Designing Securities for Scrutiny*

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Abstract

We investigate the effect of *scrutiny* (e.g., credit ratings, analyst reports, or mandatory disclosures) on the security design problem of a privately informed issuer. We show that scrutiny has important implications for both the form of security designed and the amount of inefficient retention of cash flows. The model predicts that issuers will design informationally sensitive securities (i.e., levered equity) when scrutiny is sufficiently intense. Otherwise, issuers opt for a standard debt contract. Scrutiny increases efficiency by decreasing issuers' reliance on retention to signal quality, and perhaps counterintuitively, decrease price informativeness.

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1 Introduction

It is well understood that asymmetric information between sellers and buyers can distort market outcomes (Akerlof, 1970; Spence, 1973). This issue has received a particularly large degree of attention, both from participants and regulators, in securities markets. For example, an industry of analysts is devoted to scrutinizing the financial positions of public corporations and valuing their equity; credit-rating agencies (CRAs) use sophisticated models to estimate default probabilities for bonds and asset-backed securities (ABSs); dating back to the Securities Act of 1933, US regulators have imposed reporting and disclosure requirements on entities wishing to raise capital in public markets. The aftermath of the financial crisis has seen renewed debate among policymakers about how best to regulate securities markets. Much of this debate has focused on the degree of scrutiny applied to security issuances, including disclosure requirements and regulation of CRAs.[1]

In this paper, we investigate the effect of scrutiny on security design. The model features a liquidity-constrained issuer who has existing assets that generate a random future cash flow X. To raise capital, the issuer can design and issue a security F, backed by her asset's cash flows, to a competitive market of risk-neutral investors.[2] The issuer has private information about the quality of her assets (high or low), which may hinder her ability to raise funds in the market. In addition, the security design may signal information to investors. After the security is designed, it is subject to scrutiny, modeled as an informative public signal, S, about the value of the security. Thus, there are two potential sources of information that investors receive: (i) the strategic signal, F, chosen by the issuer and (ii) the public signal, S, resulting from scrutiny. After observing F and S, the market-clearing price is determined.

As a benchmark, we first consider the model without scrutiny. In this case, an issuer

[1]For example, in implementing Section 942(b) of the Dodd-Frank act, the SEC introduced rules that require ABS issuers to provide standardized, asset-level information to potential investors prior to the offering. Section 933 of the same law increases enforcement and penalty provisions that allow the SEC to bring claims against CRAs for material misstatements or fraud.

[2]For example, the issuer could be a firm with profitable investment opportunities that raises capital by selling claims to cash flows generated by existing assets, or a bank selling ABSs in order to make more loans.

with high-quality assets chooses to perfectly signal her "type" to investors by choosing to issue a debt contract, $F = \min\{d, X\}$, and to retain the remainder of cash flows. Hence, the issuer retains a levered equity claim. The issued debt level d is determined by the minimum amount of cash flow retention needed to separate from the low type, who sells a claim to all of her cash flows, $F = X$. Because asset quality is perfectly revealed by the choice of security, prices reflect all available information. This information transmission, however, is not without cost because retaining cash flows means forgoing gains from trade.

We then analyze the model with scrutiny. We show that there exists a unique equilibrium satisfying standard refinements and provide a full characterization of the equilibrium as it depends on the statistical informativeness of scrutiny, which we take as exogenous in our baseline model.

Our first result is that scrutiny induces the seller to issue a more informationally sensitive security. Intuitively, with little to no scrutiny, the most credible signal of high quality remains the seller's willingness to retain the portion of the cash flow that is most sensitive to her private information, which is accomplished by issuing debt and retaining a (levered) equity claim. However, with enough scrutiny, the opposite is true: the most credible signal of high asset quality is to issue the most informationally sensitive portion of the cash flow. Doing so creates exposure to "scrutiny risk," signaling confidence that the issuer expects scrutiny to authenticate her private information. We characterize precisely the condition at which the issuer switches from issuing a debt security to issuing a levered equity security, and refer to this condition as α-*informativeness*.

Our second result is that scrutiny reduces inefficient retention. With little to no scrutiny, types fully separate through the securities they design. With enough scrutiny, however, the high-type issuer starts to rely (at least in part) on the resultant public signal for information transmission. Doing so requires some degree of pooling—if all information is revealed by choice of security, there is nothing left for scrutiny to uncover. Therefore, the high-type issuer retains a smaller portion of the residual cash flows, thereby reducing her amount of inefficient retention. Because the low-type issuer also chooses this level of retention (with at least some probability) as opposed to selling the entire cash flow, her inefficient retention increases. However, the first effect dominates and overall efficiency increases. We precisely characterize the condition at which retention levels switch from separating to (at least some degree of) pooling. We refer to this condition as β-*informativeness*, and show that it is strictly weaker than α-informativeness.

An implication of our findings is that introducing scrutiny decreases the total amount of information transmitted to investors and prices become *less* informative. With little to no scrutiny, full information is conveyed by the strategic signal, and thus prices are fully revealing. Because scrutiny induces pooling but is imperfect, prices convey strictly less information than without scrutiny. In the limit as scrutiny becomes fully revealing, so too do prices. Thus, price informativeness is non-monotone in the intensity of scrutiny.

In the empirical literature, it is well established that information asymmetries between firms and outsiders, as well as the presence of debt ratings or analysts reports, affect firms' financing and investment decisions (Bharath et al., 2008; Graham and Harvey, 2001; Kisgen, 2006; Chang et al., 2006). Our model is motivated by these facts, and its main predictions align with existing evidence.

First, more intense scrutiny leads the firm to issue more informationally sensitive securities. Chang et al. (2006) find that as the number of analysts that follow a firm increases, the firm is more likely to issue equity as opposed to debt. Moreover, Chang et al. (2009) find that as the quality of firms' auditors increases, companies issue more equity. Derrien and Kecskés (2013) show that an exogenous decrease in the number of analysts who follow a firm reduces the firm's external funding and subsequent investments. In addition, they find that the issuance of equity and risky debt falls by more than that of safer, short-term debt.

Second, by reducing the incentive to signal through retention, scrutiny improves firms' access to external finance. Faulkender and Petersen (2005) document that firms with a debt rating have significantly more leverage. In addition, Sufi (2007) finds that the presence of third-party certification increases firm debt issuances and subsequent investments.[3]

In our baseline model, the intensity of scrutiny is independent of the security issued and the information resulting from scrutiny is publicly observed. In practice, larger or riskier issuances may be subject to more scrutiny, and investors may obtain private signals in addition to public ones. In Section 6, we incorporate each of these two considerations.

First, we extend the model to allow for *security dependent* scrutiny, wherein a more informationally sensitive security is subject to more scrutiny thereby leading to more public information. We parameterize this degree of dependence by ρ. We first generalize our results from the baseline model by showing that levered equity (respectively, debt) emerges as the security designed in equilibrium when ρ is sufficiently high (low). However, relative to the baseline model, a high-type issuer has stronger incentive to issue an informationally sensitive security. Moreover, a novel feedback effect emerges: more scrutiny incentivizes more informationally sensitive securities and more informationally sensitive securities invite more scrutiny. For intermediate ρ, this feedback loop results in multiple equilibria, which can feature drastically different levels of retention, informativeness, and efficiency. We show that equilibria are Pareto ranked with both issuer types preferring the equilibria with less retention, more informationally sensitive securities, and more resultant scrutiny. Hence, there is a clear sense in which the feedback effects of security dependent scrutiny could lead the market to get "stuck" in a suboptimal outcome.

Second, we consider an extension of the model in which investors observe private signals instead of (or in addition to) the public signal. We generalize the two conditions that effect the security form and the level of retention (α- and β-informativeness, respectively) and show that our main results extend. What is essential for our results is that a privately informed issuer is potentially exposed to fluctuations in the price of her security due to noisy information observed by investors, which naturally captures many market and informational settings. However, when signals are privately observed, a higher level of signal precision is

[3]To the best of our knowledge, our model's prediction that introducing scrutiny can lower price informativeness has not been empirically tested.

needed to induce the issuer to shift from issuing debt to levered equity.

1.1 Related Literature

Following Myers and Majluf (1984), an extensive literature studies security design in the presence of adverse selection. There are two common modeling approaches in this literature. In the approach of DeMarzo and Duffie (1999), securities are designed before the seller observes private information (ex ante), and signaling occurs through the choice of quantity of the designed security. As a result, the *form* of the security designed cannot signal the seller's private information. This approach is also adopted in DeMarzo (2005) and Biais and Mariotti (2005). In the approach of Nachman and Noe (1994), securities are designed after the seller observes private information (ex post), but the issuer needs to raise a fixed amount of funds, which leads all seller types to offer the same security.[4] A similar approach is adopted in Boot and Thakor (1993), Fulghieri and Lukin (2001), Fulghieri et al. (2020). In both approaches, the equilibrium prediction is that the seller issues a debt security, as debt minimizes the security's sensitivity to the issuer's private information.

In contrast, we analyze an ex post security design problem where the quantity of funds raised is a strategic choice made by the seller. As a result, different seller types may issue different security forms in equilibrium. DeMarzo (2005) argues that debt is the equilibrium security design in an ex post setting without scrutiny, and DeMarzo et al. (2015) provides a formal proof of the aforementioned result for a setting with an arbitrary number of seller-types. Asriyan and Vanasco (2020) analyze an ex post security design problem in a nonexclusive competitive screening environment. Our contribution to this literature is to characterize how equilibrium security design changes in the face of scrutiny.

The use of informationally sensitive securities may also be desirable in order to extract information from investors or to induce them to acquire information (for example, Boot

[4]A recent exception is Malenko and Tsoy (2020), who show that in a setting with ambiguity aversion, different seller types may issue different security forms (debt vs. equity), even when they must raise a fixed amount from investors.

and Thakor, 1993; Yang, 2015; Yang and Zeng, 2015). Axelson (2007) considers a setting where only investors have private information. He shows that the seller may choose to issue an informationally sensitive security when competition among investors is high in order to increase the correlation between the amount raised and investors' private information. Chakraborty and Yilmaz (2011) study the role of public information revealed after the security is sold in a setting where the firm must raise a fixed amount. They show that first-best allocations can be obtained if securities can be made contingent on investors' information.

Related to our findings with security dependent scrutiny (Section 6.1), Fulghieri and Lukin (2001) show that sellers may benefit from issuing more informationally sensitive securities when investors' costs of information acquisition is relatively low.[5] A novel aspect of our model is the feedback effect that generates multiplicity, including the coexistence of equilibria with different security forms.[6]

Feedback effects between market prices and real decisions have been explored in a variety of other contexts. Broadly speaking, these models feature a decision maker (e.g., a manager or policy maker) who uses the market price to determine whether to take a certain action that effects the value of the firm and an informed trader who accounts for her influence on the manager's decision when deciding how aggressively to trade (e.g., Goldstein and Guembel 2008; Edmans et al. 2015; Boleslavsky et al. 2017). Bond et al. (2012) provide an excellent survey of this literature.

Our model builds on the framework developed in Daley and Green (2014) who study how the presence of *grades* affects equilibrium behavior in signaling games, such as the canonical models of Spence (1973) and Leland and Pyle (1977). In their model, the signal space is one dimensional. In this paper, to study security design, the signal space is an (infinite-dimensional) set of functions from realized cash flows to security payoffs.

[5] Dang et al. (2010) argue that debt promotes market liquidity as it discourages information acquisition.

[6] In a different setting, Boot et al. (2006) show that ratings can work as a coordination mechanism in situations where multiple equilibria would otherwise arise.

2 Model

There are two periods. A risk-neutral agent owns an asset that generates a stochastic cash flow X in period 2. The agent has an incentive to raise funds by issuing a claim to some portion of the cash flow due to, for example, credit constraints or capital requirements. We refer to this agent as the *seller* or *issuer* and capture her incentive in reduced form by assuming that she discounts period-2 payoffs at $\delta \in (0,1)$, whereas, there is a competitive market of risk-neutral investors, whose common discount factor is 1.[7]

At the beginning of the first period, the seller privately observes a signal $t \in \{L, H\}$, which we refer to as her *type*. We denote the distribution and density of X conditional on t by Π_t and π_t, respectively, where $\pi_t(x) > 0$ over a common support $x \in [0, \overline{x}]$. The densities satisfy the monotone likelihood ratio property (MLRP): $\frac{\pi_H(x)}{\pi_L(x)}$ is increasing in x.

After observing her type, the seller issues a *security*, $F = \phi(X)$, where $\phi : [0, \overline{x}] \to [0, \overline{x}]$.[8] Specifically, for any realization of the cash flow x, $\phi(x)$ is the amount paid to the investor who purchased the security and $x - \phi(x)$ is the amount retained by the seller. Following the literature, we restrict attention to securities in which both $\phi(x)$ and $x - \phi(x)$ are nondecreasing in x, which is typically justified on the grounds of moral hazard (e.g., Nachman and Noe, 1994). Denote the set of all such securities by $\mathcal{F}$. After the seller designs the security, the security is subject to scrutiny which generates a public signal correlated with t.

Investors share a common prior $\mu_0 \equiv \Pr(t = H) \in (0,1)$. Based upon the security offered for sale, F, and the signal realization, s, investors update their prior to a final belief $\mu_f(F, s) \equiv \Pr(t = H | F, s)$. Since the market is competitive, the price paid for the security is

$$P(F|\mu_f) = E^{\mu_f}[F] = \mu_f E[F|H] + (1 - \mu_f) E[F|L]. \quad (1)$$

[7]This approach is also used in DeMarzo and Duffie (1999), Biais and Mariotti (2005), DeMarzo (2005), and Holmström and Tirole (2011), among others.

[8]That is, $\phi(\cdot)$ is a function, whereas the security F is the random variable $\phi(X)$.

The seller's realized payoff is $U(F,P,x) \equiv P + \delta(x - \phi(x))$. Notice that because the seller is impatient, the uniquely efficient outcome is to sell the entire cash flow (i.e., $F = X$).

Solution Concept

To handle the common problems posed by the freedom of off-equilibrium-path beliefs in signaling games, our solution concept is perfect Bayesian equilibrium satisfying the D1 refinement (Banks and Sobel, 1987; Cho and Kreps, 1987), hereafter simply referred to as *equilibrium*. Essentially, D1 requires investors to attribute the offer of an unexpected security to the type who is "more likely" to gain from the offer compared to her equilibrium payoff (a formal description of the refinement is found in the appendix.).

Debt and Levered Equity

Before beginning the analysis, it will be useful to develop notation for two particular forms of securities.

Definition 1. *A **debt** security, F_d^D, is characterized by its **face value**, $d \in [0, \overline{x}]$, as $F_d^D = \min\{d, X\}$. Let $\mathcal{F}^D \equiv (F_d^D)_{d \in [0,\overline{x}]}$ be the set of all debt securities.*

If the seller issues a debt security with face value d, she retains a *levered equity* claim: $X - F_d^D = \max\{0, X - d\}$. Conversely, if the seller issues a levered equity security, she retains a debt claim.

Definition 2. *A **levered equity** security, F_a^A, is characterized by the strike cash flow, $a \in [0, \overline{x}]$, as $F_a^A = \max\{0, X - a\}$. Let $\mathcal{F}^A \equiv (F_a^A)_{a \in [0,\overline{x}]}$ be the set of all levered equity securities.*

Of course, $F_{\overline{x}}^D = F_0^A = X$. That is, selling the entire cash flow is a special case of both forms of securities.

3 The No-Scrutiny Benchmark

As a benchmark, consider the model without scrutiny, which is a signaling model in which investors only update their beliefs based on the strategic signal chosen by the issuer. In this case, we obtain the following result.

Proposition 1. *Without scrutiny, there is a unique equilibrium and it is fully separating. In it, the low type issues a claim to her entire cash flow and the high type issues a debt security with a face value* d^{LC} *given by*

$$E[X|L] = E[\min\{d^{LC}, X\}|H] + \delta E[X - \min\{d^{LC}, X\}|L]. \quad (2)$$

The first part of the result is that, of all the securities available in $\mathcal{F}$, the high type will issue a debt security. Intuitively, since high cash flow realizations are more indicative of $t = H$, the high type is more willing than the low type to retain the claim that only pays off in such realizations. Hence, issuing debt is the "least costly" way to separate from the low type. A similar result is discussed in DeMarzo (2005). We offer a new and relatively simple proof of this result, which is a special case of Theorem 1(a) (see Section 5).

Given that the high type will issue a debt security, the choice of the seller becomes one-dimensional—select $d \in [0, \overline{x}]$—and the model is similar to signaling environments that have been studied in the existing literature. Because retention is costly, the high type retains as little as possible—by setting d^{LC} as high as possible—subject to the low type weakly preferring her full-information payoff to imitating the high type's issuance, as stated in (2). Because the equilibrium is separating, the seller's information is revealed to investors and security prices accurately reflect all information.

4 Scrutiny

We model scrutiny as a process that generates a public signal, S, which is a random variable with type-dependent density function q_t on $\mathbb{R}$.[9] The information content of a realized signal, s, is captured by $\beta(s) \equiv \frac{q_L(s)}{q_H(s)}$.[10] Without loss, order the signals such that β is weakly decreasing. For convenience, we assume that q_H, q_L are continuous almost everywhere, the informativeness of the signal is bounded (i.e., $\beta(\cdot) \in (0, \infty)$), and, unless otherwise stated, signals contain non-trivial information (i.e., $\beta(s) \neq 1$ on a set of positive measure).

While $\beta(s)$ measures the informativeness of a particular signal realization, s, the informativeness of scrutiny, which depends on $\{q_L, q_H\}$, will be the critical determinant of the model's predictions. Blackwell (1951) and Lehmann (1988) provide the two predominant notions for what it means for one set of signal distributions to be unambiguously more informative than another, which each endow a partial ordering. We will show that there are two critical measures of informativeness for our analysis, each of which is based on the differing expectations of the issuer types. Both measures are strictly weaker than the notions of Blackwell and Lehmann, and each endows a complete ordering over sets of signal distributions.

Consider the market belief that $t = H$ after observing the chosen security, F, but prior to any scrutiny. We refer to this belief as the *interim* belief. An arbitrary interim belief is denoted μ, and $\mu(F)$ indicates the interim belief conditional on the seller's chosen security F. For an interim belief μ, the final market belief given $S = s$ is determined by Bayes rule:

$$\mu_f(\mu, s) = \frac{\mu q_H(s)}{\mu q_H(s) + (1-\mu) q_L(s)} = \frac{\mu}{\mu + (1-\mu)\beta(s)}. \tag{3}$$

[9] To accommodate situations with a finite support $\{s_1, s_2, \dots\}$, with probabilities $p_t(s_n)$, let $q_t(s) = p_t(s_n)$ for $s \in [n, n+1)$ and $q_t(s) = 0$ for all other s.

[10] If $q_H(s) = q_L(s) = 0$, we adopt the convention that $\beta(s) = 1$.

Let $\alpha_t(\mu)$ denote the expected posterior belief from the type t's perspective:

$$\alpha_t(\mu) \equiv E_S[\mu_f(\mu, S)|t]. \tag{4}$$

Immediately, $\alpha_H(\mu) \geq \alpha_L(\mu)$ with the inequality being strict if and only if $\mu \notin \{0, 1\}$. Also, the difference between them, denoted by $\alpha(\mu) \equiv \alpha_H(\mu) - \alpha_L(\mu)$, is continuous and single-peaked as illustrated in Figure 1(b).

In determining the form of the security designed in equilibrium under scrutiny, the relevant measure of informativeness is the maximum difference in the expected final market belief between the seller types: $\hat{\alpha} \equiv \max_{\mu \in [0,1]} \alpha(\mu)$. The key determinant will then be how this measure of scrutiny compares to the gains from trade.

