



# How stimulative are low real interest rates for intangible capital?☆

Andrea Caggese<sup>a,b,c</sup>, Ander Pérez-Orive<sup>d,\*</sup>

<sup>a</sup> Universitat Pompeu Fabra, Spain

<sup>b</sup> Centre de Recerca en Economia Internacional (CREI), Spain

<sup>c</sup> Barcelona School of Economics, Spain

<sup>d</sup> Federal Reserve Board, United States of America

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## ABSTRACT

We study theoretically and empirically how the increase in the intensity of intangible capital in production in recent decades affects the sensitivity of investment to interest rates. In contrast to tangible capital, intangible capital has low collateral value and must be financed in a larger part with accumulated corporate savings, especially in more financially constrained firms and for more lumpy investments. High-intangibles firms are, therefore, more likely to be net savers in equilibrium, and for such firms low interest rates are not as stimulative as for high-tangibles firms. We show in a realistically calibrated model that the rise of intangible capital substantially dampens the positive effects of low interest rates on investment because of this mechanism, increasing the misallocation of resources. We find strong empirical support for this effect by studying the investment decisions of U.S. firms.

## 1. Introduction

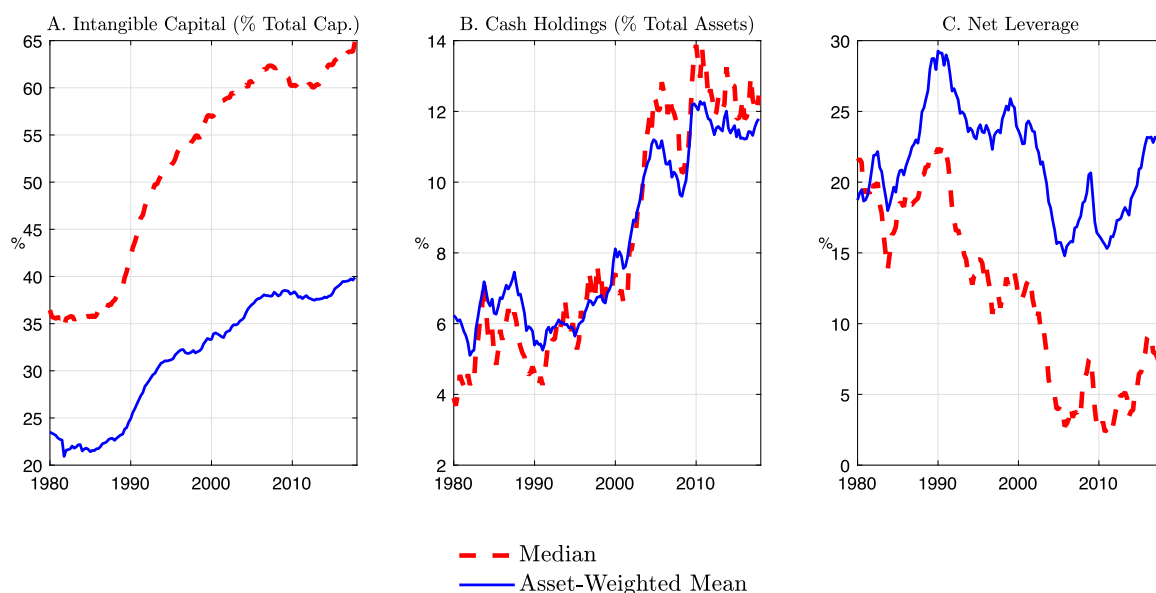
In the last 40 years, industrialized countries have experienced a sharp rise in the importance of intangible capital—such as information technology or knowledge, human, brand, or organizational capital—in production and a gradual reduction in the reliance on physical capital (see panel A in Fig. 1). In parallel, there has been a remarkable rise in corporate cash holdings, a phenomenon sometimes referred to as the “corporate savings glut” (see panel B in Fig. 1). Several recent papers argue that the increase in cash holdings is driven in large part by the rise in intangibles. Intangible capital is difficult to finance with debt because of its low collateral value, and firms planning for large intangible investments need to accumulate cash to invest.<sup>1</sup> Over this same

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\* Corresponding author.

E-mail address: [ander.perez-orive@frb.gov](mailto:ander.perez-orive@frb.gov) (A. Pérez-Orive).

<sup>1</sup> E.g., see Falato et al. (2020).



**Fig. 1.** Evolution of intangible intensity, cash holdings, and net leverage in U.S. non-financial listed firms. The sample used to compute these series consists of U.S. publicly listed firms excluding utilities and financials. Intangible capital is the sum of knowledge capital and organizational capital, measured by capitalizing, respectively, R&D expenses and a fraction of selling, general, and administrative (SG&A) expenses. Cash holdings are computed as cash and marketable securities in firms' balance sheets. Net leverage is equal to the ratio of total debt minus cash holdings to total book assets.   
 Source: Compustat.

period, firms' net leverage (debt minus cash over assets) has dropped significantly, to the point where toward the end of the 2010s debt of public firms was roughly equal to cash for the median firm (see panel C in Fig. 1). In this paper, we show, theoretically and empirically, that low interest rates are significantly less stimulative for intangible investments because of the nature of intangible capital financing, with important consequences for aggregate capital misallocation.

We start by exploring if the response of investment to changes in interest rates of tangible and intangible firms is different. We find that while the investment of tangible firms is negatively affected by increases in interest rates, as most conventional theories would predict, the investment of intangible firms responds positively to increases in interest rates. We then introduce a simple theoretical framework that can capture our ideas. In this stylized model, firms finance one-time fixed-size investment projects using both debt—the amount of which is limited by the collateral value of their assets—and retained earnings. Lower interest rates stimulate debt-financed investments by loosening borrowing constraints and allowing firms to borrow more per unit of investment, and we call this effect the “collateral channel”. However, lower interest rates *dampen* cash-financed investments by reducing the return on retained earnings and delaying firms' ability to accumulate sufficient savings to invest, and we call this effect the “savings channel”. Because tangible assets have high collateral value and can be financed mostly with debt, lower interest rates make tangibles financing easier; the collateral channel dominates the savings channel for tangible investments. However, intangible assets, which have a low collateral value, must be financed to a much larger extent with accumulated savings. In this case, lower interest rates are less expansionary for intangible investments, and could even be contractionary, because the savings channel dominates the collateral channel.

In the rest of the paper, we extend the stylized model described above—by relaxing many simplifying assumptions—to explore the robustness of our mechanism and to analyze the macroeconomic implications of the savings channel, particularly for the misallocation of capital. More specifically, we allow both financially constrained and unconstrained firms to coexist and receive stochastic investment opportunities that happen several times during their lifetime. Unconstrained firms have access to frictionless equity financing and are, therefore, unaffected by both the collateral and savings channels. Importantly, we calibrate the model to target the observed fraction of aggregate output produced by constrained firms and the observed average intensity of these constraints. We find that a reduction in the interest rate reduces financially constrained firms' ability to invest, because of the savings channel, and this effect is much stronger for intangibles than for tangibles constrained firms. However, this effect is compensated for by the expansion of unconstrained firms, for which a lower interest rate reduces the user cost of capital. Overall, in our most realistic calibration, aggregate capital always responds negatively to an increase in interest rates when the average intangibility of firms' assets is low, while such a response can be close to zero, or even positive, when the average intangibility of firms' assets is high, which is consistent with the empirical evidence. Furthermore, even though aggregate capital and output increase when interest rates decline, the expansion of unconstrained firms and the contraction of constrained ones worsen the allocation of resources, especially in an economy with high intensity of intangible capital, yielding a second testable prediction: misallocation—measured by the cross-sectional dispersion of the marginal product of capital—is larger the higher is the intangible capital share in the economy, and

more so the lower are interest rates. In the last part of the paper, we provide empirical evidence consistent with this prediction, as well as with our other main findings. In particular, we show that the lower sensitivity of intangible firms' investment to the interest rate, relative to tangible firms, is driven by those that are more likely to be financially constrained, which confirms the model's main mechanism.

Finally, the misallocation result described above also implies that lower rates have a much weaker stimulative effect on aggregate output in an economy with a higher intensity of intangible capital. Our most realistic calibration implies that, in an environment with very low interest rates, a 1% permanent fall in the interest rate in an economy with high-tangibles firms increases aggregate capital in the long run by 10.3%, and aggregate output by 2.5%. In an economy with high-intangibles firms, instead, aggregate capital increases 5% in response to the same drop in the interest rate, and output as little as 0.5%.

Overall, this paper provides new evidence that shows that the rise of intangible capital, because of the different nature of its financing, significantly alters the dynamic relationship between interest rates, aggregate investment, and the allocation of resources across productive units. This finding has important implications; in the short run, for the transmission of monetary policy to real economic activity and, in the medium and long run, for a new channel through which persistently low real interest rates can increase capital misallocation.

## Related literature

This paper is related to several strands of literature. The main motivation of our analysis is the widely documented technological change toward intangible capital (e.g., [Corrado and Hulten, 2010](#); [Corrado et al., 2012](#); [Haskel and Westlake, 2017](#); [Falato et al., 2020](#) (henceforth FKSS)). In our model, the rise of intangible capital causes a rise in corporate cash holdings and a shift in the corporate sector from a net borrower position to a net saver position. Such a shift has been documented extensively ([Armenter and Hnatkovska, 2017](#); [Quadrini, 2017](#); [Chen et al., 2017](#); [Shourideh and Zetlin-Jones, 2017](#)).<sup>2</sup>

Our finding that declining interest rates can worsen the optimal allocation of resources in intangible economies is related to [Gopinath et al. \(2017\)](#), who analyze capital misallocation in Spain during a period of falling interest rates, and, more broadly, to the literature that has documented a decline in aggregate productivity after 2000. [Fernald \(2015\)](#) and [Kahn and Rich \(2007, 2013\)](#) estimate that growth in labor productivity and total factor productivity (TFP) in the U.S. switched from a high-growth to a low-growth regime from around 2003–2004. [Cette et al. \(2016\)](#) report that Europe experienced a similar pre-crisis pattern.

Other recent papers have linked the rise of intangible capital to the productivity slowdown. [Liu et al. \(2019\)](#), [De Ridder \(2019\)](#), and [Aghion et al. \(2019\)](#) argue that intangible technologies give incumbent and highly productive firms a competitive advantage by reducing their expansion costs. This factor deters competition, creative destruction, and aggregate productivity growth. [Chiavari and Goraya \(2020\)](#) document that intangible capital features larger fixed adjustment costs than tangible capital and entails larger entry costs that reduce competition and increase concentration.

Our paper is also closely related to the research that attributes the rise in cash holdings to increasing firm-level precautionary saving motives driven by financial imperfections (e.g., see FKSS and [Begenau and Palazzo, 2021](#)).<sup>3</sup> Conversely, [Döttling et al. \(2019\)](#) argue that intangible capital requires less external finance than tangible capital because it is partly financed with deferred employee compensation (mostly in the form of stock options). In their framework, intangible firms still need to accumulate substantial cash holdings to insure the equity claims of workers. Other suggested motives for the high cash holdings in high-intangibles firms have to do with innovation in competitive markets ([Lyandres and Palazzo, 2016](#)). While in our theory we focus on corporate savings driven by financial frictions, our main results about the interaction between the rise of intangibles and the interest rate sensitivity of corporate investment only rely on the savings channel being important for firms' investment decisions, regardless of the specific factors that drive high-intangibles firms to hold large amounts of liquid assets.

Finally, our paper is related to the empirical literature that estimates the effect of monetary policy shocks on the investment of firms using micro data (e.g., [Ippolito et al., 2018](#); [Jeenas, 2019](#); [Ottoneo and Winberry, 2020](#); [Cloyne et al., 2020](#)). Among these, a very recent empirical paper by [Döttling and Ratnovski \(2020\)](#) also studies the effect of monetary policy on the investment of tangible and intangible firms and finds results in line with our empirical evidence and our theoretical results.

The rest of the paper is organized as follows. Section 2 describes the empirical evidence that motivates our work. Section 3 proposes a stylized model to illustrate our main mechanism. Section 4 develops and simulates the main model and tests its predictions. Finally, Section 5 concludes.

## 2. Motivating empirical evidence

In this section, we present a set of empirical facts that motivate our model. First, we show that the well-documented rising importance of intangible capital over time has occurred in parallel to a rise in cash holdings and a decline in net leverage. Second, we document that the investment response to an increase in interest rates of firms with a high tangibility of assets is significantly more contractionary than for firms whose assets are mostly intangible.

<sup>2</sup> Our paper is also related to the literature on the causes and consequences of falling real interest rates. [Gagnon et al. \(2021\)](#) and [Eggertsson et al. \(2019\)](#) perform a quantitative theoretical analysis based on realistic demographic changes in the U.S. in recent decades, and both conclude that demographic factors—in particular, increased life expectancy and decreased fertility rates—can account for an important share of the real interest rate fall. Similar arguments have also been made by [Baldwin and Teulings \(2014\)](#), [Rachel and Smith \(2015\)](#), and [Bean \(2016\)](#). In this paper, we abstract from the factors driving the decline in the interest rate, and therefore our analysis is robust to these different drivers.

<sup>3</sup> Recent papers investigating the relationship between financial frictions, interest rates, cash holdings, and investment and employment decisions of firms are [Bacchetta and Benhima \(2015\)](#), [Bacchetta et al. \(2019, 2020\)](#), and [Asriyan et al. \(2021\)](#).

## 2.1. Data

Our sample consists of U.S. firms covered by Compustat at a quarterly frequency between 1980 and 2018, excluding utilities (Standard Industry Classification (SIC) codes 4900–4949) and financials (SIC codes 6000–6999). We remove observations with negative revenues, missing information on total assets, or a value of total assets under \$25 million in 2014 U.S. dollar value. We winsorize all variables at the 1% level to remove outliers.

In contrast to tangible capital, intangible capital is not captured accurately in firms' balance sheets. Our measurement of intangible capital follows FKSS and [Peters and Taylor \(2017\)](#), who define intangible capital as the sum of knowledge capital and organizational capital.<sup>4</sup> We measure the former by capitalizing research and development (R&D) expenses and the latter by capitalizing selling, general, and administrative (SG&A) expenses weighted by a factor of 0.3.<sup>5</sup> The expenditures are capitalized by applying the perpetual inventory method with an annual depreciation rate of 15% for R&D and 20% for SG&A. Our measure of tangible capital is gross property, plant, and equipment. For robustness and consistency, we check that our results hold when using an alternative measure of tangible capital built in a similar way as our measure of intangible capital; by capitalizing capital expenditures, using an annual depreciation rate of 10%. Our results also hold when using the alternative measure proposed by [Peters and Taylor \(2017\)](#) that adds intangible assets booked in firms' balance sheets (Goodwill and Other Intangible Assets) to the measure described above and that is made public on their website. Our measure of investment over a horizon  $h$  is equal to the difference between the log of total capital (intangible plus tangible) in period  $t + h$  and in period  $t - 1$ . Our results are robust to using the investment rate—defined as investment over the stock of capital last period—as our measure of investment.

Other firm characteristics are defined in ways that are standard in the literature. Firm age is taken from two sources. Using Worldscope, we compute firm age as the age since foundation, unless the foundation year is missing, in which case the date of incorporation is taken into account. Next, we improve this data by using the information of firm foundation year from [Loughran and Ritter \(2004\)](#) (LR), which was updated in 2018 and provides the original incorporation date for most initial public offerings since 1975. Their coverage is smaller than Worldscope, but they conduct a careful data construction process that slightly improves the accuracy of Worldscope. Whenever a firm is covered both by Worldscope and by LR, we take the firm age according to LR. For those firms not covered by LR, we take the value from Worldscope.<sup>6</sup> We use alternative cutoffs for firms to be considered “young” or “old”. Our benchmark cutoffs are 10 and 30 years, respectively. Our results are robust to considering cutoffs of 5 or 15 years for young firms, and of 20 or 40 years for old firms. Cash holdings are computed as cash and marketable securities on firms' balance sheets. Net leverage is equal to the ratio of total debt minus cash holdings to total book assets. Firm size is proxied by firms' total assets, and real sales growth is measured as the log difference in annual sales of two consecutive years.

Finally, to capture changes in interest rates, we use the monetary policy shocks identified by [Jarocinski and Karadi \(2020\)](#). These authors follow a well-established literature that uses high-frequency financial market surprises around key monetary policy announcements to identify unexpected variations in monetary policy. The innovative aspect of [Jarocinski and Karadi's \(2020\)](#) approach is that they are able to separately identify exogenous monetary policy shocks from shocks about new information from the Federal Reserve regarding the state of the economy. These monetary policy shocks are therefore orthogonal to shocks to firms' investment opportunities. We compute the monetary surprise by adding up the monthly monetary policy shocks at the quarterly frequency.

## 2.2. Intangible capital and cash holdings

[Fig. 1](#) extends until 2018 the evidence shown by FKSS up to 2010. It shows the trends for intangible capital, cash holdings, and net leverage for the 1980–2018 period. The first panel confirms a positive trend of intangible capital as a share of total capital. A brief reversal, between 2007 and 2012, is followed by a further increase from 2012 onward. The second panel documents the increase in cash holdings as a percentage of total assets during the same period. Finally, the third panel shows net leverage, defined as total debt minus cash and short-term investments over total assets. Median net leverage decreases over time and reaches values only slightly above zero after 2005.<sup>7</sup> Our premise is that financial factors are important determinants of these parallel trends. Because intangible capital is less collateralizable than tangible capital, high-intangibles firms need to accumulate more cash to finance investment projects than high-tangibles firms.<sup>8</sup>

<sup>4</sup> FKSS also consider informational capital. However, they state that their results do not depend on its inclusion. As informational capital can be measured only at the industry level but not at the firm level using Compustat data, we choose not to include this type of capital.

<sup>5</sup> A portion of SG&A expenses captures expenditures that increase the value of intangible capital items such as brand names and knowledge capital.

<sup>6</sup> For the majority of firms, Worldscope and LR provide the same firm age. Our results are robust to only considering the age data from LR.

<sup>7</sup> The asset-weighted mean of net leverage, in contrast, does not have a clear downward sloping trend, especially because of an increase during the last 10 years, likely driven by a well-documented increase in borrowing from the larger firms in the sample in an environment of low corporate bond yields.

<sup>8</sup> Another complementary reason access to debt is limited for intangibles firms is that some intangible investments, such as R&D, face a high operating leverage because of fixed costs together with technological uncertainty, a point made in [Dockner and Siyahhan \(2015\)](#). The resulting high risk makes expected bankruptcy costs overly sensitive to additional financial leverage. All of our results extend to this reason, or any other, why intangible firms have limited access to debt financing.

### 2.3. Intangible capital and the interest rate sensitivity of investment

In this section, we provide evidence on how exogenous changes in interest rates, identified with surprise changes in the monetary policy rate, have a different impact on the investment of intangible and tangible firms. We do so by estimating the path of the cumulative growth rate of the firm-level stock of real capital using the following Jordà (2005) local projection specification as our baseline estimation framework:

$$\Delta_h \log K_{i,t+h} = \gamma_{st}^h + \sum_{g=1}^3 \left( \rho_g^h + \lambda_g^h \Delta r_t \right) I [Intan_{i,t-1} \in g] + (\alpha^h + \eta^h \Delta r_t) \text{controls}_{i,t-1} + \epsilon_{i,t+h}, \quad (1)$$

where  $\Delta_h \log K_{i,t+h}$  is the change in the log of the real stock of capital  $K$  between the end of quarter  $t-1$  and the end of quarter  $t+h$  and  $\Delta r_t$  is the monetary surprise in quarter  $t$ . We classify firms every quarter into groups of intangibility indicated by  $g$ ; we choose three groups, capturing, respectively, the bottom, middle, and top terciles of the distribution of intangible intensity (intangible capital over total capital) over the whole sample period.  $I$  is an indicator function that takes value 1 if firm  $i$  is in group  $g$  in the period preceding the monetary policy shock. Our coefficient of interest is  $\lambda_g^h$ , which gives us the interest rate sensitivity of investment for the three groups of firms. More precisely,  $\lambda_g^h$  measures the net cumulative response of investment between quarter  $t$  and quarter  $t+h$  to a monetary policy shock in quarter  $t$ . In what follows, our results show the average response of investment of the firms in the bottom tercile of intangibility (“tangible firms”) and of the firms in the top tercile (“intangible firms”), as well as the difference between both. These responses correspond, respectively, to  $\lambda_{g=\text{tangible}}^h$ ,  $\lambda_{g=\text{intangible}}^h$ , and  $\left( \lambda_{g=\text{intangible}}^h - \lambda_{g=\text{tangible}}^h \right)$ . We study the response of investment up to a horizon of  $h = 20$  quarters. Sector-time fixed effects are captured by  $\gamma_{st}^h$ .<sup>9</sup> The vector  $\text{controls}_{i,t}$  contains the time-varying demeaned firm-level controls, which are introduced independently and also interacted with  $\Delta r_t$ . Throughout, we cluster standard errors at the firm level to account for correlation within firms.<sup>10</sup> Firms in the sample are required to be active for at least five years after the monetary policy shock occurs, to cover the length of the horizon of the effects we study and ensure that effects are not driven by firm samples being different at short and long horizons.

