Endogenous Production Networks and Non-Linear Monetary Transmission

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Abstract

I develop a tractable dynamic sticky-price model, where input-output linkages are formed endogenously. The model delivers cyclical properties of networks that are consistent with those I estimate using sectoral and firm-level data, conditional on identified real and nominal shocks. A novel source of state dependence in nominal rigidities arises: the strength of complementarities in price setting and monetary non-neutrality increase in the number of suppliers optimally chosen by firms. As a result, the model simultaneously rationalizes the following observed non-linearities in monetary transmission. First, there is *cycle dependence*: the magnitude of real GDP's response to a monetary shock is procyclical. Second, there is *path dependence*: non-neutrality of real GDP is higher following previous periods of loose monetary policy. Third, there is *size dependence*: larger monetary contractions shrink the network and generate a less than proportional decrease in GDP relative to smaller contractions.

Keywords: monetary transmission; state dependence; endogenous production networks.

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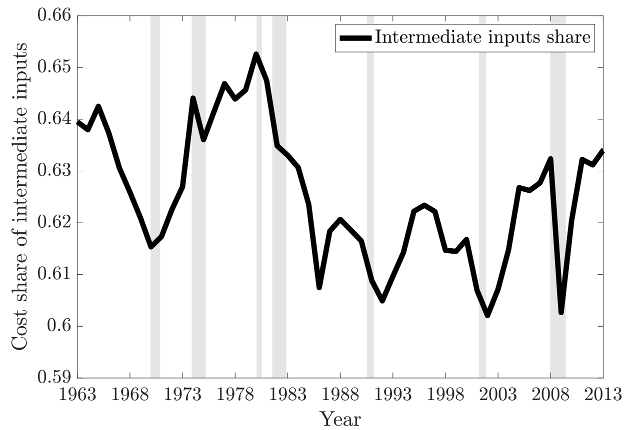
1 Introduction

Intermediate inputs, and production networks that facilitate their trade, are a cornerstone of modern economies' production structures. According to the Bureau of Economics Analysis, in the United States more than 50 per cent of all produced goods are processed as intermediate inputs. Despite intermediates being a key part of production, their relative importance falls in recessions: Figure 1 shows that the aggregate cost share of intermediates drops sharply in periods of economic slack. Moreover, one can observe that the reliance on intermediates falls both in recessions that originated in the real sector, as well as in those caused by monetary contractions, such as the Volcker Disinflation. Such network cyclicality is missed by models that take input-output linkages as given. I fill this gap by developing a dynamic general equilibrium model with sticky prices and endogenous formation of input-output linkages. The model delivers cyclical properties of production networks that are consistent with those I estimate using sectoral and firm-level data, conditional on both real and nominal shocks.

Accounting for the observed network cyclicality allows the model to rationalize multiple observed non-linearities in the transmission of monetary shocks to real variables. This is because the model links the density of the network, given by the number of suppliers each firms optimally chooses to have, with the degree of complementarities in price setting created by the roundabout production structure. Specifically, in states of the world where firms endogenously connect to more suppliers, their unit cost becomes dependent on a larger number of prices set by other firms, which strengthens pricing complementarities and amplifies monetary non-neutrality. Naturally, in states with few linkages across firms the opposite occurs: pricing complementarities weaken and monetary non-neutrality diminishes. This novel mechanism makes the strength of monetary transmission depend on the *phase of the business cycle*, *past monetary stance* and the *size of the shock* even if the probability of price adjustment is state-independent. Below I explain each of the three non-linearities step-by-step.

First, the magnitude of real GDP response to monetary shocks is *cycle-dependent*, since the degree of short-run monetary non-neutrality is stronger in expansionary states. The mechanism that generates this effect is as follows: in expansions, the level of productivity is high, which incentivizes firms to connect to more suppliers by lowering their unit costs directly through higher productivity and indirectly through lower prices charged by suppliers. The latter creates stronger non-neutrality of money due to amplified complementari-

Figure 1: **The relative importance of intermediate inputs (United States)**



Notes*: the figure documents the ratio of total expenditure on intermediate inputs and total production costs in the United States (1963-2013), based on data published by the US Bureau of Economic Analysis (BEA)*

ties in price setting. On the other hand, in recessions firms optimally decide to disconnect from some suppliers, making the economy more money-neutral, as complementarities in price setting weaken. These theoretical findings are consistent with the recent model-free econometric studies that find a weaker response of GDP to monetary shocks in recessions (Tenreyro and Thwaites, 2016; Alpanda et al., 2021; Jordà et al., 2020).

Second, there is *path dependence* in the strength of transmission, since following a monetary loosening any further monetary intervention has a stronger effect on GDP. Due to rigidities in price setting, a monetary loosening makes prices charged by supplier firms cheaper relative to the cost of in-house labor, which incentivizes firms to connect to more suppliers, hence expanding the network and strengthening complementarities in price setting. As a result, any subsequent monetary intervention has a stronger effect on GDP. Of course, the converse also holds: any monetary shocks have a weaker effect on real GDP whenever they are preceded by episodes of tight monetary policy. Such theoretical findings are consistent with the econometric evidence of stronger monetary transmission to GDP under an already loose monetary stance (Alpanda et al., 2021) and additional evidence that finds GDP to be more sensitive to monetary interventions whenever credit is loose (Jordà et al., 2020).

Third, the model produces *size dependence*, as the response of GDP does not change linearly with the size of monetary shock, even if the probability of price adjustment is state-independent. Larger monetary expansions have a disproportionally *larger* positive effect on real GDP compared to smaller monetary expansions, as the former make the network denser. As a result, achieving a given degree of real GDP expansion requires a (weakly) smaller monetary easing. On the other hand, larger monetary contractions have a disproportionally *smaller* negative effect on real GDP compared to smaller monetary contractions, as the former shrink the network. A consequence of the above is that achieving a given degree of real GDP contraction requires a (weakly) larger monetary tightening. Econometric findings in Alvarez et al. (2017) and Ascari and Haber (2022), find that large nominal shocks have a disproportionally smaller effect on real variables, compared to small nominal shocks, which is consistent with my predictions for monetary contractions.

Fourth, I provide novel reduced-form econometric evidence on network cyclicality, conditional on identified productivity and monetary shocks, which corroborates the theoretical mechanism in my model. As a first exercise, I use sectoral data from US Bureau of Economic Analysis (BEA) to construct annual time series of intermediates intensities for 65 sectors of the US economy. Consistent with the theoretical prediction, I find that intermediates intensity rises following a positive productivity shock, and following a monetary easing. Moreover, I find evidence that the effect is asymmetric: expansionary shocks, both to productivity and monetary policy, lead to disproportionally larger magnitudes of network responses. I compare estimated responses with those generated by a calibrated version of my model, and find them to be similar, both in magnitudes and in the asymmetry of responses. One limitation of using sectoral data is that it does not allow to disentangle intensive and extensive margins of network adjustment, whereas my theoretical model emphasises the extensive margin. I therefore provide additional evidence using data on firm-level linkages in Compustat, as constructed by Atalay et al. (2011), which allows to specifically test the extensive margin of network adjustment. My findings using firm-level data align with the evidence obtained using sectoral data, both in terms of directions of responses to productivity and monetary shocks, and in terms of the asymmetries detected.

Contribution to the literature. This paper makes a contribution to three strands of the literature. First, it adds to the emerging literature on endogenous production networks in macroeconomics. Seminal studies analytically characterise network formation in environments that are either entirely frictionless or feature frictions with flexible prices (Carvalho and Voigtländer, 2015; Oberfield, 2018; Taschereau-Dumouchel, 2019; Acemoglu and Azar, 2020). Subsequent studies further advance our understanding of the role of endogenous networks by adding uncertainty (Kopytov et al., 2021), or through detailed quantitative analysis that brings models with endogenous networks to the data (Lim, 2018; Huneeus, 2018). My work contributes to this literature by developing the first dynamic general equilibrium model featuring endogenous network formation under nominal rigidities. I also provide novel reduced-form econometric evidence on network cyclicality in the US, both unconditionally and conditional on identified productivity and monetary shocks, which the model successfully replicates.

Second, the paper contributes to the literature on shocks, frictions and macroeconomic policies in multi-sector models with input-output linkages. An important strand of this literature studies the dynamics of monetary transmission under *exogenous* production networks, both analytically (Ghassibe, 2021; Afrouzi and Bhattarai, 2023), quantitatively using detailed calibration (Nakamura and Steinsson, 2010; Pasten et al., 2020), as well as in fully estimated models (Carvalho et al., 2021). As for optimal monetary policy under fixed exogenous networks, it is studied in important recent papers by La'O and Tahbaz-Salehi (2022) and Rubbo (2023). A separate branch of this literature studies, both positively and normatively, fiscal policy in multi-sector models with exogenous networks (Liu, 2019; Bouakez et al., 2023). Another influential stand of this literature studies aggregation properties of any microeconomic frictions and wedges under exogenous networks (Jones, 2011; Baqaee and Farhi, 2020; Bigio and La'O, 2020). I contribute to this literature by providing novel sector-level analytical characterization of monetary transmission under *endogenous* network formation in the static version of my model. I also develop a novel numerical algorithm that allows to solve for the entire path of sectoral prices, quantities and network linkages in response to any MIT shock, nominal or real. My numerical algorithm allows me to quantify the dynamics of transmission of both productivity and monetary changes in a version of my model calibrated to 389 sectors of the US economy.

Third, my work contributes to the literature on non-linearities and state dependence in the transmission of shocks. The theoretical literature on state dependence in monetary transmission includes Santoro et al. (2014), who study asymmetric transmission of monetary policy under loss aversion; McKay and Wieland (2021) who rationalize path dependence in a framework with lumpy durable consumption demand; Alpanda et al. (2021) who explain non-linearities in a model with constraints on household borrowing and refinancing; Eichenbaum et al. (2022) who show how the effect on monetary policy depends on the distribution of savings from refinancing mortages; Berger et al. (2021) who investigates path-dependence under pre-payable mortgages; Bernstein (2021) who shows that presence of occasionally binding borrowing constraints and household heterogeneity makes responses to monetary transmission stronger in expansions and contractionary shocks more powerful than expansionary ones. There is also an important empirical strand which estimates how the strength of monetary transmission depends on the phase of the business cycle, prior monetary conditions and the size of the shock (Tenreyro and Thwaites, 2016; Jordà et al., 2020; Ascari and Haber, 2022). A separate branch of this literature studies, both theoretically and empirically, the degree of state dependence in the transmission of fiscal shocks (Auerbach and Gorodnichenko, 2012; Ramey and Zubairy, 2018; Demyanyk et al., 2019; Ghassibe and Zanetti, 2022; Jo and Zubairy, 2022). My paper contributes by developing a tractable framework that can simultaneously rationalize *multiple* non-linearities in monetary transmission through a single novel theoretical channel, which is supported by both aggregate and disaggregated data.

The remainder of the paper is structured as follows. Section 2 sets out the general theoretical framework. Section 3 develops an analytically tractable version of the model and presents the key theoretical results. Section 4 develops the numerical algorithm for solving a dynamic forward looking version of the model and quantifies the key effects. Section 5 provides novel econometric evidence that corroborates the key mechanisms of the model. Section 6 concludes.

2 Theoretical framework

I build a dynamic general equilibrium model, which unifies two environments that have so far been treated as separate in the literature. On the one hand, the model features a multi-sector input-output production structure, with firms' pricing decisions subject to sector-specific nominal rigidities. On the other hand, input-output linkages are formed *endogenously* by firms that optimize their production costs. As a result, the equilibrium production network responds to real *and* nominal disturbances, both realized and anticipated. Such cyclicality in the production network makes the strength of pricing complementarities, and hence the degree of monetary non-neutrality, depend on the underlying state of the world.

2.1 Model overview

Time is discrete, with outcomes in $t = 0$ exogenously given, and outcomes in $t \geqslant 1$ determined by agents' decisions. There are three types of agents in my model. First, a continuum of infinitely lived households. Second, a continuum of monopolistically competitive firms, owned by the households, where each firm belongs to one, and only one, of the K sectors; let the set of all firms in sector k be $\Phi_k, \forall k = 1, 2, ..., K$. Third, a government, comprising of a central bank which sets the level of money supply in the economy, and a fiscal authority which collects taxes from firms and rebates them to households as a lump-sum transfer.

A crucial feature of my economy is the presence of a production network across sectors, where the input-output linkages are formed endogenously through each firm's choice of set of suppliers, denoted by $S_k \subseteq \{1, 2, ..., K\}, \forall k, \forall j \in \Phi_k$. For every choice of supplier sectors, there is a given level of productivity pinned down by a predetermined time- and sector-specific mapping $\{\mathcal{A}_{kt}(S_{kt})\}_{t=1}^{\infty}$, and $\mathcal{A}_t \equiv [\mathcal{A}_{1t}(.), \mathcal{A}_{2t}(.), ..., \mathcal{A}_{Kt}(.)]', \forall t$. Importantly, the entire path $\{\mathcal{A}_t\}_{t=1}^{\infty}$ is known to the agents at $t = 0$, and it is expected to remain unchanged forever. For any two mappings $\underline{\mathcal{A}}$ and $\overline{\mathcal{A}}$, the convention is that $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}$ if and only if $\overline{\mathcal{A}}_k(S_k) \geqslant \underline{\mathcal{A}}_k(S_k), \forall S_k, \forall k$.

As for the nominal side of the economy, at $t = 0$ the agents know the initial level of money supply $\mathcal{M}_0$, and expect it to remain unchanged forever. At the beginning of the first period $t = 1$, they discover the future path of money supply $\{\mathcal{M}_t\}_{t=1}^{\infty}$, and there is no uncertainty from there onwards.

2.2 Firms: production and endogenous choice of suppliers

On the production side, there are K sectors, indexed by $k = 1, 2, ..., K$ with a measure one of firms in each sector; let Φ_k denote the set of all firms in sector k. The production function of firm $j \in \Phi_k$ is given by:

$$Y_{kt}(j) = \mathcal{F}_k\left[S_{kt}(j), \mathcal{A}_{kt}(S_{kt}(j)), N_{kt}(j), \{Z_{krt}(j)\}_{r \in S_{kt}(j)}\right] \tag{1}$$

where $S_{kt}(j) \subseteq \{1, 2, ..., K\}$ is the set of sectors, whose firms supply inputs to firm j in sector k at time t, $\mathcal{A}_{kt}(.)$ is a mapping from the chosen set of suppliers to the associated level of productivity at time t, $N_{kt}(j)$ is the labor input of firm j in sector k at time t, whereas $Z_{krt}(j)$ denotes purchases of intermediate inputs from sector r, which is in turn is an aggregator of purchases from all firms in that sector: $Z_{krt}(j) \equiv \left(\int_{j' \in \Phi_r} Z_{krt}(j, j')^{\frac{\theta-1}{\theta}} dj'\right)^{\frac{\theta}{\theta-1}}$, $\theta > 1$. I impose the following regularity conditions on the production function:

Assumption 1 (Production function)**.** *For every sector $k = 1, 2, ..., K$, the production function satisfies the following conditions: (a) $\mathcal{F}_k$ is strictly quasi-concave, is constant returns to scale in $(N_{kt}(j), \{Z_{krt}(j)\}_{r \in S_{kt}})$, is increasing and continuous in $\mathcal{A}_{kt}(S_{kt})$, $N_{kt}(j)$ and $\{Z_{krt}(j)\}_{r \in S_{kt}}$, and is strictly increasing in $\mathcal{A}_{kt}(S_{kt})$ when $N_{kt}(j) > 0$ and $\{Z_{krt}(j)\}_{r \in S_{kt}} > 0$; (b) labor is an essential factor of production: $\mathcal{F}_k(\cdot, \cdot, 0, \cdot) = 0$; (c) $\mathcal{A}_{kt}(\varnothing) > 0$.*

Conditional on a particular set of suppliers $S_{kt}(j)$, each firm's total cost of production at time t is given by $\left[W_t N_{kt}(j) + \sum_{r \in S_{kt}(j)} P_{rt} Z_{krt}(j)\right]$, where W_t is the nominal wage and P_{rt} is price index of sector r. Taking as given $S_{kt}(j), W_t$ and $\{P_{rt}\}_{r \in S_{kt}(j)}$, each firm chooses labor and intermediate input quantities to minimize the total cost, subject to the production function in (1). The latter delivers the following unit cost function:

$$\mathcal{Q}_{kt}(j) = \mathcal{Q}_k \left[S_{kt}(j), \mathcal{A}_{kt}(S_{kt}(j)), W_t, \{P_{rt}\}_{r \in S_{kt}(j)}\right]. \tag{2}$$

Three properties of the unit cost function should be noted. First, for a given choice of suppliers, the unit cost function is common to all firms within a given sector. Second, given the properties of the production function, $\mathcal{Q}_k$ is decreasing and continuous in $\mathcal{A}_{kt}(S_{kt}(j))$ and is increasing and homogenous of degree one in $(W_t, \{P_{rt}\}_{r \in S_{kt}(j)})$. Third, as the set of supplier sectors $S_{kt}(j)$ expands, the unit cost function becomes a function of a larger set of sectoral prices. The latter property is crucial for delivering the state-dependent degree of complementarities in price setting.

Finally, the set of suppliers is chosen optimally to minimize the unit cost of production in every period:

$$S_{kt}(j) \in \arg\min_{S_{kt}(j)} \mathcal{Q}_k \left[S_{kt}(j), \mathcal{A}_{kt}(S_{kt}(j)), W_t, \{P_{rt}\}_{r \in S_{kt}(j)}\right]. \tag{3}$$

The above minimization problem highlights the trade-off faced by firms when choosing the optimal set of suppliers: firms would like to purchase inputs from sectors whose combination delivers a high level of productivity, while at the same time avoiding those that charge high prices for their output. Notice that the optimal choice of suppliers is based on aggregate and sectoral variables only. Since all firms within a given sector are ex ante identical, they will be making the same choice of suppliers in equilibrium. To simplify notation, from now on I use S_{kt} to denote the common choice of suppliers for all firms in a sector k in time period t. As a result, the unit cost is also common for all firms within a given sector, which from now on I denote by $\mathcal{Q}_{kt}$.

2.3 Firms: pricing under nominal rigidities

Price stickiness is modeled as a modified finite-horizon version of Calvo (1983). In particular, there exists a finite, deterministic time period $T > 1$, such that in periods $1 \leqslant t \leqslant (T-1)$ firms face a constant and sector-specific probability of price non-adjustment $\alpha_k \in (0,1)$, whereas in periods $t \geqslant T$ firms face no nominal rigidities.[1] The cut-off period

[1] In subsection 3.5 I consider an extension of my model which also allows for nominal wage rigidity.

$T > 1$ is known by all agents in the economy from $t = 0$ and they expect it to stay fixed forever; naturally, as $T \to \infty$ the price setting problem collapses back to the standard Calvo (1983) pricing.[2]

More precisely, in any period $1 \leqslant t \leqslant (T-1)$ a firm in sector k has probability $(1-\alpha_k)$ of setting its price equal to its optimal value. The optimal reset price at time $1 \leqslant t \leqslant (T-1)$ is chosen to maximize expected future discounted nominal profits:

$$\max_{\hat{P}_{kt}(j)} \sum_{s=0}^{T-t-1} \alpha_k^s F_{t,t+s} \left[\hat{P}_{kt}(j)\tilde{Y}_{k,t+s}(j) - (1+\tau_k)\mathcal{Q}_{k,t+s}\tilde{Y}_{k,t+s}(j) \right], \tag{4}$$

where $\tilde{Y}_{k,t+s}(j) = \left[\frac{\hat{P}_{kt}(j)}{P_{t+s}}\right]^{-\theta} Y_{k,t+s}$, $F_{t,t+s}$ is the stochastic discount factor between periods t and $t+s$ and is defined in the next subsection; τ_k is a tax imposed by the government, revenue from which is rebated to households as a lump-sum transfer. The first order condition for the optimal reset price for any firm in sector k, $\hat{P}_{kt}$ is given by:

$$\hat{P}_{kt}(j) = \hat{P}_{kt} = (1+\mu_k)\frac{\sum_{s=0}^{T-t-1} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s} \mathcal{Q}_{k,t+s}}{\sum_{s=0}^{T-t-1} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s}}, \quad 1 \leqslant t \leqslant T-1, \quad \forall k \tag{5}$$

where $(1+\mu_k) \equiv (1+\tau_k)\frac{\theta}{\theta-1}$ is the steady-state desired markup. On the other hand, in any period $t \geqslant T$ there are no nominal rigidities, firms' maximize contemporaneous profits, and optimally set $P_{kt} = (1+\mu_k)\mathcal{Q}_{kt}, t \geqslant T$. Given that the optimal price is identical for all firms within a sector, sectoral price index can be obtained by aggregation using the ideal sectoral price index $P_{kt} \equiv \left(\int_{j\in\Phi_k} P_{kt}(j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}}$, $\forall k$:

$$P_{kt} = \begin{cases} \left[\alpha_k P_{k,t-1}^{1-\theta} + (1-\alpha_k)(\hat{P}_{kt})^{1-\theta}\right]^{\frac{1}{1-\theta}}, & 1 \leqslant t \leqslant (T-1); \\ (1+\mu_k)\mathcal{Q}_{kt}, & t \geqslant T. \end{cases} \tag{6}$$

[2] I use such modified version of Calvo (1983) pricing for two reasons. First, the special case where $T = 2$ is analytically tractable at the sector-level, as I show in Section 3. Second, given that money is neutral for all $t \geqslant T$, such formulation of pricing allows to develop a novel numerical algorithm for obtaining a sector-level solution for the forward-looking problem with endogenous network formation, implemented in Section 4.

2.4 Households

A continuum of infinitely lived households populates the economy and owns all the firms. The representative household makes choices to maximize the lifetime utility

$$\max_{\{C_{t+s},N_{t+s},B_{t+s+1}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} \beta^s \left[\log C_{t+s} - N_{t+s}\right] \tag{7}$$

subject to the budget constraint $P_t^c C_t + [F_{t,t+1}B_{t+1}] \leqslant B_t + W_t N_t + \sum_{k=1}^{K} \int_{j\in\Phi_k} \Pi_{kt}(j)dj + T_t$, where C_t is aggregate consumption, P_t^c is consumption price index (defined below), N_t is labor supply, B_{t+1} is the payoff of assets purchased at time t, $F_{t,t+1}$ is the stochastic discount factor between periods t and $t+1$, $\Pi_{kt}(j)$ denotes nominal profits of firm j in sector k, β is the discount factor for future utility and T_t are lump-sum transfers from the government.

The composite consumption index C_t is an aggregator for the final consumption of goods produced in the different sectors of my economy: $C_t \equiv u(C_{1t}, C_{2t}, ..., C_{Kt})$, where C_{kt} is in turn an aggregator for the final consumption of goods produced by firms in that sector: $C_{kt} \equiv \left(\int_{j\in\Phi_r} C_{kt}(j)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}, \forall k$. I impose the following regularity conditions on the aggregator u:

Assumption 2 (Consumption aggregation)**.** *The consumption aggregator u is continuous, differentiable, increasing, strictly quasi-concave and homogeneous of degree one in $(C_{1t}, C_{2t}, ..., C_{Kt})$, and all sectoral consumption goods are normal.*

Households choose their sectoral consumption levels by minimizing the total cost of purchases $\sum_{k=1}^{K} P_{kt}C_{kt}$ subject to the consumption aggregator u. The latter also delivers the consumption price index $P_t^c = P_t^c(P_{1t}, P_{2t}, ..., P_{Kt})$, as the minimal cost of assembling such a basket.

2.5 Monetary policy

Purchases of final goods are subject to a cash-in-advance constraint, so that $P_t^c C_t = \mathcal{M}_t, \forall t \geqslant 1$. Agents know the initial level of money supply $\mathcal{M}_0$, and at $t = 0$ anticipate it to stay at that level forever. In period $t = 1$ they discover the future path of money supply $\{\mathcal{M}_t\}_{t=1}^{\infty}$ and therefore any $\mathcal{M}_t \neq \mathcal{M}_0$ constitutes a monetary shock at time t to the agents.

2.6 Market clearing and equilibrium

In addition to the optimality conditions, budget constraints and the policy rule above, equilibrium in my economy is characterized by market-clearing conditions in the asset market: $B_t = 0$; the labor market: $N_t = \sum_{k=1}^{K} \int_{j \in \Phi_k} N_{kt}(j) dj$; and the goods markets: $Y_{kt}(j) = C_{kt}(j) + \sum_{r=1}^{K} \int_{j' \in \Phi_r} Z_{rkt}(j', j) dj', \quad \forall k, \forall j \in \Phi_k$. The equilibrium in my economy can be summarized as follows:

Definition 1 (Equilibrium)**.** *The equilibrium is a collection of prices $\{P_{kt}(j) | j \in \Phi_k\}_{k=1}^{K}$, wage W_t, allocations $\left\{ Y_{kt}(j), N_{kt}(j), C_{kt}(j), \{Z_{krt}(j, j') | j' \in \Phi_r\}_{r=1}^{K} | j \in \Phi_k \right\}_{k=1}^{K}$ and supplier choices $\{S_{kt}(j) | j \in \Phi_k\}_{k=1}^{K}$, which given the exogenous path of productivity mapping $\{\mathcal{A}_t\}_{t=1}^{\infty}$, the exogenous series of money supply $\{\mathcal{M}_t\}_{t=0}^{\infty}$ and the exogenous initial prices $\{P_{k0}\}_{k=1}^{K}$, satisfy agent optimization and market clearing in every time period $t \geqslant 1$.*

3 An analytically tractable version

In this section, I consider an analytically tractable version of the model obtained when nominal rigidities are only present in the first period. Such simplification allows to formally characterize propagation of monetary shocks to real variables, under different states of productivity and initial levels of money supply. Formal propositions establish that small monetary shocks, which do not affect the shape of the network, have an impact on GDP that is larger whenever productivity mapping improves or initial money supply rises. Further, large monetary expansions have a more than proportional positive effect on GDP than small monetary expansions; on the other hand, large monetary contractions have less than proportional negative effect on GDP than small monetary contractions.

3.1 Equilibrium in the simplified version

In this section, I focus on the version of my model with $T = 2$, so that nominal rigidities are only present at $t = 1$. I show that such simplification allows to both formally establish equilibrium existence and uniqueness properties, as well as to analytically characterize the transmission of monetary shocks under different baseline productivity mappings and levels of money supply. The assumption is formally documented below:

Assumption 3 (Horizon of stickiness)**.** *The firms cannot fully flexibly adjust prices until the horizon $T = 2$, so that nominal rigidities are only present at $t = 1$, and prices are fully flexible for all $t \geqslant 2$.*

One implication of the above assumption is that, conditional on a particular choice of suppliers, the optimal reset price at $t = 1$ is given by $\hat{P}_{k1} = (1 + \mu_k)\mathcal{Q}_{k1}, \forall k$. The latter delivers a tractable expression for the equilibrium sectoral price index in the first period, namely

$$P_{k1} = \left[\alpha_k P_{k0}^{1-\theta} + (1 - \alpha_k) \left\{ (1 + \mu_k) \min_{S_{k1}} \mathcal{Q}_{k1} \left[S_{k1}, \mathcal{A}_{k1}(S_{k1}), W_1, \{P_{r1}\}_{r \in S_{k1}} \right] \right\}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{8}$$

for $k = 1, 2, ..., K$.

