. Similarly to us, Ball (1994) proposes a non-parametric estimate of the output-inflation tradeoff using the (cumulative) trend deviations of output during disinflationary episodes, for a sample of OECD countries. More recently, Barnichon and Mesters (2020, 2021) and Galí and Gambetti (2020) exploit identified monetary shocks to obtain *conditional* estimates of the inflation-unemployment relationship, and to investigate whether the slope of the Phillips

[4]See Mavroeidis, Plagborg-Møller and Stock (2014) for a survey of earlier works.

curve has changed over time (see also Stock and Watson, 2020, and Del Negro et al., 2020). The main contribution relative to this literature is to show that the relationship between inflation and unemployment is sign- and state-dependent. In this respect our results are consistent with the evidence in Daly and Hobijn (2014) and Gagnon and Collins (2020), among others, who document the presence of important nonlinearities in the Phillips curve, using different approaches.

Our work is also related to the large literature studying the effects of monetary shocks. Most studies in this literature have relied on linear SVARs.[5] Few recent studies looked at the nonlinear effects of monetary shocks. For example, Tenreyro and Thwaites (2016) find that monetary policy is less powerful during recessions, while Barnichon and Matthes (2018) find that monetary tightening is more powerful than monetary easing.[6] Our contribution relative to those works is twofold. First, we propose a novel empirical approach to study the effects of both sign- and state-dependence, within a single framework.[7] Second, we estimate the inflation-unemployment tradeoff, rather than focusing on macroeconomic variables in isolation, which provides a different perspective about the effects of monetary interventions during booms and recessions.

3 Defining the Monetary Policy Tradeoff

Our goal is to measure the inflation cost of reducing unemployment —the tradeoff for monetary easing— and viceversa the unemployment cost of reducing inflation —the tradeoff for monetary

[5] A partial list of early contributions studying the effects of monetary policy shocks includes Bernanke and Blinder (1992), Bernanke and Gertler (1995), Bernanke and Mihov (1998), Christiano, Eichenbaum and Evans (1999), Cochrane (1994), Leeper, Sims and Zha (1996), Sims and Zha (2006) and Strongin (1995). Several advances have been made in recent years, especially in terms of shock identification, as for instance in the works of Romer and Romer (2004), Uhlig (2005), Gertler and Karadi (2015), Arias, Caldara and Rubio-Ramírez (2019), Jarocinsky and Karadi (2020), Caldara and Herbst (2019) and Miranda-Agrippino and Ricco (2021).

[6] Early contributions on the topic include Cover (1992), Karras (1996) and Weise (1999). See also Santoro et al. (2014), Angrist, Jordà and Kuersteiner (2018), Alpanda, Granziera and Zubairy (2021) and Ascari and Haber (2022) for additional evidence on the nonlinear effects of monetary shocks.

[7] Tenreyro and Thwaites (2016) use instead nonlinear local projections identifying the policy shocks as in Romer and Romer (2004), while Barnichon and Matthes (2018) estimate a nonlinear Vector Moving Average representation using Functional Approximation of Impulse Response (FAIR) approach.

tightening. In linear settings, the two measure are tightly related, as one measure is simply the inverse of the other. This is no longer the case in a nonlinear setting. It is therefore necessary to treat the two cases separately.

We define the tradeoff for monetary easing (respectively, tightening) as the average change in inflation (unemployment) forecasts induced by a monetary shock that causes a 1 percentage point change in average forecast unemployment (inflation), and where averages are taken over an horizon of H periods.[8] The tradeoffs can be calculated using the average impulse responses of unemployment (y) and inflation (π) to a one-unit monetary shock.

Formally, the tradeoffs for monetary easing ($\mathcal{T}_H^+$) and tightening ($\mathcal{T}_H^-$) are defined as

$$\mathcal{T}_H^+(s_{t-1}) \equiv \frac{\frac{1}{H}\sum_{h=0}^{H}\mathcal{R}_h^{\pi,+}(s_{t-1})}{\frac{1}{H}\sum_{h=0}^{H}\mathcal{R}_h^{y,+}(s_{t-1})} \qquad \mathcal{T}_H^-(s_{t-1}) \equiv \frac{\frac{1}{H}\sum_{h=0}^{H}\mathcal{R}_h^{y,-}(s_{t-1})}{\frac{1}{H}\sum_{h=0}^{H}\mathcal{R}_h^{\pi,-}(s_{t-1})}, \tag{1}$$

where $\mathcal{R}_h^{x,+}(s_{t-1})$ denotes the impulse response at horizon h of a generic variable x to a monetary easing, $s_{t-1} \in 0,1$ is a recession indicator, and $\mathcal{R}_h^{x,-}(s_{t-1})$ denotes instead the corresponding impulse responses for a monetary tightening.

Following this procedure allows us to mitigate well-known challenges associated with typical Phillips curve estimates (see e.g. Mavroeidis et al., 2014), for the following reasons: (i) a measure of the tradeoff can be obtained under minimal assumptions about the structure of the underlying economy, e.g. without postulating a specific Phillips curve, or other structural equations, which may lead to misspecification problems; (ii) there is no need to rely on data on inflation expectations or the “natural" rate of unemployment, which are not directly observable, and may lead to additional biases and uncertainties in coefficient estimates due to measurement error; and (iii) we

[8] In this respect, the measure of the monetary tradeoff resembles the concept of government spending multiplier in the fiscal literature, which is calculated as the ratio of the cumulative response of output and government spending, see e.g. Mountford and Uhlig (2009) and Ramey and Zubairy (2018).

obtain a measure of the tradeoff *caused* by exogenous monetary interventions which is immune from typical endogeneity problems of Phillips curve estimates.[9]

4 Methodology: a Nonlinear Proxy-SVAR

In this section we present our empirical model, the identification assumptions, and the estimation approach. We consider a model economy where macroeconomic variables react (linearly) both to the monetary shock and to nonlinear functions of that shock. We show that, under suitable conditions, such a nonlinear model admits a VARX representation that can be estimated using an external instrument, analogously to what is usually done in linear settings. To do so, we build on Forni, Gambetti and Ricco (2023), who show that in invertible linear models the shock of interest can be obtained as the projection of the instrument onto the vector of reduced form residuals of a VAR. Here we extend that result to a nonlinear context.

4.1 Representation assumptions

Let $\mathbf{x}_t$ be an n-dimensional vector of observable macroeconomic variables. We assume the following structural representation.

Assumption A0 (The structural representation). *We assume that*

(a) The vector $\mathbf{x}_t$ is wide-sense stationary and has the structural representation

$$\mathbf{x}_t \quad = \quad \boldsymbol{\nu} + \boldsymbol{\Gamma}(L)\mathbf{u}_t + \boldsymbol{\alpha}(L)u_t^r + \boldsymbol{\Phi}(L)\mathbf{g}(u_t^r, \bullet) \tag{2}$$

[9] More specifically, as is common in the monetary policy literature, we are assuming that the "natural" rate of unemployment (or output) is orthogonal to monetary shocks, so that a measure of the unemployment (or output) gap is not needed to calculate the implied monetary tradeoff.

where $\boldsymbol{\nu}$ is a vector of constants, u_t^r is the monetary policy shock, $\mathbf{u}_t$ is a m-dimensional vector including additional structural shocks (excluding monetary policy), and $\mathbf{g}(u_t^r, \bullet)$ is a k-dimensional vector of nonlinear functions of the policy shock and potentially other shocks or lagged variables. Moreover, $\boldsymbol{\alpha}(L) \equiv \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 L + \boldsymbol{\alpha}_2 L^2 \ldots$ is a n-dimensional vector representing the linear response functions to the monetary policy shock; $\boldsymbol{\Gamma}(L) \equiv \boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_1 L + \boldsymbol{\Gamma}_2 L^2 + \ldots$ is a $n \times m$ matrix of impulse response functions to the remaining structural shocks; $\boldsymbol{\Phi}(L) \equiv \boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 L + \boldsymbol{\Phi}_2 L^2 + \ldots$ is a $n \times k$ matrix of impulse response functions to the nonlinear functions $\mathbf{g}(u_t^r, \bullet)$.

(b) The shock u_t^r has zero mean, unit variance, is serially independent, independent of $\mathbf{x}_{t-\ell}$ for any lag $\ell > 0$, and $\mathbf{u}_{t-\ell}$ at all lags and leads ℓ.

(c) Each element of the vector $\mathbf{g}(u_t^r, \bullet)$ is wide-sense stationary with finite variance and uncorrelated with $\mathbf{u}_{t-\ell}$, for any lag and lead ℓ.

Assumptions A0 (a)-(b) establish that our approach accommodates a fairly general linear moving-average (MA) models augmented with nonlinear functions of the shock of interest $\mathbf{g}(u_t^r, \bullet)$ —the monetary policy shock in our case. Notably, the system (2) is not required to be square, implying that the number of shocks and nonlinear functions $m + 1 + k$ may differ from the number of observable variables n —i.e. $m + 1 + k \lesseqgtr n$. Furthermore, nonlinear functions of shocks other than monetary policy can be embedded within the vector $\mathbf{u}_t$.[10]

Assumption A0 (c) imposes specific restrictions on the admissible set of nonlinear functions of the monetary shock $\mathbf{g}(u_t^r, \bullet)$, and its specific role will be made clear in the sequel. We now present a range of illustrative examples consistent with this assumption.

First, in our empirical implementation we set $\mathbf{g}(u_t^r, \bullet) = [|u_t^r| \quad s_{t-1} u_t^r]'$, where the absolute value $|u_t^r|$ captures sign-dependent effects, and the term $s_{t-1} u_t^r$ accounts for state-dependent

[10] Notice however that Assumption A1 below imposes a restriction on the model.

effects. The variable s_{t-1} is a dummy reflecting the state of the economy prior to the arrival of current shocks, and hence depends solely on lagged values of $\mathbf{x}_t$. This specification satisfies Assumption A0 (c), as it directly follows from the independence conditions in Assumption A0 (b). Specifically, since u_t^r is independent of both $\mathbf{u}_{t-\ell}$ for any ℓ and $\mathbf{x}_{t-\ell}$ for all $\ell > 0$, it follows that: (i) $|u_t^r|$ is also independent of $\mathbf{u}_{t-\ell}$ for any ℓ; and (ii) u_t^r is independent of $s_{t-\ell}$, for all $\ell > 0$, since the latter variable only depends on lagged values of $\mathbf{x}_t$. As a result, we have $\mathbb{E}\left(u_t^r s_{t-1}\mathbf{u}_{t-\ell}\right) = \mathbb{E}\left(u_t^r\right)\mathbb{E}\left(s_{t-1}\mathbf{u}_{t-\ell}\right) = 0$, where the latter follows from the zero-mean assumption on u_t^r. Hence, $s_{t-1}u_t^r$ is also uncorrelated to $\mathbf{u}_{t-\ell}$ for any ℓ, satisfying Assumption A0 (c).

Second, consider the alternative specification $\mathbf{g}(u_t^r, \bullet) = u_t^r u_{jt}$, capturing the interaction between the monetary shock u_t^r and a non-monetary shock u_{jt} in $\mathbf{u}_t$. This specification is particularly useful for capturing the nonlinearities that emerge in a second-order approximation of a broad class of models —provided that such an approximation is valid.[11] To verify that this specification also complies with Assumption A0 (c), note that by Assumption A0 (b) we have $\mathbb{E}\left(u_t^r u_{jt}\mathbf{u}_{t-\ell}\right) = \mathbb{E}\left(u_t^r\right)\mathbb{E}\left(u_{jt}\mathbf{u}_{t-\ell}\right) = 0$ for any ℓ.[12]

Finally, all positive powers of u_t^r satisfy Assumption A0 (c), due to the independence condition specified in A0 (b).

The above examples show that Assumption A0 (c) accommodates a broad class of nonlinear functions. The absolute value can capture sign effects, i.e. potential asymmetries between positive and negative shocks. Interactions with a state variable allows for state-dependent effects.

[11]Thus, in this case eq. (2) can be interpreted as a second-order approximation of a general nonlinear mapping between the (sequence) of current and past shocks into current macro variables, as it is typically the case in structural macroeconomic models.

[12]By contrast, a function such as $\mathbf{g}(u_t^r, \bullet) = \left(u_t^r\right)^2 u_{jt}$ would not fulfill A0 (c), since $\mathbb{E}\left[\left(u_t^r\right)^2 u_{jt}\mathbf{u}_{t-\ell}\right] = \mathbb{E}\left(u_t^r\right)^2\mathbb{E}\left(u_{jt}\mathbf{u}_{t-\ell}\right)$, which cannot be zero for $\ell = 0$.

Products involving the monetary shock and other structural shocks can capture interactions — such as between monetary and fiscal policy. Finally, the square of the shock might capture a mixture of sign and size effects, while the cube might capture symmetric size effects.

The total effects of a monetary policy shock $u_t^r = \bar{u}^r$ are obtained by summing linear and nonlinear terms:

$$\boldsymbol{\mathcal{R}}(L, \bar{u}^r) \equiv \boldsymbol{\alpha}(L)\bar{u}^r + \boldsymbol{\Phi}(L)\mathbf{g}(u_t^r, \bullet). \tag{3}$$

In this nonlinear context, the total responses defined by eq. (3) correspond to the Generalized Impulse Response Functions defined as $\mathbb{E}(\mathbf{x}_{t+h}|u_t^r = \bar{u}^r) - \mathbb{E}(\mathbf{x}_{t+h}|u_t^r = 0)$, $h = 0, 1, ...$ Next, we describe a procedure to estimate the model and the implied impulse response functions.

Stationarity of $\boldsymbol{\Gamma}(L)\mathbf{u}_t$ ensures the existence of the following representation:

$$\mathbf{x}_t = \boldsymbol{\nu} + \boldsymbol{\Psi}(L)\mathbf{e}_t + \boldsymbol{\alpha}(L)u_t^r + \boldsymbol{\Phi}(L)\mathbf{g}(u_t^r, \bullet), \tag{4}$$

where $\boldsymbol{\Psi}(L)\mathbf{e}_t$ is the Wold representation of $\boldsymbol{\Gamma}(L)\mathbf{u}_t$.[13]

We further characterize the process $\boldsymbol{\Psi}(L)\mathbf{e}_t$ under the following Assumption.

