The World Income Distribution: The Effects of International Unbundling of Production*

Sergi Basco
Universitat Autònoma de Barcelona and MOVE

Martí Mestieri
Northwestern University and CEPR

February 8, 2019

Abstract

Trade in intermediates, also known as unbundling of production, and trade in capital have become increasingly important in the world economy during the last twenty-five years. To jointly study these two phenomena, we develop a dynamic, factor-proportions model with trade in final goods, intermediates and capital where countries differ in their aggregate productivity levels (TFP). Our central result is to identify a novel channel whereby trade in intermediates generates a reallocation of capital across countries that exacerbates world inequality in both income and welfare. With unbundling, high-productivity countries sort into the production of capital-intensive intermediates. They increase their capital stock (via capital imports and accumulation), and, ultimately, their real wages. This exacerbates initial productivity differences across countries and increases world income inequality. We also show that income inequality rises with unbundling *(i)* even in the case of ex-ante identical countries (symmetry breaking), *(ii)* when emerging countries start participating in trade in intermediates and *(iii)* when a labor-saving technology (computerization) is introduced. For an empirically-motivated model parametrization, middle-income countries experience the largest output decline with unbundling.

Keywords: Trade in Intermediates, Unbundling, International Capital Flows, World Income Distribution, Symmetry Breaking.
JEL Classification: F12, F43, O11, O19, O40.

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*We thank Daron Acemoglu, Pol Antràs and Jaume Ventura for useful comments and suggestions, and Guido Lorenzoni, Kiminori Matsuyama, an associate editor and three anonymous referees for comments on earlier drafts. We have also benefited from comments in our presentations at Barcelona GSE Summer Forum, Bank of Spain, Berkeley, Calgary, CEMFI, CEPR-ESSIM, Chicago Fed Trade Conference, IMF, Nottingham, Stanford, UC3M and Universitat de Barcelona. First draft: July 2014. We thank Shekhar Tomar for his excellent research assistance. Basco acknowledges financial support from Fundación Ramón Areces. Mestieri thanks the ANR for their financial support while at TSE. Contact: sergi.basco@gmail.com, marti.mestieri@northwestern.edu.

1 Introduction

Two remarkable facts of the globalization process witnessed in the last twenty-five years are the large increases in both trade in intermediate goods and capital mobility. Before the 1990s, trade in final goods accounted for most of the value of world exports and international capital mobility was relatively low. In contrast, after the 1990s, trade in intermediate goods or "unbundling of production" has become more prominent over time (e.g., see Figure 1a), and global supply chains have emerged–a phenomenon termed New Globalization by Baldwin (2016).[1] There has also been a sizable growth of both gross and net international capital flows (e.g., see Lane and Milesi-Ferretti, 2007).

We document that intermediate goods are more heterogeneous in capital intensity than final goods. Figure 1b illustrates this fact by comparing the distribution of capital intensity of intermediates and final goods according to the end-use classification of Feenstra and Jensen (2012). The dispersion in capital intensity of intermediates is 24 percent higher than final goods as measured by the standard deviation, and 43 percent higher as measured by the interquartile range.[2] This suggests that unbundling of production may lead countries to specialize in intermediates with different capital intensity.

Motivated by these facts, we develop a tractable dynamic model that combines trade in final goods, intermediates and capital to study how the unbundling of production affects the international allocation of capital and the world income distribution. We capture the higher dispersion in capital-intensity of intermediates in a stylized manner: intermediates are heterogeneous in capital-intensity, while different varieties of final goods are homogeneous in capital-intensity. This allows us to obtain a sharp characterization of the trade equilibrium and the effects of unbundling. The central result of the paper is to identify a novel channel whereby trade in intermediates (and, more broadly, trade in goods heterogeneous in capital intensity) generates a reallocation of capital towards more productive countries that exacerbates world inequality in both welfare and income per capita.

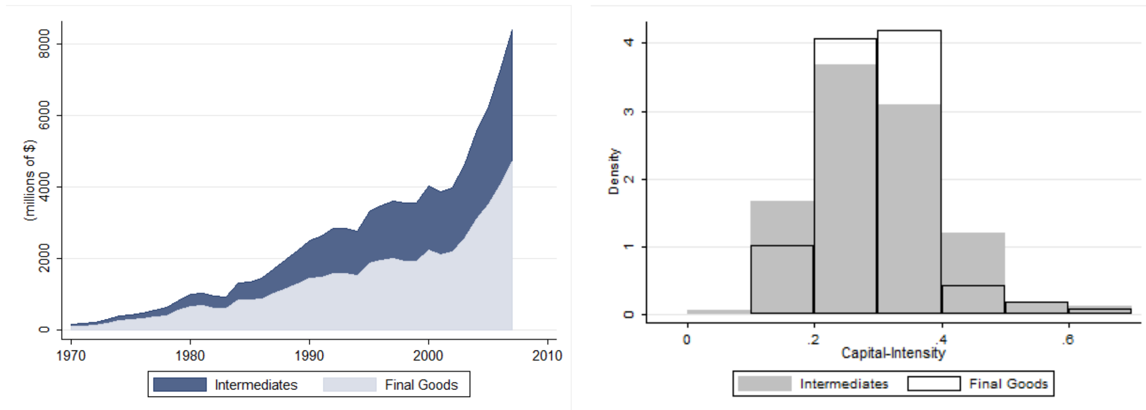
The mechanism driving our result comes from the interaction between *(i)* countries specializing in intermediates of different capital intensity and *(ii)* an elastic supply of capital. We show that, with an elastic supply of capital, high-productivity (TFP) countries have comparative advantage in capital-intensive intermediates. Intuitively, these countries have higher wages, while capital flows equalize the cost of capital across countries. Thus, high-productivity countries sort into the production of capital-intensive intermediates and the world production of capital-intensive intermediates concentrates in high-productivity countries (in accordance

[1] Antràs (2016), Johnson (2014) and Hummels et al. (2001), among others, provide evidence consistent with an increase in trade in intermediate inputs over time.

[2] We can reject the null of identical dispersion at a 5% level using the one-sided F-test for the ratio of variances. We find similar results if we define final goods as bundles of intermediates using direct requirements input-output tables (see Table A.1 in the online appendix).

Figure 1: Intermediates and Final Goods

(a) Value of Exported Intermediates and Final Goods (b) Distribution of Capital Intensity



Notes: We use the end-use classification in Feenstra and Jensen (2012) to classify goods as intermediates or final goods. See details in Appendix A.

with empirical studies).[3] As a consequence, high-productivity countries import and accumulate relatively more capital. This magnifies cross-country differences in labor productivity and real wages, implying an increase in world income and welfare inequality. We show that this mechanism also holds in the limiting case of vanishing productivity differences across countries (symmetry breaking).

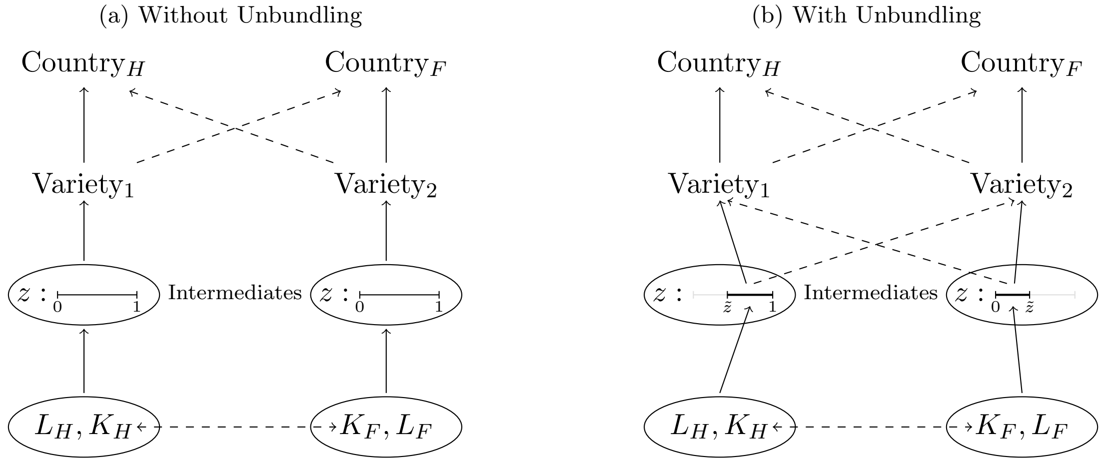
Formally, the reason why unbundling exacerbates cross-country productivity differences is that it breaks the conditional factor price equalization. In the equilibrium without unbundling, for any two countries i and j, relative wages w are proportional to their productivities, θ, i.e., $\frac{w_i}{w_j} = \frac{\theta_i}{\theta_j}$. In contrast, with unbundling, relative wages also depend on the endogenous sorting of countries into the production of intermediates with different capital intensity. In this case, we show that $\frac{w_i}{w_j} = \left(\frac{\theta_i}{\theta_j}\right)^{Z_{ij}}$, where Z_{ij} is a function that encapsulates how this endogenous sorting magnifies the effect of country TFP on relative wages and satisfies $Z_{ij} > 1$ if $\theta_i > \theta_j$. Thus, unbundling of production exacerbates cross-country wage inequality (and, ultimately, income and welfare inequality). Note that this is in stark contrast to other models of trade and capital flows, which exhibit conditional factor price equalization (e.g., Jin, 2012).

In our world economy, described in Section 2, countries only differ in their aggregate productivity (TFP). We consider a two-stage structure of production as in the seminal work by Feenstra and Hanson (1996).[4] Varieties of final goods, to which we refer as *varieties*

[3]Table B.1 (in the online appendix) provides evidence consistent with this pattern of specialization in intermediates. Moreover, it shows that it is quantitatively important. We find that, if the productivity of a country moved from the 25th to the 75th percentile, the rise in the value of exports of intermediates in the 75th percentile of capital intensity would be 40% larger than the increase in the 25th percentile. See also, among others, Baxter and Kouparitsas (2003), Hanson (2012) and Schott (2003a,b, 2004).

[4]This production structure has been subsequently used to model trade in intermediates both in the inter-

Figure 2: Production Structure for the Two-country, Two-variety Case



Note: This figure represents the production structure for two countries and two varieties under the two different trade regimes. Dashed lines indicate trade flows, z indexes intermediates.

in what follows, are differentiated by origin as in Armington (1969). They are consumed by the representative consumer of each country, who maximizes lifetime utility. We assume that the number of varieties that each country produces is proportional to its productivity level.[5] To produce each variety, a bundle of *intermediates* has to be assembled. Each of these intermediates requires capital and labor in different proportions to be produced. All technologies have constant returns to scale. Capital can freely flow across countries and can be saved across periods. Labor is fixed and cannot migrate (but can be freely reallocated within a country).

We study two trade equilibria that we take as stylized representations of the trade regimes pre- and post-unbundling. We start analyzing an equilibrium "without unbundling," in which only varieties and capital can be traded. Figure 2a represents this structure for a two-country two-variety case. The share of world output is determined by the share of varieties this country produces, which is proportional to its productivity. In the second trade regime, unbundling becomes possible. Intermediates can be freely traded and variety producers purchase intermediates from the cheapest location in the world.[6] Therefore, the location of production of intermediates becomes endogenous. In this "unbundling" equilibrium, countries sort into the production of capital-intensive intermediates according to their productivity, as depicted in Figure 2b. We also show that the world output share of a country is determined by the mass and capital-intensity of the intermediates that it produces.

national trade literature, e.g., Yi (2003), and the international macroeconomics literature, e.g., Jin (2012).

[5] Online Appendix C provides an exact microfoundation that delivers this result as an endogenous outcome.

[6] The online supplemental material A (available at the authors' websites) considers the case in which only a fraction of intermediates can be traded.

Section 3 derives the main results of the paper. Our central result is that, for any world productivity (TFP) distribution, unbundling implies an increase of world output and welfare inequality. We also find that both gross and net capital flows increase with unbundling. Section 3.1 shows that unbundling of production generates output (GDP) inequality even in the limiting case of vanishing cross-country productivity differences (symmetry breaking).[7] Section 3.2 provides an explicit characterization of the world output (GDP) distribution under the assumption that countries' productivity is log-linear in their ranking, which is a very good approximation of the data. We adopt the "continuum of countries" modelling device from Matsuyama (2013) and obtain a sharp and tractable characterization of the cross-country Lorenz curve. Under this parametrization, we also show that middle-income countries experience the largest output decline with unbundling.

