American Economic Journal: Microeconomics 2023, 15(2): 187–226
https://doi.org/10.1257/mic.20210114

The Good, the Bad, and the Complex: Product Design with Imperfect Information†

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We study the joint determination of product quality and complexity. In our model complexity affects how difficult it is for an agent to acquire information about product quality. An agent can accept or reject a product proposed by a designer, who can affect the quality and the complexity of the product. We find that complexity is not a necessary feature of low-quality products. An increase in designer–agent alignment leads to more complex but better-quality products. However, higher product demand or lower competition among designers leads to more complex and lower-quality products. We relate our findings to the existing empirical evidence. (*JEL* D82, D83, G23, L15, L51, L84)

Rapidly increasing complexity has concerned policy makers and financial market participants alike. Complex financial products and regulations have been documented to contain jargon-rich descriptions, complicated explanations (Célérier and Vallée 2017; Ghent, Torous, and Valkanov 2019) or vague provisions (McMillan 2014; Davis 2017).[1] Products or policies with these features are difficult to understand

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†Go to https://doi.org/10.1257/mic.20210114 to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.

[1] E.g., an average sentence in a Basel Committee for Banking Supervision text consists of 25.7 words, with the second sentence of the very first document including over 72 words. This is significantly longer than the average of 21 words in a sentence from the British National Corpus, a collection of texts in modern British English. This analysis was done by the Swiss newspaper *Neue Zürcher Zeitung* (Kolly and Müller 2017).

and evaluate, leading to the concern that complexity may foster the production and proliferation of lower-quality policies and products.

Central to the above concern is understanding how changes in complexity relate to changes in the quality of policies and products. A well-established literature has focused on the purposeful obfuscation of bad product attributes by firms that are faced with unsophisticated consumers (see Spiegler 2016 for a survey). In this view unsophistication is essential, as otherwise consumers would eventually stop demanding products with attributes they do not understand well (Milgrom 1981). The recently documented proliferation of complex products and policies nonetheless begs the question of whether other factors beyond unsophistication may be at work and, if so, whether complexity necessarily brings about low-quality policies and products. The goal of this paper is to shed light on these issues.

We develop a novel notion of complexity to study the joint determination of quality and complexity in a rational setting. Our starting point is that agents often receive proposals of uncertain quality, which they have to evaluate before deciding whether to accept. In many situations such proposals must contain all the information pertaining to them that is needed to make a proper evaluation. Agents have to process this information to make their acceptance decisions. Some examples include financial products proposed to retail investors and policy reforms proposed to voters. Proposal designers, in turn, not only affect the quality of their proposals, but they can also influence how difficult it is for agents to process information by complexifying or simplifying their proposals. For example, a financial product can be complexified by adding unnecessary contingencies and complicated jargon, while a policy reform can be complexified by not making an effort to be concise and clear about its applicability. Motivated by this, we model complexity as a product feature that influences the agent's ability to learn about its quality. Our approach has two distinguishing features: first, an agent can only acquire imperfect information about product quality; and second, the designer can influence but not fully control the agent's information acquisition process. These features will matter both conceptually and in terms of applied implications.

Our framework rationalizes the proliferation of complex products and policies by uncovering novel drivers of complexity. First, we break the link between bad product attributes and complexity. We show that when the information that an agent acquires is imperfect, the designer of a good product may choose to complexify it in order to reduce the agent's reliance on noisy information. Alternatively, the designer of a bad product may choose to simplify it in order to gamble on the noise of the acquired information. Second, we show that complexity can result from high product demand by agents, low competition among designers, or high alignment (e.g., little conflict of interest) between the agent and the designer. Such a rise in complexity, however, is only accompanied by worsening quality when driven by high demand or low competition. These results lead to new testable implications for the relationship between quality and complexity, which we relate to empirical findings.

We consider a setting where an agent (e.g., investor, median voter) demands a product that is supplied by a designer. First, the designer takes private actions to affect the product's quality and complexity and then proposes the product to the agent. Next, the agent acquires information about the product's quality and decides whether to accept the product or take an outside option. Whereas a product's quality (good or bad) determines the direct payoff to the agent, a product's complexity affects the precision of the information about the product's quality that the agent acquires. While the objective of the designer is to get the agent to accept his product, the agent only wants to accept a good product. For example, a bank wants to convince a retail investor to accept a savings account, or a policy maker wants to get voter support for his policy proposal. To make the problem interesting, we suppose that the designer is misaligned with the agent, as he receives a higher payoff from having a bad product accepted. Such misalignment aims to capture conflicts of interest stemming from good products being more costly to produce, career concerns, or ideological preferences.

The designer in our setting takes actions to separately affect the product's quality and complexity. For example, while the quality of a financial product can be interpreted as the net present value (NPV) that it generates for an investor, its complexity refers to features such as the number of contingencies that deliver the NPV or whether payments are linked to indices with which an investor is unlikely to be familiar. Similarly, while the quality of a policy can be interpreted as its effectiveness in addressing a particular inefficiency, the policy's complexity refers to its length, clarity, and use of complicated or vague terminology.[2] By studying both notions separately, we gain a better understanding of the incentives to produce products of good or bad quality versus complex or simple products.

In practice, an agent's ability to learn about a product depends on the product's inherent complexity (e.g., a zero-coupon bond versus an asset-backed security, a flat-rate tax versus tiered taxation with activity-specific exemptions), on her own ability to process relevant information (e.g., her opportunity cost of time, education), and on the actions toward simplification or complexification taken by the designer described above. For instance, even though a policy maker may take actions to simplify a proposed regulation, it may be hard for the median voter to learn about it if it addresses a topic that she finds complicated. We capture this by supposing that the precision of the information that the agent acquires depends on both the designer's choice to complexify or simplify the product and on some component outside of the designer's control. Importantly, we suppose that it is impossible for the agent to acquire perfect information about the product's quality.[3]

A surprising result, at first, is that the designer of a good product may choose to complexify it, or that the designer of a bad product may choose to simplify it. Intuitively, when the agent would accept the product in the absence of new information—e.g., when her prior belief about quality being good or her demand for the product are sufficiently high—the designer has an incentive to complexify the product to discourage the agent from relying on noisy information. For example, a good policy maker who has enough support to pass a tax reform may not want to incur the risk of having the median voter reading a proposal that she could misinterpret.

[2] The strategic decisions of policy makers are discussed in the literature on strategic ambiguity or noise by politicians (Alesina and Cukierman 1990; Aragonès and Postlewaite 2002; Espinosa and Ray 2018).

[3] As we show in Section IA, when product simplification ensures that the agent can acquire perfect information about product quality, then there is a unique equilibrium where only good products are designed, and they are simplified and accepted with a probability of one.

Analogously, when the agent would reject the product in the absence of new information, the designer has an incentive to simplify the product to encourage the agent to rely on noisy information. For example, a bad policy maker with no support for tax reform can only obtain such support if the median voter engages in reading and misinterprets a proposal.

When choosing his product quality, the designer anticipates the resulting probability with which his product will be accepted by the agent. He then faces a trade-off between producing a good product, which has a higher probability of acceptance, and producing a bad product, which has a higher payoff conditional on acceptance. We show that when the agent's prior belief about the product's quality is sufficiently low, the designer obtains a higher payoff from producing good products. Conversely, when the agent's prior belief is sufficiently high, the designer prefers to produce bad products. But, of course, the agent's prior belief must be consistent with the equilibrium supply of good versus bad products. Indeed, we show that there is a unique equilibrium with positive trade, and in it the designer produces good products with interior probability.

We next explore how the equilibrium quality-complexity relationship changes with various features of the economic environment. First, we consider the effect of a decrease in the agent's relative outside option, which we interpret as an increase in the agent's demand for the product. We find that this leads to lower product quality and higher product complexity. Intuitively, as the agent's relative outside option falls, she is more likely to disregard information and have a looser acceptance strategy. This in turn encourages the designer to produce worse-quality, more complex products. Second, we consider a reduction in the conflict of interest between the designer and the agent. We find that this leads to both higher product quality and higher product complexity. Intuitively, higher alignment between the designer and the agent increases the agent's trust in the designer, since product quality does indeed increase, which loosens the agent's acceptance strategy. This in turn encourages the designer to produce more complex products. We show, therefore, that the relationship between quality and complexity depends crucially on the underlying drivers of heterogeneity.

In practice, not all designers may be misaligned with the agent. To address this, we extend our analysis by supposing that the agent may meet either an aligned or a misaligned designer. Surprisingly, the introduction of aligned designers only affects equilibrium outcomes when the probability of meeting one is sufficiently high, in which case both product quality and complexity increase. This is consistent with our previous finding on the effect of a reduction in conflicts of interest. Initially, as aligned designers enter, misaligned designers respond by producing bad products with higher probability, keeping the average quality and complexity of products unchanged—until the misaligned designers produce only bad products.

Finally, we extend the analysis to study the effect of competition among designers. We consider a sequential search setting where, if the agent rejects a given product, she meets another designer with some probability, capturing the presence of search frictions. We show that as search frictions decrease, which we interpret as competition among designers intensifying, product quality increases and complexity decreases. This result is in contrast to the literature on obfuscation or price complexity, which typically finds that competition leads to more obfuscation.[4] In that literature, by obfuscating, firms reduce the consumer's ability to uncover the attributes of competing products by effectively increasing the consumer's search costs. Obfuscation in these settings counterbalances higher competition: the consumer searches fewer products and competition effectively declines, benefiting firm profits. Instead, in our model, the designer uses complexification to influence the information that an agent extracts about his own product. As agents are rational, more intense competition incentivizes each designer to supply products that are better for the agent: those that are good and simple.

We relate our model's implications to two leading applied settings in which rising complexity has been at the forefront of policy debates: financial products and regulatory policy. For financial products our model suggests that the proliferation of worse and more complex structured products documented by Célérier and Vallée (2017) could have been an optimal response of product designers to an increasing demand for relatively safe financial assets combined with an increasing trust in financial advisors. Within the context of regulation, our model's predictions provide an additional channel for the evidence presented by Gratton et al. (2021) on the worsening quality and increasing complexity of laws proposed by Italian politicians in periods of low bureaucratic effectiveness, i.e., low outside options for policy makers or, equivalently, costly status quo.

Our findings complement the growing literature that examines the incentives of firms to shroud certain product attributes from agents (Gabaix and Laibson 2006; Auster and Pavoni 2018) or to increase the opacity of their products in order to draw unsophisticated investors into the market (Pagano and Volpin 2012). In contrast to our setting, a crucial ingredient of these models is that there is a fraction of unsophisticated agents who make no inferences from the fact that they do not observe a certain product attribute. Complexity then allows sellers to extract rents from unsophisticated agents.

Our model relates to the literature on strategic information transmission in games (Milgrom 1981; Grossman 1981; Crawford and Sobel 1982; Kartik 2009) and the recent papers on the value of ignorance or opacity in incentive provision schemes (Brocas and Carrillo 2007; Boleslavsky and Cotton 2015; Ederer, Holden, and Meyer 2018). Our contribution to this literature is the joint determination of product quality and complexity, which leads to previously unexamined feedback effects: the designer's complexification strategy affects the quality of products produced in equilibrium, which in turn influences the designer's incentives to complexify in the first place. Moreover, as in Dewatripont and Tirole (2005), information transmission in our model is imperfect: the designer tries to influence the information received by the agent, but he cannot control it fully, as the agent's ability to process information depends also on external factors.

[4] See, for instance, Spiegler (2006); Ellison and Ellison (2009); Carlin (2009); Ellison and Wolitzky (2012); and Petrikaitė (2018). Some papers in this literature also highlight that obfuscation gives rise to price dispersion, which in turn allows firms to price discriminate among different consumer types (e.g., fast versus slow searchers, sophisticated versus unsophisticated agents)—a force that may be present even in the absence of competition (Salop 1977) but that is absent in our setting.

Our approach is influenced by the literature on costly information processing or rational inattention (Sims 2003; Aragonès et al. 2005; Wiederholt 2010), since our framework can be interpreted as one in which the designer, by complexifying, makes it more costly for the agent to extract information (see the online Appendix), as in Perez-Richet and Prady (2011). Through this lens, our model also relates to Roesler and Szentes (2017), who study buyer-optimal learning, but where the seller cannot affect the buyers' learning process; and to Oehmke and Zawadowski (2019), who study sellers' incentives to complexify, but where complex products give more value to buyers. Our approach, however, adds the aforementioned feedback effects generated by the joint determination of quality and complexity.

That more information may not always be desired has been pointed out in early work by Hirshleifer (1978) and more recently by Dang, Gorton, and Holmström (2012) in the context of financial markets. This idea is also at the core of the literature on Bayesian persuasion (Rayo and Segal 2010; Kamenica and Gentzkow 2011) and information design, more broadly (Bergemann and Pesendorfer 2007; Bergemann and Morris 2016; Taneva 2019). The reason is that, from an ex ante perspective (i.e., when types or states are not yet known), there may be benefits from committing to transmit imperfect information (e.g., through the design of noisy information structures or of assets that deter information acquisition) to allow worse types or states to be pooled with better ones. This is in contrast to our setting, where the decision to complexify is undertaken ex post, when the designer knows his product quality. The gains from complexification in our setting are conceptually different, as cross-subsidization is not a driver of our results. In our model the good (bad) product designer may complexify (simplify) in order to reduce (increase) the agent's reliance on information that is noisy.[5]

Finally, our paper relates to the literature in industrial organization that studies pricing and marketing strategies jointly (Saak 2006; Anand and Shachar 2009; Bar-Isaac, Caruana, and Cuñat 2010). In this literature, informative marketing strategies affect the dispersion of consumers' valuations; thus, in the language of Johnson and Myatt (2006), rotating a firm's demand curve, which may increase profits when choosing the appropriate pricing strategy. In contrast, in our setting the product designer faces one agent, and he knows her valuation of the product. As the agent acquires imperfect information, however, the designer is exposed to risk, which is key for our results.[6] Furthermore, to the best of our knowledge, our findings on the relation between quality and complexity are new to this literature.

The rest of the paper is organized as follows. In Section I we set up our baseline model. In Sections II and III, we present our main results and discuss them within the context of applications. We conclude in Section IV. All proofs are relegated to the Appendix.

[5] Furthermore, our results do not arise if the designer can choose to transmit perfect information to the consumer, as is typically the case in information design or Bayesian-persuasion settings (see Proposition 1).