Definition 3. *Scrutiny is* ***α-informative*** *if* $\hat{\alpha} > \delta$.

Notice that Definition 3 is independent of the cash flow distributions. To ease exposition, we assume that $\hat{\alpha} \neq \delta$ unless otherwise stated. The following parametric example will be used in figures to illustrate key results.

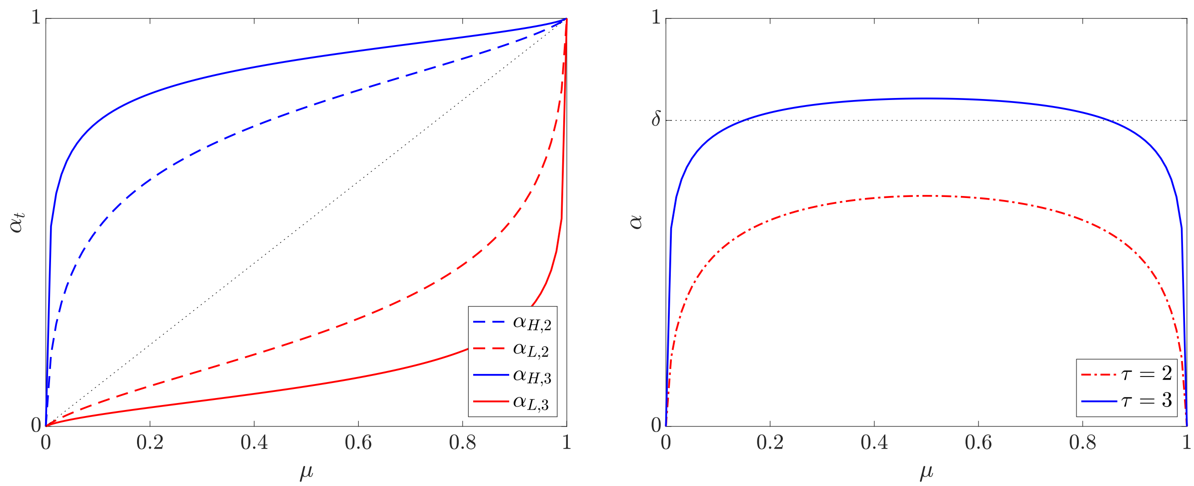
Example 1. *The cash flow, X, is distributed according to the power distribution, $\Pi_t = x^{c_t}$ on $[0, 1]$, with $c_L < c_H$. The public signal is normally distributed with type-dependent mean m_t and precision τ (larger τ corresponds to more intense scrutiny). Without loss of generality, we normalize $m_L = 0$ and $m_H = 1$.*

5 The Equilibrium with Scrutiny

5.1 Security Form

We begin with the main result.

Theorem 1 (Effect of Scrutiny on Security Design). *In the unique equilibrium,*



(a) α_L (red) and α_H (blue)

(b) $\alpha = \alpha_H - \alpha_L$

FIGURE 1: Panel (a) illustrates α_L and α_H as a function of the interim belief for precision levels of $\tau = 2$ (dashed) and $\tau = 3$ (solid). The dotted black line is the 45-degree line, which also corresponds to α_t in the model without scrutiny. Panel (b) depicts α as a function of the interim belief for the same two levels of precision. Scrutiny is (is not) α-informative when $\tau = 3$ (when $\tau = 2$). Parameters used: $c_L = 1$, $c_H = \frac{3}{2}$, $\delta = \frac{3}{4}$.

(a) If scrutiny is α-informative, then both types issue levered-equity securities.

(b) If scrutiny is <u>not</u> α-informative, then both types issue debt securities.

To arrive at this result, first recall that the final update from the interim to final belief is a straightforward application of Bayes rule. We can therefore use (1) and (4) to write the seller's expected payoff given any security F and interim belief μ as

$$\begin{aligned} u_t(F, \mu) &= E[P(F)|t, \mu] + \delta\left(E[X - F|t]\right) \\ &= \alpha_t(\mu)E[F|H] + (1 - \alpha_t(\mu))E[F|L] + \delta\left(E[X - F|t]\right). \end{aligned} \tag{5}$$

A key part of proving Theorem 1 is characterizing the solution to the following maximization problem.

$$\max_{F,\mu} \; u_H(F, \mu) \qquad \text{s.t. } u_L(F, \mu) = k. \tag{M(k)}$$

The following lemma explains why the solution to M(k) is important.

Lemma 1. *In any equilibrium, if the low type's payoff is* $u_L = k$*, then the high type issues a security* $F^*(k)$*, which results in an interim belief* $\mu(F^*(k))$*, such that the pair* $\{F^*(k), \mu(F^*(k))\}$ *solves* $\mathrm{M}(k)$*.*

Intuitively, if the high type does not select a security that solves $\mathrm{M}(k)$, then by D1, the off-path issuance of a security that does solve $\mathrm{M}(k)$ will be attributed to the high type since she stands to gain more than does the low type from this deviation. This attribution makes the deviation profitable, breaking the equilibrium.

In $\mathrm{M}(k)$, $u_L(F,\mu) = k$ by the constraint. Hence, we can subtract $u_L(F,\mu)$ from the objective in $\mathrm{M}(k)$ without changing the set of solutions. Next, from (5), we have that

$$u_H(F,\mu) - u_L(F,\mu) = (\alpha(\mu) - \delta)\left(E[F|H] - E[F|L]\right) + \delta\left(E[X|H] - E[X|L]\right).$$

Finally, $\delta\left(E[X|H] - E[X|L]\right)$ is a constant unaffected by F and μ. So, the solutions to $\mathrm{M}(k)$ are identical to the solutions to the following:

$$\max_{F,\mu} \, (\alpha(\mu) - \delta) \underbrace{\left(E[F|H] - E[F|L\right)}_{\equiv \Delta_F} \qquad \mathrm{M}^{\Delta}(k)$$
$$s.t.\ u_L(F,\mu) = k.$$

The objective in $\mathrm{M}^{\Delta}(k)$ provides insight into the factors that determine what form of security will be issued in equilibrium. The second term in its product, $\Delta_F \equiv E[F|H] - E[F|L]$, is the relevant measure of the *informational sensitivity* of the security F. Holding investors' interim belief fixed, the greater is Δ_F, the more influence scrutiny will have on the price of the security. Of course, $\Delta_F \geq 0$ for all $F \in \mathcal{F}$, with the inequality strict for all $F \neq 0$.

Given Lemma 1, the equivalence between the solutions to $\mathrm{M}(k)$ and $\mathrm{M}^{\Delta}(k)$ therefore sheds light on the importance of α-informativeness. If scrutiny is not α-informative, the objective in $\mathrm{M}^{\Delta}(k)$ is always negative, and the high type seeks to minimize informational sensitivity subject to the constraint. This is accomplished by issuing a debt security: for any k, there exists a d_k such that $F^D_{d_k}$ is the unique security in $\mathcal{F}$ that minimizes Δ_F subject to the constraint.

On the other hand, if scrutiny is α-informative, then the objective can be made positive and the high type wants to instead choose a security that maximizes Δ_F, provided that doing so leads to a belief $\mu(F)$ such that $\alpha(\mu(F)) > \delta$.[11] Intuitively, if scrutiny is sufficiently intense, the way for the high type to maximize her payoff is by designing a security whose value is most sensitive to type. By doing so, she creates more exposure to the outcome of scrutiny and demonstrates confidence that the outcome will be favorable.

Lemma 2. *For all $k \in [\underline{u}, \overline{u}) \equiv [E[X|L], E[X|H])$, the solution to* $\mathrm{M}(k)$ *is unique. If scrutiny is α-informative, then $F^*(k)$ is a levered equity security. Otherwise, $F^*(k)$ is a debt security.*

5.2 Retention and Equilibrium Characterization

The space of feasible securities that could be issued, $\mathcal{F}$, is infinite dimensional. Theorem 1 reduces the dimensionality of the relevant space: either the face value of debt (if scrutiny is not α-informative) or the strike cash flow of a levered equity claim (if scrutiny is α-informative).

Once the signaling space has been reduced to a single dimension, we can apply the framework developed in Daley and Green (2014) to fully characterize the equilibrium. They show that the measure of informativeness that determines whether the equilibrium will be separating or involve pooling depends on the expected likelihood ratios of the seller types and payoff parameters. Given the utility function of the seller in our model, the relevant condition is as follows.

Definition 4. *Scrutiny is **$\boldsymbol{\beta}$-informative** if* $\frac{E[\beta(S)|L]}{E[\beta(S)|H]} - 1 > \left(\frac{\delta}{1-\delta}\right)\left(\frac{\Pi_L(d^{LC}) - \Pi_H(d^{LC})}{1-\Pi_H(d^{LC})}\right)$.

The following lemma establishes a relation between our two measures of informativeness.

[11]For sufficiently high priors, it will not be feasible to achieve $\alpha(\mu(F)) > \delta$. In this case, both types sell their entire cash flows (see Proposition 3).

Lemma 3. *If scrutiny is α-informative then it is β-informative. The converse is not true.*

The following proposition describes equilibrium retention.

Proposition 2 (Effect on Retention)**.**

(a) If scrutiny is β-informative, in the unique equilibrium the high type retains less than is required for separation. Hence, there is at least some degree of pooling on the security issued.

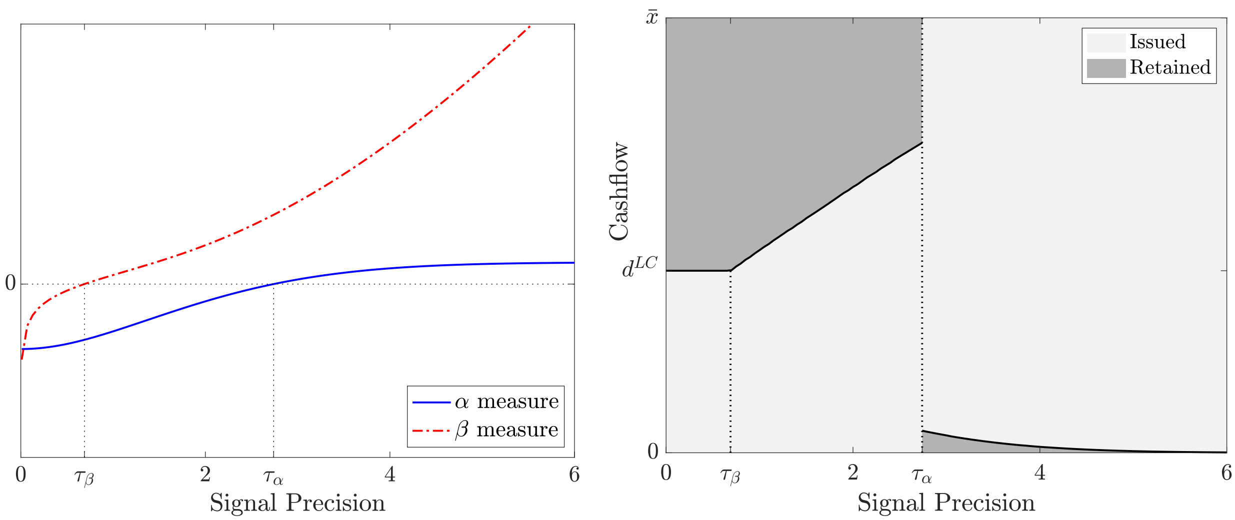
(b) If scrutiny is not β-informative, then the unique equilibrium is least-cost separating as in Proposition 1.

Recall that in the no-scrutiny benchmark, the seller issues debt because it is the most informationally insensitive form of security. In addition, equilibrium play fully reveals the seller's type to the market since the high type (inefficiently) retains enough to accomplish separation. Our results demonstrate that both of these features are upended under sufficient scrutiny, with the retention effect kicking in at a strictly lower level of signal precision than the security-form effect as shown in Lemma 3 and illustrated in Figure 2(a). Figure 2(b) illustrates the sets of cash flows that are retained and issued as a function of the signal precision. Notice that retention starts to decrease at τ_β, whereas the security form issued switches from debt to levered equity at τ_α.

To understand the argument behind Proposition 2, consider first the case where scrutiny is not α-informative, meaning the solution to $\mathrm{M}^\Delta(k)$ involves issuing debt. Hence, we can restate the problem as

$$\max_{d,\mu}\ (\alpha(\mu)-\delta)\left(E[\min\{d,X\}|H]-E[\min\{d,X\}|L]\right)$$
$$s.t.\ u_L(F_d^D,\mu)=k.$$

Since the signaling space has been reduced to a single dimension (Theorem 1), the solution can be illustrated graphically. Let us examine how the solution to $\mathrm{M}(\underline{u})$ depends on the



(a) α- and β-informativeness

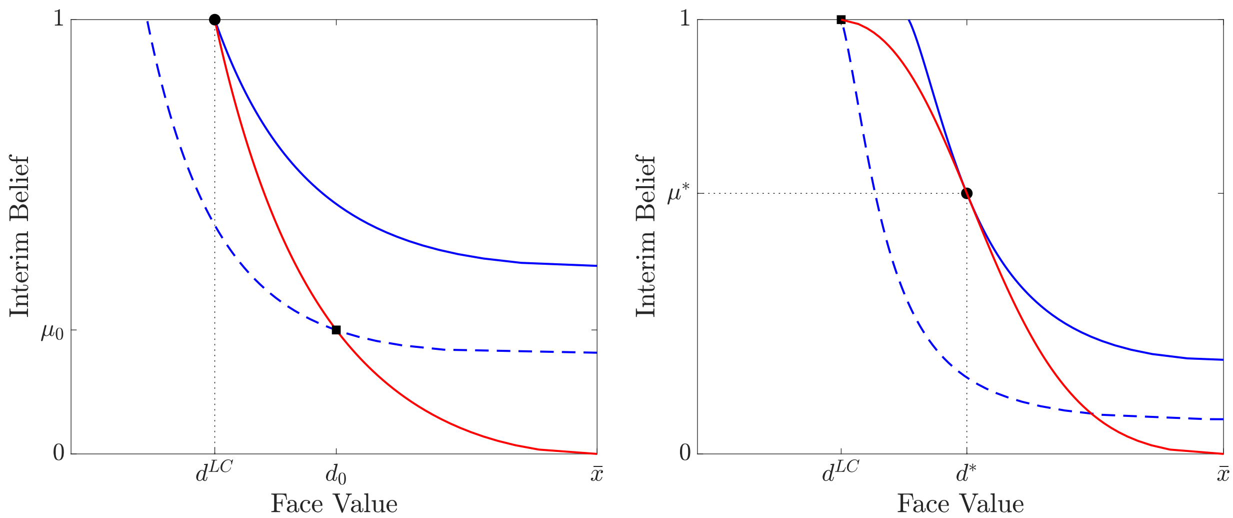
(b) Security Issued and Retention

FIGURE 2: Panel (a) illustrates when the two critical conditions for security form and retention hold as a function of the signal precision. The α measure is $\hat{\alpha} - \delta$. The β measure is the log of the ratio of the LHS to RHS of the condition in Definition 4.[12] Therefore, scrutiny is α-informative (β-informative) when the solid (dashed) curve is above zero. Panel (b) illustrates the portion of the cash flows that are issued (light gray) and retained (dark gray) by the high type as a function of the signal precision. Parameters used: $c_L = 1$, $c_H = \frac{3}{2}$, $\delta = \frac{3}{4}$.

intensity of scrutiny. Starting with the no-scrutiny case, Figure 3(a) shows the low type's indifference curve for $u_L = \underline{u}$ (in red). Therefore, the depicted (d_0, μ_0) satisfies the constraint, but it does not solve $\mathrm{M}(\underline{u})$ since shifting to points with higher μ-values on the low type's indifference curve strictly increases u_H. In fact, without scrutiny, the indifference curves for the two types satisfy the single-crossing property, meaning the unique solution to $\mathrm{M}(\underline{u})$ is the boundary solution: $(d, \mu) = (d^{LC}, 1)$. This is the property underlying Proposition 1: without scrutiny, $u_L = \underline{u}$, and the high type separates by choosing $F^D_{d^{LC}}$ which leads to the separating interim belief $\mu(F^D_{d^{LC}}) = 1$.

Recall from Figure 1 that α_H increases and α_L decreases with the introduction of scrutiny. That is, scrutiny increases (decreases) the expected posterior of the high (low) type. From (5), the type-t issuer's expected payoff is increasing $\alpha_t(\mu)$. Hence, for any (d, μ), introducing scrutiny decreases u_L and increases u_H. This is depicted in Figure 3(b). The low type's

[12] That is, the β measure is equal to $\ln\left[\left(\frac{E[\beta(S)|L]}{E[\beta(S)|H]} - 1\right) / \left(\frac{\delta}{1-\delta}\frac{\Pi_L(d^{LC}) - \Pi_H(d^{LC})}{1-\Pi_H(d^{LC})}\right)\right]$.



(a) No Scrutiny (b) β-informative Scrutiny ($\tau = 2$)

FIGURE 3: Solving M($\underline{u}$). In both panels, the red indifference curve is for the low-type $u_L(F_d^D, \mu) = \underline{u}$, the solid-blue indifference curve is for the high type at the optimal value of M($\underline{u}$), and the dashed-blue indifference curve is for the high type at a suboptimal value. Parameters used: $c_L = 1$, $c_H = \frac{3}{2}$, $\delta = \frac{3}{4}$.

indifference curve is higher than without scrutiny (i.e., a higher interim belief is needed to offset the negative impact of scrutiny and keep $u_L = \underline{u}$), whereas the high type's indifference curve is lower than without scrutiny. Whether or not the solution to M($\underline{u}$) is altered, then, is determined by whether the high type's curve falls below the low's at $(d^{LC}, 1)$. This is why Theorem 2 hinges on β-informativeness, which is the precise condition that determines whether scrutiny moves the solution to M($\underline{u}$) away from the boundary. Finally, if $(d^{LC}, 1)$ does not solve M($\underline{u}$), then separation is not possible in equilibrium: separation implies $u_L = \underline{u}$, but Lemma 1 would then imply that the high type selects a security that leads to an interim belief such that the pair is a solution to M($\underline{u}$)—a contradiction.

Intuitively, as scrutiny intensifies, the high type wishes to rely at least partially on it in equilibrium for price determination. But relying on scrutiny requires some degree of pooling—if the high type separates by choice of security, there is nothing left for scrutiny to reveal. Simultaneously, reliance on scrutiny allows the high type to reduce her degree of inefficient retention by selecting a face value of debt that is strictly higher than d^{LC}.

Appendix A provides a complete characterization of the equilibrium. To do so, we establish the properties of the solution to M(k) that allow us to apply Proposition 3.8 of Daley and Green (2014), and thus first solve M(k) for all feasible low-type utilities to then connect the solution locus to equilibria. A brief summary of these findings is as follows. First, Proposition 2(a) already describes the equilibrium when scrutiny is not β-informative. When scrutiny is β-informative, the unique equilibrium involves partial (full) pooling if the prior is below (above) a threshold. In the partial pooling equilibrium, the low type mixes between selling the entire cash flow and mimicking the high type, where the form of security issued by the high type is characterized by Theorem 1.

5.3 Price Informativeness

Perhaps counterintuitive at first pass is that scrutiny decreases the total amount of information transmitted to investors in equilibrium. Consequently, if scrutiny alters retention, securities are mispriced in that they do not reflect all information, unlike in the no-scrutiny environment. Naturally, in the limit as scrutiny becomes fully informative, so too are prices. An immediate corollary is that the total amount of information transmitted to investors (from both the strategic signal and public signal) is non-monotone in the intensity of scrutiny.