The top three panels of Fig. 2 display the results of running specification (1) without firm-level controls. Panel A contains the response of tangible firms to a 100 basis point surprise increase in interest rates (the estimates of coefficient  $\lambda_{g=\text{tangible}}^h$ ) and shows that these firms feature a significantly negative relationship between investment and interest rate shocks, as most conventional theories would suggest. In contrast, intangible firms (panel B displays the estimate of  $\lambda_{g=\text{intangible}}^h$ ) have a positive response to increases in interest rates. The difference between the response of both types of firms (panel C) is economically and statistically significant; after 10 quarters, a 1% increase in interest rates is associated, on average, with a cumulative drop in investment relative to the initial capital stock for tangible firms that is about 20% larger than for intangible firms.

Firm characteristics correlated with intangible capital intensity, if not properly controlled for, could bias our estimates and inference. We use several different specifications to reduce these concerns. As mentioned above, we control for sector-year fixed effects, which absorb important confounding factors that are likely to have an important sectorial component, such as the interest rate sensitivity of demand. Moreover, in the three bottom panels of Fig. 2, we control for observable firm characteristics that are exogenous to our model and mechanism but that are likely to be correlated both with intangibility and with the sensitivity of investment to interest rate changes. In particular, we include as additional firm-level control variables firm age, size, and investment opportunities (proxied by recent sales growth, as is common in the literature), both independently and interacted with the interest rate shock.<sup>11</sup> In this case, the investment sensitivity of tangible firms (panel D) is still strongly negative, while the sensitivity of intangible firms (panel E) comes very close to zero and is not statistically significant. Panel F, which displays the estimate of the difference between the response of both types of firms, highlights again our main finding: the capital investment of tangible firms is significantly more sensitive to an interest rate shock than that of intangible firms.

Appendix B contains additional robustness tests to address endogeneity concerns. Our results are robust to the inclusion of firm fixed effects (panel B in Fig. A.1), which absorb all time-invariant firm characteristics. Our results remain strong when we select firms into tangibles and intangibles groups based on intangibility lagged up to 3 years (unreported). Finally, our findings remain strong when introducing sector-quarter fixed effects instead of sector-year fixed effects (panel C in Fig. A.1) and when using an alternative, more granular, definition of sectors (panel D in Fig. A.1).

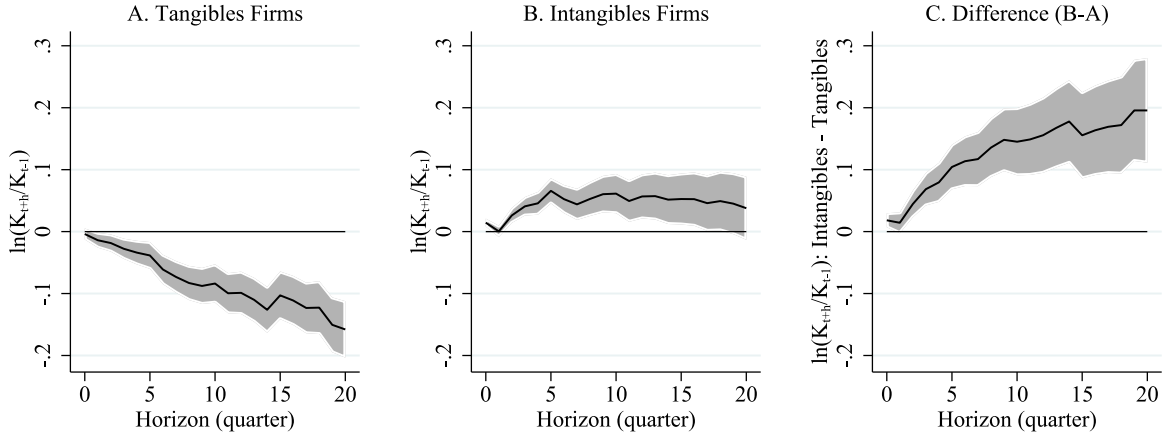
Appendix B also contains other more general robustness tests. We use an alternative measure of intangible capital from Peters and Taylor (2017), described above and shown in panel D of Fig. A.2; measure investment using the investment rate (investment flow over the stock of capital, shown in panel C in Fig. A.2); add lags of the dependent variable as regressors (panel A in Fig. A.2); cluster two ways at the firm and time levels (panel A in Fig. A.1); and use a different definition of the interest rate shock from Gertler and Karadi (2015) that does not control for the central bank information effect (panel B in Fig. A.2). Our results remain strong through all these robustness checks.

<sup>9</sup> In our benchmark specification, we group sectors into one of the following: Consumer Nondurables; Consumer Durables; Manufacturing; Energy, Oil, and Gas; Chemicals; Business Equipment; Telecommunications; Wholesale; Retail; Healthcare; and Other. In the Appendix we show that our results are very similar if we, instead, divide sectors by 2-digit SIC code.

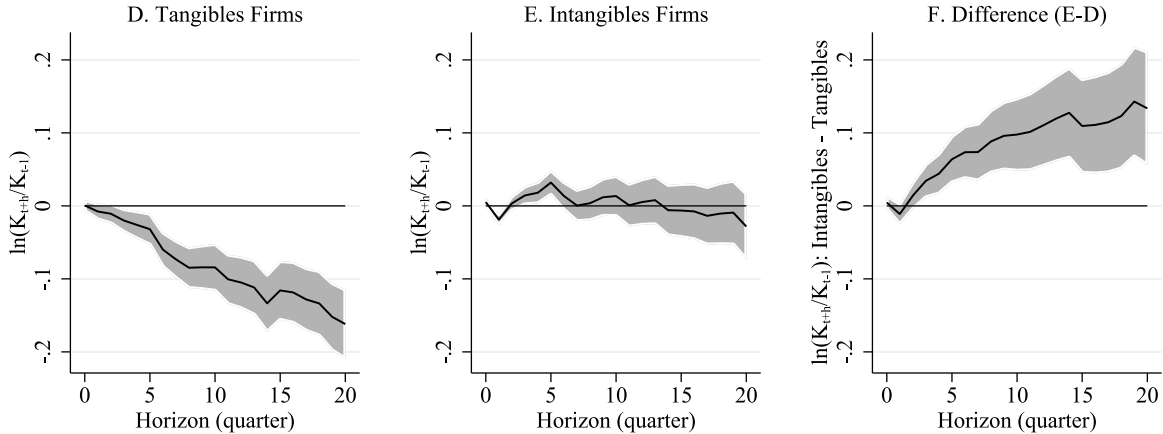
<sup>10</sup> The statistical significance of our estimates does not change meaningfully if we cluster standard errors two ways to account for correlation within firms and within quarters, as we show later.

<sup>11</sup> We do not control for other firm characteristics that are endogenous to our mechanism that links intangibility to capital structure and financing constraints, such as firm leverage, cash holdings, or measures of financial constraints.

## I. Sector-Year Fixed Effects and Error Clustering



## II. Sector-Year Fixed Effects, Error Clustering, and Controls



**Fig. 2.** Local projections: Response of investment to monetary policy shocks. The left and middle panels display the impulse response function (IRF) for investment following a 100 bps surprise increase in the interest rate that results from running the local projection specification (1). The right panels display the difference between the middle and left panels; this difference is the estimate of the effect of intangibility (top tercile of intangibility compared to the bottom tercile) on the dynamic response of investment to an increase in the interest rate. The dependent variable is the difference between the log of total capital (tangible plus intangible) in period  $t + h$  and in period  $t - 1$ . We consider tangibles (intangibles) firms to be those with an intangible intensity in the bottom (top) tercile over the whole sample period. The monetary surprise in quarter  $t$ ,  $\Delta r_t$ , is calculated by adding up the monthly monetary policy shocks obtained from [Jarocinski and Karadi \(2020\)](#). We also introduce industry-year fixed effects and clustering of standard errors at the firm level. Firms in the sample are required to be active for at least five years after the monetary policy shock occurs. The bottom three panels control for firm age, size, and lagged sales growth, both independently and interacted with the interest rate shock. Shaded areas represent the 90% confidence intervals of the estimates.

## 3. Simple and intuitive explanation of the mechanisms

In Section 4, we will illustrate a general equilibrium model that can jointly replicate the empirical evidence shown before, and that also generates additional predictions we verify in the data. However, before introducing such a framework, the aim of this section is to develop the simplest possible model that can describe our proposed mechanisms and deliver analytical results that illustrate how capital tangibility affects the response of investment to changes in interest rates. To this end, in this section we introduce a series of simplifying assumptions that will later be relaxed in Section 4.

A key element in our mechanism is the presence of large occasional investments at the firm level, which interact with financial frictions to drive firms to retain earnings and adopt a net saver position. Empirically, it is well known that individual investment



is lumpy, because of the presence of non-convex adjustment costs, arising, for example, from fixed costs of investment.<sup>12</sup> Examples of these costs are those required to develop a new production plant, to introduce a new product, or to expand into new export markets. Other examples of lumpy investments are those generated by opportunities to innovate, or to merge with or acquire another company. Our results hold regardless of the nature of the investment and apply to all investment projects that are infrequent, that are large relative to the size of the firm, and that cannot be financed mostly with external funds and need, as a result, to rely on internal finance.

### 3.1. A simple model

We consider a partial equilibrium model of an industry with a large number  $N$  of firms. Firms produce consumption goods using a nondurable factor as the only input and generate a constant net cash flow  $\pi$  at the beginning of every period. They do not distribute their earnings (an optimal choice, as will be clear later) and, instead, retain them and accumulate wealth by saving through a one-period safe financial asset  $a_t$  that generates a constant return  $1 + r$  per period, where  $r$  is the exogenous real interest rate. Firms' law of motion of wealth is thus:

$$a_{t+1} = n_t \equiv (1 + r)a_t + \pi, \quad (2)$$

where  $n_t$  is the accumulated firm wealth in period  $t$ , after the period  $t$  cash flow  $\pi$  has been realized. Firms exit with exogenous probability  $\Psi$  in any given period, and exiting firms are replaced by newborn ones with no initial endowment ( $a_0 = 0$ ).

Investment lumpiness is introduced by assuming that firms have access to a one-time investment opportunity of fixed size  $F$ . This investment is profitable, so that firms will always implement it when they have enough financial resources to do so. From now on, we identify  $F$  with an "innovation investment", for expositional convenience. A firm investing in period  $t$  can borrow ( $a_{t+1} < 0$ ) at the same constant rate  $r$ .<sup>13</sup> If the firm does not repay the debt, however, the lenders can liquidate the firm and recover a value  $\theta F$ . Therefore,  $\theta$  represents the collateral value of the investment  $F$ , and the firm faces the constraint that it can only borrow up to the present value of collateral  $\theta F$ :

$$-a_{t+1} \leq \frac{\theta}{1+r} F. \quad (3)$$

The innovation investment is feasible at the beginning of period  $t$  if current wealth  $n_t$  plus new borrowing  $-a_{t+1}$  are sufficient to finance  $F$ :

$$n_t - a_{t+1} \geq F. \quad (4)$$

We assume that  $\frac{\theta}{1+r} < 1$ , so that, even borrowing up to the limit in period 0 (i.e., borrowing  $-a_0 = \frac{\theta}{1+r} F$ ), condition (4) is not satisfied. In other words, newborn firms cannot access enough external funds to innovate, and they need to accumulate retained earnings for several periods (denoted as  $T^*$ ) to innovate. As conjectured earlier, firms choose not to pay dividends while this constraint is binding, and optimally save to finance the innovation investment, because the return to funds kept internally is larger than the return in the hands of lenders. Eq. (2) implies:

$$n_T = \pi [1 + (1+r) + \dots + (1+r)^{T-1}] = \pi \left[ \frac{(1+r)^T - 1}{r} \right]. \quad (5)$$

Substituting the binding borrowing constraint (3) into condition (4), we get  $n^*$ , the minimum wealth required to invest in  $F$ :

$$n^* = F \left[ 1 - \frac{\theta}{1+r} \right]. \quad (6)$$

Substituting  $n_T = n^*$  into (5) and rearranging gives us an explicit solution for  $T^*$ <sup>14</sup>:

$$T^* = \left\lceil \frac{\ln \left( 1 + \frac{rn^*}{\pi} \right)}{\ln(1+r)} \right\rceil. \quad (7)$$

Every period,  $N\delta$  new firms enter the industry, and the fraction of these surviving until age  $T^*$  is equal to  $(1 - \delta)^{T^*}$ . Therefore, each period, aggregate investment  $I^F$  is equal to:

$$I^F = N\delta (1 - \delta)^{T^*} F. \quad (8)$$

<sup>12</sup> Evidence of lumpy investment at the microeconomic level can be found in Doms and Dunne (1998), Caballero (1999), and Gourio and Kashyap (2007). While these papers focus on tangible investments, Kaus et al. (2020) and Chiavari and Goraya (2020) show that some types of intangible investment (R&D, software, and patents) are even lumpier than tangible investment.

<sup>13</sup> An alternative assumption could have been that borrowing and saving rates are different, which might be particularly relevant when deposit rates are constrained by the zero lower bound. On the one hand, studies show that in countries with negative nominal interest rates, often these negative rates are also passed to corporate depositors (e.g., Altavilla et al., 2021). On the other hand, if the zero lower bound was binding on deposit rates, then a fall in the borrowing rate would help tangible firms thanks to the collateral channel, while it would not penalize the intangible firms because the saving channel would not be affected. Therefore, we would still observe a differential response of tangible versus intangible firms, even though such a difference would be smaller than when the zero lower bound is not binding.

<sup>14</sup> This computation assumes that the innovation opportunity is taken up after the current period's cash flow  $\pi$  from regular operations is produced. Note that, to be more precise, the exact number of periods is the value of  $T^*$  rounded up to the nearest integer.

### 3.2. Relationship between investment, the interest rate, and intangibility

Having introduced this simple framework, we now turn to our questions of interest. How do variations in the exogenous interest rate  $r$  affect the amount of aggregate investment  $I^F$ ? Since  $0 < \delta < 1$ , it follows from Eq. (8) that a reduction in  $T^*$  increases aggregate investment  $I^F$ . Intuitively, the lower is  $T^*$ , the larger is the number of firms able to innovate before having to exit the economy. Therefore, to understand the effect of  $r$  on  $I^F$ , we must analyze the relation between  $r$  and  $T^*$ .

Eqs. (5), (6), and (7) show that the relation between  $r$  and  $T^*$  is ambiguous. On the one hand, Eq. (6) implies that a reduction in  $r$  increases borrowing and reduces the minimum wealth  $n^*$  necessary to invest:

$$\frac{\partial n^*}{\partial r} = \frac{F\theta}{(1+r)^2} > 0, \quad (9)$$

which is an effect we call the “collateral channel”. A lower interest rate increases the amount the firm can borrow with a given amount of collateral and shortens the number of periods  $T^*$  needed to have enough internal and external funds to innovate.

On the other hand, it is straightforward to see from Eq. (5) that the lower is  $r$  the more periods  $T^*$  are necessary to reach a given terminal wealth  $n^*$ . In other words, lower  $r$  reduces wealth accumulation and increases  $T^*$ , the more so the larger is  $n^*$ . We call it the “savings channel”.

Our main result in this section is that the strength of these two channels varies depending on  $\theta$ . A higher value of  $\theta$  increases  $\frac{\partial n^*}{\partial r}$  and the strength of the collateral channel. Moreover, it reduces  $n^*$  (see Eq. (6)) and therefore reduces the importance of the savings channel. We now define as a “tangible firm” a firm whose technology is based on tangible assets and therefore has a high value of  $\theta$ . Conversely, an “intangible firm” has low  $\theta$ . For tangible firms,  $n^*$  is small and very sensitive to  $r$  ( $\frac{\partial n^*}{\partial r}$  is large), so the collateral channel dominates, and a reduction in the interest rate reduces  $T^*$ . For intangible firms, the opposite is true:  $n^*$  is large and is not so sensitive to changes in  $r$  ( $\frac{\partial n^*}{\partial r}$  is small). Therefore, for intangible firms, the collateral channel is weaker than for tangible firms, the savings channel is stronger, and the stimulating effect of a lower interest rate is dampened. This is the main insight of our theoretical framework. Moreover, for sufficiently low values of  $\theta$  (sufficiently high intangibility), the effect of interest rates can be reversed and a fall in  $r$  will reduce aggregate investment.

## 4. General equilibrium model

In the previous section, we developed a very stylized model with two main assumptions. First, firms invest in lumpy investment projects. Second, intangible capital has a lower collateral value than tangible capital. Under these assumptions, two counteracting effects drive the relationship between investment and interest rates: a “collateral channel” and a “savings channel”. The savings channel dominates when capital is highly intangible; it dampens the negative relation between interest rates and investment and, if strong enough, can reverse it. This model is the simplest possible theoretical framework that can describe our ideas with clarity, but it is too stylized to derive testable predictions or general equilibrium implications. Moreover, it implies that firms invest only once, and that all firms with investment opportunities are financially constrained and are net savers, while in reality firms invest repeatedly, and many firms with investment opportunities are financially unconstrained and/or are net borrowers.

Therefore, in this section we consider a version of the model in which both financially constrained and unconstrained firms have investment opportunities, and these opportunities happen more than once during the firm’s lifetime. Because of these features, firms are heterogeneous in their asset holdings, and it is no longer the case that all firms are net savers; some firms are net borrowers, and average firm leverage increases in asset tangibility. This model is not only consistent with the well-documented observation that firms in more intangible industries hold less debt, but it also allows us to highlight another channel, which we call the “net debtor channel”: firms that are net borrowers benefit from a reduction in the interest rate because of lower interest payments. We show that the results of the simple model extend to this more realistic setting, and we are able to generate additional testable implications.

### 4.1. Firms and technology

The production sector of the economy is populated by many firms that operate a technology that uses tangible capital, intangible capital, and labor as inputs. Firms are heterogeneous in their access to external finance. Following Kiyotaki and Moore (1997, 2019) and Del Negro et al. (2017), we assume that there are two types of firms: one type faces financial imperfections and is financially constrained in equilibrium, while the other type does not face financial frictions. We call firms of the first type “constrained” and firms of the second type “unconstrained”.<sup>15</sup>

<sup>15</sup> Another approach would be, instead, to assume that all firms face the same frictions but that the presence of persistent idiosyncratic shocks and/or decreasing returns to scale implies that some firms—typically the younger ones—are endogenously more productive and financially constrained, and other firms—typically the older ones—are less productive and financially unconstrained thanks to past accumulated savings (e.g., Buera et al., 2011; Khan and Thomas, 2013). All of the results derived here could be generalized in a more complicated model following the latter approach.



