Rethinking the New Keynesian Model

Jordi Galí

CREI, UPF and BSE

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The New Keynesian Model

Key Elements

- Monopolistic competition in goods and/or labor markets
- Staggered price and/or wage setting $\Leftrightarrow$ "elastic markups"

Example: The Three Equation Model

The Three Equation Model

- *Dynamic IS Equation*

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\left[i_t - \mathbb{E}_t\{\pi_{t+1}\} - (\rho + z_t)\right] + g_t - \mathbb{E}_t\{g_{t+1}\}$$

- *New Keynesian Phillips Curve*

$$\pi_t = \beta\mathbb{E}_t\{\pi_{t+1}\} + \kappa(\widehat{y}_t - \widehat{y}_t^n)$$

where $\widehat{y}_t^n \equiv \gamma_y' \xi_t$

- *Monetary Policy Rule*

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t$$

- *Exogenous processes* for ξ_t, g_t, z_t and v_t

$\Rightarrow$ can solve for three endogenous variables $(\widehat{y}_t, \pi_t, i_t)$

The New Keynesian Model

Extensions of the Basic Model

- Sticky wages, investment, open economy, ZLB, HANK, menu costs,...

- isomorphic to basic NK in some cases
- estimated DSGE models

Applications

- Optimal monetary policy analysis
- Utility-based evaluation of alternative monetary policy rules
- Interactions between fiscal and monetary policy
- Role of monetary policy rules in shaping the economy's response to shocks

Criticisms

Rethinking the New Keynesian Model

- The purpose of macro models
  - quantitative tools: simulations, forecasts
  - tools to understand the mechanisms underlying economic fluctuations
- Two weaknesses of standard formulations of the NK model
  - focus on consumption Euler equation
  - focus on nominal interest rate rules
- A proposed reformulation of the NK model's aggregate demand block
  - focus on the consumption function
  - focus on real rate rules

Related Literature

- Consumption empirics: Campbell 1987, Campbell-Deaton 1989,...
- Models with imperfect information: Eusepi-Preston 2018, Angeletos-Lian 2018, Caballero-Simsek 2020, Roth-Wiederholt-Wohlfart 2024,...
- Models with heterogenous agents

  - TANK: Galí et al. 2007, Bilbiie 2008,...
  - HANK: Kaplan et al. 2018, Auclert 2019, Auclert-Straub-Rognlie 2024,...
  - NK-OLG models: Galí 2021, Angeletos-Lian-Wolf 2024,...

- Real interest rate rules: Romer 2000, Werning 2015, Debortoli-Galí 2024, Auclert-Straub-Rognlie 2024,...

Outline

- The Dynamic IS equation and its discontents
- A reformulation of the NK model

  - the case of a representative consumer
  - a (generalized) TANK model

- Possible extensions
- Concluding remarks

The Dynamic IS Equation and its Discontents

- Assumption: $U_{c,t} = C_t^{-\sigma}\Phi_t$
- Consumer's Euler equation

$$\widehat{c}_t = \mathbb{E}_t\{\widehat{c}_{t+1}\} - \frac{1}{\sigma}(\widehat{r}_t - z_t)$$

  where $x_t \equiv \log X_t$, $\widehat{x}_t \equiv x_t - x$, $r_t \equiv i_t - \mathbb{E}_t\{\pi_{t+1}\}$, and $z_t \equiv \phi_t - \mathbb{E}_t\{\phi_{t+1}\}$
- Goods market clearing (assuming $G = 0$)

$$\widehat{y}_t = \widehat{c}_t + \widehat{g}_t$$

  where $g_t \equiv G_t / Y$.
- Dynamic IS equation

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}(\widehat{r}_t - z_t) + \widehat{g}_t - \mathbb{E}_t\{\widehat{g}_{t+1}\}$$

- Isomorphic equation in some open economy and TANK models

The Dynamic IS Equation and its Discontents

Assuming $z_t = \rho_z z_{t-1} + \varepsilon_t^z$ we can write:

$$\widehat{c}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}\mathbb{E}_t\{\widehat{r}_{t+k}\} + \frac{1}{\sigma(1-\rho_z)}\; z_t$$

$$\widehat{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}\mathbb{E}_t\{\widehat{r}_{t+k}\} + \frac{1}{\sigma(1-\rho_z)}\; z_t + \widehat{g}_t$$

Some Unanswered Questions

- What is effect of a deficit-financed tax cut? Why?
- Where is the wealth effect on consumption from changes in g_t?
- What is the impact of a stock price bubble?
- What is the effect of an (irrational) wave of optimism about future output?
- Why is there a forward guidance puzzle?

