Q-Monetary Transmission

Priit Jeenas*
Universitat Pompeu Fabra,
CREI and Barcelona GSE

Ricardo Lagos†
NYU

April 2021

Preliminary and incomplete. For the latest version click here

Abstract

We study the effects of monetary-policy-induced changes in Tobin's q on corporate investment decisions and capital structure. We develop a theory of this q-monetary transmission mechanism, provide identification and empirical evidence for the q-channel, evaluate the ability of the quantitative theory to match the evidence, and quantify the relevance for monetary transmission to aggregate investment.

Keywords: monetary transmission, asset prices, capital structure, investment, Tobin's q.
JEL classification: D83, E22, E44, E52, G12, G31, G32.

*Jeenas is thankful for the financial support from the Juan de la Cierva - Formación Grant (FJC2019-041561) by the Spanish Ministry of Science, Innovation, and Universities.

†Lagos is thankful for the support from the C.V. Starr Center for Applied Economics at NYU.

1 Introduction

The chain of causal links that lie between monetary policy actions and their ultimate effects on macroeconomic variables is broadly referred to as *the monetary transmission mechanism*. Since the immediate effect of these policy actions is to influence an array of interest rates and prices of financial and non-financial assets, it is difficult to imagine many economic decisions that would be unaffected by monetary policy. Consequently, textbook treatments of the effects of monetary policy contain extensive taxonomies of a myriad of transmission mechanisms.[1] The broadest classification typically consists of three main transmission channels: the (*direct* or *traditional*) *interest-rate channel*, the *asset-price channel*, and the *credit channel*.

The *interest-rate channel* is best described as a *user-cost channel*: Suppose there is an unexpected increase in the nominal policy rate, and that (as is usually the case) some of the increase passes through to real rates. Then, since the real rate is a key component of the user cost of capital, and the user cost of capital is a key determinant of the demand for capital (e.g., as in Jorgenson (1963)), investment should fall as a result of the monetary policy action.[2] The *asset-price channel* is best described as a *Tobin's q channel*: Suppose an unexpected decrease in the nominal policy rate causes stock prices to rise (as is well documented empirically, e.g., Bernanke and Kuttner (2005)) relative to the replacement cost of capital. Then, since the market yield of the stock is a key determinant of the cost of external financing in capital markets, equity-financed investment should increase as a result of the monetary policy action (e.g., as conjectured by Keynes (1936) and Tobin (1969)). The *credit channel*, which includes the well-known *balance-sheet channel*, is best described as an amplification mechanism associated to the other two channels: Suppose an unexpected increase in the nominal policy rate causes asset prices to fall (e.g., through either of the previous two channels), which in turn deteriorates borrowers' net worth. Then the resulting increase in external finance premia on debt (Bernanke and Gertler (1989)) or tightening of borrowing constraints (Kiyotaki and Moore (1997)) imply debt-financed investment should fall as a result of the monetary policy action.

The user-cost channel is well-understood and present in most quantitative models used for policy analysis. The credit channel has received much attention in the past decade, and is now standard in theoretical and quantitative policy-oriented modelling. The asset-price channel was

[1] See, e.g., Mishkin (1995, 1996, 2001) and Boivin et al. (2010).

[2] Our focus here is on corporate investment, but all these channels have obvious counterparts for household spending on consumption of durables and real estate.

the key mechanism that Tobin (1969) sought to model by introducing his famous "q". This channel is described in undergraduate textbooks and discussed in policy circles, but there seems to be no academic research on it. In this paper we study the transmission of monetary policy to corporate investment through the asset-price channel activated by policy-induced changes in Tobin's q. We refer to this mechanism as *q-monetary transmission* or the *q-channel*. We develop a model of the q-monetary transmission mechanism, provide identification and empirical evidence for the q-channel, evaluate the ability of the quantitative theory to match the evidence, and quantify the relevance of q-monetary transmission to aggregate investment.

The main challenge for estimating the q-channel is that monetary policy may affect stock prices and investment through other channels. For example, a contractionary money shock may lower a firm's incentive to invest and its stock price through the user-cost channel. In this case, investment and the stock price would fall concurrently due to higher discounting. As another example, a firm's investment and stock price may fall in response to a money shock that lowers demand for the firm's output, and profit. In this case, investment and the stock price would fall concurrenlty due to lower demand. In both cases the monetary shock leads to a reduction in the stock price and investment, but the fall in the stock price is not *causing* the fall in investment. These examples illustrate we cannot hope to estimate the causal relationship between stock price and investment—the hallmark of the q-channel—simply from the comovement of investment and the stock price induced by monetary policy shocks.

We meet this empirical challenge by using *stock turnover* as a source of cross-sectional variation in the responses of stock prices to monetary shocks.[3] Specifically, our empirical strategy builds on the idea that, if the cross-sectional variation in stock turnover is uncorrelated with other sources of cross-sectional variation in the responses of stock prices to monetary shocks, then identified money shocks combined with heterogeneity in cross-sectional stock turnover can be used as a source of exogenous (policy driven) cross-sectional variation in Tobin's q. We use this cross-sectional variation in the responses of stock prices to money shocks across firms with different stock turnover to identify the effects of changes in stock prices on firms' investment and capital structure decisions. Specifically, we construct an instrument for the cross-sectional variation in Tobin's q by interacting monetary policy shocks with firm-specific stock turnover (calculated in the quarter prior to the shock). Our main exercise consists of estimating whether

[3] Lagos and Zhang (2020b) provide evidence that stock turnover is a strong predictor of the cross-sectional differences in the exposure of stock prices to monetary policy shocks.

such instrumented variation in Tobin's q has significant effects on firms' equity issuance and investment decisions. We find it does. We also find it has significant persistent effects on the capital structure of firms, both for the mix of debt and equity, as well as for the composition of assets and liabilities.

Our work makes contact with four distinct literatures. First, we contribute to the literature on monetary transmission by filling the empirical and theoretical void on the asset-price channel that operates through Tobin's q. Second, we contribute to the literature on the causal effects of changes in stock-market valuations on corporate investment decisions through an "equity financing channel" (e.g., Keynes (1936), Brainard and Tobin (1968), Tobin (1969), Tobin and Brainard (1976), Fischer and Merton (1984), Morck et al. (1990), Blanchard et al. (1993), Baker et al. (2003), Gilchrist et al. (2005)). Our marginal contribution relative to this literature is twofold. On the theory front, we develop an equilibrium model with two sectors: a productive sector where firms are managed by entrepreneurs who make investment and equity issuance decisions, and a financial sector where money and equity claims to the capital installed in the firm are traded among investors with heterogeneous valuations of the marginal product of firms' capital. Our theory highlights the roles that financial constraints (as a determinant of a firm's dependence on equity financing) and heterogeneous valuations of capital play in the transmission of monetary policy shocks to investment decisions through stock prices. On the empirical front, we propose a new instrument for variations in Tobin's q that are not caused by firm-level variation in marginal q. As mentioned above, our innovation in this regard consists of exploiting a combination of identified monetary policy shocks and the cross-sectional variation in the responses of stock prices to these shocks due to differences in stock turnover. Third, our theoretical and empirical results on the response of firms' equity issuance and capital structure to fluctuations in stock prices induced by monetary shocks contribute to the corporate finance literature that studies the relationship between firms' capital structure and macroeconomic conditions in general, and stock prices in particular (e.g., Baker and Wurgler (2002), Korajczyk and Levy (2003), Hovakimian et al. (2004)). Relative to this literature, our contribution is to identify the persistent effects of monetary policy shocks on the capital structure of public firms. Fourth, we contribute to the literature that studies new channels through which monetary policy affects macroeconomic outcomes (e.g., Lagos (2011), Lagos and Zhang (2015, 2019, 2020a,b), Rocheteau et al. (2018)).

The rest of the paper is organized as follows. Section 2 presents the basic model, Section

3 defines equilibrium, and Section 4 characterizes the equilibrium for a special case that can be solved analytically. Section 5 reports the empirical findings. In Section 6 we calibrate and simulate the general model to assess the ability of the theory to fit the empirical estimates of the effects of monetary-policy induced changes in Tobin's q on equity issuance and investment. In Section 7 we provide an estimate of the quantitative relevance of the q-channel for monetary transmission to aggregate investment.

2 Model

Time is represented by a sequence of periods indexed by $t \in \{0, 1, ...\}$. Each time period is divided into two subperiods where different activities take place. There is a continuum of agents infinitely lived agents of two types: *investors*, each identified with a point in the set $\mathcal{I} = [0, 1]$, and *brokers*, each identified with a point in the set $\mathcal{B} = [0, 1]$. There is a continuum (with unit measure) of *entrepreneurs* who live for a random number of periods. Each entrepreneur who is alive at the beginning of period t is identified with a point in the set $\mathcal{E}_t \subset \mathbb{R}_+$. A fraction $1 - \pi \in [0, 1]$ of the population of entrepreneurs in the set $\mathcal{E}_t$ dies (i.e., exits the economy) at the beginning of the second subperiod of period t. The set of entrepreneurs who die is a uniform random draw from the population, and each is immediately replaced by a newly born entrepreneur.

There are three commodities at each date: two consumption goods, called *good 1* and *good 2*, and a *capital* good. The consumption goods are perishable: good 1 and good 2 can only be consumed in the first and second subperiods, respectively. Capital is storable, but depreciates at rate $\delta \in [0, 1]$ between periods. Upon entering the economy, an entrepreneur $i \in \mathcal{E}_t$ is endowed with $w_0^i \in \mathbb{R}_+$ units of good 2 and $k_0 \in \mathbb{R}_+$ units of capital. We use a cumulative distribution function Ω to describe the heterogeneity in the initial endowment of (claims to) good 2 relative to capital, $\omega_0^i \equiv w_0^i / k_0$, across entrepreneurs. In the second subperiod of every period, investors and brokers are endowed with a resource called *labor (effort)* that they can use to produce good 2 one-for-one. There are two other production technologies that can be managed only by entrepreneurs. The first, uses capital available at the beginning of period t to produce good 1 in the first subperiod of period t. Specifically the capital stock k_t operated by an entrepreneur delivers zk_t units of good 1 at the end of the first subperiod of t, where $z \in \mathbb{R}_{++}$. The second production technology can be operated by an entrepreneur in the second subperiod of period t, and uses good 2 and the capital the entrepreneur has in place at the beginning of period t to augment the capital that the entrepreneur will have in place to produce good 1 in period $t+1$. Formally, this technology is represented by a cost function, $\mathrm{C}(x_t, k_t) \equiv x_t + \Psi(x_t/k_t) k_t$, interpreted as the cost (in terms of good 2) of producing and installing x_t units of capital for an entrepreneur whose current capital is k_t. We assume $0 < \Psi''$, and that there is a $\iota_0 \in \mathbb{R}_+$ such that $\Psi(\iota_0) = \Psi'(\iota_0) = 0$. It is convenient to define $\mathrm{c}(x_t/k_t) \equiv \mathrm{C}(x_t, k_t)/k_t$, i.e., the cost of investment per unit of installed capital. The assumptions on Ψ imply $\mathrm{c}(\iota_0) - \iota_0 = \mathrm{c}'(\iota_0) - 1 = 0 < \mathrm{c}''(\cdot)$. Once installed, capital is entrepreneur-specific, i.e., capital installed by entrepreneur $i \in \mathcal{E}_t$ is only productive when operated by entrepreneur i.

The asset structure is as follows. In the second subperiod of every period, in order to finance the cost of investing in new capital, every entrepreneur can issue identical, durable, and perfectly divisible equity claims to the future returns from the newly created capital. (Entrepreneurs are also allowed to sell equity claims on any existing capital they currently own.) An equity share issued by an entrepreneur in the second subperiod of t represents ownership of 1 unit of capital along with the stream of *dividends* produced by that unit of capital. When an entrepreneur dies, the outstanding equity claims she had previously issued disappear, and the underlying capital plus any financial assets, physical capital, or claims owned by the entrepreneur are distributed uniformly (lump sum) to the cohort of newly born entrepreneurs. There are two other financial instruments: a one-period real pure-discount government *bond*, and *money*. A unit of the bond issued in the second subperiod of t represents a risk-free claim to one unit of good 2 in the second subperiod of $t+1$. The stock of bonds outstanding at time t is B_t, and all private agents take the sequence $\{B_t\}_{t=0}^{\infty}$ as given. Money is intrinsically useless (it is not an argument of any utility or production function, and unlike equity or bonds, money does not constitute a formal claim to any resources). The nominal money supply at the beginning of period t is denoted A_t^m, and we assume $A_{t+1}^m = \mu A_t^m$, with $\mu \in \mathbb{R}_{++}$ and $A_0^m \in \mathbb{R}_{++}$ given. The government injects or withdraws money via lump-sum transfers or taxes to investors in the second subperiod of every period. At the beginning of period $t=0$, each investor is endowed with an equal portfolio of money. We assume brokers do not hold financial assets.[4]

The market structure is as follows. In the second subperiod, all agents can trade good 2, labor services, equity shares, bonds, and money, in a spot Walrasian market.[5] In the first

[4]This assumption allows us to abstract from the broker's portfolio problem in the first subperiod, which is not essential for the questions we study in this paper. See Lagos and Zhang (2015, 2020b) for a treatment of the broker's portfolio problem in a related model.

[5]Notice that equity shares (i.e., the claims on installed capital and its returns) can be traded freely, but the

subperiod, investors can trade equity shares and money in a random bilateral *over-the-counter (OTC) market* with brokers, while brokers can also trade equity shares and money with other brokers in a spot Walrasian *interbroker market*. We use $\alpha \in [0,1]$ to denote the probability that an individual investor is able to make contact with a broker in the OTC market. Once a broker and an investor have contacted each other, the pair negotiates the quantity of equity shares and money that the broker will trade in the interdealer market on behalf of the investor, and a fee for the broker's intermediation services. The terms of the trade between an investor and a broker in the OTC market are determined by Nash bargaining, where $\theta \in [0,1]$ is the investor's bargaining power. We assume the fee is negotiated in terms of good 2, and paid at the beginning of the following subperiod.[6] The timing is that the round of OTC trade takes place in the first subperiod and ends before equity pays out first-subperiod dividends.[7] Equity purchases in the OTC market cannot be financed by borrowing (e.g., due to anonymity and lack of commitment and enforcement). This assumption and the structure of preferences described below create the need for a medium of exchange in the OTC market.[8]

An individual broker's preferences are given by

$$\mathbb{E}_0^B \sum_{t=0}^{\infty} \beta^t (y_t - h_t),$$

where $\beta \in (0,1)$ is the discount factor, and y_t and h_t denote a broker's consumption of good 2, and utility cost from supplying h_t units of labor in the second subperiod of period t, respectively. The expectation operator, $\mathbb{E}_0^B$, is with respect to the probability measure induced by the random trading process in the OTC market. Dealers get no utility from good 1.[9] An individual investor's

actual physical capital created and installed by a particular entrepreneur is assumed to be non tradable. The idea is that, once installed by an entrepreneur, physical capital becomes entrepreneur-specific and cannot be operated by another entrepreneur. An entrepreneur can, however, disinvest (which entails bearing the adjustment cost, Φ) to turn installed capital into good 2, which can then be traded freely in the Walrasian market. Similarly, when the entrepreneur dies, the quantity of good 2 obtained from unistalling the capital that the entrepreneur used to manage is distributed to newly born entrepreneurs (net of adjustment costs).

[6] This is the specification used in Lagos and Zhang (2020b). Lagos and Zhang (2015) instead assume the investor must pay the intermediation fee to the broker on the spot (with money or equity). The timing convention in Lagos and Zhang (2020b) simplifies the exposition without affecting the mechanisms of interest.

[7] As in previous search models of OTC markets, e.g., Duffie et al. (2005) and Lagos and Rocheteau (2009), an investor must own the equity in order to consume the dividend flow of consumption good in the OTC round.

[8] See Lagos and Zhang (2020a, 2019) for a similar model where investors can buy equity with *margin loans*.

[9] This assumption implies that dealers have no direct consumption motive for holding the equity share. It is easy to relax, but we adopt it because it is the standard benchmark in the search-based OTC literature, e.g., see Duffie et al. (2005), Lagos and Rocheteau (2009), Lagos et al. (2011), and Weill (2007).

preferences are given by

$$\mathbb{E}_0^I \sum_{t=0}^{\infty} \beta^t \left(\varepsilon_t c_t + y_t - h_t\right),$$

where y_t and h_t denote an investor's consumption of good 2, and utility cost from supplying h_t units of labor in the second subperiod of period t, respectively, and c_t is the investor's consumption of good 1 at the end of the first subperiod of period t. The variable ε_t denotes the realization of an idiosyncratic valuation shock for good 1 that is distributed independently over time and across investors with a differentiable cumulative distribution function G with support $[\varepsilon_L, \varepsilon_H] \subseteq [0, \infty]$, and mean $\bar{\varepsilon} \equiv \int \varepsilon dG(\varepsilon)$. An investor learns the realization ε_t at the beginning of the first subperiod of period t, immediately before the OTC trading round. The expectation operator, $\mathbb{E}_0^I$, is with respect to the probability measure induced by the investor's valuation shocks, and the trading process in the OTC market.

The preferences of an entrepreneur born in the second subperiod of t are given by

$$\sum_{j=t}^{\infty} (\beta\pi)^{(j-t)} \left(y_j + \beta\varepsilon_e c_{j+1}\right),$$

where y_j is the consumption of good 2 in the second subperiod of period j, and c_{j+1} is the entrepreneur's consumption of good 1 at the end of the first subperiod of period $j+1$, and $\varepsilon_e \in \mathbb{R}_{++}$ is the entrepreneur's valuation of her own production of good 1.

3 Equilibrium

Consider the determination of the terms of trade in a bilateral meeting in the OTC round of period t between a broker and an investor with valuation ε and portfolio $\boldsymbol{a}_t = (a_t^b, a_t^m, a_t^s)$, where a_t^b denotes bond holdings, a_t^m money holdings, and a_t^s holdings of shares. Let $W_t(\boldsymbol{a}_t, \varpi_t)$ denote the maximum expected discounted payoff at the beginning of the second subperiod of period t of an investor who is holding portfolio $\boldsymbol{a}_t$ and has to pay a broker fee ϖ_t. Let $[\overline{\boldsymbol{a}}_t(\boldsymbol{a}_t, \varepsilon), \varpi_t(\boldsymbol{a}_t, \varepsilon)]$ represent the bargaining outcome in a bilateral trade at time t between a broker and an investor with portfolio $\boldsymbol{a}_t$ and valuation ε, where $\overline{\boldsymbol{a}}_t(\boldsymbol{a}_t, \varepsilon) \equiv \left(\overline{a}_t^b(\boldsymbol{a}_t, \varepsilon), \overline{a}_t^m(\boldsymbol{a}_t, \varepsilon), \overline{a}_t^s(\boldsymbol{a}_t, \varepsilon)\right)$ denotes the investor's post-trade portfolio. That is,

$$[\overline{\boldsymbol{a}}_t(\boldsymbol{a}_t, \varepsilon), \varpi_t(\boldsymbol{a}_t, \varepsilon)] = \arg \max_{(\overline{\boldsymbol{a}}_t, \varpi_t) \in \mathbb{R}_+^4} \Gamma(\overline{\boldsymbol{a}}_t, \boldsymbol{a}_t, \varepsilon)^{\theta} \varpi_t^{1-\theta} \tag{1}$$

with

$$\Gamma(\overline{\boldsymbol{a}}_t, \boldsymbol{a}_t, \varepsilon) \equiv \varepsilon z \overline{a}_t^s + W_t(\overline{a}_t^b, \overline{a}_t^m, \pi(1-\delta)\overline{a}_t^s, \varpi_t) - \varepsilon z a_t^s - W_t(a_t^b, a_t^m, \pi(1-\delta)a_t^s, 0),$$

$\overline{\boldsymbol{a}}_t \equiv (\overline{a}_t^b, \overline{a}_t^m, \overline{a}_t^s)$, and subject to

$$\begin{aligned}\overline{a}_t^m + p_t \overline{a}_t^s &\leq a_t^m + p_t a_t^s \\ 0 &\leq \Gamma\left(\overline{\boldsymbol{a}}_t, \boldsymbol{a}_t, \varepsilon\right) \\ \overline{a}_t^b &= a_t^b,\end{aligned}$$

where p_t denotes the dollar price of an equity share in the interbroker market of period t. The first and second constraints are the investor's budget, and participation constraints, respectively. The last constraint reflects the assumption that the real bond is illiquid in that it cannot be directly used as means of payment in stock-market trades.

Let $V_t\left(\boldsymbol{a}_t, \varepsilon\right)$ denote the maximum expected discounted payoff of an investor with valuation ε and portfolio $\boldsymbol{a}_t$ at the beginning of the first subperiod of period t. In the second subperiod of period t, let $\boldsymbol{\phi}_t \equiv \left(\phi_t^b, \phi_t^m, \phi_t^s\right)$, where ϕ_t^b is the real price of a newly issued government bond, ϕ_t^m, is the real price of a unit of money, and ϕ_t^s is the real price of an equity share (all in terms of good 2). At the beginning of the second subperiod the investor solves

$$W_t\left(\boldsymbol{a}_t, \varpi_t\right) = \max_{(y_t, h_t, \boldsymbol{a}_{t+1}) \in \mathbb{R}_+^5} \left[y_t - h_t + \beta \int V_{t+1}\left(\boldsymbol{a}_{t+1}, \varepsilon\right) dG(\varepsilon)\right] \tag{2}$$

$$\text{s.t. } y_t + \boldsymbol{\phi}_t \boldsymbol{a}_{t+1} \leq \boldsymbol{\phi}_t' \boldsymbol{a}_t + h_t - \varpi_t + T_t,$$

where y_t is consumption of good 2, h_t is the disutility of labor, $\boldsymbol{a}_{t+1} \equiv (a_{t+1}^b, a_{t+1}^m, a_{t+1}^s)$, $\boldsymbol{\phi}_t' \equiv (1, \phi_t^m, \phi_t^s)$, and $T_t \in \mathbb{R}$ is the real value of the lump-sum monetary transfer. The value function of an investor who enters the first subperiod of t with portfolio $\boldsymbol{a}_t$ and valuation ε is

$$\begin{aligned} V_t\left(\boldsymbol{a}_t, \varepsilon\right) &= \alpha \left\{\varepsilon z \overline{a}_t^s\left(\boldsymbol{a}_t, \varepsilon\right) + W_t\left[\overline{\boldsymbol{a}}_t'\left(\boldsymbol{a}_t, \varepsilon\right), \varpi_t\left(\boldsymbol{a}_t, \varepsilon\right)\right]\right\} \\ &\quad + (1-\alpha)\left\{\varepsilon z a_t^s + W_t\left[\boldsymbol{a}_t'\left(\boldsymbol{a}_t\right), 0\right]\right\}, \end{aligned} \tag{3}$$

where $\overline{\boldsymbol{a}}_t'\left(\boldsymbol{a}_t, \varepsilon\right) \equiv \left(\overline{a}_t^b\left(\boldsymbol{a}_t, \varepsilon\right), \overline{a}_t^m\left(\boldsymbol{a}_t, \varepsilon\right), \pi(1-\delta)\overline{a}_t^s\left(\boldsymbol{a}_t, \varepsilon\right)\right)$ and $\boldsymbol{a}_t'\left(\boldsymbol{a}_t\right) \equiv \left(a_t^b, a_t^m, \pi(1-\delta)a_t^s\right)$.

Let $J_t\left(\boldsymbol{b}_t\right)$ denote the maximum expected discounted payoff at the beginning of the second subperiod of period t, of an entrepreneur who currently has balance sheet $\boldsymbol{b}_t \equiv \left(a_t^b, k_t, s_t\right)$, composed of (claims to) a_t^b units of good 2, installed capital k_t, and s_t outstanding equity claims on installed capital. The value function satisfies

$$J_t\left(\boldsymbol{b}_t\right) = \max_{y_t, a_{t+1}^b, e_t, x_t} \left\{y_t + \beta\left[\varepsilon_e z(k_{t+1} - s_{t+1}) + \pi J_{t+1}\left(\boldsymbol{b}_{t+1}\right)\right]\right\} \tag{4}$$

$$
\begin{aligned}
\text{s.t. } y_t + \mathrm{C}\left(x_t/k_t\right)k_t + \phi_t^b a_{t+1}^b &\leq \phi_t^s e_t + a_t^b && (5)\\
k_{t+1} &= (1-\delta)\,k_t + x_t && (6)\\
s_{t+1} &= (1-\delta)\,s_t + e_t && (7)\\
0 &\leq s_{t+1} \leq k_{t+1} && (8)\\
y_t, a_{t+1}^b &\in \mathbb{R}_+, && (9)
\end{aligned}
$$

where $\boldsymbol{b}_{t+1} \equiv \left(a_{t+1}^b, k_{t+1}, s_{t+1}\right)$, y_t denotes consumption of good 2, x_t is the quantity of good 2 invested to produce new capital (net of the installation cost), and e_t is the number of newly issued equity shares. Condition (5) is the entrepreneur's budget constraint (expressed in terms of good 2), while (6) and (7) are the laws of motion for the stock of installed capital and outstanding equity shares on the entrepreneur's installed capital, respectively. The first inequality in (8) states that an entrepreneur cannot buy claims on her own dividend of good 1 issued by other agents.[10] The second inequality in (8) states that entrepreneurs cannot sell claims on capital that are not backed by capital owned by the entrepreneur, i.e., equity issuance must satisfy $e_t \leq x_t + (1-\delta)\left(k_t - s_t\right)$. The nonnegativity constraints in (9) rule out negative consumption and borrowing by shorting the government bond. The formulation (4) assumes an entrepreneur does not hold cash.[11] Let the function $\boldsymbol{g}_t : \mathbb{R}_+^3 \to \mathbb{R}_+^2 \times \mathbb{R}^2$ denote the optimal decision rule corresponding to (4), i.e., $\boldsymbol{g}_t\left(\boldsymbol{b}_t\right) \equiv \left(g_t^y\left(\boldsymbol{b}_t\right), g_t^b\left(\boldsymbol{b}_t\right), g_t^e\left(\boldsymbol{b}_t\right), g_t^x\left(\boldsymbol{b}_t\right)\right)$ gives the entrepreneur's optimal choices of second-subperiod consumption, bond holdings, equity issuance, and investment, as a function of her initial balance sheet, $\boldsymbol{b}_t$. Then, conditional on survival, the optimal path for the entrepreneur's balance sheet is described by $\boldsymbol{b}_{t+1} = \bar{\boldsymbol{g}}_t\left(\boldsymbol{b}_t\right) \equiv \left(\bar{g}_t^b\left(\boldsymbol{b}_t\right), \bar{g}_t^k\left(\boldsymbol{b}_t\right), \bar{g}_t^s\left(\boldsymbol{b}_t\right)\right)$, with $\bar{g}_t^b\left(\boldsymbol{b}_t\right) \equiv g_t^b\left(\boldsymbol{b}_t\right)$, $\bar{g}_t^k\left(\boldsymbol{b}_t\right) \equiv (1-\delta)\,k_t + g_t^x\left(\boldsymbol{b}_t\right)$, and $\bar{g}_t^s\left(\boldsymbol{b}_t\right) \equiv (1-\delta)\,s_t + g_t^e\left(\boldsymbol{b}_t\right)$.

Let $j \in \{E, I\}$ denote the agent type, i.e., "E" for entrepreneurs and "I" for investors, and let $h \in \{b, m, s\}$ denote the type of financial asset, i.e., "b" for bonds, "m" for money, and "s" for equity shares. Then let A_{It}^h denote the quantity of financial asset h held by all investors at the beginning of period t. That is, $A_{It}^h = \int a_t^h dF_{It}\left(\boldsymbol{a}_t\right)$, where F_{It} is the cumulative distribution function over portfolios $\boldsymbol{a}_t = \left(a_t^b, a_t^m, a_t^s\right)$ held by investors at the beginning of period t. Similarly, let $\bar{F}_{Et}$ denote the joint cumulative distribution function over entrepreneur's balance sheets, $\boldsymbol{b}_t = \left(a_t^b, k_t, s_t\right)$, at the beginning of the second subperiod of period t. Let

[10] Equivalently, with (7) the constraint $0 \leq s_{t+1}$ can be written as $-(1-\delta)\,s_t \leq e_t$, i.e., the entrepreneur can buy back her own equity shares, but cannot buy back more than the quantity of shares outstanding.

[11] This assumption merely simplifies the exposition. In this environment, entrepreneurs are not involed in transactions for which cash is used as a medium of exchange, so we can anticipate that an entrepreneur will never choose to carry cash given she has the option to hold interest-bearing government bonds.

A^b_{Et} denote the quantity of bonds held by entrepreneurs at the beginning of period t. Let K_t and S_t denote the beginning-of-period t capital stock managed by all entrepreneurs, and outstanding equity claims on all installed capital, respectively. Then, we have the beginning-of-period t aggregates, $A^b_{Et} = \int a^b_t dF_{Et}(\boldsymbol{b}_t)$, $K_t = \int k_t dF_{Et}(\boldsymbol{b}_t)$, and $S_t = \int s_t dF_{Et}(\boldsymbol{b}_t)$. Let $\bar{A}^m_{It}$ and $\bar{A}^s_{It}$ denote the quantities of money and shares held after the first-subperiod round of trade of period t by all the investors who are able to trade in the first subperiod. Then we have $\bar{A}^h_{It} = \alpha \int \bar{a}^h_t(\boldsymbol{a}_t, \varepsilon) dH_{It}(\boldsymbol{a}_t, \varepsilon)$ for $h \in \{m, s\}$, where H_{It} denotes the joint cumulative distribution of portfolios and valuation shocks across investors at the beginning of period t. We are now ready to define equilibrium.

Definition 1 *An equilibrium is a sequence of prices, $\{\boldsymbol{\phi}_t\}_{t=0}^{\infty}$, terms of trade in the first-subperiod, $\{\bar{\boldsymbol{a}}_t(\cdot), \varpi_t(\cdot)\}_{t=0}^{\infty}$, investor end-of-period portfolio choices, $\{\boldsymbol{a}_{t+1}\}_{t=0}^{\infty}$, and decision rules for entrepreneurs, $\{\boldsymbol{g}_t(\cdot)\}_{t=0}^{\infty}$, such that: (i) the terms of trade $\{\bar{\boldsymbol{a}}_t(\cdot), \varpi_t(\cdot)\}_{t=0}^{\infty}$ solve (1); (ii) taking prices and the bargaining protocol as given, the portfolios $\{\boldsymbol{a}_{t+1}\}_{t=0}^{\infty}$ solve the individual investor's optimization problem (2), and the decision rules $\{\boldsymbol{g}_t(\cdot)\}_{t=0}^{\infty}$ solve (4); and (iii) prices, $\{\boldsymbol{\phi}_t\}_{t=0}^{\infty}$, are such that all Walrasian markets clear, i.e., $A^b_{Et+1} + A^b_{It+1} = B_{t+1}$ (the end-of-period t Walrasian bond market clears), $A^m_{It+1} = A^m_{t+1}$ (the end-of-period t Walrasian market for money clears), $A^s_{It+1} = S_{t+1}$ (the end-of-period t Walrasian market for equity clears), $\bar{A}^m_{It} = \alpha A^m_t$ (the market for money in the first subperiod of t clears), and $\bar{A}^s_{It} = \alpha S_t$ (the market for equity in the first subperiod of t clears). An equilibrium is "monetary" if $\phi^m_t > 0$ for all t and "nonmonetary" otherwise.*

4 Theoretical results

Throughout we focus on *stationary equilibria* in which the aggregate supply of equity and aggregate real money balances are constant over time, i.e., $S_t = S$ and $\phi^m_t A^m_t \equiv \mathrm{M}_t = \mathrm{M}$ for all t, and real equity prices are time-invariant linear functions of the dividend, i.e., $\phi^s_t = \phi^s \equiv \varphi^s z$ and $p_t \phi^m_t = \bar{\varphi}^s z$, for all t.[12] In order to derive the main theoretical insights analytically, in this section we assume $\pi = 0$, i.e., entrepreneurs live for one period.[13]

To study equilibrium it is useful to define the *marginal valuation* in the stock market of the

[12]Intuitively, ϕ^s_t and $p_t \phi^m_t$ are the ex- and cum-dividend real equity prices (in terms of good 2), respectively.
[13]The general formulation with $\pi \in [0, 1]$ is analyzed quantitatively in Section 6.

first subperiod of t, $\varepsilon_t^* \equiv p_t \phi_t^m / z$, and the *nominal interest rate* between period t and $t+1$,

$$r_{t+1} \equiv \frac{\phi_t^m}{\beta \phi_{t+1}^m} - 1. \tag{10}$$

The marginal valuation ε_t^* is the one that makes an investor indifferent between holding equity or selling it for cash in the first subperiod.[14] In a stationary equilibrium with $\pi = 0$, $\varepsilon_t^* = \varepsilon^* \equiv \bar{\varphi}^s$ for all t. The nominal interest rate r_{t+1} is the nominal yield of a one-period risk-free nominal bond issued in the second subperiod of t and redeemed in the second subperiod of $t+1$ that is illiquid (in the sense that it cannot be used to purchase stocks in the first-subperiod of $t+1$). In a stationary equilibrium, $r_{t+1} = r \equiv (\mu - \beta)/\beta$ for all t, so we regard r as the *nominal policy rate*, which can be implemented by changing the growth rate in the money supply, μ.