Assumption A1 (Finite-order VARX representation). *We assume that*

(a) $\boldsymbol{\Psi}(L) = \mathbf{A}(L)^{-1}$

(b) $\boldsymbol{\Phi}(L) = \mathbf{A}(L)^{-1}\tilde{\boldsymbol{\Phi}}(L)$,

(c) $\boldsymbol{\alpha}(L) = \mathbf{A}(L)^{-1}\tilde{\boldsymbol{\alpha}}(L)$,

where $\mathbf{A}(L) = \mathbb{I}_n - \mathbf{A}_1 L - \cdots - \mathbf{A}_p L^p$ *is a matrix of polynomials of degree* p*, and* $\tilde{\boldsymbol{\alpha}}(L)$ *and* $\tilde{\boldsymbol{\Phi}}(L)$ *are polynomials of degree* $q \leq p$.

[13] If the structural representation $\boldsymbol{\Gamma}(L)\mathbf{u}_t$ is invertible, then $\boldsymbol{\Psi}(L) = \boldsymbol{\Gamma}(L)\boldsymbol{\Gamma}_0^{-1}$ and $\mathbf{e}_t = \boldsymbol{\Gamma}_0\mathbf{u}_t$, $\boldsymbol{\Gamma}_0^{-1}$ being either the inverse of $\boldsymbol{\Gamma}_0$, if $m = n$, or a left inverse of $\boldsymbol{\Gamma}_0$, if $m < n$. If the structural representation $\boldsymbol{\Gamma}(L)\mathbf{u}_t$ is not invertible (e.g. when $m > n$ or $m = n$ but $\boldsymbol{\Gamma}(L)$ vanishes within the unit disk), then $\mathbf{e}_t$ is a linear combination of the present *and past* values of $\mathbf{u}_t$ and $\boldsymbol{\Psi}(L)\mathbf{e}_t$ is just a statistical representation, devoid of economic meaning.

Assumption A1 (a) implies that the inverse of $\mathbf{\Psi}(L)$ exists, meaning its determinant does not vanish on the unit circle, and it is a finite-order polynomial matrix. This is a standard assumption in SVAR analysis. The additional requirement that $\tilde{\boldsymbol{\alpha}}(L)$ and $\tilde{\mathbf{\Phi}}(L)$ are polynomials of order $q \leq p$ is imposed to avoid collinearity issues, since the monetary policy shock is obtained as a combination of the current value and p lags of $\mathbf{x}_t$, as discussed below in more details.

Under Assumption A1, eq. (4) can be rewritten as

$$\mathbf{A}(L)\mathbf{x}_t = \boldsymbol{\mu} + \tilde{\boldsymbol{\alpha}}(L)u_t^r + \tilde{\mathbf{\Phi}}(L)\mathbf{g}(u_t^r, \bullet) + \mathbf{e}_t, \tag{5}$$

or equivalently,

$$\mathbf{x}_t = \boldsymbol{\mu} + \tilde{\mathbf{A}}(L)\mathbf{x}_{t-1} + \tilde{\boldsymbol{\alpha}}(L)u_t^r + \tilde{\mathbf{\Phi}}(L)\mathbf{g}(u_t^r, \bullet) + \mathbf{e}_t \tag{6}$$

where $\boldsymbol{\mu} \equiv \mathbf{A}(1)\boldsymbol{\nu}$, $\tilde{\mathbf{A}}(L) \equiv \mathbf{A}_1 + \mathbf{A}_2 L + \cdots + \mathbf{A}_p L^{p-1}$.

Eq. (6) represents a VARX model where the monetary policy shock and its nonlinear functions are treated as exogenous variables. A crucial property of this equation is that $\mathbf{e}_t$ is orthogonal to the regressors. This is because $\mathbf{e}_t$ is a linear combination of present and past values of $\mathbf{u}_t$, and is thus orthogonal to both u_t^r and $\mathbf{g}(u_t^r, \bullet)$ at all leads and lags by Assumptions A0 (b) and A0 (c), respectively.[14] Moreover, $\mathbf{e}_t$ is orthogonal to the past of $\mathbf{\Gamma}(L)\mathbf{u}_t$ by construction, so that it is orthogonal to $\mathbf{x}_{t-k}$, $k > 0$, see equation (2). As a consequence, for given monetary shocks u_t^r, the VARX (6) can be estimated consistently by OLS.

Unfortunately, direct estimation of the VARX (6) is unfeasible, because u_t^r is not observable.[15] However, under suitable identification assumptions described in the next subsection, we can estimate the policy shock as a linear combination of the residuals of a standard VAR. This of

[14] We have $\mathbf{e}_t = \mathbf{A}(L)\mathbf{\Gamma}(L)\mathbf{u}_t$, see equation (4) and Assumption A1 (a).

[15] If the monetary shock was observable, then eq. (6) or a local projection version of it could be estimated by OLS.

course requires the existence of a VAR representation, which in turn requires invertibility of the Wold representation of the macroeconomic variables, as established in the following Assumption.

Assumption A2 (Invertible Wold representation). *The vector* $\mathbf{x}_t$ *has an invertible Wold representation.*

Assumption A2 implies that $\mathbf{x}_t$ admits the VAR representation

$$\mathbf{x}_t = \boldsymbol{\vartheta} + \mathbf{B}(L)\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t = \boldsymbol{\vartheta} + \sum_{j=1}^{\infty} \mathbf{B}_j \mathbf{x}_{t-j} + \boldsymbol{\varepsilon}_t \tag{7}$$

where $\boldsymbol{\varepsilon}_t$ is vector white noise and orthogonal to $\mathbf{x}_{t-j}$, $j = 1, \dots, \infty$.

It is important to stress that equation (7) is perfectly compatible with our nonlinear framework. Indeed, under Assumption A0 (a), $\mathbf{x}_t$ is stationary and purely non deterministic. It must then admit a linear MA Wold representation, even if the true underlying structural model is nonlinear. If the determinant of the Wold MA matrix does not vanish on the unit circle in the complex plane, Assumption A2 is fulfilled and the VAR exists. If the Wold representation has a unit root because the variables are cointegrated —thus violating Assumption A2— the VAR still exists, but for the variables in levels.

We now explore the relationship between the VAR representation in eq. (7) and the VARX representation in eq. (6). To that end, and starting from eq. (6), consider the linear projection of $\tilde{\boldsymbol{\alpha}}(L)u_t^r + \tilde{\boldsymbol{\Phi}}(L)\mathbf{g}(u_t^r, \bullet)$ onto the constant and the past history of $\mathbf{x}_t$, i.e.

$$\tilde{\boldsymbol{\alpha}}(L)u_t^r + \tilde{\boldsymbol{\Phi}}(L)\mathbf{g}(u_t^r, \bullet) = \boldsymbol{\theta} + \mathbf{C}(L)\mathbf{x}_{t-1} + \boldsymbol{w}_t. \tag{8}$$

Substitution of (8) into (7) gives that $\boldsymbol{\vartheta} = \boldsymbol{\mu} + \boldsymbol{\theta}$, $\mathbf{B}(L) = \tilde{\mathbf{A}}(L) + \mathbf{C}(L)$ and $\boldsymbol{\varepsilon}_t = \mathbf{e}_t + \boldsymbol{w}_t$.[16] Since both u_t^r and $\mathbf{g}(u_t^r, \bullet)$ are orthogonal to the past history of $\mathbf{x}_t$, in this case we have $\mathbf{C}(L) = 0$ and

[16] An interesting special case is $\tilde{\boldsymbol{\alpha}}(L) = \tilde{\boldsymbol{\alpha}}_0$ and $\tilde{\boldsymbol{\Phi}}(L) = \tilde{\boldsymbol{\Phi}}_0$, i.e. the lags of the exogenous variables do not appear in (6).

$w_t = \tilde{\boldsymbol{\alpha}}_0 u_t^r + \tilde{\boldsymbol{\Phi}}_0 \mathbf{g}(u_t^r)$. Hence the VAR dynamics coincide with the VARX dynamics, $\mathbf{B}(L) = \tilde{\mathbf{A}}(L)$, and the terms driven by the exogenous variables enter the VAR residual, $\boldsymbol{\varepsilon}_t = \tilde{\boldsymbol{\alpha}}_0 u_t^r + \tilde{\boldsymbol{\Phi}}_0 \mathbf{g}(u_t^r) + \mathbf{e}_t$. This special case highlights that standard Proxy-SVAR identification would fail to estimate the linear component of the IRFs, unless u_t^r is orthogonal to $\mathbf{g}(u_t^r, \bullet)$. In general, the covariances of the VAR residuals $\boldsymbol{\varepsilon}_t$ with the proxy are not proportional to the covariances of the shock u_t^r with the proxy. Nevertheless, as shown below, the policy shock itself can be consistently estimated.

Clearly, due to the model's inherent nonlinearities, the VAR cannot be used to estimate the impulse response functions of the structural shock. Yet, as shown in section 4.3, the structural shock itself can be estimated from equation (7) provided that suitable conditions hold.

In the special case when $\tilde{\boldsymbol{\Phi}}(L) = 0$, the structural representation (2) reduces to a linear model, and standard SVAR analysis can be conducted using the representation (7). Hence, the linear model is nested in our model. Linearity can then be tested either considering the null hypothesis $\tilde{\boldsymbol{\Phi}}(L) = 0$ in equation (6), or the null hypothesis $\boldsymbol{\Phi}(L) = 0$ in the impulse-response functions (3).

It also follows from equation (8) that in the presence of a state-dependent nonlinear term, both w_t and the VAR residual $\boldsymbol{\varepsilon}_t$, are state-dependent and conditionally heteroskedastic —this is actually the case in our empirical application below. Indeed, the residuals $\boldsymbol{\varepsilon}_t$ depend on s_{t-1} and therefore are not independent of past $\mathbf{x}$'s. Nevertheless, $\boldsymbol{\varepsilon}_t$ is a vector of Wold residuals, which must be orthogonal to past $\mathbf{x}$'s. Hence OLS estimates of VAR (7) are consistent — and do not require gaussianity of the VAR residuals.

Given the state-dependence of VAR residuals, one may question whether the VAR coefficients should also be state-dependent, as in Threshold VAR models. For instance, consider a model where state-dependence constitutes the only source of nonlinearity, and with only two possible states. In that case, we would have two distinct MA representations, one for each possible state and, if such representations are invertible, two different VARs with state-dependent coefficients. Hence one could in principle estimate a Threshold VAR. In this respect, our method can be regarded as an alternative way to estimate state-dependent impulse response functions; but, unlike Threshold VAR, our approach can also accommodate other types of nonlinearities, such as the sign, size and interaction effects described above.[17]

Another natural question is whether a VAR with state-dependent coefficients is compatible with a fixed-coefficient VAR. The answer is affirmative. As argued above, as long as macroeconomic variables are covariance stationary, a fixed coefficient MA representation does always exist by the Wold theorem. If Assumption A2 holds, this representation is invertible and the fixed-coefficient VAR representation exists, even if the underlying dynamics are state-dependent.

4.2 Identification assumptions

Our procedure is an extension of the Proxy-SVAR identification (Mertens and Ravn, 2013 and Stock and Watson, 2018) to our nonlinear framework and involves two main steps: (i) estimating the shock by regressing a valid external instrument of the monetary policy shock onto the vector of the VAR residuals $\boldsymbol{\varepsilon}_t$; and (ii) using the estimated shock and its nonlinear function as regressors in model (6) to estimate the nonlinear impulse response functions (3).

The identification procedure relies on two assumptions. The first assumption is standard in the Proxy-SVAR literature and requires the existence of a valid instrument, as specified below.

Assumption A3 (Proxy). *The proxy z_t is given by*

$$z_t = a + bu_t^r + \delta(L)'\mathbf{x}_{t-1} + v_t = a + bu_t^r + \sum_{j=1}^{\infty} \delta_j' \mathbf{x}_{t-j} + v_t, \tag{9}$$

[17] As noticed by a referee, many papers condition on a state that stays fixed, because the coefficients of a VAR or LP depend on a state and can vary over time. The present framework just conditions on the initial state because it estimates a response to $g(u(t), s(t-1))$. It thus allows for the "usual" dynamics of $s(t)$ to unfold, such as escaping from a recession.

where $b \neq 0$ and v_t is an error independent of the structural shocks at all leads and lags.

Notice that under Assumption A3, the standard conditions for a valid instrument, i.e. $\text{cov}(z_t, u_t^r) = b \neq 0$ (relevance) and $\text{cov}(z_t, \mathbf{u}_t) = 0$ (exogeneity), are satisfied.

The second assumption ensures that the monetary shock can be estimated as a linear combination of current and past data. This is stated formally in the following Assumption.

Assumption A4 (Informational sufficiency). *The monetary policy shock can be estimated as a combination of current and past values of the variables, i.e.*

$$\sigma_r u_t^r = \boldsymbol{\gamma}(\boldsymbol{L})'\mathbf{x}_t - \psi, \tag{10}$$

with $\boldsymbol{\gamma}(L) = \boldsymbol{\gamma}_0 - \boldsymbol{\gamma}_1 L - \boldsymbol{\gamma}_2 L^2 - \cdots$.

Assumption A4 imposes specific restrictions on the VARX (6), and namely that there exists a linear combination of the variables $\gamma_0'\mathbf{x}_t \equiv r_t - \boldsymbol{\xi}'\mathbf{x}_t^{-r}$ that depends neither on the nonlinear function of the monetary shock, i.e. $\gamma_0'\tilde{\boldsymbol{\Phi}}(L) = 0$, nor on $\mathbf{e}_t$, i.e. $\gamma_0'\mathbf{e}_t = 0$ (so that the variance-covariance matrix of $\mathbf{e}_t$ must be singular).[18]

Such an assumption can be tested following the approach proposed by Forni, Gambetti and Ricco (2023). The test consists of regressing the instrument on current and future values of the VAR residuals. The shock is said to be invertible — i.e., the VAR variables are sufficiently informative for the shock— if the parameters associated to future residuals are not statistically significant. In other words, the test verifies whether the instrument Granger-causes the VAR residuals, which is equivalent to assessing whether it Granger-causes the VAR variables themselves. The intuition is as follows: if one can recover the policy shock by projecting the instrument

[18]Such restrictions however are just identifying and therefore cannot be used for testing purposes.

onto the VAR residuals, then current and lagged variables already contain the relevant information and the instrument does not cause the variables. Conversely, if the projection fails to recover the policy shock —possibly due to contamination from nonlinear terms or other confounding shocks— then the instrument contains additional information relative to current and lagged variables, and therefore Granger causes the variables themselves. In the empirical section we perform the above test and find that Assumption A4 cannot be rejected in our data.

