STATE DEPENDENCE OF FISCAL MULTIPLIERS: THE SOURCE OF FLUCTUATIONS MATTERS*

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Abstract

We develop a general theory of state-dependent fiscal multipliers in a framework featuring two empirically relevant frictions: idle capacity and unsatisfied demand. Our key novel finding is that the *source of fluctuations* determines the cyclicality of multipliers. Policies that stimulate demand, such as government spending, have multipliers that are large in demand-driven recessions, but small and possibly negative in supply-driven downturns. Conversely, policies that boost supply, such as cuts in payroll taxes, are ineffective in demand-driven recessions, but powerful if the downturn is supply-driven. Austerity, implemented by a reduction in government consumption, can be the policy with the largest multiplier in supply-side recessions and demand-driven booms, provided elasticities of labor demand and supply are sufficiently low. We obtain model-free empirical support for our predictions by developing a novel econometric specification that allows to estimate spending and tax multipliers in recessions and expansions, *conditional* on those being either demand- or supply-driven.

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Keywords: business cycle, fiscal multipliers, state dependence, search-and-matching in the goods market.

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1 Introduction

A long tradition in economics, starting with the general theory of Keynes (1936), envisages the possibility that the effect of fiscal policy on output is different in times of economic contractions and expansions.This notion of *state dependence* of fiscal multipliers has received renewed attention in recent years, when nominal interest rates reached the effective lower bound in a number of advanced economies, granting fiscal policy a chief stabilizing role. Despite the long history and recent revival, there is still no comprehensive theoretical framework to study the sources, magnitudes and policy implications stemming from state dependence of different fiscal instruments. Our study aims to fill this gap in the literature, and develops a general analytic theory of state-dependent fiscal multipliers for a broad range of spending and taxation policies. A key novel prediction of our theory is that fiscal multipliers' variation over the business cycle is pinned down by the *source of economic fluctuations*, a result that we prove in closed-form. Further, we perform model-free econometric assessment of our novel theoretical predictions and find strong empirical support in US data.

Our theoretical framework accounts for empirically relevant frictions in the goods market, which manifest themselves in idle productive capacity on the firms' side and unsatisfied demand on the side of households. We track congestion in the goods market by looking at the ratio of households' shopping visits to firms' productive capacity; intuitively, whenever the goods market is congested, there is little idle capacity and large amount of unsatisfied demand, and *vice versa*. In our model, the effectiveness of fiscal policy is pinned down by the degree of goods market congestion. Demand-side stimuli that raise the number of visits are ineffective whenever congestion is already high, as they strongly crowd out private consumption. Supply-side stimuli that expand productive capacity are ineffective whenever congestion is already low, as they weakly crowd in private consumption. In demand-side recessions, visits drop and the goods market becomes less congested; by contrast, supply-side recessions witness shrunk capacity and hence stronger congestion. Therefore, the cyclical properties of fiscal multipliers are pinned down by the *type of shocks* that drive the business cycle, and we establish the following properties for a range of spending and taxation instruments.

First, multipliers associated with fiscal instruments that stimulate aggregate demand, such as government consumption spending and consumption tax cuts, are *countercyclical* under demand-driven fluctuations and *procyclical* under supply-driven fluctuations. A recession originated by a lack of demand generates a reduction in the number of household visits, thus *lowering congestion* in the goods market. In such environment, a demand-side fiscal stimulus that boosts aggregate demand and increases the number of visits leads to an increase in production without raising congestion. Consequently, the crowding out of private consumption is small, leading to a high value of the multiplier. By contrast, a supply-driven recession, originated by a fall in productivity, generates a contraction in capacity and hence an *increase in congestion*. A demand-side fiscal stimulus that leads to an increase in the number of visits results in a further increase in congestion that crowds out private consumption and generates a small and possibly negative multiplier.

Second, multipliers associated with interventions that stimulate aggregate supply, such as reductions in taxes on firms' payroll, sales and households' labor income, are *countercyclical* under supply-driven fluctuations and *procyclical* under demand-driven fluctuations. A supply-driven recession is associated with a drop in capacity and hence a surge in congestion; in such an environment, a tax cut that expands capacity leads to a substantial drop in congestion, which leads to strong crowding in of private consumption, and hence a high multiplier. Instead, in a demand-side recession, where visits and congestion drop, expanding capacity through tax cuts leads to a further drop in congestion, thus generating a weak crowding in of private consumption, and hence a low multiplier.

Third, our theoretical framework assigns an important role to fiscal austerity, implemented by a reduction in government consumption, in severe *supply-driven recessions* and *demand-driven booms*. In particular, we show that states of the world exist where goods market congestion is sufficiently high so that a demand-driven stimulus crowds out private consumption at a ratio of more than one-to-one, and the multiplier becomes negative. Moreover, provided elasticities of labor supply and labor demand are sufficiently low so that supply-side stimuli generate a very small drop in congestion, a government consumption austerity, which reduces visits and thus crowds in private consumption, becomes the policy with the highest multiplier. Our results provide a theoretical rationale for empirical findings in Alesina et al. (2015) on the preferential properties of spending-based austerity programs, as well as an alternative justification for austerity that does not rely on government credibility to avoid default (Reinhart and Rogoff, 2010).

Fourth, we develop and estimate an econometric specification that allows for model-free testing of our novel theoretical predictions. We build on the local projections approach of Jordà (2005) and estimate spending and tax cut multipliers in recessionary and expansionary episodes, *conditional* on those being demand- or supply-driven in nature. We determine the nature of each episode by looking at the co-movement between cyclical components of economic activity and inflation. A positive co-movement is taken to be indicative of demand-driven fluctuations, whereas negative co-movement corresponds to supply-driven fluctuations. Empirical studies as early as Bayoumi and Eichengreen (1992) have widely used this approach to pinning down the source of fluctuations, but we are the first study to exploit such co-movement in a state-dependent local projections setting. In accordance with our theory, we find (cumulative) spending multipliers to be high in demand- and low in supply-side recessions, especially at horizons shorter than two years; the opposite patterns hold for tax cut multipliers.

Contribution to the literature. Our study contributes to the growing literature on theories of fiscal state dependence.[1] Early studies focus on fiscal policy at the effective lower bound, showing that fiscal multipliers rise substantially when nominal interest rates are close to zero (Christiano et al., 2011; Coenen et al., 2012; Fernández-Villaverde et al., 2015, Rendahl, 2016 and Roulleau-Pasdeloup, 2020), although more recent studies challenge such findings under fully non-linear solutions (Boneva et al., 2016; Lindé and Trabandt, 2018), under market incompleteness (Hagedorn et al., 2019) or when the liquidity trap is driven by a self-fulfilling expectations shock (Mertens and Ravn, 2014). As for multipliers away from the effective lower bound, Michaillat (2014) establishes that government employment multipliers increase in times of high unemployment, Canzoneri et al. (2016) and Faria-e-Castro (2019) show that the widening of credit spreads caused by financial frictions increases government spending multipliers during recessions, Shen and Yang (2018) show that spending multipliers become countercyclical under downward nominal wage rigidity, Boehm and Pandalai-Nayar (2020) show that firm-level capacity constraints lead to fiscal multipliers that vary with utilization and Cloyne et al. (2020) find large fiscal multipliers when monetary policy is less activist.[2] Fernández-Villaverde et al. (2019) show that search complementarities between producing firms generate multiple equilibria and that the fiscal multiplier becomes state dependent if fiscal policy is sufficiently powerful to move the economy across equilibria. In the study most related to ours, Michaillat

[1] See Ramey (2019) for a comprehensive review of recent developments in the fiscal policy literature.

[2] Hall (2009) emphasizes the importance of introducing search-driven unemployment to deliver realistic multipliers out of government consumption. Our work models search in the goods market, and considers both government consumption and a range of tax cut instruments.

and Saez (2019) conduct a normative analysis in a model with search frictions to show that the *socially optimal stock* of government spending can vary with unemployment.[3] Ziegenbein (2017) and Sims and Wolff (2018) show that multipliers out of tax cuts vary significantly across the business cycle and are larger in states in which output is high.

Compared to the aforementioned studies, we are the first study to *jointly* rationalize state dependence in a broad range of spending and taxation multipliers and to develop a tractable model with closed-form solutions. To the best of our knowledge, we are also the first study to link the state dependence of fiscal multipliers to the source of economic fluctuations. Our theoretical findings offer direct guidance for the conduct of fiscal policy, particularly to establish the effectiveness of alternative fiscal instruments in a given phase of the business cycle.

Our study also contributes to the empirical literature on fiscal state dependence, and our econometric findings offer a resolution to the debate on the degree of variation of fiscal multipliers over the business cycle. Early studies find government spending multipliers to be substantially larger in recessions compared to expansions both in the US (Auerbach and Gorodnichenko, 2012; Fazzari et al., 2014) and internationally (Auerbach and Gorodnichenko, 2013). However, more recently, Ramey and Zubairy (2018) construct a comprehensive historical dataset for government spending in the US, and find almost acyclical spending multipliers. Moreover, empirical studies do not find spending multipliers to be substantially larger at the effective lower bound, either in the UK (Crafts and Mills, 2013), the US (Ramey and Zubairy, 2018), or Japan (Miyamoto et al., 2018). Ziegenbein (2017) and Eskandari (2019) find that tax cut multipliers are highly procyclical.

Our key empirical contribution is in showing that once the estimation controls for the source of economic fluctuations, both spending and tax cut multipliers exhibit significant state dependence and that their variation over phases of the business cycle is consistent with our theory. Note that our estimation of state-dependent spending multipliers uses the same dataset as Ramey and Zubairy (2018), and our specification nests theirs as a special case where the source of fluctuations is irrelevant. We therefore offer a resolution to the empirical debate on state dependence of spending multipliers on both empirical and theoretical grounds. Barnichon et al. (2021) propose an alternative resolution to recover state dependence by controlling for the *sign* of the spending shock.

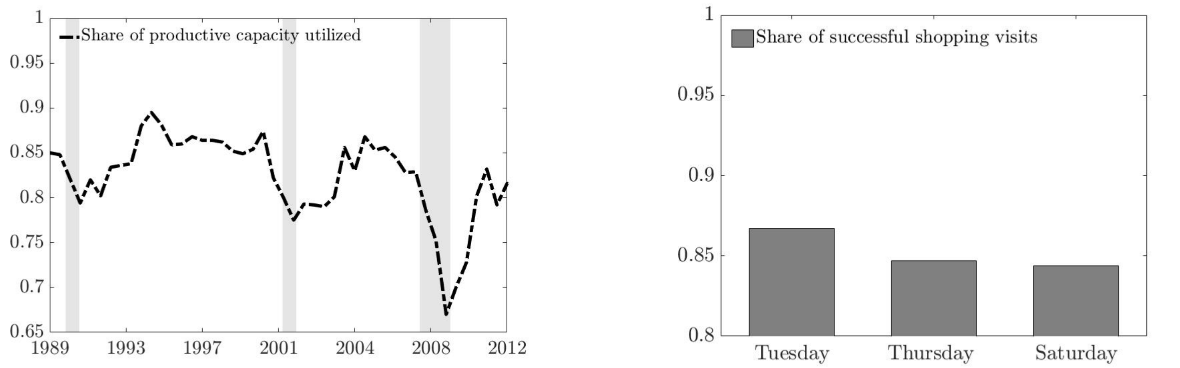
The rest of the paper is structured as follows. Section 2 develops the theoretical framework. Section 3 derives and discusses the key results on the state dependence of spending and taxation multipliers. Section 4 studies the relative effectiveness of spending and taxation multipliers and investigates the role for fiscal austerity. Section 5 quantifies the state dependence of multipliers in a dynamic version of the model. Section 6 develops an econometric model that supports our theoretical results. Section 7 concludes and outlines possible directions for future research.

[3]Though the conceptual framework used is similar, our analysis substantially differs from that in Michaillat and Saez (2019) in a number of ways. First, Michaillat and Saez (2019) do not study the role played by the source of fluctuations in determining cyclicality of fiscal multipliers. Second, the supply side of our economy is fully endogenous and allows us to study multipliers from both government consumption, which is the exclusive focus of Michaillat and Saez (2019), as well as a wider range of fiscal instruments, including government employment, distortionary taxation on consumption, labor income, and firms' sales. Third, the dynamic version of our model is set in discrete time and features transition dynamics, which allow us to study multipliers at different time horizons that can be mapped to empirical estimates in the literature. Fourth, we provide model-free econometric assessment of the predictions of our model, whereas Michaillat and Saez (2019) only perform model-based simulations.

Figure 1: Evidence on frictions in the United States goods market

(a) Share of productive capacity utilized

(b) Share of successful visits to retails stores



***Notes**: panel (a) shows a time series of the share of current capacity utilized by US firms, as calculated by the Institute for Supply Management (ISM) with NBER recessions denoted by grey shaded areas, as reported by Michaillat and Saez (2015); panel (b) shows the share of successful visits to retail stores on different days of the week, as reported by Taylor and Fawcett (2001), where Tuesday-Thursday and Tuesday-Saturday differences are statistically significant at 5% level.*

2 The theoretical framework

To motivate our framework, we begin by providing evidence of idle productive capacity on the firms' side and unsatisfied demand on the households' side. We then introduce these features to the model by embedding search-and-matching frictions into the goods market in an otherwise standard production economy. By default, proofs of results in main text are in Appendix A.

2.1 Market clearing, idle capacity and unsatisfied demand

The textbook definition of goods market clearing in a closed economy with fixed capital is:

$$Y = C + G, \tag{1}$$

where Y is the productive capacity of the economy and $C + G$ represents aggregate demand coming from households and the government. In most standard models, equation (1) implies that firms sell off their entire capacity; otherwise prices fall sufficiently to clear any excess supply. At the same time, aggregate demand is generally assumed to be satisfied frictionlessly, with no resources spent on completing the purchases.

Despite the common assumption of a frictionless goods market in macroeconomic models, the data strongly endorses the presence of frictions that generate idle productive capacity and unsatisfied demand. Panel (a) of Figure 1 uses firm-level US data collected by the Institute for Supply Management (ISM) to show that, on average, firms in manufacturing sectors sell around 80 percent of their current productive capacity, and the proportion of utilized capacity is subject to regular business cycle fluctuations, with much limited fractions utilized in recessions.

Similar frictions are present on the aggregate demand side. Several studies in the fields of business logistics and marketing research document that around 15 per cent of visits to US retail stores are unsuccessful due to stockouts (Taylor and Fawcett, 2001). Panel (b) of Figure 1 shows that demand frictions are also cyclical, with visits to stores being on average more successful on weekdays, as opposed to weekends, when shops tend to be more congested.Such frictions also are encountered in online stores, where as many as 25 per cent of online orders cannot be fulfilled due to out-of-stock items (Jing and Lewis, 2011). Finally, evidence from the American Time Use Survey (ATUS) shows that unsuccessful shopping visits are indeed costly for households, with an average American spending roughly one hour per day on queuing and searching for products.

Workhorse macroeconomic models do not jointly account for idle productive capacity and unsatisfied demand, despite their clear empirical relevance. In the rest of the section, we outline a theoretical framework that jointly models those features in an otherwise standard macroeconomic model.

2.2 A model with search frictions in the goods market

We begin our analysis with a static model that features a goods market with search-and-matching frictions in spirit of Michaillat and Saez (2015).[4] We assume a competitive labor market.[5] The economy is composed of households, firms, and the government. Firms hire labor in order to manufacture an endogenous *productive capacity* (k), and consumers and the government make a total of *v visits* in order to purchase goods. Due to search-and-matching frictions, part of productive capacity remains idle and not all visits are successful, as encapsulated by the *matching function* that maps productive capacity (k) and visits (v) into *sales* (y):

$$y = (k^{-\delta} + v^{-\delta})^{-\frac{1}{\delta}}, \tag{2}$$

where $\delta > 0$ ensures that $y < \min\{k, v\}$. We define goods market *tightness* (x) as the ratio of visits to capacity:

$$x \equiv \frac{v}{k}. \tag{3}$$

Abstracting from aggregate uncertainty, each unit of productive capacity is sold with probability:

$$f(x) \equiv \frac{y}{k} = (1 + x^{-\delta})^{-\frac{1}{\delta}}, \tag{4}$$

where $f(0) = 0, \lim_{x\to+\infty} f(x) = 1$, and $f'(x) > 0, \forall x \in [0, +\infty)$. Intuitively, the probability of selling a unit of productive capacity is higher in a tighter goods market, and *vice versa*. Similarly, a purchasing visit is successful with probability:

$$q(x) \equiv \frac{y}{v} = (1 + x^{\delta})^{-\frac{1}{\delta}}, \tag{5}$$

where $q(0) = 1, \lim_{x\to+\infty} q(x) = 0$ and $q'(x) < 0, \forall x \in [0, +\infty)$. The probability of a successful visit is lower in a tighter goods market, and *vice versa*. [6]

Within this framework, it is useful to think of productive capacity (k) as the size of the "store" and visits (v) as the length of the "queue" comprising private and government consumers. Goods market tightness (x) can be

[4] We develop and present a dynamic version of our model in Section 5. Our framework builds on the general-disequilibrium model by Barro and Grossman (1971), whose application to fiscal policy is considered in van Wijnbergen (1987). Recent studies with goods market search frictions include Bai et al. (2012), Den Haan (2013), Gourio and Rudanko (2014), Brzustowski et al. (2018) and Roldan-Blanco and Gilbukh (2020).

[5] Adding search frictions to the labor market with flexible wages leaves our results for fiscal multipliers unchanged.

[6] Petrosky-Nadeau et al. (2016) provide evidence that the average time spent shopping fell in the demand-deficient period of 2008-2010, which is consistent with the property that the probability of a successful visit rises in a less congested goods market.

interpreted as the number of queuing consumers per square meter of the store or as a measure of congestion in the goods market. This interpretation will be helpful to develop the intuition behind our results.

2.3 Households

There is a continuum of identical households of size one. Households make v^c visits in order to purchase and consume c units of the produced good. There is a cost $\rho \in (0,1)$ of the produced good per visit.[7] Total sales of the produced good to households (y^c) comprise household consumption (c) and search costs (ρv^c):

$$y^c = c + \rho v^c. \tag{6}$$

Since each visit is successful with probability $q(x)$, total sales are equal to $y^c = q(x)v^c$, and the consumption of c units of the produced good requires $c/(q(x) - \rho)$ visits. Therefore, the total number of goods that need to be purchased (inclusive of the cost of search) in order to consume c units is given by:

$$[1 + \gamma(x)]c, \tag{7}$$

where $\gamma(x) \equiv \frac{\rho x}{f(x) - \rho x}$ represents the wedge introduced by search-and-matching frictions that strictly rises in goods market tightness, such that $\gamma'(x) > 0, \forall x \in (0, x_m)$.[8] Intuitively, a tighter goods market diminishes the probability of a successful visit, increasing the expected number of visits required for a successful purchase, thus raising total search costs.

The representative household gains utility from consumption of the produced good (c), the non-produced good (m) that is in fixed exogenous supply $(\bar{m})$, and gains disutility from supplying labor (l). The non-produced good is traded in a frictionless competitive market, and we use it as the numeraire by normalizing its price to one.[9] Every household is small relative to the size of the market, and therefore takes the price (p), wage (w), goods market tightness (x) and hence the search wedge $[1 + \gamma(x)]$ as given. The representative household maximizes utility function:

$$\max_{c,m,l} \left[\chi \frac{c^{1-\sigma}}{1-\sigma} + \zeta(m) - \frac{l^{1+\psi}}{1+\psi} \right] \tag{8}$$

subject to the budget constraint

$$p[1 + \gamma(x)]c + m \leq wl + \Pi + \bar{m} - T, \tag{9}$$

[7] The visiting cost can be interpreted as the cost of time spent queuing in shops, or alternatively the (expected) cost of returning a purchased good that the customer did not like. This interpretation is supported by empirical evidence. According to the American Time Use Survey (ATUS), the average American spends approximately one hour per day on shopping and queueing. Our formulation for search costs allows us to treat government and private customers symmetrically, as we discuss in section 2.5. An alternative way to model search costs for the households is to assume that there is a *utility* cost per visit. In Appendix F, we show that results of the analysis continue to hold with this alternative approach to modelling search costs.

[8] We restrict the admissible values of tightness to $(0, x_m)$, where x_m is given by the condition $f(x_m) = \rho x_m$; this approach ensures that the aggregate supply of the produced good, net of search costs, remains positive.

[9] Introducing the numeraire good m allows us to separately pin down both the price of the produced good p, as well as the wage w, in our static framework.

where $\chi > 0$ determines the relative preference for the produced good, ψ is the inverse Frisch elasticity of labor supply, p is the price of the produced good, $[1 + \gamma(x)]$ is the search wedge, w is the wage received per unit labor supplied, Π is profits from firms owned by the representative household, and T is a lump-sum tax collected by the government. The representative household also receives an endowment of the non-produced good equal to its (fixed) supply $\bar{m}$. Finally, $\zeta(.)$ is an increasing differentiable function.

Lemmas 1 and 2 report the consumption function, $c(p, x)$, and labor supply function, $l(w)$, respectively, that solve the representative household's maximization problem.[10]

Lemma 1. *The consumption function $c(p, x)$ is the optimal consumption choice in the representative household's problem evaluated under non-produced goods market clearing ($m = \bar{m}$) and is equal to:*

$$c(p, x) = \frac{\chi}{p[1 + \gamma(x)]}, \tag{10}$$

where $\frac{\partial c}{\partial p} < 0$, $\frac{\partial c}{\partial x} < 0$ *and* $\frac{\partial c}{\partial \chi} > 0$.

Equation (10) shows that, *ceteris paribus*, higher preference parameter χ increases consumption since it generates larger utility for every unit of consumption; higher price (p) and tightness (x) increase the relative price of consumption and hence decrease consumption.

Lemma 2. *The labor supply function $l(w)$ is the optimal labor supply choice in the representative household's problem evaluated under non-produced goods market clearing ($m = \bar{m}$), and is equal to:*

$$l(w) = w^{\frac{1}{\psi}}, \tag{11}$$

where $\frac{\partial l}{\partial w} > 0$.

Equation (11) shows that, *ceteris paribus*, higher wage (w) increases the supply of labor. The elasticity of labor supply is equal to $\epsilon^s \equiv \frac{1}{\psi}$, and if $\psi \to \infty$, labor supply becomes perfectly inelastic and fixed at one.

2.4 Firms

The economy is populated by a continuum of identical, perfectly competitive firms that manufacture an identical good that is sold in a goods market characterized by search-and-matching frictions. Firms have access to a production technology that transforms labor input (n) into *productive capacity* (k):

$$k(n) = an^{\alpha}, \tag{12}$$

where $\alpha \in (0, 1]$ is returns to labor and $a > 0$ is the level of productivity. In the presence of search-and-matching frictions, every unit of productive capacity is utilized with probability $f(x)$. Abstracting from uncertainty, the representative firm achieves the following level of *sales* $y(x, n)$:

$$y(x, n) = f(x)k(n) = f(x)an^{\alpha}. \tag{13}$$

[10]For the remainder of the main text we assume log utility of consumption ($\sigma = 1$). Our choice of log utility of consumption is consistent with mean estimates of the coefficient of relative risk aversion in microeconomic studies (Chetty, 2006). Appendix E provides closed-form solutions for a generic CRRA utility of consumption. We also normalize the supply of the non-produced good so that $\zeta'(\bar{m}) = 1$.

Each firm is small relative to the size of the market and thus takes the price (p) and wage (w) as well as goods market tightness (x) and probability $f(x)$ as given. The profit maximization problem of the representative firm is:

$$\max_n \Pi = [pf(x)an^\alpha - wn(1+\tau)], \tag{14}$$

where $\tau \in [0,1)$ is a payroll tax from the government, which is a fraction τ of the representative firm's wage bill.

Lemma 3 reports the labor demand function $n(w;p,x,\tau)$ that solves the representative firm's profit-maximization problem.

Lemma 3. *The labor demand function $n(w;p,x,\tau)$ is the solution to the representative firm's profit maximization problem and is equal to:*

$$n(w;p,x,\tau) = \left[\frac{\alpha p f(x) a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}}, \tag{15}$$

where $\frac{\partial n}{\partial p} > 0$, $\frac{\partial n}{\partial x} > 0$, $\frac{\partial n}{\partial w} < 0$, $\frac{\partial n}{\partial a} > 0$ and $\frac{\partial n}{\partial \tau} < 0$.

Equation (15) shows that, *ceteris paribus*, a higher price (p) increases the revenue from every unit sold and hence incentivizes production and labor demand. Higher tightness (x) increases the probability of selling each unit produced and hence also incentivizes production and labor demand. Labor demand decreases with the cost of hiring labor, given by the wage (w). A payroll tax $\tau > 0$ increases the cost of hiring and thus lowers labor demand. Finally, higher productivity (a) increases the marginal product for each unit of labor hired, increasing labor demand. The (absolute) elasticity of labor demand is given by $|\epsilon^d| \equiv \frac{1}{1-\alpha}$, with the case of constant returns $(\alpha = 1)$ corresponding to a perfectly elastic labor demand.

2.5 Government

The government consumes an exogenous quantity G of the privately produced good and is subject to the same search-and-matching frictions as private consumers. Thus, for a given desired government consumption of the produced good G, the government must purchase $[1+\gamma(x)]G$ units, where $[1+\gamma(x)]$ is the same wedge as that faced by private consumers.[11,12] Given the rate of payroll tax τ and hence the tax revenue $wn\tau$ collected, the government imposes a lump-sum tax T on the households to balance the public budget:

$$T = p[1+\gamma(x)]G - wn\tau. \tag{16}$$

There are alternative ways of including government spending into our model. For example, government spending on defense or other public goods may be introduced in the model as government spending on employing labor to produce public goods that are offered to private consumers. Appendix C.1 extends the baseline model to allow for

[11] Our assumption of identical costs per visit for households and the government, and hence of identical search wedges, is made for algebraic simplicity and is inessential for our results. Our results hold even if we assume that the government faces a smaller $(\rho^G < \rho)$ or zero $(\rho^G = 0)$ visiting cost.

[12] Our model features a single production sector, so we abstract from any differences in sectoral compositions of government and household consumption, as documented by Ramey and Shapiro (1998) and Cox et al. (2020). However, our qualitative mechanism may still hold even if sectoral compositions are different: higher government consumption may draw factors of production away from households' sectors, thus increasing good market tightness in the latter. This mechanism is even more apparent if the government directly competes for factors of production with the private sector; we consider such extension in Appendix C.1.

government employment and production of public goods, and it shows that the cyclical properties of the multiplier out of public employment are identical to those of the government consumption multiplier.

We focus on distortionary payroll taxes in the baseline model. Appendix C.2 considers alternative distortionary taxation on firms' sales and on households' labor income, showing that the cyclical properties of multipliers out of cuts to sales tax and labor income tax are identical to those of cuts to the payroll tax. In addition, Appendix C.2 also considers distortionary taxation of households' consumption and shows that cyclical properties of the multiplier out of consumption tax cuts are identical to those of the government consumption multiplier considered in the baseline model.

2.6 Market clearing

The economy is composed of three distinct markets for the produced good (c), the non-produced good (m), and labor (n). By Walras' Law, we obtain equilibrium allocations by determining market clearing in any two markets. We focus on the markets for the produced good and labor.

Aggregate demand consists of the households' demand $c(p, x)$ and the government's exogenous demand G. Aggregate supply is given by the fraction of the firm's sales $y(x; n) = f(x)k(n; \tau)$ that is not spent on the cost of search, and hence is given by $f(x)k(x; \tau)/[1 + \gamma(x)]$. The market clearing condition in the goods market is:

$$\underbrace{\frac{f(x)}{1+\gamma(x)}k(n;\tau)}_{\text{Aggregate supply}} = \underbrace{c(p,x)+G}_{\text{Aggregate demand}} . \tag{17}$$

From Lemma 1 we know that aggregate demand is downward-sloping in tightness-quantity space. As for the aggregate supply curve, it is backward-bending: it rises in tightness between $(0, x^*)$ and falls in tightness for values (x^*, x_m). For a given productive capacity k, goods market tightness exerts two counteracting effects on aggregate supply. On the one hand, higher x increases the probability $f(x)$ of selling each produced good, thus raising aggregate supply; on the other hand, search costs, encapsulated by the wedge $[1 + \gamma(x)]$, increase in tightness. The increase in the selling probability outweighs the increase in search costs for tightness below the efficient level $(x < x^*)$. On the other hand, aggregate supply decreases for tightness above the efficient level $(x > x^*)$. Aggregate supply is maximized at $x = x^*$, where the two effects offset each other, which corresponds to the social planner's allocation in our economy, as we formally show in Appendix G.1.

Market clearing in the labor market is achieved by equating labor demand, $n(w; p, x, \tau)$, to labor supply, $l(w)$:

$$\underbrace{l(w)}_{\text{Labor supply}} = \underbrace{n(w;p,x,\tau)}_{\text{Labor demand}}. \tag{18}$$

The labor supply function is upward sloping in the wage-employment space, as established in Lemma 2, since higher wages encourage households to work more, and the labor demand function is downward sloping in the wage-employment space, as established in Lemma 3.

Equilibrium is described by price, wage, tightness, and allocations that satisfy the optimality conditions of households, firms, the government budget constraint, and the market clearing conditions. The system is indeterminate because once optimality and market clearing conditions have been combined, we are left with two equations in three unknowns (p, x, w). We therefore need a selection mechanism to choose a specific equilibrium across the infinitely many admissible combinations.[13]

2.7 Comparative statics: two polar equilibria

In the baseline model, we resolve indeterminacy by considering two polar equilibrium cases. First, in a *competitive* equilibrium, tightness is fixed at its socially efficient level x^*, formally defined in Appendix G.1, and p and w adjust fully flexibly to satisfy optimality and market clearing conditions.[14] Second, in a *fixprice* equilibrium, price p is fixed at a constant value p_0, and x and w adjust to satisfy optimality and market clearing conditions. Appendix B extends our analysis to more general equilibrium cases.

2.7.1 Comparative statics: competitive equilibrium

A competitive equilibrium is formally defined as follows:

Definition 1. *A **competitive** equilibrium is a pair (p^*, w^*) and associated allocations, such that the agents' optimality conditions and the market clearing conditions are satisfied with tightness at its efficient level $(x = x^*)$.*

Following a positive demand shock, parameterized as an increase in χ, households choose to consume more, hence increasing the number of visits; since tightness is to remain fixed at x^*, the price p has to rise to discourage further consumption and expand capacity, until markets clear with more sales in the new equilibrium.[15] On the other hand, after a positive supply shock, represented by a rise in a, capacity expands. In order to keep tightness fixed, the price has to fall, thus encouraging more consumption and more visits until markets clear with higher sales in equilibrium. Lemma 4 formally summarizes the comparative statics for the competitive equilibrium:

Lemma 4. *In a competitive equilibrium, the comparative statics of tightness (x), sales (y) and the price (p):*

$$\frac{dx}{d\chi} = 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} > 0; \qquad \frac{dx}{da} = 0, \frac{dy}{da} > 0, \frac{dp}{da} < 0. \tag{19}$$

Figure 10 in Appendix D provides graphical representation of comparative statics following demand- and supply-side shocks in a competitive equilibrium.

2.7.2 Comparative statics: fixprice equilibrium

A fixprice equilibrium is formally defined as follows:

Definition 2. *A **fixprice** equilibrium is a vector (p_0, x, w) and associated allocations, which satisfy the agents' optimality conditions and the market clearing conditions with fixed price p_0 $(p = p_0)$.*

[13] Goods market tightness (x) is taken as given by agents in a decentralized economy, and indeterminacy is a common feature of search-and-matching models. A standard approach to resolve indeterminacy in the search-and-matching framework of the labor market is adding Nash bargaining over the wage.

[14] The name *competitive equilibrium* is a reference to the directed search literature and in particular, the competitive search equilibrium in Moen (1997), where full adjustment via prices and wages leads to efficient allocations.

[15] In Appendix G.1, we show that the socially efficient level of tightness is given by the condition $f'(x^*) = \rho$ and hence x^* is invariant to changes in either χ or a.

After a positive demand shock, consumption and visits rise. Since the price p is fixed, the only way to clear such excess demand is for the tightness x to rise, which increases the cost of search, thus discouraging any further consumption until markets clear with higher sales in the new equilibrium. At the same time, following a positive supply shock, capacity expands, and the only way for such excess supply to be cleared is through a fall in tightness, which encourages consumption until markets clear.[16] Lemma 5 below formally summarizes the comparative statics in a fixprice equilibrium:

Lemma 5. *In a fixprice equilibrium, the comparative statics of tightness* (x)*, sales* (y)*, and the price* (p) *are:*

$$\frac{dx}{d\chi} > 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} = 0; \qquad \frac{dx}{da} < 0, \frac{dy}{da} = 0, \frac{dp}{da} = 0. \tag{20}$$

Figure 11 in Appendix D provides graphical representation of comparative statics following demand- and supply-side shocks in a competitive equilibrium.

3 Fiscal multipliers: key analytical results

In this section, we establish our key novel analytical results regarding cyclical properties of fiscal multipliers. In particular, we consider the *demand-side* multiplier, associated with government consumption, and the *supply-side* multiplier, associated with payroll tax cuts.[17] We show that in a competitive equilibrium, where tightness is fixed and price and wage are fully flexible, the two multipliers are *identical* and *acyclical*, and they are pinned down exclusively by the elasticities of labor supply and demand. In a fixprice equilibrium, where the price is fixed and tightness and wages clear markets, the demand-side multiplier is *countercyclical* under *demand-side fluctuations* and *procyclical* under *supply-side fluctuations*. On the other hand, the supply-side multiplier is *procyclical* under *demand-side fluctuations* and *countercyclical* under *supply-side* fluctuations. Finally, we provide evidence that equilibria featuring price rigidity and fluctuations in tightness provide a better description of the US economy at business cycle frequencies, thus endorsing equilibria featuring state-dependent fiscal multipliers, whose cyclicality depends on the source of economic fluctuations.

3.1 Definitions

A fiscal multiplier measures the effect of a marginal change in the fiscal instrument, be it government spending or a tax rate, on GDP *in equilibrium*. The following definition introduces the demand-side fiscal multiplier, associated with increases in government consumption spending: [18]

[16] Note that following a positive supply shock, the level of sales remains unchanged in a fixprice equilibrium. This is because, on the one hand, capacity k expands, but on the other, tightness the utilization rate $f(x)$ drops. The two effects cancel each other out only as long as prices remain completely fixed: as we show in Lemma 9 in Appendix B.2 In equilibria featuring rigid but not completely fixed prices, sales increase following a positive supply shock, while tightness still falls.

[17] In this section, we limit our attention to the two polar equilibria: competitive and fixprice. Appendix B studies fiscal multipliers under more general equilibrium types whereas Appendix C extends the analysis to multipliers out of government employment, as well as consumption, sales, and labor income tax cuts.

[18] For simplicity and to retain direct comparability with related studies, the multipliers are evaluated at the point where there is no government intervention, so that $G = 0$ and $\tau = 0$.

Definition 3. *The demand-side fiscal multiplier, $\varphi^d(x)$, is given by:*

$$\varphi^d(x) \equiv \frac{d\{c+G\}}{dG} = \frac{dc}{dG} + 1. \tag{21}$$

Similarly, the following definition introduces the supply-side fiscal multiplier, associated with payroll tax cuts, respectively.

Definition 4. *The supply-side fiscal multiplier, $\varphi^s(x)$, is given by:*

$$\varphi^s(x) \equiv \frac{1}{c+G}\frac{d\{c+G\}}{d[-\tau]} = -\frac{1}{c}\frac{dc}{d\tau}. \tag{22}$$

3.2 Competitive equilibrium multipliers

We first derive the fiscal multipliers in a competitive equilibrium, where p and w are fully flexible, and adjust in response to shocks to maintain tightness at the efficient level ($x = x^*$) to satisfy the equilibrium conditions:

Proposition 1. *In a competitive equilibrium, the demand- and supply-side fiscal multipliers are equal and given by:*

$$\varphi^* \equiv \frac{\alpha}{1+\psi} = \frac{1 - \frac{1}{|\epsilon^d|}}{1 + \frac{1}{\epsilon^s}}, \tag{23}$$

where $\alpha \in (0, 1]$ and $\psi > 0$ are, respectively, returns to labor and inverse Frisch elasticity, whereas $|\epsilon^d| = \frac{1}{1-\alpha}$ and $\epsilon^s = \frac{1}{\psi}$ are (absolute) elasticities of labor demand and labor supply. Hence $\varphi^* \in (0, 1]$, and it is pinned down by elasticities of labor demand and labor supply.

Proposition 1 outlines several important results. First, in a competitive equilibrium, the demand- and supply-side multipliers are equal, implying that either of the two fiscal interventions generates the same effect on GDP. However, in accordance with Definitions 3 and 4, although φ^* is identical and between zero and one for both fiscal policy instruments, consumption is crowded out ($\frac{dc}{dG} < 0$) under demand-side fiscal policy, and it is crowded in ($-\frac{dc}{d\tau} > 0$) under supply-side fiscal policy.

Second, the competitive equilibrium multiplier in Proposition 1 coincides with the government spending multiplier derived in Woodford (2011) for a New Keynesian model considered in the limit of fully flexible prices and wages, parameterized for our preferences and technology specifications. One can therefore treat φ^* as a benchmark for multipliers under fully flexible prices, in either our model with goods market search frictions or in a New Keynesian model.

Third, the competitive equilibrium multiplier is determined exclusively by the relative elasticities of labor supply and demand, equal to $\epsilon^s \equiv \frac{1}{\psi}$ and $|\epsilon^d| \equiv \frac{1}{1-\alpha}$, respectively. As labor supply becomes perfectly inelastic ($\psi \to \infty$), the multiplier decreases ($\varphi^* \to 0$). The intuition is straightforward. When the ratio of "queue length" to "store size" is to be kept at the efficient level and the "store size" is fixed, the only way to accommodate additional government customers in the queue is for the price to increase to the point where private customers in the queue are crowded-out one-for-one. Similarly, payroll tax cuts that lead to higher employment and larger "store size" result in higher wages with no change in employment and consumption when labor supply is perfectly inelastic and tightness is kept at the efficient level.

On the other hand, when the demand and supply of labor are perfectly elastic ($\alpha = 1$ and $\psi = 0$), the fiscal multiplier reaches the maximum value of one ($\varphi^* = 1$). In this case, any additional queue length from government customers generates an increase in employment and enlarges the capacity of the store, without crowding out consumption and retaining tightness at the efficient level. Any payroll tax cuts leave wages unchanged and increase the supply of labor and hence the production of goods, which enlarges the capacity of the store. The only way to retain tightness at the efficient level is for the price to fall, leading to a one-for-one crowd-in of private customers into the queue.

Given the mapping between the elasticities of labor supply and labor demand and the competitive equilibrium multipliers, we interpret φ^* as a measure of flexibility of the labor market. The more elastic the labor demand (larger α) and labor supply (smaller ψ), the larger the multiplier in a competitive equilibrium.