[6] In a setting where firms can choose advertising content, Mayzlin and Shin (2011) show that high-quality firms may choose to advertise with uninformative signals in order to induce consumers to engage in costly search to uncover (even) better information about product quality, a mechanism that is distinct from ours.

I. The Model

We consider the following interaction between a consumer and a product designer. The consumer needs a product, which only the designer can produce. The designer privately takes two actions $\{y, \kappa\}$, where $y \in \{Good, Bad\}$ affects the product's quality and $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$ affects the product's complexity. The designer then proposes the product to the consumer, who acquires information about the product's quality and decides whether to accept it $(a = 1)$ or take an outside option $(a = 0)$.

A Product's Quality.—A product's quality, y, determines the agents' payoffs. The payoff to the consumer from accepting a product with quality y (which we refer to as a y product) is $w(y)$, and her outside option if no product is accepted is w_0. The payoff to the designer from having a y product accepted is $v(y)$ and zero otherwise. We make the following assumptions:

ASSUMPTION 1: *The payoffs satisfy the following properties:*

(1) $w(G) > w_0 > w(B), w_0 \geq 0,$

(2) $v(B) > v(G) > 0.$

The first assumption states that the consumer wants to accept a G product but to reject a B product, making information about product quality relevant for the consumer's acceptance decision. The second assumption states that the designer prefers to have a B product accepted, misaligning the designer's objective with that of the consumer.[7] Since in practice not all designers may be misaligned with the consumer, we extend our analysis and introduce aligned designers in Section IIIC. As we show in the online Appendix, for some applications our payoff structure can be rationalized by introducing prices and costs of production whereby G products are costlier to produce.

A Product's Complexity.—A product's complexity, which we denote by $\chi \in [0, 1/2]$, determines the noise of the information acquired by the consumer about the product's quality. We suppose that it depends on two components, $\chi = \chi(\eta, \kappa)$. The first component, denoted by $\eta \in \mathbb{R}$, captures the product's natural (or inherent) level of complexity to the consumer, which is random and has an associated c.d.f. H. The second component is the designer's action, $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$. The consumer observes χ—i.e., she understands how complex the product is—but she does not observe η or κ; i.e., she does not know whether this

[7] In the financial products industry, misalignment may arise due to financial advisors receiving higher fees for selling products that are not necessarily the best fit for their clients (i.e., fixed versus adjustable-rate mortgages). In the policy sphere, misalignment of policy makers vis-à-vis the public may arise due to ideological differences, lobbying, or career concerns.

is due to the designer's action.[8] Let $F(\cdot\,|\,\kappa)$ denote the c.d.f. of χ conditional on κ, which is induced by the distribution H. We assume that it has an associated p.d.f. $f(\cdot\,|\,\kappa)$, which has full support and satisfies the MLRP: that is, $f(\chi\,|\,\bar{\kappa})/f(\chi\,|\,\underline{\kappa})$ is increasing in χ. Thus, we say that the designer complexifies the product when he chooses $\kappa = \bar{\kappa}$. Otherwise, we say that the designer simplifies the product.

The consumer's information acquisition technology is as follows. After the designer proposes product (y, κ), the consumer observes the complexity of the product, χ, and acquires a signal $S \in \{g, b\}$ about the product's quality with noise equal to χ, where[9]

$$\chi \equiv \Pr(S = b\,|\,y = G) = \Pr(S = g\,|\,y = B) \in \left[0, \frac{1}{2}\right]. \tag{1}$$

The timeline of the game is summarized in Figure 1. We next describe the problem of the consumer and of the designer.

The Consumer's Problem.—After observing complexity χ and signal realization s, the consumer forms a posterior belief $\mu(s, \chi) \equiv \Pr(y = G\,|\,s, \chi)$ and makes an optimal acceptance decision:

$$W(s, \chi) \equiv \max_{a \in \{0,1\}} a\{\mu(s, \chi) w(G) + [1 - \mu(s, \chi)] w(B)\} + (1 - a) w_0. \tag{2}$$

We denote the consumer's strategy by $\{a(s, \chi)\}_{s,\chi}$.[10]

The Designer's Problem.—The designer's expected payoff is given by

$$V(y, \kappa) \equiv \Pr(a = 1\,|\,y, \kappa) \cdot v(y), \tag{3}$$

where $\Pr(a = 1\,|\,y, \kappa)$ denotes the probability that product $\{y, \kappa\}$ is accepted by the consumer. The designer chooses $y \in \{G, B\}$ and $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$ to maximize (3). We denote the designer's strategy by $\{m, \sigma_G, \sigma_B\}$, where $m = \Pr(y = G)$ is the probability that the designer chooses a G product and $\sigma_y = \Pr(\kappa = \bar{\kappa}\,|\,y)$ is the probability that he chooses to complexify a y product.

[8] The imperfect link between the action κ and the product's complexity χ allows us to obtain a unique equilibrium, which facilitates rich comparative statics, by ruling out equilibrium multiplicity arising from the freedom in specifying off-equilibrium beliefs. *S*ee Matthews and Mirman (1983) for a related modeling approach. Furthermore, it has the natural interpretation that there are features of the environment unknown to the consumer (e.g., a product's natural complexity) or the designer (e.g., the consumer's opportunity cost of time) that affect the consumer's ability to acquire information.

[9] In online Appendix Sections 1.1 and 1.2, we show that our setting can be interpreted as one in which the consumer acquires costly information about the product's quality and where the cost of increasing her signal precision increases with the product's complexity, χ.

[10] As will become clear soon, focusing on pure strategies for the consumer is without loss of generality.

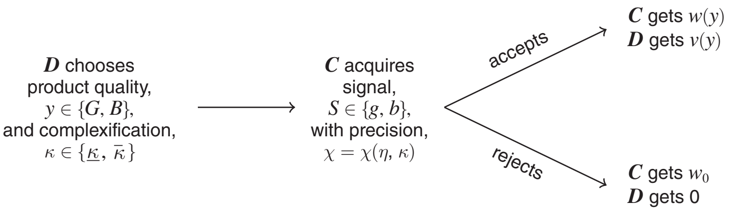

FIGURE 1. TIMELINE

Note: ***D*** denotes the designer, whereas ***C*** denotes the consumer.

Equilibrium Concept.—We use Perfect Bayesian Equilibrium (PBE) as our equilibrium concept. This has the following implications. First, given her belief, the consumer's strategy must maximize her expected payoff (consumer optimality). Second, the designer's strategy must maximize his expected payoff, given the consumer's strategy (designer optimality). Finally, the consumer's belief must be consistent with the designer's strategy and updated using Bayes' rule when possible (belief consistency).

Benchmark with Perfect Information.—Before we proceed to equilibrium analysis, it is useful to establish a benchmark against which our results can be compared. To highlight the role of imperfect information, we consider a benchmark where by simplifying the product the designer can ensure that the consumer receives a perfectly informative signal. The following proposition states that in this scenario there are no incentives to complexify products.

PROPOSITION 1: *Suppose that the consumer acquires a perfectly informative signal if and only if the designer chooses $\kappa = \underline{\kappa}$. Then, in equilibrium only the G product is produced, it is always simplified, and it is accepted with probability one.*

Intuitively, the designer of a G product does not want to expose himself to the noise of imperfect information; thus, he chooses to simplify his product by choosing $\underline{\kappa}$, which implies acceptance with probability one. As a result, the designer of a B product cannot exploit the noise of imperfect information to get his product accepted since the consumer rationally infers that the designer has chosen a B product if she observes a noisy signal. Therefore, in equilibrium only G products are produced, and they are simplified and accepted with probability one.

It follows that our results will be driven by the fact that the designer cannot ensure that the consumer acquires perfect information; i.e., the consumer's information set is imperfect.

II. Equilibrium

In this section we characterize the equilibria of our game. First, we consider the consumer's optimal strategy given her belief about the product proposed by the designer (Section IIA). Second, we analyze the designer's strategy: his optimal choice of complexification (Section IIB) and of quality (Section IIC), given the consumer's acceptance strategy. Finally, we impose belief consistency to obtain the model's equilibria (Section IID).

It is immediate that there is always a trivial equilibrium with zero trade in which the consumer correctly believes that the designer has chosen a B product and rejects it with probability one and the designer indeed chooses a B product with probability one, as he is indifferent to producing G versus B products. (Both yield zero expected payoff.) In what follows we focus on the more interesting equilibria with positive trade, where the designer chooses a G product with positive probability.

A. *The Consumer's Strategy*

From inspection of the consumer's problem in (2), it is immediate that she follows a threshold strategy: the consumer accepts the product, $a(s,\chi) = 1$, if and only if her posterior belief about the product being of good quality is sufficiently high,

$$\mu(s,\chi) \geq \frac{w_0 - w(B)}{w(G) - w(B)} \equiv \omega, \tag{4}$$

where ω captures the relative value of the consumer's outside option.[11]

To understand the consumer's optimal acceptance strategy, we need to analyze the determinants of her posterior belief. Let $\mu \equiv \Pr(y = G)$ denote the consumer's prior belief, which must be positive, as we are looking at equilibria with positive trade.[12] After the designer proposes his product, the consumer observes the product complexity χ. Since complexity is informative about the designer's action κ, it may contain information about quality y. The consumer's interim belief upon observing χ is

$$\mu(\chi) = \frac{\mu}{\mu + (1-\mu)\ell(\chi)}, \tag{5}$$

where $\ell(\chi) \equiv \Pr(\chi | y = B)/\Pr(\chi | y = G)$. Note that in equilibrium the likelihood ratio $\ell(\chi)$ will depend on the designer's complexification strategy $\{\sigma_y\}$ and the primitive likelihood ratio $f(\chi | \bar{\kappa})/f(\chi | \underline{\kappa})$. Given the interim belief in (5), the consumer observes signal s with noise χ and forms posterior belief

$$\mu(s,\chi) = \frac{\Pr(S = s | y = G) \cdot \mu(\chi)}{\Pr(S = s | y = G) \cdot \mu(\chi) + \Pr(S = s | y = B) \cdot [1 - \mu(\chi)]}. \tag{6}$$

[11] If the consumer is indifferent, we assume without loss of generality that she accepts the product. Since such an indifference will arise with probability zero, what happens in that event is inconsequential.

[12] We will abuse notation slightly by referring to μ as a prior belief, $\mu(\chi)$ as an interim belief following observation of χ, and $\mu(s,\chi)$ as a posterior belief following observation of both χ and s.

The consumer's acceptance strategy is contingent on the acquired information whenever she accepts the product after observing a good signal g but rejects it after observing a bad signal b. For this to be an optimal acceptance strategy, the signal has to be informative enough that

$$\mu(b,\chi) < \omega \leq \mu(g,\chi). \tag{7}$$

Otherwise, if the acquired information is not sufficiently precise, the consumer chooses to disregard her information. In this case she either always accepts the product (if $\mu(s,\chi) \geq \omega, \forall s$) or always rejects it (if $\mu(s,\chi) < \omega, \forall s$). To ensure that for each s the posterior $\mu(s,\chi)$ is monotonic in χ (see Lemma A.1), we impose the following regularity condition:

CONDITION II.1: $\frac{f(\chi|\underline{\kappa})}{f(\chi|\bar{\kappa})} \cdot \frac{\chi}{1-\chi}$ *is monotonic in* χ.

Given a prior belief $\mu \in (0,1)$, the posterior beliefs satisfy $\mu(g,0) = 1$, $\mu(b,0) = 0$, and $\mu(g,1/2) = \mu(b,1/2) = \mu(1/2) \in (0,1)$. That is, the signal perfectly reveals quality when complexity is zero, $\chi = 0$, and it is uninformative when complexity is maximal, $\chi = 1/2$. As χ increases from zero to one-half, the MLRP guarantees that the posterior is monotonic when the designer's equilibrium strategy satisfies $\sigma_G = \sigma_B$, whereas the MLRP combined with Condition II.1 guarantees that this is also the case when $\sigma_G \neq \sigma_B$. Intuitively, Condition II.1 ensures that the information content of the signal, s, is greater than the information content of the complexity, χ. These features of the posterior beliefs are depicted in Figure 2 where the consumer optimally chooses to disregard (or, equivalently, not to acquire) information about sufficiently complex products.

The following definition will be useful in characterizing the consumer's optimal strategy.

DEFINITION 1: *We say that the consumer is* **optimistic** *if her interim belief satisfies* $\mu(1/2) \geq \omega$, *whereas we say that she is* **pessimistic** *if* $\mu(1/2) < \omega$.

The consumer is optimistic when, upon receiving an uninformative signal, she chooses to accept the product. This will happen if her interim belief after observing a product with the highest possible complexity (i.e., $\chi = 1/2$) is higher than her relative outside option. This case is depicted in Figure 2, panel A, where the consumer only rejects the product after observing a sufficiently informative negative signal; i.e., $\mu(s,\chi) < \omega$ if and only if $s = b$ and $\chi < \bar{\chi}$. As illustrated in Figure 2, panel A, $\bar{\chi}$ is the noise level at which $\mu(b,\bar{\chi}) = \omega$. In contrast, the consumer is pessimistic when upon receiving an uninformative signal she chooses to reject the product. This will happen if her interim belief after observing a product with the highest possible complexity (i.e., $\chi = 1/2$) is lower than her relative outside option. This case is depicted in Figure 2, panel B, where the consumer only accepts the product after observing a sufficiently informative positive signal; i.e., $\mu(s,\chi) \geq \omega$ if and only if $s = g$ and $\chi \leq \bar{\chi}$. As illustrated in Figure 2, panel B, $\bar{\chi}$ is the noise level at which $\mu(g,\bar{\chi}) = \omega$. We formalize this discussion in the following lemma.

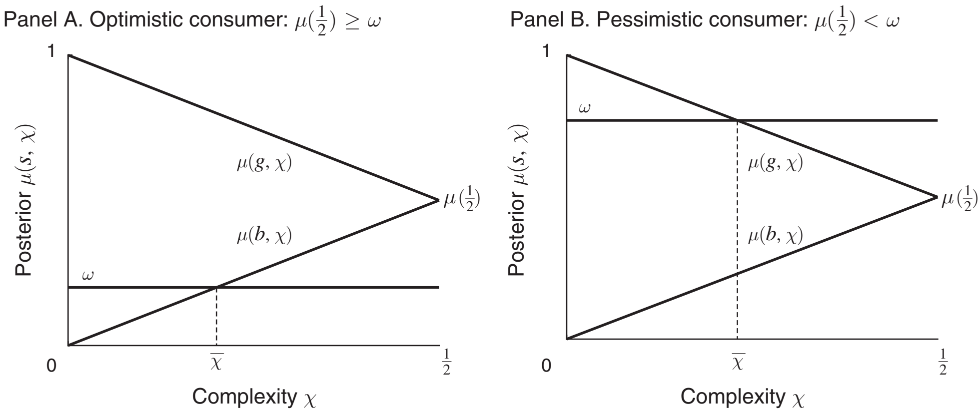

FIGURE 2

Note: The figure illustrates the behavior of posterior belief $\mu(s,\chi)$ as it depends on the signal, s, and complexity, χ.