For example, a sufficient —though not necessary— condition for Assumption A4 to hold is that the central bank follows a monetary policy rule of the type

$$r_t = \left[\psi + \boldsymbol{\xi}' \mathbf{x}_t^{-r} + \sum_{j=1}^{\infty} \boldsymbol{\gamma}_j' \mathbf{x}_{t-j} \right] + \sigma_r u_t^r. \tag{11}$$

The term in square brackets in the above equation represents the "systematic" component of the rule, where $\mathbf{x}_t^{-r}$ is the vector containing all variables in $\mathbf{x}_t$ but the interest rate, ψ is a scalar constant, while $\boldsymbol{\xi}$ and $\boldsymbol{\varphi}_j$ denote vectors of parameters with dimension $(n-1) \times 1$ and $n \times 1$, respectively. The residual $\sigma_r u_t^r$ is the non-normalized monetary policy shock. Eq. (11) nests the most standard monetary rules used in the DSGE monetary policy literature. Notably, it allows the central bank to respond contemporaneously to all variables, making it more general than the rules implied by traditional recursive (Cholesky) identification schemes, which assume no contemporaneous response to a subset of the structural shocks —see e.g. Christiano et al. (1996, 1999).[19]

Most importantly, a police rule like (11) does not rule out nonlinear responses of the policy instrument to the underlying structural shocks. While the policy instrument responds linearly

[19] Earlier empirical studies on the nonlinear effects of monetary policy (see e.g. Cover, 1992 and Karras, 1996) directly estimated a monetary rule like (10), and treated the residual as a measure of the monetary shock. As is well known, that procedure is only valid under the (restrictive) assumption that monetary shocks have no contemporaneous effects on macroeconomic variables other than the interest rate —an assumption we do not impose here.

to the variables $\mathbf{x}_t^{-r}$, these variables themselves may respond nonlinearly to (current and past) shocks of monetary or other nature. As a result, the interest rate may exhibit nonlinear dynamics even under a rule that is linear in the observable variables. In fact, a specification like (11) can be consistent with the optimal policy rule derived in a general nonlinear setting. In many such cases, the optimal rule can be expressed as a nonlinear mapping between the policy instrument and (current and past) shocks, which could be captured by our specification. As an illustrative example, Appendix A4 shows that eq. (11) implements the optimal policy in the nonlinear model presented in Section 6.

4.3 The key result

We are now ready to present the main result underpinning our empirical approach. To that end, let us consider the VAR representation (7). This representation is, in a sense, misspecified, since it does not take into account the nonlinear term. Despite this, the following Proposition shows that the VAR residuals in $\boldsymbol{\varepsilon}_t$ can be combined with the external instrument to recover the monetary policy shock.

Proposition 1. *Under Assumptions A0 to A4 the monetary policy shock is equal, up to a multiplicative constant, to the orthogonal projection of the instrument z_t onto the VAR innovations $\boldsymbol{\varepsilon}_t$.*

Proof. Let us assume, without loss of generality, that r_t is ordered first in the vector $\mathbf{x}_t$ and let $\gamma_0 \equiv [1 \quad -\boldsymbol{\xi}']'$. Rearranging equation (10) we get

$$\gamma_0' \mathbf{x}_t = \psi + \sum_{j=1}^{\infty} \gamma_j' \mathbf{x}_{t-j} + \sigma_r u_t^r, \tag{12}$$

where u_t^r is orthogonal to $\mathbf{x}_{t-j}$, $j = 1, \ldots, \infty$ by Assumption A0. On the other hand, pre-multiplying the linear VAR (7) by γ_0' we obtain

$$\gamma_0' \mathbf{x}_t = \gamma_0' \boldsymbol{\vartheta} + \gamma_0' \sum_{j=1}^{\infty} \mathbf{B}_j \mathbf{x}_{t-j} + \gamma_0' \boldsymbol{\varepsilon}_t. \tag{13}$$

Subtracting (13) from (12) and reordering terms we get

$$\psi - \gamma_0' \boldsymbol{\vartheta} + \sum_{j=1}^{\infty} \boldsymbol{\varphi}_j' \mathbf{x}_{t-j} - \gamma_0' \sum_{j=1}^{\infty} \mathbf{B}_j \mathbf{x}_{t-1} = \gamma_0' \boldsymbol{\varepsilon}_t - \sigma_r u_t^r. \tag{14}$$

Now, the right side of (14) is orthogonal to the left side, because both u_t^r and $\boldsymbol{\varepsilon}_t$ are zero-mean and orthogonal to $\mathbf{x}_{t-j}$, $j = 1, \ldots, \infty$ by Assumptions A0 and A2. Then, the term $\sigma_r u_t^r - \gamma_0' \boldsymbol{\varepsilon}_t$ is orthogonal to itself and therefore is null, implying that

$$\sigma_r u_t^r = \gamma_0' \boldsymbol{\varepsilon}_t. \tag{15}$$

Eq. (15) indicates that, at any given point in time, the monetary shock must be equal to a linear combination of the innovations of the linear VAR. It then remains to be shown that such linear combination can be estimated using the external instrument z_t.[20] Thus, let $\mathbb{P}$ denote the linear projection operator. By projecting both sides of equation (9) onto the entries of $\boldsymbol{\varepsilon}_t$ we get $\mathbb{P}(z_t | \boldsymbol{\varepsilon}_t) = \mathbb{P}(a | \boldsymbol{\varepsilon}_t) + \mathbb{P}(b u_t^r | \boldsymbol{\varepsilon}_t) + \mathbb{P}(\delta(L)' \mathbf{x}_{t-1} | \boldsymbol{\varepsilon}_t) + \mathbb{P}(v_t | \boldsymbol{\varepsilon}_t) = \mathbb{P}(b u_t^r | \boldsymbol{\varepsilon}_t)$, by the orthogonality properties in Assumptions A2 and A2. But $b u_t^r = (b/\sigma_r) \gamma_0' \boldsymbol{\varepsilon}_t$. It then follows that $\mathbb{P}(z_t | \boldsymbol{\varepsilon}_t) = (b/\sigma_r) \gamma_0' \boldsymbol{\varepsilon}_t = b u_t^r$. $\square$

[20] The recursive (Cholesky) identification often used in the literature can be viewed as a special case of our procedure where the vector γ, rather than being estimated using an external instrument, is assumed to satisfy specific restrictions, e.g. $\gamma_i = 1$ and $\gamma_j = 0$ for $j \neq i$ where i denotes the position of the interest rate in the vector $\mathbf{x}_t$.

In summary, if a suitable linear combination of variables delivers the structural shock and a valid instrument is available, the structural shock can be recovered by projecting the instrument onto the VAR residuals. Thus, unlike the standard Proxy-SVAR approach, we employ the proxy to find the shock, rather than the impulse response functions. The shock and its nonlinear function can then be used as regressors to estimate the nonlinear VARX in eq. (6), as detailed in the following subsection.

4.4 Estimation

The above result justifies the following estimation procedure.

I. Estimate the VAR (7) with OLS to obtain consistent estimates of the residual ε_t, say $\hat{\varepsilon}_t$.[21]

II. Estimate the linear regression

$$z_t = \hat{\boldsymbol{\lambda}}' \hat{\boldsymbol{\varepsilon}}_t + \hat{\eta}_t. \tag{16}$$

where $\boldsymbol{\lambda} = (b/\sigma_r)\boldsymbol{\gamma}$, as derived in Proposition 1. An estimate of the normalized shock is obtained by standardizing the fitted value of the above regression, i.e. $\hat{u}_t^r = \hat{\boldsymbol{\lambda}}' \hat{\boldsymbol{\varepsilon}}_t / \mathrm{std}(\hat{\boldsymbol{\lambda}}' \hat{\boldsymbol{\varepsilon}}_t)$.

III. Estimate equation (6) using as regressors the current value and the lags of the the estimated shock $\hat{u}_t^r$ and its nonlinear functions $\mathbf{g}(\hat{u}_t^r, \bullet)$. This gives the estimates $\widehat{\mathbf{A}(L)}$, $\widehat{\tilde{\boldsymbol{\Phi}}(L)}$ and $\widehat{\tilde{\boldsymbol{\alpha}}(L)}$. Finally, according to Assumption A1, one can estimate the IRFs of the linear and the nonlinear terms as $\widehat{\boldsymbol{\alpha}(L)} = \widehat{\mathbf{A}(L)}^{-1} \widehat{\tilde{\boldsymbol{\alpha}}(L)}$ and $\widehat{\boldsymbol{\Phi}(L)} = \widehat{\mathbf{A}(L)}^{-1} \widehat{\tilde{\boldsymbol{\Phi}}(L)}$.

IV. Compute the impulse response functions according to equation (3).

[21]Of course we have to approximate the VAR(∞) with a finite-order VAR. In view of the possible truncation bias and taking into account recent work in favour of rich dynamic specifications (see e.g. De Graeve and Westermark, 2025) we recommend not being too parsimonious when setting the number of lags.

In Appendix A.1 we describe in detail how to build confidence intervals using a bootstrapping procedure, and in Appendix A.2 we assess the validity of our empirical approach on artificial data from the VARX model (6).

One might wonder whether bypassing steps I and II —i.e. preliminary estimation of the shock— and estimating the VARX by using the instrument in place of the shock, is a valid procedure. In the linear case the answer is affirmative, since the measurement error typically associated with the instrument leads to an attenuation bias which is proportional across units and can be corrected by normalization. In the nonlinear case, however, the bias induced by the error is more complicated, and the procedure is not valid apart for special cases. In the Online Appendix we show that not applying steps I and II yields implausible results.

5 Empirical Analysis

In this section we present our main empirical results about the nonlinear transmission of monetary policy shocks and present some robustness checks.

For the nonlinear Proxy-SVAR we use a specification very similar to that used in Miranda-Agrippino and Ricco (2021). The VAR includes five variables, namely the 1-year Treasury bond rate, the growth rate of industrial production, the excess bond premium from Gilchrist and Zakrajšek (2012), the unemployment rate and CPI inflation. All data are for the U.S. economy, at a monthly frequency for the period from 1973:M1 to 2019:M6, and we include 7 lags for each variable.[22] We use 7 lags for the sake of parsimony and because 7 lags are enough to get a monetary shock which is serially uncorrelated according to the Ljung-Box Q-test with 6, 18, and 24 lags.

[22] All the variables were obtained from the Federal Reserve Economic Data (FRED) online database.

The instrument to recover the monetary shock is the one in Degasperi and Ricco (2022), which is an extended version of Miranda-Agrippino and Ricco (2021). Such instrument is relevant according to the standard first-stage F-statistic criterion, as documented in Miranda-Agrippino and Ricco (2021). We acknowledge that our nonlinear context might have special implications on the weak-instrument question. However, this is a far from trivial issue that we leave for further research.

The time span of our baseline instrument is 1991:M1—2015:M12. To get an estimate of the monetary shock for the longer sample 1973:M8—2019:M6, following Forni, Gambetti and Ricco (2023), we first regress the instrument onto the VAR residuals using the short time span, and then apply the estimated coefficients to the residuals of the longer sample.

In order to study the sign- and state- dependence of monetary shocks we set the nonlinear functions $\mathbf{g}(u_t^r) \equiv [|u_t^r| \quad s_{t-1}u_t^r]'$, s_t being an indicator of the state of the economy such that $s_{t-1} = 1$ when the average GDP growth over the previous 12 months is negative, and $s_{t-1} = 0$ otherwise.[23]

Thus, according to eq. (2), the nonlinear effects of monetary shocks are captured by the polynomial matrix $\boldsymbol{\Phi(L)} \equiv [\boldsymbol{\phi}^1(L) \quad \boldsymbol{\phi}^2(L)]$, with the first column $\boldsymbol{\phi}^1(L)$ capturing sign-dependence, and the second column $\boldsymbol{\phi}^2(L)$ capturing state-dependence.

Denoting with $\boldsymbol{\alpha}_h$ and $\boldsymbol{\phi}_h^i$, $i = 1, 2$ the h-th lag in the polynomials $\boldsymbol{\alpha}(L)$ and $\boldsymbol{\phi}^i(L)$, and normalizing the shock size to unity, the impulse responses during booms ($s_{t-1} = 0$) and recessions ($s_{t-1} = 1$), at horizon h, are given for monetary easing ($u_t^r = -1$) by

$$\mathcal{R}_h^+(s_{t-1} = 0) \equiv -\boldsymbol{\alpha}_h + \boldsymbol{\phi}_h^1 \qquad \mathcal{R}_h^+(s_{t-1} = 1) \equiv -\boldsymbol{\alpha}_h + \boldsymbol{\phi}_h^1 - \boldsymbol{\phi}_h^2$$

[23]In the Online Appendix, we also analysed the effects of sign- and state- dependence in isolation. Our results are qualitatively similar to those obtained in Barnichon and Matthes (2018) and Tenreyro and Thwaites (2016), with the main difference being the quantitative response of unemployment and prices to a monetary tightening, which is larger in our case.

Table 1. *The table reports the p-value of the test of the null hypothesis that the parameters of the leads of the residuals in the regression of the shock on current and future values of the residuals are equal to zero.*

VAR Lags	Number of leads				
	1	2	3	4	5
6	0.49	0.47	0.40	0.38	0.28
7	0.53	0.48	0.60	0.50	0.38
8	0.48	0.47	0.55	0.49	0.34
9	0.59	0.50	0.49	0.54	0.46
10	0.51	0.63	0.61	0.69	0.61
11	0.50	0.60	0.60	0.67	0.63
12	0.42	0.57	0.65	0.69	0.68

while for a monetary tightening ($u_t^r = 1$) we have

$$\mathcal{R}_h^-(s_{t-1} = 0) \equiv \boldsymbol{\alpha}_h + \boldsymbol{\phi}_h^1 \qquad \mathcal{R}_h^-(s_{t-1} = 1) \equiv \boldsymbol{\alpha}_h + \boldsymbol{\phi}_h^1 + \boldsymbol{\phi}_h^2$$

Given these impulse responses, the corresponding monetary tradeoffs can be easily calculated according to eq. (1).

5.1 The identified monetary shock

We begin by testing whether Assumption A4 holds in our data, using the invertibility test proposed by Forni, Gambetti and Ricco (2023) described in Section 4.2. The results, reported in Table 1, shows that invertibility is never rejected at the 5% level across all combinations of VAR lags and residuals leads. Thus, the evidence suggests that Assumption A4 is satisfied.