For an entrepreneur who enters with initial conditions w and k in the context of a stationary equilibrium of an economy with $\pi = 0$, (4)-(9) specialize to

$$\begin{aligned} J(w,k,0) = \max_{x,y,s_{+1}} & \; [y + \beta \varepsilon_e z (k_{+1} - s_{+1})] \\ \text{s.t. } y + \mathrm{C}(x/k)\,k & \leq \phi^s s_{+1} + w \\ k_{+1} & = (1-\delta)\,k + x \\ 0 & \leq s_{+1} \leq k_{+1} \\ 0 & \leq y. \end{aligned} \tag{11}$$

Let $g^x(w,k)$, $g^y(w,k)$, and $g^e(w,k)$ denote the levels of investment, consumption, and equity issuance that solve (11). Define $x^* \equiv g^x(w,k)/k$, $y^* \equiv g^y(w,k)/k$, $s_{+1}^* \equiv g^e(w,k)/k$, $\omega \equiv w/k$, and $\phi_e^s \equiv \beta \varepsilon_e z$. The following result characterizes (x^*, y^*, s_{+1}^*).

Lemma 1 *Let $\iota(\phi)$ denote the unique number, ι, that solves $\mathrm{C}'(\iota) = \phi$ for any $\phi \in \mathbb{R}_+$. Assume $\delta - \iota_0 \leq 1 \leq \min(\phi^s, \phi_e^s)$. (i) If $\phi_e^s \leq \phi^s$,*

$$\begin{aligned} x^* &= \iota(\phi^s) \\ s_{+1}^* &= \begin{cases} 1 - \delta + x^* & \text{if } \phi_e^s < \phi^s \\ \left[\max\left\{0, \frac{\mathrm{C}(x^*) - \omega}{\phi^s}\right\}, 1 - \delta + x^*\right] & \text{if } \phi_e^s = \phi^s. \end{cases} \end{aligned}$$

[14] In general, for $\pi \in [0,1]$, ε_t^* would be defined as $\varepsilon_t^* \equiv (p_t \phi_t^m - \pi(1-\delta)\phi_t^s)/z$. To see the role that this marginal valuation will play in the equilibrium, consider an investor with valuation ε who, in the stock market of the first subperiod of period t, is deciding whether to keep an equity share or sell it for cash. If he keeps the share, his payoff is $\varepsilon z + \pi(1-\delta)\phi_t^s$, namely his valuation of the period dividend, εz, plus the expected value (in terms of good 2) of the share of undepreciated capital in the following subperiod, $\pi(1-\delta)\phi_t^s$. If he sells it for cash, he gets payoff $p_t \phi_t^m$ (i.e., sells the share for p_t dollars, worth ϕ_t^m units of good 2 in the following subperiod). Hence, the investor keeps the share if $\varepsilon z + \pi(1-\delta)\phi_t^s > p_t \phi_t^m$, sells it for cash if $\varepsilon z + \pi(1-\delta)\phi_t^s < p_t \phi_t^m$, and is indifferent if $\varepsilon = \varepsilon_t^*$.

(ii) If $\phi^s < \phi_e^s$,

$$x^* = \begin{cases} \iota(\phi_e^s) & \text{if } \mathrm{C}(\iota(\phi_e^s)) \leq \omega \\ \mathrm{C}^{-1}(\omega) & \text{if } \mathrm{C}(\iota(\phi^s)) < \omega < \mathrm{C}(\iota(\phi_e^s)) \\ \iota(\phi^s) & \text{if } \omega \leq \mathrm{C}(\iota(\phi^s)) \end{cases}$$

$$s_{+1}^* = \begin{cases} 0 & \text{if } \mathrm{C}(\iota(\phi^s)) < \omega \\ \frac{\mathrm{C}(\iota(\phi^s))-\omega}{\phi^s} & \text{if } \omega \leq \mathrm{C}(\iota(\phi^s)) \end{cases}$$

In every case, $y^* = \omega + \phi^s s_{+1}^* - \mathrm{C}(x^*)$

In Lemma 1, ϕ_e^s is the entrepreneur's marginal value of investing capital, while ϕ^s can be interpreted as the marginal value of investing in capital to the outside investors who price the entrepreneur's equity in the market. Part (i) focuses on the case in which the market valuation of the marginal investment in capital is higher than the entrepreneur's. In this case the entrepreneur chooses the investment rate, x^*, so that the marginal cost, $\mathrm{C}'(x^*)$, equals the market value of the marginal investment, ϕ^s. Moreover, because the entrepreneur's valuation is lower than the market valuation, the entrepreneur issues equity shares on any capital she owns at the beginning of the period, and finances new investment entirely by equity issuance, i.e., she chooses $s_{+1}^* k = (1 - \delta + x^*)k$. (In the knife-edge case with $\phi_e^s = \phi^s$, the entrepreneur is indifferent between financing by equity issuance or out of her own funds, ωk.)

Part (ii) of Lemma 1 focuses on the case in which the entrepreneur's valuation of the marginal investment in capital is higher than the market valuation, i.e., $\phi^s < \phi_e^s$. In this case the investment, financing, and consumption decisions of the entrepreneur depend on her own valuation of investment, on the market valuation, and on the entrepreneur's "financial" wealth, represented by the ω endowment of good 2. First, if $\mathrm{C}(\iota(\phi_e^s)) \leq \omega$, the entrepreneur is financially unconstrained: she chooses her first-best investment rate, $\iota(\phi_e^s)$ (the x^* that equates the marginal cost of investment, $\mathrm{C}'(x^*)$, to her own marginal valuation, ϕ_e^s), finances it entirely with her own funds, i.e., $s_{+1}^* = 0$ (issues no equity), and consumes the unspent wealth, i.e., sets $y^* = \omega - \mathrm{C}(\iota(\phi_e^s))$. On the opposite extreme, if the entrepreneur's own financial wealth is very low, specifically $\omega \leq \mathrm{C}(\iota(\phi^s))$, i.e., lower than what would be needed to self-finance the level of investment that would be chosen based on outside investors' marginal valuation of investment, ϕ^s, then she chooses the investment rate $\iota(\phi^s)$ (the x^* that equates the marginal cost of investment, $\mathrm{C}'(x^*)$, to the market valuation, ϕ^s), uses all of her own funds to finance investment (sets $y^* = 0$), and resorts to equity issuance. Finally, if the entrepreneur's financial wealth is too low to self-finance her first-best investment rate but high enough to self-finance the investment rate that would be chosen based on outside investor's valuations, i.e., if $\mathrm{C}(\iota(\phi^s)) < \omega < \mathrm{C}(\iota(\phi_e^s))$, then the entrepreneur invests the maximum that can be financed with her internal funds, i.e., the investment rate x^* that satisfies $\mathrm{C}(x^*) = \omega$, sets $y^* = 0$, and issues no equity.

For what follows, let $x^*(\omega)$ and $s_{+1}^*(\omega)$ denote the optimal investment and equity issuance decisions taken by an entrepreneur who enters with a ratio of financial wealth to physical capital equal to ω, as characterized in Lemma 1. With this notation, we can write the aggregate investment chosen by all active entrepreneurs at the end of a period as

$$X^* = \int x^*(\omega)\, d\Omega(\omega), \tag{12}$$

and the aggregate stock of equity shares outstanding at the beginning of a period as

$$S^* = \int s_{+1}^*(\omega)\, d\Omega(\omega). \tag{13}$$

For the remainder of this section, we assume $\delta - \iota_0 \leq 1 \leq \min(\underline{\phi}^s, \phi_e^s)$, where $\underline{\phi}^s \equiv \beta\bar{\varepsilon}z$. The following proposition characterizes the nonmonetary equilibrium.

Proposition 1 *A nonmonetary equilibrium exists for any parametrization. In the nonmonetary equilibrium, money has no value, i.e.,* $\mathrm{M} = 0$*, and the price of an equity share is* $\underline{\phi}^s$*. Moreover: (i) If* $\phi_e^s < \underline{\phi}^s$*, then* $X^* = \iota(\underline{\phi}^s)$*, and* $S^* = 1 - \delta + \iota(\underline{\phi}^s)$*. (ii) If* $\underline{\phi}^s < \phi_e^s$*, then*

$$X^* = \Omega[\mathrm{C}(\iota(\underline{\phi}^s))]\iota(\underline{\phi}^s) + \int_{\mathrm{C}(\iota(\underline{\phi}^s))}^{\mathrm{C}(\iota(\phi_e^s))} \mathrm{C}^{-1}(\omega) d\Omega(\omega) + \{1 - \Omega[\mathrm{C}(\iota(\phi_e^s))]\}\iota(\phi_e^s),$$

and

$$S^* = \frac{1}{\underline{\phi}^s} \int_0^{\mathrm{C}(\iota(\underline{\phi}^s))} [\mathrm{C}(\iota(\underline{\phi}^s)) - \omega] d\Omega(\omega).$$

The following proposition characterizes the monetary equilibrium. Before stating the result, it is convenient to define $\overline{\phi}^s \equiv \beta\left[\bar{\varepsilon} + \alpha\theta(\varepsilon_H - \bar{\varepsilon})\right]z$ and $\bar{r} \equiv \alpha\theta(\bar{\varepsilon} - \varepsilon_L)/\varepsilon_L$.

Proposition 2 *Assume* $r \in (0, \bar{r})$*. (i) There exists a unique stationary monetary equilibrium. (ii) The equity price is*

$$\phi^s(r) = \beta\left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon)\right] z, \tag{14}$$

where $\varepsilon^* \in (\varepsilon_L, \varepsilon_H)$ *is the unique solution to*

$$\alpha\theta \int_{\varepsilon^*}^{\varepsilon_H} \frac{\varepsilon - \varepsilon^*}{\varepsilon^*} dG(\varepsilon) = r. \tag{15}$$

(iii) If $\phi_e^s \in (\underline{\phi}^s, \overline{\phi}^s)$*, let* $\hat{r} \in (0, \bar{r})$ *be defined by* $\phi^s(\hat{r}) = \phi_e^s$*. Then: (a) If* $r \in (0, \hat{r})$*, then* $X^* = \iota(\phi^s(r))$*, and* $S^* = 1 - \delta + \iota(\phi^s(r))$*. (b) If* $r \in (\hat{r}, \bar{r})$*, then*

$$X^* = \Omega[\mathrm{C}(\iota(\phi^s(r)))]\iota(\phi^s(r))) + \int_{\mathrm{C}(\iota(\phi^s(r)))}^{\mathrm{C}(\iota(\phi_e^s))} \mathrm{C}^{-1}(\omega) d\Omega(\omega) + \{1 - \Omega[\mathrm{C}(\iota(\phi_e^s))]\}\iota(\phi_e^s),$$

and

$$S^* = \frac{1}{\phi^s(r)} \int_0^{\mathrm{C}(\iota(\phi^s(r)))} [\mathrm{C}(\iota(\phi^s(r))) - \omega] d\Omega(\omega).$$

(iv) If $\phi_e^s < \underline{\phi}^s$*,* X^* *and* S^* *are as in part (iii)(a). (v) If* $\overline{\phi}^s < \phi_e^s$*,* X^* *and* S^* *are as in part (iii)(b). (vi) In every case, aggregate real money balances are given by* $\mathrm{M} = \frac{G(\varepsilon^*)\varepsilon^* z}{1-G(\varepsilon^*)} S^*$*.*

The following corollary of Proposition 2 documents how asset prices and the investment rate respond to changes in the monetary policy rate, r.

Corollary 1 *In the stationary monetary equilibrium: (i) As* $r \to \bar{r}$*,* $\mathrm{M} \to 0$*, and* $\phi^s \to \underline{\phi}^s$*. (ii) As* $r \to 0$*,* $\phi^s \to \overline{\phi}^s$*. (iii)* $d\varepsilon^*/dr < 0$ *and* $d\phi^s(r)/dr < 0$*. (iv)* $d\iota(\phi^s(r)))/dr < 0$*, (v)* $d^2\phi^s(r)/(drd\hat{\alpha}) < 0$*, where* $\hat{\alpha} \equiv \alpha\theta$*.*

The results in Corollary 1 are interpreted as follows. Part (i) states that as the opportunity cost of holding money (represented by the policy rate r) approaches $\bar{r}$, the monetary equilibrium of Proposition 2 converges to the nonmonetary equilibrium of Proposition 1. Part (ii) is a version of the celebrated *Friedman rule*: as monetary policy drives the opportunity cost of holding money toward zero, investors' liquidity needs are satiated, which implies the equilibrium equity price is set by the highest investor valuation. Part (iii) complements parts (i) and (ii) by showing that the market price of equity is decreasing in the policy rate r. Part (iv) shows that if the marginal value of the entrepreneur's investment is determined by the stock market, then increases in the nominal policy rate, r, discourage investment. Finally, part (v) states that the magnitude of the equity-price response to changes in the policy rate is increasing in the liquidity of the stock (e.g., as measured by the parameter α, which determines the frequency of trade of the stock).

4.1 Implications

In this section we derive the main implications of the theory that will guide our empirical analysis. First, we describe the relationship between an individual firm's investment rate, Tobin's marginal q, and Tobin's average q. We then discuss the role of the stock turnover rate in the transmission of monetary shocks to a firm's equity prices, and ultimately, a firm's investment rate. We then we use the equilibrium conditions of the model to derive the relationship between investment rates, equity prices, turnover, and monetary policy shocks that motivate the regression specification and guide identification strategy we use in Section 5.

4.1.1 Tobin's q, monetary policy, and investment

The following corollary of Lemma 1 establishes the conditions under which the marginal value of capital that the entrepreneur uses to make the optimal investment decision, which we denote q^*, is equal to *Tobin's q*, which in this model is equal to the stock-market price of a claim to a unit of capital installed in the firm, ϕ^s.

Corollary 2 *The optimal investment rate, x^*, satisfies $\mathrm{C}'(x^*) = q^*$. If $\phi^s_e \leq \phi^s$, $q^* = \phi^s$. If $\phi^s < \phi^s_e$,*

$$q^* = \begin{cases} \phi^s_e & \text{if } \mathrm{C}(\iota(\phi^s_e)) \leq \omega \\ \mathrm{C}'(\mathrm{C}^{-1}(\omega)) & \text{if } \mathrm{C}(\iota(\phi^s)) < \omega < \mathrm{C}(\iota(\phi^s_e)) \\ \phi^s & \text{if } \omega \leq \mathrm{C}(\iota(\phi^s)). \end{cases}$$

In well-known proposition, Hayashi (1982) showed that for a competitive firm with constant returns to scale in both production and installation, the marginal value of capital that the firm uses to make the optimal investment decision, which Hayashi refers to as *(Tobin's) marginal q*, is equal to the ratio of the market value of the capital intalled in the firm to the replacement cost of capital, i.e., *Tobin's q*, which Hayashi refers to as *(Tobin's) average q*. Corollary 2 is a generalization of this proposition to our environment, which differs from Hayashi's more traditional neoclassical setting in two ways. First, we allow for heterogeneous valuations of the fundamental marginal revenue of capital installed inside the firm: these valuations may differ across investors as well as between investors and the entrepreneur who runs the firm. Second, firms in our model face financing constraints that, if binding, will affect investment decisions.

In Corollary 2 we define q^* as the marginal value of capital that the entrepreneur uses to make the firm's optimal investment decision, so the optimal investment rate always satisfies $\mathrm{C}'(x^*) = q^*$. Thus, q^* corresponds to what Hayashi refers to as *marginal q* in his neoclassical interpretation of Keynes and Tobin. In our model, the market price of k units of capital installed in a firm is $\phi^s k$ (expressed in terms of good 2), and the replacement cost of k units of capital is k (also in terms of good 2), so *Tobin's q* (what Hayashi refers to as *average q*) is equal to ϕ^s.

The main takeaway from Lemma 1 and Corollary 2 is that, unless $\phi^s < \phi^s_e$ and $\textsc{c}(\iota(\phi^s)) < \omega$, the firm's investment and equity issuance depend on the market price of equity, which in turn depends on the monetary policy rate, r. For firms run by entrepreneurs who assign a lower value to investment than the market, as in part (i) of Lemma 1, the relationship is simple: a lower policy rate leads to a higher stock price, which in turn induces these firms to invest more and to finance investment with equity issuance. For firms run by entrepreneurs who assign a higher value to investment than the market, as in part (ii) of Lemma 1, the relationship is more nuanced. On the one hand, investment and equity issuance are increasing in the market price of equity (and therefore decreasing in the monetary policy rate) for firms run by entrepreneurs who are sufficiently financially constrained, in the sense that $\omega \leq \textsc{c}(\iota(\phi^s(r)))$. On the other hand, investment and equity issuance decisions are independent of monetary policy for firms run by entrepreneurs who are financially unconstrained in the sense that $\textsc{c}(\iota(\phi^s(r))) < \omega$. In the following section we take these theoretical predictions to the data.

4.1.2 Monetary policy and stock prices: the *turnover liquidity* channel

Corollary 1 characterizes how monetary policy affects the market prices of equity and how the effects depend on microstructure. We focus on how microstructure is determined by the parameters that regulate trading frequency and the relative bargaining strengths of traders, α and θ, respectively.[15] The real price of equity in a monetary equilibrium is in part determined by the option available to low-valuation investors to resell the equity to high-valuation investors. If the nominal interest rate r increases, the equilibrium real money balances decline and the marginal investor valuation, ε^*, decreases. This reflects the fact that under the higher nominal rate, the investor valuation that was marginal under the lower rate is no longer indifferent between carrying cash and equity out of the OTC market – he prefers equity. Since the marginal investor who prices equity in the OTC market has a lower valuation, the value of the resale option is smaller, meaning the turnover liquidity of the asset is lower, which in turn makes the real equity price smaller.

In the OTC market, α is the probability an investor is able to contact a dealer, and θ is the investor's share of the gain from trade conditional on trading with a dealer. Thus, a larger $\alpha\theta$ implies a larger expected gain from trade for low-valuation investors when they sell the asset to

[15] Lagos and Zhang (2020b) also provide a characterization of how a mean-preserving spread in the distribution of valuations $G(\varepsilon)$ affects equity prices in a similar framework.

dealers. In turn, this makes investors more willing to hold equity shares in the previous period, as they anticipate larger gains from selling the equity in case they were to draw a relatively low valuation in the following OTC round. Thus, real equity prices ϕ^s are increasing in α and θ.

Following Lemma 6, all investors with $\varepsilon < \varepsilon^*$ who have a trading opportunity in the OTC market sell all their equity holding. Therefore, in a stationary equilibrium, the quantity of assets sold by investors to dealers in the OTC market is $\alpha G(\varepsilon^*) A_I^s$. Since the OTC market has to clear, the quantity of assets purchased by investors from dealers is also $\alpha G(\varepsilon^*) A_I^s$. So the total quantity of equity shares traded in the OTC market is $2\alpha G(\varepsilon^*) A_I^s$. Or, measured as the turnover *rate*, i.e. relative to total equity outstanding, $\mathcal{T} = \frac{2\alpha G(\varepsilon^*) A_I^s}{A_I^s} = 2\alpha G(\varepsilon^*)$.

In taking our theoretical predictions to the data, we take the standpoint that the empirically relevant parametrization, as referring to parameter conditions in Proposition 2, is that $\phi_e^s \in (\underline{\phi}^s, \overline{\phi}^s)$ and $r \in (\hat{r}, \bar{r})$. The assumption $\phi_e^s < \overline{\phi}^s$ requires that there exist some outside investors who value investment sufficiently more than the entrepreneurs. In our quantitative analysis of the structural model, we will assume that ε is lognormal, so that $\varepsilon_L = 0$ and $\varepsilon_H = +\infty$, and therefore $\overline{\phi}^s = +\infty$, $\bar{r} = +\infty$, and the assumptions $\phi_e^s < \overline{\phi}^s$ and $r < \bar{r}$ are immediately satisfied. More importantly, the assumption that $\phi_e^s > \underline{\phi}^s$ is equivalent to assuming that the *average* outside investor values investment less than the entrepreneur. This is necessary for there to exist firms that are not 100 percent financed by outside equity – an empirical motivation for the assumption. In Appendix B we incorporate a simple agency problem between entrepreneurs and investors to show that, in order to have an equilibrium with $\phi^s < \phi_e^s$, one need not assume that the fundamental value of the investment is higher for entrepreneurs than for outside investors.[16] Finally, $r > \hat{r}$ is also a necessary condition for there to exist firms that are not 100 percent outside equity financed.[17]

In the following, we provide a model-based motivation of our main empirical exercises.

4.1.3 Theoretical foundation for an empirical identification strategy

According to the theory, the real equity price decreases in response to an entirely unanticipated increase in the nominal interest rate. This happens due to a reduction in the resale option value accompanied by a fall in turnover liquidity. In addition, as stated in Corollary 1, the

[16]Intuitively, the agency problem makes outside equity a relatively more costly source of financing than inside equity, a suggested by the so-called *pecking order theory* (e.g., Myers and Majluf (1984)).

[17]Note that for any given set of parameters, including $r > 0$, one can always set ε_e high enough so that $\phi_e^s \to \overline{\phi}^s$ and $\hat{r} \to 0$, yielding $r > \hat{r}$.

strength of the effect on the price depends on the turnover liquidity of the stock (as captured by $\eta \equiv \alpha\theta$). This is because the component of the asset price associated with the expected value of the resale option discussed above is increasing in the turnover of the asset.

The first order effect of unanticipated increases in the nominal rate reducing equity prices has been widely studied empirically and would be implied by various other monetary transmission channels (e.g., the traditional interest-rate channel), based on which monetary policy can affect investment also in other ways than through equity prices, as discussed in the Introduction. However, a distinguishing feature of our framework is that stocks with higher turnover liquidity are predicted to exhibit stronger price responses to monetary policy. This prediction has been studied in depth by Lagos and Zhang (2020b), and tested empirically using daily stock return data on U.S. public firms. Stock turnover can thus be considered as a measure of firms' stock price exposure to monetary policy shocks. We will identify and estimate the q-monetary transmission channel, isolating it from other transmission channels, by focusing exclusively on this cross-sectional, between-firm variation in outcomes as predicted by stock turnover. In the following, we lay out how.

Our empirical approach relies on studying how firms' investment choices are explained, first and foremost, by variation in nominal interest rates across time and in stock turnover across firms. As the empirical counterparts of unanticipated changes in the nominal interest rate, we will consider identified monetary policy shocks, as discussed in Section 5.1 below. As for cross-sectional variation, in order to first fix ideas, we will think of explicit heterogeneity across firms generated only by their idiosyncratic ω's drawn from $\Omega(\omega)$, and by differences across firms i in $\eta^i \equiv \alpha^i\theta^i$.[18] We think of these differences in η^i being driven mainly by firm-specific characteristics that are unrelated to other exogenous features and parameters of the firm, such as the distribution $\Omega(\omega)$, or the firms' productivity in operating capital z. Below, we discuss concerns that may be raised if this is not the case and the cross-sectional variation in η^i was related to other firm-specific characteristics. A firm's optimal choices in the theoretical model can still naturally depend, either directly or indirectly, on its individual η^i. Since cross-sectional variation in η^i translates directly into variation in observed turnover rates $\mathcal{T}^i \equiv 2\alpha^i G(\varepsilon^{i*})$ through ε^{i*}, we will proxy for η^i in the data using observed stock turnover rates as discussed

[18] As in Lagos and Zhang (2020b), the theoretical model can be generalized to include multiple types of stocks with different characteristics. For each stock type, corresponding versions of (14) and (15) will still determine ε^{i*} and the market stock price $\phi^{s,i}$. We think of these different types of stocks capturing firms with different stock η's.

in Section 5.1 below.[19]

In the following discussion, for brevity, we focus on firms' stock prices and investment rates as the main outcomes of interest, although our model also has predictions for equity issuance rates and we study these in our empirical exercises. As stated above, our main empirical exercises employ variation in nominal interest rates across time and in stock turnover across firms. In order to notationally emphasize these sources of empirical variation, we introduce the sub- and superscripts on r_t and η^i in the following. To make our theoretical results operational, we use the results from Proposition 2, specified to stocks heterogeneous in η^i, and define the following objects:

$$\phi^s(r_t, \eta^i) \equiv \beta \left[\bar{\varepsilon} + \mathcal{R}(r_t, \eta^i)\right] z$$
$$\text{where } \mathcal{R}(r_t, \eta^i) \equiv \eta^i \int_{\varepsilon_L}^{\varepsilon^*(r_t,\eta^i)} \left(\varepsilon^*(r_t, \eta^i) - \varepsilon\right) dG(\varepsilon),$$

with $\mathcal{R}$ referring to the resale option value and $\varepsilon^*(r_t, \eta^i)$ defined as the unique $\varepsilon^* \in (\varepsilon_L, \varepsilon_H)$ that solves

$$\eta^i \int_{\varepsilon^*}^{\varepsilon_H} \frac{\varepsilon - \varepsilon^*}{\varepsilon^*} dG(\varepsilon) = r_t$$

Following Proposition 2 and our discussion above on the empirically relevant parameter assumptions, let us also use the changes of variables $\ell(\phi) \equiv \log(\iota(\phi))$ and $\hat{\mathrm{C}}(\ell) \equiv \mathrm{C}(\exp(\ell))$ and define:

$$\begin{aligned} X^*(r_t, \eta^i) \quad \equiv \quad & \Omega[\hat{\mathrm{C}}(\ell(\phi^s(r_t, \eta^i)))]\ell(\phi^s(r_t, \eta^i))) + \int_{\hat{\mathrm{C}}(\ell(\phi^s(r_t,\eta^i)))}^{\hat{\mathrm{C}}(\ell(\phi^s_e))} \hat{\mathrm{C}}^{-1}(\omega) d\Omega(\omega) \\ & + \{1 - \Omega[\hat{\mathrm{C}}(\ell(\phi^s_e))]\}\ell(\phi^s_e) \end{aligned}$$

The objects $\phi^s(r_t, \eta^i)$ and $X^*(r_t, \eta^i)$ are simply the real equity price and average log investment rate of firms with $\alpha^i\theta^i = \eta^i$, conditional on the nominal interest rate r_t. We focus on the *average* investment rate because our baseline empirical approach employs variation in r and η, not controlling for other firm-level covariates such as the liquid financial wealth level ω. However, later we will also consider splitting firms based on an empirical proxy for ω, in which case the corresponding object of interest is the average log investment rate *conditional* on ω lying in some interval. We focus on the *log* investment rate since it will provide a better fit of the data given the skewness in the firm-level investment rates, as discussed in Abel and Eberly

[19] To be precise, this is the case as long as α^i and $\alpha^i\theta^i = \eta^i$ are not sufficiently negatively related in the cross-section.

(2002). We verify in robustness tests that all our empirical findings hold when using investment rate levels as outcomes instead.

Our identification and estimation approach relies on assessing how the effects of exogenous variations in nominal interest rates on firms' outcomes are explained by their stock turnover. That is, we will essentially be estimating the relevance of a "cross-term" in η^i and shocks to r_t, and we will let all first-order variation across time and firms be captured by time and firm fixed effects.[20] Because of this, we find it most helpful to organize the discussion of identification and drawing a parallel between the theory and our reduced form empirical regressions using a second order approximation of the functions ϕ^s and X^* in (r_t, η^i).

In what is to follow, we will use the notation $\tilde{y} \equiv y(\tilde{r}, \tilde{\eta})$, $\tilde{y}_r \equiv \frac{\partial y}{\partial r_t}(\tilde{r}, \tilde{\eta})$ or $\tilde{y}_{r\eta} \equiv \frac{\partial y}{\partial r_t \partial \eta^i}(\tilde{r}, \tilde{\eta})$ to refer to any function $y(r_t, \eta^i)$ and its derivatives, evaluated at the reference point $(\tilde{r}, \tilde{\eta})$. In the case of composite functions, such as $X^*(r_t, \eta^i) = X^*(\phi^s(r_t, \eta^i))$, in order to elaborate upon the channels of transmission, with a slight abuse of notation we also introduce terms along the lines of $\tilde{X}^*_{\phi^s} \equiv \frac{\partial X^*}{\partial \phi^s}(\phi^s(\tilde{r}, \tilde{\eta}))$. When explicit distinction between total and partial derivatives becomes necessary later on, we will use $\tilde{y}_r$ to denote the partial derivative of y with respect to r_t, and $\frac{dy}{dr_t}(\tilde{r}, \tilde{\eta})$ for the total derivative.

Lemma 2 *The second order Taylor approximations of $\phi^s(r_t, \eta^i)$ and $X^*(r_t, \eta^i)$ around a reference point of $(\tilde{r}, \tilde{\eta})$ take the following forms:*

$$\phi^s(r_t, \eta^i) \approx d_{\phi^s,t} + f^i_{\phi^s} + \tilde{\phi}^s_{r\eta} \cdot r_t \eta^i \tag{16}$$

$$X^*(r_t, \eta^i) \approx d_{X,t} + f^i_X + \tilde{X}^*_{r\eta} \cdot r_t \eta^i \tag{17}$$

where $d_{\phi^s,t}$ and $d_{X,t}$ include constants and terms that vary only in r_t, including the first order effects $\tilde{\phi}^s_r \cdot r_t$ and $\tilde{X}^*_r \cdot r_t$, respectively. $f^i_{\phi^s}$ and f^i_X include terms that vary only in η^i. The

[20] Because in the empirics of Section 5 and in the quantitative theoretical model of Section 6, firms live more than one period and the data consists of a panel of firms, there will also be firm-level controls that vary across time.

relevant coefficients are given by:

$$\begin{aligned}
\tilde{\phi}^s_{r\eta} &= \beta z \tilde{\mathcal{R}}_{r\eta} < 0 \\
\tilde{X}^*_{r\eta} &= \tilde{X}^*_{\phi^s} \times \tilde{\phi}^s_{r\eta} + \tilde{X}^*_{\phi^s\phi^s} \times \tilde{\phi}^s_{\eta} \times \tilde{\phi}^s_r \\
& \quad \textit{where} \\
\tilde{\phi}^s_r &= \beta z \tilde{\mathcal{R}}_r < 0, \quad \tilde{\phi}^s_\eta = \beta z \tilde{\mathcal{R}}_\eta > 0 \\
\tilde{X}^*_r &= \tilde{X}^*_{\phi^s} \times \tilde{\phi}^s_r \\
\tilde{X}^*_{\phi^s} &= \Omega[\hat{\mathrm{c}}(\ell(\tilde{\phi}^s))]\ell'(\tilde{\phi}^s) \\
\tilde{X}^*_{\phi^s\phi^s} &= \Omega'[\hat{\mathrm{c}}(\ell(\tilde{\phi}^s))]\hat{\mathrm{c}}'(\ell(\tilde{\phi}^s))[\ell'(\tilde{\phi}^s)]^2 + \Omega[\hat{\mathrm{c}}(\ell(\tilde{\phi}^s))]\ell''(\tilde{\phi}^s)
\end{aligned}$$

and $\tilde{\mathcal{R}}_\eta > 0$*,* $\tilde{\mathcal{R}}_r < 0$*, and* $\tilde{\mathcal{R}}_{r\eta} < 0$ *are the derivatives of the resale option value at* $(\tilde{r}, \tilde{\eta})$*. If* $\Omega[\hat{\mathrm{c}}(\ell(\tilde{\phi}^s))] > 0$*, then* $\tilde{X}^*_{\phi^s} > 0$*. If, in addition,* $\ell''(\tilde{\phi}^s) \geq 0$*, then* $\tilde{X}^*_{\phi^s\phi^s} \geq 0$ *and* $\tilde{X}^*_{r\eta} < 0$*.*

Lemma 2 formalizes how we view our reduced form empirical regressions of Section 5.2 through the lense of our model. We think of $\tilde{\phi}^s_{r\eta}$ as a theoretical analogue for the empirical regression coefficient on the interaction term between an identified shock in nominal rates r_t and a measure of firm i's stock turnover just before the rate shock, when firms' market-based q is regressed on this interaction and a collection of time and firm fixed effects. Because in our model the effective market purchase price for capital goods is constant across firms and time, fluctuations in ϕ^s correspond to fluctuations in market-based average q. $\tilde{X}^*_{r\eta}$ is the corresponding coefficient when the outcome variable of interest is the firm's log investment rate.

