Idiosyncratic Income Risk and Aggregate Fluctuations*

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Abstract

We study the role of idiosyncratic income shocks for aggregate fluctuations within a simple heterogeneous household framework with no binding borrowing constraints. We show that the presence of idiosyncratic income shocks affects the economy's response to an aggregate shock in a way that can be captured by a consumption-weighted average of the changes in uncertainty generated by the shock. We apply this framework to two example economies —an endowment economy and a New Keynesian economy— and show that under plausible calibrations the impact of idiosyncratic income shocks on aggregate fluctuations is quantitatively small, since most of the changes in uncertainty are concentrated among poorer (low consumption) households.

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1 Introduction

Most efforts at modeling and understanding aggregate fluctuations over the past decades have relied on frameworks that assume an infinitely-lived representative household. While that assumption is obviously unrealistic, its widespread adoption reflects the view that both the finite lifetimes and the pervasive heterogeneity observed in the real world (in education, wealth, income, etc.) are not important factors behind aggregate fluctuations, and thus can be safely ignored when seeking to understand the nature and causes of that phenomenon, as well as its implications for policy.[1]

But the dominance of the representative household paradigm in macroeconomics has been challenged in recent years by a number of researchers who have argued that such an assumption, while convenient on tractability grounds, is less innocuous than one may think, even when the focus is to understand aggregate fluctuations and macroeconomic policies. The growing popularity of Heterogeneous Agent New Keynesian (HANK) models are a reflection of this emerging view. HANK models up to date have focused on household heterogeneity and its implications for aggregate consumption. They commonly assume the presence of idiosyncratic shocks to households' income, together with the existence of incomplete markets and borrowing constraints. Those features are combined with the kind of nominal rigidities and monetary non-neutralities that are the hallmark of New Keynesian (henceforth, NK) models. An important focus of that recent literature has been the role of heterogeneity in the transmission and effects of monetary policy.[2]

Rather than developing a richer HANK model that accounts for a broader set of facts or innovates over existing ones in some dimension, in the present paper we take a step back and use a basic model of individual and aggregate consumption to seek to understand the mechanisms through which household heterogeneity may have an effect on aggregate fluctuations. Our model features idiosyncratic income shocks as the only exogenous source of heterogeneity, in an environment where the only asset available is a riskless one-period bond and where borrowing constraints are not binding in equilibrium. We do so in order to isolate as much as possible the role of idiosyncratic income risk, thus abstracting from the frictions often featured in heterogenous agents

[1]For instance, Krussel and Smith (1998) study the role of income and wealth heterogeneity within a real business cycle model, and find that that the behavior of macroeconomic aggregates can be almost perfectly described using only the mean of the wealth distribution. See also Heathcote, Storesletten and Violante (2009), Guvenen (2011) and Krueger, Mitman and Perri (2016) for useful surveys of this earlier literature.

[2]See, among others, Auclert (2019), Kaplan, Moll and Violante (2018), Werning (2015), Acharya and Dogra (2020), Ravn and Sterk (2021) and McKay et al. (2016).

(HA) models, namely: (i) differential (il)liquidity of across types of assets (ii) binding borrowing constraints.

At the core of our analysis is an (approximate) Euler equation for (log) aggregate consumption which we derive by aggregating the corresponding Euler equations of individual households. That aggregation is possible given our assumption of non binding borrowing constraints.

We show that the resulting Euler equation in the HA economy includes a term that captures variations in individual consumption uncertainty, averaged across households. We refer to this term as the *uncertainty shifter*, which is defined as a *consumption-weighted average of individual consumption uncertainty*. In a representative agent model, where aggregate shocks are the only source of uncertainty, the uncertainty shifter is of second order relatively to variations in aggregate consumption and, hence, usually ignored. In contrast, in a HA economy, due to the presence of (potentially large) idiosyncratic income shocks in the background, and the associated precautionary savings motive, the uncertainty shifter plays a more important role.

A central result of our analysis is that the role of idiosyncratic uncertainty on aggregate fluctuations depends on how changes in uncertainty are distributed across households. As an example, consider an aggregate shock that leads to a widespread and persistent increase in *average* consumption uncertainty. That effect, by itself, would tend to reduce aggregate consumption, due to a precautionary savings motive. But the change in average uncertainty is not enough to predict the impact of the shock on aggregate consumption: how changes in individual uncertainty are distributed across households matters. Thus, to the extent that the increase in uncertainty in concentrated among poorer (low consumption) households, the impact on aggregate consumption will be smaller. This is what we refer to as the *comovement* effect.

After deriving and discussing the Euler equation for aggregate consumption we embed that equation into two fully fledged model economies. The first economy is an endowment economy where households are subject to idiosyncratic and aggregate endowment shocks. In that context, we study the mechanisms through which heterogeneity influences the response of the (real) interest rate to aggregate endowment shocks. The second economy is described by a baseline New Keynesian model with households subject to idiosyncratic productivity shocks. Our interest lies in studying the role of heterogeneity in shaping the response of aggregate output to aggregate shocks, such as monetary policy and technology shocks. The simplicity of the models and the fact that the presence of idiosyncratic risk is the only departure from their RA counterparts allows us to isolate better their role in shaping aggregate fluctuations.

From a quantitative viewpoint, we find that idiosyncratic risk has very small net effect on aggregate fluctuations in the two calibrated model economies that we analyze, mainly because of the offsetting role of the *comovement* effect mentioned earlier.

The rest of the paper is structured as follows. Section 2 reviews the related literature. Section 3 presents the model and the corresponding Euler Equation for aggregate consumption. Section 4 and 5 embed the previous framework into an endowment economy and a New Keynesian economy, respectively, highlighting the role of the distribution of consumption uncertainty, both from a qualitative and a quantitative perspective. Section 6 concludes.

2 Related Literature

This paper belongs to a growing literature that studies the role of heterogeneity in aggregate economic fluctuations. In that literature, two main features are typically responsible for the differences in the behavior of aggregate variables relative to a representative agent economy: (i) uninsurable idiosyncratic income risk and (ii) the presence of binding borrowing constraints. However, understanding which is the exact role played by each of these factors remains a largely open question. This is what we seek to do in this paper.[3]

In this respect, our paper contributes to the literature developing *tractable* frameworks to isolate the channels through which heterogeneity operates. Following the original formulation of Campbell and Mankiw (1989), some studies in that literature (see e.g., Galí, López-Salido and Vallès (2007), Bilbiie (2008), Debortoli and Galí (2018) and Broer et. al. (2020)) have focused on the role of binding constraints, by analyzing models with two types of agents (unconstrained and hand-to-mouth), but abstracting from the presence of idiosyncratic income risk within each type. Here we do the opposite, and focus instead on the role of idiosyncratic income risk, showing how the latter may give rise to amplification/dampening of aggregate shocks, even in the absence of binding borrowing constraints.

Another branch of this literature (see e.g. Werning (2015), McKay et al. (2016), Bilbiie (2021), Ravn and Sterk (2021)) has considered economies with idiosyncratic income risk, but under assumptions that imply a degenerate wealth distribution in equilibrium.[4] That

[3] An exercise in a similar spirit, but focusing of firms' heterogeneity and the role of collateral constraints can be found in Cao and Nie (2017).

[4] For instance, economies with zero-liquidity, or with no (or limited) wealth inequality among unconstrained households. See also Challe and Ragot (2011) and Challe, Matheron, Ragot, and Rubio-Ramirez (2017) for tractable models where the wealth distribution has finite support.

literature emphasizes the role played by the cyclicality of *income* inequality and liquidity for the transmission of aggregate shocks. Our work emphasizes instead the role of *consumption* uncertainty, and uncovers a novel channel associated with how changes in uncertainty are distributed across households.

In related work, Acharya and Dogra (2020) consider an heterogenous household economy with CARA preferences and no binding borrowing constraints. In that economy, all households face the same consumption uncertainty (the marginal propensity to consume out of their cash-on-hand is identical across households), and heterogeneity mainly operates as a result of the cyclicality of *income risk*. We instead consider a framework with more standard CRRA preferences, associated with a non-trivial relationship between individual consumption, income and wealth. In our setting, the cyclical behavior of *consumption* uncertainty in response to aggregate shocks plays a crucial role for the transmission of the latter, regardless of whether the volatility of the underlying idiosyncratic risk is constant or not.

Our paper is also related to several studies in the literature proposing some "sufficient statistics" to summarize the aggregate implications of household heterogeneity (see e.g. Auclert (2019), Auclert, Rognlie and Straub (2018) and Luetticke (2021) and the references therein). Those studies have emphasized the role of the cross-sectional distribution of variables like the marginal propensity to consume, income, portfolio, etc.. Our contribution is to show that the role of idiosyncratic risk can be summarized by the cross-sectional distribution of changes in consumption uncertainty.

From a quantitative viewpoint, our finding that the net impact of heterogeneity on aggregate fluctuations is small is similar to that obtained by several authors in the literature, following Krusell and Smith (1998). In that respect, our contribution lies in highlighting that such a result is likely to prevail in economies where the largest changes in uncertainty are concentrated among poorer (low consumption) households, as is the case in our sample economies.

3 An Euler Equation for Aggregate Consumption

Throughout we assume a continuum of households indexed by $j \in [0,1]$. Preferences are common to all households and given by $\mathbb{E}_0\{\sum_{t=0}^{\infty} \beta^t U(C_t(j))$ where $C_t(j)$ denotes household j's consumption in period t, $\beta \equiv \exp\{-\rho\}$ is the discount factor and $U(C) = (1-\sigma)^{-1}(C^{1-\sigma}-1)$, with $\sigma \geq 0$. Households can borrow and lend at a (gross) riskless real rate $R_t \equiv \exp\{r_t\}$, subject to a natural debt limit. The Euler equation describing optimal consumption for an individual household is given by

$$1 = \beta R_t \mathbb{E}_t\{(C_{t+1}(j)/C_t(j))^{-\sigma}\} \tag{1}$$

which is assumed to hold for $t = 0, 1, 2,$ and for all $j \in [0, 1]$. Our objective in this section is to derive an approximate Euler equation for (log) aggregate consumption. In our approximation we include all the terms of a Taylor expansion whose variations are of the same order –which we henceforth denote as $O(|\varepsilon|)$– as variations in aggregate consumption growth or the real interest rate.

As derived in Appendix A.1, up to a second-order approximation, eq. (1) can be written as follows:

$$\mathbb{E}_t \left\{ \frac{\Delta C_{t+1}(j)}{C_t(j)} \right\} \simeq \frac{1}{\sigma}\left(1 - \frac{1}{\beta R_t}\right) + \frac{\sigma+1}{2} v_t(j) \tag{2}$$

where $v_t(j) \equiv \mathbb{E}_t\{(\Delta C_{t+1}(j)/C_t(j))^2\}$ and where in addition we have that $v_t(j) \simeq var_t\{c_{t+1}(j)\}$, with $c_t(j) \equiv \log C_t(j)$.[5] We can thus interpret $v_t(j)$ as a measure of uncertainty regarding household j's one period ahead (log) consumption, whose effect on expected consumption growth captured by (2) reflects the so called precautionary savings motive, resulting from the convexity of marginal utility.[6] Due to the presence of (potentially large) idiosyncratic income shocks in the background, we allow variations in $v_t(j)$ to be of order $\mathcal{O}(|\varepsilon|)$. This is in contrast with the representative household case, for which $v_t \equiv \mathbb{E}_t\{(\Delta C_{t+1}/C_t)^2\} \sim \mathcal{O}(|\varepsilon|^2)$, which justifies the absence of v_t from the familiar "first-order" approximations of the consumption Euler equation found in the literature. Henceforth, the approximate equalities (represented by $\simeq$) should be understood as holding up to an error term of order $\mathcal{O}(|\varepsilon|^2)$.

Next we derive the main result of the present section. Let $C_t \equiv \int C_t(j)dj$ denote aggregate consumption. Aggregating eq. (2) accross households, we get that expected

[5] The use of approximations is only made to facilitate the economic interpretation of the mathematical terms related to heterogeneity. Appendix A.2 contains an analogous representation that does not rely on any approximation, and which is actually used in our quantitative exercises.

