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Falling Interest Rates and Credit Reallocation: Lessons from General Equilibrium

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We show that in a canonical model with heterogeneous entrepreneurs, financial frictions, and an imperfectly elastic supply of capital, a fall in the interest rate has an ambiguous effect on aggregate economic activity. In partial equilibrium, a lower interest rate raises aggregate investment both by relaxing financial constraints and by prompting relatively less productive entrepreneurs to invest. In general equilibrium, however, this higher demand for capital raises its price and crowds out investment by more productive entrepreneurs. When this reallocation is strong enough, a fall in the interest rate reduces aggregate output. A numerical exploration of the model suggests that this reallocation effect is quantitatively significant and—in response to persistent changes in the interest rate—stronger than the traditional balance-sheet channel. We provide evidence of the reallocation effect using U.S. firm-level data.

Key words: Interest rates, Financial frictions, Firm heterogeneity, Allocation, Asset prices, Monetary policy

JEL codes: E22, E32, E44, E52

The editor in charge of this paper was Kurt Mitman.

1. INTRODUCTION

One distinctive feature of recent decades has been the sustained and significant decline in real interest rates across the globe. Although conventional wisdom holds that declining interest rates should stimulate economic activity, there are mounting concerns that such declines—at least when they are persistent—may have undesirable side effects, such as endangering financial stability (Rajan, 2015; Coimbra and Rey, 2017; Martinez-Miera and Repullo, 2017; Brunnermeier and Koby, 2018; Bolton *et al.*, 2021) or slowing-down the pace of technological innovation and long-term growth (Liu *et al.*, 2019; Benigno *et al.*, 2020; Quadrini, 2020). Moreover, recent evidence suggests that periods of fast credit growth fueled by low interest rates can also result in lacklustre productivity performance (Reis, 2013; Gopinath *et al.*, 2017; Doerr, 2018; García-Santana *et al.*, 2020) .

We contribute to this debate by proposing a novel mechanism through which declining interest rates can foster the proliferation of *socially* unproductive activities. We consider a canonical economy populated by entrepreneurs who have the ability to invest in capital. We make three assumptions. First, entrepreneurs are heterogeneous in their productivity, *i.e.* they differ in their effectiveness at using capital to produce consumption goods. Second, entrepreneurs face financial frictions, *i.e.* they cannot pledge the entire surplus from their activities to outsiders. Finally, the supply of capital is imperfectly elastic. The first two assumptions imply that, in equilibrium, there is heterogeneity in the marginal return to investment across entrepreneurs; the third assumption introduces general equilibrium effects. We show that, in this environment, a fall in the interest rate can further distort the allocation of capital and can therefore have an ambiguous effect on aggregate output.[1]

Our findings challenge the common notion that, in the presence of financial frictions, lower interest rates stimulate economic activity by raising both entrepreneurs' willingness and ability to invest. In our framework, the conventional view holds in partial equilibrium. In general equilibrium, however, declining interest rates may stimulate investment by the wrong mix of entrepreneurs. The logic goes as follows. Lower interest rates make it attractive for less productive entrepreneurs to invest, which raises the price of capital and crowds out investment by more productive (and financially constrained) entrepreneurs. As a result, capital is reallocated from more to less productive entrepreneurs. We formalize this general-equilibrium *reallocation effect* and show that it attenuates the stimulative effect of declining interest rates on output.

The strength of the reallocation effect depends on entrepreneurial heterogeneity, on the elasticity of the capital supply and on the severity of the financial friction. In particular, when the supply of capital is sufficiently inelastic and the financial friction is severe enough, we show that the reallocation effect becomes so strong that a fall in the interest rate is contractionary. Moreover, the reallocation effect is tied to an inefficiency.

Since the capital supply is not perfectly elastic, entrepreneurial investment affects the equilibrium price of capital. Yet, individual entrepreneurs do not internalize this effect, which gives rise to a *pecuniary externality*. In the absence of heterogeneity or financial frictions, the marginal return to investment would be equalized across entrepreneurs and this pecuniary externality would have no first-order effect on output or welfare. With heterogeneity and financial frictions, however, this is no longer the case. Simply put, less productive entrepreneurs do not

1. We focus throughout most of the paper on a small-open economy that takes the world interest rate as given. Our results extend to a closed economy where interest rate changes are driven by changes in fundamentals (see Online Appendix C.2).

internalize the crowding-out effect of their investment on that of more productive entrepreneurs. A benevolent social planner, even if she is also subject to financial frictions, would limit the investment of less productive entrepreneurs in order to free up capital and foster investment by more productive entrepreneurs. One implication of this is that a fall in the interest rate is always expansionary in the constrained efficient allocation.

We view our main contribution as conceptual: in the presence of financial frictions, the expansionary effect of a decline in the interest rate is weakened (or even overturned) by general-equilibrium reallocation effects. This mechanism requires that the price of capital depend on local economic conditions (*i.e.* domestic demand and supply for capital), which is a common feature in most macroeconomic models that emphasize the role of financial markets and balance sheets (*e.g.* Kiyotaki and Moore, 1997; Krishnamurthy, 2003; Lorenzoni, 2008; Brunnermeier and Sannikov, 2014). Indeed, just as balance-sheet effects would vanish if the price of capital were independent of local economic conditions, so would the reallocation effects that we emphasize in this paper.

In order to incorporate balance-sheet effects and study their interaction with our reallocation effects, we extend our analysis to an infinite-horizon economy. As in most macro-finance models, rising asset prices in this economy boost entrepreneurial wealth and thus relax the financial constraints of more productive entrepreneurs. This channel should amplify the stimulative effect of declining rates. We show that this intuition is only partially correct. The reason is that balance-sheet effects are driven by unexpected changes in the price of capital and are thus inherently transitory, whereas the general-equilibrium forces that drive reallocation last for as long as interest rates remain low. As a result, if the reallocation effect is strong enough, the response of output to a persistent fall in the interest rate features a transitory boom followed by a persistent bust: the balance-sheet channel temporarily raises the investment of more productive entrepreneurs, but its effect gradually wears off as the contractionary reallocation effect begins to dominate.

To explore the quantitative significance of reallocation, we calibrate the model to U.S. data. To do so, we follow the macro-finance literature and interpret capital as land or real estate (*e.g.* Kiyotaki and Moore, 1997) and set its supply elasticity to 2. This value is consistent with the average long-term real-estate supply elasticity across U.S. metropolitan areas estimated by Saiz (2010). We evaluate the effects of a gradual fall in the interest rate, from approximately 2% in the mid-1980s to 0.6% in 2018, which is consistent with the smoothed estimate of the long-term real interest rate in Bauer and Rudebusch (2020). The resulting reallocation effect is quantitatively large: whereas this fall in the interest rate would raise output by 1.8% if productivity were constant, it actually reduces output by 5.2% once the adverse general-equilibrium effects on productivity are taken into account. The key takeaway is not so much the ultimate size of the reallocation effect—which may be overestimated by assuming that all capital in the economy takes the form of land or real estate—but rather that it is of a similar magnitude as balance-sheet effects. Given that macroeconomists have devoted a substantial amount of effort to documenting the latter (Peek and Rosengren, 2000; Gan, 2007; Chaney *et al.*, 2012), our findings suggest that they should also pay close attention to the former.

These findings are consistent with recent evidence on the macroeconomic effects of credit booms driven by low interest rates. In particular, they shed further light on the experience of several Southern European economies during the early 2000s, when a reduction in interest rates coincided with local booms in credit and asset prices coupled with a decline in aggregate productivity (Gopinath *et al.*, 2017; García-Santana *et al.*, 2020). Our findings suggest that this decline in productivity may have been driven not just by the expansion of less productive activities, which is not bad in itself, but also by the crowding-out of more productive activities through general-equilibrium effects. Indeed, evidence of such adverse reallocation is found by Banerjee and Hofmann (2018), who show that—for a set of advanced economies during recent decades—an increase in the share of "zombie" (*i.e.* less productive) firms in a given sector has been associated with a decline in investment and employment by "non-zombie" (*i.e.* more productive) firms in that same sector.[2]

To provide a more direct test of our theory, we analyse the effects of interest rate changes on firm-level investment across geographical regions in the U.S. We focus on one key insight of the theory: whereas a fall in the interest rate always expands the investment of less productive firms, *i.e.* firms with low marginal product of capital (MPK), it may actually reduce the investment of more productive firms, *i.e.* firms with high MPK, due to general-equilibrium effects. We test this insight by using Compustat data to assess whether falling interest rates have a stronger impact on the investment of low-MPK (vis-à-vis high-MPK) firms in regions where general-equilibrium effects are stronger. To proxy for general-equilibrium effects at the local level, we use existing measures of real-estate supply elasticities for the U.S. This allows us to disentangle the direct effect of interest rates on firm investment from their indirect effect, which operates through real-estate prices. Our findings suggest that the general-equilibrium component is significant: the investment response to a 1.8 percentage point increase in real estate prices (equivalent to its standard deviation) is 0.23 percentage points stronger for firms that have a one standard deviation lower MPK. This is a substantial effect given that the average investment rate in the sample is 0.5%.

Our paper contributes to the literature that studies the negative side effects of credit booms on aggregate productivity (Reis, 2013; Gopinath *et al.*, 2017; Gorton and Ordonez, 2020). Gopinath *et al.* (2017), in particular, have argued that declining interest rates can lead to a fall in productivity if the ensuing rise in credit is channelled to less productive firms. However, in their setting—as in our partial-equilibrium analysis—such an expansion in credit is beneficial because even less productive firms add value from a social perspective. Our contribution is to show how declines in interest rates can actually destroy social value, potentially reducing aggregate output, once general equilibrium effects are taken into account.[3]

Our paper is also related to the growing literature on macroeconomics with heterogeneous agents. Much of this literature has studied how heterogeneity shapes an economy's response to monetary policy. Although this research focuses mostly on heterogeneity among households (*e.g.* Kaplan *et al.*, 2018; Cloyne *et al.*, 2020; Slacalek *et al.*, 2020), a growing body of work also analyses heterogeneity among firms (*e.g.* Cloyne *et al.*, 2018; Anderson and Cesa-Bianchi, forthcoming; Jeenas, 2020; Manea, 2020; Ottonello and Winberry, 2020).[4] Within this work, we are closest to Monacelli *et al.* (2018) and González *et al.* (2020), who study how interest rates shape productivity through their effects on the incentives and financial constraints of heterogeneous firms: differently from them, however, our focus is on the reallocation of resources across firms through general-equilibrium effects.

Finally, our model is related to work that stresses inefficiencies in the allocation of factors of production due to financial frictions. A recurring theme in this work is that individual firms do not internalize the effect of their demand on factor prices, which may lead to inefficient

2. For related work on "zombie lending", see also Caballero *et al.* (2008), Adalet McGowan *et al.* (2018), Tracey (2019) and Acharya *et al.* (2021).

3. Kiyotaki *et al.* (2021) have recently developed a model in which—due to financial frictions—entrepreneurs borrow predominantly against their near-term incomes, which means that a fall in the interest rate may actually tighten financial constraints.