At the same time, prices are flexible after the first period, so that the equilibrium sectoral price in $t \geqslant 2$ is pinned down by $P_{kt} = (1 + \mu_k) \min_{S_{kt}} \mathcal{Q}_{kt} \left[S_{kt}, \mathcal{A}_{kt}(S_{kt}), W_t, \{P_{rt}\}_{r \in S_{kt}} \right]$. Moreover, notice that the intratemporal consumption-labor supply condition and the cash-in-advance constraint jointly imply that $W_t = \mathcal{M}_t$, so that the nominal wage equals money supply in every period. This exogeneity of the nominal wage, combined with the fact that the only endogenous component in the pricing equations above is the unit cost function, jointly imply that prices in this simplified setting are pinned down exclusively by the exogenous supply-side factors – the productivity mapping $\mathcal{A}_t$ and desired markups $\{1 + \mu_k\}_{k=1}^{K}$ – in addition to the the exogenous level of money supply $\mathcal{M}_t$. It is the latter property which allows to represent equilibrium sectoral prices as a lattice, which in turn delivers equilibrium existence and uniqueness properties summarized below.

Proposition 1 (Equilibrium)**.** *Suppose Assumptions 1-3 hold. Then, the equilibrium introduced in Definition 1 entails the following properties: (a) it exists; (b) the equilibrium sectoral prices and final sectoral consumptions are unique; (c) the equilibrium supplier choices and remaining sectoral allocations are generically unique.*

Since Assumption 3 implies that money is neutral for $t \geqslant 2$ and given that my interest is in the transmission of monetary shocks to real variables, in the rest of this section I am going to focus exclusively on outcomes at $t = 1$. For notational simplicity, I drop time subscripts for variables at $t = 1$ for the remainder of this section.

The theoretical results in the remainder of this section are going to be presented in two steps. Before establishing each key finding, I explain it in a stylized setting featuring only two sectors. The simplified two-sector setting is summarized below.

Two-sector setting. *Consider the economy with two sectors ($K = 2$) and nominal rigidities only present in the first period ($T = 2$). For simplicity, firms are not allowed to buy inputs from other firms in their own sector ($k \notin S_k, k = 1, 2$), and the production function is*

Cobb-Douglas with the functional form: $Y_k(j) = e(k)\mathcal{A}_k(S_k)N_k(j)^{(1-\omega_{k,-k})}Z_{k,-k}(j)^{\omega_{k,-k}}$*, and* $e(k) = (1-\omega_{k,-k})^{-(1-\omega_{k,-k})}\omega_{k,-k}^{-\omega_{k,-k}}$*. The input-output shares are calibrated such that* $\omega_{k,-k} = 0.5$ *if* $-k \in S_k$ *and* $\omega_{k,-k} = 0$ *otherwise, for* $k = 1, 2$*. Price stickiness is calibrated such that Sector 1 is price flexible* $(\alpha_1 = 0)$ *and Sector 2 features nominal rigidities* $(\alpha_2 = 0.5)$*. The productivity mapping is given by* $a_k(\varnothing) \equiv \log A_k(\varnothing) = 1, k = 1, 2$ *and* $a_k(\{-k\}) \equiv \log A_k(\{-k\}) = \overline{a}, k = 1, 2$*. The exogenous initial prices are set at* $P_{k0} = 1, k = 1, 2$*. Finally, assume that* $\tau_k = -1/\theta$*, so that all market power distortions are optimally removed, and* $\theta \to 1^+$*, which allows to obtain closed-form expressions for equilibrium sectoral prices and quantities. In this setting, there are only four possibilities for supplier choices* (S_1, S_2)*, namely* $(\varnothing, \varnothing)$*,* $(\varnothing, \{1\})$*,* $(\{2\}, \varnothing)$ *and* $(\{2\}, \{1\})$*. Letting* $m \equiv \log \mathcal{M}$*, one can summarize (log) unit costs* (q_1, q_2) *associated with the four possible supplier choices as follows*:

	$S_2 = \varnothing$	$S_2 = \{1\}$
$S_1 = \varnothing$	$(m - 1, m - 1)$	$(m - 1, -\overline{a} + m - \frac{1}{2})$
$S_1 = \{2\}$	$(-\overline{a} + \frac{3}{4}m - \frac{1}{4}, m - 1)$	$\left(-\frac{10}{7}\overline{a} + \frac{5}{7}m, -\frac{12}{7}\overline{a} + \frac{6}{7}m\right)$

Another consequence of money neutrality for all $t \geqslant 2$ is that the only change in money supply relevant for real variables at $t = 1$ is that between money supply at $t = 1$ and its baseline level $\mathcal{M}_0$. For the ease of presentation of my results in this section, I introduce the following definition of a *monetary shock*:

Definition 2 (Monetary shock)**.** *Let* $\varepsilon^m \equiv \log(\mathcal{M}/\mathcal{M}_0)$*, where* $\mathcal{M}$ *is money supply at* $t = 1$ *and* $\mathcal{M}_0$ *is the baseline level of money supply, be the monetary shock in my economy.*

In the next subsection, I study properties of my economy's baseline at $t = 1$, which occurs under zero monetary shock $(\varepsilon^m = 0)$. The subsequent section studies propagation of a monetary shock $\varepsilon^m \neq 0$ conditional on different baselines.

3.2 Baseline $(\varepsilon^m = 0)$

The baseline is given by the equilibrium evaluated under $\varepsilon^m = 0$. Baseline prices, allocations and supplier choices are pinned down by the productivity mapping $\mathcal{A}$, baseline money supply $\mathcal{M}_0$ and exogenous initial prices $\{P_{k0}\}_{k=1}^K$. In this subsection, I am going to investigate how, holding the initial prices fixed, changes in the baseline pair $(\mathcal{A}, \mathcal{M}_0)$ affect the equilibrium.[3]

First, I am going to use the two-sector setting introduced earlier to build intuition on how changes in the productivity mapping affect the baseline, *ceteris paribus*.

Example 1 (Productivity mapping and baseline)**.** *In Panel (a) of Figure 2 I use the two-sector setting where, holding baseline money supply fixed at* $m_0 = 0$*, I consider three different productivity mappings by varying the parameter* $\overline{a}$*, which represents the productivity associated with using the other sector as a supplier. When* $\overline{a} = 0$*, neither sector finds it optimal to buy inputs from the other sector, and the equilibrium network is empty. As I increase* $\overline{a}$ *to 0.65, the sticky-price Sector 2 finds it optimal to purchase inputs from the flexible-price Sector 1, but not vice versa. This is because Sector 2 finds it optimal to lower its unit cost by purchasing inputs from Sector 1, whose price flexibility is associated with lower prices. Finally, as I further increase* $\overline{a}$ *to 0.8, both sectors find it optimal to connect to each other. This is because the productivity associated with the connection is now sufficiently high to spur the flexible-price Sector 1 to purchase from Sector 2, whose price stickiness prevents it from fully lowering the price. In addition, notice that as I increase the productivity parameter, equilibrium unit costs and hence sectoral prices drop, which lowers the consumption price index and through the cash-in-advance constraint implies that baseline GDP rises.*

I now use the two-sector setting to build intuition on the link between initial money supply and baseline equilibrium:

Example 2 (Money supply and baseline)**.** *In Panel (b) of Figure 2 I use the two-sector setting where, holding productivity mapping fixed at* $\overline{a} = 0$*, I consider three different initial money supplies by varying the parameter* m_0*. When* $m_0 = 0$*, neither sector finds it optimal to buy inputs from the other sector, and the equilibrium network is empty. As I increase* m_0 *to 4, the flexible-price Sector 1 finds it optimal to purchase inputs from the sticky-price Sector 2, but not vice versa. This is because the nominal wage rises one-for-one with money supply, and Sector 1 finds it cheaper to substitute in-house labor for inputs bought from Sector 2, whose price increases less than one-for-one, even though such connection lowers 1's productivity. Finally, as I further increase* m_0 *to 8, both sectors find it optimal to connect to each other. This is because the increase in money supply/nominal wage is now so large that even the sticky-price Sector 2 wishes to substitute its in-house labor for inputs from Sector 1, whose price now does not track money supply since it inherits price stickiness from Sector 2, even though such connection lowers 2's productivity. In addition, notice that as I increase money supply, the gap between money supply and equilibrium unit costs and prices, both sectoral and the aggregate consumption index, is growing, which through the cash-in-advance constraint implies that baseline GDP rises.*

[3] In subsection 3.4 I consider baselines that differ in the levels of desired markups, parameterized by exogenous changes in the sectoral cost taxes $\{\tau_k\}_{k=1}^{K}$.

Figure 2: **Equilibrium supplier choices under different baseline conditions**

(a) Equilibrium choices of suppliers under different baseline productivity mappings ($m_0 = 0$)

Recession: $\overline{a} = 0$

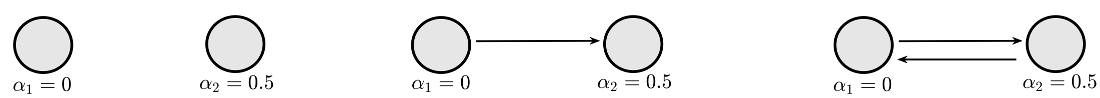
	∅	{1}
∅	**(-1,-1)**	$(-1, -\frac{1}{2})$
{2}	$(-\frac{1}{4}, -1)$	$(0, 0)$

Normal: $\overline{a} = 0.65$

	∅	**{1}**
∅	$(-1, -1)$	$(\mathbf{-1}, -\mathbf{\frac{23}{20}})$
{2}	$(-\frac{9}{10}, -1)$	$(-\frac{13}{14}, -\frac{39}{35})$

Expansion: $\overline{a} = 0.80$

	∅	**{1}**
∅	$(-1, -1)$	$(-1, -\frac{13}{10})$
{2}	$(-\frac{21}{20}, -1)$	$(-\mathbf{\frac{8}{7}}, -\mathbf{\frac{48}{35}})$



(b) Equilibrium choices of suppliers under different baseline levels of money supply ($\overline{a} = 0$)

Tight money: $m_0 = 0$

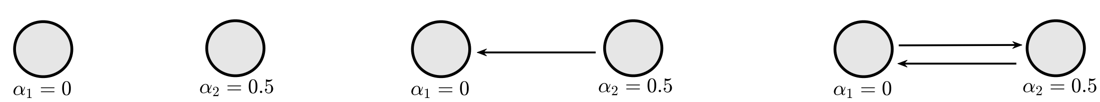
	∅	{1}
∅	**(-1,-1)**	$(-1, -\frac{1}{2})$
{2}	$(-\frac{1}{4}, -1)$	$(0, 0)$

Normal money: $m_0 = 4$

	∅	{1}
∅	$(3, 3)$	$(3, \frac{5}{2})$
{2}	$(\mathbf{\frac{11}{4}}, \mathbf{3})$	$(\frac{20}{7}, \frac{24}{7})$

Loose money: $m_0 = 8$

	∅	**{1}**
∅	$(7, 7)$	$(7, \frac{13}{2})$
{2}	$(\frac{23}{4}, 7)$	$(\mathbf{\frac{40}{7}}, \mathbf{\frac{48}{7}})$



***Notes**: the figure uses the analytically tractable version of my model under $T = 2$, calibrated for $K = 2, \omega_{kk} = 0, \forall k$, $Y_k(j) = e(k)\mathcal{A}_k(S_k)N_k(j)^{(1-\omega_{k,-k})}Z_{k,-k}(j)^{\omega_{k,-k}}$ and $e(k) = (1-\omega_{k,-k})^{-(1-\omega_{k,-k})}\omega_{k,-k}^{-\omega_{k,-k}}$, $a_{k,0} \equiv \log \mathcal{A}_{k,0}, \forall k$ and $m_0 = \log \mathcal{M}_0$, the production technology be given by $a_{1,0}(\varnothing) = 0$, $a_{1,0}(\{2\}) = \overline{a}$, $a_{2,0}(\varnothing) = 0$, $a_{2,0}(\{1\}) = \overline{a}$, $P_{10} = P_{20} = 1$, the sectoral shares are given by $\omega_{12} = \omega_{c1} = 0.5 = \omega_{21} = \omega_{c1} = 0.5$, and Calvo parameters by $\alpha_1 = 0, \alpha_2 = 0.5$. Finally, assume that $\tau_k = -1/\theta$, and $\theta \to 1^+$.*

Therefore, in my simple examples, baseline GDP and the number of suppliers of each sector (weakly) rise, *ceteris paribus*, both as one improves the productivity mapping and as one increases initial money supply. However, the mechanisms through which linkages are added are, in fact, diagonally opposite under the two scenarios. As one improves the productivity mapping, extra sectors that get adopted as suppliers are those that *adjust* their prices downwards by enough to lower the buyer's unit cost for the level of productivity they bring. On the contrary, as one increases money supply, sectors that receive additional intermediate customers are those that *do not adjust* their prices upwards enough, so that they are cheaper relative to labor for the level of productivity they bring.Having built intuition in the two-sector setting, I now formalize the relationship between the baseline pair $(\mathcal{A}, \mathcal{M}_0)$, GDP and supplier choices in equilibrium. As for the effect on GDP, no further assumptions, over and above the ones already made, are required:

Lemma 1 (Baseline GDP)**.** *Suppose Assumptions 1-3 hold. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}_0})$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ *such that either* $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ *or* $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$*. Then,* $C(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \geqslant C(\underline{\mathcal{A}}, \underline{\mathcal{M}_0})$.

The overall mechanism behind the above result is as follows. Holding initial money supply fixed, an improvement in the productivity mapping lowers each unit cost function, which in turn lowers every sectoral price. Firms re-optimize their set of suppliers, which delivers lower new unit costs, since the original set of suppliers remains available. The latter further lowers sectoral prices. The mechanism repeats until a new equilibrium is reached, which features lower sectoral prices and hence a lower consumption price index, which through the cash-in-advance constraint implies a larger GDP. Similarly, holding productivity mapping fixed, larger initial money supply decreases the ratios of sectoral prices to money supply due to sticky prices. Firms re-optimize their set of suppliers, which delivers lower new unit costs, since the original set of suppliers remains available. The latter further lowers the ratios of sectoral prices to money supply. The mechanism repeats until a new equilibrium is reached, which features lower ratios of sectoral prices to money supply and hence a lower ratio of consumption price index to money supply, which through the cash-in-advance constraint implies a larger GDP.

The link between the baseline pair $(\mathcal{A}, \mathcal{M}_0)$ and supplier choices in equilibrium can be formalized using monotone comparative statics theorems of Milgrom and Shannon (1994). Those, however, require imposing further regularity conditions. The first one puts a restriction on the relationship between the unit cost function, the choice of suppliers and the productivity mapping, holding sectoral prices and wage fixed:

Assumption 4. *For all* $W, \{P_k\}_{k=1}^K$, *the unit cost function* $\mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), W, \{P_r\}_{r\in S_k}\right]$ *is quasi-submodular in* $(S_k, \mathcal{A}_k(S_k))$, *for all* $k = 1, 2, ..., K$.

The assumption of quasi-submodularity ensures that, holding wage and prices fixed, an improvement in the productivity mapping does not encourage any sector to shrink its number of suppliers. However, prices also vary across baselines, which brings in the need for a sector regularity condition. The additional restriction is a single-crossing property on the unit cost function, relating joint variations in prices, wages and the set of suppliers:

Assumption 5. *Let* $\mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), \{\tilde{P}_r\}_{r\in S_k}\right] \equiv \frac{1}{W}\mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), 1, \{\frac{P_r}{W}\}_{r\in S_k}\right], \forall k$. *For all* $S_k \subseteq S'_k$ *and for all* $\{\tilde{P}_r, \tilde{P'}_r\}_{r=1}^K$ *such that* $\tilde{P'}_r \leqslant \tilde{P}_r, \forall r \neq k$:

$$\begin{aligned}&\mathcal{Q}_k\left[S'_k, \mathcal{A}_k(S'_k), \{\tilde{P}_r\}_{r\in S'_k}\right] - \mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), \{\tilde{P}_r\}_{r\in S_k}\right] \leqslant 0 \\ \implies \quad &\mathcal{Q}_k\left[S'_k, \mathcal{A}_k(S'_k), \{\tilde{P'}_r\}_{r\in S'_k}\right] - \mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), \{\tilde{P'}_r\}_{r\in S_k}\right] \leqslant 0, \quad \forall k. \quad (9)\end{aligned}$$

The above single crossing property ensures that a reduction in all sectoral price to wage ratios does not discourage the adoption of a larger number of suppliers by every sector. Such an assumption immediately holds under a range of widely used production functions, most notably Cobb-Douglas with Hicks-neutral technology.[4]

The additional assumptions above allow to formalize the link between the baseline pair $(\mathcal{A}, \mathcal{M}_0)$ and supplier choices in equilibrium:

Lemma 2 (Baseline supplier choices)**.** *Suppose Assumptions 1-5 hold. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ *such that either* $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ *or* $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$. *Then,* $S_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \supseteq S_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, *for all* $k = 1, 2, ..., K$.

The intuition behind the above result is as follows. An improvement in the productivity mapping, *ceteris paribus*, incentivizes firms to connect to more suppliers, as it lowers their unit costs directly through higher productivity and indirectly through lower prices charged by suppliers. Similarly, an increase in initial money supply, *ceteris paribus*, leads to a reduction in sectoral price to wage ratios, implying that it is cost-reducing to substitute in-house labor for intermediates bought from other sectors, thus leading to an increase in the number of suppliers.

[4] This is formally shown in Acemoglu and Azar (2020). See their work for a fuller characterization of families of production functions under which the single-crossing property in Assumption 5 holds.

3.3 Propagation of a monetary shock

Having established properties of the baseline, I now consider deviations from the baseline, driven by a non-zero monetary shock ε^m. The mechanics of a monetary shock in terms of its effect on the equilibrium allocations and supplier choices are isomorphic to those for changes in baseline money supply established in the previous subsection. I can therefore immediately formalize comparative statics following a monetary shock.

Lemma 3 (Comparative statics following a monetary shock)**.** *Consider a baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, which is perturbed by a monetary shock* $\varepsilon^m > 0$*. Suppose Assumptions 1-3 hold, then following the monetary shock equilibrium GDP rises relative to its baseline level:* $C(\mathcal{A}, \mathcal{M}) > C(\mathcal{A}, \mathcal{M}_0)$*. Further, suppose that in addition Assumption 4-5 also hold, then following the monetary shock the set of suppliers for each sector weakly expands:* $S_k(\mathcal{A}, \mathcal{M}) \supseteq S_k(\mathcal{A}, \mathcal{M}_0)$*, for all* $k = 1, 2, ..., K$*.*

Note that although the above lemma is stated for an expansionary monetary shock $\varepsilon^m > 0$, it naturally extends to a contractionary shock $\varepsilon^m < 0$, which leads to a reduction in GDP and a weak fall in the number of supplier for every sector.

From Lemma 3 it follows that a non-zero monetary shock can either change the set of suppliers relative to the baseline, or leave them unchanged. In light of this it is useful to formally distinguish between two such types of monetary shocks, as is done below.

Definition 3 (Small monetary shock)**.** *Define a monetary shock* ε^m *to be small with respect to the baseline* $(\mathcal{A}, \mathcal{M}_0)$ *if and only if it leaves the equilibrium supplier choices unchanged for all sectors relative to the baseline:* $S_k(\mathcal{A}, \mathcal{M}) = S_k(\mathcal{A}, \mathcal{M}_0), \ \forall k$*. Otherwise, define the monetary shock to be large with respect to the baseline* $(\mathcal{A}, \mathcal{M}_0)$*.*

Crucially, the above definition helps classify every monetary shock as either small or large *with respect to* a specific baseline pair $(\mathcal{A}, \mathcal{M}_0)$. In principle, as one changes the baseline pair, a shock of a given size can switch from being small to large, or *vice versa*. Moreover, the definition is not necessarily symmetric: for a given baseline, an expansionary shock $\varepsilon^m > 0$ being small does not necessarily imply that a contractionary shock of an equal absolute value is small with respect to the same baseline.

3.3.1 Small monetary shocks

In this section, I study how responses of prices and allocations to a small monetary shock depend on the baseline productivity mapping and initial money supply. As before, I begin by building intuition in the simplified two-sector setting before establishing general results.

First, I use the two-sector setting to consider the same small monetary expansion occurring under different baseline productivity mappings.

Example 3 (Small monetary shocks across productivity mappings)**.** *Consider the two-sector setting with the assumption that consumption aggregation is Cobb-Douglas:* $C = C_1^{\omega_{c1}} C_2^{\omega_{c2}}, \omega_{ck} = 0.5, k = 1, 2$. *Panel (a) of Figure 3 considers three baselines associated with different productivity mappings, as in Example 1. I perturb each of the baselines with the same small monetary expansion, which, by definition, does not change the equilibrium network. As one can see, in the low productivity baseline with the empty network, only the sticky-price Sector 2 responds to the shock; in the medium-productivity baseline the situation is unchanged, as Sector 1 does not buy inputs from Sector 2, and hence inherits no stickiness. However, in high-productivity state, where both sectors buy from each other, both sectors see their final consumption rise. We can see that the magnitude of both sectoral and aggregate final consumption's response to a monetary shock is weakly larger in the baselines with higher productivity.*

Similarly, I use the two-sector setting to consider the same small monetary expansion occurring under different levels of initial money supply.

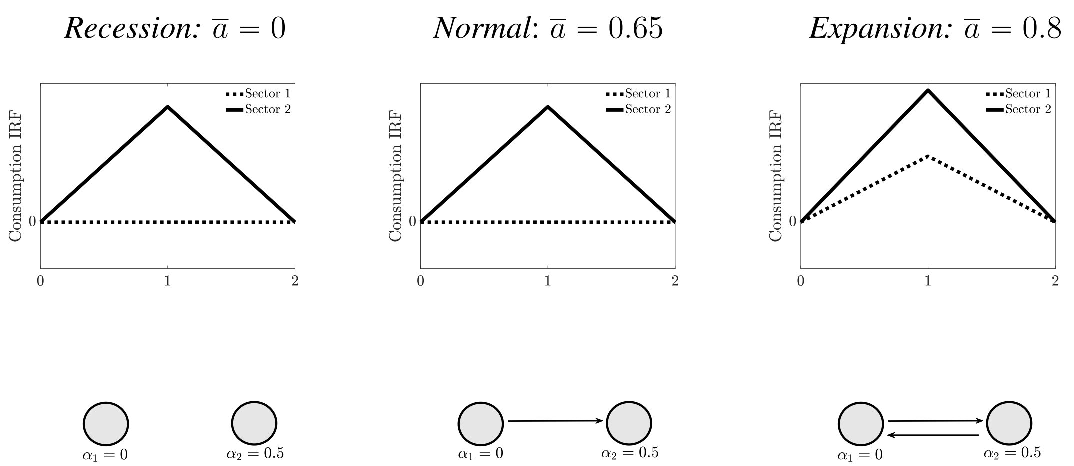
Example 4 (Small monetary shocks across initial money supplies)**.** *Consider the two-sector setting with the assumption that consumption aggregation is Cobb-Douglas:* $C = C_1^{\omega_{c1}} C_2^{\omega_{c2}}, \omega_{ck} = 0.5, k = 1, 2$. *Panel (b) of Figure 3 considers three baselines with different levels of initial money supply, as in Example 2. I perturb each of the baselines with the same small monetary expansion, which, by definition, does not change the equilibrium network. One can see that in the tight money baseline only the sticky-price Sector 2 responds to the shock; in the normal money baseline flexible-price Sector 1 buys from Sector 2 and inherits stickiness, hence responding to the shock. Finally, in the loose money baseline both sectors buy from each other and respond by even more than in the normal money state. Overall, one can see that both sectoral and aggregate consumption respond weakly stronger under the baselines with higher initial money supply.*

One can see that in the examples above, following a small monetary shock of the same size, consumption responds more strongly in states that feature more linkages across sectors. This is because whenever a sector increases the number of suppliers, its marginal cost becomes function of a larger number of other sectoral prices, which in turn strengthens complementarities in price setting and the degree of money non-neutrality.

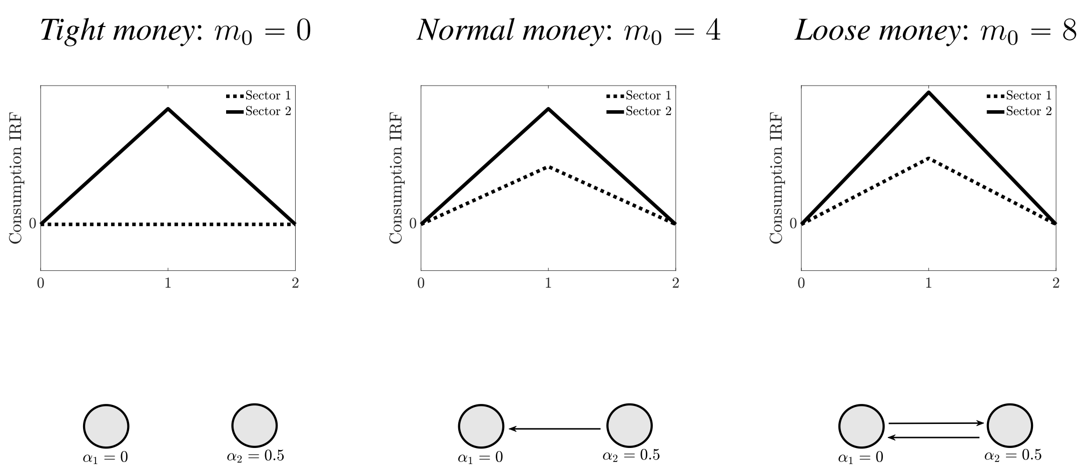
The above mechanism can be formalized by noticing that the existence of a small monetary shock around a specific baseline implies that there is a neighborhood within which variations in money supply do not affect the equilibrium set of supplier choices. The latter in turn means that such a neighborhood features no discontinuities created by formation or

Figure 3: **Final aggregate consumption following a small monetary expansion**

(a) IRFs of consumption to a small monetary expansion under recession and expansion ($m_0 = 0$)



(b) IRFs of consumption to a small monetary expansion under tight and loose initial money ($\overline{a} = 0$)



***Notes**: the figure uses the analytically tractable version of my model under $T = 2$, calibrated for $K = 2, \omega_{kk} = 0, \forall k$, $Y_k(j) = e(k)\mathcal{A}_k(S_k)N_k(j)^{(1-\omega_{k,-k})}Z_{k,-k}(j)^{\omega_{k,-k}}$ and $e(k) = (1-\omega_{k,-k})^{-(1-\omega_{k,-k})}\omega_{k,-k}^{-\omega_{k,-k}}$, $a_{k,0} \equiv \log \mathcal{A}_{k,0}, \forall k$ and $m_0 = \log \mathcal{M}_0$, the production technology be given by $a_{1,0}(\varnothing) = 0, \quad a_{1,0}(\{2\}) = \overline{a}, a_{2,0}(\varnothing) = 0, \quad a_{2,0}(\{1\}) = \overline{a}$, $P_{10} = P_{20} = 1$, the sectoral shares are given by $\omega_{12} = \omega_{c1} = 0.5 = \omega_{21} = \omega_{c1} = 0.5$, and Calvo parameters by $\alpha_1 = 0, \alpha_2 = 0.5$. Finally, assume that $\tau_k = -1/\theta$, and $\theta \to 1^+$.*

destruction of linkages, and one can appeal to local properties around the baseline. In order to establish analytically tractable local properties, I make several further assumptions. First, a vital required additional assumption is differentiability of the production function in labor and intermediate inputs:

Assumption 6. *The production function $\mathcal{F}_k$ is differentiable in $\left(N_{kt}(j), \{Z_{krt}(j)\}_{r \in S_{kt}}\right), \forall k$.*

The second additional assumption concerns the initial sectoral prices, which have so far been assumed to be completely exogenous:

Assumption 7 (Initial sectoral prices)**.** *For every sector $k = 1, 2, ..., K$, the initial sectoral price is given by $P_{k0} = (1 + \mu_k) g(\mathcal{M}_0) \mathcal{Q}_k[S_k, \mathcal{A}_k(S_k), \mathcal{M}_0, \{P_r(\mathcal{A}, \mathcal{M}_0)\}_{r \in S_k}]$, where $(\mathcal{A}, \mathcal{M}_0)$ is the baseline pair, $\mu_k = (1+\tau_k)\frac{\theta}{\theta-1}$ and $g : \mathbb{R}^+ \to (0, 1)$ and strictly decreasing on the whole domain.*

The above assumption states that for every sector the initial price is set at a fixed markup over the baseline (steady-state) unit cost, with the markup falling in the initial money supply. In this way, I tractably capture the idea that whenever the initial money supply is high, a monetary shock is occurring in an environment with low markups, representing the interaction of price stickiness with past loose money supply.[5] Crucially, baseline comparative statics properties established in Lemmas 1 and 2 continue to hold under this additional assumption about initial prices.