It is now straightforward to show that both demand- and supply-side fiscal multipliers are *acyclical* in a competitive equilibrium, as outlined formally below:

Corollary 1. *In a competitive equilibrium, both demand- and supply-side multipliers are acyclical.*

Proof. A trivial consequence of Proposition 1: in a competitive equilibrium, both multipliers are equal to $\varphi^* = \frac{\alpha}{1+\psi}$ and do not change when either preference χ or technology a varies. □

In a competitive equilibrium, prices and wages are fully flexible and adjust to ensure that tightness remains at the efficient level (x^*) in response to demand- and supply-side shocks. Hence, both the utilization probability, $f(x)$, and the search wedge, $[1 + \gamma(x)]$, remain unchanged over the business cycle, leading to a fixed level of private consumption crowding out and hence a constant multiplier.

3.3 Fixprice equilibrium multipliers

3.3.1 Demand-side fiscal multiplier

Before studying the demand-side fiscal multiplier in a generic fixprice equilibrium, we make an intermediate step and derive the multiplier in the special case of *fixed capacity*, which arises under a perfectly inelastic labor supply ($\psi \to \infty$). The properties of such *fixed capacity fiscal multiplier* will be helpful for studying fiscal multipliers in a generic fixprice equilibrium as well as for deriving a link between demand- and supply-side fiscal multipliers.

Lemma 6. *Let the fixed capacity fiscal multiplier, $\theta(x)$, be the demand-side fiscal multiplier in a fixprice equilibrium under fixed labor supply, such that*

$$\theta(x) \equiv \frac{d\{c+G\}}{dG}|_{\psi\to\infty}. \tag{24}$$

It can be shown that $\theta(x)$ has the following properties:

$$\theta(x) = \begin{cases} (0, -\infty), & \text{if} \quad x \in (x^*, x_m) \\ 0, & \text{if} \quad x = x^* \\ (0, 1), & \text{if} \quad x \in (0, x^*) \end{cases} \tag{25}$$

$$\theta'(x) < 0, \qquad \forall x \in (0, x_m),$$

where x_m is given by $f(x_m) = \rho x_m$.

Lemma 6 outlines several results. First, in an efficient fixprice equilibrium, where $x = x^*$, the First Welfare Theorem applies and the fixed capacity fiscal multiplier is equal to zero, just like the competitive equilibrium multiplier under perfectly inelastic labor supply ($\psi \to \infty$), such that $\theta(x^*) = \varphi^*_{\psi\to\infty} = 0$. Just as in a competitive equilibrium under perfectly inelastic labor supply, $\psi \to \infty$, the only way additional government spending can be accommodated under fixed labor supply and a fixed price is by crowding out private consumption, which is achieved by an increase in tightness. Moreover, in an efficient fixprice equilibrium, the crowding out of consumption is exactly one-for-one, as the increase in tightness fails to increase supply in the goods market, which is already at its maximum, given the fixed capacity.

Second, whenever $x \in (0, x^*)$, additional demand from government spending is accommodated via higher tightness, which crowds out private consumption *less than one-for-one*. This is because under under $x \in (0, x^*)$, the effect of higher tightness on increasing the fraction of capacity utilized $f(x)$ dominates the effect of increasing the search wedge $[1 + \gamma(x)]$, so that aggregate supply increases following the government consumption increase.

Third, whenever $x \in (x^*, x_m)$, higher government spending crowds out private consumption *more than one-for-one*. This is driven by the fact that under $x \in (x^*, x_m)$ the effect of higher tightness on increasing the cost of search $[1 + \gamma(x)]$ dominates the effect on increasing the fraction of capacity utilized $f(x)$, and aggregate supply falls following the rise in government consumption.

Having established the properties of $\theta(x)$, we can now provide a convenient expression for the demand-side fiscal multiplier in a generic fixprice equilibrium:

Proposition 2. *In a fixprice equilibrium, the demand-side fiscal multiplier, $\varphi^d(x)$, is given by:*

$$\varphi^d(x) = \underbrace{\varphi^*}_{\text{State-invariant component}} + \underbrace{\theta(x) \times (1 - \varphi^*)}_{\text{State-dependent component}}, \tag{26}$$

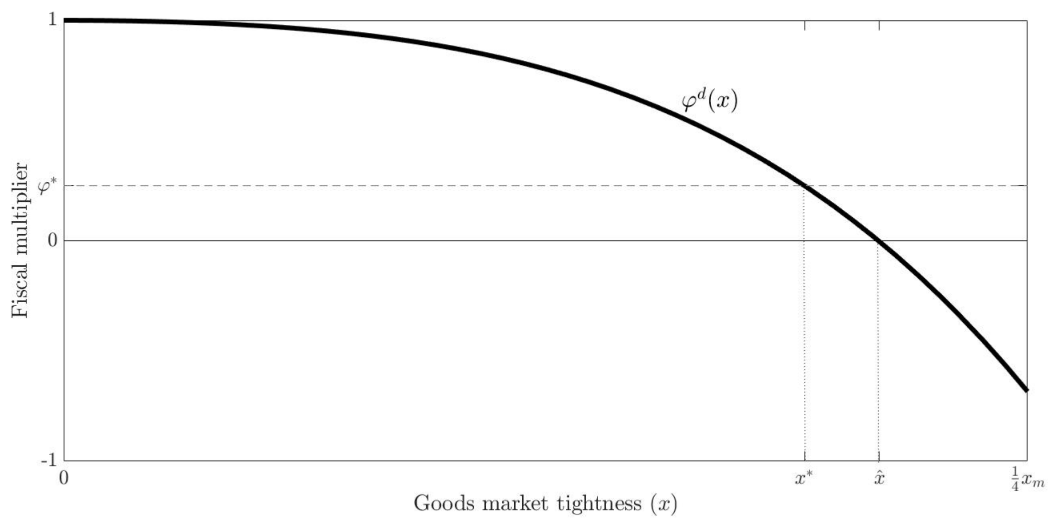
where $\varphi^* = \frac{\alpha}{1+\psi}$ is the competitive equilibrium multiplier. Hence, $\varphi^d(x) \in (-\infty, 1)$ and $\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x_m)$.

Proposition 2 establishes several important results. First, the demand-side fiscal multiplier can be represented as the sum of a state-invariant component, given by the competitive equilibrium multiplier φ^*, and a state-dependent component that is a function of the underlying goods market tightness. Moreover, in the special case of an efficient fixprice equilibrium, where $x = x^*$, the state-dependent component disappears since $\theta(x^*) = 0$, and the fixprice demand-side multiplier collapses to the multiplier in a competitive equilibrium, $\varphi^d(x^*) = \varphi^*$.

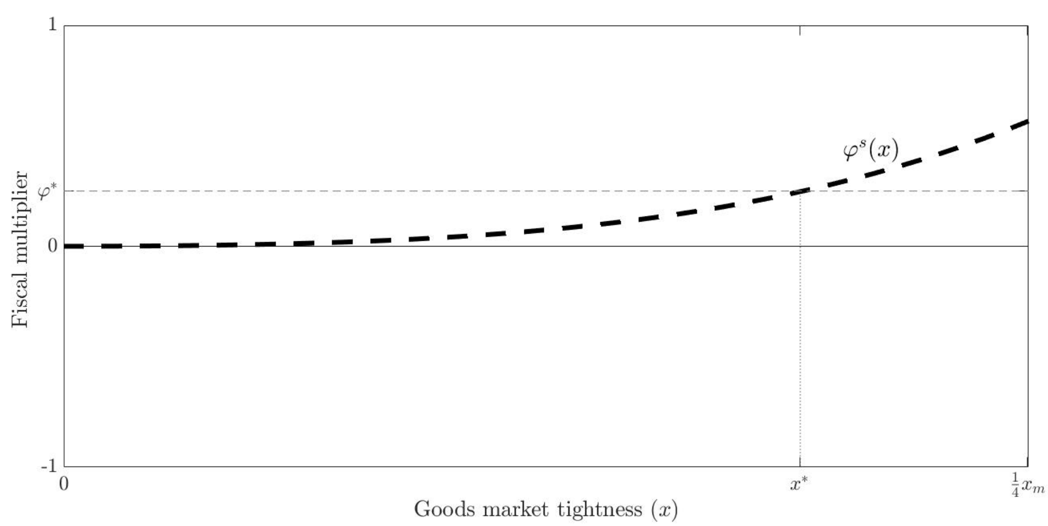
Second, from the properties of $\theta(x)$, it follows that in a fixprice equilibrium, the demand-side fiscal multiplier lies between one and negative infinity and strictly falls in tightness on the whole domain. Hence, government expenditure always crowds out private consumption, and the crowding-out effect is stronger whenever the goods market tightness is higher, as shown graphically in Panel (a) of Figure 2. To outline the intuition behind this finding, consider our narrative of "store size" and "queue length". A low goods market tightness environment is akin to a short queue for a given size of the store. When the government implements demand-side fiscal expansion

Figure 2: Fiscal multipliers in a fixprice equilibrium

(a) Demand-side fiscal multiplier, $\varphi^d(x)$



(b) Supply-side fiscal multiplier, $\varphi^s(x)$



Notes*: Panels (a) and (b) show demand-side and supply-side fiscal multipliers in a fixprice equilibrium of a calibrated version of our model* $(\alpha = 0.3, \delta = 2, \rho = 0.1, \psi = 0.2)$ *for values of goods market tightness in the range* $(0, \frac{1}{4}x_m)$ *(to avoid extreme values as* x *gets closer to* x_m*); Panel (a) shows that the demand-side fiscal multiplier* $\varphi^d(x)$ *starts at one when* $x = 0$*, then strictly falls in goods market tightness, and turns negative after* $x = \hat{x}$*; Panel (b) shows that the supply-side fiscal multiplier* φ^s *starts at zero when* $x = 0$*, then strictly rises in tightness, tending to infinity as* $x \to x_m$*.*

by sending its customers to join a short queue, it does not make the shop excessively crowded and hence achieves a relatively high multiplier since the crowding out effect is limited. Instead, in a high tightness environment, the government sends customers to join a crowded store, which results in strongly crowding out private customers and delivering a low multiplier.

Third, as seen in Panel (a) of Figure 2, under sufficiently high tightness $\hat{x}$, the demand-side fiscal multiplier turns negative, implying that private consumption gets crowded out more than one-for-one. This is despite the fact that in a generic fixprice equilibrium higher tightness increases labor demand and, *ceteris paribus*, gives a boost to capacity and aggregate supply. Moreover, as we establish in the next corollary, such threshold $\hat{x}$ always exists, regardless of the elasticities of labor demand and labor supply:

Corollary 2. *There always exists tightness $\hat{x} \in (x^*, x_m)$, such that $\varphi^d(\hat{x}) = 0$ and $\varphi^d(x) < 0, \forall x \in (\hat{x}, x_m)$, and it is equal to:*

$$\hat{x} = \theta^{-1}\left(-\frac{\varphi^*}{1-\varphi^*}\right), \tag{27}$$

where $\frac{d\hat{x}}{d\varphi^*} > 0$.

To interpret this finding, recall that Lemma 5 establishes that in a fixprice equilibrium tightness increases following a positive demand-side or a negative supply-side shock. Thus, Corollary 2 implies that in a fixprice equilibrium the demand-side multiplier can become negative either under a sufficiently strong demand-driven overheating or a sufficiently severe supply-side contraction.

Most importantly, results in Proposition 2 imply well-defined, cyclical properties of the demand-side fiscal multiplier, outlined in the next corollary:

Corollary 3. *In a fixprice equilibrium, the demand-side fiscal multiplier, $\varphi^d(x)$, is countercyclical under demand-side fluctuations and procyclical under supply-side fluctuations.*

Proof. From Lemma 5, we know that in a fixprice equilibrium $\frac{dx}{d\chi} > 0$, $\frac{dx}{da} < 0$; further, from Proposition 2, we know that in a fixprice equilibrium $\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x_m)$. Hence, $\frac{d\varphi^d(x)}{d\chi} = \frac{d\varphi^d(x)}{dx}\frac{dx}{d\chi} < 0, \forall x \in (0, x_m)$ and $\frac{d\varphi^d(x)}{da} = \frac{d\varphi^d(x)}{dx}\frac{dx}{da} > 0, \forall x \in (0, x_m)$. □

Corollary 3 establishes that the demand-side multiplier, associated with government consumption, is large in *demand-driven recessions* and *supply-driven expansions*, but small in *supply-driven recessions* and *demand-side expansions*. Intuitively, a demand-driven recession lowers the length of the "queue", whereas a supply-driven expansion increases the size of the "store"; in both cases, congestion of the store falls. In such cases, an increase in government consumption adds government customers to an uncongested store, leading to a small amount of private consumption crowding out, and hence a higher multiplier.[19] Conversely, both a supply-driven recession and a demand-driven expansion make the "store" more congested, leading to strong crowding out of private consumption, and hence a lower multiplier, following an increase in government consumption.

[19] Michaillat (2014) shows that a similar result holds in a labor market with search-and matching-frictions following shocks to public employment.

3.3.2 Supply-side fiscal multiplier

The next proposition provides an expression for the supply-side fiscal multiplier in a generic fixprice equilibrium:

Proposition 3. *In a fixprice equilibrium, the supply-side fiscal multiplier, $\varphi^s(x)$, is given by:*

$$\varphi^s(x) = \underbrace{\varphi^*}_{\text{State-invariant component}} - \underbrace{\theta(x) \times \varphi^*}_{\text{State-dependent component}}, \tag{28}$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the competitive equilibrium multiplier. Hence, $\varphi^s(x) \in (0, +\infty)$ and $\frac{d\varphi^s(x)}{dx} > 0, \forall x \in (0, x_m)$.

Proposition 3 shows that the supply-side multiplier is the sum of a state-invariant component, equal to the competitive multiplier φ^*, and a state-dependent component that depends on goods market tightness. Similarly to the demand-side multiplier, the supply-side fiscal multiplier collapses to its competitive equilibrium value in the special case of an efficient fixprice equilibrium with $x = x^*$, so that $\varphi^s(x^*) = \varphi^*$

The supply-side fiscal multiplier is always positive and strictly increases in goods market tightness on the whole domain, shown graphically in Panel (b) of Figure 2. Hence, supply-side fiscal policy always crowds in private consumption and does so more strongly in a tighter goods market. Supply-side fiscal policy in the form of a payroll tax cut encourages more labor demand for a given wage, which in turn increases capacity and reduces goods market tightness, lowering the search wedge and encouraging higher consumption; the latter positive effect on consumption through capacity expansion is stronger whenever capacity is already low, relative to the number of visits. Intuitively, the payroll tax cut increases the size of the "store" and hence reduces its congestion, thus crowding in private consumption; moreover, such crowding in of private consumption is stronger whenever the store already is very congested.

The next corollary uses the results from Proposition 3 to establish the cyclicality properties of the supply-side fiscal multiplier in a fixprice equilibrium:

Corollary 4. *In a fixprice equilibrium, the supply-side fiscal multiplier, $\varphi^s(x)$, is procyclical under demand-side fluctuations and countercyclical under supply-side fluctuations.*

Proof. From Lemma 5, we know that in a fixprice equilibrium $\frac{dx}{d\chi} > 0, \frac{dx}{da} < 0$; further, from Proposition 3, we know that in a fixprice equilibrium $\frac{d\varphi^s(x)}{dx} > 0$. Hence, $\frac{d\varphi^s(x)}{d\chi} = \frac{d\varphi^s(x)}{dx}\frac{dx}{d\chi} > 0, \forall x \in (0, x_m)$ and $\frac{d\varphi^s(x)}{da} = \frac{d\varphi^s(x)}{dx}\frac{dx}{da} < 0, \forall x \in (0, x_m)$. □

Corollary 4 establishes that the supply-side multiplier, associated with payroll tax cuts, is high in *supply-driven recessions* and *demand-driven expansions*, but low in *demand-driven recessions* and *supply-driven expansions*. Intuitively, a supply-driven recession makes the size of the "store" smaller, whereas a demand-driven expansion increases the "queue" length; in either case, the store becomes more congested. In such cases, a supply-side policy that generates an increase in capacity produces large fiscal multipliers by lowering the congestion of the "store" and strongly crowding in private consumption. On the other hand, in demand-driven recessions and supply-driven expansions the "store" becomes much less congested, so that any further increases in capacity generate only modest decreases in the cost of search, and hence a very modest crowding in of private consumption.[20]

[20] Landais et al. (2018) show that the effect of unemployment insurance on employment is weaker if recessions are driven by labor demand

3.4 Robustness of results

3.4.1 Robustness I: more general equilibrium types

In Appendix B, we solve for demand-side and supply-side multipliers under much more general equilibrium types and establish their cyclical properties, which we summarize here.

First, we show that results obtained under a competitive equilibrium fully extend to a class of *flexible* equilibria, where tightness is fixed at an arbitrary level $x^L \in (0, x_m)$ and does not respond to shocks, instead letting fully flexible prices and wages (p^L, w^L) accommodate any disturbances. Such equilibria could be obtained, for example, when the price is established by Nash bargaining between firms and households, or when the price is set at a fixed mark-up over the marginal cost. We show that any such equilibrium will have both demand- and supply-side multipliers fixed at $\varphi^* = \frac{\alpha}{1+\psi}$ and acyclical.

Second, we show that the cyclicality properties established under a fixprice equilibrium extend to a more general class of *frictional* equilibria, where prices are partially rigid. Such equilibria could occur, for example, if the price is set at an intermediate level between a fixed parameter (p_0) and the price under a flexible equilibrium (p^L), so that $p = (p_0)^{\varepsilon}(p^L)^{1-\varepsilon}$ and $\varepsilon \in (0, 1]$ pins down the degree of price rigidity. More generally, we show that in any frictional equilibrium, where the elasticity between p and p^L lies in $[0, 1)$, the demand- and supply-side multipliers still fall and rise in tightness, respectively, and hence preserve the cyclicality properties established under a fixprice equilibrium.

3.4.2 Robustness II: alternative fiscal policy instruments

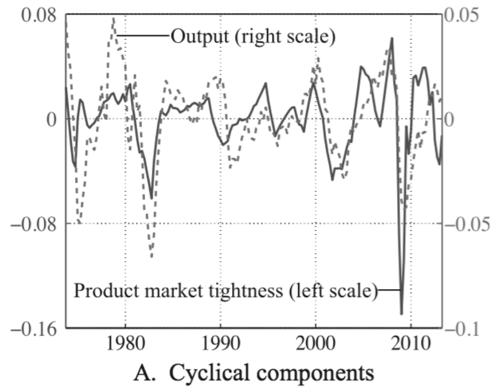
In Appendix C, we extend our analysis to alternative fiscal instruments, such as government employment, as well as distortionary taxation of households' consumption, labor income, and firms' sales. Here we briefly summarize the results.

First, we show that multipliers out of consumption tax cuts and government employment strictly fall in tightness and hence their cyclicality properties are identical to those of the government consumption multiplier established earlier. In any flexible equilibrium, both multipliers are still acyclical. In any frictional equilibrium, however, a cut in the rate of consumption tax encourages higher consumption and more visits by private households; the latter lengthens the "queue", making the shop more congested and thus raising the search cost, crowding out some of the initial increase in private consumption. Consumption tax cuts implemented under high goods market congestion are associated with stronger crowding out, and hence a lower value of the multiplier. As for government employment, whenever the government hires a fraction of the labor supply, it removes labor resources from the private sector, which shrinks the capacity of privately produced goods, making the goods market more congested and crowding out private consumption. Whenever the government employs labor under already very high tightness, congestion rises even more strongly and private consumption becomes more crowded out.

Second, we show that multipliers out of cuts in the rate of labor income tax and the rate of firms' sales tax are both identical to the multiplier out of a cut in the rate of firms' payroll tax. Hence both of them also strictly rise in goods market tightness and their cyclical properties also are just like those of the supply-side multiplier that we considered earlier. Indeed, in any flexible equilibrium, both multipliers are acyclical. In any frictional equilibrium,

shocks. The finding can be interpreted as special case of the present result that supply-side multipliers are procyclical under demand-driven cycles.

Figure 3: US goods market tightness and sales (Michaillat and Saez, 2015)



however, a cut in the rate of labor income tax increases labor supply, *ceteris paribus*, whereas lowering the rate of the sales tax encourages more labor demand. Either of those policies leads to higher equilibrium employment and hence higher capacity. The latter lowers goods market tightness and crowds in private consumption, doing so more strongly whenever goods market tightness is more congested in the first place.

3.5 Evidence on the equilibrium type

Our analysis shows that the joint dynamics of price and tightness adjustment pin down the cyclical properties of fiscal multipliers. On one hand, whenever tightness is fixed over the business cycle, and prices and wages are fully flexible, fiscal multipliers are acyclical. On the other hand, whenever tightness varies over the business cycle and prices are rigid, fiscal multipliers are state-dependent with cyclicality determined by the source of economic fluctuations.

Figure 3 plots time series for (the cyclical component of) US goods market tightness, as constructed by Michaillat and Saez (2015). It is immediately apparent that goods market tightness varies significantly over time, suggesting that equilibria that feature cyclical variations in tightness provide a better description of the US economy at business cycle frequencies. The latter also implies that equilibria featuring state-dependent fiscal multipliers, with their cyclicality determined by the source of economic fluctuations, are more empirically relevant.

Figure 3 also shows a strong co-movement between the cyclical components of goods market tightness and sales, which, according to Lemma 5, reflects the dominance of demand shocks as the primary source of fluctuations.[21,22] Combined with the cyclicality properties established in Corollary 3, the latter suggests that the demand-side multiplier is, *on average*, countercyclical. This finding is consistent with some empirical literature (Auerbach and Gorodnichenko, 2012, 2013, Fazzari et al., 2014), although Ramey and Zubairy (2018) estimate spending multipliers to be mildly countercyclical at best. The predominance of demand shocks, combined with cyclicality properties established in Corollary 4, suggest that the supply-side multiplier is, *on average*, procyclical. Such

[21] Strictly speaking, Lemma 5 only applies whenever prices remain completely fixed over the business cycle. In Lemma 9 in Appendix B.2 we show that in any equilibrium where prices are partially rigid, but not completely fixed, tightness and sales co-move under demand-driven fluctuations and counter-move under supply-driven fluctuations.

[22] Figure 13 in Appendix I.1 reports estimated impulse responses of output, inflation and goods market tightness to an identified productivity shock based on Fernald (2014). Consistently with our theory, a positive productivity shock leads to a statistically significant increase in output and a statistically significant drop in inflation and goods market tightness.

a finding is consistent with the econometric findings in Ziegenbein (2017) and Eskandari (2019), who estimate multipliers out of tax cuts to be much lower in recessions than in expansions.

The fact that spending and tax cut multipliers are, respectively, countercyclical and procyclical, *on average*, gives us no indication on their relative sizes in a *particular* recessionary or expansionary episode. Indeed, our analytical results show that depending on the *type of shock* that generates the episode in the first place, the magnitudes of both multipliers may be substantially different. In order to gain further understanding of the behavior of the *relative* magnitudes of demand- and supply-side multipliers, in the next section, we provide further analytical results regarding the particular states of the world in which the size of multipliers is different. In Section 5 we develop and calibrate a quantitative dynamic version of our model, and use a non-linear solution method to evaluate both spending and taxation multipliers in shock-specific recessionary and expansionary episodes. Subsequently, in Section 6 we develop an econometric specification that allows to estimate spending and tax cut multipliers in recessionary and expansionary episodes, *conditional* on those being either demand- or supply-driven.

4 Fiscal multipliers: additional analytical results

We now provide further analytical results that describe how the *relative* size of demand- and supply-side multipliers in a fixprice equilibrium varies with goods market tightness. First, we establish that the demand-side multiplier is lower than the supply-side multiplier whenever goods market tightness is above the socially efficient level, and *vice versa*. Second, we show that for sufficiently low elasticities of labor supply and demand, there always exists a sufficiently high level of tightness that makes *spending austerity*, implemented by a reduction in government consumption, the policy with the largest multiplier.

4.1 Link between demand- and supply-side multipliers

One can combine Propositions 2 and 3 to conveniently link the demand- and supply-side multipliers in a fixprice equilibrium:

Corollary 5. *In a fixprice equilibrium, the demand-side and supply-side fiscal multipliers are related as:*

$$\underbrace{\varphi^d(x)}_{\text{Demand-side multiplier}} = \underbrace{\theta(x)}_{\text{Fixed capacity multiplier}} + \underbrace{\varphi^s(x)}_{\text{Supply-side multiplier}} . \tag{29}$$

Hence, the demand-side multiplier is lower whenever $x \in (x^*, x_m)$, higher whenever $x \in (0, x^*)$, and equal to the supply-side multiplier when $x = x^*$.

Corollary 5 establishes that in the special case of an efficient fixprice equilibrium, where $x = x^*$ and $\theta(x^*) = 0$, the size of demand- and supply-side multipliers is the same, $\varphi^d(x^*) = \varphi^s(x^*) = \varphi^*$, in accordance with Propositions 2 and 3. When $x \in (x^*, x_m)$ and hence $\theta(x) \in (-\infty, 0)$, the demand-side multiplier is smaller than the supply-side multiplier, $\varphi^d(x) < \varphi^s(x)$, since enlarging capacity by stimulating supply lowers tightness and search costs in the inefficiently congested goods market, thus crowding in private consumption and making supply-side policies more effective. The opposite result holds when $x \in (0, x^*)$ and $\theta(x) \in (0, 1)$, making the demand-side multiplier higher than the supply-side multiplier.

4.2 Austerity multipliers

So far, we have focused on policies that either increase government consumption or cut the rate of payroll tax. However, policymakers also have reverse options at their disposal, namely spending austerity, implemented as a reduction in government consumption, and also an increase in the tax rate.

In our framework, the multiplier from austerity implemented by a reduction in government consumption is the mirror image of the demand-side multiplier, and is equal to $-\varphi^d(x)$. Similarly, the multiplier from an increase in the rate of payroll tax is equal to $-\varphi^s(x)$. From Proposition 3, we know that $\varphi^s(x) \in (0, +\infty), \forall x \in (0, x_m)$, which implies that the multiplier from an increase in the rate of payroll tax is *negative* on the whole domain of tightness, and hence in no state of the world can it be the policy option with the highest multiplier. However, the latter is not the case when it comes to spending policies. We already know from Corollary 5 that whenever $x \in (x^*, x_m)$ the supply-side multiplier exceeds the demand-side multiplier; moreover we know from Corollary 2 that there exists $\hat{x} \geq x^*$ such that whenever $x \in (\hat{x}, x_m)$, the spending multiplier is in fact negative, implying that the austerity multiplier is positive. Yet, is there an admissible level of tightness such that the austerity multiplier is sufficiently positive to exceed the supply-side multiplier from tax cuts? The next corollary establishes that the answer is yes, as long as the elasticities of labor demand ($|\epsilon^d|$) and labor supply (ϵ^s) are sufficiently low, as encapsulated by $\varphi^* = \frac{\alpha}{1+\psi} = \frac{1-\frac{1}{|\epsilon^d|}}{1+\frac{1}{\epsilon^s}}$.

Corollary 6. *For sufficiently low elasticities of labor demand and labor supply such that $\varphi^* < 0.5$, an **Austerity Threshold** $\tilde{x} \in [\hat{x}, x_m)$ exists such that:*

$$- \varphi^d(x) > \varphi^s(x) > \varphi^d(x), \quad \forall x \in (\tilde{x}, x_m). \tag{30}$$

Furthermore, $\tilde{x}$ is given by:

$$\tilde{x} = \theta^{-1}\left(-\frac{2\varphi^*}{1-2\varphi^*}\right), \quad \varphi^* < 0.5 \tag{31}$$

and hence $\frac{d\tilde{x}}{d\varphi^*} > 0$.

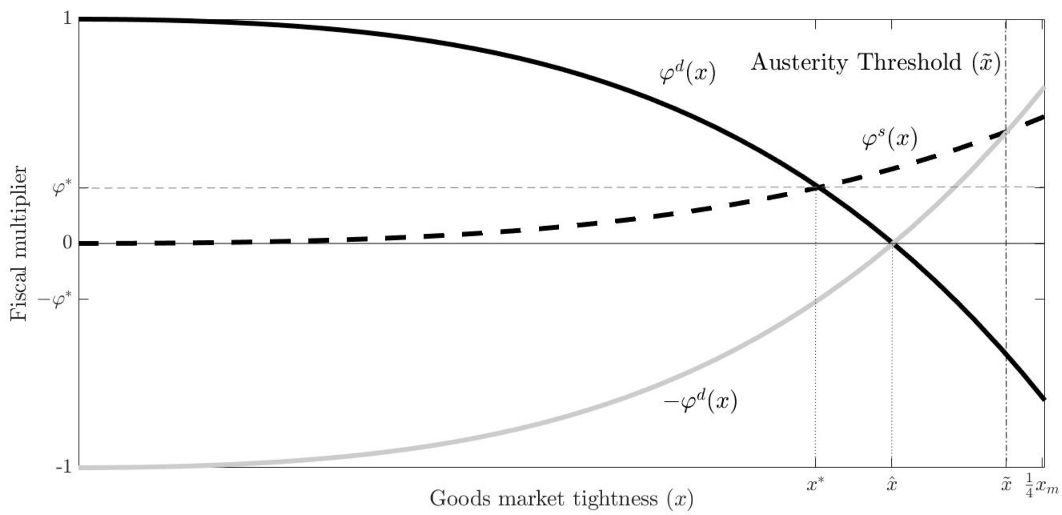
Panel (a) of Figure 4 compares the austerity multiplier against demand- and supply-side multipliers for an inelastic labor market ($\varphi^* < 0.5$). It shows that the multiplier associated with government consumption austerity exceeds the supply-side multiplier (dashed line) *and* demand side multiplier (dark-solid line), provided that tightness is larger than $\tilde{x}$.[23] Panel (b) reports that case for a flexible labor market ($\varphi^* > 0.5$), showing that if the labor market is sufficiently flexible, the supply-side multiplier is always lager than the austerity multiplier, and the Austerity Threshold does not exist.

In intuitive terms, Corollary 6 states that the store can be so congested, that decreasing tightness by removing government customers from the queue could be more effective at crowding in private consumption than enlarging the store by tax cuts. In particular, this could only be the case when the elasticities of labor demand and labor supply are sufficiently low, so that tax cuts that encourage more labor demand are not effective at increasing equilibrium employment and capacity. Based on our results about cyclical fluctuations in tightness in Lemma 5, the Austerity

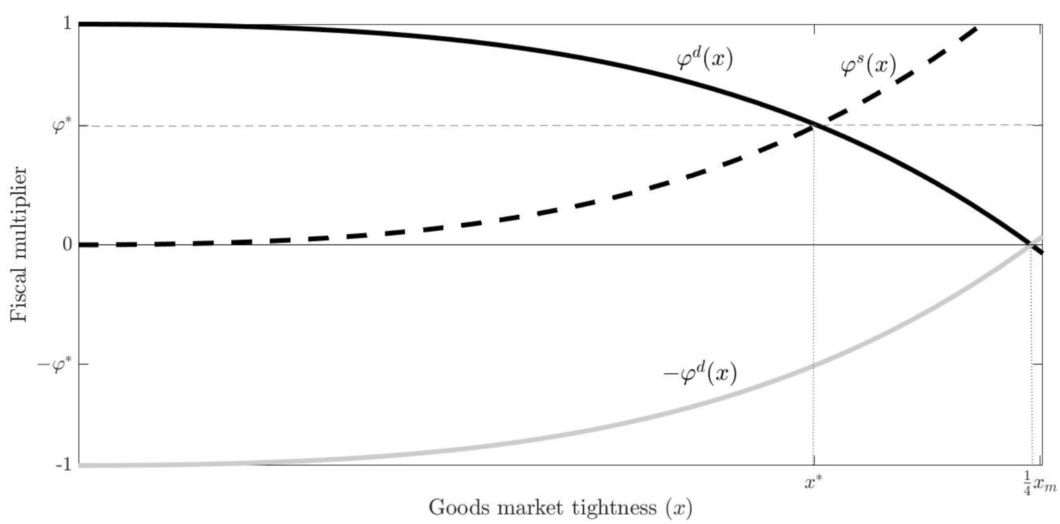
[23] Note that $\tilde{x} \geq \hat{x}$. Reaching the Austerity Threshold requires tightness to be larger than the threshold that makes the government spending multiplier negative, defined in Corollary 2.

Figure 4: Comparing fiscal multipliers in a fixprice equilibrium

(a) Inelastic labor market $(\varphi^* < 0.5)$



(b) Elastic labor market $(\varphi^* > 0.5)$



***Notes**: Panels (a) and (b) show demand-side, supply-side and spending austerity multipliers in a fixprice equilibrium of a calibrated version of our model* $(\delta = 2, \rho = 0.1, \psi = 0.2)$*; in Panel (a), we set* $\alpha = 0.3$*, so that the elasticity of labor demand is relatively low and* $\varphi^* = 0.25 < 0.5$ *– in this case one can see that Austerity Threshold* $\tilde{x}$ *exists, and for all* $x \in (\tilde{x}, x_m)$ *spending-austerity is the policy with the highest multiplier; in Panel (b), we set* $\alpha = 0.65$ *so that labor demand is relatively elastic and* $\varphi^* = 0.57 > 0.5$ *– in this case, Austerity Threshold does not exist and spending austerity is never the policy with the highest multiplier.*

Threshold could be reached in an economy with sufficiently inelastic labor markets, which is hit by either a very strong positive demand-side shock or following a severe negative supply-side shock.

Recent studies by Alesina et al. (2015) provide evidence that austerity programs based on spending reductions are more powerful than programs based on tax increases when it comes to stimulating GDP during recessions. Our results provide a theoretical underpinning for such findings, suggesting that differences between spending- and tax-based austerity programs are especially pronounced in countries with inflexible labor markets and under high goods-market tightness, arising in supply-side side recessions, or in demand-side expansions.

5 Fiscal multipliers in a quantitative dynamic model

In this section, we develop and calibrate a discrete-time dynamic version of our model and use a non-linear solution method in order to quantitatively assess cyclical properties of both spending and taxation multipliers, conditional on different sources of fluctuations.[24] Relative to our static model, we make the additional assumption of *long-term customer relationships* between households and firms, which is empirically relevant and it allows us to map our matching process to realistic goods market frictions.[25] The dynamic model corroborates the finding in our static model and shows substantial state dependence *conditional* on a particular type of shock that drives the business cycle, especially for impact multipliers.

5.1 Goods market with long-term customer relationships

Sales materialize through *long-term customer relationships* between firms and consumers from private and government sectors that are subject to an exogenous destruction rate η per period. The total number of long-term customer relationships at the end of period t is y_t. At the beginning of each period t, firms inherit $(1-\eta)y_{t-1}$ relationships that have survived destruction in the previous period $t-1$, and they hire labor n_t to yield current capacity $k_t = a_t n_t^{\alpha}$, which they utilize through the relationships carried over from last period, leaving $[k_t - (1-\eta)y_{t-1}]$ as unutilzed capacity. Households and the government make v_t visits to form new relationships that fill the unutilzed capacity. However, not every purchasing visit is successful. The number of new customer relationships formed in each period is tracked by the matching function:

$$\left[(k_t - (1-\eta)y_{t-1})^{-\delta} + v_t^{-\delta}\right]^{-\frac{1}{\delta}}, \tag{32}$$

where $\delta > 0$ ensures that not every unit of unitilized capacity is filled and not every visit is successful. Goods market tightness is defined as: $x_t \equiv \frac{v_t}{k_t-(1-\eta)y_{t-1}}$, and the probability of filling a unit of unutilized capacity is given by $f(x_t) \equiv (1 + x_t^{-\delta})^{-\frac{1}{\delta}}$, $f' > 0$, whereas the probability of a given visit yielding a new relationship is given by $q(x_t) \equiv (1 + x_t^{\delta})^{-\frac{1}{\delta}}$, $q' < 0$.

[24] In this section we limit our attention to multipliers out of government consumption and payroll tax cuts. In Appendix H.1, we study cyclical properties of multipliers out of cuts in taxes on consumption and labor supply in our dynamic model.

[25] Michaillat and Saez (2015) offer cross-country evidence that long-term customer relationships are prevalent in goods markets; for example, they report that in the US around 77 per cent of sales go to long-term customers. Moreover, Gourio and Rudanko (2014) show theoretically how such desire to accumulate long-term customers can be microfounded.

5.2 Households

As in the static version of our model, households face a cost $\rho \in (0,1)$ of consumption goods per visit. They form long-run customer relationships both to consume and purchase goods that go towards satisfying the total cost of visits. At the beginning of period t, households have $(1-\eta)y_{t-1}^c$ relationships that survived from the previous period, and the number of new relationships formed in period t is: $y_t^c - (1-\eta)y_{t-1}^c$. Since every visit is only successful with probability $q(x_t)$, the total number of visits required to form new relationships in period t is $(y_t^c - (1-\eta)y_{t-1}^c)/q(x_t)$, yielding the following expression for y_t^c:

$$y_t^c = c_t + \rho\left[\frac{y_t^c - (1-\eta)y_{t-1}^c}{q(x_t)}\right], \tag{33}$$

which can be rearranged to obtain a more familiar-looking expression for y_t^c:

$$y_t^c = [1+\gamma(x_t)]c_t - (1-\eta)\gamma(x_t)y_{t-1}^c, \tag{34}$$

where, as before, $\gamma(x_t) \equiv \frac{\rho x_t}{f(x_t)-\rho x}$ is the wedge introduced by search-and-matching frictions.

There is a continuum of identical infinitely lived households. Markets are assumed to be complete, so a full set of Arrow-Debreu securities is available. The representative household is small relative to the size of the market, and maximizes expected discounted lifetime utility, taking prices, wages and goods market tightness as given:

$$\max_{\{c_{t+s}, y_{t+s}^c, m_{t+s}, B_{t+s+1}, l_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\chi_{t+s}\frac{c_{t+s}^{1-\sigma}}{1-\sigma} + \zeta(m_{t+s}) - \nu\frac{l_{t+s}^{1+\psi}}{1+\psi}\right]$$

subject to the budget constraint and the equation of motion of their customer relationships:

$$p_t y_t^c + m_t + \mathbb{E}_t\left[F_{t,t+1}B_{t+1}\right] \leq w_t l_t + \bar{m}_t + B_t + \Pi_t - T_t, \qquad \forall t \geq 0 \tag{35}$$

$$y_t^c = [1+\gamma(x_t)]c_t - (1-\eta)\gamma(x_t)y_{t-1}^c, \qquad \forall t \geq 0 \tag{36}$$

where $\ln\chi_t = \rho_\chi \ln\chi_{t-1} + \varepsilon_t^\chi, \varepsilon_t^\chi \sim^{iid} (0, \sigma_\chi^2)$, is an exogenous process for the relative preference for consumption, β is the utility discount factor, ν captures relative labor disutility, and the rest of the notation carries over from the static version of the model. The exogenous supply of the non-produced good is assumed to be constant over time ($\bar{m}_t = \bar{m}, \forall t \geq 0$) and as before, we normalize it so that $\zeta'(\bar{m}) = 1$.