4. Relatedly, Leahy and Thapar (2019) study how age-structure shapes the impact of monetary policy: they find that monetary policy is most potent in regions with a larger share of middle-aged due to their higher propensity for entrepreneurship. On the other hand, Caggese and Pérez-Orive (2020) show how lower interest rates may become less expansionary in economies where intangible investments become more important.

outcomes in the presence of financial constraints. Biais and Mariotti (2009), Ventura and Voth (2015), Martin *et al.* (2018), Asriyan *et al.* (2022), Buera *et al.* (2021) and Lanteri and Rampini (2021) provide examples of this work. In a related vein, Coimbra and Rey (2017) study the allocation of risky capital across financial intermediaries that are subject to financial constraints and are heterogeneous in their tolerance for risk. They show how, by reallocating capital towards riskier intermediaries, a decline in the interest rate may increase financial instability.

The paper is organized as follows. Section 2 presents the baseline model. Section 3 characterizes the equilibrium effects of declining interest rates, as well as the normative properties of equilibrium. Section 4 extends the analysis to an infinite-horizon economy and performs a numerical exploration. Section 5 provides supporting evidence, and Section 6 concludes.

2. BASELINE MODEL

Consider an economy that lasts for two periods, $t=0,1$. There are two goods: a perishable consumption good (c) and capital (k). There are two sets of risk-neutral agents, entrepreneurs and capitalists, each of unit mass.

Preferences. The preferences of all agents are given by:

$$U = E_0[c_1],$$

where c_1 is the consumption at $t=1$ and $E_0[\cdot]$ is the expectations operator at $t=0$.

Endowments. Each entrepreneur is endowed with w units of the consumption good at $t=0$, while capitalists have no endowment.

Technology. Each capitalist has access to a production technology that converts $\chi(k)$ units of the consumption good into k units of capital at $t=0$, where χ is quasi-convex and weakly increasing in k.[5] Capital can be used for production by entrepreneurs. Specifically, each entrepreneur has access to a production technology that converts one unit of capital at $t=0$ into A units of the consumption good at $t=1$. We refer to A as entrepreneurial productivity and assume that it is distributed independently across entrepreneurs, with distribution G that has an associated density g with full support on the interval $[0,1]$.

Markets. The economy is small and open and there is an international financial market where agents can borrow and lend consumption goods at a world interest rate R. Here, we introduce a central friction of our analysis by supposing that an entrepreneur can always walk away with a fraction $1-\lambda$ of her output at $t=1$. This pledgeability friction will endogenously limit the borrowing and investment that each entrepreneur can undertake. There is also a competitive capital market, where agents can trade capital at a unit price q in period $t=0$. Note that, as the economy ends at $t=1$, capital is no longer valuable after production at that date.

2.1. *Optimization and equilibrium*

Since agents can borrow and lend consumption goods in the international financial market at the interest rate R, only the clearing of the capital market is crucial for equilibrium. To characterize this market clearing condition, we analyse next the demand and supply of capital.

Capital demand. Let b_A and k_A respectively denote total borrowing and capital demand by an entrepreneur with productivity A. Given prices $\{q,R\}$, the entrepreneur makes her optimal

5. The case where capitalists are simply endowed with $\bar{K}$ units of capital is captured by the following cost function: $\chi(k)=0$ for $k\leq\bar{K}$ and $\chi(k)$ is infinite thereafter.

borrowing and investment decisions to maximize her expected consumption:

$$\max_{\{b_A, k_A\}} \quad A \cdot k_A - R \cdot b_A, \tag{1}$$

subject to budget, borrowing and feasibility constraints:

$$q \cdot k_A \leq w + b_A, \tag{2}$$

$$R \cdot b_A \leq \lambda \cdot A \cdot k_A, \tag{3}$$

$$0 \leq k_A. \tag{4}$$

Note that the price of capital q enters the budget constraint (2) but not the borrowing constraint (3): the reason, as already explained, is that the price of capital at $t = 1$ equals zero. This will change when we extend the analysis to an infinite-horizon economy in Section 4.

Optimization leads to the following capital demand:

$$k_A(q, R) \begin{cases} = 0 & \text{if } \dfrac{A}{q} < R \\ \in \left[0, \dfrac{1}{q - \frac{\lambda \cdot A}{R}} \cdot w\right] & \text{if } R = \dfrac{A}{q} \\ = \dfrac{1}{q - \frac{\lambda \cdot A}{R}} \cdot w & \text{if } \dfrac{\lambda \cdot A}{q} < R < \dfrac{A}{q} \\ = \infty & \text{if } R \leq \dfrac{\lambda \cdot A}{q} \end{cases}, \tag{5}$$

which has an associated level of borrowing:

$$b_A(q, R) = q \cdot k_A(q, R) - w. \tag{6}$$

Equation (5) states that an entrepreneur's demand of capital is decreasing in the interest rate, R. When the interest rate is greater than her return to capital, A/q, an entrepreneur finds it optimal to invest in financial markets and demands no capital. When both returns are equalized, the entrepreneur is indifferent between investing in capital and not doing so. When the interest rate is smaller than the return to capital but greater than its pledgeable return, the entrepreneur demands capital until her borrowing constraint binds.

Equation (5) also implies that the demand function $k_A(q, R)$ is decreasing in q. For an entrepreneur who is unconstrained, *i.e.* $A \leq q \cdot R$, lower values of q raise the return to investing in capital. For an entrepreneur who is constrained, lower values of q relax the borrowing constraint and enable her to expand her borrowing and her purchases of capital. Finally, the demand function $k_A(q, R)$ is weakly increasing in λ, because a higher pledgeability of output enables constrained entrepreneurs to expand their borrowing and thus their purchases of capital.

We denote the aggregate demand for capital by entrepreneurs by:

$$K^D(q, R) \equiv \int_0^1 k_A(q, R) \cdot dG(A). \tag{7}$$

Capital supply. Given the price of capital, each capitalist chooses his supply of capital to maximize profits. Formally, we use $K^S(q)$ to denote a solution to:

$$\max_{k \geq 0} \; q \cdot k - \chi(k). \tag{8}$$

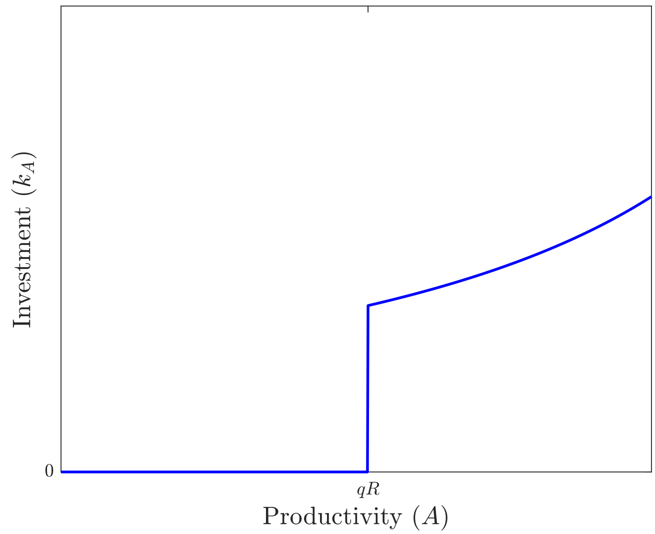


FIGURE 1

Distribution of capital across entrepreneurs given prices $\{q, R\}$

Since all capitalists are identical, $K^S(q)$ denotes both the individual and the aggregate supply of capital, which is weakly increasing in q.[6]

Market clearing. The price of capital, q, ensures that the capital market clears:

$$K^S(q) = K^D(q, R). \tag{9}$$

Aggregate domestic output is given by:

$$Y(q, R) = \int_0^1 A \cdot k_A(q, R) \cdot dG(A). \tag{10}$$

Naturally, output depends not just on the total demand of capital $K^D(q, R)$ but also on the way it is distributed across entrepreneurs.

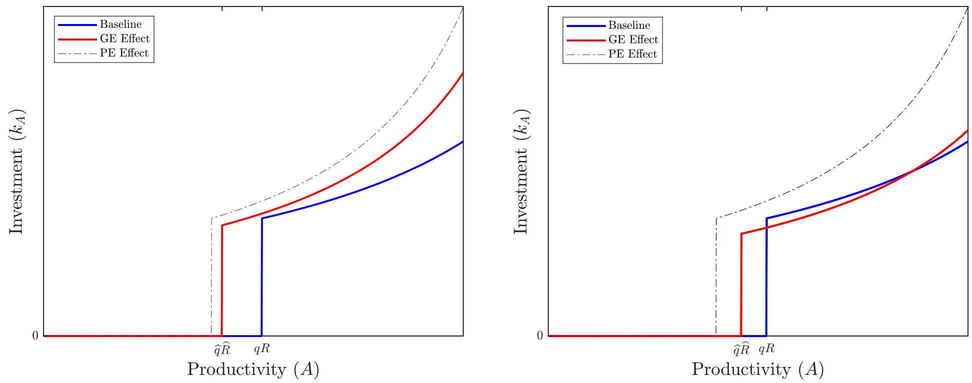
Equilibrium. Given interest rate R, an equilibrium consists of a price of capital q and allocations $\{\{k_A, b_A\}_A, K^S, Y\}$, such that $\{k_A, b_A\}_A$ satisfy equations (5) and (6), K^S is a solution to (8), the capital market clears according to equation (9), and Y satisfies equation (10).

3. EQUILIBRIUM EFFECTS OF CHANGES IN INTEREST RATES

We want to understand the equilibrium effects of changes in the interest rate R. For now, we interpret changes in R as being induced by exogenous factors, such as changes in the world interest rate or in capital inflows. In Online Appendix C.2, we show that our results also hold when R is endogenous and its decline is driven by changes in model fundamentals.

Figure 1 depicts the distribution of capital across entrepreneurs for given prices $\{q, R\}$. To determine the aggregate effects of a change in R, we need to understand how this distribution of capital responds to such a change. In what follows, we refer to those entrepreneurs who

6. *e.g.* if $\chi(\cdot)$ is increasing and convex with $\chi(0) = \chi'(0) = 0$, then $K^S(q) = \chi'^{-1}(q)$ and is increasing in q.



(a) Weak GE Effects **(b)** Strong GE Effects

FIGURE 2

Partial-and general-equilibrium effects of a fall in the interest rate

find it optimal not to invest (*i.e.* $A < q \cdot R$) as "infra-marginal", to those that invest until their borrowing constraint binds (*i.e.* $A > q \cdot R$) as "supra-marginal", and to those that are indifferent (*i.e.* $A = q \cdot R$) as "marginal" entrepreneurs.

At first sight, the effect of a change in R on investment and output seems trivial. It follows immediately from equation (5) that, for a given price q, all entrepreneurs raise their demand of capital when R falls. Supra-marginal entrepreneurs demand more capital because a lower value of R raises the present value of pledgeable output, thus relaxing their borrowing constraints. Moreover, some infra-marginal entrepreneurs start investing because a lower value of R raises the present value of their investment. This partial-equilibrium effect of a decline in R is depicted through a shift from the solid-blue (which begins at productivity qR) to the dashed curve in panels (a) and (b) of Figure 2.