Problem: incomplete characterization of the consumption-savings problem
Proposed solution: back to the consumption function

A Basic NK Model with a Representative Consumer

The Consumption-Saving Problem

- Period budget constraint

$$C_t + \mathbb{E}_t\{\Lambda_{t,t+1}\mathcal{A}_{t+1}\} = \mathcal{A}_t + W_t N_t$$

- Intertemporal budget constraint (C-IBC), given $\lim_{T\to\infty}\mathbb{E}_t\{\Lambda_{t,T}\mathcal{A}_{t+T}\} = 0$.

$$\sum_{k=0}^{\infty}\mathbb{E}_t\{\Lambda_{t,t+k}C_{t+k}\} = \mathcal{A}_t + \sum_{k=0}^{\infty}\mathbb{E}_t\{\Lambda_{t,t+k}W_{t+k}N_{t+k}\}$$

- Optimality condition:

$$\Lambda_{t,t+k} = \beta^k U_{c,t+k}/U_{c,t}$$

- *Assumption*: $U_{c,t} = C_t^{-\sigma}\Phi_t$

$$\Lambda_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\sigma}(\Phi_{t+k}/\Phi_t)$$

- In log-deviations from steady state:

$$\widehat{\lambda}_{t,t+k} = -\sigma(\widehat{c}_{t+k} - \widehat{c}_t) + \phi_{t+k} - \phi_t$$

where $\widehat{\lambda}_{t,t+k} \equiv \log(\Lambda_{t,t+k}/\beta^k)$

- Pricing of a one-period nominal bond:

$$Q_t \equiv \exp\{-i_t\} = \mathbb{E}_t\{\Lambda_{t,t+1}(P_t/P_{t+1})\}$$

$$\mathbb{E}_t\{\widehat{\lambda}_{t,t+1}\} = -\widehat{r}_t$$

implying

$$\mathbb{E}_t\{\widehat{\lambda}_{t,t+k}\} = \sum_{j=0}^{k-1} \mathbb{E}_t\{\widehat{\lambda}_{t+j,t+1+j}\} = -\sum_{j=0}^{k-1} \mathbb{E}_t\{\widehat{r}_{t+j}\}$$

- Optimality condition redux:

$$\mathbb{E}_t\{\widehat{c}_{t+k}\} = \widehat{c}_t + \frac{1}{\sigma}\sum_{j=0}^{k-1} \mathbb{E}_t\{\widehat{r}_{t+j}\} - \frac{1}{\sigma}\sum_{j=0}^{k-1} \mathbb{E}_t\{z_{t+j}\}$$

where $z_t \equiv \phi_t - \mathbb{E}_t\{\phi_{t+1}\}$

- In equilibrium:

$$\mathcal{A}_t = Q_t^S = \sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k} D_{t+k}\}$$

Implied C-IBC, letting $Y_t \equiv W_t N_t + D_t$:

$$\begin{aligned}\sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k} C_{t+k}\} &= \mathcal{A}_t + \sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k} W_{t+k} N_{t+k}\} \\ &= \sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k} Y_{t+k}\}\end{aligned}$$

- Log-linearized C-IBC (around a steady state with $Y = C$):

$$\sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{c}_{t+k}\} = \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{y}_{t+k}\}$$

- Combining C-IBC with optimality condition and $z_t \sim AR(1)$:

$$\widehat{c}_t = (1-\beta)\left(\sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{y}_{t+k}\}\right) - \frac{\beta}{\sigma}\sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\} + \frac{\beta}{\sigma(1-\beta\rho_z)} z_t$$

- Consumption function:

$$\widehat{c}_t = (1-\beta)\left(\sum_{k=0}^{\infty}\beta^k \mathbb{E}_t\{\widehat{y}_{t+k}\}\right) - \frac{\beta}{\sigma}\sum_{k=0}^{\infty}\beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\} + \frac{\beta}{\sigma(1-\beta\rho_z)} z_t$$

- Long-term yield on a (real) consol:

$$\widehat{r}_t^L \equiv (1-\beta)\sum_{k=0}^{\infty}\beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\}$$

- Letting $\widehat{x}_t \equiv \sum_{k=1}^{\infty}\beta^k \mathbb{E}_t\{\widehat{y}_{t+k}\}$

$$\widehat{c}_t = (1-\beta)\widehat{y}_t + (1-\beta)\widehat{x}_t - \frac{\beta}{\sigma(1-\beta)}\widehat{r}_t^L + \frac{\beta}{\sigma(1-\beta\rho_z)} z_t$$

- Discussion:

- static MPC: $1-\beta$, role for expected future income ($\widehat{x}_t$)
- intertemporal substitution from changes in $\widehat{r}_t$ (but no income effect)
- *direct* effect of MP: $-\frac{\beta}{\sigma(1-\beta)}\widehat{r}_t^L = -\frac{\beta}{\sigma}\sum_{k=0}^{\infty}\beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\}$ (no FG puzzle)

Equilibrium

- Goods market clearing

$$\begin{aligned}\widehat{y}_t &= \widehat{c}_t \\ &= (1-\beta)\widehat{y}_t + (1-\beta)\widehat{x}_t - \frac{\beta}{\sigma(1-\beta)}\widehat{r}_t^L + \frac{\beta}{\sigma(1-\beta\rho_z)}z_t\end{aligned}$$

- Temporary equilibrium (i.e. conditional on $\widehat{x}_t$):

$$\widehat{y}_t = \Lambda\left[(1-\beta)\widehat{x}_t - \frac{\beta}{\sigma(1-\beta)}\widehat{r}_t^L + \frac{\beta}{\sigma(1-\beta\rho_z)}z_t\right]$$

where $\Lambda \equiv \frac{1}{1-(1-\beta)} = \frac{1}{\beta} > 1$ is the aggregate demand *static* multiplier. Using $\Lambda\beta = 1$ we can write

$$\widehat{y}_t = \Lambda(1-\beta)\widehat{x}_t - \frac{1}{\sigma(1-\beta)}\widehat{r}_t^L + \frac{1}{\sigma(1-\beta\rho_z)}z_t$$