**Constrained firms** There is a continuum of mass 1 of constrained firms. The production function is Cobb–Douglas in labor and capital

$$y_{c,t} = z_{c,t} e_{c,t}^{1-\xi} \left( l_{c,t}^{1-\alpha} k_{c,t}^{\alpha} \right)^{\xi},$$

where  $0 < \alpha \leq 1$  and  $0 < \xi < 1$ . The subscript “c” is for “constrained”. Labor  $l_{c,t}$  is provided by the household sector, which can supply it to both constrained and unconstrained firms. The entrepreneurial labor input  $e_{c,t}$  is provided by a continuum of specialized entrepreneurs of mass 1, which can only provide labor to constrained firms. This additional assumption allows us to simplify aggregation and obtain a closed-form solution of the model.<sup>16</sup>

The term  $k_t$  represents capital installed in period  $t - 1$  that produces output in period  $t$ . We assume it to be a combination of tangible and intangible capital, which are complementary inputs:

$$k_t = \min \left( \frac{k_{T,t}}{1-\mu}, \frac{k_{I,t}}{\mu} \right), \quad (10)$$

where  $0 < \mu < 1$ . The terms  $k_{T,t}$  and  $k_{I,t}$  represent tangible and intangible capital, respectively. We adopt this simple Leontief structure because it implies that all firms choose the same intangible share of total capital, and this facilitates aggregation.<sup>17</sup> From the Leontief structure of the production function, it follows that  $k_{T,t} = \frac{1-\mu}{\mu} k_{I,t}$  and

$$k_{I,t} = \mu k_t \quad \text{and} \quad k_{T,t} = (1-\mu) k_t. \quad (11)$$

We assume for simplicity that the two types of capital have the same depreciation rate  $\delta$ , and therefore the law of motion of capital can be written as:

$$k_{t+1} = i_t - (1-\delta) k_t, \quad (12)$$

where  $i_{I,t} = \mu i_t$  and  $i_{T,t} = (1-\mu) i_t$ . We denote with  $\theta_I$  and  $\theta_T$  the collateral value of intangible capital and tangible capital, respectively. Condition (11) implies that:

$$\theta = \mu \theta_I + (1-\mu) \theta_T. \quad (13)$$

Our main assumption is that  $\theta_I < \theta_T$ , so that intangible capital is less collateralizable than tangible capital. This assumption is supported by a large empirical literature. Several authors have emphasized both that tangibility is important in determining firms' access to credit (e.g., Almeida and Campello, 2007) and that firms that invest in intangible projects face financial frictions. Hall (2002) documents, in an extensive survey of the literature, that “R&D-intensive firms feature much lower leverage, on average, than less R&D-intensive firms”. She concludes that “small and new innovative firms experience high costs of capital that are only partly mitigated by the presence of venture capital”. Brown et al. (2009) document that U.S. firms finance most of their R&D expenditures out of retained earnings and equity issues. Gatchev et al. (2009) document that, in addition to R&D, marketing expenses and product development are also mostly financed out of retained earnings and equity. Dell’Ariccia et al. (2021) document that the increased usage of intangible assets by firms helps explain why banks have shifted out of business lending and into residential real estate lending in the U.S. in recent decades. In contrast, tangible assets are mostly financed with debt.<sup>18</sup>

Given this assumption, the degree of intangibles intensity ( $\mu$ ) negatively affects the collateral value  $\theta$  of the composite capital input  $k_t$ . The budget constraint is given by the following dividend equation:

$$d_{c,t} = \pi_{c,t} + (1+r_t) a_{c,t} - a_{c,t+1} - (k_{c,t+1} - (1-\delta) k_{c,t}), \quad (14)$$

where

$$\pi_{c,t} \equiv y_{c,t} - w_{c,t}^e e_{c,t} - w_{c,t} l_{c,t}, \quad (15)$$

are current profits,  $w_{c,t}^e$  is the wage paid to the entrepreneur, and  $w_{c,t}$  the wage paid to a worker.  $r_t$  is the interest rate paid or received in date  $t$ . The term  $a_t > 0$  indicates that the firm is a net saver, and  $a_t < 0$  indicates that the firm is a net borrower. As in the stylized model, a constrained firm can borrow using one-period debt ( $a_{t+1} < 0$ ) at the rate  $r_{t+1}$ . If the firm does not repay the debt, however, the lenders can liquidate its depreciated capital  $(1-\delta) k_{c,t+1}$  and recover a fraction  $\theta$ . Therefore,  $\theta(1-\delta) k_{c,t+1}$  represents the collateral value of the investment, which limits the maximum face value of debt  $-(1+r_{t+1}) a_{c,t+1}$ :

$$a_{c,t+1} \geq -\theta \frac{(1-\delta) k_{c,t+1}}{1+r_{t+1}}. \quad (16)$$

<sup>16</sup> The reason is that adding entrepreneurial labor ensures that the production function is constant returns to scale, and this implies, as will be shown below, that all constrained firms chose the same optimal ratio between inputs. Eliminating specialized entrepreneurial labor would not affect any of the results but would complicate the analysis because we would not be able to solve the model analytically.

<sup>17</sup> Using a more standard Cobb–Douglas or CES function, instead of the Leontief function, would imply that the optimal ratio between tangible and intangible capital varies with the intensity of financial frictions. More constrained firms would use tangible capital more intensely, because its higher collateral value is more beneficial to them, and this would create an additional distortion in the allocation of resources across firms. See Pérez-Orive (2016) for a study of this type of distortion.

<sup>18</sup> Eisfeldt and Rampini (2009) report that a big share of machinery, equipment, buildings, and other structures is financed with debt. Inventory investment and other tangible short-term assets attract substantial debt finance in the form of trade credit and bank credit lines (Petersen and Rajan, 1997; Sufi, 2009). Finally, investment in commercial real estate is primarily financed with mortgage loans (Benmelech et al., 2005).

Furthermore, firms are unable to issue equity, which means that dividends are subject to a non-negativity constraint<sup>19</sup>:

$$d_{c,t} \geq 0. \quad (17)$$

Every period, with probability  $\gamma$ , firms can invest to expand fixed capital  $k_t$ . This can be interpreted as the opportunity to invest in a large expansion or innovation project. Otherwise, with probability  $1 - \gamma$ , they can only produce with their existing depreciated capital.<sup>20</sup> Finally, after producing, the firm's technology becomes obsolete with probability  $\psi$ . In this case, the firm liquidates all of its capital, pays out as dividends all of its savings, including the liquidation value of capital, and exits. Exiting firms are replaced with newborn ones, with initial endowment  $W_0$ .<sup>21</sup>

Firms choose their investment and savings in order to maximize the net present value of their dividends. We define the value function conditional on having an investment opportunity, denoted as  $V^+(k_t, a_t)$ , as follows:

$$V_t^+(k_t, a_t) = \max_{d_t, a_{t+1}, l_t, k_{t+1}} d_t + \frac{1}{1 + r_{t+1}} [(1 - \psi)V_{t+1}(k_{t+1}, a_{t+1}) + \psi d_t^{exit}], \quad (18)$$

subject to constraints (17) and (16), and where

$$d_t^{exit} = \pi_{c,t} + (1 + r_t)a_{f,t} + (1 - \delta)k_t, \quad (19)$$

and  $V_{t+1}(k_{t+1}, a_{t+1})$  is the value function conditional on continuation but before the investment shock is realized:

$$V_{t+1}(k_{t+1}, a_{t+1}) = \gamma V^+(k_{t+1}, a_{t+1}) + (1 - \gamma)V^-(k_{t+1}, a_{t+1}). \quad (20)$$

The value function of a non-investing firm, denoted as  $V^-(k_t, a_t)$ , is identical to  $V^+(k_t, a_t)$  but does not offer the opportunity to choose  $k_{t+1}$ . The firm solves (18) (or its non-investing counterpart) subject to (14), (16), and (17). The first order conditions for labor inputs  $e_{c,t}$  and  $l_{c,t}$  of constrained firms maximize current profits  $\pi_{c,t}$  and are as follows:

$$(1 - \xi)z_{c,t} \left( \frac{l_{c,t}^{1-\alpha} k_{c,t}^\alpha}{e_{c,t}} \right)^\xi = w_{c,t}^e, \quad (21)$$

and

$$\xi(1 - \alpha)z_{c,t} e_{c,t}^{1-\xi} l_{c,t}^{\xi(1-\alpha)-1} k_{c,t}^{\alpha\xi} = w_{c,t}. \quad (22)$$

We combine the two conditions to obtain the optimal ratios between inputs:

$$\frac{e_{c,t}}{l_{c,t}} = \frac{1 - \xi}{\xi(1 - \alpha)} \frac{w_{c,t}}{w_{c,t}^e}, \quad (23)$$

and

$$\frac{l_{c,t}}{k_{c,t}} = \left[ \frac{\xi(1 - \alpha)z_{c,t} \left( \frac{1 - \xi}{\xi(1 - \alpha)} \frac{w_{c,t}}{w_{c,t}^e} \right)^{1-\xi}}{w_{c,t}} \right]^{\frac{1}{\alpha\xi}}. \quad (24)$$

Investing in one unit of capital has an opportunity cost of  $r_{t+1}$  (the lost return on saving) plus the depreciation of capital  $\delta$ , and it has a marginal return of  $\alpha\xi z_{c,t} e_{c,t}^{1-\xi} l_{c,t}^{\xi(1-\alpha)-1} k_{c,t}^{\alpha\xi-1}$ . It follows that a constrained firm faces a binding borrowing limit (16) in equilibrium if the marginal return on capital is higher than the user cost:

$$\frac{\partial y_{c,t+1}}{\partial k_{c,t+1}} = \alpha\xi z_{c,t} e_{c,t}^{1-\xi} l_{c,t}^{\xi(1-\alpha)-1} k_{c,t}^{\alpha\xi-1} > r_{t+1} + \delta. \quad (25)$$

We claim—and check later in our calibrated simulations—that, in equilibrium, the above condition is satisfied. Since the production function is constant returns to scale in the three inputs  $e$ ,  $l$ , and  $k$ , all firms choose the same labor to capital ratio and the same entrepreneurial labor to capital ratio. It follows that individual marginal products of capital are the same across all firms and identical to the aggregate marginal product of capital. Even though an individual firm could in principle become very large if it lives very long, the assumption of an exogenous exit rate of firms and the fact that new firms are created small implies that, in equilibrium, aggregate capital converges to a finite value, determined in Eq. (38). We ensure that, for the chosen parameters,

<sup>19</sup> On the one hand, this assumption is realistic, as several papers document that firms that face borrowing constraints also face equity financing constraints (Altunkılıç and Hansen, 2000; Gomes, 2001; Belo et al., 2019). On the other hand, it is without loss of generality, because in the model we introduce unconstrained firms that have frictionless access to both equity and debt financing, and we show that our main results are confirmed for a realistic calibration of the share of output produced by constrained firms.

<sup>20</sup> An alternative approach to introduce lumpy investment decisions is to add nonconvex adjustment costs (e.g., see Gourio and Kashyap, 2007, among others). For the purpose of this paper, our assumption of exogenous investment opportunities has similar implications but is much more tractable and allows for a closed-form solution.

<sup>21</sup> We assume that newly created firms do not produce in period 0 and use their wealth  $W_0$  to invest. This is why their initial endowment is defined as  $W_0$  and not as  $a_0$ , because  $a_t$  denotes in general savings from period  $t - 1$ , that generate  $(1 + r_t)a_t$  resources in period  $t$ . The relation between  $W$  and  $a$  is derived in Appendix A.

condition (25) holds with inequality at the aggregate level in all of our simulations. Details are in Appendix A.4. The implication of condition (25) for investing firms is that the borrowing constraint (16) is binding, and that firms choose not to pay dividends, so the equity constraint (17) is also binding. Making  $d_{c,t} = 0$  in budget constraint (14), using Eq. (14) to substitute for  $a_{c,t+1}$  in Eq. (16), assuming Eq. (16) is binding, and solving for  $k_{c,t+1}$ , we obtain their level of investment:

$$(k_{c,t+1} | \text{invest}) = \frac{\pi_{c,t} + (1 + r_t)a_{c,t} + (1 - \delta)k_{c,t}}{1 - \frac{\theta}{1 + r_{t+1}}}. \quad (26)$$

The right-hand side of Eq. (26) is the maximum feasible investment for a firm. The numerator is the total wealth available to invest determined by current profits  $\pi_{c,t}$ , the net financial position from the previous period  $(1 + r_t)a_{c,t}$ , and the residual value of capital  $(1 - \delta)k_{c,t}$ . The denominator is the downpayment necessary to buy one unit of capital. Investing firms in equilibrium borrow as much as possible, and

$$(a_{c,t+1} | \text{invest}) = -\frac{\theta}{1 + r_{t+1}}k_{c,t+1} < 0. \quad (27)$$

The implication of assumption (25) for non-investing firms is that they will not sell any of their capital, and, for these firms, the law of motion of capital is

$$(k_{c,t+1} | \text{not invest}) = (1 - \delta)k_{c,t}. \quad (28)$$

Non-investing firms always retain all earnings and select  $d_{c,t} = 0$  because they face a positive probability of being financially constrained in the future and, hence, the value of cash inside the firm is always higher than its opportunity cost (see Appendix A.4 for a formal proof). Substituting  $d_{c,t} = 0$  and (28) in (14):

$$(a_{c,t+1} | \text{not invest}) = \pi_{c,t} + (1 + r_t)a_{c,t}. \quad (29)$$

Eqs. (27) and (29) determine the wealth dynamics of firms. A firm that invested in period  $t - 1$  but is not investing in period  $t$  has debt equal to  $-a_{c,t} = \frac{\theta}{1 + r_t}k_{c,t}$ . It uses current profits  $\pi_{c,t}$  to pay the interest rate on debt  $-r_t a_{c,t}$  and to reduce the debt itself. As long as the firm is not investing, the debt  $-a_{c,t}$  decreases until the firm becomes a net saver and has  $a_{c,t} > 0$ . At this point, wealth accumulation is driven both by profits  $\pi_{c,t}$  and by interest on savings  $r_t a_{c,t}$ , until the firm has an investment opportunity and its accumulated wealth  $(1 + r_t)a_{c,t}$  is used to purchase capital (see Eq. (26)). This discussion clarifies that a lower interest rate  $r_t$  helps the non-investing firm repay existing debt—an effect we call the “net debtor channel”—but slows down the accumulation of savings after the firm has repaid the debt, which is the savings channel described in the previous section.

**Unconstrained firms** There is a continuum of mass 1 of identical unconstrained firms. Their production function has the same functional form of the production function of constrained firms:

$$y_{u,t} = z_{u,t} e_{u,t}^{1-\xi} \left( l_{u,t}^{1-\alpha} k_{u,t}^\alpha \right)^\xi, \quad (30)$$

where  $e_{u,t}$  is the input provided by a mass 1 of entrepreneurs specialized in operating unconstrained firms. They finance capital with equity from the household sector and pay out all profits as dividends  $d_{u,t}^u$  to households every period:

$$d_{u,t} = \pi_{u,t} - (k_{u,t+1} - (1 - \delta)k_{u,t}), \quad (31)$$

where

$$\pi_{u,t} \equiv y_{u,t} - w_{u,t}^e e_{u,t} - w_{u,t} l_{c,t}. \quad (32)$$

Unconstrained firms are able to issue equity, so  $d_{u,t}$  is allowed to be negative. Just like constrained firms, unconstrained firms can invest with probability  $\gamma$ . The first order conditions for  $e_{u,t}$  and  $l_{u,t}$ , as well as the first order conditions for  $k_{u,t}$  for investing firms, are reported in Appendix A.1.

**Aggregation of the firm sector** We assume that all firms employ the same homogeneous labor provided by households, which is in fixed aggregate supply  $N = 1$ :

$$L_{c,t} + L_{u,t} = 1. \quad (33)$$

Therefore, the wage paid to households is equalized across sectors:

$$w_{u,t} = w_{c,t} \equiv w_t. \quad (34)$$

Furthermore, there is a measure 1 of entrepreneurial labor specialized in operating each type of firm:

$$E_{c,t} = E_{u,t} = 1. \quad (35)$$

Because firms operate a constant returns to scale production function, all firms within the constrained and unconstrained groups employ inputs in the same optimal ratio, and we can thus aggregate factors of production within each group. We denote aggregate factor values with uppercase letters. Eqs. (46)–(52) in Appendix A.2 determine, in equilibrium, the values of  $Y_{c,t}$ ,  $Y_{u,t}$ ,  $L_{u,t}$ ,  $K_{u,t}$ ,  $w_t$ ,  $w_{u,t}^e$  and  $w_{c,t}^e$  given  $K_{c,t}$ .

#### 4.1.1 Household and entrepreneurial sectors

We consider a representative household, a representative productive entrepreneur, and a representative unproductive entrepreneur. Each supplies inelastically one unit of labor and consumes. Consumption and savings are chosen to maximize:

$$V_t^j(B_t^j) = \max_{C_t^j, B_{t+1}^j} u(C_t^j) + \beta V_{t+1}(B_{t+1}^j), \quad (36)$$

subject to

$$C_t^j = D_t^j + W_t^j - (1 + r_t)B_t^j + B_{t+1}^j, \quad (37)$$

where  $j \in \{e_c, e_u, h\}$  indicates the type of agent.  $C_t^j$  is aggregate consumption,  $D_t^j$ ,  $W_t^j$  are dividends and wages, where  $W_t^{e_c} = w_{c,t}^e$ ,  $W_t^{e_u} = w_{u,t}^e$  and  $W_t^h = w_{h,t}$ , and  $B_t^j$  are aggregate borrowing (or savings if negative). The first order condition is the usual consumption Euler equation:

$$u'(C_t^j) = \beta(1 + r_{t+1})u'(C_{t+1}^j).$$

#### 4.2 Steady state equilibrium

In the steady state, household consumption is constant, implying  $r = \frac{1}{\beta} - 1$ . Furthermore, unconstrained firms distribute all their profits as dividends and do not hold any financial assets, so that the market clearing conditions for asset holdings implies that total household debt is equal to the aggregate asset holdings of the constrained firms  $A$ :

$$B^{e_c} + B^{e_u} + B^h = A.$$

The remaining steady state equilibrium conditions are described in [Appendix A.3](#). Here we focus on the aggregate capital stock  $K_c$  and the aggregate wealth  $A$  of the constrained firms.  $K_c$  can be shown (see [Appendix A.4](#) for details) to be equal to:

$$K_c = \frac{\gamma[(1 - \psi)(\Pi_c + (1 + r)A) + \psi W_0]}{\left(1 - \frac{\theta}{1+r}\right)[\delta + \psi(1 - \delta)] - \frac{\theta}{1+r}\gamma(1 - \delta)(1 - \psi)}, \quad (38)$$

where  $\Pi_c$  is the aggregate profits of constrained firms:

$$\Pi_c \equiv z_c [K_c^\alpha L_c^{(1-\alpha)}]^\xi - w_c^e - w(1 - L_u). \quad (39)$$

Eq. (38) has an intuitive explanation. The numerator is the aggregate amount of liquid resources of the measure  $\gamma$  of investing firms. For the fraction  $(1 - \psi)$  of incumbent firms, liquid resources are profits  $\Pi_c$  plus net savings  $(1 + r)A$ . For the fraction  $\psi$  of new firms, liquid resources are the initial endowment  $W_0$ . The denominator is the downpayment necessary to support one unit of capital in the steady state. It requires the replacement of the depreciated capital and the lost capital of exiting firms (a fraction  $\delta + \psi(1 - \delta)$ ) and can benefit from using existing capital held by the investing firms as collateral (fraction  $\gamma(1 - \delta)(1 - \psi)$ ). Furthermore, aggregate asset holdings of the constrained firms  $A$  can be shown to be equal to:

$$A = \frac{1}{1 - (1 - \psi)(1 + r)} \{ (1 - \psi)\Pi_c + \psi W_0 - [\psi + \delta(1 - \psi)]K_c \}. \quad (40)$$

$A$  is equal to the net earnings of the constrained firms, multiplied by a factor  $\frac{1}{1 - (1 - \psi)(1 + r)}$ , which measures the expected accumulated value of saving each period one unit of wealth at the rate  $r$  until the firm exits from the market. The net earnings are the endowment of the new firms  $\psi W_0$  plus the net earnings of continuing firms  $(1 - \psi)\Pi_c$ , minus the term  $[\psi + \delta(1 - \psi)]K$ , which is total expenditures to replace the depreciated capital of continuing firms  $\delta(1 - \psi)K$ , and the capital liquidated by exiting firms  $\psi K$ .

#### 4.3 Discussion

In this section, we briefly discuss a key feature of the equilibrium described above, namely the relation between capital tangibility and the net financial position of constrained firms. Consider the effect of a shift toward more intangible technologies (higher  $\mu$ ) that reduces  $\theta$  (see Eq. (13)). This reduces the borrowing capacity of the constrained firms by increasing the required downpayment  $\left(1 - \frac{\theta}{1+r}\right)$  in the denominator of (38). It follows that their aggregate capital  $K_c$  is lower for given net aggregate financial wealth  $A$ .

However, the reduction in aggregate capital  $K_c$  affects  $A$ , at the same time, through Eq. (40). More specifically, the multiplicative factor  $\frac{1}{1 - (1 - \psi)(1 + r)}$  in Eq. (40) is always positive. Moreover, since the endowment of new firms  $\psi W_0$  is small, the sign of the term in square brackets is determined by the difference between a positive term  $(1 - \psi)\Pi_c$ , concave in  $K_c$  (see the definition (39)) and a linear negative term  $[\psi + \delta(1 - \psi)]K_c$ . When capital is tangible and  $\theta$  is large,  $K_c$  is also large because its required downpayment is small, and in Eq. (40) the negative term dominates, making  $A$  negative. In other words, tangible constrained firms are, on aggregate, net borrowers, and lower interest rates benefit them via the net debtor channel. Conversely, in an intangible (low  $\theta$ ) economy,  $K_c$  is lower and the positive concave term dominates, making  $A$  positive and implying that a lower interest rate penalizes these firms via the savings channel. Note also that when  $A$  is positive, a reduction in the interest rate reduces investment both through a reduction in the return on savings  $rA_f$  and through a reduction in the multiplicative factor  $1/(1 - (1 - \psi)(1 + r))$ .

#### 4.4 Calibration

We use this model to perform a comparative statics exercise in which we vary the discount factor  $\beta$  from 0.94 to 1. Since  $r = \frac{1}{\beta}$  in equilibrium, this implies that the equilibrium real interest rate  $r$  changes from 6% to 0%. This increase in  $\beta$  could be considered as a shortcut for factors that, over time, have increased households' propensity to save, such as the reduction in fertility and mortality (e.g., Eggertsson et al. (2019)). We perform this exercise separately for two economies that have identical parameters except with regards to their technology. For the "tangibles economy" we set  $\mu = 0.2$ , the average share for U.S. firms at the beginning of our sample, in 1980. For the "intangibles economy" we set  $\mu = 0.65$ , the average share at the end of our sample in 2018. Given this range of values, our calibration strategy is the following: we set  $\beta$  and  $\mu$  to the intermediate values of  $\beta = 0.97$  and  $\mu = 0.4$ . Given these values, we set the remaining parameters to values commonly used in the literature or to match a set of empirical moments. In the comparative static exercise, we keep constant all parameters (except productivity  $z$ , which depends on  $\mu$ , as explained below) and we vary  $\beta$  and  $\mu$ .