LEMMA 1: *When the consumer is optimistic, her acceptance strategy satisfies*

$$a(s,\chi) = \begin{cases} \mathbf{1}\{s = g\}, & \text{if } \chi \leq \bar{\chi}; \\ 1, & \text{if } \chi > \bar{\chi}; \end{cases} \tag{8}$$

where $\mu(b,\bar{\chi}) = \omega$ and $\mathbf{1}\{s = g\}$ is the indicator equal to one when the signal is good. When the consumer is pessimistic, her acceptance strategy satisfies

$$a(s,\chi) = \begin{cases} \mathbf{1}\{s = g\}, & \text{if } \chi \leq \bar{\chi}; \\ 0, & \text{if } \chi > \bar{\chi}; \end{cases} \tag{9}$$

where $\mu(g,\bar{\chi}) = \omega$.[13]

An optimistic consumer will accept products that are sufficiently complex $(\chi > \bar{\chi})$, whereas a pessimistic consumer will reject such products. This will be essential for understanding the designer's incentives to complexify or simplify his product.

Finally, note that both the threshold level of complexity $\bar{\chi}$ and whether the consumer is optimistic or pessimistic are endogenous to equilibrium since the prior belief μ and the likelihood ratio $\ell(\chi)$, which determine the consumer's beliefs $\mu(\chi)$ and $\mu(s,\chi)$, will need to be consistent with the equilibrium strategy of the designer and Bayes' rule.

[13] When $\mu = 1$ or $\mu = 0$, we set without loss of generality $\bar{\chi} = 0$. As we will see, however, in any positive trade equilibrium $\mu \in (0,1)$.

B. *The Designer's Complexification Strategy*

Given the consumer's acceptance strategy described in the previous section, we next consider the designer's choice of κ for a y product. From the designer's objective in (3), it follows that a designer who chooses a y product also (weakly) prefers to simplify, $\underline{\kappa}$, whenever

$$\Pr\left(a = 1 \mid y, \underline{\kappa}\right) \geq \Pr\left(a = 1 \mid y, \bar{\kappa}\right). \tag{10}$$

Otherwise the designer prefers to complexify, $\bar{\kappa}$. From Lemma 1 we can compute the probability of acceptance of a y product conditional on the product's complexity χ.

When the consumer is optimistic,

$$\Pr\left(a = 1 \mid G, \chi\right) = \begin{cases} 1 - \chi, & \text{if } \chi < \bar{\chi}; \\ 1, & \text{if } \chi \geq \bar{\chi}; \end{cases} \tag{11}$$

and

$$\Pr\left(a = 1 \mid B, \chi\right) = \begin{cases} \chi, & \text{if } \chi < \bar{\chi}; \\ 1, & \text{if } \chi \geq \bar{\chi}. \end{cases}$$

Instead, when the consumer is pessimistic,

$$\Pr\left(a = 1 \mid G, \chi\right) = \begin{cases} 1 - \chi, & \text{if } \chi \leq \bar{\chi}; \\ 0, & \text{if } \chi > \bar{\chi}; \end{cases} \tag{12}$$

and

$$\Pr\left(a = 1 \mid B, \chi\right) = \begin{cases} \chi, & \text{if } \chi \leq \bar{\chi}; \\ 0, & \text{if } \chi > \bar{\chi}. \end{cases}$$

Hence, a designer who proposes a (y, κ) product to the consumer expects it to be accepted with probability

$$\Pr\left(a = 1 \mid y, \kappa\right) = \int_0^{1/2} \Pr\left(a = 1 \mid y, \chi\right) \cdot f\left(\chi \mid \kappa\right) d\chi. \tag{13}$$

The following proposition characterizes the optimal complexification strategy of a designer who has produced a y product.

PROPOSITION 2: *Let $\hat{\chi} \in \left(0, 1/2\right)$ denote the unique solution to $\int_0^{\hat{\chi}} \chi \cdot f\left(\chi \mid \underline{\kappa}\right) d\chi = \int_0^{\hat{\chi}} \chi \cdot f\left(\chi \mid \bar{\kappa}\right) d\chi$. Then, when the consumer is optimistic,*

$$\sigma_B = 1 \quad \textit{and} \quad \sigma_G \begin{cases} = 1, & \textit{if } \bar{\chi} < \hat{\chi}; \\ \in [0,1], & \textit{if } \bar{\chi} = \hat{\chi}; \\ = 0, & \textit{if } \bar{\chi} > \hat{\chi}; \end{cases} \tag{14}$$

whereas when the consumer is pessimistic,

$$\sigma_B \begin{cases} = 0, & \textit{if } \bar{\chi} < \hat{\chi}; \\ \in [0,1], & \textit{if } \bar{\chi} = \hat{\chi}; \\ = 1, & \textit{if } \bar{\chi} > \hat{\chi}; \end{cases} \quad \textit{and} \quad \sigma_G = 0. \tag{15}$$

This result says that when the consumer is optimistic, the designer has a tendency to complexify his product; conversely, when the consumer is pessimistic, the designer has a tendency to simplify it. The intuition for this result can be obtained from Figure 3, which illustrates the acceptance probability, $\Pr(a = 1 \mid y, \chi)$, as it depends on the product's quality, y, and complexity, χ. An optimistic consumer disregards information and accepts with probability one when a product is sufficiently complex. Thus, even the producer of a G product may benefit from complexifying if the consumer is sufficiently optimistic, i.e., $\bar{\chi}$ is sufficiently low (see Figure 3, panel A). Conversely, a pessimistic consumer disregards information and rejects it with probability one when a product is sufficiently complex. Thus, even the producer of a B product may benefit from simplifying if the consumer is sufficiently pessimistic—i.e., $\bar{\chi}$ is sufficiently low (see Figure 3, panel B).

Our analysis thus highlights the central trade-off faced by the designer when choosing whether to complexify his product. The designer trades off the benefit of relying on the consumer's prior belief about the product's quality against the benefit of making the consumer acquire and react to information. While the latter is higher for the designer of a good product, the former does not depend on the product quality chosen by the designer. For example, when the consumer's prior is sufficiently high (so that the consumer is optimistic and $\bar{\chi} < \hat{\chi}$), all products are complexified to reduce the consumer's reliance on information that is imperfect. Conversely, when the consumer's prior is sufficiently low (so that the consumer is pessimistic and $\bar{\chi} < \hat{\chi}$), all products are simplified to increase the consumer's reliance on information that is imperfect. Finally, for intermediate beliefs the designer of a good product wants the consumer to rely on information (and therefore simplifies), whereas the designer of a bad product wants the consumer to ignore information and rely on her prior (and therefore complexifies).

C. *The Designer's Quality Strategy*

When choosing the product's quality, the designer faces a trade-off between increasing the product's acceptance probability (by choosing $y = G$) or increasing his payoff conditional on acceptance (by choosing $y = B$). Given the consumer's acceptance strategy, the net expected payoff to the designer from choosing the G product over the B product is

$$\gamma \equiv \max_{\kappa} \Pr(a = 1 \mid G, \kappa) \cdot v(G) - \max_{\kappa} \Pr(a = 1 \mid B, \kappa) \cdot v(B). \tag{16}$$

The first term is the expected payoff from choosing the G product given the corresponding optimal choice of κ, as characterized in Proposition 2. The second term is the expected payoff from choosing the B product given the corresponding optimal choice of κ. The probabilities in each scenario are computed as in (13). Note that these probabilities and, as a result, the payoff γ depend on the consumer's belief μ and the likelihood ratio $\ell(\cdot)$, as the latter determine χ^{-}. The next result then follows immediately.

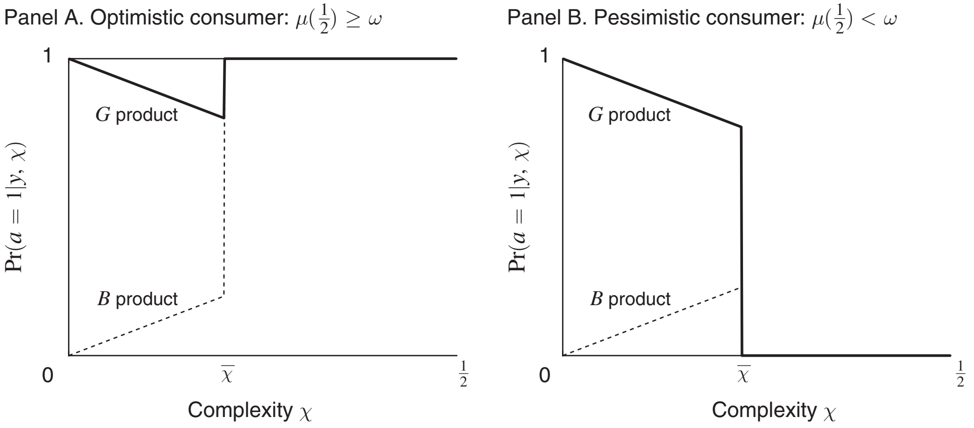

FIGURE 3

Notes: The figure illustrates the probability of acceptance of a y product as a function of complexity χ. Panel A shows the case of an optimistic consumer. Panel B shows the case of a pessimistic consumer.

PROPOSITION 3: *Given the consumer's acceptance strategy, the designer chooses the G product with probability*

$$m\begin{cases} = 1, & \text{if } \gamma > 0; \\ \in [0,1], & \text{if } \gamma = 0; \\ = 0, & \text{if } \gamma < 0; \end{cases} \tag{17}$$

where γ is given by (16).

D. *Characterization of Equilibria*

In Section IIA we characterized the consumer's strategy given her belief. In Sections IIB and IIC, we characterized the designer's quality and complexification strategy given the consumer's acceptance strategy. To characterize the equilibria of our model, we now require that the consumer's belief be consistent with the designer's strategy and Bayes' rule. We find it instructive to proceed in two steps.

Equilibrium Complexity.—In the first step we take the consumer's prior belief μ as given and find the designer's equilibrium complexification strategy $\{\sigma_y\}$ by requiring that the consumer's interim belief, $\mu(\chi)$, be consistent with it and Bayes' rule.

PROPOSITION 4: *Suppose that in equilibrium the consumer's prior belief is $\mu \in (0,1)$. Then there exist thresholds $0 < \mu_1 < \mu_2 < \mu_3 < \mu_4 < 1$ such that*

(*1*) *If $\mu \in (0, \mu_1]$, all products are simplified: $\sigma_G = \sigma_B = 0$.*

(2) *If* $\mu \in (\mu_1, \mu_2]$, *G products are simplified* $(\sigma_G = 0)$ *whereas B products are complexified with probability*

$$\sigma_B = \left[\frac{f(\hat{\chi}\,|\,\bar{\kappa})}{f(\hat{\chi}\,|\,\underline{\kappa})} - 1\right]^{-1}\left(\frac{1-\hat{\chi}}{\hat{\chi}}\frac{\mu}{1-\mu}\frac{1-\omega}{\omega} - 1\right).$$

(3) *If* $\mu \in (\mu_2, \mu_3]$, *G products are simplified* $(\sigma_G = 0)$ *whereas B products are complexified* $(\sigma_B = 1)$.

(4) *If* $\mu \in (\mu_3, \mu_4)$, *G products are complexified with probability*

$$\sigma_G \in \left\{0, 1 - \left[1 - \frac{f(\hat{\chi}\,|\,\underline{\kappa})}{f(\hat{\chi}\,|\,\bar{\kappa})}\right]^{-1}\left(1 - \frac{1-\hat{\chi}}{\hat{\chi}}\frac{1-\mu}{\mu}\frac{\omega}{1-\omega}\right), 1\right\},$$

whereas B products are complexified: $\sigma_B = 1$.

(5) *If* $\mu \in [\mu_4, 1)$, *all products are complexified*: $\sigma_G = \sigma_B = 1$.

This result is illustrated in Figure 4. As we can see, all products will be complexified (simplified) when the consumer's prior belief is sufficiently high (low), and G products will be simpler than B products for intermediate values of μ. In what follows we provide an intuition for this result by sketching the proof of the proposition.

First, let us ask whether an equilibrium in which all products are simplified—i.e., $\sigma_G = \sigma_B = 0$—exists. By Proposition 2, this can only happen if the consumer is pessimistic; i.e., $\mu(1/2) = \mu < \omega$.[14] In this case G products are always simplified, whereas B products are simplified only if $\bar{\chi} \leq \hat{\chi}$. Since the threshold $\bar{\chi}$ is given by $\mu(g, \bar{\chi}) = \omega$ (see Lemma 1) and $\mu(g, \cdot)$ is decreasing, we have that $\bar{\chi} \leq \hat{\chi}$ if and only if $\mu(g, \hat{\chi}) \leq \omega$, which is equivalent to

$$\mu \leq \frac{\omega \cdot \dfrac{\hat{\chi}}{1-\hat{\chi}}}{\omega \cdot \dfrac{\hat{\chi}}{1-\hat{\chi}} + 1 - \omega} \equiv \mu_1. \tag{18}$$

Since $\hat{\chi} \in (0, 1/2)$, $\mu_1 < \omega$. Thus, an equilibrium in which all products are simplified exists if $\mu \in (0, \mu_1]$; that is, if the consumer is sufficiently pessimistic about the product's quality.

Second, let us ask whether an equilibrium in which all products are complexified (i.e., $\sigma_G = \sigma_B = 1$) exists. By Proposition 2, this can only happen if the consumer is optimistic; i.e., $\mu(1/2) = \mu \geq \omega$. In this case B products are always complexified whereas G products are complexified only if $\bar{\chi} \leq \hat{\chi}$. Since now the threshold

[14] Note that when all products are simplified (or all products are complexified) the consumer does not make inferences upon observation of complexity; thus, $\mu(\chi) = \mu, \forall \chi$.