[6] Note that under our assumed utility function the coefficient of "relative prudence" –a measure of that convexity– is constant and given by $-(U'''/U'')C = \sigma + 1$. Appendix A.3 contains an analogous derivation for a general utility function, and also a special case for Constant-Absolute-Risk-Aversion (CARA) utility.

aggregate consumption growth is given by

$$\begin{aligned}\mathbb{E}_t\left\{\frac{\Delta C_{t+1}}{C_t}\right\} &= \mathbb{E}_t\left\{\int \frac{\Delta C_{t+1}(j)}{C_t}dj\right\} = \int \frac{C_t(j)}{C_t}\mathbb{E}_t\left\{\frac{\Delta C_{t+1}(j)}{C_t(j)}\right\} dj \\ &= \frac{1}{\sigma}\left(1-\frac{1}{\beta R_t}\right)+\frac{\sigma+1}{2}v_t \end{aligned} \tag{3}$$

where

$$v_t \equiv \int \frac{C_t(j)}{C_t}v_t(j)dj \tag{4}$$

is a *consumption-weighted average of individual consumption uncertainty*. The response of v_t to aggregate shocks will be shown to be key in understanding the role of idiosyncratic income uncertainty in aggregate fluctuations. Henceforth we refer to v_t as the *uncertainty shifter*.

Evaluating (3) at a stochastic steady state with constant consumption, we obtain the relation

$$0 = \frac{1}{\sigma}\left(1-\frac{1}{\beta R}\right)+\frac{\sigma+1}{2}v \tag{5}$$

where R and v denote the values of R_t and v_t at that steady state. Note that (5) reflects an inverse equilibrium relation between uncertainty and the real interest rate, working through precautionary savings, with $\beta R \leq 1$ and $\lim_{v\to 0}\beta R = 1$.

A first-order Taylor expansion of (3) around the stochastic steady state yields a linear Euler equation for (log) aggregate consumption $c_t \equiv \log C_t$, the object we were after:

$$c_t = \mathbb{E}_t\{c_{t+1}\} - \frac{1}{\sigma}\widehat{r}_t - \frac{\sigma+1}{2}\widehat{v}_t \tag{6}$$

where $\widehat{r}_t \equiv \frac{1}{\beta R}(\frac{R_t-R}{R})$ and $\widehat{v}_t \equiv v_t - v$. Thus, we see how the presence of idiosyncratic income shocks calls for an additional term in an otherwise familiar log-linear Euler equation for aggregate consumption. The additional term, $-\frac{\sigma+1}{2}\widehat{v}_t$, will generally vary endogenously, thus amplifying or dampening the response of consumption to aggregate shocks, conditional on a given path for the real interest rate.[7]

In order to further understand how the uncertainty shifter evolves over time we can decompose v_t as defined in eq. (4) it as follows:

$$v_t = \overline{v}_t + cov_j\left\{\frac{C_t(j)}{C_t}, v_t(j)\right\} \tag{7}$$

[7] Or, alternatively, it will amplify or dampen the response of the real interest rate to an aggregate shock, conditional on a given path for aggregate consumption, as in the endowment economy considered below.

where $\bar{v}_t \equiv \int v_t(j)dj$. The two terms on the right hand side of (**??**) capture the *average* and *comovement* effects of an aggregate shock on the uncertainty shifter v_t.

It then follows, as shown formally in Appendix A.4, that the dynamic response of the uncertainty shifter to a generic aggregate shock ε_t, denoted by $\frac{dv_{t+k}}{d\varepsilon_t}$ for $k = 0, 1, 2, ...$, can be written as

$$\frac{dv_{t+k}}{d\varepsilon_t} \simeq \frac{d\overline{v}_{t+k}}{d\varepsilon_t} + cov_j \left\{ c_{t+k}(j), \frac{dv_{t+k}(j)}{d\varepsilon_t} \right\}. \tag{8}$$

A number of results implied by the previous expressions are worth mentioning. First, note that the presence of idiosyncratic uncertainty will have an impact on aggregate consumption fluctuations only if aggregate shocks have an effect on *individual* consumption uncertainty, i.e. only if $\frac{dv_t(j)}{d\varepsilon_t}$ for a positive mass of households. Otherwise, the uncertainty shifter would not change in response to those shocks, and both terms on the right hand side of (8) would be equal to zero.

Secondly, the size of the response of the uncertainty shifter depends crucially on the cross-sectional covariance between consumption and the response of individual uncertainty. Thus, for any given increase in average uncertainty in response to an aggregate shock, the change in the uncertainty shifter (and hence the impact on aggregate consumption) will be larger the higher is the cross-sectional covariance between the level of consumption and the change in uncertainty. The intuition for the previous result is straightforward: a given change in uncertainty $\frac{\partial v_{t+k}(j)}{\partial \varepsilon_t}$ has an identical *percent* impact on the consumption of all households, independently of their initial level of wealth, consumption, etc.; however, any given percent change in the consumption of an individual household has a larger impact on *aggregate* consumption (both in absolute and relative terms) the larger is the household's initial level of consumption. Thus, how any given average increase in uncertainty is distributed across households and, in particular, how it comoves with their level of consumption is an important factor in determining the variation in the uncertainty shifter. In the limiting case, if uncertainty changes only for a subset of households with consumption close to zero, the impact on aggregate consumption would be negligible.

In the example economies considered below the change in uncertainty in response to an aggregate shock, $\frac{dv_{t+k}(j)}{d\varepsilon_t}$, tends to be larger, in absolute value, for low consumption households. As a result the comovement effect tends to dampen the impact of any change in average uncertainty, hence limiting the influence of heterogeneity on aggregate fluctuations.

3.1 Understanding Variations in Consumption Uncertainty

The discussion above has made clear the importance of changes in consumption uncertainty, current and anticipated, as well as the cross-sectional correlation of those changes with the level of consumption, in shaping aggregate fluctuations in economies where households face idiosyncratic income shocks. In the present section we try to dig further in order to shed some light on the sources of those uncertainty changes.

We assume the existence of a consumption function for household j, given by:

$$c_t(j) = \mathcal{C}(s_t(j), S_t) \tag{9}$$

where $s_t(j)$ is a vector of household-specific state variables and S_t is a vector of aggregate state variables. The state variables contain all the information available at time t that is relevant to determine $c_t(j)$ (including the distribution of household-specific variables). The existence and properties of a consumption function like (9) can be established under standard assumptions.

Let $\zeta_t(j)$ and ε_t be the vectors containing respectively *i.i.d.* idiosyncratic and aggregate shocks (i.e. mutually orthogonal innovations in the individual and aggregate *exogenous* driving variables). We can write the innovation in household j's consumption in period t as follows:

$$\begin{aligned} \xi_t(j) &\equiv c_t(j) - \mathbb{E}_{t-1}\{c_t(j)\} \\ &= f^j_{t-1}(\zeta_t(j), \varepsilon_t) \end{aligned} \tag{10}$$

where $f^j_{t-1}(\cdot)$ is a function satisfying $f^j_{t-1}(0,0) = 0$. In what follows, and in order to keep the algebra simple, we assume $\zeta_t(j)$ and ε_t are scalars.

Under our assumptions, and using (10), we can approximate individual uncertainty $v_t(j) \simeq \mathbb{E}_t\{\xi_{t+1}(j)^2\}$ in period t as

$$v_t(j) \simeq \psi_t(j)^2\sigma^2_\zeta + \varphi_t(j)^2\sigma^2_\varepsilon$$

where $\psi_t(j) \equiv \partial f^j_t(0,0)/\partial_{t+1}\zeta(j)$, and $\varphi_t(j) \equiv \partial f^j_t(0,0)/\partial\varepsilon_{t+1}$ are the (local) elasticities of individual consumption with respect to idiosyncratic and aggregate shocks, while $\sigma^2_\zeta \equiv \mathbb{E}\{\zeta_t(j)^2\}$ for all $j \in [0,1]$ and $\sigma^2_\varepsilon \equiv \mathbb{E}\{\varepsilon^2_t\}$ are, respectively, the variances of those shocks. Under our assumptions, variations in individual consumption uncertainty driven by aggregate shocks would be of second order relative to aggregate variables, i.e.

$\varphi_t(j)^2\sigma_\varepsilon^2 \sim \mathcal{O}(|\varepsilon|^2)$. Thus, for our purposes we can use the approximation

$$v_t(j) \simeq \psi_t(j)^2\sigma_\zeta^2$$

which in turn implies the following expression for v_t:

$$v_t \simeq \sigma_\zeta^2 \int \frac{C_t(j)}{C_t}\psi_t(j)^2 dj \tag{11}$$

An implication of eq. (11) is that the uncertainty shifter is proportional to the consumption-weighted average (across households) of the square elasticities of consumption with respect to the idiosyncratic shock. Letting $\Theta_t(j) \equiv \psi_t(j)^2$, which we refer to as the square elasticity, for short, we can write the dynamic response of the uncertainty shifter as follows:

$$\begin{aligned}
\frac{dv_{t+k}}{d\varepsilon_t} &\simeq \sigma_\zeta^2 \int \frac{C_{t+k}(j)}{C_{t+k}}\frac{d\Theta_{t+k}(j)}{d\varepsilon_t}dj \\
&\simeq \sigma_\zeta^2 \left[\int \frac{d\Theta_{t+k}(j)}{d\varepsilon_t}dj + \int (c_{t+k}(j) - c_{t+k})\frac{d\Theta_{t+k}(j)}{d\varepsilon_t}dj\right] \\
&= \sigma_\zeta^2 \left[\frac{d\overline{\Theta}_{t+k}}{d\varepsilon_t} + cov_j\left\{c_{t+k}(j), \frac{d\Theta_{t+k}(j)}{d\varepsilon_t}\right\}\right]
\end{aligned}$$

where $\overline{\Theta}_t \equiv \int \Theta_t(j)dj$ is the mean square elasticity. As discussed in the context of the quantitative analysis of two example economies below, the overall impact of the change in the mean square elasticity in response to an aggregate shock tends to be offset by the effect of its covariance with consumption, since the mean square elasticity is more sensitive to shocks for low consumption households. As a result the overall impact of an aggregate shock on the uncertainty shifter (and thus on aggregate endogenous variables) is often strongly muted.

Another implication of eq. (11) is that the uncertainty shifter would generally fluctuate in response to aggregate shocks, regardless of the properties of the variance of the underlying idiosyncratic risk (σ_ζ^2). Throughout our analysis, we maintain the assumption that the variance of idiosyncratic income shocks (σ_ζ^2) is constant over time —i.e. the idiosyncratic *income* risk is acyclical. In principle, cyclical income risk could be another channel through which heterogeneity affects aggregate consumption. For instance, Acharya and Dogra (2020) consider and heterogeneous agent economy with CARA preferences, where the cyclicality of income risk is the main channel through which heterogeneity affect aggregate variables. Instead, as shown formally in Appendix A.3 the comovement effect emphasized here is absent in their economy, where the sensitivity of consumption to idiosyncratic shocks is the same across households —i.e. due to CARA preferences all households have the same marginal propensity to consume.[8]

4 Idiosyncratic Risk and Aggregate Fluctuations in an Endowment Economy

Consider an endowment economy populated by a continuum of households, indexed by $j \in [0,1]$, with identical preferences given by $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t(j))$, with $U(C) \equiv \frac{C^{1-\sigma}-1}{1-\sigma}$ where $C_t(j)$ is period t consumption of the single good by household j. The household's period budget constraint is given by:

$$C_t(j) + B_t(j) \leq B_{t-1}(j) R_{t-1} + Y_t(j)$$

$$Y_t(j) = Y_t \exp\{\zeta_t(j)\}$$

for $t = 0, 1, 2...$, where $B_t(j)$ represents holdings of one-period bonds, which yield a gross riskless real return R_t. The household endowment, $Y_t(j)$, has two components (in logs): an aggregate component $y_t \equiv \log Y_t$, which is common to all households, and follows an $AR(1)$ process with autocorrelation $\rho_y \in [0,1)$; and an idiosyncratic component $\zeta_t(j) \in [\zeta_1, .., \zeta_K]$, which follows a stationary K-state Markov process, independent across households and satisfying $\mathbb{E}\{\exp\{\zeta_t(j)\}\} = 1$.[9] Note that by setting $\zeta_t(j) = 0$ for all $j \in [0,1]$ and all t, together with a uniform initial condition $B_{-1}(j) = 0$ for all $j \in [0,1]$, the previous model collapses to one with a representative household.