Our finding that scrutiny can reduce the total information transmitted is reminiscent of the "crowding out" effect (Goldstein and Yang, 2017), in which public disclosure can reduce private incentives for information acquisition. However, the reduction in price informativeness associated with scrutiny is not detrimental to welfare. In our model, the only source of inefficiency are cash flows retained by the issuer. In fact, greater scrutiny reduces the high type's incentive to strategically signal and leads to less (inefficient) retention (see Figure 2). Of course, the existing literature has identified a variety of potential reasons (outside of our model) for why more informative prices are desirable. In particular, prices may inform: managerial decisions, (Fishman and Hagerty, 1992; Leland, 1992; Dow and Gorton, 1997; Edmans, Goldstein, and Jiang, 2015), government interventions (Bond, Goldstein, and Prescott, 2009; Bond and Goldstein, 2015; Boleslavsky, Kelly, and Taylor, 2017), and monetary policy (Bernanke and Woodford, 1997).

5.4 Eliminating Inefficient Retention

As we have already shown, scrutiny decreases the reliance on inefficient retention as a strategic signal. It is natural to then ask whether scrutiny can fully eliminate signaling via retention, which is answered in the next proposition.

Proposition 3. *If scrutiny is sufficiently intense such that the public signal satisfies*

$$\frac{E[\beta(S)|L]}{E[\beta(S)|H]} - 1 > \left(\frac{\delta}{1-\delta}\right)\left(\frac{\pi_H(\overline{x}) - \pi_L(\overline{x})}{\pi_H(\overline{x})}\right),$$

then there exists $\tilde{\mu} \in (0,1)$ such that both types efficiently sell their entire cash flow (i.e., issue $F = X$) for all $\mu_0 \geq \tilde{\mu}$.

The criterion in the proposition is analogous to β-informativeness except with the RHS evaluated at the upper limit of cash flows, $\overline{x}$, rather than at d^{LC}.[13] This criterion is stronger than β-informativeness, but weaker than α-informativeness (see the proof of Lemma 3). Therefore, full efficiency is achieved for sufficiently high priors whenever scrutiny is α-informative.

6 Extensions

In this section we consider two extensions of the baseline model. In the first, we allow the intensity of scrutiny to depend on the security issued. In the second, we consider a setting in which investors observe private signals in addition to the public signal.

[13]Because $\Pi_H(\overline{x}) = \Pi_L(\overline{x}) = 1$, the condition in the proposition is from L'Hospital's rule.

6.1 Security Dependent Scrutiny

In the baseline model, the distribution of the public signal is independent of the issuer's choice of security. In this section, we assume that the more sensitive the value of a security is to information about t, the greater the scrutiny that it invites, resulting in more information for investors. Our main result is to demonstrate that this feature introduces feedback effects between investors' information and security design that can give rise to multiple equilibria, with different security forms and levels of retention.

Before discussing the feedback effects and multiplicity, we first generalize our findings from the baseline model by showing that debt and levered equity emerge as the securities designed in equilibrium. Compared to the baseline model, security dependent scrutiny provides an additional incentive to issue informationally sensitive securities, which lead to less retention in equilibrium.

To model scrutiny that depends on the security, we now suppose that after the type-t seller issues a security F, investors observe a public signal:

$$S = \rho E\left[F|t\right] + \epsilon \tag{6}$$

where the noise random variable ϵ has associated density function q_ϵ with full support on $\mathbb{R}$, and the constant $\rho > 0$ captures the *degree of dependence* of the public signal on the chosen security.[14] Recall $\Delta_F = E[F|H] - E[F|L]$ is the informational sensitivity of F and note that an H-type seller that issues security F expects to receive a public signal S with associated density $q_\epsilon(S - \rho E[F|L] - \rho\Delta_F)$, while an L-type seller that issues the same security expects to receive a signal S with density $q_\epsilon(S - \rho E[F|L])$. As $\rho E[F|L]$ shifts both signal distributions by the same constant, signal informativeness is determined by the product of ρ and Δ_F. Importantly, a more informationally sensitive security generates a more informative signal.

[14] It is straightforward to generalize to a public signal that depends also on t directly (as in the baseline model) by having $S = m_t + \rho E\left[F|t\right] + \epsilon$. Doing so would complicate the statements of results, as they would then depend on both ρ and $m_H - m_L$. Hence, for this section we focus on the new feature—security dependent scrutiny—by (implicitly) setting $m_H = m_L$.

We begin our analysis using similar techniques as in the baseline model while taking into account that expected posteriors beliefs vary both with the interim belief μ (as in the baseline model) and also now with the security issued:

$$\alpha_H(\Delta_F,\mu) = \int \frac{\mu}{\mu + (1-\mu)\frac{q_\epsilon(\tilde{s})}{q_\epsilon(\tilde{s}-\rho\Delta_F)}} q_\epsilon\left(\tilde{s}-\rho\Delta_F\right) d\tilde{s} \tag{7}$$

$$\alpha_L(\Delta_F,\mu) = \int \frac{\mu}{\mu + (1-\mu)\frac{q_\epsilon(\tilde{s})}{q_\epsilon(\tilde{s}-\rho\Delta_F)}} q_\epsilon\left(\tilde{s}\right) d\tilde{s} \tag{8}$$

where $\tilde{s} \equiv s - \rho E[F|L]$ is a transformation and $d\tilde{s} = ds = d\epsilon$. Mirroring the baseline model, we assume that $\frac{q_\epsilon(s)}{q_\epsilon(s-n)}$ is decreasing in s for any $n > 0$. The difference in expected posteriors is now denoted by $\alpha(\Delta_F,\mu) \equiv \alpha_H(\Delta_F,\mu) - \alpha_L(\Delta_F,\mu)$. Hence, we update (5) for seller's expected payoff as

$$u_t(F,\mu) = \alpha_t(\Delta_F,\mu)E[F|H] + (1-\alpha_t(\Delta_F,\mu))E[F|L] + \delta\left(E[X-F|t]\right). \tag{9}$$

Lemma 1 continues to hold using these updated expressions for u_H and u_L in $\mathrm{M}(k)$. By the same transformation, we turn to that analog of $\mathrm{M}^\Delta(k)$:

$$\max_{F,\mu} \left(\alpha(\Delta_F,\mu) - \delta\right)\cdot\Delta_F \qquad \mathrm{M}^\Delta_{SD}(k)$$
$$s.t.\ u_L(F,\mu) = k.$$

Denote the objective in $\mathrm{M}^\Delta_{SD}(k)$ by $V(\Delta_F,\mu)$, which is the product of two terms: $\alpha(\Delta_F,\mu)-\delta$ and Δ_F. The key difference from $\mathrm{M}^\Delta(k)$ is that the *first* term in the objective now depends on F (more specifically, on Δ_F). Observe that starting from $\Delta_F = 0$ (corresponding to the seller retaining the entire cash flow), $V(0,\mu) = (0-\delta)\cdot 0 = 0$, and a small increase to $\Delta_F > 0$ decreases the objective. At the other extreme, for Δ_F sufficiently large, both terms in the objective are positive and increasing in Δ_F, so increasing Δ_F increases the objective. To summarize, as Δ_F increases, the objective V is first negative and decreasing, and eventually positive and increasing. To gain analytic traction, we assume that V switches from decreasing to increasing only once, and that the low type's expected payoff function (that governs the constraint in $\mathrm{M}^{\Delta}_{SD}(k)$) is also well behaved.

Assumption 1. *For any $\mu \in (0,1)$,*

(a) *there exists a unique $\hat{\Delta} > 0$ such that sign $\left(\frac{dV}{d\Delta}\right) = sign(\Delta - \hat{\Delta})$.*

(b) $\frac{\partial(\alpha_L(\Delta,\mu)\Delta)}{\partial\Delta} \geq 0$ *for all feasible Δ.*

Assumption 1(b) says that, holding the interim belief fixed, the benefit for the low type of including more cash flows in the security more than compensates for the cost of the more intense scrutiny that results. In the case of debt securities, the condition ensures that the low type's indifference curves slope downward (as illustrated for the baseline model in Figure 3). It is straightforward to check that both conditions always hold in the baseline model.

Because of the potential for multiplicity, we will say a security form is an *equilibrium security form* for type t if there exists an equilibrium in which type t issues a security of that form. Our first result establishes the equilibrium security forms are the same as in the baseline model.

Theorem 2. *For both types, the only equilibrium security forms are debt and levered equity. Also, there exists degrees of security dependence $\underline{\rho} < \bar{\rho}$ such that, for all priors and types,*

(a) *levered equity is the unique equilibrium security form if and only if $\rho \geq \bar{\rho}$.*

(b) *debt is the unique equilibrium security form if and only if $\rho \leq \underline{\rho}$.*

We therefore have a partial analog to the baseline model: levered-equity (respectively, debt) is issued when investor information is sufficiently dependent on (divorced from) underlying value. As in the baseline model, if the objective of $\mathrm{M}^{\Delta}_{SD}(k)$ can be made positive while still satisfying the constraint, then the solution will be levered equity in order to maximize informational sensitivity. Moreover, as ρ increases, the signal is more dependent on the security's true value, and $\alpha(\Delta,\mu)$ increases for all (Δ,μ), meaning the objective can be made positive for more values of k.

In contrast to the baseline model, even if the objective cannot be made positive, the solution may still involve levered equity. This is because offering a less informationally sensitive security form (e.g., debt), now decreases α and makes the objective "even more" negative. Intuitively, the high type prefers a more informative signal. Hence, security dependent scrutiny gives the high type additional incentive to issue a more informationally sensitive security. An implication of this additional incentive is that the solution to $\mathrm{M}_{SD}^{\Delta}(k)$ involves less retention.

Proposition 4. *Let $F_{SD}^{*}(k)$ solve $\mathrm{M}_{SD}^{\Delta}(k)$ when scrutiny is security dependent. Consider now the baseline model with (security independent) scrutiny generating $S = \rho E[F_{SD}^{*}(k)|t] + \epsilon$, and let $F^{*}(k)$ solve $\mathrm{M}^{\Delta}(k)$ for this model. Then,*

1. *If $F_{SD}^{*}(k)$ is a debt security with face value $d_{SD}^{*} < \bar{x}$, then $F^{*}(k)$ is a debt security with face value $d^{*} < d_{SD}^{*}$.*

2. *If $F_{SD}^{*}(k)$ is a levered-equity security with strike value $a_{SD}^{*} > 0$, then $F^{*}(k)$ is either a levered-equity security with a strike value $a^{*} > a_{SD}^{*}$ or a debt security.*

6.1.1 Multiplicity

When scrutiny has an intermediate degree of dependence on the security designed, whether the solution to $\mathrm{M}_{SD}^{\Delta}(k)$ involves debt or levered equity depends on k. Moreover, feedback effects can sustain the issuance of securities with different levels of informational sensitivity, Δ. The feedback loop is that more intense scrutiny incents issuance of informationally sensitive securities, and more informationally sensitive securities generate more intense scrutiny.

Proposition 5. *For all $\rho \in (\underline{\rho}, \bar{\rho})$, there exists $\mu^{1} < \mu^{2} \leq \frac{1}{2} < \mu^{3}$ such that*

- *Either least-cost separating or partial pooling on a debt security with face value $d^{*} < \bar{x}$ is an equilibrium for all priors $\mu_0 \leq \mu^{3} = \mu^{*}(\underline{u})$.*

- *Full pooling on a levered equity security with $a^{*} > 0$ is an equilibrium for all priors $\mu_0 \in [\mu^{1}, \mu^{2}]$.*

- *Full pooling on issuing the entire cash flow, $F = X$, is an equilibrium for all priors $\mu_0 \geq \mu^2$.*

Therefore, for all priors $\mu_0 \in [\mu^1, \mu^3]$, there are multiple equilibria.

Perhaps the starkest feature of the proposition arises when the solution to $\mathrm{M}^{\Delta}_{SD}(\underline{u})$ involves $\mu^*(\underline{u}) = 1$. In this case, for any prior, the LCSE (in which the security is debt) is an equilibrium. In the absence of feedback effects (i.e., in the baseline model), this would imply that the solution to $\mathrm{M}(k)$ involves debt and $\mu^*(k) = 1$ for all k, and the LCSE is the unique equilibrium (Proposition 2(b)).[15]

However, to solve $\mathrm{M}^{\Delta}_{SD}(k)$ for larger k, the low type must earn a higher expected payoff, so there must be a decrease in retention. With security dependent scrutiny, a decrease in retention increases Δ_F, and therefore increases the informativeness of scrutiny. Hence, for some $k > \underline{u}$, the solution to $\mathrm{M}^{\Delta}_{SD}(k)$ can involve $\mu^*(k) < 1$ even when $\mu^*(\underline{u}) = 1$. This generates multiple equilibria: if $\mu^*(k) < 1$ for some $k > \underline{u}$, then both the LCSE and full pooling on $F^*(k)$ are equilibria when $\mu_0 = \mu^*(k)$.

Moreover, if $\rho > \underline{\rho}$, this feedback effect is strong enough such that, for some $k > \underline{u}$, the objective of $\mathrm{M}^{\Delta}_{SD}(k)$ can be made positive using levered equity. Therefore, for the prior $\mu_0 = \mu^*(k)$, the co-existing equilibria differ in their security forms. Finally, for all k large enough, $F^*(k) = X$ with $\mu^*(k)$ increasing in k continuously from $\frac{1}{2}$ to 1. It follows that for every prior $\mu_0 \in (\frac{1}{2}, 1)$, there are equilibria with drastically different levels of retention, informativeness, and efficiency: both the (very inefficient) LCSE and the efficient outcome of both types selling the entire cash flows are equilibrium outcomes.[16] Even when the differences are less extreme, the welfare ranking of any two equilibria remains unambiguous, as seen in the following proposition.

[15] When scrutiny is not security dependent, if at $(F, \mu) = (F^D_{d^{LC}}, 1)$ the high type does not want to increase the face value of the debt in exchange for the lower belief that keeps the low type's payoff at $\underline{u}$, then the high type does not want the exchange at any $(F, \mu) = (F^D_d, 1)$ with $d > d^{LC}$ and $u_L = k > \underline{u}$ because the relative cost of including an incremental cash flow in the security is increasing in the cash flow level since cash flows satisfy MLRP. Security dependent scrutiny introduces a countervailing force.

[16] One set of parameters that generates all of the properties described is: $\bar{x} = 1$, $\Pi_t(x) = x^{c_t}$ with $c_H = 0.2$ and $c_L = 0.1$, $\delta = 0.1$, $\epsilon \sim N(0, 1)$, and $\rho = 5.8$.

Proposition 6. *Equilibria are Pareto ranked, with both issuer types preferring equilibria with lower retention, and therefore, more intense scrutiny. In particular, if both a levered-equity equilibrium and a debt equilibrium exist, both issuer types prefer the levered-equity equilibrium.*

Because investors earn zero expected profit, it follows that a levered-equity equilibrium is more efficient than a debt equilibrium. Hence, there is a clear sense in which the feedback effects of security dependent scrutiny could lead the market to get "stuck" in a suboptimal outcome. This is because the debt equilibrium involves more signaling via retention, resulting in a low Δ_F, and therefore low signal informativeness (which, in turn necessitates the greater signaling via retention). In contrast, in the levered-equity equilibrium both types sell more valuable securities (i.e., $E[F|t]$ is greater for both types), which is more efficient. Under Assumption 1(b), this efficiency effect is sufficient to compensate the low type for having to face more intense scrutiny.

One might wonder why, in the debt equilibrium, deviating to the levered-equity security offered in the more efficient equilibrium is not profitable. The reason is that such a deviation has competing interpretations: should investors interpret this choice as a sign that she is unafraid of more scrutiny (indicating $t = H$), or merely that she wants to retain less cash flows (indicating $t = L$)? Because the debt equilibrium satisfies D1, it must be that the low type is "more likely" to gain from this deviation in comparison to her debt-equilibrium payoff, and the latter interpretation dominates.[17] Yet, these features do not contradict levered equity being the solution to $\mathrm{M}_{SD}^{\Delta}(k)$ for the greater low-type payoff in the levered-equity equilibrium.

[17] Specifically, given the payoffs associated with the debt equilibrium, such a deviation is profitable for the low type for a larger set of interim beliefs than for the high type. See the appendix for details regarding D1.

6.2 Dispersed Information

In our baseline model, we assume that scrutiny generates a public signal (e.g., a credit rating or analyst recommendation). Other forms of scrutiny may be conducted privately, leading to private signals (e.g., propriety research or quantitative models). We now extend our findings to a setting in which investors obtain private signals about the asset cash flows in addition to any information revealed by public scrutiny.

Dispersed Information. In addition to the public signal with properties described in Section 4, each investor $i \in \{1, 2, ..., n\}$ observes a private signal Z_i. Conditional on seller type t, investor's private signals are identically and independently distributed, with type-dependent density function ξ_t. We assume the density functions $\{\xi_H, \xi_L\}$ share the properties of the density functions for the public signal described in Section 4, $\{q_H, q_L\}$. This setting incorporates two special cases: i) our baseline model in which only the public signal is informative: if $\xi_H(z) = \xi_L(z)$ for all z; and ii) a model in which only private signals dispersed among the investors are informative: if $q_H(s) = q_L(s)$ for all s.

Sales Mechanism. The seller splits security F into $k < n$ identical units and elicits simultaneous sealed bids for a unit from each of the n (potential) investors. Each unit of security F is then allocated to the k highest bidders, who pay a price set equal to the $(k+1)^{st}$ highest bid. This mechanism has been studied extensively in the auction literature (Milgrom, 1981; Milgrom and Weber, 1982; Pesendorfer and Swinkels, 1997; Kremer, 2002), with its most well-known version being the second-price auction ($k = 1$). It also is employed, and discussed, in the security design context by Axelson (2007).

As is typical, we focus on the unique symmetric equilibrium of the auction. Let Y_m^k be the k^{th} order statistic of out of m draws of investor signals, and let Y_{-i}^k be the k^{th} order statistic of signals other than Z_i.

In any (symmetric) equilibrium of the auction, the bid of an investor who observes public signal s and private signal z is given by $E^{\mu(F)}[F|S = s, Z_i = z, Y_{-i}^k = z]$.[18] The first

[18] As first shown in Milgrom (1981).

difference with our baseline model is that now bidders condition on their private signal and on it being *pivotal*—that is, on being tied with the k^{th} highest signal among all other bidders, $Y^k_{-i} = z$. Let μ continue to denote the interim belief upon observing security F (i.e., $\mu = Pr(t = H|F)$), and let g_t be the type-dependent density of Y^k_{n-1}, and thus of Y^k_{-i}. With this notation, we can express an investor's bid as

$$b(\mu, s, z) = E[F|L] + \tilde{\mu}_f(\mu, s, z)(E[F|H] - E[F|L]), \tag{10}$$

where

$$\tilde{\mu}_f(\mu, s, z) \equiv \frac{\mu}{\mu + (1-\mu)\beta^\dagger(s, z)}, \tag{11}$$

and $\beta^\dagger(s, z) \equiv \beta(s)\frac{\xi_L(z)g_L(z)}{\xi_H(z)g_H(z)}$ is the product of the likelihood ratio of the public signal and an adjusted likelihood ratio of private signal Z that incorporates the equilibrium bidding strategies of investors. The latter measures the informativeness of signal z, conditional on z also being the k^{th} order statistic of the other $n-1$ draws.

The price of security F the seller receives is given by the $(k+1)^{st}$ highest bid, $b(\mu, s, Y^{k+1}_n)$. Thus, the seller's expected payoff given any security F and interim belief μ is

$$u_t(F, \mu) = \alpha^\dagger_t(\mu)E[F|H] + (1 - \alpha^\dagger_t(\mu))E[F|L] + \delta\left(E[X - F|t]\right), \tag{12}$$

where

$$\alpha^\dagger_t(\mu) \equiv E[\tilde{\mu}_f(\mu, S, Y^{k+1}_n)|t]. \tag{13}$$

The expression in (12) is analogous to (5) for the model with only public scrutiny, where the expected posterior $\alpha_t(\mu)$ is now adjusted to $\alpha^\dagger_t(\mu)$ to incorporate investors' equilibrium bidding strategies in the presence of dispersed information and the auction protocol. The second difference with our baseline model, seen in (13), is that the seller is exposed to the variation in price induced by the $(k+1)^{st}$ order statistic of n draws of Z, Y^{k+1}_n, in addition to the variation induced by the public signal S.