- General equilibrium, conditional on $\widehat{r}_t^L$ (recursive representation, applying $1 - \beta L_t^{-1}$ operator)

$$\begin{aligned}\widehat{y}_t &= \beta \mathbb{E}_t\{\widehat{y}_{t+1}\} + \Lambda(1-\beta)\beta\mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\widehat{r}_t + \frac{1}{\sigma}z_t \\ &= \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\widehat{r}_t + \frac{1}{\sigma}z_t\end{aligned}$$

which is the standard dynamic IS equation

- Note that the latter combines:

(i) consumption function (PE)
(ii) static multiplier effects (TE)
(iii) dynamic multiplier effects (GE)

Closing the Model: A Real Rate Rule

- Under flexible prices: DIS determines $\widehat{r}_t$, given $\widehat{y}_t^n \Rightarrow$ no role for MP
- Under "elastic markups": demand-determined output $\Rightarrow$ role for MP in determination of $\widehat{r}_t$ and $\widehat{y}_t$
- Standard assumption: a rule for the nominal rate i_t
- The case for a real rate rule: (i) relevance, (ii) convenience, (iii) robustness

Closing the Model: A *Real* Rate Rule

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- Under "elastic markups": demand-determined output $\Rightarrow$ role for MP in determination of $\widehat{r}_t$ and $\widehat{y}_t$
- Standard assumption: a rule for the nominal rate i_t
- The case for a real rate rule: (i) relevance, (ii) convenience, (iii) robustness
- A simple real rate rule

$$\widehat{r}_t = \phi_y \widehat{y}_t + \widehat{r}_t^x$$

  where $\phi_y > 0$ (uniqueness condition) and $\widehat{r}_t^x \sim AR(1)$
- Combined with DIS:

$$\widehat{y}_t = \frac{1}{\sigma(1-\rho_z)+\phi_y} z_t - \frac{1}{\sigma(1-\rho_r)+\phi_y} \widehat{r}_t^x$$

- Full stabilization of output: $\widehat{r}_t^x = z_t$ for all t
- Robust to alternative specifications of the price/wage block.

Closing the Model: A *Real* Rate Rule

- Alternative: a rule for the *long* real rate
- Example (I):

$$\widehat{r}_t^L = \phi_y \widehat{y}_t$$

implying

$$\begin{aligned}\widehat{r}_t &= \frac{\phi_y}{1-\beta}\widehat{y}_t - \frac{\phi_y \beta}{1-\beta}\mathbb{E}_t\{\widehat{y}_{t+1}\} \\ &= \phi_y \widehat{y}_t - \frac{\phi_y \beta}{1-\beta}\mathbb{E}_t\{\Delta\widehat{y}_{t+1}\}\end{aligned}$$

- Example (II)

$$\widehat{r}_t^L = \phi_\pi \pi_t$$

Combined with NKPC:

$$\widehat{r}_t = \frac{\phi_\pi \kappa}{1-\beta}(\widehat{y}_t - \widehat{y}_t^n)$$

Application (I): Direct vs. Indirect Effects of Monetary Policy Shocks

- Assumption: $\widehat{r}_t = \phi_y \widehat{y}_t + \widehat{r}_t^x$ where $\widehat{r}_t^x = \rho_r \widehat{r}_{t-1}^x + \varepsilon_t^r$
- Equilibrium:

$$\begin{aligned} \widehat{y}_t &= \widehat{c}_t = -\frac{1}{\sigma(1-\rho_r)+\phi_y}\widehat{r}_t^x \\ \widehat{r}_t &= \frac{\sigma(1-\rho_r)}{\sigma(1-\rho_r)+\phi_y}\widehat{r}_t^x \end{aligned}$$

- *Direct* effect (i.e. conditional on output) of MP shock: $-\frac{\beta}{\sigma}\sum_{k=0}^{\infty}\beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\}$

$$\frac{\partial \widehat{c}_t}{\partial \varepsilon_t^r} = -\frac{\beta}{\sigma}\sum_{k=0}^{\infty}\beta^k \mathbb{E}_t\left\{\frac{d\widehat{r}_{t+k}}{d\varepsilon_t^r}\right\} = -\frac{\beta(1-\rho_r)}{[\sigma(1-\rho_r)+\phi_y](1-\beta\rho_r)}$$

- *Total* effect of MP shock

$$\frac{d\widehat{c}_t}{d\varepsilon_t^r} = \frac{d\widehat{y}_t}{d\varepsilon_t^r} = -\frac{1}{\sigma(1-\rho_r)+\phi_y}$$

- D/T ratio:

$$\delta = \frac{\beta(1-\rho_r)}{1-\beta\rho_r} \in (0, \beta)$$

  which is decreasing in ρ_r (and independent of ϕ_y)
- Direct effect dominant for low persistence shocks only ($\delta = \beta$ if $\rho_r = 0$).
- The more persistent is the shock the more important is the GE effect, due to the more persistent response of output and the forward-lookingness of consumption ($\delta \to 0$ when $\rho_r \to 1$)
- Contrasts with claim in Kaplan et al. (2018)

Application (II): Fiscal Policy

Government

- Government period budget constraint:

$$G_t + B_t^G = \mathbb{E}_t\{\Lambda_{t,t+1} B_{t+1}^G\} + T_t$$

- Government intertemporal budget constraint (G-IBC):

$$B_t^G = \sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k}(T_{t+k} - G_{t+k}\}$$

- Assumption: $G = 0,\ B^G > 0 \Rightarrow T = (1-\beta)B^G$
- Log-linearization around the steady state:

$$\widehat{b}_t^G + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{g}_{t+k}\} = \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{t}_{t+k}\} - \frac{\beta b^G}{1-\beta}\widehat{r}_t^L$$

where $t_t \equiv T_t/Y$, $g_t \equiv G_t/Y$, and $b_t^G \equiv B_t^G/Y$.