As per Corollary 1, $\tilde{\phi}^s_{r\eta} < 0$ means that firms with higher stock turnover experience a larger drop in equity prices after exogenous nominal rate increases. This happens because higher stock turnover, as captured by η^i, implies a larger resale option value component $\mathcal{R}$ of the stock price ϕ^s, which falls more severely in response to a contractionary monetary shock ($\tilde{\mathcal{R}}_{r\eta} < 0$). Note also that the first order "level effect" of an increase in r_t that moves all stock prices down either directly or through general equilibrium effects common to all firms is captured within the term $d_{\phi^s,t}$, a theoretical analogue of a time fixed effect.

Equation (17) captures, through $\tilde{X}^*_{r\eta}$, how exogenous changes in nominal rates affect the investment of firms with different stock turnover differently. In our basic model of the q-monetary transmission channel, the only way that changes in the nominal rate r (or in η) can affect investment is by changing the stock price ϕ^s. Because of this, the partial derivatives of X^* with respect to ϕ^s, denoted as $\tilde{X}^*_{\phi^s}$ and $\tilde{X}^*_{\phi^s\phi^s}$, play a central role in monetary transmission.

Note that $\tilde{X}^*_{\phi^s}$ and $\tilde{X}^*_{\phi^s\phi^s}$ are the first and second order *causal* effects of market equity prices ϕ^s on the average log investment rate X^*. The first order effect $\tilde{X}^*_{\phi^s}$, captures the fact that in response to an increase in ϕ^s, the mass of entrepreneurs $\Omega[\hat{\mathrm{C}}(\ell(\phi^s))] = \Omega[\mathrm{C}(\iota(\phi^s))]$ for whom $\omega < \mathrm{C}(\iota(\phi^s))$, all issue equity and invest at rate $\iota(\phi^s)$, or $\ell(\phi^s)$ in logs. When ϕ^s increases, each of them increases their log investment by $\ell'(\phi^s)$. Whenever $\ell'(\phi^s) > 0$ and $\Omega[\mathrm{C}(\iota(\phi^s))] > 0$, meaning that there exists a non-zero mass of entrepreneurs issuing equity, we have that $\tilde{X}^*_{\phi^s} > 0$. The second order effect $\tilde{X}^*_{\phi^s\phi^s}$ consists of two components. First, a higher ϕ^s increases the log optimal investment rate following the outside investors' marginal valuation, $\ell(\phi^s)$, by $\ell'(\phi^s)$. This increases the financial wealth cutoff $\hat{\mathrm{C}}(\ell(\phi^s)) = \mathrm{C}(\iota(\phi^s))$ below which all entrepreneurs issue equity, in turn generating an extensive margin effect by increasing the mass of entrepreneurs $\Omega[\hat{\mathrm{C}}(\iota(\phi^s))]$ who invest at $\ell(\phi^s)$. All these entrepreneurs respond to increases in ϕ^s by $\ell'(\phi^s)$. These effects constitute the term $\Omega'[\hat{\mathrm{C}}(\ell(\tilde{\phi}^s))]\hat{\mathrm{C}}'(\ell(\tilde{\phi}^s))[\ell'(\tilde{\phi}^s)]^2 \geq 0$. The second term, $\Omega[\hat{\mathrm{C}}(\ell(\tilde{\phi}^s))]\ell''(\tilde{\phi}^s)$, appears whenever equity issuing entrepreneurs' log investment $\ell(\phi^s)$ is nonlinear in ϕ^s. As soon as $\ell''(\tilde{\phi}^s) \geq 0$, we have that $\Omega[\hat{\mathrm{C}}(\ell(\tilde{\phi}^s))]\ell''(\tilde{\phi}^s) \geq 0$ and necessarily $\tilde{X}^*_{\phi^s\phi^s} \geq 0$.

To unpack the heterogeneous effects of monetary transmission to firms' investment, let us decompose the contents of $\tilde{X}^*_{r\eta}$ in parts. First, following the above discussion, the component $\tilde{X}^*_{\phi^s} \times \tilde{\phi}^s_{r\eta} \leq 0$ captures the fact that stocks with higher turnover experience a larger drop in price when r increases ($\tilde{\phi}^s_{r\eta} < 0$). And this drop in equity prices has a first order effect decreasing average log investment ($\tilde{X}^*_{\phi^s} \geq 0$). These effects are strictly different from zero whenever there is a non-zero mass $\Omega[\mathrm{C}(\iota(\phi^s))] > 0$ of firms issuing equity. The second component, $\tilde{X}^*_{\phi^s\phi^s} \times \tilde{\phi}^s_\eta \times \tilde{\phi}^s_r$, can be easiest explained by the fact that a higher η in the cross-section implies a higher stock price ϕ^s, by $\tilde{\phi}^s_\eta > 0$. For reasons discussed above, e.g. due to a changing mass of equity issuing firms, the marginal effect of changes in the stock price ϕ^s on X^* can itself depend on the level of ϕ^s, as captured by $\tilde{X}^*_{\phi^s\phi^s}$. So when there is an increase in r that decreases stock prices by $\tilde{\phi}^s_r < 0$, the resulting effect on the investment of firms with higher η is stronger by $\tilde{X}^*_{\phi^s\phi^s} \times \tilde{\phi}^s_\eta$. Again, as soon as, $\ell''(\tilde{\phi}^s) \geq 0$, we have that necessarily, $\tilde{X}^*_{\phi^s\phi^s} \times \tilde{\phi}^s_\eta \times \tilde{\phi}^s_r \leq 0$.

Our model thus predicts that whenever there is a non-zero mass of firms issuing equity and the log investment policy function $\log(\phi^s)$ is not ("too") concave, we have that $\tilde{X}^*_{r\eta} < 0$ and the investment of firms with higher stock turnover drops relatively more in response to increases in the nominal interest rate. This happens because of their stock prices being relatively more responsive to the interest rate, and their higher stock prices making their investment more responsive to any given stock price changes.

Finally, note that for market equity prices (and monetary policy) to have an effect on firms' investment in our basic model, it must be the case that there exist firms who use equity financing for investment at the margin. Given our empirically relevant parameter assumption that entrepreneurs value investment relatively more than the average outside investor, this means that there must exist firms with sufficiently low financial wealth available for purchasing capital. If all firms in a (sub)sample with liquid financial wealth distribution $\Omega(\omega)$ had so much wealth that, at the margin, none of them were issuing equity, i.e. $\Omega[\hat{\mathrm{C}}(\ell(\tilde{\phi}^s))] = \Omega'[\hat{\mathrm{C}}(\ell(\tilde{\phi}^s))] = 0$, then by Lemma 2, $\tilde{X}^*_{r\eta} = 0$ since nominal rates have no effects on investment whatsoever. On the contrary, if one were able to focus on a (sub)sample of firms with a low level of financial wealth $\Omega(\omega)$, then for these firms, the implications for heterogeneous monetary transmission become considerably more clear-cut. This becomes most evident in the following Corollary for an especially low financial wealth distribution.

Corollary 3 *If $\Omega[\mathrm{C}(\iota(\tilde{\phi}^s))] = 1$ and $\ell''(\tilde{\phi}^s) = 0$, then the cross-term coefficient in approximation (17) is $\tilde{X}^*_{r\eta} = \tilde{X}^*_{\phi^s} \times \tilde{\phi}^s_{r\eta}$, with $\tilde{X}^*_{\phi^s} = \ell'(\tilde{\phi}^s)$.*

Corollary 3 motivates our focus on splitting firms into subsamples based on their relative liquid financial wealth in the empirical work of Section 5 below. Although the Corollary's assumption of $\Omega[\mathrm{C}(\iota(\tilde{\phi}^s))] = 1$, meaning all considered firms rely on external equity issuances due to low financial liquidity is extreme, it is an insightful limiting case for informing our empirical approach for two reasons.

First, the fact that the coefficient $\tilde{X}^*_{r\eta}$ equals exactly $\tilde{X}^*_{\phi^s} \times \tilde{\phi}^s_{r\eta}$, means that by first estimating $\tilde{\phi}^s_{r\eta}$ from the empirical equivalent of (16), the ratio $\tilde{X}^*_{r\eta}/\tilde{\phi}^s_{r\eta}$ would exactly identify the *first order causal effect* of equity prices on log investment, $\tilde{X}^*_{\phi^s}$. Based on basic two stage instrumental variables regression logic, this observation is the motivation for our IV regressions of Section 5.3, where firms' log investment rates are projected onto market-based measures of average q, instrumented with an empirical equivalent of $r_t\eta^i$. The empirical equivalent of (16) will constitute the first stage of this IV approach, and the focus on cross-sectional variation is ensured by the inclusion of time fixed effects throughout.

The second reason for why focusing on firms with relatively low liquid financial wealth can be useful in empirical work, is that by maximizing the fraction of firms that rely on equity financing $\Omega[\hat{\mathrm{C}}(\ell(\phi^s))] = \Omega[\mathrm{C}(\iota(\phi^s))]$ in a given subsample, we are increasing the first order causal effect of stock prices on the average log investment rate, i.e. $\tilde{X}^*_{\phi^s} = \Omega[\hat{\mathrm{C}}(\ell(\tilde{\phi}^s))]\iota'(\tilde{\phi}^s)$, and thus increasing the likelihood of finding statistically and economically significant estimates. Moreover, $\Omega[\hat{\mathrm{C}}(\ell(\tilde{\phi}^s))] \to 1$ should also imply that the estimated effect of stock prices on *average* log investment rates in the subsample approximates the effect stock prices on log investment *conditional* on issuing equity, $\tilde{X}^*_{\phi^s} \to \ell'(\tilde{\phi}^s)$.

4.1.4 Identifying q-monetary transmission in the presence of a traditional interest-rate channel

A fundamental part of the q-monetary transmission channel is the causal effect of stock prices on firms' investment. While the cross-sectional heterogeneity in investment responses to exogenous nominal rate changes, $\tilde{X}^*_{r\eta}$, is informative of these effects denoted by $\tilde{X}^*_{\phi^s}$ and $\tilde{X}^*_{\phi^s\phi^s}$, it might not be immediately obvious why one should necessarily focus on this cross-sectional variation. In our basic model which features *only* the q-monetary transmission channel, it would be enough to just use identified monetary policy shocks to identify and estimate the first-order effects of r on ϕ^s and on X^*, captured by $\tilde{\phi}^s_r$ and $\tilde{X}^*_r = \tilde{X}^*_{\phi^s} \times \tilde{\phi}^s_r$. The ratio $\tilde{X}^*_r/\tilde{\phi}^s_r$, or a corresponding IV coefficient, would then provide an estimate of the first order causal effect of stock prices on investment, $\tilde{X}^*_{\phi^s}$.

However, such an approach relying on first order "level effects" of monetary shocks will not suffice to identify or measure the q-channel as soon as there are other transmission channels through which nominal rate changes can affect investment, potentially through intricate general equilibrium effects. For example, by lowering demand for the firm's production and decreasing profits, a contractionary monetary shock can cause stock prices and investment to fall. But it is not necessarily the case that investment falls *because* the stock price falls. Rather, both are endogenous outcomes responding to a worsening of demand. Thus, by simply observing the comovements of q and investment in the aftermath of monetary shocks, one cannot hope to make any causal statements. We will illustrate these ideas by introducing into our basic model a notion of the most common alternative in the monetary transmission literature – the "traditional" interest-rate channel, based on which increases in nominal monetary policy rates lead to increases in real interest rates.

In our basic model, the net real interest rate is pinned down by the agents' linear utility functions and their time discount rate at $\frac{1}{\beta} - 1$. In order to introduce a notion of exogenous changes in the nominal policy rate affecting real rates, and to investigate how this would affect our identification and estimation of the q-monetary transmission channel, we will assume that the subjective time discount factor β, shared by all agents in the economy, is a decreasing function of the nominal rate r, with $\beta'(r) < 0$.[21] Thus, any exogenous increase in r will also increase the effective real interest rate, and purely through higher discounting, have an effect on both market equity prices and investment, with the causality not necessarily running from the former to the latter. This becomes clear in the following analogue of Lemma 2.

Lemma 3 *Suppose that β is a decreasing function of r, and define $\tilde{\beta} \equiv \beta(\tilde{r})$, $\tilde{\beta}_r \equiv \beta'(\tilde{r})$. The second order Taylor approximations of $\phi^s(r_t, \eta^i)$ and $X^*(r_t, \eta^i)$ around a reference point of $(\tilde{r}, \tilde{\eta})$ take the following forms:*

$$\phi^s(r_t, \eta^i) \approx d_{\phi^s,t} + f^i_{\phi^s} + \tilde{\phi}^s_{r\eta} \cdot r_t \eta^i \tag{18}$$

$$X^*(r_t, \eta^i) \approx d_{X,t} + f^i_X + \tilde{X}^*_{r\eta} \cdot r_t \eta^i \tag{19}$$

where $d_{\phi^s,t}$ and $d_{X,t}$ include constants and terms that vary only in r_t, including the first order effects $\tilde{\phi}^s_r \cdot r_t$ and $\tilde{X}^*_r \cdot r_t$, respectively. $f^i_{\phi^s}$ and f^i_X include terms that vary only in η^i. The relevant coefficients are given by:

$$\begin{aligned}
\tilde{\phi}^s_{r\eta} &= [\tilde{\beta}\tilde{\mathcal{R}}_{r\eta} + \tilde{\beta}_r \tilde{\mathcal{R}}_\eta] z < 0 \\
\tilde{X}^*_{r\eta} &= \tilde{X}^*_{\phi^s} \times \tilde{\phi}^s_{r\eta} + \tilde{X}^*_{\phi^s\phi^s} \times \tilde{\phi}^s_\eta \times \tilde{\phi}^s_r \\
&\quad \textit{where} \\
\tilde{\phi}^s_r &= [\tilde{\beta}\tilde{\mathcal{R}}_r + \tilde{\beta}_r(\bar{\varepsilon} + \tilde{\mathcal{R}})] z < 0, \quad \tilde{\phi}^s_\eta = \beta z \tilde{\mathcal{R}}_\eta > 0, \quad \tilde{\phi}^s_{e,r} = \tilde{\beta}_r z \varepsilon_e < 0 \\
\tilde{X}^*_r &= \tilde{X}^*_{\phi^s} \times \tilde{\phi}^s_r + \tilde{X}^*_{\phi^s_e} \times \tilde{\phi}^s_{e,r} \\
\tilde{X}^*_{\phi^s_e} &= \{1 - \Omega[\hat{\mathrm{c}}(\ell(\tilde{\phi}^s_e))]\} \ell'(\tilde{\phi}^s_e)
\end{aligned}$$

and $\tilde{\mathcal{R}}_\eta > 0$, $\tilde{\mathcal{R}}_r < 0$, and $\tilde{\mathcal{R}}_{r\eta} < 0$ are the derivatives of the resale option value at $(\tilde{r}, \tilde{\eta})$. The functional forms of the partial derivatives $\tilde{X}^*_{\phi^s}$ and $\tilde{X}^*_{\phi^s\phi^s}$ remain unchanged from Lemma 2. If $\Omega[\hat{\mathrm{c}}(\ell(\tilde{\phi}^s))] > 0$, then $\tilde{X}^*_{\phi^s} > 0$. If, in addition, $\ell''(\tilde{\phi}^s) \geq 0$, then $\tilde{X}^*_{\phi^s\phi^s} \geq 0$ and $\tilde{X}^*_{r\eta} < 0$. If $\Omega[\hat{\mathrm{c}}(\ell(\tilde{\phi}^s_e))] < 1$, then $\tilde{X}^*_{\phi^s_e} > 0$.

[21] Note that since β appears in the equilibrium conditions of our basic model with one-period-lived firms always alongside capital productivity as the term βz, all of the insights in this section apply if monetary policy operated through a "demand channel" with higher nominal rates decreasing the real average (and marginal) product of capital.

Several key differences between the terms in Lemmas 3 and 2 arise. First, when the nominal rate r increases, naturally due to stronger discounting, market equity prices fall through the effect of r on β, and not only due to the fall in the resale option value $\mathcal{R}$. Moreover, because β and $\mathcal{R}$ appear in the equity price ϕ^s in a multiplicative manner, there is an additional reason why stocks with higher turnover experience a larger unit drop in real price, $\tilde{\beta}_r \tilde{R}_\eta < 0$. Thus, both $\tilde{\phi}^s_r$ and $\tilde{\phi}^s_{r\eta}$ are now more negative compared to the baseline case with $\beta'(r) = 0$. However, the heterogeneity in semi-elasticities $\frac{\partial \log(\phi^s)}{\partial r}$ explained by cross-sectional variation in η^i would be identical in the two cases.

As for average log investment, note that it is still the case that X^* does not directly depend on r, η or β. However, it does depend on the entrepreneurs' fundamental valuation of investment ϕ^s_e, which is itself a function of β. If the real rate increases, i.e. β falls, entrepreneurs discount the future more and value capital less ($\tilde{\phi}^s_{e,r} < 0$). As a result, unconstrained entrepreneurs with $\omega > \mathrm{C}(\iota(\phi^s_e))$ want to reduce investment even though they do not finance themselves with equity at the margin and thus do not care about equity prices ($\tilde{X}^*_{\phi^s_e} \geq 0$). We view this as the manifestation of a traditional interest-rate channel of monetary transmission in our model. Most importantly, because of this force, whenever there exist unconstrained firms ($\Omega[\hat{\mathrm{C}}(\ell(\tilde{\phi}^s_e))] < 1$), the first order effect of r on average investment, $\tilde{X}^*_r$, is "contaminated" by the term $\tilde{X}^*_{\phi^s_e} \times \tilde{\phi}^s_{e,r} < 0$ introduced by the traditional interest-rate channel. So even if one used well-identified nominal interest rate shocks to estimate $\tilde{\phi}^s_r$ and $\tilde{X}^*_r$, or ran an IV regression of X^* on market-based q where the latter was instrumented with the monetary shocks, the resulting coefficient ratio $\tilde{X}^*_r / \tilde{\phi}^s_r = \tilde{X}^*_{\phi^s} + \tilde{X}^*_{\phi^s_e} \times \tilde{\phi}^s_{e,r} / \tilde{\phi}^s_r$ no longer allows to claim identification of a causal effect of stock prices on investment, $\tilde{X}^*_{\phi^s}$. It would capture both the effects of the q-monetary and the traditional interest-rate channel, with the latter not operating through stock prices.

The benefits of employing investment response heterogeneity as explained by stock turnover now become clear. Note that the functional forms of $\tilde{X}^*_{r\eta}$, $\tilde{X}^*_{\phi^s}$, and $\tilde{X}^*_{\phi^s\phi^s}$ in Lemma 3 are identical to those in Lemma 2. The key insight for why this happens is that the traditional interest-rate channel effect of r on investment does not depend on η, and is "differenced out" when comparing the investment responses between firms with different stock turnover. Although it is the case that for reasons discussed above, equity price responses to r are larger and more heterogeneous ($\tilde{\phi}^s_r$ and $\tilde{\phi}^s_{r\eta}$ are more negative) in the presence of the interest-rate channel, and this translates to a more negative $\tilde{X}^*_{r\eta}$, firms with higher stock turnover exhibit stronger investment responses to nominal rate changes still only due to a causal effect of equity prices on investment. And conditional on equity price responses, $\tilde{X}^*_{r\eta}$ is still equally informative about the causal effects of equity prices on investment as in the absence of the real rate channel. Most importantly, note that even if $\beta'(r) < 0$, Corollary 3 remains unchanged. And an estimate of the ratio $\tilde{X}^*_{r\eta}/\tilde{\phi}^s_{r\eta}$, or an equivalent IV regression coefficient, among low-wealth firms would still identify the *first order causal effect* of equity prices on investment, $\tilde{X}^*_{\phi^s}$.

4.1.5 Identifying q-monetary transmission with endogenous financial wealth

The above discussion has illustrated how cross-sectional variation in firms' stock turnover can be useful in the empirical identification of the q-monetary transmission channel and how it can allow to "difference out" other transmission channels, such as the traditional interest-rate channel. However, owing to the stylized nature of the model with one-period-lived firms, the prior analysis has also relied on the implicit assumption that the cross-sectional variation in stock turnover driven by η is unrelated to individual firm characteristics which could either be exogenous themselves (e.g. productivity), or the result of firms' choices made in the past (e.g. financial wealth distributed as $\Omega(\omega)$). This would be a valid empirical assumption if stock turnover at any point in time were assigned randomly across firms in the data. But as soon as stock turnover is correlated with any firm characteristics which could affect firms' investment responsiveness to nominal rate changes for reasons other than the q-monetary transmission channel, then our empirical estimates might be biased and would not provide a precise measure of the channel or the causal effects of stock prices on investment.

In our basic model of q-monetary transmission, between-firm differences in η naturally lead to heterogeneity in the levels of their equity prices ϕ^s. If firms live for more than one period, this can affect their willingness to accumulate financial wealth ω, in turn influencing their exposure to any future changes in stock prices caused by nominal rate movements. The following Lemma shows that if the entrepreneurs' distribution of liquid financial wealth ω is correlated with stock turnover η in our basic model, the interpretation of our main estimate can change slightly, but it still allows to provide a measure of the q-monetary transmission channel.

Lemma 4 *Suppose that the financial wealth of entrepreneurs with turnover parameter η is distributed according to the cumulative distribution function $\Omega(\omega, \eta)$. The second order Taylor approximation of $X^*(r_t, \eta^i)$ around a reference point of $(\tilde{r}, \tilde{\eta})$ takes the following form:*

$$X^*(r_t, \eta^i) \quad \approx \quad d_{X,t} + f^i_X + \frac{d^2 X^*}{dr_t d\eta}(\tilde{r}, \tilde{\eta}) \cdot r_t \eta^i \tag{20}$$

where $d_{X,t}$ includes constants and terms that vary only in r_t, including the first order effect $\tilde{X}^*_r \cdot r_t$. f^i_X includes terms that vary only in η^i. The relevant coefficient is given by:

$$\begin{aligned}\frac{d^2X^*}{dr_t d\eta}(\tilde{r},\tilde{\eta}) &= \tilde{X}^*_{\phi^s} \times \tilde{\phi}^s_{r\eta} + (\tilde{X}^*_{\eta\phi^s} + \tilde{X}^*_{\phi^s\phi^s} \times \tilde{\phi}^s_\eta) \times \tilde{\phi}^s_r \\ &\quad \textit{where} \\ \tilde{X}^*_{\eta\phi^s} &= \Omega_\eta[\hat{\mathrm{C}}(\ell(\tilde{\phi}^s)),\tilde{\eta}]\ell'(\tilde{\phi}^s)\end{aligned}$$

and $\Omega_\eta(\omega,\eta) \equiv \frac{\partial\Omega}{\partial\eta}(\omega,\eta)$. The functional forms of the partial derivatives $\tilde{X}^*_r$, $\tilde{X}^*_{\phi^s}$ and $\tilde{X}^*_{\phi^s\phi^s}$, and the second order Taylor approximation of $\phi^s(r_t,\eta^i)$, including the derivatives $\tilde{\phi}^s_r$, $\tilde{\phi}^s_\eta$, and $\tilde{\phi}^s_{r\eta}$ remain unchanged from Lemma 2.

The general way of how an increase in r can have heterogenous effects on the investment of firms with different η is unchanged compared to the baseline model covered by Lemma 2: a higher η leads to stronger equity prices responses ($\tilde{\phi}^s_{r\eta} < 0$) to which investment then responds (by $\tilde{X}^*_{\phi^s} \geq 0$); and a higher η predicts different investment sensitivity to equity prices (by $\tilde{X}^*_{\eta\phi^s} + \tilde{X}^*_{\phi^s\phi^s} \times \tilde{\phi}^s_\eta$) which fall in response to nominal rate increases ($\tilde{\phi}^s_r < 0$). Now, this second channel includes an additional reason for why a higher η predicts differences in the responsiveness of average investment to equity prices: because the distribution Ω directly depends on η, a change in η, all else equal, changes the mass of firms who issue equity and invest at $\ell(\phi^s)$ at any given stock price (by $\Omega_\eta[\hat{\mathrm{C}}(\ell(\tilde{\phi}^s)),\tilde{\eta}]$). So if there is a fall in ϕ^s caused by a higher r, and each equity issuer decreases log investment by $\ell'(\phi^s)$ as a result, the average log investment response is affected because of the direct effect of η on the mass of issuers. If $\Omega_\eta[\hat{\mathrm{C}}(\ell(\tilde{\phi}^s)),\tilde{\eta}] \geq 0$, meaning that higher stock turnover is associated with lower liquid financial wealth in the cross-section of firms, all else equal, then $\tilde{X}^*_{\eta\phi^s} \geq 0$ and the main coefficient of interest on the cross-term $r_t\eta^i$ is more negative than in the baseline scenario.

Even though there is now an additional force that could potentially generate a (more) negative coefficient on the cross-term $r_t\eta^i$ and make firms with higher stock turnover exhibit more responsive investment rates to changes in r, it is still the case that this force appears because stock prices have a causal effect on investment. Because of this, our basic model says that estimates of stock turnover predicting stronger investment responsiveness should provide proof of the q-monetary transmission channel even in cases when financial wealth positions are correlated with stock turnover. Moreover, if we were able to focus on an extreme subsample with such low liquid financial wealth that $\Omega[\hat{\mathrm{C}}(\ell(\tilde{\phi}^s)),\tilde{\eta}] = 1$ provides a valid approximation, then this added extensive margin effect disappears, and Corollary 3 again remains unchanged.

4.1.6 Identifying q-monetary transmission when other transmission channels are correlated with η

The discussion in Section 4.1.5 illustrates how firms' investment sensitivity to changes in the nominal rate, as explained by cross-sectional variation in stock turnover, allows to identify and assess the q-monetary transmission channel and the causal effects of equity prices on investment even if stock turnover and financial wealth are correlated in the cross-section. Yet these insights still rely on a model with only the q-monetary transmission channel being present and on the key identifying assumption that cross-sectional variation in stock turnover is not related to any reasons for investment to respond heterogeneously to changes in r, other than through equity prices.

If stock turnover was truly assigned randomly immediately prior to the realization of monetary shocks, the assumption for our approach identifying q-monetary transmission and the causality of stock prices affecting investment would be satisfied. The fact that this is unlikely to be the case in reality leads to potential concerns in the validity of a causal interpretation of our estimates. The randomness of variation in stock turnover could be violated in either of two main ways:

a) Certain firm characteristics cause their stocks to be traded relatively more or less. For example, the stocks of bigger firms may provide a more liquid market and invite higher trading activity, even relative to their potentially large market capitalizations.

b) For reasons not related to firm characteristics, stocks may experience heterogeneous trading activity. As suggested by the model, this can lead to differences in stock prices which in turn can affect firm behavior, for example their financing or portfolio decisions.

Either of these scenarios can introduce covariance between certain firm characteristics and stock turnover. If these characteristics are in turn predictive of firms' responsiveness to monetary policy shocks in their own right through other channels, as for example leverage could be (Anderson and Cesa-Bianchi, 2020), then any heterogeneous behavior predicted by turnover alone could instead be explained by reasons other than heterogeneous stock price changes, violating the assumptions necessary for identification.

To formally illustrate how such concerns would materialize in our empirical approach, and how we aim to alleviate them with additional controls in robustness analysis, suppose that there was another firm characteristic ϑ which affected the strength of the transmission of the traditional interest-rate channel to each individual firm.[22] That is, suppose that the discount rate applied to firm i's next period returns was $\beta(r, \vartheta^i)$, with $\beta_{r\vartheta} \equiv \frac{\partial^2 \beta}{\partial r \partial \vartheta} \neq 0$. Moreover, suppose that in the cross-section of firms, ϑ^i and stock turnover η^i were correlated. To formalize this idea, let us assume that firm i's ϑ^i and stock turnover η^i are related through the function $\vartheta^i = \vartheta(\eta^i)$. Finally, suppose that we can observe a proxy for ϑ^i in the firm-level data. Part (i) of the following Lemma shows how this scenario could compromise our basic empirical approach for identifying q-monetary transmission and the causal effects of stock prices on investment. Part (ii) shows how controlling for ϑ^i in empirical regressions can help in improving the identification.