In our baseline model with scrutiny, we showed that the seller's decision to *i*) design a particular *form* of security, and *ii*) to perfectly separate or (partially) pool with the other seller type, each depended on a distinct notion of the intensity of scrutiny (α- and β-informativeness, respectively). The following two notions of signal informativeness nest the notions from the baseline model. In order to do so, let $\alpha^{\dagger}(\mu) \equiv \alpha^{\dagger}_{H}(\mu) - \alpha^{\dagger}_{L}(\mu)$ and $\hat{\alpha}^{\dagger} = \max_{\mu \in [0,1]} \alpha^{\dagger}(\mu)$.

Definition 5 (Generalized Measures of Informativeness)**.**

- *Investors' signals are* $\boldsymbol{\alpha}^{\dagger}$**-informative** *if* $\hat{\alpha}^{\dagger} > \delta$.
- *Investors' signals are* $\boldsymbol{\beta}^{\dagger}$**-informative** *if* $\frac{E[\beta^{\dagger}(S,Y_n^{k+1})|L]}{E[\beta^{\dagger}(S,Y_n^{k+1})|H]} - 1 \geq \left(\frac{\delta}{1-\delta}\right)\left(\frac{\Pi_L(d^{LC})-\Pi_H(d^{LC})}{1-\Pi_H(d^{LC})}\right)$.

The notion of $\alpha^{\dagger}$-informativeness generalizes the notion of α-informativeness, and $\beta^{\dagger}$-informativeness generalizes β-informativeness. Analogously, $\alpha^{\dagger}$-informativeness implies $\beta^{\dagger}$-informativeness, but not the converse (generalizing Lemma 3). First, we re-establish our main finding on security form.

Theorem 3 (Effect of Signals on Security Design)**.** *In the unique equilibrium,*

(a) If signals are $\alpha^{\dagger}$*-informative, then both types issue levered-equity securities.*

(b) If signals are <u>*not*</u> $\alpha^{\dagger}$*-informative, then both types issue debt securities.*

Next, we re-establish our main finding on the effect of signals on equilibrium retention:

Proposition 7 (Effect of Signals on Retention)**.**

(a) If signals are $\beta^{\dagger}$*-informative, in the unique equilibrium the high type retains less than is required for separation. Hence, there is at least some degree of pooling on the security issued.*

(b) If signals are <u>*not*</u> $\beta^{\dagger}$*-informative, then the unique equilibrium is least-cost separating as in Proposition 1.*

Theorem 3 and Proposition 7 show that our results do not rely on the information observed by investors being exclusively public. What matters is the degree to which the privately informed seller can rely on the price to move based on information received by the investor side of the market. If investors will receive very noisy information, its impact on price will be too low, and the seller chooses to perfectly convey her type through the issuance of debt securities. If, instead, investor information will be accurate enough to meaningfully impact prices, the seller exposes herself to this force first by pooling (according to the β or $\beta^\dagger$ criterion), and second by issuing levered equity (according to the α or $\alpha^\dagger$ criterion).

6.2.1 Example—The Effect of Dispersed Information

In this example we illustrate the impact of investors receiving common versus dispersed information on the form of the issued security. To do so, we analyze the level of signal precision required to achieve $\alpha^\dagger$-informativeness, which as we have shown shifts the security form from debt to levered equity. We will analyze two settings. In the first setting, there are two signals and each signal is publicly observed. In the second, there are two investors and each investor observes a private signal. In both cases, as in Example 1, the signals are normally distributed with type-dependent mean and type-independent precision. Figure 4 illustrates the minimum signal precision needed to achieve $\alpha^\dagger$-informativeness as a function of the discount factor.

Notice that, regardless of the discount factor, the model with privately dispersed signals requires "more" total information (in terms of signal precision) in order for the form of the security to shift from debt to levered equity. For example, for a security with a cash flow duration of 5 years and when the difference between the seller and investors' discount rate is 1 percentage point per year (corresponding to $\delta \approx 0.95$), the seller issues levered equity when the signal precision is above 3 with public signals, but only when the signal precision is above 3.4 when signals are private. Because the auction does not perfectly aggregate the private information of all investors, privatizing the signals has a similar effect to lowering

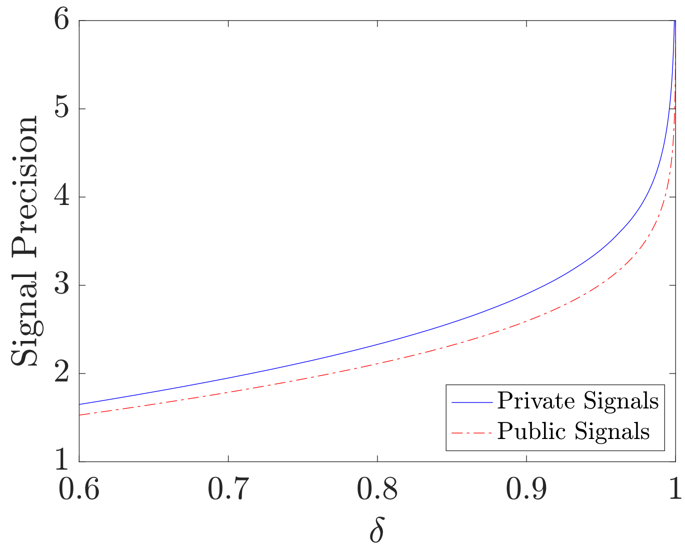


FIGURE 4: Plot of the threshold signal precision as it depends on δ and whether signals are public or private. Above the threshold, the equilibrium security form shifts from debt to levered equity.

their precision.

7 Conclusion

We have analyzed the effect of scrutiny on a general security design problem with a privately informed seller. Scrutiny incentivizes high-value sellers to issue informationally sensitive securities and to decrease their inefficient retention. Consequently, low-value sellers are induced to pool with high-value ones with positive probability, and prices are less informative than in the (separating) equilibrium of the no-scrutiny environment. When the intensity of scrutiny depends on the security issued, we demonstrate that feedback effects can lead to multiple equilibrium including ones that are Pareto dominated. Our main results persist when investors observe private signals.

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A Detailed Equilibrium Characterization

We here provide a complete equilibrium characterization for the baseline model. To do so, we solve $\mathrm{M}(k)$ for all $k \in [\underline{u}, \overline{u})$. We then connect the solutions of $\mathrm{M}(k)_{k\in[\underline{u},\overline{u})}$ to equilibrium, following Daley and Green (2014). Technically, the payoffs in our model do not satisfy the assumptions in Daley and Green (2014), so in what follows we establish the properties of the solutions to $\mathrm{M}(k)_{k\in[\underline{u},\overline{u})}$ that are sufficient to apply Proposition 3.8 therein.

We begin by analyzing the case when scrutiny is not α-informative. The properties of the solution in this case are recorded in the following lemma and illustrated in Figure A.1. Let $\underline{d}(k)$ be the unique solution to $u_L(F^D_{\underline{d}(k)}, 1) = k$. That is, $\underline{d}(k)$ is the face value of a debt security required for $u_L = k$ given that it engenders a belief that $t = H$ with probability one (e.g., $\underline{d}(\underline{u}) = d^{LC}$).

Lemma A.1. *Suppose scrutiny is not α-informative. Then,*

(a) The solution to $\mathrm{M}(k)$, denoted $(F^*(k), \mu^*(k))$, is unique for all $k \in [\underline{u}, \overline{u})$.

(b) $F^*(k)$ is a debt security, with face value $d^*(k)$, for all $k \in [\underline{u}, \overline{u})$.

(c) $\mu^*(k) < 1$ if and only if $\frac{E[\beta(S)|L]}{E[\beta(S)|H]} - 1 > \left(\frac{\delta}{1-\delta}\right)\left(\frac{\Pi_L(\underline{d}(k)) - \Pi_H(\underline{d}(k))}{1-\Pi_H(\underline{d}(k))}\right)$.

(d) d^* and μ^* are continuous and strictly increasing in k (modulo boundary conditions).

As seen in Theorem 1, if we further increase the informativeness of scrutiny, the seller no longer issues debt, but instead its complement: levered equity. We again begin by characterizing the solutions to $\mathrm{M}(k)_{k\in[\underline{u},\overline{u})}$, this time for the case when scrutiny is α-informative.

Lemma A.2. *Suppose scrutiny is α-informative. Then,*

(a) The solution to $\mathrm{M}(k)$, denoted $(F^*(k), \mu^*(k))$, is unique for all $k \in [\underline{u}, \overline{u})$.

(b) $F^*(k)$ is a levered equity security with strike cash flow $a^*(k)$, for all $k \in [\underline{u}, \overline{u})$.

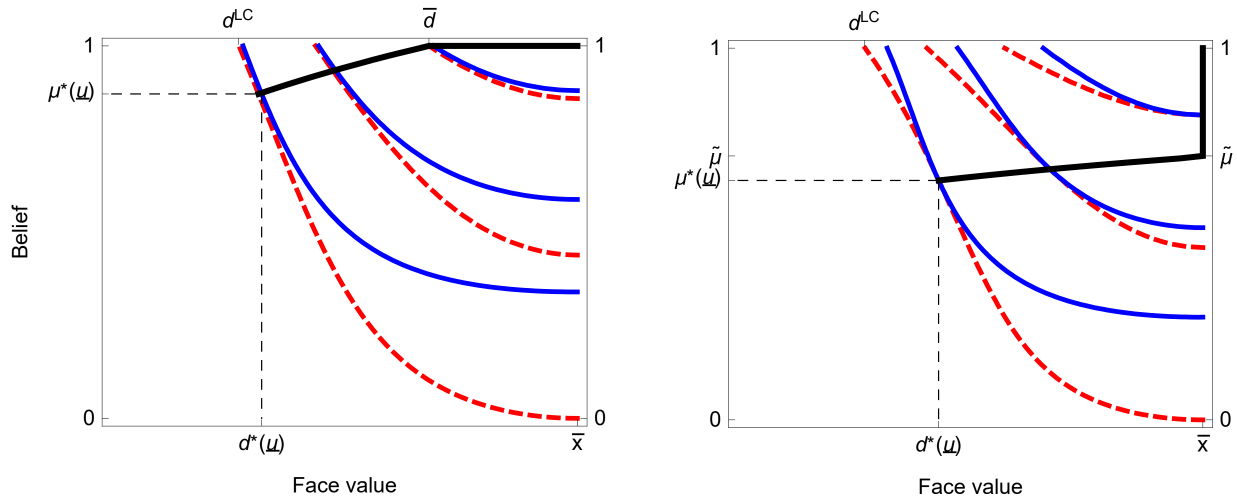
(c) $\mu^*(k) < 1$ for all $k \in [\underline{u}, \overline{u})$.

(d) a^* is strictly decreasing, and μ^* strictly increasing, in k (modulo boundary conditions).

The final component is connecting the solutions of $\mathrm{M}(k)_{k\in[\underline{u},\overline{u})}$ to equilibrium. For each prior belief μ_0, only a single value for the low type's payoff is consistent with equilibrium. The result established in the following Proposition combines the results in Lemmas A.1 and A.2 with those in Proposition 3.8 in Daley and Green (2014).

Proposition A.1. *In the unique equilibrium, we have that*

(a) if $\mu_0 < \mu^*(\underline{u})$, there is some degree of pooling, as the high type issues security $F^*(\underline{u})$ and the low type mixes between security $F^*(\underline{u})$ and selling the entire cash flow, X, with probability $\frac{\mu_0(1-\mu^*(\underline{u}))}{\mu^*(\underline{u})(1-\mu_0)} \in (0,1)$ and the complementary probability, respectively.



(a) β-informative, but positive retention for all priors. (b) β-informative, and zero retention for high priors.

FIGURE A.1: The effect of signal informativeness on $(d^*(\cdot), \mu^*(\cdot))$, depicted as heavy-black curve. Low type's indifference curves in dashed-red, high type's in solid-blue.

(b) if $\mu_0 \geq \mu^*(\underline{u})$, there is full pooling as both types select the unique security $F^*(k)$ such that $\mu^*(k) = \mu_0$.

Figure A.1 illustrates Proposition A.1, as well Proposition 3, for two cases in which scrutiny is β-informative, but not α-informative. Hence, debt is the equilibrium security design. In Panel (a), β-informativeness implies pooling, but with positive levels of retention for all priors. That is, for all μ_0, the high type issues debt with face value $d \leq \overline{d} < \overline{x}$. Panel (b) increases the informativeness of scrutiny such that the criterion in Proposition 3 is satisfied, and the seller efficiently sells her entire cash flow for high priors.

B Proofs for Baseline Model

B.1 Preliminaries and Definitions

Fact B.1. *For any $t \in \{L, H\}$ and $F \in \mathcal{F} \backslash \{0\}$,*

1. *$E[F|H] > E[F|L]$.*
2. *$u_t(F, \mu)$ is strictly increasing in μ.*
3. *There exists unique $d, a \in [0, \overline{x}]$ such that $E[F|t] = E[F_d^D|t] = E[F_a^A|t]$.*
4. *For $\eta \in [0, 1]$, let $F^\eta \equiv (1 - \eta)F + \eta X$. Then $F^\eta \in \mathcal{F}$, and if $F \neq X$, then $E[F^\eta|t]$ and $u_t(F^\eta, 1)$ are strictly increasing in η.*

Fact B.2. *In any PBE, $u_t \in [\underline{u}, \overline{u})$ for any $t \in \{L, H\}$.*

Lemma B.1. *The expected posterior beliefs have the following properties:*

1. $\alpha_t(\cdot)$ *is strictly increasing for any $t \in \{H, L\}$.*

2. $\alpha_H(\mu) \geq \alpha_L(\mu)$ *with the inequality being strict if and only if $\mu \notin \{0, 1\}$.*

3. $\alpha(\mu) \equiv \alpha_H(\mu) - \alpha_L(\mu)$ *is continuous and single-peaked.*

4. $\frac{d}{d\mu} \frac{\alpha'_H(\mu)}{\alpha'_L(\mu)} < 0$.

The D1 Refinement

Fix $k \in [\underline{u}, \overline{u})$ and $F \in \mathcal{F}$, and consider the equation $u_t(F, \mu) = k$. By Fact B.1(2), there is at most one solution for μ. If it exists, denote it by $b_t(F, k)$—that is, $u_t(F, b_t(F, k)) = k$. Next, let $B_t(F, k) \equiv \{\mu : u_t(F, \mu) > k\}$. From Fact B.1(2), the connection between b_t and B_t is immediate: if $b_t(F, k)$ exists, then $B_t(F, k) = (b_t(F, k), 1]$. If $b_t(F, k)$ fails to exist, then either $B_t(F, k) = [0, 1]$ or $B_t(F, k) = \emptyset$.

In our model, the D1 refinement can be stated as follows. Fix an equilibrium endowing expected payoffs $\{u_L, u_H\}$. Consider a security F that is not in the support of either type's strategy. If $B_L(F, u_L) \subset B_H(F, u_H)$, then D1 requires that $\mu(F) = 1$ (where $\subset$ denotes strict inclusion). If $B_H(F, u_H) \subset B_L(F, u_L)$, then D1 requires that $\mu(F) = 0$.

B.2 Proofs of Lemmas

The lemmas are proved in the following order: B.1, A.1, A.2, 1, 3. Notice that Lemmas A.1 and A.2 together subsume Lemma 2.

Proof of Lemma B.1. Note that

$$\alpha(\mu) = \int \frac{\mu}{\mu + (1 - \mu)\beta(s)} (q_H(s) - q_L(s)) ds$$

is bounded, twice continuously differentiable and meets the criteria for exchanging the order of integration and differentiation by the functional form of the integrand, which has bounded first and second partial derivatives with respect to μ. Thus, result 1 follows inmediately from $\mu_f(\mu, \cdot)$ being strictly increasing in μ, and result 2 from $\mu_f(\cdot, s)$ being weakly increasing in s together with the MLRP property that $\frac{q_H(s)}{q_L(s)}$ is weakly increasing in s. For result 3, note that $\alpha(0) = \alpha(1) = 0$, and since the signal is informative, $\alpha(\mu) > 0$ for all $\mu \in (0, 1)$. Thus, it must be that $\alpha'(0) > 0$ and that $\alpha'(1) < 0$. Finally, note that:

$$\alpha'(\mu) = \int \frac{(1 - \beta(s))}{(\mu + (1 - \mu)\beta(s))^2} q_L(s) ds$$
$$\alpha''(\mu) = -2 \int \frac{(1 - \beta(s))^2}{(\mu + (1 - \mu)\beta(s))^3} q_L(s) ds$$

with $\alpha''(\mu) < 0$ for all $\mu \in (0,1)$. Thus, $\alpha(\mu)$ is single-peaked in μ. Finally, result 4 is established in Daley and Green,2014 (Lemma A.1) and Karlin, 1968, (Chapter 3, Proposition 5.1). □

Proof of Lemma A.1. Recall that the solutions to $\mathrm{M}(k)$ are identical to the solutions to $\mathrm{M}^{\Delta}(k)$, which we show are characterized by (a)-(d).

Starting with (b), fix $k \in [\underline{u}, \overline{u})$ and let $\{F^*, \mu^*\}$ be a solution to $\mathrm{M}^{\Delta}(k)$, where $F^* = \phi^*(X)$ and $F^* \notin \mathcal{F}^D$. By Fact B.1(3), let d be the unique solution to $E\left[F^*|L\right] = E[F_d^D|L]$, and $\phi_d(x) = \min\{d, x\}$. Since $F^* \in \mathcal{F}$, ϕ^* is non-decreasing and $\phi^*(x) \leq x$ for all x. Thus, there exists an $\tilde{x} \in (0, \overline{x})$ such that $\phi^*(x) \leq \phi_d(x)$ for all $x \leq \tilde{x}$ with strict inequality for a positive measure set of $x < \tilde{x}$, and $\phi^*(x) \geq \phi_d(x)$ for all $x \geq \tilde{x}$ with strict inequality for a positive measure set of $x > \tilde{x}$. Next,

$$
\begin{aligned}
&E[F^* - F_d^D|H] - E[F^* - F_d^D|L] \\
&\quad = \int_0^{\overline{x}} (\phi^*(x) - \phi_d(x))(\pi_H(x) - \pi_L(x))dx \\
&\quad = \int_0^{\tilde{x}} (\phi^*(x) - \phi_d(x))(\pi_H(x) - \pi_L(x))dx + \int_{\tilde{x}}^{\overline{x}} (\phi^*(x) - \phi_d(x))(\pi_H(x) - \pi_L(x))dx \\
&\quad = \int_0^{\tilde{x}} (\phi^*(x) - \phi_d(x)) \left(\frac{\pi_H(x)}{\pi_L(x)} - 1\right) \pi_L(x)dx + \int_{\tilde{x}}^{\overline{x}} (\phi^*(x) - \phi_d(x)) \left(\frac{\pi_H(x)}{\pi_L(x)} - 1\right) \pi_L(x)dx
\end{aligned}
$$

$$
\begin{aligned}
&= \int_0^{\tilde{x}} (\phi^*(x) - \phi_d(x)) \left(\frac{\pi_H(x)}{\pi_L(x)}\right) \pi_L(x)dx + \int_{\tilde{x}}^{\overline{x}} (\phi^*(x) - \phi_d(x)) \left(\frac{\pi_H(x)}{\pi_L(x)}\right) \pi_L(x)dx \\
&> \int_0^{\tilde{x}} (\phi^*(x) - \phi_d(x)) \left(\frac{\pi_H(\tilde{x})}{\pi_L(\tilde{x})}\right) \pi_L(x)dx + \int_{\tilde{x}}^{\overline{x}} (\phi^*(x) - \phi_d(x)) \left(\frac{\pi_H(\tilde{x})}{\pi_L(\tilde{x})}\right) \pi_L(x)dx \\
&= \left(\frac{\pi_H(\tilde{x})}{\pi_L(\tilde{x})}\right) \int (\phi^*(x) - \phi_d(x))\pi_L(x)dx = \left(\frac{\pi_H(\tilde{x})}{\pi_L(\tilde{x})}\right) \left(E[F^*|L] - E[F_d^D|L]\right) = 0,
\end{aligned}
$$

where the last inequality follows from MLRP: $\frac{\pi_H(\tilde{x})}{\pi_L(\tilde{x})} = \max_{x \leq \tilde{x}} \frac{\pi_H(x)}{\pi_L(x)}$ and that $\frac{\pi_H(\tilde{x})}{\pi_L(\tilde{x})} = \min_{x \geq \tilde{x}} \frac{\pi_H(x)}{\pi_L(x)}$. Thus, the last inequality results from maximizing the weights assigned to the negative points and minimize the weights assigned to the positive ones.

