Structural Change with Long-run Income and Price Effects

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June 12, 2020*

Abstract

We present a new multi-sector growth model that features nonhomothetic, constant-elasticity-of-substitution preferences, and accommodates long-run demand and supply drivers of structural change for an arbitrary number of sectors. The model is consistent with the decline in agriculture, the hump-shaped evolution of manufacturing, and the rise of services over time. We estimate the demand system derived from the model using household-level data from the U.S. and India, as well as historical aggregate-level panel data for 39 countries during the postwar period. The estimated model parsimoniously accounts for the broad patterns of sectoral reallocation observed among rich, miracle, and developing economies. Our estimates support the presence of strong nonhomotheticity across time, income levels, and countries. We find that income effects account for the bulk of the within-country evolution of sectoral reallocation.

Keywords: Structural Transformation, Nonhomothetic CES preferences, Implicitly Additively Separable Preferences.
JEL Classification: E2, O1, O4, O5.

*We thank Robert Barro, Sam Kortum, and Kiminori Matsuyama for comments on previous drafts. We thank the editor, six anonymous referees, Gadi Barlevy, Timo Boppart, Paco Buera, Iván Canay, Berthold Herrendorf, Bart Hobijn, Chad Jones, Joe Kaboski, Sam Kortum, Pete Klenow, Dirk Krueger, Robert Z. Lawrence, Erzo Luttmer, Serguei Maliar, Alex Monge-Naranjo, Martin Rotemberg, Orie Shelef, Chris Snyder, Tomasz Swiecki, Chris Tonetti, Kei-Mu Yi, Chris Udry, Jing Zhang, and participants in several seminar and conference presentations for useful comments and feedback. We thank Emily Breza and Cynthia Kinnan for their help with the Indian NSS data. We thank Ana Danieli for her outstanding assistance in a revision of an earlier draft. We also thank Clement Bohr, Marco Errico, Matt Song, and Emre Enes Yavuz for their assistance polishing the draft. Comin acknowledges the generous support of the National Science Foundation, the Institute for New Economic Thinking, and the European Commission through the H2020 grant to the FRAME project. Mestieri acknowledges the generous support of the Agence Nationale de la Recherche (JJCC - GRATE program) while at TSE. All remaining errors are our own. Comin: diego.comin@dartmouth.edu, Lashkari: danial.lashkari@bc.edu, Mestieri: marti.mestieri@northwestern.edu.

1 Introduction

Economies undergo large scale sectoral reallocations of employment and capital as they develop, in a process commonly known as structural change (Kuznets, 1973; Maddison, 1980; Herrendorf et al., 2014; Vries et al., 2014). These reallocations lead to a gradual fall in the relative size of the agricultural sector and a corresponding rise in manufacturing. As income continues to grow, services eventually emerge as the largest sector in the economy. Leading theories of structural change attempt to understand these sweeping transformations through mechanisms involving either supply or demand. Supply-side theories focus on differences across sectors in the rates of technological growth and capital intensities, which create trends in the composition of consumption through price (substitution) effects (Baumol, 1967; Ngai and Pissarides, 2007; Acemoglu and Guerrieri, 2008). Demand-side theories, in contrast, emphasize the role of heterogeneity in income elasticities of demand across sectors (nonhomotheticity in preferences) in driving the observed reallocations accompanying income growth (Kongsamut et al., 2001).

The differences in the shapes of Engel curves across sectors play a crucial role in determining the contribution of supply and demand channels to structural change.[1] If relative sectoral demand shows a strong and stable dependence on income, the demand channel can readily explain the reallocation of resources toward sectors with higher income elasticities. For instance, rising demand for services and falling demand for agriculture, when both are compared against manufacturing, may give rise to sizable shifts of employment from agriculture toward services. However, demand-side theories have generally relied on specific classes of nonhomothetic preferences, e.g., generalized Stone-Geary preferences, that imply relative Engel curves that level off quickly as income grows. Because of this rapid flattening-out of the slopes of relative Engel curves across sectors, these specifications limit the explanatory power of the demand channel in the long-run.