Lastly, Section 4 takes advantage of the tractability of our framework to investigate how unbundling interacts with other relevant features of the world economy in the last decades. First, we show that if emerging countries (the "South") join the "global supply chain" (i.e., start participating in trade in intermediates), the world output share of Northern countries and the most productive Southern countries will increase. Second, we find that the adoption of a labor-saving technology (computerization) exacerbates the effects of unbundling on world inequality. Third, we show that a first-order-stochastic-dominant shift of world technology distribution always leads to convergence in GDP but, with unbundling, the mass of low-productivity countries catching up is larger. Fourth, we explain how unbundling generates a relative decline in the labor share of high-productivity countries and provide empirical evidence supporting this result. Finally, we show that the dynamics of world aggregate outcomes are analogous to a standard Ramsey model and characterize the transitional dynamics once unbundling becomes possible.

Related Literature. This paper relates to different strands of the literature on trade, economic growth, offshoring and capital flows. Despite the increasing prominence of international trade and capital flows, the attention given to the interaction between both has been relatively scant. Earlier work investigated how capital mobility affects the determination of optimal tariffs in a static two-country, two-commodities, two-factor world economy (Kemp, 1966 and Jones, 1967). Some important recent contributions have analyzed the role of financial development (e.g., Antràs and Caballero, 2009, Matsuyama, 2005), the effects on the skill premium (Feenstra and Hanson, 1996) and, within dynamic stochastic general equilibrium models, global imbalances (Jin, 2012) and the international co-movement of investment over the business cycle (Li and Jin, 2012).[8]

[7] Even though output inequality emerges in equilibrium, consumption (and welfare) remains the same across countries. The reason is that the asymmetric allocation of capital does not change factor prices. Thus, total factor payments remain the same in all countries.

[8] Recent working papers by Reyes-Heroles (2017) and Sposi et al. (2017) have analyzed the interaction between Ricardian trade and capital flows driven by changes in trade costs. These models abstract from

Among this set of papers, the closest to ours is Jin (2012). In both papers, countries that specialize in capital-intensive intermediates increase their capital imports. However, the economic forces driving this result are different. In Jin (2012), an increase in a country's productivity induces a production shift towards labor-intensive goods, which translates into capital outflows. In contrast, in our model, consistent with the empirical evidence, high productivity countries have comparative advantage in capital-intensive goods. Second, and more importantly, in our model, trade in intermediates exacerbates the effect of productivity differences on wages. In our model, trade in intermediates breaks conditional factor price equalization and an increase in productivity has a more than one-for-one effect on wages. In contrast, Jin (2012) features conditional factor price equalization and, thus, exacerbation-type results are precluded by assumption.

There exists a growing international trade literature analyzing the effects of unbundling. Nonetheless, this literature has not studied the interaction between trade in goods heterogeneous in capital intensity and an elastic supply of capital, which is the main mechanism of our theory. Moreover, our paper is the first to study equilibria pre- and post-unbundling in a unified framework. For example, Acemoglu et al. (2015), Baldwin and Robert-Nicoud (2014), Baldwin and Venables (2013), Grossman and Rossi-Hansberg (2008) and Jones (2000) revisit the standard trade theorems in the presence of trade in intermediates.[9] A closer paper is Costinot et al. (2013). They derive a static free trade equilibrium with global supply chains. There are two key differences with our paper: *(i)* they consider production with only one factor of production, which is in fixed supply (cannot be accumulated or traded), and *(ii)* there is no heterogeneity in the production technology across intermediates. Thus, the economic forces driving our results are entirely different.

Even though we derive our main results in a world in which countries differ in terms of productivity, the intuition of our results is similar to dynamic Heckscher-Ohlin models. For instance, Stiglitz (1970) considers a two-country two-good model and assumes that countries have different time preferences. Given this difference in time preferences, the integrated economy ends up outside the factor price equalization region. Trade leads to either convergence or divergence in income depending on the capital elasticity of the good in which the patient country specializes in. In contrast, in our model, countries only differ in productivity. The level of productivity uniquely determines the pattern of specialization, the stock of capital and income distribution in the long run. We show that world income inequality unambiguously increases with trade in intermediates.

Our baseline dynamic model also relates to the large number of models that study the interaction between economic growth and trade. Our model structure for production is similar

heterogeneity in capital intensity across intermediates, which is at the heart of our results.

[9]The sequential production process in which intermediates are first produced and then used to assemble each variety pioneered in Dixit and Grossman (1982) is also assumed in, among others, Antràs and Chor (2013), Caliendo and Parro (2012), Deardorff (1998, 2001a) and Kohler (2004).

to Acemoglu and Ventura (2002). The most important difference is that we introduce the additional layer of intermediates in the production process. This allows us to study the effect of unbundling on the world income distribution. Other papers that study how trade in goods affects economic growth include Bajona and Kehoe (2010), Baxter (1992), Cunat and Maffezzoli (2004), Deardorff (2001b, 2013) and Ventura (1997). These papers make different assumptions on the number of goods and whether factor prices equalize. However, they do not consider trade in intermediates.[10] Yi (2003) calibrates a two-country, two-stage Ricardian model to show that vertical specialization is needed to explain how small trade cost reductions resulted in the observed growth in exports.

In our model, unbundling of production generates symmetry breaking of ex-ante identical countries. Krugman and Venables (1995) and Matsuyama (2004, 2013), among others, have also analyzed how trade can generate symmetry breaking. Our mechanism is different from these papers because it does not rely on economies of scale or credit market imperfections. In our model, countries become ex-post asymmetric because in the unbundling equilibrium they specialize in intermediates with different capital intensity. This triggers capital flows (or differential saving rates across countries) that reinforce the initial differences in the specialization pattern. Thus, our framework provides a novel environment in which symmetry breaking emerges: trade regimes in which traded goods are heterogeneous in their input requirements and there is an elastic supply of one of these inputs. It is important to emphasize that while we have symmetry breaking in terms of production and trade patterns, welfare remains the same across countries in the unbundling equilibrium because factor prices equalize. This is in contrast with the previous literature in which, due to increasing returns (or borrowing constraints), they have symmetry breaking also in welfare.[11]

2 A Dynamic Model of Trade and Capital Flows

In this section, we present our baseline model. We characterize the world GDP distribution, the demand for assets and the transitional dynamics of the world economy. All omitted proofs in the paper can be found in Appendix B and the online appendix.

Our baseline model is dynamic to endogenize the savings of agents and to allow for different margins of an elastic capital supply (i.e., trade in capital and capital accumulation). In the

[10] We note that for the case in which countries have identical productivities, if we eliminated the varieties stage of production, we would have a standard (dynamic) Heckscher-Ohlin model. In this case, we can also derive the unique steady-state world income distribution (using the symmetry breaking equilibrium refinement).This is in contrast with the indeterminacy of equilibria in the case of a finite number of traded goods (e.g., Bajona and Kehoe, 2010). We focus on the case of trade in intermediates because they are more heterogeneous in capital intensity.

[11] In terms of techniques, the characterization of the unbundling equilibrium relies in solving for the equilibrium assignment in a similar manner to Matsuyama (2013). Our solution differs from the Ricardo-Roy assignment models as used in Costinot and Vogel (2010) and Grossman and Helpman (2014) because our production functions are not linear and factors of production are homogeneous.

online appendix, we present two extreme versions of our baseline model to illustrate that our results only hinge on an elastic supply of capital (rather than allowing simultaneously for trade in capital and endogenous savings). The first version is a static model, which rules out the possibility of accumulating capital through saving but allows for capital to be reallocated across countries. The second version is a dynamic model that allows for capital accumulation but rules out trade in capital. We show that unbundling has the same qualitative effects with these alternative formulations of elastic capital supply.

2.1 Model

We consider a world economy with J countries, indexed by j. Countries only differ in their level of productivity θ_j. Without loss of generality, we assume that $\theta_1 \geq \theta_2 \geq \cdots \geq \theta_J$.

A final composite good is produced by aggregating varieties. There is a mass N of varieties in the world, indexed by $v \in [0, N]$. Varieties are differentiated by origin, as in Armington (1969). Each country j produces a subset $\mathcal{V}_j$ with measure μ_j of these differentiated varieties, such that $\sum_{j=1}^{J} \mu_j = N$. In the baseline model, we assume that the number of varieties is exogenously given by $\mu_j = \kappa\theta_j$, with $\kappa > 0$. This implies that more productive countries produce a larger number of varieties. Online Appendix C provides an exact microfoundation for this production function of varieties.[12]

The final composite good is produced aggregating varieties according to the constant returns to scale technology

$$y_j(t) = \exp\left(\int_0^N \frac{1}{N} \ln x_j(v,t)dv\right), \tag{1}$$

where $y_j(t)$ denotes the final composite good produced in j at time t and $x_j(v,t)$ denotes the amount of variety v used in country j and time t.

The production of varieties requires a bundle of intermediates, indexed by $z \in [0, 1]$,

$$x_j(v,t) = \exp\left[\int_0^1 \beta(z) \ln a_j(z,v,t)dz\right], \tag{2}$$

where $a_j(z,v,t)$ denotes the amount of intermediate z used to produce variety v in country j at time t.[13] $\beta(z)$ reflects the relative importance of intermediate z in the production of variety

[12] The microfoundation is based on Jones (1995). There exists an innovation sector that produces varieties using the final good. Innovators sell patents to the producer of varieties and extract all the surplus of the producer, who has monopoly rights on the production of the variety. With this microfoundation, the Armington assumption is not needed as firms endogenously choose to produce different products. We also note that this particular microfoundation for μ_j plays a simplifying role. However, the mechanism driving our results is independent of this microfoundation. We could carry $\{\mu_j\}_{j=1}^{J}$ as additional parameters or simply set $\mu_j = 1$ for all j and obtain the same qualitative results.

[13] This formulation has been used, among others, by Dornbusch et al. (1980) in the context of Heckscher-Ohlin models.

v. We initially assume, for simplicity, that $\beta(z) = 1$. In Section 4.2 we study the effects of computerization and conduct comparative statics on $\beta(z)$.

Intermediates are produced using labor l and capital k in different proportions,

$$a_j(z,t) = \theta_j \left(\frac{k_j(z,t)}{z}\right)^z \left(\frac{l_j(z,t)}{1-z}\right)^{1-z}, \qquad z \in [0,1]. \tag{3}$$

z parametrizes the capital intensity of intermediates and θ_j is a country-specific Hicks-neutral productivity term.

The lifetime utility of the representative household in country j is

$$V_j = \int_0^\infty e^{-\rho t} \frac{c_j(t)^{1-\gamma}}{1-\gamma} dt, \tag{4}$$

where $c_j(t)$ is consumption in country j at time t and $\gamma > 0$. For $\gamma = 1$, we take $u(c) = \ln(c)$. The budget constraint of the representative household in country j is

$$\dot{b}_j(t) + c_j(t) = r(t)b_j(t) + w_j(t), \tag{5}$$

where $\dot{b}_j$ denotes the saving decision of the household. We take the final composite good as the numéraire (its price is the same across countries). We assume that each country is endowed with a fixed stock of labor, normalized to one, and an initial amount of assets $b_j(0)$ proportional to its productivity. That is, $b_j(0) \equiv \frac{\theta_j}{\Theta} B(0)$, where $B(0) \equiv \sum_{j=1}^J b_j(0)$ denotes the total initial stock of assets and $\Theta \equiv \sum_{j=1}^J \theta_j$.[14] Given that all technologies are constant returns to scale in our economy, the normalization of population to one is inconsequential to our results and it allows us to refer interchangeably to countries' output and output per capita.[15] We also impose the standard no-Ponzi condition. Finally, the trade balance condition of country j is $TB_j(t) = r(t)\left[k_j(t) - b_j(t)\right] + \dot{b}_j(t) - \dot{k}_j(t)$.

2.2 The Two Trade Regimes

This subsection analyzes the competitive equilibrium under two trade regimes. In the first trade regime, varieties and capital are freely and costlessly traded but intermediates cannot be traded. We denote this trade equilibrium as "without unbundling." In the second trade

[14] This assumption is made to simplify the exposition of the results. In the next section, we show that it corresponds to the steady-state of the equilibrium without unbundling. Importantly, our results on changes of relative GDP per capita and welfare do not depend on this assumption. See also footnote 18.

[15] If countries had different labor endowments, we would obtain the same qualitative results. In this case, all variables would need to be interpreted in per capita terms. In particular, θ_j would be the number of varieties per capita (i.e., $\mu_j = \kappa\theta_j L_j$). Under this assumption, we can compute the labor market clearing condition following the same steps as in the main text and check that the same exacerbation results hold (even though the exact value of the thresholds governing the endogenous selection of intermediates in the unbundling equilibrium would change). For example, we can use an analogous argument to the one below to show that unbundling increases real wage differences.

regime, which we refer to as "unbundling," intermediates become freely tradeable at zero cost.[16]

2.2.1 Equilibrium Without Unbundling

This subsection characterizes the trade equilibrium without unbundling of production, which is defined as follows.