Lemma 5 *Suppose that $\beta(r, \vartheta)$ is a decreasing function of r and the effects of r on β depend on a firm-specific characteristic ϑ. Suppose that ϑ is a function of the firm's stock turnover η. Define $\tilde{\vartheta} \equiv \vartheta(\tilde{\eta})$, $\tilde{\vartheta}_\eta \equiv \vartheta'(\tilde{\eta})$, $\tilde{\beta} \equiv \beta(\tilde{r}, \tilde{\vartheta})$, $\tilde{\beta}_r \equiv \frac{\partial \beta}{\partial r}(\tilde{r}, \tilde{\vartheta})$, $\tilde{\beta}_\vartheta \equiv \frac{\partial \beta}{\partial \vartheta}(\tilde{r}, \tilde{\vartheta})$, $\tilde{\beta}_{r\vartheta} \equiv \frac{\partial^2 \beta}{\partial r \partial \vartheta}(\tilde{r}, \tilde{\vartheta})$, etc.*

(i) The second order Taylor approximations of the composite functions $\phi^s(r_t, \eta^i)$ and $X^*(r_t, \eta^i)$ around a reference point of $(\tilde{r}, \tilde{\eta})$ take the following forms:

$$\phi^s(r_t, \eta^i) \approx d_{\phi^s,t} + f^i_{\phi^s} + \tilde{\phi}^s_{r\eta} \cdot r_t \eta^i \tag{21}$$

$$X^*(r_t, \eta^i) \approx d_{X,t} + f^i_X + \tilde{X}^*_{r\eta} \cdot r_t \eta^i \tag{22}$$

where $d_{\phi^s,t}$ and $d_{X,t}$ include constants and terms that vary only in r_t, including the first order effects $\tilde{\phi}^s_r \cdot r_t$ and $\tilde{X}^*_r \cdot r_t$, respectively. $f^i_{\phi^s}$ and f^i_X include terms that vary only in η^i. The relevant coefficients are given by:

$$\begin{aligned}
\tilde{\phi}^s_{r\eta} &= \{[\tilde{\beta}_{r\vartheta}(\bar{\varepsilon} + \tilde{\mathcal{R}}) + \tilde{\beta}_\vartheta \tilde{\mathcal{R}}_r]\tilde{\vartheta}_\eta + \tilde{\beta}\tilde{\mathcal{R}}_{r\eta} + \tilde{\beta}_r \tilde{\mathcal{R}}_\eta\} z \\
\tilde{X}^*_{r\eta} &= \tilde{X}^*_{\phi^s} \times \tilde{\phi}^s_{r\eta} + \tilde{X}^*_{\phi^s\phi^s} \times \tilde{\phi}^s_\eta \times \tilde{\phi}^s_r + \tilde{X}^*_{\phi^s_e} \times \tilde{\phi}^s_{e,r\eta} + \tilde{X}^*_{\phi^s_e\phi^s_e} \times \tilde{\phi}^s_{e,\eta} \times \tilde{\phi}^s_{e,r} \\
&\quad \textit{where} \\
\tilde{\phi}^s_r &= [\tilde{\beta}\tilde{\mathcal{R}}_r + \tilde{\beta}_r(\bar{\varepsilon} + \tilde{\mathcal{R}})]z, \quad \tilde{\phi}^s_\eta = [\tilde{\beta}\tilde{\mathcal{R}}_\eta + \tilde{\beta}_\vartheta \tilde{\vartheta}_\eta(\bar{\varepsilon} + \tilde{\mathcal{R}})]z \\
\tilde{\phi}^s_{e,r} &= \tilde{\beta}_r z \varepsilon_e, \quad \tilde{\phi}^s_{e,\eta} = \tilde{\beta}_\vartheta \tilde{\vartheta}_\eta z \varepsilon_e, \quad \tilde{\phi}^s_{e,r\eta} = \tilde{\beta}_{r\vartheta} \tilde{\vartheta}_\eta z \varepsilon_e
\end{aligned}$$

(ii) Treating ϑ^i as an explicit observable argument of all relevant functions, the second order Taylor approximations of the functions $\phi^s(r_t, \eta^i, \vartheta^i)$ and $X^*(r_t, \eta^i, \vartheta^i)$ around a reference point

[22] For brevity, we are currently illustrating these ideas by assuming a heterogeneous pass-through of nominal rate changes to the effective discount rate applied to firms' future payoffs. As pointed out in Footnote 21, in our most basic model the analysis would be identical if nominal rate changes instead had heterogeneous effects on firms' effective productivity z, e.g. due to heterogeneous effects on firms' demand dictated by ϑ.

of $(\tilde{r}, \tilde{\eta}, \tilde{\vartheta})$ *take the following forms:*

$$\phi^s(r_t, \eta^i, \vartheta^i) \approx d_{\phi^s,t} + f^i_{\phi^s} + \tilde{\phi}^s_{r\eta} \cdot r_t\eta^i + \tilde{\phi}^s_{r\vartheta} \cdot r_t\vartheta^i \quad (23)$$

$$X^*(r_t, \eta^i, \vartheta^i) \approx d_{X,t} + f^i_X + \tilde{X}^*_{r\eta} \cdot r_t\eta^i + \tilde{X}^*_{r\vartheta} \cdot r_t\vartheta^i \quad (24)$$

where $d_{\phi^s,t}$ *and* $d_{X,t}$ *include constants and terms that vary only in* r_t*, including the first order effects* $\tilde{\phi}^s_r \cdot r_t$ *and* $\tilde{X}^*_r \cdot r_t$*, respectively.* $f^i_{\phi^s}$ *and* f^i_X *include terms that vary only across firms* i*, including terms in* η^i*,* ϑ^i*, and* $\eta^i\vartheta^i$*. The relevant coefficients are given by:*

$$\begin{aligned}
\tilde{\phi}^s_{r\eta} &= \{\tilde{\beta}\tilde{\mathcal{R}}_{r\eta} + \tilde{\beta}_r\tilde{\mathcal{R}}_\eta\}z \\
\tilde{\phi}^s_{r\vartheta} &= \{\tilde{\beta}_{r\vartheta}(\bar{\varepsilon} + \tilde{\mathcal{R}}) + \tilde{\beta}_\vartheta\tilde{\mathcal{R}}_r\}z \\
\tilde{X}^*_{r\eta} &= \tilde{X}^*_{\phi^s} \times \tilde{\phi}^s_{r\eta} + \tilde{X}^*_{\phi^s\phi^s} \times \tilde{\phi}^s_\eta \times \tilde{\phi}^s_r \\
\tilde{X}^*_{r\vartheta} &= \tilde{X}^*_{\phi^s} \times \tilde{\phi}^s_{r\vartheta} + \tilde{X}^*_{\phi^s\phi^s} \times \tilde{\phi}^s_\vartheta \times \tilde{\phi}^s_r + \tilde{X}^*_{\phi^s_e} \times \tilde{\phi}^s_{e,r\vartheta} + \tilde{X}^*_{\phi^s_e\phi^s_e} \times \tilde{\phi}^s_{e,\vartheta} \times \tilde{\phi}^s_{e,r} \\
& \quad \textit{where} \\
\tilde{\phi}^s_r &= [\tilde{\beta}\tilde{\mathcal{R}}_r + \tilde{\beta}_r(\bar{\varepsilon} + \tilde{\mathcal{R}})]z, \quad \tilde{\phi}^s_\eta = \tilde{\beta}\tilde{\mathcal{R}}_\eta z, \quad \tilde{\phi}^s_\vartheta = \tilde{\beta}_\vartheta(\bar{\varepsilon} + \tilde{\mathcal{R}})z \\
\tilde{\phi}^s_{e,r} &= \tilde{\beta}_r z\varepsilon_e, \quad \tilde{\phi}^s_{e,\vartheta} = \tilde{\beta}_\vartheta z\varepsilon_e, \quad \tilde{\phi}^s_{e,r\vartheta} = \tilde{\beta}_{r\vartheta} z\varepsilon_e
\end{aligned}$$

In both parts (i) and (ii) above, $\tilde{X}^*_{\phi^s_e\phi^s_e}$ *and* $\tilde{X}^*_r$ *take the functional forms:*

$$\begin{aligned}
\tilde{X}^*_{\phi^s_e\phi^s_e} &= -\Omega'[\hat{\mathrm{C}}(\ell(\tilde{\phi}^s_e))]\hat{\mathrm{C}}'(\ell(\tilde{\phi}^s_e))[\ell'(\tilde{\phi}^s_e)]^2 + \{1 - \Omega[\mathrm{C}(\ell(\tilde{\phi}^s_e))]\}\ell''(\tilde{\phi}^s_e) \\
\tilde{X}^*_r &= \tilde{X}^*_{\phi^s} \times \tilde{\phi}^s_r + \tilde{X}^*_{\phi^s_e} \times \tilde{\phi}^s_{e,r}
\end{aligned}$$

$\tilde{\mathcal{R}}_\eta > 0$*,* $\tilde{\mathcal{R}}_r < 0$*, and* $\tilde{\mathcal{R}}_{r\eta} < 0$ *are the derivatives of the resale option value at* $(\tilde{r}, \tilde{\eta})$*. And the functional forms of the partial derivatives* $\tilde{X}^*_{\phi^s}$*,* $\tilde{X}^*_{\phi^s\phi^s}$*, and* $\tilde{X}^*_{\phi^s_e}$ *remain unchanged from Lemmas 2 and 3.*

Part (i) of Lemma 5 shows that if one does not control for monetary transmission operating through other channels in a manner correlated with ϑ^i (and thus with η^i), then any observed response heterogeneity predicted by stock turnover (as captured by $\tilde{\phi}^s_{r\eta}$) could reflect these other channels instead of q-monetary transmission. For example, if it was the case that $\tilde{\beta}_{r\vartheta} < 0$ and $\tilde{\vartheta}_\eta > 0$, then firms with higher stock turnover would exhibit more responsive equity prices because they have a higher ϑ^i. And this in turn implies a stronger pass-through of the traditional interest-rate channel to their equity prices. Also in this case, the valuation of investment by entrepreneurs whose equity has a higher turnover falls relatively more in response to nominal rate increases ($\tilde{\phi}_{e,r\vartheta} < 0$). This makes the average log investment rate of entrepreneurs with higher stock turnover fall relatively more for reasons completely unrelated to equity prices or issuance. The unconstrained high-η (and high-ϑ) entrepreneurs would simply be reducing investment due to the stronger impact of the traditional interest-rate channel on their effective discount rate, even though they are not using equity issuance to finance investment.

Part (ii) of Lemma 5 illustrates the basic regression logic that if we could observe proxies for ϑ^i in the data, we can add controls $r_t\vartheta^i$ to capture the ways that ϑ^i predicts response heterogeneity through channels other than q-monetary transmission. The functional forms in part (ii) show that after doing this, the main coefficients of interest, $\tilde{\phi}^s_{r\eta}$ and $\tilde{X}^*_{r\eta}$, on the $r_t\eta^i$-interaction have exactly the form as in Lemma 3, thus allowing to identify q-monetary transmission and the causal effects of equity prices on investment.

Motivated by the existing literature on heterogeneous monetary transmission to firms' investment and our discussion above on why stock turnover is likely correlated with other firm-level covariates because a) turnover is affected by other firm characteristics, and b) exogenous variation in turnover affects firms' portfolio and financing decisions, we include firm size and age (point a) and leverage and liquid asset holdings (point b) as the empirical counterparts of ϑ^i in the robustness analysis of our regressions.

In the following Section, we will take our predictions and the empirical approach motivated by our theoretical model to the data. We will first investigate whether firms with different stock turnover exhibit heterogeneous responsiveness to monetary shocks both in q and in their equity issuance and investment behavior. We do so by estimating 'reduced form' OLS regressions that are the dynamic empirical analogues of equations (16) and (17), providing estimates of the coefficients $\tilde{\phi}^s_{r\eta}$ and $\tilde{X}^*_{r\eta}$. The corresponding OLS regression for q will also serve to illustrate the first stage of our IV approach. Having established whether and when firms with higher turnover exhibit differential equity issuance and investment responses to identified monetary shocks, we move to IV regressions which, based on the discussion above, provide an assessment of the causal effects of equity prices on firms' issuances and investment.

5 Empirical analysis

5.1 Data

Our empirical baseline uses firm-level measures of Tobin's q, equity issuances, and investment rates, financial-market data on trade volume for individual firms' stocks, and a short-term nominal interest rate proxy for the monetary policy rate.

Our sample consists of the Compustat universe of publicly listed U.S. incorporated non-financial firms, and covers the period 1990Q1–2016Q4.[23] For each individual common stock from the Center for Research in Security Prices (CRSP), we construct the daily *turnover rate* as the ratio of daily trade volume (total number of shares traded) to the number of outstanding shares. We average the daily turnover rate into a quarterly turnover series for firm i in quarter t (denoted x_t^i), and merge it with the corresponding firm-level quarterly data from Compustat.

The key objects of interest from Compustat are: investment rates, equity issuances, and Tobin's q. We compute Tobin's q as the book value of total assets ($\mathcal{V}_A^B$) plus the market value of common equity ($\mathcal{V}_E^M$) minus the book value of common equity ($\mathcal{V}_E^B$), scaled by the book value of total assets, i.e., $q = 1 + (\mathcal{V}_E^M - \mathcal{V}_E^B)/\mathcal{V}_A^B$.[24] Our measure of *(net) equity issuances* for firm i in quarter t (denoted E_t^i), consists of the equity sales minus equity purchases reported in Compustat. We normalize these quarterly net issuances by the total balance sheet size of firm i at the beginning of quarter t (denoted B_t^i).[25] For investment of firm i in quarter t (denoted I_t^i) we use capital expenditures reported by Compustat. We normalize this measure of investment for firm i in quarter t with Compustat's net property, plant, and equipment at the beginning of the quarter (denoted K_t^i). In robustness analysis, we also employ measures of firms' *size*, *age*, *leverage*, and *liquidity ratios* as additional controls.[26]

As a proxy for the nominal policy rate we use the tick-by-tick nominal interest rate implied by the 3-month ahead fed funds futures contract with nearest maturity after each regular monetary-policy announcement of the Federal Open Market Committee (FOMC). The use of a

[23] Since our regression specifications include simple firm fixed effects in a dynamic panel setting, we only include firms that are in the dataset for at least 40 consecutive quarters during the sample period.

[24] This is the definition of *average* q in Kaplan and Zingales (1997), except that as in Baker et al. (2003) and Cloyne et al. (2018), we do not subtract deferred taxes from the numerator (due to many missing values in our data). We follow Eberly et al. (2012) and use the natural logarithm of q in our regressions. This provides a better fit given the skewness in the firm-level data, as discussed in Abel and Eberly (2002).

[25] We measure the "beginning of quarter t" values of firms' stock variables with the values reported in Compustat as of the end of quarter $t-1$.

[26] To construct a measure of firm age, we follow the approach of Cloyne et al. (2018) and use data from Thomson Reuters' WorldScope database to infer time since the firm's incorporation.

futures rate allows us to focus on the unanticipated component of the interest rate change on FOMC policy announcements dates, which we regard as monetary-policy shocks.[27]

In order to identify the impact of these monetary shocks, we follow the event-study methodology that consists of estimating the changes that occur in a 30-minute window around the time of the FOMC announcement.[28] The identification assumption is that in such a narrow window around the press release, futures rates are not affected by variables or news other than the FOMC announcement. We consider alternative shock series in our robustness analysis.[29]

Since the firm-level data from Compustat is quarterly, we sum up the high-frequency changes in the federal funds futures rate by quarter to arrive at a quarterly series of monetary policy shocks for quarter t (denoted ε_t^m). We interpret a positive value of ε_t^m as a contractionary monetary shock, i.e., an unexpected policy-induced increase in the nominal interest rate.[30]

5.2 Results from reduced-form regressions

Our empirical analysis builds on local projections in the spirit of Jordà (2005), applied in a panel setting. As mentioned, we first estimate what we refer to as 'reduced form' specifications. The main goal of these regressions is to verify whether in our sample, firms with different stock turnover, as measured prior to monetary policy shocks, exhibit differential responses in q,

[27] The importance of focusing on the unanticipated component of policy announcements in order to identify the response of asset prices to monetary policy was originally pointed out by Kuttner (2001) and has been emphasized by the literature since then, e.g., Bernanke and Kuttner (2005) and Rigobon and Sack (2004).

[28] In the context of monetary policy, the event-study approach was originally used by Cook and Hahn (1989), and has been followed by a large number of papers, e.g., Bernanke and Kuttner (2005), Cochrane and Piazzesi (2002), Kuttner (2001), and Thorbecke (1997). Most recent applications follow Gürkaynak et al. (2005), who use time windows consisting of only a few minutes before and after the announcement (rather than consisting of the whole announcement day, as was common in earlier work).

[29] High-frequency movements in federal funds futures rates may encode additional information about the conduct of monetary policy, such as the implicit revelation of the monetary authority's information about economic fundamentals imperfectly observed by the private sector. See, for example, Nakamura and Steinsson (2018), Miranda-Agrippino and Ricco (2019), and Jarociński and Karadi (2020). To contemplate this possibility, we also carry out our main estimations with a proxy for the monetary shock computed using the method proposed by Jarociński and Karadi (2020). Their approach employs a structural vector autoregression that uses high-frequency changes in federal funds futures rates alongside sign restrictions imposing that conventional monetary policy shocks generate opposite-signed surprises in futures rates and returns in the S&P500 index. The idea is that this sign restriction would purge the proxy series from informational components that generate positive high-frequency comovement between interest rates and stock returns.

[30] To construct the various measures of ε_t^m we use the dataset used by Jarociński and Karadi (2020), which is in turn based on an updated version of the dataset used by Gürkaynak et al. (2005). Since ε_t^m is possibly a noisy measure of the true monetary shocks, it should be used as an instrument in IV regressions (see, e.g., Stock and Watson (2018)). In our reduced-form specifications (Section 5.2) we treat ε_t^m as if it were an accurate measure of the true monetary shocks. In our main empirical IV specifications (Section 5.3), we instead use ε_t^m to construct an instrument for changes in stock prices.

equity issuances, and investment.[31] Given that in our theoretical model of the turnover channel, stock prices affect investment because firms adjust their equity issuances to fluctuations in the former, we also analyze equity issuances to test whether there is evidence of such "market timing" behavior (Baker and Wurgler, 2002). As our baseline, we estimate panel regression specifications of the following form on our full sample of firm-level data:

$$y^i_{t+h} = f^i_h + d_{s,h,t+h} + \rho_h y^i_{t-1} + \beta_h \mathcal{T}^i_{t-1} + \gamma_h \mathcal{T}^i_{t-1} \varepsilon^m_t + u^i_{h,t+h} \tag{25}$$

$h = 0, 1, \ldots, H$ denotes the horizon at which the shock impact effects are being estimated. y^i_t refers to firm i's outcome variable of interest in quarter t. Based on the notation introduced above, y^i_t is one of $\log(q^i_t)$, e^i_t/b^i_t, or $\log(x^i_t/k^i_t)$.

f^i_h denotes firm i's fixed effect in the projection at horizon h. $d_{s,h,t+h}$ is shorthand for industry-quarter dummies at the SIC 2-digit level, given the h-quarter projection horizon and the outcome variable being measured in period $t+h$. ε^m_t is a measure of the quarterly monetary policy shock as discussed above. $u^i_{h,t+h}$ is the error term in the projection of the outcome variable in period $t+h$, given the h-quarter projection horizon. $\rho_h, \beta_h, \gamma_h$ are regression coefficients. The main object of interest is the estimate for γ_h which captures any heterogeneity in shock responsiveness predicted by stock turnover.

We lag firm controls to ensure they are unaffected by the realization of ε^m_t and can be thought of as measures of shock-exposure. As long as there is persistence in stock turnover from one quarter to another, the turnover measured in $t-1$ proxies for turnover immediately before the FOMC announcement in quarter t. As discussed above, our focus is on cross-sectional differences in how firms' stock turnover predicts their responsiveness to monetary policy shocks. Including a detailed industry-time dummy $d_{s,h,t+h}$ allows for a flexible way to isolate this cross-sectional variation. Thus, the identification of the mechanism of interest is driven by *within-industry between-firm* variation across time.

We multiply all the y^i_t considered by 100 for convenience, so the coefficients for changes in q and investment ratio can be interpreted in percentage terms and the issuance ratio in percentage points. We standardize the turnover measure $\mathcal{T}^i_t$ by the standard deviation of turnover in the cross-section of firms, averaged across time over our sample. And we standardize the monetary shock measures ε^m_t by their standard deviation between 1990Q1–2016Q4 of approximately 9.66 bp, as measured by changes in federal funds futures rates.

[31]In doing so, we are also providing a test for whether the main empirical findings of Lagos and Zhang (2020b) hold when measuring equity valuations based on q at a quarterly frequency, instead of using daily stock returns.

Figure 1 presents the point estimates and 95% confidence intervals for γ_h given the three outcome variables of interest. As one would expect based on financial markets incorporating the FOMC announcements virtually immediately, the heterogeneity in stock price responses predicted by turnover is strongest in the quarter of the monetary policy shock. The point estimate of approximately -0.5 says that an increase in stock turnover by 1 sd predicts a 0.5% stronger contraction in the firm's q in the quarter of a 1 sd contractionary monetary policy shock.[32] And the predicted differences between stock prices persist for about up to a year after the shock.

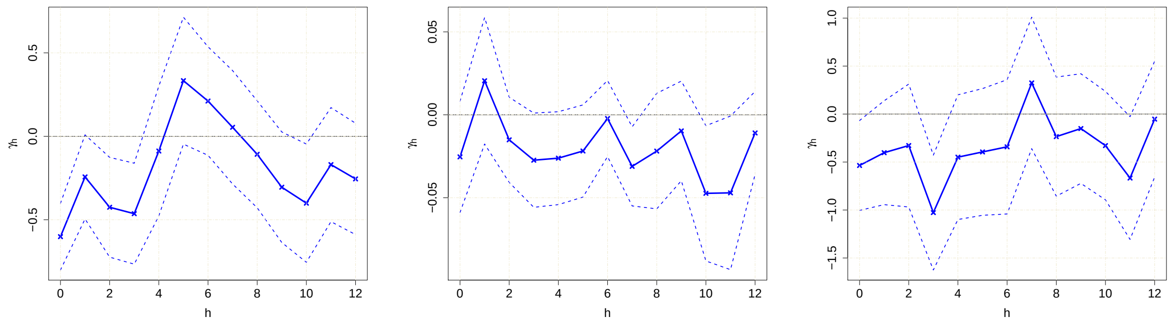
Heterogeneity in the responsiveness of equity issuances predicted by stock turnover appears about three quarters after the shock. Yet the estimated heterogeneity is just marginally statistically insignificant at the 95% confidence level. In terms of quantitative magnitudes, the coefficient of approximately -0.03 implies that a 1 sd increase in stock turnover predicts a 0.03 pp larger drop in net equity issuances relative to book assets three quarters after a 1 sd contractionary monetary shock. The negative predictive effect of turnover for quarterly issuances in the full sample of firm-quarters appears to be persistent and yields statistically significant estimates 7 and 10 quarters after the shock. Finally, the estimates in the last panel of Figure 1 indicate that firms with higher stock turnover do exhibit relatively lower investment rates after contractionary monetary shocks. The differences are statistically significant in the first and third quarter after the shock, with a 1sd increase in stock turnover predicting an approximately 1% larger drop in investment rates after a 1 sd contractionary monetary shock.

The theoretical model presented above provides a stark prediction about which firms' choices should be affected by the q-monetary transmission mechanism. Firms which have few liquid resources available, relative to their size, are more likely to rely on external equity financing and expose themselves to fluctuations in stock prices. Firms that do not issue equity are isolated from these fluctuations. So even though among such firms stock prices respond to monetary policy shocks, and more so for those with high turnover, their choices of equity issuance and investment are unaffected by this. And therefore, no heterogeneous responses of issuances and investment conditional on turnover should be observed.

To test the empirical validity of these predictions and allow for differences in the strength of the q-channel across groups of firms, we define the indicator $\mathbb{I}^i_{L,t}$ which equals 1 if firm i belongs in the bottom half of the liquidity ratio distribution of the cross-section of firms in quarter t,

[32] More precisely, given specification (25), a negative γ_h only allows to infer a drop in q *relative* to other firms.

Figure 1: Heterogeneity in responses to monetary policy shock conditional on stock turnover



(a) $y_t^i = \log(q_t^i)$ (b) $y_t^i = e_t^i/b_t^i$ (c) $y_t^i = \log(x_t^i/k_t^i)$

Notes: Point estimates and 95% confidence intervals for γ_h from estimating specification (25). Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

and 0 otherwise. We define the *liquidity ratio* for firm i in quarter t as the ratio of Compustat reported *cash and short-term investments* to i's total assets in t, meant to capture the holdings of various assets that firms use to manage their liquidity and financial savings. And we estimate the following specification:

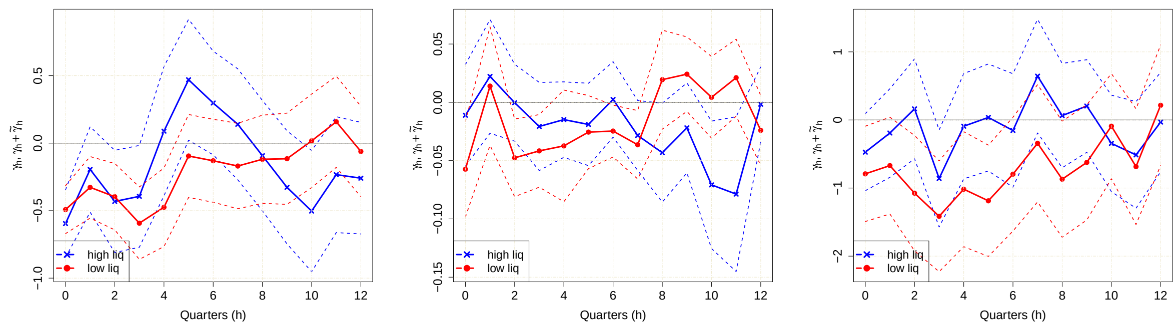
$$\begin{aligned} y_{t+h}^i =& f_h^i + d_{s,h,t+h} + \alpha_h \mathbb{I}_{L,t-1}^i + \left(\rho_h + \tilde{\rho}_h \mathbb{I}_{L,t-1}^i\right) y_{t-1}^i \\ &+ \left(\beta_h + \tilde{\beta}_h \mathbb{I}_{L,t-1}^i\right) \mathcal{T}_{t-1}^i + \left(\gamma_h + \tilde{\gamma}_h \mathbb{I}_{L,t-1}^i\right) \mathcal{T}_{i,t-1} \varepsilon_t^m + u_{h,t+h}^i \end{aligned} \tag{26}$$

In this case, γ_h measures the predictive power of turnover heterogeneity for firms with high liquidity ratios prior to the shock, and $\gamma_h + \tilde{\gamma}_h$ for those with low liquidity ratios. Figure 2 presents the point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ from the estimation of (26) for the three outcome variables of interest.

The first panel in Figure 2 indicates that the predictive power of turnover heterogeneity for stock price responses is similar across the two liquidity ratio groups. As predicted by the model, the turnover-liquidity channel is operative for all stocks, with the high-turnover ones responding relatively more in the quarter of a monetary shock, independently of the firms' liquid asset positions. The point estimates for γ_0 are close to the estimates of the full sample of firms in specification (25). While the differences in stock prices persist for about a year after the shock, the statistical significance of the estimates for the high liquidity group in quarters after the shock is weakened slightly.

The middle panel of Figure 2 shows that the relation between turnover and equity issuance

Figure 2: Heterogeneity in responses to monetary policy shock conditional on stock turnover, across liquidity ratio groups



(a) $y_t^i = \log(q_t^i)$ (b) $y_t^i = e_t^i/b_t^i$ (c) $y_t^i = \log(x_t^i/k_t^i)$

Notes: Point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ from estimating specification (26). Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

responses to monetary policy shocks during the first two years is driven by firms with low liquid asset holdings. Among firms with below-median liquidity ratios prior to the shock, higher stock turnover predicts significantly stronger contractions in equity issuance both in the shock impact quarter and two and three quarters after a contractionary shock. Among such firms, an increase of 1 sd in turnover predicts an approximately 0.05 pp larger decrease in equity issuances, measured as a fraction of total assets, both immediately and two and three quarters after a 1 sd contractionary monetary shock. Also, among firms with high liquidity ratios, higher turnover predicts lower equity issuances in the aftermath of policy rate increases, although this relation is weaker over the two-year horizon, and becomes more pronounced at longer horizons. The extended version of our model with long-lived firms below provides a potential rationale for why firms with high liquidity ratios at the time of the shock may respond with a considerable delay. This happens whenever they draw down their liquid assets and engage in equity financing in subsequent quarters, while the effects of the shock on stock prices have not yet dissipated. Also, this empirical finding is not robust in our IV regressions when controlling for other firm-level covariates.

Finally, the estimates in the last panel of Figure 2 indicate that among firms with below median liquid asset holdings, those with higher turnover exhibit relatively lower investment rates after a contractionary monetary policy shock. For these firms, a 1 sd higher stock turnover predicts approximately 1.4% lower investment rates, three quarters after a 1 sd contractionary monetary policy shock. As for equity issuances, the differences in investment rate responses predicted by turnover are persistent, with higher turnover predicting statistical differences in investment responses up to six quarters after a monetary shock. Yet among firms with high liquidity ratios, heterogeneity in stock turnover does not predict any differential responses in investment, apart from a marginally significant effect 3 quarters after the shock.

5.3 Results from IV regressions

We now turn to our main exercise of interest. We combine the cross-sectional heterogeneity in the monetary shock responses of Tobin's q, equity issuances and investment explained by stock turnover into an instrumental variables specification, in order to evaluate the effects of stock price fluctuations on equity issuances and investment. To do so, we construct the analogue of specification (25) by replacing the interaction term between turnover and the monetary shock $\mathcal{T}^i\varepsilon^m$ with the firm's measure of q, which is then instrumented with the $\mathcal{T}^i\varepsilon^m$-term.

As suggested by the OLS estimates for the reduced form specification, the heterogeneity in the monetary shock responses of q, equity issuances, and investment as explained by turnover heterogeneity can materialize at different horizons. Because of this, we consider allowing for the possibility that the variation in q instrumented by turnover and the monetary shock in period t is measured in period $t+h_q$, and the predicted effects on issuances and investment measured in period $t+h$, with $0 \leq h_q \leq h$. For example, if the effect of a monetary shock on stock prices required 1 quarter to fully materialize, yet the effects of stock price fluctuations on investment take 3 quarters to transmit, the main interest would be to study how heterogeneous variation in q in period $t+1$ explains investment in $t+4$ after a monetary policy shock in t. However, given that the heterogeneity in stock prices appears strongest in the impact quarter, as seen in Figures 1a and 2a, we focus the main estimations below on the case of $h_q = 0$.

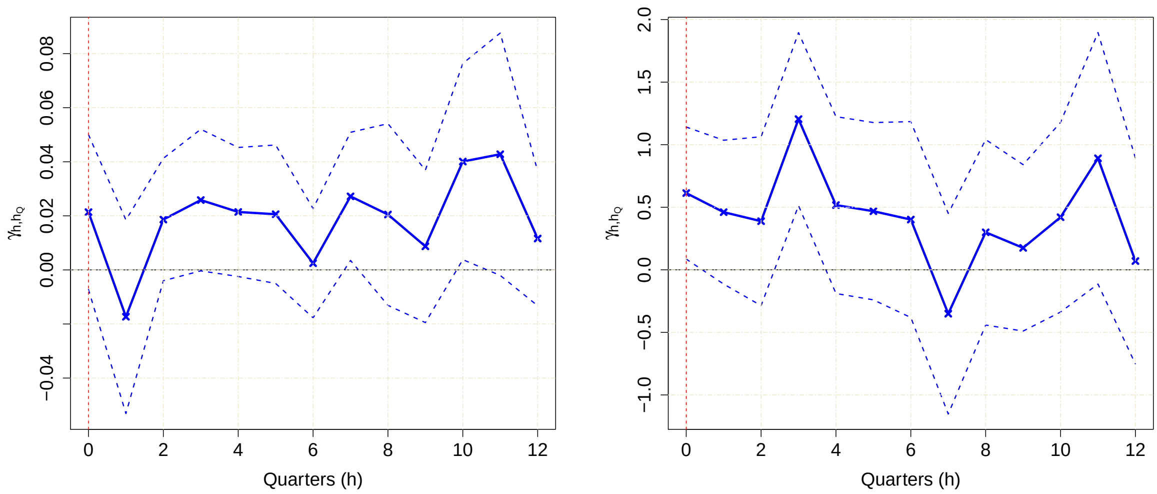
Our baseline instrumental variable specification is as follows:

$$y^i_{t+h} = f^i_h + d_{s,h,t+h} + \rho_h y^i_{t-1} + \beta_h \mathcal{T}^i_{t-1} + \gamma_{h,h_q} \log\left(q^i_{t+h_q}\right) + u^i_{h,t+h} \tag{27}$$

where $\log\left(q^i_{t+h_q}\right)$ is instrumented with $\mathcal{T}_{i,t-1}\varepsilon^m_t$, and $0 \leq h_q \leq h$ for some h_q, with $h = 0, 1, \ldots, H$.

Figure 3 depicts the point estimates and 95% confidence intervals for γ_{h,h_q} from the estimation of (27) with 2SLS, given $h_q = 0$, for equity issuances and investment as dependent variables. The IV estimates are in line with what one would expect based on the reduced form OLS results in Section 5.2. The cross-sectional variation in q instrumented with turnover-based monetary shock exposure predicts higher equity issuances after increases in q caused by monetary shocks. Or, in light of the identification assumptions and formalization discussed in Section 4.1, this suggests that firms' equity issuances respond positively to exogenous increases in Tobin's q. The point estimates, although just barely statistically insignificant, indicate that a 1% increase in q leads to an approximately 0.02 pp increase in equity issuances relative to total assets three quarters later. The instrumented variation in q predicts statistically significantly higher issuances 7 and 10 quarters later, although these estimates are not robust to all specifications. As expected based on the reduced form estimates, cross-sectional variation in q instrumented with monetary shocks and stock turnover does predict slightly higher investment in the sample of all firm-quarters, with estimates being statistically significant in the impact quarter and three quarters after.