- (Negative) Income effect of interest rate changes: $-\frac{\beta b^G}{1-\beta}\widehat{r}_t^L$

Consumers

- Consumer's period budget constraint:

$$C_t + \mathbb{E}_t\{\Lambda_{t,t+1}\mathcal{A}_{t+1}\} = \mathcal{A}_t + W_t N_t - T_t$$

- Consumer's intertemporal budget constraint (C-IBC)

$$\begin{aligned}\sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k} C_{t+k}\} &= \mathcal{A}_t + \sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k}(W_{t+k} N_{t+k} - T_t)\} \\ &= B_t^G + \sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k}(Y_{t+k} - T_{t+k})\}\end{aligned}$$

using the fact that $\mathcal{A}_t = B_t^G + Q_t^S$ in equilibrium.

- Log-linearizing the IBC around the steady state:

$$\sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{c}_{t+k}\} = \widehat{b}_t^G + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{y}_{t+k} - \widehat{t}_{t+k}\} + \frac{\beta b^G}{1-\beta}\widehat{r}_t^L$$

- Combining it with the optimality condition yields:

$$\widehat{c}_t = (1-\beta)\left(\widehat{b}_t^G + \sum_{k=0}^{\infty}\beta^k \mathbb{E}_t\{\widehat{y}_{t+k} - \widehat{t}_{t+k}\}\right) + \left[(1-\beta)b^G - \frac{\Gamma}{\sigma}\right]\frac{\beta}{1-\beta}\widehat{r}_t^L$$

- Discussion: substitution and income effects of $\widehat{r}_t^L$
- Combined with the G-IBC

$$\begin{aligned}\widehat{c}_t &= (1-\beta)\sum_{k=0}^{\infty}\beta^k \mathbb{E}_t\{\widehat{y}_{t+k} - \widehat{g}_{t+k}\} - \frac{\beta}{\sigma}\sum_{k=0}^{\infty}\beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\} \\ &= (1-\beta)\widehat{y}_t + (1-\beta)\widehat{x}_t - \widehat{g}_t^P - \frac{\beta}{\sigma(1-\beta)}\widehat{r}_t^L\end{aligned}$$

where $\widehat{g}_t^P \equiv (1-\beta)\sum_{k=0}^{\infty}\beta^k \mathbb{E}_t\{\widehat{g}_{t+k}\}$.

- Discussion

- income effects of $\widehat{r}_t^L$ cancel out
- irrelevance of $\{\widehat{t}_t, \widehat{b}_t^G\}$ (Ricardian equivalence)
- reflects the negative wealth effect of government purchases (working through implied taxes).
- discounted impact of an anticipated changes in $\widehat{g}_t$ is decreasing with the horizon
- from a PE perspective an increase in $\widehat{g}_t$ is consistent with higher consumption

Equilibrium

- Goods market clearing:

$$\begin{aligned}\widehat{y}_t &= \widehat{c}_t + \widehat{g}_t \\ &= (1-\beta)\widehat{y}_t + (1-\beta)\widehat{x}_t - \frac{\beta}{\sigma(1-\beta)}\widehat{r}_t^L + (\widehat{g}_t - \widehat{g}_t^P)\end{aligned}$$

- Temporary equilibrium

$$\widehat{y}_t = \Lambda\left[(1-\beta)\widehat{x}_t - \frac{\beta}{\sigma(1-\beta)}\widehat{r}_t^L + (\widehat{g}_t - \widehat{g}_t^P)\right]$$

where $\Lambda \equiv \frac{1}{1-(1-\beta)} = \frac{1}{\beta} > 1$

- General equilibrium, given $\widehat{r}_t$:

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\widehat{r}_t + \widehat{g}_t - \mathbb{E}_t\{\widehat{g}_{t+1}\}$$

- Assuming an exogenous $AR(1)$ process for $\widehat{g}_t$, and a simple rule $\widehat{r}_t = \phi_y \widehat{y}_t$:

$$\widehat{y}_t = \frac{\sigma(1-\rho_g)}{\sigma(1-\rho_g)+\phi_y}\widehat{g}_t \quad ; \quad \widehat{c}_t = -\frac{\phi_y}{\sigma(1-\rho_g)+\phi_y}\widehat{g}_t$$

$$\widehat{y}_t = \frac{\sigma(1-\rho_g)}{\sigma(1-\rho_g)+\phi_y}\widehat{g}_t \qquad ; \qquad \widehat{c}_t = -\frac{\phi_y}{\sigma(1-\rho_g)+\phi_y}\widehat{g}_t$$

- G-multiplier $\Phi_g \equiv \frac{\sigma(1-\rho_g)}{\sigma(1-\rho_g)+\phi_y}$ bounded above by one. Why?
- If $\phi_y = 0$, then $\Phi_g = 1$ and no crowding out of consumption. No intertemporal substitution ($\widehat{r}_t = 0$) and the negative wealth effect of higher taxes is exactly offset by the positive wealth effect caused by the persistently higher output, i.e. $\sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{y}_{t+k} - \widehat{g}_{t+k}\} = 0$
- If $\phi_y > 0$, then $\Phi_g < 1$ and crowding out of consumption. Intertemporal substitution dampens consumption, preventing output from increasing as much as $\widehat{g}_t$ (negative wealth effect).