Armed with the two additional assumptions, I can now begin to formalize the differences in responses to a small monetary shock that arrives under different baselines. As a first step, the lemma below documents baseline-specific local responses of sectoral prices to a monetary shock:

Lemma 4. *Suppose Assumptions 1-3 and 6-7 hold. For any baseline pair $(\mathcal{A}, \mathcal{M}_0)$, let $p_k(\mathcal{A}, \mathcal{M}_0)$ be a first order approximation of $\log P_k(\mathcal{A}, \mathcal{M})$ around $\log P_k(\mathcal{A}, \mathcal{M}_0)$, given $S_k = S_k(\mathcal{A}, \mathcal{M}_0), \forall k$. Consider any two baseline pairs $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ and a monetary shock ε^m which is small with respect to both baselines, then:*

$$\mathbb{p}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathbb{p}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = -\left[\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathcal{L}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)\right] \mathcal{E}^m \tag{10}$$

where $\mathbb{p} \equiv [p_1, p_2, ..., p_K]'$, $\mathcal{E}^m \equiv [\varepsilon^m, \varepsilon^m, ..., \varepsilon^m]'$ and $\mathcal{L}$ is a Leontief inverse given by:

$$\mathcal{L}(\mathcal{A}, \mathcal{M}_0) \equiv [I - (I - A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)]^{-1} [I - (I - A)\Gamma(\mathcal{M}_0)] \tag{11}$$

[5] With this additional assumption on initial prices, the baseline sectoral markups are given by $(1 + \mu_k)[\alpha_k g(\mathcal{M}_0))^{1-\theta} + (1 - \alpha_k)]^{\frac{1}{1-\theta}}, \forall k$, which fall in $\mathcal{M}_0$. Crucially, I do not make this assumption in the fully dynamic forward-looking setting considered in Section 4.

where $A \equiv diag(\alpha_1, ..., \alpha_K)$, $\Gamma(\mathcal{M}_0) \equiv diag(\gamma_1(\mathcal{M}_0), ..., \gamma_K(\mathcal{M}_0))$, $\gamma_k \equiv \frac{1}{\alpha_k (g(\mathcal{M}_0))^{1-\theta} + 1 - \alpha_k}$ *and* $[\Omega(\mathcal{A}, \mathcal{M}_0)]_{kr} = \frac{\partial \log \mathcal{Q}_k \left[S_k, \mathcal{A}_k(S_k), W, \{P_{k'}\}_{k' \in S_k} \right]}{\partial \log P_r}|_{\mathcal{M}=\mathcal{M}_0}, \forall k, r.$

The above result details channels through which changes in the baseline affect local properties of price responses to a monetary shock. Baselines with either a better productivity mapping or a higher money supply, *ceteris paribus*, feature more suppliers for each sector and hence more non-zero elements in the $\Omega(\mathcal{A}, \mathcal{M}_0)$ matrix, whose entries denote cross-elasticities of a sector's unit cost to sectoral prices. The latter affects price responses through two margins. At the extensive margin, having more suppliers strengthens complementarities in price setting and unambiguously delivers smaller increases in sectoral prices following a small monetary expansion. At the intensive margin, however, the effect of a change in the baseline on the unit cost elasticity to prices of already existing suppliers is ambiguous. Therefore, the effect of a change in baseline on the responses of sectoral prices to small monetary shock depends on the relative quantitative importance of the three channels described above.

However, a sufficient condition exists under which the ambiguity described above is resolved. In particular, it occurs in the special case when the intensive margin adjustment described above is absent. The latter happens when, conditional on a supplier relationship existing, the elasticity of a sector's unit cost to that supplier's price is fixed across baselines, which is the case under Cobb-Douglas production function with Hicks-neutral technology:

Assumption 8. *For every sector* $k = 1, 2, ..., K$ *the production function is Cobb-Douglas with Hicks-neutral technology:* $Y_{kt}(j) = \mathcal{A}_{kt}(S_{kt}) N_{kt}(j)^{1-\sum_{r \in S_{kt}} \omega_{kr}} \prod_{r \in S_{kt}} Z_{krt}(j)^{\omega_{kr}}$, $\omega_{kr} \geqslant 0, \quad \sum_{r=1}^{K} \omega_{kr} < 1.$

Adding the above assumption ensures that under a baseline with either a better productivity mapping or a higher money supply, the same small monetary expansion delivers smaller increases in all sectoral prices, as is formalized below:

Proposition 2. *Suppose Assumptions 2-5 and 7-8 hold. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ *such that either* $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ *or* $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$. *For a monetary shock* $\varepsilon^m > 0$ *that is small with respect to both baselines, it follows that* $p_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \leqslant p_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$ *for all* $k = 1, 2, ..., K$.

Importantly, though the above proposition is formulated for a small monetary expansion $\varepsilon^m > 0$, it trivially extends to a small monetary contraction $\varepsilon^m < 0$. In particular, it would follow that under a baseline with either a better productivity mapping or a higher money supply, the same small monetary contraction delivers smaller decreases in all sectoral prices.

I now use properties of price responses established above to study how changes in the baseline affect consumption responses to small monetary shocks. From the cash-in-advance constraint, it follows that the change in aggregate consumption can be written as $[\log C(\mathcal{A},\mathcal{M}) - \log C(\mathcal{A},\mathcal{M}_0)] = \varepsilon^m - [\log P^c(\mathcal{A},\mathcal{M}) - \log P^c(\mathcal{A},\mathcal{M}_0)]$. It follows that local properties of GDP around the baseline can be inferred from local properties of the consumption price index. For a monetary shock ε^m that is small with respect to a baseline pair $(\mathcal{A},\mathcal{M}_0)$ one can write the following local approximation:

$$\log P^c(\mathcal{A},\mathcal{M}) - \log P^c(\mathcal{A},\mathcal{M}_0) \approx \sum_{k=1}^{K} \frac{\partial \log P^c(\mathcal{A},\mathcal{M}_0)}{\partial \log P_k} \left[\log P_k(\mathcal{A},\mathcal{M}) - \log P_k(\mathcal{A},\mathcal{M}_0)\right]. \tag{12}$$

From Proposition 2 we know local properties of (log-)deviations of sectoral prices as one varies the baseline. However, the variation in elasticities of the consumption price index with respect to sectoral prices as one varies the baseline is ambiguous. In this sense, the local properties of aggregate consumption/GDP around different baselines remain on relative quantitative properties of movements in sectoral prices and the elasticities that are used to aggregate them.

However, a sufficient condition exists under which the ambiguity described above is resolved. In particular, it occurs in the special case when the elasticities of the consumption price index with respect to sectoral prices remain fixed across all baselines. The latter occurs when the consumption aggregator takes the Cobb-Douglas form, as is detailed below:

Assumption 9. *The consumption aggregator $u(.)$ is Cobb-Douglas: $C_t = \prod_{k=1}^{K} C_{kt}^{\omega_{ck}}$, $\omega_{ck} \geqslant 0, \sum_{k=1}^{K} \omega_{ck} = 1$.*

With the additional assumption above, I can now formalize non-linear transmission of a small monetary shock to both sectoral and aggregate consumption, the latter being equivalent to GDP in my economy. First, whenever a small monetary expansion arrives under a baseline with a better productivity mapping, it triggers a consumption increase of a larger magnitude:

Theorem 1 (Cycle dependence)**.** *Suppose Assumptions 3-5 and 7-9 hold. For any baseline pair $(\mathcal{A},\mathcal{M}_0)$, let $c_k(\mathcal{A},\mathcal{M}_0) \equiv \log C_k(\mathcal{A},\mathcal{M}) - \log C_k(\mathcal{A},\mathcal{M}_0), \forall k$. Consider any two baseline pairs $(\underline{\mathcal{A}},\mathcal{M}_0)$, $(\overline{\mathcal{A}},\mathcal{M}_0)$ such that $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}$. For a monetary shock $\varepsilon^m > 0$ which is small with respect to both baselines it follows that $c_k(\overline{\mathcal{A}},\mathcal{M}_0) \geqslant c_k(\underline{\mathcal{A}},\mathcal{M}_0)$ for all $k = 1, 2, ..., K$, and $c(\overline{\mathcal{A}},\mathcal{M}_0) \geqslant c(\underline{\mathcal{A}},\mathcal{M}_0)$.*

Three points should be noted. First, although the theorem is stated for small monetary expansions, it trivially extends to small monetary contractions. In particular, it follows that if a small monetary contraction arrives in a state with, *ceteris paribus*, higher productivity, the resulting fall in GDP is (weakly) smaller. Second, the additional assumption of Cobb-Douglas consumption aggregation implies cycle dependence not only at the level of aggregate GDP, but also at the level of final consumptions of individual sectors. Third, my cycle dependence result provides a theoretical rationale for empirical finds of procyclical magnitude of impulse response of GDP to monetary shocks (Tenreyro and Thwaites, 2016, Alpanda et al., 2021, Jordà et al., 2020).

In a similar way, I can formalize that whenever a small monetary expansion arrives under a baseline with higher money supply, it triggers a consumption increase of a larger magnitude:

Theorem 2 (Path dependence)**.** *Suppose Assumptions 3-5 and 7-9 hold. For any baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, let* $c_k(\mathcal{A}, \mathcal{M}_0) \equiv \log C_k(\mathcal{A}, \mathcal{M}) - \log C_k(\mathcal{A}, \mathcal{M}_0), \forall k$*. Consider any two baseline pairs* $(\mathcal{A}, \underline{\mathcal{M}}_0)$*,* $(\mathcal{A}, \overline{\mathcal{M}}_0)$ *such that* $\overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$*. For a monetary shock* $\varepsilon^m > 0$ *which is small with respect to both baselines it follows that* $c_k(\mathcal{A}, \overline{\mathcal{M}}_0) \geqslant c_k(\mathcal{A}, \underline{\mathcal{M}}_0)$ *for all* $k = 1, 2, ..., K$*, and* $c(\mathcal{A}, \overline{\mathcal{M}}_0) \geqslant c(\mathcal{A}, \underline{\mathcal{M}}_0)$*.*

As before, the theorem trivially extends to small monetary contractions: if a small monetary contraction arrives in a state with, *ceteris paribus*, lower money supply, the resulting fall in GDP is (weakly) smaller. My path dependence result provides a theoretical rationale for empirical finds of monetary transmission to GDP being stronger under already loose monetary stance (Alpanda et al., 2021) and additional evidence that finds that GDP is more sensitive to monetary interventions in states of the world with already loose credit (Jordà et al., 2020).

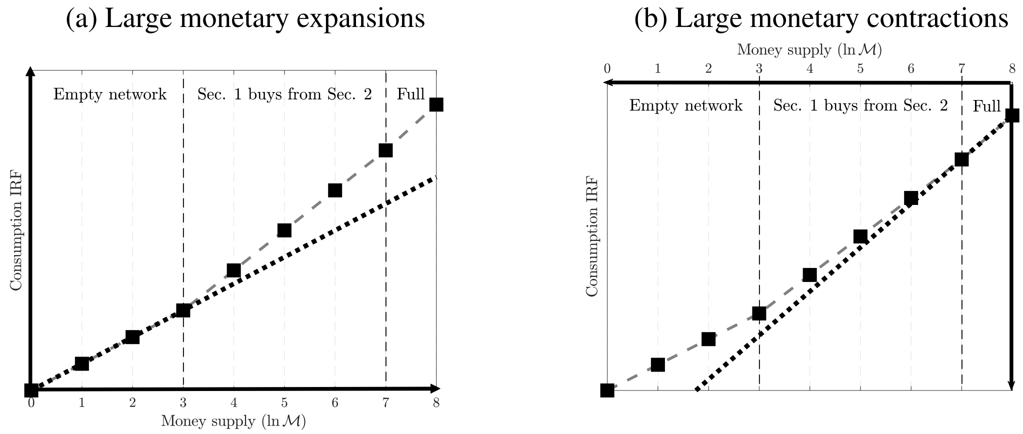
The intuition behind both theorems is very similar. Baselines with a better productivity mapping or higher initial money supply, *ceteris paribus*, feature more suppliers for every sector in equilibrium. The latter strengthens complementarities in price setting, and hence deliver more money non-neutrality and consumption response of a larger magnitude.

3.3.2 Large monetary shocks

I now move to studying properties of large monetary shocks which, by definition, are able to change the equilibrium set of suppliers relative to the baseline. As before, I begin with building intuition using the two-sector setting before establishing formal results.

First, I consider larger monetary expansions in the two-sector setting:

Figure 4: **Aggregate consumption response to large monetary shocks (two sectors)**



***Notes**: the figure uses the analytically tractable version of the model under $T = 2$, calibrated for $K = 2, \omega_{kk} = 0, \forall k$, $Y_k(j) = e(k)\mathcal{A}_k(S_k)N_k(j)^{(1-\omega_{k,-k})}Z_{k,-k}(j)^{\omega_{k,-k}}$ and $e(k) = (1-\omega_{k,-k})^{-(1-\omega_{k,-k})}\omega_{k,-k}^{-\omega_{k,-k}}$, $a_{k,0} \equiv \log \mathcal{A}_{k,0}, \forall k$ and $m_0 = \log \mathcal{M}_0$, the production technology be given by $a_{1,0}(\varnothing) = 0$, $a_{1,0}(\{2\}) = \overline{a}, a_{2,0}(\varnothing) = 0$, $a_{2,0}(\{1\}) = \overline{a}$, $P_{10} = P_{20} = 1$, the sectoral shares are given by $\omega_{12} = \omega_{c1} = 0.5 = \omega_{21} = \omega_{c1} = 0.5$, and Calvo parameters by $\alpha_1 = 0, \alpha_2 = 0.5$. Finally, assume that $\tau_k = -1/\theta$, and $\theta \to 1^+$. Throughout exercises $\overline{a}_0 = 0$.*

Example 5 (Large monetary expansions)**.** *Panel (a) of Figure 4 considers large monetary shocks in the context of the two-sector setting with an additional assumption on consumption aggregation: $C = C_1^{\omega_{c1}}C_2^{\omega_{c2}}, \omega_{ck} = 0.5, k = 1, 2$. Starting with an initial state with $\overline{a} = m_0 = 0$, I subject my economy to progressively larger monetary shocks. One can see that shocks smaller than $\varepsilon^m = 3$ keep the initially empty network unchanged, and the changes in the aggregate consumption impulse response are exactly proportional to the size of the shock. However, as I consider larger expansions, they turn out to be large enough to expand the network by encouraging the flexible-price Sector 1 to buy from the sticky-price Sector 2, also increasing the slope of the relationship between the monetary shock size and consumption response.*

Similarly, I use the two-sector setting to study large monetary contractions:

Example 6 (Large monetary contractions)**.** *Panel (b) of Figure 4 considers large monetary contractions in the context of the same two-sector setting with an additional assumption on consumption aggregation: $C = C_1^{\omega_{c1}}C_2^{\omega_{c2}}, \omega_{ck} = 0.5, k = 1, 2$ I start from a baseline with $\overline{a} = 0$ and $m_0 = 8$. Initial reductions in money supply leave the baseline full network unchanged and the magnitude of aggregate consumption contraction is exactly proportional to the size of the monetary shock. However, larger contractions break the network, in this case by discouraging Sector 2 from buying from Sector 1, and lower the slope of the relationship between the monetary shock size and consumption response.*

One can see that in Example 5 above, large monetary expansions deliver an increase in GDP that is larger than the one that would occur under fixed networks. In this sense, large monetary expansions deliver more than proportional increases in GDP by expanding the set of suppliers in each sector. On the other hand, in Example 6 above, large monetary contraction deliver reductions in GDP that are smaller in magnitude that those under exogenous networks. Therefore, large monetary contractions break supplier relationships and in this way deliver less than proportional decreases in GDP, relative to the outcome under exogenous networks.

In order to formalize the above intuition, I introduce several new concepts. First, I let the exact impulse response of a sectoral price to monetary shock ε^m be given by $\tilde{P}_k(\varepsilon^m) \equiv P_k(\mathcal{A}, \mathcal{M})/P_k(\mathcal{A}, \mathcal{M}_0), \forall k$. Second, in order to facilitate comparison with a setting where supplier choices are exogenous, I let $\tilde{P}^e_k(\varepsilon^m; S) \equiv P_k(\mathcal{A}, \mathcal{M}; S)/P_k(\mathcal{A}, \mathcal{M}_0; S), \forall k$ denote exact impulse responses of sectoral prices to monetary shock ε^m in a version of my economy with an *exogenous* production network S. I am now ready to state a proposition which establishes bounds on how the exact impulse responses of sectoral prices grow with the size of the monetary shock:

Proposition 3. *Suppose Assumption 1-3 and 6 hold. For a baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, and the baseline equilibrium network* S^0*, let* $\varepsilon^m > 0$ *be a small monetary shock and* $E^m > 0$ *be a large monetary shock. Further, let* S^L *be the equilibrium network following the large monetary shock. The ratios of exact impulse responses of sectoral prices satisfy:*

$$\frac{\tilde{P}^e_k(E^m; S^L)}{\tilde{P}^e_k(\varepsilon^m; S^L)} \leqslant \frac{\tilde{P}_k(E^m)}{\tilde{P}_k(\varepsilon^m)} \leqslant \frac{\tilde{P}^e(E^m; S^0)}{\tilde{P}^e_k(\varepsilon^m; S^0)}, \tag{13}$$

for all $k = 1, 2, ..., K$*, where* $\tilde{P}_k(.)$ *is the exact impulse response under endogenous networks, whereas* $\tilde{P}^e_k(.; S)$ *is the exact impulse response under the exogenous network* S*.*

Intuitively, as one moves from a small monetary expansion to a large monetary expansion, the rate at which the exact responses of sectoral prices increase is smaller than what would occur if the network stayed fixed at its baseline level. This is because the opportunity to endogenously select the supplier sectors creates an extra margin along which units costs, and hence prices, can be minimized. This implies that prices increase at most as quickly as under the network fixed at its baseline level. At the same time, this drag on price increases is bounded from below by what would occur if the network was fixed at the level delivered by the large monetary expansion. Crucially, although the above proposition is stated for monetary expansions it trivially extends to monetary contractions by reversing the signs of inequalities. In particular, as one moves from a small monetary contraction to a large monetary contraction, the rate at which the exact magnitudes of prices drops change is larger than what would occur if the network stayed fixed at its baseline level. Once again, this is because the opportunity to optimally select the suppliers creates an extra margin along which unit costs and prices are minimised.

Since the above proposition applies to all sectoral prices, it implies corresponding properties for the aggregate consumption price index. The latter combined with the cash-in-advance in turn delivers properties of aggregate consumption, which also represents GDP in my model:

Theorem 3 (Size dependence)**.** *Suppose Assumption 1-3 and 6 hold. For a baseline pair $(\mathcal{A}, \mathcal{M}_0)$, and the baseline equilibrium network S^0, let $\varepsilon^m > 0$ be a small monetary shock and $E^m > 0$ be a large monetary shock. Further, let S^L be the equilibrium network following the large monetary shock. The ratios of exact impulse responses of GDP satisfy:*

$$\frac{\tilde{C}^e(E^m; S^0)}{\tilde{C}^e(\varepsilon^m; S^0)} \leqslant \frac{\tilde{C}(E^m)}{\tilde{C}(\varepsilon^m)} \leqslant \frac{\tilde{C}^e(E^m; S^L)}{\tilde{C}^e(\varepsilon^m; S^L)}. \tag{14}$$

where $\tilde{C}(.)$ is the exact impulse response under endogenous networks, whereas $\tilde{C}^e(.; S)$ is the exact impulse response under the exogenous network S.

The theorem above formally establishes size-dependence in the response of GDP to monetary shocks in my model. In particular, as one moves from a small monetary expansion to a large monetary expansion, the rate at which the exact impulse response of GDP rises is larger than what would occur if the set of suppliers was fixed at the baseline level. This is due to the fact that, relative to the world where networks are fixed at the baseline level, the opportunity to optimally select suppliers puts a drag on how quickly prices increase following monetary expansions. At the same time, the above theorem also implies that following a small monetary contraction the rate at which the magnitude of exact impulse response of GDP changes is smaller than what would occur if the set of suppliers was fixed at the baseline level.

3.4 Discussion: business cycles driven by exogenous markup shocks

I so far consider baselines that vary in their productivity mappings $\mathcal{A}$ and initial levels of money supply $\mathcal{M}_0$. That said, my model also permits that baselines differ in the levels of exogenous desired markups $\{1 + \mu_k\}_{k=1}^K$, which I discuss in this section. In particular, given that $1 + \mu_k = (1 + \tau_k)\frac{\theta}{\theta - 1}$, shifts in the sectoral tax rates $\{\tau_k\}_{k=1}^K$ represent exogenous changes in baseline desired sectoral markups.

In terms of the effect on real GDP and supplier choices, exogenous decreases in baseline markups are isomorphic to exogenous improvements in the productivity mapping. In particular, *ceteris paribus*, an exogenous decrease in each sectoral desired markup lowers each sectoral price. Firms re-optimize their set of suppliers, which delivers lower new unit costs, since the original set of suppliers remains available. The latter further lowers sectoral prices. The mechanism repeats until a new equilibrium is reached, which features lower sectoral prices and hence a lower consumption price index, which implies a larger GDP. As for the effect on equilibrium supplier choices, an exogenous decrease in each sectoral desired markup incentivizes firms to connect to more suppliers, as lower desired markups decrease the prices charged by potential suppliers, while leaving the nominal wage unchanged. Online Appendix B.1 formally establishes the above results for exogenous changes in sectoral desired markups.

3.5 Discussion: nominal wage rigidity

The key results of the analysis so far are established in a setting where price setting is subject to nominal rigidities, while the nominal wage is fully flexible. However, my framework can be easily extended to allow for nominal rigidities in both price and wage setting, while leaving the key results qualitatively unchanged.

In particular, consider a version of my model with $K + 1$ sectors, where the additional sector is populated by firms which act as labor unions.[6] More specifically, they buy labor directly from households at rate W_t and sell those labor services to firms in the remaining K sectors of the economy. Moreover, labor union firms do not use any inputs other than labor and do not sell any output to households. Nominal rigidities in the price setting of firms in the labor union sector are then isomorphic to aggregate nominal wage rigidity. In particular, suppose each labor union firm faces an exogenous probability of price adjustment $(1-\alpha_u) \in (0, 1)$. Then under $T = 2$, the price index of the labor union sector at $t = 1$ is given by $P_u = \left[\alpha_u P_{u,0}^{1-\theta} + (1-\alpha_u)W^{1-\theta}\right]^{\frac{1}{1-\theta}}$.[7] The combination of the cash-in-advance constraint and the consumption-labor supply condition still implies that $W = \mathcal{M}$, meaning that the price index of the labor union sector is pinned down exclusively by the exogenous money supply $\mathcal{M}$. However, an exogenous increase/decrease in $\mathcal{M}$ implies an increase/decrease in P_u that is less than one-for-one, implying a degree of nominal rigidity in the cost of

[6] Such approach to introducing nominal wage rigidity has been used in multi-sector models with exogenous production networks, notably Rubbo (2023).

[7] I normalize the sectoral productivity of all firms in the union sector to one in all periods: $\mathcal{A}_{u,t} = 1, \forall t$. Further, I assume that the tax rate in the union sector is set to exactly offset any distortions coming from market power: $(1 + \tau_u)\frac{\theta}{\theta-1} = 1$. Those assumptions are without loss of generality, the comparative statics of real GDP and supplier choices remain qualitatively unchanged when those assumptions are relaxed.

purchasing labor services.

Firms in the remaining K sectors of the economy cannot purchase labor directly from households, instead buying it from the labor union sector. Then the unit cost function for any firm in a non-union sector k is given by $\mathcal{Q}_{kt}\left[S_{kt}, \mathcal{A}_{kt}(S_{kt}), P_{ut}, \{P_{rt}\}_{r\in S_{kt}}\right]$, and the optimal choice of supplier sectors S_{kt} reduces to deciding whether to use more labor at the price P_{ut}, or instead buy from a non-union sector r at P_{rt}. Then, the comparative statics following changes in the productivity mapping, exogenous desired markups and money supply are qualitatively unchanged relative to the results under flexible wages. In particular, either an improvement in the productivity mapping or a fall in desired markups of non-union sectors leaves the nominal price of labor P_u unchanged, while lowering the prices of the non-union sectors. The latter leads to a larger real GDP and encourages the adoption of more suppliers, as they are cheaper relative to labor, just as in the flexible-wage case. An increase in money supply delivers a less than one-for-one increase in the nominal price of labor P_u, and (weakly) an *even smaller* increase in the prices of the non-union sectors. Intuitively, any nominal rigidities in the price of labor become part of the unit cost function, and any nominal rigidities in price setting emerge over and above the rigidity in the unit cost function. Put simply, wage stickiness make sectoral prices *even stickier* in terms of the magnitude of response to the monetary expansion. As a result, prices in the non-union sectors rise by less than the nominal price of labor, which in turn rises by less than the money supply. The latter implies an increase in real GDP, as well as an expanded set of suppliers, as they are now cheaper relative to labor. Online Appendix B.2 formally establishes the qualitative equivalence of the cases with and without nominal wage rigidity.[8]

4 A quantitative dynamic version

After establishing the key properties of my model in a simplified static version, I now quantify the effects in a forward-looking setting calibrated to 389 sectors of the US economy. I develop a novel numerical algorithm for solving a dynamic version of my multi-sector model with sticky prices and endogenous network formation, and use it to determine how changes in baseline productivity and money supply affect equilibrium supplier choices, and how the latter creates non-linearities in transmission of monetary shocks.

[8] The equivalence is trivial for changes in the productivity mapping or changes in the desired markups of non-union sectors, since those do not affect the price of labor P_u. Online Appendix B.2 therefore only considers changes in money supply under nominal wage rigidity.