Definition 1 ***(Equilibrium Without Unbundling)*** *A competitive equilibrium without unbundling is defined by a set of prices* $\{r(t), w_j(t), p_j(t), p_j(v,t), p_j(z,t)\}$ *and allocations* $\{l_j(z,t), k_j(z,t), c_j(t), a_j(z,v,t), a_j(z,t), x_j(v,t)\}$ *for* $j = 1, \ldots, J$, $v \in [0, N]$ *and* $t \geq 0$ *such that, given an initial distribution of assets* $\{b_j(0)\}$*: (i) the representative agent maximizes utility subject to the budget constraint, (ii) final good, variety and intermediate producers maximize profits given prices, (iii) the labor market clears and (iv) net exports of varieties equal net imports of capital.*

From the problem of the final good producer and noting that all varieties are traded, it follows that the world demand for variety v at time t is

$$x(v,t) = \sum_{i=1}^{J} x_i(v,t) = \frac{1}{p_j(v,t)} \frac{Y(t)}{N}, \tag{6}$$

where $Y(t) \equiv \sum_{i=1}^{J} y_i(t)$ denotes world output. We have used that the price of the final good is the same across countries (and normalize it to one). Note also that, given our Armington assumption, variety v is only produced in j and thus we carry the subindex j in $p_j(v,t)$ only for completeness.

Since there is no trade in intermediates, the demand for each intermediate comes only from the producers of domestic varieties. Thus, the demand of intermediate z in country j at time t is

$$a_j(z,t) = \int_{v \in \mathcal{V}_j} a_j(z,v,t)dv = \mu_j a_j(z,v,t) = \mu_j \frac{Y(t)}{N p_j(z,t)},$$

where μ_j is the measure of varieties produced in country j. The domestic labor market and international capital market clearing conditions for each date t are

$$\begin{aligned} 1 &= \int_0^1 l_j(z,t)dz = \frac{1}{w_j(t)} \int_0^1 (1-z) p_j(z,t) a_j(z,v,t) dz = \frac{\mu_j}{2} \frac{Y(t)}{N} \frac{1}{w_j(t)}, && (7) \\ B(t) &= \sum_{i=1}^{J} k_i(t), && (8) \end{aligned}$$

[16]In Section A in the online supplemental material (available at the authors' websites), we consider the intermediate case in which only a fraction α of the intermediates is traded and perform comparative statics on α.

where $k_j(t)= \frac{1}{r(t)} \int_0^1 z p_j(z,t) a_j(z,t) dz = \frac{\mu_j}{2} \frac{Y(t)}{N} \frac{1}{r(t)}$ for $j = 1, \cdots, J$.

From the market clearing conditions and the number of varieties produced in each country, the world production share of country j at time t, $s_j^y(t)$, is

$$s_j^y(t) \equiv \frac{y_j(t)}{Y(t)} = \frac{\theta_j}{\Theta},$$

where $\Theta \equiv \sum_{i=1}^{J} \theta_i$. Note that the production share of country j depends only on the relative productivity of the country.

Before solving the problem of the household, note that we can write the wage of the representative household in country j as $\pi_j w(t)$ where $\pi_j \equiv \frac{\mu_j}{N}$ and $w(t) = \frac{Y(t)}{2}$. Let us also define the share of assets owned by households in country j at time t as $b_j^R(t) \equiv b_j(t)/B(t)$. Then, the solution of the problem is given by the differential equations

$$\begin{aligned} \dot{c}_j(t) &= \frac{r(t)-\rho}{\gamma} c_j(t), \\ \dot{b}_j^R(t) &= \delta(t)(\pi_j - b_j^R(t)), \end{aligned}$$

where $\delta(t) \equiv \frac{w}{B} - \xi(t) \int_t^\infty \frac{w}{B} e^{-\int_t^\tau r(v)dv} d\tau$ and $\xi(t)^{-1} \equiv \int_t^\infty \rho^{-1} e^{\int_t^\tau \frac{r(v)(1-\gamma)-\rho}{\gamma} dv} d\tau$.

The Euler equation implies that consumption inequality at time 0 is a sufficient statistic for welfare inequality. This is the case because all countries face the same interest rate and have the same utility function.[17] Integrating forward the differential equations, we find that consumption and relative asset ownership are

$$\begin{aligned} c_j(t) &= \xi(t) \left[b_j(t) + \int_t^\infty \pi_j w(\tau) e^{-\int_t^\tau r(v)dv} d\tau \right], \\ b_j^R(t) &= b_j^R(0) e^{-\int_0^t \delta(\tau)d\tau} + \pi_j \int_0^t \delta(\tau) e^{-\int_\tau^t \delta(v)dv} d\tau. \end{aligned}$$

Our choice for the initial distribution of assets implies that there is no change in the relative saving decision $\dot{b}_j^R(t) = 0$, and, thus, consumption inequality at any point in time between two arbitrary countries is proportional to the productivity of the countries

$$\frac{c_j(t)}{c_i(t)} = \frac{\theta_j C(t)}{\theta_i C(t)} = \frac{\theta_j}{\theta_i},$$

where $C(t) \equiv \Theta \xi(t) \left[B(t) + \int_t^\infty w(\tau) e^{-\int_t^\tau r(v)dv} d\tau \right]$. We note that this result would hold in steady-state for any initial distribution of asset holdings.

Finally, the world consumption share of country j at any time t is given by

[17] These results are a direct application of the Ramsey model, see Section A in Caselli and Ventura (2000). Note that $V_j = \int_0^\infty \frac{1}{1-\gamma} c_j(t)^{1-\gamma} e^{-\rho t} dt = \int_0^\infty \frac{1}{1-\gamma} \left[c_j(0) e^{\int_0^t \frac{r(v)-\rho}{\gamma} dv} \right]^{1-\gamma} e^{-\rho t} dt$. Thus, $\frac{V_j}{V_i} = \left[\frac{c_j(0)}{c_i(0)} \right]^{1-\gamma}$.

$$s_j^c(t) \equiv \frac{c_j(t)}{C(t)} = \frac{\theta_j}{\Theta}.$$

The consumption share is the same as the production share, which implies that the trade balance is zero in all countries at any t. This completes the characterization of the model without unbundling.[18]

2.2.2 Equilibrium With Unbundling

Next, we characterize the trade equilibrium with unbundling, in which varieties, intermediates and capital are freely traded.

Definition 2 ***(Equilibrium With Unbundling)*** *A competitive equilibrium with unbundling is defined by a set of prices* $\{r(t), w_j(t), p_j(t), p_j(v,t), p_j(z,t)\}$ *and allocations* $\{l_j(z,t), k_j(z,t), c_j(t), a_j(z,v,t),\ a_j(z,t),\ x_j(v,t)\}$ *for* $j = 1, \ldots, J$, $v \in [0, N]$ *and* $t \geq 0$ *such that, given an initial distribution of assets* $\{b_j(0)\}$*: (i) the representative agent maximizes utility subject to the budget constraint, (ii) final good, variety and intermediate producers maximize profits given prices, (iii) the labor market clears and (iv) net exports of varieties* ***and*** *intermediates equal net imports of capital.*

To derive the equilibrium, we repeat the same steps as above. The demand of varieties is given by (6), as in the previous section. The key difference is that, since intermediates are now costlessly traded, the producer of variety v purchases intermediates from the cheapest location. Thus, the price of variety v at time t is given by

$$\ln p_j(v,t) = \int_0^1 \ln\left(\min_{i\in\{1,\cdots,J\}}\{p_i(z,t)\}\right) dz.$$

This implies that the aggregate demand of intermediate z in country j, rather than coming from the domestic demand as in the equilibrium without unbundling, now comes from the entire world, provided that country j produces z at the cheapest world price.[19] Thus, the mass of intermediates that each country produces is endogenously determined. Denoting by $\mathcal{Z}_j(t)$ the measure of intermediates that country j produces in the unbundling equilibrium at time t, we have that

$$a_j(z,t) = \sum_{i=1}^{J} \mu_i a_j(z,v,t) = \frac{Y(t)}{p_j(z,t)}, \qquad \text{if } z \in \mathcal{Z}_j(t),$$

[18] If the initial asset distribution were different, the trade balance would also be zero in the steady-state because $b_j^R = \pi_j$ in the steady-state (but generically would be non-zero along the transition path).

[19] We are implicitly assuming that each intermediate is produced only by one country, which is indeed true almost everywhere in equilibrium.

and zero otherwise. Note that the price of the final good is the same across countries and we also set it equal to one.

The expressions for the domestic labor market and the international capital market clearing conditions are the same as in the equilibrium without unbundling, (7) and (8), adjusting for the fact that each country only produces a measure $\mathcal{Z}_j(t)$ of the intermediates,

$$\begin{aligned} 1 &= \int_{z\in\mathcal{Z}_j(t)}(1-z)dz\frac{Y(t)}{w_j(t)}, & (9)\\ B(t) &= \sum_{i=1}^{J}k_i(t), & (10)\end{aligned}$$

where $k_j(t)=\int_{z\in\mathcal{Z}_j(t)}zdz\frac{Y(t)}{r(t)}$ for $j=1,\cdots,J$.

To complete the equilibrium characterization, we derive the measure of intermediates $\mathcal{Z}_j(t)$ that each country produces.

Lemma *Country $j\in\{1,\ldots,J\}$ specializes in the production of intermediates $z\in[z_j,z_{j-1})$, where $\{z_i\}_{i=0}^J$ is given by the solution of the recurrence equation (12).*

First note that given that the production of intermediates is Cobb-Douglas, it satisfies the Inada conditions. This implies that all countries employ their labor in equilibrium. Abusing notation and denoting the unit cost of producing intermediate z in country j at time t by $c_j(z,t)$ (this variable is different from the time path of consumption, $c_j(t)$), we have that

$$c_j(z,t)=\theta_j^{-1}w_j(t)^{1-z}r(t)^z.$$

The equilibrium price of an intermediate z is $p(z,t)=\min_{i\in\{1,\cdots,J\}}\{c_i(z,t)\}$. This equation implies that the most capital-intensive intermediate ($z=1$) is produced by country 1, since $c_1(1,t)=\theta_1^{-1}r(t)=\min_i\{c_i\}$ for $\theta_1>\theta_2>...>\theta_J$.[20]

Next, consider an intermediate $\tilde{z}<1$ and suppose that it is produced in countries j and $j+1$ (recall that all countries employ their labor, producing some intermediate with $\tilde{z}<1$). This implies that the production cost is the same in the two countries, $c_j(\tilde{z},t)=c_{j+1}(\tilde{z},t)$. Rearranging this expression, we find that

$$\frac{w_j(t)}{w_{j+1}(t)}=\left(\frac{\theta_j}{\theta_{j+1}}\right)^{\frac{1}{1-\tilde{z}}}. \tag{11}$$

Since $\frac{1}{1-z}$ is an increasing function of z when $\theta_j>\theta_{j+1}$, equation (11) implies that the set of intermediates that country j produces is more capital-intensive than country $j+1$.

[20] In the case that $\theta_1=\theta_2=...=\theta_J$, this is still an equilibrium but there exist other equilibria. See Section 3.1.

To see this result, notice that the cost of producing intermediate z in country j relative to country $j+1$ is decreasing with z, i.e., $\frac{c_j(z,t)}{c_{j+1}(z,t)} = \left(\frac{\theta_j}{\theta_{j+1}}\right)^{\frac{\tilde{z}-z}{1-\tilde{z}}}$ is decreasing in z. Given that the cost of producing intermediate $\tilde{z}$ is the same in country j and $j+1$, it must be the case that the intermediate $\tilde{z}+\varepsilon$ is cheaper to produce in country j if $\varepsilon > 0$. Thus, more productive countries specialize in the production of relatively more capital-intensive intermediates. Online Appendix B shows that this prediction is consistent with trade data in intermediates and reviews the related literature providing additional supporting evidence.

Combining the market clearing condition (9) with (11), we find that the second-order difference equation that determines the pattern of specialization of intermediates is

$$\left(\frac{\theta_j}{\theta_{j+1}}\right)^{\frac{1}{1-z_j}} = \frac{\Delta_j}{\Delta_{j+1}}, \qquad \text{where} \quad \Delta_j(t) \equiv \int_{z_j}^{z_{j-1}} (1-z)dz, \tag{12}$$

with terminal conditions $z_0 = 1$ and $z_J = 0$.