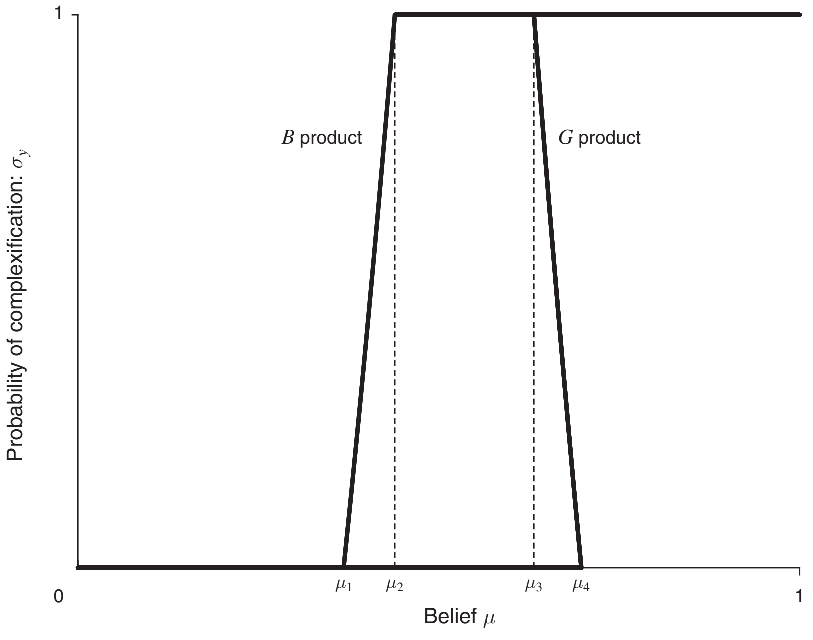

FIGURE 4

Note: The figure illustrates how the complexification strategy of the designer who chooses a *y* product varies with the consumer's belief μ.

$\bar{\chi}$ is given by $\mu(b, \bar{\chi}) = \omega$ (see Lemma 1) and $\mu(b, \cdot)$ is increasing, we have that $\bar{\chi} \leq \hat{\chi}$ if and only if $\mu(b, \hat{\chi}) \geq \omega$, which is equivalent to

$$\text{(19)} \qquad \mu \geq \frac{\omega \cdot \frac{1-\hat{\chi}}{\hat{\chi}}}{\omega \cdot \frac{1-\hat{\chi}}{\hat{\chi}} + 1 - \omega} \equiv \mu_3.$$

Since $\hat{\chi} \in (0, 1/2)$, $\mu_3 > \omega$. Thus, an equilibrium in which all products are complexified exists if $\mu \in [\mu_3, 1)$—that is, if the consumer is sufficiently optimistic about the product's quality.

Third, let us ask whether an equilibrium in which G products are simplified and B products are complexified—i.e., $\sigma_G = 0$ and $\sigma_B = 1$—exists. Note that in this case the consumer makes an inference about the product's quality upon observing complexity, χ, since more complex products are more likely to be B products. By Proposition 2, there are two cases to consider depending on whether the consumer is optimistic or pessimistic. If the consumer is pessimistic (i.e., $\mu(1/2) < \omega$), then G products are always simplified, whereas B products are complexified only if $\bar{\chi} \geq \hat{\chi}$. Both conditions hold if and only if

$$\text{(20)} \qquad \mu_2 \equiv \frac{\omega \cdot \ell(\hat{\chi}) \cdot \frac{\hat{\chi}}{1-\hat{\chi}}}{\omega \cdot \ell(\hat{\chi}) \cdot \frac{\hat{\chi}}{1-\hat{\chi}} + 1 - \omega} \leq \mu \leq \frac{\omega \cdot \ell\left(\frac{1}{2}\right)}{\omega \cdot \ell\left(\frac{1}{2}\right) + 1 - \omega},$$

where $\ell(\cdot) = f(\cdot\,|\,\bar{\kappa})/f(\cdot\,|\,\underline{\kappa})$. Instead, if the consumer is optimistic (i.e., $\mu(1/2) \geq \omega$), then *B* products are always complexified, whereas *G* products are simplified only if $\bar{\chi} \geq \hat{\chi}$. Both conditions hold if and only if

$$\text{(21)} \qquad \frac{\omega \cdot \ell\left(\frac{1}{2}\right)}{\omega \cdot \ell\left(\frac{1}{2}\right) + 1 - \omega} < \mu \leq \frac{\omega \cdot \ell(\hat{\chi}) \cdot \frac{1-\hat{\chi}}{\hat{\chi}}}{\omega \cdot \ell(\hat{\chi}) \cdot \frac{1-\hat{\chi}}{\hat{\chi}} + 1 - \omega} \equiv \mu_4.$$

Thus, an equilibrium in which *G* products are simplified and *B* products are complexified exists provided that $\mu \in [\mu_2, \mu_4]$; that is, the consumer is neither too optimistic nor too pessimistic about the product's quality.

Last, because the designer's incentive to complexify is always greater for *B* products than for *G* products, there cannot be an equilibrium in which *G* products are complexified and *B* products are simplified (see Proposition 2). Moreover, note that the ranking of the belief thresholds follows by inspection of (18)–(21) since $\hat{\chi} \in (0, 1/2)$ and the MLRP implies that $f(\hat{\chi}\,|\,\bar{\kappa})/f(\hat{\chi}\,|\,\underline{\kappa}) > 1$. Such ranking implies that for $\mu \in (\mu_1, \mu_2)$ there cannot exist equilibria that involve either pooling or separation on κ, whereas for $\mu \in (\mu_3, \mu_4)$ both pooling and separation on κ are possible. In these regions multiple complexification strategies can be consistent with a given belief μ, as illustrated in Figure 4.

We have thus characterized the designer's equilibrium complexification-strategy, $\{\sigma_y\}_\mu$, that is consistent with a given prior belief μ, where it is now convenient to make the dependence of the strategy on μ explicit.

Equilibrium Quality.—In the second step we pin down the equilibrium prior belief, which we will denote by μ^*. That is, we need to show that the complexification strategies and designer payoffs implied by μ^* are consistent with the designer producing *G* products with probability $m = \mu^*$. To this end, we use equation (16) to compute the designer's net payoff $\gamma(\mu, \{\sigma_y\}_\mu)$ from producing *G* versus *B* products as a function of the prior belief, μ, and the corresponding complexification strategy, $\{\sigma_y\}_\mu$, from Proposition 4.

The following result establishes the existence and uniqueness of the positive trade equilibrium.

PROPOSITION 5: *There is, generically, a unique equilibrium with positive trade, and in it* $m = \mu^* = \psi \in (0,1)$, *where* ψ *and* $\{\sigma_y\}_\psi$ *are solutions to*

$$\text{(22)} \qquad \gamma\left(\psi, \{\sigma_y\}_\psi\right) = 0.$$

First, it is easy to rule out an equilibrium with $\mu^* = 1$, since in that case the consumer would accept the product with probability one, making it optimal for the designer to only produce a *B* product, contradicting belief consistency. As discussed before, an equilibrium with $\mu^* = 0$ always exists, but in it products are rejected with probability one; thus, there is no trade. Therefore, in any equilibrium with positive trade, it must be that $\mu^* \in (0,1)$. Such an equilibrium exists if there is

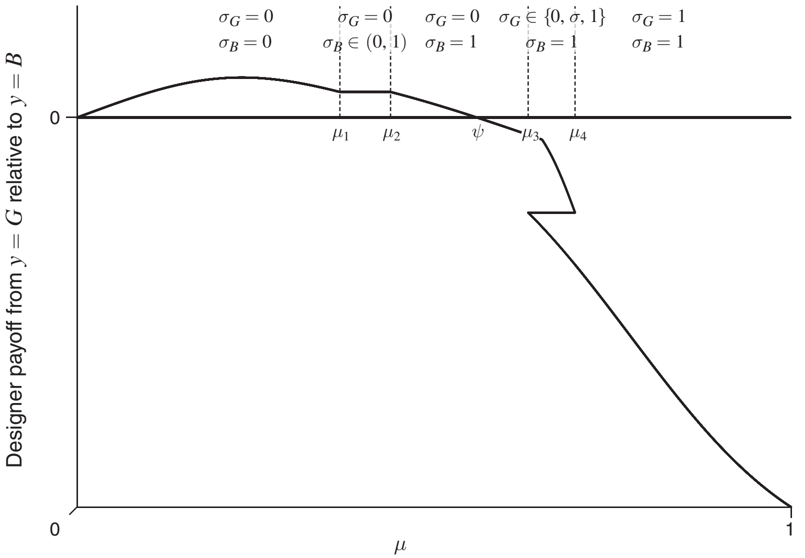

Figure 5

Note: The figure illustrates how the designer's net payoff from choosing the *G* product varies with the consumer's belief μ.

belief, ψ, and corresponding complexification strategy, $\{\sigma_y\}_{\psi}$, that make the designer indifferent between producing *G* versus *B* products.

Figure 5 illustrates the generic behavior of the designer's net payoff from producing *G* versus *B* products, as it depends on μ. First, it is positive for μ that is small and becomes negative for μ that is large. Intuitively, when μ is small the consumer is pessimistic and thus accepts products with low probability. The designer then expects a higher payoff from producing a *G* product, given its higher probability of acceptance. Second, as μ increases the difference between the probabilities of acceptance for *G* versus *B* products increases to further favor producing the *G* product, since the consumer becomes more likely to rely on information. Third, as μ increases further the consumer becomes sufficiently optimistic, which reduces her reliance on information, and the gap between the probabilities of acceptance of the two products begins to shrink as a result. Finally, as μ becomes sufficiently large, the probabilities of acceptance become large enough that the designer obtains a higher expected payoff from *B* products. (See the proof of Proposition 5 for formal details.) Using continuity arguments, we can then show that an intersection ψ exists and that it can, generically, lie in anywhere outside of the interval (μ_1, μ_2) where the designer's payoff is independent of μ.

We have provided a full characterization of the equilibrium of our signaling game. We note that (positive-trade) equilibrium uniqueness is obtained due to two distinguishing features of our model. First, the designer can influence but not fully control the product complexity for the consumer, which ensures that multiplicity due to the freedom of specifying off-equilibrium beliefs does not arise in our setting. This formulation is not only analytically convenient, but it is also reasonable within our applications, where a number of factors can determine the consumer's ability to process information about a product. Second, the endogeneity of product quality rules out multiplicity arising from several complexification strategies being consistent with a given prior belief (as stated in Proposition 4 for $\mu \in (\mu_3, \mu_4)$). In particular, there is, generically, a single value of ψ and a single complexification strategy, $\{\sigma_y\}_\psi$, such that the designer's strategy is optimal and the consumer's belief is consistent.

Next, we exploit the uniqueness of our equilibrium to obtain sharp comparative statics and to discuss how our results lead to applied implications for the relationship between product quality and complexity.

III. The Quality-Complexity Relationship

Our model generates sharp predictions about the relationship between quality and complexity of products and how this relationship varies with features of the environment. We now explore these predictions through comparative statics and several model extensions. In doing so, we say that a product's expected quality increases when the probability that a product has good quality, $\mu = \Pr(y = G)$, increases and that a product's expected complexity increases when the probability that a product is complexified, $\Pr(\kappa = \bar{\kappa}) = \mu\sigma_G + (1-\mu)\sigma_B$, increases.

A. *Relative Outside Option*

We begin by considering the effect of a decrease in ω, which reflects an increase in the relative payoff of the product to the consumer. Such an increase could result from an increase in the consumer's direct payoff from the product—i.e., $w(G)$ or $w(B)$—which we interpret as higher product demand or a fall in the consumer's outside option, w_0.

PROPOSITION 6: *As ω decreases, μ decreases while σ_G and σ_B do not change. Expected quality of products falls while the expected complexity of products rises as the consumer's relative outside option falls.*

Figure 6 illustrates the effect of a change in ω on expected product quality and complexity.[15] Intuitively, as the consumer's relative outside option falls, she is more likely to accept the product as her acceptance decision becomes less strict. This makes the *B* product more likely to be accepted, increasing the designer's incentives to produce a *B* product, lowering expected quality. Interestingly, the designer's equilibrium complexification strategy is independent of ω, as all of the adjustment happens through changes in average quality, leaving the consumer's acceptance strategy unchanged. Finally, as complexification is always weakly higher for a

[15] Throughout this section, unless stated otherwise, the figures are produced under the following parameterization: $\chi \sim$ Truncated Normal$(\mu_\kappa, \sigma, 0, 0.5)$ with means $\mu_{\underline{\kappa}} = 0.2$, $\mu_{\bar{\kappa}} = 0.3$ and standard deviation $\sigma = 0.2$ on interval $[0, 0.5]$, $\omega = 0.5$, $v(G) = 0.3$, and $v(B) = 1$.

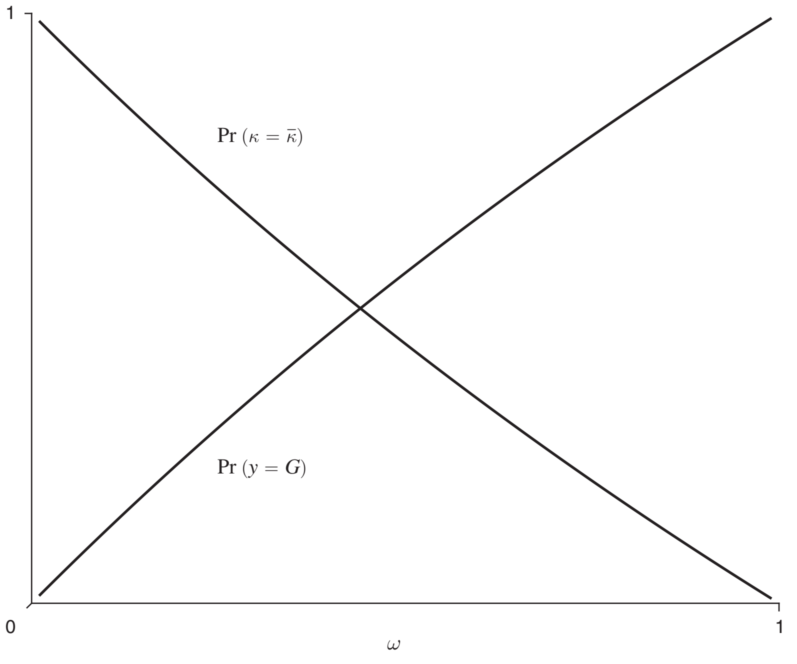

FIGURE 6

Note: The figure illustrates how expected product quality and complexity vary with the relative outside option, ω.

B than a G product, a decrease in expected quality results in an increase in expected complexity.