4.1 Forward-looking setting

I now quantify the effects established analytically in the previous section using a dynamic forward-looking version of the model. In particular, in this section I do not make the simplifying assumptions needed to obtain closed-form results, and solve the model numerically. Numerical simulations still require functional form assumptions and I therefore maintain the assumption of Cobb-Douglas production function and Cobb-Douglas consumption aggregation. Moreover, I need to specify the functional form of the productivity mapping, where I augment the mapping of Acemoglu and Azar (2020) with an aggregate productivity term:

Assumption 10 (Productivity mapping)**.** *For every sector $k = 1, 2, ..., K$ the productivity mapping $\mathcal{A}_{kt}(S_{kt})$ takes the following form:*

$$\mathcal{A}_{kt}(S_{kt}) = \mathcal{Z}_t e(S_{kt}) B_0 \prod_{r \in S_{kt}} B_{kr}, \tag{15}$$

where $e(S_{kt})$ is a term given by $e(S_{kt}) \equiv (1 - \sum_{r\in S_{kt}} \omega_{kr})^{-(1-\sum_{r\in S_{kt}}\omega_{kr})} \prod_{r\in S_{kt}} \omega_{kr}^{-\omega_{kr}}$, $\mathcal{Z}_t$ is aggregate productivity which follows an AR(1) process in logs: $\log \mathcal{Z}_t = \rho_z \log \mathcal{Z}_{t-1} + \zeta_t$, and B_0, $\{B_{kr}\}_{kr}$ are parameters.

Two points should be noted regarding the productivity mapping above. First, it delivers an equilibrium unit cost function given by $\mathcal{Q}_k = \left[\mathcal{Z}_t B_0 \prod_{r\in S_{kt}} B_{kr}\right]^{-1} W_t \prod_{r\in S_{kt}} \left(\frac{P_{rt}}{W_t}\right)^{\omega_{kr}}$, or, writing in logs, $-z_t - b_0 + w_t + \sum_{r\in S_{kt}}[\omega_{kr}(p_{rt} - w_t) - b_{kr}]$. The latter implies a tractable rule for whether or not it is optimal to buy inputs from a particular supplier. Namely, given the levels of sectoral prices and the nominal wage, sector k should only buy inputs from sector r if $\omega_{kr}(p_{rt} - w_t) < b_{kr}$. Second, the entire path of aggregate productivity is known to the agents, so that they are aware of $\mathcal{Z}_0$ and $\{\zeta_t\}_{t=1}^{\infty}$ at $t = 0$.[9]

I also need to specify the functional form for the money supply process, which I assume to follow an AR(1) process in log-differences:

Assumption 11. *[Money supply] For a given initial money supply $\mathcal{M}_0$, the money supply in $t \geqslant 1$ takes the following form: $\Delta \log \mathcal{M}_t = \rho_m \Delta \log \mathcal{M}_{t-1} + \varepsilon_t^m$, $\rho_m \in (0, 1)$.*

The agents are aware of $\mathcal{M}_0$ at $t = 0$ and at the beginning of $t = 1$ discover the entire future path of monetary shocks $\{\varepsilon_t^m\}_{t=1}^{\infty}$, facing no uncertainty beyond that point.

My modified, finite-horizon version of Calvo (1983) pricing, combined with the simple rule for inclusion of suppliers described above, allows to use backward induction to solve

[9] Note that it is trivial to extend the productivity mapping to allow for sector-specific productivity shocks, or even productivity shocks specific to a particular buyer-supplier sectoral pair.

my model at the sector-level. Most, specifically, I am interested in solving for equilibrium for periods $t < T$, which feature nominal rigidities and hence real variables responding to monetary shocks. For $t \geqslant T$, the problem is static from firms' pricing perspective, and money is neutral, which can therefore be used as a terminal condition in the backward induction algorithm. Here I outline my method, which numerically solves for sectoral prices, supplier choices and allocations:

Algorithm 1 (Model with sticky prices and endogenous production networks)**.** *Start from a guess for sectoral prices, supplier choices and allocations; let $\mathcal{X}_t^-$, $\mathcal{X}_t$ and $\mathcal{X}_t^+$ be, respectively, the full set of past, present and future prices, supplier choices and allocations at time t for any $1 \leqslant t \leqslant T-1$. Let $\left\{\{P_{kt}^0\}_{k=1}^K\right\}_{t=1}^{T-1}$ be sectoral prices from the initial guess. Taking as given exogenous paths for money supply and the productivity mapping, follow the steps below, starting from $t = T-1$*

(i) Taking as given sectoral supplier choices, as well as past and future variables $\mathcal{X}_t^-$, $\mathcal{X}_t^+$, solve for prices $\{P_{kt}\}_{k=1}^K$;

(ii) Given prices $\{P_{kt}\}_{k=1}^K$, update supplier choices according to the following rule: sector k should only buy inputs from sector r if $\omega_{kr}(p_{rt} - m_t) < b_{kr}$, where $p_{rt} \equiv \log P_{kt}$ and $m_t \equiv \log \mathcal{M}_t$;

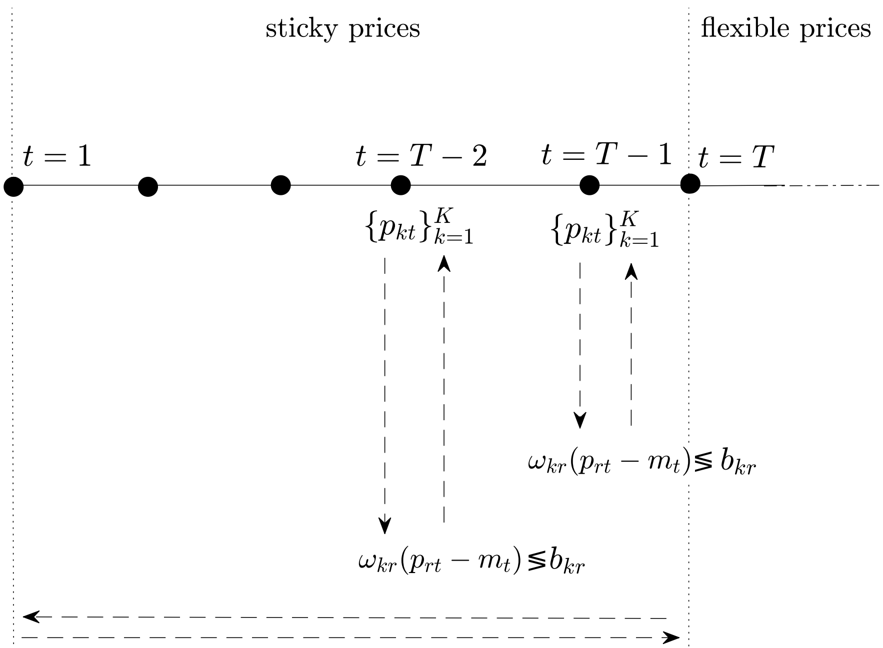
(iii) Taking as given $\mathcal{X}_t^-$, $\mathcal{X}_t^+$, $\{P_{kt}\}_{k=1}^K$ and $\{S_{kt}\}_{k=1}^K$, update $\mathcal{X}_t$;

(iv) Repeat (i)-(iii) until convergence within the time period;

(v) If $t > 1$, decrease t by one and go back to (i). Otherwise, compare $\{\{P_{kt}^0\}_{k=1}^K\}_{t=1}^{T-1}$ with $\{\{P_{kt}\}_{k=1}^K\}_{t=1}^{T-1}$; if they are equal, stop the algorithm; if they are not equal, set $P_{kt}^0 = P_{kt}, \forall k, 1 \leqslant t \leqslant T-1$, set $t = T-1$ and return to (i).

Figure 5 provides a graphical summary of the novel numerical algorithm. Notice that step (i) involves solving for sectoral prices and allocations, taking the production network *as given*. This allows to approximate the solution in step (i) by considering a log-linear approximation around the deterministic steady state, holding the choice of suppliers fixed. The supplier choices are then computed in step (ii) using those approximate solutions for sectoral prices. This decoupling of the "smooth" optimal pricing problem and the "non-smooth" supplier choice problem allows to substantially speed up the algorithm, while preserving the non-linearity coming from the extensive margin of production network formation.

Figure 5: **The numerical algorithm: a graphical representation**



***Notes**: the figure provides a graphical representation of the numerical approach used to solve the dynamic forward-looking version of my model with sticky prices and endogenous production networks, formally introduced in Algorithm 1.*

4.2 Calibration

I calibrate my model for the United States an annual frequencies. Calibration of aggregate parameters is standard. I set $\beta = 0.99$ to target annualized real interest rate of 1% in steady state. The within-sector elasticity of substitution is set at $\theta = 6$. The persistence parameters of productivity and money supply growth are set at $\rho_z = 0.86$ and $\rho_m = 0.80$, respectively. As for the threshold beyond which there are no nominal rigidities, I choose $T = 50$, so that after fifty years all firms can adjust their prices with certainty. Finally, I normalize $\mathcal{Z}_0 = 1$.

As for sector-specific parameters, those are selected for the US Bureau of Economic Analysis (BEA) Detail level of disaggregation featuring $K = 389$ sectors. Sector-specific Calvo parameters $\{\alpha_k\}_{k=1}^{K}$ are set as one minus the sector-specific frequency of price adjustment from Pasten et al. (2020). The long-run markups in my economies are given by $\{(1 + \tau_k)\frac{\theta}{\theta-1}\}_{k=1}^{K}$ and sectoral tax rates $\{\tau_k\}_{k=1}^{K}$ are chosen to match the estimated sectoral markups from De Loecker et al. (2020). The observed input-output shares $\{\omega_{kr}\}_{kr}$ are calibrated using the 2007 BEA Input-Output table, whereas input-output shares for linkages not observed in the data are imputed following Acemoglu and Azar (2020) by setting each unobserved share equal to 0.95 of the cost share of labor of the buying sector divided by the number of potential additional suppliers for that sector. Finally, B_0 and $\{B_{kr}\}_{kr}$ are estimated to ensure the steady-state equilibrium of my economy under $\mathcal{M}_t = \mathcal{Z}_t = 1, \forall t$ simultaneously matches observed input-output linkages and real GDP in 2007.

4.3 Simulation results

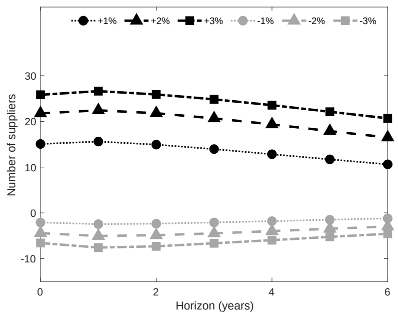
4.3.1 Baseline economies

Variations in the baseline of my economy are driven by changes in the initial money supply $\mathcal{M}_0$ and changes in the sequence $\{\zeta_t\}_{t=1}^{\infty}$, which pins down the path of aggregate productivity. I consider two sets of variations in the baseline. First, I hold $\mathcal{M}_0 = 1$ and $\zeta_t = 0, \forall t \geqslant 2$, and consider values of $\zeta_1 = \{-3\%, -2\%, -1\%, 0, 1\%, 2\%, 3\%\}$. In this way, I look at baselines with both low and high productivity paths, all of which eventually converge to the initial value of $\mathcal{Z}_0 = 1$; the case of $\zeta_1 = 0$ where aggregate productivity remains fixed at the initial value is set as a benchmark against which the other baselines are compared. Second, I hold $\zeta_t = 0, \forall t \geqslant 1$ and consider different values of $\mathcal{M}_0$. In particular, I assume that in the unmodelled past before $t = 0$ money supply starts from $\mathcal{M}_{-\infty} = 1$ and follows the process in Assumption 11, experiencing a one-time never repeating shock $\varepsilon^m_{-\infty}$, eventually converging to the value of $\mathcal{M}_0$. I consider values of $\varepsilon^m_{-\infty} = \{-6\%, -4\%, -2\%, 0, 2\%, 4\%, 6\%\}$, where the case of $\varepsilon^m_{-\infty} = 0$ is treated as a benchmark against which all the other baselines are compared.

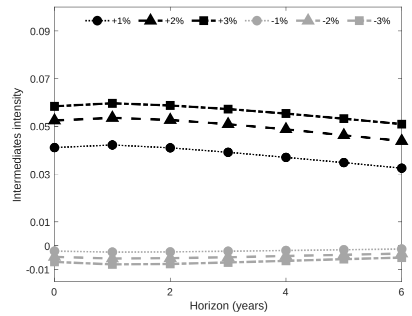
Panels (a) and (b) of Figure 6 consider baselines with different aggregate productivity paths and their associated paths of average number of suppliers and average intermediates intensity, which measures the average share of total costs that goes to suppliers. Relative to the fixed-productivity baseline ($\zeta_1 = 0$), letting $\zeta_1 = 1\%$ is associated with a long-lived increase in the average number of suppliers by around 15 per sector, which translates into an increase in average intermediates intensity by around 0.04. Similarly, setting $\zeta_1 = 3\%$, which delivers a baseline with an even better aggregate productivity, increases the average number of suppliers per sector by around 27, or an approximately 0.06 increase in average intermediates intensity. Importantly, the results suggest that the effect of changing the baseline productivity path is not symmetric, which is a quantitative result specific to the parameters of the productivity mapping $\{B_{kr}\}_{kr}$ I have estimated. Specifically, a low-productivity baseline with $\zeta_1 = -1\%$ decreases the number of suppliers only by ap-

Figure 6: **Numbers of suppliers and intermediates intensities across baselines**

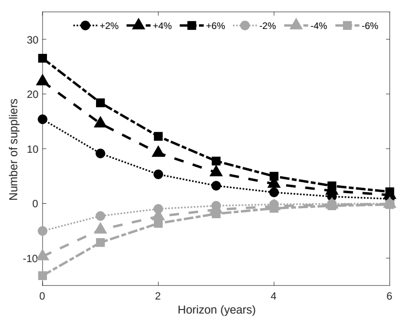
(a) Average number of suppliers



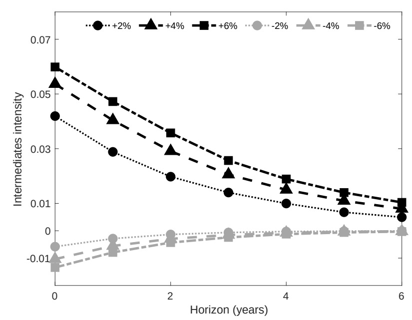
(b) Average intermediates intensity



(c) Average number of suppliers



(d) Average intermediates intensity



***Notes**: Panels (a) and (b) show the average number of suppliers and average intermediates intensities across baselines where $\mathcal{M}_0 = 1$ and $\zeta_t = 0, \forall t \geqslant 2$, and consider values of $\zeta_1 = \{-3\%, -2\%, -1\%, 1\%, 2\%, 3\%\}$, relative to $\zeta_1 = 0$; Panels (c) and (d) show the average number of suppliers and average intermediates intensities across baselines where $\zeta_t = 0, \forall t \geqslant 1$ and values of $\varepsilon^m_{-\infty} = \{-6\%, -4\%, -2\%, 2\%, 4\%, 6\%\}$, relative to the case where $\varepsilon^m_{-\infty} = 0$.*

proximately 2 per sector, or a small decrease of 0.002 in average intermediates intensity. Under an even worse productivity baseline with $\zeta_1 = -3\%$, the average number of suppliers falls by around 8 per sector, equivalent to a 0.01 reduction in average intermediates intensity.

As for results under baselines with different initial levels of money supply, those are reported in Panels (c) and (d) of Figure 6. Relative the benchmark case ($\varepsilon^m_{-\infty} = 0, \mathcal{M}_0 = 1$), a looser money supply baseline under $\varepsilon^m_{-\infty} = 2\%$ features an initial rise in the average number of suppliers by around 16, or an increase in the average intermediates intensity by around 0.045. A baseline with an even looser initial money supply ($\varepsilon^m_{-\infty} = 6\%$) is associated with an initial increase in the average number of suppliers by around 28, equivalent to a 0.06 increase in average intermediates intensity. Just like with variations in baseline aggregate productivity, the effect of variations in initial money supply is not symmetric. In particular, a baseline with tighter money supply ($\varepsilon^m_{-\infty} = -2\%$) has an initial drop in the average number of suppliers by 5, or a 0.005 reduction in average intermediates intensity. An even tighter initial money supply ($\varepsilon^m_{-\infty} = -6\%$) features a drop of 15 in the average number of suppliers, or a 0.012 reduction in the average intermediates intensity.

Note that the paths of the number of suppliers and intermediates intensity are generally very persistent. In the case of baselines with different levels of aggregate productivity, this persistence has two sources. First, there is the exogenous persistence, coming from the autoregressive process assumed for aggregate productivity, with the AR(1) parameter ρ_z set equal to 0.86. Second, there is the endogenous persistence coming from price stickiness: since changes in the price level are staggered, so are the optimal choices of supplier sectors. In order to separately assess the degree of endogenous persistence in production network dynamics, Online Appendix C reports the dynamics of average number of suppliers and intermediate intensities when productivity changes across baselines are purely transitory, i.e. $\rho_z = 0$. One can see that the amount of persistence in this case remains substantial, with purely transitory shifts in productivity generating changes in network configurations that last for years. This highlights that price stickiness simultaneously plays two roles in my model: it generates monetary non-neutrality *and* it produces endogenous persistence in optimal supplier choices at the extensive margin, despite the fact that dropping and adopting new suppliers is costless.

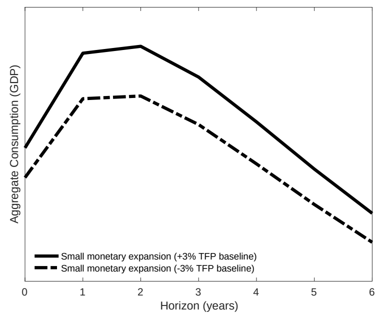
4.3.2 Small monetary shocks

I now perturb each of the baselines considered in the previous subsection with the same monetary shock, which is small in the sense that it leaves the equilibrium set of suppliers unchanged relative to the baseline in every period. The aim is to study how variations in the baseline affect the magnitude of monetary transmission to GDP.

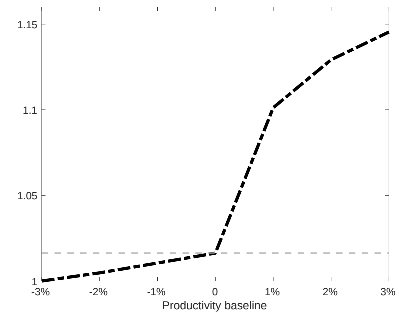
Figure 7(a) shows IRFs of GDP to the same small monetary expansionary shock under baselines with low ($\zeta_1 = -3\%$) and high ($\zeta_1 = 3\%$) aggregate productivity. One can see that under high productivity the response of GDP is persistently higher across horizons, despite the fact that the size of the shock is the same across the two baselines. This is because, as seen in Figure 6(a), under the high productivity baseline the average number of suppliers is higher, which strengthens complementarities in price setting and amplifies monetary non-neutrality. Moreover, Figure 6(a) also shows that under the high productivity

Figure 7: **Impulse responses of GDP to a small monetary expansion across baselines**

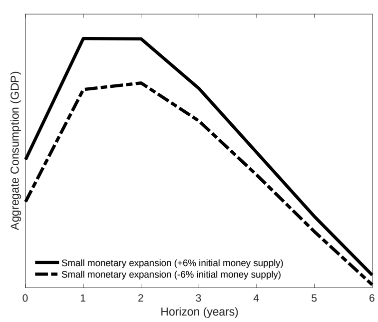
(a) IRFs under expansion and recession



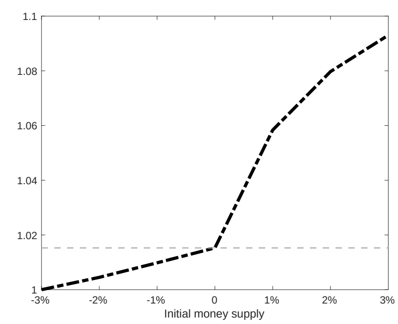
(b) Peaks of IRFs across productivities



(c) IRFs under tight and loose money



(d) Peaks of IRFs across money supplies



***Notes**: Panel (a) shows IRFs of GDP to the same small monetary expansion around two baselines, with* $(\mathcal{M}_0 = 1, \zeta_1 = -3\%)$ *and* $(\mathcal{M}_0 = 1, \zeta_1 = 3\%)$*; Panel (b) shows peaks of IRFs to the same small monetary expansion around two baselines, with* $\mathcal{M}_0 = 1$ *and* $\zeta_1 = \{-3\%, -2\%, 0, -1\%, 1\%, 2\%, 3\%\}$*, where that peak for* $\zeta_1 = -3\%$ *is normalized to one. Panel (c) shows IRFs of GDP to the same small monetary expansion around two baselines, with* $(\zeta_t = 0, \forall t \geqslant 1, \varepsilon^m_{-\infty} = \{-6\%\})$ *and* $(\zeta_t = 0, \forall t \geqslant 1, \varepsilon^m_{-\infty} = \{6\%\})$*; Panel (d) shows peaks of IRFs to the same small monetary expansion around two baselines, with* $\zeta_t = 0, \forall t \geqslant 1$ *and* $\varepsilon^m_{-\infty} = \{-6\%, -4\%, -2\%, 0, 2\%, 4\%, 6\%\}$ *, where that peak for* $\varepsilon^m_{-\infty} = -6\%$ *is normalized to one.*

baseline the increase in the average number of suppliers is persistent, which is what creates persistently higher non-neutrality of money, and explains why the gap between the two IRFs holds across horizons.

In order to further quantify such interplay between baseline productivity and monetary non-neutrality, Figure 7(b) plots peaks of IRFs of GDP to the same small monetary expansion across baselines with different aggregate productivities. For the ease of interpretation, the peak under the lowest productivity baseline $(\zeta_1 = -3\%)$ is normalized to one. One can see that peak response of GDP under the highest productivity baseline is almost one-sixth larger than under the lowest productivity baseline. However, the rate at which monetary non-neutrality varies with baseline productivity is highly non-linear. In particular, the change in non-neutrality between the highest productivity baseline ($\zeta_1 = 3\%$) and the benchmark case ($\zeta_1 = 0$) is almost seven times larger than the change between the benchmark case and the lowest productivity baseline ($\zeta_1 = -3\%$). In this sense, in my quantitative model monetary policy becomes powerful in expansions relative to normal times, but its effectiveness does not wane by nearly as much in recessions. The latter is explained by the asymmetry in the relationship between the average number of suppliers and baseline productivity, as documented in Figure 6(a).

I now repeat the above exercise for baselines that differ in their initial money supply. Figure 7(c) shows IRFs of GDP to the same small monetary expansionary shock under baselines with tight ($\varepsilon^m_{-\infty} = -6\%$) and loose ($\varepsilon^m_{-\infty} = 6\%$) initial money supply. One can see that under loose initial money supply the response of GDP is higher across horizons. As before, this is because under the loose initial money supply baseline the average number of suppliers is higher, which strengthens complementarities in price setting and amplifies monetary non-neutrality. Notice that according to Figure 6(c), the change in average number of suppliers triggered by change in initial money supply is much shorter lived that that caused by changes in baseline productivity. As a result, additional monetary non-neutrality created by high initial money supply is shorter lived and, as can be seen in Figure 7(c), IRFs of GDP under different baseline money supplies converge to each other faster than in Figure 7(a).

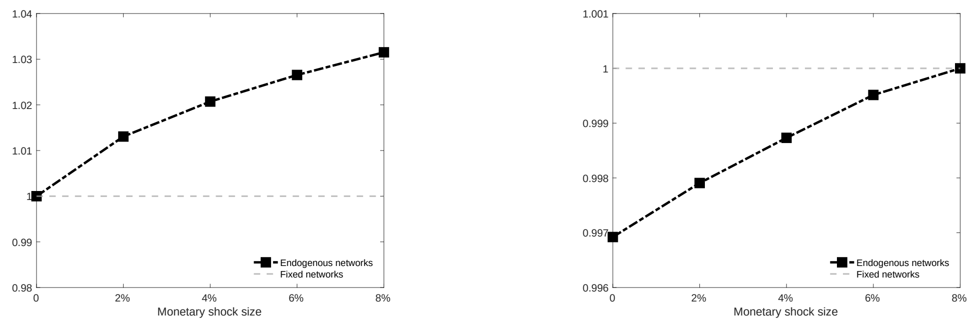
In Figure 7(d) I plot peaks of GDP responses to the same small monetary expansion across baselines with different initial money supply. One can see that peak response of GDP under the baseline with most loose money supply ($\varepsilon^m_{-\infty} = 6\%$) is almost one-tenth larger than under the baseline with tightest money supply ($\varepsilon^m_{-\infty} = -6\%$). As in Figure 7(b) the relationship between peak GDP and changes in the baseline is highly non-linear: the drop in potency of monetary policy under tight initial money supply is much smaller than the rise in potency under loose initial money. And just as before, this is explained by the asymmetry in the relationship between the average number of suppliers and baseline money supply, as documented in Figure 6(c).

4.3.3 Large monetary shocks

Having studied properties of small monetary shocks, I now turn to monetary shocks that are large enough to affect equilibrium supplier choices. Recall that in the previous section I formally established that large monetary expansions expand the set of suppliers which

Figure 8: **IRFs of GDP to large monetary shock relative to a fixed network benchmark**

(a) Large monetary expansions (b) Large monetary contractions



***Notes**: Panel (a) shows peaks of IRFs to large monetary expansions around the baseline with $\zeta_t = 0, \forall t \geqslant 1$ and $\mathcal{M}_0 = 1$, for shock values $\varepsilon_1^m = \{0, 2\%, 4\%, 6\%, 8\%\}$, where that peak for $\varepsilon_1^m = 0$ is normalized to one; Panel (b) shows peaks of IRFs to large monetary contractions around the baseline with $\zeta_t = 0, \forall t \geqslant 1$ and $\mathcal{M}_0 = 1$, for shock values $\varepsilon_1^m = \{-8\%, -6\%, -4\%, -2\%\, 0\}$, where that peak for $\varepsilon_1^m = 0$ is normalized to one.*

leads to a larger GDP response relative to the case where networks are fixed. On the contrary, large monetary contractions reduce the number of linkages and thus deliver a smaller contraction in GDP relative to what would happen under exogenous networks.

In Figure 8 I report results, where I fix the baseline at $\mathcal{M}_0 = \mathcal{Z}_t = 1, \forall t$, and subject my economy to one-time large monetary expansions $\varepsilon_1^m = \{2\%, 4\%, 6\%, 8\%\}$ and large monetary contractions $\varepsilon_1^m = \{-8\%, -6\%, -4\%, -2\%\}$. For each shock size, I normalize GDP IRFs with the response of GDP that would occur if the same shock hit an otherwise identical economy with fixed exogenous input-output linkages.

Panel (a) reports normalized peaks of GDP IRFs to large monetary expansions. Relative to an economy with fixed networks, the peak response of GDP to a 2% monetary expansion is larger by just over 1 percent, whereas for an 8% shock the response is 3 percent larger. Consistently with my formal results in the previous section, large monetary expansions deliver more than proportional response of GDP, despite fully time-dependent probabilities of price adjustment.

Results for large monetary contractions are reported in Panel (b). Relative to an economy with exogenous production networks, an 8% monetary contraction delivers a drop in GDP that is 0.3 percent smaller at the trough, with the gap shrinking down to just over 0.1 percent for a 4% contraction. Overall, the effects are consistent with formal results from the previous section, though their quantitative relevance is much smaller than under large monetary expansions. This is a consequence of asymmetric sensitivity of equilibrium supplier changes to changes in money supply that was documented in Figures 6(c)-(d), where reductions in money supply do not eliminate as many linkages as what is added under increase in money supply of the same magnitude.