We make the important observation that the solution of the recurrence equation (12) depends only on $\{\theta_j\}_{j=1}^J$. Thus, the equilibrium thresholds $\{z_j\}_{j=0}^J$ are independent of time. This result implies that the distribution of world output across countries is constant over time. In fact, the world output share of country j at time t is simply

$$s_j^y(t) = \frac{y_j(t)}{Y(t)} = Z_j \equiv \int_{z_j}^{z_{j-1}} dz.$$

Finally, to solve the consumer's problem, we write wages of households in country j as $\pi_j w(t)$, with $\pi_j = 2\Delta_j$. We then take the initial distribution of assets to correspond to the steady-state of the equilibrium without unbundling, $b_j(0) = b_j^{without}(ss) = \frac{\theta_j}{\Theta} B^{without}(ss) = \frac{\theta_j}{\Theta} K^{without}(ss)$, where ss denotes steady-state and $K^{without}$ is the aggregate capital stock in the equilibrium without unbundling. Consumption at time 0 and the evolution of relative asset holdings are given by

$$\begin{aligned} c_j(0) &= \xi(0)\left[\frac{\theta_j}{\Theta} K^{without}(ss) + \Delta_j \int_0^\infty Y(t) e^{-\int_0^t r(v)dv} dt\right], && (13) \\ b_j^R(t) &= \frac{\theta_j}{\Theta} K^{without}(ss) e^{-\int_0^t \delta(v)dv} + 2\Delta_j \int_0^t \delta(\tau) e^{-\int_\tau^t \delta(v)dv} d\tau. && (14) \end{aligned}$$

We can also characterize the steady-state trade balance in the unbundling equilibrium. Since in steady-state it is satisfied that $b_j^R = \pi_j$, the trade balance of country j in the steady-state is

$$\frac{TB_j(ss)}{Y(ss)} = Z_j - 2\Delta_j.$$

This equation means that countries that produce a large share of intermediates (high Z_j) have a trade surplus, which they use to pay for the capital imported to produce these goods.

3 Main Results

This section derives the main results of the paper by comparing the equilibria with and without unbundling. We show that GDP and welfare inequality increase with unbundling. Then, we show in Section 3.1 that unbundling generates GDP inequality also in the case of ex-ante identical countries (symmetry breaking). Finally, in Section 3.2, we fully characterize the changes in the world GDP distribution and compute the Lorenz curve for a distribution of productivity that approximates well the observed world distribution.

Welfare, GDP Inequality and Capital Flows The change in consumption inequality between any two countries at time 0 is

$$\left[\frac{c_j(0)}{c_i(0)}\right]^{with} - \left[\frac{c_j(0)}{c_i(0)}\right]^{without} = \frac{\frac{\theta_j}{\Theta} + a\Delta_j}{\frac{\theta_i}{\Theta} + a\Delta_i} - \frac{\frac{\theta_j}{\Theta}}{\frac{\theta_i}{\Theta}},$$

where $a \equiv \frac{\int_0^\infty Y(t)e^{-\int_t^\tau r(v)dv}dt}{K^{without}(ss)}$ and, as reminder, $\Theta \equiv \sum_{i=1}^{J}\theta_i$. From the discussion of the unbundling equilibrium, it is immediate to check that this difference is positive for any pairs of adjacent countries j and $j+1$ since $\Delta_j > \Delta_{j+1}$. The same reasoning applies for any pairs of countries i and j that are non-adjacent.Therefore, since consumption inequality at time 0 increases and welfare is proportional to initial consumption, welfare inequality strictly increases with unbundling of production.

The change in relative output (GDP) between two countries at time t is

$$\left[\frac{y_j(t)}{y_i(t)}\right]^{with} - \left[\frac{y_j(t)}{y_i(t)}\right]^{without} = \frac{Z_j}{Z_i} - \frac{\theta_j}{\theta_i},$$

where Z_j is the equilibrium mass of intermediates produced by country j. Following a similar argument as above, it is immediate to show that this difference is positive for any two adjacent countries, j and $j+1$. Thus, it applies also to non-adjacent pairs countries.

We can also characterize the change in the steady-state distribution of assets between any two countries i and j,

$$\left[\frac{b_j(ss)}{b_i(ss)}\right]^{with} - \left[\frac{b_j(ss)}{b_i(ss)}\right]^{without} = \frac{\Delta_j}{\Delta_i} - \frac{\theta_j}{\theta_i}.$$

From the threshold equilibrium, equation (16), we know that this difference is positive for any two adjacent countries. Thus, the distribution of assets in steady-state becomes more unequal.

Finally, note that the steady-state distribution of assets implies that capital flows increase.

The trade balance in country j is

$$\left[\frac{TB_j(ss)}{Y(ss)}\right]^{with} = Z_j - 2\Delta_j. \tag{15}$$

This trade balance is generically different from zero. In contrast, the trade balance in the steady-state without unbundling is zero. Thus, the more productive countries have a positive trade balance in the steady-state with unbundling. That is, relatively high productivity countries are exporters of goods and importers of capital.[21] We summarize these results in the next proposition.

Proposition 1 *Welfare and GDP inequality increase with unbundling of production. The distribution of assets in the steady state equilibrium with unbundling is more unequal than without unbundling. In steady state, more productive countries are net exporters of goods and importers of capital. Capital flows increase with unbundling.*

As previously discussed, the key mechanism driving these results is that, with unbundling of production, high-productivity countries specialize in capital-intensive intermediates. This increases their demand for capital and triggers capital inflows and/or internal capital accumulation. Unbundling has two opposing effects on labor demand in high-productivity countries. First, the capital intensity of the intermediates produced by high-productivity countries increases. Ceteris paribus, this increases their capital share and reduces labor demand. Second, there is an increase in the capital stock. The resulting higher capital-labor ratio makes labor more productive and enables high-productivity countries to produce more intermediates, which increases labor demand. We have shown in Proposition 1 that the second effect always dominates.[22] This increase in labor demand raises wages and asset holdings in high-productivity countries. As a result, total factor payments increase relatively more in high-productivity countries, increasing GDP inequality. The increase in wages in high-productivity countries also implies a relative increase in domestic consumption and welfare, which increases welfare inequality in the world economy.

The predictions of the model are in line with the behavior of world inequality and capital flows in the last decades. First, world GDP and consumption inequality increased post-1990 (the unbundling period), according to the data constructed by Feenstra et al. (2015). For example, the average top-bottom GDP inequality was 21.2 during the 1980s and it increased

[21] If we had not made the assumption that each country has an initial stock of capital proportional to its productivity, capital flows would be different than zero in the equilibrium without unbundling. In this case, the results discussed in this paragraph would simply imply that high productivity countries import relatively more capital in the unbundling equilibrium.

[22] Section B in the online supplemental material (available at the authors' website) shows that this result does not depend on the Cobb-Douglas assumption in the production function for intermediates. We show that it extends to a general CES production function.

up to 27.8 in the post-1990 period (up to 2008).[23] The same pattern emerges for world consumption inequality. Indeed, the average top-bottom consumption inequality was 15.9 in the 1980s and it increased up to 21.1 in the post-1990 period. Second, the prediction that capital flows increase with unbundling is consistent with the documented boom in international capital flows since the 1990s (see, for example, Lane and Milesi-Ferretti, 2007). In addition, the predicted reallocation of capital towards countries that export capital-intensive intermediates is also consistent with the evidence presented in Jin (2012). To be clear, many other factors have affected the world income distribution and capital flows during this period and empirically disentangling the effects of unbundling from other confounding factors is beyond the scope of this paper.

3.1 The Vanishing-Productivity-Differences World Economy

This section shows that unbundling generates GDP inequality also in the extreme case of ex-ante identical countries (symmetry breaking). An additional contribution of this section is to derive a closed-form solution to a dynamic Heckscher-Ohlin model with J countries as a limit case of our baseline economy.

Consider the particular case of our baseline economy in which we take the limit of all countries' productivity converging to the same level (to be made more precise below). Denoting the common level of productivity across countries, $\theta_j = \theta$, the equilibrium threshold equation (12) reduces to

$$\Delta_j = \Delta_{j+1} \qquad \text{for all} \qquad j = 1, \ldots, J-1.$$

Using the boundary conditions $z_0 = 1$ and $z_J = 0$, we obtain the following results.

Proposition 2 *Consider J identical countries except for their productivities, which are $\theta_j = \theta - j\varepsilon$, with $\varepsilon > 0$, $j = 1, \ldots, J$. Select the equilibrium resulting from taking the limit $\varepsilon \to 0^+$.*[24] *Without unbundling of production, the world production and consumption share of each country are identical and equal to $1/J$. With unbundling of production, country j specializes in the set of intermediates $(z_j, z_{j-1}]$ with $z_j = 1 - \sqrt{\frac{j}{J}}$. This implies symmetry breaking in production. Low index-j countries are net exporters of goods and importers of capital. Welfare inequality and the distribution of assets do not change in the unbundling equilibrium.*

[23] Top-bottom inequality is the ratio between the 90th percentile of GDP per capita (PPP) and the 10th percentile. We choose to finish in 2008 to exclude the effects of the Great Recession. If we include all available information (up to 2014), the top-bottom GDP inequality is 27.1 in the post-1990 period. The same qualitative results hold if we define top-bottom inequality as the ratio between the 95th and 5th percentile.

[24] We discuss in more detail this refinement concept in Section F of the online supplemental material (available at the authors' websites).

The equilibrium threshold z_j is a decreasing and convex function of j. Thus, while all countries produce an equal share of world output in the equilibrium without unbundling, inequality emerges among ex-ante identical countries in the equilibrium with unbundling. Recall that the world production share with unbundling is

$$s_j^y(t) = Z_j = z_{j-1} - z_j = \sqrt{\frac{j}{J}} - \sqrt{\frac{j-1}{J}}.$$

This term is decreasing and convex, which means that countries that specialize in capital-intensive intermediates have a higher output share.[25] Since all countries have the same productivity, changing the allocation of intermediates does not change aggregate world production.[26] Thus, unequal world production shares imply that the level of GDP falls with unbundling for all countries with index $j > \underline{j}$ where $\underline{j}$ is implicitly defined by $\sqrt{\frac{\underline{j}}{J}} - \sqrt{\frac{\underline{j}-1}{J}} = 1/J$.

We can also characterize the Lorenz curve for world output. Using the expression of the world production shares,

$$L_j^y(t) = \sum_{i=J}^{j} s_i^y = 1 - \sqrt{\frac{j-1}{J}}, \qquad j = 1, \ldots, J,$$

which is an increasing and convex function. Note that the ordering of countries for the Lorenz curve is descending in the country index j. It starts with $j = J$, with $L_j^y(t) = 1 - \sqrt{1 - 1/J}$, and it ends at $j = 1$, with $L(1) = 1$. These heterogeneous production shares are in contrast with the complete equality benchmark in the equilibrium without unbundling, which has a linear Lorenz curve, $L_j^y(t) = j/J$.

Using (13), the consumption of country j at time 0 in the unbundling equilibrium is

$$c_j(0) = \xi(0) \left[\frac{1}{J} K^{without}(ss) + \Delta_j \int_0^\infty Y(t) e^{-\int_0^t r(v)dv} dt \right].$$

Since the equilibrium threshold is constant and $\{\Delta_j\}$ are equalized across counties, it follows that initial consumption and overall welfare are the same in all countries. The reason is that unbundling of production changes the allocation of capital but it does not affect relative wages (as implied by equation 12).

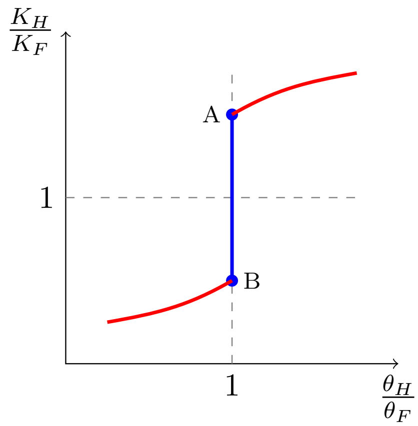
Figure 3 provides a graphical intuition for the symmetry breaking result for the two-country case.[27] It depicts the equilibrium correspondence of k_H/k_F as a function of the relative

[25] The first derivative is proportional to $j^{-1/2} - (j-1)^{-1/2}$, which is negative for $j > 1$. The second derivative is proportional to $-j^{-3/2} + (j-1)^{-3/2}$, which is positive for $j > 1$.

[26] World output with and without unbundling is $Y^{\text{World}} = 2\theta \left(BJ\right)^{\frac{1}{2}}$.