B. *Conflicts of Interest*

We next analyze the effect of an increase in the designer's alignment with the consumer given by an increase in the designer's payoff from producing a G product, $v(G)$.[16]

PROPOSITION 7: *As $v(G)$ increases σ_G and σ_B weakly increase, while μ can be nonmonotonic, but it goes to one as $v(G)$ goes to $v(B)$. Expected quality and expected complexity of products rise when the designer becomes sufficiently aligned with the consumer.*

Figure 7, panel A illustrates the effect of a change in the designer's alignment with the consumer on expected product quality and complexity. Intuitively, an increase in $v(G)$ increases the net payoff to the designer from producing a G product. Therefore, unsurprisingly, expected product quality increases. Since in equilibrium

[16] Note that the effect of an increase in $v(G)$ is the same as the effect of a decrease in $v(B)$.

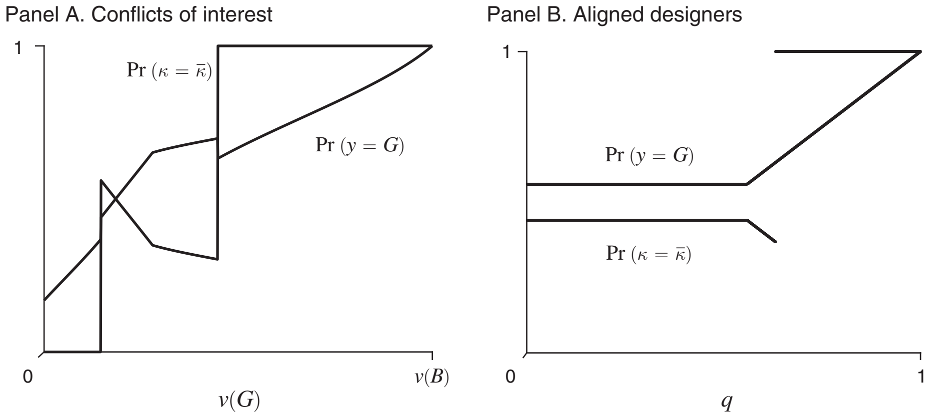

Figure 7

Notes: The figure illustrates how expected product quality and complexity vary with the consumer-designer alignment. Panel A shows the comparative statics on the designer's payoff from producing a G product, $v(G)$. Panel B shows the comparative statics on the probability of finding an aligned designer, q.

the consumer's prior belief μ must increase as well, she becomes less selective in accepting products, which increases the designer's incentive to complexify.

More precisely, an increase in $v(G)$ generates an upward shift of the correspondence γ, depicted in Figure 5, while leaving the thresholds $\mu_1 - \mu_4$ unchanged. Generically, this leads to an increase in ψ and thus to an improvement in overall product quality. There are, however, two values of $v(G)$ at which ψ jumps, generating the two kinks in Figure 7, panel A. First, as $v(G)$ increases, ψ gradually approaches μ_1 and then jumps up to μ_2. The reason for this is that the equilibrium switches from pooling at simplification to separation (i.e., $\sigma_G = 0$ and $\sigma_B = 1$), generating a discontinuous change in the consumer's interim belief as she begins inferring product quality from observed complexity, which incentivizes the supply of good products. Second, as $v(G)$ increases further, μ gradually approaches μ_4 and then jumps down to μ_3. The reason for this is that the equilibrium switches from separation to pooling at complexification, generating a discontinuous change in the consumer's interim belief as she stops inferring product quality from observed complexity, which incentivizes the supply of bad products. As we show next, this result is consistent with other mechanisms that reduce the misalignment between the designer and the consumer.

C. *Aligned Designers*

We next study the implications of introducing a designer whose payoffs are aligned with the consumer's. We suppose that with probability $q \in [0,1]$ the consumer encounters an aligned designer who obtains a higher payoff from having a G product being accepted, $\bar{v}(G) > \bar{v}(B) > 0$. With probability $1 - q$, however, the consumer meets the misaligned designer, as in the baseline model. Finally, whether the designer is aligned or misaligned is not observable to the consumer. Our baseline model corresponds to the case of $q = 0$.

An aligned designer takes private actions $\{y, \kappa\}$ to maximize his expected payoff, $\Pr(a = 1 \mid y, \kappa) \cdot \bar{v}(y)$. As both his probability of acceptance and payoff conditional on acceptance are higher for a G product, the aligned designer always produces a G product. Moreover, the complexification strategy for the G product is given by Proposition 2. Thus, even an aligned designer chooses to complexify his G product when the consumer is sufficiently optimistic.

The presence of an aligned designer only affects the equilibrium analyzed in Section II through the belief that a G product is offered, which is now given by $\mu = q + (1 - q) \cdot m$. As before, m is the probability with which the misaligned designer produces a G product. The following proposition characterizes the main effects of introducing an aligned designer.

PROPOSITION 8: *As q increases, μ, σ_G, and σ_B weakly increase. Expected quality and expected complexity of products rise as the fraction of aligned designers becomes sufficiently large.*

When the probability of meeting an aligned designer is sufficiently small $(q < \psi)$, there is no effect on equilibrium outcomes, as the presence of an aligned designer is fully offset by an increase in the misaligned designer's incentives to produce a B product. As a result, expected product quality and complexity are as in the baseline model, with $\mu = \psi$. When q is sufficiently large, however, the misaligned designer only produces a B product, $m = 0$, and further increases in q lead to higher product quality and thus to more complexification (see Proposition 4). These effects are depicted in Figure 7, panel B. The upward jump in expected complexity illustrated in the figure arises due to an equilibrium switch from separation to pooling at complexification.[17]

D. *Sequential Search and Competition*

Finally, we study the effects of competition among designers. In particular, we suppose that if the consumer rejects a product, she searches for a new designer, whom she finds with probability $\beta \in (0,1)$. The new designer proposes a product to the consumer, and the game repeats until the consumer accepts an offered product. In this setting higher β corresponds to lower search frictions and, hence, more intense competition among designers.

In a stationary equilibrium, in which U denotes the consumer's equilibrium value, we have

$$U = E\left[\max_{a \in \{0,1\}} \left\{ a \cdot \left[\mu(s,\chi) \cdot w(G) + \left(1 - \mu(s,\chi)\right) \cdot w(B)\right] + (1 - a) \cdot \beta U \right\}\right]. \tag{23}$$

[17] Since for $q > \psi$ the expected quality is effectively exogenous, there may be multiple equilibria whenever $q \in (\mu_3, \mu_4)$ due to multiple complexification strategies being consistent with belief $\mu = q$ (see Proposition 4). Figure 7, panel B is produced for the equilibrium where $\sigma_G = 0$ in this region so that the increase in expected complexity occurs at $q = \mu_4$ rather than sooner.

Note that for $w_0 = \beta U$, the equilibrium is fully characterized in Section IID. Thus, the main difference with our baseline model is that the consumer's outside option is now endogenous. Further, we assume that $w(B) < 0$ (but maintain that $w(G) > 0$), which ensures that the consumer prefers to search rather than accept a bad product; this rules out an uninteresting equilibrium where B products are produced and accepted with probability one.[18]

PROPOSITION 9: *An equilibrium exists, and in it* $\beta U \in \big(0, w(G)\big)$. *Furthermore,* βU *is increasing in* β. *Expected quality rises while expected complexity falls as competition among designers intensifies.*

As βU is the consumer's outside option in our search setting, comparative statics with respect to β are as those with respect to the relative outside option, ω, in Proposition 6. We find that competition has the desirable effect of increasing incentives to design products that the consumer wants: those that are good and simple. This prediction is in contrast to the literature on obfuscation and price complexity (Spiegler 2006; Ellison and Ellison 2009; Carlin 2009; Ellison and Wolitzky 2012), which finds that higher competition leads to more obfuscation, as obfuscation effectively increases the producer's market power by making it harder for the consumer to observe the attributes of competing products, e.g., by effectively increasing search costs. The consumer searches fewer products and competition effectively declines, which benefits firm profits. This channel is not present in our setting. Instead, here complexification influences the information the consumer extracts about the product. More intense competition effectively increases the consumer's outside option. This in turn incentivizes the designer to offer products that are more likely to be accepted by the consumer: those that are good and simple.[19]

E. *Empirical Implications*

In this section we summarize the main empirical implications and discuss them within the context of our two leading applications.

REMARK 1: *The model generates the following testable implications:*

(*1*) *As product demand by the consumer increases*, *products become more complex and of worse quality* (*Proposition 6*).

(*2*) *As the consumer–designer alignment increases*, *products become more complex and of better quality* (*Propositions 7 and 8*).

[18] Note that in such an equilibrium the consumer's outside option would be $w_0 = \beta U = \beta w(B) < w(B)$.

[19] A related result—though in a Bayesian-persuasion setting—is found in Au and Kawai (2020), who study competition in Bayesian persuasion where senders disclose information about their respective qualities. They find that competition (i.e., a higher number of senders) induces each sender to disclose information more aggressively.

(3) *As competition among the designers increases, products become less complex and of better quality* (*Proposition 9*).

We now discuss these predictions within the context of our two leading applications: financial products and regulatory design.

Financial Products.—We interpret the product designer as a financial advisor and the consumer as a retail investor. Financial advisors design or select financial products to offer to investors, such as investment funds, credit cards, and securitized products. Investors then decide whether they are willing to accept the offer or not. Within this context our model suggests that financial products are more likely to be complex and of low quality when demand for these products is high. This suggests that the proliferation of low-quality and complex structured financial products documented by Jaffee et al. (2009) and Célérier and Vallée (2017) could have been driven by an increase in the demand for such products.[20]

Our model also highlights the importance of financial advisors' compensation structures or career concerns, as they determine the level of alignment between financial advisors and retail investors. In particular, our model suggests that when the trust in financial advisors is high due to perceived alignment, we should observe more complex products being offered. This is consistent with the observation that the proliferation of complex products in recent decades has been accompanied by a period of increased trust in financial advisors, which may have culminated with the financial crisis. In this scenario, unlike in our model, the perceived alignment may have been unjustified and accompanied by the proliferation of bad-quality products.[21]

Recent empirical work has examined the effects of policies aimed at reducing conflicts of interest between financial advisors and clients. Bhattacharya, Illanes, and Padi (2019) exploit state-level variation in fiduciary duty laws in the United States and find that broker-dealers bound by fiduciary duty propose higher-quality products. Our model provides an additional prediction that is so far untested: although such policies can be effective in improving product quality, they may have the side effect of increasing product complexity. This insight is relevant given current debates about expanding fiduciary duty in the financial advice industry.[22]

Regulatory Policies.—In the regulatory sphere, we interpret the designer as a policy maker and the consumer as the median voter who has to accept a policy

[20] It is by now conventional wisdom that the last two decades have witnessed an unprecedented increase in the demand for stores of value produced by the United States due to the so-called "global savings glut" (Bernanke 2005). Structured financial products were perceived as a class of safer assets that could satisfy this demand.

[21] For example, in 2011 the Federal Housing Finance Agency filed lawsuits against some of the largest US financial institutions involving allegations of securities law violations and fraud in the packaging and sale of mortage-backed securities. For a detailed description, see https://www.fhfa.gov/SupervisionRegulation/LegalDocuments/Pages/Litigation.aspx.

[22] For instance, the Securities and Exchange Commission (SEC) has proposed to forbid the use of the term *financial advisor* for those managing brokerage accounts (particularly, retirement funds) unless the broker has formally accepted a fiduciary duty to act in the investor's best interest. See Dave Michaels, "'Fiduciary Rule' Poised for Second Life Under Trump Administration," *Wall Street Journal*, January 10, 2018: https://www.wsj.com/articles/fiduciary-rule-poised-for-second-life-under-trump-administration-1515580200.

proposal. Within this context we interpret a good (bad) policy proposal as one that is ideologically aligned (misaligned) with the median voter's preferences and q as the fraction of policy makers who are aligned with the median voter (as in Section IIIC). Finally, a lower median-voter outside option corresponds to a costlier status quo or, equivalently, to a greater urgency to pass a policy.

In a study of Italian legislative proposals, Gratton et al. (2021) show that legislation that is of worse quality and more complex is proposed by politicians when the bureaucracy is less effective and the expected duration of the legislative sessions is shorter. The authors argue that these conditions reduce the voters' ability to gather information on the competence of politicians. This in turn incentivizes bad politicians to pool with good politicians in producing legislation, which results in many low-quality legislative proposals. Our model suggests an additional channel for these outcomes. Bureaucratic ineffectiveness and short legislative sessions could map to a costly status quo and urgency to pass policies given substantial pressure for reforms in a short time interval. The results of Proposition 6 suggest that laws passed under such time pressure would indeed be more complex and of lower quality.

In the legislative context alignment between voters and policy makers is an important factor, and it is likely to arise due to similar ideological or political preferences. Such alignment between policy makers and the median voter is reflected in public opinion data, which provides politicians with real-time information about voters' support. Thus, our results suggest that policy makers who face high public opinion are more likely to propose policies that are complex but aligned with the median voter's preferences (and vice versa). Indeed, legal scholars have argued that in policy domains where public opinion is low (e.g., financial services or pharmaceuticals), policy proposals from the US Congress tend to be simpler, leaving it to federal agencies to draft additional rules (Stiglitz 2017). Our results provide a theoretical basis for further exploring this empirical observation.

IV. Concluding Remarks

In this paper we explore the incentives of product designers to produce complex products and the resulting implications for overall product quality. Our novel framework examines the joint determination of a product's quality and its complexity in a setting with only rational agents. We view our approach and our results as complementary to those studied by the literature on obfuscation and shrouded attributes.

We find that product complexity is not necessarily a feature of low-quality products. In particular, complexification or simplification may be used strategically by designers of both high- and low-quality products. Exploring the model's implications, we highlight the importance of understanding the underlying drivers of product heterogeneity for deriving empirical predictions regarding the relationship between product quality and complexity.

We focus our model's implications on two domains in which increasing complexity has received close scrutiny: financial products and regulatory policies. We provide a new rationale for the observed proliferation of complex financial products and regulatory policies. In the context of the financial products industry, we argue that high demand for safe assets may have been an important driver of the increasing complexity and worsening quality of structured products. In the context of regulatory design, we argue that increased complexity and worsening quality of regulatory proposals may be driven by the high urgency of passing regulatory reform or the high cost of inaction (maintaining the status quo).

Finally, our model contributes to the policy debate on whether policy makers should be concerned by rising complexity and whether they should act to reduce it. If the policy makers' worries are about the effect of complexity on the quality of products offered to consumers, then our results help to isolate which features of the environment should be monitored for signs that rising complexity will lead to lower quality.