Our benchmark calibration is illustrated in Table 1. We set the degree of returns to scale to capital  $k$  and labor  $l$  to  $\xi = 0.85$ , based on estimates surveyed in Restuccia and Rogerson (2008), and the elasticity of output with respect to capital  $\alpha$  to 0.4, a common value used in most of the literature.<sup>22</sup> The quarterly depreciation rate of capital is set at  $\delta = 0.025$ , which is a standard value for quarterly Real Business Cycle models. The pledgeability parameters of tangible capital  $\theta^T$  and intangible capital  $\theta^I$  are equal to 1 and 0.2, respectively.<sup>23</sup> Regarding productivity, we assume that  $z_u$ , the productivity of unconstrained firms, is related to intangible intensity according to the following relation:

$$z_u = [1 + (\mu - 0.2)\eta], \quad (41)$$

where the parameter  $\eta$  measures the productivity gains of more intangible technologies. We set the value of  $\eta$  to 0.85, meaning that in the tangibles economy  $z_u$  is normalized to 1, while it is equal to 1.3825 in the intangibles economy. Since the increase from  $\mu = 0.2$  to  $\mu = 0.65$  corresponds to the rise of the intangible share in the US from 1980 to 2018, assuming  $\eta = 0.85$  is equivalent to assuming that the shift towards more intangible technologies generated a yearly productivity growth equal to 0.8% in that period, which is in line with the estimates reported by the Federal Reserve Bank of San Francisco.<sup>24</sup> Note that the results presented in the next sections would be qualitatively unaffected if we simply assumed that the productivity of tangible and intangible technologies was the same. However, we would then have the following problem: if more intangible technologies are less collateralizable, why do firms facing financial frictions adopt them? In reality, the emergence of more intangible technologies observed in the data is also accompanied with higher productivity, which presumably explains why such technologies were adopted in the first place. To make sure the model is consistent with this view, we introduce the positive correlation between intangibility and productivity described in Eq. (41). In Appendix A.5 we demonstrate that the chosen value of  $\eta$  ensures that a firm in the intangibles economy does not have the incentive to deviate and adopt a tangible technology instead.

Then, given the value of  $z_u$ , we calibrate  $z_c$  and  $\psi$  to jointly match the share of output produced by constrained firms and the average intensity of financial frictions in the economy. In the model, unconstrained firms have access to frictionless financing. Therefore, we match their share with the share of output produced by the Compustat firms with the best credit ratings. The share of output produced by AAA- and AA-rated firms is around 15%. Adding also A-rated firms gives a value of 45%. We choose an intermediate value of 30%. This implies a share of output produced by firms facing some form of financial imperfections equal to 70%. Regarding the average intensity of financial frictions, Gilchrist et al. (2013) document the bond spreads of the cross-section of US firms and report the average spreads for the 10th percentile. We interpret the firms with the 10% lowest spreads as within our group of unconstrained firms defined above, and we consider the difference in spread between them and all the other firms, obtaining an average spread of 2.5%. In the model, we compute the spread of constrained firms  $\Delta^c$  as the interest rate premium that the firms would be willing to pay over  $r$  to access additional credit, which is given by:

$$\Delta^c = \alpha \xi z_c L_c^{\xi(1-\alpha)} K_c^{\alpha\xi-1} - (r + \delta). \quad (42)$$

Since such a spread is 0 for unconstrained firms, to obtain an average spread equal to 2.5% it must be that  $\Delta^c * 0.7 = 2.5\%$ , which implies that  $\Delta^c = 3.6\%$ .

The quarterly probability of receiving an investment opportunity,  $\gamma$ , is set to 0.1. This value is a bit low compared to empirical studies of tangible capital. For example, Doms and Dunne (1998) document that around half of the plants in their sample perform a capital adjustment of at least 37% in a given year. However, Chiavari and Goraya (2020) document that intangible capital is considerably lumpier than tangible capital. Nonetheless, selecting a higher value of  $\gamma$  would not significantly change our results. The initial endowment of newborn firms  $W_0$  is equal to 0.3 and is a parameter not calibrated to match a specific moment due to a lack of a clear empirical counterpart. It corresponds to 2% of average firm annual output. Our results show very little sensitivity to variations in our choice of  $W_0$  in the range 0.1% – 20%.

<sup>22</sup> See King and Rebelo (1999) or Corrado et al. (2009).

<sup>23</sup> FKSS argue, based on data on syndicated loans from LPC DealScan and on corporate debt structure from Capital IQ, that  $\theta^I$  for all intangible assets except patents and brands should be set to 0. They estimate  $\theta^T$ , on the other hand, to be between 0.9 and 1. Unlike in the model of FKSS, in our model we do not allow for any equity issuance, even though it would be realistic to assume that financially constrained firms have some ability—even if limited—to issue equity. Rather than complicating the model further, we compensate for the lack of equity financing by assuming a larger values of  $\theta^I$  than FKSS assume. A positive value of  $\theta^I$  is also supported by evidence that some forms of intellectual capital have collateral value (e.g., see Amable et al., 2010). The overall tangibility of capital in the industry depends on these two values and  $\mu$ , according to Eq. (13).

<sup>24</sup> See <https://www.frbfsf.org/economic-research/indicators-data/total-factor-productivity-tfp/>.

**Table 1**  
Benchmark calibration.

Parameter	Symbol	Value
Discount factor	$\beta$	0.97
Intangibles share	$\mu$	0.4
Capital share	$\alpha$	0.4
Relative TFP	$z_c/z_u$	1.25
$\partial z/\partial \mu$	$\eta$	0.85
Decreasing returns to scale	$\xi$	0.85
Prob. of investment opportunity (quarterly)	$\gamma$	0.1
Collateral value: tangible	$\theta^T$	1
Collateral value: intangible	$\theta^I$	0.2
Exit probability (quarterly)	$\psi$	0.01
Depreciation rate (quarterly)	$\delta$	0.025
Initial endowment of firms	$W_0$	0.3

One caveat of this calibration strategy is that by targeting 70% of output produced by firms facing some type of financing imperfection, we might overestimate aggregate financial frictions. It is plausible to assume that most firms in the data do not have frictionless access to finance, but for many of these firms, frictions take the form of an interest rate premium rather than a quantity borrowing constraint as in the model. Another caveat is that we impose the same depreciation rate  $\delta$  for both types of capital. We do so to simplify the model, but it is well known that in reality most types of intangible capital depreciate faster than tangible capital. Therefore, in a robustness section we verify that the results are robust to lowering the fraction of output produced by constrained firms and to assuming a higher depreciation rate of intangible capital than of tangible capital.

#### 4.5 Simulation results

##### 4.5.1 Benchmark exercise

Fig. 3 analyzes the tangibles economy first, and therefore the parameters are the benchmark ones except that we set  $\mu = 0.2$ , so tangible capital is 80% of total capital. The lines represent different simulations for different values of the real interest rate  $r$  (displayed in the top left graph), ranging from 6% (when  $\beta = 0.94$ ) to 0% (when  $\beta = 1$ ) in annualized terms. All other parameters are kept constant at their benchmark value. Since we perform a comparative static analysis, when we talk about the effect of changes in  $r$  for aggregate capital, we are implicitly talking about long run effects.

The savings channel, which tends to generate a positive relation between  $r$  and capital, is weak in this economy, because the constrained firms as a whole are net borrowers (panel C). Even though some constrained investing firms are net savers (those firms that receive an investment opportunity after many periods from the previous investment opportunity), many firms are instead net borrowers and benefit from the reduction in  $r$ . However, the savings channel is still present and implies that, as the interest rate falls from 6% to 0%, the capital in constrained firms falls slightly, by 3% (panel D). Conversely, capital in unconstrained firms expands strongly, by up to 250% (panel E), driving up aggregate output (panel I). One reason for this large expansion in aggregate capital is that we assume that the supply of capital is fully elastic and its price is always equal to one. Assuming a less elastic capital supply would dampen the expansion of unconstrained firms when  $r$  falls. Furthermore, the equilibrium increase in the price of capital would increase the downpayment constrained firms need to pay for one unit of capital, and would make them even more constrained. In other words, our assumption that capital is fully elastic dampens the misallocation implications of the drop in interest rates and its negative consequences for aggregate capital and output.

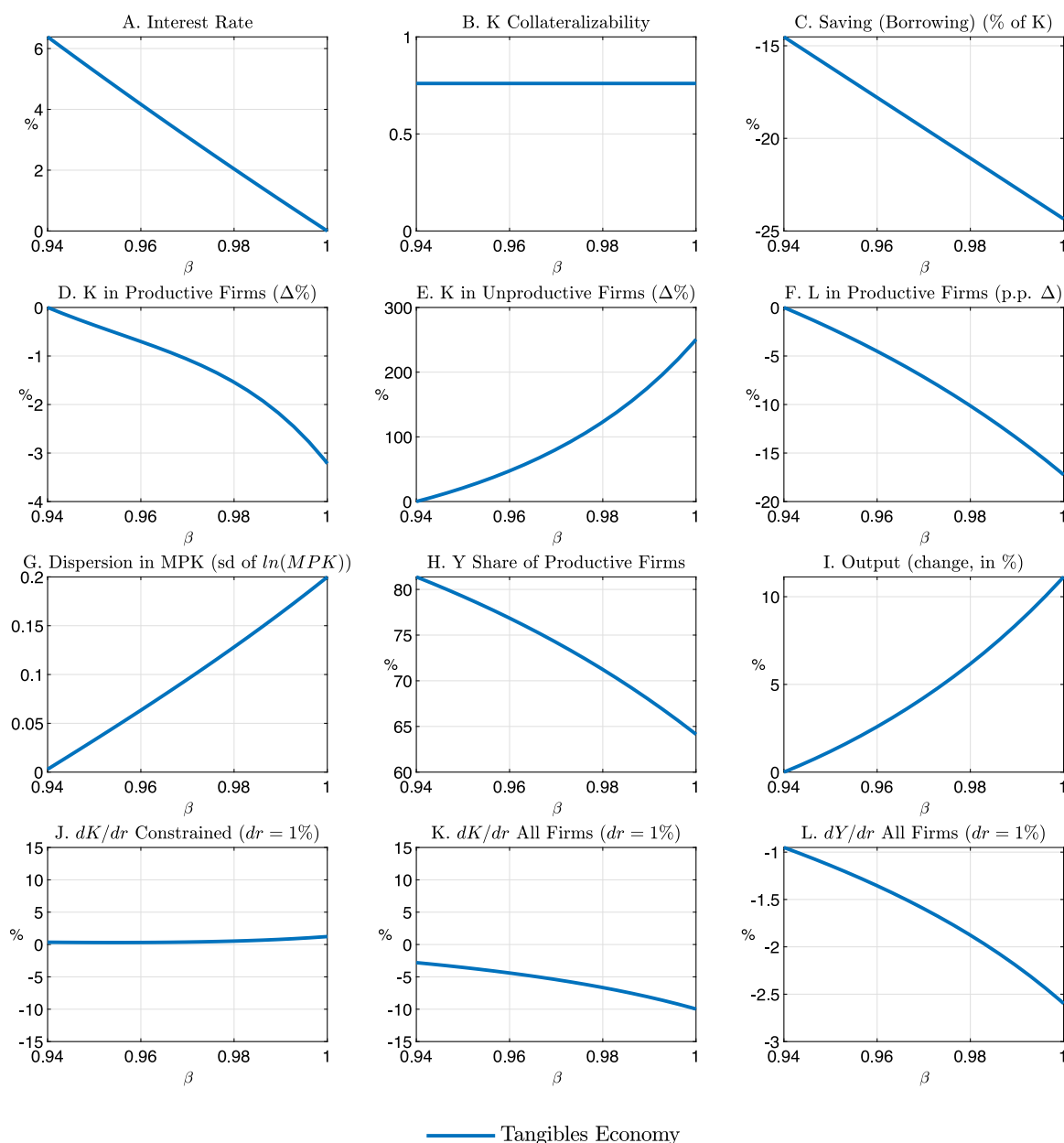
The last row of Fig. 3 shows the implications for  $\frac{dK}{dr}$  and  $\frac{dY}{dr}$ , the sensitivity of aggregate capital and output to a marginal change in the interest rate. Panel J shows that  $\frac{dK}{dr}$  for constrained firms, which is equivalent to the slope of the line in panel D with inverted sign, is positive and small in absolute value. Panel K shows  $\frac{dK}{dr}$  for aggregate capital. This is negative, because the negative value of  $\frac{dK}{dr}$  for unconstrained firms more than compensates for the positive value of  $\frac{dK}{dr}$  for constrained ones. Hence,  $\frac{dY}{dr}$  is also negative for all firms (panel L). In other words, a reduction in  $r$  stimulates investment and, hence, output in the economy because the expansionary effect for unconstrained firms dominates the small contractionary effect for constrained firms.

Furthermore, panel K shows that  $\frac{dK}{dr}$  becomes even more negative with a lower level of  $r$ , reflecting the fact that the overall weight in the economy of unconstrained firms is larger. However, this expansion of the unconstrained firms happens at the cost of a worsening allocation of resources, as shown by a larger dispersion in the marginal product of capital across all firms (panel G).

Fig. 4 presents the same simulations shown above and compares them with the simulation of an intangibles-intensive economy ( $\mu = 65\%$ ), represented by the red dashed lines. Because intangible firms borrow less when they have an investment opportunity, overall constrained firms are net savers (panel C).<sup>25</sup> As a result, the savings channel is much stronger in this economy and implies a sharp reduction in capital in constrained firms as  $r$  falls (panel D). This significantly reduces the output expansion caused by

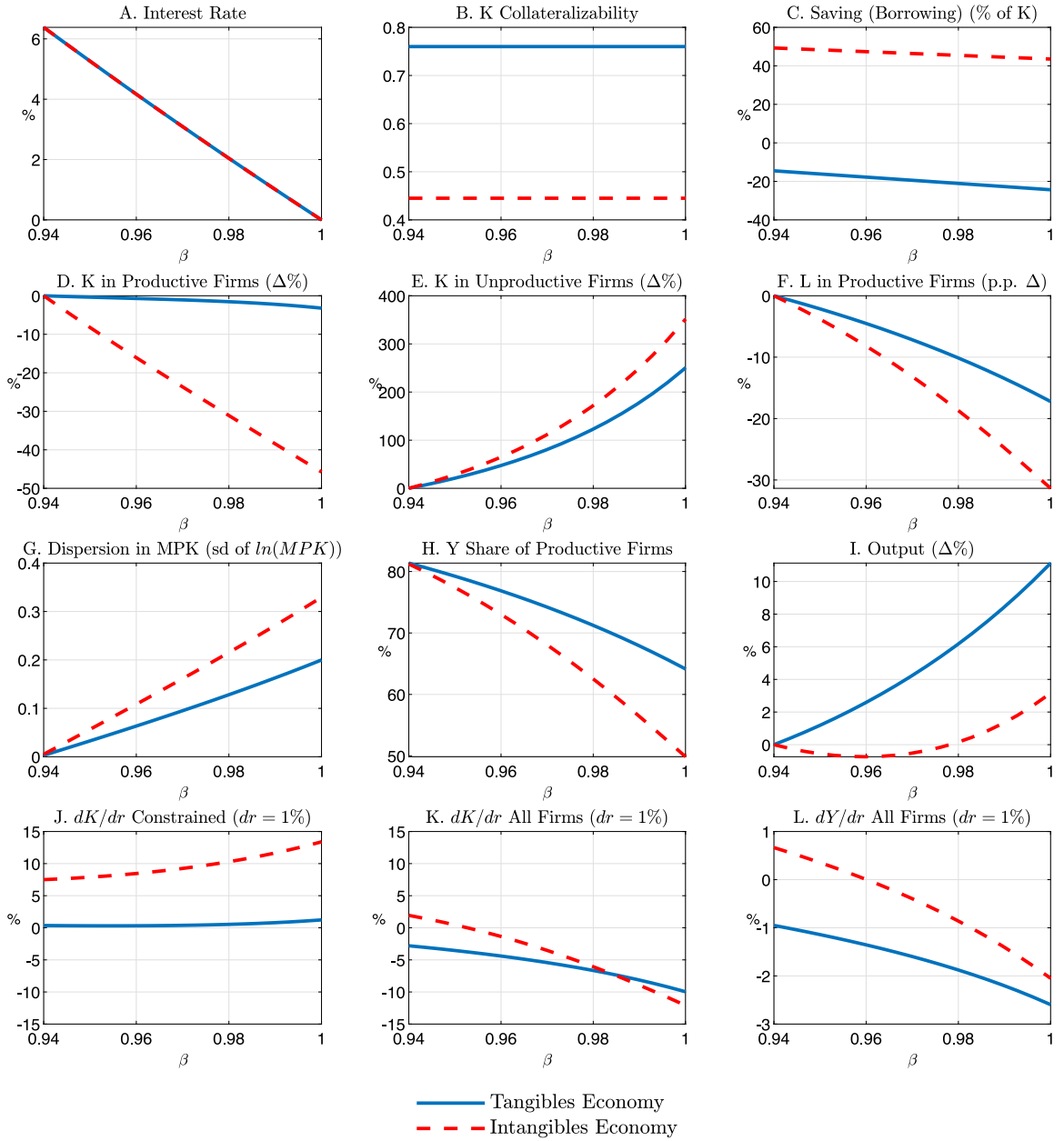
<sup>25</sup> Since we assume that the unconstrained firms finance investment with equity, their net borrowing is always zero. It follows that when the constrained firms are net savers, so is the corporate sector as a whole. If we assumed that unconstrained firms financed investment with short term debt instead, the total debt of the corporate sector would increase, but our qualitative results would be unchanged. We would still find that the rise of intangible capital implies lower aggregate borrowing, as a percentage of assets, in the corporate sector.





**Fig. 3.** Simulations of a tangibles economy for different values of the discount rate  $\beta$ . Comparative static analysis of the effects of a reduction in  $r$  in a tangibles economy. The curves capture steady state equilibrium values for different values of the real interest rate  $r$  (where  $r = 1/\beta - 1$ ). All other parameters are kept constant at their benchmark value.

lower rates (panel I). The larger misallocation between constrained and unconstrained firms is reflected also in the sharper increase in marginal product of capital dispersion relative to the tangibles economy (panel G). Finally, panel J shows that capital held by constrained firms responds positively and very strongly to  $r$ , implying that the sensitivity of aggregate capital to  $r$  is less negative than in the tangibles economy for most values of  $r$  (panel K), roughly those in the 1.75%–6% interval. For very low values of  $r$ , the amount of capital held by constrained productive firms drops so much in the intangibles economy that the sensitivity of aggregate capital to  $r$  is actually more negative than in the tangibles economy. However, the increase in misallocation implies a lower sensitivity of aggregate output. In other words, when  $r$  is very low, a 1% fall in  $r$  in the tangibles economy increases aggregate capital by 10% and aggregate output by 2.5% (panels K and L, respectively). Conversely, in the intangibles economy, the rise in aggregate capital is slightly larger (12%), but the increase in aggregate output is lower (2%).



**Fig. 4.** Simulations of tangibles and intangibles economies for different values of the discount rate  $\beta$ . Comparative static analysis of the effects of a reduction in  $r$  in a tangibles economy and an intangibles economy. The curves capture steady state equilibrium values for different values of the real interest rate  $r$  (where  $r = 1/\beta - 1$ ). All other parameters are kept constant at their benchmark value.

#### 4.5.2 Robustness checks

In this section, we provide two robustness checks of the above results. First, in Fig. 5 we change the calibration to reduce the fraction of constrained firms from 70% to 35%.<sup>26</sup> The results are qualitatively similar to those shown earlier. We still find that, compared to the tangibles economy, the intangibles economy has a much stronger contraction in the capital of constrained firms as  $r$  falls and a larger increase in marginal product of capital dispersion. However, the aggregate implications of this increased misallocation are smaller, and the sensitivity of aggregate capital to  $r$  is almost identical in the two economies.

<sup>26</sup> Notice that we still match the overall intensity of frictions, as measured by the average spread of 2.5%. Therefore, the spread between constrained and unconstrained firms is  $2.5\%/0.35 = 7.1\%$ .