In equilibrium, the bonds and goods markets must clear, which implies $\int_0^1 B_t(j)dj = 0$ and $\int_0^1 C_t(j)dj = Y_t$. We can use the Euler equation for (log) aggregate consumption (6) to derive an expression for the equilibrium real interest rate:

$$\widehat{r}_t = -\sigma(1-\rho_y) y_t - \frac{\sigma+1}{2} \widehat{v}_t \tag{12}$$

The first term on the right hand side of (12) is the equilibrium real rate in the corresponding representative agent economy, and captures the well known effect on the interest rate of the desire to smooth consumption in the face of short-run output fluctu-

[8] See also Bayer et. al. (2019) and Ravn and Sterk (2020), among others, for examples of heterogenous household economies with cyclical idiosyncratic risk.

[9] The previous normalization guarantees that $Y_t = \int Y_t(j)dj$, for all t.

ations. The impact of heterogeneity on the interest rate is captured by the second term, which moves in proportion to the uncertainty shifter $\widehat{v}_t$. Thus, an increase in the latter variable tends to increase the demand for precautionary savings, leading to a reduction in the equilibrium interest rate.

In summary, equation (12) implies that the impact of idiosyncratic risk on the response of the real interest rate to an aggregate endowment shock is determined by the response of the uncertainty shifter. In particular, the sign and size of that response determines the extent to which the effect of the aggregate endowment shock on the interest rate is amplified or dampened. Next we turn to a quantitative assessment of these effects in a calibrated version of the above economy.

4.1 Calibration and Solution Method

The baseline calibration of our endowment economy is summarized in Table 1. Each period is assumed to correspond to a quarter. We set the coefficient of risk aversion $\sigma = 1$, which corresponds to log utility. We set the discount factor $\beta = 0.9937$, which implies a real risk-free rate of 2 percent (in annual terms) in the steady state.

We calibrate the parameters of the K-state Markov process for idiosyncratic income using the Rouwenhorst method in order to match the volatility and persistence of an AR(1) process $\zeta_t(j) = \rho_\zeta \zeta_{t-1}(j) + \zeta_t(j)$, where $\zeta_t(j) \sim N(0, \sigma_\zeta \sqrt{1-\rho_\zeta^2})$, with $\rho_\zeta = 0.966$ and $\sigma_\zeta = 0.5$ as in Auclert et. al. (2021).[10] Finally, we set the autoregressive coefficient in the AR(1) process for the (log) aggregate endowment to $\rho_y = 0.9$.

Regarding the numerical solution method, we build a grid for individual assets of 500 points, equally distanced (in logs) between a lower bound (which corresponds to the natural debt limit, as discussed below) and an upper bound set to 300 times quarterly income. We impose a borrowing constraint of the form

$$B_t(j) \geq \underline{B} \tag{13}$$

for all t. We set $\underline{B} = -Y \exp\{\zeta_1\}/r$, which constitutes the "natural debt limit," given aggregate output and interest rate at their the steady state values (Y, r). The desire to avoid zero consumption (given that $\lim_{c\to 0} U_c = +\infty$) guarantees that $B_t(j) > \underline{B}$ for all t when aggregate output and the interest rate are at their steady state levels. Given

[10] As a robustness check, Appendix B considers an alternative income process which combines a transitory and persistent component, and is a discrete-time (quarterly) version of the continuous-time process in Kaplan, Moll and Violante (2018).

sufficiently small fluctuations in the previous two variables, the fraction of constrained households in equilibrium can be made arbitrarily close to zero.[11]

For given values of the real interest rate and the aggregate endowment, we solve for the households' policy functions using the endogenous gridpoints method described in Carroll (2006), which are then used to calculate the implied equilibrium asset distribution. We solve for the steady state iterating on the value of the discount factor β so that the stationary assets distribution implied by the households' choices satisfies the market clearing condition $\int B_t(j)dj = 0$.

For the transition dynamics, we adopt the Sequence-Space Jacobian approach described in Auclert et. al. (2021). This amounts to finding the first-order approximation of the equilibrium responses to arbitrary sequences of anticipated shocks to the aggregate endowment (i.e. under perfect foresight) over a finite horizon (set to $T = 300$ quarters). Due to certainty equivalence, the resulting dynamics are equivalent to the ones that would be obtained solving the linearized rational expectations model, e.g. as in Reiter (2009) and Ahn et. al. (2018).[12] Also, by construction, the approximate responses to positive and negative aggregate shocks are fully symmetric, and proportional to the size of the shocks.

Most importantly, the assumption of perfect-foresight (or certainty equivalence) with respect to aggregate shocks implies that idiosyncratic shocks are the only source of individual (and aggregate) uncertainty. As shown above, such uncertainty is a key determinant of the impact of heterogeneity on aggregate consumption. In all our numerical exercises, and in order to accurately capture the quantitative role of idiosyncratic risk, we do not rely on the approximation described in Section 1, but instead on the the exact representation contained in Appendix A.2.

4.2 Findings

We focus our discussion on the dynamic response of the real interest rate to a positive aggregate endowment shock. Figure 1 shows the responses of the real interest rate and (log) aggregate output to a 1 percent positive shock in the latter variable. The response of the real interest rate (expressed in annual terms) is plotted on the left panel for both our baseline model with heterogeneity (red line with circles) and for the corresponding representative agent model (blue line with crosses). The real rate declines persistently in

[11]In our simulations, the fraction of constrained consumer is negligible (below 0.1 percent) both in steady state, and in response to aggregate shocks.

[12]See also Boppart, Krusell and Mitman (2018) for a related perfect-foresight sequence-based approach.

both models. Finally, the same Figure displays (green dashed line) the real rate response to the same shock under the assumption that the response of the uncertainty shifter corresponds to that of average uncertainty, i.e. $\frac{\partial v_{t+k}}{\partial \varepsilon_t} = \frac{\partial \overline{v}_{t+k}}{\partial \varepsilon_t}$, thus implicitly turning off the comovement effect by setting $cov_j \left\{ c_{t+k}(j), \frac{\partial v_{t+k}(j)}{\partial \varepsilon_t} \right\} = 0$.

The overall effect of idiosyncratic risk on the response of the real interest rate is positive, i.e. it dampens the decline in the interest rate relatively to a representative agent model, but quantitatively small (less than 5 basis points at all horizons). That positive impact is a consequence of a decline in the uncertainty shifter. Note, however, that there are two distinct forces operating in opposite directions. On the one hand, the increase in aggregate output leads to a reduction in average uncertainty $\overline{v}_t$, which lowers the demand for savings and tends to increase the interest rate. This is captured by the green dashed line, which lies considerably higher than the response implied by the representative agent model. On the other hand, the gap between the green dashed line and the red circled line captures the comovement effect, which nearly fully offsets the effect of average uncertainty, making the overall impact on the uncertainty shifter (and, hence, of idiosyncratic risk) very small.[13] This phenomenon is clearly visible in Figure 2, which displays the (small) response of the uncertainty shifter, as well as of its two components related to average and comovement effects going in opposite directions.

As mentioned in section 3.1, the behavior of average consumption uncertainty is related to the distribution of the change in the (square) elasticity of consumption with respect to the idiosyncratic shock. This is illustrated in Figure 3, which shows the steady state relationship between (log) consumption and the corresponding (square) elasticity of consumption $\psi_t^2(j)$.[14] As it can be seen, there is a negative relationship between these two variables, since households with higher consumption have more buffer to absorb unexpected changes in income, and thus their consumption is less sensitive to idiosyncratic shocks. Thus, an increase in aggregate income, which in and of itself causes an increase in consumption for most households, leads to a decline in the average elasticity of consumption. At the same time, the figure shows that the relationship between consumption and the (square) elasticity of consumption is convex. Intuitively, the elasticity of consumption varies substantially as households get closer to their natural debt limit, but roughly constant (and small) for households with high income and wealth, who behave

[13] This result is consistent with earlier findings in the asset pricing literature, see e.g. Heaton and Lucas (1996) and Marcet and Singleton (1999), showing that household heterogeneity and market incompleteness have small effects on the volatility of returns.

[14] More precisely, the figure displays the range of (square) elasticities $\psi_t^2(j)$ as well as the corresponding median for each value of consumption. The existence of a range is due to the fact that, a given level consumption could be associated with different combinations of the two individual state variables, namely wealth and idiosyncratic shocks, giving rise to different elasticities.

almost as permanent-income consumers. This explains why an increase in aggregate income generates a larger uncertainty reduction among low consumption households, thus accounting for the offsetting comovement effect on the uncertainty shifter.

Figure 4 show the results for a calibration with higher coefficient of risk aversion ($\sigma = 3$). In this case, as it can be seen in the left panel, the overall effects of idiosyncratic risk remain relatively small, even though a bit larger than in the baseline calibration. This is mainly because under this calibration the (square) elasticity of consumption is less convex (see Figure 5), and thus the offsetting comovement channel is weaker.

5 Idiosyncratic Risk and Aggregate Fluctuations in a New Keynesian Economy

Next we analyze the role of idiosyncratic risk and aggregate fluctuations in a version of the New Keynesian model. The economy is populated by a continuum of households, indexed by $j \in [0,1]$, with identical preferences given by $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t(j), \mathcal{N}_t(j))$. The term $C_t(j) \equiv \left(\int_0^1 C_t(i,j)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$ is a consumption aggregator. $C_t(i,j)$ denoting the quantity of good i consumed by household j. $\mathcal{N}_t(j)$ denotes work hours. We assume $U(C, \mathcal{N}) = \left(\frac{C^{1-\sigma}-1}{1-\sigma} - \frac{\mathcal{N}^{1+\varphi}}{1+\varphi}\right)$.

Optimal allocation of expenditures requires that $C_t(i,j) = (P_t(i)/P_t)^{-\epsilon} C_t(j)$, where $P_t(i)$ is the price of good i and $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ is the aggregate price index. This in turn implies that total expenditures are given by $\int_0^1 P_t(i) C_t(i,j) di = P_t C_t(j)$. The household's period budget constraint can thus be written as follows:

$$C_t(j) + B_t(j) \leq B_{t-1}(j) R_t + W_t \mathcal{N}_t(j) \exp\{\zeta_t(j)\} + D_t(j)$$

where $B_t(j)$ denotes holdings of real bonds (fully indexed to inflation) yielding a riskless real return R_t, W_t is the real wage (per efficiency unit of labor), $D_t(j)$ are real dividends, and $\zeta_t(j)$ is an idiosyncratic productivity shifter which follows a stationary K-state Markov process identical to the one assumed in the previous section, satisfying $\mathbb{E}\{\exp\{\zeta_t(j)\}\} = 1$.[15] Firms' shares are assumed to be nontradable and to be held in equal amounts by all households. As a result dividends are distributed uniformly to all households, i.e. $D_t(j) = D_t$. As in the endowment economy analyzed in the previous

[15]The assumption of a riskless real bond implies that we are abstracting from the redistributive effects due to inflation (Fisher's debt deflation channel). Changes in the real interest rate, however, still have differential income effects on households, depending on their individual net wealth positions.

section we assume that the borrowing constraint is not binding in equilibrium, so that an Euler equation like (1) holds for all households at all times.

The supply side of the economy is kept as simple as possible, and such that it remains insulated from the effects of idiosyncratic risk. This allows us to focus on the impact of the latter on aggregate demand (which coincides with aggregate consumption in our simple model), in the spirit of Werning (2015).