The optimization problem yields the following first-order conditions for the intertemporal choice of consumption:

$$\underbrace{\chi_t c_t^{-\sigma} + \beta(1-\eta)\mathbb{E}_t\left[\chi_{t+1}c_{t+1}^{-\sigma}\frac{[1+\gamma(x_t)]}{[1+\gamma(x_{t+1})]}\gamma(x_{t+1})\right]}_{\text{Expected marginal benefit}} = \underbrace{p_t[1+\gamma(x_t)]}_{\text{Marginal cost}}, \tag{37}$$

and for the labor supply and the stochastic discount factor:

$$l_t = [w_t/\nu]^{\frac{1}{\psi}}, \qquad F_{t,t+s} = \beta^s. \tag{38}$$

Note that for $\eta = 1$, the consumption function in (37) nests equation (10) in the static model, with the comparative statics intuition preserved. However, when $\eta \in (0, 1)$, the marginal utility from consuming an extra unit of the good consists of both the contemporaneous component $(\chi_t c_t^{-\sigma})$ as well as a forward-looking component, which stems from the fact that a fraction $(1 - \eta)$ of relationships in period t will be preserved in period $t + 1$, leading to further consumption. The intuition behind the labor supply function in (38) remains unchanged from the static case.

5.3 Firms

There is a continuum of identical perfectly competitive producing a homogenous good. At the beginning of each period t, firms have $(1 - \eta)y_{t-1}$ customer relationships that have survived from the previous period $t - 1$. Firms hire labor n_t that yields current capacity $k_t = a_t n_t^\alpha$, leaving $[a_t n_t^\alpha - (1 - \eta)y_{t-1}]$, a fraction $f(x_t)$ of which is then utilized. The latter gives the following equation of motion for firms' sales:

$$y_t = (1 - \eta)y_{t-1} + f(x_t)\left[a_t n_t^\alpha - (1 - \eta)y_{t-1}\right], \qquad \forall t \geq 0 \tag{39}$$

where $\ln a_t = \rho_a \ln a_{t-1} + \varepsilon_t^a$, and $\varepsilon_t^a \sim^{iid} (0, \sigma_a^2)$, is an exogenous process for productivity.

The representative firm is small relative to the size of the market, and therefore maximizes its lifetime discounted profits taking prices, wages, and tightness as given:

$$\max_{\{y_{t+s}, n_{t+s}\}_{s=0}^\infty} \mathbb{E}_t \sum_{s=0}^{\infty} F_{t,t+s}\left[p_{t+s}y_{t+s} - w_{t+s}n_{t+s}(1 + \tau_{t+s})\right]$$

subject to equation (39). The optimization problem yields the following optimality condition:

$$\underbrace{p_t + (1 - \eta)\mathbb{E}_t\left[F_{t,t+1}\frac{w_{t+1}(1 + \tau_{t+1})}{\alpha f(x_{t+1})a_{t+1}n_{t+1}^{\alpha-1}}[1 - f(x_{t+1})]\right]}_{\text{Expected marginal benefit}} = \underbrace{\frac{w_t(1 + \tau_t)}{\alpha f(x_t)a_t n_t^{\alpha-1}}}_{\text{Marginal cost}}. \tag{40}$$

Note that under $\eta = 1$, the above expression collapses to the labor demand function in the static problem given in equation (15). However, under $\eta \in (0, 1)$ the marginal benefit from selling an extra unit in period t consists both of the contemporaneous marginal increase in revenue (p_t) as well as the (expected) marginal increases in future revenue that come from retained customer relationships.

5.4 Government

In the baseline version of our model, government consumption is isomorphic to private consumption. Given a sequence of government spending $\{G_t\}_{t=0}^\infty$, the government's customer relationships evolve according to:

$$y_t^G = G_t + \rho\left[\frac{y_t^G - (1 - \eta)y_{t-1}^G}{q(x_t)}\right], \tag{41}$$

or alternatively

$$y_t^G = [1 + \gamma(x_t)]G_t - (1 - \eta)\gamma(x_t)y_{t-1}^G. \tag{42}$$

The government levies a lump-sum tax T_t on households to run balanced budgets every period, given a sequence of payroll tax rates $\{\tau_t\}_{t=0}^{\infty}$:

$$T_t = p_t y_t^G - w_t n_t \tau_t. \tag{43}$$

In the baseline version of our model, we assume exogenous autoregressive paths for government spending and the payroll tax rates:

$$G_t = (1 - \rho_G) g + \rho_G G_{t-1} + \varepsilon_t^G, \qquad \forall t \geq 0 \tag{44}$$

$$\tau_t = (1 - \rho_\tau) \tau + \rho_\tau \tau_{t-1} + \varepsilon_t^\tau, \qquad \forall t \geq 0, \tag{45}$$

where $\{\varepsilon^G\}_t \sim^{iid} (0, \sigma_G^2)$ and $\{\varepsilon^\tau\}_t \sim^{iid} (0, \sigma_\tau^2)$ are exogenous government spending and payroll taxation shocks, respectively.

5.5 Market clearing

The equilibrium of the system is described by optimality conditions (39), (40), (42), the equations of motion for customer relationships in (36), (41) and (44) as well as market clearing conditions in the goods market:

$$y_t = y_t^c + y_t^G, \qquad \forall t \geq 0 \tag{46}$$

in the labor market:

$$l_t = n_t, \qquad \forall t \geq 0, \tag{47}$$

and the market for the non-produced good:

$$m_t = \bar{m}, \qquad \forall t \geq 0. \tag{48}$$

With the market clearing in asset market ($B_t = 0$) omitted by Walras Law, they give ten equations in eleven endogenous variables $\{y_t, y_t^c, y_t^G, l_t, n_t, c_t, m_t, x_t, p_t, w_t, F_{t,t+1}\}_{t=0}^{\infty}$. Just like in the static model, this indeterminacy is intrinsic in any model with search-and-matching frictions. The next subsection outlines our strategy for closing the model.

5.6 Closing the model: pricing rule

Recall that in the main text we introduced broad classes of flexible and frictional equilibria, which differ in the way the model is closed. We are going to follow a similar approach in our dynamic model. As an example of a flexible equilibrium, one could consider our equilibrium conditions, augmented by a sequence of prices $\{\tilde{p}_t\}_{t=0}^{\infty}$ that would ensure that resulting equilibrium tightness is at the same level that would be chosen by a social planner.[26] Hence, closing our model with the pricing equation $p_t = \tilde{p}_t, \forall t \geq 0$ gives an example of a flexible equilibrium.

As an example of a frictional equilibrium, one could close the model with a pricing equation that describes persistent adjustment to the price $\tilde{p}_t$:

$$p_t = p_{t-1}^{\varepsilon} \tilde{p}_t^{1-\varepsilon}, \qquad \forall t \geq 0 \tag{49}$$

[26] Appendix G.2 provides the solution to the social planner's problem in the dynamic model.

Table 1: Parameter Calibrations (United States, annual frequency)

Parameter	Description	Value	Source/Target
Household parameters			
β	Time discount factor	0.96	Annual real rate of 4 per cent
σ	Relative risk aversion	1.00	Chetty (2006)
ψ	Inverse Frisch elasticity of labor supply	0.50	Standard
ν	Disutility of supplying labor	2.13	Target $l = 1/3$
Firm parameters			
α	Returns to labor	0.60	Standard
ε	Price rigidity	0.70	Standard
Goods market parameters			
η	Rate of destruction of customer relationships	0.40	Mattersion (2001)
ρ	Goods cost per visit	0.41	Target $q(x) = 0.77$
δ	Elasticity parameter of the matching function	3.62	Target $\left[\frac{f(x)}{\eta+f(x)(1-\eta)}\right]^{\frac{1}{\alpha}} = 0.91$
Fiscal policy parameters			
g	Steady state government spending	0.07	Target $g/(c+g)$ = 0.18
τ	Steady state firms' payroll tax rate	0.20	—
Exogenous processes parameters			
$\rho_X = \rho_a = \rho_G = \rho_\tau$	Persistence of exogenous processes	0.90	Standard

where $\varepsilon \in (0,1]$ pins down the degree of price rigidity. Given the evidence in Subsection 3.5 that equilibria featuring price rigidity are a better description of the US economy at business cycle frequencies, we assume a degree of price rigidity instead of fully flexible prices.

5.7 Calibration

We calibrate the model on US data at annual frequencies. Most of the calibration is standard. We set $\beta = 0.96$ (to produce a real interest rate of 4 per cent in steady state), $\sigma = 1.00$ (log utility of consumption, based on mean estimates in Chetty, 2006), $\psi = 0.50$ (Frisch elasticity of labor supply equal to 2) and $\alpha = 0.60$ (labor share of income equal to 0.6). The labor disutility parameter $\nu = 2.33$ is set to target a steady-state employment rate of 1/3 ($l = 1/3$). The degree of price rigidity is calibrated at $\varepsilon = 0.70$. We set $\eta = 0.40$, which yields the rate of destruction of customer relationships equal to 40 per cent per year (based on US customer attrition evidence in Mattersion, 2001).

The cost per visit ρ and elasticity of marching function δ are non-standard parameters. We calibrate them by targeting the steady-state rate of current labor utilization $\left[\frac{f(x)}{\eta+f(x)(1-\eta)}\right]^{\frac{1}{\alpha}}$ (estimated at 0.91 as the long-run average of labor utilization rate, reported by the Institute for Supply Management)[27] and the steady-steady probability of a successful shopping visit $q(x)$ (estimated at 0.77 as one minus the average stock-out rate, reported by Taylor and Fawcett (2001) and Jing and Lewis (2011).

Steady state of government spending parameter g is set to match the spending-to-GDP ratio equal to 18% in steady state. The steady-state payroll tax rate τ is calibrated to be equal to 0.20. Finally, all autoregressive

[27] From the equation of motion of firms' sales in (41) it follows that in steady state $y = \frac{f(x)}{\eta+f(x)(1-\eta)} a n^\alpha = a \left\{ \left[\frac{f(x)}{\eta+f(x)(1-\eta)}\right]^{\frac{1}{\alpha}} n \right\}^\alpha$, so that $\left[\frac{f(x)}{\eta+f(x)(1-\eta)}\right]^{\frac{1}{\alpha}}$ is the steady-state labor utilization rate.

parameters are set equal at 0.9, so that $\rho_\chi = \rho_a = \rho_G = \rho_\tau = 0.9$. Table 1 summarizes the calibration.

5.8 *Conditional* state-dependent fiscal multipliers

To quantitatively assess the degree of state-dependence of fiscal multipliers and its variation with the source of business cycle fluctuations, we compute spending and tax cut multipliers in recessionary and expansionary episodes, conditioning on whether a particular episode was generated by a demand or supply shock. We consider a fully non-linear solution to our model under perfect foresight.

We first compute the impulse response of GDP to a one-time preference/technology shock, $\{GDP_j^{shock}\}_{j=0}^H$, where $shock \in \{\varepsilon^\chi, \varepsilon^a\}$; we then compute the impulse response of GDP subject to the same shock, combined with either a spending shock $\{\varepsilon^G > 0\}$ or a tax cut shock $\{-\varepsilon^\tau < 0\}$, to obtain time series that embed the interaction between the fundamental shock driving the business cycle and the shock related to the expansion in government spending, $\{GDP_j^{shock+\varepsilon^G}\}_{j=0}^H$, or reduction in taxes, $\{GDP_j^{shock-\varepsilon^\tau}\}_{j=0}^H$. We construct the *conditional* government spending multiplier as follows:

$$\varphi^G(shock) = \frac{\sum_{j=0}^H \left[GDP_j^{shock+\varepsilon^G} - GDP_j^{shock}\right]}{\sum_{j=0}^H \left[G_j^{\varepsilon^G} - g\right]}, \tag{50}$$

where $\left\{G_j^{\varepsilon^G}\right\}_{j=0}^H$ is the impulse response of government spending to the spending shock $\{\varepsilon^G > 0\}$ and H is the horizon of the impulse response, so that $\sum_{j=0}^H \left[G_j^{\varepsilon^G} - g\right]$ denotes a cumulative increase in government spending compared to its steady-state value.

Similarly, we construct horizon-H *conditional* tax cut multipliers out of cutting the rate of tax τ_t as:

$$\varphi^\tau(shock) = \frac{\left[GDP_H^{shock-\varepsilon^\tau} - GDP_H^{shock}\right]/\overline{GDP}}{\varepsilon^\tau} \tag{51}$$

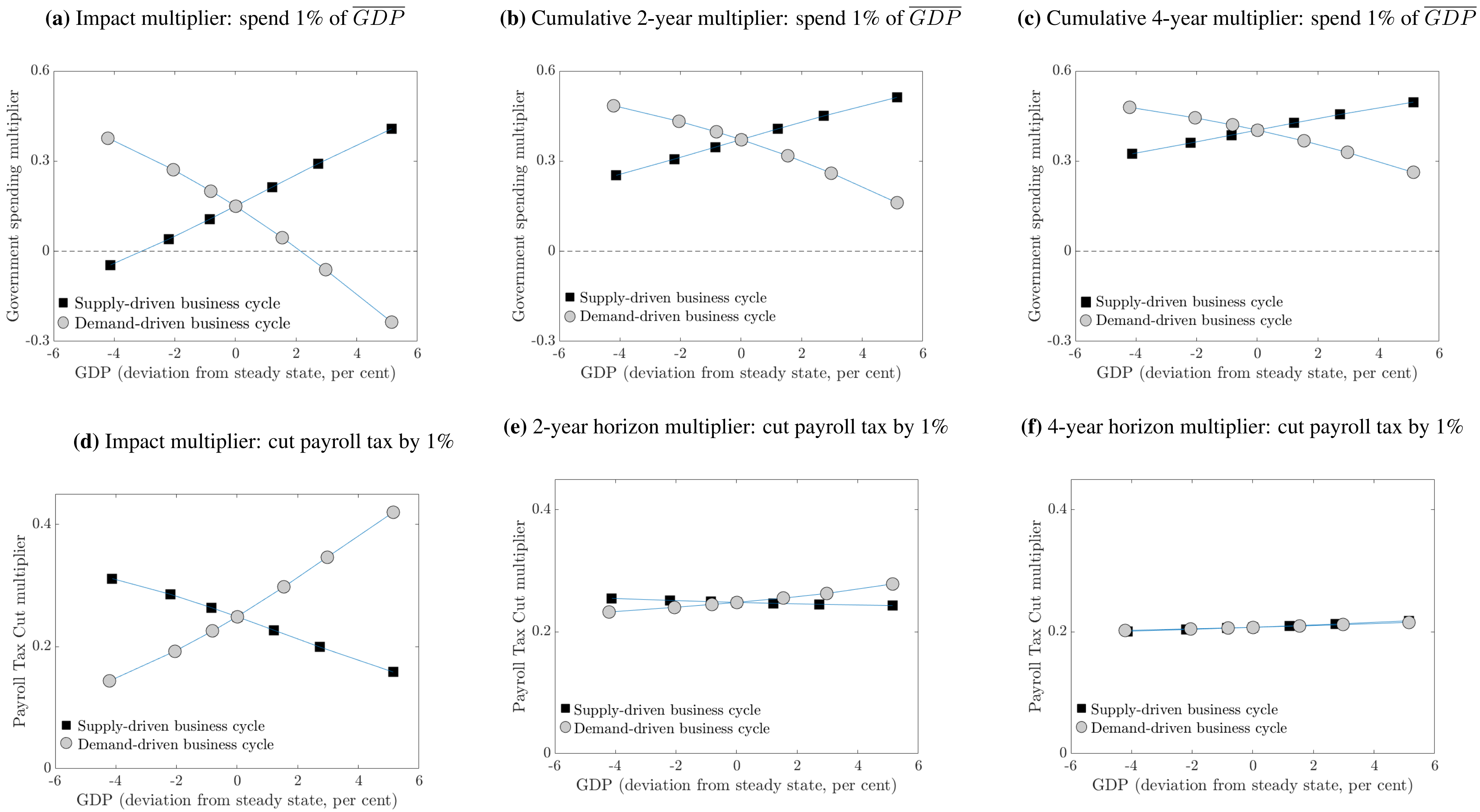
where $\overline{GDP}$ is steady-state level of GDP.

We calibrate $\varepsilon^G = 0.01\overline{GDP}$ to consider a one-period spending shock equal to 1 per cent of steady-state GDP; further, we set $\varepsilon^\tau = 0.01$, so that we consider a 1 percentage point cut in the rate of the payroll tax.

To investigate the link between state dependence and the horizon of the response, we distinguish between the *impact* multipliers ($H = 0$), and *cumulative* 2-year ($H = 2$) and 4-year ($H = 4$) multipliers, following the convention in the empirical literature that considers a two- and four-year horizons (Ramey and Zubairy, 2018).

Figure 5 shows our constructed *conditional* government spending and payroll tax cut multipliers. Panel (a) plots the impact of the government spending multiplier, which is equal to around 0.15 when GDP is at the steady-state value. A strong demand-driven recession that takes GDP 4 per cent below the steady state, raises the spending multiplier to 0.40. On the other hand, a demand-driven expansion that raises output 4 per cent above the steady state, decreases the multiplier to -0.2. Under supply-driven fluctuations, the cyclicality of spending multipliers is reversed. In a supply-driven recession, where GDP drops 4 per cent below the steady state, the spending multiplier drops to around -0.05, and in a 4 per cent supply-driven expansion the multiplier increases to around 0.30. From Panels (b) and (c), the above properties are preserved for cumulative spending multipliers, although the degree of

Figure 5: Conditional state-dependent fiscal multipliers



(a) Impact multiplier: spend 1% of $\overline{GDP}$

(b) Cumulative 2-year multiplier: spend 1% of $\overline{GDP}$

(c) Cumulative 4-year multiplier: spend 1% of $\overline{GDP}$

(d) Impact multiplier: cut payroll tax by 1%

(e) 2-year horizon multiplier: cut payroll tax by 1%

(f) 4-year horizon multiplier: cut payroll tax by 1%

Notes: *Panel (a) shows impact multipliers following a one-time innovation to the government spending process equal to 1% of steady-state GDP, in recessionary and expansionary episodes caused by different types of shocks; Panels (b) and (c) repeat the exercise for cumulative multipliers, computed over a 2-year and 4-year horizon, respectively.*
Panel (d) shows impact multipliers following a one-time innovation to the payroll tax rate process equal to negative 1 percentage point, in recessionary and expansionary episodes caused by different types of shocks; Panels (e) and (f) repeat the exercise for the 2-year and 4-year horizon tax cut multipliers, respectively.

state dependence is weaker. This is because price rigidity is crucial for state dependence in our model, and over the five-year horizon a higher fraction of firms gets to adjust prices.

Panel (d) of Figure 5 shows that the impact multiplier out of payroll tax cuts is close to 0.25 in steady state, but almost doubles in size in a 4 per cent demand-driven expansion, and increases to 0.30 in a 2 per cent supply-side recession. However, it drops to almost 0.15 in a 4 per cent demand-side recession and a 4 per cent supply-side expansion. As before, Panel (e) and (f) show that the two- and four-year horizon multipliers out of payroll tax cuts preserve the properties of their impact counterparts, although state dependence is significantly muted.

5.9 Implications for the empirical debate on fiscal state dependence

Results in Figure 5 offer an interpretation to the recent empirical debate on the degree of fiscal state dependence. In particular, studies such Auerbach and Gorodnichenko (2012, 2013) and Fazzari et al. (2014) point to much larger spending multipliers in recessions compared to expansions, whereas Ramey and Zubairy (2018) construct a longer historical dataset and estimate the degree of state dependence to be much more modest. All of the aforementioned studies distinguish between recessionary and expansionary episodes with unconditional thresholds, such as unemployment, GDP growth, or the output gap. However, as shown in Panels (a)-(c) of Figure 6, a given level of GDP could be consistent with both low and high levels of the government spending multiplier, depending on the *type of shock* that caused that level of GDP in the first place. As a result, econometric techniques that distinguish between recessions and expansions with unconditional thresholds can estimate spending multipliers to be either countercyclical or procyclical, depending on whether the estimation sample covers periods with predominantly demand-driven fluctuations or supply-driven fluctuations.

In the next section, we develop and estimate an econometric model that allows us to evaluate *conditional* state-dependent fiscal multipliers, where one explicitly accounts for the source of economic fluctuations.

6 Econometric evidence

This section develops and estimates a novel econometric model that allows to perform reduced-form estimation of *conditional* state-dependent fiscal multipliers, controlling for either the demand- or supply-driven nature of a given recessionary or expansionary state. We find strong empirical support to our theoretical predictions: the estimated spending multipliers in demand-side recessions are substantially higher than those in supply-side recessions, particularly at shorter horizons. At the same time we find tax cut multipliers to be significantly higher in supply-side recessions compared to the demand-side ones.

6.1 Conditional state-dependent fiscal multipliers

Our theory establishes that fiscal multipliers are state-dependent and cyclicality is determined by the source of economic fluctuations. Conditional on demand-driven fluctuations, demand-side multipliers are countercyclical and supply-side multipliers are procyclical. Conditional on supply-driven fluctuations, the exact opposite hold.

We estimate two econometric models using the local projection methodology in Jordà (2005) and evaluate spending and tax cut multipliers in recessionary and expansionary episodes, *conditional* on those being demand-or supply-driven in nature. Our approach identifies demand- and supply-driven fluctuations using observed co-movement between cyclical components of economic activity and inflation. In accordance with the insights of a wide range of models, a demand-side recession is characterized by a joint fall in economic activity and inflation while a supply-driven recession is characterized by a fall in economic activity and a rise in inflation.[28] This study is the first one to estimate state-dependent fiscal multipliers controlling for the source of fluctuations.[29]

6.1.1 Conditional state-dependent spending multipliers

We extend the one-step IV procedure from Ramey and Zubairy (2018) to account for the source of economic fluctuations. Instead of distinguishing between expansions and recessions using an unconditional unemployment threshold $\bar{U}$, we split recessionary states, where $U_t \geq \bar{U}$, into those where inflation is below its trend value, $\pi_t < \bar{\pi}_t$, corresponding to demand-side recessions, and those where inflation is above trend, $\pi_t \geq \bar{\pi}_t$, corresponding to supply-side recessions. We could additionally split expansionary states, $U_t < \bar{U}$, into those where inflation is below its trend value, $\pi_t < \bar{\pi}_t$, corresponding to supply-side expansions, and those where inflation is above trend, $\pi_t \geq \bar{\pi}_t$, corresponding to demand-side expansions; we perform this exercise in Appendix I.

Our baseline specification to estimate cumulative spending multipliers at horizon H is:

$$\begin{aligned}\sum_{s=t}^{t+H}\left(\frac{GDP}{GDP^*}\right)_s =& \mathbf{1}\{U_{t-1} < \bar{U}\}\left[\alpha_H^E + \beta_H^E \sum_{s=t}^{t+H}\left(\frac{G}{GDP^*}\right)_s + \gamma_H^E \mathbf{z}_{t-1}\right] + \\ & \mathbf{1}\{U_{t-1} \geq \bar{U}; \pi_{t-1} < \bar{\pi}_{t-1}\}\left[\alpha_H^{DR} + \beta_H^{DR} \sum_{s=t}^{t+H}\left(\frac{G}{GDP^*}\right)_s + \gamma_H^{DR} \mathbf{z}_{t-1}\right] + \\ & \mathbf{1}\{U_{t-1} \geq \bar{U}; \pi_{t-1} \geq \bar{\pi}_{t-1}\}\left[\alpha_H^{SR} + \beta_H^{SR} \sum_{s=t}^{t+H}\left(\frac{G}{GDP^*}\right)_s + \gamma_H^{SR} \mathbf{z}_{t-1}\right] + \varepsilon_{t+H},\end{aligned} \tag{52}$$

where $\sum_{s=t}^{t+H}\left(\frac{GDP}{GDP^*}\right)_s$ and $\sum_{s=t}^{t+H}\left(\frac{G}{GDP^*}\right)_s$ are cumulative real GDP and real government expenditures, both normalized by trend real GDP (GDP^*), [30] $\mathbf{z}_{t-1}$ is a vector of controls, [31] and $\mathbf{1}\{.\}$ is the indicator variable. The above equation is estimated by 2SLS, where the instrument set includes exogenous government spending shocks, such as narrative military spending news shocks from Ramey and Zubairy (2018) or VAR-based shocks from Blanchard and Perotti (2002) , interacted with the state-specific indicator variable.

[28] Recent work by Guerrieri et al. (2020) introduces the notion of ”Keynesian supply shocks”, which can lead to demand-deficiency, and hence create a positive co-movement between inflation and activity, unlike traditional supply shocks. Though such shocks are not a feature of our model, the demand-deficiency they create is akin to a state with low goods market tightness. In this sense, even if Keynesian supply shocks are present in the data, our strategy of using co-movement between inflation and activity still allows to identify periods with high and low goods market tightness, and test our theory.

[29] Ramey and Zubairy (2018) and Ziegenbein (2017) distinguish between recessions and expansions with an unconditional unemployment threshold whereas Auerbach and Gorodnichenko (2013) employ smooth state transitions based on the unconditional rate of economic growth.

[30] Following Ramey and Zubairy (2018), this normalization is to ensure cumulative GDP and government spending are measured in the same units, which avoids the need to covert estimates in logs to levels.

[31] The precise set of variables used as controls is outlined in description to the relevant regression tables.

An advantage of this approach is that our estimates for β_H^E, β_H^{DR}, and β_H^{SR} directly give us values for horizon-H cumulative spending multipliers in, respectively, expansions (E), demand-side recessions (DR), and supply-side recessions (SR). Our theory predicts that spending multipliers in demand-side recessions are higher that those in supply-side recessions, so that $\beta_H^{DR} > \beta_H^{SR}$, and we can test this prediction.

6.1.2 Conditional state-dependent tax cut multipliers

In a similar fashion, we use local projections to estimate *conditional* state dependence for tax cut multipliers. We extend the approach in Eskandari (2019) that distinguishes between recessionary and expansionary episodes using an unconditional unemployment threshold by further differentiating between demand-side and supply-side recessions. One also can split expansionary states, $U_t < \bar{U}$, into those where inflation is below its trend value, $\pi_t < \bar{\pi}_t$, representing supply-side expansions, and those where inflation is above trend, $\pi_t \geq \bar{\pi}_t$, representing demand-side expansions; we perform this exercise in Appendix I.

Our baseline specification to estimate tax cut multipliers at horizon H is given by:

$$\begin{aligned} \ln GDP_{t+H} - \ln GDP_{t-1} =& \mathbf{1}\{U_{t-1} < \bar{U}\} \left[\alpha_H^E + \beta_H^E \tau_t + \gamma_H^E \mathbf{z}_{t-1}\right] + \\ & \mathbf{1}\{U_{t-1} \geq \bar{U}; \pi_{t-1} < \bar{\pi}_{t-1}\} \left[\alpha_H^{DR} + \beta_H^{DR} \tau_t + \gamma_H^{DR} \mathbf{z}_{t-1}\right] + \\ & \mathbf{1}\{U_{t-1} \geq \bar{U}; \pi_{t-1} \geq \bar{\pi}_{t-1}\} \left[\alpha_H^{SR} + \beta_H^{SR} \tau_t + \gamma_H^{SR} \mathbf{z}_{t-1}\right] + \varepsilon_{t+H}, \end{aligned} \tag{53}$$

where τ_t is an exogenous shock to the average tax rate in the economy, and the rest of the notation carries over from the spending multiplier regressions. Given a time series for exogenous tax rate shocks, such as narrative tax shocks from Romer and Romer (2010), the above specification is estimated by OLS.

The estimates for β_H^E, β_H^{DR}, and β_H^{SR} directly provide values for horizon-H tax cut multipliers in, respectively, expansions (E), demand-side recessions (DR), and supply-side recessions (SR). Our theory predicts that tax cut multipliers in demand-side recessions are lower than those in supply-side recessions, so that $\beta_H^{DR} < \beta_H^{SR}$, and we test the prediction.

6.2 Data

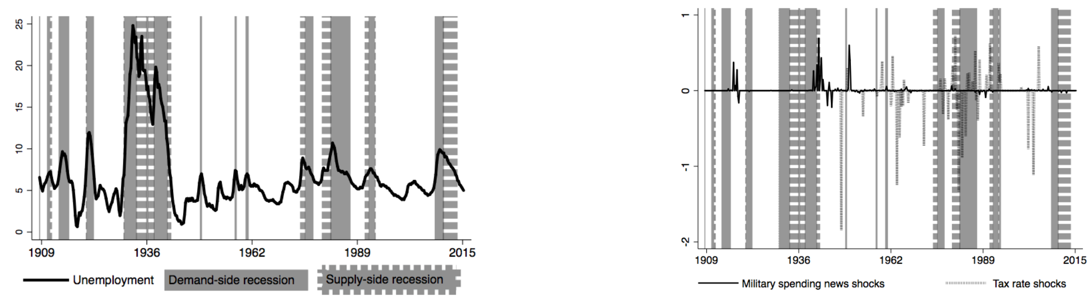
We estimate the model using quarterly US data. We use the series for real GDP (GDP), civilian unemployment (U), and government consumption and fixed capital formation (G) data that extend back to 1889 by Ramey and Zubairy (2018). Trend GDP (GDP^*) is measured as sixth-order polynomial exponential trend of real GDP, following Gordon and Krenn (2010). We measure quarterly inflation (π_t) as year-on-year change in (log) GDP deflator, and trend inflation ($\bar{\pi}_t$) is obtained by HP-filtering the raw inflation series with a smoothing parameter $\lambda = 1600$ for quarterly data. The baseline unemployment threshold is set at $\bar{U} = 6.5\%$, consistent with Ramey and Zubairy (2018).

Our baseline measure of the government spending shock is the narrative military spending news shocks in Ramey and Zubairy (2018), and for tax rate shocks, we use the narrative measure in Romer and Romer (2010).

Figure 6: Demand- and supply-side recessions and fiscal shocks

(a) Demand- and supply-side recessions

(b) Fiscal shocks and sources of recessions



Notes: *Panel (a) shows the unemployment rate in the US between 1909-2015, as well as demand-side recessions, identified by the indicator variable* $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$*, and supply-side recessions, identified by the indicator variable* $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$*; Panel (b) additionally plots time series of military spending news shocks from Ramey and Zubairy (2018) and narrative tax rate shocks from Romer and Romer (2010).*

Appendix I shows that results hold using VAR-based spending and tax shocks constructed following Blanchard and Perotti (2002).

For spending multipliers, the sample period is 1909:Q1-2015:Q4; for tax cut multipliers, we use the shorter time sample 1947:Q1-2007:Q4, with the shorter sample driven by the available time series of tax shocks in Romer and Romer (2010).[32]

6.3 Demand-side and supply-side recessions: a closer look

Panel (a) in Figure 6 shows historical periods of demand-side recessions characterized by a negative co-movement between unemployment and the cyclical component of inflation (solid shaded area), and supply-side recession characterized by positive comovement between these variables (striped shaded area). The majority of the US Great Depression is identified as a demand-side recession; the oil shocks of the 1970s start off as a supply-side recession, evolving into a demand-side recession. In the case of the late 1970s/early 1980s recession, this could be due to Volcker disinflation that immediately followed the second wave of oil shocks. The Great Recession, on the other hand, originates as a demand-side recession, evolving into a supply-side recession. One explanation is that initial negative effect on households wealth and income evolved into a supply-side constraint as firms were unable to access capital due to the distorted financial system.

Our identification strategy relies on having enough spending and taxation shocks in each of the three states of the world considered in the baseline specification. Panel (b) in Figure 6 plots the time series for military spending news shocks from Ramey and Zubairy (2018) and narrative tax rate shocks from Romer and Romer (2010) against our definition of states. The figure shows that spending and tax rate shocks are spread fairly evenly across expansions, demand- and supply-side recessions.[33] Formally, 28% of quarters identified as an expansion,

[32]In principle, the narrative military spending news shocks series from Ramey and Zubairy (2018) goes back to 1889:Q1, but we exclude the first twenty years of their sample due to excessive volatility of inflation in that time period, as the latter complicates our strategy of separating out episodes with inflation above and below trend; however, our baseline results are robust to considering the full sample and are available upon request.

[33]A bulk of variation in our spending and tax rate shocks overlaps with periods of price controls around World War II and the Korean War. In order to check that such price controls do not pose a challenge to our strategy of using co-movement between inflation and activity,

16% of quarters identified as a demand-side recession and 16% of quarters identified as a supply-side recession contain a non-zero military spending news shock. Similarly, 14% of quarters identified as an expansion, 31% of quarters identified as a demand-side recession and 32% of quarters identified as a supply-side recession contain a non-zero narrative tax shock.

6.4 Empirical results

6.4.1 Conditional state-dependent spending multipliers

Table 2 shows baseline estimation results for spending multipliers. Column (1) shows that the 2-year cumulative spending multiplier is equal to 0.70 without any conditioning on the source of fluctuations. Column (2) replicates the exercise in Ramey and Zubairy (2018), by distinguishing between recessions and expansions based on an unconditional unemployment threshold. The 2-year cumulative multiplier is equal to 0.68 in expansions, which is larger than the estimated recession multiplier equal to 0.54, although the difference is not statistically significant. These estimates blend demand- and supply-driven episodes, while our theory shows that the source of fluctuations is crucial to establish an estimate for the spending multiplier. To test our theoretical prediction, column (3) separately estimates 2-year cumulative spending multipliers in demand- and supply-driven recessions. Consistent with our theoretical findings, the spending multiplier in demand-driven recessions is equal to 0.86, which is larger than the multiplier in supply-driven recessions, which equals 0.32.

Columns (4)-(6) repeat the exercise for 4-year cumulative multipliers. As before, conditioning on recessions and expansions delivers spending multipliers that are slightly higher in expansions (0.76) than in recessions (0.65), although the difference is not statistically significant. Instead, controlling for whether recessions are generated by demand- or supply-side shocks corroborates our theory: spending multipliers are higher in demand-side recessions (0.71) than in supply-side recessions (0.63), although the difference is smaller than in the case of 2-year multipliers. The finding that conditional state dependence becomes weaker at longer horizons is consistent with our theory, as verified in the quantitative dynamic model. At longer time horizons, prices adjust to shocks and tightness plays a smaller role in business cycle adjustment, bringing the multiplier closer to its value under flexible prices, determined by the elasticities of labor demand and labor supply.

To investigate the relationship between the degree of conditional state dependence and the horizon of cumulation, Figure 7 repeats the exercise for horizons from 4 to 20 quarters. In Panel (a), we do not condition on the source of fluctuations and simply distinguish between recessions and expansions; consistently with Ramey and Zubairy (2018), very limited state dependence is detected, with formal statistical tests showing no significant differences at any horizon. However, when we condition on the source of fluctuations in Panel (b), we find that, consistent with our theory, spending multipliers in demand-side recessions are higher than those in supply-side recessions at all horizons. Moreover, the degree of such conditional state dependence is strongest at shorter horizons: 4 quarters after a spending shock, the cumulative spending multiplier is close to one in demand-driven recessions, but close to zero in supply-side recessions. On the other hand, and again consistent with our theory, multipliers in demand- and supply-side recessions become very similar after 12 quarters. We formally test the restriction $\beta_H^{DR} = \beta_H^{SR}$, which implies that the source of fluctuations does not matter as in Ramey and Zubairy (2018), and can reject it at

Figure 14 in Appendix I.2 reports estimated responses of inflation to our spending and tax rate shocks. Reassuringly, we find that both a positive spending shock and a positive tax rate shock produce a statistically significant rise in inflation.

the 10% level at 6- and 7-quarter horizons, and at 32% level for all horizons between 4 and 11 quarters.

6.4.2 Conditional state-dependent tax cut multipliers

Table 3 shows baseline estimation results for tax cut multipliers. Column (1) shows that 2-year tax cut multiplier is 1.50 without any conditioning on the state of the economy, and not significantly different from zero. The lack of significance could be explained by the fact that tax cuts affect GDP through expansions in capacity, a very gradual process that is difficult to detect within 2 years. Column (3) conditions the estimates on recessions, and it shows that the tax cut multiplier is equal to 1.81 in expansions and 0.98 in recessions, although again neither are significantly different from zero. As before, the recessionary states blend demand- and supply-driven episodes, and our theory predicts that tax cut multipliers are larger in supply-side recessions. Results in column (3), derived by controlling for the source of fluctuations, support our theoretical prediction: in demand-driven recessions, the tax cut multiplier is 1.49 and not significantly different from zero, whereas in supply-side recessions it is 4.29, and statistically significant at 10% level.

Columns (4)-(6) repeat the tax-cut estimation exercise for the 4-year horizon. The unconditional tax cut multiplier in column (4) is 1.71 and statistically significant at 5% level. The fact that the multiplier is significant at 4-year horizon, and not at 2-year horizon, is consistent with the fact that capacity expansion is considered to be a gradual process. Column (5) reports that the tax cut multiplier is equal to 2.37, and significant at 5 per cent level, in expansions, but lower and equal to 1.24 and insignificant in recessions. Column (6) shows estimates that control for demand- and supply-side recessions, showing that in supply-side recessions the multiplier is higher at 1.80, and significant at 10 per cent, whereas it is negative at -1.98 and highly insignificant in demand-side recessions.

Figure 8 investigates the relationship between conditional state dependence of tax cut multipliers and the horizon considered. In Panel (a), we do not condition on the source of fluctuations and simply distinguish between recessions and expansions: as one can see, very limited state dependence is detected in this case. However, once we condition on the source of fluctuations in Panel (b), the tax cut multiplier in supply-side recessions is consistently higher than the multiplier in demand-side recessions, as our theory predicts, except for 4- and 5-quarter horizons. Unlike the spending multiplier, conditional state dependence of tax cut multipliers is not at its maximum at shorter horizons and instead is close to uniform after approximately 8 quarters. We formally test the restriction $\beta_H^{DR} = \beta_H^{SR}$, which implies that the source of fluctuations does not matter, and can reject it at the 10% level at 11- and 13-quarter horizons, and at 32% level for all horizons between 8 and 16 quarters, as well as at the 20-quarter horizon.

6.4.3 Robustness checks

In Appendix I, we perform further robustness checks, briefly outlined here. In particular, in Appendix I.3, we show that once one further distinguishes between demand- and supply-side expansions, our theory receives further empirical support: spending multipliers are higher in supply-side expansions, whereas multipliers out of tax cuts are larger in demand-driven economic upturns. In Appendix I.4, we show that our results are robust to using VAR-based fiscal shocks, following Blanchard and Perotti (2002). Finally, in Appendix I.5 we provide results where instead of measuring economic activity using unemployment, we are using detrended real GDP, a measure more consistent with our theory.