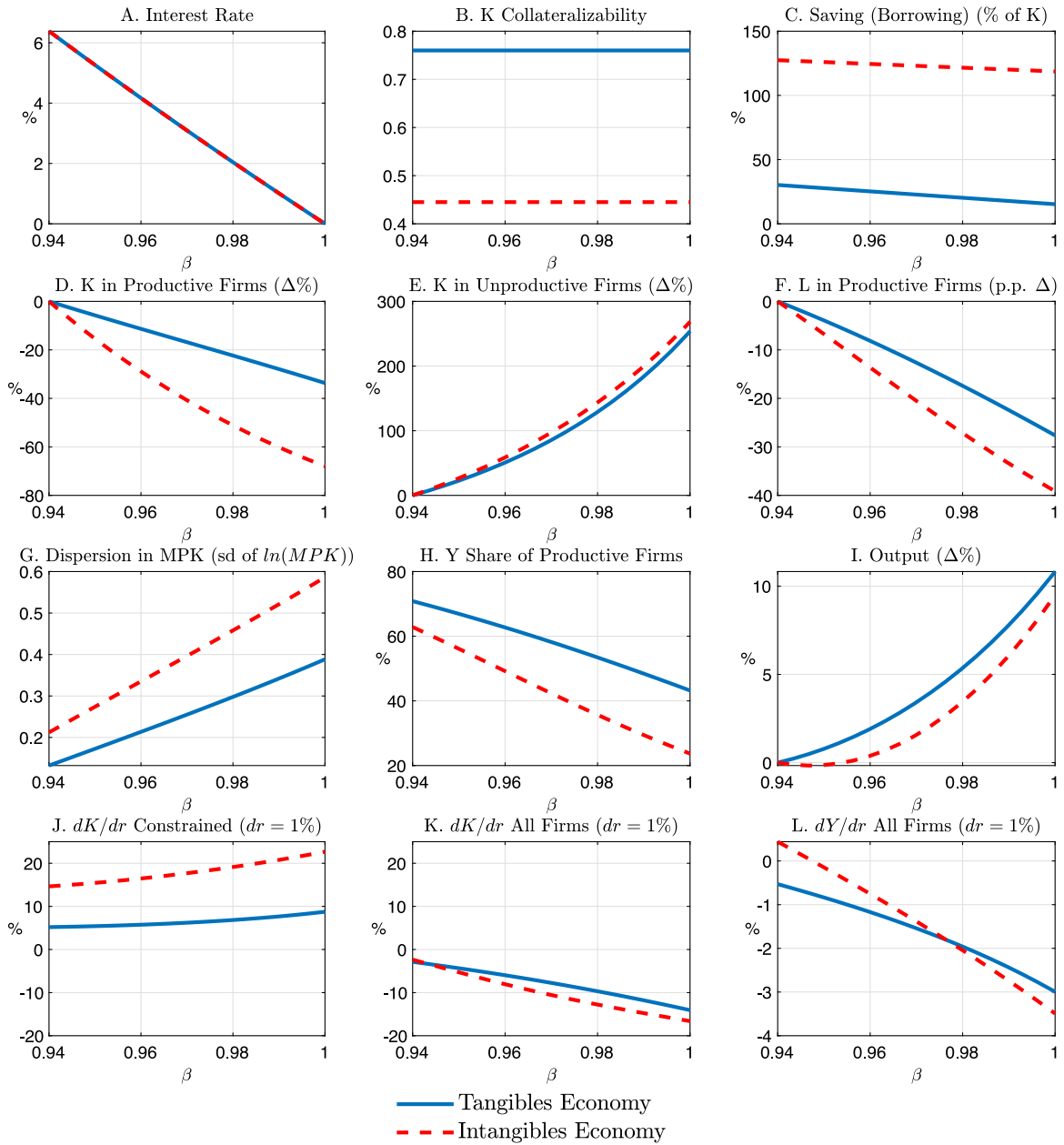
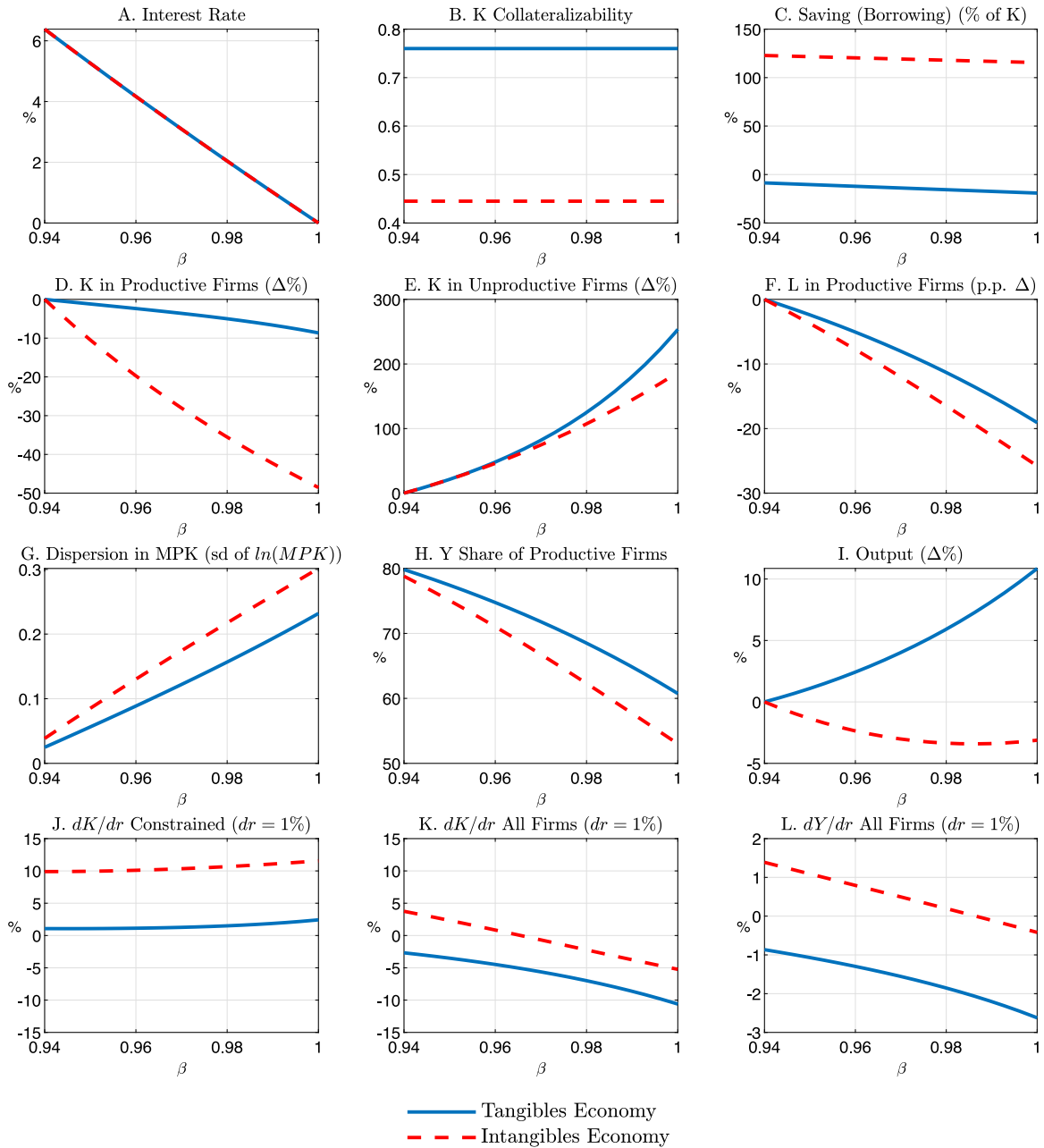


Fig. 5. Simulations of tangibles and intangibles economies for different values of the discount rate  $\beta$ : Alternative calibration with lower share of output from constrained firms. Comparative static analysis of the effects of a reduction in  $r$  in a tangibles economy and an intangibles economy. The curves capture steady state equilibrium values for different values of the real interest rate  $r$  (where  $r = 1/\beta - 1$ ). All other parameters are kept constant at their benchmark value. These simulations result from changing the calibration to reduce the fraction of constrained firms from the benchmark level of 70% to a lower level of 35%.

Second, in Fig. 6 we reduce to 35% the fraction of output produced by constrained firms, as before, and, in addition, select different depreciation rates for tangible and intangible capital. More precisely, we choose the realistic values computed by FKSS: yearly depreciation rates  $\delta$  are equal to 10% and 19% for tangible and intangible capital, respectively. Interestingly, the simulations in Fig. 6 show larger differences between tangibles and intangibles economies as  $r$  falls, closer to the results in the benchmark simulations in Fig. 4, and thus confirm our main findings both qualitatively and quantitatively. The intuition is simple. A higher depreciation rate dampens the sensitivity of unconstrained firms to  $r$  and therefore increases the importance of the constrained firms in determining the behavior of aggregate capital and output, even when their share of output is relatively small. The implications of this more realistic calibration are that interest rates are much less expansionary, both in terms of capital and output, in the intangibles economy than in the tangibles economy (see panels K and L). This happens also when  $r$  is very low. In this case, a 1%



**Fig. 6.** Simulations of tangibles and intangibles economies for different values of the discount rate  $\beta$ : Alternative calibration with lower share of output from constrained firms and different depreciation rates of tangible and intangible capital. Comparative static analysis of the effects of a reduction in  $r$  in a tangibles economy and an intangibles economy. The curves capture steady state equilibrium values for different values of the real interest rate  $r$  (where  $r = 1/\beta - 1$ ). All other parameters are kept constant at their benchmark value. These simulations result from changing the calibration to (1) reduce the fraction of constrained firms from the benchmark level of 70% to a lower level of 35% and (2) increasing the depreciation rate of intangible capital from the benchmark level of 10% to a level of 19% as in FKSS.

fall in  $r$  in the tangibles economy increases aggregate capital by 10.3% and aggregate output by 2.5%. Conversely, in the intangibles economy, the rise in capital is 5% and the rise in output is as little as 0.5% (panel L).

#### 4.6 Testing the model predictions: Empirical evidence

In Section 2, we showed how the remarkable increase in intangible intensity in recent decades has coincided with a strong reduction in the net borrower position of the U.S. corporate sector. The model is consistent with this finding, since the net financial

position of firms is negative in the intangible economy and positive in the tangible economy. Furthermore, the model also predicts that the sensitivity of capital to  $r$  is more negative in tangible than in intangible firms, thus providing an explanation to the finding in Fig. 2. In this section, we provide additional empirical evidence in support of the model's main mechanism.

#### 4.6.1 Intangible intensity, interest rates, and financial constraints

In the model, the sensitivity of capital to  $r$  is less negative in the intangible economy because intangible constrained firms have a sensitivity that is positive and larger than that of tangible constrained firms (see panel J of Fig. 4). To verify this prediction, we run local projection (1) on different subsets of firms determined by their financially constrained status. We consider 5 proxies for the intensity of financial frictions, widely used in the finance literature.

First, we consider the age of the firm, where the “young” are firms no more than 10 years old, and the “mature” are firms at least 30 years old.<sup>27</sup> As argued by Hadlock and Pierce (2010) and Cloyne et al. (2020), young firms are more likely to be financially constrained than mature ones both because they suffer from more intense informational frictions with their lenders and because they have had less time to accumulate internal funds to overcome these problems. Fig. 7 confirms the model's predictions. In panels D–F, we find that among old firms, there is little difference between the interest rate sensitivity of investment of intangibles and tangible firms, while we find a large and significant difference among young firms in panels A–C. Fig. A.3 in Appendix B shows that this result is confirmed for alternative age thresholds. Moreover, Figs. A.4 and A.5 confirm the results using the same battery of alternative specifications used in Figs. A.1 and A.2 to provide robustness to the results in Fig. 2, and described at the end of Section 2.3.

In Fig. 8, we consider four additional selection criteria to identify financially constrained firms: firm size, credit rating, net leverage, and dividend payer status. A large body of empirical literature has found that financially constrained firms are more likely to be smaller and unrated, have low net leverage, and be non-dividend payers, in addition to being younger as discussed above (Hadlock and Pierce, 2010; Farre-Mensa and Ljungqvist, 2016). These firm characteristics arise endogenously in our model: constrained firms are smaller than unconstrained ones, since they operate below their optimal unconstrained size; only unconstrained firms distribute dividends while in operation; and intangible constrained firms in equilibrium have low net leverage because of financial frictions.<sup>28</sup> Finally, the credit rating status can be considered as the empirical counterpart of belonging to the unconstrained group of firms in the model. All the panels in Fig. 8 display the difference in the interest rate sensitivity of investment between intangible and tangible firms. Such a difference is always large and significant for likely constrained firms, as predicted by the model (panels B, D, F, and H). In contrast, it is smaller and sometimes not statistically significant for likely unconstrained firms (panels A, C, E, and G).

#### 4.6.2 Productivity dispersion

Another important implication of our model is that economies with a higher intensity of intangible capital suffer from a higher degree of misallocation of capital between financially constrained and unconstrained firms, compared to more tangibles economies, which implies a higher dispersion in marginal products in intangibles economies (see panel G of Fig. 4). It follows that we expect a larger dispersion in the marginal product of capital in intangible industries than in tangible industries.

We verify this prediction using the data described in Section 2.1 and following the conventions in the most recent literature on the measurement of capital misallocation (e.g., Kehrig and Vincent, 2020). We consider sectors at the 2-digit SIC level and drop those with less than 50 observations, on average, per year. We measure output by sales and total capital by the sum of tangible and intangible capital as described in Section 2.1. Capital productivity  $mpk_{sit}$  of firm  $i$  in industry  $s$  in period  $t$  is measured as the log difference between firm output and the total capital stock, and productivity dispersion is computed as the standard deviation of  $mpk_{sit}$  in each industry  $s$  each period  $t$ . This measure of productivity dispersion is comparable to the one used in the model simulations shown in Figs. 3–6.<sup>29</sup> To control for outliers, we drop firms below and above the 1st and 99th percentiles, respectively, of the distribution of capital productivity.

The left panel of Fig. 9 plots the dispersion of capital productivity in 2-digit SIC industries. The dashed line displays the sales-weighted mean of the dispersion measure across high-intangible industries, defined as the industries in the top 33% of the

<sup>27</sup> Age is computed as described in Section 2.1. We find that 12% of all firm year observations belong to young firms, roughly equally distributed among tangible and intangible sectors. An exception is the dotcom bubble period of 1995–2000, during which there was a surge in the stock market entry of young firms in more intangible sectors. However, eliminating this period from our analysis does not significantly change the results.

<sup>28</sup> In our model, unconstrained firms are indifferent between financing with debt or equity, so they have higher leverage than constrained firms only if we assume that they prefer to use debt. Most corporate finance models assume, with empirical support, that debt has some benefits (such as a tax advantage or the ability to discipline managers) that justify that unconstrained firms are able to sustain higher levels of leverage.

<sup>29</sup> The standard approach in the literature at least since Hsieh and Klenow (2009) is to estimate separate elasticities for intangible capital and tangible capital and then compute the revenue TFP as a residual. This method is not appropriate in our setting because of two main shortcomings. First, Chiavari and Goraya (2020) show that the elasticity of output to intangible capital is not constant over time during our sample period (1980–2018): it gradually increased from around 0.05 to around 0.1. Second, using the dispersion in MPK instead of in revenue TFP to measure misallocation is more appropriate given our model. Since in our model there is perfect competition and firms have constant returns to scale, all firms have the same ratio across inputs and revenue TFP is not distorted across firms. In other words, revenue TFP is equal to efficiency  $z$  for all firms, both constrained and unconstrained. In contrast, MPK is distorted by the intensity of financial frictions.

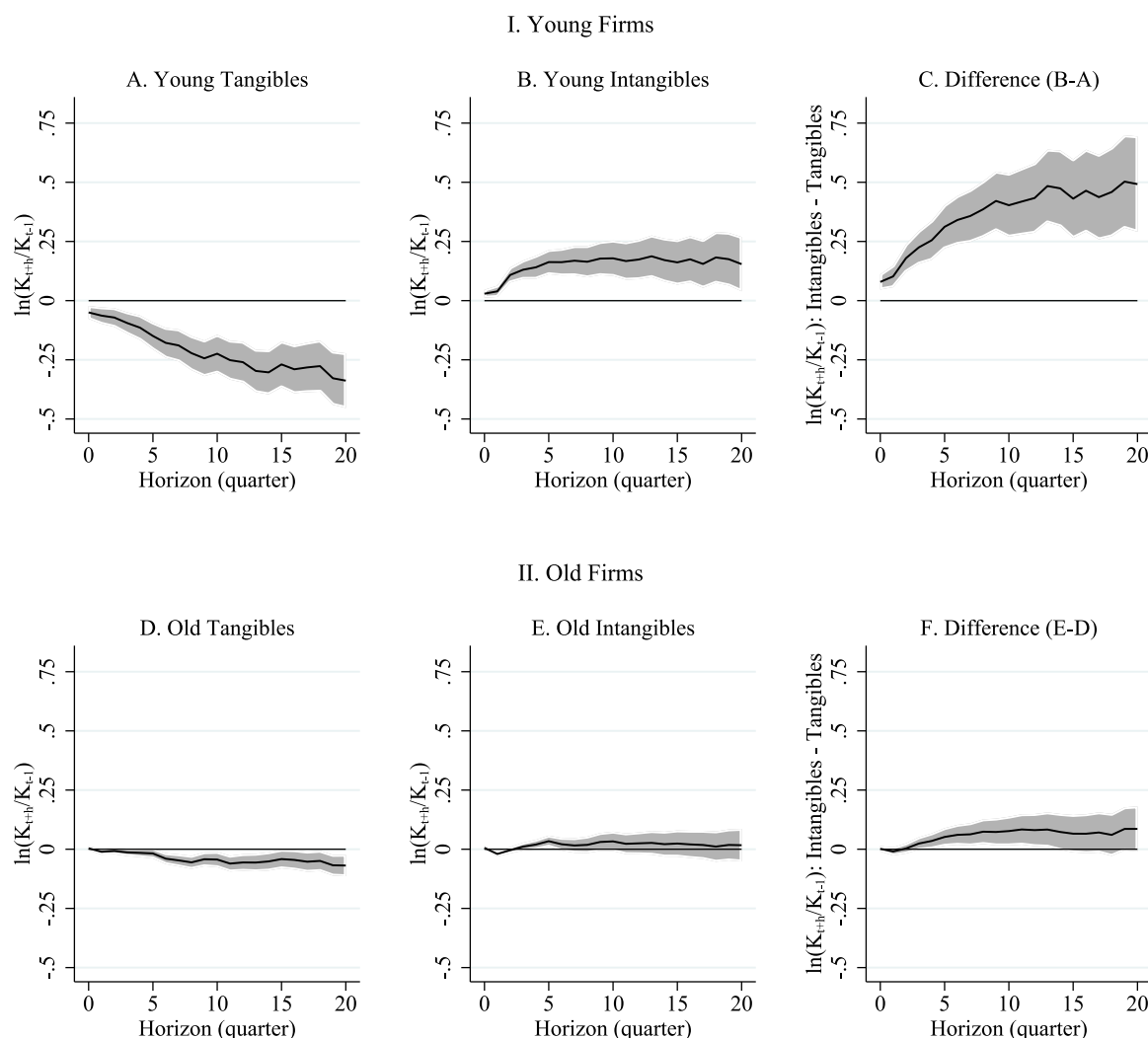


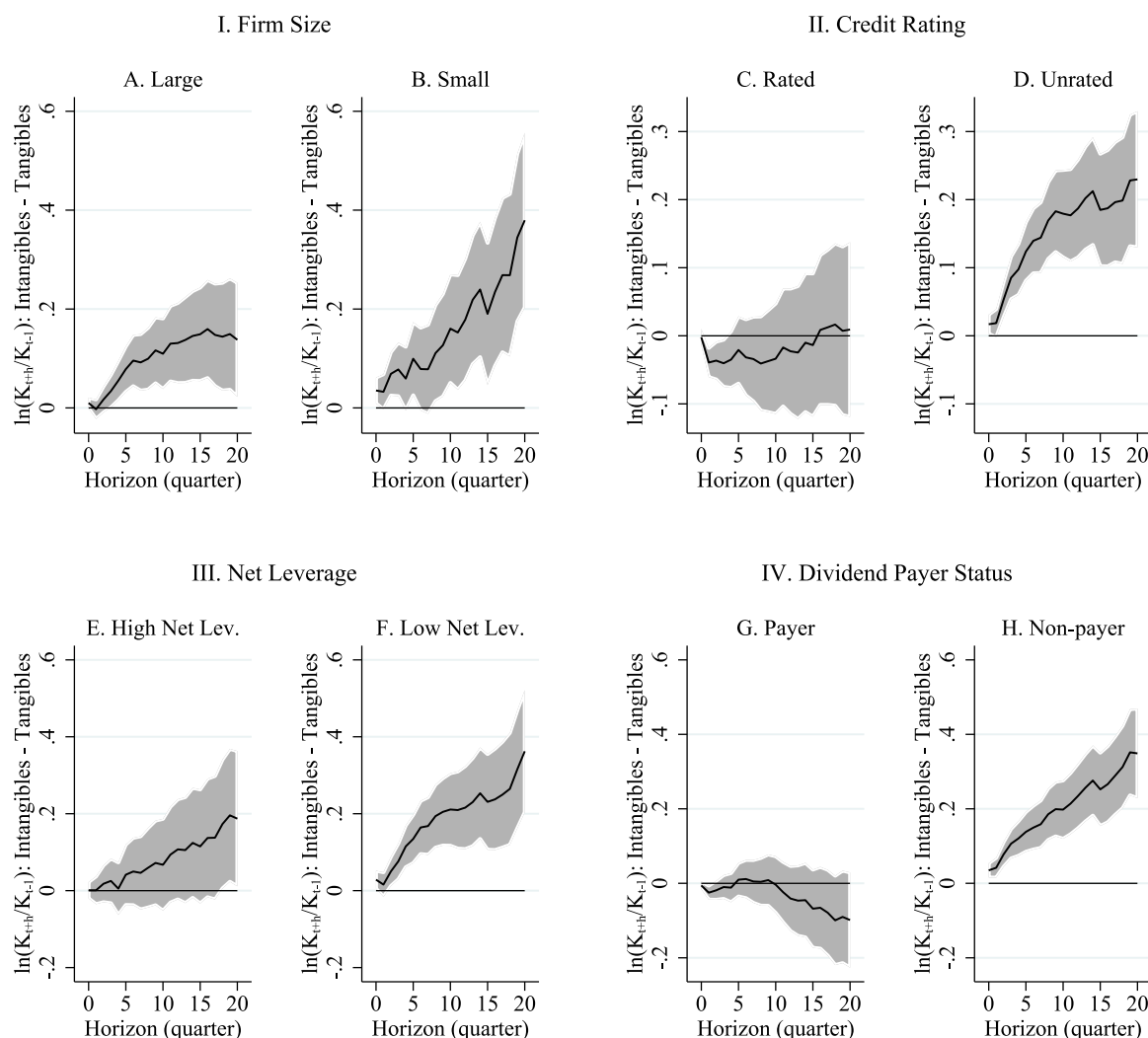
Fig. 7. Local projections: Response of investment by firm age. The left and middle panels display the impulse response function (IRF) for investment following a 100 bps surprise increase in the interest rate that results from running the local projection specification (1). The right panels display the difference between the middle and left panels; this difference is the estimate of the effect of intangibility (top tercile of intangibility compared to the bottom tercile) on the dynamic response of investment to an increase in the interest rate. The dependent variable is the difference between the log of total capital (tangible plus intangible) in period  $t + h$  and in period  $t - 1$ . Young firms are those that at the time of the interest rate shock are younger than 10 years, while mature firms are those that at the time of the shock are older than 30 years. We consider tangibles (intangibles) firms to be those with an intangible intensity in the bottom (top) tercile of their age group over the whole sample period. The monetary surprise in quarter  $t$ ,  $\Delta r_t$ , is calculated by adding up the monthly monetary policy shocks obtained from Jarocinski and Karadi (2020). We also introduce industry-year fixed effects and clustering of standard errors at the firm level. Firms in the sample are required to be active for at least five years after the monetary policy shock occurs. Shaded areas represent the 90% confidence intervals of the estimates.