On the production side, we assume a continuum of firms, indexed by $i \in [0,1]$. Each firm produces a differentiated good with the linear technology

$$Y_t(i) = A_t N_t(i) \tag{14}$$

where $N_t(i)$ is the quantity of labor (expressed in efficiency units) hired by firm i, and $A_t \equiv \exp\{a_t\}$ is an exogenous technology parameter common to all firms. Each firm sets the price of its good optimally each period, subject to a quadratic adjustment cost $\frac{\xi}{2} P_t Y_t \left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2$ where $\xi > 0$, and a sequence of demand constraints $Y_t(i) = (P_t(i)/P_t)^{-\epsilon} Y_t$, where Y_t denotes aggregate output. Profit maximization, combined with the symmetric equilibrium conditions $P_t(i) = P_t$ and $Y_t(i) = Y_t$ for all $i \in [0,1]$, implies:

$$\Pi_t(\Pi_t - 1) = \mathbb{E}_t\left\{\Lambda_{t,t+1}\left(\frac{Y_{t+1}}{Y_t}\right)\Pi_{t+1}(\Pi_{t+1} - 1)\right\} + \frac{\epsilon}{\xi}\left(\frac{W_t(1-\tau)}{A_t} - \frac{1}{\mathcal{M}_p}\right) \tag{15}$$

where $\Pi_t \equiv P_t/P_{t-1}$ is (gross) price inflation rate and $\mathcal{M}_p \equiv \epsilon/(\epsilon-1) > 1$ is the desired (or flexible price) price markup. The term τ denotes a proportional labor subsidy, which is set to eliminate all the steady-state distortions due to monopolistic power in the goods and labor markets, and is financed with lump-sum taxes on firms.[16] Aggregate profits are then given by $D_t = Y_t \Delta^p(\Pi_t) - W_t N_t$ where $\Delta^p(\Pi_t) \equiv 1 - (\xi/2)(\Pi_t - 1)^2$.

We assume a wage schedule

$$W_t = \mathcal{M}_w C_t^{\sigma} N_t^{\varphi} \tag{16}$$

where $C_t \equiv \int_0^1 C_t(j)dj$ and $N_t \equiv \int_0^1 N_t(i)di$ denote aggregate consumption and employment, respectively, and where $\mathcal{M}_w > 1$ is a constant (gross) average wage markup.

Combining eqs. (16)-(15), and taking a first-order approximation around the zero-inflation steady state gives the well-known New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa \tilde{y}_t \tag{17}$$

[16] Formally, the subsidy is chosen such that $\mathcal{M}^p \mathcal{M}^w (1-\tau) = 1$.

where $\kappa \equiv (\sigma + \phi)(\epsilon - 1)/\xi$, and where $\tilde{y}_t \equiv y_t - y_t^n$ denotes the output gap, which is the difference between (log) output y_t and its natural (i.e. flexible price) counterpart $y_t^n \equiv a_t (1+\varphi) / (\sigma + \varphi)$. Note that the latter is independent from monetary policy and, importantly, is unaffected by idiosyncratic risk.

Regarding monetary policy, we assume the central bank controls directly the real interest rate, i.e. that $\widehat{r}_t$ follows an exogenous $AR(1)$ process $\widehat{r}_t = \rho_r \widehat{r}_{t-1} + \varepsilon_{m,t}$, where $\mathbb{E}_t\{\varepsilon_{m,t+1}\} = 0$. This specification allows us to isolate the (direct) effects of heterogeneity on aggregate demand, abstracting from the potential (indirect) effects due to the differential behavior of aggregate variables, which in turn may lead to a different endogenous monetary policy response. In Appendix C, we also consider a case where the central bank follows a Taylor-type rule for the real interest rate, and show that our main conclusions remain unaltered.

In the symmetric equilibrium $Y_t(i) = Y_t$ and $C_t(i) = C_t$ for all $i \in [0,1]$. Thus, market clearing in the goods market requires

$$C_t = Y_t \Delta^p(\Pi_t) \tag{18}$$

Market clearing in the bonds markets implies that $\int_0^1 B_t(j)dj = 0$ for all t. Aggregate employment is given by $N_t = Y_t/A_t$. We assume firms distribute their demand for work hours uniformly across households, i.e. $\mathcal{N}_t(j) = N_t$ for all $j \in [0,1]$.[17] Clearing of the labor market $N_t = \int_0^1 \mathcal{N}(j) \exp\{\zeta_t(j)\}dj$ is then guaranteed by the fact that $\int_0^1 \exp\{\zeta_t(j)\}dj = 1$.

Up to a first-order approximation and in a neighborhood of the zero inflation steady state (18) can be written as

$$c_t = y_t$$

Combining the previous condition with the Euler equation for aggregate consumption derived in Section 3 we obtain a version of the dynamic IS equation:

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\sigma}\widehat{r}_t - \frac{\sigma+1}{2}\widehat{v}_t$$

Iterating forward the previous condition and imposing $\lim_{T\to\infty} \mathbb{E}_t\{y_{t+T}\} = y$ and

[17] Thus, we implicitly assume $W_t \exp\{\zeta_t(j)\} \geq C_t(j)^\sigma N_t^\varphi$ for all $j \in [0,1]$ and all t, so that all households are willing to supply the work hours demanded by firms at a wage W_t (per efficiency unit).

$\lim_{T\to\infty} \mathbb{E}_t\{x_{t+T}\} = x$ we obtain the following expression for (log) aggregate output

$$\widehat{y}_t = \underbrace{-\frac{1}{\sigma}\sum_{k=0}^{\infty}\mathbb{E}_t\{\widehat{r}_{t+k}}_{\text{RA model}} \ \underbrace{-\frac{\sigma+1}{2}\sum_{k=0}^{\infty}\mathbb{E}_t\{\widehat{v}_{t+k}\}}_{\text{HA component}} \tag{19}$$

The first term in the previous expression corresponds to equilibrium output in the RA version of the New Keynesian model. The second term reflects instead the impact of idiosyncratic risk on equilibrium output, which is decreasing in current and anticipated uncertainty shifter —through its effects on precautionary savings. As discussed in section 3, the response of the uncertainty shifter to an aggregate shock is given by a consumption-weighted average of the responses of individual consumption uncertainty. Formally, letting $\widehat{y}_t^H \equiv -\frac{\sigma+1}{2}\sum_{k=0}^{\infty}\mathbb{E}_t\{\widehat{v}_{t+k}\}$ denote the component of aggregate output fluctuations associated with heterogeneity, we can write:

$$\begin{aligned}
\frac{dy_{t+k}^H}{d\varepsilon_t} &= -\frac{\sigma+1}{2}\sum_{k=0}^{\infty}\frac{dv_{t+k}}{d\varepsilon_t} \\
&\simeq -\frac{\sigma+1}{2}\sum_{k=0}^{\infty}\int \frac{C_{t+k}(j)}{C_{t+k}}\frac{dv_{t+k}(j)}{d\varepsilon_t}dj \\
&\simeq -\frac{\sigma+1}{2}\sum_{k=0}^{\infty}\left[\frac{d\overline{v}_{t+k}}{d\varepsilon_t} + cov_j\left\{c_{t+k}(j), \frac{dv_{t+k}(j)}{d\varepsilon_t}\right\}\right].
\end{aligned} \tag{20}$$

In the numerical simulations shown below for a calibrated version of our model, the dynamic response of uncertainty to an aggregate shock is larger for low consumption households. As a result the impact of the shock on average uncertainty is muted by the comovement effect, leading to a small aggregate impact.

5.1 Calibration

We set $\beta = 0.9937$ and $\sigma = 1$ as in the endowment economy analyzed above, and consider the same calibration for the idiosyncratic shock $\zeta_t(j)$. In addition, we set the (inverse) Frisch elasticity of substitution to unity ($\varphi = 1$). Also, we set the elasticity of substitution among good varieties $\epsilon = 11$, which implies an average price markup of about 10 percent and the price adjustment cost parameter ξ so that the resulting slope of the Phillips Curve is $\kappa = 0.10$, in line with available estimates. Regarding the persistence of aggregate shocks, we assume that $\rho_a = 0.9$ and $\rho_r = 0.5$. We adopt the same numerical solution method described in Section 4.1.

5.2 Findings

We now analyze how idiosyncratic risk affects the response of our New Keynesian economy to monetary policy and technology shocks. For concreteness we focus on the response of aggregate output, and assume that the monetary policy rule takes the form of an exogenous process for the real rate, as introduced above. In Appendix C we show results are similar when considering a standard Taylor rule.

Figure 6 shows the response of aggregate output to a 25 basis point expansionary monetary shock, which leads to a 100 basis point reduction in the (annualized) real interest rate. The figure displays that response for three economies: our baseline model with heterogeneity (red line with circles), and economy with heterogeneity but no comovement effects (green dashed line) and a representative agent economy (blue line with crosses). The companion Figure 7 shows the response to the real rate shock of the uncertainty shifter as well as its two components (average and comovement).

Note that the presence of idiosyncratic risk tends to amplify the output effects of the monetary policy shock. The effects are stronger on impact, and more persistent. However, from a quantitative viewpoint, the magnitude of this amplification seems very small —less that 0.05 percentage points at all horizons. That small effect arises despite the quantitatively large change in the average uncertainty component, as captured by the green dashed line. As confirmed by Figure 7 the reason for the difference between the latter effect and the total effect of uncertainty lies in the offsetting impact of the comovement effect: the decrease in uncertainty is concentrated on low consumption households, which tends to mute the overall impact on aggregate consumption and output.

Finally, Figures 8 and 9 show the dynamic responses to a positive technology shock. Again the difference between the models with and without heterogeneity in terms of the responses of output and inflation is quantitatively negligible, due to the offsetting comovement effect.[18]

6 Concluding Remarks

The objective of the present paper was to study the role of idiosyncratic income risk for aggregate fluctuations within a simple heterogeneous household framework with no binding borrowing constraints. We derive analytically an Euler equation for (log) aggre-

[18] Note that output remains unchanged in response to the technology shock. This is due to the constancy of the real rate implied by our baseline monetary policy rule. See Appendix C for corresponding results under a standard Taylor rule.

gate consumption, which helps us shed some light on the differential behavior of such an economy relative to its representative agent counterpart. In particular, we show that those differences are related to how changes in consumption uncertainty are distributed among households, as captured by a consumption-weighted average of changes in uncertainty.

Our findings raise several issues that are relevant to current efforts to introduce heterogeneity in models of aggregate fluctuations.

Firstly, an implication of our findings is that idiosyncratic risk may have to be combined with other ingredients in order for household heterogeneity to have a significant impact on aggregate fluctuations. The assumption of financial frictions in the form of binding borrowing constraints is a prominent candidate to play that role. From that viewpoint, our findings can be interpreted as providing a rationale for the widespread adoption of that assumption in the recent literature, in additional to its arguable realism. On the other hand, our findings may also be read as suggesting that one may want to ignore altogether idiosyncratic risk when introducing heterogeneity in macro models, focusing instead on the presence of a binding borrowing constraint. This is the approach adopted in models with a constant fraction of hand-to-mouth households (as exemplified by the TANK models of Galí et al. (2007), Bilbiie (2008, 2021), and Broer et al. (2019)). In a companion paper (Debortoli and Galí (2018)), we analyze the extent to which the predictions of richer heterogenous agent models can be approximated by two-agent models that abstract from idiosyncratic risk.

Secondly, an implication of our findings is that idiosyncratic risk is likely to have a small impact on aggregate fluctuations in economies where fluctuations in consumption uncertainty are concentrated among poorer (low consumption) households, as is the case in the quantitative example economies studied above –an endowment economy and a New Keynesian economy. Conversely, such idiosyncratic risk may be more relevant in economies where rich (i.e. high consumption) households experience large fluctuations in consumption uncertainty, as it is likely to be the case in recent models in which a fraction of wealthy households behave in a hand-to-mouth fashion, possibly as a result of the low liquidity of their wealth (e.g. Kaplan et al. (2018)). Thus, and even though changes in uncertainty resulting from aggregate shocks will not (directly) impinge on the consumption of currently constrained households (wealthy or not), it will still be the case that those changes in uncertainty will be significant for households "close to the constraint," which in the context of those models also include relatively wealthy (high consumption) households, with a consequent larger impact on aggregate consumption.

Thirdly, it should be clear that how aggregate shocks affect uncertainty for different types of households is ultimately an empirical question, and one which we plan to address in future work using micro data.