As long as the supply of capital is not perfectly elastic, however, the effects of a decline in R do not end here. There is also a general-equilibrium effect because the price of capital q must increase to ensure capital-market clearing. This effect makes capital less attractive and reduces investment along all margins, but it cannot be so strong as to raise the productivity of the marginal entrepreneur, $q \cdot R$: otherwise, all entrepreneurs would reduce their demand of capital, which is a contradiction. This general-equilibrium effect of a decline in R is depicted through a shift from the dashed to the solid-red curve (which begins at productivity $\widehat{q}\widehat{R}$) in panels (a) and (b) of Figure 2.

This discussion suggests that a decline in the interest rate must necessarily raise the investment of some infra-marginal entrepreneurs, although it may reduce the investment of some supra-marginal entrepreneurs. Formally, the change in the investment of a supra-marginal entrepreneur with productivity A is given by:

$$\frac{dk_A}{dR} = \frac{\left|\frac{dq}{dR}\right| - \frac{\lambda \cdot A}{R^2}}{q - \frac{\lambda \cdot A}{R}} \cdot k_A, \tag{11}$$

which has both a partial- and a general-equilibrium component. The second term in the numerator represents the partial-equilibrium effect, by which a decline in R increases the pledgeable return to capital and thus entrepreneurs' ability to invest. The first term in the numerator captures instead the general-equilibrium effect, by which a decline in R raises the price of capital thereby reducing entrepreneurial demand of capital. Equation (11) suggests that the investment of supra-marginal entrepreneurs may decline when the interest rate falls, provided that the general-equilibrium effect is strong and the partial-equilibrium effect is weak enough.

The strength of the general-equilibrium effect depends largely on the elasticity of capital supply, while the strength of the partial-equilibrium effect depends on the tightness of the financial friction. As $\lambda \to 0$, for instance, the partial-equilibrium effect disappears altogether. In panels (a) and (b) in Figure 2, the overall effect of a fall in the interest rate on investment is captured by the shift from the solid blue (dark) to the solid red (light) line. In panel (a), general-equilibrium effects are weak and all supra-marginal entrepreneurs invest more when the interest rate falls; in panel (b), instead, general-equilibrium effects are strong and some supra-marginal entrepreneurs invest less when the interest rate declines.

The effect of a change in the interest rate on output is shaped by the behaviour of investment across all entrepreneurs. Formally, we can combine equations (9) and (10) to obtain:

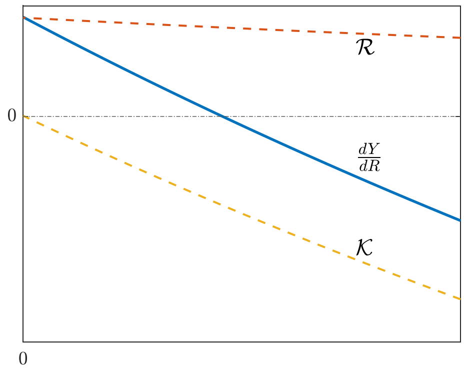
$$\frac{dY}{dR} = \underbrace{\int_{q\cdot R}^{1} (A - q\cdot R)\cdot \frac{dk_A}{dR}\cdot dG(A)}_{\text{Capital-reallocation effect} \equiv \mathcal{R}} + \underbrace{q\cdot R\cdot \frac{dK^S(q)}{dR}}_{\text{Capital-supply effect} \equiv \mathcal{K}}, \tag{12}$$

where dk_A/dR is given by equation (11). The first term in equation (12), which we denote by $\mathcal{R}$, captures the capital-reallocation effect: the change in output driven by changes in the investment of supra-marginal entrepreneurs. As we have already noted, this effect can be positive or negative depending on the relative strength of the general- and partial-equilibrium effects. In what follows, we say that the capital-reallocation effect is *stronger* when $\mathcal{R}$ is more positive. The second term in equation (12), which we denote by $\mathcal{K}$, captures instead the capital-supply effect: the change in output driven by adjustments in the aggregate capital stock. This effect is always (weakly) negative, since lower interest rates raise the demand for capital and thus the equilibrium stock of capital. In what follows, we say that the capital-supply effect is *stronger* when $\mathcal{K}$ is more negative.

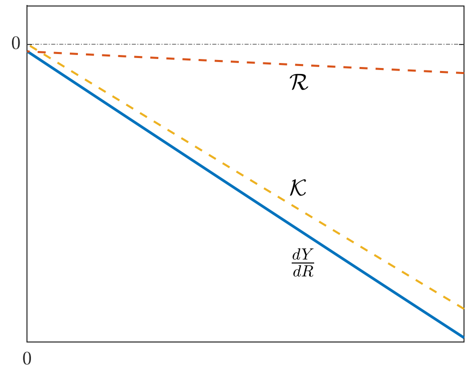
Equation (12) illustrates the role of both heterogeneity and financial frictions in shaping the aggregate response of output to changes in the interest rate. In the absence of heterogeneity, all entrepreneurs would have the same productivity; in the absence of financial frictions, only the most productive entrepreneur would invest. In either case, the capital-reallocation effect would disappear and the response of output to the interest rate would be negative and driven only by the capital-supply effect, *i.e.* on the economy's ability to adjust the capital supply to the shifting demand. With heterogeneity and financial frictions, however, the response of output to changes in the interest rate depends not just on the behaviour of aggregate investment but also on its reallocation across entrepreneurs. In fact, the capital reallocation effect can be so strong that falling interest rates may become contractionary.

To illustrate this possibility, consider a simple example in which there is no borrowing (*i.e.* $\lambda = 0$) and the capital stock is fixed (*i.e.* $K^S(q) = \bar{K}$). The lack of borrowing means that the investment of supra-marginal entrepreneurs is equal to w/q, and thus independent of the interest rate: this maximizes the strength of the reallocation effect, $\mathcal{R}$. The fixed capital supply, in turn, completely eliminates the capital supply effect, $\mathcal{K}$. Therefore, a decline in the interest rate must necessarily reduce aggregate output. By boosting the investment of unconstrained infra-marginal entrepreneurs, a lower interest rate raises the equilibrium price of capital and thus reduces supra-marginal investment, which is productive but constrained.

This example is of course stark but, as the next proposition shows, the result is more general and does not rely on such extreme scenarios.



(a) Low λ



(b) High λ

FIGURE 3

Effect of the elasticity of capital supply on the response of output to a change in the interest rate

Proposition 1. *Consider two economies that have the same equilibrium allocations and are identical in all respects except for the capital supply schedule. Let ε denote the elasticity of capital supply with respect to the price of capital q in equilibrium. Then, in the economy with lower ε:*

- *the capital-reallocation effect, $\mathcal{R}$, is stronger;*
- *the capital-supply effect, $\mathcal{K}$, is weaker;*
- *the response of output to a change in the interest rate, dY/dR, is greater;*

moreover, for low enough ε, there is a threshold $\bar{\lambda}_\varepsilon > 0$ such that dY/dR is positive if $\lambda < \bar{\lambda}_\varepsilon$.

Proposition 1 states that the response of output to the interest rate is decreasing in the elasticity of the capital supply, ε, which governs the strength of the general-equilibrium effect.[7] This is illustrated in Figure 3, which plots dY/dR against ε for low and high values of λ, respectively. Both panels show that dY/dR increases as ε decreases. Lower values of ε weaken the capital-supply effect and, by reinforcing the general-equilibrium response of the price of capital, strengthen the capital-reallocation effect. When λ is low, moreover, the reallocation effect becomes positive and—for low values of ε—a fall in the interest rate leads to a decline in aggregate output (see panel (a)).[8]

Proposition 1 shows that, in general-equilibrium, the interaction of heterogeneous productivity and financial frictions gives rise to capital-reallocation effect that can mitigate or even overturn the expansionary effects of declining interest rates. Although it has been derived in a fairly stylized environment, we discuss next how it extends to more general settings.

3.1. *Robustness*

In our baseline model, entrepreneurs operate a linear production technology. As a result, an entrepreneur's marginal productivity is constant and coincides with her average productivity. Moreover, entrepreneurs who operate larger firms (*i.e.* those with higher A) are also those who are more financially constrained. We next show that our main insights extend naturally to settings

7. The parametrization of the cost function $\chi(\cdot)$ used for Figure 3 is provided in the proof of Proposition 1.

8. The result in Proposition 1 is local, in the sense that it characterizes dY/dR at a given equilibrium. In particular, if the equilibrium changes—*e.g.* due to a change in R—so does the strength of the reallocation effect and thus the threshold $\bar{\lambda}_\varepsilon$. This implies that, for given parameter values, the sign of dY/dR need not be the same for all levels of R.

with diminishing returns, where marginal and average productivity need not coincide, and where large productive firms may be unconstrained.

We make the following two adjustments to the setup in Section 2. First, we assume that entrepreneurs have a diminishing-returns technology $k \mapsto A \cdot f(k)$, with $f'(\cdot) > 0 > f''(\cdot)$ and $\lim_{k\to 0} f'(k) = \infty$. Second, we allow for the possibility of large, unconstrained firms by letting endowments vary across entrepreneurs. We denote the joint distribution of (A, w) by G.

Given prices (q, R), let $k_A^*(q, R)$ denote the first-best investment scale for an entrepreneur with productivity A, which satisfies $\frac{A\cdot f'(k_A^*)}{q} = R$. Optimization leads to the following capital demand:

$$k_{(A,w)}(q,R) = \begin{cases} k_A^*(q,R) & \text{if } q\cdot k_A^*(q,R) \le w + \dfrac{\lambda\cdot A\cdot f\left(k_A^*(q,R)\right)}{R} \\ k : q\cdot k = w + \dfrac{\lambda \cdot A\cdot f(k)}{R} & \text{otherwise} \end{cases}. \tag{13}$$

This economy features two sets of entrepreneurs: those who are financially unconstrained, with an MPK, $A \cdot f'(k)$, *equal* to $q \cdot R$; and those who are financially constrained, with MPK *above* $q \cdot R$.[9] Note that an entrepreneur is unconstrained if her endowment, w, is large relative to her first-best investment scale, k_A^*.[10]

The capital price q ensures market clearing:

$$K^S(q) = \int k_{(A,w)}(q,R)\cdot dG, \tag{14}$$

and aggregate output is given by:

$$Y = \int A\cdot f\left(k_{(A,w)}(q,R)\right)\cdot dG. \tag{15}$$

The response of output to a change in the interest rate can be expressed as follows:

$$\frac{dY}{dR} = \int_{(A,w)\in\mathcal{C}} \left(A\cdot f'(k_{(A,w)}(q,R)) - q\cdot R\right)\cdot \frac{dk_{(A,w)}(q,R)}{dR}\cdot dG + q\cdot R\cdot \frac{dK^S(q)}{dR}, \tag{16}$$

where $\mathcal{C} = \{(A,w) : A\cdot f'(k_{(A,w)}(q,R)) > q\cdot R\}$ is the set of constrained entrepreneurs.

Equation (16) is essentially the same as equation (12), with the only difference that now entrepreneurs' marginal productivity of capital, $A \cdot f'(k)$, depends on their scale of production. It is then straightforward to extend the result in Proposition 1, where the reallocation effect now refers to the reallocation of capital from higher MPK (*i.e.* more constrained) towards lower MPK (*i.e.* less constrained) entrepreneurs in response to a fall in the interest rate. This setting generalizes our baseline model and allows for the presence of large-unconstrained firms (as in Khan and Thomas (2013)) if we suppose that entrepreneurs with high productivity have sufficiently high endowments (*i.e.* there is a positive correlation between A and w). However, these productive, large entrepreneurs have a low-MPK because they are unconstrained and thus invest until their marginal product of capital equals $q \cdot R$.