Decomposition of Consumption Response

- Negative wealth effect from higher current and anticipated taxes:

$$-(1-\beta)\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{g}_{t+k}\} = \frac{1-\beta}{1-\beta\rho_g}\widehat{g}_t$$

- Positive wealth effect from higher current and anticipated output

$$(1-\beta)\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{y}_{t+k}\} = \frac{\sigma(1-\rho_g)(1-\beta)}{[\sigma(1-\rho_g)+\phi_y](1-\beta\rho_g)}\widehat{g}_t$$

- Intertemporal substitution from higher interest rates:

$$-\frac{\beta}{\sigma}\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t\{\widehat{r}_{t+k}\} = -\frac{(1-\rho_g)\beta\phi_y}{[\sigma(1-\rho_g)+\phi_y](1-\beta\rho_g)}\widehat{g}_t$$

- Contrast with the conventional approach:

$$\begin{aligned}\widehat{c}_t &= \mathbb{E}_t\{\widehat{c}_{t+1}\} - \frac{1}{\sigma}\widehat{r}_t \\ &= -\frac{1}{\sigma}\sum_{k=0}^{\infty}\mathbb{E}_t\{\widehat{r}_{t+k}\}\end{aligned}$$

which suggests only direct intertemporal substitution effects are at work, while mismeasuring them.

Application (III): Animal Spirits

- Consumption function

$$\widehat{c}_t = (1-\beta)\sum_{k=0}^{\infty}\beta^k \mathbb{E}_t^*\{\widehat{y}_{t+k}\} - \frac{\beta}{\sigma}\sum_{k=0}^{\infty}\beta^k \mathbb{E}_t^*\{\widehat{r}_{t+k}\}$$

Equivalently:

$$\widehat{c}_t = (1-\beta)\sum_{k=0}^{\infty}\beta^k \mathbb{E}_t\{\widehat{y}_{t+k}\} - \frac{\beta}{\sigma}\sum_{k=0}^{\infty}\beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\} + s_t$$

where

$$s_t \equiv \sum_{k=1}^{\infty}\beta^k [\mathbb{E}_t^*\{\Delta y_{t+k}\} - \mathbb{E}_t\{\Delta y_{t+k}\}] - \frac{\beta}{\sigma}\sum_{k=1}^{\infty}\beta^k [\mathbb{E}_t^*\{\widehat{r}_{t+k}\} - \mathbb{E}_t\{\widehat{r}_{t+k}\}]$$

captures waves of optimism or pessimism regarding future output growth and interest rates

- Assumption

$$\mathbb{E}_t^*\{\Delta y_{t+k}\} - \mathbb{E}_t\{\Delta y_{t+k}\} = \alpha^{k-1}\zeta_t$$

$$\mathbb{E}_t^*\{\widehat{r}_{t+k}\} - \mathbb{E}_t\{\widehat{r}_{t+k}\} = -\varpi\alpha^{k-1}\zeta_t$$

for $k = 1, 2, 3, ...$where $\zeta_t \sim AR(1)$ is an exogenous expectational bias ("animal spirits"). Implication:

$$s_t \equiv \beta\chi\zeta_t$$

where $\chi \equiv \frac{\sigma+\beta\varpi}{\sigma(1-\beta\alpha)} > 0$.

- Implied dynamic IS equation

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+k}\} - \frac{1}{\sigma}\widehat{r}_t + \chi(1-\beta\rho_\zeta)\zeta_t$$

- Combined with $\widehat{r}_t = \phi_y\widehat{y}_t$ we obtain:

$$\widehat{y}_t = \frac{\sigma\chi(1-\beta\rho_\zeta)}{\sigma(1-\rho_\zeta)+\phi_y}\zeta_t$$

$\Rightarrow$ fluctuations driven by "animal spirits"

Application (IV): (Irrational) Bubbles

- Stock valuation:

$$\begin{aligned} Q_t^S &= Q_t^F + Q_t^B \\ &= \sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k} D_{t+k}\} + Q_t^B \end{aligned}$$

- Implied C-IBC:

$$\sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k} C_{t+k}\} = Q_t^B + \sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k} Y_{t+k}\}$$

- Consumption function

$$\widehat{c}_t = (1-\beta) q_t^B + (1-\beta) \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{y}_{t+k}\} - \frac{\beta}{\sigma} \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{r}_{t+k}\}$$

where $q_t^B \equiv Q_t^B / Y$.