The empirical evidence suggests that the relationship between relative sectoral expenditure shares and income is stable, and the slopes of relative Engel curves do not level off rapidly as income grows. Using aggregate data from a sample of OECD countries, Figure 1 plots the residual (log) expenditure share in agriculture (Figure 1a) and services (Figure 1b) relative to manufacturing on the y-axis and residual (log) income on the x-axis after controlling for relative prices.[2] The depicted fit shows that a constant slope captures a considerable part of

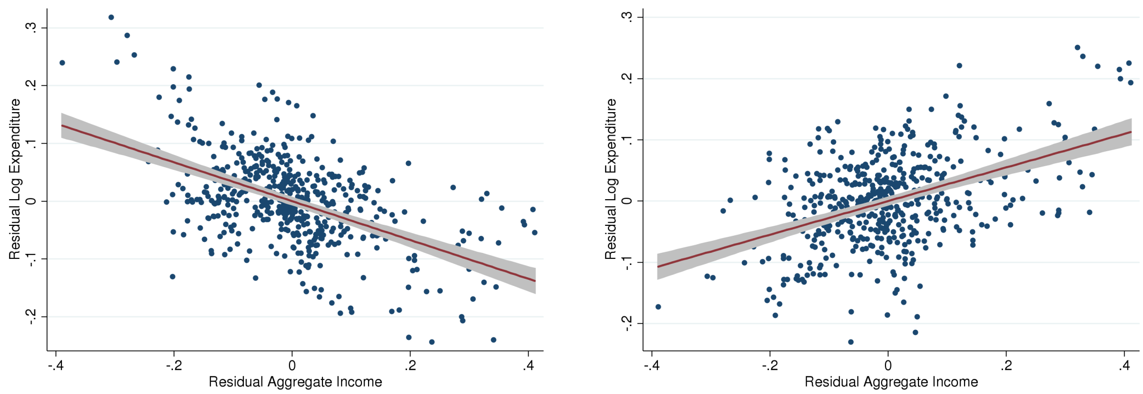
[1] Engel curves are functions characterizing how consumption expenditure on a given good varies with income under constant prices (Lewbel, 2008). More specifically, here we focus on *relative* Engel curves, which we define as the relationship between the logarithm of the relative of sectoral consumption between two sectors on the logarithm of the aggregate real consumption, holding prices constant.

[2] Residual Aggregate Income is constructed by taking the residuals of the following OLS regresssion: $\log Y_t^n = \alpha \log p_{at}^n + \beta \log p_{mt}^n + \gamma \log p_{st}^n + \xi^n + \nu_t^n$ where superscript n denotes country, and subscript t, time. Y_t^n, p_{at}^n, p_{mt}^n, and p_{st}^n denote aggregate income, the prices of agriculture, manufacturing, and services, respectively. ξ^n denotes a country fixed effect and ν_t^n the error term. Residual log-expenditures are constructed

Figure 1: Partial Correlations of Sectoral Expenditure and Aggregate Consumption

(a) Agriculture relative to Manufacturing

(b) Services Relative to Manufacturing



Notes: Data for OECD countries, 1970-2005. Each point corresponds to a country-year observation after partialling-out sectoral prices and country fixed effects. The red line depicts the OLS fit, the shaded regions, the 95% confidence interval.

the variation in the data and that it does not appear that the relationship levels off as aggregate consumption grows.[3] As discussed below, in this paper we complement this aggregate-level evidence with micro-level household data from the Consumption Expenditure survey (CEX) from the US and the National Sample Survey (NSS) from India. We analyze the relationship between relative shares and expenditure in these data, and show that sectoral differences in the estimated slopes do not level off and remain stable across households with different expenditure levels.[4]

Motivated by this evidence, we develop a multi-sector model of structural change that ac-

in an analogous manner using the log of relative sectoral expenditures as dependent variables. Table F.1 in the online appendix reports the estimates of the regression. Section 5 shows how the slopes of the regressions depicted in Figure 1 are connected to our theory.

[3]The partial R^2 of the regressions shown in Figure 1 are 27% and 20%, respectively. In fact, if we split the sample into observations below and above the median income in the sample and estimate the relative Engel curves separately, we cannot reject the hypothesis of identical slopes of the Engel curves. See Table F.1 in the online appendix. If we reported separately the Engel curves for agriculture, manufacturing and services, we would find a negative, zero and positive slope, respectively.

[4]A number of recent papers have similarly used log-linear specifications of Engel curves in analyzing micro-level expenditure data. Aguiar and Bils (2015) use the U.S. Consumer Expenditure Survey (CEX) to estimate Engel curves for 20 different consumption categories. Their estimates for the income elasticities are different from unity and vary significantly across consumption categories. Young (2012) employs the Demographic and Health Survey (DHS) to infer the elasticity of real consumption of 26 goods and services with respect to income for 29 sub-Saharan and 27 other developing countries. He estimates the elasticity of consumption for the different categories with respect to the education of the household head and then uses the estimates of the return to education from Mincerian regressions to back out the income elasticity of consumption. Young also uses a log-linear formulation for the Engel curves and finds that the slopes of Engel curves greatly differ across consumption categories but appear stable over time. Olken (2010) discusses Young's exercise using Indonesia survey data and finds similar results for a small sample of three goods and services. Young (2013) also makes use of log-linear Engel curves to infer consumption inequality.