9. In our baseline model, the marginal entrepreneurs are also unconstrained, as their MPK is $q \cdot R$. However, in contrast to this setting, they have an infinitesimal mass.

10. Alternatively, we could allow entrepreneurs to differ in the severity of the financial friction that they face, *i.e.* heterogeneous λ's (capturing, *e.g.* more or less reputable firms), where those with sufficiently high λ are unconstrained.

In the Online Appendix, we explore other extensions of our baseline setting. Online Appendix C.1 explores an extension with credit risk, in which entrepreneurs differ in their probability of success and can thus default. Our main result is unaffected, although it is now the heterogeneity in entrepreneurs' *expected* productivity that matters for the capital-reallocation effect. Online Appendix C.2 explores a closed-economy version of our setting, in which the interest rate is determined endogenously and its fall is prompted by an increased desire to save (*i.e.* a savings glut). Thus, our findings are consistent with one of the most common hypothesis to explain the sustained decline in interest rates over the past several decades (*e.g.* Bernanke, 2005; Caballero *et al.*, 2008).

Perhaps the main limitation of the baseline economy is that it is essentially static, as it lasts for only two periods, and entrepreneurial wealth is exogenous. In a fully dynamic economy, entrepreneurial wealth would naturally be endogenous to (i) productivity, as more productive entrepreneurs may accumulate wealth faster; and potentially (ii) asset prices, due to the well-known balance-sheet effects à la Kiyotaki and Moore (1997). Section 4 extends the analysis to an infinite-horizon economy and shows how the reallocation effect highlighted here interacts with wealth accumulation and balance-sheet effects. Before moving to the fully dynamic model, however, we turn to the normative properties of equilibrium.

3.2. *Normative properties*

We now analyse the extent to which the competitive equilibrium is constrained (in)efficient. As we show, the general-equilibrium induced reallocation that we identified in the previous section is closely linked to an externality, by which the investment of some less productive entrepreneurs is excessive from a social point of view.

Consider the problem of a social planner whose objective is to maximize aggregate consumption at $t = 1$.[11] The planner is constrained to only choosing investment $\{k_A\}$ for entrepreneurs, but agents make all the other decisions on their own. This implies, in particular, that the planner must respect individual budget and financial constraints. To simplify the exposition, we assume throughout that $\chi(\cdot)$ is strictly convex.

Formally, the social planner solves the following maximization problem:

$$\max_{\{k_A\}} \int_0^1 A \cdot k_A \cdot dG(A) - R \cdot [\chi(K^S) - w], \tag{17}$$

subject to:

$$R \cdot (q \cdot k_A - w) \leq \lambda \cdot A \cdot k_A \quad \text{and} \quad 0 \leq k_A \quad \forall A, \tag{18}$$

and to capitalists' optimization and market clearing:

$$\chi'^{-1}(q) = K^S = \int_0^1 k_A \cdot dG(A). \tag{19}$$

The objective function of the planner in (17) says that aggregate consumption at $t = 1$ is equal to aggregate output net of repayments to international lenders, which are in turn given by R times the difference between the cost of capital production and aggregate endowment at $t = 0$. Equations (18) and (19) state that the planner must respect individual budget and financial

11. Since preferences are linear, such an objective is equivalent to maximizing the equally weighted aggregate welfare. We thus abstract from distributional considerations.

constraints, feasibility, and market clearing. In particular, the planner is not able to make transfers so as to overcome financial frictions.

To understand the solution to the planner's problem, consider the (social) net present value (NPV_A^{SP}) of a unit of investment, k_A, by entrepreneur with productivity A:

$$\text{NPV}_A^{SP} \equiv \frac{A}{R} - q - \left[\chi''\left(K^S\right) \cdot \int \gamma_{\widehat{A}} \cdot k_{\widehat{A}} \cdot dG(\widehat{A})\right] \tag{20}$$

where γ_A denotes the multiplier on the borrowing constraints of entrepreneurs with productivity A. The first observation is that, since NPV_A^{SP} is linear and increasing in A, there exists a marginal entrepreneur $\widetilde{A}$ with $\text{NPV}_{\widetilde{A}}^{SP} = 0$, such that only entrepreneurs with productivity above $\widetilde{A}$ invest and they do so until their borrowing constraints bind. The second observation is that, since the term in brackets is positive, the planner perceives a higher social cost (or a lower social benefit) of investment than individual entrepreneurs, who only compare A/R to q. This is because the planner internalizes that each additional unit of investment raises the equilibrium price of capital (as $\chi''(K^S) > 0$) and thus tightens the borrowing constraints of all entrepreneurs with productivity above $\widetilde{A}$. Since the borrowing constraints bind for all such entrepreneurs (as $\gamma_A > 0$ $\forall A > \widetilde{A}$), this entails a first-order welfare loss. Consequently, the planner restricts investment by some entrepreneurs by setting $\widetilde{A} > q \cdot R$.

The following proposition summarizes the above discussion and also states its main implication for the response of output to changes in interest rates.

Proposition 2. *Let $\widetilde{A}$ denote the productivity of the marginal entrepreneur at the social planner allocation, and q^{CE} denote the price of capital in the competitive equilibrium. Then:*

$$\widetilde{A} > R \cdot q^{CE},$$

i.e. relative to the competitive equilibrium, the planner restricts investment by some supra-marginal entrepreneurs. Moreover, letting Y^{SP} denote output in the social planner allocation, it holds that:

$$\frac{dY^{SP}}{dR} < 0,$$

i.e. a fall in the interest rate is always expansionary in the social planner's allocation.

The first part of Proposition 2 follows directly from our previous discussion. It illustrates that the planner forbids some entrepreneurs from investing altogether: by doing so, she enables more productive entrepreneurs to expand their investment, as their constraints are relaxed when capital becomes less scarce (*i.e.* when the price of capital falls). The second part of Proposition 2 follows directly from the first. To see this, simply note that a fall in the interest rate can only reduce output if it reallocates capital from supra- to infra-marginal entrepreneurs (see equation (12)). But the planner can always keep these reallocation effects under control by adjusting the productivity of the marginal entrepreneur, $\widetilde{A}$, when the interest rate changes.

This type of planner intervention, which prevents some entrepreneurs from investing altogether, may seem far-fetched and informationally demanding for the planner (*i.e.* she needs to know which entrepreneurs to exclude). However, it is straightforward to show that the planner allocation can be decentralized through a simple Pigouvian subsidy τ on savings, financed with lump-sum taxes on capitalists. By choosing the subsidy appropriately, the planner can ensure that all entrepreneurs with productivity lower than $\widetilde{A}$ prefer to save their endowments at the market interest rate and collect the subsidy rather than investing in capital.

These results are reminiscent of the literature on "zombie" firms, which emphasizes that low interest rates incentivize unproductive activities (Caballero *et al.*, 2008; Adalet McGowan *et al.*, 2018; Banerjee and Hofmann, 2018; Tracey, 2019; Acharya *et al.*, 2021). In much of that literature, however, the emphasis is on distortions that provide incentives to keep some firms operational even though they are not profitable (*e.g.* evergreening by banks). We show instead that investment can be socially excessive despite having a positive net present value from a private standpoint, because individual entrepreneurs do not internalize the crowding-out effect that they have on more productive investment.

3.3. *How do we interpret capital?*

The main insight of our paper is that, in the presence of heterogeneity and financial frictions, the expansionary effect of a decline in the interest rate is weakened (or even overturned) by general-equilibrium reallocation effects. This mechanism requires the price of capital to depend on local economic conditions (*i.e.* domestic demand and supply for capital), which is a common feature in most macroeconomic models that emphasize the role of financial markets and balance sheets (*e.g.* Kiyotaki and Moore, 1997; Krishnamurthy, 2003; Lorenzoni, 2008; Brunnermeier and Sannikov, 2014). Indeed, just as balance-sheet effects would vanish if the price of capital were independent of local economic conditions, so would our reallocation effect.

Going beyond the conceptual insight, how can we interpret capital in the data? The literature has often thought of physical assets, which are owned by firms and give rise to balance sheet effects (*e.g.* Kiyotaki and Moore, 1997). Within these assets, moreover, the literature has mostly focused on land or real estate, where the importance of general equilibrium effects (*e.g.* as induced by an imperfectly elastic supply) seems most natural. This is in opposition to machinery, for instance, whose price is less dependent on local economic conditions because it can be traded internationally. In the remainder of the paper, we follow the literature and interpret capital as land or real estate. By doing so, we can incorporate the classic balance-sheet effect and show how it interact with the reallocation effect that we emphasize here.

Our key insight is not limited to physical assets, however, and could arise in relation to any factor of production that requires credit. One example is skilled labour, which (i) requires working capital insofar as firms pay wages before output is realized, and (ii) is relatively scarce, at least in the medium-term. Under this interpretation, there are no balance sheet effects (as firms do not own their workers) but there still are reallocation effects: by increasing the demand for skilled labour by relatively less productive firms, a decline in interest rates would raise skilled wages thereby crowding out labour demand by relatively more productive firms.[12]

4. DYNAMICS OF WEALTH ACCUMULATION

We now extend the analysis to a dynamic economy. There are two main reasons for doing so. The first is conceptual: in a dynamic setting, changes in the interest rate are likely to have additional effects through wealth accumulation. In particular, productive entrepreneurs may stand to gain from a falling interest rate, as they benefit both through lower costs of borrowing and—potentially—through higher asset prices and their associated balance-sheet effects. The

12. Moreover, these two interpretations are not mutually exclusive. Even firms that do not use real estate directly as an input of production do so indirectly as the value of residential real estate is a key determinant of wages. Under this interpretation, a decline in the interest rate raises labour demand by relatively less productive firms, boosting the local demand for residential real estate and prompting an increase in wage that crowds out relatively more productive firms.

second reason is quantitative: a dynamic setting lends itself to a numerical exploration that can inform us about the quantitative relevance of the reallocation effect highlighted in the theory.

Throughout this section, we follow the literature and adopt a continuous-time setting. The advantage of doing so is that it enables us to obtain analytical expressions for the stationary distribution of wealth, which is a key component of equilibrium characterization. We show, however, that all of our results go through in a discrete-time economy à la Kiyotaki and Moore (1997) in Online Appendix C.3.

4.1. *A dynamic economy*

Suppose that time t is continuous. To simplify notation, we omit time subscripts whenever possible. As in the baseline model of Section 3, the economy is populated by a continuum of entrepreneurs with mass one. The key difference relative to the baseline is that entrepreneurs differ not only in their productivity, which varies stochastically, but also in their wealth, which evolves endogenously.