We next obtain an estimate of monetary shocks regressing the external instrument on the VAR residuals, as described in Section 4.3. Figure 1 reports the resulting series, smoothed by using a moving average of 12 months. Over the sample period, monetary easing and tightening shocks show no systematic correlation with the business cycle. For instance, the largest easing

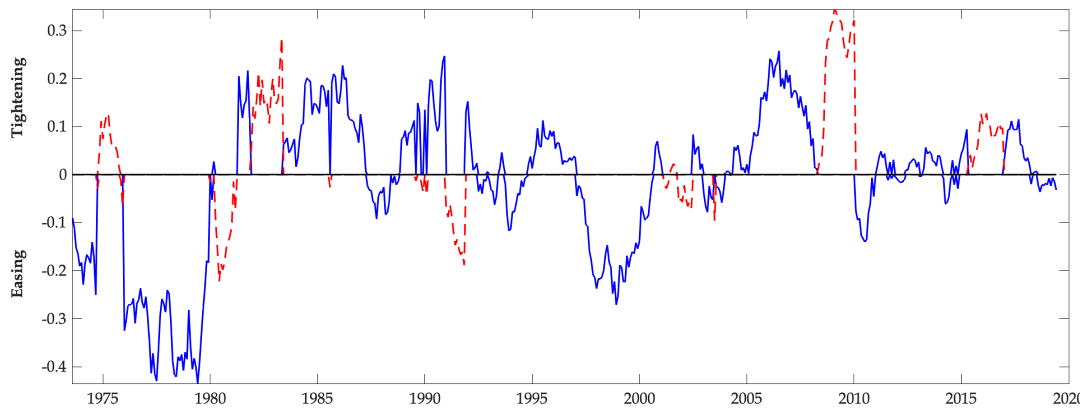

Figure 1. *Time-series of identified monetary shocks, smoothed using a moving average of 12 months, for monetary easing (negative values), tightening (positive values), during booms (blue solid line) and recessions (red dashed line).*

episode (a negative shock) is identified during 1975-1979, a period characterized by high GDP growth and elevated inflation. Also, monetary policy appears contractionary (positive shock) in the 1982 recession under the Volcker mandate, and the largest tightening occurred during the 2008-09 recession. The latter result aligns with the view that US Federal Reserve deviated from its conventional policy rule, due to a binding zero lower bound constraint on the short-term interest rate.[24] In contrast, during the Great Moderation period, the identified monetary shocks are smaller, and predominantly countercyclical, with the period 1998-99 standing out as the main exception.

5.2 The impulse-response functions

As a preliminary step, we estimate a *linear* Proxy-SVAR, thus ignoring the presence of nonlinear terms.

[24]When the zero lower bound is binding, the interest rate cannot be lowered in response to the falling inflation and output. As a result, the discretionary component (shock) must increase in order to keep the interest rate at zero.

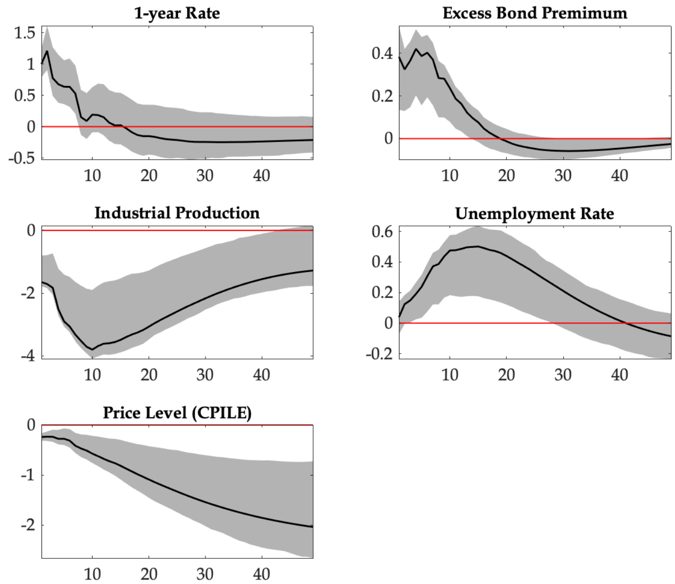

Figure 2. *Impulse responses to a monetary shocks in the linear model. Solid lines represent point estimates, the grey areas are 68% confidence bands. The shock size is normalized so that the response on impact of the 1-year rate equals 1%.*

The results are summarized in Figure 2, which plots the point estimates of the impulse response functions (solid lines) together with their 68% confidence bands (grey area), and where the shock size is normalized so that the response of the nominal interest rate (top left panel) equals 1% on impact.

Consistent with conventional results in the literature, we find that an increase in the interest rate is associated with a significant decline in inflation and industrial production, and a significant increase in the unemployment rate and in the excess bond premium.

Figure 3 plots the impulse responses for the *nonlinear* model, distinguishing between the effects of the linear component $\boldsymbol{\alpha}(L)$ (first column), the absolute value term $\boldsymbol{\phi}^1(L)$ (second column), and the state-dependent term $\boldsymbol{\phi}^2(L)$ (third column). An important caveat is the width

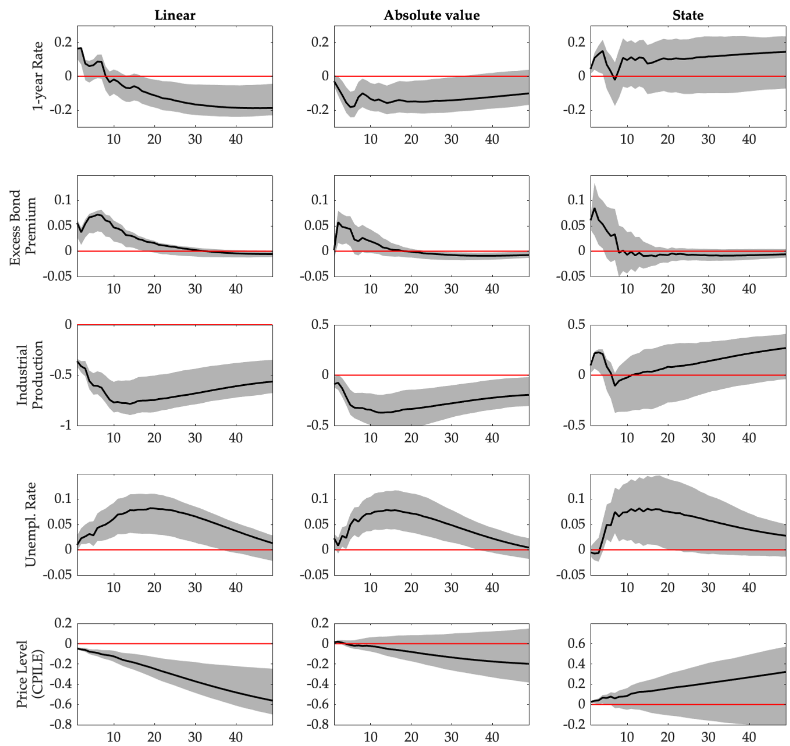

Figure 3. *Impulse response to a monetary shock of the linear term (first column), the absolute value term (second column) and the state-dependent term (third column). Solid lines represent point estimates, the grey areas are 68% confidence bands, constructed with the wild bootstrap procedure described in Appendix A.1.*

of the confidence bands, signalling the presence of substantial uncertainty around the point estimates. Nonetheless, several interesting patterns emerge from the analysis. The responses of the linear component closely resemble those displayed in Figure 2 for the linear model, up to a scaling factor.[25] More notably, for all variables the magnitude of the responses of the nonlinear components are similar to those of the linear counterparts and statistically significant in most

[25] Differently from Figure 2, in Figure 3 the impulse responses are not normalized. This choice facilitates the comparison of the impulse responses of three components (linear, absolute value and state-dependent component), as normalization would require applying a different scaling factor to each component.

cases. Formally, we conduct a Likelihood Ratio test for the null hypothesis $\tilde{\mathbf{\Phi}}(L) = 0$ (see Section 4.1), which is rejected at the 1% significance level.[26]

These results suggest that nonlinear effects play a central role for the propagation of monetary shocks. In particular, both the sign (second column) and the state (third column) components have persistent and significant positive effects on unemployment (fourth row), implying that monetary policy leads to larger changes in unemployment if a tightening is implemented during a recession. Instead, for the case of prices (last column) the sign and state components operate in opposite directions, implying that the largest changes in inflation are associated with monetary tightening during an expansion.

This can be seen more clearly in Figure 4, which compares the total effects of monetary easing and tightening during booms (first two columns) and recessions (last two columns), and where in each case the impulse response is normalized so that the response on impact of the 1-year rate equals 1% (in absolute value).

While the estimates remain subject to substantial uncertainty —particularly for easing episodes in recessions— a few interesting results emerge. For real variables such as unemployment and industrial production, a monetary tightening generates large and significant effects, whereas the effects of a monetary easing are more modest. In this respect, our result aligns with previous findings by Tenreyro and Thwaites (2016) and Barnichon and Matthes (2018), and may suggest that monetary policy is not very effective as a stimulus tool. Yet, a muted response of economic activity, provided it is statistically significant, does not it itself undermine the effectiveness of monetary policy. It rather implies that stronger policy interventions may be needed to counteract recessions. Whether such an aggressive stance is desirable ultimately depends on the

[26]When performing the test we did not consider the equation of the interest rate to avoid singularity of the residuals (see footnote 16).

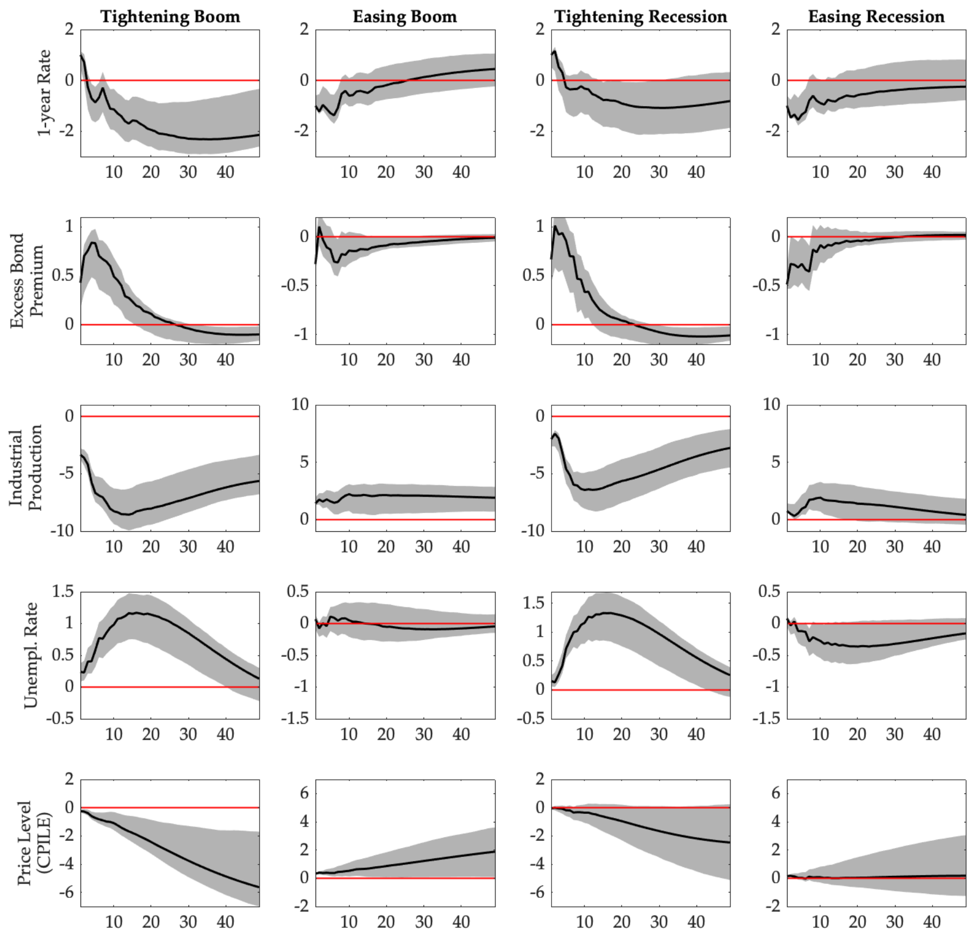

Figure 4. *Impulse response to a monetary shock during booms (first two columns) and recessions (last two columns), for monetary tightening (first and third column) and easing (second and fourth column). Solid lines represent point estimates, the grey areas are 68% confidence bands, constructed with the wild bootstrap procedure described in Appendix A.1. In each case, the shock size is normalized so the response on impact of the 1-year rate equals 1% (in absolute value).*

inflation-unemployment tradeoff facing the central bank: stronger measures are likely to entail higher inflationary costs.