Figure 3: Issuances and investment predicted by instrumented q



(a) $y_t^i = e_t^i / b_t^i$

(b) $y_{i,t} = \log(x_t^i / k_t^i)$

Notes: Point estimates and 95% confidence intervals for γ_{h,h_q} from estimating specification (27). Vertical red dashed line marks the value of $h_q = 0$. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

Following the predictions of our model and the evidence presented in Section 5.2, we finally turn to estimating the IV specification by allowing for differences in coefficient estimates for firms with high versus low liquid asset holdings. Employing the indicator $\mathbb{I}^i_{L,t}$ of having a below-median liquidity ratio in t, defined in Section 5.2, we consider the following specification:

$$\begin{aligned} y^i_{t+h} =& f^i_h + d_{s,h,t+h} + \alpha_h \mathbb{I}^i_{L,t-1} + \left(\rho_h + \tilde{\rho}_h \mathbb{I}^i_{L,t-1}\right) y^i_{t-1} \\ &+ \left(\beta_h + \tilde{\beta}_h \mathbb{I}^i_{L,t-1}\right) \mathcal{T}^i_{t-1} + \left(\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q} \mathbb{I}^i_{L,t-1}\right) \log\left(q^i_{t+h_q}\right) + u^i_{h,t+h} \end{aligned} \tag{28}$$

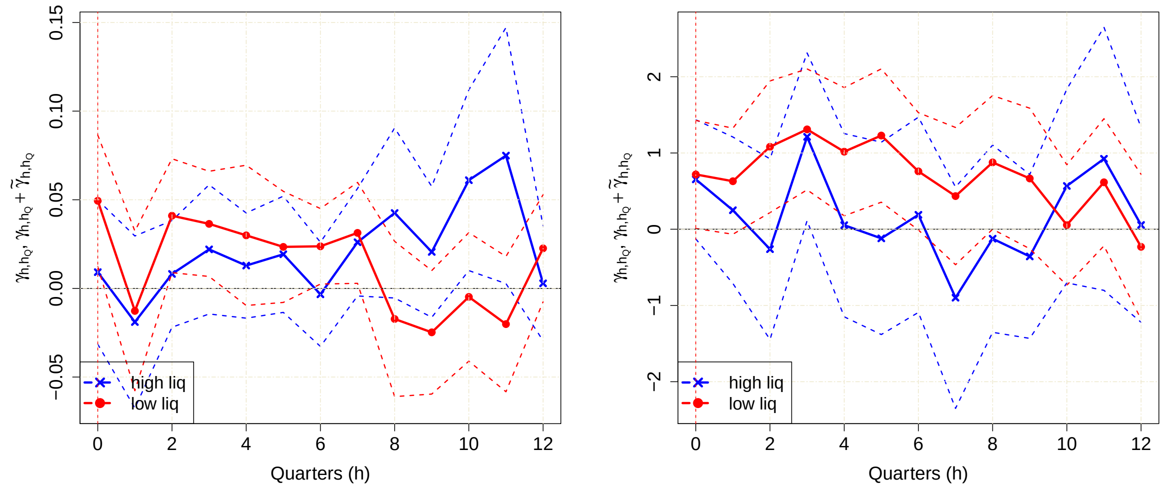
where the vector $\left[\log\left(q^i_{t+h_q}\right), \mathbb{I}^i_{L,t-1} \log\left(q^i_{t+h_q}\right)\right]$ is instrumented with $\left[\mathcal{T}^i_{t-1}\varepsilon^m_t, \mathbb{I}^i_{L,t-1}\mathcal{T}^i_{t-1}\varepsilon^m_t\right]$.

Figure 4 presents the point estimates and 95% confidence intervals for γ_{h,h_q} and $\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q}$ from the estimation of (28) given $h_q = 0$, for equity issuances and investment as dependent variables. Again, the IV estimates confirm the findings from the reduced form regressions. Among firms with low liquid asset holdings, the cross-sectional variation in q instrumented with turnover and identified monetary policy shocks predicts significant heterogeneity in equity issuances. A 1% increase in q leads to an approximately 0.05 pp increase in equity issuances relative to total assets in the same quarter. Similar, statistically significant point estimates are implied for equity issuances also two and three quarters later. For firms with high liquidity ratios, the positive relation between instrumented variation in q and equity issuances is weaker at the two year horizon, but becomes more evident later on.

Finally, increases in instrumented q predict higher investment for firms with low liquid asset holdings. For these firms, a 1% increase in q implies an elevated investment rate for up to six quarters after, with the peak effect of approximately 1.2% higher investment rate at the three quarter horizon. For firms with liquidity ratios above the median, instrumented variation in q predicts higher investment only 3 quarters later, with the coefficient just marginally statistically significant.

Robustness. In Appendix C.1, Figure 7 we include additional firm-level controls interacted with the monetary shock measure in specification (28) to verify that the predicted heterogeneity in issuance and investment responsiveness is not in fact explained by other firm-level covariates. Based on the discussion in Section 4.1.6, we consider as the main controls measures of size, leverage and liquidity ratios. Comparing the results in Figures 4 and 7, it is clear that while the confidence intervals on the estimates of γ_{h,h_q} and $\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q}$ widen due to cross-sectional correlation between stock turnover and the various firm-level controls, the point estimates are in large part unchanged and our main results hold. In Figure 8 in Appendix C.1, we further add firm age as a control. Because of worse coverage of the age variable, we lose almost a fifth of the firm-quarter observations from the full sample behind the results in Figure 4, so

Figure 4: Issuances and investment predicted by instrumented q, across liquidity ratio groups



(a) $y_t^i = e_t^i / b_t^i$ (b) $y_t^i = \log(x_t^i / k_t^i)$

Notes: Point estimates and 95% confidence intervals for γ_{h,h_q} and $\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q}$ from estimating specification (28). Vertical red dashed line marks the value of $h_q = 0$. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

the estimates are again slightly less precise. But the main finding remains. An increase in firms' q, as instrumented by stock turnover and monetary policy shocks, leads to significantly higher investment among low-liquidity firms, and this finding cannot be explained by the other firm-level covariates predicting heterogeneous responsiveness to monetary shocks.

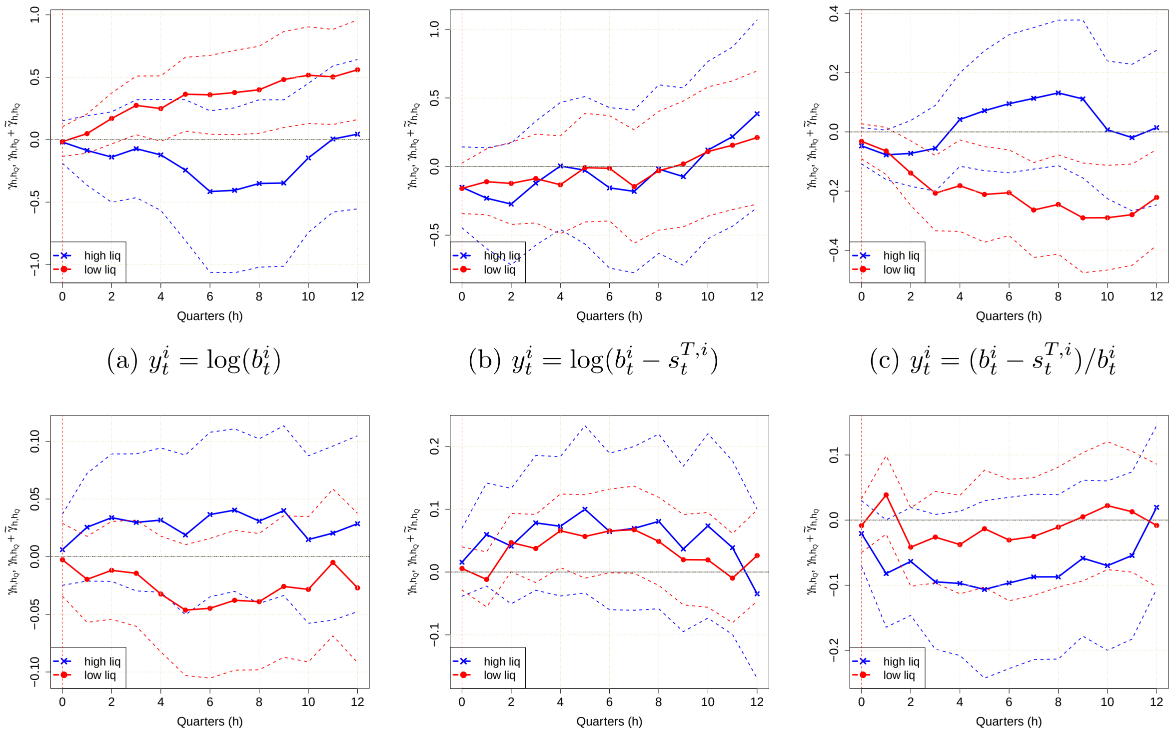
In Figure 9 in Appendix C.1, we present the main OLS and IV coefficient estimates for an alternative ε_t^m series identified based on the 'poor man's sign restrictions' by Jarociński and Karadi (2020).[33] As seen from the figures, our main findings hold when potential informational effects of policy announcements are purged from the monetary shock series. Figure 10 in Appendix C.1 illustrates that our main findings are robust to alternative variable transformations, such as using q_t^i instead of $\log\left(q_t^i\right)$ as the measure of q, or the level of investment rates as outcome, instead of log investment rates.

[33]We focus on the 'poor man's sign restrictions' series by Jarociński and Karadi (2020) since their benchmark identification approach relies on (set-)identification with a linear model which can lead to further imprecisions during the financial crisis and zero lower bound periods after 2008 during which nonlinear dynamics most likely played a central role in the economy.

5.3.1 Capital structure dynamics

By applying our empirical IV approach, we can not only study how instrumented changes in Tobin's q lead firms to adjust their equity issuances and investment, but also trace out the effects on firms' capital structure and the allocation of funds in assets other than productive capital. The corresponding estimates, resulting from the estimation of specification (28) with various broad balance sheet entries as outcomes are shown in Figure 5.

Figure 5: Capital structure and assets predicted by instrumented q, across liquidity ratio groups



(a) $y_t^i = \log(b_t^i)$ (b) $y_t^i = \log(b_t^i - s_t^{T,i})$ (c) $y_t^i = (b_t^i - s_t^{T,i})/b_t^i$

(d) $y_t^i = k_t^i/b_t^i$ (e) $y_t^i = a_t^{b,i}/b_t^i$ (f) $y_t^i = (b_t^i - k_t^i - a_t^{b,i})/b_t^i$

Notes: Point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ from estimating specification (28). b_t^i refers to *Total assets*; $(b_t^i - s_t^{T,i})$ refers to *Total liabilities*, with $s_t^{T,i}$ being *Total stockholders' equity*; k_t^i refers to *Net property, plant, and equipment*; $a_t^{b,i}$ refers to *Cash and short-term investments*; $(b_t^i - k_t^i - a_t^{b,i})$ refers to all other assets. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

Firms with below median liquid asset holdings respond to an increase in Tobin's q by increasing their size, as measured by their total asset holdings (panel (a)), suggesting that the higher equity issuances seen in Figure 4 do not immediately flow out of the firms. Also, firms do not seem to do any active rebalancing or readjusting of the level of their liabilities (panel (b)), naturally implying that the share of funding by liabilities falls (panel (c)), and the firms become more equity-financed in terms of their capital structure.[34] On the assets side of the firms' balance sheets, no significant shifts in the relative allocation of funds across asset classes appear, apart from a slight increase in the share of cash and other liquid financial assets (panel (e)). This suggests that firms use the additional funds raised from equity issuances at higher Tobin's q to scale up their operations. Initially, the raised funds are held mostly in the form of liquid financial assets, allocated as investments into physical capital and other assets over time, and eventually returning to an asset structure similar to before the increase in q, only now at a larger scale (panel (a)). Firms with high liquidity ratios do not exhibit any statistically significant responses in capital structure nor asset holdings to the instrumented changes in Tobin's q.

6 Quantitative analysis

In this section we use a quantitative version of the model presented in Section 2 to assess the ability of the theory to match the dynamic responses of investment documented in Section 5. The theory consists of two building blocks: an asset-pricing block that determines equity prices given monetary policy, and an investment block that determines the capital structure and investment decisions of firms.

We extend the theory of Section 2 and introduce monetary policy shocks in the form of an unexpected change in the path of the nominal interest rate (10). The shock we consider is an unexpected increase of ε^m in r_{t+1} in period $t = 0$, after which the nominal rate follows an autoregressive path back to its steady state value according to: $r_{t+1} = \bar{r} + \rho_n (r_t - \bar{r})$ where $\bar{r}$ is the steady state net nominal rate. We choose ε^m so as to generate a 1% increase in firms' stock prices at the time of the announcement of the shock, conditional on other parameter values.

We extend the problem of the entrepreneurs seen in Section 2 in two ways. First, we introduce stochastic fixed equity issuance costs. More specifically, we assume that for an entrepreneur with capital stock k_t^i to issue new equity in period t, i.e. choose $e_t^i > 0$ in the second subperiod of t, he must exert effort and suffer a disutility of $\xi_t^i k_t^i$. The stochastic cost ξ_t^i is i.i.d. across entrepreneurs and time, distributed uniformly $\xi_t^i \sim U\left[0, \bar{\xi}\right], \forall (i, t)$, and is drawn at the

[34]The dynamics of *Total debt*, *Long-term debt*, and *Short-term debt*, both in log-levels and relative to total assets (currently not presented) are very similar to those of *Total liabilities* in panels (b) and (c) of Figure 4.

beginning of the second subperiod of t by each entrepreneur.

Second, we assume that a unit of capital available at the beginning of period t also delivers $a \in \mathbb{R}_+$ units of good 2 in the second subperiod of t, in addition to producing z units of good 1 at the end of the first subperiod. A share issued by entrepreneur i in the second subperiod of t now represents ownership of 1 unit of capital along with the stream of *dividends* of both good 1 and good 2. That is, holding 1 share in period t constitutes the right to receive z units of good 1 in subperiod 1, and a units of good 2 in subperiod 2. We make an additional simplifying assumption by imposing that instead of receiving the a units of good 2, an investor who owns a share of entrepreneur i's capital at the beginning of subperiod 2 is instead given new shares $\tilde{e}_t^i$ in i's capital stock. And the size of this equity distribution is such that the market value of the new shares is equal to the market value of the dividends, i.e. $\phi_t^i \tilde{e}_t^i = a$, and thus the investor is indifferent. This means that we are implicitly imposing that the returns of capital in subperiod 2 remain within the firm and shareholders are compensated by an increase in the value of their shares they own, i.e. the firm simply retains all earnings in subperiod 2.[35] Since this operation constitutes a firm growing by retaining earnings, the distribution of $\tilde{e}_t^i > 0$ is not subject to fixed equity issuance costs.

As illustrated by the analytical example in Section 4, the introduction of the equity issuance costs and capital's ability to produce good 2 in subperiod 2 are in no way necessary to produce our main qualitative results. Rather, their main purpose to, in a straighforward manner, improve the quantitative characteristics and flexibility of the model. The equity issuance cost allows the model to yield a realistic fraction of firms issuing equity at any given point in time and a nontrivial stationary distribution of liquid asset holdings. With the productivity $a > 0$, firms in the quantitative model can invest and grow not only using new equity issuances, but also using retained earnings, thus allowing the model to yield realistic average investment rates while matching the empirical frequency and size of equity issuances by public firms.

As for the remaining functional forms in the model, we assume quadratic capital adjustment costs $\Psi(x/k) = \frac{\kappa}{2}\left(\frac{x}{k}\right)^2$, and a lognormal distribution G for $\varepsilon \sim \log\mathcal{N}(\mu_\varepsilon, \sigma_\varepsilon)$. For the entrepreneur's problem, the relevant idiosyncratic state variable is the ratio of bond holdings to its capital stock.[36] For the exercises relevant below, because of this, the characteristics of newly

[35]It can be shown that in the stationary equilibrium of the model, retaining earnings and distributing equity is strictly preferred by all entrepreneurs over paying out any of the dividends a to outside investors.

[36]The entrepreneur's problem is homogeneous of degree 1 in k_t^i. So choices of investment rates, equity issuance rates, and bond holdings relative to capital are independent of the incoming capital stock.

born entrepreneurs simply need to be specified in terms of the distribution of initial endowment of good 2 to capital. As *ex post* heterogeneity of entrepreneurs is generated by the stochastic equity issuance costs and the occurrence of death, we simply assume that all entrepreneurs are born with a given $\omega_0 \equiv w_0/k_0 \in \mathbb{R}_{++}$, dictating the liquidity of the entrepreneurs' balance sheets at birth.

As the main exercise, we compare the impulse responses of log investment rates for firms with high and low liquidity ratios in the stationary distribution of our model to the estimates from the data. The empirical IV coefficients from Section 5.3 estimate how much log investment rates respond to a 1% increase in q generated by a monetary policy shock. In our stylized model, the *only* channel of monetary transmission from nominal rates to stock prices and investment is the turnover-liquidity channel. Therefore, we can simply study the impulse responses of *ex ante* identical firms (who also have identical stock turnover), with *ex post* heterogeneous liquidity positions. In contrast, in the data, it was necessary to employ cross-sectional variation in the monetary shock responses of firms with different stock turnover in order to identify and isolate the turnover-liquidity channel.

The parameter values we currently use for the quantitative exercise are chosen based on one time period being a quarter: $\beta = 0.995$, $\delta = 0.025$, $1 - \pi = 0.017$ (exit rate targeted by Begenau and Salomao (2019)), $\alpha = \theta = 1$, $\rho_n = 0.5$, $\bar{r} = 0.04/4$. As Lagos and Zhang (2020b), we consider a baseline calibration $\theta = 1$ to abstract from micro-level pricing frictions induced by bargaining. Also, given that the current quantitative exercise does not rely on turnover heterogeneity among firms in the model, we consider a baseline $\alpha = 1$. We calibrate σ_ε in the distribution of ε so that the stock price sensitivity of the firms (with $\alpha = 1$) in the model matches the impact effects of monetary policy shocks on the prices of the 10% highest turnover stocks in our empirical work.[37] And we normalize $\mu_\varepsilon = -\frac{\sigma_\varepsilon^2}{2}$. We use $\omega_0 = 2/3$ which is consistent with the approximate average cash-to-assets ratio of 0.40 for firms "entering" Compustat, i.e. engaging in an IPO and entering our sample of public firms during the period that we study, following Begenau and Palazzo (2020).

We calibrate the values of the remaining parameters ε_e, z, a, $\bar{\xi}$, and κ to match moments yielded by the stationary equilibrium of our model to the sample of Compustat firms used in our empirical analysis of Section 5. More specifically, we target: the median liquidity (cash-

[37]Given that in this simple model, the only real effect of nominal rate shocks works through their effect on firms' stock prices and we normalize the monetary shock size in our main exercise so that stock prices respond by 1%, this choice simply governs the size of the required nominal rate shock in the background.

to-assets) ratio, the average investment rates for firms with below-median and above-median liquidity, the unconditional frequency of equity issuance across firms and time, and the average ratio of equity issuance relative to total assets conditional on positive equity issuance.[38] Table 1 provides an overview of the employed structural parameter values and calibration targets.

Table 1: Calibrated parameter values and calibration targets

Parameter	Value	Target / Source
Externally calibrated		
β	0.995	2% annual real rate
$\bar{r}$	0.04/4	4% annualized nominal rate
δ	0.025	Conventional
$1-\pi$	0.017	Compustat exit (Begenau and Salomao, 2019)
σ_ε	2.56	Top 10% turnover ϕ_t^i response to MP
$(\alpha, \theta, \mu_\varepsilon)$	$\left(1, 1, -\frac{\sigma_\varepsilon^2}{2}\right)$	Normalization (Lagos and Zhang, 2020b)
ω_0	2/3	Average cash-to-assets at IPO (Begenau and Palazzo, 2020)
Internally calibrated		
z	0.031	$\text{med}\left(\frac{\text{cash}^i}{\text{assets}^i}\right) = 7.96\%$ (**model: 7.81%**)
a	0.038	$\text{avg}(x^i/k^i)\vert_{\mathbb{I}_{L,t-1}=1} = 2.74\%$ (**2.80%**)
ε_e	4.21	$\text{avg}(x^i/k^i)\vert_{\mathbb{I}_{L,t-1}=0} = 3.69\%$ (**3.68%**)
$\bar{\xi}$	0.244	$\text{freq}(e^i/b^i > 0.01) = 0.0714$ (**0.0719**)
κ	27.80	$\text{avg}(e^i/b^i)\vert_{e^i/b^i>0.01} = 9.57\%$ (**9.73%**)

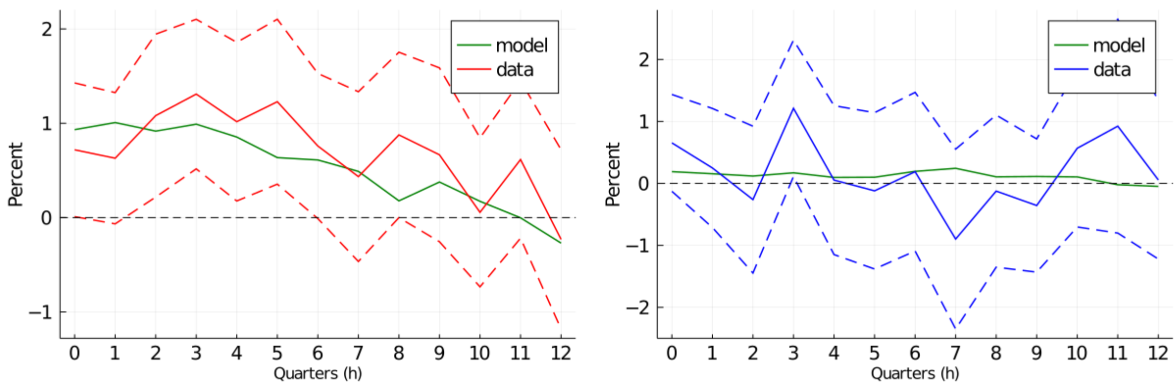
Figure 6 depicts the model impulse responses of log investment rates alongside the corresponding point estimates and confidence intervals already presented in panel (b) of Figure 4 in Section 5.3. Low-liquidity firms increase their investment rates by roughly 1% in response to a monetary shock that increases q by 1%. And both qualitatively and quantitatively, the whole path of the average response of investment rates is very similar in the model and the data.

For high-liquidity firms, the average investment rate response in the model is considerably smaller, consistent with no evidence of the q-channel affecting such firms' investment in the data. Although, their investment response in the model is not exactly zero. This happens for two

[38]We follow convention in the corporate finance literature and define the incidence of a firm issuing equity in our sample as the net equity issuance to asset ratio e_t^i/b_t^i exceeding some chosen cutoff. For example Leary and Roberts (2005) use a cutoff of 5% when working with annual Compustat data. For our analysis with quarterly data, we consider the cutoff of 1%. One reason for employing such a cutoff rule is that, as McKeon (2015) points out, the proceeds from sales of stock reported on firms' statements of cash flows often come from employees excercising options, rather than a managerial decision to sell stock as we are interested in identifying. Since firm-initiated issuances tend to be large and infrequent, he shows that using relative issuance size can with high reliability identify equity issuance proceeds that contain a firm-initiated component.

main reasons. First, in any given period, some firms designated as "high-liquidity" may get low enough draws of the equity issuance cost ξ_t^i and take advantage of the beneficial circumstances to issue equity. Although, they are significantly less likely to issue equity than the low-liquidity firms. But if they happen to do so exactly at the time of the monetary shock, their investment will respond to the change in the price of equity and is thus not "isolated" from the shock. Second, whenever not issuing equity, high-liquidity firms draw down their liquid assets, slowly becoming "low-liquidity", and experience an increase in their probability of issuing equity over time. If the monetary shock is persistent and its effect on stock prices lasts for several periods, the direct effect on these firms simply appears with a lag and to a smaller extent, depending on the shock's persistence. Moreover, because the high-liquidity firms anticipate this immediately when the shock is revealed, and they want to smooth investment due to the convex adjustment costs, they respond already at shock impact. To do so, they invest more out of their liquid asset holdings, even though they are not yet accessing the equity market.

Figure 6: Comparison of investment rate responses from model and data estimates



(a) Low liquidity ratio firms

(b) High liquidity ratio firms

Notes: Data refers to point estimates and 95% confidence intervals for γ_{h,h_q} and $\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q}$ from estimating specification (28) with $y_t^i = \log(x_t^i/k_t^i)$ as the outcome variable. *Model* response is computed as the average firm-level impulse response of log investment rates, averaged over a large panel of firms drawn from the stationary distribution of the model. High and low liquidity ratios are defined as above or below the cross-sectional median cash-to-assets ratio in both model and the data.

7 Aggregate relevance in monetary transmission

Having established that our empirical estimates of the q-monetary channel are both qualitatively and quantitatively consistent with the calibrated theoretical model, we proceed to provide a back-of-the-envelope assessment of the importance of the turnover-q channel for monetary transmission to aggregate investment.[39] We do so by directly employing our empirical regression estimates from Section 5, instead of relying on the calibrated structural model.

We first provide a brief discussion on how our empirical estimates based on between-firm variation can allow us to take the extra step and give an assessment of the overall effect of monetary transmission working through the turnover-q channel. To fix notation, let us use $\frac{dy}{dx}\Big|^{\text{TC}}$ to denote the effect of variable x on y *through the turnover-q channel.* This is in contrast to $\frac{dy}{dx}$ by which we mean the effect of x on y *through all possible transmission channels.* For concreteness, let us first focus the discussion on the effects that monetary policy shocks have on q. The estimates of γ_h from the reduced form OLS regressions of Section 5.2 provide an estimate of $\frac{d^2 \log\left(q^i_{t+h}\right)}{d\varepsilon^m_t d\mathcal{T}^i_{t-1}}$. That is, $\widehat{\gamma}_h$ captures how the estimated effect of ε^m_t on $\log\left(q^i_{t+h}\right)$ differs conditional on past turnover $\mathcal{T}^i_{t-1}$. By the identifying assumption that differences in firms' responses, as predicted by turnover, appear *only* because of the turnover-q channel, we can attribute these differences in full to the turnover-q channel, i.e. $\frac{d^2 \log\left(q^i_{t+h}\right)}{d\varepsilon^m_t d\mathcal{T}^i_{t-1}} = \frac{d^2 \log\left(q^i_{t+h}\right)}{d\varepsilon^m_t d\mathcal{T}^i_{t-1}}\Big|^{\text{TC}}$, estimated by γ_h.

However, without further moment restrictions or identifying assumptions, regressions relying on between-firm differences in responses only allow to identify these cross-derivatives: they tell us how the monetary shock affects the q of firms with different turnover differently (through the turnover channel). Yet the ultimate goal is to evaluate how the monetary shock affects firms' q through the turnover channel. That is, we would like to identify the first derivative $\frac{d \log\left(q^i_{t+h}\right)}{d\varepsilon^m_t}\Big|^{\text{TC}}$. Continuing with imposing linearity, one could "integrate out" $\mathcal{T}^i_{t-1}$ from the cross-derivatives and write:

$$\frac{d \log\left(q^i_{t+h}\right)}{d\varepsilon^m_t}\Big|^{\text{TC}} = \bar{\gamma}^i_h + \gamma_h \mathcal{T}^i_{t-1}$$

where $\bar{\gamma}^i_h$ could be thought of as a "missing intercept", referring to the (potentially firm specific) effect of monetary shocks on q^i_{t+h} through the turnover channel that is not explained by variation in turnover. $\bar{\gamma}^i_h$ cannot be identified solely based on our empirical regressions. But it can

[39] Here, we refer to the "turnover-q channel" as monetary transmission operating through reducing stock prices by reducing turnover liquidity in stock markets, which in turn has an effect on investment through the q-channel.

be identified based on our theoretical model of the turnover channel: for all stocks with zero turnover, the turnover channel is inactive. So the effect of monetary shocks on the corresponding firms through the channel must be zero. And we can pin down the missing empirical "intercept" as:

$$\left.\frac{d\log\left(q^i_{t+h}\right)}{d\varepsilon^m_t}\right|^{\mathrm{TC}}_{\mathcal{T}^i_{t-1}=0} = 0 \Longrightarrow \bar{\gamma}^i_h = 0 \quad \text{and} \quad \left.\frac{d\log\left(q^i_{t+h}\right)}{d\varepsilon^m_t}\right|^{\mathrm{TC}} = \gamma_h \mathcal{T}^i_{t-1}$$

Note, importantly, that due to the stylized nature of our theoretical model, there exist no general equilibrium effects through which responsive firms can affect market prices, which in turn influence firms with zero stock turnover. So based on our model, this additional moment restriction is precise. In reality, its validity depends on whether such general equilibrium feedback effects are negligible or not.

Having established that $\left.\frac{d\log\left(q^i_{t+h}\right)}{d\varepsilon^m_t}\right|^{\mathrm{TC}}$ can be gauged using $\widehat{\gamma}_h \mathcal{T}^i_{t-1}$, we compute that in our Compustat sample, across time and firms, the average effect of a 25 bp contractionary shock in ε^m_t, as measured by quarterly aggregated 3m Federal funds futures rate changes, is to decrease q^i_t by 1.65% at impact through the turnover-q channel.

Next, we can use the IV estimates from Section 5.3 to evaluate the effects of monetary shocks through the turnover channel on firms' investment. For illustration, let us first consider the specification that does not split the sample into firm-quarters with high and low liquidity ratios. Based on our identification assumption, the IV coefficient $\gamma_{h,0}$ in specification (27) for $y^i_t = \log(x^i_t/k^i_t)$ provides an estimate of $\left.\frac{d\log\left(x^i_{t+h}/k^i_{t+h}\right)}{d\log\left(q^i_t\right)}\right|^{\mathrm{TC}}$. By the chain rule, we can therefore write:

$$\left.\frac{d\log\left(x^i_{t+h}/k^i_{t+h}\right)}{d\varepsilon^m_t}\right|^{\mathrm{TC}} = \left.\frac{d\log\left(x^i_{t+h}/k^i_{t+h}\right)}{d\log\left(q^i_t\right)}\right|^{\mathrm{TC}} \cdot \left.\frac{d\log\left(q^i_t\right)}{d\varepsilon^m_t}\right|^{\mathrm{TC}} = \gamma_{h,0} \cdot \gamma_0 \mathcal{T}^i_{t-1}$$

where $\gamma_{h,0}$ refers to the coefficient on the instrumented $\log\left(q^i_{t+h_q}\right)$ in specification (27) for $y^i_t = \log(x^i_t/k^i_t)$, with $h_q = 0$. And γ_0 refers to the coefficient on $\mathcal{T}^i_{t-1}\varepsilon^m_t$ in specification (25) for $y^i_t = \log\left(q^i_t\right)$. More precisely, since our estimates indicate that monetary shocks transmit to investment through the q-channel only for firms with low liquid asset holdings, we use the following calculation to condition on the liquidity positions:

$$\left.\frac{d\log\left(x^i_{t+h}/k^i_{t+h}\right)}{d\varepsilon^m_t}\right|^{\mathrm{TC}} = \left[(1-\mathbb{I}^i_{L,t-1}) \cdot \gamma_{h,0} \cdot \gamma_0 + \mathbb{I}^i_{L,t-1} \cdot (\gamma_{h,0} + \tilde{\gamma}_{h,0}) \cdot (\gamma_0 + \tilde{\gamma}_0)\right] \cdot \mathcal{T}^i_{t-1}$$

where $\gamma_{h,0}$ and $\tilde{\gamma}_{h,0}$ are estimated in specification (28), for $y^i_t = \log(x^i_t/k^i_t)$ with $h_q = 0$. And γ_0 and $\tilde{\gamma}_0$ come from estimating specification (26) for $y^i_t = \log\left(q^i_t\right)$. Based on this, we compute that in our Compustat sample, across firms and time, the average effect of a 25 bp contractionary shock in ε_t^m is to decrease x_{t+4}^i/k_{t+4}^i, i.e. the investment rate four quarters after the shock, by 0.65% through the turnover-q channel.

Finally, to assess the relevance of the turnover-q channel in monetary transmission to aggregate investment, we employ the implied semi-elasticity of firm i's quarterly investment rate x_{t+4}^i/k_{t+4}^i with respect to ε_t^m, i.e. the estimate of $\frac{d\log\left(x_{t+4}^i/k_{t+4}^i\right)}{d\varepsilon_t^m}\Big|^{\text{TC}}$, as a proxy for the semi-elasticity of the firm's quarterly investment level x_{t+4}^i. For each quarter t, we compute the cross-sectional average of these semi-elasticities, weighted by firms' capital expenditures x_{t-1}^i, in our Compustat panel, to get an estimate of the semi-elasticity of aggregate public firm investment in quarter $t+4$ with respect to a monetary shock in t. Taking the average of these aggregate semi-elasticities across time and adjusting for the share of approximately 46.5% of US aggregate nonresidential investment being done by public firms (Asker et al., 2011), we find that in response to a 25 bp unexpected increase in the Federal funds rate, aggregate investment drops by 0.14% four quarters later due to the turnover-q channel. For comparison, the corresponding peak effect on aggregate investment estimated by Christiano et al. (2005) is approximately 0.45%. We can thus conclude that the effects of monetary policy shocks on public firms' investment due to equity price responses have the potential to explain a considerable fraction of overall monetary transmission to aggregate investment in the US.