Table 2: Conditional state-dependent spending multipliers ($\bar{U} = 6.5\%$; US military spending news shocks

US data: 1909:Q1-2015:Q4		2-year horizon			4-year horizon	
State	(1)	(2)	(3)	(4)	(5)	(6)
β_H: Linear	**0.70*****			**0.75*****		
	(0.06)			*(0.06)*		
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		**0.68*****	**0.68*****		**0.76*****	**0.76*****
		(0.10)	*(0.09)*		*(0.13)*	*(0.12)*
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		**0.54*****			**0.65*****	
		(0.13)			*(0.08)*	
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			**0.86*****			**0.72*****
			(0.33)			*(0.12)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			**0.32*****			**0.63*****
			(0.11)			*(0.09)*
$\beta_H^E = \beta_H^R$ (p-value)		0.37			0.44	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.14			0.54
T	416	416	416	408	408	408

***Notes**: HAC standard errors are reported in parentheses, with ***(**,*) denoting statistical significance at 1%(5%, 10%) level; all regressions include a set of controls, consisting of four lags of real GDP, real government spending and military spending news shocks, all normalized by trend real GDP as well as a constant (coefficients on controls are allowed to be state-specific).*

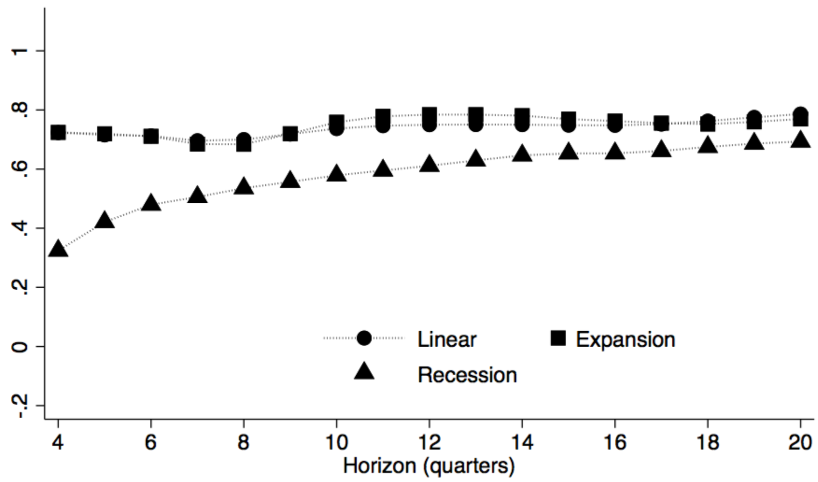
Table 3: Conditional state-dependent tax cut multipliers ($\bar{U} = 6.5\%$; US Romer-Romer narrative tax shocks)

US data: 1947:Q1-2007:Q4		2-year horizon			4-year horizon	
State	(1)	(2)	(3)	(4)	(5)	(6)
β_H: Linear	1.50			**1.71****		
	(1.14)			*(0.82)*		
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		1.81	1.81		**2.37****	**2.37****
		(1.17)	*(1.12)*		*(0.99)*	*(0.99)*
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		0.98			1.24	
		(1.07)			*(0.87)*	
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			1.49			-1.98
			(1.04)			*(2.75)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			**4.29***			**1.80***
			(2.18)			*(1.00)*
$\beta_H^E = \beta_H^R$ (p-value)		0.48			0.39	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.25			0.20
T	240	240	240	240	240	240

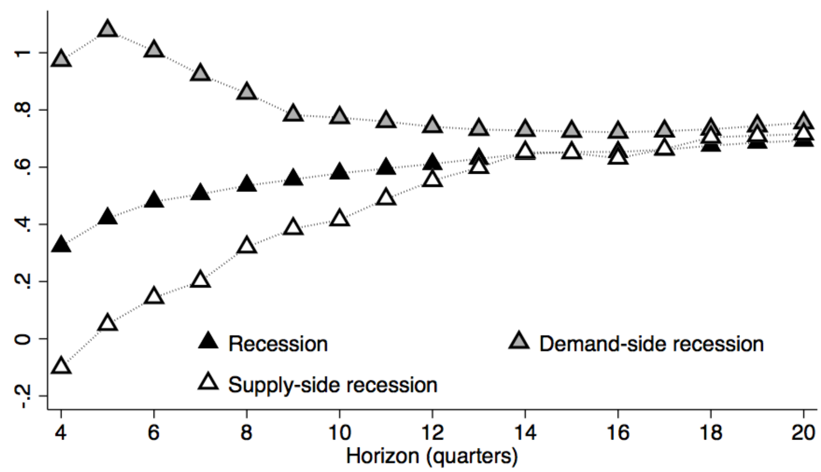
***Notes**: HAC standard errors are reported in parentheses, with ***(**,*) denoting statistical significance at 1%(5%, 10%) level; all regressions include a set of controls, consisting of four lags of (log) real GDP as well as a constant (coefficients on controls are allowed to be state-specific).*

Figure 7: Government Spending Multipliers across Horizons (US military spending news shocks, 1909-2015)

(a) Cumulative government spending multipliers in recessions and expansions



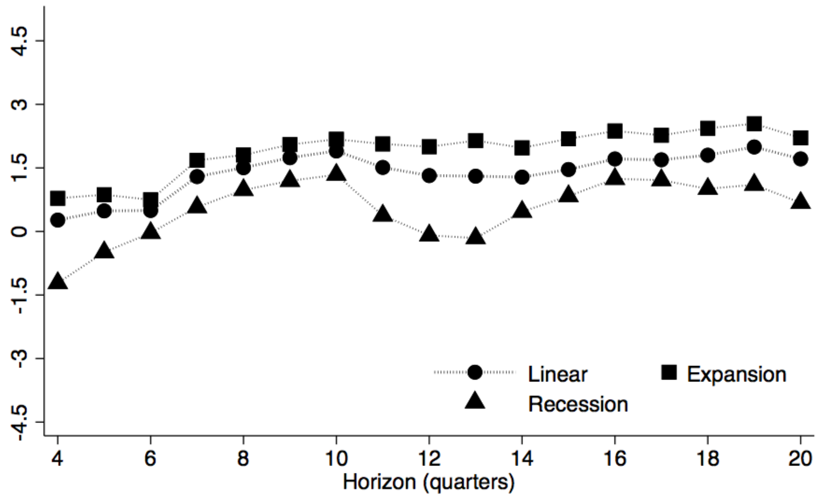
(b) Cumulative government spending multipliers in demand-side and supply-side recessions



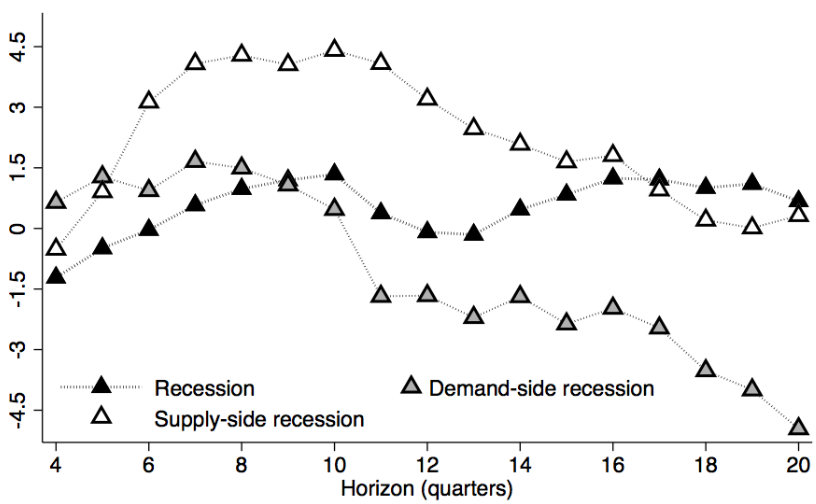
Notes*: Panel (a) shows cumulative government spending multipliers estimated in recessionary* $\mathbf{1}\{U_t \geq \bar{U}\}$ *and expansionary* $\mathbf{1}\{U_t < \bar{U}\}$ *episodes as well as linear benchmarks for different cumulation horizons* $4 \leq H \leq 20$*;*
Panel (b) shows cumulative government spending multipliers estimated in demand-side recessionary episodes $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$ *and supply-side recessionary episodes* $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$ *as well as unconditional recessions* $\mathbf{1}\{U_t \geq \bar{U}\}$ *for different horizons* $4 \leq H \leq 20$*; we set* $\bar{U} = 6.5\%$ *in all estimations.*

Figure 8: Tax Cut Multipliers across Horizons (US Romer-Romer narrative tax shocks, 1947-2007)

(a) Tax cut multipliers in recessions and expansions



(b) Tax cut multipliers in demand-side and supply-side recessions



Notes: *Panel (a) shows tax cut multipliers estimated in recessionary* $\mathbf{1}\{U_t \geq \bar{U}\}$ *and expansionary* $\mathbf{1}\{U_t < \bar{U}\}$ *episodes as well as linear benchmarks for different horizons* $4 \leq H \leq 20$*;*
Panel (b) shows tax cut multipliers estimated in demand-side recessionary episodes $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$ *and supply-side recessionary episodes* $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$ *as well as unconditional recessions* $\mathbf{1}\{U_t \geq \bar{U}\}$*, for different horizons* $4 \leq H \leq 20$*; we set* $\bar{U} = 6.5\%$ *in all estimations.*

7 Conclusion

This paper develops a general theory of state-dependent fiscal multipliers for a broad range of spending and taxation policies. The framework accounts for empirically relevant goods market frictions by incorporating idle productive capacity and unsatisfied households' demand into an otherwise standard general equilibrium setup. Our key novel finding is that cyclicality of fiscal multipliers is pinned down by the *source of economic fluctuations*, and we provide model-free econometric evidence that strongly supports our predictions.

Crucially, we establish that multipliers associated with fiscal instruments which stimulate aggregate demand, such as government spending and consumption tax cuts, are *countercyclical* under demand-driven fluctuations and *procyclical* under supply-driven fluctuations. On the other hand, multipliers associated with interventions that stimulate aggregate supply, such as reductions in taxes on firms' payroll, sales and households' labor income, are *countercyclical* under supply-driven fluctuations and *procyclical* under demand-driven fluctuations. In addition, our theoretical results establish a relevant role for fiscal austerity, implemented by a reduction in government consumption in severe *supply-driven recessions* and *demand-driven booms*, provided elasticities of labor supply and labor demand are sufficiently low.

Further, we develop and estimate a novel econometric specification that allows us to perform model-free evaluation of both spending and taxation multipliers in recessionary and expansionary episodes, *conditional* on those being either demand- or supply-driven in nature. Our empirical results detect substantial state dependence, *conditional* on the source of fluctuations, which is in line with the predictions of our theory. Such findings offer a resolution to the debate on state dependence of fiscal policy, on both empirical and theoretical grounds, and they provide guidance for the conduct of fiscal policy in the different phases of the business cycle.

Our analysis opens fruitful avenues for future research. First, in our dynamic framework, current changes in fiscal policy determine the future path of goods market tightness and thus constrain the effectiveness of policy in the future. Our framework can therefore be extended to study the intertemporal trade-offs and the *path dependence* of fiscal policy. Second, by extending the model to include heterogeneity in the goods markets, one can study how composition of spending and taxation policies may generate spillover effects of fiscal policy across sectors and the socially optimal distribution of such policies. We plan to investigate these issues in future research.

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Appendix

(not for publication)

A Proofs of results in main text

Lemma 1. *The consumption function $c(p,x)$ is the optimal consumption choice in the representative household's problem evaluated under non-produced goods market clearing($m = \bar{m}$) and is given by:*

$$c(p,x) = \frac{\chi}{p[1+\gamma(x)]},$$

where $\frac{\partial c}{\partial p} < 0$, $\frac{\partial c}{\partial x} < 0$ *and* $\frac{\partial c}{\partial \chi} > 0$.

Proof. The Lagrangian of the representative household's problem is given by:

$$\mathcal{L} = \left[\chi \frac{c^{1-\sigma}}{1-\sigma} + \zeta(m) - \frac{l^{1+\psi}}{1+\psi} + \lambda(wl + \Pi + \bar{m} - T - p[1+\gamma(x)]c - m)\right]. \tag{54}$$

First-order conditions with respect to consumption of the produced and non-produced good, as well as the labor supply are given by:

$$\frac{d\mathcal{L}}{dc} = \chi c^{-\sigma} - \lambda p[1+\gamma(x)] = 0, \tag{55}$$
$$\frac{d\mathcal{L}}{dm} = \zeta'(m) - \lambda = 0, \tag{56}$$
$$\frac{d\mathcal{L}}{dl} = -l^{\psi} + \lambda = 0. \tag{57}$$

Combining the first-order conditions for consumption of the produced and the non-produced good, and evaluating it under $m = \bar{m}$ and our baseline assumption of $\zeta'(\bar{m}) = 1$ delivers the following consumption function:

$$c(p,x)^{\sigma} = \frac{\chi}{p[1+\gamma(x)]}. \tag{58}$$

Further, in the main text we assume $\sigma = 1$:

$$c(p,x) = \frac{\chi}{p[1+\gamma(x)]}. \tag{59}$$

It then follows that $\frac{\partial c}{\partial p} = -\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p} < 0, \quad \frac{\partial c}{\partial x} = -\frac{\chi}{p[1+\gamma(x)]}\frac{1}{[1+\gamma(x)]} < 0, \quad \frac{\partial c}{\partial \chi} = \frac{1}{p[1+\gamma(x)]} > 0.$ □

Lemma 2. *The labor supply function l(w) is the optimal labor supply choice in the representative household's problem evaluated at $m = \bar{m}$, and is given by:*

$$l(w) = w^{\frac{1}{\psi}},$$

where $\frac{\partial l}{\partial w} > 0$.

Proof. Combing the first-order conditions for consumption of the non-produced good (56) and labor supply (57), and evaluating it under $m = \bar{m}$ and our baseline assumption of $\zeta'(\bar{m}) = 1$ delivers the following labor supply function

$$l(w) = w^{\frac{1}{\psi}}.$$

It then follows that $\frac{\partial l}{\partial w} = \frac{1}{\psi} w^{\frac{1}{\psi}-1} > 0.$ □

Lemma 3. *The labor demand function* $n(p, x, w)$ *is the solution to the representative firm's profit maximisation problem and is given by:*

$$n(p, x, w) = \left[\frac{\alpha p f(x) a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}},$$

where $\frac{\partial n}{\partial p} > 0$, $\frac{\partial n}{\partial x} > 0$, $\frac{\partial n}{\partial w} < 0$, $\frac{\partial n}{\partial a} > 0$ *and* $\frac{\partial n}{\partial \tau} < 0$.

Proof. The first-order condition of the representative firm's profit maximization problem is given by:

$$\frac{\partial \Pi}{\partial n} = \alpha p f(x) a n^{\alpha-1} - w(1+\tau) = 0. \tag{60}$$

Solving for n from the above first-order condition gives the following labor demand function:

$$n(p, x, w) = \left[\frac{\alpha p f(x) a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}}.$$

It then follows that $\frac{\partial n}{\partial p} = \frac{1}{1-\alpha}\left[\frac{\alpha p f(x) a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}-1} \frac{\alpha f(x) a}{w(1+\tau)} > 0$, $\frac{\partial n}{\partial x} = \frac{1}{1-\alpha}\left[\frac{\alpha p f(x) a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}-1} \frac{\alpha p f'(x) a}{w(1+\tau)} > 0$, $\frac{\partial n}{\partial w} = -\frac{1}{1-\alpha}\left[\frac{\alpha p f(x) a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}-1} \frac{\alpha p f(x) a}{w(1+\tau)} \frac{1}{w} < 0$, $\frac{\partial n}{\partial a} = \frac{1}{1-\alpha}\left[\frac{\alpha p f(x) a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}-1} \frac{\alpha p f(x)}{w(1+\tau)} > 0$, $\frac{\partial n}{\partial \tau} = -\frac{1}{1-\alpha}\left[\frac{\alpha p f(x) a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}-1} \frac{\alpha p f(x) a}{w(1+\tau)} \frac{1}{(1+\tau)} < 0$. □

Lemma 4. *In a competitive equilibrium, the following are the comparative statics of tightness* (x)*, sales* (y) *and the price* (p)*:*

$$\frac{dx}{d\chi} = 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} > 0; \qquad \frac{dx}{da} = 0, \frac{dy}{da} > 0, \frac{dp}{da} < 0$$

Proof. Combining the labor supply function $l(w)$ in Lemma 2 and the labor demand function $n(p, x, w)$ in Lemma 3 with the labor market clearing condition delivers the following expression for equilibrium employment:

$$l = n = (\alpha p f(x) a)^{\frac{1}{1-\alpha+\psi}} (1+\tau)^{-\frac{1}{1-\alpha+\psi}}. \tag{61}$$

Inserting equilibrium employment level into goods market clearing condition:

$$\frac{f(x)}{1+\gamma(x)} a \left[(\alpha p f(x) a)^{\frac{1}{1-\alpha+\psi}} (1+\tau)^{-\frac{1}{1-\alpha+\psi}}\right]^{\alpha} = c(p, x) + G, \tag{62}$$

$$\frac{f(x)}{1+\gamma(x)} a \left[(\alpha p f(x) a)^{\frac{1}{1-\alpha+\psi}} (1+\tau)^{-\frac{1}{1-\alpha+\psi}}\right]^{\alpha} = \frac{\chi}{p[1+\gamma(x)]} + G, \tag{63}$$

$$p f(x) a \alpha^{\frac{\alpha}{1-\alpha+\psi}} (p f(x) a)^{\frac{\alpha}{1-\alpha+\psi}} (1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \chi + p[1+\gamma(x)]G \tag{64}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} (p f(x) a)^{\frac{1+\psi}{1-\alpha+\psi}} (1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \chi + p[1+\gamma(x)]G. \tag{65}$$

By definition of the competitive equilibrium, $x = x^*$, and so $\frac{dx}{d\chi} = \frac{dx}{da} = 0$. The latter implies the following comparative statics for p (for simplicity, evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} (f(x)a)^{\frac{1+\psi}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} p^{\frac{\alpha}{1-\alpha+\psi}} \frac{dp}{d\chi} = 1 \tag{66}$$

$$\frac{dp}{d\chi} = \alpha^{-\frac{\alpha}{1-\alpha+\psi}} (f(x)a)^{-\frac{1+\psi}{1-\alpha+\psi}} \frac{1-\alpha+\psi}{1+\psi} p^{-\frac{\alpha}{1-\alpha+\psi}} = \frac{1-\alpha+\psi}{1+\psi} \frac{p}{\chi} > 0. \tag{67}$$

$$\tag{68}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} (f(x))^{\frac{1+\psi}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} (pa)^{\frac{\alpha}{1-\alpha+\psi}} (p + a\frac{dp}{da}) = 0 \tag{69}$$

$$\frac{dp}{da} = -\frac{p}{a} < 0. \tag{70}$$

Finally, the above implies the following comparative statics for sales $y = [1+\gamma(x)](c(p,x)+G) = \chi/p + G[1+\gamma(x)]$ (also evaluated at $G = \tau = 0$):

$$py = \chi + p[1+\gamma(x)]G \tag{71}$$

$$\frac{dp}{d\chi} y + p\frac{dy}{d\chi} = 1 \tag{72}$$

$$\frac{dy}{d\chi} = \frac{1}{p}\left(1 - \frac{dp}{d\chi} y\right) = \frac{1}{p}\frac{\alpha}{1+\psi} > 0. \tag{73}$$

$$\frac{dp}{da} y + p\frac{dy}{da} = 0 \tag{74}$$

$$\frac{dy}{da} = -\frac{dp}{da}\frac{y}{p} > 0. \tag{75}$$

□

Lemma 5. *In a fixprice equilibrium, the following are the comparative statics of tightness (x), sales (y) and the price (p):*

$$\frac{dx}{d\chi} > 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} = 0; \qquad \frac{dx}{da} < 0, \frac{dy}{da} = 0, \frac{dp}{da} = 0$$

Proof. Condition (65) remains unchanged in a fixprice equilibrium. However, now the price is a parameter, so that $\frac{dp}{d\chi} = \frac{dp}{da} = 0$. The latter implies the following comparative statics for x (for simplicity, evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} (pa)^{\frac{1+\psi}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} f(x)^{\frac{\alpha}{1-\alpha+\psi}} f'(x) \frac{dx}{d\chi} = 1 \tag{76}$$

$$\frac{dx}{d\chi} = \alpha^{-\frac{\alpha}{1-\alpha+\psi}} (f(x)a)^{-\frac{1+\psi}{1-\alpha+\psi}} \frac{1-\alpha+\psi}{1+\psi} f(x)^{-\frac{\alpha}{1-\alpha+\psi}} \frac{1}{f'(x)} > 0. \tag{77}$$

$$\tag{78}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} (p)^{\frac{1+\psi}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} (f(x)a)^{\frac{\alpha}{1-\alpha+\psi}} (f(x) + af'(x)\frac{dx}{da}) = 0$$

$$\frac{dx}{da} = -\frac{f(x)}{a}\frac{1}{f'(x)} < 0. \tag{79}$$

The above implies the following comparative statics for sales $y = [1+\gamma(x)](c(p,x)+G) = \chi/p + G[1+\gamma(x)]$

(also evaluated at $G = \tau = 0$):

$$py = \chi + p[1+\gamma(x)]G \tag{80}$$
$$p\frac{dy}{d\chi} = 1 \tag{81}$$
$$\frac{dy}{d\chi} = \frac{1}{p} > 0. \tag{82}$$

$$p\frac{dy}{da} = 0 \tag{83}$$
$$\frac{dy}{da} = 0. \tag{84}$$

□

Proposition 1. *In a competitive equilibrium, the demand-side and the supply-side fiscal multipliers are equal and given by:*

$$\varphi^* \equiv \frac{\alpha}{1+\psi} = \frac{1-\frac{1}{|\epsilon^d|}}{1+\frac{1}{\epsilon^s}},$$

where $\alpha \in (0,1]$ and $\psi > 0$ are, respectively, returns to labor and inverse Frisch elasticity, whereas $|\epsilon^d| = \frac{1}{1-\alpha}$ and $\epsilon^s = \frac{1}{\psi}$ are (absolute) elasticities of labor demand and labor supply. Hence $\varphi^* \in (0,1]$ and it is pinned down by elasticities of labor demand and labor supply.

Proof. First differentiate (65) with respect to G (evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1} \left[\frac{dp}{dG} f(x)a + pf'(x)\frac{dx}{dG}a\right] = p[1+\gamma(x)], \tag{85}$$
$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}} \left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = p[1+\gamma(x)], \tag{86}$$
$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{p[1+\gamma(x)]}{\frac{1+\psi}{1-\alpha+\psi}\underbrace{\alpha^{\frac{\alpha}{1-\alpha+\psi}}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}}_{=\chi \text{(by (67) under } G=\tau=0)}}, \tag{87}$$
$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{p[1+\gamma(x)]}{\frac{1+\psi}{1-\alpha+\psi}\chi} \tag{88}$$
$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)}. \tag{89}$$

From definition of demand-side fiscal multiplier:

$$\varphi^d = \frac{d\{c+G\}}{dG} = \frac{dc}{dG} + 1 = \frac{\partial c}{\partial p}\frac{dp}{dG} + \frac{\partial c}{\partial x}\frac{dx}{dG} + 1 \tag{90}$$

In a competitive equilibrium, $x = x^*$, so that $\frac{dx}{dG} = 0$, which combined with (88) implies the following:

$$\varphi^d = \frac{\partial c}{\partial p}\frac{dp}{dG} + 1 = -\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p}\frac{dp}{dG} + 1 = -c(p,x)\frac{1}{p}p\frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)} + 1 \tag{91}$$

$$= -\frac{1-\alpha+\psi}{1+\psi} + 1 \tag{92}$$

$$= \frac{\alpha}{1+\psi} \equiv \varphi^*. \tag{93}$$

Similarly, differentiate (65) with respect to τ (evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[\frac{1+\psi}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1}\left(\frac{dp}{d\tau}f(x)a + pf'(x)\frac{dx}{d\tau}a\right) - \frac{\alpha}{1-\alpha+\psi}(-1)(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}\right] = 0 \tag{94}$$

$$\left[\frac{1}{p}\frac{dp}{d\tau} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau}\right] = \frac{\alpha}{1+\psi}. \tag{95}$$

From definition of supply-side fiscal multiplier:

$$\varphi^s = \frac{d\{c+G\}/\{c+G\}}{d[-\tau]} = -\frac{1}{c}\frac{dc}{d\tau} = -\frac{1}{c}\left[\frac{\partial c}{\partial p}\frac{dp}{d\tau} + \frac{\partial c}{\partial x}\frac{dx}{d\tau}\right]. \tag{96}$$

In a competitive equilibrium, $x = x^*$, so that $\frac{dx}{d\tau} = 0$, which combined with (95) implies the following:

$$\varphi^s = -\frac{1}{c}\frac{\partial c}{\partial p}\frac{dp}{d\tau} = -\frac{1}{c}\left[-\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p}\right]\frac{dp}{d\tau} = \frac{1}{c}c\frac{1}{p}p\frac{\alpha}{1+\psi} \tag{97}$$

$$= \frac{\alpha}{1+\psi} = \varphi^d = \varphi^*. \tag{98}$$

□

Lemma 6. *Define the fixed capacity fiscal multiplier $\theta(x)$ to be the demand-side fiscal multiplier in a fixprice equilibrium under fixed labor supply, so that*

$$\theta(x) \equiv \frac{d\{c+G\}}{dG}|_{\psi\to\infty}$$

then $\theta(x)$ has the following properties:

$$\theta(x) = \begin{cases} (-\infty, 0), & \text{if} \quad x \in (x^*, x_m) \\ 0, & \text{if} \quad x = x^* \\ (0,1), & \text{if} \quad x \in (0, x^*) \end{cases}$$

$$\theta'(x) < 0, \qquad \forall x \in (0, x_m),$$

where x_m is given by $f(x_m) = \rho x_m$.

Proof. Under $\psi \to \infty$, (89) can be written as:

$$\lim_{\psi\to\infty}\left[\frac{1}{p}\frac{dp}{dG}+\frac{f'(x)}{f(x)}\frac{dx}{dG}\right]=\lim_{\psi\to\infty}\frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)}. \tag{99}$$

$$\lim_{\psi\to\infty}\left[\frac{1}{p}\frac{dp}{dG}+\frac{f'(x)}{f(x)}\frac{dx}{dG}\right]=\lim_{\psi\to\infty}\frac{1}{c(p,x)}=\frac{1}{\lim_{\psi\to\infty}c(p,x)}. \tag{100}$$

From definition of $\theta(x)$:

$$\theta(x)\equiv\frac{d\{c+G\}}{dG}|_{\psi\to\infty}=\lim_{\psi\to\infty}\frac{dc}{dG}+1=\lim_{\psi\to\infty}\left[\frac{\partial c}{\partial p}\frac{dp}{dG}+\frac{\partial c}{\partial x}\frac{dx}{dG}\right]+1 \tag{101}$$

In a fixprice equilibrium $p=p_0$ is a parameter, so that $\frac{dp}{dG}=0$, which combined with (101) implies the following:

$$\theta(x)=\lim_{\psi\to\infty}\frac{\partial c}{\partial x}\frac{dx}{dG}+1=\lim_{\psi\to\infty}\frac{\partial c}{\partial x}\lim_{\psi\to\infty}\frac{dx}{dG}+1 \tag{102}$$

$$=-\lim_{\psi\to\infty}\frac{\chi}{p[1+\gamma(x)]}\frac{\gamma'(x)}{[1+\gamma(x)]}\frac{f(x)}{f'(x)}\frac{1}{\lim_{\psi\to\infty}c(p,x)}+1 \tag{103}$$

$$=1-\frac{\gamma'(x)}{[1+\gamma(x)]}\frac{f(x)}{f'(x)}\frac{\lim_{\psi\to\infty}c(p,x)}{\lim_{\psi\to\infty}c(p,x)} \tag{104}$$

$$=1-\frac{\gamma'(x)}{[1+\gamma(x)]}\frac{f(x)}{f'(x)}. \tag{105}$$

Recall that $\gamma(x)\equiv\frac{\rho x}{f(x)-\rho x}$, so that $\gamma'(x)=\frac{\rho(f(x)-\rho x)-(f'(x)-\rho)\rho x}{(f(x)-\rho x)^2}=\frac{\rho(f(x)-f'(x)x)}{(f(x)-\rho x)^2}$, and $\theta(x)$ may be rewritten as:

$$\theta(x)=1-\frac{\frac{\rho(f(x)-f'(x)x)}{(f(x)-\rho x)^2}}{\frac{f(x)}{f(x)-\rho(x)}}\frac{f(x)}{f'(x)}=1-\frac{\rho(f(x)-f'(x)x)}{(f(x)-\rho x)f(x)}\frac{f(x)}{f'(x)} \tag{106}$$

$$=1-\frac{\rho(f(x)-f'(x)x)}{f'(x)(f(x)-\rho x)}=\frac{f'(x)f(x)-\rho f(x)}{f'(x)f(x)-f'(x)\rho x}=\frac{1-\frac{\rho}{f'(x)}}{1-\frac{\rho x}{f(x)}}. \tag{107}$$

We can now show that $\theta(x)$ possesses several convenient properties. Firstly, $\theta'(x)<0,\quad\forall x\in(0,x_m)$, where x_m is given by $f(x_m)=\rho x_m$. In order to show this, notice that $q(x)=\frac{f(x)}{x}$ and $f'(x)=q(x)^{1+\delta}$, which allows us to rewrite $\theta(x)$ as follows:

$$\theta(x)=\frac{1-\frac{\rho}{q(x)^{1+\delta}}}{1-\frac{\rho}{q(x)}}=\frac{q(x)^{1+\delta}-\rho}{q(x)^{1+\delta}-\rho q(x)^{\delta}}, \tag{108}$$

and $\theta'(x)$ is now given by:

$$\theta'(x)=\frac{(1+\delta)q(x)^{\delta}q'(x)[q(x)^{1+\delta}-\rho q(x)^{\delta}]-[(1+\delta)q(x)^{\delta}q'(x)-\delta\rho q(x)^{\delta-1}q'(x)](q(x)^{1+\delta}-\rho)}{(q(x)^{1+\delta}-\rho)^2}. \tag{109}$$

Given that $q(x) > 0, q'(x) < 0, (q(x)^{1+\delta} - \rho)^2 > 0, \quad \forall x \in (0, \infty)$, a sufficient condition for $\theta'(x) < 0$ is:

$$(1+\delta)q(x)^\delta[q(x)^{1+\delta} - \rho q(x)^\delta] - [(1+\delta)q(x)^\delta - \delta\rho q(x)^{\delta-1}](q(x)^{1+\delta} - \rho) > 0 \tag{110}$$

$$-\rho q(x)^{2\delta} + \rho(1+\delta)q(x)^\delta - \delta\rho^2 q(x)^{\delta-1} > 0 \tag{111}$$

$$\rho q(x)^{\delta-1}[q(x) - q(x)^{\delta+1}] + \delta[q(x) - \rho] > 0. \tag{112}$$

Finally, $q(0) = 1$ and $q(x_m) = \rho$, and since $q'(x) < 0, \forall x \in (0, \infty)$ it follows that $q(x) \in (\rho, 1), \quad \forall x \in (0, x_m)$; it is clear that for all $q(x) \in (\rho, 1)$ the sufficient condition above is satisfied. Hence, $\theta'(x) < 0, \quad \forall x \in (0, x_m)$. Secondly, it follows directly from (107) that $\theta(x^*) = 0$, since $f'(x^*) = \rho$. At the extremes:

$$\theta(0) = \frac{q(0)^{1+\delta} - \rho}{q(0)^{1+\delta} - \rho q(0)^\delta} = \frac{1^{1+\delta} - \rho}{1^{1+\delta} - \rho 1^\delta} = 1, \tag{113}$$

$$\lim_{x \to x_m^-} \theta(x) = \lim_{h \to 0} \frac{q(x_m - h)^{1+\delta} - \rho}{q(x_m - h)^{1+\delta} - \rho q(x_m - h)^\delta} \tag{114}$$

$$= \frac{\rho^{1+\delta} - \rho}{\rho^{1+\delta} - \rho\rho^\delta} = \frac{\rho(\rho^\delta - 1)}{0} = -\infty. \tag{115}$$

Since $\theta(0) = 1, \theta(x^*) = 0$ and $\lim_{x \to x_m^-} \theta(x) = -\infty$, and $\theta'(x) < 0, \forall x \in (0, x_m)$ it follows that $\theta(x) \in (0, 1), \forall x \in (0, x^*)$ and $\theta(x) \in (-\infty, 0), \forall x \in (x^*, x_m)$. ☐

Proposition 2. *In a fixprice equilibrium, the demand-side fiscal multiplier $\varphi^d(x)$ is given by*

$$\varphi^d(x) = \underbrace{\varphi^*}_{\text{State-invariant component}} + \underbrace{\theta(x) \times (1 - \varphi^*)}_{\text{State-dependent component}},$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the competitive equilibrium multiplier. Hence, $\varphi^d(x) \in (-\infty, 1)$ and $\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x_m)$.

Proof. From (89) we know that:

$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{1 - \alpha + \psi}{1 + \psi}\frac{1}{c(p, x)} = (1 - \varphi^*)\frac{1}{c(p, x)}. \tag{116}$$

Further, in a fixprice equilibrium $p = p_0$ is a parameter, so that $\frac{dp}{dG} = 0$ and it follows that:

$$\frac{dx}{dG} = (1 - \varphi^*)\frac{f(x)}{f'(x)}\frac{1}{c(p, x)}. \tag{117}$$

From the definition of the demand-side fiscal multiplier:

$$\varphi^d(x) = \frac{d\{c + G\}}{dG} = \frac{dc}{dG} + 1 = \frac{\partial c}{\partial x}\frac{dx}{dG} + 1 = -\frac{\chi}{p[1 + \gamma(x)]}\frac{\gamma'(x)}{[1 + \gamma(x)]}(1 - \varphi^*)\frac{f(x)}{f'(x)}\frac{1}{c(p, x)} + 1, \tag{118}$$

$$= 1 - (1 - \varphi^*)\underbrace{\frac{\gamma'(x)}{[1 + \gamma(x)]}\frac{f(x)}{f'(x)}}_{1 - \theta(x)}\frac{c(p, x)}{c(p, x)} = 1 - (1 - \varphi^*)(1 - \theta(x)), \tag{119}$$

$$= \varphi^* + \theta(x) \times (1 - \varphi^*). \tag{120}$$

Since $\frac{d\varphi^d(x)}{dx} = \theta'(x)(1 - \varphi^*)$ and $\theta'(x) < 0, \forall x \in (0, x_m)$ it follows that $\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x_m)$. Further,

$\varphi^d(0) = \varphi^* + \theta(0) \times (1 - \varphi^*) = 1$ and $\lim_{x \to x_m^-} \varphi^d(x) = \varphi^* + \lim_{x \to x_m^-} \theta(x) \times (1 - \varphi^*) = -\infty$, so that $\varphi^d(x) \in (-\infty, 1), \forall x \in (0, x_m)$. □

Corollary 1. *There always exists tightness $\hat{x} \in (x^*, x_m)$ such that $\varphi^d(\hat{x}) = 0$ and $\varphi^d(x) < 0, \forall x \in (\hat{x}, x_m)$, and it is given by:*

$$\hat{x} = \theta^{-1}\left(-\frac{\varphi^*}{1 - \varphi^*}\right),$$

where $\frac{d\hat{x}}{d\varphi^*} > 0$.

Proof. Suppose there exists $\hat{x} \in (0, x_m)$, such that $\varphi^d(\hat{x}) = 0$; then it should satisfy the following condition:

$$\varphi^d(\hat{x}) = \varphi^* + (1 - \varphi^*) \times \theta(\hat{x}) = 0, \tag{121}$$

$$\theta(\hat{x}) = -\frac{\varphi^*}{1 - \varphi^*}. \tag{122}$$

We know that $\theta(x)$ lies between $(-\infty, 1)$ and is differentiable and strictly decreasing on $(0, x_m)$; hence the inverse function $\theta^{-1}(.)$ exists on $(-\infty, 1)$ and returns values in $(0, x_m)$. Moreover, since $-\frac{\varphi^*}{1-\varphi^*} \in (-\infty, 1), \forall \varphi^* \in (0, 1)$, then $\hat{x} \in (0, x_m)$ always exists and is given by:

$$\hat{x} = \theta^{-1}\left(-\frac{\varphi^*}{1 - \varphi^*}\right). \tag{123}$$

Since $\varphi^d(x^*) = \varphi^* \in (0, 1)$ and $\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x_m)$ it must be that $\hat{x} \in (x^*, x_m)$; further, since $\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x_m)$ and $\varphi^d(\hat{x}) = 0$, it follows that $\varphi^d(x) < 0, \forall x \in (\hat{x}, x_m)$. It is also true that:

$$\theta'(\hat{x})\frac{d\hat{x}}{d\varphi^*} = \frac{d}{d\varphi^*}\left(-\frac{\varphi^*}{1 - \varphi^*}\right) = -\frac{1}{(1 - \varphi^*)^2}, \tag{124}$$

$$\frac{d\hat{x}}{d\varphi^*} = -\frac{1}{\theta'(\hat{x})}\frac{1}{(1 - \varphi^*)^2} > 0. \tag{125}$$

□

Proposition 3. *In a fixprice equilibrium, the supply-side fiscal multiplier $\varphi^s(x)$ is given by*

$$\varphi^s(x) = \underbrace{\varphi^*}_{\text{State-invariant component}} - \underbrace{\theta(x) \times \varphi^*}_{\text{State-dependent component}},$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the competitive equilibrium multiplier. Hence, $\varphi^d(x) \in (0, +\infty)$ and $\frac{d\varphi^d(x)}{dx} > 0, \forall x \in (0, x_m)$.

Proof. From (95) we know that:

$$\left[\frac{1}{p}\frac{dp}{d\tau} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau}\right] = \frac{\alpha}{1 + \psi} = \varphi^*. \tag{126}$$

In a fixprice equilibrium, $p = p_0$ is a parameter, so that $\frac{dp}{d\tau} = 0$, and it follows that:

$$\frac{dx}{d\tau} = \varphi^*\frac{f(x)}{f'(x)}. \tag{127}$$

From the definition of the supply-side multiplier:

$$\varphi^s(x) = \frac{d\{c+G\}/\{c+G\}}{d[-\tau]} = -\frac{1}{c}\frac{dc}{d\tau} = -\frac{1}{c}\frac{\partial c}{\partial x}\frac{dx}{d\tau} = \frac{1}{c}\frac{\chi}{p[1+\gamma(x)]}\underbrace{\frac{\gamma'(x)}{[1+\gamma(x)]}\frac{f(x)}{f'(x)}}_{1-\theta(x)}\varphi^* \tag{128}$$

$$= \frac{c}{c}(1-\theta(x))\varphi^* = \varphi^* - \theta(x)\times\varphi^*. \tag{129}$$

Since $\frac{d\varphi^s(x)}{dx} = -\theta'(x)\varphi^*$ and $\theta'(x) < 0, \forall x \in (0, x_m)$ it follows that $\frac{d\varphi^s(x)}{dx} > 0, \forall x \in (0, x_m)$. Further, $\varphi^s(0) = \varphi^* - \theta(0)\times\varphi^* = 0$ and $\lim_{x\to x_m^-}\varphi^s(x) = \varphi^* - \lim_{x\to x_m^-}\theta(x)\times\varphi^* = \infty$, so that $\varphi^s(x) \in (0,\infty), \forall x \in (0, x_m)$. □

Corollary 5. *In a fixprice equilibrium, the demand-side and supply-side fiscal multipliers are related as*

$$\underbrace{\varphi^d(x)}_{\text{Demand-side multiplier}} = \underbrace{\theta(x)}_{\text{Fixed capacity multiplier}} + \underbrace{\varphi^s(x)}_{\text{Supply-side multiplier}},$$

so that the demand-side multiplier is higher in slack equilibria, lower in tight equilibria and exactly equal to the supply-side multiplier in an efficient fixprice equilibrium.