5 Empirical evidence

Having established and quantified my theoretical results, I now turn to empirical assessment of the key mechanisms. In this section I use both sectoral and firm-level input-output data to estimate network cyclicality conditional on identified technology and monetary shocks. I show that my novel empirical results strongly support the theoretical predictions of the model.

5.1 Evidence using sector-level data

5.1.1 Data

In this subsection I use sector-level data on intermediates intensity in order to perform econometric evaluation of the predictions of my model regarding business cycle fluctuations in the shape of the production network. In particular, I use US data on sector-level employee compensation and expenditure on intermediate inputs, published by the Bureau of Economics Analysis (BEA), to construct the share of total costs going to intermediate inputs:

$$\delta_{kt} = \frac{\text{Intermediate inputs costs}_{kt}}{\text{Compensation of Employees}_{kt} + \text{Intermediate inputs costs}_{kt}}. \tag{16}$$

I construct such measure of intermediates intensity for 65 three-digit sectors of the US economy at annual frequency between 1987-2017. In the case of a Cobb-Douglas production function, such measure maps directly into the model-based measure of sector-specific intermediates intensity in a particular time period, given by $\sum_{r \in S_{k,t}} \omega_{kr}, \forall k$.

5.1.2 Econometric strategy

Once those measures have been constructed, I use the local projection approach of Jordà (2005) in order to estimate horizon-specific impulse responses of sectoral intermediates intensity to productivity and monetary shocks:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta_H s_t + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H}, \tag{17}$$

for $H = 0, 1, ..., \overline{H}$, where $\delta_{k,t}$ is intermediate intensity of sector k, s_t is an identified exogenous shock, $x_{k,t-1}$ is a vector of control variables and $\alpha_{k,H}$ represents horizon-specific sectoral fixed effects.[10] The estimated value of β_H gives the horizon-specific response of intermediate inputs intensity to an identified exogenous shock; according to my theory, intermediates intensity should, *ceteris paribus*, rise following expansionary productivity and monetary shocks, and *vice versa*. The specification above allows to econometrically test such predictions in a transparent way using horizon-by-horizon fixed-effects regressions.

A feature of my theory is that changes in productivity and monetary conditions can affect the equilibrium set of suppliers in a non-linear fashion. For example, in the previous section I documented how improvements in productivity and money supply affect the use of intermediates much more strongly than deteriorations of the same magnitude. In order to test for presence of such non-linearities in the data, I consider the following augmented local projection specification:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta_H^{linear} s_t + \beta_H^{sign} s_t \times \mathbf{1}\{s_t > 0\} + \beta_H^{size} s_t \times |s_t| + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H}, \quad (18)$$

for $H = 0, 1, ..., \overline{H}$, where it follows that $\beta_H^{sign} > 0$ indicates that positively-valued shocks make the impulse response more positive, whereas $\beta_H^{size} > 0$ indicates that shocks which are large in magnitude make the response scale up more than proportionally.

In the baseline results presented in this subsection, I use identified annual productivity shocks from Fernald (2014) and identified annual monetary shocks from Romer and Romer (2004) that have been extended by Wieland and Yang (2016).

5.1.3 Estimation results

Figure 9 shows estimation results using linear local projections in equation (17). Panel (a) shows that following a positive 1% shock to aggregate productivity, the intermediates intensity responds positively on impact, reaching peak increase of 0.004 after one year, and then gradually declines back to zero; such positive response is consistent with my theory. As for a monetary expansion, Panel (b) shows that a surprise one-time 100bp easing leads to a slight decline in the intermediates intensity on impact, after which it gradually increases, reaching a peak of 0.005 increase after four years; the response at longer horizons is consistent with theoretical predictions of the model.

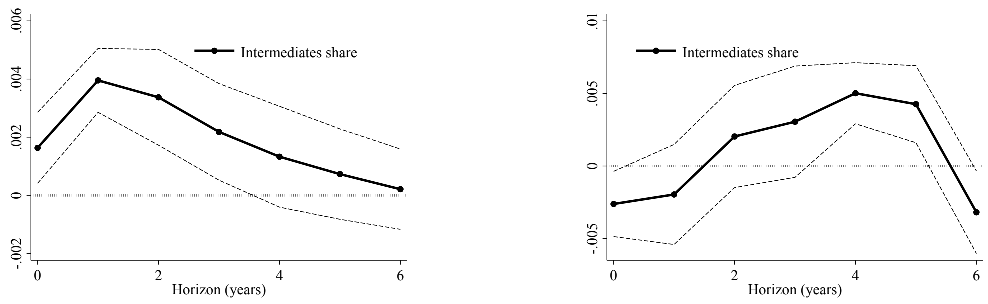
Figure 10 reports the results of estimation using non-linear local projections in (18).[11]

[10] The set of controls is specified in the descriptions to figures presenting each of the estimation results.

[11] Formal significance tests of the horizon-specfic non-linear effects are reported in Online Appendix D.1

Figure 9: **Linear IRFs of intermediates intensity to productivity and monetary shocks**

(a) Productivity expansion (+1%) (b) Monetary expansion (-100bp)



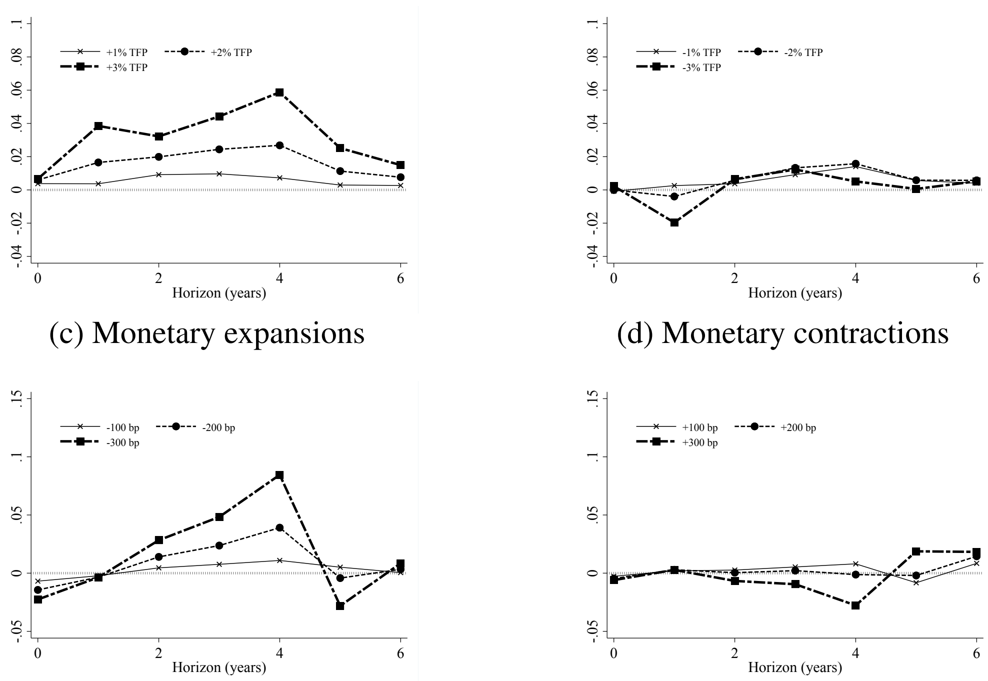
***Notes**: Panel (a) shows estimated IRFs of intermediates intensity to a productivity shock based on Fernald (2014), using specification in (17); the set of controls includes one lead of productivity shock and one lag of intermediates intensity, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows estimated IRFs of intermediates intensity to a monetary shock based on Romer and Romer (2004), using specification in (17); the set of controls includes one lead of monetary shock and one lag of intermediates intensity, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence bands.*

Panel (a) shows that following a 1% productivity expansion, intermediates intensity rises by around 0.01 at the peak, whereas a 3% productivity expansion increases intermediates intensity by around 0.06 at the peak. Importantly, these differences imply that the responses estimated allowing for non-linearities are not equal to linear IRFs scaled by the size of the shock, suggesting an important role for size effects. Another important aspect is that the responses estimated allowing for non-linearities match the magnitudes of responses generated by the quantitative model; recall that in Figure 6(b) I reported that a baseline generated by a one-time 3% aggregate productivity improvement also generates a rise in average intermediates intensity of around 0.06.

Figure 10(b) considers productivity contractions. A 1% productivity contraction does not generate a drop in intermediates intensity, whereas a 3% contraction generates a modest drop in intermediates intensity of 0.02 at the trough. The latter suggest presence of sign effects: productivity contractions do not produce drops in intermediates intensity of the same magnitude as the rises in intensity produced by productivity expansions. Importantly, the quantitative version of my model produces size effects of the same direction and magnitude; as can be seen in Figure 6(b), a baseline generated by a one-time 3% aggregate productivity contraction has average intermediates intensity drop by just around 0.01 at the trough.

In Figure 10(c) I turn to the effect of monetary expansions on intermediates intensity. A 100bp monetary easing raises intermediates intensity by around 0.01 at the peak, whereas

Figure 10: **Non-linear IRFs of intermediates intensity to shocks**



(a) Productivity expansions (b) Productivity contractions

(c) Monetary expansions (d) Monetary contractions

***Notes**: Panels (a)-(b) show estimated non-linear IRFs of intermediates intensity to a productivity shock based on Fernald (2014), using specification in (18); the set of controls includes one lead of productivity shock and one lag of intermediates intensity, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panels (c)-(d) show estimated non-linear IRFs of intermediates intensity to a monetary shock based on Romer and Romer (2004), using specification in (18); the set of controls includes one lead of monetary shock and one lag of intermediates intensity, monetary shock, (log) real GDP, federal funds rate, as well as a time trend.*

a 300bp easing produces a rise of close to 0.08. Once again, size effects are present. As for monetary contractions, Panel (d) shows that a 100bp tightening produces close to no drop in intermediates intensity, while a larger 300bp tightening delivers a drop of around 0.02 at the trough. Just like in the case of productivity changes, sign effects are detected: monetary contractions produce drops in intermediates intensity that are much smaller in magnitude than the increases produced by easings of the same size. Finally, recall that in Figure 6(d) I documented presence of similar sign effects in the quantitative version of my model as one varies money supply.

5.1.4 Discussion: margin of adjustment

The cyclicality of intermediates intensity established in this subsection is consistent with the predictions of my model. However, given that the cyclicality is established using sectoral data, it does not say anything about the margin of adjustment. At the same time, my theoretical model emphasizes the extensive margin of network adjustment.

Before formally testing for the extensive margin in the next subsection, I would like to present an argument, which rationalizes why at least some of the observed variation in intermediates intensity is likely to occur at the extensive margin. Consider a version of my model which does not have the supplier choice at the extensive margin, but instead has variation in intermediates intensity occurring at the intensive margin only. The standard way to obtain that would be to consider a constant elasticity of substitution (CES) production function in labor and intermediates inputs: $Y_{kt}(j) = \mathcal{A}_{kt}\left[(1-\overline{\delta}_k)^{\frac{1}{\eta}} N_{kt}^{\frac{\eta-1}{\eta}}(j) + \overline{\delta}_k^{\frac{1}{\eta}} Z_{kt}^{\frac{\eta-1}{\eta}}(j)\right]^{\frac{\eta}{\eta-1}}$, where $\eta > 0$ is the elasticity of substitution, $Z_{kt}(j)$ is the composite intermediate input given some exogenous set of supplier sectors. Then equilibrium intermediates intensity is given by $\delta_{kt} = \overline{\delta}_k \times \left[(1-\overline{\delta}_k)(W_t/P_t^k)^{1-\eta} + \overline{\delta}_k\right]^{-1}$, where P_t^k is the price index for the composite intermediate input. In my setup with sticky prices, following either a monetary expansion or an exogenous productivity improvement (W_t/P_t^k) rises. Therefore, having a procyclical intermediates intensity in this setup requires $\eta > 1$. However, this is inconsistent with available micro estimates of η, which find it to be close to one from below (Atalay, 2017). Therefore, matching the observed procyclicality of intermediates intensity using purely the intensive margin of adjustment is likely to require a strongly counterfactual elasticity of substitution between labor and intermediates.

5.2 Evidence using firm-level data

5.2.1 Data

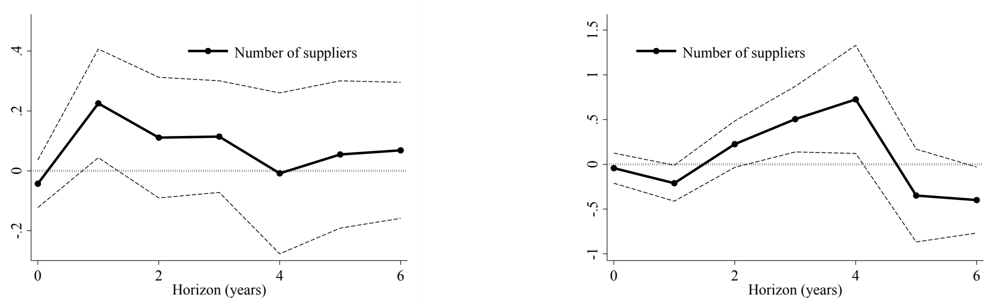
A unique prediction of my model is that the adjustment in the relative reliance on intermediate inputs can happen at the extensive margin. In this subsection, I use firm-level data on the number of suppliers of US publicly listed firms in order to perform econometric evaluation of the *extensive margin* of network adjustment over the business cycle. In particular, I use the dataset constructed by Atalay et al. (2011) based on US Compustat data, which contains a time series for the firm-level *indegree*, which is the number of suppliers in the dataset. The dataset covers a large number of US publicly listed firms.

5.2.2 Econometric strategy

Using the indegree measure, I once again follow the local projection approach of Jordà (2005) in order to estimate horizon-specific impulse responses of a firm's number of sup-

Figure 11: **Linear IRFs of number of suppliers to productivity and monetary shocks**

(a) Productivity expansion (+1%) (b) Monetary expansion (-100bp)



***Notes**: Panel (a) shows estimated IRFs of the number of suppliers to a productivity shock based on Fernald (2014), using specification in (19); the set of controls includes four lags of the number of suppliers, one lag of productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows estimated IRFs of the number of suppliers to a monetary shock based on Romer and Romer (2004), using specification in (19); the set of controls includes one lead of monetary shock, four lags of the number of suppliers, one lag of monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence bands.*

pliers to productivity and monetary shocks:

$$indeg_{j,t+H} = \alpha_{j,H} + \beta_H s_t + \gamma_H x_{j,t-1} + \varepsilon_{j,t+H}, \tag{19}$$

for $H = 0, 1, ..., \overline{H}$, where $indeg_{j,t}$ is the indegree (number of suppliers) of firm j in year t, s_t is an identified exogenous shock, $x_{j,t-1}$ is a vector of control variables and $\alpha_{j,H}$ represents horizon-specific firm fixed effect. The estimated value of β_H gives the horizon-specific response of the number of suppliers to an identified exogenous shock; according to my theory, the number of suppliers should, *ceteris paribus*, rise following expansionary productivity and monetary shocks, and *vice versa*. The specification above allows to test such predictions in a transparent model-free way using horizon-by-horizon fixed-effects regressions.

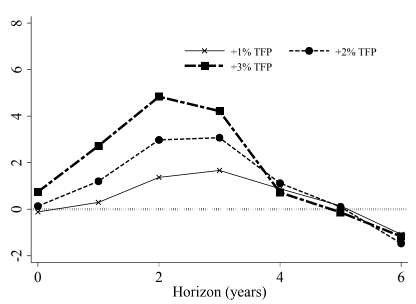
As discussed earlier, my model predicts a non-linear relationship between changes in productivity and monetary conditions, and the equilibrium set of suppliers. In order to test for such non-linear adjustment at the extensive margin, I once again extend the previous specification to include additional terms:

$$indeg_{j,t+H} = \alpha_{j,H} + \beta_H^{linear} s_t + \beta_H^{sign} s_t \times \mathbf{1}\{s_t > 0\} + \beta_H^{size} s_t \times |s_t| + \gamma_H x_{j,t-1} + \varepsilon_{j,t+H}, \tag{20}$$

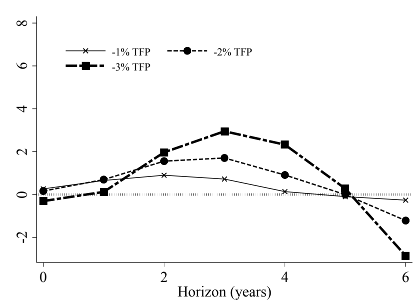
for $H = 0, 1, ..., \overline{H}$, where it follows that $\beta_H^{sign} > 0$ indicates that positively-valued shocks make the impulse response more positive, whereas β_H^{size} indicates that larger shocks make the response more positive.

Figure 12: **Non-linear IRFs of number of suppliers to shocks**

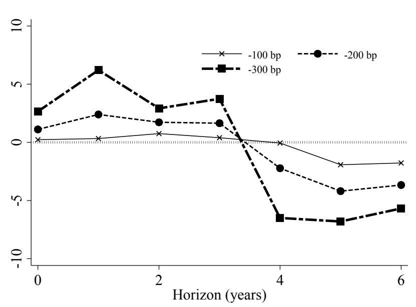
(a) Productivity expansions



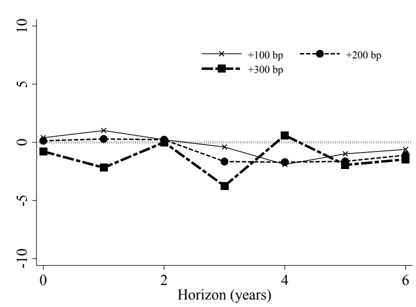
(b) Productivity contractions



(c) Monetary expansions



(d) Monetary contractions



***Notes**: Panels (a)-(b) show estimated non-linear IRFs of the number of suppliers to a productivity shock based on Fernald (2014), using specification in (20); the set of controls includes one lead of productivity shock, four lags of the number of suppliers, one lag of productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panels (c)-(d) show estimated non-linear IRFs of the number of suppliers to a monetary shock based on Romer and Romer (2004), using specification in (20); the set of controls includes one lead of monetary shock, four lags of the number of suppliers, one lag of monetary shock, (log) real GDP, federal funds rate, as well as a time trend.*

In the baseline results presented in this subsection, I once again use identified annual productivity shocks from Fernald (2014) and identified annual monetary shocks from Romer and Romer (2004) that have been extended by Wieland and Yang (2016).

5.2.3 Estimation results

Figure 11 shows estimation results using linear local projections in (19). As one can see, following a positive 1% shock to aggregate productivity, the (average) number of suppliers responds positively on impact, reaching peak increase of around 0.20 after one year, and then gradually declines back to zero; such positive response is consistent with my theory. As for a monetary expansion, a surprise one-time 100bp easing first leads to a slight insignificant decline, after which in gradually increases, reaching a peak of 0.85 increase after four years; once again, the response at longer horizons is both statistically significant and consistent with theoretical predictions.

Figure 12 reports the results obtained using non-linear local projections in (20).Panel (a) shows that following a 1% productivity expansion, the (average) number of suppliers goes up by almost 2 at the peak, whereas a 3% productivity expansion increases the number of suppliers per firm by approximately 5. At the same time, Panel (b) suggests that productivity contractions are unable to deliver drops in the number of suppliers. The latter suggests that sign effects, documented both in estimation using sectoral data and in the quantitative version of my model, are similarly found in firm-level data.

Similarly, Figure 12(c) shows that following monetary easings the (average) number of suppliers per firm rises: by around 1 following a 100bp easing, and by close to 6 after a 300bp loosening. And once again, monetary contractions are unable to generate a drop in the number of suppliers per firm of a comparable magnitude, giving another evidence in favor of sign effects, previously documented for monetary interventions both using estimation with sectoral data, as well as using a quantitative version of my theoretical model.

5.2.4 Further results and robustness checks

In Online Appendix D, I consider an alternative approach to modeling non-linearity in my local projections setting. In particular, I re-estimate (18) and (20), where instead of adding interactions with a sign dummy and absolute value of the shock, I add quadratic and cubic shocks, which similarly capture possible sign and size effects. Reassuringly, results remain virtually unchanged, both in terms of magnitudes of effects, as well as when it comes to sign and size effects and their statistical significance.

6 Conclusion

I develop a novel dynamic multi-sector general equilibrium model with sticky prices, where input-output linkages are formed endogenously through firms' optimizing decisions. I provide novel empirical evidence on the cyclical behavior of production networks and show that the model replicates the observed variation, conditional on either productivity or monetary shocks driving the cycle. In particular, in the data the reliance on intermediates rises following productivity improvements and monetary easings. In the model, technological improvements also increase the reliance on intermediates, as expanding the set of suppliers lowers firms' unit costs directly through higher productivity and indirectly through lower prices charged by suppliers. Similarly, nominal easings also expand the production network in the model, as they make prices charged by supplier firms cheaper relative to the cost of in-house labor.

The model delivers a novel mechanism for state-dependence in monetary transmission, which is generated by cyclical variation in the strength of complementarities in price setting. This mechanism makes the strength of monetary transmission depend on the *phase of the business cycle*, *past monetary stance* and the *size of the shock* even if the probability of price adjustment is state-independent. In particular, in periods of high productivity and loose monetary policy, firms optimally decide to connect to more suppliers, which strengthens pricing complementarities and amplifies monetary non-neutrality. On the contrary, episodes of low productivity and tight monetary policy lead to sparse networks and weaker complementarities in price setting, which diminishes monetary non-neutrality. At the same time, larger monetary expansions have a disproportionally *larger* positive effect on GDP compared to smaller monetary expansions, as the former expand the production network. Larger monetary contractions instead have a disproportionally *smaller* negative effect on GDP, as they shrink the network. The non-linearities in the transmission of monetary shocks in the theoretical model are consistent with the econometric evidence (Tenreyro and Thwaites, 2016; Jordà et al., 2020; Alpanda et al., 2021; Ascari and Haber, 2022).

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Online Appendix

Endogenous production networks and non-linear monetary transmission

(Mishel Ghassibe, *CREi, UPF & BSE*)

A Detailed proofs

Proposition 1 (Equilibrium)**.** *Suppose Assumptions 1-3 hold. Then equilibrium introduced in Definition 1 possesses the following properties: (a) it exists; (b) equilibrium sectoral prices and final sectoral consumptions are unique; (c) equilibrium supplier choices and remaining sectoral allocations are generically unique.*

Proof. (a) Existence;

In periods $t \geqslant 2$ prices are flexible, and firms' problem is static and collapses to that studied by Acemoglu and Azar (2020). Hence, it suffices to show existence at $t = 1$, where prices are sticky. For convenience, let $\overline{P}_k \equiv \frac{P_k}{\mathcal{M}}$ and $\overline{\mathcal{Q}}_k \equiv \frac{\mathcal{Q}_k}{\mathcal{M}}$ be ratios of sectoral prices and unit costs to money supply.

As a first step, I am going to establish existence of equilibrium prices and supplier choices. Subsequently, I am going to show how existence of equilibrium prices and supplier choices implies existence of equilibrium quantities.

Rewrite equation for sectoral price aggregation, using the fact that every sectoral production function exhibits constant returns to scale:

$$P_k^{1-\theta} = \alpha_k P_{k,0}^{1-\theta} + (1-\alpha_k)\left[\mathcal{M}(1+\mu_k)\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k})\right]^{1-\theta}$$

$$\overline{P}_k = \left[\alpha_k\left[\frac{P_{k,0}}{\mathcal{M}}\right]^{1-\theta} + (1-\alpha_k)\left[(1+\mu_k)\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k})\right]^{1-\theta}\right]^{\frac{1}{1-\theta}}, \quad (21)$$

or $\overline{P}_k = f_k[\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k}); \mathcal{M}]$, where $f_k' > 0, f_k'' < 0 \forall k$, and $f_k(.;\overline{\mathcal{M}}) < f_k(.;\underline{\mathcal{M}}), \forall k : \alpha_k > 0$ for any $\overline{\mathcal{M}} > \underline{\mathcal{M}}$. Define the following function:

$$\kappa(\{\overline{P}_k\}_{k=1}^K) =$$
$$= \left[f_1[\min_{S_1}\overline{\mathcal{Q}}_1(S_1, \mathcal{A}_1(S_1), \{\overline{P}_r\}_{r\in S_1}); \mathcal{M}], ..., f_K[\min_{S_K}\overline{\mathcal{Q}}_K(S_K, \mathcal{A}_K(S_K), \{\overline{P}_r\}_{r\in S_K}); \mathcal{M}]\right]' \quad (22)$$

One can show that $\kappa(.)$ has a fixed point, and that its fixed point corresponds to equilibrium ratios of sectoral prices to money supply. This follows from the fact that the set $\mathbb{L} = \{\overline{P}_k \geqslant 0, \forall k : \overline{P}_k = f_k[\min_{S_k}\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k}); \mathcal{M}]\}$ is a complete lattice with respect to operations

$$\mathbb{P}\bigwedge\mathbb{Q} = [\min(P_1, Q_1), ..., \min(P_K, Q_K)]'$$

and

$$\mathbb{P}\bigvee\mathbb{Q} = [\max(P_1, Q_1), ..., \max(P_K, Q_K)]'.$$

I now formally establish the latter point.

First, a producer in any sector k can produce using labor only, incurring a unit cost of $\overline{\mathcal{Q}}_k(\varnothing, \mathcal{A}_k(\varnothing), \varnothing)$, which delivers the price to money supply ratio that is given by $\overline{P}_k' \equiv f_k[\overline{\mathcal{Q}}_k(\varnothing, \mathcal{A}_k(\varnothing), \varnothing); \mathcal{M}], \forall k$, which does not depend on sectoral prices, and so

$\kappa(\{\overline{P}_k\}_{k=1}^K) \leqslant [\overline{P}_1', ..., \overline{P}_K']'$ for all $\{\overline{P}_k\}_{k=1}^K$. Since labor is an essential input by assumption, it follows that at zero prices the unit cost is positive for any choice of suppliers: $\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), 0_{|S_k|\times 1}) > 0, \forall S_k, k$.

Let $\overline{P}_k'' \equiv \kappa_k(0_{|S_k|\times 1}) = f_k[\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), 0_{|S_k|\times 1}); \mathcal{M}], \forall k$. Since the unit cost function is increasing in sectoral prices and $f_k' > 0, \forall k$, it follows that $\kappa(\{\overline{P}_k\}_{k=1}^K) \geqslant \kappa(0_{|S_k|\times 1}) = [\overline{P}_1'', ..., \overline{P}_K'']'$ for all $\{\overline{P}_k\}_{k=1}^K$. Hence, $\mathbb{O} \equiv \times_{k=1}^K [\overline{P}_k', \overline{P}_k'']$ is a complete lattice and κ maps $\mathbb{O}$ to $\mathbb{O}$.