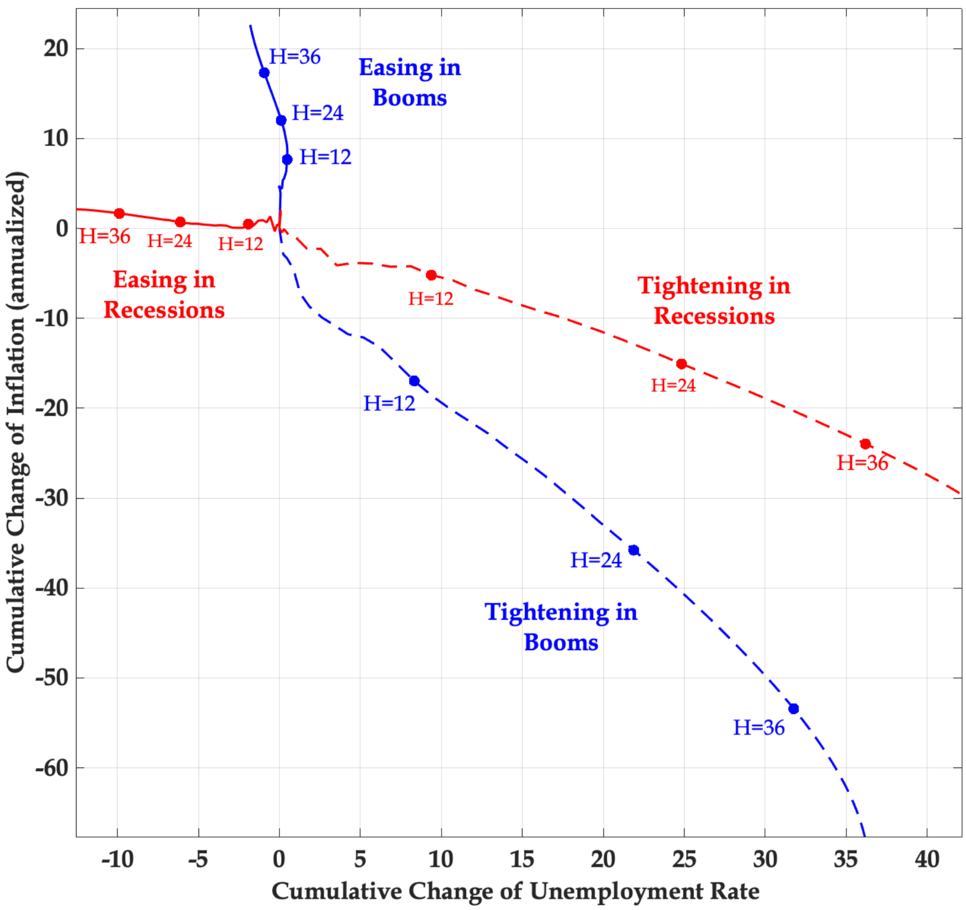

Figure 5. *Monetary Tradeoffs: the figure plots the relationship between the cumulative change in the unemployment rate (x-axis) and in the inflation rate (y-axis) at different horizons (*$H = 12, 24, 36$*), in response to monetary easing (solid lines) and tightening (dashed lines), during booms (blue lines) and recessions (red lines).*

To get a sense of the magnitude of the inflation-unemployment tradeoff, Figure 5 displays a scatterplot of the impulse responses of unemployment (horizontal axis) and inflation (vertical axis), and where each point represents the cumulative effects of monetary shocks over different horizons (e.g., $H = 12, 24, 36$ months). The figures delivers the following insights. First, when the

Table 2. *Monetary Tradeoffs: the table reports the size of the monetary tradeoff at different horizons ($H = 12, 24, 36, 48$ months) associated with monetary easing during recessions —i.e. the inflation cost of reducing unemployment ($\mathcal{T}_H^+(s_{t-1} = 1)$— and monetary tightening during a boom —i.e. the unemployment cost of reducing inflation ($\mathcal{T}_H^+(s_{t-1} = 1)$). Column 2 and 3 refers to the full-sample (1973:M1-2019:M6), while the last two columns refer to the pre-2009 sample. 68% confidence bands are reported in brackets. The bands are constructed with the wild bootstrap procedure described in Appendix A.1.*

	Full-Sample (1973:M1-2019:M6)		Pre-2009	
Horizon	Easing in Recessions $\mathcal{T}_H^+(s_{t-1} = 1)$	Tightening in Booms $\mathcal{T}_H^-(s_{t-1} = 0)$	Easing in Recessions $\mathcal{T}_H^+(s_{t-1} = 1)$	Tightening in Booms $\mathcal{T}_H^-(s_{t-1} = 0)$
$H = 12$	-0.03	-0.51	0.30	-0.63
	(-5.31, 2.81)	(-1.10, -0.26)	(-4.88, 3.77)	(-1.52, 0.32)
$H = 24$	-0.12	-0.61	-0.36	-0.70
	(-4.04, 1.98)	(-1.33, -0.34)	(-3.07, 2.03)	(-1.50, 0.23)
$H = 36$	-0.17	-0.59	-0.59	-0.63
	(-3.64, 2.15)	(-1.30, -0.30)	(-3.31, 1.70)	(-1.31, 0.32)
$H = 48$	-0.17	-0.53	-0.70	-0.54
	(-3.40, 2.68)	(-1.23, -0.24)	(-3.75, 1.99)	(-1.15, 0.61)

central bank eases monetary policy during a recession (red solid line), unemployment falls significantly and persistently, while inflation remains largely unchanged —i.e. a nearly flat Phillips curve. Thus, our results suggest that even though the real effects of monetary shocks are muted during a recession, it is still possible to stimulate output through large policy interventions, with modest costs in terms of inflation.

Second, a monetary tightening during expansions (blue dashed line) has sizable costs in terms of unemployment. Those costs, however, are substantially smaller than the costs associated with contractionary policies during recessions (red dashed) or those implied by a flat Phillips curve (which would be infinite). Third, easing during an expansion (blue solid) line is extremely inflationary with virtually no effects on the unemployment rate —i.e. an extremely large inflation-unemployment tradeoff.

Table 2 reports the estimated values of the monetary tradeoffs, together with the corresponding 68% confidence intervals.[27] The inflation cost of reducing unemployment during a recession (second column) is small, ranging between 0.03 and 0.17 (in absolute value) depending on the horizon considered, and generally insignificant. In a linear model, the unemployment cost of reducing inflation would be given by $\mathcal{T}^- = 1/\mathcal{T}^+$. Thus, our estimates for $\mathcal{T}^+$ would imply that for each percentage point reduction in inflation, the unemployment rate should fall between 6 and 33 percentage points —i.e. the inverse of 0.17 and 0.03, respectively. Instead, we find that during a boom (third column) the unemployment cost is an order of magnitude smaller, with estimates ranging between 0.5 and 0.6 percentage points.

Similar results are obtained if we consider the pre-2009 sample (last two columns), to exclude the period when the zero-lower bound was binding. The monetary tradeoff is instead much bigger in other situations —e.g. a tightening during recessions— as could be seen in Figure 5.[28] All in all, these results suggest that both monetary easing during recessions and tightening during expansions are associated with relatively favourable inflation-unemployment tradeoffs.

5.3 Robustness checks

This subsection present three robustness checks. First, we replace our baseline state dummy —based on the industrial production index— with a recession dummy based on NBER recession dates. Second, we use the Federal Funds rate and the 6-month Treasury Bill rate in place of the 1-year government bond rate. Third, we use the instrument of Jarociński and Karadi (2020), both as it is and cleaned up, regressing it on six lags of the VAR variables.

[27] A caveat: the proposed bootstrap procedure can in principle fail to produce meaningful confidence bands for the tradeoffs, if the denominator is too close to zero.

[28] The corresponding values are not reported in Table 2, since in many instances calculating the tradeoff would require dividing by values close to zero, giving rise to uninformative results.

The corresponding results are summarized in Figures 6, 7 and 8, where for ease of comparison we also include the impulse response from the baseline specification shown in Figure 4. As before, monetary tightening during booms leads to larger real effects than easing during recessions. However, because price responses are also more muted during recessions, the central bank still appears to face a relatively favourable tradeoff. Overall, this suggest that our main findings are reasonably robust to these alternative specifications.

The Online Appendix also reports results obtained using the unemployment rate in place of the industrial production index as the business cycle state indicator and adding variables to the baseline specification. Results are qualitatively similar to the benchmark case.

6 A model with downward nominal wage rigidities

This section illustrates a simple theoretical model with downward nominal wage rigidities that gives rise to a sign- and state-dependent monetary tradeoff.

This model is used for two main purposes: first, to provide a simple framework to interpret the evidence discussed in the previous section; second, to assess, by means of a Monte Carlo simulation, whether our empirical approach is able to capture the nonlinearities featured by the theoretical model.[29]

[29]For this reason, the model purposefully abstracts from considering the presence of endogenous propagation mechanisms (e.g., capital accumulation, adjustment costs), alternative sources of nonlinearities and state-dependence (e.g., occasionally binding financial constraints), and sources of booms and recessions (demand vs supply), or other factors that would be needed to fully account for the impulse responses obtained in the previous sections, which is beyond the scope of this paper.

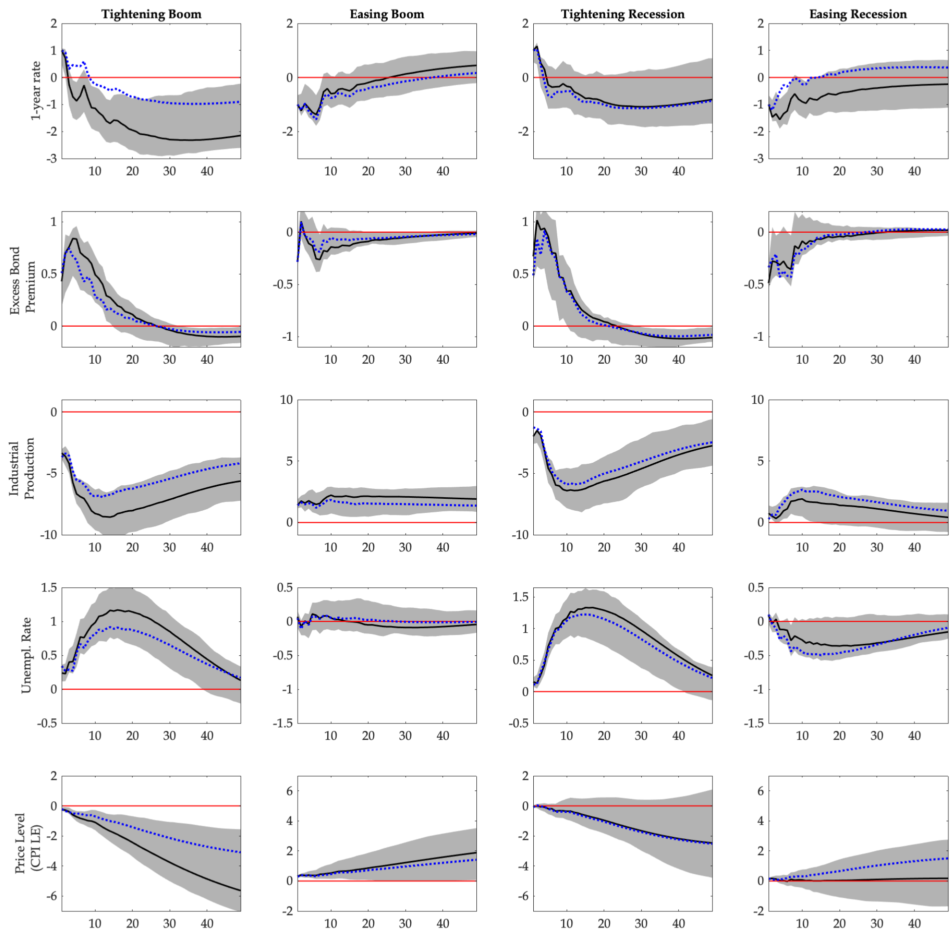

Figure 6. *NBER recession dates: impulse responses to a monetary shock during booms (first two columns) and recessions (last two columns), for monetary tightening (first and third column) and easing (second and fourth column). Solid lines represent the benchmark point estimates, the grey areas are 68% confidence bands for the benchmark; the blue dotted line is the point estimate obtained with the NBER recession dates in place of our state dummy.*

6.1 Preferences, Technology and Monetary Policy

The economy is populated by a large number of identical households with preferences described by the objective function $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$, where C_t denotes consumption and $\beta \in (0,1)$ is the

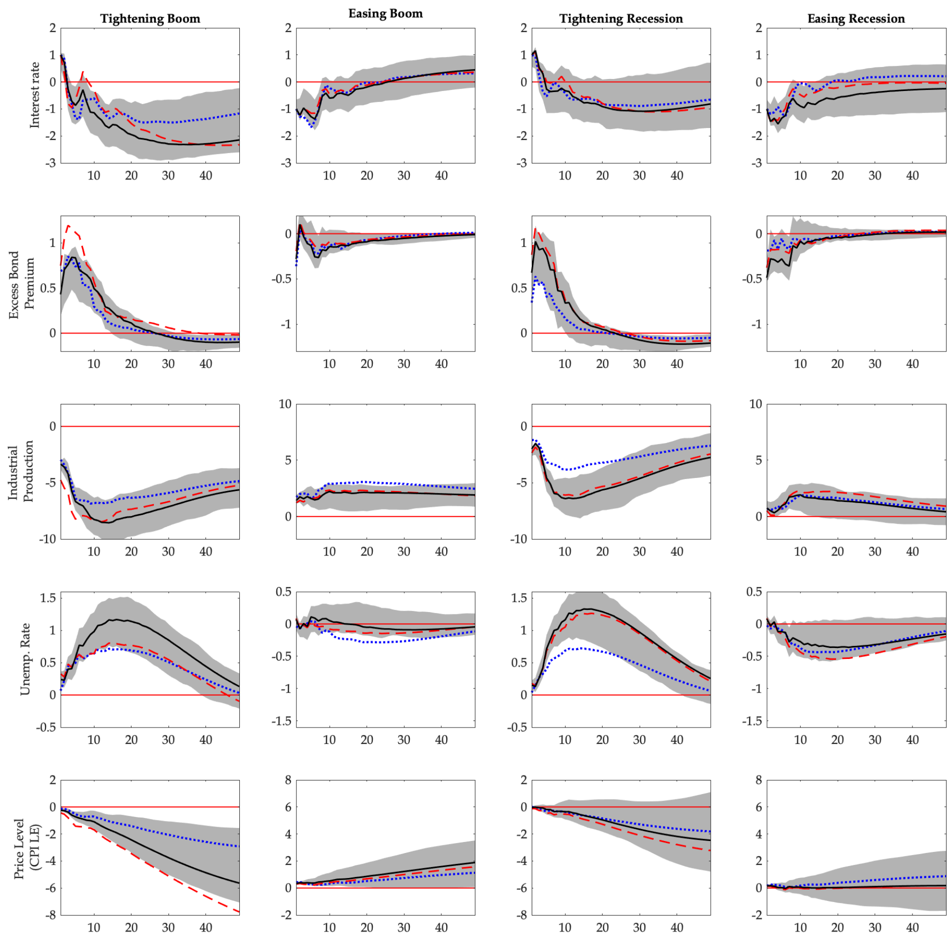

Figure 7. *Alternative interest rate measures: impulse responses to a monetary shock during booms (first two columns) and recessions (last two columns), for monetary tightening (first and third column) and easing (second and fourth column). Solid lines represent the benchmark point estimates, the grey areas are 68% confidence bands for the benchmark; the blue dotted line is the point estimate obtained with the Federal Funds rate; the red dashed line is the point estimate obtained with the 6-month T-Bill rate.*