distribution of the industry-wide ratio of intangible capital to total capital averaged across years.<sup>30</sup> The solid line displays the sample of low-intangible (bottom 33%) industries. The figure shows that capital productivity dispersion in recent decades has increased in intangibles sectors much more than in tangibles sectors. Since during the same period there has been a gradual fall in the real

<sup>30</sup> The sectors with high shares of intangible capital are: Chemicals and Allied Products; Industrial and Commercial Machinery and Computer Equipment; Electronic & Other Electrical Equipment & Components; Transportation Equipment; Measuring, Photographic, Medical, & Optical Goods, & Clocks; Miscellaneous Manufacturing Industries; Wholesale Trade - Durable Goods; Home Furniture, Furnishings and Equipment Stores; Miscellaneous Retail Business Services; and Engineering, Accounting, Research, and Management Services.

The sectors with low shares of intangible capital are: Oil and Gas Extraction; Food and Kindred Products; Paper and Allied Products; Rubber and Miscellaneous Plastic Products; Stone, Clay, Glass, and Concrete Products; Primary Metal Industries; Fabricated Metal Products; Wholesale Trade - Nondurable Goods; General Merchandise Stores; Food Stores; Apparel and Accessory Stores; and Eating and Drinking Places.



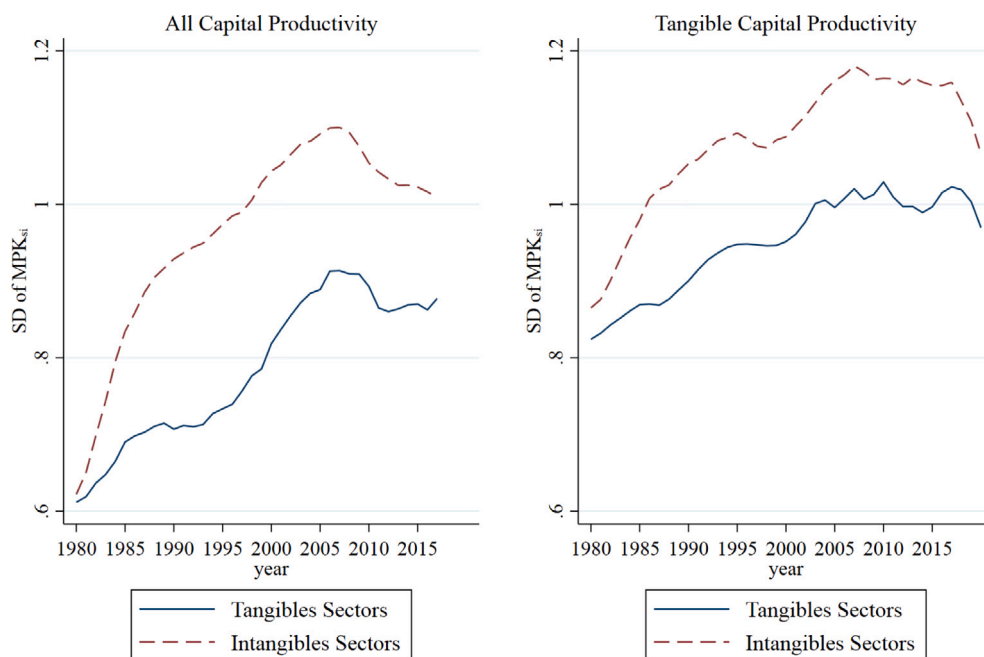


**Fig. 8.** Local projections: Effect of intangibility on the response of investment by degree of financial constraints (alternative measures). The charts display the estimate of the effect of intangibility on the dynamic response of investment to a 100 bps surprise increase in the interest rate in different subsets of firms based on the local projection specification (1). They plot the estimate of the coefficient on the term that interacts the interest rate shock with the dummy that takes value 1 when a firm is in the top tercile of intangibility, which represents the differential response of investment of a firm in the top tercile of intangibility compared to a firm in the bottom tercile. The dependent variable is the difference between the log of total capital (tangible plus intangible) in period  $t + h$  and in period  $t - 1$ . We consider tangibles (intangibles) firms to be those with an intangible intensity in the bottom (top) tercile of their age group over the whole sample period. The monetary surprise in quarter  $t$ ,  $\Delta r_t$ , is calculated by adding up the monthly monetary policy shocks obtained from Jarocinski and Karadi (2020). We also introduce industry-year fixed effects and clustering of standard errors at the firm level. Firms in the sample are required to be active for at least five years after the monetary policy shock occurs. Panels I: Large firms (panel A) are those in the top tercile of the distribution of firm real assets and small firms (panel B) are those in the bottom tercile. Panels II: Rated firms (panel C) are those that, at the time of the monetary policy shock, have a rating from either Moody's or Standard and Poor's (S&P) (excluding distressed rated firms with a rating equivalent to S&P's CCC rating or below). Unrated firms (panel D) are those without a rating from either Moody's or S&P. Panels III: Low (high) net leverage firms (respectively, panels E and F) are those in the bottom (top) tercile of the net leverage distribution. Panels IV: Firms are dividend payers (panel G) if they have paid dividends at some point in the year before the monetary policy shock, and non-payers (panel H) otherwise. Shaded areas represent the 90% confidence intervals of the estimates.

interest rate, from values of around 5% in the early 1980s to values around zero today, this finding is consistent with the model's simulations.<sup>31</sup>

One possible shortcoming of this finding is that intangible capital is likely to be mismeasured (Corrado et al., 2009; McGrattan and Prescott, 2010). Therefore, the rise in intangibles might itself cause an increase in dispersion if measurement errors are heterogeneous across firms (Bils et al., 2020). To control for this possibility, the right panel of Fig. 9 considers the dispersion of the marginal product of tangible capital only instead. Our model implies that using tangible capital instead of total capital is an appropriate way

<sup>31</sup> In unreported results, we found similar trends also for labor productivity and TFP. Details are available from the authors.



**Fig. 9.** Dispersion of the marginal product of capital in tangibles and intangibles industries. The figure explores the relationship between the rise in intangibles and dispersion in the marginal product of capital. The sample consists of U.S. firms covered by Compustat between 1980 and 2018, excluding utilities (SIC codes 4900–4949) and financials (SIC codes 6000–6999). We define intangible capital as the sum of knowledge capital and organizational capital. Intangibles (tangibles) sectors are those with average intangibility in the top (bottom) tercile of the distribution of sectoral intangibility during the sample period. We consider sectors at the 2-digit Standard Industrial Classification (SIC) level and drop those with less than 50 annual observations on average over the sample period. We measure output by sales. Capital productivity is measured as the log difference between firm output and the total capital stock (left panel) and as the log difference between firm output and the tangible capital stock (right panel). Misallocation is computed as the standard deviation of the marginal product of capital in each industry each period.

to measure the misallocation caused by the rise of intangibles. Furthermore, another advantage of using tangible capital is that, as argued by [Chiavari and Goraya \(2020\)](#), over time, the share of tangible capital in production has been roughly constant for publicly listed firms in the U.S. (our sample).<sup>32</sup> We compute the marginal product of tangible capital applying their estimated tangible capital elasticity of output of 0.215. The results using the misallocation of tangible capital confirm our findings using the misallocation of total capital.

#### 4.6.3 Additional robustness checks

In [Fig. A.7](#) in [Appendix B](#), we verify that our main results are robust to measuring intangibility at the industry level. We confirm that interest rate shocks are less contractionary for firms in more intangible industries than for firms in more tangible industries, and that this result is stronger for younger firms. In [Fig. A.6](#), we provide a robustness check in which we estimate the sensitivity of the intangible to total capital ratio to the interest rate to verify whether the interest rate shocks affect differentially the two types of capital. In our model, the intangibles to tangibles ratio is fixed by assumption. Nonetheless, if we assumed a more flexible production function—for example, a Cobb–Douglas function in the two types of capital—then such a ratio would decrease in the intensity of financial frictions, measured as the wedge  $\Delta^c$  in [Eq. \(42\)](#). An increase in the interest rate reduces the wedge, thus implying that the intangible ratio should increase. However, we expect this effect, if present, to be small, because  $r$  is small relative to  $\delta$  and because intangible capital is more sticky than tangible capital, and this factor counteracts the previous effect. [Fig. A.6](#) shows that in response to an interest rate shock, the intangible ratio changes very little, increasing by around 0.5 percentage point. Overall, this result is consistent with the mechanism of the model.

## 5 Conclusion

The widespread emergence of intangible technologies in recent decades and the associated changes in corporate financing patterns may have significantly affected the relationship between interest rates and corporate investment. In our theoretical framework, as in reality, a shift toward intangible capital in production is followed by a shift in the corporate sector toward a net saving position, because intangible capital has a low collateral value. We show that, as a result, firms' ability to purchase intangible

<sup>32</sup> Their findings suggest that the increased factor share of intangible capital has occurred in part at the expense of the labor share.

capital is impaired by low interest rates because low rates slow down the accumulation of savings, and our empirical analysis strongly supports this prediction. Furthermore, we also present empirical evidence consistent with the misallocation implications of our model.

Our insights have relevant policy implications. On the one hand, the mechanisms described in this paper suggest that the rise in intangibles dampens the effectiveness of expansionary monetary policy shocks. On the other hand, the negative externality in households' and firms' excessive saving decisions might introduce a role for fiscal policy to discourage such saving.

## Appendix A. Solution of the general equilibrium model

### A.1. Optimization of unconstrained firms

The first order conditions for unconstrained firms are:

$$\frac{e_{u,t}}{l_{u,t}} = \frac{1-\xi}{\xi(1-\alpha)} \frac{w_{u,t}}{w_{u,t}^e}, \quad (43)$$

and

$$\frac{l_{u,t}}{k_{u,t}} = \left[ \frac{\xi(1-\alpha)z_{u,t} \left( \frac{1-\xi}{\xi(1-\alpha)} \frac{w_{u,t}}{w_{u,t}^e} \right)^{1-\xi}}{w_{u,t}} \right]^{\frac{1}{\alpha\xi}}. \quad (44)$$

As for the constrained firms, these firms are also able to invest with probability  $\gamma$ , and the first order condition for capital for investing firms is:

$$\alpha\xi z_{u,t} e_{u,t}^{1-\xi} l_{u,t}^{\xi(1-\alpha)} k_{u,t}^{\alpha\xi-1} = r_{t+1} + \delta. \quad (45)$$

### A.2. Definition of equilibrium

Using (33), (35), (34) and the aggregate versions of (23), (24), (43), (44) and (45) we obtain:

$$L_{u,t} = \frac{\xi(1-\alpha)}{1-\xi} \frac{w_{u,t}^e}{w_t}, \quad (46)$$

$$L_{u,t} = \left[ \frac{\xi(1-\alpha)z_{u,t} \left( \frac{1-\xi}{\xi(1-\alpha)} \frac{w_t}{w_{u,t}^e} \right)^{1-\xi}}{w_t} \right]^{\frac{1}{\alpha\xi}} K_{u,t}, \quad (47)$$

$$1 - L_{u,t} = \frac{\xi(1-\alpha)}{1-\xi} \frac{w_{c,t}^e}{w_t}, \quad (48)$$

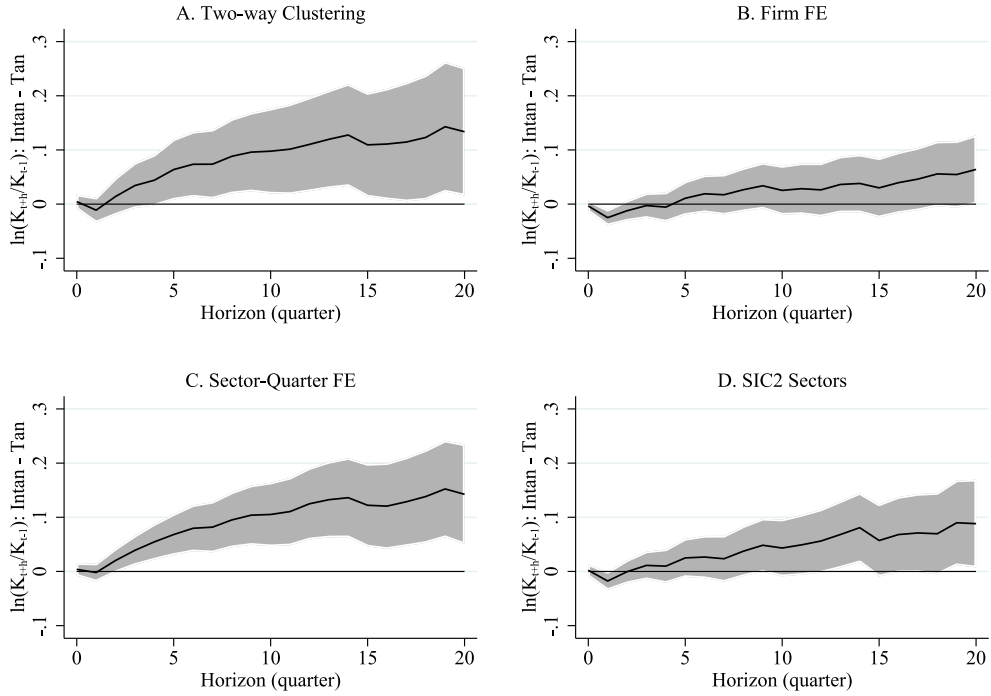
$$1 - L_{u,t} = \left[ \frac{\xi(1-\alpha)z_{c,t} \left( \frac{1-\xi}{\xi(1-\alpha)} \frac{w_t}{w_{c,t}^e} \right)^{1-\xi}}{w_t} \right]^{\frac{1}{\alpha\xi}} K_{c,t}, \quad (49)$$

$$\alpha\xi z_{u,t} L_{u,t}^{\xi(1-\alpha)} K_{u,t}^{\alpha\xi-1} = r_{t+1} + \delta, \quad (50)$$

$$Y_{c,t} = z_{c,t} \left[ K_{c,t}^\alpha L_{c,t}^{(1-\alpha)} \right]^\xi, \quad (51)$$

and

$$Y_{u,t} = z_{u,t} \left[ K_{u,t}^\alpha L_{u,t}^{(1-\alpha)} \right]^\xi. \quad (52)$$



**Fig. A.1.** Local projections: Robustness of results in Fig. 2. The charts display the estimate of the effect of intangibility on the dynamic response of investment to a 100 bps surprise increase in the interest rate in different subsets of firms based on the local projection specification (1). They plot the estimate of the coefficient on the term that interacts the interest rate shock with the dummy that takes value 1 when a firm is in the top tercile of intangibility, which represents the differential response of investment of a firm in the top tercile of intangibility compared to a firm in the bottom tercile. The dependent variable is the difference between the log of total capital (tangible plus intangible) in period  $t + h$  and in period  $t - 1$ . We consider tangibles (intangibles) firms to be those with an intangible intensity in the bottom (top) tercile of their age group over the whole sample period. The monetary surprise in quarter  $t$ ,  $\Delta r_t$ , is calculated by adding up the monthly monetary policy shocks obtained from Jarocinski and Karadi (2020). Panel A introduces two-way clustering at the firm and year level. Panel B replaces sector-year fixed effects with firm fixed effects. Panel C replaces sector-year fixed effects with sector-quarter fixed effects. Panel D considers sectors as defined by 2-digit SIC codes. All regressions control for firm age, size, and lagged sales growth, both independently and interacted with the interest rate shock. Firms in the sample are required to be active for at least five years after the monetary policy shock occurs. Shaded areas represent the 90% confidence intervals of the estimates.

### A.3. Steady state

From Eqs. (46)–(52), without the time subscript, we obtain:

$$L_u = \frac{\xi(1-\alpha)}{1-\xi} \frac{w_u^e}{w}, \quad (53)$$

$$L_u = \left[ \frac{\xi(1-\alpha)z_u \left( \frac{1-\xi}{\xi(1-\alpha)} \frac{w}{w_u^e} \right)^{1-\xi}}{w} \right]^{\frac{1}{\alpha\xi}} K_u, \quad (54)$$

$$1 - L_u = \frac{\xi(1-\alpha)}{1-\xi} \frac{w_c^e}{w}, \quad (55)$$

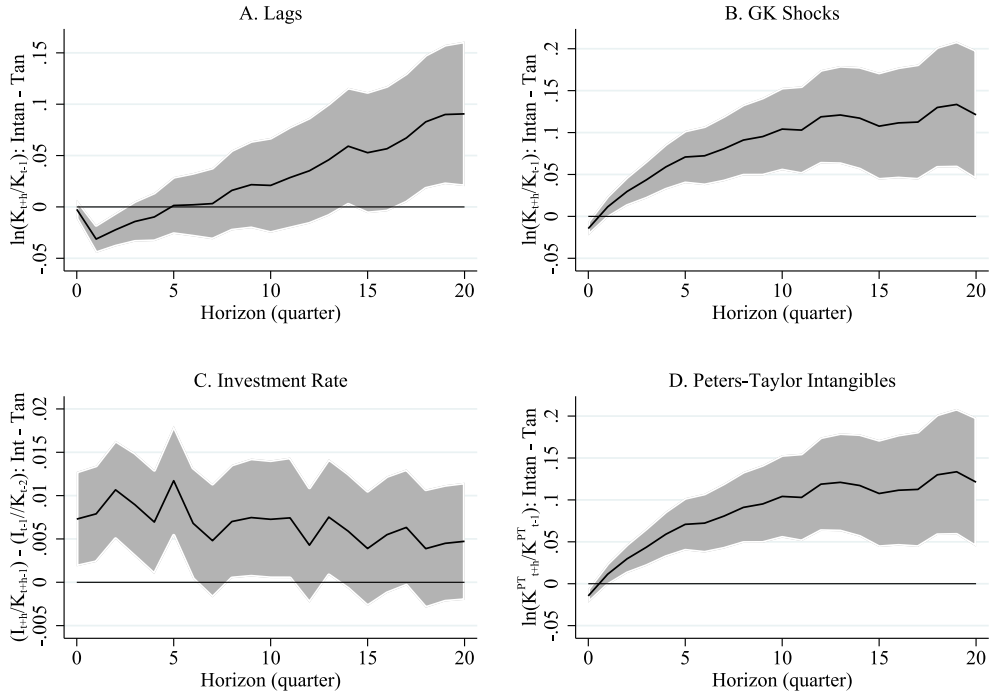
$$1 - L_u = \left[ \frac{\xi(1-\alpha)z_c \left( \frac{1-\xi}{\xi(1-\alpha)} \frac{w}{w_c^e} \right)^{1-\xi}}{w} \right]^{\frac{1}{\alpha\xi}} K_c, \quad (56)$$

$$\alpha\xi z_u L_u^{\xi(1-\alpha)} K_u^{\alpha\xi-1} = r + \delta, \quad (57)$$

$$Y_c = z_c [K_c^\alpha L_c^{(1-\alpha)}]^\xi, \quad (58)$$

and

$$Y_u = z_u [K_u^\alpha L_u^{(1-\alpha)}]^\xi. \quad (59)$$



**Fig. A.2.** Local projections: Robustness of results in Fig. 2 (Continued). The charts display the estimate of the effect of intangibility on the dynamic response of investment to a 100 bps surprise increase in the interest rate in different subsets of firms based on the local projection specification (1). They plot the estimate of the coefficient on the term that interacts the interest rate shock with the dummy that takes value 1 when a firm is in the top tercile of intangibility, which represents the differential response of investment of a firm in the top tercile of intangibility compared to a firm in the bottom tercile. The dependent variable in panels A, B, and D is the difference between the log of total capital (tangible plus intangible) in period  $t + h$  and in period  $t - 1$ . We consider tangibles (intangibles) firms to be those with an intangible intensity in the bottom (top) tercile of their age group over the whole sample period. In panels A, C, and D, the monetary surprise in quarter  $t$ ,  $\Delta r_t$ , is calculated by adding up the monthly monetary policy shocks obtained from Jarocinski and Karadi (2020). Panel A introduces lags of the dependent variable (the same number as the horizon of each estimate of the local projection). Panel B considers an alternative measure of monetary policy shocks as in Gertler and Karadi (2015). Panel C uses as dependent variable the change in the investment rate (defined as investment over previous period's capital stock). Panel D considers an alternative measure of intangible intensity as in Peters and Taylor (2017), which adds the balance sheet measure of intangible capital and goodwill to the measure obtained using the perpetual inventory method. All regressions control for firm age, size, and lagged sales growth, both independently and interacted with the interest rate shock. Firms in the sample are required to be active for at least five years after the monetary policy shock occurs. Shaded areas represent the 90% confidence intervals of the estimates.