References

Abel, A. B. and J. C. Eberly (2002): "Investment and q with fixed costs: An empirical analysis," Working paper, University of Pennsylvania.

Anderson, G. and A. Cesa-Bianchi (2020): "Crossing the Credit Channel: Credit Spreads and Firm Heterogeneity," Working Paper.

Asker, J., J. Farre-Mensa, and A. Ljungqvist (2011): "Comparing the Investment Behavior of Public and Private Firms," NBER Working Paper No. 17394.

Baker, M., J. C. Stein, and J. Wurgler (2003): "When Does the Market Matter? Stock Prices and the Investment of Equity-Dependent firms," *Quarterly Journal of Economics*, 118, 969–1005.

Baker, M. and J. Wurgler (2002): "Market Timing and Capital Structure," *Journal of Finance*, 57, 1–32.

Begenau, J. and B. Palazzo (2020): "Firm Selection and Corporate Cash Holdings," *Journal of Financial Economics*, forthcoming.

Begenau, J. and J. Salomao (2019): "Firm Financing Over the Business Cycle," *Review of Financial Studies*, 32, 1235–1274.

Bernanke, B. and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79, 14–31.

Bernanke, B. S. and K. N. Kuttner (2005): "What Explains the Stock Market's Reaction to Federal Reserve Policy?" *Journal of Finance*, 60, 1221–1257.

Blanchard, O., C. Rhee, and L. Summers (1993): "The Stock Market, Profit, and Investment," *Quarterly Journal of Economics*, 108, 115–136.

Boivin, J., M. T. Kiley, and F. S. Mishkin (2010): "How has the Monetary Transmission Mechanism Evolved Over Time?" in *Handbook of Monetary Economics*, Elsevier, vol. 3, 369–422.

Brainard, W. C. and J. Tobin (1968): "Pitfalls in Financial Model Building," *American Economic Review*, 58, 99–122.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113, 1–45.

Cloyne, J., C. Ferreira, M. Froemel, and P. Surico (2018): "Monetary Policy, Corporate Finance and Investment," NBER Working Paper No. 25366.

Cochrane, J. H. and M. Piazzesi (2002): "The Fed and Interest Rates: a High-Frequency Identification," *American Economic Review*, 92, 90–95.

Cook, T. and T. Hahn (1989): "The Effect of Changes in the Federal Funds Rate Target on Market Interest Rates in the 1970s," *Journal of Monetary Economics*, 24, 331–351.

Duffie, D., N. Gârleanu, and L. H. Pedersen (2005): "Over-the-Counter Markets," *Econometrica*, 73, 1815–1847.

Eberly, J., S. Rebelo, and N. Vincent (2012): "What explains the lagged-investment effect?" *Journal of Monetary Economics*, 59, 370–380.

Fischer, S. and R. C. Merton (1984): "Macroeconomics and Finance: The Role of the Stock Market," in *Carnegie-Rochester Conference Series on Public Policy*, Elsevier, vol. 21, 57–108.

Gilchrist, S., C. P. Himmelberg, and G. Huberman (2005): "Do Stock Price Bubbles Influence Corporate Investment?" *Journal of Monetary Economics*, 52, 805–827.

Gürkaynak, R. S., B. Sack, and E. T. Swansonc (2005): "Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements," *International Journal of Central Banking*, 1, 55–94.

Hayashi, F. (1982): "Tobin's Marginal q and Average q: A Neoclassical Interpretation," *Econometrica*, 213–224.

Hovakimian, A., G. Hovakimian, and H. Tehranian (2004): "Determinants of Target Capital Structure: The Case of Dual Debt and Equity Issues," *Journal of Financial Economics*, 71, 517–540.

Jarociński, M. and P. Karadi (2020): "Deconstructing Monetary Policy Surprises – The Role of Information Shocks," *American Economic Journal: Macroeconomics*, 12, 1–43.

Jordà, O. (2005): "Estimation and inference of impulse responses by local projections," *American Economic Review*, 95, 161–182.

Jorgenson, D. W. (1963): "Capital Theory and Investment Behavior," *American Economic Review*, 53, 247–259.

Kaplan, S. N. and L. Zingales (1997): "Do Investment-Cash Flow Sensitivities Provide Useful Measures of Financing Constraints?*," *Quarterly Journal of Economics*, 112, 169–215.

Keynes, J. M. (1936): *The General Theory of Employment, Interest and Money*, Macmillan, 14th edition, 1973.

Kiyotaki, N. and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211–248.

Korajczyk, R. A. and A. Levy (2003): "Capital Structure Choice: Macroeconomic Conditions and Financial Constraints," *Journal of Financial Economics*, 68, 75–109.

Kuttner, K. N. (2001): "Monetary Policy Surprises and Interest Rates: Evidence from the Fed Funds Futures Market," *Journal of Monetary Economics*, 47, 523–544.

Lagos, R. (2011): "Asset Prices, Liquidity, and Monetary Policy in an Exchange Economy," *Journal of Money, Credit and Banking*, 43, 521–552.

Lagos, R. and G. Rocheteau (2009): "Liquidity in Asset Markets With Search Frictions," *Econometrica*, 77, 403–426.

Lagos, R., G. Rocheteau, and P.-O. Weill (2011): "Crises and Liquidity in Over-the-Counter Markets," *Journal of Economic Theory*, 146, 2169–2205.

Lagos, R. and S. Zhang (2015): "Monetary Exchange in Over-the-Counter Markets: A Theory of Speculative Bubbles, the Fed Model, and Self-Fulfilling Liquidity Crises," Working Paper 21528, NBER.

——— (2019): "The Limits of onetary Economics: On Money as a Medium of Exchange in Near-Cashless Credit Economies," Working Paper 25803, NBER.

——— (2020a): "The Limits of onetary Economics: On Money as a Latent Medium of Exchange," Working Paper 26756, NBER.

——— (2020b): "Turnover Liquidity and the Transmission of Monetary Policy," *American Economic Review*, 110, 1635–1672.

Leary, M. T. and M. R. Roberts (2005): "Do Firms Rebalance Their Capital Structures?" *The Journal of Finance*, 60, 2575–2619.

McKeon, S. B. (2015): "Employee Option Exercise and Equity Issuance Motives," Working Paper.

Miranda-Agrippino, S. and G. Ricco (2019): "The Transmission of Monetary Policy Shocks," Working Paper.

Mishkin, F. S. (1995): "Symposium on the Monetary Transmission Mechanism," *Journal of Economic Perspectives*, 9, 3–10.

——— (1996): "The Channels of Monetary Transmission: Lessons for Monetary Policy," Working Paper 5464, NBER.

——— (2001): "The Transmission Mechanism and the Role of Asset Prices in Monetary Policy," Working Paper 8617, NBER.

Morck, R., A. Shleifer, R. W. Vishny, M. Shapiro, and J. M. Poterba (1990): "The Stock Market and Investment: Is the Market a Sideshow?" *Brookings Papers on Economic Activity*, 1990, 157–215.

Myers, S. C. and N. S. Majluf (1984): "Corporate Financing and Investment Decisions When Firms Have Information that Investors Do Not Have," *Journal of Financial Economics*, 13, 187–221.

Nakamura, E. and J. Steinsson (2018): "High Frequency Identification of Monetary Non-Neutrality: The Information Effect," *Quarterly Journal of Economics*, 133, 1283–1330.

Rigobon, R. and B. Sack (2004): "The Impact of Monetary Policy on Asset Prices," *Journal of Monetary Economics*, 51, 1553–1575.

Rocheteau, G., R. Wright, and C. Zhang (2018): "Corporate Finance and Monetary Policy," *American Economic Review*, 108, 1147–86.

Stock, J. H. and M. W. Watson (2018): "Identification and estimation of dynamic causal effects in macroeconomics using external instruments," *The Economic Journal*, 128, 917–948.

Thorbecke, W. (1997): "On Stock Market Returns and Monetary Policy," *Journal of Finance*, 52, 635–654.

Tobin, J. (1969): "A General Equilibrium Approach to Monetary Theory," *Journal of Money, Credit and Banking*, 1, 15–29.

Tobin, J. and W. C. Brainard (1976): "Asset Markets and the Cost of Capital," Tech. rep., Cowles Foundation for Research in Economics, Yale University.

Weill, P.-O. (2007): "Leaning Against the Wind," *Review of Economic Studies*, 74, 1329–1354.

A Proofs

A.1 Investor's portfolio and bargaining problems

Lemma 6 *Let*

$$\varepsilon_t^* \equiv \frac{p_t \phi_t^m - \pi(1-\delta)\phi_t^s}{z} \tag{29}$$

and define the correspondence $\chi : \mathbb{R}^2 \rightrightarrows [0,1]$ *as*

$$\chi\left(\varepsilon_t^*, \varepsilon\right) \begin{cases} = 1 & \text{if } \varepsilon_t^* < \varepsilon \\ \in [0,1] & \text{if } \varepsilon_t^* = \varepsilon \\ = 0 & \text{if } \varepsilon < \varepsilon_t^*. \end{cases}$$

Consider a bilateral meeting in the first subperiod of period t between a dealer and an investor with portfolio $\boldsymbol{a}_t$ *and valuation* ε*. The investor's post-trade portfolio,*

$$\bar{\boldsymbol{a}}\left(\boldsymbol{a}_t, \varepsilon\right) \equiv \left(\overline{a}_t^b\left(\boldsymbol{a}_t, \varepsilon\right), \overline{a}_t^m\left(\boldsymbol{a}_t, \varepsilon\right), \overline{a}_t^s\left(\boldsymbol{a}_t, \varepsilon\right)\right),$$

is given by

$$\begin{aligned} \overline{a}_t^b\left(\boldsymbol{a}_t, \varepsilon\right) &= a_t^b \\ \overline{a}_t^m\left(\boldsymbol{a}_t, \varepsilon\right) &= \left[1 - \chi\left(\varepsilon_t^*, \varepsilon\right)\right]\left(a_t^m + p_t a_t^s\right) \\ \overline{a}_t^s\left(\boldsymbol{a}_t, \varepsilon\right) &= a_t^s + \frac{1}{p_t}\left[a_t^m - \overline{a}_t^m\left(\boldsymbol{a}_t, \varepsilon\right)\right], \end{aligned}$$

and the intermediation fee charged by the dealer is

$$\varpi_t\left(\boldsymbol{a}_t, \varepsilon\right) = (1-\theta)\left(\varepsilon_t^* - \varepsilon\right) z \frac{1}{p_t}\left[\overline{a}_t^m\left(\boldsymbol{a}_t, \varepsilon\right) - a_t^m\right].$$

Proof. The value function (2) can be written as

$$\begin{aligned} W_t\left(\boldsymbol{a}_t, \varpi_t\right) &= \boldsymbol{\phi}_t' \boldsymbol{a}_t - \varpi_t + \bar{W}_t \\ &= a_t^b + \phi_t^m a_t^m + \phi_t^s a_t^s - \varpi_t + \bar{W}_t, \end{aligned} \tag{30}$$

where

$$\bar{W}_t \equiv T_t + \max_{\boldsymbol{a}_{t+1} \in \mathbb{R}_+^3} \left[-\boldsymbol{\phi}_t \boldsymbol{a}_{t+1} + \beta \int V_{t+1}\left(\boldsymbol{a}_{t+1}, \varepsilon\right) dG(\varepsilon)\right]. \tag{31}$$

With (30) we can write

$$\begin{aligned} \Gamma\left(\overline{\boldsymbol{a}}_t, \boldsymbol{a}_t, \varepsilon\right) = \; & \overline{a}_t^b + \phi_t^m \overline{a}_t^m + \left(\varepsilon z + \pi(1-\delta)\phi_t^s\right) \overline{a}_t^s \\ & - \left[a_t^b + \phi_t^m a_t^m + \left(\varepsilon z + \phi_t^s \pi(1-\delta)\right) a_t^s\right] - \varpi_t, \end{aligned}$$

so the solution to (1) is

$$\begin{aligned}
\overline{a}_t^b(\boldsymbol{a}_t,\varepsilon) &= a_t^b \\
\overline{a}_t^s(\boldsymbol{a}_t,\varepsilon) &= a_t^s + \frac{1}{p_t}[a_t^m - \overline{a}_t^m(\boldsymbol{a}_t,\varepsilon)] \\
\varpi_t(\boldsymbol{a}_t,\varepsilon) &= (1-\theta)(\varepsilon_t^* - \varepsilon) z \frac{1}{p_t}[\overline{a}_t^m(\boldsymbol{a}_t,\varepsilon) - a_t^m] \\
\overline{a}_t^m(\boldsymbol{a}_t,\varepsilon) &= \arg\max_{0\leq \overline{a}_t^m \leq p_t a_t^s + a_t^m} \left[(\varepsilon_t^* - \varepsilon) z \frac{1}{p_t}(\overline{a}_t^m - a_t^m)\right].
\end{aligned}$$

This concludes the proof. ■

Lemma 7 *Let* $\left(a_{t+1}^b, a_{t+1}^m, a_{t+1}^s\right)$ *denote the portfolio chosen by an investor in the second subperiod of period* t*. This portfolio must satisfy the following first-order necessary and sufficient conditions:*

$$\phi_t^b \geq \beta, \text{ with } “=” \text{ if } a_{t+1}^b > 0 \tag{32}$$

$$\phi_t^m \geq \beta\left[\phi_{t+1}^m + \alpha\theta\int_{\varepsilon_{t+1}^*}^{\varepsilon_H}\left(\varepsilon - \varepsilon_{t+1}^*\right) z dG(\varepsilon)\frac{1}{p_{t+1}}\right], \text{ with } “=” \text{ if } a_{t+1}^m > 0 \tag{33}$$

$$\phi_t^s \geq \beta\left[\bar{\varepsilon}z + \pi(1-\delta)\phi_{t+1}^s + \alpha\theta\int_{\varepsilon_L}^{\varepsilon_{t+1}^*}\left(\varepsilon_{t+1}^* - \varepsilon\right) z dG(\varepsilon)\right], \text{ with } “=” \text{ if } a_{t+1}^s > 0. \tag{34}$$

Proof. With (30) and the bargaining outcome described in the statement of Lemma 6, (3) can be written as

$$\begin{aligned}
V_t(\boldsymbol{a}_t,\varepsilon) &= a_t^b + (\varepsilon z + \pi(1-\delta)\phi_t^s) a_t^s + \phi_t^m a_t^m + \bar{W}_t \\
&\quad + \alpha\theta(\varepsilon - \varepsilon_t^*) z \frac{1}{p_t}[a_t^m - \overline{a}_t^m(\boldsymbol{a}_t,\varepsilon)].
\end{aligned}$$

Hence, using the expression for $\overline{a}_{t+1}^m(\boldsymbol{a}_{t+1},\varepsilon)$ from Lemma 6,

$$\begin{aligned}
\int V_{t+1}(\boldsymbol{a}_{t+1},\varepsilon) dG(\varepsilon) &= a_{t+1}^b + \left[\bar{\varepsilon}z + \pi(1-\delta)\phi_{t+1}^s + \alpha\theta\int_{\varepsilon_L}^{\varepsilon_{t+1}^*}\left(\varepsilon_{t+1}^* - \varepsilon\right) z dG(\varepsilon)\right] a_{t+1}^s \\
&\quad + \left[\phi_{t+1}^m + \alpha\theta\frac{1}{p_{t+1}}\int_{\varepsilon_{t+1}^*}^{\varepsilon_H}\left(\varepsilon - \varepsilon_{t+1}^*\right) z dG(\varepsilon)\right] a_{t+1}^m + \bar{W}_{t+1}
\end{aligned}$$

.

Thus, the necessary and sufficient first-order conditions corresponding to the maximization problem in (31) are as in the statement of the lemma. ■

A.2 Stock-market clearing

Lemma 8 *In period t, the first-subperiod market-clearing condition for equity is*

$$[1 - G(\varepsilon_t^*)] \frac{1}{p_t} A_t^m = G(\varepsilon_t^*) S_t. \tag{35}$$

Proof. Recall that $\bar{A}_{It}^s = \alpha \int \bar{a}_t^s(\boldsymbol{a}_t, \varepsilon) dH_{It}(\boldsymbol{a}_t, \varepsilon)$, so using the bargaining outcomes in Lemma 6, we have

$$\bar{A}_{It}^s = \alpha [1 - G(\varepsilon_t^*)] \left(S_t + \frac{1}{p_t} A_t^m \right).$$

With this expression, the market-clearing condition for equity in the first subperiod of period t, i.e., $\bar{A}_{It}^s = \alpha S_t$, can be written as (35). ■

A.3 Equilibrium characterization: stock prices and real money balances

The following result characterizes the equilibrium paths $\{\mathrm{M}_t\}_{t=0}^{\infty}$ and $\{\phi_t^s\}_{t=0}^{\infty}$ taking as given the path for the outstanding aggregate quantity of stocks, $\{S_t\}_{t=0}^{\infty}$.

Corollary 4 *In equilibrium, aggregate real money balances, $\{\mathrm{M}_t\}_{t=0}^{\infty}$, and the real price of equity shares, $\{\phi_t^s\}_{t=0}^{\infty}$, satisfy the following conditions:*

$$\mathrm{M}_t \geq \frac{\beta}{\mu} \left[1 + \alpha\theta \int_{\varepsilon_{t+1}^*}^{\varepsilon_H} \frac{\varepsilon - \varepsilon_{t+1}^*}{\varepsilon_{t+1}^* z + \pi(1-\delta)\phi_{t+1}^s} z dG(\varepsilon) \right] \mathrm{M}_{t+1}, \text{ with “$=$” if } \mathrm{M}_{t+1} > 0 \tag{36}$$

$$\phi_t^s = \beta \left[\bar{\varepsilon} z + \pi(1-\delta)\phi_{t+1}^s + \alpha\theta \int_{\varepsilon_L}^{\varepsilon_{t+1}^*} \left(\varepsilon_{t+1}^* - \varepsilon \right) z dG(\varepsilon) \right], \tag{37}$$

where for all $t \geq 0$, ε_t^* satisfies

$$\frac{1 - G(\varepsilon_t^*)}{\varepsilon_t^* z + \pi(1-\delta)\phi_t^s} \mathrm{M}_t = G(\varepsilon_t^*) S_t. \tag{38}$$

Proof. Conditions (36), (37), and (38) follow from (33), (34), and (35), respectively, using $\mathrm{M}_t \equiv \phi_t^m A_t^m$, $A_{t+1}^m / A_t^m = \mu$, and (29). ■

The following result characterizes the equilibrium paths $\{\mathrm{M}_t\}_{t=0}^{\infty}$ and $\{\phi_t^s\}_{t=0}^{\infty}$ taking as given the path for the outstanding aggregate quantity of stocks, $\{S_t\}_{t=0}^{\infty}$—in the context of a stationary equilibrium.

Corollary 5 *In a stationary equilibrium,* $S_t = S$, $\varepsilon_t^* = \varepsilon^*$, $\phi_t^s = \varphi^s z$, *and* $\mathrm{M}_t = \mathrm{M}$ *for all* t, *and* $(\varepsilon^*, \varphi^s, \mathrm{M})$ *satisfy the following conditions:*

$$r \geq \alpha\theta \int_{\varepsilon^*}^{\varepsilon_H} \frac{\varepsilon - \varepsilon^*}{\varepsilon^* + \pi(1-\delta)\varphi^s} dG(\varepsilon), \text{ with "= " if } \mathrm{M} > 0 \tag{39}$$

$$\varphi^s = \frac{\beta}{1 - \beta\pi(1-\delta)} \left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon) \right], \tag{40}$$

where ε^* *satisfies*

$$\frac{1 - G(\varepsilon^*)}{[\varepsilon^* + \pi(1-\delta)\varphi^s]\, z} \mathrm{M} = G(\varepsilon^*)\, S. \tag{41}$$

Proof. Conditions (36)-(38) follow immediately from (39)-(41) imposing the stationarity conditions described in the statement. ■

Lemma 9 *Let* $S > 0$ *be given. Then:*

(i) There always exists a solution to (39)-(41) in which money is not valued, i.e., $\mathrm{M} = 0$, $\varepsilon^* = \varepsilon_L$, *and* $\varphi^s = \frac{\beta}{1-\beta\pi(1-\delta)}\bar{\varepsilon}$.

(ii) Let

$$\bar{r} \equiv \frac{\alpha\theta\,(\bar{\varepsilon} - \varepsilon_L)}{\varepsilon_L + \frac{\beta\pi(1-\delta)}{1-\beta\pi(1-\delta)}\bar{\varepsilon}}.$$

If $r \in (0, \bar{r})$ *there exists a unique solution to (39)-(41) with* $\mathrm{M} > 0$, *i.e.,*

$$\mathrm{M} = \frac{G(\varepsilon^*)\,[\varepsilon^* + \pi(1-\delta)\varphi^s]\, z}{1 - G(\varepsilon^*)} S \tag{42}$$

$$\varphi^s = \frac{\beta}{1 - \beta\pi(1-\delta)} \left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon) \right], \tag{43}$$

where $\varepsilon^* \in (\varepsilon_L, \varepsilon_H]$ *is the unique solution to*

$$\frac{\alpha\theta \int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^*)\, dG(\varepsilon)}{\varepsilon^* + \frac{\beta\pi(1-\delta)}{1-\beta\pi(1-\delta)} \left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon) \right]} = r. \tag{44}$$

Moreover:

(a) As $r \to \bar{r}$, $\varepsilon^* \to \varepsilon_L$, $\mathrm{M} \to 0$, *and* $\varphi^s \to \frac{\beta}{1-\beta\pi(1-\delta)}\bar{\varepsilon}$.

(b) As $r \to 0$, $\varepsilon^* \to \varepsilon_H$ *and* $\varphi^s \to \frac{\beta}{1-\beta\pi(1-\delta)} [\bar{\varepsilon} + \alpha\theta\,(\varepsilon_H - \bar{\varepsilon})]$.

(c) $\frac{\partial \varepsilon^*}{\partial r} < 0$, $\frac{\partial \mathrm{M}}{\partial r} < 0$, *and* $\frac{\partial \varphi^s}{\partial r} < 0$.

Proof. To establish part (i), simply set M $= 0$ in (39)-(41). To establish part (ii), proceed as follows. Assume M > 0; then (39) holds with equality, and using (40) to substitute φ^s from (39) gives $T(\varepsilon^*; r) = 0$, where

$$T(\varepsilon^*; r) \equiv \frac{\alpha\theta \int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^*)\, dG(\varepsilon)}{\varepsilon^* + \frac{\beta\pi(1-\delta)}{1-\beta\pi(1-\delta)} \left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon)\right]} - r.$$

First, notice that

$$\frac{\partial T(\varepsilon^*; r)}{\partial \varepsilon^*} = -\frac{[1-G(\varepsilon^*)]\left\{\varepsilon^* + \frac{\beta\pi(1-\delta)}{1-\beta\pi(1-\delta)}\left[\bar{\varepsilon} + \alpha\theta\int_{\varepsilon_L}^{\varepsilon^*}(\varepsilon^*-\varepsilon)dG(\varepsilon)\right]\right\} + \left[\int_{\varepsilon^*}^{\varepsilon_H}(\varepsilon-\varepsilon^*)dG(\varepsilon)\right]\left[1+\frac{\beta\pi(1-\delta)}{1-\beta\pi(1-\delta)}\alpha\theta G(\varepsilon^*)\right]}{\frac{1}{\alpha\theta}\left\{\varepsilon^* + \frac{\beta\pi(1-\delta)}{1-\beta\pi(1-\delta)}\left[\bar{\varepsilon} + \alpha\theta\int_{\varepsilon_L}^{\varepsilon^*}(\varepsilon^*-\varepsilon)dG(\varepsilon)\right]\right\}^2} < 0.$$

Assume $r \in (0, \bar{r})$. Then

$$T(\varepsilon_H; r) = -r < 0 < T(\varepsilon_L; r) = \bar{r} - r. \tag{45}$$

Since T is a continuous function of ε^*, $\partial T(\varepsilon^*; r)/\partial\varepsilon^* < 0$ and (45) imply that for any $r \in (0, \bar{r})$ there exists a unique ε^* that solves $T(\varepsilon^*; r) = 0$ on the interval $(\varepsilon_L, \varepsilon_H)$. Given the ε^* that solves $T(\varepsilon^*; r) = 0$, M and ϕ_t^s are given by (42) and (43), respectively.

Part $(ii)(a)$ is immediate from (42) and (43), and the observation that $T(\varepsilon_L; \bar{r}) = 0$. Part $(ii)(b)$ is immediate from (43), and the observation that $T(\varepsilon_H; 0) = 0$. Part $(ii)(c)$, follows from

$$\begin{aligned}
\frac{\partial \text{M}}{\partial r} &= \frac{G'(\varepsilon^*)}{[1 - G(\varepsilon^*)]^2} S \frac{\partial \varepsilon^*}{\partial r} + \frac{G(\varepsilon^*)}{1 - G(\varepsilon^*)} \frac{\partial S}{\partial r} \\
\frac{\partial \varphi^s}{\partial r} &= \alpha\theta \frac{\beta}{1 - \beta\pi(1-\delta)} G(\varepsilon^*) \frac{\partial \varepsilon^*}{\partial r}
\end{aligned}$$

together with the fact that

$$\frac{\partial \varepsilon^*}{\partial r} = \frac{1}{\frac{\partial T(\varepsilon^*; r)}{\partial \varepsilon^*}}$$

and $\partial T(\varepsilon^*; r)/\partial\varepsilon^* < 0$. ■

A.4 Economy with $\pi = 0$

A.4.1 Entrepreneur's choice of investment and capital structure

Proof of Lemma 1. The Lagrangian for the optimization problem of the one-period-lived entrepreneur at entry, i.e., (11), is

$$\begin{aligned}
\mathcal{L} &= y + \phi_e^s [(1-\delta) k + x - s_{+1}] \\
&\quad + \xi [\phi^s s_{+1} + w - y - \text{C}(x/k) k] \\
&\quad + \zeta_L^e s_{+1} + \zeta_H^e [(1-\delta) k + x - s_{+1}] + \zeta_L^c y,
\end{aligned}$$

where ξ, ζ_L^e, ζ_H^e, and ζ_L^c are the Lagrange multipliers on the entrepreneur's budget constraint, nonnegativity constraint on equity issuance, upper bound on equity issuance, and nonnegativity constraint on consumption, respectively.

The first-order conditions are

$$0 = 1 - \xi + \zeta_L^c \tag{46}$$

$$0 = \phi_e^s - \xi \mathrm{C}'(x/k) + \zeta_H^e \tag{47}$$

$$0 = -\phi_e^s + \xi \phi^s + \zeta_L^e - \zeta_H^e \tag{48}$$

$$0 = \xi \left[\phi^s s_{+1} + w - y - \mathrm{C}(x/k)\, k\right] \tag{49}$$

$$0 = \zeta_L^c y \tag{50}$$

$$0 = \zeta_L^e s_{+1} \tag{51}$$

$$0 = \zeta_H^e \left[(1-\delta)\, k + x - s_{+1}\right]. \tag{52}$$

Conditions (46)-(48) are the first-order conditions with respect to y, x, and s_{+1}, respectively. Condition (46) implies $\xi = 1 + \zeta_L^c > 0$, so (49) implies

$$0 = \phi^s s_{+1} + w - y - \mathrm{C}(x/k)\, k. \tag{53}$$

There are potentially eight cases depending on whether the multipliers $(\zeta_L^c, \zeta_L^e, \zeta_H^e)$ are positive or equal to zero. We consider each in turn. Recall ι_0 is the investment rate that satisfies $\mathrm{C}'(\iota_0) = 1$, so $\mathrm{C}'' > 0$ and the assumption in the statement of the lemma imply

$$\delta - 1 \leq \iota_0 \leq \min\{\iota(\phi^s), \iota(\phi_e^s)\}. \tag{54}$$

Case 1: $\zeta_L^e = \zeta_H^e = 0 < \zeta_L^c$. In this case condition (50) implies

$$y = 0,$$

condition (53) implies

$$\phi^s s_{+1} = \mathrm{C}(x/k)\, k - w, \tag{55}$$

and conditions (47) and (48) imply

$$\mathrm{C}'(x/k) = \phi^s.$$

For this case to be a solution we need three conditions to be satisfied. First, $0 < \zeta_L^c$, which by (46) is equivalent to $\xi > 1$, which by (48) is equivalent to

$$\phi^s < \phi_e^s.$$

Second, since the solution must satisfy the constraints $0 \le s_{+1} \le (1-\delta)\,k + x$, (55) implies we must have

$$\Xi\left(\iota\left(\phi^s\right)\right) \le \omega \le \mathrm{C}\left(\iota\left(\phi^s\right)\right),$$

where

$$\Xi\left(\iota\right) \equiv \mathrm{C}\left(\iota\right) - \mathrm{C}'\left(\iota\right)\left(1-\delta+\iota\right). \tag{56}$$

Notice $\Xi\left(\iota_0\right) = \delta - 1 \le 0$ and $\Xi'\left(\iota\right) = -\,\mathrm{C}''\left(\iota\right)\left(1-\delta+\iota\right) \le 0$ for all $\iota \ge \iota_0$, so (54) implies the condition $\Xi\left(\iota\left(\phi^s\right)\right) \le \omega$ is satisfied for any $\omega \ge 0$.

Case 2: $\zeta_L^c = \zeta_H^e = 0 < \zeta_L^e$. In this case (46) implies $\xi = 1$, (47) implies

$$\mathrm{C}'\left(x/k\right) = \phi_e^s,$$

(48) implies

$$\zeta_L^e = \phi_e^s - \phi^s, \tag{57}$$

(51) implies

$$s_{+1} = 0,$$

and (53) implies

$$y = w - \mathrm{C}(\iota(\phi_e^s))k. \tag{58}$$

For this case to be a solution we need three conditions to be satisfied. First, $0 < \zeta_L^e$, which by (57) is equivalent to

$$\phi^s < \phi_e^s.$$

Second, $0 \le y$, which by (58) is equivalent to

$$\mathrm{C}(\iota(\phi_e^s))k \le w.$$

Third, $0 \le k_{+1} - s_{+1}$, is equivalent to

$$0 \le 1 - \delta + \iota(\phi_e^s).$$

This condition is implied by (54).

Case 3: $\zeta_L^c = \zeta_L^e = 0 < \zeta_H^e$. In this case (46) implies $\xi = 1$, (48) implies

$$\zeta_H^e = \phi^s - \phi_e^s, \tag{59}$$

and this together with (47) implies

$$\mathrm{C}'(x/k) = \phi^s.$$

Then condition (52) implies

$$s_{+1} = [1 - \delta + \iota(\phi^s)]\, k \tag{60}$$

and (53) implies

$$y = \{\phi^s [1 - \delta + \iota(\phi^s)] + \omega - \mathrm{C}(\iota(\phi^s))\}\, k. \tag{61}$$

For this case to be a solution we need three conditions to be satisfied. First, $0 < \zeta_H^e$, which by (59) is equivalent to

$$\phi_e^s < \phi^s.$$

Second, $0 \leq s_{+1}$, which by (60) is equivalent to

$$0 \leq 1 - \delta + \iota(\phi^s).$$

This condition is implied by (54). Third, $0 \leq y$, which by (61) is equivalent to

$$\Xi(\iota(\phi^s)) \leq \omega, \tag{62}$$

where $\Xi(\cdot)$ is as defined in (56). Notice $\Xi(\iota_0) = \delta - 1 \leq 0$ and $\Xi'(\iota) = -\,\mathrm{C}''(\iota)(1 - \delta + \iota) \leq 0$ for all $\iota \geq \iota_0$, so (54) implies (62) is satisfied for any $\omega \geq 0$.