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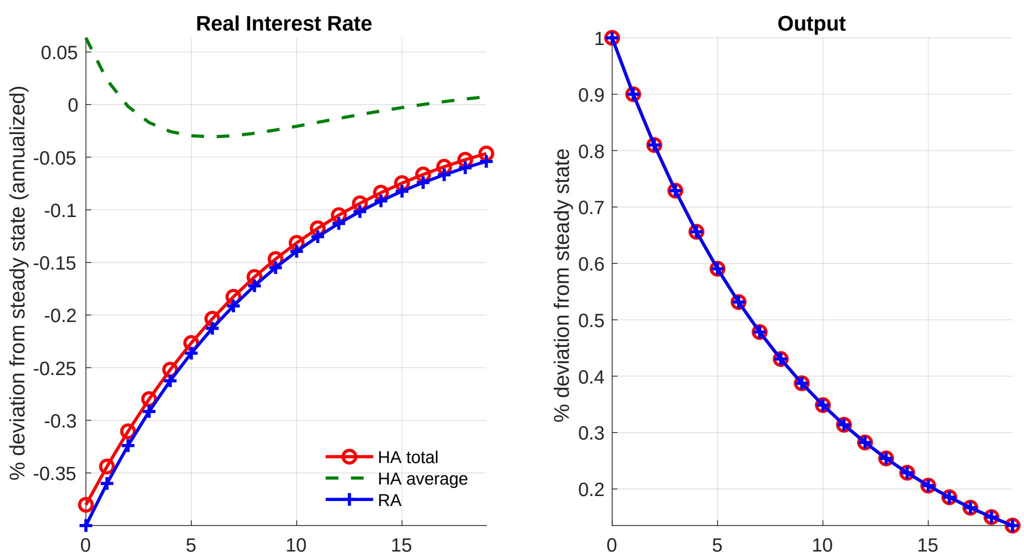
Tables and Figures

Table 1: Calibration of the Endowment Economy

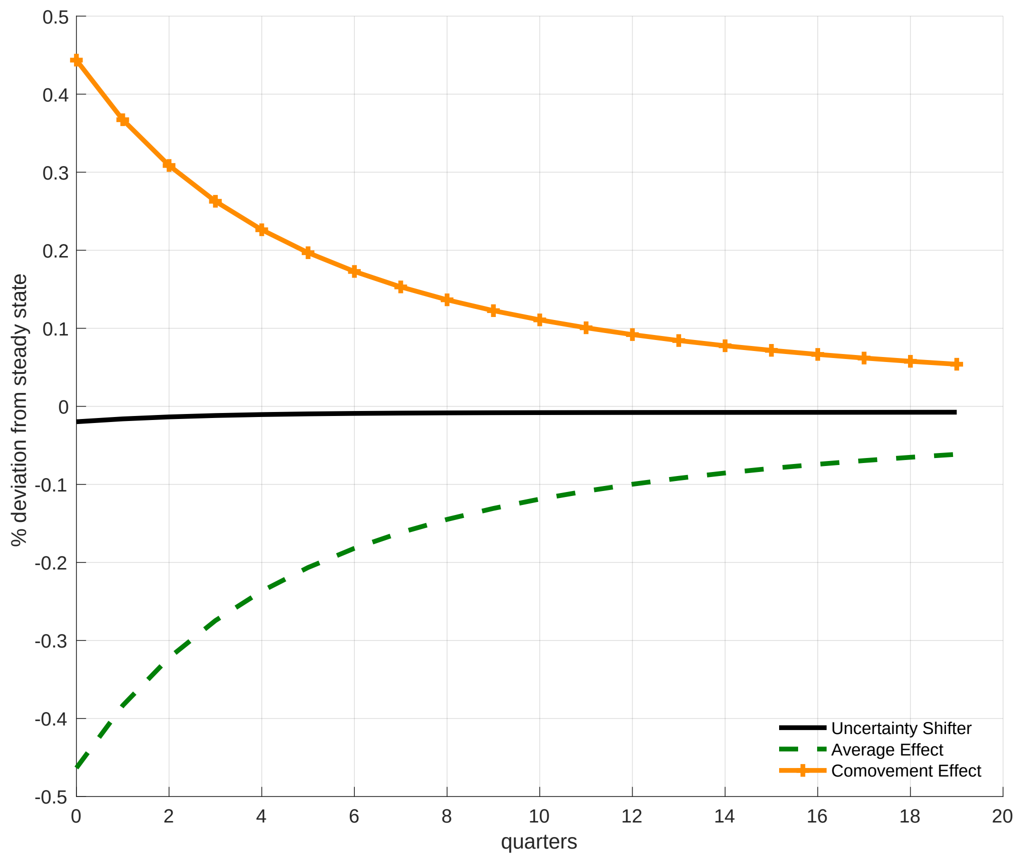
Parameter	Meaning	Value
Model parameters		
σ	Coefficient of Risk Aversion	1
$\bar{r}$	Steady State Interest Rate (annualized)	0.02
ρ_y	Autocorr. of agg. endowment shocks	0.9
ρ_ζ	Autocorr. of idiosyncratic earnings	0.966
σ_ζ	Std. dev. of idiosyncratic earnings	0.5
Discretization		
n_ζ	Points in Markov Chain for ζ	11
n_a	Points in Markov Chain for Asssets	500

Figure 1: The Effects of an Aggregate Endowment Shock



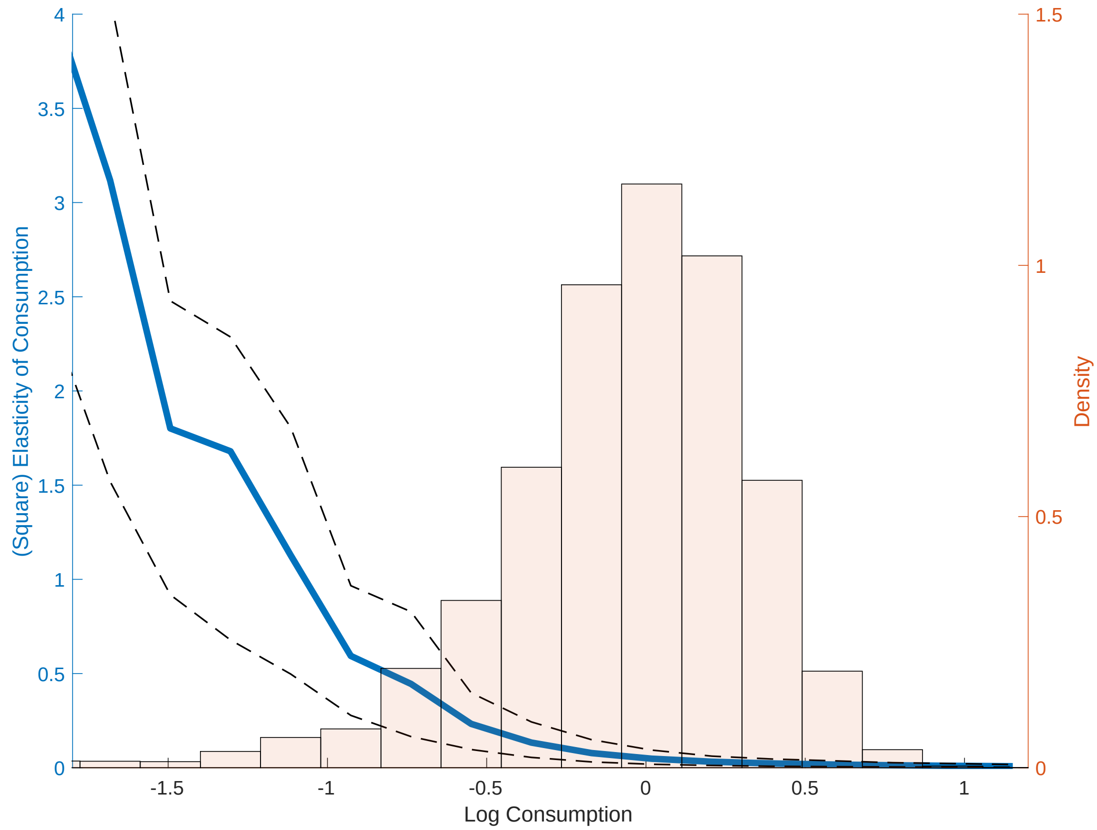
Notes: The figure shows the response of the annualized real interest rate (left panel) to a positive aggregate endowment shock (right panel) in a representative agent model (blue line with crosses), in the baseline model with heterogeneity (red line with circles), and in a model with heterogeneity but considering only the effect of average uncertainty (dashed green line).

Figure 2: The Role of Heterogeneity in an Endowment Economy



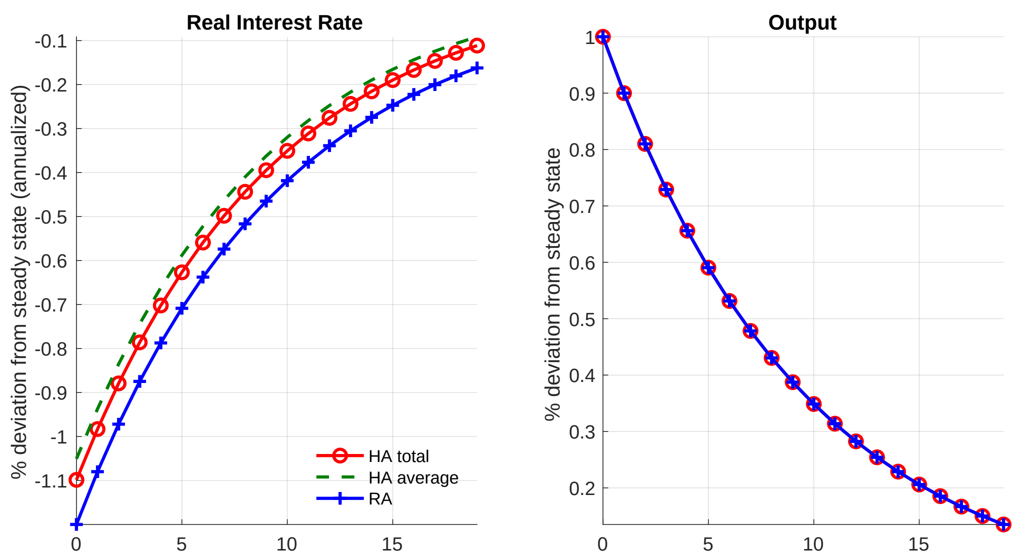
Notes: The figure illustrates the impulse responses to a 1 percent increase in aggregate income shocks for the uncertainty shifter (solid black line) and its two components, i.e. average uncertainty (dashed green line) and the comovement effect (orange line with crosses). All figures are annualized.

Figure 3: Elasticity of Consumption in Steady State



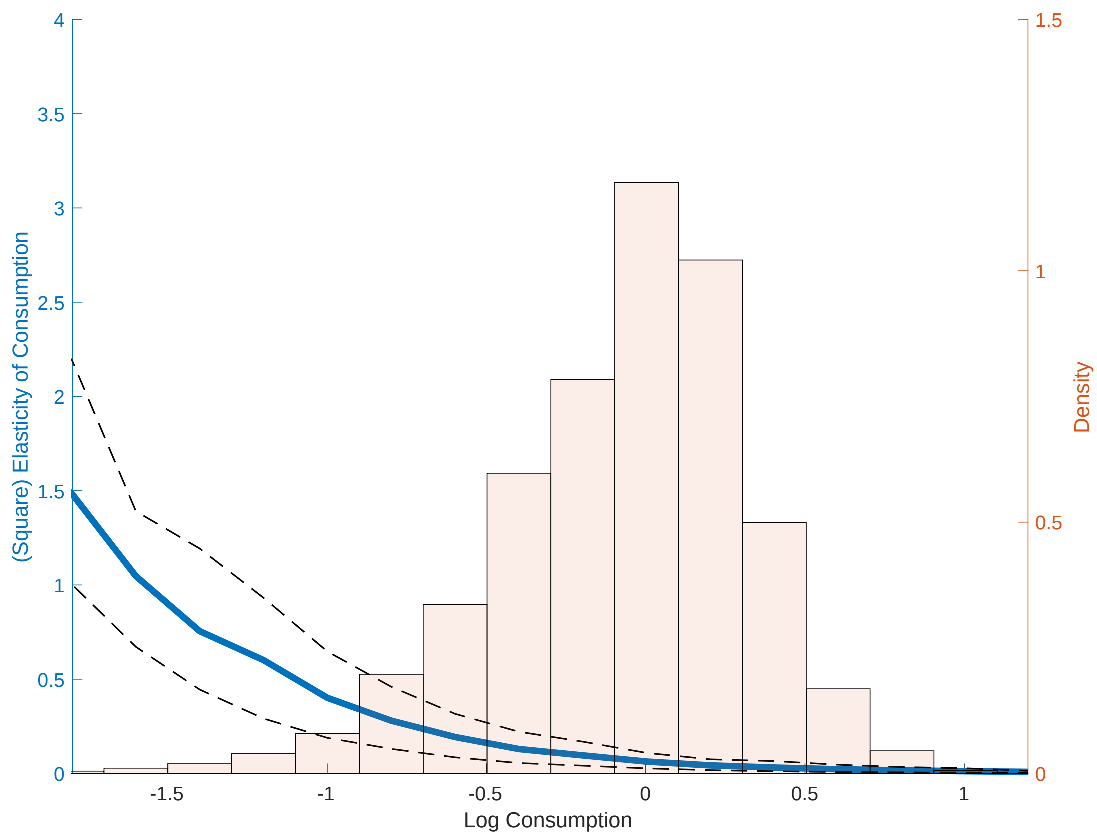
Notes: The figure shows the relationship between log consumption (horizontal axis), and the elasticity of consumption (left vertical axis) in steady state. For each value of consumption, the figure reports the average elasticity (solid blue line), the 5% - 95% interval of the distribution (black dashed lines), while the histogram indicate the steady state distribution (right vertical axis).