It follows that $E\left[F_d^D|H\right] - E\left[F_d^D|L\right] < E\left[F^*|H\right] - E\left[F^*|L\right]$, and

$$
u_L(F_d^D, \mu^*) = \alpha_L(\mu^*) \left(E[F_d^D|H] - E[F_d^D|L]\right) + (1-\delta)E[F_d^D|L] + \delta E[X|L] < k.
$$

Since $u_L(F_d^D, \mu^*)$ is continuous and increasing in d, there exists security $F_{d'}^D \in \mathcal{F}^D$, with $d' > d$, such that $u_L(F_{d'}^D, \mu^*) = k$, satisfying the constraint for $\mathrm{M}^{\Delta}(k)$. Further, because $E[F_{d'}^D|L] > E[F_d^D|L] = E[F^*|L]$ and $u_L(F_{d'}^D, \mu^*) = k = u_L(F^*, \mu^*)$, it must be that $E[F_{d'}^D|H] - E[F_{d'}^D|L] < E[F^*|H] - E[F^*|L]$. But then the objective in $\mathrm{M}^{\Delta}(k)$ attains a higher value at $\{F_{d'}^D, \mu^*\}$ than at $\{F^*, \mu^*\}$ since $\alpha(\mu^*) - \delta < 0$. This contradicts that $\{F^*, \mu^*\}$ solves $\mathrm{M}^{\Delta}(k)$. Hence, any solution to $\mathrm{M}^{\Delta}(k)$ must be a debt security.

To establish (a), we show that there is a unique solution to the following re-statement of $\mathrm{M}^{\Delta}(k)$:

$$\max_{d,\mu} \ v^D(d,\mu) \quad s.t. \ \ h^D(d,\mu) = k$$

where

$$v^D(d,\mu) \equiv (\alpha(\mu) - \delta)\left(E[\min\{d,X\}|H] - E[\min\{d,X\}|L]\right)$$
$$h^D(d,\mu) \equiv \alpha_L(\mu)\left(E[\min\{d,X\}|H] - E[\min\{d,X\}|L]\right) + (1-\delta)E[\min\{d,X\}|L] + \delta E[X|L].$$

Define $\mu_\ell(k)$ to be the unique solution to $u_L(X, \mu_\ell(k)) = k$. Since $X = F^D_{\overline{x}}$ and $u_L(F^D_d, \mu)$ is increasing in both d and μ, any $\{d, \mu\}$ that satisfies the constraint in $\mathrm{M}^{\Delta}(k)$ must have $\mu \in [\mu_\ell(k), 1]$.

Let us look first for a solution, $\{d^*, \mu^*, \lambda^*\}$, with interior $\mu^* \in (\mu_\ell(k), 1)$, where λ denotes the multiplier on the constraint. Such a solution is characterized by the first-order conditions (second-order conditions are verified at the end of this proof):

$$\left(\alpha'(\mu) - \lambda\alpha'_L(\mu)\right)\left(E[\min\{d,X\}|H] - E[\min\{d,X\}|L]\right) = 0 \qquad \text{(FOC-}\mu\text{)}$$
$$\left(\alpha(\mu) - \delta - \lambda\alpha_L(\mu)\right)\int_d^{\overline{x}} \left(\pi_H(x) - \pi_L(x)\right)dx - \lambda(1-\delta)\int_d^{\overline{x}} \pi_L(x)\,dx = 0 \qquad \text{(FOC-}d\text{)}$$

In any solution, $d^* > 0$, as $u_L(F^D_0, \mu) = \delta E[X|L] < \underline{u} \leq k$, in violation of the constraint. From Fact B.1(1), the second term in the LHS of FOC-μ is positive, and thus $\lambda^* = \frac{\alpha'(\mu^*)}{\alpha'_L(\mu^*)}$. The second condition, FOC-d, requires $\lambda^* < 0$, since $\alpha(\mu) - \delta < 0$ for all μ. Thus, $\alpha'(\mu^*) < 0$ which implies $\mu^* \in [\hat{\mu}, 1]$ (Lemma B.1(3)). Combining these two conditions, we obtain the system of equations that determines $\{d^*, \mu^*\}$:

$$\frac{1 - \Pi_H(d^*)}{1 - \Pi_L(d^*)} - 1 = \frac{1-\delta}{(\alpha(\mu^*) - \delta)\frac{\alpha'_L(\mu^*)}{\alpha'(\mu^*)} - \alpha_L(\mu^*)} \qquad \text{(B.1)}$$
$$h^D(d^*, \mu^*) = k. \qquad \text{(B.2)}$$

Let $d_1 : \mu \mapsto d_1(\mu)$ denote the mapping from beliefs to debt levels implied by (B.1) and $d_2^k : \mu \mapsto d_2^k(\mu)$ denote the mapping implied by (B.2). First, note that $d_1(\cdot)$ is continuous and strictly increasing in μ since the LHS of (B.1) is strictly increasing in d^* due to hazard rate dominance (implied by MLRP), and the RHS of (B.1) is strictly increasing in μ when $\alpha(\mu) - \delta < 0$:

$$\frac{d}{d\mu}RHS = \frac{1-\delta}{\left((\alpha(\mu) - \delta)\frac{\alpha'_L(\mu)}{\alpha'(\mu)} - \alpha_L(\mu)\right)^2}\left(\frac{\alpha(\mu) - \delta}{\alpha'(\mu)^2}\left(\alpha''_H(\mu)\alpha'_L(\mu) - \alpha'_H(\mu)\alpha''_L(\mu)\right)\right) > 0,$$

since $\frac{d}{d\mu}\frac{\alpha'_H(\mu)}{\alpha'_L(\mu)} < 0$ from Lemma B.1(4).

Second, $d_2^k(\cdot)$ is continuous and strictly decreasing in μ for all k since $\alpha'_L(\cdot) > 0$ (Lemma B.1(1)). Therefore, there is at most one pair $\{d^*(k), \mu^*(k)\}$ such that $d_1(\mu^*(k)) = d_2^k(\mu^*(k)) = d^*(k)$ (i.e., solves (B.1) and (B.2)). If this pair exits, it is then the unique solution to $\mathrm{M}^{\Delta}(k)$. If it fails to exists, the unique solution to $\mathrm{M}^{\Delta}(k)$ is a boundary solution. In this case: (i) if $d_1(1) < d_2^k(1)$ the unique solution is $\{d^*(k), \mu^*(k)\} = \{d_2^k(1) = \underline{d}(k), 1\}$; and (ii) if instead $d_1(1) > d_2^k(1)$, then given there is no intersection, $d_1(\mu_\ell(k)) > d_2^k(\mu_\ell(k))$ as well, and the unique solution is $\{d^*(k), \mu^*(k)\} = \{\overline{x}, \mu_\ell(k)\}$.

Next, (c) is a matter of direct calculation. For any $k \in [\underline{u}, \overline{u})$, $\mu^*(k) < 1$ if and only if $d_1(1) > d_2^k(1) = \underline{d}(k)$. This holds if and only if

$$\frac{1 - \Pi_H\left(\underline{d}(k)\right)}{1 - \Pi_L\left(\underline{d}(k)\right)} - 1 < \frac{\left(\frac{\alpha'(1)}{\alpha'_L(1)}\right)(1-\delta)}{\alpha(1) - \delta - \left(\frac{\alpha'(1)}{\alpha'_L(1)}\right)\alpha_L(1)}. \tag{B.3}$$

By straightforward calculations $\alpha(1) = 0$, $\alpha_L(1) = 1$, $\alpha'_t(1) = E[\beta(S)|t]$, and $E[\beta(S)|H] = 1$. So (B.3) becomes

$$E[\beta(S)|L] > \frac{\left(1 - \Pi_H\left(\underline{d}(k)\right)\right) - \delta\left(1 - \Pi_L\left(\underline{d}(k)\right)\right)}{1 - \Pi_H\left(\underline{d}(k)\right) - \delta\left(1 - \Pi_H\left(\underline{d}(k)\right)\right)} = \frac{\delta\left(\Pi_L(\underline{d}(k)) - \Pi_H(\underline{d}(k))\right)}{(1-\delta)(1 - \Pi_H(\underline{d}(k)))} + 1.$$

Finally, for (d), note that changes in k do not impact the mapping $d_1(\cdot)$. For any two $k, k' > 0$ such that $k' > k$ we have that $d_2^{k'}(\mu^*(k)) > d_2^k(\mu^*(k)) = d_1(\mu^*(k))$. Since we have shown that $d_1(\cdot)$ is strictly increasing, it must be that (modulo boundary solutions) $\mu^*(k') > \mu^*(k)$ and thus $d^*(k') > d^*(k)$. Finally, both $\mu^*(k)$ and $d^*(k)$ are continuous in k since d_1 and d_2^k are continuous in μ and k.

Verifying Second-Order Conditions. We now verify that the solution given by the first-order conditions (B.1)-(B.2) is, in fact, a solution to $\mathrm{M}^{\Delta}(k)$. We verify that the determinant of the Bordered Hessian is negative at our interior critical point:

$$BH = \begin{bmatrix} 0 & h_d^D & h_\mu^D \\ h_d^D & L_{dd} & L_{d\mu} \\ h_\mu^D & L_{\mu d} & L_{\mu\mu} \end{bmatrix}$$

where $L(d, \mu) = v^D(d, \mu) - \lambda\left(h^D(d, \mu) - k\right)$.

$$\begin{aligned}
h_d^D &= \alpha_L(\mu^*) \int_{d^*}^{\overline{x}} \left(\pi_H(x) - \pi_L(x)\right) dx + (1-\delta) \int_{d^*}^{\overline{x}} \pi_L(x)\, dx && > 0 \\
d_\mu^D &= \alpha'_L(\mu^*) \left[E[\min\{d^*, X\}|H] - E[\min\{d^*, X\}|L]\right] && > 0 \\
L_{dd} &= -\left[\left(\alpha(\mu^*) - \delta - \lambda^* \alpha_L(\mu^*)\right)\left(\pi_H(d^*) - \pi_L(d^*)\right) - \lambda^*(1-\delta)\pi_L(d^*)\right] && < 0 \\
L_{\mu\mu} &= \left(\alpha''(\mu^*) - \lambda^* \alpha''_L(\mu^*)\right)\left[E[\min\{d^*, X\}|H] - E[\min\{d^*, X\}|L]\right] && < 0 \\
L_{d\mu} = L_{\mu d} &= \left(\alpha'(\mu^*) - \lambda^* \alpha'_L(\mu^*)\right) \int_{d^*}^{\overline{x}} \left(\pi_H(x) - \pi_L(x)\right) dx && = 0
\end{aligned}$$

Where $L_{dd} < 0$ since hazard rate dominance implies $\frac{1-\Pi_H(d)}{1-\Pi_L(d)} > \frac{\pi_H(d)}{\pi_L(d)}$ which combined with the

FOC implies:

$$\frac{\pi_H(d^*)}{\pi_L(d^*)} - 1 < \frac{\lambda^*(1-\delta)}{\alpha(\mu^*) - \delta - \lambda^* \alpha_L(\mu^*)}$$

$$(\alpha(\mu^*) - \delta - \lambda^* \alpha_L(\mu^*))(\pi_H(d^*) - \pi_L(d^*)) - \pi_L(d^*)\lambda^*(1-\delta) > 0$$

where the inequality changes sign since $(\alpha(\mu^*) - \delta - \lambda^* \alpha_L(\mu^*)) < 0$. Finally, $L_{\mu\mu}(\mu^*, d^*, \lambda^*) < 0$ since $\frac{d}{d\mu}\left(\frac{\alpha'_H(\mu)}{\alpha'_L(\mu)}\right) < 0$. A sufficient condition for our solution to be a local maximum is that the bordered Hessian is negative definite. That is, $|BH_1| < 0$ and $|BH_2| > 0$. It is easy to see that $|BH_1| = -(h_d^D)^2 < 0$ and that $|BH_2| = -\left(h_d^D\right)^2 L_{\mu\mu} - \left(h_\mu^D\right)^2 L_{dd} > 0$. □

Proof of Lemma A.2. Recall that the solutions to $\mathrm{M}(k)$ are identical to the solutions to $\mathrm{M}^{\Delta}(k)$, which we show are characterized by (a)-(d). Lemma B.1 and α-informativeness imply that there are exactly two solutions to $\alpha(\mu) = \delta$, which we denote $\underline{\mu}, \overline{\mu}$, and that $\underline{\mu} < \hat{\mu} < \overline{\mu}$. For all $\mu \in (\underline{\mu}, \overline{\mu})$, $\alpha(\mu) > \delta$, and for all $\mu \notin [\underline{\mu}, \overline{\mu}]$, $\alpha(\mu) < \delta$. As in the proof of Lemma A.1, define $\mu_\ell(k)$ to be the unique solution to $u_L(X, \mu_\ell(k)) = k$. Because, for any μ and $F \neq X$, $u_L(X, \mu) > u_L(F, \mu)$, and u_L is increasing μ, any $\{F, \mu\}$ that satisfies the constraint in $\mathrm{M}^{\Delta}(k)$ must have $\mu \in [\mu_\ell(k), 1]$.

Case 1: $\mu_\ell(k) \in [0, \overline{\mu})$.

First, in any solution it must be that $\mu^*(k) \in (\underline{\mu}, \overline{\mu})$. To see this, recall that $E[F|H] - E[F|L] \geq 0$ for any $F \in \mathcal{F}$ (Fact B.1(1)). Hence, if $\mu \notin (\underline{\mu}, \overline{\mu})$, then the objective in $\mathrm{M}^{\Delta}(k)$ is weakly negative. However, the objective can attain positive value. For example, select arbitrary $\mu \in (\mu_\ell(k), \overline{\mu})$ and let $\nu > 0$ solve $u_L(\nu X, \mu) = k$ (it is straightforward to show such a ν always exists, and is positive). Then,

$$\underbrace{(\alpha(\mu) - \delta)}_{>0}\underbrace{(E[(\nu X|H] - E[\nu X|L])}_{>0} > 0.$$

Because any solution must do at least this well, $\mu^*(k) \in (\underline{\mu}, \overline{\mu})$, and $\alpha(\mu^*(k)) - \delta > 0$. Notice that this establishes claim (c) of the lemma.

To establish (b), fix $k \in [\underline{u}, \overline{u})$ and let $\{F^*, \mu^*\}$ be a solution to $\mathrm{M}^{\Delta}(k)$, where $F^* = \phi^*(X)$ and $F^* \notin \mathcal{F}^A$. By Fact B.1(3), let a be the unique solution to $E[F^*|L] = E[F_a^A|L]$, and $\phi_a(x) = \max\{0, x - a\}$. Since $F^* \in \mathcal{F}$, $\phi^*(x) - x$ is non-decreasing and $\phi^*(x) \geq 0$ for all x. Thus, there exists an $\tilde{x} \in (0, \overline{x})$ such that $\phi^*(x) \geq \phi_a(x)$ for all $x \leq \tilde{x}$ with strict inequality for a positive measure set of $x < \tilde{x}$, and $\phi^*(x) \leq \phi_a(x)$ for all $x \geq \tilde{x}$ with strict inequality for a positive measure set of $x > \tilde{x}$. From here the calculations run analogously to those in the proof of Lemma A.1(b), to show that $E\left[F_a^A|H\right] - E\left[F_a^A|L\right] > E[F^*|H] - E[F^*|L]$, and

$$u_L(F_a^A, \mu^*) = \alpha_L(\mu^*)\left(E[F_a^A|H] - E[F_a^A|L]\right) + (1-\delta)E[F_a^A|L] + \delta E[X|L] > k.$$

Since $u_L(F_a^A, \mu^*)$ is continuous and decreasing in a, there exists $F_{a'}^A \in \mathcal{F}^A$, with $a' > a$, such that $u_L(F_{a'}^A, \mu^*) = k$, satisfying the constraint for $\mathrm{M}^{\Delta}(k)$. Further, because $E[F_{a'}^A|L] < E[F_a^A|L] = E[F^*|L]$ and $u_L(F_{a'}^A, \mu^*) = k = u_L(F^*, \mu^*)$, it must be that $E[F_{a'}^A|H] - E[F_{a'}^A|L] > E[F^*|H] - E[F^*|L]$. But then the objective in $\mathrm{M}^{\Delta}(k)$ attains a higher value at $\{F_{a'}^A, \mu^*\}$ than at $\{F^*, \mu^*\}$ since $\alpha(\mu^*) - \delta > 0$. This contradicts that $\{F^*, \mu^*\}$ solves $\mathrm{M}^{\Delta}(k)$. Hence, any solution to $\mathrm{M}^{\Delta}(k)$

must be a levered equity security.

To establish (a), we show that there is a unique solution to the following re-statement of $\mathrm{M}^{\Delta}(k)$:

$$\max_{a,\mu}\ v^{A}(a,\mu) \quad s.t.\ \ h^{A}(a,\mu)=k,$$

where

$$\begin{aligned} v^{A}(a,\mu) &\equiv (\alpha(\mu)-\delta)\left(E[\max\{0,X-a\}|H]-E[\max\{0,X-a\}|L]\right), \\ h^{A}(a,\mu) &\equiv \alpha_L(\mu)\left(E[\max\{0,X-a\}|H]-E[\max\{0,X-a\}|L]\right) \\ &\qquad\qquad\qquad + (1-\delta)E[\max\{0,X-a\}|L]+\delta E[X|L]. \end{aligned}$$

Again, the constraint implies that in any solution $\mu^{*}\geq\mu_{\ell}(k)$, and we have already established that $\mu^{*}\in(\underline{\mu},\overline{\mu})$.