Appendix

We now provide the technical proofs for Sections I–III. Robustness exercises can be found in the online Appendix.

PROOF OF PROPOSITION 1:

Observe that in any equilibrium a G product must be accepted with probability one, since the G product designer can always choose $\underline{\kappa}$ and effectively reveal the product's quality to the consumer, and the consumer would accept it since $w(G) > w_0$.

Next, let κ_y denote the complexification choice of the y product designer, and suppose for contradiction that in equilibrium the designer of a G product chooses $\kappa_G = \bar{\kappa}$ with positive probability. Since the product has to be accepted with probability one, it must be that the consumer accepts it independently of her signal. But then the designer of a B product can also set $\kappa_B = \bar{\kappa}$ and get his product accepted with probability one. If in equilibrium, however, both G and B products were accepted with probability one, then the designer would only produce a B product since he gets a higher payoff with that product: $v(B) > v(G)$. But then the consumer would reject all products with probability one since $w_0 > w(B)$, a contradiction.

Therefore, in any equilibrium the designer of a G product must choose $\kappa_G = \underline{\kappa}$ and perfectly reveal his product quality to the consumer. Hence, there does not exist an equilibrium in which the designer produces a B product with positive probability since then the consumer would find this out and would reject such a product with probability one. ∎

LEMMA A.1: *Suppose that Condition II.1 holds. Then, in equilibrium the posterior belief $\mu(g,\chi)$ is increasing in χ, the posterior belief $\mu(b,\chi)$ is increasing in χ, and the threshold $\bar{\chi}$ defined in Definition 1 is unique.*

PROOF:

From (6), the posterior belief $\mu(b,\chi)$ is increasing in χ if and only if the likelihood ratio $\frac{\sigma_G f(\chi|\bar{\kappa}) + (1-\sigma_G) f(\chi|\underline{\kappa})}{\sigma_B f(\chi|\bar{\kappa}) + (1-\sigma_B) f(\chi|\underline{\kappa})} \cdot \frac{\chi}{1-\chi}$ is increasing in χ. But the latter follows from the MLRP and Condition II.1. Analogously, the posterior belief $\mu(g,\chi)$ is decreasing in χ if and only if the likelihood ratio $\frac{\sigma_B f(\chi|\bar{\kappa}) + (1-\sigma_B) f(\chi|\underline{\kappa})}{\sigma_G f(\chi|\bar{\kappa}) + (1-\sigma_G) f(\chi|\underline{\kappa})} \cdot \frac{\chi}{1-\chi}$ is increasing in χ. But the latter also follows from the MLRP and Condition II.1. Finally, the uniqueness of the threshold $\bar{\chi}$ follows from the monotonicity of the posteriors combined with the facts that $\mu(g,0) = 1$, $\mu(b,0) = 0$, and $\mu(g,1/2) = \mu(b,1/2) = \mu(1/2) \in (0,1)$.

We will use the result in Lemma A.1 in the proofs that follow. ∎

PROOF OF LEMMA 1:

See text. ∎

PROOF OF PROPOSITION 2:

We begin by studying the designer's optimal choice of κ in the case where the consumer is optimistic (see Definition 1).

Case 1 (Consumer Is Optimistic)**:** In this case the designer's product is accepted with probability one when complexity is high enough: $\chi \geq \bar{\chi}$. So, his optimal choice of κ solves

$$\max_{\kappa \in \{\underline{\kappa}, \bar{\kappa}\}} \int_0^{\bar{\chi}} \Pr(S = g|y) \cdot f(\chi|\kappa) d\chi + \int_{\bar{\chi}}^{1/2} f(\chi|\kappa) d\chi. \tag{A1}$$

Thus, it is optimal for the designer of a *B* product to choose $\bar{\kappa}$ if

$$\int_0^{\bar{\chi}} \chi \cdot f(\chi|\bar{\kappa}) d\chi + \int_{\bar{\chi}}^{1/2} f(\chi|\bar{\kappa}) d\chi \geq \int_0^{\bar{\chi}} \chi \cdot f(\chi|\underline{\kappa}) d\chi + \int_{\bar{\chi}}^{1/2} f(\chi|\underline{\kappa}) d\chi, \tag{A2}$$

and it is uniquely optimal if the inequality is strict. The expression above is equivalent to

$$\int_0^{\bar{\chi}} (1-\chi) \cdot \left[f(\chi|\underline{\kappa}) - f(\chi|\bar{\kappa})\right] d\chi \geq 0. \tag{A3}$$

Suppose $\bar{\chi} > 0$, as will be the case in any equilibrium, and let χ^* be the maximum $\chi \in (0, \bar{\chi}]$ for which $f(\chi|\underline{\kappa})/f(\chi|\bar{\kappa}) \geq 1$, and note that it exists and is unique from the MLRP. With this, we can rewrite the left-hand side of condition (A3) as follows:

$$\begin{aligned} &\int_0^{\chi^*} (1-\chi) \cdot \left[\frac{f(\chi|\underline{\kappa})}{f(\chi|\bar{\kappa})} - 1\right] f(\chi|\bar{\kappa}) d\chi \\ &\quad + \int_{\chi^*}^{\bar{\chi}} (1-\chi) \cdot \left[\frac{f(\chi|\underline{\kappa})}{f(\chi|\bar{\kappa})} - 1\right] f(\chi|\bar{\kappa}) d\chi \\ &> (1-\chi^*) \cdot \int_0^{\chi^*} \left[\frac{f(\chi|\underline{\kappa})}{f(\chi|\bar{\kappa})} - 1\right] f(\chi|\bar{\kappa}) d\chi \end{aligned} \tag{A4}$$

$$+ (1-\chi^*) \cdot \int_{\chi^*}^{\bar{\chi}} \left[\frac{f(\chi|\underline{\kappa})}{f(\chi|\bar{\kappa})} - 1 \right] f(\chi|\bar{\kappa}) d\chi$$

$$= (1-\chi^*) \cdot \left[F(\bar{\chi}|\underline{\kappa}) - F(\bar{\chi}|\bar{\kappa}) \right] \geq 0,$$

where both inequalities follow from the MLRP. As condition (A3) is satisfied with strict inequality, it is uniquely optimal for the designer of the B product to choose $\bar{\kappa}$.

On the other hand, it is optimal for the designer of the G product to choose $\bar{\kappa}$ if

$$\text{(A5)} \quad \int_0^{\bar{\chi}} (1-\chi) \cdot f(\chi|\bar{\kappa}) d\chi + \int_{\bar{\chi}}^{1/2} f(\chi|\bar{\kappa}) d\chi \geq \int_0^{\bar{\chi}} (1-\chi) \cdot f(\chi|\underline{\kappa}) d\chi + \int_{\bar{\chi}}^{1/2} f(\chi|\underline{\kappa}) d\chi,$$

and it is uniquely optimal if the inequality is strict. This is equivalent to

$$\text{(A6)} \quad \int_0^{\bar{\chi}} \chi \cdot \left[f(\chi|\underline{\kappa}) - f(\chi|\bar{\kappa}) \right] d\chi \geq 0.$$

Condition (A5) is satisfied if $\bar{\chi} \leq \hat{\chi}$ and holds with strict inequality if $\bar{\chi} < \hat{\chi}$. Thus, if $\bar{\chi} < \hat{\chi}$, it is uniquely optimal for the designer of the G product to choose $\bar{\kappa}$. Otherwise, if $\bar{\chi} = \hat{\chi}$ the designer is indifferent to the choice of κ, and if $\bar{\chi} > \hat{\chi}$ it is uniquely optimal to choose $\underline{\kappa}$.

Next, we study the designer's choice of κ in the case when the consumer is pessimistic.

Case 2 (Consumer Is Pessimistic)**:** In this case the designer's product is rejected if complexity is too high: $\chi > \bar{\chi}$. So, his optimal choice of κ solves

$$\text{(A7)} \quad \max_{\kappa \in \{\underline{\kappa}, \bar{\kappa}\}} \int_0^{\bar{\chi}} \Pr(S = g|y) \cdot f(\chi|\kappa) d\chi.$$

Thus, it is optimal for the designer of the B product to choose $\underline{\kappa}$ if

$$\text{(A8)} \quad \int_0^{\bar{\chi}} \chi \cdot f(\chi|\bar{\kappa}) d\chi \leq \int_0^{\bar{\chi}} \chi \cdot f(\chi|\underline{\kappa}) d\chi,$$

and it is uniquely optimal if the inequality is strict. This is equivalent to

$$\text{(A9)} \quad \int_0^{\bar{\chi}} \chi \cdot \left[f(\chi|\underline{\kappa}) - f(\chi|\bar{\kappa}) \right] d\chi \geq 0.$$

Condition (A8) is satisfied if $\bar{\chi} \leq \hat{\chi}$ and holds with strict inequality if $\bar{\chi} < \hat{\chi}$. Thus, if $\bar{\chi} < \hat{\chi}$ it is uniquely optimal for the designer of the B product to choose $\underline{\kappa}$. Otherwise, if $\bar{\chi} = \hat{\chi}$ the designer is indifferent to the choice of κ, and if $\bar{\chi} > \hat{\chi}$ it is uniquely optimal to choose $\bar{\kappa}$.

On the other hand, it is optimal for the designer of the G product to choose $\underline{\kappa}$ if

$$\text{(A10)} \quad \int_0^{\bar{\chi}} (1-\chi) \cdot f(\chi|\bar{\kappa}) d\chi \leq \int_0^{\bar{\chi}} (1-\chi) \cdot f(\chi|\underline{\kappa}) d\chi,$$

and it is uniquely optimal if the inequality is strict. This is equivalent to

$$\text{(A11)} \qquad \int_0^{\bar{\chi}} (1-\chi)\cdot\left[f(\chi\,|\,\underline{\kappa}) - f(\chi\,|\,\bar{\kappa})\right]d\chi \ \geq\ 0,$$

which always holds with strict inequality by (A4) for all $\bar{\chi} > 0$, as will be the case in any equilibrium. Thus, it is uniquely optimal for the designer of the G product to choose $\underline{\kappa}$. ∎

PROOF OF PROPOSITION 3:

The proof is straightforward. ∎

PROOF OF PROPOSITION 4:

Suppose that in equilibrium the consumer's belief that the designer has produced a G product is $\mu \in (0,1)$.

Pooling on κ.—Consider first the candidate equilibrium in which $\sigma_B = \sigma_G = 0$. By Proposition 2, this requires that the consumer is pessimistic—i.e., $\mu(1/2) = \mu \leq \omega$ (see Definition 1)—and that $\bar{\chi} \leq \hat{\chi}$. On the equilibrium path the consumer does not update upon observing χ; thus, threshold $\bar{\chi}$ is given by $\mu(g,\bar{\chi}) = \omega$, which is equivalent to

$$\text{(A12)} \qquad \bar{\chi} = \frac{(1-\omega)\cdot\mu}{(1-\omega)\cdot\mu + \omega\cdot(1-\mu)}.$$

This is an equilibrium if and only if $\bar{\chi} \leq \hat{\chi}$, which is equivalent to

$$\text{(A13)} \qquad \mu \leq \frac{\omega\cdot\frac{\hat{\chi}}{1-\hat{\chi}}}{\omega\cdot\frac{\hat{\chi}}{1-\hat{\chi}} + 1 - \omega} \equiv \mu_1.$$

Consider next the candidate equilibrium in which $\sigma_B = \sigma_G = 1$. By Proposition 2, this requires that the consumer is optimistic—i.e., $\mu(1/2) = \mu \geq \omega$—and that $\bar{\chi} \leq \hat{\chi}$. On the equilibrium path the consumer does not update upon observing χ; thus, threshold $\bar{\chi}$ is given by $\mu(b,\bar{\chi}) = \omega$, which is equivalent to

$$\text{(A14)} \qquad \bar{\chi} = \frac{\omega\cdot(1-\mu)}{(1-\omega)\cdot\mu + \omega\cdot(1-\mu)}.$$

This is an equilibrium if and only if $\bar{\chi} \leq \hat{\chi}$, which is equivalent to

$$\text{(A15)} \qquad \mu \geq \frac{\omega\cdot\frac{1-\hat{\chi}}{\hat{\chi}}}{\omega\cdot\frac{1-\hat{\chi}}{\hat{\chi}} + 1 - \omega} \equiv \mu_3.$$

Therefore, $\sigma_B = \sigma_G = 0$ is an equilibrium if and only if $\mu \in (0,\mu_1]$, whereas $\sigma_B = \sigma_G = 1$ is an equilibrium if and only if $\mu \in [\mu_3, 1)$.

Separation on κ.—Consider the candidate equilibrium in which $\sigma_B = 1$ and $\sigma_G = 0$. There are two cases to consider, depending on whether the consumer is optimistic or pessimistic.

First, suppose that the consumer is pessimistic—i.e.,

$$\mu\left(g, \frac{1}{2}\right) = \mu\left(b, \frac{1}{2}\right) = \frac{\mu}{\mu + (1-\mu) \cdot \ell\left(\frac{1}{2}\right)} \leq \omega, \tag{A16}$$

where $\ell(\cdot) = f(\cdot \mid \bar{\kappa})/f(\cdot \mid \underline{\kappa})$. On the equilibrium path the consumer updates upon observing χ; thus, threshold $\bar{\chi}$ is given by

$$\mu(g, \bar{\chi}) = \frac{\mu}{\mu + (1-\mu) \cdot \ell(\bar{\chi}) \cdot \frac{\bar{\chi}}{1-\bar{\chi}}} = \omega. \tag{A17}$$

This is an equilibrium if and only if $\bar{\chi} \geq \hat{\chi}$, i.e.,

$$\mu_2 \equiv \frac{\omega \cdot \ell(\hat{\chi}) \cdot \frac{\hat{\chi}}{1-\hat{\chi}}}{\omega \cdot \ell(\hat{\chi}) \cdot \frac{\hat{\chi}}{1-\hat{\chi}} + 1 - \omega} \leq \mu \leq \frac{\omega \cdot \ell\left(\frac{1}{2}\right)}{\omega \cdot \ell\left(\frac{1}{2}\right) + 1 - \omega} \equiv \tilde{\mu}. \tag{A18}$$

Second, suppose that the consumer is optimistic, i.e.,

$$\mu\left(g, \frac{1}{2}\right) = \mu\left(b, \frac{1}{2}\right) = \frac{\mu}{\mu + (1-\mu) \cdot \ell\left(\frac{1}{2}\right)} > \omega. \tag{A19}$$

The threshold $\bar{\chi}$ is now given by

$$\mu(b, \bar{\chi}) = \frac{\mu}{\mu + (1-\mu) \cdot \ell(\bar{\chi}) \cdot \frac{1-\bar{\chi}}{\bar{\chi}}} = \omega. \tag{A20}$$

This is an equilibrium if and only if $\bar{\chi} \geq \hat{\chi}$, i.e.,

$$\tilde{\mu} = \frac{\omega \cdot \ell\left(\frac{1}{2}\right)}{\omega \cdot \ell\left(\frac{1}{2}\right) + 1 - \omega} < \mu \leq \frac{\omega \cdot \ell(\hat{\chi}) \cdot \frac{1-\hat{\chi}}{\hat{\chi}}}{\omega \cdot \ell(\hat{\chi}) \cdot \frac{1-\hat{\chi}}{\hat{\chi}} + 1 - \omega} \equiv \mu_4. \tag{A21}$$

Therefore, $\sigma_B = 0$ and $\sigma_G = 1$ is an equilibrium if and only if $\mu \in [\mu_2, \mu_4]$.