Figure 4: The Effects of an Aggregate Endowment Shock: $\sigma = 3$



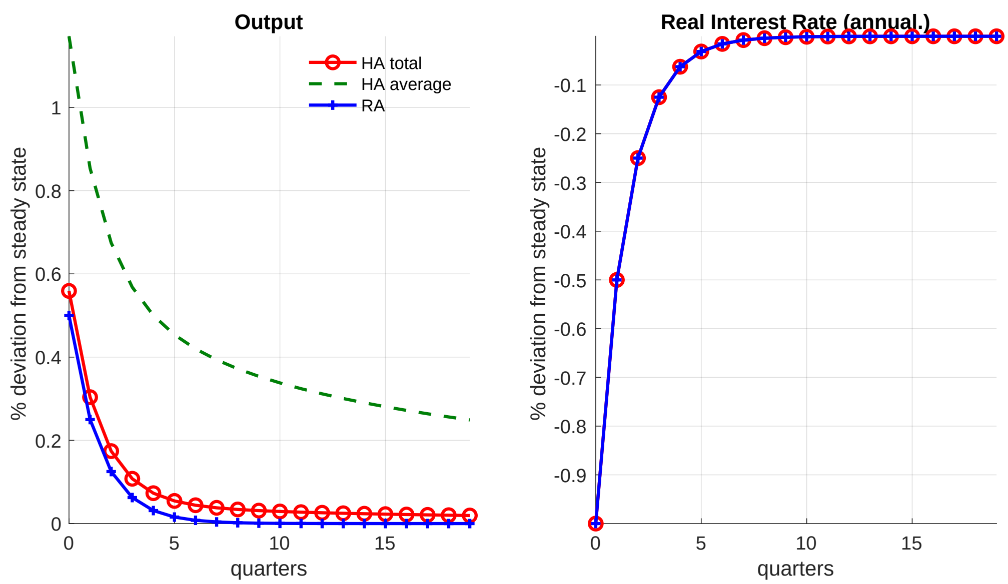
Notes: The figure shows the response of the annualized real interest rate (left panel) to a positive aggregate endowment shock (right panel) in a representative agent model (blue line with crosses), in the baseline model with heterogeneity (red line with circles), and in a model with heterogeneity but considering only the effect of average uncertainty (dashed green line).

Figure 5: Elasticity of Consumption in Steady State: $\sigma = 3$



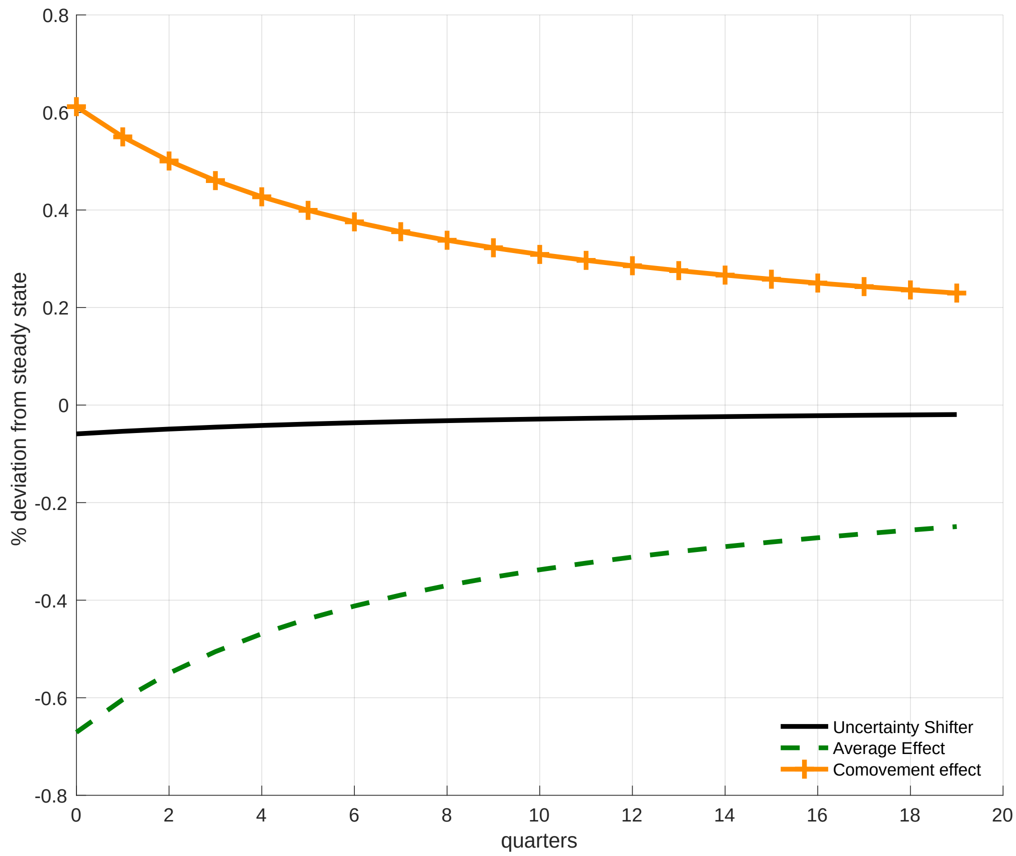
Notes: The figure shows the relationship between log consumption (horizontal axis), and the elasticity of consumption (left vertical axis) in steady state. For each value of consumption, the figure reports the average elasticity (solid blue line), the 5% - 95% interval of the distribution (black dashed lines), while the histogram indicate the steady state distribution (right vertical axis).

Figure 6: The Effects of a Monetary Policy Shock



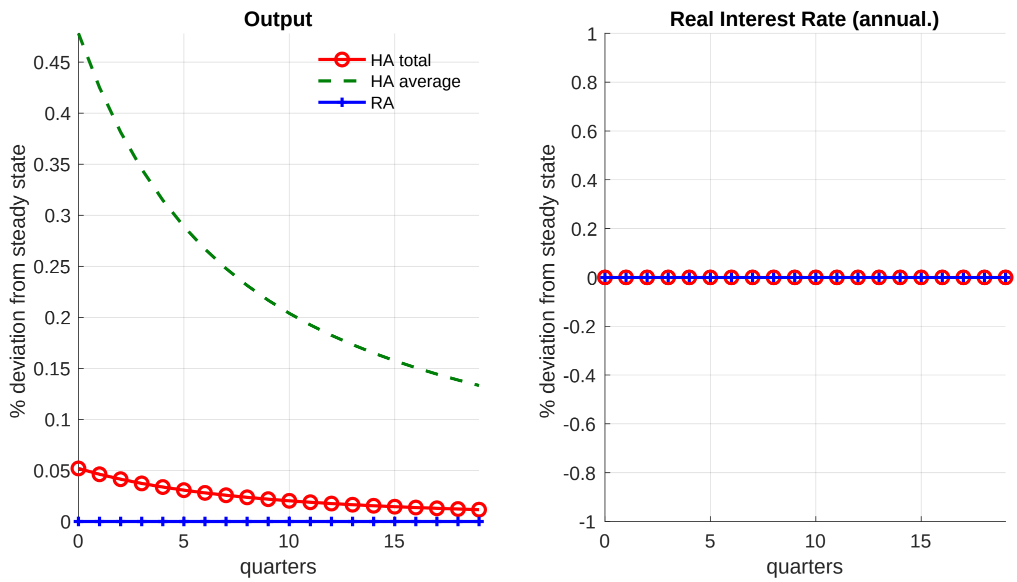
Notes: The figure shows the response of output to a 1 percent decrease in the (annualized) real interest rate, in a representative agent model (blue line with crosses), in the baseline model with heterogeneity (red line with circles), and in a model with heterogeneity but considering only the effect of average uncertainty (dashed green line).

Figure 7: The Role of Heterogeneity in a New Keynesian Economy (Monetary Shock)



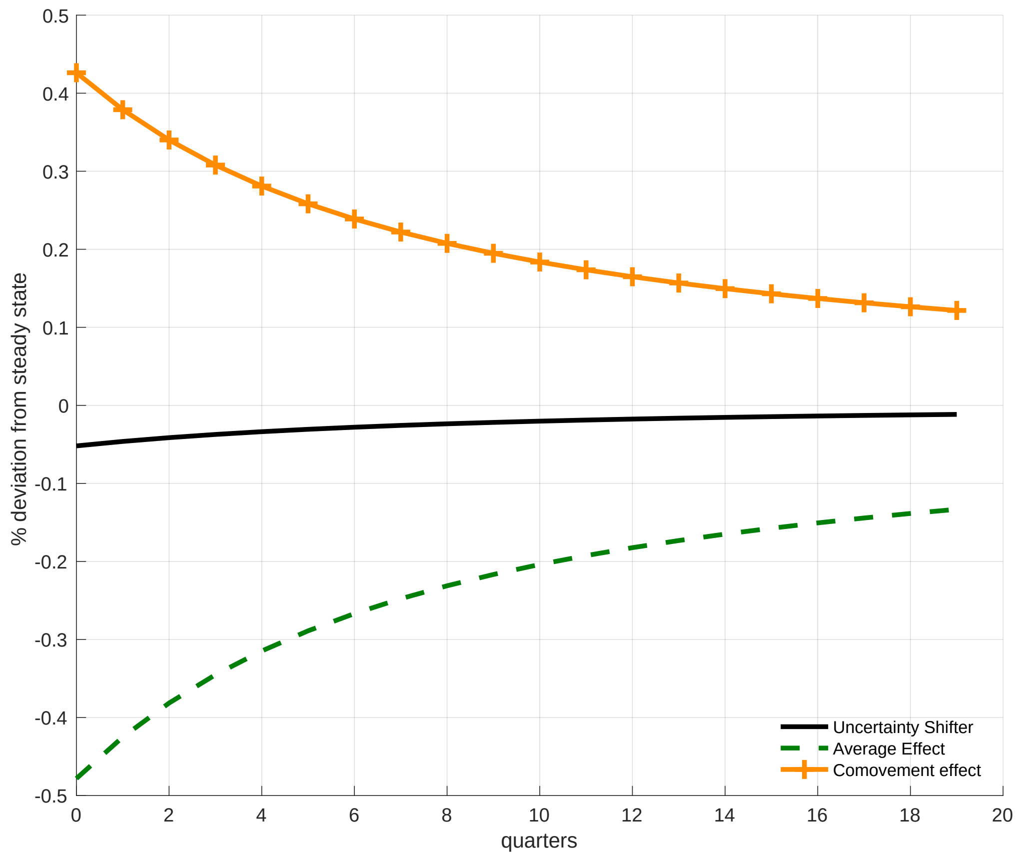
The figure illustrates the (cumulated) effects of a 1 percent reduction of the (annualized) real interest rate on the uncertainty shifter (solid black line) and its two componenents, i.e. average uncertainty (dashed green line) and the comovement effect (orange line with crosses).

Figure 8: The Effects of a Technology Shock



Notes: The figure shows the responses of output and the real interest rate to a 1 percent positive technology shock, in a representative agent model (blue line with crosses), in the baseline model with heterogeneity (red line with circles), and in a model with heterogeneity but considering only the effect of average uncertainty (dashed green line).

Figure 9: The Role of Heterogeneity in a New Keynesian Economy (Technology Shock)



Notes: The figure illustrates the (cumulated) effects of a 1 percent positive technology shock on the uncertainty shifter (solid black line) and its two components, i.e. average uncertainty (dashed green line) and the comovement effect (orange line with crosses).