Let us look first for a solution, $\{a^{*},\mu^{*},\lambda^{*}\}$, with interior $\mu^{*}\in(\mu_{\ell}(k),1)$, where λ denotes the multiplier on the constraint. Such a solution is characterized by the following first-order conditions (second-order conditions are verified at the end of this proof):

$$\left(\alpha'(\mu)-\lambda\alpha_L'(\mu)\right)\left(E[\max\{0,X-a\}|H]-E[\max\{0,X-a\}|L]\right)=0 \qquad \text{(FOC-}\mu\text{)}$$

$$\left(\alpha(\mu)-\delta-\lambda\alpha_L(\mu)\right)\int_a^{\overline{x}}\left(\pi_L(x)-\pi_H(x)\right)dx+\lambda(1-\delta)\int_a^{\overline{x}}\pi_L(x)\,dx=0. \qquad \text{(FOC-}a\text{)}$$

In any solution, $a^{*}<\overline{x}$, as $u_L(0,\mu)=\delta E[X|L]<\underline{u}\leq k$, in violation of the constraint. From Fact B.1(1) then, the second term in the LHS of FOC-μ is positive, and $\lambda^{*}=\frac{\alpha'(\mu^{*})}{\alpha_L'(\mu^{*})}$. The second condition, FOC-a, requires $\lambda^{*}>0$ because we have already established that $\mu^{*}\in(\underline{\mu},\overline{\mu})$, meaning $\alpha(\mu^{*})-\delta>0$. Thus, $\alpha'(\mu^{*})>0$ which implies $\mu^{*}<\hat{\mu}$. Combining the two conditions, we obtain the following system of equations that determines $\{a^{*},\mu^{*}\}$:

$$\frac{1-\Pi_H(a^{*})}{1-\Pi_L(a^{*})}-1=\frac{1-\delta}{\left(\alpha(\mu^{*})-\delta\right)\frac{\alpha_L'(\mu^{*})}{\alpha'(\mu^{*})}-\alpha_L(\mu^{*})} \qquad \text{(B.4)}$$

$$h^{A}(a^{*},\mu^{*})=k. \qquad \text{(B.5)}$$

Let $a_1:\mu\mapsto a_1(\mu)$ denote the mapping from beliefs to residual debt levels implied by (B.4) and $a_2^{k}:\mu\mapsto a_2^{k}(\mu)$ denote the mapping implied by (B.5). First, note that $a_1(\cdot)$ is continuous and strictly decreasing in $\mu\in(\underline{\mu},\overline{\mu})$ since the LHS of (B.4) is strictly increasing in a due to hazard rate dominance (implied by MLRP), whereas the RHS of (B.4) is strictly decreasing in μ when $\alpha(\mu)-\delta>0$, as shown in the proof of Lemma A.1(a). Second, $a_2^{k}(\cdot)$ is strictly increasing in μ for all k since $\alpha_L'(\cdot)>0$. Therefore, there is at most one pair $\{a^{*}(k),\mu^{*}(k)\}$ such that $a_1(\mu^{*}(k))=a_2^{k}(\mu^{*}(k))=a^{*}(k)$ (i.e., solves (B.4) and (B.5)). If this pair exits, it is then the unique solution to $\mathrm{M}^{\Delta}(k)$. If it fails to exists, the unique solution to $\mathrm{M}^{\Delta}(k)$ is a boundary solution: $\mu^{*}(k)\in\{\mu_{\ell}(k),1\}$. Since we established at the outset that $\mu^{*}(k)\in(\underline{\mu},\overline{\mu})$, if the solution is boundary it must be that $\mu^*(k) = \mu_\ell(k)$ and (by definition of $\mu_\ell(k)$) $a^*(k) = 0$ (i.e., $F^*(k) = X$).

Finally, The argument for (d) is analogous to that provided for Lemma A.1(d).

Case 2: $\mu_\ell(k) \in [\overline{\mu}, 1)$.

To begin, let $\mu_\ell(k) = \overline{\mu}$. We claim that $\{F^*, \mu^*\} = \{X, \mu_\ell(k)\}$ is the unique solution to $\mathrm{M}^\Delta(k)$. To see this, note that it is feasible (by definition of $\mu_\ell(k)$) and produces a value of 0 for the objective since $\alpha(\overline{\mu}) = \delta$. Consider now any other candidate $\{F, \mu\}$. First, if $\mu = \mu_\ell(k)$ but $F \neq X$, then $u_L(F, \mu_\ell(k)) < u_L(X, \mu_\ell(k)) = k$, in violation of the problem's constraint. Second, if $F = 0$, then for any μ, $u_L(0, \mu) = \delta E[X|L] < \underline{u} \leq k$, also in violation of the constraint. The only remaining possibility is that $F \neq 0$ and $\mu \neq \mu_\ell(k)$. In order to satisfy the constraint, it must be that $\mu \in (\mu_\ell(k), 1] = (\overline{\mu}, 1]$. But then the objective attains a negative value, establishing the claim. Notice that $X \in \mathcal{F}^A \cap \mathcal{F}^D$.

Now let $\mu_\ell(k)$ be arbitrary in $[\overline{\mu}, 1]$, and consider the restricted version of $\mathrm{M}(k)$ in which only *debt* securities can be offered:

$$\begin{aligned} &\max_{d,\mu} u_H(F_d^D, \mu) \\ &s.t.\ u_L(F_d^D, \mu) = k \end{aligned} \qquad (\mathrm{M}_d(k))$$

For this problem, claims (a), (c), and (d) of Lemma A.1 remain true (whereas (b) is simply assumed). Because $X = F^D_{\overline{x}} \in \mathcal{F}^D$, for $\tilde{k}$ such that $\mu_\ell(\tilde{k}) = \overline{\mu}$ the unrestricted solution is feasible in the restricted problem, so it must remain the solution in the restricted problem. Lemma A.1(d) then implies that, in $\mathrm{M}_d(k)$, $d^*(k) = \overline{x}$ for all $k > \tilde{k}$ as well. Note that $k > \tilde{k}$ is equivalent to $\mu_\ell(k) > \overline{\mu}$. Thus we have that if $\mu_\ell(k) > \overline{\mu}$, the optimal debt security to offer has face value $d^* = \overline{x}$.

Turing back now to the unrestricted problem $\mathrm{M}^\Delta(k)$, for any $\mu_\ell(k) > \overline{\mu}$, since any feasible μ is in $[\mu_\ell(k), 1]$, $\alpha(\mu) - \delta < 0$ for all feasible μ. The same argument given for Lemma A.1 implies that any solution must be a debt security. So restricting to debt securities is without loss, and the solution to $\mathrm{M}^\Delta(k)$ is the same as the solution to $\mathrm{M}_d(k)$, which is $\{F^*(k), \mu^*(k)\} = \{X, \mu_\ell(k)\}$. Claims (a)-(d) follow immediately.

Verifying Second-Order Conditions. We now verify that the solution given by the first-order conditions (B.4)-(B.5) is, in fact, a solution to $\mathrm{M}^\Delta(k)$. We verify that the determinant of the Bordered Hessian is negative at our interior critical point:

$$BH = \begin{bmatrix} 0 & h_a^A & h_\mu^A \\ h_a^A & L_{aa} & L_{a\mu} \\ h_\mu^A & L_{\mu a} & L_{\mu\mu} \end{bmatrix}$$

where $L(a,\mu) = v^A(a,\mu) - \lambda\left(h^A(a,\mu) - k\right)$.

$$h_a^A = -\alpha_L(\mu^*)\int_{a^*}^{\infty}(\pi_H(x) - \pi_L(x))\,dx - (1-\delta)\int_{a^*}^{\infty}\pi_L(x)\,dx \qquad < 0$$

$$h_\mu^A = \alpha_L'(\mu^*)\left[E[\max\{0, X - a^*\}|H] - E[\max\{0, X - a^*\}|L]\right] \qquad > 0$$

$$L_{aa} = \left(\alpha(\mu^*) - \delta - \lambda^*\alpha_L(\mu^*)\right)(\pi_H(a^*) - \pi_L(a^*)) + \lambda^*(1-\delta)\pi_L(a^*) \qquad < 0$$

$$L_{\mu\mu} = \left(\alpha''(\mu^*) - \lambda^*\alpha_L''(\mu^*)\right)\left[E[[\max\{0, X - a^*\}|H] - E[[\max\{0, X - a^*\}|L]\right] \qquad < 0$$

$$L_{a\mu} = L_{\mu a} = -\left(\alpha'(\mu^*) - \lambda^*\alpha_L'(\mu^*)\right)\int_{a^*}^{\infty}(\pi_H(x) - \pi_L(x))\,dx \qquad = 0$$

Where $L_{aa} < 0$ since hazard rate dominance implies $\frac{1-\Pi_H(d)}{1-\Pi_L(d)} > \frac{\pi_H(a)}{\pi_L(a)}$ which combined with the FOC implies:

$$\frac{\pi_H(a^*)}{\pi_L(a^*)} - 1 < \frac{\lambda^*(1-\delta)}{\alpha(\mu^*) - \delta - \lambda^*\alpha_L(\mu^*)}$$

$$(\alpha(\mu^*) - \delta - \lambda^*\alpha_L(\mu^*))(\pi_H(a^*) - \pi_L(a^*)) - \pi_L(d^*)\lambda^*(1-\delta) < 0$$

Finally, $L_{\mu\mu}(\mu^*, a^*, \lambda^*) < 0$ since $\frac{d}{d\mu}\left(\frac{\alpha_H'(\mu)}{\alpha_L'(\mu)}\right) < 0$. A sufficient condition for our solution to be a local maximum is that the bordered Hessian is negative definite. That is, $|BH_1| < 0$ and $|BH_2| > 0$. It is easy to see that $|BH_1| = -(h_a^A)^2 < 0$ and that $|BH_2| = -\left(h_a^A\right)^2 L_{\mu\mu} - \left(h_\mu^A\right)^2 L_{aa} > 0$. □

Proof of Lemma 1. Fix an equilibrium with arbitrary $u_H = \hat{u}_H$ and $u_L = k \in [\underline{u}, \overline{u})$. Let S_t be the support of the type t's strategy. Since the low type has the option to choose the same security as the high type, $u_L(F, \mu(F)) \leq k$ for all $F \in S_H$. Fix now $F \in S_H$ and suppose that $u_L(F, \mu(F)) < k$. Then $F \notin S_L$, so $\mu(F) = 1 = b_H(F, \hat{u}_H)$ and $B_L(F, k) = \emptyset$. Further, it must be that $F \neq X$ since $u_L(X, 1) = \overline{u} > k$. Then for $\eta \in (0, 1)$ small enough $b_H(F^\eta, \hat{u}_H) \in (0, 1)$ and $B_L(F^\eta, k) = \emptyset$. Therefore, $F^\eta \notin S_L$ and $\mu(F^\eta) = 1$ (by belief consistency if $F^\eta \in S_H$, by D1 if not). Since $u_H(F^\eta, 1) > u_H(F, 1) = \hat{u}_H$, the high type would gain by deviating to F^η, breaking the equilibrium. Therefore, $u_L(F, \mu(F)) = k$, or equivalently $\mu(F) = b_L(F, k)$, for all $F \in S_H$.

Suppose now there exists $F \in S_H$ such that $F \neq F^*(k)$. Then

$$u_H(F, \mu(F)) = u_H(F, b_L(F, k)) < u_H(F^*(k), \mu^*(k)) = u_H(F^*(k), b_L(F^*(k), k)),$$

and thus $b_H(F^*(k), \hat{u}_H) < \mu^*(k) = b_L(F^*(k), k)$. D1 then implies that $\mu(F^*(k)) = 1$, meaning that deviating to $F^*(k)$ is profitable for the high type and breaking the equilibrium. Hence, if the low type's equilibrium payoff is k, then $S_H = \{F^*(k)\}$ and $\mu(F^*(k)) = \mu^*(k)$. □

Proof of Lemma 3. First, generalize the notion of β-informativeness to "β-informativeness at x" by replacing d^{LC} with arbitrary $x \in [0, \overline{x}]$ in Definition 4. See now that β-informativeness at x implies β-informativeness at all $x' \leq x$, but not the converse. The statement holds because $\frac{\Pi_L(x) - \Pi_H(x)}{1 - \Pi_H(x)}$ is nondecreasing in x, as taking the derivative yields

$$\frac{(1-\Pi_H(x))\pi_L(x)-(1-\Pi_L(x))\pi_H(x)}{(\Pi_H(x)-1)^2}\geq 0$$
$$\iff \quad (1-\Pi_H(x))\pi_L(x)-(1-\Pi_L(x))\pi_H(x)\geq 0 \iff \quad \frac{\pi_L(x)}{1-\Pi_L(x)}\geq\frac{\pi_H(x)}{1-\Pi_H(x)},$$

where the last inequality is the definition of hazard rate dominance, which holds due to MLRP.

Now suppose that scrutiny is α-informative. Then, from the proof of Lemma A.2, there exists $\tilde{k}$ such that $F^*(k)=X$ and $\mu^*(k)=\mu_\ell(k)<1$ for all $k\geq\tilde{k}$. Consider now the restricted version of M(k) in which only debt securities can be offered:

$$\max_{d,\mu} u_H(F^D_d,\mu) \qquad s.t.\ u_L(F^D_d,\mu)=k. \tag{M$_d(k)$}$$

For this problem, claims (a), (c), and (d) of Lemma A.1 remain true (whereas (b) is simply assumed). Because $X\in\mathcal{F}^D$, for all $k\geq\tilde{k}$ the unrestricted solution is feasible in the restricted problem, so it must remain the solution in the restricted problem. From Lemma A.1(d), then, in M$_d(k)$, $\mu^*(k)\leq\mu_\ell(\tilde{k})<1$ for all $k\leq\tilde{k}$. Lemma A.1(d) then implies that scrutiny is β-informative for any value of $d^{LC}=x\in[0,\overline{x}]$.

That the converse does not hold can be shown with a counterexample in which scrutiny is β-informative at all $x\in[0,\overline{x}]$, but not α-informative. Let the signal be binary and symmetric (see footnote 9): $S\in\{l,h\}$ and $\Pr(S=h|t=H)=\Pr(S=l|t=L)\equiv p\in(\frac{1}{2},1)$. A sufficient condition for β-informativeness at any x is

$$\frac{E[\beta(S)|L]}{E[\beta(S)|H]}-1>\frac{\delta}{1-\delta}$$

which, using $E[\beta(S)|H]=1$, is equivalent to

$$\frac{E[\beta(S)|L]-1}{E[\beta(S)|L]}=\frac{(1-2p)^2}{1-3p(1-p)}>\delta.$$

Next, α-informativeness requires $\alpha(\hat{\mu})>\delta$. For binary-symmetric signals, $\hat{\mu}=\frac{1}{2}$ for all p, and the requirement is $\alpha(\frac{1}{2})=(1-2p)^2>\delta$. Since, for all $p\in(0,1)$,

$$0<(1-2p)^2<\frac{(1-2p)^2}{1-3p(1-p)}<1,$$

β-informativeness holds for all x, while α-informativeness fails, when $\delta\in\left((1-2p)^2,\frac{(1-2p)^2}{1-3p(1-p)}\right)$, producing the counterexample. □

B.3 Proofs of Propositions

Proof of Proposition 1. No scrutiny means that scrutiny is not β-informative at any x. Hence, by Lemma A.1(c), $\mu^*(k) = 1$ and $d^*(k) = \underline{d}(k)$ for all $k \in [\underline{u}, \overline{u})$. The proposition then follows directly from Proposition A.1. □

Proof of Proposition A.1. From Lemmas A.1 and A.2 we have that $F^*(k)$ and $\mu^*(k)$ are unique for all $k \in [\underline{u}, \overline{u})$. Let S_t be the support of the type t's strategy. In the proposed unique equilibrium, the high type plays a pure strategy, denoted it F_H, so $S_H = \{F_H\}$, and $S_L \subseteq \{X, F_H\}$. For completeness, we must specify the off-path beliefs: $\mu(F) = 0$ for all $F \neq F_H$.

Verifying that the proposed profile is a PBE is straightforward. To see that it satisfies D1, fix a μ_0 and consider the proposition's unique equilibrium candidate. Denote the high type's equilibrium payoff $\hat{u}_H$ and low type's equilibrium payoff k, so $F_H = F^*(k)$. Let F be an arbitrary security in $\mathcal{F}$ such that $F \neq F^*(k)$. First, if $B_L(F, k) = [0, 1]$, then the low type could deviate to F and obtain a payoff strictly greater than k, regardless of $\mu(F)$, breaking the PBE. Hence, either $b_L(F, k) \in [0, 1]$ exits or $u_L(F, 1) < k$. If $b_L(F, k)$ exits, then since $\{F^*(k), \mu^*(k)\}$ is the unique solution to M(k), $u_H(F, b_L(F, k)) < u_H(F^*(k), \mu^*(k)) = \hat{u}_H$. By Fact B.1(2) then, $b_H(F, \hat{u}_H) > b_L(F, k)$ (or $B_H(F, \hat{u}_H) = \emptyset$) implying $B_H(F, \hat{u}_H) \subseteq B_L(F, k)$. So, $\mu(F) = 0$ is consistent with D1. If instead $u_L(F, 1) < k$ (so $B_L(F, k) = \emptyset$), then there exists unique $\eta \in (0, 1)$ such that $u_L(F^\eta, 1) = k$. Since $\{F^*(k), \mu^*(k)\}$ solves M(k), $u_H(F^*(k), \mu^*(k)) \geq u_H(F^\eta, 1) > u_H(F, 1)$. Hence, $B_H(F, \hat{u}_H) = \emptyset$ as well, and D1 places no restriction on $\mu(F)$.

We now establish uniqueness. By Lemma 1, if the low type's equilibrium payoff is k, then $S_H = \{F^*(k)\}$ and $\mu(F^*(k)) = \mu^*(k)$. Further, if the low type selects $F \notin S_H \cup \{X\}$, then $\mu(F) = 0$, and $u_L(F, 0) < u_L(X, 0) \leq u_L(X, \mu(X))$ for any value of $\mu(X)$. She could therefore profitably deviate to X. Hence, $S_L \subseteq S_H \cup \{X\}$.

The final step is to characterize which values of $u_L = k$ are consistent with equilibrium, which depends on the prior, μ_0. Recall from Lemmas A.1(d) and A.2(d) that μ^* is continuous and strictly increasing in k. First, let $\mu_0 < \mu^*(\underline{u})$, and let $u_L = k$. Therefore, $S_H = \{F^*(k)\}$ and $\mu(F^*(k)) = \mu^*(k) > \mu_0$. For this belief to be consistent with seller strategies, $S_L \neq \{F^*(k)\}$. Hence, $S_L = \{F^*(k), X\}$ and $k = \underline{u}$. The precise mixing probabilities given in the proposition are required for the Bayesian consistency: $\mu(F^*(\underline{u})) = \mu^*(\underline{u})$.

Second, let $\mu_0 \geq \mu^*(\underline{u})$. Hence, there exists unique $k_0 \in [\underline{u}, \overline{u})$ such that $\mu^*(k_0) = \mu_0$. Suppose that $u_L = k > k_0$. Then $S_H = \{F^*(k)\}$ and $\mu(F^*(k)) = \mu^*(k) > \mu^*(k_0) = \mu_0$. But then for this belief to be consistent with seller strategies, $S_L \neq \{F^*(k)\}$. Hence, $S_L = \{F^*(k), X\}$ and $k = \underline{u}$, which contradicts $k > k_0$. Suppose instead that $u_L = k < k_0$. Then $S_H = \{F^*(k)\}$ and $\mu(F^*(k)) = \mu^*(k) < \mu^*(k_0) = \mu_0$. But then $\mu(F) < \mu_0$ for all F on the equilibrium path, which violates belief consistency. Hence, $k = k_0$, and $S_H = S_L = \{F^*(k_0)\}$, exactly as given in the proposition. □

Proof of Proposition 2. The result is a direct implications of the properties established in Lemmas A.1 and A.2 and Proposition A.1. □

Proof of Proposition 3. First, consider the case when scrutiny is β- but not α-informative. Using the proof of Lemma A.1(a) and the equilibrium characterization in Proposition A.1, it is sufficient to show that $d_1(\mu_\ell(k)) > d_2^k(\mu_\ell(k))$ for all k such that $\mu_\ell(k)$ is sufficiently close to 1. Recalling that $d_2^k(\mu_\ell(k))$ satisfies $u_L(F^D_{d_2^k(\mu_\ell(k))}, \mu_\ell(k)) = k$, it follows that $d_2^k(\mu_\ell(k)) = \overline{x}$ by definition of $\mu_\ell(k)$. For any $k \in [\underline{u}, \overline{u})$, $d_1(\mu_\ell(k)) > \overline{x}$ if and only if

$$\frac{1-\Pi_H\left(\overline{x}\right))}{1-\Pi_L\left(\overline{x}\right))} - 1 < \frac{\left(\frac{\alpha'(\mu_\ell(k))}{\alpha'_L(\mu_\ell(k))}\right)(1-\delta)}{\alpha(\mu_\ell(k)) - \delta - \left(\frac{\alpha'(\mu_\ell(k))}{\alpha'_L(\mu_\ell(k))}\right)\alpha_L(\mu_\ell(k))}.$$

Because the RHS is continuous in μ, we can take the limit as $\mu_\ell(k) \to 1$, at which point the calculations are analogous to those in the proof of Lemma A.1(c), establishing the result.