[27] With two ex-ante identical countries H and F, the threshold equilibrium is $z_1 = 1 - \sqrt{1/2}$. It is readily verified that this equilibrium implies that the relative world production share of country H becomes $\left(\frac{s_H^y}{s_F^y}\right)^{with} = \frac{Z_H}{Z_F} = \frac{1-z_1}{z_1} = \frac{1}{\sqrt{2}-1} > 1 = \left(\frac{s_H^y}{s_F^y}\right)^{without}$.

Figure 3: Equilibrium Correspondence for the Two-country Case



productivity, θ_H/θ_F. It shows that the equilibrium is unique for all $\theta_H/\theta_F \neq 1$. The blue segment $\overline{AB}$ corresponds to all possible equilibria for $\theta_H/\theta_F = 1$ (which also corresponds to the factor price equalization set). Thus, the equilibrium correspondence is upper-hemicontinuous but not lower-hemicontinuous at $\theta_H/\theta_F = 1$. Our refinement selects the extremes of the blue segment, denoted by blue dots. In particular, taking the limit $\varepsilon \to 0^+$ for $\theta_H = \theta_F + \varepsilon$, it selects point A in the figure. This makes apparent that our equilibrium selection is right-continuous in θ_H/θ_F at $\theta_H/\theta_F = 1$.[28]

Intuitively, we can see that the equilibrium is unique by realizing that any other interior point in the segment $\overline{AB}$ is unstable to small perturbations to productivity (which is our refinement criterion). For example, suppose that we start with a symmetric equilibrium in which both countries produce all intermediates in the same amount. Consider a small positive perturbation to the productivity of H. By being slightly more productive, H gains comparative advantage in capital-intensive intermediates and it specializes in the production of capital-intensive intermediates, thereby raising the demand for capital. Thus, capital flows from F to H, which further reinforces the comparative advantage of H in capital-intensive intermediates, and so on until point A is reached.

The mechanism of our symmetry breaking result is different from previous results in the literature, which have emphasized economies of scale in production or credit market imperfections as key mechanisms, e.g., Krugman and Venables (1995) and Matsuyama (2004, 2013). The key insight from our model is that the interaction of trade in goods heterogeneous in capital intensity and an elastic capital supply generates symmetry breaking in production. We note that perfect capital mobility (as we have assumed in this baseline model) is not necessary for the result. Section E in the online supplemental material (available at the

[28]There exists an analogous equilibrium that corresponds to the permutation of H and F, $\theta_F = \theta_H + \varepsilon$, $\varepsilon \to 0^+$, which corresponds to point B in the figure.

authors' website) shows that the steady-state of a dynamic model without capital flows also delivers symmetry breaking in production. Differently from Krugman and Venables (1995) or Matsuyama (2013), this model does not need a positive trade cost or non-traded goods to have symmetry breaking in production.[29]

In sum, this symmetry breaking result illustrates how the combination of trade in goods heterogeneous in capital intensity and an elastic capital supply magnifies arbitrarily small productivity differences across countries. This result underlines how trade in intermediates begets inequality in the allocation of capital and production in the world economy

3.2 The World GDP Distribution Under a Parametric Example

In order to fully describe the world output distribution and compute the world Lorenz curve, we need to fully characterize the pattern of specialization in intermediates. Equation (12) defines the equilibrium assignment of countries to the production of intermediates. Unfortunately, this equation is not analytically solvable in general. To make progress, we take the same approach as in Matsuyama (2013) and approximate the solution to the case in which the number of countries is very large, $J \to \infty$, so that we effectively have a continuum of countries $j \in [0, \bar{j}]$ (see details in online Appendix G). In this case, equation (12) converges to the second-order differential equation

$$(1 - z(j))\frac{z''(j)}{z'(j)} - z'(j) = \frac{\theta'(j)}{\theta(j)}, \tag{16}$$

with terminal conditions $z(0) = 1$ and $z(\bar{j}) = 0$. It can be verified that the assignment function $z(j)$ is decreasing and convex.[30]

Equation (16) governing the assignment process is a non-linear differential equation, which cannot be characterized in analytic form without making parametric assumptions on $\theta(j)$. We specialize $\theta(j)$ to be a function that approximates the data well. Our theory suggests that θ_j can be obtained by studying how TFP varies across countries or, alternatively, the world GDP per capita distribution without unbundling, which is also proportional to θ_j. Figure 4 shows the relationship of log TFP and log GDP per capita in 1988 as a function of the country ranking index j. The figure also depicts the linear fit between them. We find that this linear fit is remarkably good, which implies an exponential relationship of both variables with the country index. The R^2 of log TFP on the country ranking is .97, and .98 for log GDP per capita.[31] Thus, we proceed making the following assumption.

[29] Section A in the online supplemental material (available at the authors' websites) shows that the symmetry breaking result also extends to the intermediate case in which only a fraction $\alpha > 0$ of intermediates are traded.

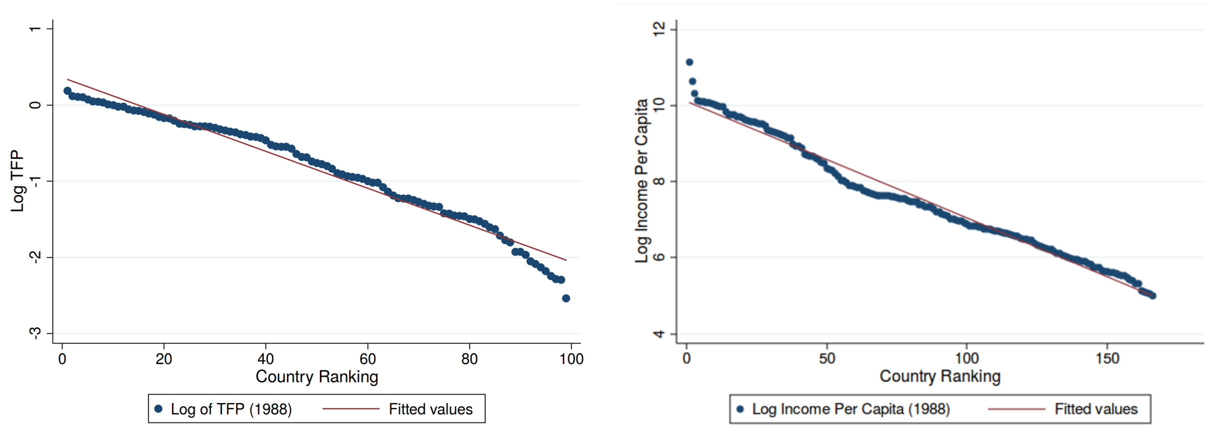
[30] From the equilibrium assignment in the discrete case, we know that more productive countries specialize in capital-intensive (higher index z) intermediates, $z'(j) < 0$. Thus, $\theta'(j)z'(j) > 0$. Rearranging (16),we find that $z(j)$ is convex, as $z''(j) = (1 - z(j))^{-1}(\theta'(j)z'(j)/\theta(j) + z'^2(j)) > 0$.

[31] This fit is better than a "power law" where we regress the log of TFP or country GDP per capita on the

Figure 4: Log TFP and Log GDP per capita on Country Ranking

(a) Log TFP

(b) Log GDP per capita



Notes: TFP and GDP per capita (PPP adjusted) are from Hall and Jones (1999) and the World Bank.

Assumption 1 *Countries' productivity $\theta(j)$ is related to the country index by*

$$\theta(j) = \lambda \exp(-\lambda j), \qquad j \in [0, \infty), \qquad \lambda > 0. \tag{17}$$

Note that this assumption implies a linear relationship between the country index j and the logarithm of θ as in the linear fit depicted in Figure 4. The most productive country, $j = 0$, has productivity level $\theta(0) = \lambda$ and productivity is decreasing in j. Given this particular functional form, the differential equation (16) becomes

$$(1 - z(j))\frac{z''(j)}{z'(j)} - z'(j) = -\lambda,$$

with terminal conditions $z(0) = 1$ and $z(\infty) = 0$.[32] We can characterize the inverse of the assignment function,

$$j(z) = \frac{z - \ln z - 1}{\lambda}, \tag{18}$$

which is monotonically decreasing in z. It is possible to invert this function and obtain $z(j)$,

log of the country ranking. In this case, the log-log regression yields an R^2 of 0.86 and 0.69, respectively, see Table I.1 in the online appendix for the estimation results. We can also compute the solution of the differential equation for the power-law case, i.e., $\theta(j) \propto j^{-\upsilon}$ for some $\upsilon > 0$. In this case, the solution is not more tractable than with an exponential. The choice of 1988 is given by our data source, Hall and Jones (1999), which report TFP data for this year. We choose this source because it roughly coincides with the increase in trade in intermediates documented in Figure 1a.

[32] We would obtain the same differential equation if we allowed the constant in equation (17) to be different from the exponential decay λ, i.e., $\theta(j) = \tilde{\lambda} e^{-\lambda j}$. Using $\tilde{\lambda} = \lambda$ allows us to economize on notation in the computation of the Lorenz curves below.

although the expression involves a transcendental function,

$$z(j) = -W(-\exp(-1-\lambda j)), \tag{19}$$

where $W(z)$ is the Lambert $W-$function, defined as the real solution of $z = xe^x$ for x. We summarize the properties of the assignment function in the next remark.

Remark 1 *The assignment function $z(j;\lambda)$ is continuously decreasing and convex in j and λ. The cross-partial derivative $z_{j,\lambda}$ is negative for all $j < \tilde{j}(\lambda)$ and positive for $j > \tilde{j}(\lambda)$.*

After deriving the solution for the assignment function, we compare the distribution of world GDP in the two trade equilibria. In the equilibrium without unbundling, the world production share of country j is $\mu(j)/\int \mu(j)dj$ and, in the equilibrium with unbundling, it is $-z'(j)$. Integrating these shares, we obtain the Lorenz curves in the equilibrium with and without unbundling

$$L_j^y(t)^{without} = \int_j^\infty \lambda e^{-\lambda j} dj = e^{-\lambda j},$$
$$L_j^y(t)^{with} = \int_j^\infty -z'(j)dj = z(j),$$

where $z(j)$ is given by (19). Comparing the two Lorenz curves, we find that $L_j^y(t)^{without} > L_j^y(t)^{with}$ for all j.[33] Thus, the world distribution of output is more unequal with unbundling, as measured by the Lorenz curve. An analogous result follows if we analyze the ratio of world output shares in the equilibrium with and without unbundling.[34]

To better understand how the world output distribution changes throughout its support, we analyze the change in production shares country by country. Rewriting the change in production shares as a function of the equilibrium assignment of intermediates $j(z)$, (18), we obtain

$$\Delta s^y(z) = z\lambda\left(\frac{1}{1-z} - e^{1-z}\right).$$

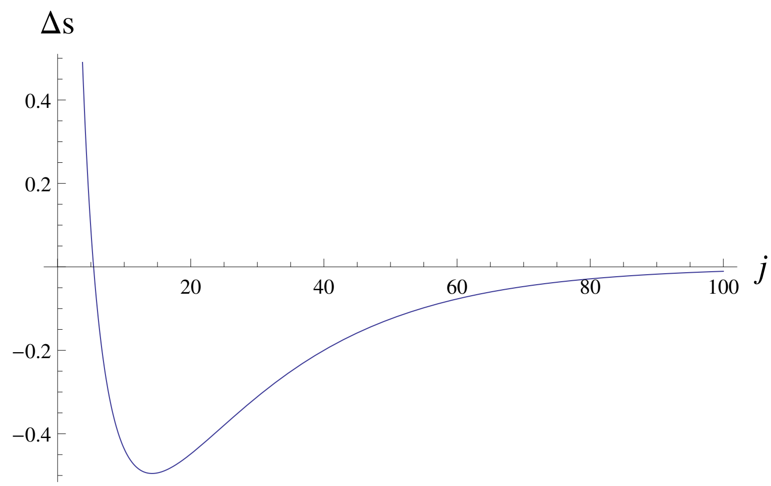
The change in world output share is negative for $z \in (0,\bar{z})$ and positive for $z \in (\bar{z},1]$. Thus, the world output share declines in the countries assigned to the intermediates $z < \bar{z}$ and it increases in the rest. The next proposition characterizes the change in the world output distribution as a function of fundamentals, rather than the endogenous variable z.

Proposition 3 *The change in the output share from the equilibrium without unbundling to the*

[33] To see this, rewrite the Lorenz curves in terms of the assignment $j(z)$, (18), so that $L(z)^{without} = ze^{1-z}$ and $L(z)^{with} = z$, and the result follows.

[34] In this case, the ratio $s^{y,\text{with}}(j)/s^{y,\text{without}}(j)$ is a monotonically decreasing function, positive for j lower than a threshold and negative thereafter. This means that there is a strict ranking in the percentage increase of world output share, being highest for the most productive country and decreasing thereafter.