Appendices

A.1 Derivation of the approximate individual Euler equation

Our starting point is the individual Euler equation

$$C_t(j)^{-\sigma} = \beta R_t \mathbb{E}_t\{C_{t+1}(j)^{-\sigma}\}$$

A second order approximation of $C_{t+1}(j)^{-\sigma}$ around $C_t(j)$ yields

$$C_t(j)^{-\sigma} \simeq \beta R_t \mathbb{E}_t \left\{ C_t(j)^{-\sigma} - \sigma C_t(j)^{-\sigma} \left(\frac{\Delta C_{t+1}(j)}{C_t(j)} \right) + \frac{\sigma(\sigma+1)}{2} C_t(j)^{-\sigma} \left(\frac{\Delta C_{t+1}(j)}{C_t(j)} \right)^2 \right\}.$$

Rearranging terms,

$$\mathbb{E}_t \left\{ \frac{\Delta C_{t+1}(j)}{C_t(j)} \right\} \simeq \frac{1}{\sigma} \left(1 - \frac{1}{\beta R_t} \right) + \frac{\sigma+1}{2} v_t(j)$$

where $v_t(j) \equiv \mathbb{E}_t \left\{ \left(\frac{\Delta C_{t+1}(j)}{C_t(j)} \right)^2 \right\}$, which corresponds to eq. (2) in the main text.

Letting $c_t(j) \equiv \log C_t(j)$ and using the Taylor expansion $\frac{\Delta C_{t+1}(j)}{C_t(j)} \simeq \Delta c_{t+1}(j) + \frac{1}{2}(\Delta c_{t+1}(j))^2$, we can rewrite the Euler equation in terms of (log) consumption:

$$\mathbb{E}_t \{\Delta c_{t+1}(j)\} \simeq \frac{1}{\sigma} \left(1 - \frac{1}{\beta R_t} \right) + \frac{\sigma}{2} v_t(j) \tag{A.1}$$

Evaluating the previous equation at the stochastic steady state (with $R_t = R$), and taking unconditional expectations we have

$$0 \simeq \frac{1}{\sigma} \left(1 - \frac{1}{\beta R} \right) + \frac{\sigma}{2} \mathbb{E} \{v_t(j)\} \tag{A.2}$$

Subtracting (A.2) from (A.1) and taking a first-order Taylor expansion of the resulting expression yields:

$$\mathbb{E}_t \{\Delta c_{t+1}(j)\} \simeq \frac{1}{\sigma} \widehat{r}_t + \frac{\sigma}{2} \widehat{v}_t(j), \tag{A.3}$$

where $\widehat{r}_t \equiv \frac{1}{\beta R}(\frac{R_t - R}{R})$ and $\widehat{v}_t(j) \equiv v_t(j) - \mathbb{E}\{v_t(j)\}$. Thus, it follows that $(\mathbb{E}_t \{\Delta c_{t+1}(j)\})^2 \sim \mathcal{O}(|\varepsilon|^2)$ thus implying $\mathbb{E}_t \{\Delta c_{t+1}(j)^2\} \simeq \mathbb{E}_t\{\xi_{t+1}(j)^2\}$, where $\xi_t(j) \equiv c_t(j) - \mathbb{E}_{t-1} \{\Delta c_t(j)\}$

is the innovation in household j's (log) consumption. Accordingly, we have

$$v_t(j) \equiv \mathbb{E}_t \left\{ \left(\frac{\Delta C_{t+1}(j)}{C_t(j)} \right)^2 \right\} \simeq \mathbb{E}_t\{\xi_{t+1}(j)^2\}.$$

A.2 Derivation of an exact Euler equation for aggregate consumption

Starting from the individual Euler equation

$$C_t(j)^{-\sigma} = \beta R_t \mathbb{E}_t\{C_{t+1}(j)^{-\sigma}\}$$

and multiplying and dividing the RHS by $[\mathbb{E}_t\{C_{t+1}(j)\}]^{-\sigma}$ we have

$$C_t(j)^{-\sigma} = \beta R_t [\mathbb{E}_t\{C_{t+1}(j)\}]^{-\sigma} \frac{\mathbb{E}_t\left\{C_{t+1}(j)^{-\sigma}\right\}}{[\mathbb{E}_t\{C_{t+1}(j)\}]^{-\sigma}}$$

or equivalently

$$C_t(j)\, V_t(j) = (\beta R_t)^{-\frac{1}{\sigma}}\, \mathbb{E}_t\{C_{t+1}(j)\} \tag{A.4}$$

where $V_t(j) \equiv \left[\frac{\mathbb{E}_t\left\{C_{t+1}(j)^{-\sigma}\right\}}{[\mathbb{E}_t\{C_{t+1}(j)\}]^{-\sigma}}\right]^{\frac{1}{\sigma}} \geq 1$ captures the effects of individual uncertainty on individual consumption choices, i.e. the "wedge" relative to the certainty-equivalence case.

Next, dividing an multiplying the LHS of (A.4) by aggregate consumption C_t, and integrating across households (and abstracting from aggregate uncertainty, as we do in our quantitative exercises) we get

$$C_t \int \frac{C_t(j)}{C_t} V_t(j)\, dj = (\beta R_t)^{-\frac{1}{\sigma}} \int \mathbb{E}_t\{C_{t+1}(j)\}\, dj$$

which implies that

$$C_t = (\beta R_t)^{-\frac{1}{\sigma}}\, C_{t+1} V_t^{-1} \tag{A.5}$$

where $V_t \equiv \int \frac{C_t(j)}{C_t} V_t(j)\, dj$.

Finally, in terms of log-deviations from steady state we have

$$\hat{c}_t = \hat{c}_{t+1} - \frac{1}{\sigma}\hat{r}_t - \hat{v}_t \tag{A.6}$$

which is analogous to eq. (6) in the main text.

A.3 Derivation of the approximate individual Euler equation for a general utility function

The individual Euler equation under a general utility function $U(\cdot)$ is given by

$$U'(C_t(j)) = \beta R_t \mathbb{E}_t\{U'(C_{t+1}(j))\}.$$

Define $\sigma_t(j) \equiv -U''(C_t(j))C_t(j)/U'(C_t(j))$ (relative risk aversion) and $\varkappa_t(j) \equiv -U'''(C_t(j))C_t(j)/U''(C_t(j))$ (relative prudence). Approximating $U'(C_{t+1}(j))$ around $C_t(j)$ gives

$$U'(C_{t+1}(j)) \simeq U'(C_t(j)) + U''(C_t(j))\Delta C_{t+1}(j) + \frac{1}{2}U'''(C_t(j))(\Delta C_{t+1}(j))^2.$$

Substituting for $U'(C_{t+1}(j))$ in the Euler equation using the previous approximation we obtain

$$1 \simeq \beta R_t \mathbb{E}_t \left\{ 1 - \sigma_t(j)\frac{\Delta C_{t+1}(j)}{C_t(j)} + \frac{1}{2}\sigma_t \varkappa_t \left(\frac{\Delta C_{t+1}(j)}{C_t(j)}\right)^2 \right\}.$$

which gives the approximate Euler equation for aggregate consumption

$$\mathbb{E}_t \Delta C_{t+1}(j) \simeq -\frac{U'(C_t(j))}{U''(C_t(j))}\left(1 - \frac{1}{\beta R_t}\right) - \frac{1}{2}\frac{U'''(C_t(j))}{U''(C_t(j))}\mathbb{E}_t \left[\Delta C_{t+1}(j)\right]^2 \tag{A.7}$$

Dividing by $C_t(j)$ and using our definitions of relative risk aversion and relative prudence gives

$$\mathbb{E}_t \left\{ \frac{\Delta C_{t+1}(j)}{C_t(j)} \right\} \simeq \frac{1}{\sigma_t(j)}\left(1 - \frac{1}{\beta R_t}\right) + \frac{\varkappa_t}{2}\mathbb{E}_t \left\{ \left(\frac{\Delta C_{t+1}(j)}{C_t(j)}\right)^2 \right\}.$$

Note that a CRRA utility implies $\sigma_t(j) = \sigma$ and $\varkappa_t(j) = \sigma + 1$, so the previous expression collapses to eq. (3) in the main text.

Alternatively, denoting with $\tilde{\sigma}_t(j)$ and $\tilde{\varkappa}_t(j)$ the coefficients of absolute risk aversion and prudence gives

$$\mathbb{E}_t \Delta C_{t+1}(j) \simeq \frac{1}{\tilde{\sigma}_t(j)}\left(1 - \frac{1}{\beta R_t}\right) - \frac{\tilde{\varkappa}_t(j)}{2}\mathbb{E}_t \left\{ \Delta C_{t+1}(j)^2 \right\}.$$

For example, the special case of CARA preferences implies that $\tilde{\sigma}_t(j) = \tilde{\varkappa}_t(j) \equiv \tilde{\sigma}$, so the previous expression collapses to

$$\mathbb{E}_t \Delta C_{t+1}(j) \simeq \frac{1}{\tilde{\sigma}}\left(1-\frac{1}{\beta R_t}\right)-\frac{\tilde{\sigma}}{2}\tilde{v}_t(j)$$

where $\tilde{v}_t(j) \equiv \mathbb{E}_t\left\{\Delta C_{t+1}(j)^2\right\} \simeq \mathbb{E}_t\left\{\left[C_{t+1}(j)-\mathbb{E}_t C_{t+1}(j)\right]^2\right\} \equiv \mathbb{E}_t\left\{\xi_{t+1}(j)^2\right\}$, which is analogous to eq. (3) in the main text. In this economy, under the assumption of i.i.d. idiosyncratic income shocks $y_t(j) \sim N(0, \sigma_{y,t})$, it can be shown that individual consumption is a linear function of cash-on-hand $x_t(j)$, i.e. $C_t(j) = C_t + \mu_t x_t(j)$, where μ_t denotes the marginal propensity to consume, and is constant across households (see Acharya and Dogra (2020)). It then follows that uncertainty $\tilde{v}_t(j) = \bar{v}_t = \mu_{t+1}^2 \sigma_{y,t}^2$, is common across households, and thus the comovement effect described in the main text is absent.

A.4 Derivation of the dynamic response of v_t

Recalling that $v_t \equiv \int \frac{C_t(j)}{C_t} v_t(j) dj$ we have

$$\begin{aligned}
\frac{dv_{t+k}}{d\varepsilon_t} &= \int \frac{d[C_{t+k}(j)/C_{t+k}]}{d\varepsilon_t} v_t(j) dj + \int \frac{C_{t+k}(j)}{C_{t+k}} \frac{dv_{t+k}(j)}{d\varepsilon_t} dj \\
&= \int \frac{d\exp\{c_{t+k}(j)-c_{t+k})}{d\varepsilon_t} v_t(j) dj + \int \frac{C_{t+k}(j)}{C_{t+k}} \frac{dv_{t+k}(j)}{d\varepsilon_t} dj \\
&= \int \frac{d[c_{t+k}(j)-c_{t+k}]}{d\varepsilon_t} \frac{C_{t+k}(j)}{C_{t+k}} v_t(j) dj + \int \frac{C_{t+k}(j)}{C_{t+k}} \frac{dv_{t+k}(j)}{d\varepsilon_t} dj
\end{aligned}$$

Next we derive an approximate expression for $\frac{d[c_{t+k}(j)-c_{t+k}]}{d\varepsilon_t}$. Combining the previous equation with (3) in the text and rearranging terms yields the difference equation

$$c_t(j) - c_t = \mathbb{E}_t\left\{(c_{t+1}(j) - c_{t+1})\right\} - \frac{\sigma}{2} v_t(j) + \frac{\sigma+1}{2} v_t$$

which can be solved forward to obtain

$$c_t(j) - c_t = -\sum_{k=0}^{\infty}\left[\frac{\sigma}{2}\mathbb{E}_t\{v_{t+k}(j)\} + \frac{\sigma+1}{2}\mathbb{E}_t\{v_{t+k}\}\right] + \mathbb{E}\left\{c_t(j) - c_t\right\} \qquad \text{(A.8)}$$

where we have used the fact that $\lim_{T\to\infty}\mathbb{E}_t\left\{c_{t+T}(j)\right\} = \mathbb{E}\left\{c_t(j)\right\}$ and $\lim_{T\to\infty}\mathbb{E}_t\left\{c_{t+T}\right\} = \mathbb{E}\left\{c_t\right\}$.