Individual productivity alternates stochastically according to an idiosyncratic Poisson process with common arrival rate θ. If an entrepreneur with productivity A is hit by the shock, she draws a new productivity A' from the distribution $g(A' \mid A)$, where we denote $g(A)$ as the corresponding stationary density that has full support on some interval $[\underline{A}, \bar{A}]$. Otherwise, her productivity remains unchanged. An individual's wealth w in turn evolves endogenously according to her equilibrium rate of return and her consumption choices. Absent the Poisson shock, the law of motion of entrepreneurial wealth is given by:

$$\dot{w} = y + \dot{q} \cdot k - \delta \cdot q \cdot k - r \cdot b - c, \tag{21}$$

where $y = A \cdot k$ is the output flow obtained from operating capital stock $k \geq 0$; $\dot{q}$ is the rate of change of the price of capital $q > 0$; $\delta \geq 0$ is the depreciation rate of physical capital; $r > 0$ is the interest rate on debt b, where $b < 0$ denotes savings; and $c \geq 0$ is the consumption flow.

As in the baseline economy, entrepreneurs may be financially constrained. We follow Moll (2014) and assume that, immediately after issuing debt, an entrepreneur can default, walk away with a portion $1 - \lambda \in (0, 1)$ of her capital and re-enter financial markets.[13] Formally, this gives rise to the following financial constraint:

$$b \leq \lambda \cdot q \cdot k. \tag{22}$$

Capital is supplied by a unit mass of capitalists. In particular, they have the ability to produce $I \geq 0$ units of capital per unit of time at a cost of $\chi(I) \geq 0$ in terms of the consumption good, where $\chi(\cdot)$ is increasing and quasi-convex. Thus, the aggregate capital stock K evolves according to:

$$\dot{K} = I(q) - \delta \cdot K, \tag{23}$$

where $I(q)$ is derived from the capitalists' optimization problem and is given by:

$$I(q) \equiv \underset{I \geq 0}{\arg\max} \left\{ q \cdot I - \chi(I) \right\}. \tag{24}$$

13. As in the literature, switching to a continuous-time setting requires a slight change in the nature of the financial constraint in order to maximize tractability. As we mentioned earlier, however, Online Appendix C.3 extends our results to a discrete-time setting à la Kiyotaki and Moore (1997) where the financial constraint is analogous to the one in the baseline model of Section 2.

All agents have logarithmic preferences for consumption and discount future consumption at rate $\rho > r$.[14] Since in this small open economy capitalists only affect equilibrium dynamics through the supply of capital, we ignore their consumption-savings choice in what follows.

4.2. *Equilibrium*

In any period t, entrepreneurs choose their consumption c, their capital stock k, and their debt b, given the path of asset prices and the interest rate. The optimal choice of entrepreneurs with productivity A is:

$$k = \begin{cases} \frac{1}{1-\lambda} \cdot \frac{w}{q} & \text{if } A + \dot{q} \geq (r+\delta) \cdot q \\ 0 & \text{otherwise} \end{cases}, \tag{25}$$

and

$$b = q \cdot k - w, \tag{26}$$

$$c = \rho \cdot w. \tag{27}$$

Equation (25) says that, just as in the baseline model, there is a threshold entrepreneur who is indifferent between saving or investing in capital. The only difference is that this threshold is now given by $(r+\delta) \cdot q - \dot{q}$, as part of the cost of capital is in the form of depreciation and part of the return to capital accrues in the form of capital gains. Entrepreneurs above the threshold (*i.e.* supra-marginals) borrow and invest as much as possible, whereas those below (*i.e.* infra-marginals) save at the interest rate r. Equation (27) says that, due to log-preferences, entrepreneurs consume a portion ρ of their wealth at each instant.

Substituting these optimal choices into law of motion (21), it follows that individual wealth w evolves according to:

$$\dot{w} = \begin{cases} \left[\left(\frac{A+\dot{q}}{q} - \delta - \lambda \cdot r\right) \cdot \frac{1}{1-\lambda} - \rho\right] \cdot w & \text{if } A + \dot{q} \geq (r+\delta) \cdot q \\ (r-\rho) \cdot w & \text{otherwise} \end{cases}. \tag{28}$$

Equation (28) captures the endogeneity of wealth accumulation in the dynamic economy, which depends on the interest rate and the return to capital. In particular, more productive entrepreneurs accumulate wealth at a faster pace due to their higher return on capital. Moreover, a lower interest rate has a positive effect on the wealth accumulation of supra-marginal entrepreneurs (who are borrowers) and a negative effect on that of infra-marginal entrepreneurs (who are savers). Lastly, higher capital gains boost the wealth accumulation of those entrepreneurs that invest more and thus have a higher exposure to capital.

To characterize the equilibrium, it is convenient to aggregate all entrepreneurs with the same productivity A and define their aggregate wealth as:

$$W_A \equiv \int w \cdot f(A, w) \cdot dw, \tag{29}$$

14. As is standard, entrepreneurs must be impatient relative to international lenders because otherwise they would eventually outgrow financial constraints.

where $f(A, w)$ is the share of entrepreneurs with productivity A and wealth w. We denote aggregate entrepreneurial wealth by:

$$W \equiv \int W_A \cdot dA. \tag{30}$$

By combining equation (29) with the stochastic structure of productivity shocks, it follows that W_A evolves according to:

$$\dot{W}_A = \int \dot{w} \cdot f(A, w) \cdot dw + \theta \cdot \int \left[g\left(A \mid A'\right) \cdot W_{A'} - g\left(A' \mid A\right) \cdot W_A \right] \cdot dA' .$$

The first term in this expression aggregates the evolution of individual wealth across all entrepreneurs with productivity A. The second term reflects how this pool of entrepreneurs evolves due to productivity shocks. Together with equation (28), we obtain:

$$\dot{W}_A = \begin{cases} \left[\left(\frac{A + \dot{q}}{q} - \delta - \lambda \cdot r \right) \cdot \frac{1}{1-\lambda} - \rho - \theta \right] \cdot W_A & \\ \quad + \theta \cdot \int g\left(A \mid A'\right) \cdot W_{A'} \cdot dA' & \text{if } A + \dot{q} \geq (r + \delta) \cdot q \\ (r - \rho - \theta) \cdot W_A + \theta \cdot \int g\left(A \mid A'\right) \cdot W_{A'} \cdot dA' & \text{otherwise} \end{cases} . \tag{31}$$

Per-capita investment of entrepreneurs with productivity A is then given by:

$$k_A = \begin{cases} \frac{1}{1-\lambda} \cdot \frac{W_A}{q} \cdot \frac{1}{g(A)} & \text{if } A + \dot{q} \geq (r + \delta) \cdot q \\ 0 & \text{otherwise} \end{cases} , \tag{32}$$

so that the market-clearing condition for capital can be expressed as:

$$K = \int_{A \geq (r+\delta) \cdot q - \dot{q}} k_A \cdot g(A) \cdot dA, \tag{33}$$

where K satisfies equation (23), and aggregate output is:

$$Y = \int_{A \geq (r+\delta) \cdot q - \dot{q}} A \cdot k_A \cdot g(A) \cdot dA. \tag{34}$$

Given a path of interest rates $\{r\}$, an equilibrium consists of paths of prices $\{q\}$ and allocations $\{\{W_A, k_A\}_A, W, K, Y, I\}$ such that equations (23)–(24) and (30)–(34) are satisfied in all periods.

4.3. *An illustration: reallocation and balance-sheet effects*

To illustrate how a fall in the interest rate affects the equilibrium in the dynamic economy, we focus first on a special case in which the capital supply is fixed at $\bar{K}$ (thus $\delta = 0$) and the productivity process satisfies $g(A' \mid A) = g(A')$ for all A' and A, *i.e.* the likelihood that an entrepreneur transitions to productivity A' is independent of her current productivity. This enables us to analytically characterize the steady-state effects of a permanent fall in the interest rate. It also allows us to isolate the interplay between the capital-reallocation effect—the novel transmission channel of real interest rates in our analysis—and the traditional balance-sheet effect.

4.3.1. Reallocation effects between steady states. In steady state, prices and aggregate variables are constant over time, *i.e.* $\dot{q}=0$ and $\dot{W}_A=0$ for all A. Given our assumptions on productivity, we can express the wealth of entrepreneurs with productivity A as a share of aggregate wealth W:

$$\frac{W_A}{W}=\begin{cases}\dfrac{\theta}{\theta+\rho-\frac{1}{1-\lambda}\cdot\left(\frac{A}{q}-\lambda\cdot r\right)}\cdot g(A) & \text{if } A\geq r\cdot q\\ \dfrac{\theta}{\theta+\rho-r}\cdot g(A) & \text{otherwise}\end{cases}. \tag{35}$$

Equation (35) shows that the positive link between productivity and wealth extends to the steady state. Since wealth shares must add up to one, it follows that:

$$\int_{r\cdot q}^{1}\frac{\theta}{\theta+\rho-\frac{1}{1-\lambda}\cdot\left(\frac{A}{q}-\lambda\cdot r\right)}\cdot g(A)\cdot dA=1-\int_{0}^{r\cdot q}\frac{\theta}{\theta+\rho-r}\cdot g(A)\cdot dA, \tag{36}$$

while the market clearing condition for capital can be written as:

$$\bar{K}=\left[\int_{r\cdot q}^{1}\frac{\theta}{\theta+\rho-\frac{1}{1-\lambda}\cdot\left(\frac{A}{q}-\lambda\cdot r\right)}\cdot g(A)\cdot dA\right]\cdot\frac{1}{1-\lambda}\cdot\frac{W}{q}. \tag{37}$$

Equations (35)–(37) jointly determine the steady-state values of $\{W_A\}$, W, and q as a function of r.

Equation (35) shows that, for given aggregate wealth W and price of capital q, a permanent fall in r reduces the relative wealth of infra-marginal entrepreneurs: lower interest rates transfer wealth from creditors to debtors. Together with equation (36), this implies that the productivity of the marginal entrepreneur, $r\cdot q$, must fall as savings become less attractive relative to investment. Note that equation (36) pins down the price of capital q to ensure that wealth shares are consistent with a stationary equilibrium. Finally, equation (37) implies that—given the wealth share of entrepreneurs—aggregate wealth in terms of capital (W/q) must adjust to preserve market clearing.

As in the baseline economy, the reallocation effect is driven by general-equilibrium forces in the capital market. The main difference is that aggregate wealth W is endogenous in the dynamic economy, so that reallocation is not just driven by the price of capital q but also by the ratio W/q.[15]

The effect of a fall in r on steady-state output can be obtained by combining equations (33) and (34):

$$\frac{dY}{dr}=\int_{r\cdot q}^{1}(A-r\cdot q)\cdot\frac{dk_A}{dr}\cdot g(A)\cdot dA, \tag{38}$$

which is the equivalent of equation (12) when the capital supply is fixed. As in the baseline model, the capital-reallocation effect can be positive or negative because it captures both the reallocation of capital from supra- to infra-marginal entrepreneurs (which reduces aggregate

15. In fact, the effect of a fall in r on the price of capital q is in general ambiguous in this economy. For instance, if the losses inflicted on infra-marginal entrepreneurs by a lower value of r are not compensated by the gains of supra-marginals (*e.g.* if λ is small), then equation (36) implies that q must fall in response to a fall in r. Online Appendix C.4 displays the steady state in closed form for a simpler case in which productivity is uniformly distributed on the unit interval across entrepreneurs.