Figure 5: Change in World Output Shares



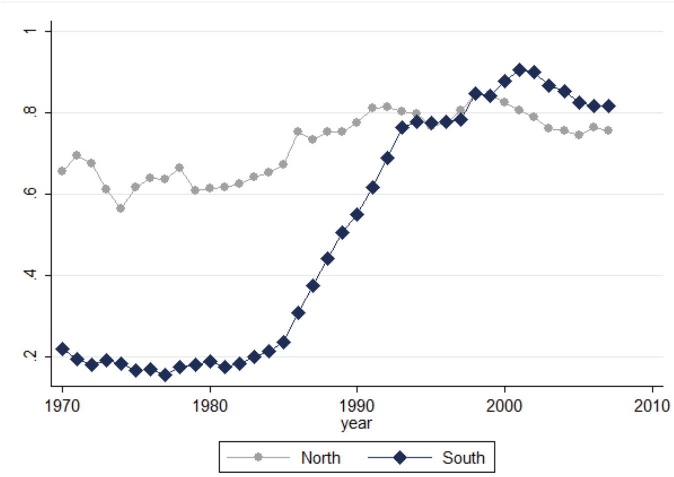
equilibrium with unbundling, $\Delta s^y_j(t)$*, (i) is independent of time, (ii) it is continuous in* j*, (iii) it is decreasing in* j *for* $j < j_-$ *and increasing thereafter, with* $j_- = \lambda^{-1}\left(-3W(3) - \ln\left(1 - 3W(3)\right)\right)$*, (iv) it is convex for* $j < j_c$ *and concave thereafter, with* $j_c < j_-$ *and (v)* $\Delta s^y(0) = \infty$*,* $\Delta s^y(\infty) = 0$ *and* $\Delta s^y(\lambda^{-1}\left(-W(1) - \ln\left(1 - W(1)\right)\right)) = 0$*.*

This proposition implies that *(i)* top-bottom inequality increases with unbundling and *(ii)* the production share falls relatively more in middle-productivity countries. Figure 5 illustrates a generic case. Without unbundling of production, the demand of capital is determined by the number of varieties a country produces. In contrast, with unbundling, the demand of capital is determined by the intermediates in which the country specializes. The most productive country increases its capital stock the most (through capital inflows and internal accumulation) and total output. Low-productivity countries specialize in low-capital-intensive intermediates. In absolute terms, they are not the ones that lose the most, because they have a small amount of capital to begin with. The main losers are middle-productivity countries. These countries operated a sizable amount of capital in the equilibrium without unbundling. However, they compete against more productive countries for the production of intermediates in the equilibrium with unbundling. They end up specializing in relatively low-capital-intensive intermediates and, thus, receive and accumulate less capital and have a lower output share. Finally, we note that this mechanism provides a rationale for why globalization can make middle-income countries deindustrialize prematurely, as recently suggested by Rodrik (2016).

4 Applications

In this last section, we use our model to explore how unbundling interacts with other relevant features of the world economy. Section 4.1 studies the effects of southern countries joining the

Figure 6: Ratio of Value of Exported Intermediates to Final Goods.



Source: Feenstra World Trade Database. To classify goods as intermediates, we use the end-use classification of Feenstra and Jensen (2012). Southern countries are defined as countries with GDP per capita (PPP) lower than 50 percent of the United States in 2000.

global supply chain. Section 4.2 analyzes the effects of a labor saving technology, computerization, on inequality. Section 4.3 studies how the interaction between cross-country distribution of technology and unbundling affects the world output distribution. Section 4.4 analyzes how unbundling is related to changes in the labor share and TFP accounting. Finally, Section 4.5 discusses the aggregation properties of our model and the transition between steady-states.

4.1 South Joins the Global Supply Chain

One important factor in the increasing prominence of trade in intermediates is that southern countries have joined the global supply chain (e.g., Baldwin, 2012). Figure 6 reports evidence supporting this view. It decomposes the ratio of exported intermediates to final goods between northern and southern countries. Trade in intermediates increased in both northern and southern countries after the late 1980s, but most of the aggregate increase comes from southern countries. For southern countries, the ratio was roughly constant around .2 before the 1990s. Then, it sharply increased and it has converged to around .8 in the late 2000s.

Motivated by this evidence, we analyze the effect on the world output distribution of southern countries joining the global supply chain. Consider a world of J countries and define as South the set of countries with a productivity level θ below $\underline{\theta}$. We compare two equilibria. *(i) Before* the South joins the global supply chain: all countries trade in varieties but only countries with productivity θ above $\underline{\theta}$ can trade intermediates. *(ii) After* the South joins the global supply chain: all countries trade in both varieties and intermediates.

The equilibrium *after* southern countries join the global supply chain is the same as in the baseline model. The output share of each country j is given by $s_j^{after} = -dz^{after}/dj$, where the assignment of intermediates to countries is given by equation (19).

The equilibrium output share *before* the South joins the global supply chain is a piece-wise continuous function that specifies the production share for northern and southern countries separately. Southern countries are those with low productivity levels, that is, countries $j > \underline{j}$, where $\underline{j} \equiv \frac{1}{\lambda} \ln\left(\frac{\lambda}{\underline{\theta}}\right)$. As southern countries only trade varieties, their output shares, implied by the trade balance condition, are

$$s_j^{before} = \theta(j) = \lambda \exp(-\lambda j), \qquad \text{for } j > \underline{j}.$$

Northern countries trade both varieties and intermediates. The trade balance of each northern country $j < \underline{j}$ implies that[35]

$$s_j^{before} = -\frac{dz^{before}}{dj}\left(1 - \frac{\int_{\underline{j}}^{\infty} \mu(j)dj}{\int_0^{\infty} \mu(j)dj}\right), \qquad \text{for } j < \underline{j}, \tag{20}$$

where z^{before} is the equilibrium assignment of intermediates when only northern countries trade intermediates.

Therefore, we need to derive the equilibrium assignment of intermediates to compute the output share of northern countries before the South joins the global supply chain. To derive the assignment, we proceed in an analogous way as in Section 3 and solve equation (16) with the terminal condition $z(\underline{j}) = 0$. That is, the South (countries with $j > \underline{j}$) does not participate in intermediates trade. The equilibrium assignment is given by

$$j = -\frac{1 - z^{before}}{\lambda} - \frac{C_1^*(\underline{j})}{\lambda^2} \ln\left(1 - \frac{\lambda(1 - z^{before})}{C_1^*(\underline{j})}\right),$$

where $C_1^*(\underline{j})$ is an integrating constant. We show in Online Appendix H.1 that $z^{before}(j; \underline{j})$ is decreasing in $\underline{j}$. This is illustrated in Figure 7a for two different $\underline{j}$. It means that if there are more countries participating in intermediates trade ($\underline{j}$ larger), each northern country specializes in more capital-intensive intermediates (higher z). Finally, note that, by definition, $z^{before}(j; \underline{j} = \infty) = z^{after}(j)$.

We can write the change in the world output distribution when the South joins the global

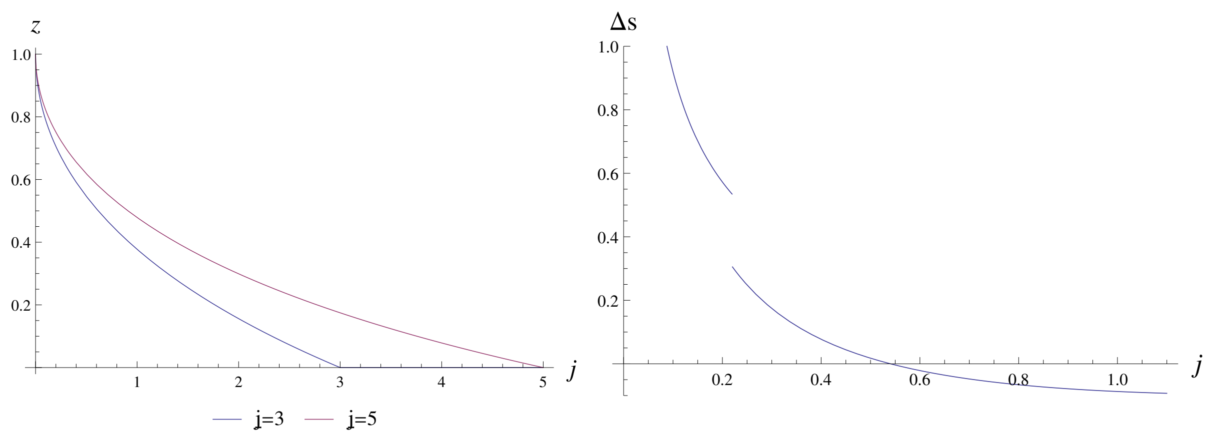
[35] For the case of a discrete number of countries, denoting by η_ω the amount of varieties produced by southern countries (i.e., $\eta_\omega = \sum_{\omega=\underline{j}}^{J} \mu_\omega$), the trade balance of northern countries j becomes

$$\frac{\mu_\omega}{N}(Y - y_\omega) + Z_\omega \frac{N - \eta_\omega - \mu_\omega}{N} Y = \frac{N - \mu_\omega}{N} y_\omega + (1 - Z_\omega)\frac{\mu_\omega}{N} Y.$$

Rearranging, $s_\omega^{before} = Z_\omega^{before}\left(1 - \frac{\eta_\omega}{N}\right)$. Taking the limit to a continuum of countries, the expression becomes (20).

Figure 7: South joins the Global Supply Chain

(a) Change in Equilibrium Assignment with $\underline{j}$ (b) Change in the World Output Distribution



supply chain as

$$\Delta s_j = \begin{cases} -z'(j) - \lambda e^{-\lambda j} & \text{if } j > \underline{j} \text{ (Southern country)}, \\ -z'(j) + z'(j;\underline{j})(1 - e^{-\lambda \underline{j}}) & \text{if } j < \underline{j} \text{ (Northern country)}. \end{cases}$$

Proposition 4 *When the South joins the global supply chain, all northern countries increase their output shares. If $\underline{j} < j^*$, southern countries with $j \in [\underline{j}, j^*]$ increase their output share and the rest decrease their share, where $j^* \equiv -\lambda^{-1}(W(1) + \ln(1 - W(1)))$. If $\underline{j} > j^*$, the output share of all southern countries declines.*

The reason for these results is as follows. For southern countries, we have the same comparison as in Section 3. Their world output shares increase if they specialize in intermediates yielding to more capital accumulation. Therefore, if the country is productive enough, it produces enough intermediates and accumulates more capital participating in the global supply chain, thereby increasing its production share. For northern countries, there are two effects that go in the same direction. First, a selection effect: they produce less intermediates but they are more capital-intensive (which we already argued increases labor demand). Second, a market size effect: northern countries sell intermediates to all countries, not only in the North. Thus, the overall effect is positive because northern countries specialize in more capital-intensive intermediates and sell them to a bigger market. Figure 7b illustrates the change in the world output distribution.

In this section, we have assumed, for simplicity, that southern countries either fully participated or did not participate in intermediates trade. In Section C of the online supplemental material (available at the authors' website), we relax this assumption and we assume that a fraction $\alpha(j)$ of a country participates in intermediates trade, where $\alpha(j)$ is a decreasing function of j. We show numerically that the same qualitative results hold. As $\alpha(j)$ increases, the world output share increases in countries with $j < j^*$ and it decreases in the rest. Note that this section could explain why relatively productive Southern countries, like China or India, can benefit from joining the global supply chain.

4.2 Computerization

The adoption of Information Technologies has been pointed out as one important reason behind the unbundling of production (see, for example, Basco and Mestieri, 2013). This extension analyzes how the effects of computerization on the world output distribution depend on the trade regime.

As discussed in equation (2), a bundle of intermediates of different labor intensity must be assembled to produce a variety v,

$$x_j(v) = \exp\left[\int_0^1 \beta(z) \ln a_j(z,v) dz\right],$$

where $\beta(z)$ is a weight on intermediate z, with $\int_0^1 \beta(z)dz = 1$. We model computerization as a shift in the weighting function $\beta(z)$ that reduces the weight of labor-intensive (low z) intermediates.

More precisely, we assume that the distribution $\beta(z)$ has a monotonically decreasing probability ratio (MPR), where the probability ratio is defined as

$$\mathcal{I}(z) \equiv \frac{\beta(z)}{ß(z)},$$

and $ß(z)$ denotes the cumulative distribution of z.[36] We assume that computerization induces a shift in $\beta(z)$ that can be ranked in terms of the probability ratio. Supposing that χ indexes computerization, we assume that $\mathcal{I}(z;\chi)$ is monotonically increasing in χ. Eeckhoudt and Gollier (1995) show that a monotone increase in the probability ratio implies a first-order stochastic dominant shift.[37] Accordingly, we define computerization as an increase in χ. That is, an increase in χ implies that, ceteris paribus, relatively less labor-intensive intermediates are needed to produce each variety.