Finally, consider the case when scrutiny is α-informative. First, for any $k \in [\underline{u}, \overline{u})$, in order to satisfy the constraint in M(k), $a^*(k) = 0$ if and only if $\mu^*(k) = \mu_\ell(k)$. Second, the proof of Lemma A.2 establishes that, for all $k \in [\underline{u}, \overline{u})$, if $\mu^*(k) \neq \mu_\ell(k)$ then $\mu^*(k) < \hat{\mu}$ and that a^* is continuous and decreasing in k. Hence, there exists $\tilde{k}$ such that $\{a^*(k), \mu^*(k)\} = \{0, \mu_\ell(k)\}$ for all $k \geq \tilde{k}$, and that $\mu_\ell(\tilde{k}) \leq \hat{\mu}$. The proposition then follows from the equilibrium characterization in Proposition A.1. □

B.4 Proof of Theorem 1

Proof of Theorem 1. The result is a direct implications of the properties established in Lemmas A.1 and A.2 and Proposition A.1. □

C Proofs for Model Extensions

C.1 Proofs for Section 6.1

Lemma C.1. *A solution to* $\mathrm{M}^{\Delta}_{SD}(k)$ *exists for all* $k \in [\underline{u}, \overline{u})$. *Moreover, any solution,* $(F^*(k), \mu^*(k))$, *is either*

1. $F^*(k) = \min\{d, X\}$ *for some* $d \in (0, \bar{x}]$, $\mu^*(k) \geq \frac{1}{2}$ *and* $\lambda^*(k) \leq 0$ *or*

2. $F^*(k) = \max\{0, X - a\}$ *for some* $a \in [0, \bar{x})$, $0 < \mu^*(k) \leq \frac{1}{2}$ *and* $\lambda^*(k) \geq 0$,

where $\lambda^*(k)$ *is the multiplier on the constraint in* $\mathrm{M}^{\Delta}_{SD}(k)$.

Proof. We begin by ruling out two trivial candidates. First, if $\mu = 0$ then $u_L(F, 0) = k \geq \underline{u}$ if and only if $F = X$ and $k = \underline{u}$. However, since $F = X$ maximizes Δ and $\mu = 0$ minimizes $\alpha - \delta < 0$, setting $(F, \mu) = (X, 0)$ *minimizes* V subject to $k = \underline{u}$, so is not the solution to $\mathrm{M}^{\Delta}_{SD}(k)$ for any k. Second, note that the security, $F = 0 = F^D_0 = F^A_{\bar{x}}$ is never a solution to $\mathrm{M}^{\Delta}_{SD}(k)$, as $u_L(0, \mu) < k$ for all μ and $k \in [\underline{u}, \overline{u})$.

As a feasible security $F > 0$ consists of a payment $\phi(x)$ to be made after the realization of cash flow x, we re-state problem $\mathrm{M}^{\Delta}_{SD}(k)$ as one in which the contingent $\phi(x)$ is optimally chosen, subject to the feasibility constraints implied by $F \in \mathcal{F}$.

$$\max_{\left(\{\phi(x)\}_{x\in[0,\bar{x}]},\mu\right)} (\alpha(\Delta_F,\mu)-\delta)\cdot\Delta_F \tag{C.1}$$

subject to, for all $x \in [0,\bar{x}]$:

$$\begin{aligned}
&k = \alpha_L(\Delta_F,\mu)\Delta_F + (1-\delta)E[F|L] + \delta E[X|L]\\
&0 \le \phi(x)\\
&x \ge \phi(x)\\
&0 \le \lim_{o\to 0}\frac{\phi(x+o)-\phi(x)}{o}\\
&1 \ge \lim_{o\to 0}\frac{\phi(x+o)-\phi(x)}{o}.
\end{aligned}$$

Omitting the dependence on k, let λ be the multiplier on the first constraint. It is convenient to scale the x-contingent multipliers on the remaining four (feasibility) constraints by $\pi_L(x)$: writing them as $(\pi_L(x)\cdot\ell^n_x)$, for $n = 1,2,3,4$ respectively.

As in the baseline model, the solution to the above problem can be characterized by its first-order conditions. For the FOC on μ, we have

$$\left(\frac{\partial\alpha(\Delta_F,\mu)}{\partial\mu} - \lambda\frac{\partial\alpha_L(\Delta_F,\mu)}{\partial\mu}\right)\Delta_F = 0 \Rightarrow \lambda = \frac{\partial\alpha(\Delta_F,\mu)/\partial\mu}{\partial\alpha_L(\Delta_F,\mu)/\partial\mu}. \tag{C.2}$$

Using that $\frac{\partial\Delta}{\partial\phi(x)} = \pi_H(x) - \pi_L(x)$, we take the first-order condition for each contingent payment $\phi(x)$ and divide by $\pi_L(x)$ to derive

$$\begin{aligned}
&\left(\left(\frac{\partial\alpha}{\partial\Delta} - \lambda\frac{\partial\alpha_L}{\partial\Delta}\right)\Delta_F + \alpha - \delta - \lambda\alpha_L\right)\left(\frac{\pi_H(x)}{\pi_L(x)} - 1\right)\\
&\qquad - \lambda(1-\delta) + \ell^1_x - \ell^2_x - \lim_{o\to 0}\frac{\ell^3_x - \ell^4_x}{o} + \lim_{o\to 0}\frac{\ell^3_{x-o} - \ell^4_{x-o}}{o} = 0 \quad \forall x.
\end{aligned} \tag{C.3}$$

Note that both the problem's objective and its first constraint are linear in contingent payment $\phi(x)$ for all x. For any given solution candidate, (F^*,μ^*), let $\lambda^* = \frac{\partial\alpha(\Delta_{F^*},\mu^*)/\partial\mu}{\partial\alpha_L(\Delta_{F^*},\mu^*)/\partial\mu}$ and

$$\begin{aligned}
T_1(F^*,\mu^*) &\equiv \frac{\partial\alpha(\Delta_{F^*},\mu^*)}{\partial\Delta}\Delta_{F^*} + \alpha(\Delta_{F^*},\mu^*) - \delta - \lambda^*\left(\frac{\partial\alpha_L(\Delta_{F^*},\mu^*)}{\partial\Delta}\Delta_{F^*} + \alpha_L(\Delta_{F^*},\mu^*)\right)\\
T_2(F^*,\mu^*) &\equiv \lambda^*(1-\delta).
\end{aligned}$$

So, the FOC in (C.3) can be re-written:

$$T_1(F,\mu)\left(\frac{\pi_H(x)}{\pi_L(x)} - 1\right) - T_2(F,\mu) + \ell^1_x - \ell^2_x - \lim_{o\to 0}\frac{\ell^3_x - \ell^4_x}{o} + \lim_{o\to 0}\frac{\ell^3_{x-o} - \ell^4_{x-o}}{o} = 0. \tag{C.4}$$

As the marginal gain of increasing the security's payment in realization x is a constant, it is immediate that, generically, at least one feasibility constraint will bind. To understand which feasibility constraint binds, we proceed as follows. Given a solution (F^*, μ^*), by MLRP of cash flows, it follows that there is at most one x-value, $\hat{x} \in [0, \bar{x}]$, for which the the marginal benefit of increasing ϕ is zero (i.e., (C.4) holds and feasibility constraints do not bind):

$$T_1(F^*, \mu^*)\left(\frac{\pi_H(\hat{x})}{\pi_L(\hat{x})} - 1\right) - T_2(F^*, \mu^*) = 0.$$

Thus, the above expression is positive or negative for all $x \neq \hat{x}$. If the expression is positive (negative) for all $x \in [0, \bar{x}]$, then assign $\hat{x} > \bar{x}$ (respectively, $\hat{x} < 0$). Therefore, if $T_1(F^*, \mu^*) > 0$, then

$$sign\left(T_1(F^*, \mu^*)\left(\frac{\pi_H(x)}{\pi_L(x)} - 1\right) - T_2(F^*, \mu^*)\right) = sign(x - \hat{x}).$$

This implies (i) that $\phi(x) = 0$ and $\ell_x^1 > 0, \ell_x^2 = \ell_x^3 = \ell_x^4 = 0$, for all $x < \hat{x}$; and (ii) that $\phi(x) = x - \hat{x}$ and $\ell_x^4 > 0, \ell_x^1 = \ell_x^2 = \ell_x^3 = 0$, for all $x \geq \hat{x}$. Thus, in this case F^* must be a levered-equity security: $F^* = \phi(X) = \max\{0, X - \hat{x}\}$. An analogous argument establishes that if $T_1(F^*, \mu^*) < 0$, then F^* must be a debt security: $F^* = \phi(X) = \min\{\hat{x}, X\}$.

If $T_1(F^*, \mu^*) = 0$ and $T_2(F^*, \mu^*) < (>) 0$, then it is immediate that $F^* = X$ ($F^* = 0$), as the marginal benefit from increasing $\phi(x)$ is strictly positive (negative) for all x. Note that both solutions are special cases of debt and levered equity. If instead $T_2(F^*, \mu^*) = 0$, then $\lambda^* = 0$ and $\mu^* = \frac{1}{2}$. In this case, $T_1(F^*, \mu^*) = 0$ if and only if $\frac{\partial \alpha(\Delta_{F^*}, \mu^*)}{\partial \Delta}\Delta_{F^*} + \alpha(F^*, \mu^*) - \delta = 0$. That is, the derivative of the objective of (C.1) with respect to Δ_F is zero. But by Assumption 1(a) this derivative can be zero only at the minimum of the objective (constrained to $\mu^* = \frac{1}{2}$), which cannot be a solution to the $\mathrm{M}_{SD}^{\Delta}(k)$. Thus, we conclude that if $T_1(F^*, \mu^*) = 0$, it must be that $T_2(F^*, \mu^*) \neq 0$.

Having established that any solution to $\mathrm{M}_{SD}^{\Delta}(k)$ must involve either a debt or a levered-equity contract, we turn to the claims regarding μ^* and λ^*. To do so, first consider the constrained problem, in which only levered-equity securities are permitted (recall $F_a^A \equiv \max\{0, X - a\}$),

$$\max_{a,\mu} \left(\alpha(\Delta_{F_a^A}, \mu) - \delta\right)\Delta_{F_a^A} \tag{C.5}$$
$$s.t. \quad k = \alpha_L(\Delta_{F_a^A}, \mu)\Delta_{F_a^A} + (1-\delta)E\left[F_a^A | L\right] + \delta E[X|L].$$

For any k, a solution exists since the problem is continuous and the set of (a, μ)-pairs satisfying the constraint is compact. Let (a^*, μ^*) be a solution. First suppose that $T_1\left(F_{a^*}^A, \mu^*\right) \neq 0$ and $\mu^* < 1$, meaning (a^*, μ^*) must satisfy first-order conditions:

$$\frac{\Pi_L(a^*) - \Pi_H(a^*)}{1 - \Pi_L(a^*)} = \frac{\lambda^*(1-\delta)}{T_1\left(F_{a^*}^A, \mu^*\right)} \tag{C.6}$$
$$k - \delta E[X|L] = \alpha_L(\Delta_{F_{a^*}^A}, \mu^*)\Delta_{F_{a^*}^A} + (1-\delta)E\left[F_{a^*}^A | L\right]. \tag{C.7}$$

The relevant case is when $(F^A_{a^*}, \mu^*)$ also solves $\mathrm{M}^{\Delta}_{SD}(k)$, which from above implies that $T_1\left(F^A_{a^*}, \mu^*\right) > 0$. Because both the LHS of (C.6) and $(1-\delta)$ are always weakly positive, this implies that $\lambda^* \geq 0$. Finally, recall that

$$\lambda^* \equiv \frac{\partial \alpha\left(F^A_{a^*}, \mu^*\right)/\partial \mu}{\partial \alpha_L\left(F^A_{a^*}, \mu^*\right)/\partial \mu} \geq 0 \iff \mu^* \leq \frac{1}{2}.$$

Second, suppose that $T_1\left(F^A_{a^*}, \mu^*\right) = 0$. Above, we established that if $T_1(F^*, \mu^*) = 0$ for any solution to $\mathrm{M}^{\Delta}_{SD}(k)$, then $F^* = X$, which always satisfies either claims 1 or 2 of the lemma. Finally, suppose that the solution to (C.5) involves $\mu^* = 1$. Then $\lambda^* < 0$, which implies that also that $T_1\left(F^A_{a^*}, \mu^*\right) < 0$, meaning that the solution to $\mathrm{M}^{\Delta}_{SD}(k)$ is not levered equity. This establishes the lemma's claims regarding levered-equity solutions. An analogous argument establishes the lemma's claims regarding debt solutions. □

Lemma C.2. *For each ρ, there exists $\hat{k} \in [\underline{u}, \overline{u}]$ such that any solution to $\mathrm{M}^{\Delta}_{SD}(k)$ involves a debt security for all $k < \hat{k}$, and any solution to $\mathrm{M}^{\Delta}_{SD}(k)$ involves a levered-equity security for all for $k > \hat{k}$.*

Proof. By Lemma C.1, any solution to $\mathrm{M}^{\Delta}_{SD}(k)$, involves an F^* that is either a debt or a levered-equity security. In addition, any instances in which $F^* = X$ have no effect on the claim in the lemma, so we focus on debt solutions with $d^* < \bar{x}$ and levered-equity solutions with $a^* > 0$ for the remainder. We proceed to show the existence of a unique threshold $\hat{k}$. First, suppose all solutions to $\mathrm{M}^{\Delta}_{SD}(k)$ are levered equity solutions, and consider one of them: $F^* = F^A_{a^*}$. Then,

$$\frac{\partial V(\Delta_{F^*}, \mu^*)}{\partial \Delta} > 0. \tag{C.8}$$

To see this, recall that by Assumption 1(a), for all $\mu \in (0,1)$, there exists a unique $\hat{\Delta}(\mu)$ such that $\frac{dV(\Delta,\mu)}{d\Delta} < 0$ if and only if $\Delta < \hat{\Delta}(\mu)$. Moreover, there always exists F^D_d that, together with μ^*, also satisfies the constraint and $\Delta_{F^D_d} < \Delta_{F^A_{a^*}}$. Hence, if the LHS of (C.8) were non-positive at (F^A_a, μ), then $V(F^A_a, \mu) < V(F^D_d, \mu)$. Now let $V^*(k) \equiv V(\Delta_{F^*(k)}, \mu^*(k))$. For levered-equity securities, let $a(k,\mu)$ be such that $u_L(F^A_{a(k,\mu)}, \mu) = k$. So in this case, $V^*(k) = V^A(k) = V(\Delta_{a(k,\mu^*(k))}, \mu^*(k))$. Consider a marginal increase in k,

$$\frac{dV^*}{dk} = \frac{dV^A}{dk} = \frac{\partial V(\Delta_{a(k,\mu^*)}, \mu^*)}{\partial \Delta}\frac{\partial \Delta_{a(k,\mu^*)}}{\partial k} + \frac{dV(\Delta_{a(k,\mu^*)}, \mu^*)}{d\mu}\frac{\partial \mu^*}{\partial k} > 0, \tag{C.9}$$

where the inequality follows from $\frac{\partial V(\Delta_{a(k,\mu^*)},\mu^*)}{\partial \Delta} > 0$ (just established), $\frac{\partial \Delta_{a(k,\mu^*)}}{\partial k} > 0$ (by Assumption 1(b)), and $\frac{dV(\Delta_{a(k,\mu^*)},\mu^*)}{d\mu} = 0$ (see first-order condition (C.2)). Thus, no debt solutions arise by marginally increasing k, as V^* and Δ_{a^*} marginally increase with k. An analogous argument establishes that if all solutions involve debt, then it will remain so if k is marginally decreased.

Finally, suppose that there exists a k_{ind} such that there is both a debt and a levered-equity solution to $\mathrm{M}^{\Delta}_{SD}(k)$. Then it must be that $V^* = V(\Delta^*_d, \mu^*_d) = V(\Delta^*_a, \mu^*_a)$. As we have shown that $\frac{dV(\Delta^*_a,\mu^*_a)}{dk} > 0$ while $\frac{dV(\Delta^*_d,\mu^*_d)}{dk} < 0$, it is immediate that after a marginal increase in k, the debt solution becomes inferior to the levered equity solution, and all solutions at the higher k thus involve levered equity securities. Combining with the results above, it follows that for all $k < k_{ind}$, the unique solution is given by a debt security while for all $k > k_{ind}$, the unique solution is given by a levered equity security.

Thus, for each ρ, there exists a unique, $\hat{k} \in [\underline{u}, \overline{u})$, such that the solution security form changes from debt to levered equity. Specifically, $\hat{k} = \underline{u}$ when the solution is given by levered equity securities for all k; $\hat{k} = \overline{u}$ when the solution is given by debt securities for all k, and $\hat{k} = k_{ind}$ otherwise. □

Proof of Theorem 2. From Lemmas 1 and C.1, the only equilibrium security forms for the high type are debt and levered equity. If the low type issues any security that is not in the support of the high type's strategy, then her type is revealed. For this to be part of an equilibrium, the low type must be issuing her entire cash flow, $F = X$, which is a special case of debt and levered equity.

For claim (a), we first show that for ρ large enough, any solution to $\mathrm{M}_{SD}^{\Delta}(k)$ must be a levered equity solution. From immediately above, this only entails showing that the solution cannot be a debt security with $d \in (0, \bar{x})$. From the proof of Lemma C.1, recall that at a debt solution (F^*, μ^*), it must be that $T_1(F^*, \mu^*) \leq 0$. Consider now any $(F, \mu) \in \mathcal{F} \times (0, 1)$. Using $\lambda(F, \mu) = \frac{\partial \alpha(\Delta_F, \mu)/\partial \mu}{\partial \alpha_L(\Delta_F, \mu)/\partial \mu}$ and writing out the expression for $T_1(F, \mu)$, then taking limit as $\rho \to \infty$, we have

$$\lim_{\rho \to \infty} T_1(F, \mu) = \underbrace{\frac{\partial \alpha\left(\Delta_F, \mu\right)}{\partial \Delta} \Delta_F}_{\to 0 \cdot \Delta_F \,=\, 0} + \underbrace{\alpha\left(F, \mu\right) - \delta}_{\to 1 - \delta} - \underbrace{\lambda(F, \mu) \cdot \left(\frac{\partial \alpha_L\left(\Delta_F, \mu\right)}{\partial \Delta} \Delta_F + \alpha_L\left(F, \mu\right) \right)}_{\to \left(\lim_{\rho \to \infty} \lambda(F, \mu) \,\in\, \mathbb{R} \right) \cdot 0 \,=\, 0} = 1 - \delta > 0.$$

By continuity of T_1, if $(F, \mu) \in \mathcal{F} \times (0, 1)$ solves $\mathrm{M}_{SD}^{\Delta}(k)$ with ρ large enough, then F must be a levered equity-security. Consider instead now any $(F, 1)$. Then, $\alpha_L = \alpha_H = 1$, and therefore,

$$T_1(F, 1) = -\delta - \lambda(F, 1) = 1 - \delta - \frac{\partial \alpha_H(1, \Delta_F)/\partial \mu}{\partial \alpha_L(1, \Delta_F)/\partial \mu} = 1 - \delta - \frac{1}{E\left[\frac{q_\epsilon(\tilde{s})}{q_\epsilon(\tilde{s} - \rho \Delta_F)} | L\right]}.$$

Hence, as in the first case, $\lim_{\rho \to \infty} T_1(F, 1) = 1 - \delta$ since $\lim_{\rho \to \infty} E\left[\frac{q_\epsilon(\tilde{s})}{q_\epsilon(\tilde{s} - \rho \Delta_F)} | L\right] = \infty$, establishing one direction of claim (a): there exists $\bar{\rho}$ such that the unique security form is levered-equity for all $\rho \geq \bar{\rho}$. The proof for the analogous direction of claim (b) is symmetric, as for all candidate solutions (F, μ): $\lim_{\rho \to 0} T_1(F, \mu) = -\delta < 0$.