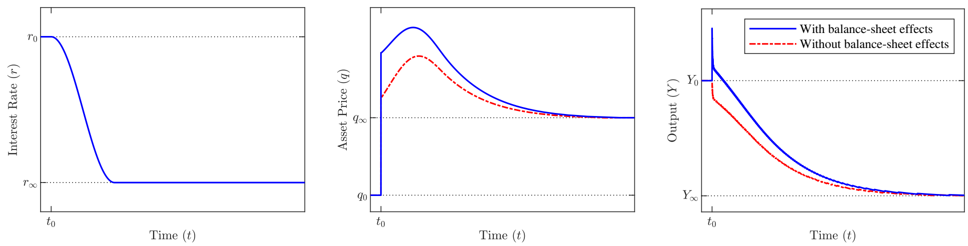


FIGURE 4

A permanent fall in the interest rate

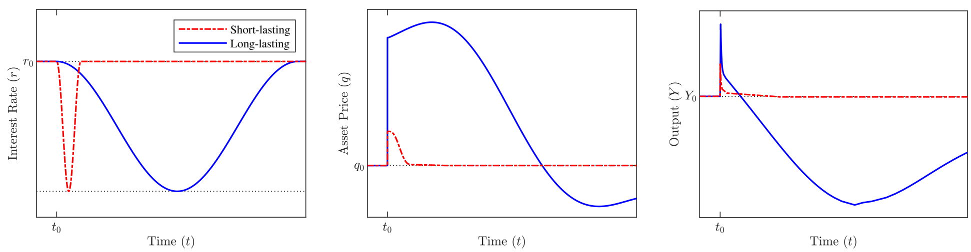


FIGURE 5

A temporary fall in the interest rate

output) and the reallocation of capital among supra-marginal entrepreneurs (which may increase aggregate output).

4.3.2. Reallocation effects in non-stationary dynamics. We consider an economy that is in steady state until time t_0, when it is hit by an unanticipated shock to the path of the interest rate. The subsequent path is deterministic and eventually converges to a stationary value.[16] We examine the economy's response to both a permanent (Figure 4) and a temporary (Figure 5) fall in the interest rate.

On impact, a fall in the interest rate increases the price of capital, which in turn raises the wealth of supra-marginal entrepreneurs—who owned the capital before the shock—while leaving the wealth of infra-marginal entrepreneurs unchanged. Formally, letting t_0 and t_0^+ denote the instants immediately before and after the shock, respectively, the wealth of entrepreneurs at time t_0^+ is given by:

$$W_{A,t_0^+} = \begin{cases} \left(\frac{q_{t_0^+}}{q_{t_0}} - \lambda\right) \cdot \frac{1}{1-\lambda} \cdot W_{A,t_0} & \text{if } A \geq \left(r_{t_0} + \delta\right) \cdot q_{t_0} \\ W_{A,t_0} & \text{otherwise} \end{cases}. \tag{39}$$

Combining equation (39) with market-clearing condition (33) at time t_0^+, it is possible to characterize the jump in the price of capital on impact,

$$\frac{q_{t_0^+}}{q_{t_0}} = \lambda \cdot \frac{\int_{A \geq r \cdot q_{t_0^+} - \dot{q}_{t_0^+}} W_{A,t_0} \cdot dA}{\int_{A \geq r \cdot q_{t_0^+} - \dot{q}_{t_0^+}} W_{A,t_0} \cdot dA - (1-\lambda) \cdot \int_{A \geq r \cdot q_{t_0}} W_{A,t_0} \cdot dA}, \tag{40}$$

16. Similar results follow if the shock is instead anticipated, but we keep our approach for tractability.

where from time t_0^+ onward, the evolution of aggregate variables $\{\{W_A, k_A\}_A, W, q, Y\}$ is characterized by equations (30)–(34).[17]

Equations (39)–(40) capture the balance-sheet effect and, together with equations (32)–(34), characterize its implications for the allocation of capital and for aggregate output. On impact, a fall in the interest rate increases the wealth of supra-marginal entrepreneurs more than proportionally to the change in the price of capital, which improves the allocation of capital and expands output.[18] This *positive* reallocation eventually vanishes, as there are no more unexpected changes to the price of capital, while the *negative* reallocation effect emphasized in our baseline model persists as long as the interest rate remains depressed.

Figure 4 illustrates these dynamics by depicting the evolution of asset prices (middle panel) and output (right panel) in response to a decline in the path of the interest rate (left panel). As the solid lines show, asset prices jump on impact and they continue to rise for a while after the shock due to the expectation of further interest-rate declines going forward. Output first expands relative to the baseline as the balance-sheet effect dominates, but it eventually contracts as the negative reallocation effect takes over.

To further isolate the effects of the balance-sheet channel, the dashed lines in Figure 4 depict a counterfactual economy in which the balance-sheet channel is fully shut down by assuming that the wealth of entrepreneurs increases proportionally to the price of capital (*i.e.* $W_{A,t_0^+}/q_{t_0^+} = W_{A,t_0}/q_{t_0}$ for all supra-marginal entrepreneurs). In this case, the lower path of the interest rate still generates an upward jump in the price of capital but this now has no effect on output. The reason is intuitive: given that the wealth of supra-marginal entrepreneurs increases alongside the price of capital, their demand of capital is unaffected by the shock.[19]

The same forces are at work if the fall in the interest rate is instead transitory, but the overall response of output may be qualitatively different. This is illustrated in Figure 5, which depicts the evolution of asset prices and output in response to a short- versus a long-lived decline in the interest rate. If the decline is sufficiently short-lived, balance-sheet effects dominate throughout and thus output expands at all horizons. As the shock becomes more persistent, the balance-sheet effect eventually vanishes, generating a boom–bust response of output.

4.4. *A numerical exploration*

We explore the model's quantitative implications by calibrating it to an annual frequency. The functional form for the production costs of capital is:

$$\chi(I) = \frac{\delta^{-\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} \cdot I^{1+\frac{1}{\varepsilon}},$$

where $\varepsilon \geq 0$ is the price-elasticity of the capital supply. Under this functional form, $I(q) = \delta \cdot q^{\varepsilon}$ and thus, in steady state, $K = q^{\varepsilon}$.

17. We omit variables $\{K, I\}$ and equations (23)–(24) in this characterization because the capital stock is fixed. To solve the dynamics from time t_0^+ onward, one needs to guess a value for $\dot{q}_{t_0^+}$ and verify that the obtained solution using (30)–(34) and (39)–(40) converges to the final steady state.

18. This positive reallocation induced by the balance-sheet channel critically depends on leverage: it becomes arbitrarily small with $\lambda \simeq 0$, as the wealth of supra-marginal entrepreneurs becomes proportional to the value of capital.

19. On impact, output is unaffected in the absence of balance-sheet effects although it adjusts very fast given the parameter values. This finding is in line with the literature on the balance-sheet channel, which shows that balance-sheet effects rely critically on entrepreneurs' inability to hedge their positions from asset-price fluctuations (see *e.g.* Krishnamurthy, 2003; Di Tella, 2017; Asriyan, 2021).

TABLE 1
The parameter values used for the numerical exercise

Parameter	Value	Source
Frequency of transition of log productivity	$\theta = 1$	Yearly frequency
Persistence of disturbance to log productivity	$\gamma_A = 0.8$	Foster *et al.* (2008)
Volatility of disturbance to log productivity	$\sigma_A = 0.45$	Bloom *et al.* (2018)
Borrowing limit (ratio of debt to assets)	$\lambda = 0.65$	Kalemli-Ozcan *et al.* (2012)
Elasticity of capital-supply	$\varepsilon = 2$	Saiz (2010)
Depreciation rate of capital	$\delta = 6.5\%$	Khan and Thomas (2008)
Subjective time discount rate	$\rho = 4\%$	Lustig *et al.* (2013)

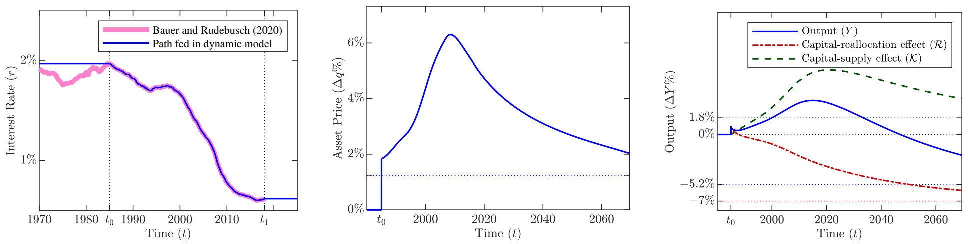


FIGURE 6

Quantitative effects from declining real interest rates

We set parameter values to match empirical estimates of variables that are directly affected by them (see Table 1 for details), and feed into the model a path of the interest rate consistent with empirical estimates in the literature (see the left-hand panel of Figure 6). We also perform a sensitivity analysis to assess the relative contribution of the three key ingredients of our model: heterogeneous productivity, financial frictions, and the elasticity of the capital supply.

In the model, idiosyncratic productivity changes when a Poisson shock arrives, in which case productivity evolves according to the transition density $g(A' \mid A)$. We set the grid of idiosyncratic productivity and its transition density to match the evolution of firm-level productivity in the data, which is well captured in logarithmic form by an AR(1) process with a persistence of $\gamma_A = 0.8$ and a volatility of disturbance $\sigma_A = 0.45$ (Foster *et al.*, 2008; Bloom *et al.*, 2018). The arrival rate of the Poisson shock is set to $\theta = 1$, so that productivity transitions every year. In the sensitivity analysis, we allow γ_A to vary between 0.7 and 0.9, values that are respectively closer to the estimates used by Khan and Thomas (2008, 2013). We also allow σ_A to vary between 0.4 and 0.5, which respectively correspond to the estimates of the interquartile range and the standard deviation of the disturbance to log of productivity in Bloom *et al.* (2018). Given parameter value $\gamma_A = 0.8$, the former value is also closer to the value required to match the cross-sectional dispersion of log of productivity estimated by Asker *et al.* (2014).[20]

For the severity of the financial friction, we set $\lambda = 0.65$ to match the ratio of liabilities to assets for non-financial companies estimated by Kalemli-Ozcan *et al.* (2012). We allow λ to vary between 0.6 and 0.7 in the sensitivity analysis to accommodate these authors' estimates for the U.S. economy and the euro area, respectively. The former value is closer to the estimate of 0.56

20. Under the AR(1) process, the cross-sectional dispersion of log of productivity is given by $\sigma_A/\sqrt{1-\gamma_A^2}$. Using firm-level data comparable to those in Foster *et al.* (2008), Bloom *et al.* (2018) and Asker *et al.* (2014) estimate a cross-sectional dispersion for the US economy of 0.63. Then, given $\gamma_A = 0.8$, $\sigma_A = 0.63\sqrt{1-0.8^2} \simeq 0.38$ is required to match the estimated dispersion.

for non-financial companies in emerging market economies by Alter and Elekdag (2020), while the latter value is closer to the leverage multiple of 4 commonly targeted in macroeconomic models with a financial sector (*e.g.* Gertler and Kiyotaki, 2010 and Gertler and Karadi, 2011).