commodates non-vanishing nonhomotheticities. The model builds on the standard framework used in recent empirical work on structural transformation (e.g., Buera and Kaboski, 2009; Herrendorf et al., 2013). Our key departure from the standard framework is the introduction of a class of utility functions that generates nonhomothetic sectoral demands for all levels of income, including when income grows toward infinity. These preferences, which we will refer to as nonhomothetic Constant Elasticity of Substitution (CES) preferences, allow for an arbitrary number of goods, include good-specific nonhomotheticity parameters that control *relative* income elasticities, and feature a constant elasticity of substitution. They have been studied by Gorman (1965), Hanoch (1975), Sato (1975), and Blackorby and Russell (1981) in the context of static, partial-equilibrium models. Our theory embeds these preferences into a general equilibrium model of economic growth. The framework predicts log-linear relations between relative sectoral allocation, relative sectoral prices, and income. Thus, it lends itself to the task of decomposing the contributions of the demand and supply channels to structural change. As part of our contributions, we also derive a strategy for structurally estimating the parameters of these preferences, using both micro and aggregate data. Finally, we use the estimated model parameters to quantify the contributions of income and price effects to structural change across countries.

We characterize the equilibrium paths of our growth model in the long-run and derive the dynamics of the economy along the transition path. The equilibrium in our model asymptotically converges to a path of constant real consumption growth. The asymptotic growth rate of real consumption depends on parameters characterizing both the supply and demand channels; it is a function of the sectoral income elasticities as well as sectoral growth rates of TFP and sectoral factor intensities. In this respect, our model generalizes the results of Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008) to the case featuring nonhomothetic CES demand. We also show that our theory produces similar evolutions for nominal and real sectoral measures of economic activity, which is a robust feature of the data.[5] This is a consequence of the role of income elasticities in generating sectoral reallocation patterns. Our framework can generate hump-shaped patterns for the evolution of manufacturing consumption shares, which is a well-documented feature in the data (Buera and Kaboski, 2012a).

In the empirical part of the paper, we first provide household-level evidence in favor of the stable effect of nonhomotheticities implied by nonhomothetic CES preferences. We estimate our demand system using household-level data from the Consumption Expenditure survey (CEX) from the US. We group household expenditures into three broad categories of products: agriculture, manufacturing, and services. The estimated nonhomotheticity parameters imply that income elasticities are highest in services, lowest in agriculture, and fall in-between for manufacturing. We also show that the estimated nonhomotheticity parameters are similar

[5] Herrendorf et al., 2014 show that supply-side driven structural transformation cannot account for the similar evolution of nominal and real sectoral measures of activity.

for households across different income brackets and time periods. As mentioned above, the theory also implies a log-linear relationship between relative sectoral consumption and the real consumption index (derived from nonhomothetic CES). We show that this log-linear relationship approximately holds in our data.

We then empirically evaluate the implications of our growth model for structural transformation at the macroeconomic level. We estimate the elasticities that characterize our demand function using cross-country sectoral data in a panel of 39 countries for the postwar period. The countries in our sample substantially vary in terms of their stages of development and growth experiences (e.g., our sample includes countries such as Ghana, Taiwan, and the US). We find that the estimated nonhomotheticity parameters are similar across different measures of sectoral activity (employment and output) and country groupings (OECD and Non-OECD countries).

Armed with the estimated parameters of our model, we turn to the analysis of the drivers of structural change. We use our model to decompose the within-country evolution of relative sectoral employment into income and price effects. We find that income effects are the main contributors to structural transformation. They account for over 73% of the within-country sectoral reallocation in employment predicted by the estimated model. This finding contrasts with previous studies (e.g., Dennis and Iscan, 2009, Boppart, 2014a). A key reason for this discrepancy is that in our framework, income effects are not hard-wired to have only transitory effects on the structural transformation (as in Stone-Geary preferences) or to be correlated with price effects. Without these constraints on income effects, our estimates are consistent with a predominant role of income effects in accounting for the structural transformation.

We further present two important empirical analyses that illustrate the robustness and generality of our baseline empirical results. We first investigate the robustness of our identification strategy and the results of our decomposition with respect to a number of alternative parameterizations of the nonhomothetic CES preferences. We show that, as predicted by the theory, the estimated parameters and the resulting decomposition of the patterns of structural change are similar across these different specifications. We then provide an additional specification of demand suitable for estimation of household-level data in cases where sectoral price data is not available. This specification, which identifies the rank-ordering and the relative magnitude of nonhomotheticity parameters across sectors, allows us to extend our micro-analysis to the National Sample Survey (NSS) data from India. Despite the vast differences in the distribution of household-level income and the sectoral composition of consumption between the US and India, we find that the nonhomotheticity parameters estimated using NSS data are very similar to those estimated using US CEX data.