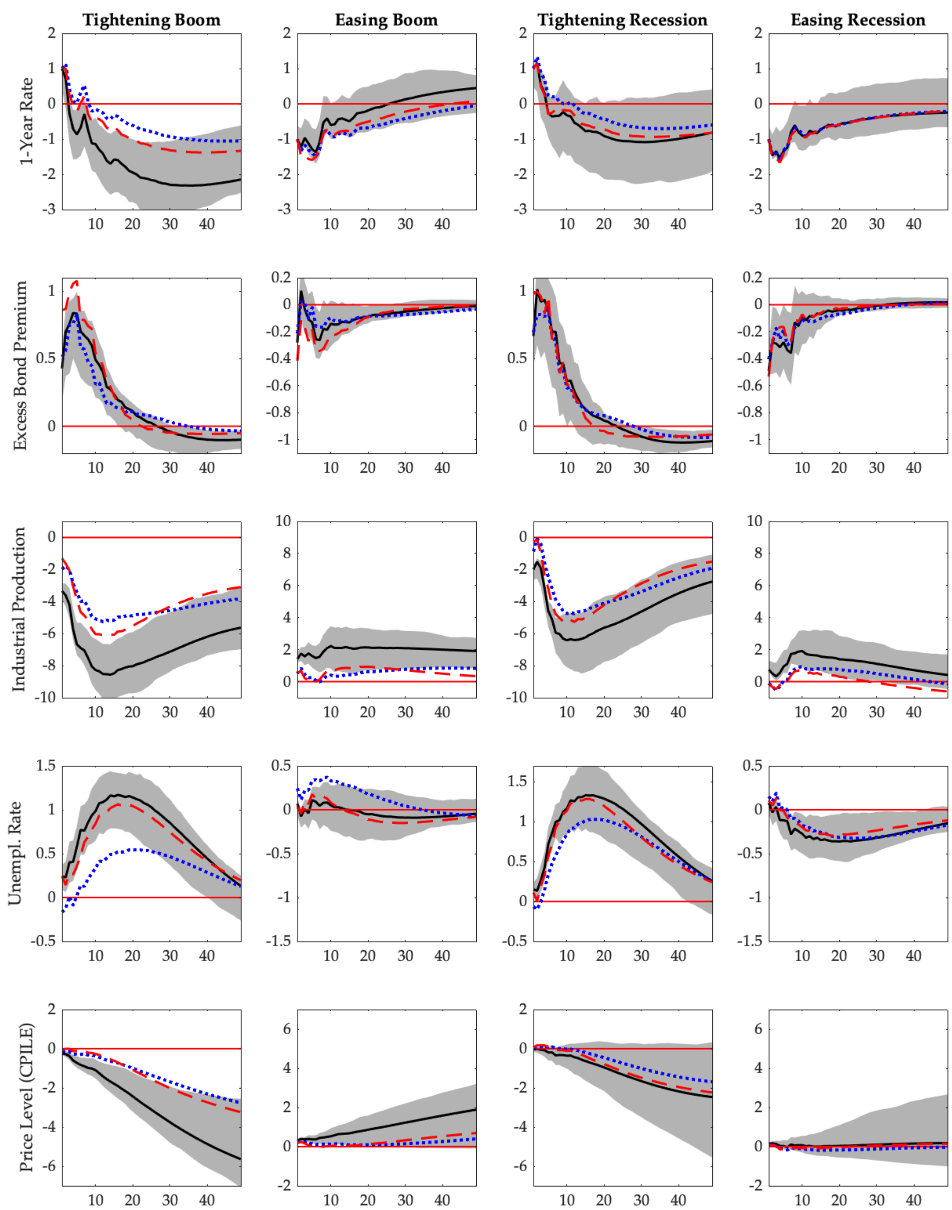

Figure 8. *Alternative instrument: impulse responses to a monetary shock during booms (first two columns) and recessions (last two columns), for monetary tightening (first and third column) and easing (second and fourth column). Solid lines represent the benchmark point estimates, the grey areas are 68% confidence bands for the benchmark; the blue dotted line is the point estimate obtained with the instrument used in Jarociński and Karadi (2020); the red dashed line is the point estimate obtained with the same instrument, cleaned by regressing onto six lags of the VAR variables.*

subjective discount factor. The household budget constraint is given by

$$P_t C_t + B_t = W_t N_t + r_{t-1} B_{t-1}, \tag{17}$$

where P_t denotes the price level, B_t denote nominal one-period riskless bonds, and r_t is the gross nominal interest rate between period t and $t+1$.

Each household supplies inelastically one unit of labour $\bar{N} = 1$. However, the labour market features downward nominal wage rigidities, so that $W_t \geq \theta W_{t-1}$, where $\phi \leq 1$ is a parameter measuring the severity of the rigidity. Whenever the latter constraint is binding, only a fraction $N_t \leq \bar{N} = 1$ of households is employed, and the remaining $1 - N_t$ households remain unemployed. In other words, the presence of downward nominal wage rigidities may give rise to "involuntary" unemployment.

Output of the single good (Y_t) is produced by perfectly competitive firms using labour as the only input according to the linear technology $Y_t = \exp\{a_t\} N_t$, where a_t denotes total factor productivity, which is assumed to follow the exogenous random-walk process $a_t = a_{t-1} + u_t^a$, with $u_t^a \sim N\left(-\sigma_a^2/2, \sigma_a^2\right)$. Firms' profit maximization implies that real wages $W_t/P_t = \exp\{a_t\}$ in every period. Also, it follows that the "natural" level of output (i.e. the level of output prevailing when the economy operates at full employment) is given by $Y_t^n \equiv \exp\{a_t\}$.

Monetary policy is conducted according to a Taylor-type interest rate rule

$$r_t = \bar{R} \Pi_t^{\phi_\pi} \exp\{m_t\} \tag{18}$$

where $\bar{R}$ is the steady-state interest rate, $\phi_\pi > 1$ is a parameter measuring the central bank's response to inflation, and m_t is a monetary policy shocks, following the AR(1) process $m_t = \rho_m m_{t-1} + u_t^r$, where $u_t^r \sim N\left(-\sigma_r^2/2, \sigma_r^2\right)$.

6.2 Equilibrium

The competitive equilibrium of this economy is fully characterized by the following two equations, summarizing the relationship between output and inflation:

$$1 = \Pi_t^{\phi_\pi} \exp\{m_t\} \mathbb{E}_t \left\{ \left(Y_{t+1}/Y_t\right)^{-\sigma} \Pi_{t+1}^{-1} \right\} \tag{19}$$

$$\left(Y_t / \exp\{a_t\} - 1\right) \left(\exp\{u_t^a\} - \phi \Pi_t^{-1}\right) = 0 \tag{20}$$

Equation (19) is an aggregate demand (AD) relationship, and is obtained combining the consumption Euler equation from the household's optimal consumption/savings decision with the monetary policy rule (18) and the market clearing condition $Y_t = C_t$. Equation (20) describes instead an aggregate supply (AS) relationship, and is obtained combining the production function, the household's labour supply subject to the downward nominal wage rigidity, and the firms' labour demand implying that the real wage $W_t/P_t = \exp\{a_t\}$.

Figure 9 provides a graphical illustration of the main mechanism of the model. It plots the aggregate demand (AD) and aggregate supply (AS) curves, for a given level of expected output and inflation. Note that the presence of downward wage rigidities introduce a "kink" in the aggregate supply relationship, and for this reason the real effects of monetary policy shocks are asymmetric. Suppose for instance that the economy is initially in a situation where technology is at its steady-state level, the economy is at full-employment, i.e. $Y_t^n = \exp\{a_t\} = 1$, and (gross) inflation $\Pi = 1$, so that the downward wage rigidity is not binding (point A in the graph). Starting from that situation, an expansionary monetary shock stimulates aggregate demand (i.e. the AD shifts to the right, to point B) putting upward pressures on nominal wages and prices, meaning that the downward wage rigidity is not binding (i.e. the economy lies in the vertical portion of the AS curve). Thus, the only effect of the monetary shock is an increase in inflation, with no effect on output. On the contrary, a contractionary monetary shock that reduces aggregate demand (the AD shifts to the left) makes the downward wage rigidity binding (i.e. the economy moves to the horizontal part of the AS curve), which implies a reduction in output, with no effect on inflation (point C).

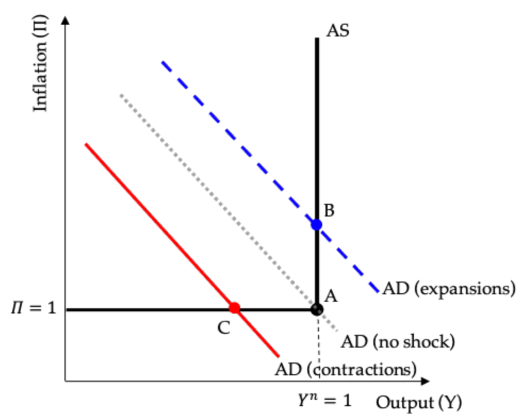

Figure 9. *A Simple Model with Downward Nominal Wage Rigidities: the figure shows the Aggregate Supply (AS) and Aggregate Demand (AD) curves. Point (A) denotes the steady-state equilibrium (no monetary shock). Point (B) denotes the equilibrium with an expansionary monetary shock, and point (C) is the equilibrium with a contractionary shock.*

More generally, within this model the effects of monetary easing and tightening depends on whether the economy is or not at full-employment. Thus, conditional on economic activity remaining below full-employment, the effects of monetary policies would be completely symmetric, as the economy moves along a flat portion of the supply curve, with no effect on prices. Yet, when looking at the *average* effects of monetary shocks across periods with full-employment and periods with "involuntary" unemployment, a monetary tightening has larger effects on output and weaker effects on prices than monetary easing. This is because, other things equal, in response to a monetary tightening the economy remains below full-employment for a longer period of time than in response to a monetary easing.[30]

6.3 Quantitative results

In order to provide a quantitative illustration of the described asymmetries, we adopt a quarterly calibration of the model, where the discount factor $\beta = 0.99$, the intertemporal elasticity of substitution $\sigma = 1$, the downward wage rigidity parameter $\theta = 1$, the monetary policy coefficient $\phi_\pi = 1.5$. Regarding the two shock processes, in line with existing empirical estimates (see e.g. Smets and Wouters, 2007), we set the autocorrelation of the monetary shock $\rho_m = 0.5$ and the standard deviations $\sigma_r = 0.25$ percent, while the standard deviation of (permanent) innovations to technology is $\sigma_a = 0.45$. The model is solved and simulated using a (nonlinear) global projection method, where the expectation term in the aggregate demand (19) is approximated with a Chebyshev polynomial on a coarse grid for the monetary policy shock (see Appendix A.3 for more details).

We then perform a Monte Carlo simulation using the model discussed above to validate our empirical procedure. This exercise is particularly important since the empirical specification (6) is possibly misspecified when the data are generated by a nonlinear model, but still could

[30]For this reason, monetary tightening has larger real effects also when controlling for the state of the economy before the monetary shock hits s_{t-1}, as we did in the empirical exercise.

represent a good approximation of the nonlinear dynamics embedded in the DSGE. In particular, we generate 1000 realizations of the technology shock and the monetary policy shock from the model, and calculate the implied series for output, inflation and the interest rate. For every realization of the monetary policy shock, we also construct an instrument which is equal to the shock plus an independent measurement error with standard deviation equal to 0.025 (the same standard deviation of the monetary policy shock). We then apply our econometric procedure using output, prices and the interest rate. First, we estimate the monetary policy shock (the VAR used to estimate the residuals has two lags and includes the three variables output, inflation and the interest rate). Second, we estimate the nonlinear impulse response functions from equation (6) (the VARX is estimated with two lags for both endogenous and exogenous variables). The exogenous variables in the VARX are: the estimated shock itself, its absolute value, and the interaction between the shock and a dummy taking value one if the technology shock in the previous time period was negative and one if positive. Then, we compute the average impulse response functions across the 1000 realizations.

Figure 10 displays the average impulse responses to a monetary shock, where averages are taken across different histories of technology shocks. The first column displays the theoretical impulse responses from the model, and shows that the presence of downward nominal wage rigidities can rationalize (at least qualitatively) the sign- and state-dependence of monetary trade-offs found in the data. For instance, monetary easing during a recession (red solid line) leads on impact to a 0.3 percent increase in output, at a cost of only a 0.1 percentage point increase on inflation. Instead, an equal size monetary tightening during a boom (blue dotted line) reduces inflation by roughly 0.4 percentage points, with essentially no cost in terms of output.

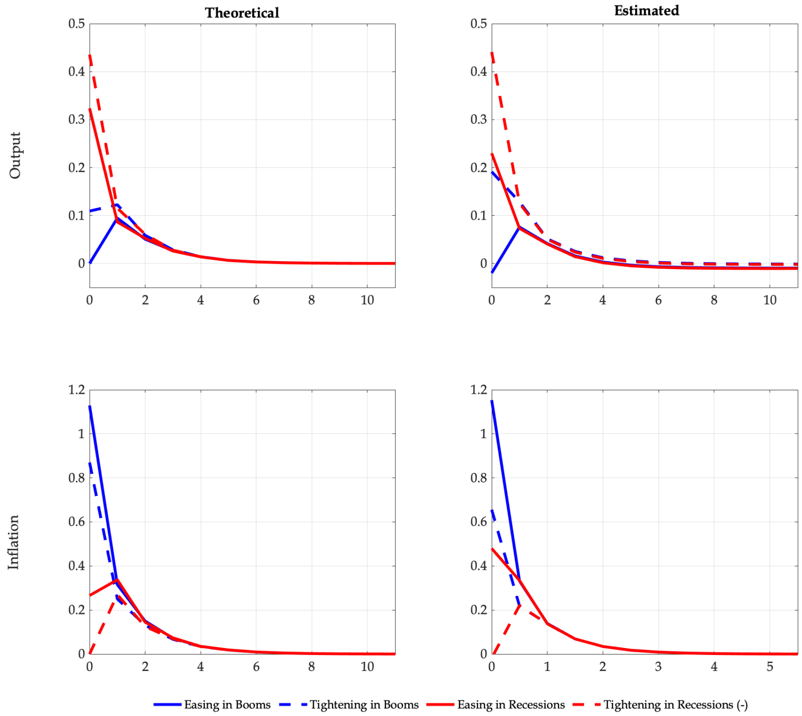

Figure 10. *Monte Carlo exercise: the figure show the average impulse responses of output (first row) and annualized inflation (second row) to a monetary shock. The first column reports the average generalized impulse responses from the theoretical model, calculated as the difference between the path of a variable in the presence of a monetary shock, and the corresponding path without a monetary shock, and where averages are taken across 1000 histories of technology shocks. The second column correspond to the estimated impulse responses using the empirical approach described in Section 4, and where averages are taken with respect to 1000 histories of technology and monetary shocks. To facilitate the comparison, the responses to a monetary tightening are multiplied by minus 1.*

The second column of Figure 10 displays the impulse responses obtained by applying our econometric procedure on the artificial data generated by the model. Such responses are very similar to their theoretical counterparts. This result suggests that the empirical nonlinear representation (6) together with the Proxy-SVAR identification works remarkably well in approximating the nonlinearities arising from the theoretical model. We believe this is an important result since it sheds some light on the linkages between DSGE models and empirical models with relevant nonlinearities. This is a relatively unexplored issue in the literature which we plan to study further in future research.