- Dynamic IS equation:

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\widehat{r}_t + \frac{1-\beta}{\beta}(q_t^B - \beta\mathbb{E}_t\{q_{t+1}^B\})$$

- Assumptions:

$$q_t^B = \rho_b q_{t-1}^B + \varepsilon_t^B$$

$$\widehat{r}_t = \phi_y \widehat{y}_t$$

- Equilibrium output

$$\widehat{y}_t = \frac{\sigma(1-\beta)(1-\rho_b)}{\beta[\sigma(1-\rho_b)+\phi_y]} q_t^B$$

$\Rightarrow$ bubble-driven fluctuations

Heterogeneity: A Generalized TANK Model

- Debortoli-Galí 2025: HANK = TANK + background noise

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- But need for a suitably designed TANK

Heterogeneity: A Generalized TANK Model

- Debortoli-Galí 2025: HANK = TANK + background noise
- But need for a suitably designed TANK
- Objective: to understand the channels through which heterogeneity shapes aggregate fluctuations

A Generalized TANK Model

Hand-to-Mouth Consumers

- Constant fraction λ
- Binding borrowing constraint: $B^H_{t+1} \geq -\upsilon_t Y$
- Consumption function:

$$C^H_t = \Xi^H W_t N_t + \Theta^H D_t - T^H_t + \left(\frac{\upsilon_t}{R_t} - \upsilon_{t-1}\right) Y$$
$$= \Omega^H_t Y_t + \left(\frac{\upsilon_t}{R_t} - \upsilon_{t-1} - t^H_t\right) Y$$

where $t^H_t \equiv \frac{T^H_t}{Y}$ and $\Omega^H_t \equiv \Xi^H \frac{1}{\mathcal{M}_t} + \Theta^H \left(1 - \frac{1}{\mathcal{M}_t}\right)$.

- Log-linearized consumption function (I):

$$\Gamma^H \widehat{c}_t^H = \Omega^H \widehat{y}_t - \Sigma^H \widehat{\mu}_t + \beta \widehat{v}_t - \widehat{v}_{t-1} - v\beta \widehat{r}_t - \widehat{t}_t^H$$

where $\Gamma^H \equiv \frac{C^H}{Y}$ and $\Sigma^H \equiv \frac{\Xi^H - \Theta^H}{\mathcal{M}}$ (income composition bias)

- Assumption:

$$\widehat{\mu}_t = -\mu_y \widehat{y}_t + \widehat{\mu}_t^x$$

Example: technology $Y_t = A_t N_t$, wage equation $W_t = \mathcal{M}^w C_t^\sigma N_t^\varphi$ and $Y_t = C_t + G_t$:

$$\widehat{\mu}_t = -(\sigma + \varphi)\widehat{y}_t + (1 + \varphi) a_t + \sigma g_t$$

- Log-linearized consumption function (II):

$$\Gamma^H \widehat{c}_t^H = \chi^H \widehat{y}_t - \Sigma^H \widehat{\mu}_t^x + \beta \widehat{v}_t - \widehat{v}_{t-1} - v\beta \widehat{r}_t - \widehat{t}_t^H$$

where $\chi^H \equiv \Omega^H + \Sigma^H \mu_y$

Unconstrained Consumers

- Constant fraction $1-\lambda$
- Intertemporal budget constraint:

$$\sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k} C_{t+k}^U\} = B_t^U + \sum_{k=0}^{\infty} \mathbb{E}_t\{\Lambda_{t,t+k}(Y_{t+k}^U - T_{t+k}^U)\}$$

where

$$\begin{aligned} Y_t^U &\equiv \Xi^U W_t N_t + \Theta^U D_t \\ &= \Omega_t^U Y_t \end{aligned}$$

with $\Omega_t^U \equiv \Xi^U \frac{1}{\mathcal{M}_t} + \Theta^U \left(1 - \frac{1}{\mathcal{M}_t}\right)$

- Log-linearized consumption function

$$\begin{aligned} \Gamma^U \widehat{c}_t^U &= (1-\beta)\left(\widehat{b}_t^U + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\chi^U \widehat{y}_{t+k} - \Sigma^U \widehat{\mu}_{t+k}^x - \widehat{t}_t^U\}\right) \\ &\quad + \left[(1-\beta) b^U - \frac{\Gamma^U}{\sigma}\right] \frac{\beta}{1-\beta} \widehat{r}_t^L + \frac{\Gamma^U \beta}{\sigma(1-\beta\rho_z)} z_t \end{aligned}$$

where $\Gamma^U \equiv \frac{C^U}{Y}$, $\chi^U \equiv \Omega^U + \Sigma^U \mu_y$, and $\Sigma^U \equiv \frac{\Xi^U - \Theta^U}{\mathcal{M}}$

Government

- G-IBC:

$$\widehat{b}_t^G + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{g}_{t+k}\} = \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{t}_{t+k}\} - \frac{\beta b^G}{1-\beta}\widehat{r}_t^L$$

where $t_t \equiv (1-\lambda)t_t^U + \lambda t_t^H$, $g_t \equiv G_t / Y$, and $b_t^G \equiv B_t^G / Y$.

- Assumption: $\widehat{t}_t^U = \widehat{t}_t^C = \widehat{t}_t$

Aggregate Consumption

$$C_t = \lambda C_t^H + (1-\lambda) C_t^U$$

- Log-linearized version:

$$\begin{aligned}
\widehat{c}_t \quad = \quad & [\lambda \chi^H + (1-\lambda)(1-\beta)\chi^U]\ \widehat{y}_t + (1-\lambda)(1-\beta)\chi^U\ \widehat{x}_t \\
& - \lambda \Sigma^H \widehat{\mu}_t^x - (1-\lambda)(1-\beta)\Sigma^U \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{\widehat{\mu}_{t+k}^x\} \\
& - \lambda \widehat{t}_t - (1-\lambda)\widehat{g}_t^P + \lambda(1-\beta)\widehat{b}_t^G \\
& - \lambda \upsilon \beta \widehat{r}_t + \left(\lambda(1-\beta)(\upsilon + b^G) - \frac{(1-\lambda)\Gamma^U}{\sigma} \right) \frac{\beta}{1-\beta} \widehat{r}_t^L \\
& + \lambda \Delta \widehat{\upsilon}_t + \frac{(1-\lambda)\Gamma^U \beta}{\sigma(1-\beta\rho_z)} z_t
\end{aligned}$$

where $\lambda \chi^H + (1-\lambda)(1-\beta)\chi^U$ is the static aggregate MPC.