Using (A.8) as a reference, we can derive the dynamic response of (log) consumption differential to an aggregate shock in period t:

$$\frac{d[c_{t+k}(j) - c_{t+k}]}{d\varepsilon_t} = -\sum_{h=k}^{\infty} \left[\frac{\sigma}{2} \frac{dv_{t+h}(j)}{d\varepsilon_t} + \frac{\sigma+1}{2} \frac{dv_{t+h}}{d\varepsilon_t} \right] \sim \mathcal{O}(|\varepsilon|)$$

Accordingly, $\int \frac{d[c_{t+k}(j) - c_{t+k}]}{d\varepsilon_t} \frac{C_{t+k}(j)}{C_{t+k}} v_t(j) dj \sim \mathcal{O}(|\varepsilon|^2)$ and can thus be ignored in our approximation. Thus, it follows that

$$\frac{dv_{t+k}}{d\varepsilon_t} \simeq \int \frac{C_{t+k}(j)}{C_{t+k}} \frac{dv_{t+k}(j)}{d\varepsilon_t} dj$$

as found in the text.

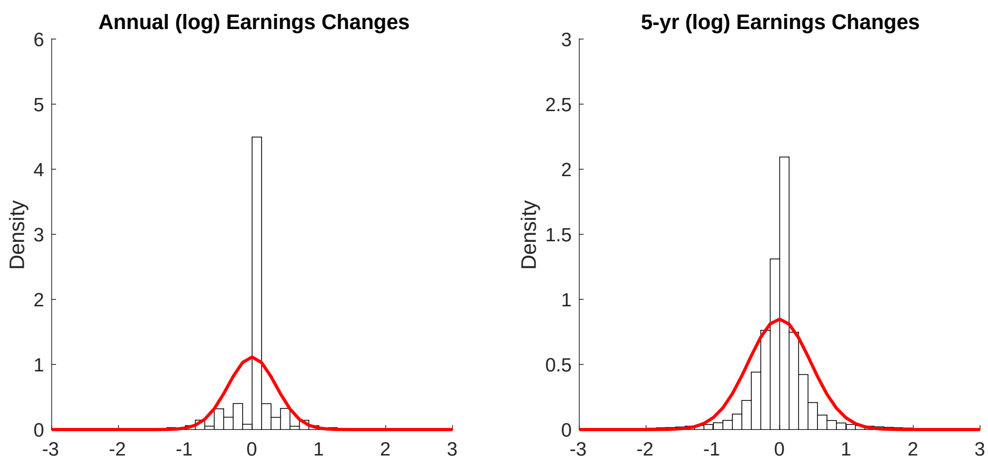
B Robustness: Alternative Process for Idiosyncratic Income Shocks

In this section, we study the role of heterogeneity in the New Keynesian economy described in Section 5, but considering an alternative process for the idiosyncratic income shocks $\zeta_t(i)$. In particular, we consider a discrete-time quarterly version of the continous-time process used in Kaplan, Moll and Violante (2018), which is the sum of two independent components $\zeta_t(i) = \zeta_{1,t}(i) + \zeta_{2,t}(i)$. Both components evolve according to a "jump-drift" process, where jumps arrive at a Poisson rate $\lambda_1 = 0.080$ and $\lambda_2 = 0.007$ and where, conditionally on a jump, innovations are drawn from a normal distribution with mean zero and standard deviations $\sigma_1 = 1.74$ and $\sigma_2 = 1.53$. Between jumps, the processes drift toward zero at rates $\beta_1 = 0.0761$ and $\beta_2 = 0.009$, respectively. The two continous-time components are discretized with 3 grid points for ζ_1 (transitory component) and 11 points for ζ_2 (persistent component) — see Section 4.2.2 and Appendix D in Kaplan, Moll, Violante (2018) for more details.

We calculate the corresponding Markov transition matrix at a quarterly frequency. The resulting discretized process gives rise to a leptokurtic distribution of income changes, as shown in Figure B.1. In particular, the values of the kurtosis are 14.8 for annual income changes, and 12.6 for 5-year changes, which are close to the empirical counterparts using data U.S. male earnings as in Guvenen et. al. (2015). We then recalibrate the discount factor to $\beta = 0.982$ so that the steady state real interest rate equal 2 percent per year, as in our baseline case.

Figure B.2 shows that the response of output to a monetary shock in this economy (green line with diamonds) is remarkably close to the response obtained in our baseline calibration (red line with circles), and in turn similar its counterpart in a representative agent economy (blue line with crosses). A similar result is obtained in response to other shocks (results are omitted for brevity, and available from the authors upon request).

Figure B.1: Distribution of (Log) Income Shocks in the Alternative Calibration



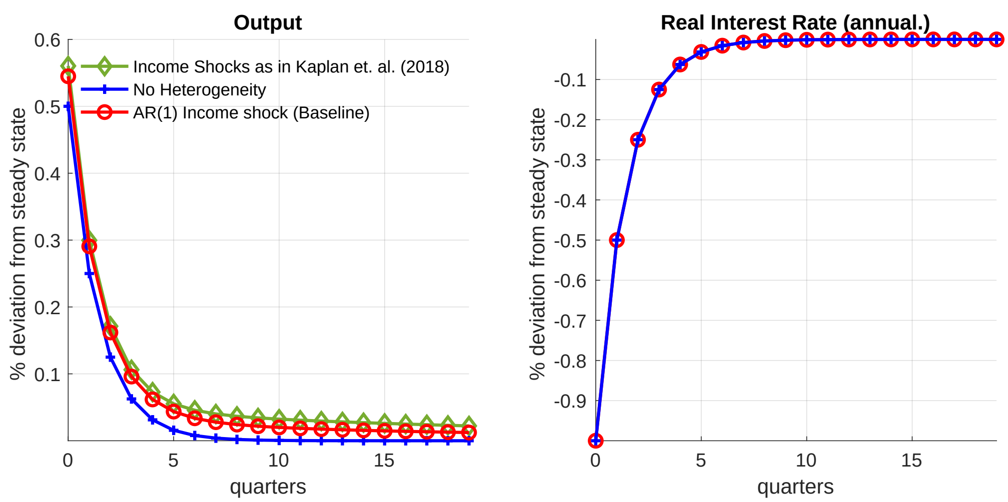
Notes: The figure shows the distribution of (log) earning changes at annual frequency (left panel) and at a 5yr frequency (right panel). In each panel, the histograms correspond to the distribution resulting from the (discretized) process with a transitory and a persistent component, while the solid line indicates the normal distribution with the same mean and variance.

C Robustness: Monetary Policy Rule

In this appendix, we study the role of heterogeneity in the New Keynesian economy described in Section 5, assuming that the central bank follows a Taylor-type rule for the real interest rate $\widehat{r}_t = \phi_\pi \pi_t + m_t$, where m_t is a monetary shock, which is assumed to follow an AR(1) process, with auto-correlation coefficient $\rho_m = 0.5$. We set the coefficient $\phi_\pi = 0.5$, in line with the original estimates of Taylor (1999).

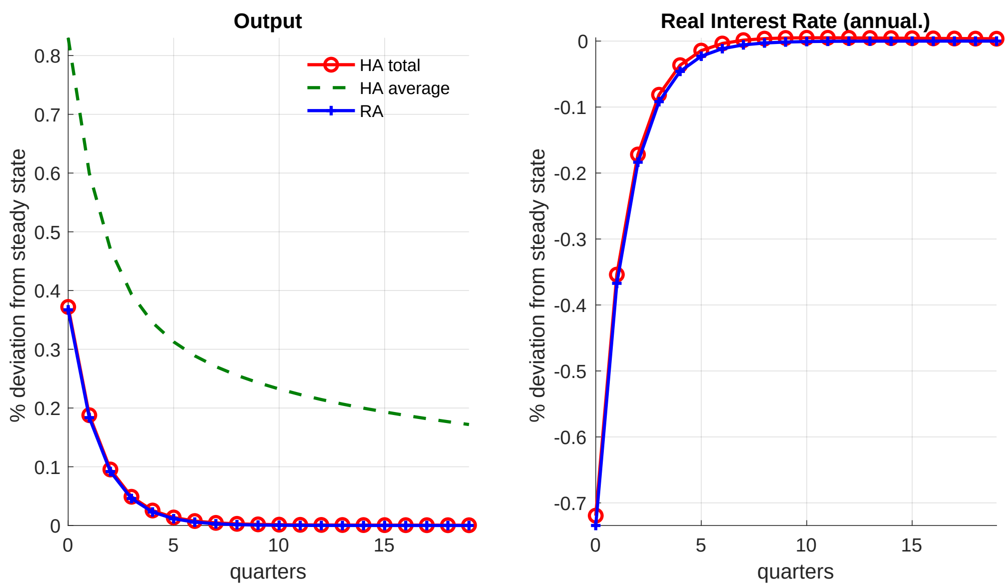
Figure C.3-C.4 report the response of aggregate variables to monetary and technology shocks, respectively. In response to all these shocks, and analogously to what shown in Figures 6 and 8 in the main text, the responses of aggregate variables in an heterogeneous agent economy (red lines with circles) are similar to those obtained in the corresponding model with a representative agent (blue line with crosses).

Figure B.2: The Effects of Monetary Shocks with Alternative Idiosyncratic Risk Process



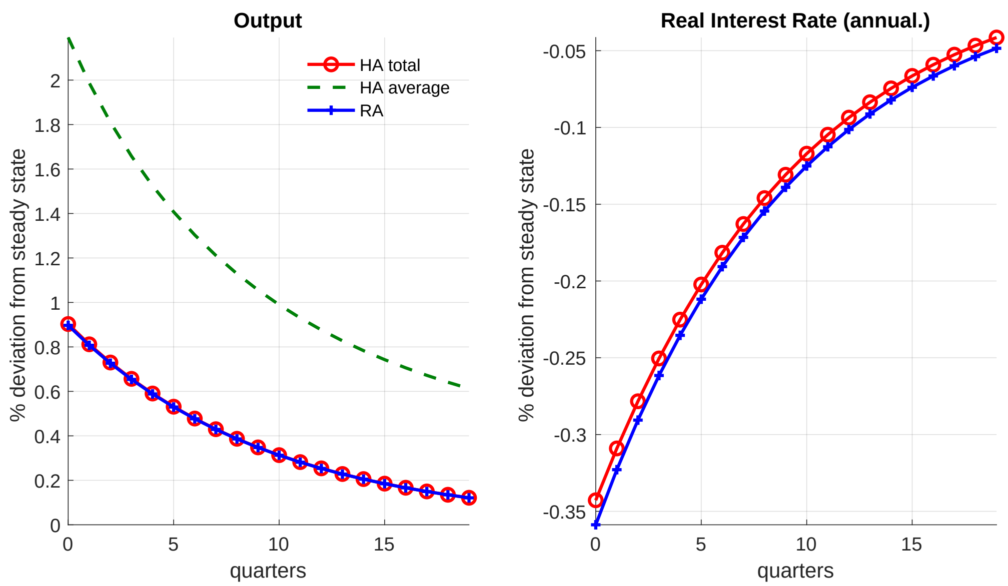
Notes: The figure shows the response of output and (annualized) real interest rate to a 25 basis points expansionary monetary shock. The figure compares the responses in a model without heterogeneity (blue line with crosses), in the baseline heterogeneous household model with AR(1) idiosyncratic income shocks (red line with circles), and in a model with idiosyncratic shocks with a transitory and a persistent component as in Kaplan, Moll and Violante (2018) (green line with diamonds).

Figure C.3: The Effects of a Monetary Shock (Monetary Rule)



Notes: The figure shows the response of output and (annualized) real interest rate to a 25 basis point monetary shock, in a representative agent model (blue line with crosses), in the baseline model with heterogeneity (red line with circles), and in a model with heterogeneity but considering only the effect of average uncertainty (dashed green line).

Figure C.4: The Effects of a Technology Shock (Monetary Rule)



Notes: The figure shows the response of output and (annualized) real interest rate to a 1 percent technology shock, in a representative agent model (blue line with crosses), in the baseline model with heterogeneity (red line with circles), and in a model with heterogeneity but considering only the effect of average uncertainty (dashed green line).