Proof. From the expression for the demand-side fiscal multiplier is a fixprice equilibrium in Proposition 2:

$$\varphi^d(x) = \varphi^* + \theta(x)(1-\varphi^*) = \theta(x) + \underbrace{\varphi^* - \theta(x)\varphi^*}_{\varphi^s(x)}, \tag{130}$$

$$\varphi^d(x) = \theta(x) + \varphi^s(x). \tag{131}$$

□

Corollary 6. *Suppose that elasticities of labor demand and labor supply are sufficiently low so that $\varphi^* < 0.5$; then there always exists tightness $\tilde{x} \in [\hat{x}, x_m)$ such that:*

$$-\varphi^d(x) > \varphi^s(x) > \varphi^d(x), \quad \forall x \in (\tilde{x}, x_m).$$

Furthermore, $\hat{x}$ is given by:

$$\tilde{x} = \theta^{-1}\left(-\frac{2\varphi^*}{1-2\varphi^*}\right), \quad \varphi^* < 0.5$$

and hence $\frac{d\tilde{x}}{d\varphi^*} > 0$.

Proof. It is apparent that the austerity threshold for tightness cannot be below $\hat{x}$, as in that case $\varphi^d(x) > 0 > -\varphi^d(x), \forall x \in (0, \hat{x})$. However, suppose there exists $\tilde{x} \in (\hat{x}, x_m)$ such that $-\varphi^d(\tilde{x}) = \varphi^s(\tilde{x}) > \varphi^d(\tilde{x})$. Then it must satisfy the following:

$$-\varphi^* - \theta(\tilde{x})(1-\varphi^*) = \varphi^* - \theta(\tilde{x})\varphi^*, \tag{132}$$

$$\theta(\tilde{x}) = -\frac{2\varphi^*}{1-2\varphi^*}. \tag{133}$$

As established earlier, $\theta(x)$ is differentiable and strictly decreasing on $(0, x_m)$, taking values in $(-\infty, 1)$. Therefore, the inverse function $\theta^{-1}(.)$ exists on the domain $(-\infty, 1)$. Hence, as long as $\varphi < 0.5$, $-\frac{2\varphi^*}{1-2\varphi^*} \in (-\infty, 1)$, the austerity threshold $\tilde{x}$ exists and is given by:

$$\tilde{x} = \theta^{-1}\left(-\frac{2\varphi^*}{1-2\varphi^*}\right), \quad \varphi^* < 0.5. \tag{134}$$

Further, if $\varphi^* < 0.5$, $\frac{d[-\varphi^d(x)-\varphi^s(x)]}{dx} = -\theta'(x)(1-2\varphi^*) > 0$, so that $-\varphi^d(x) > \varphi^s(x) > \varphi^d(x), \forall x \in (\tilde{x}, x_m)$. It also follows that:

$$\theta'(\tilde{x})\frac{d\tilde{x}}{d\varphi^*} = \frac{d}{d\varphi^*}\left[-\frac{2\varphi^*}{1-2\varphi^*}\right], \tag{135}$$

$$\frac{d\tilde{x}}{d\varphi^*} = -\frac{1}{\theta'(\tilde{x})}\frac{2}{(1-2\varphi^*)^2} > 0. \tag{136}$$

□

B Fiscal multipliers: (more) general cases

In this section we show that the results derived earlier hold in much more general settings. In particular, we introduce the class of *flexible* equilibria, which is a superset of the competitive equilibria. We then show that in any flexible equilibrium that has tightness fixed over the business cycle, both demand-side and supply-side multipliers are equal and acyclical, just like in the competitive equilibrium. On the other hand, we show that the cyclicality results established under fixprice equilibria extend to the more general class of *frictional* equilibria, where part of the adjustment happens via tightness.

B.1 Flexible equilibria multipliers

In the previous section we started off be considering a competitive equilibrium, where tightness was fixed at the efficient level x^* and all adjustment happened via prices and wages. However, this is not the only way to pin down tightness. Below we consider two common alternatives found in search-and-matching literature (Nash bargaining, fixed markup pricing), before introducing a much more general *Tightness Determination Mapping (TDM)*.

B.1.1 Nash bargaining

One alternative, very common in the search-and-matching literature, is to consider Nash bargaining over the price between consumers and firms in order to get an extra equilibrium condition needed to close the model. In our case, the surplus to consumers from buying an additional unit of the produced good at price $\tilde{p}$ after a match is made is given by:

$$\mathcal{B}(\tilde{p}) = \frac{\chi}{c} - \tilde{p}, \tag{137}$$

whereas the firms' surplus from selling an extra unit at price $\tilde{p}$ is

$$\mathcal{S}(\tilde{p}) = \tilde{p} - pf(x). \tag{138}$$

Assuming the consumers' bargaining power is given by $\beta \in (0, 1)$, the solution to Nash bargaining is given by:

$$(1-\beta)\mathcal{S}(p) = \beta\mathcal{B}(p). \tag{139}$$

Combining the above with agents' optimality conditions obtained earlier, one gets:

$$\frac{1-\beta}{\beta} = \frac{\gamma(x^L)}{1-f(x^L)}, \quad \frac{dx^L}{d\beta} < 0. \tag{140}$$

As one can see, the condition above pins down tightness at $x = x^L$, and we can even get the equivalent of the Hosios (1990) condition for the bargaining power β^* that delivers the socially efficient allocation, $\beta^* = \frac{1}{1+\frac{\gamma(x^*)}{1-f(x^*)}}$.

B.1.2 Fixed markup pricing

An alternative way to pin down tightness is to assume that the equilibrium price p is set as a fixed markup over the marginal cost, so that:

$$p = \mu \times mc, \tag{141}$$

where $\mu \geq 1$ is a markup parameter and mc is the marginal cost. From firms' optimisation problem one gets that the effective selling price $pf(x)$ is set equal to the marginal cost:

$$pf(x) = mc. \tag{142}$$

Combining the above two equations one gets the following condition for pinning down the level of tightness:

$$f(x^L) = \frac{1}{\mu}, \quad \frac{dx^L}{d\mu} < 0. \tag{143}$$

As before, the equivalent of the Hosios (1990) condition here is the markup μ^* that delivers the socially efficient allocation, n amely $\mu^* = \frac{1}{f(x^*)}$.

B.1.3 Generalization: Tightness Determination Mapping

In fact, the above approaches to pinning down tightness can be generalized by introducing the notion of a Tightness Determination Mapping (TDM):

Definition 5. *A Tightness Determination Mapping (TDM) $\mathcal{M}$ is given by:*

$$\mathcal{M}: \quad \left\{\Omega^M, \Omega^S, \Omega^T\right\} \to x^L, \tag{144}$$

*where $\Omega^M = \{\rho, \gamma, \psi, \alpha\}$ is the set of model structural parameters, $\Omega^S = \{\chi, a, G, \tau\}$ is the set of shock parameters, Ω^T is the set of parameters specific to the TDM and x^L is the resulting tightness. Further, a TDM $\mathcal{M}$ is said to be **shock invariant** if and only if*

$$\frac{d\mathcal{M}\left(\Omega^M, \Omega^S, \Omega^T\right)}{d\tilde{s}} = 0, \quad \forall \tilde{s} \in \Omega^S. \tag{145}$$

so that changes in shock parameters do not affect the determination of tightness.

It is easy to see that the TDM used in the competitive equilibrium is simply $x^L = x^*$. Also, Nash bargaining is a particular TDM with $\Omega^T = \{\beta\}$, which pins down tightness according to $\frac{1-\beta}{\beta} = \frac{\gamma(x^L)}{1-f(x^L)}$; similarly, fixed markup pricing is a TDM with $\Omega^T = \{\mu\}$, which pins down tightness according to $f(x^L) = \frac{1}{\mu}$. Note that all of the above are also shock invariant TDMs, as none of the shock parameters enter the conditions that pin down the level of tightness.

We can now define a class of *flexible* equilibria that is a superset of the competitive equilibrium considered before:

Definition 6. *A **flexible** equilibrium is a vector $(p^L, w^L, \mathcal{M})$, and associated allocations, such that the agents' optimality conditions and the market clearing conditions are satisfied with tightness pinned down at a level $x^L = \mathcal{M}\left(\Omega^M, \Omega^S, \Omega^T\right)$.*

Clearly, the competitive equilibrium considered earlier is just a flexible equilibrium with $x^L = x^*$ as the TDM. In fact, it can be shown that all of the comparative statics results established for the competitive equilibrium in Lemma 4 hold in the exact same way for any flexible equilibrium with a shock invariant TDM:

Lemma 7. *In any flexible equilibrium generated by a shock-invariant TDM, the following are the comparative statics of tightness (x), sales (y) and the price (p):*

$$\frac{dx}{d\chi} = 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} > 0; \qquad \frac{dx}{da} = 0, \frac{dy}{da} > 0, \frac{dp}{da} < 0 \tag{146}$$

Proof. Note that in this more generalized setting, condition (65) still holds:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \left(pf(x)a\right)^{\frac{1+\psi}{1-\alpha+\psi}} (1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \chi + p[1+\gamma(x)]G. \tag{147}$$

In a flexible equilibrium, $x = x^L = \mathcal{M}(\Omega^M, \Omega^S, \Omega^T)$; further, since the TDM $\mathcal{M}$ is shock-invariant it follows that $\frac{dx^L}{d\chi} = \frac{dx^L}{da} = 0$. The latter implies the following comparative statics for p (for simplicity, evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \left(f(x)a\right)^{\frac{1+\psi}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} p^{\frac{\alpha}{1-\alpha+\psi}} \frac{dp}{d\chi} = 1 \tag{148}$$

$$\frac{dp}{d\chi} = \alpha^{-\frac{\alpha}{1-\alpha+\psi}} \left(f(x)a\right)^{-\frac{1+\psi}{1-\alpha+\psi}} \frac{1-\alpha+\psi}{1+\psi} p^{-\frac{\alpha}{1-\alpha+\psi}} = \frac{1-\alpha+\psi}{1+\psi}\frac{p}{\chi} > 0. \tag{149}$$

$$\tag{150}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \left(f(x)\right)^{\frac{1+\psi}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} (pa)^{\frac{\alpha}{1-\alpha+\psi}} (p + a\frac{dp}{da}) = 0$$

$$\frac{dp}{da} = -\frac{p}{a} < 0. \tag{151}$$

Finally, the above implies the following comparative statics for sales $y = [1+\gamma(x)](c(p,x)+G) = \chi/p + G[1+\gamma(x)]$ (also evaluated at $G = \tau = 0$):

$$py = \chi + p[1+\gamma(x)]G \tag{152}$$

$$\frac{dp}{d\chi}y + p\frac{dy}{d\chi} = 1 \tag{153}$$

$$\frac{dy}{d\chi} = \frac{1}{p}\left(1 - \frac{dp}{d\chi}y\right) = \frac{1}{p}\frac{\alpha}{1+\psi} > 0. \tag{154}$$

$$\frac{dp}{da}y + p\frac{dy}{da} = 0 \tag{155}$$

$$\frac{dy}{da} = -\frac{dp}{da}\frac{y}{p} > 0. \tag{156}$$

□

Of our particular interest, however, is the fact that all the results that we established for demand-side and supply-side fiscal multipliers under the competitive equilibrium remain true for any flexible equilibrium with a shock invariant TDM:

Proposition 4. *In any flexible equilibrium generated by a shock-invariant TDM, the demand-side fiscal multiplier and the supply-side fiscal multiplier are equal and given by:*

$$\varphi^* \equiv \frac{\alpha}{1+\psi} = \frac{1-\frac{1}{|\epsilon^d|}}{1+\frac{1}{\epsilon^s}}, \tag{157}$$

where $\alpha \in (0,1]$ and $\psi > 0$ are, respectively, returns to labor and inverse Frisch elasticity, whereas $|\epsilon^d| = \frac{1}{1-\alpha}$ and $\epsilon^s = \frac{1}{\psi}$ are (absolute) elasticities of labor demand and labor supply. Hence $\varphi^* \in (0,1]$ and it is pinned down by elasticities of labor demand and labor supply.

Proof. Note that in this more generalized setting, condition (88) still holds, so that:

$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)}. \tag{158}$$

Further, the definition of demand-side fiscal multiplier also remains unchanged:

$$\varphi^d = \frac{d\{c+G\}}{dG} = \frac{dc}{dG} + 1 = \frac{\partial c}{\partial p}\frac{dp}{dG} + \frac{\partial c}{\partial x}\frac{dx}{dG} + 1 \tag{159}$$

In a flexible equilibrium, $x = x^L = \mathcal{M}(\Omega^M, \Omega^S, \Omega^T)$; further, since the TDM $\mathcal{M}$ is shock-invariant it follows that $\frac{dx^L}{dG} = 0$, which combined with (159) implies the following:

$$\varphi^d = \frac{\partial c}{\partial p}\frac{dp}{dG} + 1 = -\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p}\frac{dp}{dG} + 1 = -c(p,x)\frac{1}{p}p\frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)} + 1 \tag{160}$$

$$= -\frac{1-\alpha+\psi}{1+\psi} + 1 \tag{161}$$

$$= \frac{\alpha}{1+\psi} \equiv \varphi^*. \tag{162}$$

Similarly, condition (95) also holds in this more generalised setting

$$\left[\frac{1}{p}\frac{dp}{d\tau} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau}\right] = \frac{\alpha}{1+\psi}. \tag{163}$$

And the definition of supply-side fiscal multiplier also stays the same:

$$\varphi^s = \frac{d\{c+G\}/\{c+G\}}{d[-\tau]} = -\frac{1}{c}\frac{dc}{d\tau} = -\frac{1}{c}\left[\frac{\partial c}{\partial p}\frac{dp}{d\tau} + \frac{\partial c}{\partial x}\frac{dx}{d\tau}\right]. \tag{164}$$

In a flexible equilibrium, $x = x^L = \mathcal{M}(\Omega^M, \Omega^S, \Omega^T)$; further, since the TDM $\mathcal{M}$ is shock-invariant it follows that $\frac{dx^L}{d\tau} = 0$, which combined with (163) implies the following:

$$\varphi^s = -\frac{1}{c}\frac{\partial c}{\partial p}\frac{dp}{d\tau} = -\frac{1}{c}\left[-\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p}\right]\frac{dp}{d\tau} = \frac{1}{c}c\frac{1}{p}p\frac{\alpha}{1+\psi} \tag{165}$$

$$= \frac{\alpha}{1+\psi} = \varphi^d = \varphi^*. \tag{166}$$

□

In other words, any equilibrium where tightness remains fixed over the business cycle will have demand-side and supply-side fiscal multipliers both fixed at φ^* and acyclical.

B.2 Frictional equilibria multipliers

As an alternative to the competitive equilibrium, last section considered a fixprice equilibrium, where all adjustment is happening via tightness and wages. Here we start off by considering a slightly more general *rigid price* equilibrium, that allows for an arbitrary degree of price rigidity and nests fixprice equilibrium as a special case. Subsequently, we introduce a much more general notion of a *Frictional Mapping (FM)*.

B.2.1 Rigid price equilibrium

A rigid price equilibrium is formally introduced as:

Definition 7. *A **rigid price** equilibrium is a vector $(p_0, x, w, \varepsilon, \mathcal{M})$, and associated allocations, such that the agents' optimality conditions and the market clearing conditions are satisfied with price given by:*

$$p = (p_0)^{\varepsilon}(p^L)^{1-\varepsilon}, \quad \varepsilon \in (0,1] \tag{167}$$

where ε is the degree of price rigidity and p_0 is a parameter and p^L is the price from the flexible equilibrium $(p^L, w^L, \mathcal{M})$.

Clearly, the fixprice equilibrium is just a special case under $\varepsilon = 1$. In fact, a non-fixprice rigid price equilibrium $(p_0, x, w, \varepsilon, \mathcal{M})$ where $\mathcal{M}$ is a shock invariant TDM shares a lot in common with the corresponding fixprice equilibrium. Particularly, the comparative statics to demand-side and supply-side shocks are given by:

Lemma 8. *In a non-fixprice ($\varepsilon \in (0,1)$) rigid price equilibrium $(p_0, x, w, \varepsilon, \mathcal{M})$ where $\mathcal{M}$ is a shock invariant TDM, the following are the comparative statics of tightness (x), sales (y) and the price (p):*

$$\frac{dx}{d\chi} > 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} > 0; \qquad \frac{dx}{da} < 0, \frac{dy}{da} > 0, \frac{dp}{da} < 0 \tag{168}$$

Proof. Special case of Lemma 9 under $\mathcal{T}(p^L; \{p_0, \varepsilon\}) = (p_0)^{\varepsilon}(p^L)^{1-\varepsilon}$. □

As one can see, the only difference compared to a fixprice equilibrium is that the price co-moves with tightness and supply-side shocks have an effect on the level of sales.

Of a greater interest to us, however, are the properties of fiscal multipliers under rigid price equilibria. The following proposition establishes the demand-side multiplier is a rigid price equilibrium:

Proposition 5. *In a rigid price equilibrium $(p_0, x, w, \varepsilon, \mathcal{M})$, where $\mathcal{M}$ is a shock invariant TDM, the demand-side fiscal multiplier $\varphi^d(x)$ is given by*

$$\varphi^d(x) = \varphi^* + \theta(x) \times \left[(1-\varphi^*)\{1-(1-\varepsilon)g(x,x^L)\}\right] \tag{169}$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the flexible equilibrium multiplier and the function $g(x, x^L)$ is given by:

$$g(x, x^L) = \frac{f(x) - \rho x}{f(x^L) - \rho x^L}. \tag{170}$$

Hence, $\varphi^d(x) \in (-\infty, 1]$ and $\frac{d\varphi^d(x)}{dx}|_{x=x^L} < 0$.

Proof. Special case of Proposition 7 under $\mathcal{T}(p^L; \{p_0, \varepsilon\}) = (p_0)^{\varepsilon}(p^L)^{1-\varepsilon}$. □

Note that for $\varepsilon = 1$ the expression above collapses back to the fixprice equilibrium demand-side multiplier from Proposition 2. We can also see that the rigid price equilibrium demand-side multiplier above maintains a lot of the properties of its fixprice equilibrium counterpart. In particular, it also collapses back to φ^* if the equilibrium tightness happens to coincide with the socially efficient one ($x = x^*$), it also lies between $-\infty$ and one, so that consumption always gets crowded out; it is also guaranteed to fall in tightness, although only in the neighbourhood of the corresponding flexible equilibrium allocation. The role played by ε here is in determining the relative magnitude of the state-dependent component. The upper panel of Figure 9 (drawn for simplicity under $x^L = x^*$) shows that as the degree of price rigidity ε falls, the multiplier becomes flatter around x^* at the level equal to φ^* suggesting that the degree of state-dependence falls as well.

Similarly, one can derive the supply-side multiplier under rigid price equilibrium:

Proposition 6. *In a rigid price equilibrium $(p_0, x, w, \varepsilon, \mathcal{M})$, where $\mathcal{M}$ is a shock invariant TDM, the supply-side fiscal multiplier $\varphi^s(x)$ is given by*

$$\varphi^s(x) = \varphi^* - \theta(x) \times \varepsilon\varphi^*, \tag{171}$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the long-run equilibrium multiplier. Hence, $\varphi^d(x) \in (0, +\infty)$ and $\frac{d\varphi^d(x)}{dx} > 0, \forall x \in (0, x_m)$.

Proof. Special case of Proposition 8 under $\mathcal{T}(p^L; \{p_0, \varepsilon\}) = (p_0)^\varepsilon (p^L)^{1-\varepsilon}$. □

Again, under $\varepsilon = 1$ the expression above collapses back to the fixprice equilibrium supply-side multiplier. One can see that the expression above shares every single property with the fixprice equilibrium counterpart, the only difference being the magnitude of state-dependence, which increases in the degree of price rigidity ε, as shown in the bottom panel of Figure 9 (again, for simplicity drawn for $x^L = x^*$).

B.2.2 Generalization: Frictional Mapping

One can in fact show that the properties of fiscal multipliers that we have established for the rigid price equilibrium hold more generally, and not for the particular parametric form of frictions that we have considered so far. We generalize our findings by introducing the notion of a Frictional Mapping:

Definition 8. *For a given flexible equilibrium $(p^L, w^L, \mathcal{M})$, a Frictional Mapping (FM) $\mathcal{T}$ is given by:*

$$\mathcal{T}: \quad \{p^L, \Omega^F\} \to p^F, \tag{172}$$

*where Ω^F is the set of parameters specific to the FM and p^F is the resulting price. Moreover, the Frictional Mapping $\mathcal{T}(p^L; \Omega^F)$ is said to be **contractionary** if and only if*

$$\frac{d \ln p^F}{d \ln p^L} = \frac{d\mathcal{T}(p^L; \Omega^F)}{dp^L} \frac{p^L}{p^F} \in [0, 1). \tag{173}$$

Having defined a Frictional Mapping (FM), one can now define a *frictional* equilibrium:

Definition 9. *A **frictional** equilibrium is a vector $(p^F, x^F, w^F, \mathcal{T}, \mathcal{M})$, and associated allocations, such that the agents' optimality conditions and the market clearing conditions are satisfied with price given by:*

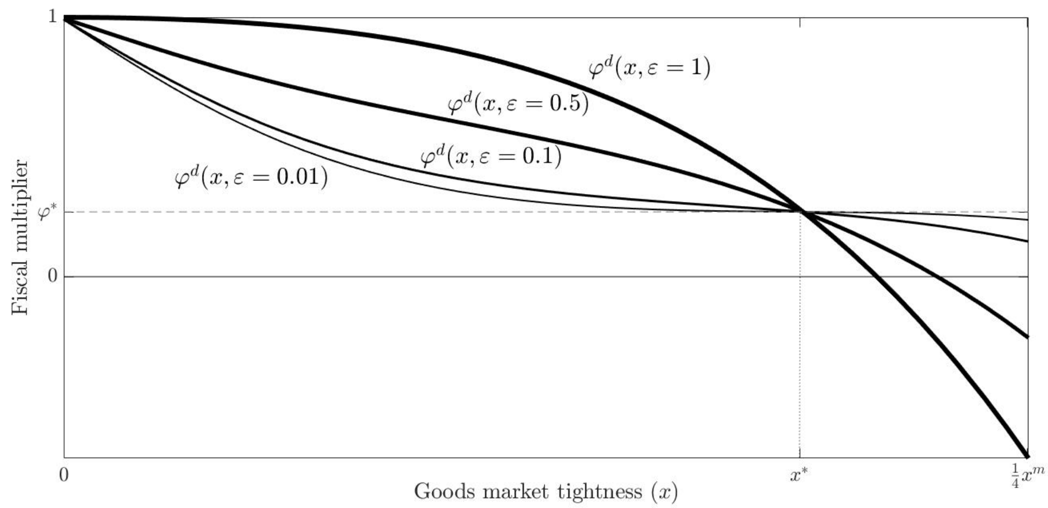
$$p^F = \mathcal{T}(p^L) \tag{174}$$

where $\mathcal{T}$ is the Frictional Mapping and p^L is the price from the flexible equilibrium $(p^L, w^L, \mathcal{M})$.

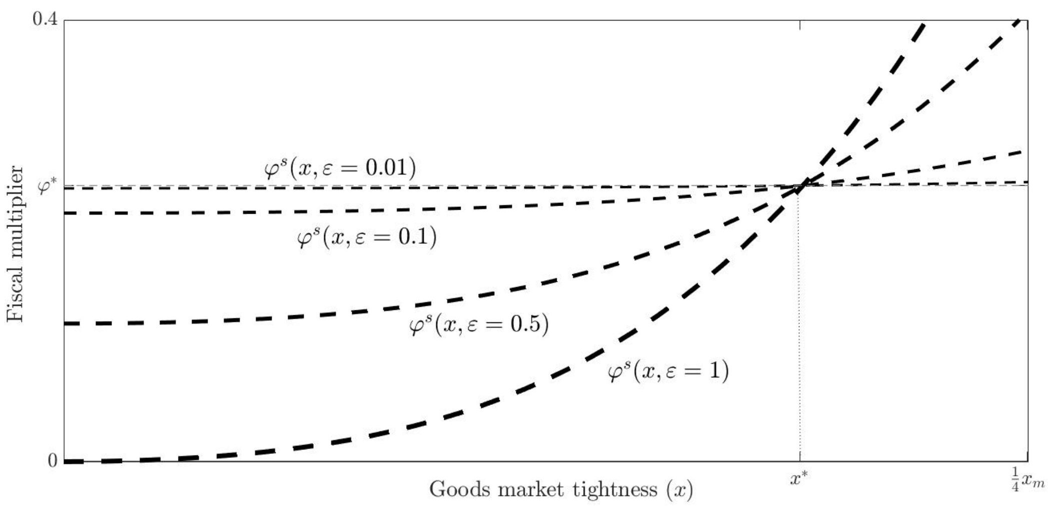
Rigid price equilibrium is a special case of a frictional equilibrium for $\mathcal{T}(z) = (p_0)^\varepsilon (z)^{1-\varepsilon}$, $\Omega^F = \{p_0, \varepsilon\}$, $\varepsilon \in (0, 1]$. Further, the above frictional mapping associated with a rigid price equilibrium is indeed contractionary,

Figure 9: Fiscal multipliers in a rigid price equilibrium

(a) Demand-side fiscal multiplier $\varphi^d(x)$



(b) Supply-side fiscal multiplier $\varphi^s(x)$



***Notes**: Panels (a) and (b) show demand-side and supply-side fiscal multipliers in a rigid price equilibrium of a calibrated version of our model ($\alpha = 0.3, \delta = 2, \rho = 0.1, \psi = 0.2, x^L = x^*$). Panel (a) shows demand-side fiscal multipliers for different values of the price rigidity parameter ε – one case see that $\varphi^d(x)$ strictly falls in tightness for all considered values of ε, but the degree of state-dependence rises in the degree of price rigidity; in Panel (b) we can see that the supply-side fiscal multiplier strictly rises in tightness for all values of ε considered, but again the degree if state-dependence falls as we allow for more price flexibility.*

since

$$\frac{d\mathcal{T}(z;\Omega^F)}{dz}\frac{z}{p^F} = (1-\varepsilon) \in [0,1), \tag{175}$$

as $\varepsilon \in (0,1]$.

We can now derive and discuss properties of demand-side and supply-side multipliers in a generic frictional equilibrium. Firstly, note that the comparative statics to demand-side and supply-side shocks established in a rigid price equilibrium extend to a generic frictional equilibrium generated by a contractionary frictional mapping:

Lemma 9. *In a non-fixprice $\left(\frac{d\mathcal{T}(p^L;\Omega^F)}{dp^L}\frac{p^L}{p^F} \neq 0\right)$ frictional equilibrium $(p^F, x^F, w^F, \mathcal{T}, \mathcal{M})$ where $\mathcal{T}$ is a contractionary FM and $\mathcal{M}$ is a shock invariant TDM, the following are the comparative statics of tightness (x), sales (y) and the price (p):*

$$\frac{dx}{d\chi} > 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} > 0; \qquad \frac{dx}{da} < 0, \frac{dy}{da} > 0, \frac{dp}{da} < 0 \tag{176}$$

Proof. Note that condition (65) still holds in this more generalized setting:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \left(pf(x)a\right)^{\frac{1+\psi}{1-\alpha+\psi}} (1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \chi + p[1+\gamma(x)]G. \tag{177}$$

Differentiate both sides with respect to χ (evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} [pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}} \left[\frac{1}{p}\frac{dp}{d\chi} + \frac{f'(x)}{f(x)}\frac{dx}{d\chi}\right] = 1, \tag{178}$$

$$\frac{dx}{d\chi} = \frac{f(x)}{f'(x)}\left[\alpha^{-\frac{\alpha}{1-\alpha+\psi}}\frac{1-\alpha+\psi}{1+\psi}[pf(x)a]^{-\frac{1+\psi}{1-\alpha+\psi}} - \frac{1}{p}\frac{dp}{d\chi}\right], \tag{179}$$

$$\frac{dx}{d\chi} = \frac{f(x)}{f'(x)}\left[\frac{1-\alpha+\psi}{1+\psi}\frac{1}{\chi} - \frac{1}{p}\frac{dp}{d\chi}\right]. \tag{180}$$

Since $p = \mathcal{T}(p^L)$, it follows that $\frac{dp}{d\chi} = \frac{d\mathcal{T}(p^L)}{dp^L}\frac{dp^L}{d\chi}$; further, from Lemma 7 we know that $\frac{dp^L}{d\chi} = \frac{1-\alpha+\psi}{1+\psi}\frac{p^L}{\chi}$ and it follows that:

$$\frac{dp}{d\chi} = \underbrace{\frac{d\mathcal{T}(p^L)}{dp^L}\frac{p^L}{p}}_{\in(0,1)\text{ as }\mathcal{T}\text{ contractionary FM}} \frac{1-\alpha+\psi}{1+\psi}\frac{p}{\chi} > 0. \tag{181}$$

$$\frac{dx}{d\chi} = \frac{f(x)}{f'(x)}\left[\frac{1-\alpha+\psi}{1+\psi}\frac{1}{\chi}\underbrace{\left(1 - \frac{d\mathcal{T}(p^L)}{dp^L}\frac{p^L}{p}\right)}_{\in(0,1)\text{ as }\mathcal{T}\text{ contractionary FM}}\right] > 0 \tag{182}$$

Given that $py = \chi$, it follows that $\frac{dy}{d\chi} = \frac{1}{p}\left[1 - \frac{dp}{d\chi}\frac{\chi}{p}\right]$ and hence:

$$\frac{dy}{d\chi} = \frac{1}{p}\left[1 - \underbrace{\frac{d\mathcal{T}(p^L)}{dp^L}\frac{p^L}{p}\frac{1-\alpha+\psi}{1+\psi}}_{\in(0,1)\text{ as }\mathcal{T}\text{ contractionary FM}}\right] > 0. \tag{183}$$

Similarly, differentiate both sides of (177) with respect to a (evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} [pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}} \left[\frac{1}{p}\frac{dp}{da} + \frac{f'(x)}{f(x)}\frac{dx}{da} + \frac{1}{a}\right] = 0, \tag{184}$$

$$\frac{dx}{da} = -\frac{f(x)}{f'(x)}\left[\frac{1}{a} + \frac{1}{p}\frac{dp}{da}\right] = 0. \tag{185}$$

Since $p = \mathcal{T}(p^L)$, it follows that $\frac{dp}{da} = \frac{d\mathcal{T}(p^L)}{dp^L}\frac{dp^L}{da}$; further, from Lemma 7 we know that $\frac{dp^L}{da} = -\frac{p^L}{a}$ and it follows that:

$$\frac{dp}{da} = -\underbrace{\frac{d\mathcal{T}(p^L)}{dp^L}\frac{p^L}{p}}_{\in(0,1) \text{ as } \mathcal{T} \text{ contractionary FM}} \frac{p}{a} < 0. \tag{186}$$

$$\frac{dx}{da} = -\frac{f(x)}{f'(x)}\left[\frac{1}{a}\underbrace{\left(1 - \frac{d\mathcal{T}(p^L)}{dp^L}\frac{p^L}{p}\right)}_{\in(0,1) \text{ as } \mathcal{T} \text{ contractionary FM}}\right] < 0 \tag{187}$$

Given that $py = \chi$, it follows that:

$$\frac{dy}{da} = -\frac{dp}{da}\frac{y}{p} > 0. \tag{188}$$

□

Moreover, one can also solve for the demand-side fiscal multiplier under a generic frictional equilibrium and see that under certain properties it also falls in tightness:

Proposition 7. *In a frictional equilibrium $(p^F, x^F, w^F, \mathcal{T}, \mathcal{M})$, where $\mathcal{T}$ is a contractionary FM and $\mathcal{M}$ is a shock invariant TDM, the demand-side fiscal multiplier $\varphi^d(x)$ is given by*

$$\varphi^d(x) = \varphi^* + \theta(x) \times \left[(1-\varphi^*)\left\{1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)} g(x, x^L)\right\}\right] \tag{189}$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the flexible equilibrium multiplier and the function $g(x, x^L)$ is given by:

$$g(x, x^L) = \frac{f(x) - \rho x}{f(x^L) - \rho x^L}. \tag{190}$$

Hence, $\varphi^d(x) \in (-\infty, 1)$ and $\frac{d\varphi^d(x)}{dx}|_{x=x^L} < 0$.

Proof. Note that (89) still holds in this more general setting:

$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)} = (1-\varphi^*)\frac{1}{c(p,x)}. \tag{191}$$

From the definition of the demand-side fiscal multiplier:

$$\varphi^d(x) = \frac{d\{c+G\}}{dG} = \frac{dc}{dG} + 1 = \frac{\partial c}{\partial p}\frac{dp}{dG} + \frac{\partial c}{\partial x}\frac{dx}{dG} + 1 = \tag{192}$$

$$= -c(p,x)\left[\frac{1}{p}\frac{dp}{dp^L}\frac{dp^L}{dG}\right] - c(p,x)\underbrace{\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}_{1-\theta(x)}\left[(1-\varphi^*)\frac{1}{c(p,x)} - \frac{1}{p}\frac{dp}{dp^L}\frac{dp^L}{dG}\right]. \tag{193}$$

From Proposition 4 we know that $\frac{dp^L}{dG} = p^L(1-\varphi^*)\frac{1}{c(p^L,x^L)}$, hence:

$$\varphi^d(x) = -c(p,x)\frac{dp}{dp^L}\frac{p^L}{p}(1-\varphi^*)c(p^L,x^L) - c(p,x)(1-\theta(x))\left[(1-\varphi^*)\frac{1}{c(p,x)} - \frac{dp}{dp^L}\frac{p^L}{p}(1-\varphi^*)\frac{1}{c(p^L,x^L)}\right], \tag{194}$$

$$= 1 - (1-\varphi^*)(1-\theta(x)) - \theta(x)(1-\varphi^*)\frac{dp}{dp^L}\frac{p^L}{p}\frac{c(p,x)}{c(p^L,x^L)} \tag{195}$$

$$= \varphi^* + \theta(x)(1-\varphi^*)\left[1 - \frac{dp}{dp^L}\frac{p^L}{p}\frac{c(p,x)}{c(p^L,x^L)}\right] \tag{196}$$

$$= \varphi^* + \theta(x)(1-\varphi^*)\left[1 - \frac{dp}{dp^L}\frac{p^L}{p}\frac{p^L[1+\gamma(x^L)]}{p[1+\gamma(x)]}\right] \tag{197}$$

$$= \varphi^* + \theta(x)(1-\varphi^*)\left[1 - \frac{dp}{dp^L}\frac{p^L}{p}\frac{\frac{p^L f(x^L)}{f(x^L)-\rho x^L}}{\frac{p f(x)}{f(x)-\rho x}}\right] \tag{198}$$

$$= \varphi^* + \theta(x)(1-\varphi^*)\left[1 - \frac{dp}{dp^L}\frac{p^L}{p}\frac{f(x)-\rho x}{f(x^L)-\rho x^L}\right] \tag{199}$$

$$= \varphi^* + \theta(x)(1-\varphi^*)\left[1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}g(x,x^L)\right], \tag{200}$$

where $g(x,x^L) = \frac{f(x)-\rho x}{f(x^L)-\rho x^L}$. Further, notice that:

$$\frac{d\varphi^d(x)}{dx}|_{x=x^L} = \theta'(x^L)(1-\varphi^*)\left[1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right] - \theta(x^L)(1-\varphi^*)\frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\frac{f'(x^L)-\rho}{f(x^L)-\rho x^L} < 0, \forall x^L \in (0,x_m) \tag{201}$$

since $\theta'(x^L) < 0, \theta(x^L)(f'(x^L)-\rho) > 0, \forall x \in (0,x_.)$. Also, it follows that $\varphi^d(0) = \varphi^* + \theta(0)(1-\varphi^*)[1-0] = 1$, and $\lim_{x\to x_m^-}\varphi^d(x) = \varphi^* + \lim_{x\to x_m^-}(1-\varphi^*)[1-\frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}] = -\infty$, so that $\varphi^d(x) \in (-\infty,1), \forall x \in (0,x_m)$. □

Similarly, one can also solve for the supply-side fiscal multiplier in a generic frictional equilibrium and establish its properties:

Proposition 8. *In a frictional equilibrium $(p^F, x^F, w^F, \mathcal{T}, \mathcal{M})$, where $\mathcal{T}$ is a contractionary FM and $\mathcal{M}$ is a shock-invariant TDM, the supply-side fiscal multiplier $\varphi^s(x)$ is given by*

$$\varphi^s(x) = \varphi^* - \theta(x) \times \left(1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right)\varphi^*, \tag{202}$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the flexible equilibrium multiplier. Hence, $\varphi^d(x) \in (0,+\infty)$ and $\frac{d\varphi^d(x)}{dx} > 0, \forall x \in (0,x_m)$.