7 Conclusions

We propose a novel empirical approach to show that, for the US economy, the inflation-unemployment tradeoff varies substantially depending on the sign of the intervention (easing and tightening) and the state of the economy (booms and recessions). In particular, we find small (or no) inflation costs of reducing unemployment during recessions, and moderate unemployment costs of reducing inflation during booms, while the tradeoff is much larger in other cases. We also show that the empirical findings can be rationalized by a simple model with downward nominal wage rigidities.

Taken together, our findings suggest that monetary policy could be an effective instrument for controlling inflation while supporting full employment, both during booms and recessions. This conclusion is subject to two main caveats. First, the inflation-unemployment tradeoff appears to vary significantly with the state of the economy. As such, monetary interventions could entail considerable risk in situations of high uncertainty regarding the underlying economic conditions —e.g. a disinflationary policy could be very costly if output growth turns out weaker than anticipated. Second, our estimates reflect the effects of *average* monetary interventions during the sample period. Clearly, the tradeoff may vary if the central bank adopts unusual policies, in terms of magnitude, persistence, or instrument (conventional vs unconventional tools), and could be influenced by the accompanying fiscal policy measures. In this respect, our empirical approach may constitute a useful tool to study additional sources of nonlinearities, and explore the link between nonlinear theoretical models and empirical evidence.

Affiliations

[1]ICREA-UPF, CREi, BSE and CEPR

[2]Università di Modena e Reggio Emilia, CEPR and RECent

[3]UAB, BSE, Università di Torino and Collegio Carlo Alberto

[4]Università Bocconi, IGIER and Baffi Carefin

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Appendix

A.1 Inference

To draw the confidence bands we use a wild bootstrap procedure that takes into account the fact that we have generated regressors and conditional heteroscedasticity. The bootstrap works as follows:

1. Let S_t be a random sequence assuming the values -1 and 1 with equal probability. Construct the artificial sequences $\mathbf{u}_t^1 = S_t \hat{\mathbf{u}}_t$, $u_t^{r,1} = S_t \hat{u}_t^r$, $\mathbf{g}_t^1 = \hat{\mathbf{g}}(u_t^{r,1})$, $\mathbf{e}_t^1 = S_t \hat{\mathbf{e}}_t$, for $t = 1, \ldots, T$.

2. With the sequences obtained in step 1 compute an artificial dataset $\mathbf{x}_t^1$, $t = 1, ..., T$, using the VARX equation (6) with the initial conditions, $\mathbf{x}_t$, $t = 1, ..., p$, and possibly, if $q > 0$, $u_t^{1,r}$ and $\mathbf{g}_t^1$, $t = p - q + 1, \ldots, p$.

3. Using $\mathbf{x}_t^1$, $u_t^{r,1}$ and $\hat{v}_t^1 = S_t \hat{v}_t$, generate z_t^1 according to the equation of the instrument (9), for the time span of z_t.[31]

4. With the new dataset, repeat the estimation procedure. In particular:

   (a) Estimate the VAR of equation (7) and get the new residuals.

   (b) Estimate the new shock from equation (16), by regressing the bootstrapped instrument obtained in Step 3 onto the residuals obtained in Step 4(a).

   (c) Use the dataset obtained in Step 2, the shock estimated in Step 4(b) and its nonlinear function to estimate equation (6) and the impulse response functions.

[31] If the $\hat{\delta}_j$'s are not significant, as is the case for our application, $\mathbf{x}_t^1$ is not needed.

5. Repeat steps 1-4 $J-1$ times to obtain $J-1$ datasets $\mathbf{x}_t^j$, $j = 2, ..., J$ and the related impulse response functions. Compute the confidence band as usual, by taking appropriate pointwise percentiles.

A.2 Simulations

In this Appendix we run a simulation to assess the validity of the empirical procedure when the data generating process is given by eq. (6). To keep things tractable we consider the following simplified version of the model:

$$\mathbf{x}_t = \mathbf{A}_1\mathbf{x}_{t-1} + \boldsymbol{\mu} + \tilde{\boldsymbol{\alpha}} u_t^r + \tilde{\boldsymbol{\Phi}} g(u_t^r) + \mathbf{e}_t. \tag{A.1}$$

We set

$$\mathbf{A}_1 = \begin{pmatrix} 0.2 & 0.4 & 0.2 \\ 0.3 & 0.7 & -0.1 \\ 0.3 & -0.2 & 0.6 \end{pmatrix}.$$

We fix the matrix coefficients (rather than randomly generating them) to ensure stability. We set $m = n - 1$ so that $\mathbf{e}_t = \boldsymbol{\Gamma}_0\mathbf{u}_t$ where $\boldsymbol{\Gamma}_0$ is 3×2 matrix whose coefficients are randomly generated from a uniform distribution in $[-1, 1]$ and $\mathbf{u}_t$ is a 2×1 vector with distribution $N(0, I)$. The monetary policy shock is also generated from a standardized Normal, $u_t^r \sim N(0, 1)$. We consider a scalar nonlinear function $g(u_t^r) = |u_t^1|$. In this case $\tilde{\boldsymbol{\Phi}} \equiv [\tilde{\boldsymbol{\phi}}^1]$ is just a column vector. The elements of $\tilde{\boldsymbol{\alpha}}$ and the elements two and three of $\tilde{\boldsymbol{\phi}}^1$. i.e. $\tilde{\boldsymbol{\phi}}_2^1$ and $\tilde{\boldsymbol{\phi}}_3^1$, are also uniform in $[-1, 1]$. The element on the other hand $\tilde{\boldsymbol{\phi}}_1^1$ is obtained by imposing that the first entry of $\boldsymbol{\phi}^1$ is zero, i.e $\boldsymbol{\phi}_1^1 = 0$, so that assumption A4 is satisfied. We generate 1000 dataset of length $T = 300$

observations as the length of the instrument in the empirical application. For each dataset the econometrican observes $z_t = u_t^r + v_t$ where $v_t \sim N(0,1)$.

In a second simulation we use the same setting but assuming an asymmetric distribution for u_t^r. Indeed we set $u_t^r = u_t^1$ and $u_t^1 \sim \chi_2^2$.

Panel (a) of Figure A.1 plots the results of the first simulation. The black lines are the average across the point estimates, the grey areas are the 68% and 95% bands. The blue dotted lines are the true impulse response functions. The black and blue line are essentially identical suggesting that the approach succeeds at estimating the true responses. Panel (b) plots the results of the second simulation. As before the average response and the true responses are almost identical confirming the validity of the procedure even the distribution of the policy shock is not normal. In the last section of the paper we preform a simulation using a DSGE model.

A.3 Solution of the Theoretical Model

Solving the theoretical model of Section 6 amounts to solve the following system of equations

$$1 = \beta \bar{R} \Pi_t^{\phi_\pi} \tilde{Y}_t^{\sigma} \exp\{m_t\} \mathbb{E}_t \left\{ \exp\{-\sigma u_{a,t+1}\} \tilde{Y}_{t+1}^{-\sigma} \Pi_{t+1}^{-1} \right\}$$

$$\left(\tilde{Y}_t - 1\right) \left(\exp\{u_t^a\} - \Pi_t^{-1} \right) = 0$$

where $\tilde{Y}_t \equiv Y_t / \exp\{a_t\}$ is detrended output.

To solve the model, we approximate the expectation term on the RHS of the aggregate demand through a Chebyshev polynomial on a coarse grid for the monetary shocks, i.e. we approximate the function

$$X(m_t) \equiv \mathbb{E}_t \left\{ \exp\{-\sigma u_{t+1}^a\} \left(\tilde{Y}_{t+1}\right)^{-\sigma} \Pi_{t+1}^{-1} \right\}.$$

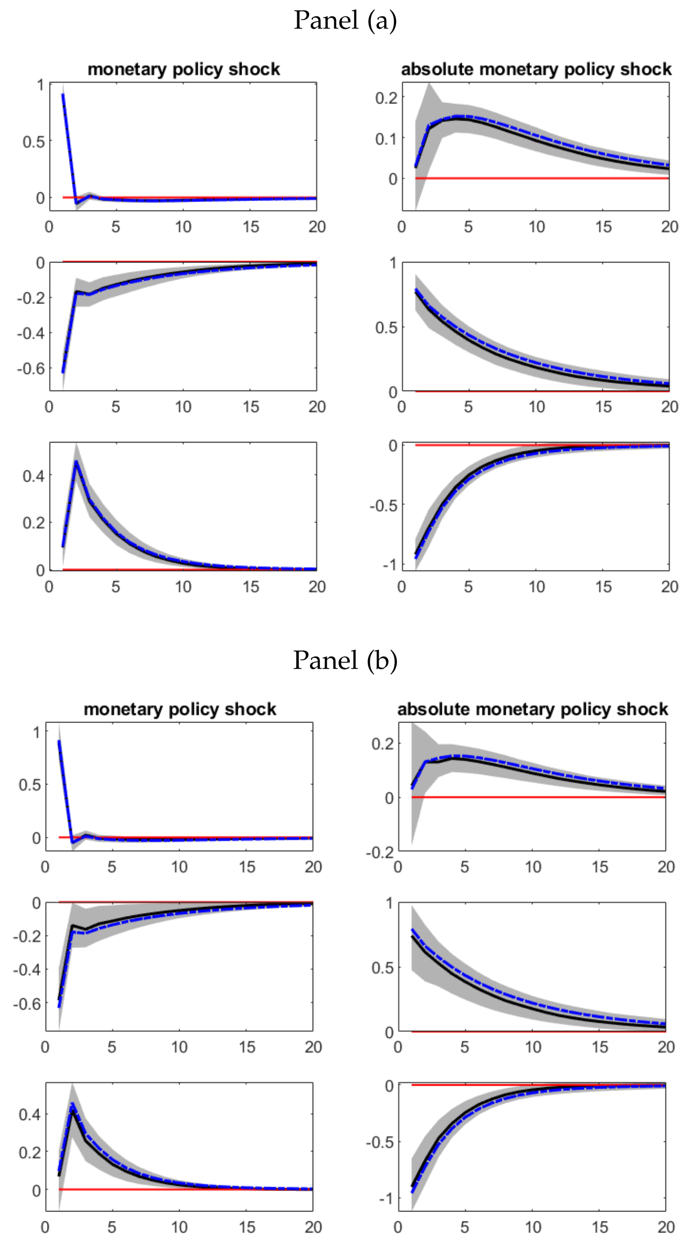

Figure A.1. *Impulse response functions from the Proxy-SVAR using 1000 data sets generated from (6). Panel (a) simulation with standardized Normal structural shocks. Panel (b) the policy shock has a chi-square distribution with two degrees of freedom. The black lines are the average across the point estimates, the grey areas are the 68% and 95% bands. The blue dotted lines are the true impulse response functions.*

Note that, since the technology innovation u_t^a is assumed to be *i.i.d.*, it does not affect future expectations and thus it does not constitute an argument of the function $X(\cdot)$. The advantage of this procedure is that the expectation function $X(\cdot)$ is a smooth function of the monetary shock, while the policy functions of inflation and output are not, due to the "kink" related to downward wage rigidities.

For a given guess of the function $X(m_t)$, the solution of the model can be obtained analytically as

$$
\begin{aligned}
&\tilde{y}_t = 0,\ \pi_t = -\frac{1}{\phi_\pi}\left[x_t + m_t + (\bar{r} - \rho)\right] && \text{if } m_t \leq \phi_\pi u_t^a - x_t - (\bar{r} - \rho) \\
&\tilde{y}_t = -\frac{1}{\sigma}\left[x_t + m_t - \phi_\pi u_t^a + (\bar{r} - \rho)\right],\ \pi_t = -u_t^a && \text{if } m_t > \phi_\pi u_t^a - x_t - (\bar{r} - \rho)
\end{aligned}
$$

where lower-case variables denote the log of upper case variables, which can be used to calculate $\tilde{y}_{t+1}$ and π_{t+1} for all realizations of future shocks. The initial guess constitutes an equilibrium if it satisfies

$$
X(m_t) = \mathbb{E}_t\left[\exp\{-\sigma(u_{a,t+1} + \tilde{y}_{t+1}) - \pi_{t+1}\}\right].
$$

A.4 Optimal Monetary Policy in the Theoretical Model

This appendix shows that in the theoretical model of Section 6 can be implemented through a (log) linear interest rate rule. For given nominal interest rate R_t, the competitive equilibrium of the economy is characterized by the following equations:

$$
1 = \beta R_t \tilde{Y}_t^{\sigma} \mathbb{E}_t\left\{\exp\{-\sigma u_{a,t+1}\}\tilde{Y}_{t+1}^{-\sigma}\Pi_{t+1}^{-1}\right\} \tag{A.2}
$$

$$
\left(\tilde{Y}_t - 1\right)\left(\exp\{u_t^a\} - \theta\Pi_t^{-1}\right) = 0, \tag{A.3}
$$

where $\tilde{Y}_t \equiv Y_t / \exp\{a_t\}$ is detrended output, or equivalently the output-gap, defined as the the deviation of output from its full-employment counterpart.

The optimal policy consists in achieving full-employment, i.e. $\tilde{Y}_t = 1$ in all periods, which is the equilibrium that would prevail in the absence of nominal rigidities, and coincides with the first-best equilibrium. Also, in that equilibrium output growth equals the rate of technological change, i.e. $Y_t / Y_{t-1} = \exp\{u_t^a\}$.

To achieve the optimal equilibrium, monetary policy should be designed such that the downward nominal wage rigidity constraint is not binding. From, eq. (A.3), this requires that the (gross) inflation rate $\Pi_t \geq \exp\{-u_t^a\}/\theta$, $\forall t$.