Case 4: $\zeta_H^e = 0 < \min(\zeta_L^c, \zeta_L^e)$. In this case (50) implies

$$y = 0,$$

(51) implies

$$s_{+1} = 0,$$

and hence (53) implies

$$x/k = \mathrm{C}^{-1}(\omega).$$

Conditions (46) and (47) imply

$$\zeta_L^c = \frac{\phi_e^s - \mathrm{C}'(\mathrm{C}^{-1}(\omega))}{\mathrm{C}'(\mathrm{C}^{-1}(\omega))}, \tag{63}$$

and conditions (47) and (48) imply

$$\zeta_L^e = \frac{\mathrm{C}'(\mathrm{C}^{-1}(\omega)) - \phi^s}{\mathrm{C}'(\mathrm{C}^{-1}(\omega))}\phi_e^s. \tag{64}$$

For this case to be a solution we need three conditions to be satisfied. First, $0 < \zeta_L^c$, which by (63) is equivalent to

$$\mathrm{C}'(\mathrm{C}^{-1}(\omega)) < \phi_e^s \Leftrightarrow \mathrm{C}^{-1}(\omega) < \iota(\phi_e^s) \tag{65}$$

Second, $0 < \zeta_L^e$, which by (64) is equivalent to

$$\phi^s < \mathrm{C}'(\mathrm{C}^{-1}(\omega)) \Leftrightarrow \iota(\phi^s) < \mathrm{C}^{-1}(\omega). \tag{66}$$

Notice that conditions (65) and (66) can both be satisfied only if

$$\phi^s < \phi_e^s.$$

The third condition that needs to be satisfied for this case to be a solution is $0 \leq k_{+1} - s_{+1}$, which by (52) is equivalent to

$$0 \leq 1 - \delta + \mathrm{C}^{-1}(\omega). \tag{67}$$

From (66), we know that $\mathrm{C}(\iota(\phi^s)) < \omega$, which together with (54) implies

$$\iota_0 = \mathrm{C}(\iota_0) \leq \mathrm{C}(\iota(\phi^s)) < \omega.$$

Hence, $\iota_0 < \mathrm{C}^{-1}(\omega)$, which implies condition (67) is satisfied.

Case 5: $\zeta_L^e = 0 < \min(\zeta_L^c, \zeta_H^e)$. In this case (50) implies

$$y = 0,$$

and conditions (47) and (48) imply

$$\mathrm{C}'(x/k) = \phi^s.$$

Then (52) implies

$$s_{+1} = [1 - \delta + \iota(\phi^s)]\, k. \tag{68}$$

For this case to be a solution we need four conditions to be satisfied. First, $0 \leq s_{+1}$, which with (68) is equivalent to

$$0 \leq 1 - \delta + \iota(\phi^s).$$

This condition is implied by (54). Second, (49) and (52) require that

$$\omega = \Xi(\iota(\phi^s)) \tag{69}$$

with $\Xi(\cdot)$ as defined in (56). As argued in Case 3, the assumptions in the statement of the lemma imply $\Xi(\iota(\phi^s)) \leq 0$. Since $\omega \geq 0$, (69) implies this case is only possible if $\omega = 0$. Third, $0 < \zeta_L^c$ requires that $1 < \xi$. Fourth, $0 < \zeta_H^e$ requires that $\zeta_H^e = \xi\phi^s - \phi_e^s > 0$. There exist values of ξ that satisfy both these conditions.

Case 6: $\zeta_L^c = 0 < \min(\zeta_L^e, \zeta_H^e)$. In this case (51) implies

$$s_{+1} = 0$$

and then (52) implies

$$x/k = \delta - 1,$$

and condition (49) implies

$$y = [\omega - \mathrm{C}(\delta - 1)]\, k. \tag{70}$$

Conditions (47) and (48) imply

$$\zeta_L^e = \mathrm{C}'(\delta - 1) - \phi^s \tag{71}$$

$$\zeta_H^e = \mathrm{C}'(\delta - 1) - \phi_e^s. \tag{72}$$

For this case to be a solution, we need three conditions to hold. First, $0 \leq y$, which by (70) is equivalent to

$$\mathrm{C}(\delta - 1) \leq \omega.$$

And $0 < \min(\zeta_L^e, \zeta_H^e)$, which by (71) and (72) are equivalent to

$$\max(\phi^s, \phi_e^s) < \mathrm{C}'(\delta - 1). \tag{73}$$

Notice (54) implies

$$\mathrm{C}'(\delta - 1) \leq \mathrm{C}'(\iota_0) \leq \mathrm{C}'(\min\{\iota(\phi^s), \iota(\phi_e^s)\}) = \min(\phi^s, \phi_e^s), \tag{74}$$

which contradicts (73), so this case cannot be a solution.

Case 7: $0 < \min(\zeta_L^c, \zeta_L^e, \zeta_H^e)$. In this case (50)-(52) imply

$$y = 0$$

$$s_{+1} = 0$$

$$x/k = \delta - 1.$$

For this to be a solution, we need the following conditions to hold

$$\begin{aligned} w &= \text{C}(\delta-1)k \\ 1 &< \xi \\ \zeta_L^e &= \xi\left[\text{C}'(\delta-1)-\phi^s\right]>0 \quad (75)\\ \zeta_H^e &= \xi\left[\text{C}'(\delta-1)\right]-\phi_e^s>0. \quad (76)\end{aligned}$$

The first is implied by (49), the second by the condition $0<\zeta_L^c$, and the third and fourth by the conditions (47) and (48), and the requirement that $0<\min(\zeta_L^e,\zeta_H^e)$. Notice (54) implies (74), which contradicts (75) and (76), so this case cannot be a solution.

Case 8: $\zeta_L^c=\zeta_L^e=\zeta_H^e=0$. In this case conditions (47) and (48) imply

$$\text{C}'(x/k)=\phi_e^s=\phi^s,$$

condition (53) implies

$$y=\phi^s s_{+1}+\left[\omega-\text{C}(\iota(\phi^s))\right]k,$$

and s_{+1} is any number that satisfies that satisfies

$$\max\left\{0,\frac{\text{C}(\iota(\phi^s))-\omega}{\phi^s}k\right\}\le s_{+1}\le\left[1-\delta+\iota(\phi^s)\right]k.$$

Cases 1, 2, and 4, are summarized in part (ii) of the statement of the lemma, while part (i) summarizes cases 3, 5, and 8. This concludes the proof. ■

Proof of Corollary 2. The Lagrangian for (11) can be written as

$$\begin{aligned}\mathcal{L} &= y+\phi_e^s(k_{+1}-s_{+1})\\ &\quad+\hat{q}\left[(1-\delta)k+x-k_{+1}\right]\\ &\quad+\xi\left[\phi^s s_{+1}+w-y-\text{C}(x/k)k\right]\\ &\quad+\zeta_L^e s_{+1}+\zeta_H^e(k_{+1}-s_{+1})+\zeta_L^c y,\end{aligned}$$

where ξ, ζ_L^e, ζ_H^e, and ζ_L^c are the Lagrange multipliers on the entrepreneur's budget constraint, nonnegativity constraint on equity issuance, upper bound on equity issuance, and nonnegativity constraint on consumption, respectively. The Lagrange multiplier $\hat{q}$ is associated to the law of motion of the capital stock, and is interpreted as the shadow price of a margial unit of capital to the entrepreneur. The first-order conditions with respect to y, x, s_{+1}, and k_{+1} are, respectively,

$$0 = 1 - \xi + \zeta_L^c \tag{77}$$

$$0 = \hat{q} - \xi \mathrm{C}'(x/k) \tag{78}$$

$$0 = -\phi_e^s + \xi \phi^s + \zeta_L^e - \zeta_H^e \tag{79}$$

$$0 = \phi_e^s - \hat{q} + \zeta_H^e. \tag{80}$$

Condition (80) implies the shadow price of capital to the entrepreneur, $\hat{q}$, is at least as large as the discounted value that she assigns to the return on capital, ϕ_e^s, but could exceed it if the entrepreneur is facing a binding financing constraint, i.e., in the form of a binding upper bound on equity issuance ($0 < \zeta_H^e$). If we use (80) to substitute $\hat{q}$ in (78), then (77)-(79) become identical to (46)-(48) in the proof of Lemma 1. For what follows, it is convenient to define

$$q \equiv \frac{\hat{q}}{\xi}. \tag{81}$$

Intuitively, ξ is the shadow price to the entrepreneur of a unit of good 2 (in terms of second-subperiod marginal utility). Since the entrepreneur's utility for good 2 is linear, this shadow price equals 1 in an interior solution. But it will exceed 1 if the entrepreneur is financially constrained in the sense that it would like to be able to borrow good 2 to invest but is unable to do so. This "binding financial constraint" manisfests itself with $0 < \zeta_L^c$, i.e., a situation in which the nonnegativity constraint on consumption binds. In sum, the q defined in (81) is the *return* (gross of adjustment costs) to the entrepreneur from investing an additional unit good 2 into capital. When investing an additional unit of good 2, the entrepreneur pays utility cost ξ to get payoff $\hat{q}$. Condition (78) then says that at an optimum, $\mathrm{C}'(x/k) = q$, i.e., the marginal (technological) cost of investing, $\mathrm{C}'(x/k)$, must equal the marginal return to investing, q. Next, we derive the value of q corresponding to every case in Lemma 1.

Case 1. This case corresponds to the lowest endowment range (i.e., $\omega \leq \mathrm{C}(\iota(\phi^s))$) in part (*ii*) of Lemma 1. In this case the Lagrange multipliers are:

$$\begin{aligned} \zeta_L^e &= \zeta_H^e = 0 < \zeta_L^c = \frac{\phi_e^s}{\phi^s} - 1 = \xi - 1 \\ \hat{q} &= \phi_e^s \\ q &= \phi^s, \end{aligned}$$

and the optimal investment rate, x^*, satisfies

$$\text{C}'(x^*) = \phi^s.$$

Case 2. This case corresponds to the highest endowment range (i.e., $\text{C}(\iota(\phi_e^s)) \leq \omega$) in part (ii) of Lemma 1. In this case the Lagrange multipliers are:

$$\begin{aligned} \zeta_L^c &= \zeta_H^e = 0 = \xi - 1 < \phi_e^s - \phi^s = \zeta_L^e \\ \hat{q} &= \phi_e^s \\ q &= \phi_e^s, \end{aligned}$$

and the optimal investment rate, x^*, satisfies

$$\text{C}'(x^*) = \phi_e^s.$$

Case 4. This case corresponds to the intermediate endowment range (i.e., $\text{C}(\iota(\phi^s)) < \omega < \text{C}(\iota(\phi_e^s)))$ in part (ii) of Lemma 1. In this case the Lagrange multipliers are:

$$\begin{aligned} 0 &= \zeta_H^e \\ 0 &< \zeta_L^c = \xi - 1 = \frac{\phi_e^s}{\text{C}'(x^*)} - 1 \\ 0 &< \zeta_L^e = \left[1 - \frac{\phi^s}{\text{C}'(x^*)}\right]\phi_e^s \\ \hat{q} &= \phi_e^s \\ q &= \text{C}'(x^*), \end{aligned}$$

and the optimal investment rate, x^*, satisfies

$$\text{C}(x^*) = \omega.$$

Case 3. This case corresponds to the case with $\phi_e^s < \phi^s$ in part (i) of Lemma 1. In this case the Lagrange multipliers are:

$$\begin{aligned} \zeta_L^c &= \zeta_L^e = 0 < \zeta_H^e = \phi^s - \phi_e^s \\ \xi &= 1 \\ q &= \hat{q} = \phi^s, \end{aligned}$$

and the optimal investment rate, x^*, satisfies

$$\mathrm{C}'(x^*) = \phi^s.$$

Case 5. This case corresponds to the case with $y^* = 0 < \phi^s - \phi_e^s$ in part (i) of Lemma 1. In this case the Lagrange multipliers are:

$$\begin{aligned} 0 &< \xi - 1 = \zeta_L^c \\ 0 &< \hat{q} - \phi_e^s = \zeta_H^e \\ q &= \phi^s, \end{aligned}$$

and the optimal investment rate, x^*, satisfies

$$\mathrm{C}'(x^*) = \phi^s.$$

Case 8. This case corresponds to the case with $\phi^s = \phi_e^s$ in part (i) of Lemma 1. In this case the Lagrange multipliers are:

$$\begin{aligned} 0 &= \zeta_L^c = \zeta_L^e = \zeta_H^e = \xi - 1 \\ q &= \hat{q} = \phi^s = \phi_e^s, \end{aligned}$$

and the optimal investment rate, x^*, satisfies

$$\mathrm{C}'(x^*) = \phi^s.$$

By collecting all cases we obtain the expressions in the statement. ■

Corollary 6 *The value function (11) can be written as*

$$J(w,k,0) = [y^* + \phi_e^s(1 - \delta + x^* - s_{+1}^*)]k,$$

with (x^*, y^*, s_{+1}^*) *as given in Lemma 1.*

(i) If $\phi_e^s \leq \phi^s$,

$$\frac{J(w,k,0)}{k} = \phi^s[1 - \delta + \iota(\phi^s)] + \omega - \mathrm{C}(\iota(\phi^s)).$$

(ii) If $\phi^s < \phi_e^s$,

$$\frac{J(w,k,0)}{k} = \begin{cases} \phi_e^s[1 - \delta + \iota(\phi_e^s)] + \omega - \mathrm{C}(\iota(\phi_e^s)) & \textit{if } \mathrm{C}(\iota(\phi_e^s)) \leq \omega \\ \phi_e^s[1 - \delta + \mathrm{C}^{-1}(\omega)] & \textit{if } \mathrm{C}(\iota(\phi^s)) < \omega < \mathrm{C}(\iota(\phi_e^s)) \\ \phi_e^s[1 - \delta + \iota(\phi^s) - \frac{\mathrm{C}(\iota(\phi^s)) - \omega}{\phi^s}] & \textit{if } \omega \leq \mathrm{C}(\iota(\phi^s)). \end{cases}$$

In every case, the value function can be written as $\mathcal{J}(\omega)k$, *where* $\mathcal{J}(\omega) \equiv J(\omega k, k, 0)/k$.

A.4.2 Equilibrium characterization

Proof of Proposition 1. In a stationary nonmonetary equilibrium, we know from Lemma 9 that M $= 0$, $\varepsilon^* = \varepsilon_L$, and $\phi^s = \varphi^s z$, with $\varphi^s = \frac{\beta}{1-\beta\pi(1-\delta)}\bar{\varepsilon}$. In this case $\pi = 0$, so $\phi^s = \beta\bar{\varepsilon}z \equiv \underline{\phi}^s$. The expressions for X^* and S^* in parts (i) and (ii) follow from (12) and (13), and the expressions in parts (i) and (ii) of Lemma 1. ■

Proof of Proposition 2. The existence and uniqueness claim in part (i) follows from the fact that there exists a unique ε^* that satisfies (15), as established in Lemma 9. Parts (ii) and (vi) also follow from Lemma 9. To establish parts (iii), (iv), and (v) we again rely on Lemma 9, which shows that $\varphi^s(r)$ is continuous, with $\frac{\partial\varphi^s(r)}{\partial r} < 0$, $\phi^s(0) = \overline{\phi}^s$, and $\phi^s(\bar{r}) = \underline{\phi}^s$. From this it follows that for every $\phi^s_e \in (\underline{\phi}^s, \overline{\phi}^s)$ there exists a unique $\hat{r} \in (0, \bar{r})$ that satisfies $\phi^s(\hat{r}) = \phi^s_e$, with $\phi^s(r) > \phi^s_e$ for all $r \in (0, \hat{r})$, and $\phi^s(r) < \phi^s_e$ for all $r \in (\hat{r}, \bar{r})$. Given this, the expressions for X^* and S^* then follow from (12), (13), and Lemma 1. ■

B Adverse selection

In this section we formalize a simple agency problem between entrepreneurs and investors to show that in order to have an equilibrium with $\phi^s < \phi^s_e$, one need not assume that the fundamental value of the dividend of good 1 is higher for entrepreneurs than for outside investors.

Consider a generalization of the model of Section 4 in which the productivity of a unit of capital created in the second subperiod of period t is a random variable $Z_t \in \{0, z\}$. A fraction $1-\lambda$ of the entrepreneurs draw productivity $Z_t = 0$, while the remaining draw $Z_t = z > 0$.[40] The timing of information is that an entrepreneur makes the investment and equity issuance decisions at the end of period t, having observed the realization of Z_t, while outside investors learn this realization at the beginning of period $t+1$ (before the round of stock-market trades in the first subperiod). We maintain the assumption of competitive trade in the second subperiod, so the stock price in the second subperiod of period t (i.e., at the time the investment in physical capital is made and equity claims on these units of capital are issued) is determined in a competitive market in which all shares trade at the same price.[41]

[40] The model of Section 4 corresponds to the special case with $\lambda = 1$. For simplicity, we assume this random variable is independent across entrepreneurs and uncorrelated with the entrepreneur's characteristics (e.g., her capital, k, and claims to good 2, w).

[41] This would be a natural market outcome in a context in which investors know the probability distribution over Z_t but have no way of obtaining entrepreneur-specific information. One could instead set the model up as

As in Section 4, we focus on stationary equilibria, and maintain the assumption $\pi = 0$ (entrepreneurs live for one period). In addition, to simplify the exposition, in this section we assume $\delta = 1$ (capital only lasts one period), and $\iota_0 = 0$. Under these conditions, the entrepreneur's problem (analogous to (11)) is:

$$\max_{x,y,s_{+1}} \left[y + \beta\varepsilon_e Z(x - s_{+1})\right] \tag{82}$$

$$\begin{aligned} \text{s.t. } y + \text{C}(x/k)\,k &\leq \phi^s s_{+1} + w && (83) \\ 0 &\leq s_{+1} \leq x && (84) \\ 0 &\leq y. && (85) \end{aligned}$$

Let $\tilde{g}^x(Z, w, k)$, $\tilde{g}^y(Z, w, k)$, and $\tilde{g}^e(Z, w, k)$ denote the levels of investment, consumption of good 2, and equity issuance that solve (82)-(85) for an entrepreneur with productivity realization $Z \in \{0, z\}$. Define $x^* \equiv g^x(Z, w, k)/k$, $y^* \equiv g^y(Z, w, k)/k$, $s^*_{+1} \equiv g^e(Z, w, k)/k$, and $\omega \equiv w/k$. The following result, which is analogous to Lemma 1, characterizes (x^*, y^*, s^*_{+1}) as a function of the entrepreneur's marginal valuation, $\phi^s_e \equiv \beta\varepsilon_e Z$, and the market valuation, ϕ^s.

Lemma 10 *Consider the economy with adverse selection, and assume $\pi = 1 - \delta = \iota_0 = 0$. Let $\iota(\phi)$ denote the unique number, ι, that solves $\text{C}'(\iota) = \phi$ for any $\phi \in \mathbb{R}_+$.*

(i) If $\max(\phi^s_e, \phi^s) < 1$, then $x^* = s^*_{+1} = 0$.

(ii) If $1 \leq \max(\phi^s_e, \phi^s)$ and $\phi^s_e \leq \phi^s$, then

$$\begin{aligned} x^* &= \iota(\phi^s) \\ s^*_{+1} &= \begin{cases} x^* & \text{if } \phi^s_e < \phi^s \\ \left[\max\left\{0, \frac{\text{C}(x^*)-\omega}{\phi^s}\right\}, x^*\right] & \text{if } \phi^s_e = \phi^s. \end{cases} \end{aligned}$$

(iii) If $1 \leq \phi^s < \phi^s_e$, then

$$\begin{aligned} x^* &= \begin{cases} \iota(\phi^s_e) & \text{if } \text{C}(\iota(\phi^s_e)) \leq \omega \\ \text{C}^{-1}(\omega) & \text{if } \text{C}(\iota(\phi^s)) < \omega < \text{C}(\iota(\phi^s_e)) \\ \iota(\phi^s) & \text{if } \omega \leq \text{C}(\iota(\phi^s)) \end{cases} \\ s^*_{+1} &= \begin{cases} 0 & \text{if } \text{C}(\iota(\phi^s)) < \omega \\ \frac{\text{C}(\iota(\phi^s))-\omega}{\phi^s} & \text{if } \omega \leq \text{C}(\iota(\phi^s)). \end{cases} \end{aligned}$$

a signalling game in which entrepreneurs play the role of senders and investors play the role of receivers.

(iv) If $\phi^s < 1 \leq \phi_e^s$, then

$$\begin{aligned} x^* &= \begin{cases} \iota(\phi_e^s) & \text{if } \mathrm{C}(\iota(\phi_e^s)) \leq \omega \\ \mathrm{C}^{-1}(\omega) & \omega < \mathrm{C}(\iota(\phi_e^s)) \end{cases} \\ s_{+1}^* &= 0. \end{aligned}$$

(v) In every case, $y^* = \omega + \phi^s s_{+1}^* - \mathrm{C}(x^*)$.

Proof. Since the constraint (83) will bind at an optimum, the problem (82)-(85) implies

$$(x^*, s_{+1}^*) = \arg\max_{x, s_{+1}} \left[\phi_e^s x - \mathrm{C}(x) + (\phi^s - \phi_e^s)\, s_{+1}\right] \tag{86}$$

$$\text{s.t. } \max\left(0, \frac{\mathrm{C}(x) - \omega}{\phi^s}\right) \leq s_{+1} \leq x \tag{87}$$

and

$$y^* = \omega + \phi^s s_{+1}^* - \mathrm{C}(x^*). \tag{88}$$

The Lagrangian for (86)-(87) is

$$\mathcal{L} = \phi_e^s x - \mathrm{C}(x) + (\phi^s - \phi_e^s)\, s_{+1} + \zeta_L^e s_{+1} + \zeta_H^e (x - s_{+1}) + \zeta_L^c \left[\omega + \phi^s s_{+1} - \mathrm{C}(x)\right],$$

where ζ_L^e, ζ_H^e, and ζ_L^c are the Lagrange multipliers on the nonnegativity constraint on equity issuance, the upper bound on equity issuance, and the nonnegativity constraint on consumption of good 2, respectively. The first-order conditions are

$$0 = \phi_e^s - (1 + \zeta_L^c)\, \mathrm{C}'(x) + \zeta_H^e \tag{89}$$

$$0 = (1 + \zeta_L^c)\, \phi^s - \phi_e^s + \zeta_L^e - \zeta_H^e \tag{90}$$

$$0 = \zeta_L^e s_{+1} \tag{91}$$

$$0 = \zeta_H^e (x - s_{+1}) \tag{92}$$

$$0 = \zeta_L^c \left[\omega + \phi^s s_{+1} - \mathrm{C}(x)\right]. \tag{93}$$

There are eight cases depending on whether the multipliers $(\zeta_L^e, \zeta_H^e, \zeta_L^c)$ are positive or equal to zero. We consider each in turn. In every case, we suppose $0 < \min(\phi^s, \phi_e^s)$.

Case 1: $\zeta_L^e = \zeta_H^e = 0 < \zeta_L^c$. In this case (89)-(93) imply the optimum is characterized by

$$\mathrm{C}'(x^*) = \phi^s < \phi_e^s \tag{94}$$

$$s_{+1}^* = \frac{\mathrm{C}(x^*) - \omega}{\phi^s}. \tag{95}$$

Recall that $\textsc{c}'(0) = 1$ and $\textsc{c}'' > 0$, so $1 \leq \textsc{c}'(x)$ for all $x \geq 0$ (with "=" only if $x = 0$). Hence for (94) to hold it is necessary that

$$1 \leq \phi^s < \phi^s_e. \tag{96}$$

Also, for (94)-(95) to be a solution it must satisfy $0 \leq s^*_{+1} \leq x^*$, or equivalently,

$$\textsc{c}(x^*) - \textsc{c}'(x^*)\, x^* \leq \omega \leq \textsc{c}(x^*). \tag{97}$$

The second inequality in (97) is equivalent to

$$\omega \leq \textsc{c}(\iota(\phi^s)). \tag{98}$$

Next, we show that the first inequality in (97) is redundant. Since $\textsc{c}$ is strictly convex, we have

$$\textsc{c}(x) \geq \textsc{c}(x^*) + \textsc{c}'(x^*)(x - x^*), \tag{99}$$

with "=" only if $x = x^*$. Since $\textsc{c}(0) = 0$, evaluating (99) at $x = 0$ implies

$$\textsc{c}(x^*) - \textsc{c}'(x^*)\, x^* \leq 0, \tag{100}$$

so the first inequality in (97) is satisfied for all $\omega \in \mathbb{R}_+$.

Case 2: $\zeta^c_L = \zeta^e_H = 0 < \zeta^e_L$. In this case (89)-(93) imply the optimum is characterized by

$$\phi^s < \phi^s_e = \textsc{c}'(x^*) \tag{101}$$

$$s^*_{+1} = 0. \tag{102}$$

Recall that $\textsc{c}'(0) = 1$ and $\textsc{c}'' > 0$, so $1 \leq \textsc{c}'(x)$ for all $x \geq 0$ (with "=" only if $x = 0$). Hence for (101) to hold it is necessary not only that $\phi^s < \phi^s_e$, but also that $1 \leq \phi^s_e$, which together can be written as

$$\max(1, \phi^s) \leq \phi^s_e, \text{ with " < " if } \max(1, \phi^s) = \phi^s. \tag{103}$$

Also, for (101)-(102) to be a solution, it must satisfy

$$\frac{\textsc{c}(x^*) - \omega}{\phi^s} \leq s^*_{+1} \leq x^*,$$

or equivalently,

$$\frac{\textsc{c}(x^*) - \omega}{\phi^s} \leq 0 \leq x^*. \tag{104}$$

The first inequality in (104) is equivalent to

$$\text{C}\left(\iota\left(\phi_e^s\right)\right)\leq\omega, \tag{105}$$

and the second inequality in (104) is implied by (103).

Case 3: $\zeta_L^c=\zeta_L^e=0<\zeta_H^e$. In this case (89)-(93) imply the optimum is characterized by

$$\begin{aligned}\phi_e^s &< \phi^s=\text{C}'\left(x^*\right) & (106)\\ s_{+1}^* &= x^*. & (107)\end{aligned}$$

Recall that $\text{C}'(0)=1$ and $\text{C}''>0$, so $1\leq\text{C}'(x)$ for all $x\geq0$ (with "=" only if $x=0$). Hence for (106) to hold it is necessary not only that $\phi_e^s<\phi^s$, but also that $1\leq\phi^s$, which together can be written as

$$\max\left(1,\phi_e^s\right)\leq\phi^s,\text{ with " < " if }\max\left(1,\phi_e^s\right)=\phi_e^s. \tag{108}$$

Also, for (106)-(107) to be a solution, it must satisfy

$$0\leq s_{+1}^*\text{ and }0\leq\omega+\phi^s s_{+1}^*-\text{C}\left(x^*\right),$$

which using (106) and (107) are equivalent to

$$0\leq x^*\text{ and }0\leq\omega+\text{C}'\left(x^*\right)x^*-\text{C}\left(x^*\right). \tag{109}$$

The first inequality in (109) is redundant since it follows from (106), (108), $\text{C}'(0)=1$, and $\text{C}''>0$, which imply

$$\text{C}'(0)-\phi^s=1-\phi^s\leq0=\text{C}'\left(x^*\right)-\phi^s.$$

The second inequality in (109) is satisfied for all $\omega\in\mathbb{R}_+$ since the maintained assumptions $\text{C}(0)=0<\text{C}''$ imply (100).

Case 4: $\zeta_H^e=0<\min\left(\zeta_L^c,\zeta_L^e\right)$. In this case (89)-(93) imply the optimum is characterized by

$$\begin{aligned}\omega &= \text{C}\left(x^*\right) & (110)\\ s_{+1}^* &= 0. & (111)\end{aligned}$$

For (110)-(111) to be a solution, it must also satisfy $s_{+1}^*\leq x^*$ and

$$\phi^s<\text{C}'\left(x^*\right)<\phi_e^s. \tag{112}$$

With (111), the condition $s^*_{+1} \leq x^*$ is equivalent to $0 \leq x^*$, which is implied by (110) for any $\omega \in \mathbb{R}_+$, since $\text{C}(0) = 0 < \text{C}'$. For (112) to hold it is necessary that

$$\max\left(1, \phi^s\right) < \phi^s_e. \tag{113}$$

Under assumption (113) we can write $\phi^s_e = \text{C}'\left(\iota\left(\phi^s_e\right)\right)$, and write the second inequality in (112) as

$$\text{C}'\left(x^*\right) < \text{C}'\left(\iota\left(\phi^s_e\right)\right),$$

which is equivalent to

$$\omega < \text{C}\left(\iota\left(\phi^s_e\right)\right). \tag{114}$$

If $\phi^s < 1$, then the first inequality in (112) holds for all $\omega \in \mathbb{R}_+$. Conversely, if $1 \leq \phi^s$, then we can write $\phi^s = \text{C}'\left(\iota\left(\phi^s\right)\right)$ and the first inequality in (112) can be written as

$$\text{C}'\left(\iota\left(\phi^s\right)\right) < \text{C}'\left(x^*\right),$$

which is equivalent to

$$\text{C}\left(\iota\left(\phi^s\right)\right) < \omega \qquad \text{if } 1 \leq \phi^s. \tag{115}$$

Conditions (114) and (13) can be written jointly as

$$\omega \in \begin{cases} \left(\text{C}\left(\iota\left(\phi^s\right)\right), \text{C}\left(\iota\left(\phi^s_e\right)\right)\right) & \text{if } 1 \leq \phi^s \\ \left(-\infty, \text{C}\left(\iota\left(\phi^s_e\right)\right)\right) & \text{if } \phi^s < 1. \end{cases} \tag{116}$$

Case 5: $\zeta^e_L = 0 < \min\left(\zeta^c_L, \zeta^e_H\right)$. In this case (89)-(93) imply

$$\text{C}'\left(x^*\right) = \phi^s \tag{117}$$

$$s^*_{+1} = x^*. \tag{118}$$

For (117)-(118) to be a solution, it must also satisfy

$$\omega = \text{C}\left(x^*\right) - \text{C}'\left(x^*\right) x^* \tag{119}$$

and $0 \leq x^*$. Also, $1 \leq \phi^s$ is necessary for (117) to hold (since $\text{C}'\left(x\right) \geq 1$ for all $x \geq 0$). The maintained assumptions $\text{C}(0) = 0 < \text{C}''$ imply (100), so (119) can only hold if $x^* = 0$ and

$$\omega = 0. \tag{120}$$

Together with (117), $x^{*}=0$ implies we must also have

$$\phi^{s}=1, \tag{121}$$

while ϕ_{e}^{s} can take any nonnegative value. To summarize, if (120) and (121) hold, the solution for this case is

$$s_{+1}^{*}=x^{*}=0. \tag{122}$$

Case 6: $\zeta_{L}^{c}=0<\min\left(\zeta_{L}^{e},\zeta_{H}^{e}\right)$. In this case (89)-(93) imply the optimum is characterized by

$$s_{+1}^{*}=x^{*}=0 \tag{123}$$

provided

$$\max\left(\phi_{e}^{s},\phi^{s}\right)<1. \tag{124}$$

Case 7: $0<\min\left(\zeta_{L}^{c},\zeta_{L}^{e},\zeta_{H}^{e}\right)$. In this case (89)-(93) imply the optimum is characterized by

$$s_{+1}^{*}=x^{*}=0 \tag{125}$$

provided

$$\omega=0 \tag{126}$$

and

$$\phi^{s}<1 \tag{127}$$

while ϕ_{e}^{s} can take any nonnegative value.