Proof. Note that (95) still holds in this more general setting:

$$\left[\frac{1}{p}\frac{dp}{d\tau} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau}\right] = \varphi^*. \tag{203}$$

From the definition of the supply-side fiscal multiplier:

$$\varphi^s(x) = \frac{d\{c+G\}/\{c+G\}}{d[-\tau]} = -\frac{1}{c(p,x)}\frac{dc}{d\tau} = -\frac{1}{c(p,x)}\left[\frac{\partial c}{\partial p}\frac{dp}{d\tau} + \frac{\partial c}{\partial x}\frac{dx}{d\tau}\right] \tag{204}$$

$$= \frac{1}{c(p,x)}\left[c(p,x)\frac{1}{p}\frac{dp}{d\tau} + c(p,x)\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}\left(\varphi^* - \frac{1}{p}\frac{dp}{d\tau}\right)\right] \tag{205}$$

$$= (1-\theta(x))\varphi^* + \frac{1}{p}\frac{d\mathcal{T}(p^L)}{dp^L}\frac{dp^L}{d\tau}\theta(x). \tag{206}$$

From Lemma 4 we know that $\frac{dp^L}{d\tau} = p^L\varphi^*$:

$$\varphi^s(x) = \varphi^* - \theta(x)\left(1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right)\varphi^*. \tag{207}$$

Further, $\frac{d\varphi^s(x)}{dx} = -\theta(x)\left(1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right)\varphi^* > 0, \forall x \in (0, x_m)$ since $\theta'(x) < 0, \forall x \in (0, x_m)$. Also, $\varphi^s(0) = \varphi^*\frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)} \in [0,1)$, and $\lim_{x\to x_m^-}\varphi^s(x) = \varphi^* - \lim_{x\to x_m}\theta(x)\left(1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right) = +\infty$, so that $\varphi^s(x) \in (0,+\infty), \forall x \in (0, x_m)$. □

B.3 Cyclicality of fiscal multipliers

Firstly , note that our result of equal and acyclical demand-side and supply-side multipliers in a competitive equilibrium extends more generally to *any* flexible equilibrium generated by a shock-invariant TDM:

Corollary 10. *In any flexible equilibrium generated by policy-invariant TDM both demand-side and supply-side multipliers are acyclical.*

Proof. Trivial consequence of Proposition 4: in any flexible equilibrium generated by a shock-invariant TDM, both multipliers are equal to $\varphi^* = \frac{\alpha}{1+\psi}$ and do not change as either preference χ or technology a varies. □

In the words, any equilibrium that sees tightness fixed over the business cycle, will see both multipliers fixed at φ^* and hence acyclical.

Further, our state-dependence result for the demand-side multiplier in a fixprice equilibrium still holds in *any* frictional equilibrium as long as the elasticity between frictional and flexible price is in $[0,1)$ and the flexible equilibrium around which the friction is defined is generated by a shock-inariant TDM:

Corollary 11. *In any frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM, in the local neighbourhood of the flexible equilibrium allocation, the demand-side multiplier is countercyclical under demand-driven fluctuations, and procyclical under supply-driven fluctuations.*

Proof. From Lemma 9 we know that in any frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM, $\frac{dx}{d\chi} > 0$, $\frac{dx}{da} < 0$; further, from Proposition 7 we know that in a frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM $\frac{d\varphi^d(x)}{dx}|_{x=x^L} < 0$. Hence, $\frac{d\varphi^d(x)}{d\chi}|_{x=x^L} = \frac{d\varphi^d(x)}{dx}|_{x=x^L}\frac{dx}{d\chi}|_{x=x^L} < 0$ and $\frac{d\varphi^d(x)}{da}|_{x=x^L} = \frac{d\varphi^d(x)}{dx}|_{x=x^L}\frac{dx}{da}|_{x=x^L} > 0$. □

Similarly for the supply-side multiplier:

Corollary 12. *In any frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM, the supply-side multiplier is procyclical under demand-driven fluctuations, and countercyclical under supply-driven fluctuations.*

Proof. From Lemma 9 we know that in any frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM, $\frac{dx}{d\chi} > 0$, $\frac{dx}{da} < 0$; further, from Proposition 8 we know that in a frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM $\frac{d\varphi^s(x)}{dx} > 0$. Hence, $\frac{d\varphi^s(x)}{d\chi} = \frac{d\varphi^s(x)}{dx}\frac{dx}{d\chi} > 0$ and $\frac{d\varphi^s(x)}{da} = \frac{d\varphi^s(x)}{dx}\frac{dx}{da} < 0$. □

C Alternative fiscal instruments

C.1 Government employment

Let the government employ a fraction $h \in [0, 1)$ of the households' labor supply, so that government labor demand is given by $n^G = hl$ and the government collects additional lump-sum taxes to finance public sector wages, so that $T = p[1 + \gamma(x)]G - wn\tau - wn^G$. Then labor market clearing condition becomes:

$$n(w; p, x, \tau) + n^G = l(w) \tag{208}$$

$$n(w; p, x, \tau) = (1 - h)l(w) \tag{209}$$

Without loss of generality, assume $\tau = 0$, and substitute our solutions for n and l:

$$[\alpha pf(x)a]^{\frac{1}{1-\alpha}} w^{-\frac{1}{1-\alpha}} = (1 - h)w^{\frac{1}{\psi}} \tag{210}$$

$$[\alpha pf(x)a]^{\frac{1}{1-\alpha}} (1 - h)^{-1} = w^{\frac{1-\alpha+\psi}{\psi(1-\alpha)}} \tag{211}$$

$$w = [\alpha pf(x)a]^{\frac{\psi}{1-\alpha+\psi}} (1 - h)^{-\frac{\psi(1-\alpha)}{1-\alpha+\psi}} \tag{212}$$

$$n = (1 - h)l(w) = (1 - h)w^{\frac{1}{\psi}} = [\alpha pf(x)a]^{\frac{1}{1-\alpha+\psi}} (1 - h)^{\frac{\psi}{1-\alpha+\psi}}. \tag{213}$$

Substituting the solution for n above into the goods market clearing condition:

$$\frac{f(x)an^{\alpha}}{1 + \gamma(x)} = c(p, x) + G \tag{214}$$

$$pf(x)an^{\alpha} = \chi + p[1 + \gamma(x)]G \tag{215}$$

$$pf(x)a[\alpha pf(x)a]^{\frac{\alpha}{1-\alpha+\psi}} (1 - h)^{\frac{\alpha\psi}{1-\alpha+\psi}} \tag{216}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} [apf(x)]^{\frac{1+\psi}{1-\alpha+\psi}} (1 - h)^{\frac{\alpha\psi}{1-\alpha+\psi}} = \chi + p[1 + \gamma(x)]G \tag{217}$$

Differentiating with respect to h (for simplicity, where $G = h = 0$):

$$\frac{1+\psi}{1-\alpha+\psi}[apf(x)]^{\frac{1+\psi}{1-\alpha+\psi}-1}\left(a\frac{dp}{dh}f(x)+apf'(x)\frac{dx}{dh}\right)(1-h)^{\frac{\alpha\psi}{1-\alpha+\psi}} = [apf(x)]^{\frac{1+\psi}{1-\alpha+\psi}}\frac{\alpha\psi}{1-\alpha+\psi}(1-h)^{\frac{\alpha\psi}{1-\alpha+\psi}-1} \tag{218}$$

$$\frac{1+\psi}{1-\alpha+\psi}[apf(x)]^{\frac{1+\psi}{1-\alpha+\psi}}\left(\frac{1}{p}\frac{dp}{dh}+\frac{f'(x)}{f(x)}\frac{dx}{dh}\right) = [apf(x)]^{\frac{1+\psi}{1-\alpha+\psi}}\frac{\alpha\psi}{1-\alpha+\psi} \tag{219}$$

$$\frac{1}{p}\frac{dp}{dh}+\frac{f'(x)}{f(x)}\frac{dx}{dh} = \frac{\alpha\psi}{1+\psi}. \tag{220}$$

Define the government employment multiplier:

$$\varphi^h(x) \equiv \frac{d\{c+G\}/\{c+G\}}{dh} = \frac{1}{c}\frac{c}{h}. \tag{221}$$

In a competitive equilibrium $x = x^*$ so that:

$$\varphi^h = \frac{1}{c}\left[-\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p}\frac{dp}{dh}\right] = -\frac{1}{p}p\frac{\alpha\psi}{1+\psi} = \frac{\alpha\psi}{1+\psi}. \tag{222}$$

Therefore, just like the multipliers studied in the main text, the government employment multiplier is acyclical in the competitive equilibrium and is pinned down exclusively by the relative elasticities of labor demand and labor supply.

In a fixprice equilibrium, $p = p_0$ so that:

$$\varphi^h(x) = -\frac{\gamma'(x)}{1+\gamma(x)}\frac{dx}{dh} = -\underbrace{\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}_{1-\theta(x)}\frac{\alpha\psi}{1+\psi} = (\theta(x)-1)\frac{\alpha\psi}{1+\psi}. \tag{223}$$

Note that $\frac{d\varphi^h(x)}{dx} = \theta'(x)\frac{\alpha\psi}{1+\psi} < 0, \forall x \in (0, x_m)$; therefore, in a fixprice equilibrium, the government employment multiplier strictly falls in tightness and has the same cyclicality properties as the government consumption spending multiplier considered in the main text.

C.2 Distortionary taxes on consumption, labor income and firms' sales

We introduce taxes on households' consumption and labor income, so that the budget constraint of the representative households is given by:

$$p(1+\tau^c)[1+\gamma(x)] + m \leq w(1-\tau^l)l + \bar{m} + \Pi - T, \tag{224}$$

where τ^c is the consumption tax rate, τ^l is the labor income tax rate. The consumption function and the labor supply function become:

$$c(p,x) = \frac{\chi}{p(1+\tau^c)[1+\gamma(x)]}, \qquad l(w) = [w(1-\tau^l)]^{\frac{1}{\psi}}. \tag{225}$$

Further, we introduce taxes on firms' payroll, so that firms' profits are given by:

$$\Pi = p(1-\tau^s)f(x)an^\alpha - wn(1+\tau), \tag{226}$$

where τ^s is the rate of tax on firms' sales. The labor demand function resulting from profit maximisation is then given by:

$$n(w; p, x, \tau, \tau^s) = \left[\frac{\alpha p(1-\tau^s)f(x)a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}}. \tag{227}$$

Combining the labor demand function with labor market clearing condition delivers the following equilibrium employment:

$$n = [\alpha p f(x) a]^{\frac{1}{1-\alpha+\psi}} (1-\tau^s)^{\frac{1}{1-\alpha+\psi}} (1+\tau)^{-\frac{1}{1-\alpha+\psi}} (1-\tau^l)^{\frac{1}{1-\alpha+\psi}}. \tag{228}$$

Using the goods market clearing condition:

$$\frac{f(x)an^\alpha}{1+\gamma(x)} = c(p,x) + G \tag{229}$$

$$pf(x)an^\alpha = \frac{\chi}{1+\tau^c} + p[1+\gamma(x)]G \tag{230}$$

$$\alpha^{\frac{\alpha}{1\alpha+\psi}} [pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}} (1-\tau^s)^{\frac{\alpha}{1-\alpha+\psi}} (1-\tau)^{-\frac{\alpha}{1-\alpha+\psi}} (1-\tau^l)^{\frac{\alpha}{1-\alpha+\psi}} = \frac{\chi}{1+\tau^c} + p[1+\gamma(x)]G. \tag{231}$$

C.2.1 Consumption tax cut multiplier

Differentiate with respect to τ^c (at $\tau = \tau^c = \tau^l = \tau^s = G = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \left[\frac{1+\psi}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1} \left\{\frac{dp}{d\tau^c} f(x)a + pf'(x)\frac{dx}{d\tau^c}\right\}\right] = -\frac{\chi}{(1+\tau^c)^2} \tag{232}$$

$$\underbrace{\alpha^{\frac{\alpha}{1-\alpha+\psi}} [pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}}}_{\chi} \left\{\frac{1}{p}\frac{dp}{d\tau^c} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau^c}\right\} = -\chi\frac{1-\alpha+\psi}{1+\psi} \tag{233}$$

$$\frac{1}{p}\frac{dp}{d\tau^c} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau^c} = \varphi^* - 1. \tag{234}$$

Define the consumption tax cut multiplier:

$$\varphi^{\tau^c}(x) \equiv \frac{d\{c+G\}/\{c+G\}}{d[-\tau^c]} = -\frac{1}{c}\frac{dc}{d\tau^c}. \tag{235}$$

In a competitive equilibrium $x = x^*$, so that:

$$\varphi^{\tau^c} = -\frac{1}{c}\left[-c\frac{1}{p}\frac{dp}{d\tau^c} + \frac{\partial c}{\partial \tau^c}\right] = -\frac{1}{c}\left[-c\frac{1}{p}p(\varphi^* - 1) - c\right] = \varphi^*, \tag{236}$$

so that in a competitive equilibrium the consumption tax cut multiplier is acyclical and pinned down exclusively by elasticities of labor supply and labor demand.

In a fixprice equilibrium $p = p_0$, so that:

$$\varphi^{\tau^c}(x) = -\frac{1}{c}\left[-c\frac{\gamma'(x)}{1+\gamma(x)}\frac{dx}{d\tau^c} + \frac{\partial c}{\partial \tau^c}\right] = -\frac{1}{c}\left[-c\underbrace{\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}_{1-\theta(x)}(\varphi^* - 1) - c\right] = \varphi^* + \theta(x)(1-\varphi^*) = \varphi^d)(x), \tag{237}$$

so that in a fixprice equilibrium the consumption tax cut multiplier is identical to the government consumption spending multiplier, and thus shares all of the properties of the latter.

C.2.2 Labor income tax cut multiplier

Differentiate the goods market clearing condition with respect to τ^l (at $\tau = \tau^c = \tau^l = \tau^s = G = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[\frac{1+\psi}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1}\left\{\frac{dp}{d\tau^l}f(x)a + pf'(x)\frac{dx}{d\tau^l}a\right\} - \frac{\alpha}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}\right] = 0 \tag{238}$$

$$\frac{1}{p}\frac{dp}{d\tau^l} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau^l} = \frac{\alpha}{1+\psi} = \varphi^*. \tag{239}$$

Define the labor income tax cut multiplier:

$$\varphi^{\tau^l}(x) \equiv \frac{d\{c+G\}/\{c+G\}}{d[-\tau^l]} = -\frac{1}{c}\frac{dc}{d\tau^l}. \tag{240}$$

In a competitive equilibrium $x = x^*$, so that:

$$\varphi^{\tau^l} = -\frac{1}{c}\frac{\partial c}{\partial p}\frac{dp}{d\tau^l} = -\frac{1}{c}[-c\frac{1}{p}]p\varphi^* = \varphi^*, \tag{241}$$

so that in a competitive equilibrium the labor income tax cut multiplier is acyclical and pinned down exclusively by the elasticities of labor demand and labor supply.

In a fixprice equilibrium $p = p_0$, so that:

$$\varphi^{\tau^l}(x) = -\frac{1}{c}\frac{\partial c}{\partial x}\frac{dx}{d\tau^l} = -\frac{1}{c}[-c\underbrace{\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}_{1-\theta(x)}]\varphi^* = \varphi^* - \theta(x)\varphi^* = \varphi^s(x), \tag{242}$$

so that in a fixprice equilibrium the labor income tax cut multiplier is identical to the payroll tax cut multiplier considered in the main text, and shares all of its properties.

C.2.3 Firms' sales tax cut multiplier

Differentiate the goods market clearing condition with respect to τ^s (at $\tau = \tau^c = \tau^l = \tau^s = G = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[\frac{1+\psi}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1}\left\{\frac{dp}{d\tau^s}f(x)a + pf'(x)\frac{dx}{d\tau^s}a\right\} - \frac{\alpha}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}\right] = 0 \tag{243}$$

$$\frac{1}{p}\frac{dp}{d\tau^s} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau^s} = \frac{\alpha}{1+\psi} = \varphi^*. \tag{244}$$

Define the sales tax cut multiplier:

$$\varphi^{\tau^s}(x) \equiv \frac{d\{c+G\}/\{c+G\}}{d[-\tau^s]} = -\frac{1}{c}\frac{dc}{d\tau^s}. \tag{245}$$

In a competitive equilibrium $x = x^*$, so that:

$$\varphi^{\tau^s} = -\frac{1}{c}\frac{\partial c}{\partial p}\frac{dp}{d\tau^s} = -\frac{1}{c}[-c\frac{1}{p}]p\varphi^* = \varphi^*, \tag{246}$$

so that in a competitive equilibrium the sales tax cut multiplier is acyclical and pinned down exclusively by the elasticities of labor demand and labor supply.

In a fixprice equilibrium $p = p_0$, so that:

$$\varphi^{\tau^s}(x) = -\frac{1}{c}\frac{\partial c}{\partial x}\frac{dx}{d\tau^s} = -\frac{1}{c}[-c\underbrace{\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}_{1-\theta(x)}]\varphi^* = \varphi^* - \theta(x)\varphi^* = \varphi^s(x), \tag{247}$$

so that in a fixprice equilibrium the sales tax cut multiplier is identical to the payroll tax cut multiplier considered in the main text, and shares all of its properties.

D Comparative statics

D.1 Competitive equilibrium

Panel (a) of Figure 10 shows comparative statics following a positive demand-side shock, parameterized as a permanent increase in χ. The aggregate demand curve shifts out, exercising upward pressure on goods market tightness. In order to retain tightness at the socially efficient level, the price must increase to lower private consumption to offset the rise in aggregate demand. Higher price increases labor demand, which expands capacity until the goods market reaches the efficient level of tightness (x^*). The new equilibrium features tightness at the efficient level, with higher price and sales compared to the original equilibrium.

Panel (b) of Figure 10 shows comparative statics following a positive supply shock, parameterized as a permanent increase in a. In response to the shock, aggregate supply curve shifts out, putting downward pressure on goods market tightness. In order to retain tightness at the socially efficient level, the price decreases in order to increase private consumption, until there are no more pressures on tightness to deviate from x^*. Eventually, tightness remains at the socially efficient level, sales increase and price falls.

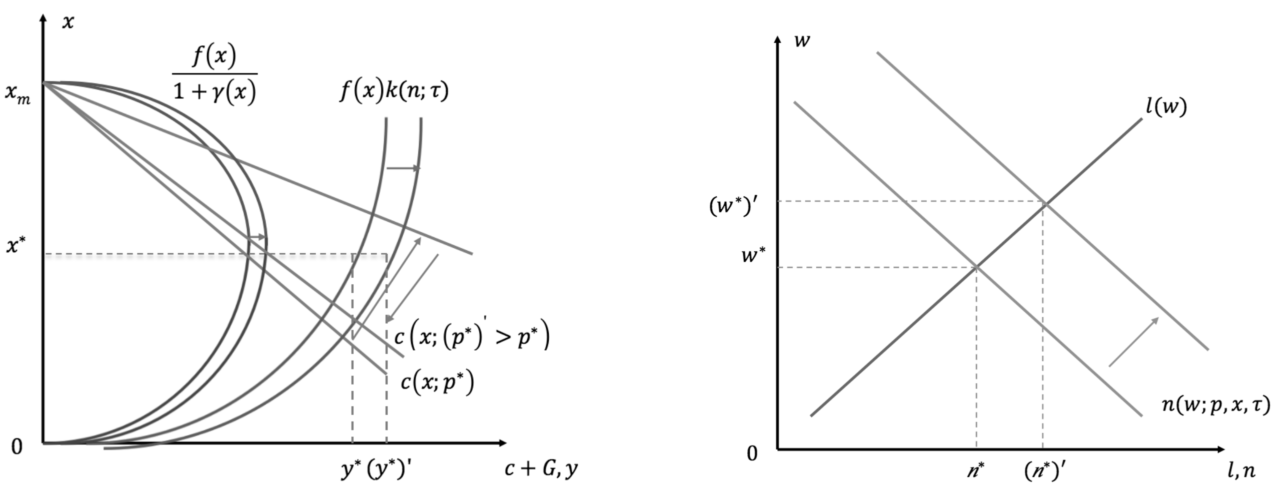
D.2 Fixprice equilibrium

Panel (a) of Figure 11 shows comparative statics following a positive demand-side shock, parameterized as a permanent increase in χ. The aggregate demand curve shifts out, creating excess demand that under the fixed price is cleared out by rising tightness increase the cost of search and decreasing private consumption; higher tightness also encourages more labor demand as the effective price from the firms' perspective increases. The latter effect causes an outward shift of the aggregate supply curve, but tightness remains above the initial level, similarly to sales. By construction, the price remains unchanged.

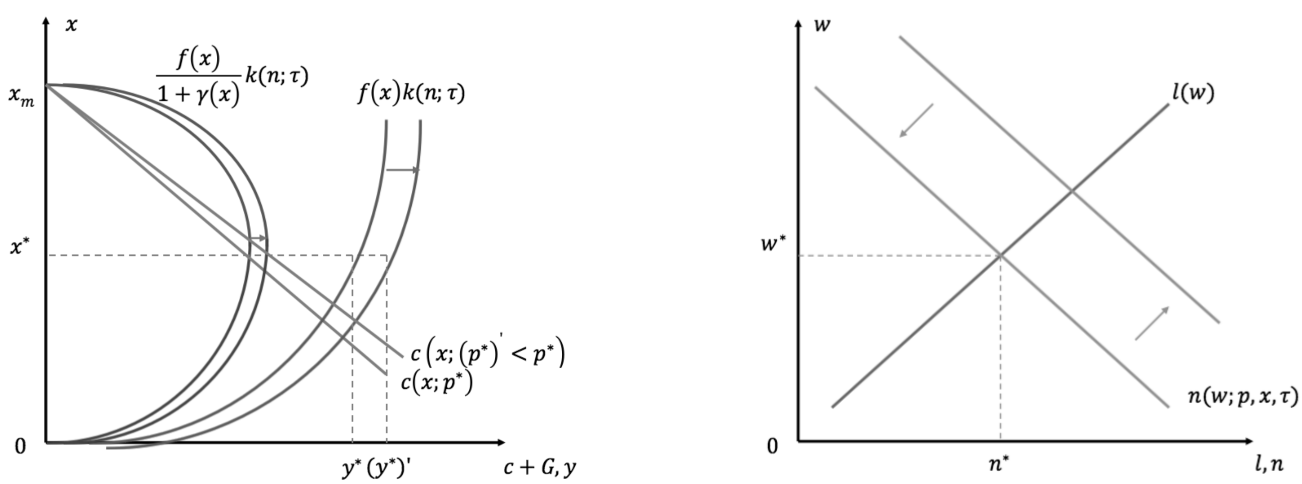
Panel (b) of Figure 11 shows comparative statics for a positive supply-side shock, parameterized as a permanent increase in a. The aggregate supply curve shifts out, putting downward pressure on tightness via excess supply, and under the fixed price tightness falls to clear the market by lowering the cost of search for households and increasing

Figure 10: Comparative statics in a competitive equilibrium

(a) Positive demand-side shock (increase in χ)



(b) Positive supply-side shock (increase in a)

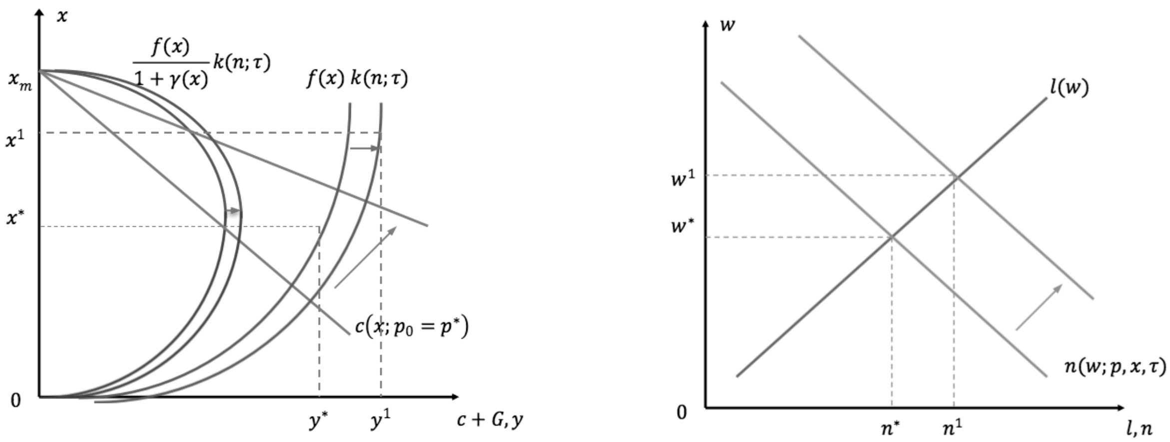


***Notes**: Panel (a) shows comparative statics in a competitive equilibrium, following a positive demand-side shock, parameterised as an increase in the preference parameter χ; following the shock, price increases to clear excess demand created by the shock and keep tightness at x^*, and equilibrium labor and sales also increase.*

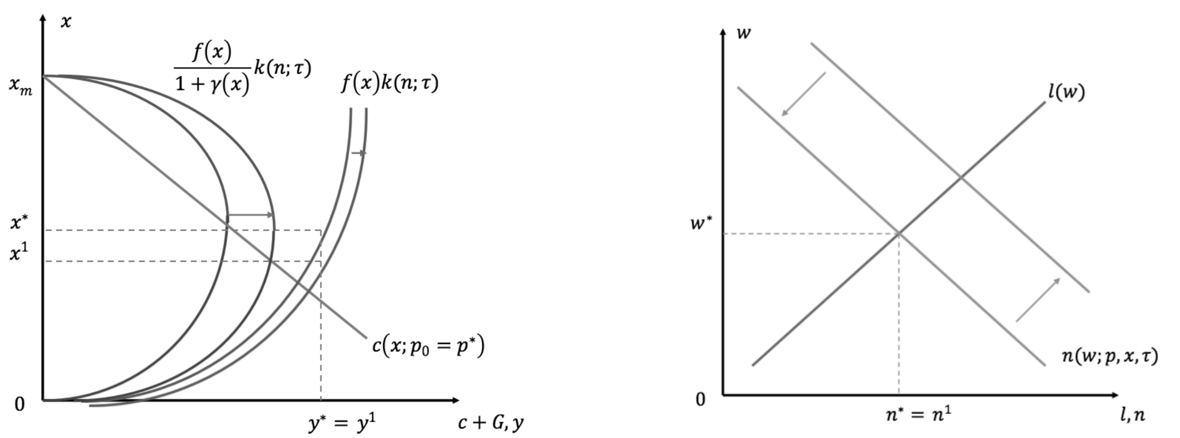
Panel (b) shows comparative statics in a competitive equilibrium, following a positive supply-side shock, parameterised as an increase in the technology parameter a; following the shock, price falls to clear excess supply caused by the shock and keep tightness at x^*, equilibrium labor remains unchanged and sales increase, as every unit of labor is now more productive, leading to higher capacity and higher sales, due to unchanged level of tightness.

Figure 11: Comparative statics in a fixprice equilibrium

(a) Positive demand-side shock (increase in χ)



(b) Positive supply-side shock (increase in a)



***Notes**: Panel (a) shows comparative statics in a fixprice equilibrium that initially coincides with the socially efficient allocation, and is hit by a positive demand-side shock, parameterised as an increase in the preference parameter χ; following the shock, goods market tightness increases to clear excess demand created by the shock, and equilibrium labor and sales also increase.*

Panel (b) shows comparative statics in a fixprice equilibrium that initially coincides with the socially efficient allocation, and is hit by a positive supply-side shock, parameterised as an increase in the technology parameter a; following the shock, goods market tightness falls to clear excess supply caused by the shock, whereas equilibrium labor and sales remain unchanged, since the effects of lower tightness and higher level of technology exactly offset each other.

private consumption. In equilibrium, tightness fall and sales remain unchanged.[34] By construction, the price also remains unchanged.

E Results under general CRRA utility of consumption

E.1 Comparative statics

Goods market clearing under general CRRA utility is given by:

$$\frac{f(x)an^{\alpha}}{1+\gamma(x)} = \frac{\chi^{\frac{1}{\sigma}}}{(p[1+\gamma(x)])^{\frac{1}{\sigma}}} + G. \tag{248}$$

Combining with the equilibrium labor expression:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}(1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \chi^{\frac{1}{\sigma}}(p[1+\gamma(x)])^{1-\frac{1}{\sigma}} + p[1+\gamma(x)]G. \tag{249}$$

E.1.1 Competitive equilibrium

In a competitive equilibrium $x = x^*$, so that $\frac{dx}{d\chi} = 0$; differentiate (249) (at $G = \tau = 0$) with respect to χ to find the comparative statics to a demand shock in a competitive equilibrium:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\frac{1+\psi}{1-\alpha+\psi}[pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}-1}\frac{dp}{d\chi}f(x)a = \frac{1}{\sigma}\chi^{\frac{1}{\sigma}-1}(p[1+\gamma(x)])^{1-\frac{1}{\sigma}} + \left(1-\frac{1}{\sigma}\right)\chi^{\frac{1}{\sigma}}(p[1+\gamma(x)])^{-\frac{1}{\sigma}}\frac{dp}{d\chi}[1+\gamma(x)] \tag{250}$$

$$\frac{dp}{d\chi} = \left[\frac{\alpha}{1-\alpha+\psi} + \frac{1}{\sigma}\right]^{-1}\frac{p}{\sigma\chi} > 0. \tag{251}$$

$$\frac{dc}{d\chi} = \frac{d}{d\chi}\left[\chi^{\frac{1}{\chi}}p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}}\right] = \frac{1}{\sigma}\chi^{\frac{1}{\sigma}-1}p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}} - \frac{1}{\sigma}\chi^{\frac{1}{\sigma}}p^{-\frac{1}{\sigma}-1}\frac{dp}{d\chi}[1+\gamma(x)]^{-\frac{1}{\sigma}} \tag{252}$$

$$\frac{dc}{d\chi} = \frac{c}{\sigma\chi}\left[\frac{1}{\sigma}\left(\frac{\alpha}{1+\psi}\right)^{-1} + \left(1-\frac{1}{\sigma}\right)\right]^{-1} > 0. \tag{253}$$

$$\frac{dy}{d\chi} = \frac{d}{d\chi}(c[1+\gamma(x)]) = [1+\gamma(x)]\frac{dc}{d\chi} > 0. \tag{254}$$

[34] Note that sales remain the same due to two countervailing forces: on the one hand, productivity increases, expanding capacity and leading to more sales, *ceteris paribus*; on the other hand, tightness falls, lowering $f(x)$, which decreases sales, *ceteris paribus*. In the special case of log utility of consumption and a fixprice equilibrium these two effects exactly offset each other. However, once one considers equilibria with rigid, but not fully fixed prices, as we do in Appendix B, the first effect dominates and sales rise following a positive technology shock.

Similarly, differentiate (249) (at $G = \tau = 0$) with respect to a to find the comparative statics to a supply shock in a competitive equilibrium:

$$\frac{1+\psi}{1-\alpha+\psi}\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[pf(x)a\right]^{\frac{1+\psi}{1-\alpha+\psi}-1}\left[\frac{dp}{da}f(x)a+pf(x)\right]=\left(1-\frac{1}{\sigma}\right)\chi^{\frac{1}{\sigma}}\left(p[1+\gamma(x)]\right)^{-\frac{1}{\sigma}}\frac{dp}{da}[1+\gamma(x)] \tag{255}$$

$$\frac{dp}{da}=-\frac{p}{a}\frac{\sigma(1+\psi)}{1+\psi+(\sigma-1)\alpha}<0. \tag{256}$$

$$\tag{257}$$

$$\frac{dc}{da}=\frac{d}{da}\left[\chi^{\frac{1}{\chi}}p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}}\right]=-\frac{1}{\sigma}\chi^{\frac{1}{\sigma}}p^{-\frac{1}{\sigma}-1}\frac{dp}{da}[1+\gamma(x)]^{-\frac{1}{\sigma}} \tag{258}$$

$$\frac{dc}{da}=\frac{c}{a}\frac{\frac{1}{\sigma}}{\frac{1}{\sigma}+\left(1-\frac{1}{\sigma}\right)\frac{\alpha}{1+\psi}}>0. \tag{259}$$

$$\tag{260}$$

$$\frac{dy}{da}=\frac{d}{da}(c[1+\gamma(x)])=[1+\gamma(x)]\frac{dc}{da}>0. \tag{261}$$

E.1.2 Fixprice equilibrium

In a fixprice equilibrium $p = p_0$ is a parameter, so that $\frac{dp}{d\chi} = 0$; differentiate (249) (at $G = \tau = 0$) with respect to χ to find the comparative statics to a demand shock in a fixprice equilibrium:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi}[pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}-1} pf'(x)\frac{dx}{d\chi}a = \frac{1}{\sigma}\chi^{\frac{1}{\sigma}-1}(p[1+\gamma(x)])^{1-\frac{1}{\sigma}} + \left(1-\frac{1}{\sigma}\right)\chi^{\frac{1}{\sigma}}(p[1+\gamma(x)])^{-\frac{1}{\sigma}}p\gamma'(x)\frac{dx}{d\chi} \tag{262}$$

$$\left[\frac{1+\psi}{1-\alpha+\psi} - \left(1-\frac{1}{\sigma}\right)\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}\right]\frac{f'(x)}{f(x)}\frac{dx}{d\chi} = \frac{1}{\sigma\chi} \tag{263}$$

$$\frac{dx}{d\chi} = \frac{1}{\sigma\chi}\left[\frac{1+\psi+(\sigma-1)\alpha}{\sigma(1-\alpha+\psi)} + \left(1-\frac{1}{\sigma}\right)\theta_{\sigma=1}(x)\right]^{-1}\frac{f(x)}{f'(x)} \tag{264}$$

$$\frac{dx}{d\chi}|_{x=x^*} = \frac{1}{\sigma\chi}\left[\frac{1+\psi+(\sigma-1)\alpha}{\sigma(1-\alpha+\psi)}\right]^{-1}\frac{f(x^*)}{f'(x^*)} > 0. \tag{265}$$

$$\frac{dc}{d\chi} = \frac{d}{d\chi}\left[\chi^{\frac{1}{\chi}}p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}}\right] = \frac{1}{\sigma}\chi^{\frac{1}{\sigma}-1}p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}} - \frac{1}{\sigma}\chi^{\frac{1}{\sigma}}p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}-1}\gamma'(x)\frac{dx}{d\chi} \tag{266}$$

$$\frac{dc}{d\chi} = \frac{c}{\sigma\chi}\left[\frac{1}{\sigma}[\varphi^d_{\sigma=1}(x)]^{-1} + \left(1-\frac{1}{\sigma}\right)\right]^{-1}. \tag{267}$$

$$\frac{dc}{d\chi}|_{x=x^*} = \frac{c}{\sigma\chi}\frac{\sigma\alpha}{1+\psi+(\sigma-1)\alpha} > 0. \tag{268}$$

$$\frac{dy}{d\chi} = \frac{d}{d\chi}(c[1+\gamma(x)]) = \frac{dc}{dx}[1+\gamma(x)] + c\gamma'(x)\frac{dx}{d\chi} \tag{269}$$

$$\frac{dy}{d\chi}|_{x=x^*} = \frac{dc}{dx}|_{x=x^*}[1+\gamma(x^*)] + c\gamma'(x^*)\frac{dx}{d\chi}|_{x=x^*} > 0. \tag{270}$$

Similarly, differentiate (249) (at $G = \tau = 0$) with respect to a to find the comparative statics to a supply shock in a fixprice equilibrium:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} [pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}-1} \left[pf'(x)\frac{dx}{dx}a + pf(x)\right] = \left(1 - \frac{1}{\sigma}\right) \chi^{\frac{1}{\sigma}} (p[1+\gamma(x)])^{-\frac{1}{\sigma}} p\gamma'(x)\frac{dx}{da} \tag{271}$$

$$\frac{dx}{da} = -\frac{1}{a}\frac{1+\psi}{1-\alpha+\psi}\frac{f(x)}{f'(x)}\left[\frac{1+\psi+(\sigma-1)\alpha}{\sigma(1-\alpha+\psi)} + \left(1 - \frac{1}{\sigma}\theta_{\sigma=1}(x)\right)\right]^{-1} \tag{272}$$

$$\frac{dx}{da}|_{x=x^*} = -\frac{1}{a}\frac{1+\psi}{1-\alpha+\psi}\frac{f(x^*)}{f'(x^*)}\left[\frac{1+\psi+(\sigma-1)\alpha}{\sigma(1-\alpha+\psi)}\right]^{-1} < 0. \tag{273}$$

$$\frac{dc}{da} = \frac{d}{da}\left[\chi^{\frac{1}{\chi}} p^{-\frac{1}{\sigma}} [1+\gamma(x)]^{-\frac{1}{\sigma}}\right] = -\frac{1}{\sigma}\chi^{\frac{1}{\sigma}} p^{-\frac{1}{\sigma}} [1+\gamma(x)]^{-\frac{1}{\sigma}-1}\gamma'(x)\frac{dx}{da} \tag{274}$$

$$\frac{dc}{da} = \frac{c}{a}\frac{1+\psi}{\alpha}\frac{\frac{1}{\sigma}\varphi^s_{\sigma=1}(x)}{\frac{1}{\sigma} + \left(1-\frac{1}{\sigma}\right)\varphi^d_{\sigma=1}(x)} \tag{275}$$

$$\frac{dc}{da}|_{x=x^*} = \frac{c}{a}\left[1 + (\sigma-1)\frac{\alpha}{1+\psi}\right]^{-1} > 0. \tag{276}$$

$$\frac{dy}{da} = \frac{dc}{da}[1+\gamma(x)] + c\gamma'(x)\frac{dx}{da} \tag{277}$$

$$\frac{dy}{da} = (1-\sigma)\frac{dc}{da}[1+\gamma(x)] \tag{278}$$

$$\frac{dy}{da}|_{x=x^*} = (1-\sigma)\frac{dc}{da}_{x=x^*}[1+\gamma(x^*)]. \tag{279}$$

E.2 Demand-side fiscal multiplier

Differentiate (249) with respect to G (at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi}\left(\frac{dp}{dG}f(x)a + \frac{f'(x)}{f(x)}\frac{dx}{dG}pa\right)(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1} = \tag{280}$$

$$\chi^{\frac{1}{\sigma}}(p[1+\gamma(x)])^{-\frac{1}{\sigma}}(1-\frac{1}{\sigma})\left([1+\gamma(x)]\frac{dp}{dG} + p\gamma'(x)\frac{dx}{dG}\right) + p[1+\gamma(x)]. \tag{281}$$

In a competitive equilibrium $x = x^*$, so that $\frac{dx}{dG} = 0$; the demand-side multiplier is thus given by:

$$(\varphi^d_\sigma)^* = 1 + \frac{\partial c}{\partial p}\frac{dp}{dG} = \frac{\frac{1+\psi}{1-\alpha+\psi} - 1}{\frac{1+\psi}{1-\alpha+1} - (1-\frac{1}{\sigma})} = \frac{\frac{\alpha}{1+\psi}}{\frac{1}{\sigma} + (1-\frac{1}{\sigma})\frac{\alpha}{1+\psi}} = \left[\frac{1}{\sigma}\times\left(\frac{\alpha}{1+\psi}\right)^{-1} + \left(1-\frac{1}{\sigma}\right)\times 1^{-1}\right]^{-1}. \tag{282}$$

(283)

or just a weighted harmonic average between $\frac{\alpha}{1+\psi}$ and one, with weights given by $\left(\frac{1}{\sigma}, 1-\frac{1}{\sigma}\right)$. Hence, in a competitive equilibrium the demand-side multiplier remains acyclical under a general CRRA utility of consumption.