Semi-separation on κ.—Consider the candidate equilibrium in which $\sigma_B \in (0,1)$. By Proposition 2 such an equilibrium requires that the consumer be pessimistic; thus, $\sigma_G = 0$. On the equilibrium path there is updating from observing χ, and threshold $\bar{\chi}$ must equal $\hat{\chi}$ so that the designer of the *B* product is indifferent to the choice of κ (Proposition 2) and is willing to mix

$$\mu(g, \hat{\chi}) = \frac{\mu}{\mu + (1-\mu) \cdot \left[\sigma_B \cdot \ell(\hat{\chi}) + 1 - \sigma_B\right] \cdot \frac{\hat{\chi}}{1-\hat{\chi}}} = \omega, \tag{A22}$$

which in turn implies that

$$\text{(A23)} \qquad \sigma_B = \frac{\frac{1-\hat{\chi}}{\hat{\chi}} \cdot \frac{\mu}{1-\mu} \cdot \frac{1-\omega}{\omega} - 1}{\frac{f(\hat{\chi}\,|\,\bar{\kappa})}{f(\hat{\chi}\,|\,\underline{\kappa})} - 1}.$$

Since the posterior belief $\mu(g, \hat{\chi})$ is continuous and decreasing in σ_B (and the MLRP implies that $\ell(\hat{\chi}) > 1$), this equilibrium exists if and only if

$$\text{(A24)} \qquad \mu(g, \hat{\chi})|_{\sigma_B = 1} < \omega < \mu(g, \hat{\chi})|_{\sigma_B = 0},$$

which is equivalent to

$$\text{(A25)} \qquad \mu_1 = \frac{\omega \cdot \frac{\hat{\chi}}{1-\hat{\chi}}}{\omega \cdot \frac{\hat{\chi}}{1-\hat{\chi}} + 1 - \omega} < \mu < \frac{\omega \cdot \ell(\hat{\chi}) \cdot \frac{\hat{\chi}}{1-\hat{\chi}}}{\omega \cdot \ell(\hat{\chi}) \cdot \frac{\hat{\chi}}{1-\hat{\chi}} + 1 - \omega} = \mu_2.$$

Therefore, $\sigma_G = 0$ and $\sigma_B \in (0,1)$ is an equilibrium if and only if $\mu \in (\mu_1, \mu_2)$.

Consider the candidate equilibrium in which $\sigma_G \in (0,1)$. By Proposition 2 such an equilibrium requires that the consumer be optimistic, and so $\sigma_B = 1$. On the equilibrium path there is updating from observing χ, and threshold $\bar{\chi}$ must equal $\hat{\chi}$ so that the designer of the *G* product is indifferent to the choice of κ and is willing to mix

$$\text{(A26)} \qquad \mu(b, \hat{\chi}) = \frac{\mu}{\mu + (1-\mu) \cdot \frac{1-\hat{\chi}}{\hat{\chi}} \cdot \frac{1}{\sigma_G + (1-\sigma_G) \cdot \ell(\hat{\chi})^{-1}}} = \omega,$$

which in turn implies that

$$\text{(A27)} \qquad \sigma_G = 1 - \frac{1 - \frac{1-\hat{\chi}}{\hat{\chi}} \cdot \frac{1-\mu}{\mu} \cdot \frac{\omega}{1-\omega}}{1 - \frac{f(\hat{\chi}\,|\,\underline{\kappa})}{f(\hat{\chi}\,|\,\bar{\kappa})}}.$$

Since the posterior belief $\mu(b, \hat{\chi})$ is continuous and increasing in σ_G, this equilibrium exists if and only if

$$\text{(A28)} \qquad \mu(b, \hat{\chi})|_{\sigma_G = 0} < \omega < \mu(b, \hat{\chi})|_{\sigma_G = 1},$$

which is equivalent to

$$\text{(A29)} \qquad \mu_3 = \frac{\omega \cdot \frac{1-\hat{\chi}}{\hat{\chi}}}{\omega \cdot \frac{1-\hat{\chi}}{\hat{\chi}} + 1 - \omega} < \mu < \frac{\omega \cdot \ell(\hat{\chi}) \cdot \frac{1-\hat{\chi}}{\hat{\chi}}}{\omega \cdot \ell(\hat{\chi}) \cdot \frac{1-\hat{\chi}}{\hat{\chi}} + 1 - \omega} = \mu_4.$$

Therefore, $\sigma_G \in (0,1)$ and $\sigma_B = 1$ is an equilibrium if and only if $\mu \in (\mu_3, \mu_4)$.

We have thus characterized all the possible equilibria $\{\sigma_y\}$ as a function of belief μ:

(1) If $\mu \in (0, \mu_1]$, then $\sigma_G = \sigma_B = 0$.

(2) If $\mu \in (\mu_1, \mu_2)$, then $\sigma_G = 0$ and $\sigma_B = \dfrac{\frac{1-\hat{\chi}}{\hat{\chi}} \cdot \frac{\mu}{1-\mu} \cdot \frac{1-\omega}{\omega} - 1}{\frac{f(\hat{\chi}|\bar{\kappa})}{f(\hat{\chi}|\underline{\kappa})} - 1}$.

(3) If $\mu \in [\mu_2, \mu_3]$, then $\sigma_G = 0$ and $\sigma_B = 1$.

(4) If $\mu \in (\mu_3, \mu_4)$, then $\sigma_G \in \left\{0, 1 - \dfrac{1 - \frac{1-\hat{\chi}}{\hat{\chi}} \cdot \frac{1-\mu}{\mu} \cdot \frac{\omega}{1-\omega}}{1 - \frac{f(\hat{\chi}|\underline{\kappa})}{f(\hat{\chi}|\bar{\kappa})}}, 1\right\}$ and $\sigma_B = 1$.

(5) If $\mu \in [\mu_4, 1]$, then $\sigma_G = \sigma_B = 1$.

This establishes the stated result. ∎

PROOF OF PROPOSITION 5:

The designer's net expected payoff from choosing a G product relative to a B product is defined in (16). Define the correspondence $\Gamma : [0,1] \to 2^{\mathbb{R}}$, where $\Gamma(\mu)$ is the set of designer net payoffs $\gamma(\mu)$ implied by all the $\{\sigma_y\}$, which are consistent with an equilibrium in which the consumer's prior belief is μ, given in Proposition 4. We now make explicit the dependence of the net payoff on the consumer's belief μ.

First, note that $0 \in \Gamma(0)$ and that $\Gamma(0)$ is a singleton, since the product is rejected with probability one when the consumer's belief is $\mu = 0$. Second, consider $\mu \in (0,1]$. Note that $\Gamma(\mu)$ is a singleton for $\mu \notin (\mu_3, \mu_4)$, since $\{\sigma_y\}$ corresponding to such μ are unique. On the other hand, $\Gamma(\mu)$ consists of three elements if $\mu \in (\mu_3, \mu_4)$, since either $\sigma_G = 0$ and $\sigma_B = 1$, $\sigma_G \in (0,1)$ and $\sigma_B = 1$, or $\sigma_G = 1$ and $\sigma_B = 1$. We consider each case next.

Case $\mu \in (0, \mu_1]$.—By Proposition 4, equilibrium must have $\sigma_G = \sigma_B = 0$, and it must be that the consumer is pessimistic, since $\mu_1 < \tilde{\mu}$ (defined in (A18)). Furthermore, $\Gamma(\mu)$ is a singleton with

$$\text{(A30)} \quad \gamma(\mu) = v(G) \cdot \int_0^{\bar{\chi}(\mu)} (1-\chi) f(\chi|\underline{\kappa}) d\chi - v(B) \cdot \int_0^{\bar{\chi}(\mu)} \chi f(\chi|\underline{\kappa}) d\chi,$$

since the product is rejected whenever $\chi > \bar{\chi}(\mu)$ and accepted following signal $s = g$ otherwise. Note that

$$\text{(A31)} \quad \gamma'(\mu) = \left\{v(G) - [v(G) + v(B)] \cdot \bar{\chi}(\mu)\right\} \cdot f(\chi|\underline{\kappa}) \cdot \frac{d\bar{\chi}(\mu)}{d\mu}$$

where $\bar{\chi}(\mu)$ is given by (A12) and thus satisfies $\frac{d\bar{\chi}(\mu)}{d\mu} > 0$, $\bar{\chi}(0) = 0$, and $\bar{\chi}(\mu_1) = \hat{\chi}$. As a result, for μ sufficiently small, $\gamma'(\mu) > 0$ and $\gamma(\mu) > 0$. Next, consider the value μ_v such that

$$\text{(A32)} \quad \bar{\chi}(\mu_v) = \frac{v(G)}{v(G) + v(B)} \Rightarrow \mu_v \equiv \frac{v(G) \cdot \omega}{v(G) \cdot \omega + v(B) \cdot (1 - \omega)}.$$

If $\mu_v \geq \mu_1$, then $\gamma'(\mu) > 0, \forall \mu \in (0, \mu_1)$. Otherwise, $\gamma'(\mu) > 0$ for $\mu \in (0, \mu_v)$ and $\gamma'(\mu) < 0$ for $\mu \in (\mu_v, \mu_1)$.

Case $\mu \in (\mu_1, \mu_2]$.—Equilibrium must have $\sigma_G = 0$ and $\sigma_B \in (0,1)$, and it must be that the consumer is pessimistic, since $\mu_2 < \tilde{\mu}$. In this case, $\bar{\chi}(\mu) = \hat{\chi}$ and $\Gamma(\mu)$ is singleton with

$$\text{(A33)} \quad \gamma(\mu) = v(G) \cdot \int_0^{\hat{\chi}} (1 - \chi) f(\chi \,|\, \underline{\kappa}) d\chi - v(B) \cdot \int_0^{\hat{\chi}} \chi f(\chi \,|\, \underline{\kappa}) d\chi,$$

since the product is rejected whenever $\chi > \bar{\chi}(\mu) = \hat{\chi}$ and accepted following signal $s = g$ otherwise, and where we use the fact that the B product designer is indifferent between the choice of $\bar{\kappa}$ and $\underline{\kappa}$. It therefore follows that $\gamma(\mu)$ is constant on interval (μ_1, μ_2) and equal to $\gamma(\mu_1)$.

Case $\mu \in (\mu_2, \mu_3]$.—Equilibrium must have $\sigma_G = 0$ and $\sigma_B = 1$. The consumer is pessimistic if $\mu < \tilde{\mu}$, and she is optimistic otherwise. Here, again, $\Gamma(\mu)$ is a singleton.

Suppose that $\mu < \tilde{\mu}$. Then, the consumer is still pessimistic, and we have

$$\text{(A34)} \quad \gamma(\mu) = v(G) \cdot \int_0^{\bar{\chi}(\mu)} (1 - \chi) f(\chi \,|\, \underline{\kappa}) d\chi - v(B) \cdot \int_0^{\bar{\chi}(\mu)} \chi f(\chi \,|\, \bar{\kappa}) d\chi,$$

since the product is rejected whenever $\chi > \bar{\chi}(\mu)$ and accepted following a $s = g$ signal otherwise. Therefore,

$$\text{(A35)} \quad \gamma'(\mu) = \Big\{ v(G) \cdot \big[1 - \bar{\chi}(\mu)\big] \cdot f\big(\bar{\chi}(\mu) \,|\, \underline{\kappa}\big) - v(B) \cdot \bar{\chi}(\mu) \cdot f\big(\bar{\chi}(\mu) \,|\, \bar{\kappa}\big) \Big\} \cdot \frac{d\bar{\chi}(\mu)}{d\mu},$$

where $\bar{\chi}(\mu)$ is now given by (A17); thus, $\frac{d\bar{\chi}(\mu)}{d\mu} > 0$. Note that since $\bar{\chi}(\mu_2) = \hat{\chi}$, $\gamma(\mu)$ is continuous at μ_2. Furthermore, $\gamma'(\mu) \geq 0$ if and only if

$$\text{(A36)} \quad \bar{\chi}(\mu) \leq \frac{v(G)}{v(G) + v(B) \cdot \ell(\bar{\chi}(\mu))} \Leftrightarrow \frac{(1 - \mu) \cdot \omega}{\mu \cdot (1 - \omega)} \geq \frac{v(B)}{v(G)} \Leftrightarrow \mu \leq \mu_v,$$

with strict inequalities if and only if $\mu < \mu_v$. Since $\frac{(1 - \mu) \cdot \omega}{\mu \cdot (1 - \omega)}$ is decreasing in μ and equal to $\ell(1/2) < 1$ when $\mu = \tilde{\mu}$, it follows that $\mu_v < \tilde{\mu}$; thus, $\gamma'(\tilde{\mu}) < 0$.