As is well known, there could be many policy rules implementing the optimal equilibrium. We now conjecture and verify that the optimal policy equilibrium can be implemented with a simple interest rate rule of the form

$$R_t = \bar{R}\Pi_t^{\phi_\pi} \left(\theta \frac{Y_t}{Y_{t-1}}\right)^{\phi_y}, \tag{A.4}$$

where $\bar{R}$, ϕ_π and ϕ_y are policy coefficients to be determined. Indeed, replacing eq. (A.4) into eq. (A.2) we get that

$$\Pi_t = \left[\beta \bar{R} \bar{X} \tilde{Y}_t^{\sigma} \left(Y_t / Y_{t-1}\right)^{\phi_y}\right]^{-\frac{1}{\phi_\pi}} = [\beta \bar{X} \bar{R}]^{-\frac{1}{\phi_\pi}} \left(\exp\{-u_t^a\}/\theta\right)^{\frac{\phi_y}{\phi_\pi}} \tag{A.5}$$

where $\bar{X} \equiv \mathbb{E}_t \left\{\exp\{-\sigma u_{a,t+1}\} \tilde{Y}_{t+1}^{-\sigma} \Pi_{t+1}^{-1}\right\}$ which is constant given our i.i.d. assumption. Setting $\bar{R} = (\beta \bar{X})^{-1}$ and $\phi_y = \phi_\pi$, eq. (A.5) implies that in any period, inflation is equal

$$\Pi_t = \exp\{-u_t^a\}/\theta \tag{A.6}$$

which implies that the downward wage rigidity constraint is not binding, and thus verifies the conjecture. Finally, taking logs of (A.4), the optimal simple rule take the (log) linear form

$$r_t = constant + \phi_\pi \pi_t + \phi_y (y_t - y_{t-1}), \tag{A.7}$$

which is consistent with the assumption made in our empirical application —see eq. (11) in the main text.

Online Appendix
Nonlinear Monetary Policy Tradeoff

D. Debortoli, M. Forni, L. Gambetti, L. Sala

November 2025

OA.1 Model with Only Sign-Dependence

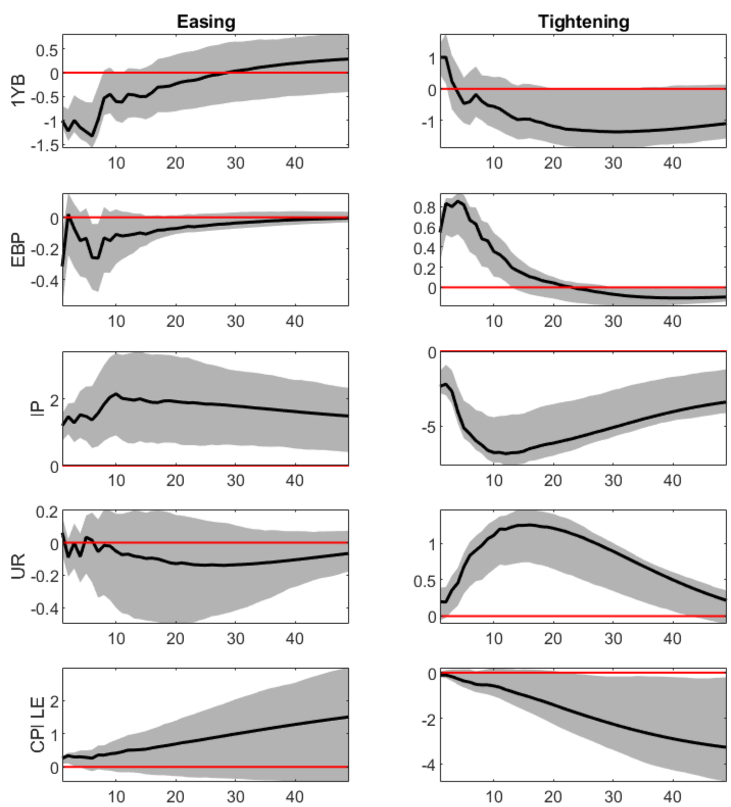

Figure OA.1. *Only sign-dependence. Impulse response functions. Solid lines are the point estimates, the gray areas are 68% confidence bands, constructed with the wild bootstrap procedure described in Appendix A.1.*

OA.2 Model with only State-Dependence

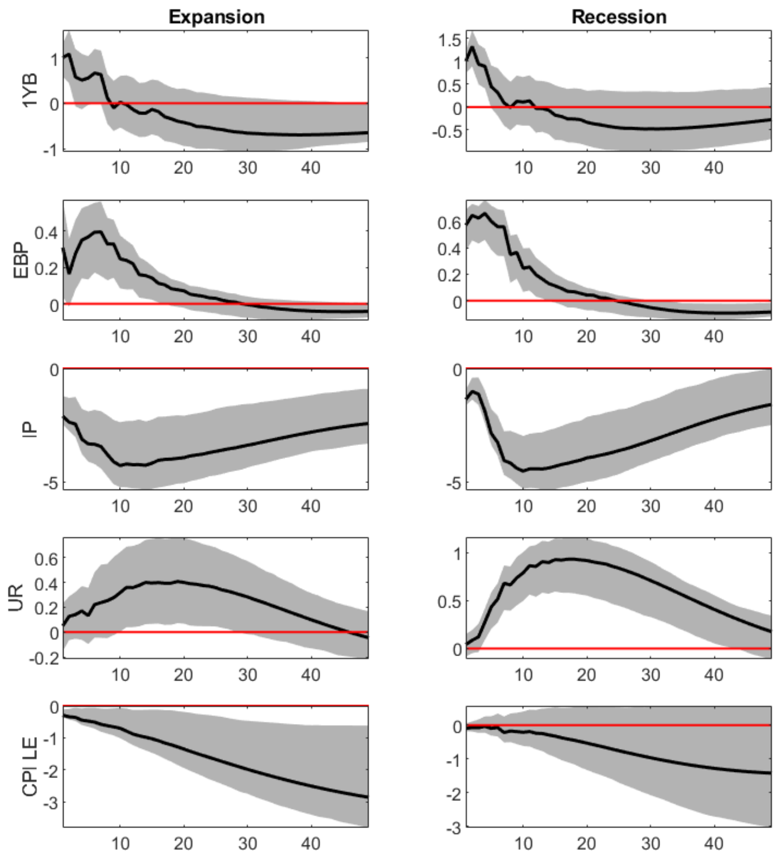

Figure OA.2. *Only state-dependence: impulse response functions. Solid lines are the point estimates, the gray areas are 68% confidence bands, constructed with the wild bootstrap procedure described in Appendix A.1.*

OA.3 Unemployment Rate as a State Variable

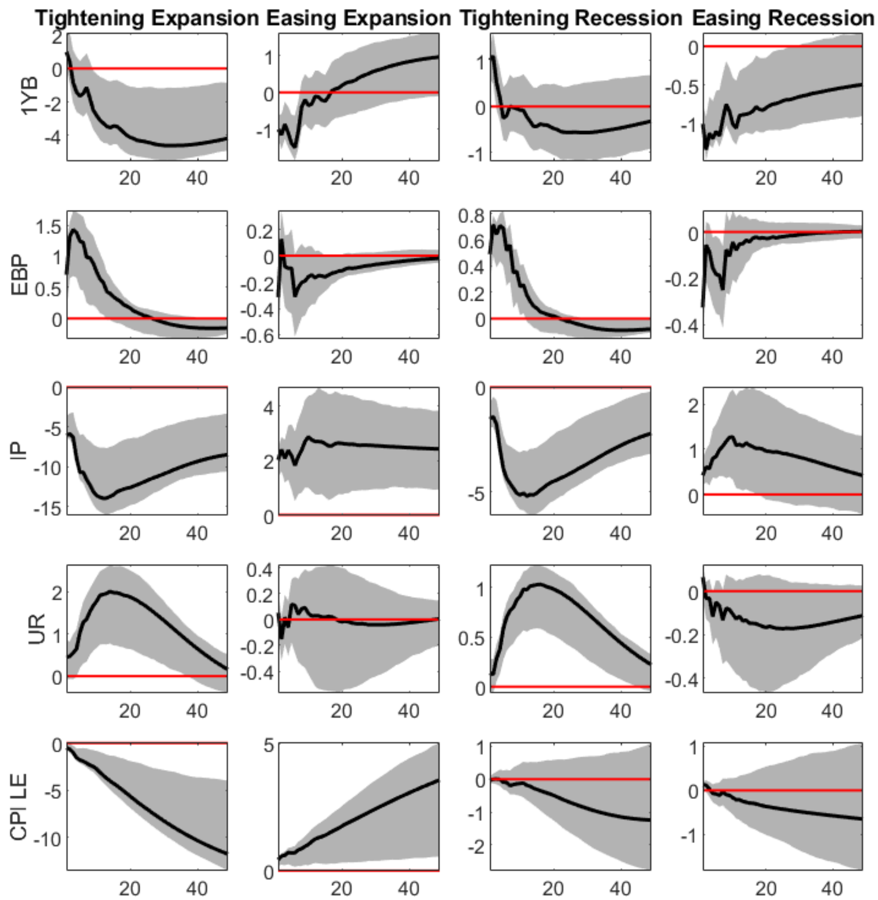

Figure OA.3. *Unemployment rate as state variable, long sample: impulse response functions of easing and contractions in booms and recessions. Solid lines are the point estimates, the gray areas are 68% confidence bands, constructed with the wild bootstrap procedure described in Appendix A.1.*

OA.4 Estimating the VARX with the instrument in place of the estimated shock

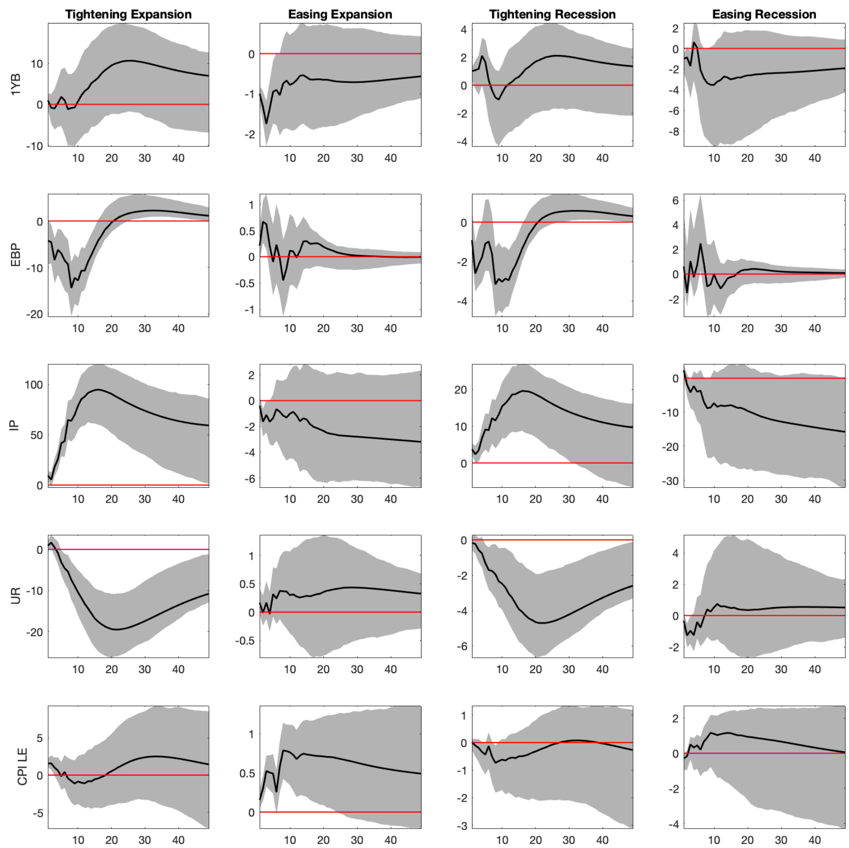

Figure OA.4. *The instrument in place of the estimated shock: impulse response functions of easing and contractions in expansions and recessions. Solid lines are the point estimates, the gray areas are 68% confidence bands, constructed with the wild bootstrap procedure described in Appendix A.1.*

OA.5 Adding variables to the VAR specification

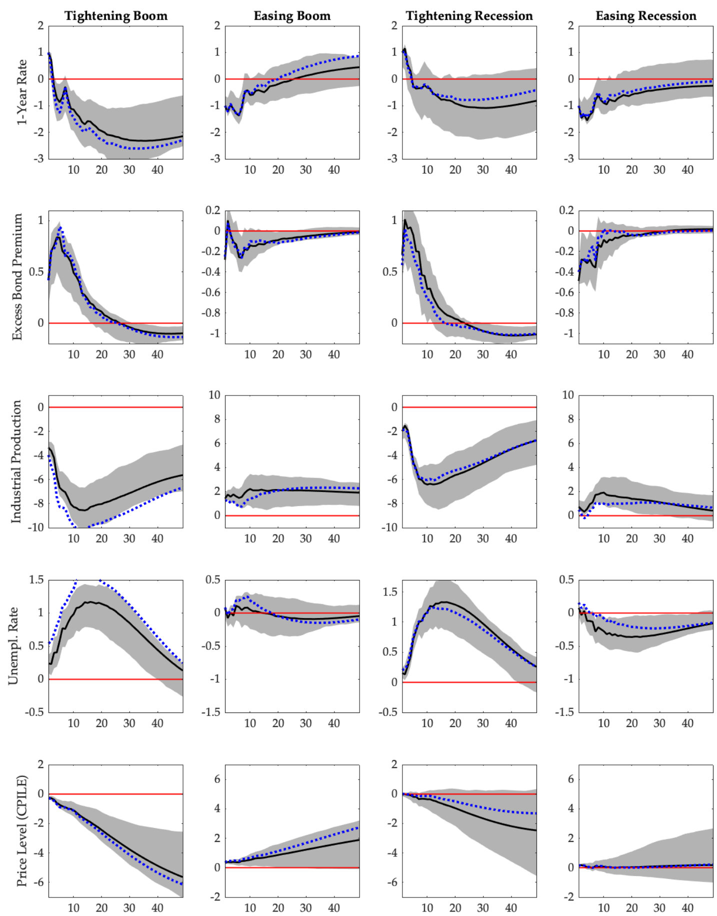

Figure OA.5. *Both the VAR and the VARX include the 10-years treasury bill rate and the M2 monetary aggregate, in addition to the variables of the benchmark. Impulse response to a monetary shock during booms (first two columns) and recessions (last two columns), for monetary tightening (first and third column) and easing (second and fourth column). Solid lines represent the benchmark point estimates, the gray areas are 68% confidence bands for the benchmark.*