Since we think of capital as representing land or real estate, we set the supply elasticity to $\varepsilon = 2$. This value lies between the population-weighted average (1.75) and the simple average (2.5) of the long-term real-estate supply elasticity across U.S. metropolitan areas estimated by Saiz (2010). In the sensitivity analysis, we allow this elasticity to vary between 0 and 4, the latter value being the highest in the range of the estimated elasticity distribution that results from removing its original 2%-upper tail. Lastly, we set $\delta = 6.5\%$, following Khan and Thomas (2008, 2013), and $\rho = 4\%$, which is consistent with the price-dividend ratio on equity (26.0) and the discount rate of a claim to aggregate consumption (3.51%) estimated by Lustig *et al.* (2013).[21]

We feed into the model a declining path of the interest rate based on the smoothed estimate of the long-term real interest rate in Bauer and Rudebusch (2020). We use a long-term estimate because the model is not suited for analysis at the business cycle frequency (the only aggregate shock is an unanticipated one to the interest rate). In the numerical exercise, therefore, we interpret changes in the interest rate as changes in its long-term trend.

The left panel in Figure 6 illustrates the path for the interest rate that we consider. Before the mid-1980s, the interest rate is fixed at a stationary value slightly below 2%; from the mid-1980s to the late-2010s, it follows the estimated series in Bauer and Rudebusch (2020); after the late 2010s, it is fixed again at a stationary value slightly above 0.6%. The initial and final moments of the transition between the two stationary values respectively correspond to $t_0 = 1985:\text{Q}3$—the moment in which the estimated series from 1980:Q1 onwards reaches its maximum,—and to $t_1 = 2018:\text{Q}3$—the endpoint of the series.[22] This path is consistent with historical estimates of the long-term trend in the world real interest rate by Del Negro *et al.* (2019)—who find that this trend has hovered slightly below 2% in the 1880–1980 period,—and with the widespread view first espoused by Bernanke (2005) that the world real interest rate has experienced a steady and persistent decline starting in the mid-1980s. In the exercise, we assume that the economy is at its initial steady state at $t_0 = 1985:\text{Q}3$, at which time it is hit by an unanticipated shock that reveals the entire path of the interest rate going forward. Thus, this specification implies an unanticipated, permanent decline in the interest rate of around 1.4 percentage points with a gradual transition of approximately four decades.[23]

We find that the reallocation effect is quantitatively significant, as shown in Figure 6. The fall in the interest rate initially leads to an expansion in output, with a positive reallocation of capital across entrepreneurs, a positive and increasing capital-supply effect, and a high and increasing price of capital. Eventually, however, the expansion turns into a contraction, with a negative and decreasing reallocation effect, a positive but decreasing capital-supply effect, and a high but decreasing asset price. In the long-run, the capital-reallocation effect leads to a 7% decline in

21. In the model, wealth-to-consumption ratio $W/C = 1/\rho$ can be interpreted as the price-dividend ratio on equity, because wealth W only includes internal equity of corporate businesses and the personal consumption of entrepreneurs can be thought of as dividend distributions (as in Bianchi and Bigio, 2022). The stochastic discount factor (SDF) of a representative entrepreneur whose consumption is given by C_t can be constructed as $\Lambda_t \equiv e^{-\rho \cdot t}/C_t$. Under this SDF, in steady state, the discount rate of a claim to aggregate consumption is $-\dot{\Lambda}/\Lambda = \rho$.

22. The stationary values themselves correspond to these moments, $r_0 = 1.97\%$ and $r_1 = 0.62\%$.

23. In Online Appendix C.5, we perform two sensitivity exercises. First, we allow for the same overall decline in the interest rate but with an instantaneous transition. Second, we allow for the same overall decline in the interest rate but assume that it is transitory, in the sense that it is expected to return to its original stationary value according to a path that for simplicity is symmetric at t_1.

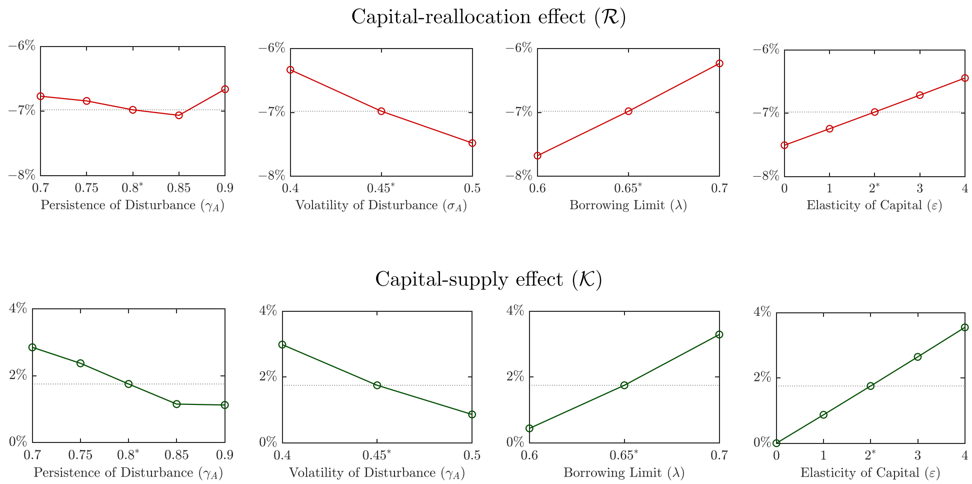


FIGURE 7

Sensitivity analysis

output, although this decline is partly offset by the increase in the capital supply.[24] Moreover, this finding is robust to changes in key parameter values, as discussed below.

We first analyse the sensitivity to changes in the distribution of entrepreneurial productivity as captured by persistence (γ_A) and volatility (σ_A). The key takeaway is that, while volatility is quantitatively relevant for the size of the reallocation effect, persistence is not (see first two columns of Figure 7). Whereas both persistence and volatility strengthen the reallocation effect by increasing the dispersion of productivity, persistence has a countervailing effect by allowing productive entrepreneurs to accumulate wealth faster.

Second, we consider the impact of changes in the financial friction (λ) and the elasticity of the capital supply (ε). As expected, a more severe financial friction and a lower elasticity of capital supply strengthen the reallocation effect and weaken the capital-supply effect (see third and fourth columns of Figure 7). However, the quantitative effect of the financial friction on reallocation is substantially stronger than that of the elasticity. This is partly explained by the fact that, in steady state, the price of capital and hence the distribution of wealth is independent of the capital-supply elasticity (see equations (35)–(36)).

5. SUPPORTING EVIDENCE

The main insight of the theoretical analysis is that, in the presence of financial frictions, the expansionary effect of a decline in the interest rate is weakened (or even overturned) by

24. As the change in the interest rate considered in this numerical exercise is discrete, we adjust equation (12) to compute the reallocation and capital-supply effects by replacing the productivity of the marginal entrepreneur, $q \cdot r$, by the average productivity of those infra-marginal entrepreneurs that demand capital after the shock:

$$\frac{Y_t - Y_0}{Y_0} = \underbrace{\frac{\int_{A \geq \hat{A}_0} \left(A - \bar{A}_t\right) \cdot \left(k_{A,t} - k_{A,0}\right) \cdot g\left(A\right) \cdot dA}{Y_0}}_{\equiv \mathcal{R}_t} + \underbrace{\bar{A}_t \cdot \frac{K_t - K_0}{Y_0}}_{\equiv \mathcal{K}_t}, \tag{41}$$

where $\bar{A}_t \equiv \int_{\hat{A}_t}^{\hat{A}_0} A \cdot k_{A,t} \cdot g(A) \cdot dA / \int_{\hat{A}_t}^{\hat{A}_0} k_{A,t} \cdot g(A) \cdot dA$ and $\hat{A}_t = (r_t + \delta) \cdot q_t - \dot{q}_t$.

general-equilibrium reallocation effects. Whereas a fall in the interest rate always expands the investment of unconstrained (low-MPK) firms, it may actually reduce the investment of constrained (high-MPK) firms due to the general-equilibrium increase in the price of capital.[25] We test this mechanism in U.S. firm-level data by assessing whether falling interest rates have a stronger impact on the investment of low-MPK (vis-à-vis high-MPK) firms when general-equilibrium effects are stronger.

5.1. *Empirical strategy*

To fix ideas, consider the following regression on firm-level investment:

$$\Delta \log k_{it+1} = \alpha_i + \alpha_{jt} + \beta \Delta \log q_{kt} + \gamma \Delta r_t + \varepsilon_{it}, \tag{42}$$

where $\Delta \log k_{it+1}$ is the investment of firm i during period t, α_i is a firm fixed effect, α_{jt} is a sector-time fixed effect, $\Delta \log q_{kt}$ is a measure of asset-price growth in region k at time t, and Δr_t is the interest rate change at time t. The effect of Δr_t is absorbed by the sector-time fixed effects.

Ideally, we would want γ in regression model (42) to capture the direct effect of interest-rate changes on investment, and β to capture the general-equilibrium effect that operates through changes in asset prices. Bringing regression model (42) to the data immediately raises two questions. First, what is the right asset on which to measure general-equilibrium effects? Second, how can we isolate the changes in asset prices that are induced by changes in the interest rate?

To address these questions, we follow the macro-finance literature and focus on real estate (Chaney *et al.*, 2012). The main advantage of doing so is that there are readily available measures of real-estate supply elasticities for the U.S. (Saiz, 2010), which we can use to obtain exogenous variation for the strength of general-equilibrium effects across regions. This allows us to estimate equation (42) using two-stage least squares (2SLS) estimation, where the first-stage regression is:

$$\Delta \log q_{kt} = \alpha_k + \alpha_t + \rho H_k \Delta r_t + \varepsilon_{kt}, \tag{43}$$

where H_k is the real estate supply elasticity in region k. We expect $\rho > 0$ and significant, implying that real estate prices are more sensitive to interest rate changes in regions where real estate supply is less elastic. The second-stage regression then replaces real estate prices with their predicted values from the first-stage regression.

To test directly the prediction of our model, we estimate the following extension of regression model (42) using 2SLS:

$$\begin{aligned}\Delta \log k_{it+1} = {} & \alpha_i + \alpha_{jt} + \beta_1 \Delta \log q_{kt} + \gamma_1 \Delta r_t \\ & + \beta_2 mpk_{it-1} \Delta \log q_{kt} + \gamma_2 mpk_{it-1} \Delta r_t + \Gamma' Z_{it-1} + \varepsilon_{it},\end{aligned} \tag{44}$$

where mpk_{it-1} is the log of the MPK of firm i at time $t-1$ and Z_{it-1} is a vector of firm-level controls including log MPK, sales growth, size, current assets as a share of total assets, total debt as a share of total assets, and the interaction of log MPK with lagged real GDP growth. The reallocation effect emphasized by the theory, by which changes in the interest rate affect lower- and higher-MPK firms differentially, is captured by β_2 and γ_2. Moreover, the theory predicts that $\beta_2 < 0$, so that the general-equilibrium component of the reallocation effect raises the relative investment of lower-MPK (vis-à-vis higher-MPK) firms.

25. This is captured in our baseline model by equation (11).

5.2. *Data*

To estimate regression models (42) and (44), we use firm-level data from the U.S. Compustat database of Standard and Poor's (2023) at quarterly frequency. The main advantages of this dataset is that it offers quarterly data for a long time period and that it contains detailed data on firm financials. A drawback is that it only covers publicly listed firms and may therefore underestimate the relevance of financial frictions in the overall economy. We provide next a brief description of the main variables used in the analysis, and relegate a more detailed explanation to Online Appendix B.