Case 8: $\zeta_{L}^{c}=\zeta_{L}^{e}=\zeta_{H}^{e}=0$. In this case (89)-(93) imply the optimum is characterized by

$$\mathrm{C}'\left(x^{*}\right) = \phi^{s} \tag{128}$$

$$s_{+1}^{*} \in \left[\max\left\{0,\frac{\mathrm{C}\left(x^{*}\right)-\omega}{\phi^{s}}\right\},x^{*}\right] \tag{129}$$

provided

$$1\leq\phi_{e}^{s}=\phi^{s}. \tag{130}$$

Part (i) in the statement of the lemma corresponds to Case 6 and Case 7. Part (ii) corresponds to Case 3 and Case 8. Part (*iii*) corresponds to Case 1, Case 2, and Case 4. Part (*iv*) corresponds to Case 2 and Case 4. Part (*v*) is the same as (88). ■

In this model, the outside investor's Euler equations for money and equity analogous to (33) and (34) are

$$\phi_t^m \geq \beta \left[\phi_{t+1}^m + \alpha\theta \int_{\varepsilon_{t+1}^*}^{\varepsilon_H} \left(\varepsilon - \varepsilon_{t+1}^*\right) z dG(\varepsilon) \frac{1}{p_{t+1}} \right], \text{ with " = " if } a_{t+1}^m > 0 \quad (131)$$

$$\phi_t^s \geq \beta\Lambda \left[\bar{\varepsilon} z + \alpha\theta \int_{\varepsilon_L}^{\varepsilon_{t+1}^*} \left(\varepsilon_{t+1}^* - \varepsilon\right) z dG(\varepsilon) \right], \text{ with " = " if } a_{t+1}^s > 0, \quad (132)$$

where $\Lambda \in [0,1]$ is the investor's belief that a traded equity share represents a claim to a *productive* unit of capital. The stock-market clearing condition in the first subperiod (analogous to (38)) is

$$\frac{1 - G(\varepsilon^*)}{\varepsilon^* z} \mathrm{M}_t = G(\varepsilon^*) \Lambda S_t.$$

As in Section 4, we focus on *stationary equilibria* in which the aggregate supply of equity and aggregate real money balances are constant over time, i.e., $S_t = S$ and $\phi_t^m A_t^m \equiv \mathrm{M}_t = \mathrm{M}$ for all t, and real equity prices are time-invariant linear functions of the (*expected*) dividend, i.e., $\phi_t^s = \phi^s \equiv \varphi^s z$ for all t. Thus (again imposing $\pi = 0$), the stationary-equilibrium conditions in Corollary 5 become

$$r \geq \alpha\theta \int_{\varepsilon^*}^{\varepsilon_H} \frac{\varepsilon - \varepsilon^*}{\varepsilon^*} dG(\varepsilon), \text{ with " = " if } \mathrm{M} > 0 \quad (133)$$

$$\varphi^s = \beta\Lambda \left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) dG(\varepsilon) \right] \quad (134)$$

$$\mathrm{M} = \frac{G(\varepsilon^*)}{1 - G(\varepsilon^*)} \varepsilon^* z \Lambda S. \quad (135)$$

The equilibrium conditions (133)-(135) for the economy with adverse selection are a simple generalization of conditions (39)-(41) (with $\pi = 0$) for the economy without adverse selection (both sets of conditions coincide if $\Lambda = 1$). The following result is analogous to Lemma 9, but for an economy with $\pi = 0$ and adverse selection.

Lemma 11 *Let $S > 0$ and $\Lambda \in [0,1]$ be given. Then:*

(i) There always exists a solution to (133)-(135) in which money is not valued, i.e., $\mathrm{M} = 0$, $\varepsilon^* = \varepsilon_L$, *and* $\varphi^s = \Lambda\beta\bar{\varepsilon}$.

(ii) Let $\bar{r} \equiv \alpha\theta\left(\bar{\varepsilon} - \varepsilon_L\right)/\varepsilon_L$*. If* $r \in (0, \bar{r})$*, there exists a unique solution to (133)-(135) with* $\mathrm{M} > 0$*, i.e.,*

$$\begin{aligned} \mathrm{M} &= \frac{G\left(\varepsilon^*\right)\varepsilon^* z}{1 - G\left(\varepsilon^*\right)}\Lambda S \\ \varphi^s &= \Lambda\beta\left[\bar{\varepsilon} + \alpha\theta\int_{\varepsilon_L}^{\varepsilon^*}\left(\varepsilon^* - \varepsilon\right)dG(\varepsilon)\right], \end{aligned} \tag{136}$$

where $\varepsilon^* \in (\varepsilon_L, \varepsilon_H]$ *is the unique solution to*

$$\alpha\theta\int_{\varepsilon^*}^{\varepsilon_H}\frac{\varepsilon - \varepsilon^*}{\varepsilon^*}dG(\varepsilon) = r. \tag{137}$$

Moreover:

(a) As $r \to \bar{r}$*,* $\varepsilon^* \to \varepsilon_L$*,* $\mathrm{M} \to 0$*, and* $\varphi^s \to \Lambda\beta\bar{\varepsilon}$*.*

(b) As $r \to 0$*,* $\varepsilon^* \to \varepsilon_H$ *and* $\varphi^s \to \Lambda\beta\left[\bar{\varepsilon} + \alpha\theta\left(\varepsilon_H - \bar{\varepsilon}\right)\right]$*.*

(c) $\frac{\partial\varepsilon^*}{\partial r} < 0$*,* $\frac{\partial\mathrm{M}}{\partial r} < 0$*, and* $\frac{\partial\varphi^s}{\partial r} < 0$*.*

(d) $\frac{\partial\mathrm{M}}{\partial\Lambda} > 0$*, and* $\frac{\partial\varphi^s}{\partial\Lambda} > 0$*.*

Proof. Immediate from the equilibrium conditions (133)-(135) by following steps similar to those in the proof of Lemma 9. ∎

Part (*i*), and parts (*ii*), (*a*), (*b*), and (*c*), of Lemma 11 are results analogous to their counterparts in Lemma 9. Part (*ii*) (*d*) shows how real money balances and the equity price change with the investor's belief about the proportion of outstanding shares that are claims to the productive capital.

For what follows, let $x_Z^*(\omega)$ and $s_Z^*(\omega)$ denote the optimal investment and equity issuance decisions of an entrepreneur with productivity realization $Z \in \{0, z\}$ and a balance sheet with financial wealth per unit of own capital equal to ω. We can write the aggregate investment chosen at the end of a period by all entrepreneurs with productivity $Z \in \{0, z\}$, as

$$X_Z^* = \lambda_Z\int x_Z^*(\omega)\,d\Omega(\omega),$$

and the aggregate stock of equity claims on the capital of entrepreneurs with productivity $Z \in \{0, z\}$ outstanding at the beginning of a period as

$$S_Z^* = \lambda_Z\int s_Z^*(\omega)\,d\Omega(\omega),$$

where $\lambda_Z \equiv \lambda \mathbb{I}_{\{Z=z\}} + (1-\lambda)\,\mathbb{I}_{\{Z=0\}}$ for $Z \in \{0, z\}$. The following lemma characterizes the behavior of the entrepreneurs' optimal investment and equity issuance decisions as a function of the market belief Λ, for a given policy rate, r. To state the result it is convenient to make explicit the dependence of the equity price on the belief, Λ, and the nominal rate, r, by defining the price function $\phi^s(\Lambda, r) \equiv \varphi^s z$, where φ^s is given in Lemma 11, i.e.,

$$\phi^s(\Lambda, r) = \begin{cases} \Lambda\beta \left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon)\right] z, \text{ with } \varepsilon^* \text{ given by (137)} & \text{if } 0 \leq r \leq \bar{r} \\ \Lambda\beta\bar{\varepsilon}z & \text{if } \bar{r} < r. \end{cases} \tag{138}$$

Lemma 12 *Assume* $1 < \max\{\phi_z^s, \phi^s(1, \bar{r})\}$, *where* $\phi_Z^s \equiv \beta\varepsilon_e Z$ *for* $Z \in \{0, z\}$. *For any* $r \in \mathbb{R}_+$, *let* $\Lambda' \in (0,1)$ *be the number that satisfies* $\phi^s(\Lambda', r) = 1$.

(i) If $\phi^s(1, r) < \phi_z^s$, *then:*

(a) If $\Lambda' \leq \Lambda$,

$$\begin{aligned} X_z^* &= \lambda \left[\Omega\left[\mathrm{C}\left(\iota\left(\phi^s(\Lambda, r)\right)\right)\right] \iota(\phi^s(\Lambda, r)) + \int_{\mathrm{C}(\iota(\phi^s(\Lambda,r)))}^{\mathrm{C}(\iota(\phi_z^s))} \mathrm{C}^{-1}(\omega) d\Omega(\omega) + \{1 - \Omega[\mathrm{C}(\iota(\phi_z^s))]\} \iota(\phi_z^s) \right] \\ S_z^* &= \lambda \int_0^{\mathrm{C}(\iota(\phi^s(\Lambda,r)))} \frac{\mathrm{C}(\iota(\phi^s(\Lambda, r))) - \omega}{\phi^s(\Lambda, r)} d\Omega(\omega) \end{aligned}$$

and

$$X_0^* = S_0^* = (1-\lambda)\,\iota(\phi^s(\Lambda, r)).$$

(b) If $\Lambda < \Lambda'$,

$$\begin{aligned} X_z^* &= \lambda \left[\int_0^{\mathrm{C}(\iota(\phi_z^s))} \mathrm{C}^{-1}(\omega) d\Omega(\omega) + \{1 - \Omega[\mathrm{C}(\iota(\phi_z^s))]\} \iota(\phi_z^s) \right] \\ S_z^* &= 0 \end{aligned}$$

and

$$X_0^* = S_0^* = 0.$$

(ii) If $\phi_z^s < \phi^s(1, r)$, *let* $\Lambda'' \in (\Lambda', 1)$ *be the number that satisfies* $\phi^s(\Lambda'', r) = \phi_z^s$. *Then:*

(a) If $\Lambda'' \leq \Lambda$, $X_z^* = \lambda\iota(\phi^s(\Lambda, r))$, $X_0^* = S_0^* = (1-\lambda)\,\iota(\phi^s(\Lambda, r))$, *and*

$$S_z^* \begin{cases} = X_z^* & \text{if } \Lambda'' < \Lambda \\ \in \left[\lambda \int_0^{\mathrm{C}(\iota(\phi^s(\Lambda,r)))} \frac{\mathrm{C}(\iota(\phi^s(\Lambda,r))) - \omega}{\phi^s} d\Omega(\omega), X_z^*\right] & \text{if } \Lambda = \Lambda''. \end{cases}$$

(b) If $\Lambda' \leq \Lambda < \Lambda''$, X_Z^* *and* S_Z^* *for* $Z \in \{0, z\}$ *are as in part (i)(a).*

(c) If $\Lambda < \Lambda'$, X_Z^* *and* S_Z^* *for* $Z \in \{0, z\}$ *are as in part (i)(b).*

Proof. (i) (a) The expressions for $x_z^*(\omega)$ and $s_z^*(\omega)$ used to compute X_z^* and S_z^* are from part (iii) of Lemma 10, and the expressions for $x_0^*(\omega)$ and $s_0^*(\omega)$ used to compute X_0^* and S_0^* are from part (ii) of Lemma 10.

(i) (b) The expressions for $x_z^*(\omega)$ and $s_z^*(\omega)$ used to compute X_z^* and S_z^* are from part (iv) of Lemma 10, and the expressions for $x_0^*(\omega)$ and $s_0^*(\omega)$ used to compute X_0^* and S_0^* are from part (i) of Lemma 10.

(ii) (a) The expressions for $x_Z^*(\omega)$ and $s_Z^*(\omega)$ used to compute X_Z^* and S_Z^* for $Z \in \{0, z\}$ are from part (ii) of Lemma 10.

(ii) (b) The expressions for $x_z^*(\omega)$ and $s_z^*(\omega)$ used to compute X_z^* and S_z^* are from part (iii) of Lemma 10, and the expressions for $x_0^*(\omega)$ and $s_0^*(\omega)$ used to compute X_0^* and S_0^* are from part (ii) of Lemma 10.

(ii) (c) The expressions for $x_z^*(\omega)$ and $s_z^*(\omega)$ used to compute X_z^* and S_z^* are from part (iv) of Lemma 10, and the expressions for $x_0^*(\omega)$ and $s_0^*(\omega)$ used to compute X_0^* and S_0^* are from part (i) of Lemma 10. ■

The assumption $1 < \max\{\phi_z^s, \phi^s(1, \bar{r})\}$, or equivalently, $1 < \max\{\varepsilon_e, \bar{\varepsilon}\}\beta z$, in the statement of Lemma 12 ensures that, in the absence of adverse selection, entrepreneurs and outside investors would want to invest a positive amount under any monetary policy (i.e., even in the nonmonetary equilibrium that obtains for $r > \bar{r}$).[42]

To be part of an equilibrium, an investor's belief, Λ, that a traded equity share represents a claim to *productive* unit of capital that yields dividend $z > 0$ (as opposed to a claim to an unproductive unit of capital that yields zero dividend) must satisfy $\Lambda \in [0, 1]$ and

$$\Lambda = \Upsilon(\Lambda),$$

where

$$\Upsilon(\Lambda) \equiv \frac{S_z^*}{S_0^* + S_z^*}, \tag{139}$$

with S_Z^* for $Z \in \{0, z\}$ as described in Lemma 12. Next, we provide a more explicit characterization of the mapping $\Upsilon(\cdot)$.

[42]The condition $1 < \phi_z^s$ says that the entrepreneur with the high productivity realization has an incentive to invest because the entrepreneur's private return from investing a marginal unit of capital is higher than the price of capital (in terms of good 2, which equals 1). The condition $1 < \phi^s(1, \bar{r})$ says that in the absence of adverse selection, an outside investor's discounted expected marginal return from investment under no equity trade, i.e., $\beta\bar{\varepsilon}$, is higher than the price of capital (in terms of good 2, which equals 1).

Lemma 13 *Let $\phi^s(\Lambda, r)$ be given by (138), define $\phi^s_Z \equiv \beta\varepsilon_e Z$ for $Z \in \{0, z\}$, and assume $1 < \max\{\phi^s_z, \phi^s(1, \bar{r})\}$. For any $r \in \mathbb{R}_+$, let $\Lambda'(r) \in (0,1)$ be the number that satisfies $\phi^s(\Lambda', r) = 1$, and for any $(\Lambda, r) \in [\Lambda'(r), 1] \times \mathbb{R}_+$ define*

$$\Theta(\Lambda, r) \equiv \int_0^{\mathrm{C}(\iota(\phi^s(\Lambda,r)))} \frac{\mathrm{C}(\iota(\phi^s(\Lambda, r))) - \omega}{\phi^s(\Lambda, r)\,\iota(\phi^s(\Lambda, r))} d\Omega(\omega).$$

(i) If $\phi^s(1, r) < \phi^s_z$,

$$\Upsilon(\Lambda) = \begin{cases} \frac{\lambda}{\lambda + (1-\lambda)\frac{1}{\Theta(\Lambda,r)}} & \text{for } \Lambda'(r) < \Lambda \\ 0 & \text{for } \Lambda = \Lambda'(r), \end{cases}$$

and equity is not issued if $\Lambda < \Lambda'(r)$.

(ii) If $\phi^s_z < \phi^s(1, r)$, let $\Lambda''(r) \in (\Lambda'(r), 1)$ be the number that satisfies $\phi^s(\Lambda'', r) = \phi^s_z$. Then:

$$\Upsilon(\Lambda) \begin{cases} = \lambda & \text{for } \Lambda''(r) < \Lambda \\ \in \left[\frac{\lambda}{\lambda + (1-\lambda)\frac{1}{\Theta(\Lambda,r)}}, \lambda\right] & \text{for } \Lambda''(r) = \Lambda \\ = \frac{\lambda}{\lambda + (1-\lambda)\frac{1}{\Theta(\Lambda,r)}} & \text{for } \Lambda'(r) < \Lambda < \Lambda''(r) \\ = 0 & \text{for } \Lambda = \Lambda'(r), \end{cases}$$

and equity is not issued if $\Lambda < \Lambda'(r)$.

Proof. The expression for $\Upsilon(\Lambda)$ for the case with $\Lambda'(r) < \Lambda$ in part (i) follows from (139) and parts $(i)(a)$ and $(i)(b)$ of Lemma 12. The expression for $\Upsilon(\Lambda)$ in part (ii) for the cases with $\Lambda'(r) < \Lambda$ follow from (139) and parts $(ii)(a)$, $(ii)(b)$, and $(ii)(c)$ of Lemma 12. To show that $\Upsilon(\Lambda'(r)) = 0$, both for $\Lambda'(r) < \Lambda$ in part (i), and for $\Lambda \in (\Lambda'(r), \Lambda''(r))$ in part (ii), proceed as follows. Write $\Theta(\Lambda, r)$ as

$$\Theta(\Lambda, r) = \frac{\int_0^{\mathrm{C}(\iota(\phi^s(\Lambda,r)))} [\mathrm{C}(\iota(\phi^s(\Lambda, r))) - \omega]\, d\Omega(\omega)}{\phi^s(\Lambda, r)\,\iota(\phi^s(\Lambda, r))},$$

notice that

$$\lim_{\Lambda \downarrow \Lambda'(r)} \phi^s(\Lambda, r) - 1 = \lim_{\Lambda \downarrow \Lambda'(r)} \iota(\phi^s(\Lambda, r)) = \lim_{\Lambda \downarrow \Lambda'(r)} \mathrm{C}(\iota(\phi^s(\Lambda, r))) = 0, \tag{140}$$

so

$$\lim_{\Lambda \downarrow \Lambda'(r)} \int_0^{\mathrm{C}(\iota(\phi^s(\Lambda,r)))} [\mathrm{C}(\iota(\phi^s(\Lambda, r))) - \omega]\, d\Omega(\omega) = \lim_{\Lambda \downarrow \Lambda'(r)} \phi^s(\Lambda, r)\,\iota(\phi^s(\Lambda, r)) = 0.$$

By L'Hôpital's rule,

$$\begin{aligned}
\lim_{\Lambda \downarrow \Lambda'(r)} \Theta(\Lambda, r) &= \lim_{\Lambda \downarrow \Lambda'(r)} \frac{\frac{d}{d\Lambda}\int_0^{\mathrm{C}(\iota(\phi^s(\Lambda,r)))} [\mathrm{C}(\iota(\phi^s(\Lambda,r))) - \omega]\, d\Omega(\omega)}{\frac{d}{d\Lambda}[\phi^s(\Lambda,r)\,\iota(\phi^s(\Lambda,r))]} \\
&= \lim_{\Lambda \downarrow \Lambda'(r)} \frac{\int_0^{\mathrm{C}(\iota(\phi^s(\Lambda,r)))} \mathrm{C}'(\iota(\phi^s(\Lambda,r)))\frac{\partial \iota(\phi^s(\Lambda,r))}{\partial \phi^s(\Lambda,r)}\frac{\partial \phi^s(\Lambda,r)}{\partial \Lambda} d\Omega(\omega)}{\iota(\phi^s(\Lambda,r))\frac{\partial \phi^s(\Lambda,r)}{\partial \Lambda} + \phi^s(\Lambda,r)\frac{\partial \iota(\phi^s(\Lambda,r))}{\partial \phi^s(\Lambda,r)}\frac{\partial \phi^s(\Lambda,r)}{\partial \Lambda}} \\
&= \frac{\lim_{\Lambda \downarrow \Lambda'(r)} \left[\mathrm{C}'(\iota(\phi^s(\Lambda,r)))\frac{\partial \iota(\phi^s(\Lambda,r))}{\partial \phi^s(\Lambda,r)}\Omega(\mathrm{C}(\iota(\phi^s(\Lambda,r))))\right]}{\lim_{\Lambda \downarrow \Lambda'(r)} \left[\iota(\phi^s(\Lambda,r)) + \phi^s(\Lambda,r)\frac{\partial \iota(\phi^s(\Lambda,r))}{\partial \phi^s(\Lambda,r)}\right]} \\
&= \lim_{\Lambda \downarrow \Lambda'(r)} \Omega(\mathrm{C}(\iota(\phi^s(\Lambda,r)))) = 0,
\end{aligned}$$

where the last two equalities follow from (140) and

$$\lim_{\Lambda \downarrow \Lambda'(r)} \mathrm{C}'(\iota(\phi^s(\Lambda,r))) - 1 = 0.$$

■

The following proposition considers an economy in which the equilibrium market valuation of marginal investment would be higher than the entrepreneur's valuation if there were no adverse selection, and shows that the presence of adverse selection causes the equilibrium market valuation of marginal investment to fall below the entrepreneur's valuation.

Proposition 3 *For any $\Lambda \in [0,1]$, let $\phi^s(\Lambda,r)$ be given by (138), and define $\phi^s_Z \equiv \beta\varepsilon_e Z$ for $Z \in \{0,z\}$. Assume $1 < \max\{\phi^s_z, \phi^s(1,\bar{r})\}$ and $\phi^s_z < \phi^s(1,r)$. For any $r \in \mathbb{R}_+$, let $\Lambda'(r) \in (0,1)$ be the number that satisfies $\phi^s(\Lambda',r) = 1$, and let $\Lambda''(r) \in (\Lambda'(r),1)$ be the number that satisfies $\phi^s(\Lambda'',r) = \phi^s_z$. If $\Lambda''(r) < \lambda < 1$, there exists an equilibrium with equity issuance, $(\phi^s(\Lambda^*,r), \Lambda^*)$, with $\Lambda^* \in (\Lambda'(r), \Lambda''(r)]$, that is characerized by (138) and $\Lambda^* = \Upsilon(\Lambda^*)$ (with Υ as specified in Lemma 13), provided $\Omega[\mathrm{C}(\iota(\phi^s(\Lambda^*,r)))] > 0$. Moreover, $\phi^s(\Lambda^*,r) \leq \phi^s_z$, with "<" if*

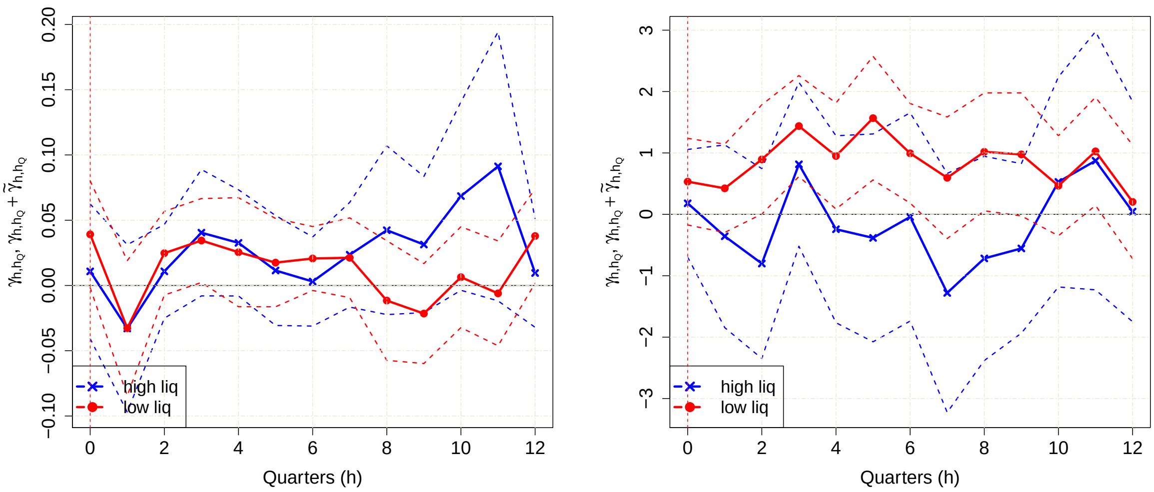
$$\Lambda''(r) < \frac{\lambda}{\lambda + (1-\lambda)\frac{1}{\Theta(\Lambda''(r),r)}}. \tag{141}$$

Proof. In an equilibrium with equity issuance, the equity price is given by (138) (by Lemma 11), and the equilibrium belief, Λ^*, satisfies $\Lambda^* = \Upsilon(\Lambda^*)$ (with Υ as specified in Lemma 13). The assumption $1 < \max\{\phi^s_z, \phi^s(1,\bar{r})\}$ ensures that investment is always positive. The assumption $\phi^s_z < \phi^s(1,r)$ means that in the absence of adverse selection, the equilibrium market valuation of marinal investment would be higher than the entrepreneur's valuation of marginal investment, as in part (ii) of Lemma 13. In this case, it is immediate from part (ii) of Lemma 13, that if $\Lambda''(r) < \lambda < 1$, then there exists at least one value $\Lambda^* \in (\Lambda'(r), \Lambda''(r)]$ that satisfies $\Lambda^* = \Upsilon(\Lambda^*)$. Condition (141) implies $\Lambda^* < \Lambda''(r)$, which is equivalent to $\phi^s(\Lambda^*, r) < \phi^s_z$. ■

C Data and robustness of empirical findings

C.1 Robustness of regression estimates

Figure 7: Issuances and investment predicted by instrumented q, across liquidity ratio groups, with additional firm controls



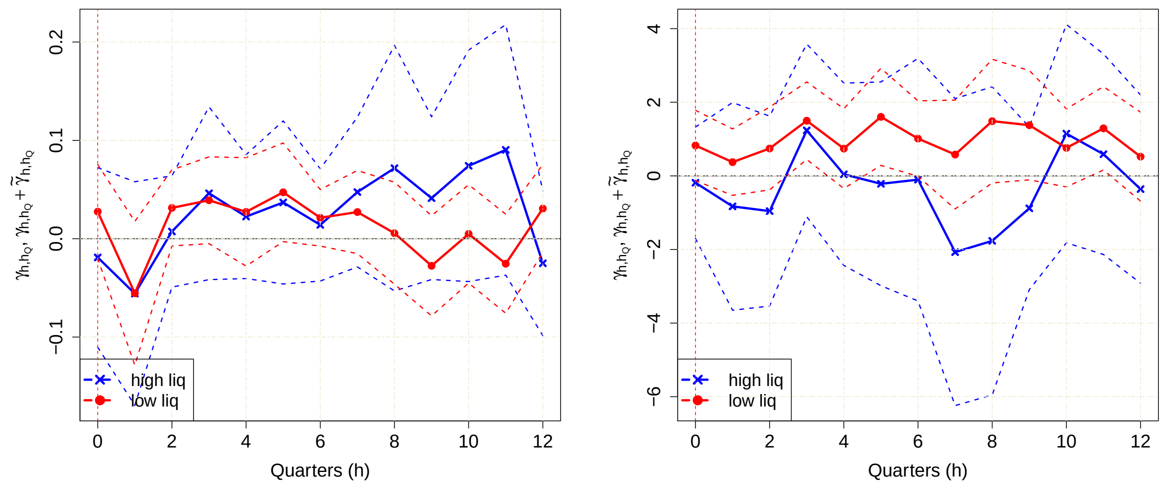
(a) $y^i_t = e^i_t / b^i_t$ (b) $y^i_t = \log(x^i_t / k^i_t)$

Notes: Point estimates and 95% confidence intervals for γ_{h,h_q} and $\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q}$ from estimating specification

$$\begin{aligned} y^i_{t+h} =& f^i_h + d_{s,h,t+h} + \alpha_h \mathbb{I}^i_{L,t} + \left(\rho_h + \tilde{\rho}_h \mathbb{I}^i_{L,t-1}\right) y^i_{t-1} + \left(\Lambda_h + \tilde{\Lambda}_h \mathbb{I}^i_{L,t-1}\right) Z^i_{t-1} + \left(\Psi_h + \tilde{\Psi}_h \mathbb{I}^i_{L,t-1}\right) Z^i_{t-1}\varepsilon^m_t \\ &+ \left(\beta_h + \tilde{\beta}_h \mathbb{I}^i_{L,t-1}\right) \mathcal{T}^i_{t-1} + \left(\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q} \mathbb{I}^i_{L,t-1}\right) \log\left(q^i_{t+h_q}\right) + u^i_{h,t+h} \end{aligned}$$

where Z^i_t is a vector containing the firm's liquidity ratio, log total assets as a measure of firm size, and $\frac{\text{total debt}^i_t}{\text{total assets}^i_t}$ as a measure of leverage. $\log\left(q^i_{t+h_q}\right)$ is instrumented with $\mathcal{T}^i_{t-1}\varepsilon^m_t$. Vertical red dashed line marks the value of $h_q = 0$. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

Figure 8: Issuances and investment predicted by instrumented q, across liquidity ratio groups, with additional firm controls including age



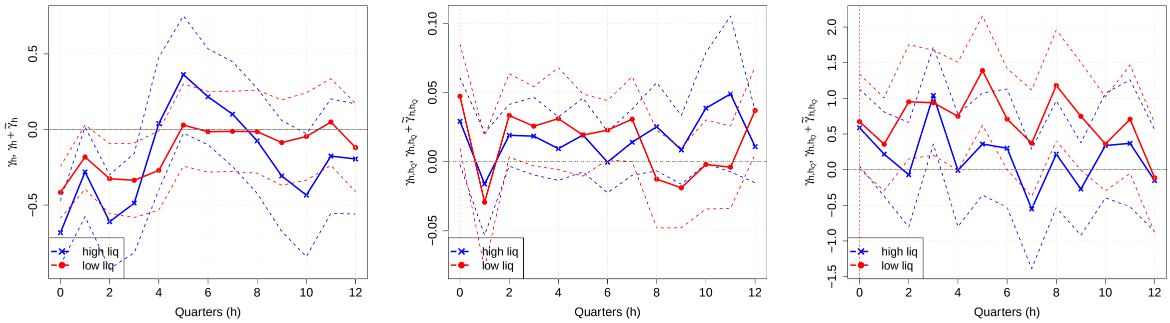
(a) $y_t^i = e_t^i / b_t^i$ (b) $y_t^i = \log(x_t^i / k_t^i)$

Notes: Point estimates and 95% confidence intervals for γ_{h,h_q} and $\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q}$ from estimating specification

$$\begin{aligned} y_{t+h}^i =& f_h^i + d_{s,h,t+h} + \alpha_h \mathbb{I}_{L,t}^i + \left(\rho_h + \tilde{\rho}_h \mathbb{I}_{L,t-1}^i\right) y_{t-1}^i + \left(\Lambda_h + \tilde{\Lambda}_h \mathbb{I}_{L,t-1}^i\right) Z_{t-1}^i + \left(\Psi_h + \tilde{\Psi}_h \mathbb{I}_{L,t-1}^i\right) Z_{t-1}^i \varepsilon_t^m \\ &+ \left(\beta_h + \tilde{\beta}_h \mathbb{I}_{L,t-1}^i\right) \mathcal{T}_{t-1}^i + \left(\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q} \mathbb{I}_{L,t-1}^i\right) \log\left(q_{t+h_q}^i\right) + u_{h,t+h}^i \end{aligned}$$

where Z_t^i is a vector containing the firm's liquidity ratio, log total assets as a measure of firm size, $\frac{\text{total debt}_t^i}{\text{total assets}_t^i}$ as a measure of leverage, and time since incorporation as a measure of age. $\log\left(q_{t+h_q}^i\right)$ is instrumented with $\mathcal{T}_{t-1}^i \varepsilon_t^m$. Vertical red dashed line marks the value of $h_q = 0$. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

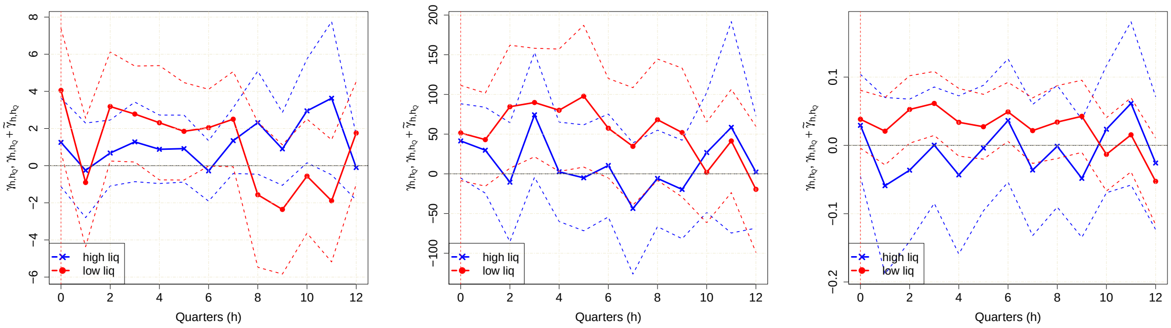
Figure 9: OLS and IV regression estimates, across liquidity ratio groups, given Jarociński and Karadi (2020) ‘poor man’s sign restrictions’



(a) $y_t^i = \log(q_t^i)$ (b) $y_t^i = e_t^i/b_t^i$ (c) $y_t^i = \log(x_t^i/k_t^i)$

Notes: Point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ from estimating specification (26) in panel (a), and specification (28) in panels (b) and (c) with $y_{i,t+h}$ as dependent variable. ε_t^m is the shock series inferred based on the ‘poor man’s sign restrictions’ of Jarociński and Karadi (2020), for 1990Q1–2016Q4. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

Figure 10: IV regression estimates with alternative variable transformations



(a) $y_t^i = e_t^i/b_t^i$, on q_t^i (b) $y_t^i = \log(x_t^i/k_t^i)$, on q_t^i (c) $y_t^i = x_t^i/k_t^i$, on $\log(q_t^i)$

Notes: Point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ from estimating specification (28) with y_{t+h}^i as dependent variable. In panels (a) and (b), q_t^i is included in the regression in levels, in panel (c) as $\log(q_t^i)$. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.