In a fixprice equilibrium $p = p_0$, so that $\frac{dp}{dG} = 0$; the demand-side multiplier is thus given by:

$$\varphi_\sigma^d(x) = 1 + \frac{\partial c}{\partial x}\frac{dx}{dG} = \frac{\frac{1+\psi}{1-\alpha+\psi} - \frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}{\frac{1+\psi}{1-\alpha+\psi} - \left(1-\frac{1}{\sigma}\right)\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}} = \frac{\varphi_{\sigma=1}^d(x)}{\frac{1}{\sigma} + \left(1-\frac{1}{\sigma}\right)\varphi_{\sigma=1}^d(x)}$$
$$= \left[\frac{1}{\sigma} \times \left\{\varphi_{\sigma=1}^d(x)\right\}^{-1} + \left(1-\frac{1}{\sigma}\right) \times 1^{-1}\right]^{-1} \quad (284)$$

or just a weighted harmonic average between $\varphi_{\sigma=1}^d(x)$, which is the demand-side multiplier under $\sigma = 1$ considered in the main text, and one, with weights given by $\left(\frac{1}{\sigma}, 1-\frac{1}{\sigma}\right)$. Hence, $\varphi_\sigma^d(0) = 1$ and $\frac{d\varphi_\sigma^d(x)}{dx} = \frac{\frac{1}{\sigma}\frac{d\varphi_{\sigma=1}^d(x)}{dx}}{\left[\frac{1}{\sigma}+\left(1-\frac{1}{\sigma}\right)\varphi_{\sigma=1}^d(x)\right]^2} < 0, \quad \forall x \in (0, x_m)$, so that $\frac{d\varphi_\sigma^d(x)}{d\chi}|_{x=x^*} = \frac{d\varphi_\sigma^d(x)}{dx}|_{x=x^*}\frac{dx}{d\chi}|_{x=x^*} < 0$ and $\frac{d\varphi_\sigma^d(x)}{da}|_{x=x^*} = \frac{d\varphi_\sigma^d(x)}{dx}|_{x=x^*}\frac{dx}{da}|_{x=x^*} > 0$, establishing that the cyclicality properties of the demand-side multiplier found in the main text are preserved under a general CRRA utility of consumption in the local neighborhood of the efficient allocation.

E.3 Supply-side fiscal multiplier

Differentiate (249) with respect to τ (at $G = \tau = 0$):

$$\left[\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)} - \left(1-\frac{1}{\sigma}\right)\frac{\gamma'(x)}{1+\gamma(x)}\right]\frac{dx}{d\tau} + \left[\frac{1+\psi}{1-\alpha+\psi}\frac{1}{p} - \left(1-\frac{1}{\sigma}\right)\frac{1}{p}\right]\frac{dp}{d\tau} = \frac{\alpha}{1-\alpha+\psi}. \quad (285)$$

In a competitive equilibrium $x = x^*$, so that $\frac{dx}{d\tau} = 0$; the supply-side multiplier is thus given by:

$$(\varphi_\sigma^s)^* = -\frac{1}{c}\frac{\partial c}{\partial p}\frac{dp}{d\tau} = \frac{\frac{1}{\sigma}\frac{\alpha}{1-\alpha+\psi}}{\frac{1+\psi}{1-\alpha+\psi} - \left(1-\frac{1}{\sigma}\right)} = \frac{\frac{1}{\sigma}\frac{\alpha}{1+\psi}}{\frac{1}{\sigma} + \left(1-\frac{1}{\sigma}\right)\frac{\alpha}{1+\psi}} = \frac{1}{\sigma}(\varphi^d)^*, \quad (286)$$

so that in a competitive equilibrium the supply-side multiplier remains acyclical under a general CRRA utility of consumption.

In a fixprice equilibrium $p = p_0$, so that $\frac{dp}{d\tau} = 0$; the supply-side multiplier is thus given by:

$$\varphi_\sigma^s(x) = -\frac{1}{c}\frac{\partial c}{\partial x}\frac{dx}{d\tau} = \frac{\frac{1}{\sigma}\frac{\alpha}{1-\alpha+\psi}\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}{\frac{1+\psi}{1-\alpha+\psi} - \left(1-\frac{1}{\sigma}\right)\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}} = \frac{\frac{1}{\sigma}\varphi_{\sigma=1}^s(x)}{\frac{1}{\sigma} + \left(1-\frac{1}{\sigma}\right)\varphi_{\sigma=1}^d(x)}, \quad (287)$$

where $\varphi_{\sigma=1}^s(x)$ is the supply-side multiplier under $\sigma = 1$ considered in the main text. Hence, $\varphi_\sigma^s(0) = 0$ and $\frac{d\varphi_\sigma^s(x)}{dx} = -\frac{\frac{1}{\sigma}\frac{\alpha}{1+\psi}\theta'(x)}{\left[\frac{1}{\sigma}+\left(1-\frac{1}{\sigma}\right)\varphi_{\sigma=1}^d(x)\right]^2} > 0, \quad \forall x \in (0, x_m)$, so that $\frac{d\varphi_\sigma^s(x)}{d\chi}|_{x=x^*} = \frac{d\varphi_\sigma^s(x)}{dx}|_{x=x^*}\frac{dx}{d\chi}|_{x=x^*} > 0$ and $\frac{d\varphi_\sigma^s(x)}{da}|_{x=x^*} = \frac{d\varphi_\sigma^s(x)}{dx}|_{x=x^*}\frac{dx}{da}|_{x=x^*} < 0$, establishing that the cyclicality properties of the supply-side multiplier found in the main text are preserved under a general CRRA utility of consumption in the local neighborhood of the efficient allocation.

F Results under utility cost per visit

F.1 Household optimization

In this version of the model, the setup is remains unchanged compared to the baseline case in main text, except households now face a *utility* cost $\iota > 0$ per visit; given that every visit is successful with probability $q(x)$, the total number of visits required to purchase c units of the produced good is $v = c/q(x)$, and the total utility cost of search is $\iota \times v = \iota \times c/q(x)$. Households' optimization problem is now given by:

$$\max_{c,m,l} \left[\chi \frac{c^{1-\sigma}}{1-\sigma} + \zeta(m) - \frac{l^{1+\psi}}{1+\psi} - \iota \frac{c}{q(x)} \right] \qquad \text{s.t.} \tag{288}$$

$$pc + m \leq wl + \bar{m} + \Pi - T. \tag{289}$$

As before, here we normalize $\bar{m}$ so that $\zeta'(\bar{m}) = 1$, and focus on the special case of log utility of consumption ($\sigma = 1$). The solution to the above problem delivers a labor supply function identical to the one in the baseline model; however, the consumption function now takes a different form:

$$c(p, x) = \frac{\chi}{p + \varsigma(x)}, \tag{290}$$

where $\varsigma(x) \equiv \frac{\iota x}{f(x)} = \frac{\iota}{q(x)} > 0, \quad \varsigma'(x) > 0, \forall x \in (0, +\infty)$ summarizes the total cost of search, which is now additive to the price, and strictly increases in tightness on the whole domain. It thus follows that $\frac{\partial c}{\partial p} = -\frac{\chi}{[p+\varsigma(x)]^2} < 0$ and $\frac{\partial c}{\partial x} = -\frac{\chi\varsigma'(x)}{[p+\varsigma(x)]^2} < 0$.

The firms' problem remains unchanged, hence labor market equilibrium remains unaffected; goods market clearing can now be written as:

$$f(x)an^{\alpha} = c(p, x) + G \tag{291}$$

$$f(x)a\left[\alpha p f(x) a\right]^{\frac{\alpha}{1-\alpha+\psi}} (1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \frac{\chi}{p+\varsigma(x)} + G \tag{292}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \left[p f(x) a\right]^{\frac{1+\psi}{1-\alpha+\psi}} [p + \varsigma(x)] = p\chi + pG[p + \varsigma(x)]. \tag{293}$$

Given the evidence that fixprice equilibrium is more empirically relevant at business cycle frequencies, we will continue our analysis in this section under the assumption of fixprice equilibrium, so that $p = p_0$ is a parameter.

F.2 Comparative statics

Differentiate (293) with respect to χ (at $G = \tau = 0$) in order to find comparative statics after a demand shock:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \left[\frac{1+\psi}{1-\alpha+\psi} (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1} pf'(x) \frac{dx}{d\chi} a[p+\varsigma(x)] + (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}} \varsigma'(x) \frac{dx}{d\chi} \right] = p \tag{294}$$

$$\frac{dx}{d\chi} = \frac{1}{\chi} \left[\frac{1+\psi}{1-\alpha+\psi} \frac{f'(x)}{f(x)} + \frac{\varsigma'(x)}{p+\varsigma(x)} \right]^{-1} > 0. \tag{295}$$

$$\frac{dc}{d\chi} = \frac{1}{p+\varsigma(x)} - \frac{\chi\varsigma'(x)}{[p+\varsigma(x)]^2} \frac{dx}{d\chi} \tag{296}$$

$$\frac{dc}{d\chi} = \frac{\frac{1+\psi}{1-\alpha+\psi}}{\left[\frac{1+\psi}{1-\alpha+\psi} + \frac{\varsigma'(x)}{p+\varsigma(x)}\right][p+\varsigma(x)]} > 0. \tag{297}$$

Similarly, differentiate (293) with respect to a (at $G = \tau = 0$) in order to find comparative statics after a supply shock:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \left[\frac{1+\psi}{1-\alpha+\psi} (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1} (pf'(x)\frac{dx}{da}a + pf(x))[p+\varsigma(x)] + (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}} \varsigma'(x)\frac{dx}{da} \right] = 0 \quad (298)$$

$$\frac{dx}{da} = -\frac{1+\psi}{1-\alpha+\psi}\frac{1}{a}\left[\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)} + \frac{\varsigma'(x)}{p+\varsigma(x)}\right]^{-1} < 0. \quad (299)$$

$$\frac{dc}{da} = -\frac{\chi\varsigma'(x)}{[p+\varsigma(x)]^2}\frac{dx}{da} \quad (300)$$

$$\frac{dc}{da} = \frac{\frac{\chi\varsigma'(x)}{[p+\varsigma(x)]^2}\frac{1+\psi}{1-\alpha+\psi}\frac{1}{a}}{\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)} + \frac{\varsigma'(x)}{p+\varsigma(x)}} > 0. \quad (301)$$

F.3 Demand-side fiscal multiplier

Differentiating (293) with respect to G (at $G = \tau = 0$) delivers the following:

$$\frac{dx}{dG} = \frac{1}{c}\left[\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)}\frac{\varsigma'(x)}{p+\varsigma(x)}\right]^{-1}. \quad (302)$$

From the definition of the demand-side fiscal multiplier:

$$\varphi^d(x) = 1 + \frac{\partial c}{\partial x}\frac{dx}{dG} \quad (303)$$

$$= \left[1 + \frac{1-\alpha+\psi}{1+\psi}\frac{\varsigma'(x)}{p+\varsigma(x)}\frac{f(x)}{f'(x)}\right]^{-1}. \quad (304)$$

After some algebra it can be shown that:

$$\frac{d\varphi^d(x)}{dx} = \frac{\iota\frac{1-\alpha+\psi}{1+\psi}q(x)^\delta q'(x)(\delta[p+\varsigma(x)] + p(1-q(x)^\delta))}{[[p+\varsigma(x)]q(x)^{1+\delta} + \frac{1-\alpha+\psi}{1+\psi}\iota(1-q(x)^\delta)]^2} < 0, \quad (305)$$

since $\varsigma(x) > 0, q(x) \in (0,1), q'(x) < 0, \forall x \in (0,+\infty)$ and $\delta > 0$. Hence, $\frac{\varphi^d(x)}{d\chi} = \frac{d\varphi^d(x)}{dx}\frac{dx}{d\chi} < 0$ and $\frac{d\varphi^d(x)}{da} = \frac{d\varphi^d(x)}{dx}\frac{dx}{da} > 0$, so the cyclicality properties of the demand-side fiscal multiplier found in the main text are preserved.

F.4 Supply-side fiscal multiplier

Differentiating (293) with respect to τ (at $G = \tau = 0$) delivers the following:

$$\frac{dx}{d\tau} = \frac{\alpha}{1-\alpha+\psi}\left[\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)} + \frac{\varsigma'(x)}{p+\varsigma(x)}\right]^{-1}. \quad (306)$$

From the definition of the supply-side fiscal multiplier:

$$\varphi^s(x) = -\frac{1}{c}\frac{\partial c}{\partial x}\frac{dx}{d\tau} \tag{307}$$

$$= \frac{\alpha}{1+\psi}\varsigma'(x)f(x)\left[1+\frac{1-\alpha+\psi}{1+\psi}\frac{\varsigma'(x)}{p+\varsigma(x)}\frac{f(x)}{f'(x)}\right]^{-1}. \tag{308}$$

After some algebra it can be shown that:

$$\frac{\varphi^s(x)}{dx} = -\frac{\iota\frac{\alpha}{1+\psi}q(x)^{\delta}q'(x)(\delta(p+\varsigma(x))+p(1-q(x)^{\delta}))}{[[p+\varsigma(x)]q(x)^{1+\delta}+\frac{1-\alpha+\psi}{1+\psi}\iota(1-q(x)^{\delta})]^2} > 0, \tag{309}$$

since $\varsigma(x) > 0, q(x) \in (0,1), q'(x) < 0, \forall x \in (0,+\infty)$ and $\delta > 0$. Hence, $\frac{\varphi^s(x)}{d\chi} = \frac{d\varphi^s(x)}{dx}\frac{dx}{d\chi} > 0$ and $\frac{d\varphi^s(x)}{da} = \frac{d\varphi^s(x)}{dx}\frac{dx}{da} < 0$, so the cyclicality properties of the supply-side fiscal multiplier found in the main text are preserved.

G Social planner's allocation

G.1 Static model

The social planner's problem is given by:

$$\max_{c,l,v,m}\left[\chi\frac{c^{1-\sigma}}{1-\sigma}+\zeta(m)-\frac{l^{1+\psi}}{1+\psi}\right] \qquad \text{s.t.} \tag{310}$$

$$c+G+\rho v = \left[(al^{\alpha})^{-\delta}+v^{-\delta}\right]^{-\frac{1}{\delta}}, \qquad m=\bar{m}. \tag{311}$$

Inserting $m=\bar{m}$, the associated Lagrangian becomes:

$$\mathcal{L} = \left[\chi\frac{c^{1-\sigma}}{1-\sigma}+\zeta(\bar{m})-\frac{l^{1+\psi}}{1+\psi}\right]+\lambda\left(\left[(al^{\alpha})^{-\delta}+v^{-\delta}\right]^{-\frac{1}{\delta}}-(c+G+\rho v)\right) \tag{312}$$

The first-oder conditions are:

$$\frac{\partial\mathcal{L}}{\partial c} = \chi c^{-\delta}-\lambda = 0 \tag{313}$$

$$\frac{\partial\mathcal{L}}{\partial v} = \lambda\left(-\frac{1}{\delta}\left[(al^{\alpha})^{-\delta}+v^{-\delta}\right]^{-\frac{1}{\delta}-1}(-\delta)v^{-\delta-1}-\rho\right) = 0 \tag{314}$$

$$\frac{\partial\mathcal{L}}{\partial v} = -l^{\psi}+\lambda\left(-\frac{1}{\delta}\left[(al^{\alpha})^{-\delta}+v^{-\delta}\right]^{-\frac{1}{\delta}-1}(-\delta)\alpha a l^{\alpha-1}\right) = 0 \tag{315}$$

Note that $x \equiv \frac{v}{al^{\alpha}}$ and $f'(x) = (1 + x^{\delta})^{-\frac{1}{\delta}-1}$, the social planner's allocation $\{c^*, l^*, v^*, m^*, x^*\}$ is given by:

$$f'(x^*) = \rho \tag{316}$$

$$x^* = \frac{v^*}{a(l^*)^{\alpha}} \tag{317}$$

$$\chi(c^*)^{-\sigma} = \frac{\left[(a(l^*)^{\alpha})^{-\delta} + (v^*)^{-\delta}\right]^{-\frac{1}{\delta}-1} \alpha a(l^*)^{\alpha-1}}{(l^*)^{\psi}} \tag{318}$$

$$c^* + G + \rho v^* = \left[(a(l^*)^{\alpha})^{-\delta} + (v^*)^{-\delta}\right]^{-\frac{1}{\delta}} \tag{319}$$

$$m^* = \bar{m}. \tag{320}$$

G.2 Dynamic model

The social planner's problem is given by:

$$\max_{\{c_{t+s}, l_{t+s}, m_{t+s}, v_{t+s}, y_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\chi_{t+s} \frac{c_{t+s}^{1-\sigma}}{1-\sigma} + \zeta(m_{t+s}) - \nu \frac{l_{t+s}^{1+\psi}}{1+\psi}\right] \qquad \text{s.t.} \tag{321}$$

$$y_t = (1-\eta)y_{t-1} + \left[v_t^{-\delta} + (a_t l_t^{\alpha} - (1-\eta)y_{t-1})^{-\delta}\right]^{-\frac{1}{\delta}}, \qquad \forall t \geq 0 \tag{322}$$

$$y_t = c_t + G_t + \rho v_t, \qquad m_t = \bar{m}, \qquad \forall t \geq 0. \tag{323}$$

The associated Lagrangian is given by:

$$\mathcal{L}_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s [\chi_{t+s} \frac{c_{t+s}^{1-\sigma}}{1-\sigma} + \zeta(\bar{m}) - \frac{l_{t+s}^{1+\psi}}{1+\psi} + \lambda_{t+s}\left(y_{t+s} - (1-\eta)y_{t+s-1} - \left[v_{t+s}^{-\delta} + (a_{t+s} l_{t+s}^{\alpha} - (1-\eta)y_{t+s-1})^{-\delta}\right]^{-\frac{1}{\delta}}\right)$$
$$+\mu_{t+s}(y_{t+s} - c_{t+s} - G_{t+s} - \rho v_{t+s})]. \tag{324}$$

The first order conditions are given by:

$$\lambda_t + \frac{\nu l_t^{\psi}}{\left[x_t^{-\delta} + 1\right]^{-\frac{1}{\delta}-1} \alpha a_t l_t^{\alpha-1}} = 0, \qquad \forall t \geq 0 \tag{325}$$

$$\lambda_t [1 + x_t^{\delta}]^{-\frac{1}{\delta}-1} + \rho \chi_t c_t^{-\sigma} = 0, \qquad \forall t \geq 0 \tag{326}$$

$$\lambda_t + \chi_t c_t^{-\sigma} + \beta(1-\eta)\mathbb{E}_t \left[\lambda_{t+1}\left((x_{t+1}^{-\delta} + 1)^{-\frac{1}{\delta}-1} - 1\right)\right] = 0, \qquad \forall t \geq 0 \tag{327}$$

which together with the definition of tightness $x_t = \frac{v_t}{a_t l_t^{\alpha} - (1-\eta)y_{t-1}}$ and the feasibility constraints describe the social planner's allocation.

H Dynamic model: further results and steady state

H.1 Alternative fiscal instruments in the dynamic model

In this subsection we consider two additional fiscal instruments in the context of the our dynamic model: distortionary taxation on consumption (τ_t^c) and households' labor income (τ_t^l). Compared to the baseline model in the main text, the representative household's per-period budget constraint becomes:

$$p_t(1+\tau_t^c)y_t^c + m_t + \mathbb{E}_t\left[F_{t,t+1}B_{t+1}\right] \le w_t(1-\tau_t^l)l_t + \bar{m}_t + B_t + \Pi_t - T_t, \qquad \forall t \ge 0, \tag{328}$$

and the first order conditions for the choice of consumption and labor supply become:

$$\chi_t c_t^{-\sigma} + \beta(1-\eta)\mathbb{E}_t\left[\chi_{t+1}c_{t+1}^{-\sigma}\frac{[1+\gamma(x_t)]}{[1+\gamma(x_{t+1})]}\gamma(x_{t+1})\right] = p_t(1+\tau_t^c)[1+\gamma(x_t)], \tag{329}$$

$$l_t = [w_t(1-\tau_t^l)/\nu]^{\frac{1}{\psi}}. \tag{330}$$

We further assume that the two additional tax rates follow exogenous autoregressive processes:

$$\tau_t^i = (1-\rho_\tau)\tau^i + \rho_\tau\tau_{t-1}^i + \varepsilon_t^{\tau^i}, \qquad \forall t \ge 0, \quad \tau_t^i \in \{\tau_t^l, \tau_t^c\}, \tag{331}$$

and the lump sum tax raised by the government is now given by:

$$T_t = p_t y_t^G - w_t n_t \tau_t - w_t n_t \tau_t^l - p_t y_t^c \tau_t^c. \tag{332}$$

Steady state consumption and labor income taxes ($\tau^c = 0.05, \tau^l = 0.28$) follow calibrations in Trabandt and Uhlig (2011) and Zanetti (2012). The rest of the model remains unchanged.

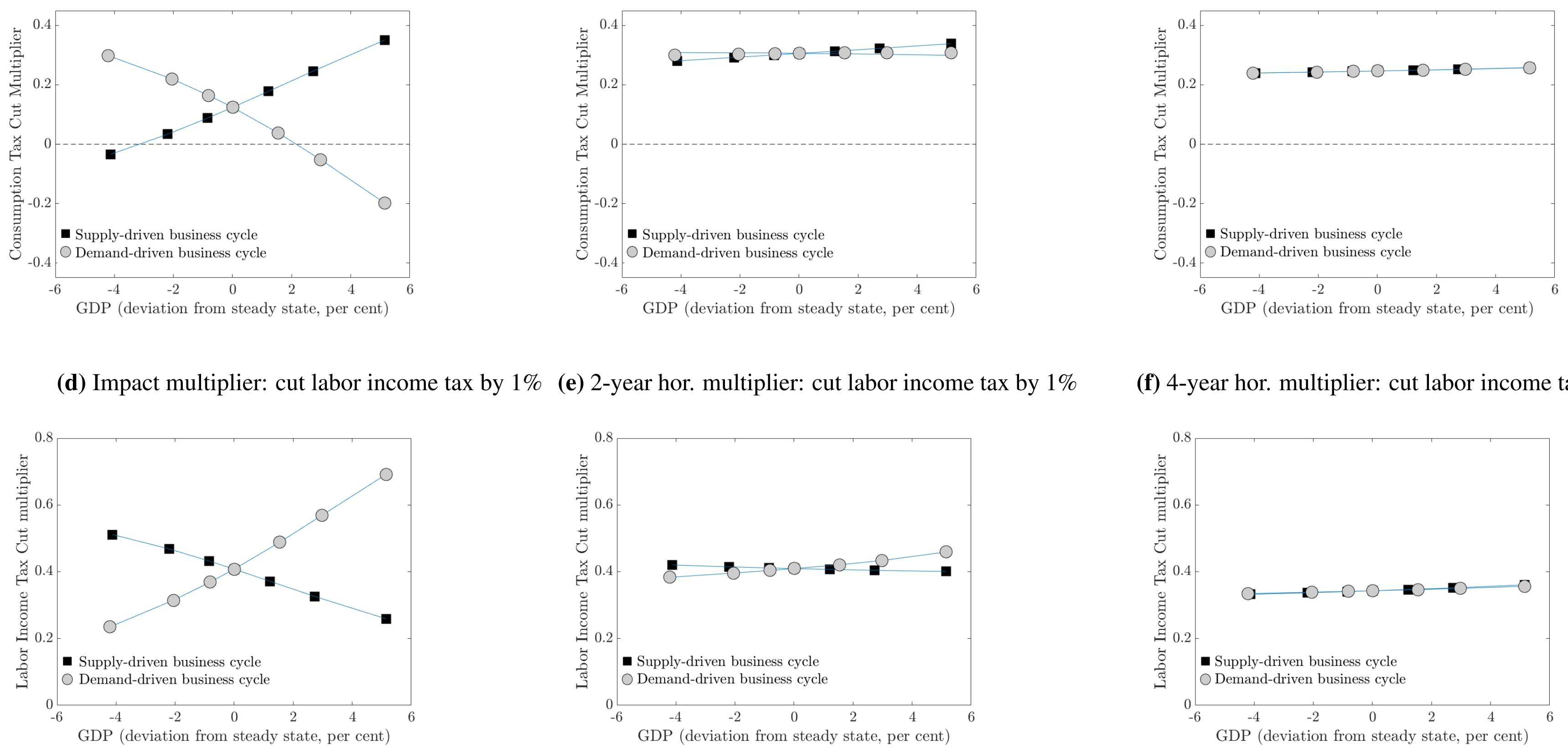
We compute horizon-specific conditional state-dependent multipliers out of cuts in taxes on consumption and labor income, following the methodology used for multipliers out of payroll taxes, as detailed in the main text. Figure 12 shows the results for impact, 2-year and 4-year horizon multipliers. As one can see, multipliers out of cuts in consumption taxes exhibit cyclicality properties that are similar to those of government consumption multipliers, as detailed in the main text; in particular, compared to the steady state, consumption tax cut multipliers rise in demand-side recessions and supply side recessions, but fall in demand-side expansions and supply-side recession, with the magnitude of state dependence falling at further horizons.

As for multipliers out of cuts in taxes on labor income, those have cyclicality properties identical to those of payroll tax cut multipliers. Indeed, compared to steady state, labor income tax cut multipliers are high in supply-side recession and demand-side expansions, whereas they are low in demand-side recessions and supply-side expansions. As before, the magnitude of state-dependence falls with the horizon considered.

Figure 12: Conditional state-dependent fiscal multipliers

(a) Impact multiplier: cut consumption tax by 1% **(b)** 2-year hor. multiplier: cut consumption tax by 1% **(c)** 4-year hor. multiplier: cut consumption tax by 1%

(d) Impact multiplier: cut labor income tax by 1% **(e)** 2-year hor. multiplier: cut labor income tax by 1% **(f)** 4-year hor. multiplier: cut labor income tax by 1%



Notes: *Panel (a) shows impact multipliers following a one-time innovation to the consumption tax rate process equal to negative 1 percentage point, in recessionary and expansionary episodes caused by different types of shocks; Panels (b) and (c) repeat the exercise for the 2-year horizon and the 4-year horizon tax cut multipliers, respectively.*
Panel (d) shows impact multipliers following a one-time innovation to the labor income tax rate process equal to negative 1 percentage point, in recessionary and expansionary episodes caused by different types of shocks; Panels (e) and (f) repeat the exercise for the 2-year horizon and the 4-year horizon tax cut multipliers, respectively.

H.2 Decentralized equilibrium: steady state

$$c^{-\sigma} = \frac{p(1+\tau^c)[1+\gamma(x)]}{1+\beta(1-\eta)\gamma(x)} \tag{333}$$

$$y^c = \frac{[1+\gamma(x)]c}{1+(1-\eta)\gamma(x)} \tag{334}$$

$$l^\psi = w(1-\tau^l)/\nu \tag{335}$$

$$y = \frac{f(x)l^\alpha}{1-(1-\eta)(1-f(x))} \tag{336}$$

$$p = \frac{w(1+\tau)}{\alpha f(x) l^{\alpha-1}}[1-(1-\eta)\beta(1-f(x))] \tag{337}$$

$$y^G = \frac{[1+\gamma(x)]g}{1+(1-\eta)\gamma(x)} \tag{338}$$

$$y = y^c + y^G \tag{339}$$

$$x = \frac{v}{l^\alpha - (1-\eta)y} \tag{340}$$

$$m = \bar{m} \tag{341}$$

$$\gamma(x) = \frac{\rho x}{f(x) - \rho x} \tag{342}$$

$$f(x) = (1+x^{-\delta})^{-\frac{1}{\delta}}. \tag{343}$$

H.3 Social planner's allocation: steady state

$$c^{-\sigma} = \frac{\nu l^{1+\psi-\alpha}}{\rho}\left[\frac{1+x^\delta}{1+x^{-\delta}}\right]^{-\frac{1}{\delta}-1} \tag{344}$$

$$y = \frac{1}{\eta}\left[v^{-\delta} + (l^\alpha - (1-\eta)y)^{-\delta}\right]^{-\frac{1}{\delta}} \tag{345}$$

$$\frac{1}{\rho}[1+x^\delta]^{-\frac{1}{\delta}-1} = 1+\beta(1-\eta)[(1+x^{-\delta})^{-\frac{1}{\delta}-1} - 1] \tag{346}$$

$$y = c + g + \rho v \tag{347}$$

$$x = \frac{v}{l^\alpha - (1-\eta)y} \tag{348}$$

$$m = \bar{m}. \tag{349}$$

I Econometric evidence: additional results and robustness checks

I.1 Responses to an identified productivity shock

In this subsection we would like to test whether following an identified productivity shock, output, tightness and inflation move in the same direction as predicted by our theory. We run the following sequence of local projections:

$$variable_{t+H} = \alpha_H + \beta_H \times a_t + \gamma_H \mathbf{z}_{t-1} + \varepsilon_{t+H}, \tag{350}$$

with $variable_t \in \left\{\frac{GDP_t}{GDP_t^*}, x_t, \pi_t\right\}$, where $\frac{GDP_t}{GDP_t^*}$ is real GDP over its polynomial trend, x_t is goods market tightness series from Michaillat and Saez (2015), π_t is inflation based on GDP deflator, and $a_t \equiv \ln TFP_t - \ln TFP_{t-1}$, where TFP is utilization-adjusted TFP series from Fernald (2014); the set of controls $\mathbf{z}_{t-1}$ is specified in the description to the results figure.

In Figure 13 one can see that consistently with our theory, following a positive productivity shock, one obtains a statistically significant increase in (cyclical) output, as well as a statistically significant reduction in inflation and goods market tightness.

I.2 Inflation responses to fiscal shocks

A bulk of variation in our spending and tax rate shocks overlaps with periods of price controls around World War II and the Korean War. In order to check that such price controls do not pose a challenge to our strategy of using co-movement between inflation and activity to identify sources of fluctuations, we estimate the response of inflation to our fiscal shocks:

$$\pi_{t+H} = \alpha_H + \beta_H \times shock_t + \gamma_H \mathbf{z}_{t-1} + \varepsilon_{t+H}, \tag{351}$$

where π_t is inflation based on GDP deflator, $shock_t$ is either military spending news shock from Ramey and Zubairy (2018) or tax rate shocks from Romer and Romer (2010); the set of controls $\mathbf{z}_{t-1}$ is specified in the description to the results figure.

As can be seen in Figure 14, both a positive spending shock and a positive shock to the tax rate produce a statistically significant increase in inflation. In this sense, even though the bulk of variation in our spending and tax rate shocks overlaps with periods of price controls, they produce a response in inflation that is consistent with our theory. The latter gives extra support to our strategy of using co-movement of inflation and activity to identify periods driven by either demand or supply shocks.

I.3 Demand-side and supply-side expansions

In Table 4 we repeat estimation of conditional state-dependent spending multipliers, but extend our baseline exercise by further splitting expansionary states, where $U_t < \bar{U}$, into those where inflation is above trend, $\pi_t \geq \bar{\pi}_t$, corresponding to demand-side expansions, and those where inflation is below trend, $\pi_t < \bar{\pi}_t$, corresponding to supply-side expansions. Consistently with our theory, we find the 2-year horizon cumulative spending multiplier in supply-side expansions (0.77) to be higher than in demand-side expansions (0.64); however, the 4-year horizon spending multiplier is very imprecisely estimated in supply-side expansions, making it hard to test our predictions. In Figure 15 we report conditional state-dependent spending multipliers at horizons ranging from 4 to 20 quarters; Panel (b) confirms our earlier finding: our prediction of higher multipliers in supply-side expansions finds confirmation only at shorter horizons, up to 8 quarters.

In Table 5 we lower the unemployment threshold down to $\bar{U} = 4.5\%$, so that our expansionary states, where $U_t < \bar{U}$ now pick up more severe overheating episodes, potentially making our identification sharper and helping test our theoretical predictions regarding spending multipliers in demand- and supply-side expansions. Once again, we find strong confirmation of our theory at the 2-year horizon: in supply-side expansions the multiplier is at 1.12, as opposed to 0.85 in demand-side expansions; at the 4-year horizon we still find supply-side expansion multipliers to be higher, although the demand-side expansion multiplier is very imprecisely estimated. Figure 16 confirms that most robust confirmation of our theory for expansions is indeed found at shorter horizons, up to 8 quarters.

Table 6 extends our analysis of conditional state-dependent tax cut multipliers to demand- and supply-side expansions. Our theory predicts that tax cut multipliers should be higher in demand-side recessions, and we find empirical support for this at the 4-year horizon, but not at the 2-year horizon; moreover Figure 17 shows that our prediction for expansions holds at longer horizons, above 10 quarters, but not at shorter ones. One reason behind this could be income effects associated with tax cuts that our model does not capture very well.

I.4 Blanchard and Perotti (2002) shocks

In Tables 7 and 8 as well as Figures 18 and 19 we repeat the conditional state-dependent spending multiplier estimation using VAR-based spending shocks following Blanchard and Perotti (2002), for both demand- and supply-side recessions and expansions. Overall, when we set $\bar{U} = 6.5\%$, we find confirmation to our theory for both expansions and recessions at shorter horizons, up to 5-quarters, whereas the results at longer horizons are less precise and deliver mixed evidence; when we set $\bar{U} = 4.5\%$, the results are consistent with our theoretical predictions across all horizons, but still most quantitatively significant at shorter horizons, up to 8 quarters. Therefore, our theoretical predictions for spending multiplier find most robust econometric confirmation at shorter cumulation horizons, regardless of whether one performs estimation with military spending news shocks or Blanchard-Perotti (2002) shocks; this horizon-dependence is in fact consistent with our dynamic simulations: at longer horizons more firms set prices optimally, less adjustment happens via tightness, state-dependence of multipliers is weaker and hence harder to detect econometrically.

I.5 Economic activity threshold based on detrended real GDP

Our baseline analysis uses unemployment as the measure of economic activity, which is done to be consistent with Ramey and Zubairy (2018). However, our theoretical model does not feature (involuntary) unemployment, and a measure of activity most consistent with our model is the cyclical component of real GDP. In this subsection we describe the results of performing estimation with an activity threshold based on detrended real GDP. [35]

In Table 9 we show results for 2- and 4-year horizon cumulative spending multipliers, where we define a recession as an episode where real GDP drops more than 3% below trend. Our classification of demand- and supply-driven recessions and expansions based on cyclical component of inflation remains unchanged. Consistently with our theory, we find that spending multipliers in demand-driven recessions are larger than spending multipliers in supply-driven recessions: 0.55 vs. 0.11 at the 2-year horizon, and 0.60 vs. 0.48 at the 4-year horizon. In panel (c) of Figure 20 we show that the pattern of higher multipliers in demand-driven recessions holds consistently across horizons, with the effect most pronounced at earlier horizons, again in line with our theory.

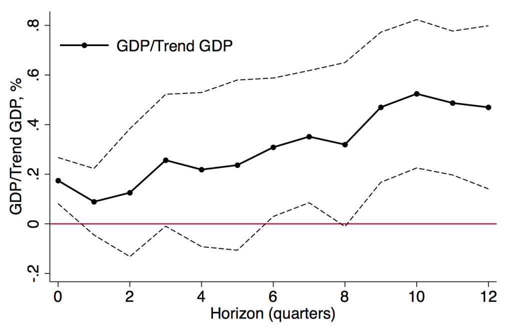
In Table 10 we change the threshold, so that only episodes where real GDP is more than 3% above trend counts as an expansion (and anything else counts as a recession). In this way we can focus on the most substantial episodes of overheating and have more power to test our theoretical predictions for expansions. Consistently with our theory we find that spending multipliers are higher in supply-driven expansions relative to demand-driven expansions: 0.68 vs 0.38 at the 2-year horizon and 0.69 vs 0.40 at the 4-year horizon. In panel (b) of Figure 21 we show that the pattern of higher multipliers in supply-driven expansions holds consistently across horizons, with the effect most pronounced at earlier horizons, again in line with our theory.

In Table 11 we again define a recession as an episode where real GDP drops for than 3% below trend, but this time estimate our specification for tax shocks. Further, panels (b) and (c) of Figure 22 exhibit estimation results for a broader set of horizons. At horizons beyond 8 quarters we find results consistent with our theory: tax cut multipliers are larger in demand-side expansions and supply-side recessions.