Application (I): Transmission of Monetary Policy Shocks

- Assumption: $\widehat{r}_t = \widehat{r}_t^x \sim AR(1)$
- Aggregate consumption function (using $\lambda\chi^H + (1-\lambda)\chi^U = 1$)

$$\widehat{c}_t = [1 - \beta(1 - \lambda\chi^H)]\ \widehat{y}_t + (1-\beta)(1-\lambda\chi^H)\ \widehat{x}_t - \Psi_r \widehat{r}_t$$

where

$$\Psi_r = \lambda v\beta + (1-\lambda)\left(\frac{\Gamma^U}{\sigma} - \frac{(1-\beta)\lambda}{1-\lambda}(v + b^G)\right)\frac{\beta}{1-\beta\rho_r}$$

measures the *direct* effect of MP.

- Discussion
  - MPCs
  - comparison of direct effects:

$$\frac{\Psi_r^{TA}}{\Psi_r^{RA}} = 1 + \lambda\sigma\left[v\beta(1-\rho_r) - \frac{\Gamma^H}{\sigma} - (1-\beta)b^G\right]$$

  - "sign switch" ($\Psi_r^{TA} < 0$) if:

$$\lambda(1-\beta)b^G > \lambda v\beta(1-\rho_r) + \frac{(1-\lambda)\Gamma^U}{\sigma}$$

- Goods market clearing

$$\widehat{y}_t = [1 - \beta(1 - \lambda\chi^H)]\ \widehat{y}_t + (1 - \beta)(1 - \lambda\chi^H)\ \widehat{x}_t - \Psi_r \widehat{r}_t$$

where KC slope: $1 - \beta(1 - \lambda\chi^H) > 1 - \beta$

- Temporary equilibrium

$$\widehat{y}_t = \Lambda(1 - \beta)(1 - \lambda\chi^H)\ \widehat{x}_t - \Lambda\Psi_r \widehat{r}_t$$

with static multiplier

$$\Lambda \equiv \frac{1}{\beta(1 - \lambda\chi^H)} > \frac{1}{\beta} \equiv \Lambda^{RA}$$

- General equilibrium, given $\widehat{r}_t$ (recursive representation)

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \Lambda\Psi_r(1 - \beta\rho_r)\widehat{r}_t$$

$\Rightarrow$ modified dynamic IS equation
$\Rightarrow$ isomorphic to RANK

- Equilibrium output

$$\widehat{y}_t = -\frac{\Lambda\Psi_r(1-\beta\rho_r)}{1-\rho_r}\,\widehat{r}_t$$

- Limiting case (RA model): $\lambda = 0$, $\Lambda^{RA} = \frac{1}{\beta}$ and $\Psi_r^{RA} = \frac{\beta}{\sigma(1-\beta\rho_r)}$, implying

$$\widehat{y}_t^{RA} = -\frac{1}{\sigma(1-\rho_r)}\,\widehat{r}_t$$

- D/T ratio

$$\delta^{TA} = \frac{1-\rho_r}{\Lambda(1-\beta\rho_r)} = (1-\lambda\chi^H)\delta^{RA} < \delta^{RA}$$

- Total amplification TA vs RA: $\frac{d\widehat{y}_t^{TA}}{d\widehat{r}_t} > \frac{d\widehat{y}_t^{RA}}{d\widehat{r}_t}$ if and only if

$$\Sigma^H\mu_y + v[1-\beta+\sigma\beta(1-\rho_r)] + t^H > \sigma(1-\beta)b^G$$

Application (II): Fiscal Policy

- Assumption: $\widehat{r}_t = 0$ for all t, only fiscal shocks
- Tax rule

$$\widehat{t}_t = \widehat{t}_t^x + \phi_b \widehat{b}_t^G$$

with $\widehat{t}_t^x \sim AR(1)$ and $1 - \beta < \phi_b < 1$, implying stable debt dynamics:

$$\widehat{b}_{t+1}^G = \frac{1 - \phi_b}{\beta} \widehat{b}_t^G + \frac{1}{\beta} (\widehat{g}_t - \widehat{t}_t^x)$$

- Linearized aggregate consumption function:

$$\begin{aligned} \widehat{c}_t \quad = \quad & [1 - \beta(1 - \lambda\chi^H)]\ \widehat{y}_t + (1 - \beta)(1 - \lambda\chi^H)\ \widehat{x}_t \\ & - \lambda \widehat{t}_t^x - \lambda \Sigma^H \mu_g\ \widehat{g}_t - (1 - \lambda)(1 + \Sigma^U \mu_g) \widehat{g}_t^P \\ & - \lambda[\phi_b - (1 - \beta)]\ \widehat{b}_t^G \end{aligned}$$