For example, one family of distributions satisfying the MPR ordering is given by

$$\beta(z) = \begin{cases} 0 & \text{if } z < \chi, \\ \frac{1}{1-\chi} & \text{if } z \in [\chi, 1], \end{cases} \tag{21}$$

[36] This condition has been applied in other economic contexts, see Hopkins and Kornienko (2004) and the references therein. The normal, uniform and exponential distribution among other distributions satisfy this condition.

[37] Moreover, they also show that a Monotone Likelihood Ratio (MLRP) order implies MPR. Thus, MPR is more stringent than first-order stochastic dominance but less stringent than MLRP.

where χ is the index of computerization. When $\chi = 0$, there is no computerization and $\beta(z) = 1$, as we assumed in the baseline model. For $\chi > 0$, the most labor-intensive intermediates $z < \chi$ are no longer required to produce varieties.

In the equilibrium without unbundling, the output share of each country depends only on the number of varieties and it is given by $s_j^y = \mu_j / \int_{j \in \mathcal{J}} \mu_j dj$, which is independent from the weighting function $\beta(z)$.

To analyze the equilibrium with unbundling, note that computerization changes the equilibrium assignment. Proceeding as in Section 3.2, the assignment function is characterized by the following differential equation

$$(1 - z(j)) \left(\frac{z''(j)}{z'(j)} + z'(j) \frac{\beta'(z(j))}{\beta(z(j))} \right) - z'(j) = \frac{\theta'(j)}{\theta(j)} = -\lambda.$$

Note that $\beta(z)$ enters into the assignment function through its semi-elasticity, $z'(j)\beta'(z)/\beta(z)$.

The solution to this differential equation with boundary conditions $z(0) = 1$ and $z(\infty) = 0$ is given by[38]

$$j(z) = \frac{1}{\lambda} \int_z^1 \mathcal{I}(x, \chi)(1 - x) dx.$$

The output share in terms of z is

$$s^y(z) = \frac{\lambda}{\mathcal{I}(z, \chi)(1 - z)}. \tag{22}$$

With unbundling, computerization changes the world output distribution. From equation (22), we see that χ affects the output shares through the inverse probability ratio, $\mathcal{I}(z, \chi)$, and the equilibrium assignment $z(j(\chi))$. On the one hand, by assumption, $\mathcal{I}(z, \chi)$ is increasing in χ, which reduces the output share. On the other hand, $z(j(\chi))$ increases with χ, each country j is now assigned to a higher z intermediate, which raises the output share.[39] Therefore, the overall effect on the output share (22) is ambiguous. The next proposition shows how it depends on the country ranking.

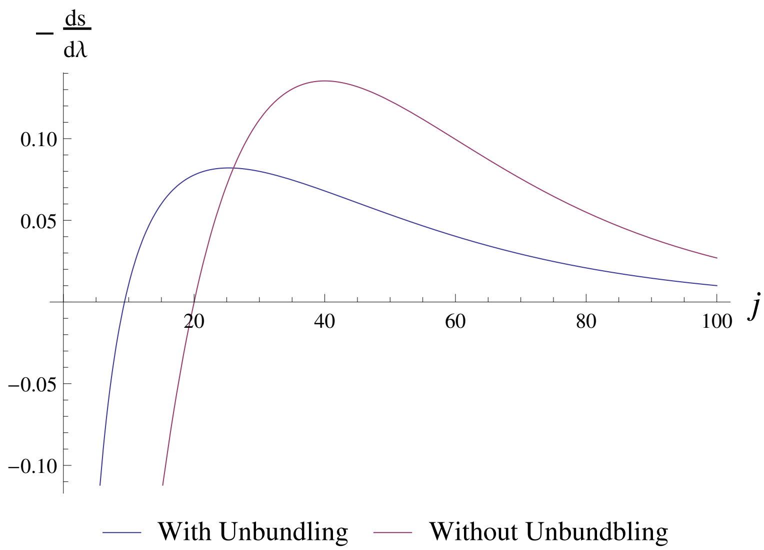
Proposition 5 *In the equilibrium without unbundling, computerization does not affect output inequality between countries. In the equilibrium with unbundling, computerization increases the output share for countries with $j \in [0, j_1)$ and decreases it for countries with $j > j_2$ where $j_1 \leq j_2$. If $\beta(z)$ is given by equation (21), an increase in χ increases (decreases) the output share of countries $j < (>) j^*$ for some $j^* \in (0, \infty)$.*

Proposition 5 implies that top-bottom inequality unambiguously increases. The reason is

[38]Note that if $\beta(z) = 1$, we obtain that $j(z) = \lambda^{-1}(z - \ln z - 1)$ as in the baseline model. Also, note that, for simplicity, we are reporting the case in which the support of intermediates remains $[0, 1]$. Online Appendix H.2 discusses the case when $\beta(z)$ takes the form of (21), in which the support changes with computerization.

[39]Note that $j(z, \chi)$ increases monotonically with an increase in $\mathcal{I}(z, \chi)$.

Figure 8: Change in the World GDP Distribution with a change in λ



that all countries specialize in more capital-intensive intermediates. However, this shift in the pattern of specialization disproportionately favors the most productive countries, which can now specialize in even more capital-intensive intermediates. This is the reason why the output share rises at the top. The least productive countries do not benefit from computerization because $\beta(z)$ does not change much at the extreme of the distribution.

To sum up, in this section we have shown that the effects of computerization on the world output distribution depend on the trade regime. Without unbundling, computerization does not change the relative output of countries. In contrast, with unbundling, computerization leads all countries to specialize in more capital-intensive intermediates, which exacerbates GDP differences between countries.

4.3 Cross-country Distribution of Technology

The source of comparative advantage in our model is technology. In the baseline model, we assumed that technology is exogenous and constant. However, technology diffuses over time and low-productivity countries eventually learn the innovations that the countries in the technological frontier make. In this section we analyze how the changes in the distribution of technology affect the world GDP distribution with and without unbundling.

We assumed, consistent with the data, that productivity decays exponentially on the country index,

$$\theta(j) = \lambda \exp(-\lambda j).$$

We model technological catch-up of low-productivity countries as a decline in the parameter

λ from λ_1 to $\lambda_2 < \lambda_1$. This implies a first-order-stochastic-dominant (FOSD) shift in the distribution of productivities in the world.

Proposition 6 *The first-order-stochastic-dominant shift induced by a decline in the technology parameter λ leads to convergence in GDP with and without unbundling. Moreover, the GDP share increases more in low-productivity countries when there is unbundling of production.*

These results are illustrated in Figure 8. Without unbundling of production, the output share of country j is $s_j^{without} = \theta(j)$. Note that changes in productivity directly affect the output share. It is then straightforward to see that the output share increases in low-productivity countries ($j > \bar{j}$) and declines in the rest ($j < \bar{j}$).[40] Therefore, we have convergence in GDP shares.

With unbundling of production, the output share of country j is $s_j^{with} = -z'(j)$. It means that productivity affects the share through the endogenous assignment of intermediates. To understand the effect of a change in the world distribution of technology, first notice how the assignment function changes,

$$\Delta j = \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)(z - \ln z - 1) > 0.$$

This change in the assignment function implies that low-productivity countries are "climbing up the ladder of global supply chains" by producing higher z intermediates. This new selection of intermediates results in an increase in the output share of low-productivity countries ($j > j^\dagger$) and a decline in the rest ($j < j^\dagger$). The reason is that, due to the FOSD shift of technology, low-productivity countries can now produce more intermediates, thereby increasing their output share. This result implies that FOSD shifts of technology lead to GDP convergence. Finally, we compare the changes in the world GDP distribution under the two trade regimes. It can be checked that[41]

$$\frac{\partial s_j^{with}}{\partial \lambda}\Big|_{j=\bar{j}} < 0.$$

This inequality implies that $\bar{j} > j^\dagger$, which means that in the equilibrium with unbundling the output share increases for a larger mass of low-productivity countries. In particular, the output share of countries with $j \in \left(j^\dagger, \bar{j}\right)$ rises with unbundling but falls without unbundling. The intuition is that, in the equilibrium with unbundling, the relative productivity (not the absolute level) determines the assignment of intermediates. The slope of the distribution of productivities flattens with the FOSD shift of technology, which results in countries

[40] Using that $\theta(j)$ follows an exponential decay (equation (17)), the threshold $\bar{j}$ is $\bar{j} = \frac{\ln\left(\frac{\lambda_1}{\lambda_2}\right)}{\lambda_1 - \lambda_2}$.

[41] See online appendix H.3 for a derivation and further characterization of the change in the world output distribution.

with productivity $j \in \left(j^{\dagger}, \overline{j}\right)$ gaining comparative advantage against nearby more-productive countries. This allows them to climb the supply chain ladder and produce relatively more capital-intensive intermediates.

To sum up, in this section we have shown that FOSD shifts of technology lead to convergence in GDP under the two trade regimes. However, the mass of low-productivity countries benefiting from technological catch-up is larger in the trade equilibrium with unbundling.

4.4 TFP Accounting and the Labor Share in a World with Unbundling

An important question in the development and economic growth literature is how much of the cross-country variation in output can be explained by differences in endowments and aggregate productivity (TFP). In the traditional TFP accounting exercises, e.g., Caselli (2005), it is assumed that the capital share is constant and identical across countries. Our model underlines the importance of allowing for heterogeneous capital and labor shares across countries when performing TFP accounting in a world with an increasing fragmentation of production.

In our model, the aggregate production function of countries changes with unbundling[42]

$$\begin{aligned} y_j^{without} &= f(\theta_j) k_j^{\alpha}, \\ y_j^{with} &= g(\theta_j) k_j^{\alpha_j}. \end{aligned}$$

Thus, through the lens of our model, the traditional TFP accounting exercise would only be correct in a world without unbundling. The reason is that, when only varieties are traded, the capital share of the aggregate production function is the same in all countries because each country produces all intermediates. However, when there is unbundling, the aggregate production function of each country is different and depends on the endogenous selection of intermediates. The most productive country has a larger capital share, α_j. If we assumed homogeneous capital shares to perform a growth accounting exercise, we would be underestimating how much of the output variation can be accounted for by differences in capital stocks and overestimating the dispersion in TFP. [43]

Using data from the Penn World Table (Feenstra et al., 2015), we find that the correlation between labor shares and aggregate TFP is consistent with the prediction of the unbundling equilibrium: countries with high TFP levels experience a larger decline in the labor share.

[42] Online Appendix E derives these aggregate production functions. We show that $f(\theta_j) \equiv 2\theta_j$, $\alpha = 1/2$, $g(\theta_j) \equiv \theta_j \left[\exp \int_{z \in \mathcal{Z}_j} \ln \left[\frac{\Delta_j^{1-z}}{\left(Z_j - \Delta_j\right)^z}\right] dz\right]$ and $\alpha_j \equiv Z_j - \Delta_j$. We also show that α_j is increasing with the productivity of the country, which directly follows from Proposition 1.

[43] This accounting exercise with heterogeneous capital shares has recently been done by Feenstra et al. (2015), who document sizable differences from traditional TFP accounting. For example, the fraction of the variance of GDP explained by the variance of inputs (in 2005) sharply increases when allowing for heterogeneous shares from 0.25 to 0.34.

Table 1: Labor Share and TFP, 1990-2008

Dep. Var.: Log Labor share	(1)	(2)	(3)	(4)
Log TFP	-0.13	-0.10	-0.11	-0.09
	(0.05)	(0.03)	(0.03)	(0.04)
Log Income				-0.03
				(0.05)
Country FE	N	Y	Y	Y
Year FE	N	N	Y	Y
Observations	2203	2203	2203	2203
Within R^2	0.06^a	0.05	0.07	0.07

Standard errors clustered at the country level shown in parenthesis. *a:* this number corresponds to the R^2 of the OLS regression.