Second, for any $\{\overline{P}_k\}_{k=1}^K$ and $\{\overline{\overline{P}}_k\}_{k=1}^K$ such that $\overline{P}_k \leqslant \overline{\overline{P}}_k \forall k$ we have

$$\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k}) \leqslant \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{\overline{P}}_r\}_{r\in S_k}), \forall S_k, \quad \forall k$$

and hence

$$\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k}) \leqslant \min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{\overline{P}}_r\}_{r\in S_k}), \quad \forall k$$

which further implies

$$f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k})] \leqslant f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{\overline{P}}_r\}_{r\in S_k})], \quad \forall k$$

and

$$\kappa(\{\overline{P}_k\}_{k=1}^K) \leqslant \kappa(\overline{\{\overline{P}}_k\}_{k=1}^K).$$

Hence, κ is an order-preserving function which maps a complete lattice $\mathbb{O}$ onto itself. By Knaster-Tarski theorem, it follows that κ has a minimal fixed point, which yields equilibrium ratios of sectoral prices to money supply $\{\overline{P}_k\}_{k=1}^K$ and supplier choices $\{S_k\}_{k=1}^K$. Trivially, equilibrium sectoral prices are then recovered as $P_k = \mathcal{M} \times \overline{P}_k, \forall k$. As for equilibrium firm-level prices within a particular sector, those are given by $P_{k,0}$ for non-adjusters at $t = 1$ and by $(1 + \mu_k)\mathcal{Q}_k(S_k, \mathcal{A}_k(S_k), \mathcal{M}, \{P_r\}_{r\in S_k})$ for adjusters at $t = 1$.

Having established existence of equilibrium prices and supplier choices, I now show that their existence allows to recover equilibrium allocations. First, the cash-in-advance constraint allows to recover aggregate final consumption $C_t = \mathcal{M}_t / P_t^c$, where $P_t^c = P_t^c[P_{1t}, ..., P_{Kt}]'$ is the consumption price index. Given aggregate final consumption, sectoral consumptions $C_{kt} = C_{kt}(\{P_{kt}\}_{k=1}^K, C_t)$ are pinned down uniquely from properties of the aggregator $u(.)$. As for firm-level consumptions, those are pinned down as $C_{kt}(j) = \left(\frac{P_{kt}(j)}{P_{kt}}\right)^{-\theta} C_{kt}$. Let $z_{krt} \equiv \frac{Z_{krt}(j)}{Y_{kt}(j)}$ and $n_{kt} \equiv \frac{N_{kt}(j)}{Y_{kt}(j)}$ be firm-level ratios of factor demand to output; given assumed properties of the production function, those are common to all firms within a particular sector, and independent of output. From goods market clearing condition:

$$C_{kt}(j) + \sum_{r=1}^{K} \int_{j' \in \Phi_r} Z_{rkt}(j', j) dj' = Y_{kt}(j), \quad \forall k, \forall j \in \Phi_k \tag{23}$$

$$C_{kt} + \sum_{r=1}^{K} z_{rkt} \int_{j' \in \Phi_r} \left(\frac{P_{rt}(j')}{P_{rt}} \right)^{-\theta} Y_{rt} dj' = Y_{kt}, \quad \forall k, \forall j \in \Phi_k \tag{24}$$

$$C_{kt} + \sum_{r=1}^{K} z_{rkt} \Delta_{rt} Y_{rt} = Y_{kt}, \quad \forall k, \forall j \in \Phi_k \tag{25}$$

where $\Delta_{rt} = \int_{j \in \Phi_k} \left(\frac{P_{rt}(j)}{P_{rt}} \right)^{-\theta} dj$ is price dispersion in sector r. The expression above allows to recover sectoral outputs:

$$[Y_{1t}, ..., Y_{Kt}]' = (I - \Omega_t')^{-1} [C_{1t}, ..., C_{Kt}]', \tag{26}$$

where Ω_t is a matrix whose entries are given by $[\Omega_t]_{kr} = z_{rkt} \Delta_{rt}$. Given sectoral outputs, firm-level outputs can be recovered as $Y_{kt}(j) = \left(\frac{P_{kt}(j)}{P_{kt}} \right)^{-\theta} Y_{kt}$, further allowing to pin down factor demands: $Z_{krt}(j) = z_{krt} Y_{kt}(j)$, $N_{kt}(j) = n_{kt} Y_{kt}(j)$.

(b) Uniqueness of sectoral prices and final consumptions;

As before, uniqueness of sectoral prices under no nominal rigidities ($t \geqslant 2$) is shown in Acemoglu and Azar (2020). Here I only need to establish uniqueness of equilibrium prices under nominal rigidities in $t = 1$.

Let $\{\overline{P}_k\}_{k=1}^K$ be the minimal element of complete lattice $\mathbb{L}$ introduced in (a); suppose $\{\overline{P}_k^*\}_{k=1}^K$ is another set of equilibrium ratios of sectoral prices to money supply; then it must be that $\overline{P}_k^* > \overline{P}_k, \forall k$. Below I show by contradiction that the latter implies $\{\overline{P}_k\}_{k=1}^K$ is the unique equilibrium set of ratios of sectoral prices to money supply.

Given assumed properties of the production function, it follows that $\overline{\mathcal{Q}}_k$ is concave in $\{\overline{P}_k\}_{k=1}^K$. Moreover, minimum of a collection of concave functions is also concave, so that $\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r \in S_k})$ is also concave. Recall that $f_k' > 0, f_k'' < 0, \forall k$ so that f_k is increasing and concave, and hence

$$\kappa_k(\{\overline{P}_k\}_{k=1}^K) = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r \in S_k})]$$

is also concave for all $k = 1, ..., K$.

Let $\nu \in (0, 1)$ be such that $\nu \overline{P}_k^* = \overline{P}_k, \forall k$ with at least one $r = 1, 2, ..., K$ such that $\overline{P}_k^* = \overline{P}_k$. Then:

$$\kappa_r(\overline{P}_r) - \overline{P}_r \geqslant \kappa_r(\nu \overline{P}_r^*) - \nu \overline{P}_r^* \geqslant \kappa_r(0)(1 - \nu) + \underbrace{\kappa_r(\overline{P}_r^*)\nu - \nu \overline{P}_r^*}_{=0} \geqslant (1 - \nu)\kappa_r(0) > 0.$$

The final inequality implies $\kappa_r(\overline{P}_r) > \overline{P}_r$, which contradicts that $\{\overline{P}_k^*\}_{k=1}^K$ is a fixed point. Hence, $\{\overline{P}_k\}_{k=1}^K$ is the unique fixed point, and equilibrium sectoral prices are unique.

Uniqueness of sectoral prices implies uniqueness of the aggregate consumption price index, and through the cash-in-advance constraint, it follows that equilibrium aggregate final consumption is also unique. Finally, properties of the consumption aggregator u imply that given unique sectoral prices and aggregate consumption, sectoral consumptions are also unique.

(c) Generic uniqueness of supplier choices and remaining quantities;

Generic uniqueness of sectoral supplier choices follows directly from the proof in Acemoglu and Azar (2020), since the possibility of multiplicity in equilibrium networks is unrelated to the degree of nominal rigidities.

From (29) one can see that equilibrium sectoral outputs are function of the amount of inputs sectors are buying from each other. Since supplier choices are generically unique, so are sectoral outputs in equilibrium. Finally, since factor demand are pinned down, among other things, by sectoral outputs, sectoral labor demands and desired purchases of intermediate inputs are also generically unique.

□

Lemma 1 (Baseline GDP)**.** *Suppose Assumptions 1-3 hold. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}_0})$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ *such that either* $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ *or* $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$*. Then,* $C(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \geqslant C(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$.

Proof. (a) $\overline{\mathcal{A}} > \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0 = \mathcal{M}_0$;

Let $\{\overline{P}_k^0\}_{k=1}^K$ be equilibrium ratios of sectoral prices to money supply under the baseline pair $(\underline{\mathcal{A}}, \mathcal{M}_0)$. Naturally, they satisfy the following fixed point condition:

$$\overline{P}_k^0 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \underline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}); \mathcal{M}_0], \quad \forall k \tag{27}$$

where functions f_k were introduced in (21).

Suppose the productivity mapping changes to $\overline{\mathcal{A}} > \underline{\mathcal{A}}$, while baseline money supply remains unchanged. Define $\{\overline{P}_k^1\}_{k=1}^K$ such that they satisfy:

$$\overline{P}_k^1 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}); \mathcal{M}_0], \quad \forall k \tag{28}$$

Since $\overline{\mathcal{Q}}_k$ is decreasing in $\mathcal{A}_k$ and $f'_k > 0$, it follows that:

$$\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}) < \min_{S_k} \overline{\mathcal{Q}}_k(S_k, \underline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}), \quad \forall k \tag{29}$$

$$\Rightarrow f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}); \mathcal{M}_0] < f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \underline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}); \mathcal{M}_0], \quad \forall k \tag{30}$$

$$\Rightarrow \overline{P}_k^1 < \overline{P}_k^0, \quad \forall k. \tag{31}$$

As before, define $\kappa_k(\{\overline{P}_k\}_{k=1}^K) = f_k[\min_{S_k} \overline{\mathcal{Q}_k}(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_k\}_{k=1}^K; \mathcal{M}_0], \forall k$. The new equilibrium ratios of sectoral prices to money supply are the minimal fixed point of $\kappa(.)$. For $\tau \geqslant 1$, define $\overline{P}_k^\tau = \kappa_k(\{\overline{P}^{\tau-1}\}_{k=1}^K)$. Since $\kappa(.)$ is increasing, it follows that:

$$\overline{P}_k^1 < \overline{P}_k^0, \forall k \Rightarrow \kappa(\{\overline{P}^1\}_{k=1}^K) < \kappa(\{\overline{P}^0\}_{k=1}^K) \tag{32}$$

$$\Rightarrow \overline{P}_k^2 < \overline{P}_k^1, \forall k \Rightarrow \kappa(\{\overline{P}^2\}_{k=1}^K) < \kappa(\{\overline{P}^1\}_{k=1}^K) \tag{33}$$

$$\Rightarrow \overline{P}_k^3 < \overline{P}_k^2, \forall k \Rightarrow ... \Rightarrow \overline{P}_k^t < \overline{P}_k^{t-1}, \forall k \tag{34}$$

$$\Rightarrow \lim_{t \to \infty} \overline{P}_k^t < \overline{P}_k^0, \forall k. \tag{35}$$

Since $\kappa(.)$ is continuous, $\lim_{t\to\infty} \overline{P}_k^t$ is a fixed point of $\kappa(.)$. Further, since the new equilibrium is a minimal fixed point, it must be that $\overline{P}_k(\overline{\mathcal{A}}, \mathcal{M}_0) \leqslant \lim_{t\to\infty} \overline{P}_k^t < \overline{P}_k^0 = \overline{P}_k(\underline{\mathcal{A}}, \mathcal{M}_0), \forall k$.

From the definition of the aggregate consumption price index it can be deduced that

$$\overline{P}^c(\overline{\mathcal{A}}, \mathcal{M}_0) = P^c(\frac{P_1(\overline{\mathcal{A}}, \mathcal{M}_0)}{\mathcal{M}_0}, ..., \frac{P_K(\overline{\mathcal{A}}, \mathcal{M}_0)}{\mathcal{M}_0}) =$$
$$= P^c(\overline{P}_1(\overline{\mathcal{A}}, \mathcal{M}_0), ..., \overline{P}_K(\overline{\mathcal{A}}, \mathcal{M}_0)) < P^c(\overline{P}_1(\underline{\mathcal{A}}, \mathcal{M}_0), ..., \overline{P}_K(\underline{\mathcal{A}}, \mathcal{M}_0)) = \overline{P}^c(\underline{\mathcal{A}}, \mathcal{M}_0).$$

From the cash-in-advance constraint, $C(\mathcal{A}, \mathcal{M}_0) = 1/P^c(\mathcal{A}, \mathcal{M}_0)$, and hence $C(\overline{\mathcal{A}}, \mathcal{M}_0) > C(\underline{\mathcal{A}}, \mathcal{M}_0)$.

(b) $\overline{\mathcal{A}} = \underline{\mathcal{A}} = \mathcal{A}, \overline{\mathcal{M}}_0 > \underline{\mathcal{M}}_0$;

Similarly, let $\{\overline{P}_k^0\}_{k=1}^K$ be equilibrium ratios of sectoral prices to money supply under the baseline pair $(\mathcal{A}, \underline{\mathcal{M}}_0)$. Suppose $\underline{\mathcal{M}_0}$ rises to $\overline{\mathcal{M}_0} > \underline{\mathcal{M}_0}$, and define $\{\overline{P}_k^1\}_{k=1}^K$ such that they satisfy:

$$\overline{P}_k^1 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}); \overline{\mathcal{M}_0}], \quad \forall k \tag{36}$$

Since $f_k[\quad . \quad ; \overline{\mathcal{M}_0}] < f_k[\quad . \quad ; \underline{\mathcal{M}_0}], \forall k$, it follows that:

$$f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \overline{\mathcal{M}_0}] < f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \underline{\mathcal{M}_0}], \quad \forall k \tag{37}$$

$$\Rightarrow \overline{P}_k^1 < \overline{P}_k^0, \quad \forall k. \tag{38}$$

The rest of the proof follows from (a), where it is similarly established that $\overline{P}_k(\mathcal{A}, \overline{\mathcal{M}_0}) < \overline{P}_k(\mathcal{A}, \underline{\mathcal{M}_0}), \forall k$, and hence $\overline{P}^c(\mathcal{A}, \overline{\mathcal{M}_0}) < \overline{P}^c(\mathcal{A}, \underline{\mathcal{M}_0})$ and $C(\mathcal{A}, \overline{\mathcal{M}_0}) > C(\mathcal{A}, \underline{\mathcal{M}_0})$. □

Lemma 2 (Baseline supplier choices). *Suppose Assumptions 1-5 hold. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}_0})$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}_0})$ *such that either* $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}, \overline{\mathcal{M}_0} = \underline{\mathcal{M}_0}$ *or* $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}_0} \geqslant \underline{\mathcal{M}_0}$*. Then,* $S_k(\overline{\mathcal{A}}, \overline{\mathcal{M}_0}) \supseteq S_k(\underline{\mathcal{A}}, \underline{\mathcal{M}_0})$*, for all* $k = 1, 2, ..., K$.

Proof. (a) $\overline{\mathcal{A}} > \underline{\mathcal{A}}, \overline{\mathcal{M}_0} = \underline{\mathcal{M}_0} = \mathcal{M}_0$; Let $S_k^0 = S_k(\underline{\mathcal{A}}, \mathcal{M}_0), \forall k$, be the original set of supplier choices in equilibrium, which satisfy:

$$\begin{aligned} S_k^0 \in \quad & \arg\min_{S_k} \mathcal{Q}_k(S_k, \underline{\mathcal{A}}_k(S_k), \mathcal{M}_0, \{P_r(\underline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k}) = \\ & = \arg\min_{S_k} \mathcal{M}_0 \overline{\mathcal{Q}}_k(S_k, \underline{\mathcal{A}}_k(S_k), \{\overline{P}_r(\underline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k}) \\ & = \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \underline{\mathcal{A}}_k(S_k), \{\overline{P}_r(\underline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k}), \quad \forall k. \end{aligned}$$

Suppose $\underline{\mathcal{A}}$ improves to $\overline{\mathcal{A}} > \underline{\mathcal{A}}$, and define supplier choices S^1 such that it is true that $S_k^1 \in \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r(\underline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k}), \forall k$. From Theorem 4 of Milgrom and Shannon (1994) it follows that $S_k^1 \supseteq S_k^0, \forall k$.

From the proof of Lemma 1 it is known that $\overline{P}_k(\overline{\mathcal{A}}, \mathcal{M}_0) < \overline{P}_k(\underline{\mathcal{A}}, \mathcal{M}_0), \forall k$, and hence

$$\arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r(\overline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k}) \supseteq \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r(\underline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k})$$

$$\Rightarrow S_k(\overline{\mathcal{A}}, \mathcal{M}_0) \supseteq S_k^1 \supseteq S_k(\underline{\mathcal{A}}, \mathcal{M}_0), \quad \forall k. \tag{39}$$

(b) $\overline{\mathcal{A}} = \underline{\mathcal{A}} = \mathcal{A}, \overline{\mathcal{M}_0} > \underline{\mathcal{M}_0}$;
Let $S^0 = S(\mathcal{A}, \underline{\mathcal{M}_0})$ be the original equilibrium network, which satisfies

$$S_k^0 \in \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r(\mathcal{A}, \underline{\mathcal{M}_0})_{r\in S_k}\}), \quad \forall k. \tag{40}$$

Suppose $\underline{\mathcal{M}_0}$ changes to $\overline{\mathcal{M}_0} > \underline{\mathcal{M}_0}$. From the proof of Lemma 1 it is known that $\overline{P}_k(\mathcal{A}, \overline{\mathcal{M}_0}) < \overline{P}_k(\mathcal{A}, \underline{\mathcal{M}_0}), \forall k$, and hence by Theorem 4 of Milgrom and Shannon (1994):

$$\arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r(\mathcal{A}, \overline{\mathcal{M}_0})\}_{r\in S_k}) \supseteq \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r(\mathcal{A}, \underline{\mathcal{M}_0})\}_{r\in S_k}),$$

$$\Rightarrow S_k(\mathcal{A}, \overline{\mathcal{M}_0}) \supseteq S_k(\mathcal{A}, \underline{\mathcal{M}_0}), \quad \forall k. \tag{41}$$

□

Lemma 3 (Comparative statics following a monetary shock)**.** *Consider a baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, which is perturbed by a monetary shock* $\varepsilon^m > 0$*. Suppose Assumptions 1-3 hold, then following the monetary shock equilibrium GDP rises (falls) relative to its baseline level:* $C(\mathcal{A}, \mathcal{M}) > C(\mathcal{A}, \mathcal{M}_0)$*. Further, suppose that in addition Assumption 4-5 also hold, then following the monetary shock the set of suppliers for each sector weakly expands:* $S_k(\mathcal{A}, \mathcal{M}) \supseteq S_k(\mathcal{A}, \mathcal{M}_0)$*, for all* $k = 1, 2, ..., K$*.*

Proof. Proof follows directly from Lemmas 1 and 2, where variation in money supply is driven by the monetary shock, as opposed to changes in baseline money supply, which is instead kept fixed. □

Lemma 4. *Suppose Assumptions 1-3 and 6-7 hold. For any baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, let* $p_k(\mathcal{A}, \mathcal{M}_0)$ *be a first order approximation of* $\log P_k(\mathcal{A}, \mathcal{M})$ *around* $\log P_k(\mathcal{A}, \mathcal{M}_0)$*, given* $S_k = S_k(\mathcal{A}, \mathcal{M}_0), \forall k$*. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$*,* $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ *and a monetary shock* ε^m *which is small with respect to both baselines, then:*

$$\mathbb{p}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathbb{p}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = -\left[\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathcal{L}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)\right] \mathcal{E}^m \tag{42}$$

where $\mathbb{p} = [p_1, p_2, ..., p_K]'$*,* $\mathcal{E}^m = [\varepsilon^m, \varepsilon^m, ..., \varepsilon^m]'$ *and* $\mathcal{L}$ *is a Leontief inverse given by:*

$$\mathcal{L}(\mathcal{A}, \mathcal{M}_0) = \left[I - (I - A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)\right]^{-1}\left[I - (I - A)\Gamma(\mathcal{M}_0)\right] \tag{43}$$

where $A = diag(\alpha_1, ..., \alpha_K)$*,* $\Gamma(\mathcal{M}_0) = diag(\gamma_1(\mathcal{M}_0), ..., \gamma_K(\mathcal{M}_0))$*,* $\gamma_k = \frac{1}{\alpha_k (g(\mathcal{M}_0))^{1-\theta} + 1 - \alpha_k}$ *and* $[\Omega(\mathcal{A}, \mathcal{M}_0)]_{kr} = \frac{\partial \log \mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), W, \{P_{k'}\}_{k' \in S_k}\right]}{\partial \log P_r}|_{\mathcal{M} = \mathcal{M}_0}, \forall k, r.$

Proof. First, notice that under the assumptions used in this lemma, if a shock $\varepsilon^m > 0$ is small with respect to a baseline pair $(\mathcal{A}, \mathcal{M}_0)$, then any other monetary shock $\bar{\varepsilon}^m \in (0, \varepsilon^m)$ is also small with respect to the baseline pair $(\mathcal{A}, \mathcal{M}_0)$. The proof of that follows by contradiction. Suppose there exists $\bar{\varepsilon}^m \in (0, \varepsilon^m)$ such that $S_k(\mathcal{A}, \mathcal{M}_0 \exp(\bar{\varepsilon}^m)) \supset S_k(\mathcal{A}, \mathcal{M}_0)$ for at least one k. Then for those k it follows $S_k(\mathcal{A}, \mathcal{M}_0 \exp(\bar{\varepsilon}^m)) \supset S_k(\mathcal{A}, \mathcal{M}_0 \exp(\varepsilon^m)) = S_k(\mathcal{A}, \mathcal{M}_0)$ where $\varepsilon^m > \bar{\varepsilon}^m$, which implies that for those sectors the set of suppliers shrinks as money supply increases, which contradicts Theorem 4 of Milgrom and Shannon (1994). Similarly, suppose there exists $\bar{\varepsilon}^m \in (0, \varepsilon^m)$ such that $S_k(\mathcal{A}, \mathcal{M}_0 \exp(\bar{\varepsilon}^m)) \subset S_k(\mathcal{A}, \mathcal{M}_0)$ for at least one k. Then for those k it follows $S_k(\mathcal{A}, \mathcal{M}_0 \exp(\bar{\varepsilon}^m)) \subset S_k(\mathcal{A}, \mathcal{M}_0)$ where $\varepsilon^m > 0$, which once again implies that for those sectors the set of suppliers shrinks as money supply increases, which contradicts Theorem 4 of Milgrom and Shannon (1994).

Hence, starting from the baseline $(\mathcal{A}, \mathcal{M}_0)$, the solution to my model for any monetary shock $\bar{\varepsilon}^m \in (0, \varepsilon^m]$ coincides with the solution to a restricted version of my model where the set of suppliers is kept fixed at $S(\mathcal{A}, \mathcal{M}_0)$. In the rest of this proof I consider such restricted version of my model for an unrestricted value of monetary shock, and ultimately evaluate solution to that model under $\bar{\varepsilon}^m \in (0, \varepsilon^m]$. Write sectoral price aggregation in such restricted model:

$$P_k^{1-\theta} = \alpha_k P_{k,0}^{1-\theta} + (1 - \alpha_k)\left[(1 + \mu_k)\mathcal{Q}_k(S_k, \mathcal{A}_k(S_k), \mathcal{M}, \{P_r\}_{r \in S_k}\right]^{1-\theta} \tag{44}$$

For a given baseline $(\mathcal{A}, \mathcal{M}_0)$ and model parameters, equilibrium prices are pinned down by the value of the monetary shock ε^m. The model with a fixed network features no discontinuities and since the unit cost function is differentiable by assumption, one can write a first order Taylor approximation around $\varepsilon^m = 0$:

$$\log P_k(\varepsilon^m) = \log P_k(0) + \frac{(1-\alpha_k)\left[(1+\mu_k)\mathcal{Q}_k(0)\right]^{1-\theta}}{\alpha_k P_{k,0}^{1-\theta} + (1-\alpha_k)\left[(1+\mu_k)\mathcal{Q}_k(0)\right]^{1-\theta}}[\log \mathcal{Q}_k(\varepsilon^m) - \log \mathcal{Q}_k(0)] \tag{45}$$

By assumption, $P_{k,0} = (1+\mu_k)g(\mathcal{M}_0)\mathcal{Q}_k(0)$:

$$\log P_k(\varepsilon^m) = \log P_k(0) + \frac{(1-\alpha_k)}{\alpha_k g(\mathcal{M}_0)^{1-\theta} + (1-\alpha_k)}[\log \mathcal{Q}_k(\varepsilon^m) - \log \mathcal{Q}_k(0)]. \tag{46}$$

$$= (1-\alpha_k)\gamma_k(\mathcal{M}_0)\left[\varepsilon^m + \log \overline{\mathcal{Q}}(\varepsilon^m) - \log \overline{\mathcal{Q}}(0)\right] \tag{47}$$

where $\gamma_k(\mathcal{M}_0) \equiv \frac{1}{\alpha_k g(\mathcal{M}_0)^{1-\theta} + 1 - \alpha_k}, \forall k$. Further,

$$\log \overline{P}_k(\varepsilon^m) - \log \overline{P}_k(0) + \varepsilon^m = (1-\alpha_k)\gamma_k(\mathcal{M}_0)\varepsilon^m + + (1-\alpha_k)\gamma(\mathcal{M}_0)\sum_{r\in S_k}\frac{\partial \log \mathcal{Q}_k(0)}{\partial \log P_r}\left[\log \overline{P}_r(\varepsilon^m) - \log \overline{P}_r(0)\right]. \tag{48}$$

Letting $\overline{p}_k \equiv [\log \overline{P}_k(\varepsilon^m) - \log \overline{P}_k(0)]$ and $\overline{\mathbb{p}} = [\overline{p}_1, \overline{p}_2, ..., \overline{p}_K]'$, one can re-write the above conveniently in matrix form:

$$\overline{\mathbb{p}} = [(I-A)\Gamma(\mathcal{M}_0) - I]\mathcal{E}^m + (I-A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)\overline{\mathbb{p}} \tag{49}$$

$$\overline{\mathbb{p}} = -[I - (I-A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)]^{-1}[I - (I-A)\Gamma(\mathcal{M}_0)]\mathcal{E}^m \tag{50}$$

where $A = diag(\alpha_1, ..., \alpha_K), \Gamma(\mathcal{M}_0) = diag(\gamma_1(\mathcal{M}_0), ..., \gamma_K(\mathcal{M}_0)), \mathcal{E}^m = [\varepsilon^m, \varepsilon^m, ..., \varepsilon^m]'$ and $[\Omega(\mathcal{A}, \mathcal{M}_0)]_{kr} = \frac{\partial \log \mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), W, \{P_{k'}\}_{k'\in S_k}\right]}{\partial \log P_r}|_{\mathcal{M}=\mathcal{M}_0}, \forall k, r$.

Return to the original version of my model, where network formation is endogenous. For a monetary shock $\varepsilon^m > 0$ that is small with respect to two baselines $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$, the expression in (50) holds locally around both baselines. Further, for those two baselines $\overline{p}_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \overline{p}_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = p_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) + \varepsilon^m - p_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}) - \varepsilon^m = p_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - p_k(\underline{\mathcal{A}}, \underline{\mathcal{M}})$, and so $\overline{\mathbb{p}}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \overline{\mathbb{p}}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = \mathbb{p}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathbb{p}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$. Combining this with (50) delivers the final result:

$$\mathbb{p}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathbb{p}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = -\left[\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathcal{L}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)\right]\mathcal{E}^m \tag{51}$$

where $\mathcal{L}(\mathcal{A}, \mathcal{M}_0) = [I - (I-A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)]^{-1}[I - (I-A)\Gamma(\mathcal{M}_0)]$. □

Proposition 2. *Suppose Assumptions 2-5, 7-8 hold. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ *such that either* $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ *or* $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$. *For a monetary shock* $\varepsilon^m > 0$ *that is small with respect to both baselines, it follows that*

$p_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \leqslant p_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$ *for all* $k = 1, 2, ..., K$.