We measure investment $\Delta \log k_{it+1}$ as the logarithmic change in k_{it+1}, where k_{it+1} is the real book value of the tangible capital stock of firm i at the end of period t. We measure the real capital stock using the perpetual inventory method and changes in net property, plant and equipment.[26]

We obtain quarterly real estate prices from FHFA (2023) for each Metropolitan Statistical Area (MSA), and compute $\Delta \log q_{kt}$ as the logarithmic change in real estate prices in each MSA k in period t. We use real-estate supply elasticities at the MSA-level from Saiz (2010). Firms are assigned to MSAs based on headquarter location.

To proxy for the change in the interest rate Δr_t, we use the real interest rate on 30-year mortgages from the Primary Mortgage Market Survey of Mac (2023), as in Chaney *et al.* (2012). The advantage of this measure is that long-term interest rates are arguably more relevant for borrowing decisions of firms than short-term rates. In Online Appendix B, we also use the monetary shocks of Jarociński and Karadi (2020), who identify monetary surprises based on high-frequency data around monetary policy announcements. The advantage of this variable is that it is by construction exogenous to investment.

We compute MPK at the firm level as the product of the output-elasticity of capital times the ratio of the firm's real value added and its real capital stock. To obtain the output-elasticity of capital at the sector (one-digit SIC industry) level, we estimate production functions using Wooldridge (2009)'s generalized method of moments.

Our final dataset covers the 1990:Q1-2019:Q2 period.

5.3. *Empirical results*

We estimate equations (42) and (44) and report the results in Table 2. To ease interpretation, all explanatory variables except real estate price growth and the interest rate change are standardized over the entire sample. Standard errors are two-way clustered by firms and quarter. This increases the standard errors compared to not clustering and should therefore be seen as a conservative estimate.

Column (1) and (2) respectively present the OLS and 2SLS estimations of regression model (42) without an extended set of control variables. In the first stage, we instrument real estate price growth using the interaction between the supply elasticity and the change in the real mortgage rate, as in equation (43). The first-stage results are presented in column (1) of Online Appendix Table 3. The strong F-test supports the choice of our instrument and $\rho > 0$ is significant, indicating that—as expected—real estate prices are more sensitive to changes in interest rates in regions with a lower supply elasticity. We find that, on average, real estate price growth

26. We measure investment on a net basis by taking the log difference in the capital stock. We could have in principle constructed the net investment series using data on capital expenditures and depreciation but, as pointed out by Clementi and Palazzo (2019), this is problematic because it requires subtracting sales of capital which are under-reported in Compustat.

TABLE 2

Investment response to real estate (RE) prices and interest rates

	(1)	(2)	(3)	(4)	(5)
RE price growth	0.078***	0.066	0.058**	−0.145	−0.195
	(0.023)	(0.876)	(0.023)	(0.884)	(0.800)
MPK x RE price growth			−0.053***	−0.127***	−0.095***
			(0.017)	(0.039)	(0.037)
MPK x Interest rate change			0.021	0.022	0.004
			(0.052)	(0.058)	(0.058)
Observations	129,027	100,263	129,027	100,263	100,263
R-squared	0.149	0.151	0.169	0.170	0.179
Estimation	OLS	2SLS	OLS	2SLS	2SLS
Firm controls	no	no	no	no	yes
Time sector FE	yes	yes	yes	yes	yes
Time clustering	yes	yes	yes	yes	yes

Notes: In Columns (1) and (2), we report regression results from estimating the following specification: $\Delta \log k_{it+1} = \alpha_i + \alpha_{jt} + \beta \Delta \log q_{kt} + \varepsilon_{it}$ where α_i is a firm fixed effect, α_{jt} is a sector-time fixed effect, $\Delta \log q_{kt}$ is real RE price growth in MSA k at time t, using OLS and 2SLS respectively. In Columns (3) and (4), we report regression results from estimating the following specification: $\Delta \log k_{it+1} = \alpha_i + \alpha_{jt} + \beta_1 \Delta \log q_{kt} + \beta_2 mpk_{it-1} \Delta \log q_{kt} + \gamma_2 mpk_{it-1} \Delta r_t + \Gamma' Z_{it-1} + \varepsilon_{it}$ where α_i is a firm fixed effect, α_{jt} is a sector-time fixed effect, $\Delta \log q_{kt}$ is real RE price growth in MSA k at time t, mpk_{it-1} is the log of the MPK of firm i at time $t-1$, Δr_t is the change in real mortgage rates at time t, and Z_{it-1} is a vector containing mpk_{it-1}, using OLS and 2SLS, respectively. In column (5), we re-estimate the 2SLS regression in column (4) with an enlarged set of firm-level control variables, Z_{it-1}, containing MPK, sales growth, size, current assets as a share of total assets, total debt as a share of total assets, and the interaction of MPK with lagged real GDP growth. All explanatory variables (notably MPK) except RE price growth and the interest rate change are standardized over the entire sample. Standard errors are two-way clustered by firms and quarter. Standard errors in columns (2), (4), and (5) are bootstrapped based on 500 simulations. *** Significant at the 1 percent level. ** Significant at the 5 percent level.

is positively associated with firm investment, although not significantly so in the 2SLS estimation. One possible interpretation of this correlation is that balance-sheet effects are dominant for the average firm.

Column (3) presents the OLS estimation results of regression model (44), also without an extended set of control variables. Consistent with our theory, general-equilibrium effects as captured by real estate prices appear to hurt the relative investment of higher-MPK firms. In contrast, the direct effect of changes in interest rates appears to be similar for lower- and higher-MPK firms.

Column (4) presents our main specification, a 2SLS estimation of regression model (44). The first-stage specification is the same as that used in column (2), as presented in column (1) of Online Appendix Table 3. In the second stage, we continue to find that the relative investment of higher-MPK firms falls in response to real estate price increases. If anything, the estimated effect is larger once the potential bias in the OLS estimate is accounted for. Column (5) shows that these results are robust to the inclusion of firm controls including sales growth, size, current assets as a share of total assets, total debt as a share of total assets, and MPK and its interactions with lagged real GDP growth.[27] Finally, Table 5 in Online Appendix B shows that results are also robust to using high-frequency monetary shocks from Jarociński and Karadi (2020).

27. Online Appendix Table 4 reports the estimated coefficients of the control variables that are omitted from Table 2 for brevity.

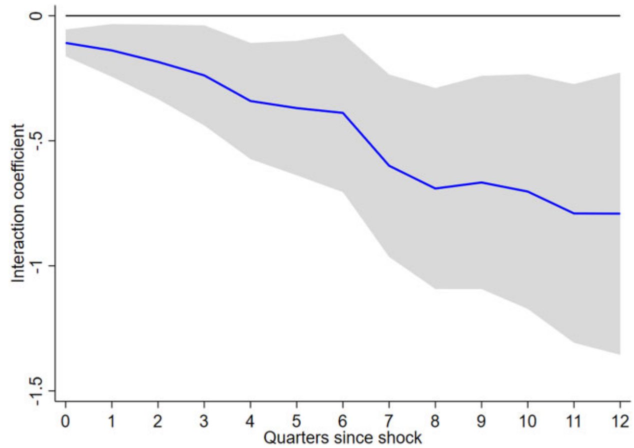


FIGURE 8

Dynamics of differential response of investment to real estate price changes

Notes: The figure displays the regression coefficients β_{2h} (the solid line) obtained when estimating the following specification: $\log k_{it+h} - \log k_{it-1} = \alpha_{ih} + \alpha_{jth} + \beta_{1h}\Delta\log q_{kt} + \beta_{2h} mpk_{it-1}\Delta\log q_{kt} + \gamma_{1h}\Delta r_t + \gamma_{2h} mpk_{it-1}\Delta r_t + \Gamma_h' Z_{it-1} + \varepsilon_{ith}$ for $h = 0, 1, 2, \ldots, 12$ and Z_{it-1} is a vector containing mpk_{it-1}. Regression is estimated using 2SLS with the interaction between the real estate supply elasticity and the real mortgage rate as instrument for real estate price growth in the first stage. The shaded area depicts the 90% confidence interval. All variables are as defined in Table 2. Standard errors are two-way clustered by firms and quarter.

These findings suggest that the general-equilibrium component plays an important role in shaping reallocation effects.[28] In particular, the estimates in column (4) imply that the investment response to a 1.8 percentage point increase in real estate prices (equivalent to its standard deviation) is 0.23 percentage points stronger for firms that have a one standard deviation lower MPK. This is a substantial effect given that the average investment rate in the sample is 0.5%. This effect is reduced to 0.18 percentage points when adding all the firm controls in column (5), which is still substantial.

Finally, we analyse the dynamic impact of the general-equilibrium effect on investment by estimating a type of local projection of equation (44) á la Jordà (2005):

$$\log k_{it+h} - \log k_{it-1} = \alpha_{ih} + \alpha_{jth} + \beta_{1h}\Delta\log q_{kt} + \gamma_{1h}\Delta r_t + \beta_{2h} mpk_{it-1}\Delta\log q_{kt} + \gamma_{2h} mpk_{it-1}\Delta r_t + \Gamma_h' Z_{it-1} + \varepsilon_{ith}, \quad (45)$$

for $h \geq 0$, where we use our 2SLS procedure. The regression coefficients β_{2h} indicate the cumulative general-equilibrium effect of changing interest rates, over the period t to $t+h$, on the relative investment of higher-MPK firms. We include the full set of control variables as in column (5) of Table 2.

The results of the local projection exercise are depicted in Figure 8. Crucially, the main finding reported in Table 2—that the general-equilibrium effect of a fall in the interest rate reduces the relative investment of higher-MPK firms—is persistent and remains significant up to twelve quarters after the shock. The strength of this cumulative effect increases over time to around −0.8, though it is estimated with large standard errors.

28. Note that our methodology does not allow us to measure the total effect (either directly or through general equilibrium) of interest rate changes on investment.

6. CONCLUSIONS

In a canonical economy with heterogeneous entrepreneurs, financial frictions, and an imperfectly elastic supply of capital, a fall in the interest rate has ambiguous effects on economic activity. In partial equilibrium, a lower interest rate raises aggregate investment both by relaxing borrowing constraints and by prompting relatively less productive entrepreneurs to increase their investment. In general equilibrium, however, this higher demand for capital raises its price and crowds-out investment by more productive (financially constrained) entrepreneurs: ultimately, there is a reallocation of capital from more to less productive entrepreneurs. When this general-equilibrium effect is strong enough, a fall in the interest rate can become contractionary.

Our mechanism requires that the price of capital depend on local economic conditions, a common feature in most macroeconomic models that emphasize the role of financial markets and balance sheets. Like most of the literature, we have focused here on real estate to show that the reallocation effect is quantitatively significant and to provide empirical evidence using U.S. firm-level data. The theory is more general, however, as our key insight could arise in relation to any factor of production that requires credit and that is in scarce supply (*e.g.* skilled labour that requires working capital).

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Supplementary Data

Supplementary data are available at *Review of Economic Studies* online.

Data Availability Statement

The data and code from this article are available in Zenodo: https://doi.org/10.5281/zenodo.10966336. Wharton Research Data Services (WRDS) was used in to access Standard and Poor's Compustat data, which is a commercial database that requires a subscription. This service and the data available thereon constitute valuable intellectual property and trade secrets of WRDS and/or its third-party suppliers.

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