Table 1 illustrates this correlation by running the following regression for the period 1990-2008,

$$\log(\text{labor share})_{ct} = \beta \log(TFP)_{ct} + \delta_c + \delta_t + \Upsilon X_{ct} + \epsilon_{ct}, \tag{23}$$

where $\log(TFP)_{ct}$ denotes the logarithm of TFP in country c at time t, δ_c and δ_t denote country and year fixed effects, X_{ct} denotes additional controls and ϵ_{ct} is the error term. We find that the coefficient on log-TFP is negative and significant across the board. For example, the specification in column (4), which includes year and country fixed effects and controls for aggregate income (GDP), implies an elasticity of -9%. This elasticity implies that an increase in one standard deviation of TFP (0.31) corresponds to a 28% fall of the labor share.[44]

4.5 World Aggregation and Transitional Dynamics

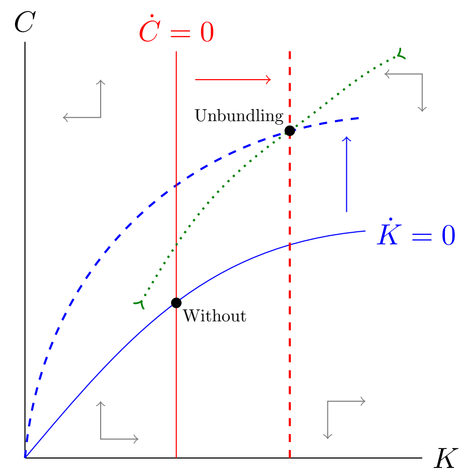
In this section, we show that world aggregates in our multi-country dynamic model behave as a particular case of the Ramsey model. We use this result to then analyze the transition between steady-states for world aggregates and individual countries when unbundling of production becomes possible.

We show in Online Appendix F that world output can be expressed as a production function, $Y = \sum_{j=1}^{J} y_j$, that depends on the world stock of capital, $K = \sum_{j=1}^{k} k_j$, and the world distribution of TFP, $\{\theta_j\}_{j=1}^{J}$. The expression for world output *with* and *without* unbundling can be written as

$$\begin{aligned} Y^{without}(t) &= \phi(\theta) K(t)^{\frac{1}{2}}, \\ Y^{with}(t) &= \psi(\theta, z^*) K(t)^{\frac{1}{2}}, \end{aligned}$$

[44]We obtain similar results if we use long differences of 5-years or if we use the initial TFP level in 1990 as a regressor instead of $\log(TFP)_{ct}$.

Figure 9: Transitional Dynamics of the World Aggregate



where $\phi(\theta) \equiv 2\Pi_{j=1}^{J}\theta_j^{\frac{\theta_j}{2}}$ and $\psi(\theta, z^*) \equiv 2^{\frac{1}{2}}\Pi_{j=1}^{J}\theta_j^{z_j}\Delta_j^{-\Delta_j}$.

We want to emphasize three properties of these aggregate production functions. First, note that they only differ on the aggregate productivity term $\phi(\cdot)$ and $\psi(\cdot)$. Second, this productivity term is constant. The reason is that without unbundling all countries produce all intermediate goods and, with unbundling, the endogenous selection of intermediates is independent of time. Third, unbundling of production is akin to an increase in world productivity, i.e., $\phi(\theta) \leq \psi(\theta, z^*)$. The intuition for this last result is that trade in intermediates allows a more efficient allocation of production.[45]

We can also aggregate the demand side since all households face the same prices (this step is analogous to Caselli and Ventura, 2000). Using these two aggregation results, the dynamics for aggregate consumption and capital are given by

$$\begin{aligned} \frac{\dot{C}(t)}{C(t)} &= \frac{Y_K(t) - \rho}{\gamma}, \\ \dot{K}(t) &= Y(t) - C(t), \end{aligned}$$

where $Y_K(t) \equiv \frac{\partial Y(.)}{\partial K}$. These equations are a particular case of the Ramsey model, whose dynamics and saddle-path stability are well understood. Figure 9 reports an example of the phase diagram. The steady-state is the point in which the red and blue lines intersect, $\dot{C} = \dot{K} = 0$. The vertical solid line corresponds to $\dot{C} = 0$, which determines the world capital steady-state. The concave solid line corresponds to the condition $\dot{K} = 0$. There exists a unique saddle-path stable arm that connects any initial allocation of capital with the steady-state

[45] The proof is immediate and follows from the fact that our competitive equilibrium replicates the social planner allocation. If the social planner is allowed a thinner geographical allocation of inputs, aggregate output cannot decrease.

(denoted by the dotted line in the figure).

In the same figure, we represent the transition between steady-states when unbundling becomes possible. Suppose that when unbundling becomes possible, the economy is in the steady-state without unbundling (denoted by the dot labeled "without" in the figure). Graphically, unbundling represents an outward shift to the $\dot{K} = 0$ line and a parallel shift to the right of $\dot{C} = 0$ (denoted by arrows in the figure). The new long-run steady state is determined by the intersection of the dashed lines (denoted by the dot labeled "unbundling" in the figure). The transition path is given by the dotted line. As in the standard Ramsey model, the effect on consumption at impact depends on the elasticity of intertemporal substitution. However, it is unambiguous that in the steady-state with unbundling both consumption and capital are higher than in the steady-state without unbundling.

Finally, we turn to the transitional dynamics of individual countries. From the capital market clearing condition, it is immediate to show that the evolution of capital in country j with and without unbundling follows

$$\begin{aligned} k_j^{without}(t) &= \frac{1}{2}\frac{\mu_j}{N}K(t), \\ k_j^{with}(t) &= 2\sigma_j K(t), \end{aligned}$$

where $\sigma_j \equiv \int_{z_j}^{z_{j-1}} zdz$. This implies that the evolution of capital for any individual country is proportional to the evolution of capital in the world economy. This is because the number of varieties and endogenous selection of intermediates are constant over time. The evolution of consumption is also proportional to the world consumption because the growth rate of individual consumption depends only on the interest rate, which is common in all countries. Thus, this implies that the transition of capital and consumption for any country is the same (in growth, not levels) as the phase diagram we described for the world economy in Figure 9.[46]

To conclude, it is worth mentioning the similarities between our model and Acemoglu and Ventura (2002). The main finding in their paper is that trade in goods results in a stable world GDP distribution, even though countries have different AK production technologies. Without unbundling, our model is essentially Acemoglu and Ventura allowing for capital flows. Countries have different productivity but they have the same capital share (which, in our case, is less than one). If we had that all countries experienced a common Hicks-neutral growth rate, $\theta_j(t) = \theta_j e^{gt}$ for some $g > 0$, we would obtain a stable world GDP distribution and all countries would grow at some common positive rate in the long-run. Interestingly, in

[46] From the steady-state level condition of world capital and $\dot{b}_j(t) = 0$, we can find the amount of assets hold by agents in country j. However, there exists an indeterminacy in the composition of the asset portfolio of agents. This is inconsequential for our purposes given that the interest rate is equalized across countries and, thus, the composition of the asset portfolio does not affect income nor consumption.

this extended model, when unbundling becomes possible, the capital shares of the countries would change (as discussed in Section 4.4). However, all countries would grow at the same rate in the new steady-state and the long-run world GDP distribution would remain stable, but more unequal.

5 Conclusions

In this paper, we have developed a dynamic trade model to study how the international unbundling of production has affected the world income distribution. The key novelty of our framework is to consider trade in goods heterogeneous in capital intensity and an elastic supply of capital.

The central result of the paper is that trade in intermediates heterogeneous in capital intensity generates a reallocation of capital across countries that exacerbates world inequality in both GDP per capita and welfare. GDP per capita inequality also arises in the extreme case of ex-ante identical countries (symmetry breaking). We used our model to investigate the interaction of unbundling with relevant features of the world economy. We showed that inequality increases when southern countries join the global supply chain (participate in trade in intermediates) and when a labor-saving technology (computerization) is introduced. We also showed that low-productivity countries benefit more from a FOSD shift of technology with unbundling. Lastly, we discussed the prediction that unbundling generates a relative decline in the labor share of high-productivity countries and provided evidence supporting this result.

At a more theoretical level, this paper underscores that the interaction between *(i)* trade in goods heterogeneous in factor intensity and *(ii)* an elastic supply of one of these factors magnifies productivity differences across countries. We have applied our theory to unbundling of production and an elastic supply of capital. However, we believe that these two conditions apply to a broader set of environments. For example, a world economy in which goods are heterogeneous in skill intensity and skill can be accumulated through education and/or skilled workers can more easily migrate across countries.

Lastly, the unbundling of production is exogenous in the model. Nonetheless, in practice, firms adopt technologies (for example, computers and the internet) to be able to offshore part of the production process. We plan on extending our framework to analyze the interdependence between technology adoption and trade. It would also be interesting to quantitatively investigate the effects of changes in the fraction of intermediates traded on different trade elasticities.

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A Data Appendix: Construction of Figure 1b

To construct the histogram for final goods we weight the capital share of each 6-digit NAICS by its final-use content. The final-use content is a weighted average of the different end-uses that each 6-digit NAICS has. The weight is computed as a function of the U.S. imports in 1990. Capital shares of each 6-digits NAICS are computed from the NBER CES Manufacturing database definition. The histogram for intermediates is computed in an analogous way. We use the end-use classification of Feenstra and Jensen (2012). Table A.1 in the online appendix reports additional measures of capital intensity dispersion. It also documents the same fact defining final goods as bundles of intermediates.

B Proofs

This section presents the proofs to some of the claims in the paper. The rest can be found in the online appendix.

Derivation of Trade Balance in steady state The trade balance in the steady-state is $TB_j = r(k_j - b_j)$. From the Euler Equation, $r = \rho$. Thus, from the demand of capital in country j, $rk_j = \rho k_j = [Z_j - \Delta_j] Y$. Lastly, from the evolution of asset holdings, $b_j = \pi_j B = 2\Delta_j B = 2\Delta_j K = \Delta_j \frac{2}{2\rho} Y = \frac{\Delta_j}{\rho} Y$. Thus, $TB_j = [Z_j - 2\Delta_j] Y$.

Proof of claim on change in relative welfare Note that $\left[\frac{c_j(0)}{c_{j+1}(0)}\right]^{with} > \left[\frac{c_j(0)}{c_{j+1}(0)}\right]^{without}$ if $\frac{\Delta_j}{\Delta_{j+1}} > \frac{\mu_j}{\mu_{j+1}} = \frac{\theta_j}{\theta_{j+1}}$. This is true because, from the threshold equilibrium, equation (16), $\frac{\Delta_j}{\Delta_{j+1}} = \left(\frac{\theta_j}{\theta_{j+1}}\right)^{\frac{1}{1-z}} > \frac{\theta_j}{\theta_{j+1}}$ given that $z < 1$.

Proof of claim on change in relative GDP To prove this result, we use the equality $\frac{\Delta_j}{\Delta_{j+1}} = \frac{Z_j}{Z_{j+1}} \frac{a_j}{a_{j+1}}$ where $a_j \equiv 1 - \frac{1}{2}(z_{j-1} + z_j)$. First, notice that $\frac{a_j}{a_{j+1}} < 1$ since z_j is decreasing with j. Second, we know from the threshold equilibrium, equation (16), that $\frac{\Delta_j}{\Delta_{j+1}} = \left(\frac{\theta_j}{\theta_{j+1}}\right)^{\frac{1}{1-z}} > 1$. These two statements imply that $\frac{Z_j}{Z_{j+1}} > \frac{\Delta_j}{\Delta_{j+1}}$. This last inequality means that production inequality increases because $\frac{Z_j}{Z_{j+1}} > \frac{\Delta_j}{\Delta_{j+1}} = \left(\frac{\theta_j}{\theta_{j+1}}\right)^{\frac{1}{1-z}} > \frac{\theta_j}{\theta_{j+1}}$, where we have used the threshold equilibrium and that $0 < z < 1$.

Derivation of production shares as a function of the assignment of intermediates To derive this expression note that $s^{y,\text{without}}(z) = \lambda e^{-\lambda\left(-\frac{1+\ln\left(ze^{-z}\right)}{\lambda}\right)} = \lambda z e^{1-z}$. To express the output share with unbundling, note that $s^{y,\text{with}}(j) = -\frac{dz}{dj} \iff s^{y,\text{with}}(z) = -\frac{1}{\frac{dj}{dz}}$. Using that $\frac{dj}{dz} = -\frac{1-z}{\lambda z}$,we have that $s^{y,\text{with}}(z) = \frac{\lambda z}{1-z}$. The change in output share in terms of j is $\Delta s^y_j = \frac{\lambda W(-\exp(-1-\lambda j))}{1+W(-\exp(-1-\lambda j))} - \lambda \exp(-\lambda j)$.

Proof of claim that the change in world output share is negative for $z \in (0, \bar{z})$ and positive for $z \in (\bar{z}, 1]$. Note that $\Delta s^y(z)$ is continuous, increasing for $z \in (1 - 3W(1/3), 1]$ and decreasing otherwise. Moreover, $\Delta s^y(0) = 0$, $\frac{d\Delta s^y}{dz}(0) < 0$, $\frac{d\Delta s^y}{dz}(1) = \infty$ and the result follows. We note in passing that $\Delta s^y(z)$ is convex for all z.