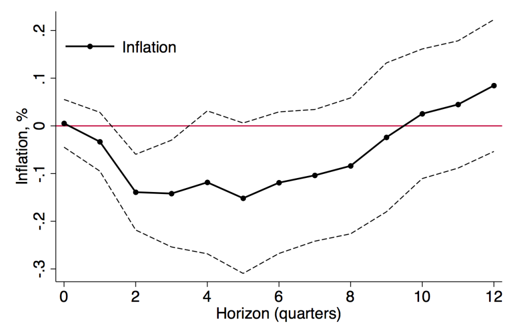
[35] We use the same polynomial trend as in Gordon and Krenn (2010)

Figure 13: Impulse response functions to a +1% productivity shock (utilization-adjusted shocks from Fernald (2014))

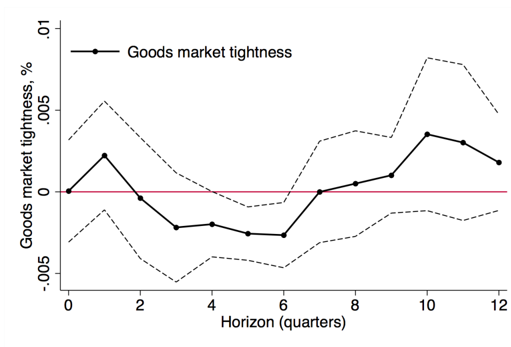
(a) Response of cyclical component of GDP



(b) Response of inflation (based on GDP deflator)



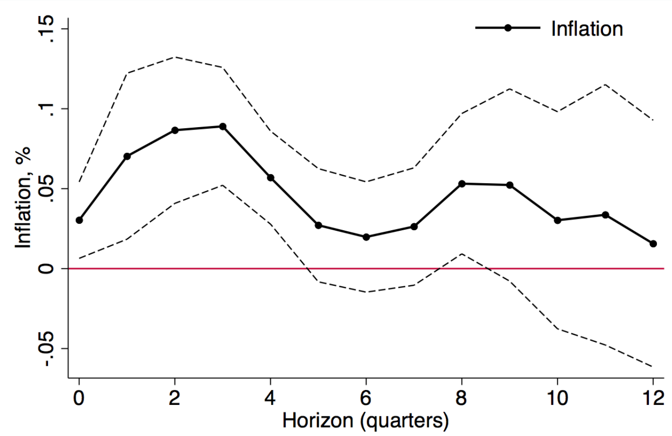
(c) Response of goods market tightness measure of Michaillat and Saez (2015)



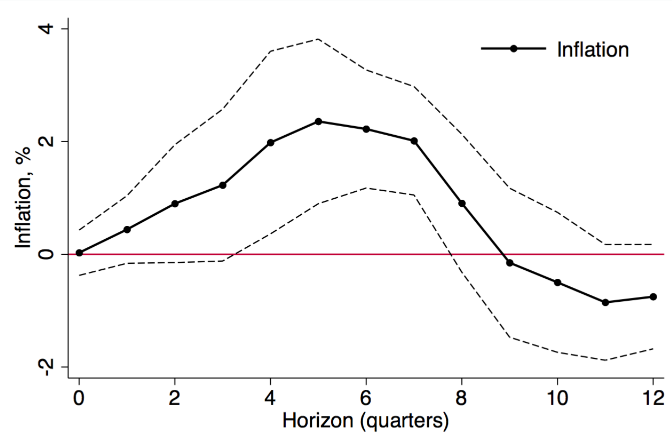
***Notes**: the figure presents estimation using local projections based on econometric specification outlined in Appendix I.1 over the sample period 1973:Q4-2013:Q2; the set of controls includes one lag of cyclical GDP, inflation and goods market tightness. The dotted lines represent 90% confidence bands based on standard errors robust to autocorrelation and heteroskedasticity.*

Figure 14: Impulse response functions of inflation to government spending and tax shocks

(a) Response to +1% military spending shock of Ramey and Zubairy (2018)



(b) Response to +1% tax rate shock of Romer and Romer (2010)



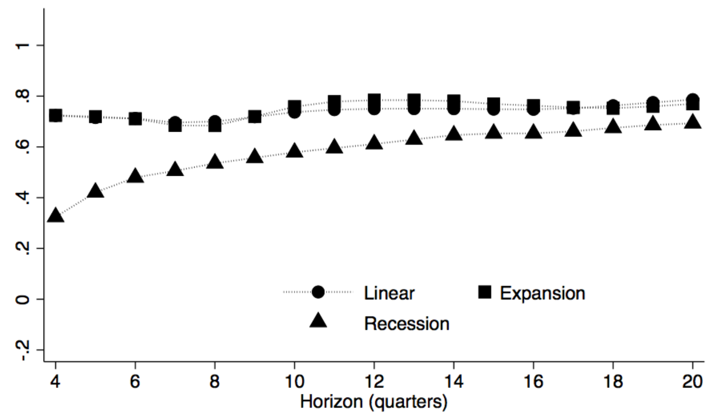
Notes: *the figure presents estimation using local projections based on econometric specification outlined in Appendix I.2 over the sample period 1909:Q1-2015:Q4 (panel (a)) and 1947:Q1-2007:Q4 (panel (b)) ; the set of controls includes four lags of cyclical GDP, inflation and spending shock (panel (a)) and four lags of cyclical GDP, inflation and average tax rate (panel (b)). The dotted lines represent 90% confidence bands based on standard errors robust to autocorrelation and heteroskedasticity.*

Table 4: Conditional state-dependent spending multipliers ($\bar{U} = 6.5\%$; US military spending news shocks)

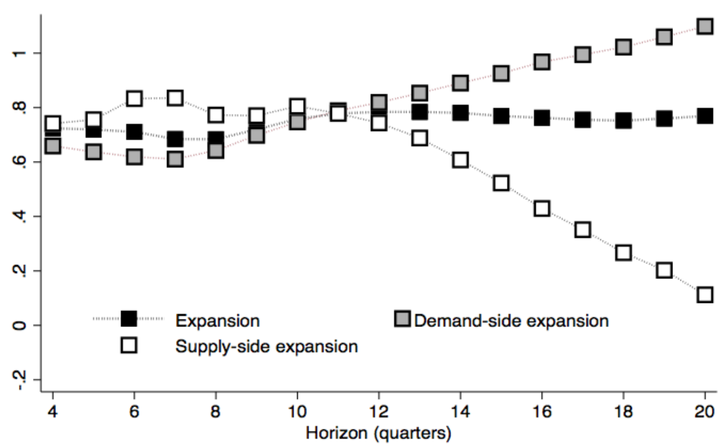
US data: 1909-2015			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.70***				0.75***			
	(0.06)				*(0.06)*			
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		0.68***	0.68***			0.76***	0.76***	
		(0.10)	*(0.09)*			*(0.13)*	*(0.12)*	
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		0.54***				0.65***		
		(0.13)				*(0.08)*		
β_H^{DE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t \geq \bar{\pi}_t\}$				0.64***				0.97***
				(0.06)				*(0.20)*
β_H^{SE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t < \bar{\pi}_t\}$				0.77***				0.43
				(0.29)				*(0.47)*
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			0.86***	0.86***			0.72***	0.72***
			(0.33)	*(0.33)*			*(0.12)*	*(0.12)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			0.32***	0.32***			0.63***	0.63***
			(0.11)	*(0.11)*			*(0.09)*	*(0.09)*
$\beta_H^E = \beta_H^R$ (p-value)		0.37				0.44		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.62				0.81	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.01				0.40	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.14	0.14			0.54	0.54
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.63				0.29
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure 15: Government Spending Multipliers across Horizons (US military spending news shocks, 1909-2015)

(a) Government spending multipliers in recessions and expansions across horizons ($\bar{U} = 6.5\%$)



(b) Government spending multipliers in demand-side and supply-side expansions across horizons



(c) Government spending multipliers in demand-side and supply-side recessions across horizons

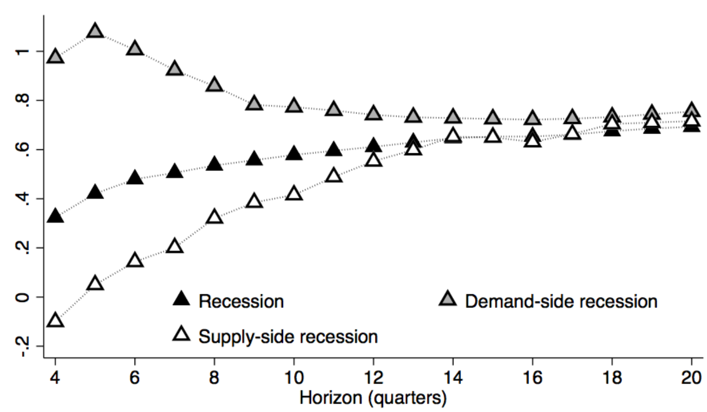
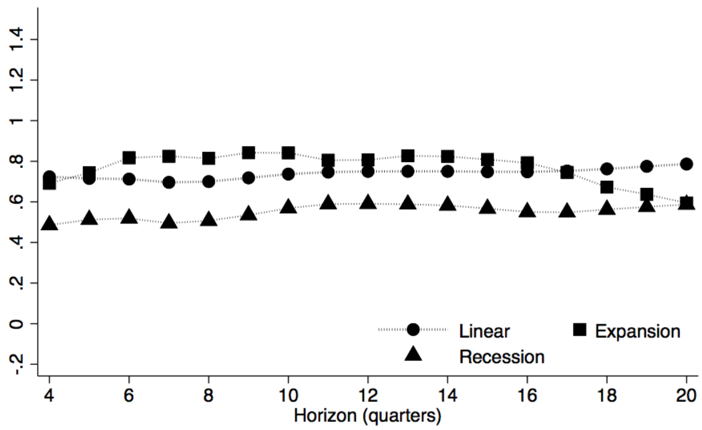


Table 5: Conditional state-dependent spending multipliers ($\bar{U} = 4.5\%$; US military spending news shocks)

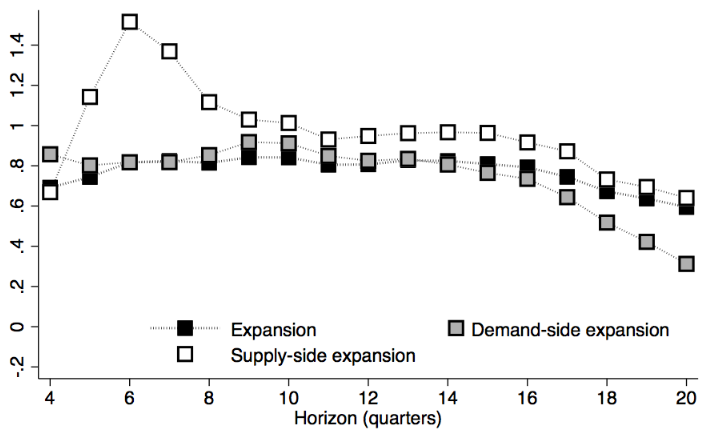
US data: 1909-2015			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.70***				0.75***			
	(0.06)				*(0.06)*			
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		0.81***	0.81***			0.79***	0.79***	
		(0.20)	*(0.20)*			*(0.28)*	*(0.30)*	
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		0.51***				0.55***		
		(0.12)				*(0.12)*		
β_H^{DE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t \geq \bar{\pi}_t\}$				0.85**				0.74
				(0.34)				*(0.66)*
β_H^{SE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t < \bar{\pi}_t\}$				1.12*				0.92*
				(0.61)				*(0.55)*
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			0.69**	0.69**			0.51**	0.51**
			(0.32)	*(0.32)*			*(0.23)*	*(0.25)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			0.31***	0.31***			0.55***	0.55***
			(0.09)	*(0.09)*			*(0.10)*	*(0.10)*
$\beta_H^E = \beta_H^R$ (p-value)		0.11				0.41		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.73				0.43	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.02				0.48	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.22	0.22			0.88	0.88
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.72				0.83
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure 16: Government Spending Multipliers across Horizons (US military spending news shocks, 1909-2015)

(a) Government spending multipliers in recessions and expansions across horizons ($\bar{U} = 4.5\%$)



(b) Government spending multipliers in demand-side and supply-side expansions across horizons



(c) Government spending multipliers in demand-side and supply-side recessions across horizons

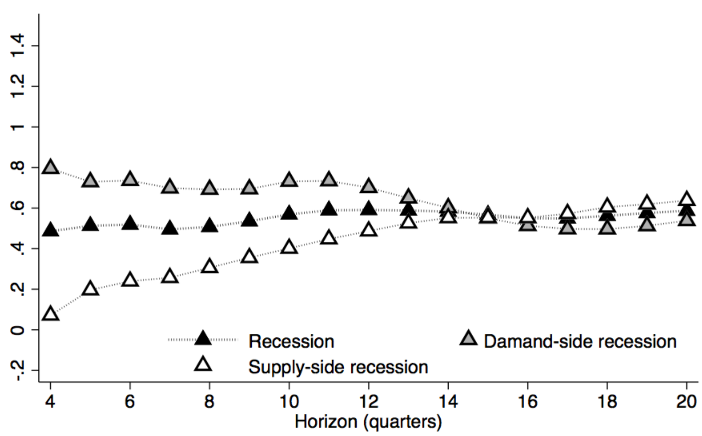
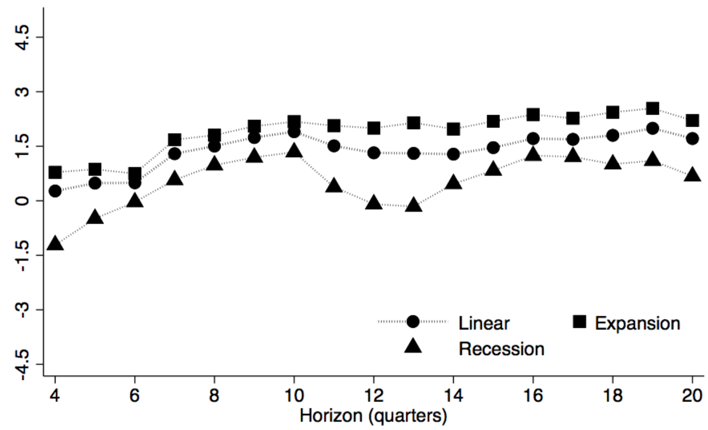


Table 6: Conditional state-dependent tax cut multipliers ($\bar{U} = 6.5\%$; US Romer-Romer narrative tax shocks)

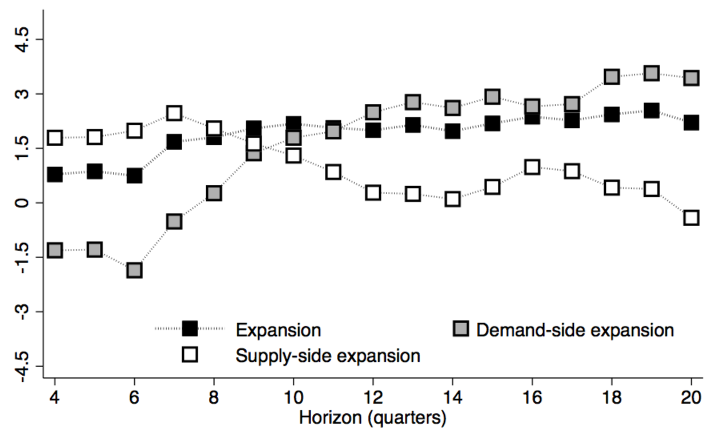
US data: 1947-2097			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	1.50				1.71**			
	(1.14)				*(0.82)*			
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		1.81	1.81			2.37**	2.37**	
		(1.17)	*(1.16)*			*(0.99)*	*(0.99)*	
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		0.98				1.24		
		(1.07)				*(0.87)*		
β_H^{DE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t \geq \bar{\pi}_t\}$				0.27				2.65*
				(1.08)				*(1.50)*
β_H^{SE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t < \bar{\pi}_t\}$				2.05				0.99
				(1.34)				*(1.89)*
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			1.49	1.49			-1.98	-1.98
			(1.04)	*(1.04)*			*(2.75)*	*(2.77)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			4.29*	4.29*			1.80*	1.80*
			(2.18)	*(2.18)*			*(1.00)*	*(1.00)*
$\beta_H^E = \beta_H^R$ (p-value)		0.48				0.39		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.84				0.12	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.28				0.70	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.25	0.25			0.20	0.20
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.32				0.51
Paap-Kleibergen LM-test								
T	240	240	240	240	240	240	240	240

Figure 17: Tax Cut Multipliers across Horizons (US Romer-Romer narrative tax shocks, 1947-2007)

(a) Tax cut multipliers in recessions and expansions across horizons ($\bar{U} = 6.5\%$)



(b) Tax cut multipliers in demand-side and supply-side expansions across horizons



(c) Tax cut multipliers in demand-side and supply-side recessions across horizons

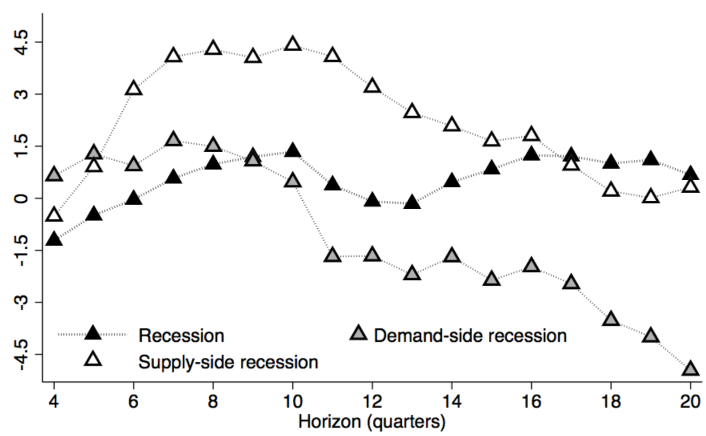
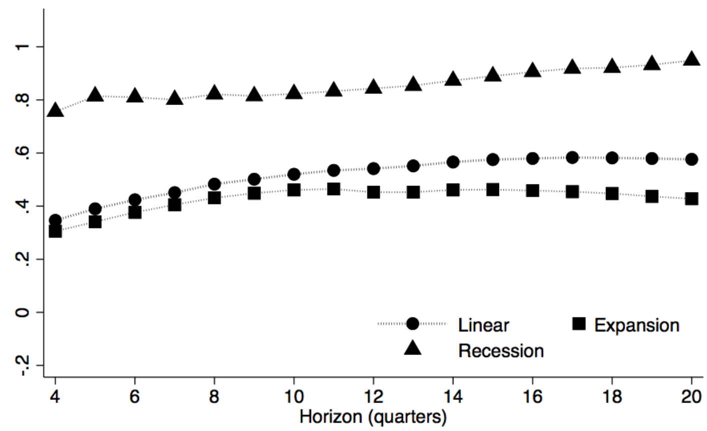


Table 7: Conditional state-dependent spending multipliers ($\bar{U} = 6.5\%$; US Blanchard-Perotti spending shocks)

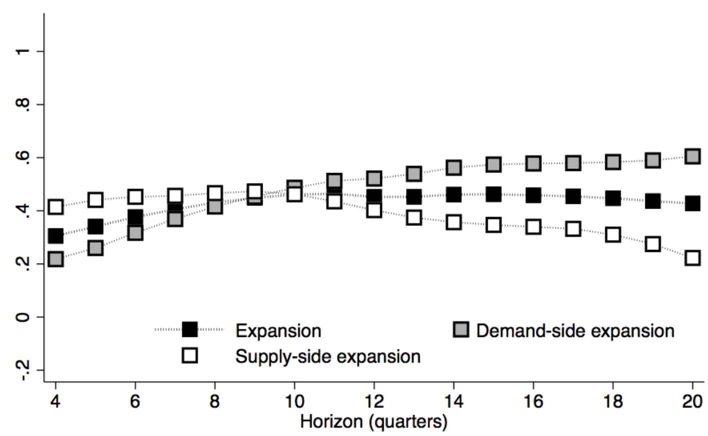
US data: 1909-2015		2y horizon				4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.48***				0.58***			
	(0.09)				*(0.10)*			
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		0.43***	0.42***			0.46***	0.44***	
		(0.08)	*(0.09)*			*(0.12)*	*(0.13)*	
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		0.82***				0.91***		
		(0.11)				*(0.07)*		
β_H^{DE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t \geq \bar{\pi}_t\}$				0.42***				0.58***
				(0.08)				*(0.14)*
β_H^{SE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t < \bar{\pi}_t\}$				0.47**				0.34
				(0.19)				*(0.25)*
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			0.54***	0.56***			0.62***	0.65***
			(0.16)	*(0.15)*			*(0.13)*	*(0.14)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			0.84***	0.84***			0.92***	0.91***
			(0.14)	*(0.14)*			*(0.21)*	*(0.21)*
$\beta_H^E = \beta_H^R$ (p-value)		0.00				0.00		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.50				0.36	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.01				0.05	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.19	0.22			0.34	0.37
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.81				0.35
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure 18: Government Spending Multipliers across Horizons (US Blanchard-Perotti spending shocks, 1909-2015)

(a) Government spending multipliers in recessions and expansions across horizons ($\bar{U} = 6.5\%$)



(b) Government spending multipliers in demand-side and supply-side expansions across horizons



(c) Government spending multipliers in demand-side and supply-side recessions across horizons

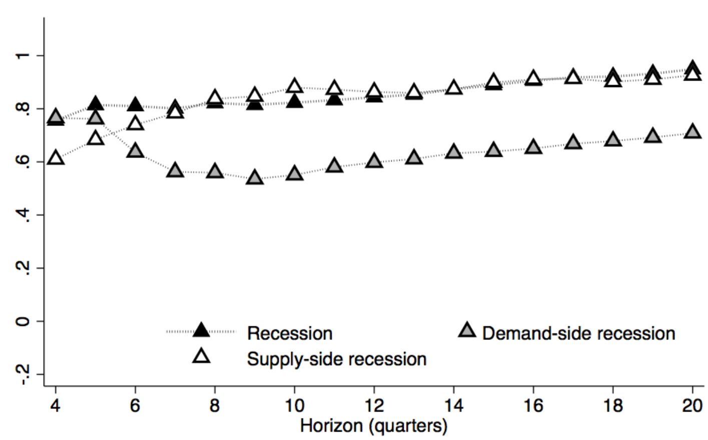
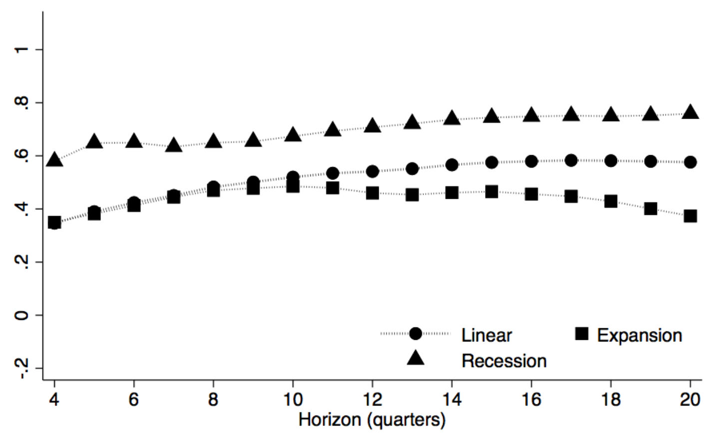


Table 8: Conditional state-dependent spending multipliers ($\bar{U} = 4.5\%$; US Blanchard-Perotti spending shocks)

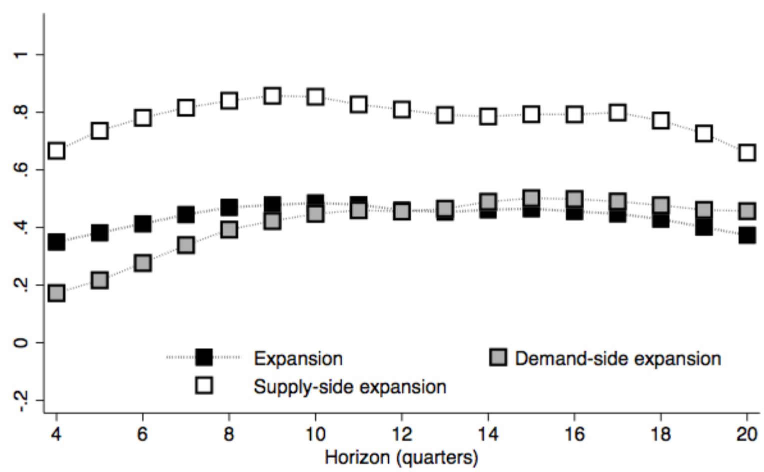
US data: 1889-2015			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.48***				0.58***			
	(0.09)				*(0.10)*			
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		0.47***	0.47***			0.46**	0.46**	
		(0.15)	*(0.15)*			*(0.18)*	*(0.18)*	
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		0.65***				0.75***		
		(0.10)				*(0.10)*		
β_H^{DE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t \geq \bar{\pi}_t\}$				0.39***				0.50***
				(0.09)				*(0.14)*
β_H^{SE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t < \bar{\pi}_t\}$				0.84***				0.79***
				(0.11)				*(0.26)*
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			0.75***	0.76***			0.74***	0.76***
			(0.16)	*(0.16)*			*(0.10)*	*(0.09)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			0.54***	0.51***			0.73***	0.70***
			(0.15)	*(0.15)*			*(0.12)*	*(0.12)*
$\beta_H^E = \beta_H^R$ (p-value)		0.25				0.08		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.22				0.12	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.70				0.09	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.38	0.33			0.99	0.68
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.00				0.31
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure 19: Government Spending Multipliers across Horizons (US Blanchard-Perotti spending shocks, 1909-2015)

(a) Spending multipliers in recessions and expansions across horizons ($\bar{U} = 4.5\%$)



(b) Spending multipliers in demand-side and supply-side expansions across horizons



(c) Spending multipliers in demand-side and supply-side recessions across horizons

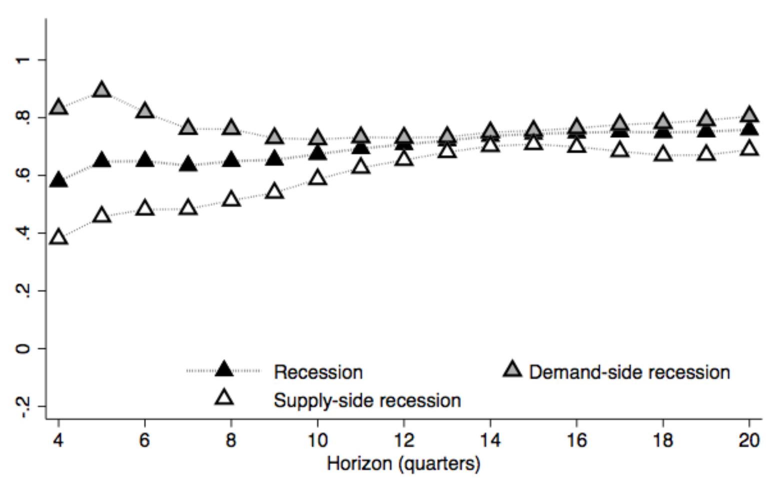
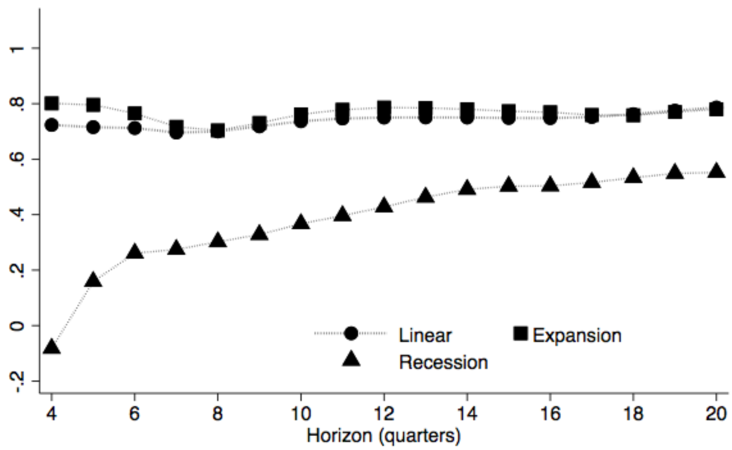


Table 9: Conditional state-dependent spending multipliers (cyclical GDP-based threshold $G\hat{D}P_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP; US military spending news shocks)

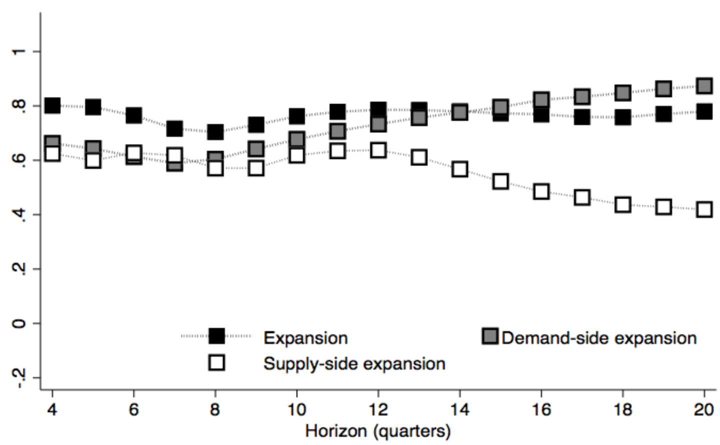
US data: 1909-2015	2y horizon				4y horizon			
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.70***				0.75***			
	(0.06)				*(0.06)*			
β_H^E : $\mathbf{1}\{G\hat{D}P_t \geq -3\%\}$		0.70***	0.70***			0.77***	0.77***	
		(0.11)	*(0.11)*			*(0.08)*	*(0.08)*	
β_H^R : $\mathbf{1}\{G\hat{D}P_t < -3\%\}$		0.30				0.50***		
		(0.24)				*(0.17)*		
β_H^{DE} : $\mathbf{1}\{G\hat{D}P_t \geq -3\%; \pi_t \geq \bar{\pi}_t\}$				0.60***				0.82***
				(0.07)				*(0.11)*
β_H^{SE} : $\mathbf{1}\{G\hat{D}P_t \geq -3\%; \pi_t < \bar{\pi}_t\}$				0.57***				0.49**
				(0.18)				*(0.19)*
β_H^{DR} : $\mathbf{1}\{G\hat{D}P_t < -3\%; \pi_t < \bar{\pi}_t\}$			0.55**	0.55**			0.60***	0.60***
			(0.25)	*(0.24)*			*(0.12)*	*(0.12)*
β_H^{SR} : $\mathbf{1}\{G\hat{D}P_t < -3\%; \pi_t \geq \bar{\pi}_t\}$			0.11	0.11			0.48***	0.48***
			(0.29)	*(0.31)*			*(0.15)*	*(0.15)*
$\beta_H^E = \beta_H^R$ (p-value)		0.13				0.20		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.56				0.28	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.06				0.13	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.26	0.28			0.48	0.48
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.85				0.12
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure 20: Government Spending Multipliers across Horizons (US military spending news shocks, 1909-2015, cyclical GDP-based threshold $\hat{GDP}_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP, threshold of -3%)

(a) Spending multipliers in recessions and expansions (recession if GDP more than 3% below trend)



(b) Spending multipliers in demand-side and supply-side expansions



(c) Spending multipliers in demand-side and supply-side recessions

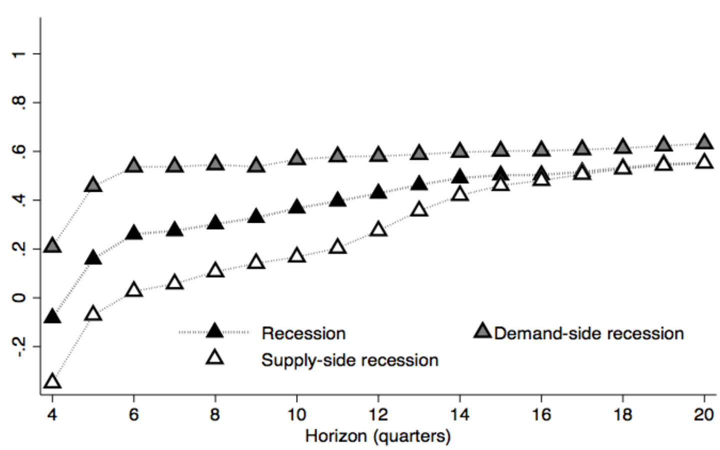
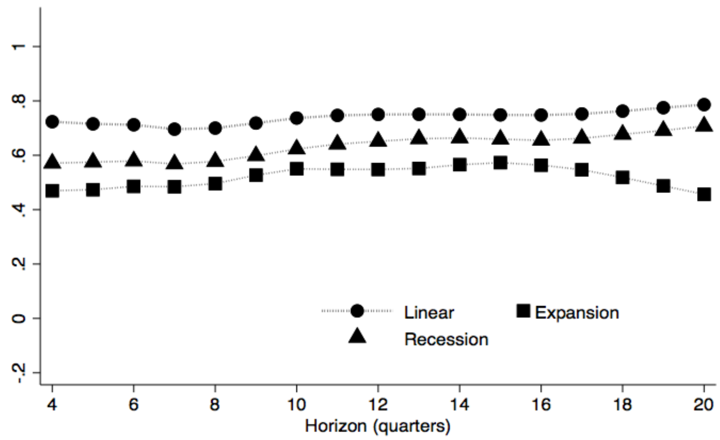


Table 10: Conditional state-dependent spending multipliers (cyclical GDP-based threshold $\hat{GDP}_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP; US military spending news shocks)

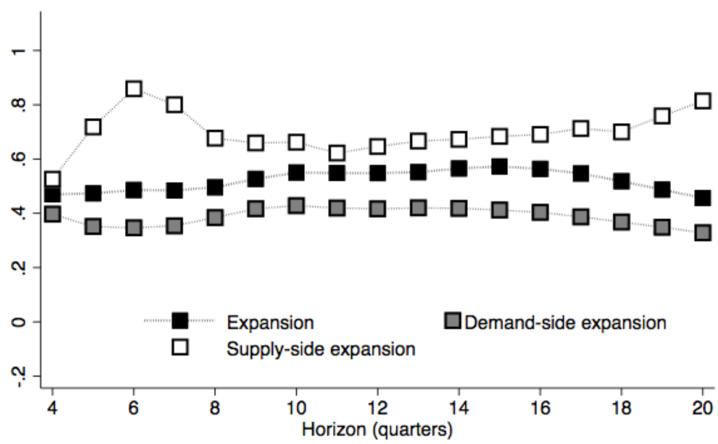
US data: 1909-2015			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.70***				0.75***			
	(0.06)				*(0.06)*			
β_H^E : $\mathbf{1}\{\hat{GDP}_t \geq 3\%\}$		0.50***	0.50***			0.56***	0.56***	
		(0.08)	*(0.08)*			*(0.09)*	*(0.10)*	
β_H^R : $\mathbf{1}\{\hat{GDP}_t < 3\%\}$		0.58***				0.66***		
		(0.07)				*(0.07)*		
β_H^{DE} : $\mathbf{1}\{\hat{GDP}_t \geq 3\%; \pi_t \geq \bar{\pi}_t\}$				0.38***				0.40***
				(0.08)				*(0.12)*
β_H^{SE} : $\mathbf{1}\{\hat{GDP}_t \geq 3\%; \pi_t < \bar{\pi}_t\}$				0.68**				0.69**
				(0.30)				*(0.32)*
β_H^{DR} : $\mathbf{1}\{\hat{GDP}_t < 3\%; \pi_t < \bar{\pi}_t\}$			0.33	0.33			0.32	0.32
			(0.28)	*(0.27)*			*(0.26)*	*(0.40)*
β_H^{SR} : $\mathbf{1}\{\hat{GDP}_t < 3\%; \pi_t \geq \bar{\pi}_t\}$			0.57***	0.57***			0.76***	0.76***
			(0.07)	*(0.06)*			*(0.12)*	*(0.10)*
$\beta_H^E = \beta_H^R$ (p-value)		0.42				0.48		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.54				0.33	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.50				0.28	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.43	0.39			0.11	0.28
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.36				0.41
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure 21: Government Spending Multipliers across Horizons (US military spending news shocks, 1909-2015, cyclical GDP-based threshold $\hat{GDP}_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP, threshold of 3%)

(a) Spending multipliers in recessions and expansions (expansion if GDP more than 3% above trend)



(b) Spending multipliers in demand-side and supply-side expansions



(c) Spending multipliers in demand-side and supply-side recessions

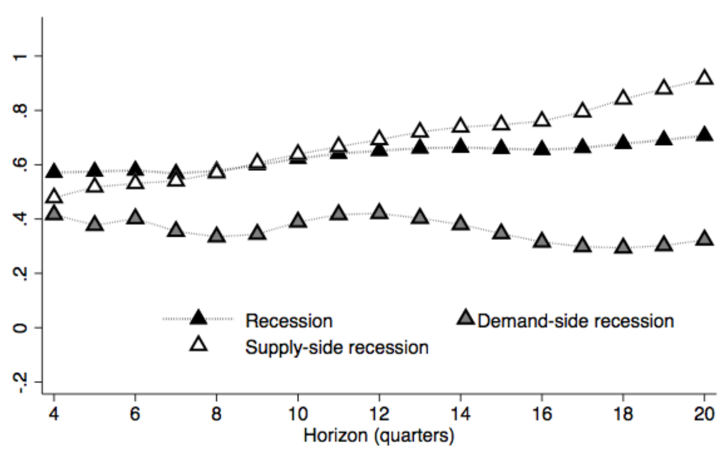
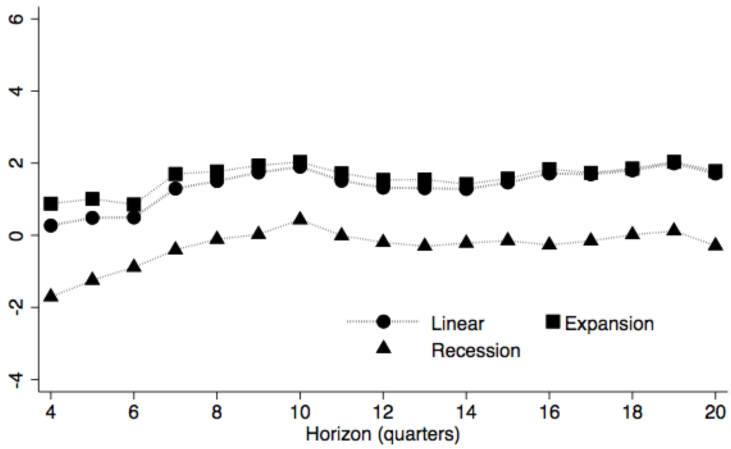


Table 11: Conditional state-dependent tax cut multipliers (cyclical GDP-based threshold $G\hat{D}P_t \equiv (GDP_t - \overline{GDP_t})/\overline{GDP_t}$ where $\overline{GDP_t}$ is trend GDP; US Romer-Romer narrative tax shocks)

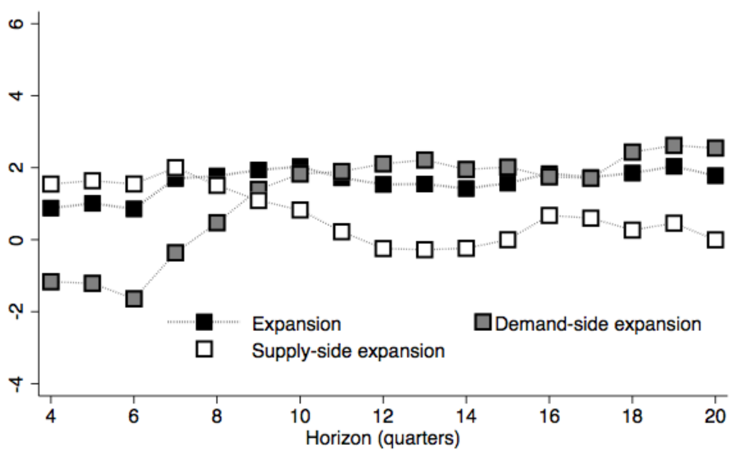
US data: 1909-2015			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	1.50				1.71**			
	(1.14)				*(0.82)*			
β_H^E : $\mathbf{1}\{G\hat{D}P_t \geq -3\%\}$		1.77*	1.77*			1.84*	1.84*	
		(1.01)	*(1.01)*			*(0.95)*	*(1.01)*	
β_H^R : $\mathbf{1}\{G\hat{D}P_t < -3\%\}$		-0.11				-0.27		
		(0.60)				*(0.45)*		
β_H^{DE} : $\mathbf{1}\{G\hat{D}P_t \geq -3\%; \pi_t \geq \bar{\pi}_t\}$				0.47				1.75
				(1.56)				*(1.25)*
β_H^{SE} : $\mathbf{1}\{G\hat{D}P_t \geq -3\%; \pi_t < \bar{\pi}_t\}$				1.51				0.67
				(1.16)				*(2.13)*
β_H^{DR} : $\mathbf{1}\{G\hat{D}P_t < -3\%; \pi_t < \bar{\pi}_t\}$			1.05	1.05			1.51	1.51
			(1.45)	*(1.45)*			*(1.35)*	*(1.38)*
β_H^{SR} : $\mathbf{1}\{G\hat{D}P_t < -3\%; \pi_t \geq \bar{\pi}_t\}$			0.81	0.81			2.27*	2.27*
			(0.89)	*(0.89)*			*(1.30)*	*(1.30)*
$\beta_H^E = \beta_H^R$ (p-value)		0.04				0.04		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.70				0.85	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.45				0.78	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.88	0.88			0.69	0.69
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.61				0.66
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure 22: Tax Cut Multipliers across Horizons (US Romer-Romer narrative tax shocks, 1947-2007, cyclical GDP-based threshold $G\hat{D}P_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP, threshold of -3%)

(a) Tax cut multipliers in recessions and expansions (recession if GDP more than 3% below trend)



(b) Tax cut multipliers in demand-side and supply-side expansions



(c) Tax cut multipliers in demand-side and supply-side recessions