Finally, we provide a number of additional extensions as well as comparisons between our framework and prior work. We compare the predictive power of our model with the two most prominent demand systems that feature nonhomothetic preferences: the generalized

Stone-Geary (Buera and Kaboski, 2009) and the price-independent generalized-linear (PIGL) preferences (Boppart, 2014a). We find that nonhomothetic CES preferences provide a better account for the patterns of structural transformation across agriculture, manufacturing and services in our cross-country sample. We further extend our analysis by shifting the focus from sectoral shares of final good consumption expenditure or employment to those of value added, following, e.g., Herrendorf et al. (2013). We find similar results when we repeat our analysis in terms of value added both for household and aggregate level data. We also discuss the connection between the proxies of real consumption implied by nonhomothetic CES with off-the-shelf indices of real consumption. As another noteworthy extension, we take advantage of the fact that nonhomothetic CES can accommodate an arbitrary number of goods. We extend our empirical analysis to a richer sectoral disaggregation and document substantial heterogeneity in income elasticity within manufacturing and services.

Our paper relates to a large literature that aims to quantify the role of nonhomotheticity of demand on growth and development (see, among others, Matsuyama (1992), Echevarria, 1997, Gollin et al., 2002, Duarte and Restuccia, 2010, Alvarez-Cuadrado and Poschke, 2011).[6] Buera and Kaboski (2009) and Dennis and Iscan (2009) have noted the limits of the generalized Stone-Geary utility function to match long time series or cross-sections of countries with different income levels. More recently, Boppart (2014a) has studied the evolution of consumption of goods relative to services by introducing a sub-class of PIGL preferences that also yield non-vanishing income effects in the long-run. PIGL preferences also presuppose specific parametric correlations for the evolution of income and price elasticities over time (Gorman, 1965), and only accommodate two goods with distinct income elasticities. In contrast, our framework features a constant elasticity of substitution and allows for an arbitrary number of goods.[7] The differences between the two models are further reflected in their empirical implications. Whereas we find a larger contribution for demand nonhomotheticity in accounting for structural change, Boppart concludes that supply and demand make roughly similar contributions.[8]

[6]An alternative formulation that can reconcile demand being asymptotically nonhomothetic with balanced growth path is given by hierarchical preferences (e.g., Foellmi and Zweimüller, 2006, 2008 and Foellmi et al., 2014). Swiecki (2017) estimates a demand system that features non-vanishing income effects in combination with subsistence levels à la Stone-Geary. However, this demand system also imposes a parametric relation between income and price effects. In subsequent work, Duernecker et al. (2017b) use a nested structure of nonhomothetic CES to study structural change within services. Sáenz (2017) extends our framework to time-varying capital intensities across production sectors and calibrates his model to South Korea. Matsuyama (2015, 2017) embeds nonhomothetic CES preferences in a monopolistic competition framework with international trade à la Krugman to study the patterns of structural change in a global economy and endogenizes the pattern of specialization of countries through the home market effect. Sposi (2018) and Lewis et al. (2018) incorporate nonhomothetic CES in a quantitative trade model of structural change.

[7]One can extend PIGL preferences to more than two goods by nesting other functions as composites within the two-good utility function (Boppart, 2014a), e.g., CES aggregators (this is how we proceed to estimate them in our empirical analysis). However, the resulting utility function does not allow for heterogeneity in income elasticity among the goods within each nested composite.

[8]In terms of the scope of the empirical exercise, while Boppart (2014a) estimates his model with U.S. data

The remainder of the paper is organized as follows. Section 2 introduces the properties of the nonhomothetic CES preferences and presents the model. Section 3 presents the estimation of the model using the household-level and aggregate data. Section 4 uses the model estimates to investigate the relative quantitative importance of price and income effects for explaining the observed patterns of structural transformation. Section 5 presents the robustness analysis of our benchmark empirical specification and the analysis of the Indian NSS micro-level data. Section 6 discusses the comparison of our model with prior accounts of nonhomothetic preferences in models of structural change, as well as a number of additional extensions. Section 7 presents a calibration exercise where we investigate the transitional dynamics of the model, and Section 8 concludes. Appendix A presents some general properties of nonhomothetic CES preferences. All proofs are in Appendix B.