Next, we use Lemma C.2 and make the dependence of $\hat{k}$ on ρ explicit. The argument above established that $\hat{k}(\rho) = \underline{u}$ (respectively, $\hat{k}(\rho) = \overline{u}$) for all $\rho \geq \bar{\rho}$ (respectively, $\rho \leq \underline{\rho}$). It is also immediate, by the continuity of $V(F, \mu)$ in ρ, that $\underline{\rho} < \bar{\rho}$.

To prove the other direction of claim (a), we will show that if $\rho < \bar{\rho}$, then debt is an equilibrium security form. Recall from the proof of Lemma C.2 that debt solutions correspond to $\frac{dV(\Delta^*, \mu^*)}{d\Delta} < 0$. It is easy to check that, for all μ^*, $\frac{\partial^2 V(\Delta^*, \mu^*)}{\partial \Delta \partial \rho} \geq 0$ (with strict inequality for $\mu \in (0, 1)$). Hence, by the same arguments used in the proof of Lemma C.2, $\hat{k}$ is decreasing in ρ. It follows that for all $\rho < \bar{\rho}$ there exists at least some $k \in [\underline{u}, \overline{u})$ with solution $F^*(k)$ that is a debt security. Linking back to equilibrium, recall that for any k and solution $(F^*(k), \mu^*(k))$, full pooling on $F^*(k)$ is an equilibrium when the prior is $\mu_0 = \mu^*(k)$, establishing the result. The argument is analogous for why levered equity is an equilibrium security form when $\rho > \underline{\rho}$. □

Proof of Proposition 4. Because k is fixed throughout, we do not write the dependence on k for this proof. First, suppose that $F^*_{SD} = \min\{d^*_{SD}, X\}$ for some $d^*_{SD} < \bar{x}$. As F^*_{SD} is a debt contract, from the Proof of Lemma C.1 we know that: (i) interior debt level d^*_{SD} is given by the following first-order condition (analogous to (C.6), but for a debt solution):

$$\frac{\Pi_L\left(d^*_{SD}\right) - \Pi_H\left(d^*_{SD}\right)}{1 - \Pi_L\left(d^*_{SD}\right)} = \frac{\lambda^*_{SD}\left(1-\delta\right)}{T_1(F^*_{SD}, \mu^*_{SD})}. \tag{C.10}$$

where $\mu^*_{SD} > \frac{1}{2}$ is pinned down by the binding constraint $u_L(F^D_{d^*_{SD}}, \mu^*_{SD}) = k$, and $\lambda^*_{SD} < 0$ is then from (C.2), and (ii) $T_1(F^*_{SD}, \mu^*_{SD}) < 0$.

Moreover, using that $\lambda^*_{SD} = \frac{\partial\alpha_H\left(\Delta^*_{F_{SD}}, \mu^*_{SD}\right)/\partial\mu}{\partial\alpha_L\left(\Delta^*_{F_{SD}}, \mu^*_{SD}\right)/\partial\mu}$ we have,

$$\begin{aligned} T_1(F^*_{SD}, \mu^*_{SD}) = {} & \alpha\left(\Delta_{F^*_{SD}}, \mu^*_{SD}\right) - \delta - \lambda^*_{SD}\alpha_L\left(\Delta_{F^*_{SD}, \mu^*_{SD}}\right) \\ & + \underbrace{\left(\frac{\partial\alpha\left(\Delta_{F^*_{SD}}, \mu^*_{SD}\right)}{\partial\Delta} - \frac{\partial\alpha_H\left(\Delta^*_{F_{SD}}, \mu^*_{SD}\right)/\partial\mu}{\partial\alpha_L\left(\Delta^*_{F_{SD}}, \mu^*_{SD}\right)/\partial\mu} \cdot \frac{\partial\alpha_L\left(\Delta_{F^*_{SD}}, \mu^*_{SD}\right)}{\partial\Delta}\right)}_{>0} \Delta_{F^*_{SD}} \end{aligned} \tag{C.11}$$

where the inequality is because α_H increases, while α_L decreases, in Δ for all $\mu \in (0,1)$, and both increase in μ for all Δ. Combining this inequality with $T_1(F^*_{SD}, \mu^*_{SD}) < 0$, we have that $\alpha(\Delta^*_{F_{SD}}, \mu^*_{SD}) - \delta < 0$.

Next we show that $\alpha(\Delta_{F^*_{SD}}, \mu) < \delta$ for all μ. Suppose not, then because α is maximized at $\mu = \frac{1}{2}$ for fixed Δ, we have $\alpha(\Delta_{F^*_{SD}}, \frac{1}{2}) > \delta$. Then, the objective of $\mathrm{M}^{\Delta}_{SD}(k)$ is improved by moving to $(F^*_{SD}, \frac{1}{2})$, but because $\frac{1}{2} < \mu^*_{SD}$, $u_L(F^*_{SD}, \frac{1}{2}) < k$. By Assumption 1(b), to compensate the low type for decreasing μ, we can increase the debt level to $d' > d^*_{SD}$ such that $u_L(d', \frac{1}{2}) = k$. But $\Delta_{d'} > \Delta_{d^*_{SD}}$, which implies that $\alpha(\Delta_{F^D_{d'}}, \frac{1}{2}) > \alpha(\Delta_{F^*_{SD}}, \frac{1}{2}))$, and the objective in $\mathrm{M}^{\Delta}_{SD}(k)$ improves even further. Hence, $(F^D_{d'}, \frac{1}{2})$ is a feasible candidate to $\mathrm{M}^{\Delta}_{SD}(k)$ that does better than the solution (F^*_{SD}, μ^*_{SD}), a contradiction.

Turning now to the baseline model with security independent scrutiny determined as in the statement of the proposition, we have $\alpha^{base}_t(\mu) \equiv \alpha_t(\Delta_{F^*_{SD}}, \mu)$, which is not α-informative. By Theorem 1, F^* must be a debt contract. In addition, the FOC for a solution with interior debt level d^* is:

$$\frac{\Pi_L\left(d^*\right) - \Pi_H\left(d^*\right)}{1 - \Pi_L\left(d^*\right)} = \frac{\lambda^*\left(1-\delta\right)}{\alpha\left(, \Delta^*_F, \mu^*\right) - \delta - \lambda^*\alpha_L\left(\Delta^*_F, \mu^*\right)}. \tag{C.12}$$

Let us try setting $d^* = d^*_{SD}$. As now the constraint is the same as with security-dependent scrutiny, it must be that $\mu^* = \mu^*_{SD}$, and thus $\lambda^* = \lambda^*_{SD}$. But then, the only difference between (C.10) and (C.12) is the positive term in (C.11), so d^*_{SD} is not the solution in the baseline model. As the LHS is increasing in d (see proof of Lemma A.1), it follows that $d^* < d^*_{SD}$.

Suppose instead that $F^*_{SD}(k) = \max\{X - a^*_{SD}, 0\}$ for some $a^*_{SD} > 0$. As F^*_{SD} is a levered-equity contract, we know that (i) $\lambda^*_{SD} > 0$, and (ii) $T_1(F^*_{SD}, \mu^*_{SD}) > 0$. However, now the sign of $\alpha(\Delta_{F^*_{SD}}, \mu^*_{SD}) - \delta - \lambda^*_{SD}\alpha_L(\Delta_{F^*_{SD}}, \mu^*_{SD})$ cannot be determined. There are (generically) two possibilities:

1. If $\alpha(\Delta_{F^*_{SD}}, \mu^*_{SD}) - \delta > 0$, then the scrutiny in the baseline model is α-informative, and F^* is a levered-equity security.

2. If $\alpha(\Delta_{F^*_{SD}}, \mu^*_{SD}) - \delta < 0$, then the scrutiny in the baseline model is not α-informative, and F^* is a debt security.

If the first case is true, then an analogous argument to that used in the debt case (above) comparing the first-order conditions in both problems shows that $a^* > a^*_{SD}$. □

Proof of Proposition 5. First, by the same arguments used in the proof of Proposition A.1, *i*) for any solution $(F^*(\underline{u}), \mu^*(\underline{u}))$, if $\mu_0 < \mu^*(\underline{u})$, then there exists a partial pooling equilibrium in which $S_H = \{F^*(\underline{u})\}$ and $S_L = \{F^*(\underline{u}), X\}$ if $\mu^*(\underline{u}) < 1$ or the LCSE is an equilibrium if $\mu^*(\underline{u}) = 1$; and *ii*) for any k and solution $(F^*(k), \mu^*(k))$, if $\mu_0 = \mu^*(k)$, then there exists a full pooling equilibrium in which $S_H = S_L = \{F^*(k)\}$.

Next, combining $\rho \in (\underline{\rho}, \bar{\rho})$ with Lemmas C.1 and C.2 has the following implications. First, for all $k < \hat{k}$ all solutions to $\mathrm{M}^{\Delta}_{SD}(k)$ involve debt with $d^*(k) < \bar{x}$. Moreover, from the proof of Lemma C.1, any solution involving debt with $d^*(k) < \bar{x}$ has $\mu^*(k) > \frac{1}{2}$, establishing the first bulleted claim. Second, there exists $k > \hat{k}$ with solutions to $\mathrm{M}^{\Delta}_{SD}(k)$ that involve levered equity with $a^*(k) > 0$ and $\mu^*(k) < \frac{1}{2}$. Third, letting $\tilde{k} \equiv u_L(X, \frac{1}{2})$, it must be that $F^*(k) = X$ in the unique solution to $\mathrm{M}^{\Delta}_{SD}(k)$ for all $k \geq \tilde{k} > \hat{k}$. For this third implication, by Assumption 1(b), for $k > \tilde{k}$, any (F, μ) for which $u_L(F, \mu) = k$ will involve $\mu > \frac{1}{2}$. For such k the only way to simultaneously satisfy Lemma C.1 and C.2 when $\rho \in (\underline{\rho}, \bar{\rho})$, is for $F^*(k) = X$. The existence of $\mu^1 < \mu^2$ follows from the continuity of the levered-equity solution. □

Proof of Proposition 6. As in earlier proofs, we let $V^*(k)$ be the optimum value attained in $\mathrm{M}^{\Delta}_{SD}(k)$. By Lemma 1, in any equilibrium with low-type payoff $u_L = k$, the high type's payoff is $u_H = V^*(k)$. Hence the Pareto ranking of any two equilibria follows if V^* is increasing in k, which we now establish. Recall that $F^\eta = (1-\eta)F + \eta X \in \mathcal{F}$, and notice that Δ_{F^η} is increasing in η. Similarly, let $\mu^\eta \equiv (1-\eta)\mu + \eta$. For both types, $u_t(F^\eta, \mu^\eta)$ is increasing in both η (where u_L increasing in the F^η-term is by 1(b)). Now let $u_L(F_1, \mu_1) = k_1$ and $u_H(F_1, \mu_1) = V^*(k_1)$. By continuity of u_L, for any $k_2 > k_1$, there exists $\eta > 0$ such that $u_L(F_1^\eta, \mu_1^\eta) = k_2$. So we have $V^*(k_2) \geq u_H(F_1^\eta, \mu_1^\eta) > u_H(F_1, \mu_1) = V^*(k_1)$, establishing the Pareto ranking of equilibria. Next, Assumption 1(b) implies that the Pareto superior equilibrium is the one with greater Δ. Finally, Lemma C.2 guarantees that any debt equilibrium has a lower k-value than does any levered-equity equilibrium. □

C.2 Proofs for Section 6.2

Lemma C.3. *With dispersed investor information (Section 6.2), for all $k \geq 1$, adjusted expected posteriors, $\alpha_t^\dagger(\cdot)$, preserve the following properties of $\alpha_t(\cdot)$:*

1. $\alpha_t^\dagger(\cdot)$ *is strictly increasing for any $t \in \{H, L\}$.*
2. $\alpha_H^\dagger(\mu) \geq \alpha_L^\dagger(\mu)$, *with the inequality being strict if and only if $\mu \notin \{0, 1\}$.*
3. $\alpha^\dagger(\mu) \equiv \alpha_H^\dagger(\mu) - \alpha_L^\dagger(\mu)$ *is continuous and single-peaked.*
4. $\frac{d}{d\mu} \frac{\alpha_H^{\dagger\prime}(\mu)}{\alpha_L^{\dagger\prime}(\mu)} < 0$.

Proof. As $g_t(z)$ is the density of the k^{th} order statistic among $m = n - 1$ remaining bidders, we have that

$$g_t(z) = \frac{m!}{(k-1)!(m-k)!} \xi_t(z) \Xi_t(z)^{k-1} (1 - \Xi_t(z))^{m-k},$$

where Ξ_t is the c.d.f. with corresponding p.d.f. ξ_t. Thus, the adjusted likelihood ratio is given by

$$\beta^\dagger(s, z) \equiv \frac{q_L(s)\xi_L(z)g_L(z)}{q_H(s)\xi_H(z)g_H(z)} = \frac{q_L(s)\xi_L(z)^2 \Xi_L(z)^{k-1}(1 - \Xi_L(z))^{m-k}}{q_H(s)\xi_H(z)^2 \Xi_H(z)^{k-1}(1 - \Xi_H(z))^{m-k}}.$$

We also need the likelihood ratio of Y_n^{k+1}, because this is the random variable over which the seller will compute the expected posterior. Let $h_t(z)$ denote the density of Y_n^{k+1} conditional on t. And define

$$\tilde{\beta}^\dagger(s, z) \equiv \beta(s) \frac{h_L(z)}{h_H(z)} = \frac{q_L(s)\xi_L(z)\Xi_L(z)^{m-k}(1 - \Xi_L(z))^k}{q_H(s)\xi_H(z)\Xi_H(z)^{m-k}(1 - \Xi_H(z))^k}$$

We now show that adjusted likelihood ratios $\beta^\dagger$ and $\tilde{\beta}^\dagger$ preserve the properties of the baseline model likelihood ratio, β: mainly, that they are both weakly decreasing in s and in z.

As $\beta(s)$ is decreasing in s, so too are $\beta^\dagger$ and $\tilde{\beta}^\dagger$, so it remains to show that $\frac{h_L(z)}{h_H(z)}$ decreases in z, which we show in what follows in two steps.

First, note that $\frac{\Xi_L(z)}{\Xi_H(z)}$ is weakly decreasing in z. To see this, note that

$$sign\left(\frac{d}{dz}\left(\frac{\Xi_L(z)}{\Xi_H(z)}\right)\right) = sign\left(\xi_L \Xi_H - \xi_H \Xi_L\right).$$

So it suffices to show that $\frac{\xi_L(z)}{\xi_H(z)} \geq \frac{\Xi_L(z)}{\Xi_H(z)}$. Note that we can write $\Xi_L(z) = \int_0^z \frac{\xi_L(x)}{\xi_H(x)} \xi_H(x) dx$ and since $\frac{\xi_L}{\xi_H}$ is monotonic, we have that

$$\Xi_L(z) = \int_0^z \frac{\xi_L(x)}{\xi_H(x)} \xi_H(x) dx \leq \frac{\xi_L(z)}{\xi_H(z)} \int \xi_H(x) dx = \frac{\xi_L(z)}{\xi_H(z)} \Xi_H(z).$$

Second, we have that $\frac{1-\Xi_L(z)}{1-\Xi_H(z)}$ is weakly decreasing in z. To see this, $sign\left(\frac{d}{dz}\left(\frac{1-\Xi_L(z)}{1-\Xi_H(z)}\right)\right) =$

$sign\left(-\xi_L(1-\Xi_H)+\xi_H(1-\Xi_L)\right)$ so it suffices to show that

$$\frac{\xi_L(z)}{\xi_H(z)} \leq \frac{1-\Xi_L(z)}{1-\Xi_H(z)}$$

Then, note that

$$\frac{1-\Xi_L(z)}{1-\Xi_H(z)} = \frac{\int_z^1 \frac{\xi_L(x)}{\xi_H(x)}\xi_H(x)dx}{\int_z^1 \xi_H(x)dx} \geq \frac{\frac{\xi_L(z)}{\xi_H(z)}\int_z^1 \xi_H(x)dx}{\int_z^1 \xi_H(x)dx} = \frac{\xi_L(z)}{\xi_H(z)}.$$

It follows that both $\beta^\dagger$ and $\tilde{\beta}^\dagger$ are weakly decreasing in z. With this result, we can proceed to the proof of results 1. to 4.

First, result 1 is immediate as

$$\alpha^{\dagger\prime}(\mu) = \int\int \frac{\beta^\dagger(s,z)}{\left(\mu+(1-\mu)\beta^\dagger(s,z)\right)^2} h_t(z)\, q_t(s)dzds > 0 \tag{C.13}$$

Second, as $\mu_f(\mu,s,z)$ is weakly increasing both s and z, and $\frac{h_H(z)}{h_L(z)}$ is weakly increasing in z while $\frac{q_H(s)}{q_L(s)}$ is weakly increasing in s, result 2 follows.

Third, as signals are informative, and $\alpha^\dagger(0)=\alpha^\dagger(1)=0$ and $\alpha^\dagger(\mu)\in(0,1)$ for all $\mu\in(0,1)$, it must be that $\alpha^{\dagger\prime}(\mu)(0)>0$ and $\alpha^{\dagger\prime}(1)<0$. In addition, note that for all $\mu\in(0,1)$

$$\frac{d^2\alpha^\dagger}{d\mu^2}(\mu) = -2\int\int \frac{\beta^\dagger(s,z)\left(1-\beta^\dagger(s,z)\right)}{\left(\mu+(1-\mu)\beta^\dagger(s,z)\right)^3}\left(h_H(z)\,q_H(s)-h_L(z)\,q_L(s)\right)dzds \tag{C.14}$$

$$= -2\int\int \frac{\left(1-\beta^\dagger(s,z)\right)^2}{\left(\mu+(1-\mu)\beta^\dagger(s,z)\right)^3} h_L(z)\,q_L(s)dzds < 0 \tag{C.15}$$

Thus, result 3 follows. Finally, result 4 is established in Lemma A.1 in Daley and Green (2014) (and also in Karlin, 1968, (Chapter 3, Proposition 5.1)). For the Daley and Green (2014) proof to go through, we need it to be the case that for any two private signals, z,z', $\frac{\xi_L(z)g_L(z)}{\xi_H(z)g_H(z)} \geq \frac{\xi_L(z')g_L(z')}{\xi_H(z')g_H(z')} \iff \tilde{\beta}^\dagger(\cdot,z)\geq\tilde{\beta}^\dagger(\cdot,z')$. Because we have already established that both are decreasing in z, the result follows. □

Proof of Theorem 3. Using Lemma C.3 in place of Lemma B.1, the proof follows the same steps used to establish Theorem 1. □

Proof of Proposition 7. Using Lemma C.3 in place of Lemma B.1, the proof follows the same steps used to establish Proposition 2. □