- Discussion

- Goods market clearing:

$$
\begin{aligned}
\widehat{y}_t &= \widehat{c}_t + \widehat{g}_t \\
&= \left[1-\beta(1-\lambda\chi^H)\right] \widehat{y}_t + (1-\beta)(1-\lambda\chi^H)\ \widehat{x}_t - \lambda\widehat{t}_t^x - \lambda[\phi_b - (1-\beta)]\ \widehat{b}_t^G \\
&\quad + \left(1-\lambda\Sigma^H\mu_g\right)\ \widehat{g}_t - (1-\lambda)(1+\Sigma^U\mu_g)\widehat{g}_t^P
\end{aligned}
$$

- Temporary equilibrium

$$
\begin{aligned}
\widehat{y}_t &= \Lambda(1-\beta)(1-\lambda\chi^H)\ \widehat{x}_t - \Lambda\lambda\widehat{t}_t^x - \Lambda\lambda[\phi_b - (1-\beta)]\ \widehat{b}_t^G \\
&\quad + \Lambda\left(1-\lambda\Sigma^H\mu_g\right)\ \widehat{g}_t - \Lambda(1-\lambda)(1+\Sigma^U\mu_g)\widehat{g}_t^P
\end{aligned}
$$

where $\Lambda \equiv \frac{1}{\beta(1-\lambda\chi^H)}$

- General equilibrium

$$
\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \Lambda\lambda\Psi_t\widehat{t}_t^x - \Lambda\lambda\Psi_b\widehat{b}_{t+1}^G + \Lambda\Psi_g\widehat{g}_t
$$

where

$$
\Psi_t \equiv \frac{\beta[1-\rho_t(1-\phi_b)]}{1-\phi_b} \qquad ; \qquad \Psi_b \equiv \frac{\beta[\phi_b-(1-\beta)]\phi_b}{1-\phi_b}
$$

$$
\Psi_g \equiv \left(1-\lambda\Sigma^H\mu_g\right)(1-\beta\rho_g) + \frac{\lambda[\phi_b-(1-\beta)]}{1-\phi_b} - (1-\lambda)(1+\Sigma^U\mu_g)(1-\beta)
$$

- Equilibrium dynamics

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \Lambda\lambda\Psi_t \widehat{t}_t^x - \Lambda\lambda\Psi_b \widehat{b}_{t+1}^G + \Lambda\Psi_g \widehat{g}_t$$

$$\widehat{b}_{t+1}^G = \frac{1-\phi_b}{\beta}\widehat{b}_t^G + \frac{1}{\beta}(\widehat{g}_t - \widehat{t}_t^x)$$

- Solution

$$\widehat{y}_t = -\frac{\lambda\phi_b}{1-\lambda\chi^H}\widehat{b}_t^G + \frac{1-\lambda\Sigma^H\mu_g}{1-\lambda\chi^H}\widehat{g}_t - \frac{\lambda}{1-\lambda\chi^H}\widehat{t}_t^x$$

- RANK: $\lambda = 0$ (Woodford 2011)

$$\widehat{y}_t = \widehat{g}_t$$

- No income composition bias: $\Xi^H = \Theta^H = \Omega^H \Rightarrow \Sigma^H = 0$, $\chi^H = \Omega^H$

$$\widehat{y}_t = \frac{1}{1-\lambda\Omega^H}\widehat{g}_t$$

- Tax-financed increase government purchases ($\widehat{g}_t = \widehat{t}_t^x$):

$$\widehat{y}_t = \frac{1 - \lambda(1 + \Sigma^H \mu_g)}{1 - \lambda \chi^H} \widehat{g}_t$$

where $\Phi_g \equiv \frac{1-\lambda(1+\Sigma^H \mu_g)}{1-\lambda\chi^H}$ is the "balanced-budget multiplier" ($\lesseqgtr \Phi_g^{RA}$)

(i) $\Phi_g > 0$ if and only if $\lambda(1 + \Sigma^H \mu_g) < 1$

(ii) $\Phi_g > 1$ if and only $\Omega^H + \Sigma^H(\mu_y - \mu_g) > 1$

- Standard TANK (Bilbiie 2008): $\Xi^H = 1$, $\Theta^H = 0$, $\Sigma^H = \frac{1}{\mathcal{M}}$, $\chi^H = \frac{1+\mu_y}{\mathcal{M}}$

$$\widehat{y}_t = \frac{\mathcal{M} - \lambda \mu_g}{\mathcal{M} - \lambda(1 + \mu_y)} \widehat{g}_t$$

with multiplier larger than one if $1 + \mu_y > \mu_g$

Possible Extensions

- Investment: explicit role for anticipated output ("animal spirits")
- Open economy: wealth effects of terms of trade on consumption ("real income channel")
- ...

Concluding Remarks

- Standard formulations/expositions of the aggregate demand block in the NK model (RANK and TANK) are not fully satisfactory:

  (i) Dinamic IS equation: combines PE and GE, hiding underlying mechanisms
  (ii) Interest rate rule: does not focus on the relevant interest rate
  Proposed reformulation:
  (i) Focus on the consumption function, explicit PE/GE distinction
  (ii) Real interest rate rule

- Proof of concept through several applications
- Unexpected outcome: NK model with no reference to inflation (nor to money)

$\Rightarrow$ *Real (New) Keynesian Model*

- Advantage: no need to take a stand on the right model of inflation
- Advantage vs KC or IS/LM models: no need to assume constant prices
- Inflation in Keynes' *General Theory*