Aggregate profits of constrained and unconstrained firms are, respectively:

$$\Pi_c = z_c [K_c^\alpha L_c^{(1-\alpha)}]^\xi - w_c^e - w(1 - L_u), \quad (60)$$

and

$$\Pi_u = z_u [K_u^\alpha L_u^{(1-\alpha)}]^\xi - w_u^e - wL_u. \quad (61)$$

The dividends of unconstrained firms are their profits, while the aggregate dividends of constrained firms are equal to the savings distributed by the fraction  $\psi$  of exiting firms minus the endowment of new firms  $\psi W_0$ . Therefore:

$$D = \Pi_u + \psi (\Pi_c + (1 + r)A + K_c) - \psi W_0. \quad (62)$$

Summing up,  $Y_c, Y_u, L_u, K_u, K_c, w, w_u^e, w_c^e, A$  and  $D$  are jointly determined by (53)–(59), (38), (40) and (62). Given  $K_c$  and  $K_u$ , Eq. (11) determines aggregate holdings of tangible and intangible capital:

$$K_{c,I} = \mu K_c; \quad K_{c,T} = (1 - \mu) K_c \quad (63)$$

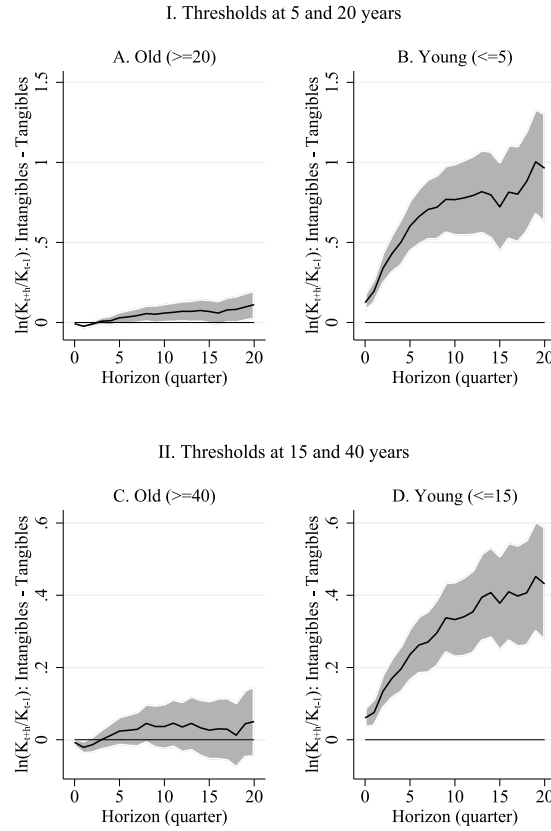
$$K_{u,I} = \mu K_u; \quad K_{u,T} = (1 - \mu) K_u \quad (64)$$

And labor in the productive sector is:

$$L_c = 1 - L_u.$$

Finally, aggregate borrowing from the representative household  $B$  is equal to aggregate savings of the corporate sector, or

$$B = A. \quad (65)$$



**Fig. A.3.** Local projections: Response of investment by firm age using different age thresholds. The charts display the estimate of the effect of intangibility on the dynamic response of investment to a 100 bps surprise increase in the interest rate in different subsets of firms based on the local projection specification (1). They plot the estimate of the coefficient on the term that interacts the interest rate shock with the dummy that takes value 1 when a firm is in the top tercile of intangibility, which represents the differential response of investment of a firm in the top tercile of intangibility compared to a firm in the bottom tercile. The dependent variable is the difference between the log of total capital (tangible plus intangible) in period  $t + h$  and in period  $t - 1$ . Young firms in panel B (D) are those that at the time of the interest rate shock are younger than 5 (15) years, while mature firms in panel A (C) are those that at the time of the shock are older than 20 (40) years. We consider tangibles (intangibles) firms to be those with an intangible intensity in the bottom (top) tercile of their age group over the whole sample period. The monetary surprise in quarter  $t$ ,  $\Delta r_t$ , is calculated by adding up the monthly monetary policy shocks obtained from Jarocinski and Karadi (2020). We also introduce industry-year fixed effects and clustering of standard errors at the firm level. Firms in the sample are required to be active for at least five years after the monetary policy shock occurs. Shaded areas represent the 90% confidence intervals of the estimates.

Household consumption for each type of agent is determined by Eq. (37):

$$C^j = \frac{1}{3} D + W^j - rB^j, \quad (66)$$

where we assume that total dividends  $D$  are divided in equal parts between households, entrepreneurs in constrained firms, and entrepreneurs in unconstrained firms.

#### A.4. Derivation of the aggregate capital of constrained firms

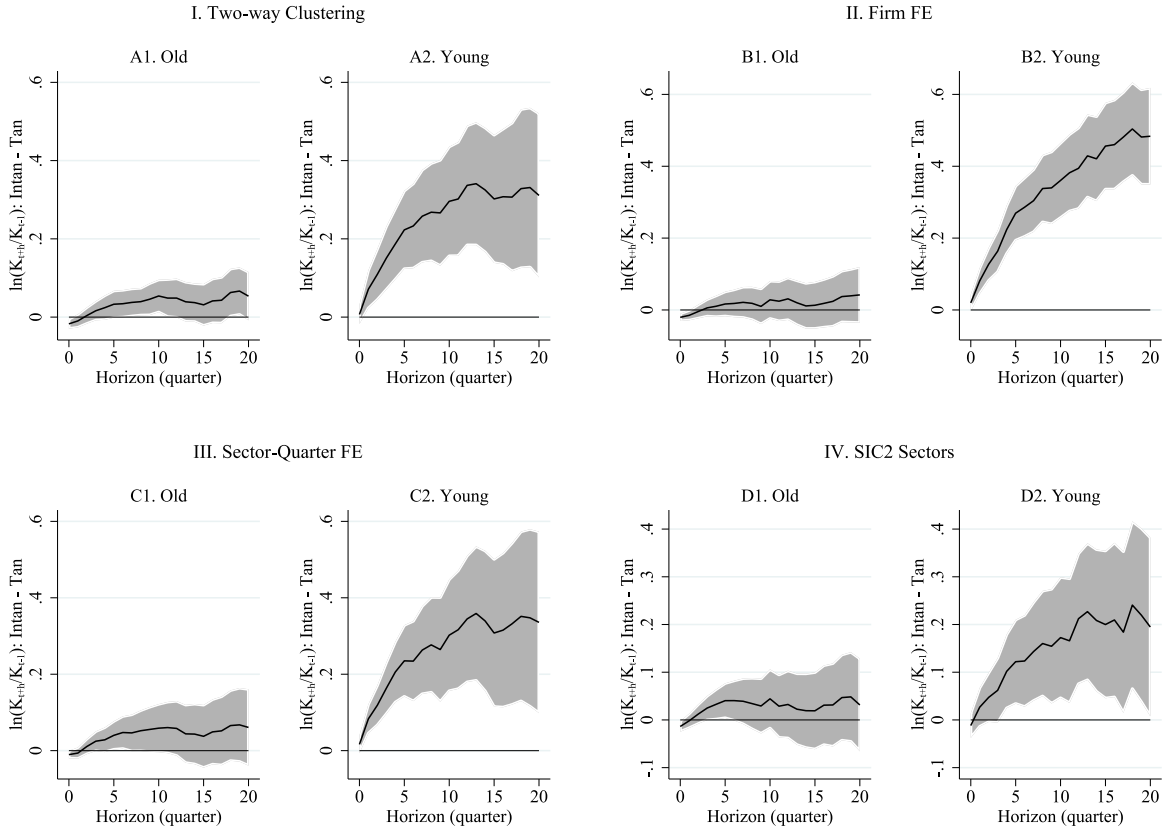
Aggregate wealth  $W_t$  of the constrained firms at the beginning of period  $t$  is their initial financial wealth  $(1 + r_t)A_t$  plus profits  $\Pi_{c,t}$  as defined in Eq. (39) plus the residual value of their capital:

$$W_t \equiv \Pi_{c,t} + (1 + r_t)A_t + (1 - \delta)K_{c,t}. \quad (67)$$

Aggregate capital is determined as follows. A fraction  $(1 - \psi)$  of constrained firms continue activity, and a fraction  $\gamma$  of those have an investment opportunity. They have a fraction  $(1 - \psi)\gamma$  of total wealth  $W_t$ , which they use to buy capital. A fraction  $\psi$  of constrained firms exit, and are replaced by an equal number of firms with an initial endowment of  $W_0$  and no capital. A fraction  $\gamma$  of new entrants invest. Therefore, we define total capital in the hands of investing agents at the end of period  $t$ , expressed in aggregate terms, as  $\gamma K_{I,t+1}^{INV}$ , where  $K_{I,t+1}^{INV}$  is

$$K_{I,t+1}^{INV} = \frac{(1 - \psi)W_t + \psi W_0}{1 - \frac{\theta}{1 + r_{t+1}}}. \quad (68)$$





**Fig. A.4.** Local projections: Robustness of results in Fig. 3. The charts display the estimate of the effect of intangibility on the dynamic response of investment to a 100 bps surprise increase in the interest rate in different subsets of firms based on the local projection specification (1). They plot the estimate of the coefficient on the term that interacts the interest rate shock with the dummy that takes value 1 when a firm is in the top tercile of intangibility, which represents the differential response of investment of a firm in the top tercile of intangibility compared to a firm in the bottom tercile. The dependent variable is the difference between the log of total capital (tangible plus intangible) in period  $t + h$  and in period  $t - 1$ . Young firms are those that at the time of the interest rate shock are younger than 10 years, while mature firms are those that at the time of the shock are older than 30 years. We consider tangibles (intangibles) firms to be those with an intangible intensity in the bottom (top) tercile of their age group over the whole sample period. The monetary surprise in quarter  $t$ ,  $\Delta r_t$ , is calculated by adding up the monthly monetary policy shocks obtained from Jarocinski and Karadi (2020). Panels A1 and A2 introduce two-way clustering at the firm and year level. Panels B1 and B2 replace sector-year fixed effects with firm fixed effects. Panels C1 and C2 replace sector-year fixed effects with sector-quarter fixed effects. Panels D1 and D2 consider sectors as defined by 2-digit SIC codes. All regressions control for firm age, size, and lagged sales growth, both independently and interacted with the interest rate shock. Firms in the sample are required to be active for at least five years after the monetary policy shock occurs. Shaded areas represent the 90% confidence intervals of the estimates.

The  $(1 - \gamma)$  fraction of surviving firms that do not have an investment opportunity continue to hold their depreciated capital. Therefore, aggregate capital for the next period is equal to

$$K_{c,t+1} = \gamma K_{I,t+1}^{INV} + (1 - \delta)(1 - \psi)(1 - \gamma) K_{c,t}. \quad (69)$$

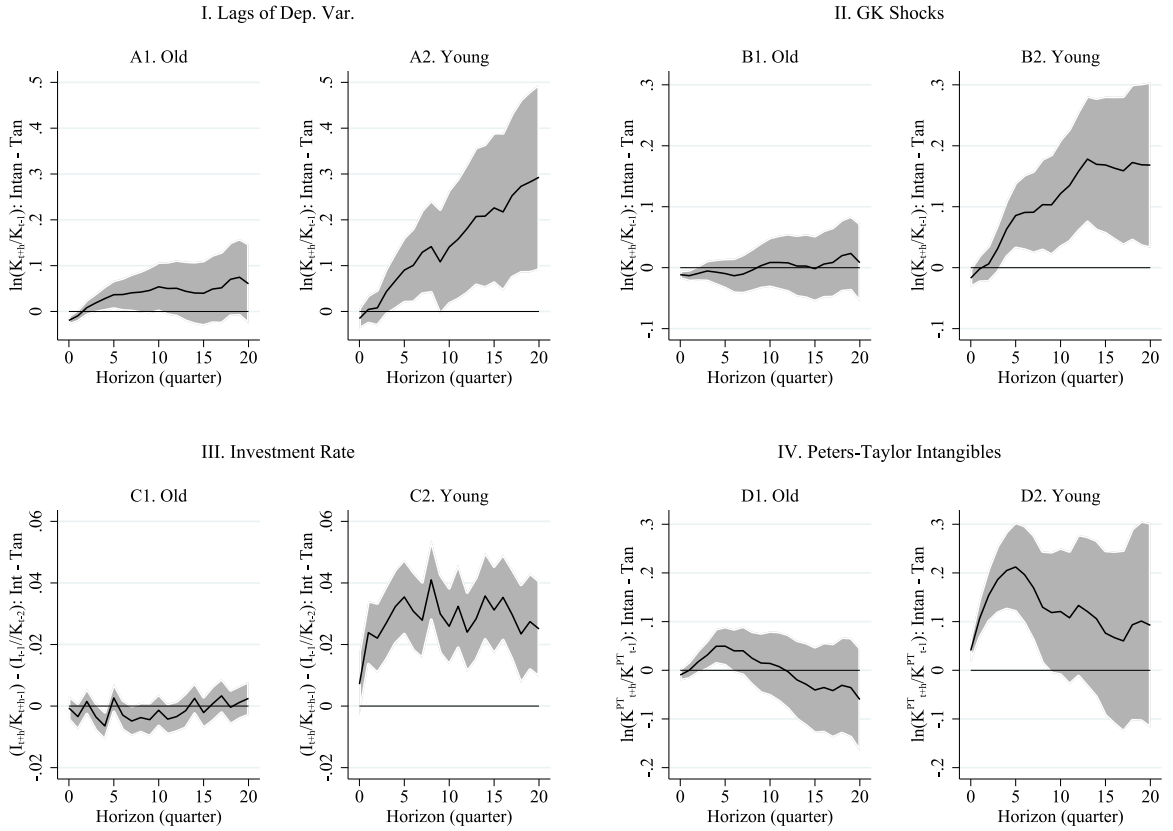
By substituting  $K_{I,t+1}^{INV}$  and  $W_t$  into  $K_{c,t+1}$  and evaluating it in the steady state we obtain:

$$K_{c,t+1} = \frac{\gamma(1 - \psi) [\Pi_{c,t} + (1 + r_t)A_t + (1 - \delta) K_{c,t}] + \psi W_0}{1 - \frac{\theta}{1+r_{t+1}}} + (1 - \delta)(1 - \psi)(1 - \gamma) K_{c,t}. \quad (70)$$

Finally, rearranging, we get:

$$K_c = \frac{\gamma [(1 - \psi) (\Pi_c + (1 + r)A) + \psi W_0]}{\left(1 - \frac{\theta}{1+r}\right) [\delta + \psi(1 - \delta)] - \frac{\theta}{1+r} \gamma(1 - \delta)(1 - \psi)}. \quad (71)$$

We claim in the text that constraint (25) holds at the aggregate as well as at the individual level. As shown in Eqs. (23) and (24), all constrained firms choose the same ratios of inputs, even if their levels of inputs are different. This is a straightforward implication of the assumption of Cobb–Douglas production functions and unconstrained labor choices, as is the fact that the marginal products



**Fig. A.5.** Local projections: Robustness of results in Fig. 3 (Continued). The charts display the estimate of the effect of intangibility on the dynamic response of investment to a 100 bps surprise increase in the interest rate in different subsets of firms based on the local projection specification (1). They plot the estimate of the coefficient on the term that interacts the interest rate shock with the dummy that takes value 1 when a firm is in the top tercile of intangibility, which represents the differential response of investment of a firm in the top tercile of intangibility compared to a firm in the bottom tercile. The dependent variable in panels A1/A2, B1/B2, and D1/D2 is the difference between the log of total capital (tangible plus intangible) in period  $t + h$  and in period  $t - 1$ . Young firms are those that at the time of the interest rate shock are younger than 10 years, while mature firms are those that at the time of the shock are older than 30 years. We consider tangibles (intangibles) firms to be those with an intangible intensity in the bottom (top) tercile of their age group over the whole sample period. In panels A1/A2, C1/C2, and D1/D2, the monetary surprise in quarter  $t$ ,  $\Delta r_t$ , is calculated by adding up the monthly monetary policy shocks obtained from Jarocinski and Karadi (2020). Panels A1 and A2 introduce lags of the dependent variable (the same number as the horizon of each estimate of the local projection). Panels B1 and B2 consider an alternative measure of monetary policy shocks as in Gertler and Karadi (2015). Panels C1 and C2 use as dependent variable the change in the investment rate (defined as investment over previous period's capital stock). Panels D1 and D2 consider an alternative measure of intangible intensity as in Peters and Taylor (2017), which adds the balance sheet measure of intangible capital and goodwill to the measure obtained using the perpetual inventory method. All regressions control for firm age, size, and lagged sales growth, both independently and interacted with the interest rate shock. Firms in the sample are required to be active for at least five years after the monetary policy shock occurs. Shaded areas represent the 90% confidence intervals of the estimates.

of capital across constrained firms equalize. The aggregate marginal product of capital can be expressed as

$$\frac{\partial Y_c}{\partial K_c} = \alpha \xi z_c L_c^{\xi(1-\alpha)} K_c^{\alpha\xi-1}. \quad (72)$$

The requirement that inequality (25) holds at the individual firm level requires that condition

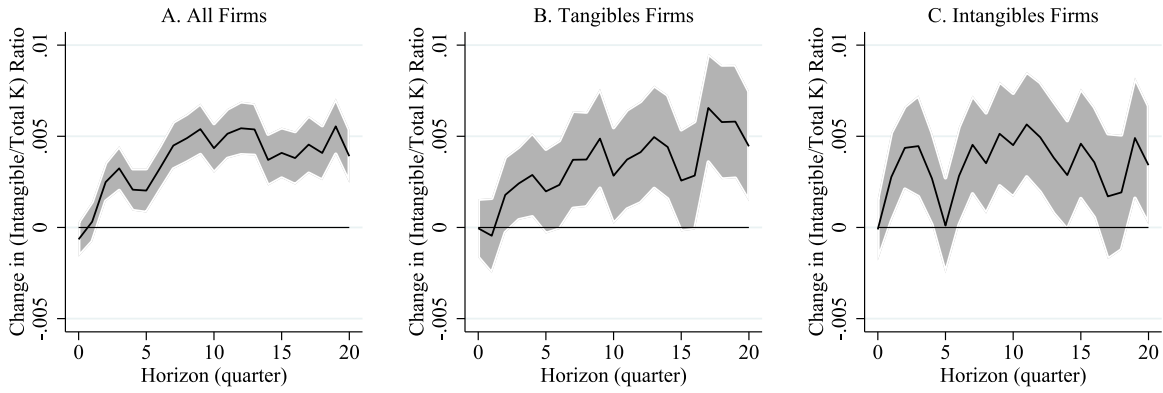
$$\alpha \xi z_c L_c^{\xi(1-\alpha)} K_c^{\alpha\xi-1} > r + \delta \quad (73)$$

is satisfied at the aggregate level. The fact that firms exit the economy at an exogenous rate  $\psi$  and new firms enter with a small amount of wealth (lower than the average sector wealth) ensures that the constrained sector as a whole never accumulates enough net worth to grow out of their binding borrowing constraint. We check in all of our simulations that parameter values are such that  $K_c$  is low enough and  $r + \delta$  high enough to ensure that this is the case.