Proof. Let $D \equiv \Omega(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \Omega(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, then one can rewrite the difference between the Leontief Inverses as follows:

$$\begin{aligned}\mathcal{L}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) - \mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) =&(I - A)\Gamma(\underline{\mathcal{M}}_0)\Omega(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)\mathcal{L}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) \\ &- (I - A)\Gamma(\overline{\mathcal{M}}_0)\Omega(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \\ &- (I - A)\Gamma(\overline{\mathcal{M}}_0)D\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \\ &+ (I - A)(\Gamma(\overline{\mathcal{M}}_0) - \Gamma(\underline{\mathcal{M}}_0)). \end{aligned} \tag{52}$$

Let $\Xi \equiv diag\left(\frac{\gamma_1(\overline{\mathcal{M}}_0)}{\gamma_1(\underline{\mathcal{M}}_0)}, ..., \frac{\gamma_K(\overline{\mathcal{M}}_0)}{\gamma_K(\underline{\mathcal{M}}_0)}\right)$, then the above can be re-written as:

$$\begin{aligned}\mathcal{L}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) - \mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) = -[I - (I - A)\Gamma(\underline{\mathcal{M}}_0)\Omega(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)]^{-1}\times \\ \times [(I - A)\Gamma(\overline{\mathcal{M}}_0)D\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) + (I - A)(I - \Xi)\Gamma(\underline{\mathcal{M}}_0)(I - \Omega(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0))]. \end{aligned} \tag{53}$$

From the definition of $\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$:

$$\begin{aligned}\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) &= [I - (I - A)\Gamma(\overline{\mathcal{M}})\Omega(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)]^{-1}[I - (I - A)\Gamma(\overline{\mathcal{M}}_0)] \\ &= I + [I - (I - A)\Gamma(\overline{\mathcal{M}})\Omega(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)]^{-1}(I - A)\Gamma(\overline{\mathcal{M}}_0)(\Omega(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - I) \\ &\Rightarrow I - \Omega(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) = [I - \Omega(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)] + \\ &+ \Omega(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)[I - (I - A)\Gamma(\overline{\mathcal{M}}_0)\Omega(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)]^{-1}(I - A)\Gamma(\overline{\mathcal{M}}_0)(I - \Omega(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)). \end{aligned} \tag{54}$$

Suppose we have any two baselines pairs $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ such that either $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ or $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$. Under Cobb-Douglas production function entries of the Ω matrix in the Leontief Inverse are given by shares in the production function, so that $[\Omega]_{kr} = \omega_{kr}$ if sector k buys inputs from sector r and $[\Omega]_{kr} = 0$ otherwise. As a result, $D = \Omega(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \Omega(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$ is a $K \times K$ matrix with non-negative entries, since from Lemma 2 the first baseline will have (weakly) more suppliers for every sector. Further, from the properties of γ_k, it follows that all entries of the Ξ matrix are positive and less than or equal to one. Finally, since labor is an essential input, it follows that sum of rows of $(I - \Omega(\mathcal{A}, \mathcal{M}))$ for any baseline pair are positive and less than or equal to one.

Combining the above observations with the expressions in (53) and (54), it follows that for any two baselines pairs $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ such that either $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ or $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$, and a monetary shock $\varepsilon^m > 0$ that is small with respect to both baselines, it holds that:

$$\mathbb{p}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathbb{p}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = -\left[\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathcal{L}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)\right]\mathcal{E}^m \leqslant 0_{K\times 1}. \tag{55}$$

or $p_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \leqslant p_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0), \forall k$ in scalar form. □

Theorem 1 (Cycle dependence)**.** *Suppose Assumptions 3-5, 7-9 hold. For any baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, let* $c_k(\mathcal{A}, \mathcal{M}_0) \equiv \log C_k(\mathcal{A}, \mathcal{M}) - \log C_k(\mathcal{A}, \mathcal{M}_0), \forall k$*. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \mathcal{M}_0)$*,* $(\overline{\mathcal{A}}, \mathcal{M}_0)$ *such that* $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}$*. For a monetary shock* $\varepsilon^m > 0$ *which is small with respect to both baselines it follows that* $c_k(\overline{\mathcal{A}}, \mathcal{M}_0) \geqslant c_k(\underline{\mathcal{A}}, \mathcal{M}_0)$ *for all* $k = 1, 2, ..., K$*, and* $c(\overline{\mathcal{A}}, \mathcal{M}_0) \geqslant c(\underline{\mathcal{A}}, \mathcal{M}_0)$*.*

Proof. Under Cobb-Douglas aggregation of sectoral consumption it follows that sectoral consumption demand takes the following form:

$$C_k = \omega_{ck} \left[\frac{P_k}{P^c} \right]^{-1} C = \omega_{ck} \frac{P^c C}{P_k} = \omega_{ck} \frac{\mathcal{M}}{P_k}. \tag{56}$$

From the above it follows that $\log C_k = \log \omega_{ck} + \log \mathcal{M} - \log P_k$ and evaluating the latter under baseline and following the shock yields:

$$\begin{aligned} \log C_k(\mathcal{A}, \mathcal{M}) - \log C_k(\mathcal{A}, \mathcal{M}_0) &= \log \mathcal{M} - \log P_k(\mathcal{A}, \mathcal{M}) - \log \mathcal{M}_0 + \log P_k(\mathcal{A}, \mathcal{M}_0) \\ c_k(\mathcal{A}, \mathcal{M}_0) &= \varepsilon^m - p_k(\mathcal{A}, \mathcal{M}_0). \end{aligned} \tag{57}$$

For the two baselines $(\underline{\mathcal{A}}, \mathcal{M}_0)$, $(\overline{\mathcal{A}}, \mathcal{M}_0)$ such that $\overline{\mathcal{A}} \geqslant \underline{\mathcal{A}}$, and a monetary shock $\varepsilon^m > 0$ that is small with respect to both baselines it follows:

$$c_k(\overline{\mathcal{A}}, \mathcal{M}_0) - c_k(\underline{\mathcal{A}}, \mathcal{M}_0) = -\left[p_k(\overline{\mathcal{A}}, \mathcal{M}_0) - p_k(\underline{\mathcal{A}}, \mathcal{M}_0) \right] \geqslant 0, \quad \forall k \tag{58}$$

where the last part follows from Proposition 2.

Consider deviation of aggregate consumption from baseline:

$$c_k(\overline{\mathcal{A}}, \mathcal{M}_0) - c_k(\underline{\mathcal{A}}, \mathcal{M}_0) = \sum_{k=1}^{K} \omega_{ck} \left[c_k(\overline{\mathcal{A}}, \mathcal{M}_0) - c_k(\underline{\mathcal{A}}, \mathcal{M}_0) \right] \geqslant 0. \tag{59}$$

□

Theorem 2 (Path dependence)**.** *Suppose Assumptions 3-5, 7-9 hold. For any baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, let* $c_k(\mathcal{A}, \mathcal{M}_0) \equiv \log C_k(\mathcal{A}, \mathcal{M}) - \log C_k(\mathcal{A}, \mathcal{M}_0), \forall k$*. Consider any two baseline pairs* $(\mathcal{A}, \underline{\mathcal{M}}_0)$*,* $(\mathcal{A}, \overline{\mathcal{M}}_0)$ *such that* $\overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$*. For a monetary shock* $\varepsilon^m > 0$ *which is small with respect to both baselines it follows that* $c_k(\mathcal{A}, \overline{\mathcal{M}}_0) \geqslant c_k(\mathcal{A}, \underline{\mathcal{M}}_0)$ *for all* $k = 1, 2, ..., K$*, and* $c(\mathcal{A}, \overline{\mathcal{M}}_0) \geqslant c(\mathcal{A}, \underline{\mathcal{M}}_0)$*.*

Proof. Recall from the proof of Theorem 1 it is true that $c_k(\mathcal{A}, \mathcal{M}_0) = \varepsilon^m - p_k(\mathcal{A}, \mathcal{M}_0), \forall k$. For the two baselines $(\mathcal{A}, \underline{\mathcal{M}}_0)$, $(\mathcal{A}, \overline{\mathcal{M}}_0)$ such that $\overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$, and a monetary shock $\varepsilon^m > 0$ that is small with respect to both baselines it follows:

$$c_k(\mathcal{A}, \overline{\mathcal{M}}_0) - c_k(\mathcal{A}, \underline{\mathcal{M}}_0) = -\left[p_k(\mathcal{A}, \overline{\mathcal{M}}_0) - p_k(\mathcal{A}, \underline{\mathcal{M}}_0) \right] \geqslant 0 \tag{60}$$

where the last part follows from Proposition 2.

Consider deviation of aggregate consumption from baseline:

$$c_k(\mathcal{A}, \overline{\mathcal{M}}_0) - c_k(\mathcal{A}, \underline{\mathcal{M}}_0) = \sum_{k=1}^{K} \omega_{ck} \left[c_k(\mathcal{A}, \overline{\mathcal{M}}_0) - c_k(\mathcal{A}, \underline{\mathcal{M}}_0) \right] \geqslant 0. \tag{61}$$

□

Proposition 3. *Suppose Assumption 1-3 and 6 hold. For a baseline pair $(\mathcal{A}, \mathcal{M}_0)$, and the baseline equilibrium network S^0, let $\varepsilon^m > 0$ be a small monetary shock and $E^m > 0$ be a large monetary shock. Further, let S^L be the equilibrium network following the large monetary shock. The ratios of exact impulse responses of sectoral prices satisfy:*

$$\frac{\tilde{P}^e_k(E^m; S^L)}{\tilde{P}^e_k(\varepsilon^m; S^L)} \leqslant \frac{\tilde{P}_k(E^m)}{\tilde{P}_k(\varepsilon^m)} \leqslant \frac{\tilde{P}^e(E^m; S^0)}{\tilde{P}^e_k(\varepsilon^m; S^0)}, \tag{62}$$

for all $k = 1, 2, ..., K$, where $\tilde{P}_k(.)$ is the exact impulse response under endogenous networks, whereas $\tilde{P}^e_k(.; S)$ is the exact impulse response under the exogenous network S.

Proof. Let $\{\overline{P}^0_k\}_{k=1}^K$ be equilibrium ratios of sectoral prices to money supply under the initial level of money supply $\mathcal{M}_0$. As shown in the proof of Lemma 1, they satisfy:

$$\overline{P}^0_k = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}^0_r\}_{r \in S_k}); \mathcal{M}_0], \quad \forall k \tag{63}$$

where f_k is an increasing function. Further, we know that the equilibrium network under the initial level of money supply $\mathcal{M}_0$ is S^0.

Suppose that following a large monetary shock E^m the level of money supply rises to $\mathcal{M}^L > \mathcal{M}_0$. Define $\{\overline{P}^1_k\}_{k=1}^K$ such that they satisfy:

$$\overline{P}^1_k = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}^0_r\}_{r \in S_k}); \mathcal{M}^L], \quad \forall k \tag{64}$$

Let $\{\overline{P}^1_k(S^0)\}_{k=1}^K$ be the equivalent of $\{\overline{P}^1_k\}_{k=1}^K$ in an economy with an exogenous production network, given by S^0, then it follows that:

$$\begin{aligned}\overline{P}^1_k(S^0) = f_k[\overline{\mathcal{Q}}_k(S^0_k, \mathcal{A}_k(S^0_k), \{\overline{P}^0_r\}_{r \in S^0_k}); \mathcal{M}^L] \geqslant & f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}^0_r\}_{r \in S_k}); \mathcal{M}^L] \\ &= \overline{P}^1_k, \quad \forall k\end{aligned} \tag{65}$$

given that the economy with endogenous networks has an extra layer of minimization.

As in the proof of Lemma 1, we can define $\{\overline{P}^2_k\}_{k=1}^K$ and $\{\overline{P}^2_k(S^0)\}_{k=1}^K$ as the outcome of the next stage of fixed point iteration (with endogenous and exogenous network, respectively), and it follows that:

$$\overline{P}_k^2(S^0) = f_k[\overline{\mathcal{Q}}_k(S_k^0, \mathcal{A}_k(S_k^0), \{\overline{P}_r^1(S^0)\}_{r\in S_k^0}); \mathcal{M}^L] \geqslant f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^1\}_{r\in S_k}); \mathcal{M}^L]$$
$$= \overline{P}_k^2, \quad \forall k \tag{66}$$

where the inequality follows from two sources: first, $\overline{P}_k^1(S^0) \geqslant \overline{P}_k^1, \forall k$; second, the economy with endogenous networks has an extra layer of minimization. By induction, it then follows that for any stage $t \geqslant 1$ of the fixed point iteration:

$$\overline{P}_k^t(S^0) = f_k[\overline{\mathcal{Q}}_k(S_k^0, \mathcal{A}_k(S_k^0), \{\overline{P}_r^{t-1}(S^0)\}_{r\in S_k^0}); \mathcal{M}^L] \geqslant f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^{t-1}\}_{r\in S_k}); \mathcal{M}^L]$$
$$= \overline{P}_k^t, \quad \forall k$$

$$\Rightarrow \lim_{t\to\infty} \overline{P}_k^t(S^0) \geqslant \lim_{t\to\infty} \overline{P}_k^t, \quad \forall k$$
$$\Rightarrow \overline{P}_k(\mathcal{M}^L; S^0) \geqslant \overline{P}_k(\mathcal{M}^L), \quad \forall k \tag{67}$$
$$\Rightarrow P_k(\mathcal{M}^L; S^0)/\mathcal{M}^L \geqslant P_k(\mathcal{M}^L)/\mathcal{M}^L, \quad \forall k$$
$$\Rightarrow P_k(\mathcal{M}^L; S^0) \geqslant P_k(\mathcal{M}^L), \quad \forall k \tag{68}$$

where $P_k(\mathcal{M}^L)$ is the equilibrium price index of sector k under money supply $\mathcal{M}^L$ and endogenous network, whereas $P_k(\mathcal{M}^L; S^0)$ is the equilibrium price index of sector k under money supply $\mathcal{M}^L$ and exogenous network S^0. We can further see that:

$$P_k(\mathcal{M}^L) \leqslant P_k(\mathcal{M}^L; S^0), \quad \forall k$$
$$\Rightarrow \frac{P_k(\mathcal{M}^L)}{P_k(\mathcal{M}_0; S^0)} \leqslant \frac{P_k(\mathcal{M}^L; S^0)}{P_k(\mathcal{M}_0; S^0)}, \quad \forall k$$
$$\Rightarrow \tilde{P}_k(E^m) \leqslant \tilde{P}_k^e(E^m; S^0), \quad \forall k$$
$$\Rightarrow \frac{\tilde{P}_k(E^m)}{\tilde{P}_k(\varepsilon^m)} \leqslant \frac{\tilde{P}_k^e(E^m; S^0)}{\tilde{P}_k^e(\varepsilon^m; S^0)}, \quad \forall k \tag{69}$$

where we know that $\tilde{P}_k(\varepsilon^m) = \tilde{P}_k^e(\varepsilon^m; S^0)$ since, by definition, the small monetary shock ε^m leaves the equilibrium network unchanged at S^0. This establishes the first inequality in the proposition.

In order to prove the second inequality, return to our definition of $\{\overline{P}_k^0\}_{k=1}^K$, which gives the equilibrium ratios of sectoral prices to money supply under the initial level of money supply $\mathcal{M}_0$. Suppose that following a small monetary shock ε^m the level of money supply rises to $\mathcal{M}^S > \mathcal{M}_0$. Define $\{\overline{P}_k^1\}_{k=1}^K$ such that they satisfy:

$$\overline{P}_k^1 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \mathcal{M}^S], \quad \forall k \tag{70}$$

Recall that S^L is the equilibrium network under money supply $\mathcal{M}^L$. Let $\{\overline{P}^1_k(S^L)\}_{k=1}^K$ be the equivalent of $\{\overline{P}^1_k\}_{k=1}^K$ in an economy with an exogenous production network, given by S^L, then:

$$\begin{aligned}\overline{P}^1_k(S^L) = f_k[\overline{\mathcal{Q}}_k(S^L_k, \mathcal{A}_k(S^L_k), \{\overline{P}^0_r\}_{r\in S^L_k}); \mathcal{M}^S] \geqslant f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}^0_r\}_{r\in S_k}); \mathcal{M}^S] \\ = \overline{P}^1_k, \quad \forall k \end{aligned} \tag{71}$$

given that the economy with endogenous networks has an extra layer of minimization.

As before, by induction, for any stage $t \geqslant 1$ of the fixed point iteration:

$$\begin{aligned}\overline{P}^t_k(S^L) = f_k[\overline{\mathcal{Q}}_k(S^L_k, \mathcal{A}_k(S^L_k), \{\overline{P}^{t-1}_r(S^L)\}_{r\in S^L_k}); \mathcal{M}^S] \geqslant f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}^{t-1}_r\}_{r\in S_k}); \mathcal{M}^S] \\ = \overline{P}^t_k, \quad \forall k\end{aligned}$$

$$\begin{aligned}
&\Rightarrow \lim_{t\to\infty} \overline{P}^t_k(S^L) \geqslant \lim_{t\to\infty} \overline{P}^t_k, \quad \forall k \\
&\Rightarrow \overline{P}_k(\mathcal{M}^S; S^L) \geqslant \overline{P}_k(\mathcal{M}^S), \quad \forall k \qquad (72)\\
&\Rightarrow P_k(\mathcal{M}^S; S^L)/\mathcal{M}^S \geqslant P_k(\mathcal{M}^S)/\mathcal{M}^S, \quad \forall k \\
&\Rightarrow P_k(\mathcal{M}^S; S^L) \geqslant P_k(\mathcal{M}^S), \quad \forall k \qquad (73)
\end{aligned}$$

where $P_k(\mathcal{M}^S)$ is the equilibrium price index of sector k under money supply $\mathcal{M}^S$ and endogenous network, whereas $P_k(\mathcal{M}^S; S^L)$ is the equilibrium price index of sector k under money supply $\mathcal{M}^S$ and exogenous network S^L. Further, we know that since ε^m is a small monetary shock, it does not change the equilibrium network relative to S^0. Therefore, $P_k(\mathcal{M}^S) = P_k(\mathcal{M}^S; S^0), \forall k$, where $P_k(\mathcal{M}^S; S^0)$ is the the equilibrium price index of sector k under money supply $\mathcal{M}^S$ and exogenous network S^0. We can therefore see that:

$$\begin{aligned}
&P_k(\mathcal{M}^S; S^0) = P_k(\mathcal{M}^S) \leqslant P_k(\mathcal{M}^S; S^L), \quad \forall k \\
&\Rightarrow \frac{1}{P_k(\mathcal{M}^S; S^0)} \geqslant \frac{1}{P_k(\mathcal{M}^S; S^L)}, \quad \forall k \\
&\Rightarrow \frac{P_k(\mathcal{M}^L; S^L)}{P_k(\mathcal{M}^S; S^0)} \geqslant \frac{P_k(\mathcal{M}^L; S^L)}{P_k(\mathcal{M}^S; S^L)}, \quad \forall k \\
&\Rightarrow \frac{P_k(\mathcal{M}^L; S^L)/P_k(\mathcal{M}^0; S^0)}{P_k(\mathcal{M}^S; S^0)/P_k(\mathcal{M}^0; S^0)} \geqslant \frac{P_k(\mathcal{M}^L; S^L)/P_k(\mathcal{M}^0; S^L)}{P_k(\mathcal{M}^S; S^L)/P_k(\mathcal{M}^0; S^L)}, \quad \forall k \\
&\Rightarrow \frac{\tilde{P}_k(E^m)}{\tilde{P}_k(\varepsilon^m)} \geqslant \frac{\tilde{P}^e_k(E^m; S^L)}{\tilde{P}^e_k(\varepsilon^m; S^L)}, \quad \forall k \qquad (74)
\end{aligned}$$

which gives us the second inequality in the proposition.

Combining (69) and (74):

$$\frac{\tilde{P}_k^e(E^m;S^L)}{\tilde{P}_k^e(\varepsilon^m;S^L)} \leqslant \frac{\tilde{P}_k(E^m)}{\tilde{P}_k(\varepsilon^m)} \leqslant \frac{\tilde{P}^e(E^m;S^0)}{\tilde{P}_k^e(\varepsilon^m;S^0)}, \tag{75}$$

for all $k = 1, 2, ..., K$. □

Theorem 3 (Size dependence)**.** *Suppose Assumption 1-3 and 6 hold. For a baseline pair $(\mathcal{A}, \mathcal{M}_0)$, and the baseline equilibrium network S^0, let $\varepsilon^m > 0$ be a small monetary shock and $E^m > 0$ be a large monetary shock. Further, let S^L be the equilibrium network following the large monetary shock. The ratios of exact impulse responses of GDP satisfy:*

$$\frac{\tilde{C}^e(E^m;S^0)}{\tilde{C}^e(\varepsilon^m;S^0)} \leqslant \frac{\tilde{C}(E^m)}{\tilde{C}(\varepsilon^m)} \leqslant \frac{\tilde{C}^e(E^m;S^L)}{\tilde{C}^e(\varepsilon^m;S^L)}. \tag{76}$$

where $\tilde{C}(.)$ is the exact impulse response under endogenous networks, whereas $\tilde{C}^e(.;S)$ is the exact impulse response under the exogenous network S.

Proof. Recall that $\overline{P}_k(\mathcal{M}^L) \leqslant \overline{P}_k(\mathcal{M}^L;S^0)\forall k$. Hence,

$$\begin{aligned}\frac{P^c(\mathcal{M}^L)}{\mathcal{M}^L} = P^c(\overline{P}_1(\mathcal{M}^L), ..., \overline{P}_K(\mathcal{M}^L)) &\leqslant P^c(\overline{P}_1(\mathcal{M}^L;S^0), ..., \overline{P}_K(\mathcal{M}^L;S^0)) \\ &= \frac{P^c(\mathcal{M}^L;S^0)}{\mathcal{M}^L}.\end{aligned} \tag{77}$$

Combined with the cash-in-advance constraint:

$$\begin{aligned}\frac{C(\mathcal{M}^L)}{C(\mathcal{M}^L;S^0)} &= \frac{P^c(\mathcal{M}^L;S^0)}{P^c(\mathcal{M}^L)} \geqslant 1 \\ \Rightarrow C(\mathcal{M}^L) &\geqslant C(\mathcal{M}^L;S^0) \\ \Rightarrow \frac{C(\mathcal{M}^L)}{C(\mathcal{M}^S;S^0)} &\geqslant \frac{C(\mathcal{M}^L;S^0)}{C(\mathcal{M}^S;S^0)} \\ \Rightarrow \frac{C(\mathcal{M}^L)/C(\mathcal{M}_0;S^0)}{C(\mathcal{M}^S;S^0)/C(\mathcal{M}_0;S^0)} &\geqslant \frac{C(\mathcal{M}^L;S^0)/C(\mathcal{M}_0;S^0)}{C(\mathcal{M}^S;S^0)/C(\mathcal{M}_0;S^0)} \\ \Rightarrow \frac{\tilde{C}(E^m)}{\tilde{C}(\varepsilon^m)} &\geqslant \frac{\tilde{C}^e(E^m;S^0)}{\tilde{C}^e(\varepsilon^m;S^0)}.\end{aligned} \tag{78}$$

Recall that $\overline{P}_k(\mathcal{M}^S) = \overline{P}_k(\mathcal{M}^S;S^0) \leqslant \overline{P}_k(\mathcal{M}^S;S^L)\forall k$. Hence,

$$\begin{aligned}\frac{P^c(\mathcal{M}^S;S^0)}{\mathcal{M}^S} = P^c(\overline{P}_1(\mathcal{M}^S;S^0), ..., \overline{P}_K(\mathcal{M}^S;S^0)) &\leqslant P^c(\overline{P}_1(\mathcal{M}^S;S^L), ..., \overline{P}_K(\mathcal{M}^S;S^L)) \\ &= \frac{P^c(\mathcal{M}^S;S^L)}{\mathcal{M}^S}.\end{aligned} \tag{79}$$

Combined with the cash-in-advance constraint:

$$
\begin{aligned}
\frac{C(\mathcal{M}^S;S^0)}{C(\mathcal{M}^S;S^L)} &= \frac{P^c(\mathcal{M}^S;S^L)}{P^c(\mathcal{M}^S;S^0)} \geqslant 1 \\
\Rightarrow \frac{1}{C(\mathcal{M}^S;S^0)} &\leqslant \frac{1}{C(\mathcal{M}^S;S^L)} \\
\Rightarrow \frac{C(S^L,\mathcal{M}^L)}{C(S^0,\mathcal{M}^s)} &\leqslant \frac{C(S^L,\mathcal{M}^L)}{C(S^L,\mathcal{M}^s)} \\
\Rightarrow \frac{C(\mathcal{M}^L;S^L)/C(\mathcal{M}_0;S^0)}{C(\mathcal{M}^S;S^0)/C(\mathcal{M}_0;S^0)} &\leqslant \frac{C(\mathcal{M}^L;S^L)/C(\mathcal{M}_0;S^L)}{C(\mathcal{M}^S;S^L)/C(\mathcal{M}_0;S^L)} \\
\Rightarrow \frac{\tilde{C}(E^m)}{\tilde{C}(\varepsilon^m)} &\leqslant \frac{\tilde{C}^e(E^m;S^L)}{\tilde{C}^e(\varepsilon^m;S^L)}. \qquad (80)
\end{aligned}
$$

Combining (78) and (80):

$$
\frac{\tilde{C}^e(E^m;S^0)}{\tilde{C}^e(\varepsilon^m;S^0)} \leqslant \frac{\tilde{C}(E^m)}{\tilde{C}(\varepsilon^m)} \leqslant \frac{\tilde{C}^e(E^m;S^L)}{\tilde{C}^e(\varepsilon^m;S^L)}. \qquad (81)
$$

□

B Proofs of additional results

B.1 Business cycles driven by exogenous markups shocks

Lemma A1 (Baseline GDP)**.** *Suppose Assumptions 1-3 hold. Consider two otherwise identical baselines, which differ only in the exogenous sectoral tax rates given by* $\{\underline{\tau}_k\}_{k=1}^K$ *and* $\{\overline{\tau}_k\}_{k=1}^K$*, respectively. Suppose that* $\underline{\tau}_k \leqslant \overline{\tau}_k, \forall k$*, so that one of the baselines has (weakly) larger tax rates in every sector, and hence (weakly) larger desired markups. Then,* $C\left(\{\underline{\tau}_k\}_{k=1}^K\right) \geqslant C\left(\{\overline{\tau}_k\}_{k=1}^K\right)$.

Proof. Let $\{\overline{P}_k^0\}_{k=1}^K$ be equilibrium ratios of sectoral prices to money supply under the baseline sectoral tax rates $\{\overline{\tau}_k\}_{k=1}^K$. Naturally, they satisfy the following fixed point condition:

$$
\overline{P}_k^0 = \left[\alpha_k \left[\frac{P_{k,0}}{\mathcal{M}}\right]^{1-\theta} + (1-\alpha_k)\left[(1+\overline{\tau}_k)\frac{\theta}{\theta-1}\min_{S_k}\overline{\mathcal{Q}}_k(S_k,\mathcal{A}_k(S_k),\{\overline{P}_r^0\}_{r\in S_k})\right]^{1-\theta}\right]^{\frac{1}{1-\theta}} \qquad (82)
$$

or

$$
\overline{P}_k^0 = f_k[\min_{S_k}\overline{\mathcal{Q}}_k(S_k,\mathcal{A}_k(S_k),\{\overline{P}_r^0\}_{r\in S_k});\overline{\tau}_k], \quad \forall k \qquad (83)
$$

where $f[\quad . \quad ; \overline{\tau}_k] \geqslant f[\quad . \quad ; \underline{\tau}_k]$, for any $\overline{\tau}_k \geqslant \underline{\tau}_k$, $\forall k$.