Suppose that $\mu > \tilde{\mu}$. Now, the consumer is optimistic and $\bar{\chi}(\mu) \geq \hat{\chi}$ is given by (A20), with $\frac{d\bar{\chi}(\mu)}{d\mu} < 0$. Therefore, we have

$$\text{(A37)} \quad \gamma(\mu) = v(G) \cdot \left[\int_0^{\bar{\chi}(\mu)} (1-\chi) f(\chi \,|\, \underline{\kappa}) d\chi + 1 - F\big(\bar{\chi}(\mu) \,|\, \underline{\kappa}\big)\right] - v(B) \cdot \left[\int_0^{\bar{\chi}(\mu)} \chi f(\chi \,|\, \bar{\kappa}) d\chi + 1 - F\big(\bar{\chi}(\mu) \,|\, \bar{\kappa}\big)\right],$$

since the product is now accepted whenever $\chi > \bar{\chi}(\mu)$. Thus,

$$\text{(A38)} \quad \gamma'(\mu) = \Big\{v(B) \cdot \big[1 - \bar{\chi}(\mu)\big] \cdot f\big(\bar{\chi}(\mu) \,|\, \bar{\kappa}\big) - v(G) \cdot \bar{\chi}(\mu) \cdot f\big(\bar{\chi}(\mu) \,|\, \underline{\kappa}\big)\Big\} \cdot \frac{d\bar{\chi}}{d\mu} < 0,$$

where the inequality follows from the observation that $\ell\big(\bar{\chi}(\mu)\big) \geq \ell(\hat{\chi}) > 1$; thus, the term in brackets is always positive. Recall that $\tilde{\mu}$ is the threshold between the region where the consumer is pessimistic and the region where she is optimistic. Since $\bar{\chi}(\tilde{\mu}) = 1/2$, and $F\big(\bar{\chi}(\tilde{\mu}) \,|\, \kappa\big) = 1$ for $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$, it is easy to check that $\gamma(\mu)$ is continuous at $\tilde{\mu}$.

Case $\mu \in [\mu_4, 1]$.—Equilibrium must have $\sigma_G = \sigma_B = 1$, and it must be that the consumer is optimistic since $\mu_4 > \tilde{\mu}$. Here again, $\Gamma(\mu)$ is a singleton. Moreover, $\bar{\chi}(\mu)$ given by (A14), $\frac{d\bar{\chi}(\mu)}{d\mu} < 0$, and

$$\text{(A39)} \quad \gamma(\mu) = v(G) \cdot \left[\int_0^{\bar{\chi}(\mu)} (1-\chi) f(\chi \,|\, \bar{\kappa}) d\chi + 1 - F\big(\bar{\chi} \,|\, \bar{\kappa}\big)\right] - v(B) \cdot \left[\int_0^{\bar{\chi}(\mu)} \chi f(\chi \,|\, \bar{\kappa}) d\chi + 1 - F\big(\bar{\chi} \,|\, \bar{\kappa}\big)\right],$$

since the product is accepted whenever $\chi > \bar{\chi}(\mu)$. It follows that $\gamma(\mu)$ is decreasing in μ since

$$\text{(A40)} \quad \gamma'(\mu) = \big[v(B) \cdot (1-\bar{\chi}) - v(G) \cdot \bar{\chi}\big] \cdot f\big(\chi \,|\, \bar{\kappa}\big) \cdot \frac{d\bar{\chi}}{d\mu} < 0.$$

Finally, note that $\gamma(1) = v(G) - v(B) < 0$.

Case $\mu \in (\mu_3, \mu_4)$.—By Proposition 4, equilibrium must have either (i) $\sigma_G = 0$ and $\sigma_B = 1$, (ii) $\sigma_G \in (0,1)$ and $\sigma_B = 1$, or (iii) $\sigma_G = \sigma_B = 1$, and it must be that the consumer is optimistic, since $\mu_3 > \tilde{\mu}$. Thus, $\Gamma(\mu)$ consists of three elements, and we let $\gamma^j(\mu) \in \Gamma(\mu)$ for $j \in \{1,2,3\}$ denote the net expected payoff to the L type of choosing the G product when the equilibrium $\{\sigma_y\}$ is in region (i), (ii), and (iii), respectively.

We have already shown that the functions $\gamma^1(\mu)$ and $\gamma^3(\mu)$ are decreasing in μ. (See Case $\mu \in [\mu_2, \mu_3)$ when $\mu > \tilde{\mu}$ and Case $\mu \in [\mu_4, 1]$.) Let us consider $\gamma^2(\mu)$, which is given by

$$\text{(A41)} \qquad \gamma^2(\mu) = v(G) - v(G) \cdot \int_0^{\hat{\chi}} \chi f(\chi \mid \bar{\kappa}) d\chi - v(B) \cdot \int_0^{\hat{\chi}} \chi f(\chi \mid \bar{\kappa}) d\chi - v(B) \cdot [1 - F(\hat{\chi} \mid \bar{\kappa})],$$

and is thus constant on the interval (μ_3, μ_4). Furthermore, it is easy to check that $\lim_{\mu \downarrow \mu_3} \gamma^1(\mu) = \gamma(\mu_3)$ where $\gamma(\mu_3)$ is defined in Case $\mu \in (\mu_2, \mu_3]$, $\lim_{\mu \uparrow \mu_4} \gamma^1(\mu) = \lim_{\mu \uparrow \mu_4} \gamma^2(\mu) = \lim_{\mu \downarrow \mu_3} \gamma^2(\mu) = \lim_{\mu \downarrow \mu_3} \gamma^3(\mu)$, and $\lim_{\mu \uparrow \mu_4} \gamma^3(\mu) = \gamma(\mu_4)$ where $\gamma(\mu_4)$ is defined in Case $\mu \in [\mu_4, 1]$.

Therefore, we have shown that (i) $\Gamma(\mu)$ is a singleton for $\mu \in [0, \mu_3]$, with $\lim_{\mu \to 0} \gamma(\mu) = \gamma(0) = 0$ where $\gamma(\mu)$ is continuous, and increasing on $[0, \mu_v]$ but decreasing on $[\mu_v, \mu_3]$; (ii) $\Gamma(\mu)$ is a singleton, where $\gamma(\mu)$ is continuous and decreasing on $[\mu_4, 1]$, with $\gamma(1) < 1$; and finally, (iii) $\Gamma(\mu)$ has three elements on (μ_3, μ_4), where $\{\gamma^j(\mu)\}$ are continuous and (weakly) decreasing, with $\lim_{\mu \downarrow \mu_3} \gamma^1(\mu) = \gamma(\mu_3)$, $\lim_{\mu \uparrow \mu_4} \gamma^1(\mu) = \lim_{\mu \uparrow \mu_4} \gamma^2(\mu)$, $\lim_{\mu \downarrow \mu_3} \gamma^2(\mu) = \lim_{\mu \downarrow \mu_3} \gamma^3(\mu)$, $\lim_{\mu \uparrow \mu_4} \gamma^3(\mu) = \gamma(\mu_4)$, and where $\gamma^2(\mu)$ is constant (see Figure 5 for illustration). Hence, it must be that (generically) there is a unique μ on $(0,1)$ denoted by ψ such that $0 \in \Gamma(\psi)$. We conclude that there is (generically) a unique positive trade equilibrium. In it, the designer produces the *G* product with probability $\psi \in (0,1)$, and his complexification strategy $\{\sigma_y\}$ is given by Proposition 4, where the consumer's prior belief is $\mu = \psi$. ∎

PROOF OF PROPOSITION 6:

By inspection of the designer's net payoff γ from producing a *G* product relative to a *B* product, we see that μ and ω only affect it through their effect on threshold complexity $\bar{\chi}$ (see proof of Proposition 5), which determines whether the consumer's acceptance decision is contingent on the signal or not (see Lemma 1). As a result, any change in ω must be fully offset by a corresponding change in $\mu = \psi$ so as to keep the designer indifferent between producing a *G* versus a *B* product. It is easy to check that an increase in ω increases the thresholds $\mu_1 - \mu_4$. Therefore, in equilibrium, it must be that ψ increases in ω but the designer's complexification strategy $\{\sigma_y\}$ does not change. As a result, an increase in ω increases expected quality, and it decreases expected complexity since $\sigma_G \leq \sigma_B$. ∎

PROOF OF PROPOSITION 7:

An increase in $v(G)$ affects the equilibrium complexification strategy $\{\sigma_y\}$ only to the extent that it affects the consumer's equilibrium belief μ (see proof of Proposition 4 and note that the thresholds $\mu_1 - \mu_4$ are independent of $v(G)$). Now consider the designer's net payoff γ from producing a G product relative to producing a B product, as given by

$$\gamma = \max_{\kappa} \Pr\big(a = 1 \,|\, G, \kappa; \{\sigma_y\}, \mu\big) \cdot v(G) - \max_{\kappa} \Pr\big(a = 1 \,|\, B, \kappa; \{\sigma_y\}, \mu\big) \cdot v(B), \tag{A42}$$

where we now make explicit that the equilibrium probability of acceptance of a y product, $\Pr\big(a = 1 \,|\, y, \kappa; \{\sigma_y\}, \mu\big)$, will depend on the equilibrium belief μ and complexification strategy $\{\sigma_y\}$. By our previous argument, for a given belief μ, $\max_\kappa \Pr\big(a = 1 \,|\, y, \kappa; \{\sigma_y\}, \mu\big)$ is independent of $v(G)$; therefore, γ must be increasing in $v(G)$. Thus, if equilibrium features $\mu = \psi \notin [\mu_3, \mu_4]$, it must be that ψ increases with $v(G)$. (See proofs of Proposition 5 and Figure 5.) The same holds if $\psi \in [\mu_3, \mu_4]$ and the change in $v(G)$ is large enough that the new equilibrium ψ is greater than μ_4. In particular, it is easy to check that $\mu = \psi$ goes to one as $v(G)$ goes to $v(B)$; however, when $\psi \in (\mu_3, \mu_4)$ it is possible that an increase in $v(G)$ implies that ψ falls as the equilibrium jumps from separation on κ to pooling at $\kappa = \bar{\kappa}$ (see Figure 7). From Proposition 4, since the thresholds $\mu_1 - \mu_4$ are unchanged, as $\mu = \psi$ increases, $\{\sigma_y\}$ increase. As a result, a large-enough change in $v(G)$ increases both expected quality and expected complexity—although locally, the effect may be nonmonotonic. ∎

PROOF OF PROPOSITION 8:

Note that the aligned designer's net benefit from choosing the G product is

$$\gamma = \max_{\kappa} \Pr\big(a = 1 \,|\, G, \kappa\big) \cdot \bar{v}(G) - \max_{\kappa} \Pr\big(a = 1 \,|\, B, \kappa\big) \cdot \bar{v}(B). \tag{A43}$$

Since the probability of acceptance is always higher for a G product—i.e., $\Pr\big(a = 1 \,|\, G, \kappa\big) \geq \Pr\big(a = 1 \,|\, B, \kappa\big) > 0$ for all κ and $\bar{v}(G) > \bar{v}(B)$—the aligned designer produces a G product with probability one. In turn, this designer's complexification strategy is given by σ_G, as characterized in Proposition 2. Thus, the presence of an aligned designer will affect the equilibrium outcomes only by affecting the probability of a G product being produced, captured by the fact that belief consistency now requires that $\mu = q + (1 - q) \cdot m$.

Consider the case of $q > \psi$. First, note that in equilibrium $\mu \geq q$, since $m \geq 0$. Second, note that it must be that $m = 0$, since the misaligned designer's net payoff from producing a G product relative to a B product is always negative for $\mu > \psi$ (see proof of Proposition 5); thus, we have that in equilibrium $\mu = q \geq \psi$; thus, expected quality is higher in the presence of an aligned designer. It follows from Proposition 4 that $\{\sigma_y\}$ are higher as well, since the presence of an aligned designer simply increases μ.

Next, consider the case of $q \leq \psi$. We now show that the equilibrium μ and $\{\sigma_y\}$ need not change in the presence of an aligned designer; their presence is simply offset by the misaligned designer producing a G product with smaller probability. If $q < \mu_3$, then the misaligned designer's payoff from producing a

G product is strictly positive if equilibrium had $\mu = q$ (see proof of Proposition 5 and Figure 5), which is inconsistent with an equilibrium; thus, it must be that the misaligned designer produces a G product with positive probability $m = (\psi - q)/(1 - q)$ so that the equilibrium belief is $\mu = \psi$ and he is indifferent to producing a G versus a B product. Now suppose that $q \in [\mu_3, \psi]$. If $\min\{\gamma : \gamma \in \Gamma(\mu_3)\} > 0$, then the equilibrium is as the one described above, since the misaligned designer's net payoff from producing a G product is still strictly positive if equilibrium had $\mu = q$, which cannot be consistent with equilibrium. If instead $\min\{\gamma : \gamma \in \Gamma(\mu_3)\} \leq 0$, then multiple equilibria exist. In particular, the equilibrium where the misaligned designer produces a G product with probability $m = (\psi - q)/(1 - q)$ (so that $\mu = \psi$) still exists, since $0 \in \Gamma(\psi)$. However, there is also an equilibrium where $\mu = q \in [\mu_3, \psi]$, since there exists a $\gamma < 0$ such that $\gamma \in \Gamma(q)$. ■

PROOF OF PROPOSITION 9:

For each $U \in [0, w(G)]$, consider map $T_\beta : U \mapsto \mathbb{R}$ defined by

$$T_\beta(U) = E\Big[\max_{a \in \{0,1\}}\Big\{a \cdot \big[\mu(s,\chi) \cdot w(G) + \big(1 - \mu(s,\chi)\big) \cdot w(B)\big] + (1 - a) \cdot \beta U\Big\}\Big], \tag{A44}$$

where, recall, $\mu(s,\chi)$ is the consumer's equilibrium belief that the proposed product has quality G, given signal s and complexity χ. For an exogenously given value of U, which pins down the consumer's outside option $w_0 = \beta U$, this map gives us the consumer's ex ante value $T_\beta(U)$. An equilibrium is a fixed point of this map, and we denote it by U^*. Clearly, $w(B) < T_\beta(U) \leq w(G)$, which implies that in equilibrium $w_0 = \beta U^* \in \big(w(B), w(G)\big)$, satisfying Assumption 1. As in our baseline model, we focus on positive trade equilibria, in which good products are produced with positive probability. And, for the same reason as in the baseline model, bad products must be produced with positive probability. Therefore, to show that an equilibrium exists, it suffices to show that $T_\beta(\cdot)$ is increasing. But note that an increase in the outside option increases the consumer's ex ante welfare directly and indirectly through its effects on equilibrium μ and $\{\sigma_y\}$. The latter follows from Proposition 6, where we have shown that $\mu = \psi$ increases in the outside option, whereas $\{\sigma_y\}$ are independent of it.

For comparative statics, note that for a given U an increase from β to some β' is equivalent to an increase in the consumer's outside option. Thus, it must be that $T_\beta(U) < T_{\beta'}(U)$. The fixed point must therefore be higher at β' than at β, since $T_\beta(\cdot)$ is increasing. If there are multiple fixed points, then the statement holds locally for the maximal one. ■

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