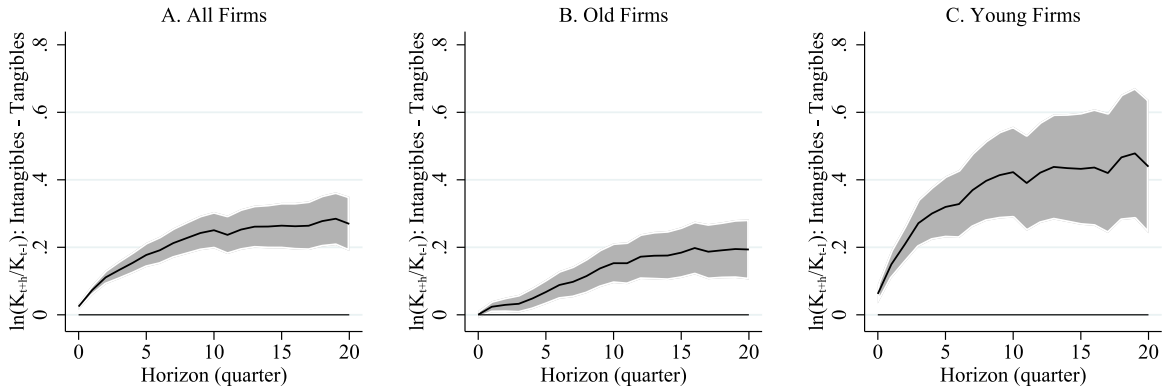
#### A.5. Proof that a firm does not have an incentive to deviate and adopt a tangible technology in the intangibles economy

Consider the production function of a generic firm:

$$y_{j,t} = z_{j,t} e^{1-\xi} \left( l_{j,t}^{1-\alpha} k_{j,t}^{\alpha} \right)^{\xi},$$



**Fig. A.6.** Local projections: Response of the intangible share to monetary policy shocks. The panels display the impulse response function (IRF) for the intangible share following a 100 bps surprise increase in the interest rate. The IRFs result from running the local projection specification (1) but using as dependent variable the change in firm-level intangible capital over total capital. Regressions control for firm age, size, and lagged sales growth, both independently and interacted with the interest rate shock. We consider tangibles (intangibles) firms to be those with an intangible intensity in the bottom (top) tercile of their age group over the whole sample period. The monetary surprise in quarter  $t$ ,  $\Delta r_t$ , is calculated by adding up the monthly monetary policy shocks obtained from Jarocinski and Karadi (2020). We also introduce industry-year fixed effects and clustering of standard errors at the firm level. Firms in the sample are required to be active for at least five years after the monetary policy shock occurs. Shaded areas represent the 90% confidence intervals of the estimates.



**Fig. A.7.** Local projections: Intangibility measured at the industry level. The charts display the estimate of the effect of intangibility on the dynamic response of investment to a 100 bps surprise increase in the interest rate in different subsets of firms based on the local projection specification (1). They plot the estimate of the coefficient on the term that interacts the interest rate shock with the dummy that takes value 1 when a firm is in the top tercile of intangibility, which represents the differential response of investment of a firm in the top tercile of intangibility compared to a firm in the bottom tercile. The dependent variable is the difference between the log of total capital (tangible plus intangible) in period  $t + h$  and in period  $t - 1$ . We consider tangibles (intangibles) firms to be those that belong to an industry with an average intangible intensity in the bottom (top) tercile of the industry average intangibility distribution over the whole sample period. Young firms are those that at the time of the interest rate shock are younger than 10 years, while mature firms are those that at the time of the shock are older than 30 years. The monetary surprise in quarter  $t$ ,  $\Delta r_t$ , is calculated by adding up the monthly monetary policy shocks obtained from Jarocinski and Karadi (2020). We also introduce industry-year fixed effects and clustering of standard errors at the firm level. Firms in the sample are required to be active for at least five years after the monetary policy shock occurs. Shaded areas represent the 90% confidence intervals of the estimates.

where  $j \in \{c, u\}$  indicates the type (constrained or unconstrained). Since both types choose labor inputs optimally, deriving the first order condition with respect to profits  $\pi_{j,t+1} = y_{j,t+1} - w_j^e e_{j,t+1} - w_j^l l_{j,t+1}$ , we obtain the optimal choices of  $e_{j,t+1}$  and  $l_{j,t+1}$  given the choice of  $k_{t+1}$  and the equilibrium wages  $w_j^e$  and  $w_j^l$ . Substituting these value into the profit function  $\pi_{c,t+1}$ , we obtain an expression for the profits per unit of capital:

$$\frac{\pi_{c,t+1}}{k_{t+1}} = z_j^{\frac{1}{\alpha\xi}} \Theta,$$

where  $\Theta$  is a nonlinear function of  $\alpha, \xi$ , and equilibrium wages  $w_j^e$  and  $w_j^l$ . Consider an unconstrained firm first. In our benchmark calibration, Eq. (41) evaluated at the benchmark value of  $\eta = 0.85$  implies that productivity  $z_u$  is 38.25% higher if a firm adopts an intangible technology than if it adopts a tangible one, and therefore, given that  $\xi = 0.85$  and  $\alpha = 0.4$ , the term  $z_u^{\frac{1}{\alpha\xi}}$  is 61.4% higher. In other words, for an unconstrained firm in the intangibles economy, deviating and adopting a tangible technology would reduce profits per unit of capital  $\frac{\pi_{c,t+1}}{k_{t+1}}$  by 61.4%, and would reduce total profits by even more, since optimal capital would also fall. For a constrained firm in the intangibles economy, the calculation is different, because we need to take into account that investment

capacity  $k_{c,t+1}$  is constrained. In other words, switching to a tangible technology would have the same reduction in  $\frac{\pi_{c,t+1}}{k_{t+1}}$ , but would also allow this firm to borrow more, since  $\theta$ , the collateral value of total capital, would increase. To evaluate this trade-off, consider a constrained firm that has net worth  $W_t$ . If this firm has an investment opportunity, then:

$$k_{c,t+1} = \frac{1}{1 - \frac{\theta}{1+r}} W_t,$$

which implies:

$$\frac{k_{c,t+1}}{W_t} = \frac{1+r}{1+r-\theta}.$$

It follows that:

$$\frac{\pi_{c,t+1}}{k_{t+1}} \frac{k_{c,t+1}}{W_t} = \frac{\pi_{c,t+1}}{W_t} = z_j^{\frac{1}{\alpha\bar{\epsilon}}} \Theta \frac{1+r}{1+r-\theta}. \quad (74)$$

Eq. (74) shows profits per units of liquid wealth  $W_t$ . The term  $\frac{1+r}{1+r-\theta}$  represents the advantage of tangible capital in allowing an higher leverage and reducing the downpayment necessary to buy capital. It is the highest when  $r$  is lowest, so assuming  $r = 0$  (the lowest value we consider in our comparative static exercise):

$$\frac{\pi_{c,t+1}}{W_t} = \Theta \left[ z_j^{\frac{1}{\alpha\bar{\epsilon}}} \frac{1}{1-\theta} \right].$$

From our calibration we obtain that  $\theta$ , the collateral value of total capital, is equal to 0.76 for the tangible technology and to 0.445 for the intangible technology. Plugging in these values in Eq. (74) above we obtain that the term  $\left[ z_j^{\frac{1}{\alpha\bar{\epsilon}}} \frac{1}{1-\theta} \right]$  falls from 4.67 to 4.17 for a constrained firm in the intangibles economy that deviates and adopts a tangible technology. This happens because the fall in  $z_j^{\frac{1}{\alpha\bar{\epsilon}}}$  more than compensates for the increase in  $\frac{1}{1-\theta}$ . Summing up, under the equilibrium allocation conditional on the chosen parameter values, no firm in the intangibles economy would find it optimal to deviate and adopt a tangibles technology instead.

## Appendix B. Additional empirical results

In this Appendix, Fig. A.1-Fig. A.7 show several robustness checks of the results presented in the paper, as explained in Sections 2.3, 4.6.1, and 4.6.3.

## Appendix C. Supplementary data

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.euroecorev.2021.103987>.

## References

- Aghion, P., Bergeaud, A., Clette, G., Lecat, R., Maghin, H., 2019. Coase lecture-the inverted-u relationship between credit access and productivity growth. *Economica* 86 (341), 1–31.
- Almeida, H., Campello, M., 2007. Financial constraints, asset tangibility, and corporate investment. *Rev. Financ. Stud.* 20 (5), 1429–1460.
- Altavilla, C., Burlon, L., Giannetti, M., Holton, S., 2021. Is there a zero lower bound? The effects of negative policy rates on banks and firms. *J. Financ. Econ.* Forthcoming.
- Altunkılıç, O., Hansen, Robert S., 2000. Are there economies of scale in underwriting fees? Evidence of rising external financing costs. *Rev. Financ. Stud.* 13 (1), 191–218. <http://dx.doi.org/10.1093/rfs/13.1.191>, 2000.
- Amable, B., Chatelain, J.B., Ralf, K., 2010. Patents as collateral. *J. Econom. Dynam. Control* 34 (6), 1092–1104.
- Armenter, R., Hnatkovska, V., 2017. Taxes and capital structure: Understanding firms' savings. *J. Monetary Econ.* 87, 13–33.
- Asriyan, V., Laeven, L., Martin, A., der Ghote, A. Van, Vanasco, V., 2021. Falling Interest Rates and Credit Misallocation: Lessons from General Equilibrium. Mimeo.
- Bacchetta, P., Benhima, K., 2015. The demand for liquid assets, corporate saving, and international capital flows. *J. Eur. Econom. Assoc.* 13 (6), 1101–1135.
- Bacchetta, P., Benhima, K., Kalantzis, Y., 2020. Money and capital in a persistent liquidity trap. *J. Monetary Econ.* 116, 70–87.
- Bacchetta, P., Benhima, K., Poilly, C., 2019. Corporate cash and employment. *Amer. Econ. J.: Macroecon.* 11 (3), 30–66.
- Baldwin, R., Teulings, C., 2014. Secular Stagnation: Facts, Causes and Cures. Centre for Economic Policy Research-CEPR, London.
- Bean, C., 2016. Living with low for long. *Econ. J.* 126 (592), 507–522.
- Begenau, J., Palazzo, B., 2021. Firm selection and corporate cash holdings. *J. Financ. Econ.* 139 (3), 697–718.
- Belo, Frederico, Lin, Xiaojie, Yang, Fan, 2019. External equity financing shocks, financial flows, and asset prices. *Rev. Financ. Stud.* 32 (9), 3500–3543.
- Benmelech, E., Garmaise, M.J., Moskowitz, T.J., 2005. Do liquidation values affect financial contracts? Evidence from commercial loan contracts and zoning regulation. *Q. J. Econ.* 120 (3), 1121–1154.
- Bils, Mark, Klenow, Peter J., Ruane, Cian, 2020. Misallocation Or Mismeasurement?. NBER working paper n.26711, <http://dx.doi.org/10.3386/w26711>.
- Brown, J.R., Fazzari, S.M., Petersen, B.C., 2009. Financing innovation and growth: Cash flow, external equity, and the 1990s R & D boom. *J. Finance* 64 (1), 151–185.
- Buera, F.J., Kaboski, J.P., Shin, Y., 2011. Finance and development: A tale of two sectors. *Amer. Econ. Rev.* 101 (5), 1964–2002.
- Caballero, R., 1999. Aggregate investment. *Handb. Macroecon.* 1 (1), 12, *Handbook of Macroeconomics*, Edition.
- Cette, G., Fernald, J., Mojon, B., 2016. The pre-great recession slowdown in productivity. *Eur. Econ. Rev.* 88, 3–20.
- Chen, P., Karabarbounis, L., Neiman, B., 2017. The global rise of corporate saving. *J. Monetary Econ.* 89, 1–19.

- Chiavari, A., Goraya, S., 2020. The Rise of Intangible Capital and the Macroeconomic Implications. Working Paper.
- Cloyne, J., Ferreira, C., Froemel, M., Surico, P., 2020. Monetary Policy, Corporate Finance and Investment (No. W25366). National Bureau of Economic Research.
- Corrado, C., Haskel, J., Jona-Lasinio, C., Iommi, M., 2012. Intangible Capital and Growth in Advanced Economies: Measurement Methods and Comparative Results (No. 6733). IZA Discussion Papers.
- Corrado, C.A., Hulten, C.R., 2010. How do you measure a technological revolution? *Amer. Econ. Rev.* 100 (2), 99–104.
- Corrado, C., Hulten, C., Sichel, D., 2009. Intangible capital and US economic growth. *Rev. Income Wealth* 55 (3), 661–685.
- De Ridder, M., 2019. Intangibles Investment and the Persistent Effect of Financial Crises on Output. In Technical Report.
- Del Negro, M., Eggertsson, G., Ferrero, A., Kiyotaki, N., 2017. The great escape? A quantitative evaluation of the Fed's liquidity facilities. *Amer. Econ. Rev.* 107 (3), 824–857.
- Dell'Ariccia, G., Kadyrzhanova, K., Minoiu, C., Ratnovski, L., 2021. Bank lending in the knowledge economy. *Rev. Financ. Stud.* 34 (10), 5036–5076.
- Dockner, E.J., Siyahhan, B., 2015. Value and risk dynamics over the innovation cycle. *J. Econom. Dynam. Control* 61, 1–16.
- Doms, M., Dunne, T., 1998. Capital adjustment patterns in manufacturing plants. *Rev. Econ. Dyn.* 1 (2), 409–429.
- Döttling, R., Ladika, T., Perotti, E., 2019. Creating Intangible Capital. Working Paper.
- Döttling, R., Ratnovski, L., 2020. Monetary Policy and Intangible Investment. Working Paper.
- Eggertsson, G.B., Mehrotra, N.R., Robbins, J.A., 2019. A model of secular stagnation: Theory and quantitative evaluation. *Amer. Econ. J.: Macroecon.* 11 (1), 1–48.
- Eisfeldt, A.L., Rampini, A.A., 2009. Leasing, ability to repossess, and debt capacity. *Rev. Financ. Stud.* 22 (4), 1621–1657.
- Falato, A., Kadyrzhanova, D., Sim, J., Steri, R., 2020. Rising intangible capital, shrinking debt capacity, and the US corporate savings glut. In: *Shrinking Debt Capacity, and the US Corporate Savings Glut*. Working Paper.
- Farre-Mensa, J., Ljungqvist, A., 2016. Do measures of financial constraints measure financial constraints? *Rev. Financ. Stud.* 29 (2), 271–308.
- Fernald, J.G., 2015. Productivity and potential output before, during, and after the Great Recession. *NBER Macroecon. Annu.* 29 (1), 1–51.
- Gagnon, E., Johannsen, B.K., Lopez-Salido, D., 2021. Understanding the new normal: The role of demographics. *IMF Econ. Rev.* 69 (2), 357–390.
- Gatchev, V.A., Spindt, P.A., Tarhan, V., 2009. How do firms finance their investments?: The relative importance of equity issuance and debt contracting costs. *J. Corp. Financ.* 15 (2), 179–195.
- Gertler, Mark, Karadi, Peter, 2015. Monetary policy surprises, credit costs, and economic activity. *Amer. Econ. J.: Macroecon.* 7 (1), 44–76.
- Gilchrist, S., Sim, J.W., Zakrajšek, E., 2013. Misallocation and financial market frictions: Some direct evidence from the dispersion in borrowing costs. *Rev. Econ. Dyn.* 16 (1), 159–176.
- Gomes, J.F., 2001. Financing investment. *Amer. Econ. Rev.* 91 (5), 1263–1285.
- Gopinath, G., Kalemli-Özcan, S., Karabarbounis, L., Villegas-Sanchez, C., 2017. Capital allocation and productivity in South Europe. *Q. J. Econ.* 132 (4), 1915–1967.
- Gourio, F., Kashyap, A.K., 2007. Investment spikes: New facts and a general equilibrium exploration. *J. Monetary Econ.* 54, 1–22.
- Hadlock, C.J., Pierce, J.R., 2010. New evidence on measuring financial constraints: Moving beyond the KZ index. *Rev. Financ. Stud.* 23 (5), 1909–1940.
- Hall, B.H., 2002. The financing of research and development. *Oxf. Rev. Econ. Policy* 18 (1), 35–51.
- Haskel, J., Westlake, S., 2017. *Capitalism Without Capital*. Princeton University Press.
- Hsieh, Chang-Tai, Klenow, Peter J., 2009. Misallocation and manufacturing TFP in China and India. *Q. J. Econ.* 124 (4), 1403–1448, JSTOR, [www.jstor.org/stable/40506263](http://www.jstor.org/stable/40506263).
- Ippolito, F., Ozdagli, A.K., Perez-Orive, A., 2018. The transmission of monetary policy through bank lending: The floating rate channel. *J. Monetary Econ.* 95, 49–71.
- Jarocinski, M., Karadi, P., 2020. Deconstructing monetary policy surprises—the role of information shocks. *Amer. Econ. J.: Macroecon.* 12 (2), 1–43.
- Jeenas, P., 2019. Firm Balance Sheet Liquidity, Monetary Policy Shocks, and Investment Dynamics. Technical report, Working paper.
- Jordà, Òscar, 2005. Estimation and inference of impulse responses by local projections. *Amer. Econ. Rev.* 95 (1), 161–182.
- Kahn, J.A., Rich, R.W., 2007. Tracking the new economy: Using growth theory to detect changes in trend productivity. *J. Monetary Econ.* 54 (6), 1670–1701.
- Kahn, J.A., Rich, R.W., 2013. Productivity model update. [http://www.newyorkfed.org/research/national\\_economy/richkahn\\_prodmod.pdf](http://www.newyorkfed.org/research/national_economy/richkahn_prodmod.pdf).
- Kaus, W., Slavtchev, V., Zimmermann, M., 2020. Intangible Capital and Productivity: Firm-Level Evidence from German Manufacturing (No. 1/2020). IWH Discussion Papers.
- Kehrig, M., Vincent, N., 2020. Good Dispersion, Bad Dispersion. CEPR Discussion Paper No. 13772.
- Khan, A., Thomas, J.K., 2013. Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *J. Polit. Econ.* 121 (6), 1055–1107.
- King, R.G., Rebelo, S.T., 1999. Resuscitating real business cycles. *Handb. Macroecon.* 1, 927–1007.
- Kiyotaki, N., Moore, J., 1997. Credit cycles. *J. Polit. Econ.* 105 (2), 211–248.
- Kiyotaki, N., Moore, J., 2019. Liquidity, business cycles, and monetary policy. *J. Polit. Econ.* 127 (6), 2926–2966.
- Liu, E., Mian, A., Sufi, A., 2019. Low Interest Rates, Market Power, and Productivity Growth (No. W25505). National Bureau of Economic Research.
- Loughran, T., Ritter, J., 2004. Why has IPO underpricing changed over time? *Financ. Manag.* 5–37.
- Lyandres, E., Palazzo, B., 2016. Cash holdings, competition, and innovation. *J. Financ. Quant. Anal.* 1823–1861.
- McGrattan, Ellen R., Prescott, Edward C., 2010. Unmeasured investment and the puzzling US boom in the 1990s. *Amer. Econ. J.: Macroecon.* 2 (4), 88–123.
- Ottoneo, P., Winberry, T., 2020. Financial heterogeneity and the investment channel of monetary policy. *Econometrica* 88 (6), 2473–2502.
- Pérez-Orive, A., 2016. Credit constraints, firms' precautionary investment, and the business cycle. *J. Monetary Econ.* 78, 12–131.
- Peters, R.H., Taylor, L.A., 2017. Intangible capital and the investment-q relation. *J. Financ. Econ.* 123 (2), 251–272.
- Petersen, M.A., Rajan, R.G., 1997. Trade credit: theories and evidence. *Rev. Financ. Stud.* 10 (3), 661–691.
- Quadrini, V., 2017. Bank liabilities channel. *J. Monetary Econ.* 89, 25–44.
- Rachel, L., Smith, T., 2015. Secular Drivers of the Global Real Interest Rate. Bank of England Staff Working Paper No. 571.
- Restuccia, D., Rogerson, R., 2008. Policy distortions and aggregate productivity with heterogeneous establishments. *Rev. Econ. Dyn.* 11 (4), 707–720.
- Shourideh, A., Zetlin-Jones, A., 2017. External financing and the role of financial frictions over the business cycle: Measurement and theory. *J. Monetary Econ.* 92, 1–15.
- Sufi, A., 2009. Bank lines of credit in corporate finance: An empirical analysis. *Rev. Financ. Stud.* 22 (3), 1057–1088.