Suppose each $\overline{\tau}_k$ falls to $\underline{\tau}_k \leqslant \overline{\tau}_k, \forall k$, and define $\{\overline{P}_k^1\}_{k=1}^K$ such that they satisfy:

$$\overline{P}_k^1 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}); \underline{\tau}_k], \quad \forall k \tag{84}$$

Since $f_k[\quad . \quad ; \underline{\tau}_k] \leqslant f_k[\quad . \quad ; \overline{\tau}_k], \forall k$, it follows that:

$$f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}); \underline{\tau}_k] \leqslant f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}); \overline{\tau}_k], \quad \forall k \tag{85}$$

$$\Rightarrow \overline{P}_k^1 \leqslant \overline{P}_k^0, \quad \forall k. \tag{86}$$

The rest of the proof follows directly from the proof of part (a) of Lemma 1, where from fixed point iteration it follows that $\overline{P}_k(\{\overline{\tau}_k\}_{k=1}^K) \geqslant \overline{P}_k(\{\underline{\tau}_k\}_{k=1}^K), \forall k$, and hence $\overline{P}^c(\{\overline{\tau}_k\}_{k=1}^K) \geqslant \overline{P}^c(\{\underline{\tau}_k\}_{k=1}^K)$ and $C(\{\underline{\tau}_k\}_{k=1}^K) \geqslant C(\{\overline{\tau}_k\}_{k=1}^K)$. □

Lemma A2 (Baseline supplier choices)**.** *Suppose Assumptions 1-5 hold. Consider two otherwise identical baselines, which differ only in the exogenous sectoral tax rates given by $\{\underline{\tau}_k\}_{k=1}^K$ and $\{\overline{\tau}_k\}_{k=1}^K$, respectively. Suppose that $\underline{\tau}_k \leqslant \overline{\tau}_k, \forall k$, so that one of the baselines has (weakly) larger tax rates in every sector, and hence (weakly) larger desired markups. Then, $S_k\left(\{\underline{\tau}_k\}_{k=1}^K\right) \supseteq S_k\left(\{\overline{\tau}_k\}_{k=1}^K\right)$, for all $k = 1, 2, ..., K$.*

Proof. Let $S^0 = S(\{\overline{\tau}_k\}_{k=1}^K)$ be the equilibrium network under the sectoral tax rates $\{\overline{\tau}_k\}_{k=1}^K$. Naturally, it satisfies:

$$S_k^0 \in \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r(\{\overline{\tau}_k\}_{k=1}^K)\}_{r \in S_k}), \quad \forall k. \tag{87}$$

Suppose each $\overline{\tau}_k$ falls to $\underline{\tau}_k \leqslant \overline{\tau}_k, \forall k$. From the proof of Lemma A1 it is known that $\overline{P}_k(\{\overline{\tau}_k\}_{k=1}^K) \geqslant \overline{P}_k(\{\underline{\tau}_k\}_{k=1}^K), \forall k$, and hence by Theorem 4 of Milgrom and Shannon (1994):

$$\arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r(\{\underline{\tau}_k\}_{k=1}^K)\}_{r \in S_k}) \supseteq \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r(\{\overline{\tau}_k\}_{k=1}^K)\}_{r \in S_k}),$$

$$\Rightarrow S_k(\{\underline{\tau}_k\}_{k=1}^K) \supseteq S_k(\{\overline{\tau}_k\}_{k=1}^K), \quad \forall k. \tag{88}$$

□

B.2 Nominal wage rigidity

Lemma A3 (Baseline GDP)**.** *Suppose Assumptions 1-3 hold. Consider the extended version of the model with $K+1$ sectors, where the additional sector contains a continuum of labor unions, whose price setting is subject to a lottery with an adjustment probability $(1-\alpha_u) \in (0,1)$. Further, consider any two otherwise identical baselines which differ only* *in the level of money supply given by $\underline{\mathcal{M}}_0$ and $\overline{\mathcal{M}}_0$, respectively, where $\underline{\mathcal{M}}_0 \leqslant \overline{\mathcal{M}}_0$. Then, $C(\overline{\mathcal{M}}_0) \geqslant C(\underline{\mathcal{M}}_0)$.*

Proof. The price index of the union sector is given by

$$P_u = \left[\alpha_u P_{u,0}^{1-\theta} + (1-\alpha_u) W^{1-\theta}\right]^{\frac{1}{1-\theta}} = \left[\alpha_u P_{u,0}^{1-\theta} + (1-\alpha_u)\mathcal{M}^{1-\theta}\right]^{\frac{1}{1-\theta}} = P_u(\mathcal{M}) \quad (89)$$

where $P_u(\mathcal{M})$ strictly rises in $\mathcal{M}$, whereas $\frac{P_u(\mathcal{M})}{\mathcal{M}}$ strictly falls in $\mathcal{M}$.

For any non-union sector $k = 1, 2, ..., K$, define $\check{P}_k \equiv \frac{P_k}{P_u}$ to be the ratio of the sectoral price index to the price index of the labor union sector. Let $\{\check{P}_k^0\}_{k=1}^K$ be equilibrium ratios of sectoral prices to labor union price index under the baseline money supply $\underline{\mathcal{M}}_0$. Naturally, they satisfy the following fixed point condition:

$$\check{P}_k^0 = \left[\alpha_k \left[\frac{P_{k,0}}{P_u(\underline{\mathcal{M}}_0)}\right]^{1-\theta} + (1-\alpha_k)\left[(1+\mu_k)\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r^0\}_{r\in S_k})\right]^{1-\theta}\right]^{\frac{1}{1-\theta}} \quad (90)$$

or

$$\check{P}_k^0 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r^0\}_{r\in S_k}); P_u(\underline{\mathcal{M}}_0)], \quad \forall k \quad (91)$$

where $f_k[\quad . \quad ; \overline{P_u}] \leqslant f_k[\quad . \quad ; \underline{P_u}], \forall k$ for any $\overline{P_u} \geqslant \underline{P_u}$.

Suppose the baseline level of money supply rises from $\underline{\mathcal{M}}_0$ falls to $\overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$, and define $\{\overline{P}_k^1\}_{k=1}^K$ such that they satisfy:

$$\check{P}_k^1 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r^0\}_{r\in S_k}); P_u(\overline{\mathcal{M}}_0)], \quad \forall k \quad (92)$$

Since $P_u(\overline{\mathcal{M}}_0) \geqslant P_u(\underline{\mathcal{M}}_0)$, then $f_k[\quad . \quad ; P_u(\overline{\mathcal{M}}_0)] \leqslant f_k[\quad . \quad ; P_u(\underline{\mathcal{M}}_0)], \forall k$, it follows that:

$$f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r^0\}_{r\in S_k}); P_u(\overline{\mathcal{M}}_0)] \leqslant f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r^0\}_{r\in S_k}); P_u(\underline{\mathcal{M}}_0)] \quad (93)$$

$$\Rightarrow \check{P}_k^1 \leqslant \check{P}_k^0 = \check{P}_k(\underline{\mathcal{M}}_0), \quad \forall k. \quad (94)$$

In the same way as in proof of part (a) of Lemma 1, from fixed point iteration it then follows that $\check{P}_k(\overline{\mathcal{M}}_0) \leqslant \check{P}_k(\underline{\mathcal{M}}_0), \forall k$. From the cash-in-advance constraint:

$$C(\overline{\mathcal{M}}_0) = \left(\frac{P^c(P_1, ..., P_K)}{\overline{\mathcal{M}}_0}\right)^{-1} = \left(P^c\left(\frac{P_1}{\overline{\mathcal{M}}_0}, ..., \frac{P_K}{\overline{\mathcal{M}}_0}\right)\right)^{-1}$$
$$= \left(\frac{P_u(\overline{\mathcal{M}}_0)}{\overline{\mathcal{M}}_0} P^c(\check{P}_1(\overline{\mathcal{M}}_0), ..., \check{P}_K(\overline{\mathcal{M}}_0))\right)^{-1}. \quad (95)$$

Since $\check{P}_k(\overline{\mathcal{M}}_0) \leqslant \check{P}_k(\underline{\mathcal{M}}_0), \forall k$ and $\frac{P_u(\overline{\mathcal{M}}_0)}{\overline{\mathcal{M}}_0} \leqslant \frac{P_u(\underline{\mathcal{M}}_0)}{\underline{\mathcal{M}}_0}$, it follows that

$$C(\overline{\mathcal{M}}_0) \geqslant C(\underline{\mathcal{M}}_0). \tag{96}$$

□

Lemma A4 (Baseline supplier choices)**.** *Suppose Assumptions 1-5 hold. Consider the extended version of the model with $K+1$ sectors, where the additional sector contains a continuum of labor unions, whose price setting is subject to a lottery with an adjustment probability $(1-\alpha_u) \in (0,1)$. Further, consider any two otherwise identical baselines which differ only in the level of money supply given by $\underline{\mathcal{M}}_0$ and $\overline{\mathcal{M}}_0$, respectively, where $\underline{\mathcal{M}}_0 \leqslant \overline{\mathcal{M}}_0$. Then, $S_k(\overline{\mathcal{M}}_0) \supseteq S_k(\underline{\mathcal{M}}_0)$, for all the non-union sectors $k = 1, 2, ..., K$.*

Proof. Let $S^0 = S(\underline{\mathcal{M}}_0)$ be the equilibrium network under the baseline money supply $\underline{\mathcal{M}}_0$. Naturally, for every non-union sector $k = 1, 2, .., K$ it satisfies:

$$\begin{aligned} S_k^0 \in \arg\min_{S_k} \mathcal{Q}_k(S_k, \mathcal{A}_k(S_k), P_u(\underline{\mathcal{M}}_0), \{P_r(\underline{\mathcal{M}}_0)\}_{r\in S_k}) \\ = \arg\min_{S_k} P_u(\underline{\mathcal{M}}_0)\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r(\underline{\mathcal{M}}_0)\}_{r\in S_k}) \\ = \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r(\underline{\mathcal{M}}_0)\}_{r\in S_k}), \quad \forall k. \end{aligned} \tag{97}$$

Suppose the baseline level of money supply rises from $\underline{\mathcal{M}}_0$ falls to $\overline{\mathcal{M}}_0 \geqslant \underline{\mathcal{M}}_0$. From the proof of Lemma A3 it is known that $\check{P}_k(\overline{\mathcal{M}}_0) \leqslant \check{P}_k(\underline{\mathcal{M}}_0), \forall k$, and hence by Theorem 4 of Milgrom and Shannon (1994):

$$\arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r(\overline{\mathcal{M}}_0)\}_{r\in S_k}) \supseteq \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\check{P}_r(\underline{\mathcal{M}}_0)\}_{r\in S_k}), \quad \forall k \tag{98}$$

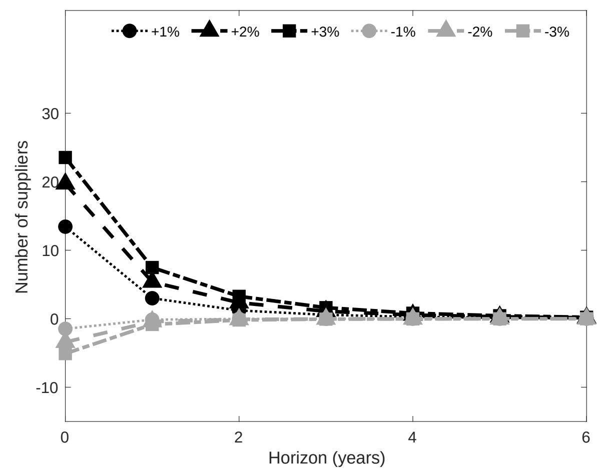
$$\Rightarrow S_k(\overline{\mathcal{M}}_0) \supseteq S_k(\underline{\mathcal{M}}_0), \quad \forall k. \tag{99}$$

□

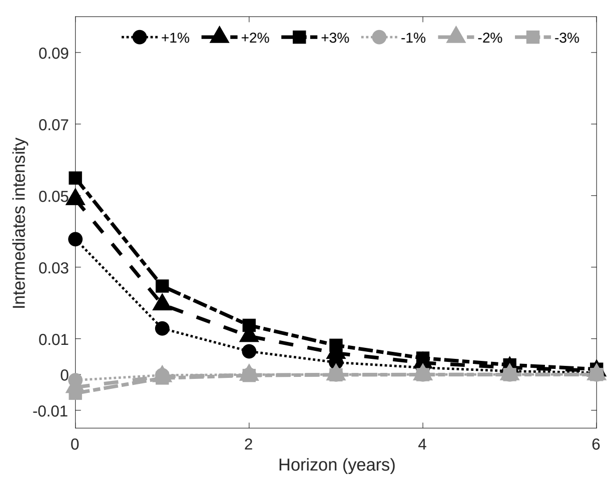
C Endogenous persistence of production networks

Figure 13: **Production networks under baselines driven by transitory productivity changes** ($\rho_z = 0$)

(a) Average number of suppliers



(b) Average intermediates intensity



Notes*: Panels (a) and (b) show the average number of suppliers and average intermediates intensities across baselines where* $\mathcal{M}_0 = 1$ *and* $\zeta_t = 0, \forall t \geqslant 2$*, and consider values of* $\zeta_1 = \{-3\%, -2\%, -1\%, 1\%, 2\%, 3\%\}$ *and* $\rho_z = 0$ *relative to* $\zeta_1 = 0$

D Additional econometric exercises

D.1 Format tests for significance of non-linear effects

Figure 14 shows p-values for formal tests of significance of non-linear (sign and size) effects present in specifications (18) and (20). Panels (a) and (b) show that for the response of intermediates intensity to productivity and monetary shocks, both sign and size effects are highly significant at the horizons exhibiting largest responses of intermediates intensity. Panels (c) and (d) show that the patterns are similar for the responses of average number of suppliers.

D.2 Alternative non-linear estimation using sectoral data

Here I consider an alternative approach to modeling non-linearity in my local projections setting. In particular, I re-estimate (18), where instead of adding interactions with a sign dummy and absolute value of the shock, I add quadratic and cubic shocks, which similarly capture possible sign and size effects:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta_H^{lin} s_t + \beta_H^{sign} s_t^2 + \beta_H^{size} s_t^3 + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H}, \tag{100}$$

for $H = 0, 1, ..., \overline{H}$, where it follows that $\beta_H^{sign} > 0$ indicates that positively-valued shocks make the impulse response more positive, whereas β_H^{size} indicates that larger shocks make the response more positive. Figure 15 reports estimation results using this alternative specification. Reassuringly, results remain virtually unchanged relative to the non-linear specification in (18), both in terms of magnitudes of effects, as well as in when it comes to sign and size effects and their statistical significance.

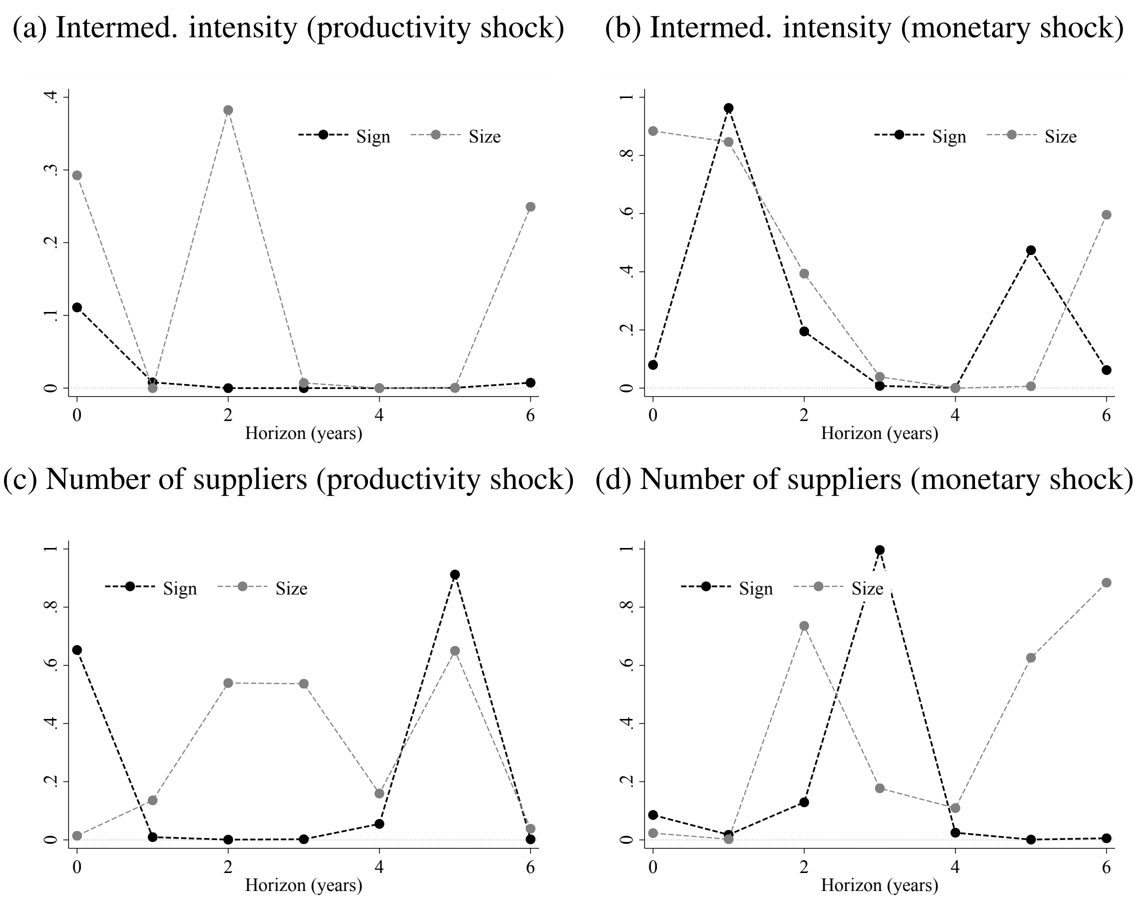
D.3 Alternative non-linear estimation using firm-level data

Here I once again consider the alternative approach to modeling non-linearity in my local projections setting, this time applied to the firm-level data on the number of suppliers.

$$indeg_{j,t+H} = \alpha_{j,H} + \beta_H^{linear} s_t + \beta_H^{sign} s_t^2 + \beta_H^{size} s_t^3 + \gamma_H x_{j,t-1} + \varepsilon_{j,t+H}, \tag{101}$$

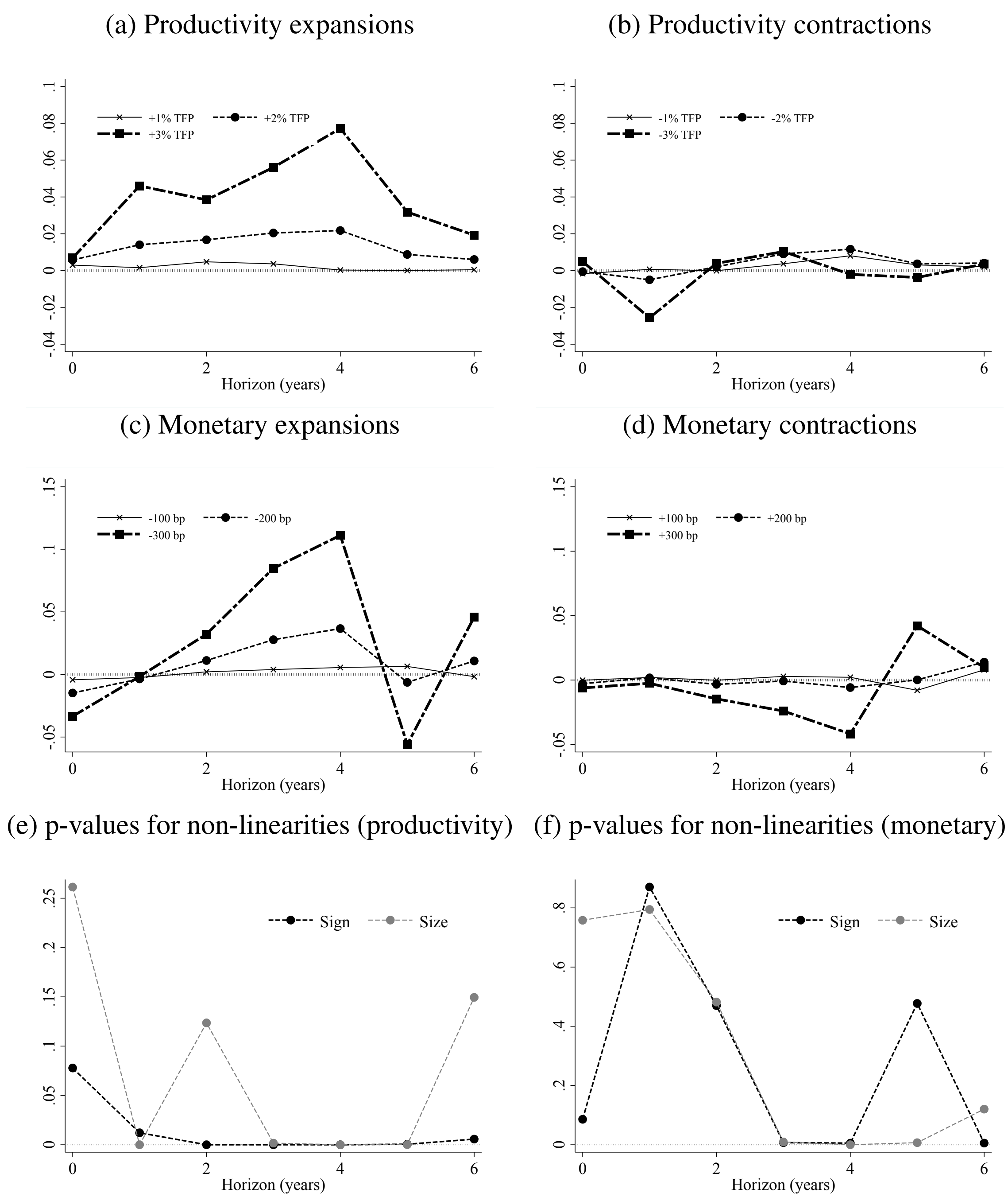
for $H = 0, 1, ..., \overline{H}$. Figure 16 reports the estimation results. One can see that results relative to the specification (20), results remain virtually unchanged, both in terms of magnitudes of effects, as well as in when it comes to sign effects and their statistical significance.

Figure 14: **P-values for tests of significance of non-linear effects**



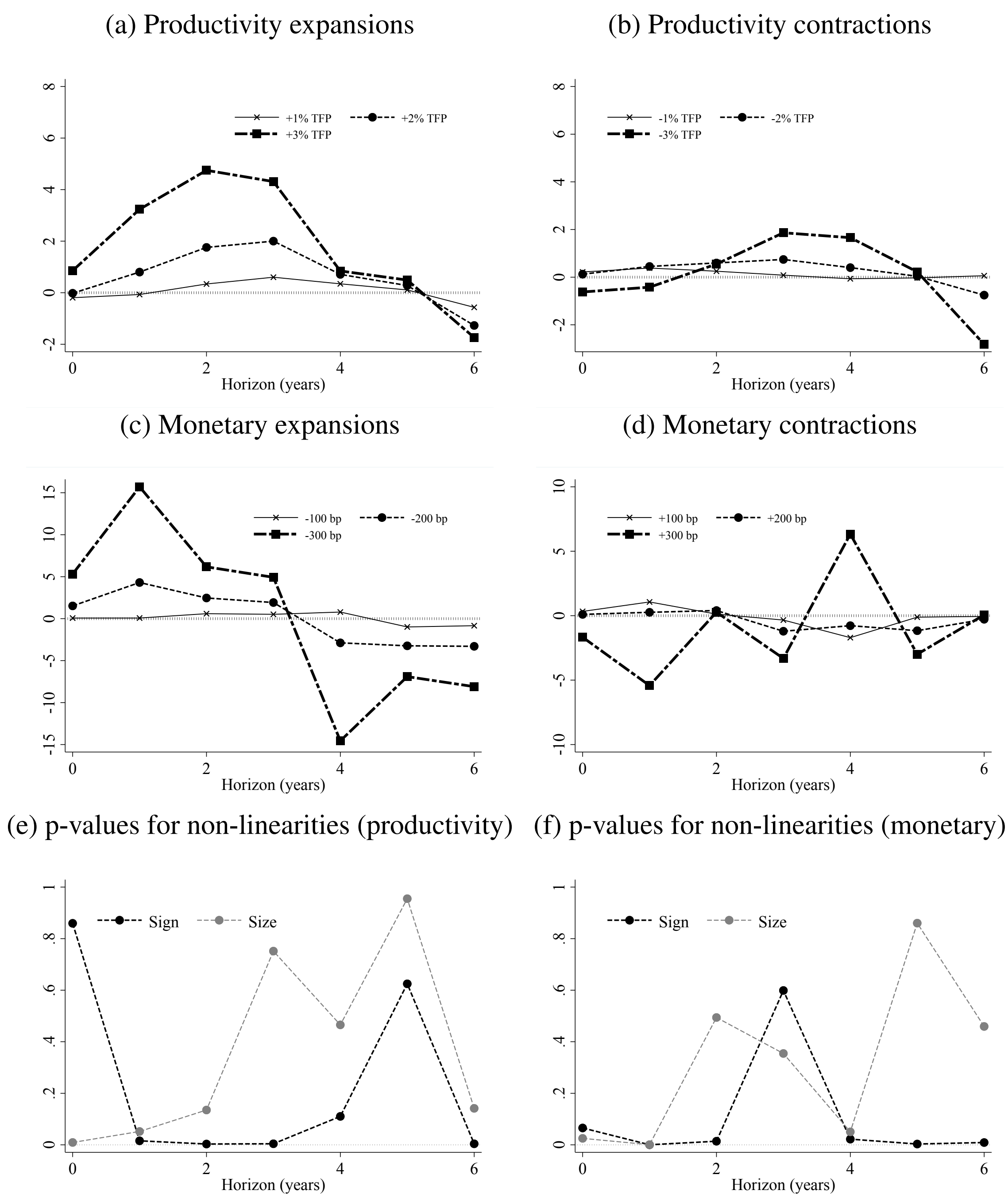
Notes: *Panels (a)-(b) show p-values for the test that the horizon-specific coefficients on non-linear sign and size effects from specification (18) (β_H^{sign}, β_H^{size}) are significantly different from zero, for Fernald (2014) productivity and Romer and Romer (2004) monetary shocks, respectively. Panels (c)-(d) show p-values for the test that the horizon-specific coefficients on non-linear sign and size effects from specification (20)(β_H^{sign}, β_H^{size}) are significantly different from zero, for Fernald (2014) productivity and Romer and Romer (2004) monetary shocks, respectively.*

Figure 15: **Non-linear IRFs of intermediates intensity to shocks**



Notes: *Panels (a)-(b) show estimated non-linear IRFs of intermediates intensity to a productivity shock based on Fernald (2014), using specification in (100); the set of controls includes one lead of productivity shock and one lag of intermediates intensity, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panels (c)-(d) show estimated non-linear IRFs of intermediates intensity to a monetary shock based on Romer and Romer (2004), using specification in (100); the set of controls includes one lead of monetary shock and one lag of intermediates intensity, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Panels (e) and (f) shock p-values for the test that non-linear terms are zero.*

Figure 16: **Non-linear IRFs of number of suppliers to shocks**



Notes: *Panels (a)-(b) show estimated non-linear IRFs of the number of suppliers to a productivity shock based on Fernald (2014), using specification in (101); the set of controls includes one lead of productivity shock, four lags of the number of suppliers, one lag of productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panels (c)-(d) show estimated non-linear IRFs of the number of suppliers to a monetary shock based on Romer and Romer (2004), using specification in (101); the set of controls includes one lead of monetary shock, four lags of the number of suppliers, one lag of monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Panels (e) and (f) shock p-values for the test that non-linear terms are zero.*