2 Theory

In this section, we present a class of preferences that rationalize the empirical regularities on relative sectoral consumption expenditures discussed in the Introduction. We then incorporate these preferences in a multi-sector growth model and show how we can use them to account for the patterns of structural transformation across countries. The growth model closely follows workhorse models of structural transformation (Buera and Kaboski, 2009; Herrendorf et al., 2013, 2014, e.g.,). The only difference with these is that we replace the standard aggregators of sectoral consumption goods with a nonhomothetic CES aggregator. This single departure from the standard workhorse model delivers the main theoretical results of the paper and the demand system later used in the estimation.

2.1 Nonhomothetic CES Preferences

Consider preferences over a bundle of goods $\boldsymbol{C} \equiv (C_i)_{i=1}^{I}$ characterized by a utility function $U = F(\boldsymbol{C})$ implicitly defined through the constraint

$$\sum_{i=1}^{I} \Upsilon_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g\left(U\right)^{\varepsilon_i}} \right)^{\frac{\sigma-1}{\sigma}} = 1, \tag{1}$$

for a positive-valued, continuously differentiable, and monotonically increasing function $g(\cdot)$. We impose the parametric restrictions that $\sigma \in (0,1) \cup (1,\infty)$, $\Upsilon_i > 0$ and $\varepsilon_i > 0$ for all $i \in \mathcal{I} \equiv \{1, \cdots, I\}$.[9] Each sectoral good i is identified with a nonhomotheticity parameter ε_i.

and considers two goods, the empirical evaluation of our model includes, in addition to the U.S., a wide range of other rich and developing countries and more than two goods. The variable price elasticity implied by PIGL is also quantitatively important in accounting for the difference in the decomposition results (see Section 6).

[9] We can show that under these parameter restrictions the function $U(\cdot)$ introduced in Equation (1) is globally monotonically increasing and quasi-concave, yielding a well-defined utility function over the bundle

As we will see below, parameters $\boldsymbol{\varepsilon} \equiv (\varepsilon_i)_{i=1}^I$ control relative income (expenditure) elasticities of demand across different goods.[10,11]

Next, we first characterize the demand implied by the utility function $U = F(\boldsymbol{C})$, and then discuss how to use expenditure and price data to identify these preferences.

2.1.1 Nonhomothetic CES Demand

To characterize the demand system, we begin with the Hicksian demand, which has a straightforward derivation for nonhomothetic CES, and further illustrates the key properties of the preferences. We then discuss the Marshallian demand and the sector-level income elasticities implied by the preferences.

Hicksian Demand Consider the expenditure minimization problem with the set of prices $\boldsymbol{p} \equiv (p_i)_{i=1}^I$ and preferences defined as in Equation (1). The nonhomothetic CES Hicksian demand function is given by

$$C_i = \Upsilon_i \left(\frac{p_i}{E}\right)^{-\sigma} g(U)^{(1-\sigma)\varepsilon_i}, \qquad \forall i \in \mathcal{I}, \tag{2}$$

where we have defined E as the expenditure $E = \sum_{i=1}^I p_i C_i$. The associated expenditure function is

$$E(U;\boldsymbol{p}) \equiv \left[\sum_{i=1}^I \Upsilon_i g(U)^{(1-\sigma)\varepsilon_i} p_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{3}$$

and gives the cost $E = \sum_{i=1}^I p_i C_i$ of achieving utility U. Note that substituting the demand for C_i from (2) in the definition of nonhomothetic CES (1), we find that each summand in (1) corresponds to the equilibrium expenditure share. Denoting expenditure shares by $\omega_i \equiv p_i C_i / E$, Equation (1) simply implies $\sum_{i=1}^I \omega_i = 1$.

Properties of the Demand System Two unique features of nonhomothetic CES demand make these preferences suitable for capturing the patterns discussed in the introduction:

of goods $\boldsymbol{C}$, see Hanoch (1975). In the case of $\sigma = 1$, the only globally well-defined CES preferences are homothetic and correspond to Cobb-Douglas preferences (Blackorby and Russell, 1981).

[10] In Appendix A, we discuss the most general formulation of nonhomothetic CES. In this case, instead of using an isoelastic function $g(U)^{\varepsilon_i}$ in the definition of the utility aggregator (1), an i-specific function $g_i(U)$ is allowed and U is implicitly defined by

$$\sum_{i=1}^I \Upsilon_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)}\right)^{\frac{\sigma-1}{\sigma}} = 1.$$

This formulation allows for more general patterns of nonhomotheticity and nests our baseline formulation. The advantage of our baseline formulation is that it delivers an estimating equation that is log-linear in terms of observables while preserving non-vanishing nonhomotheticity.

[11] In a previous version of the paper, we used the functional form $g(U) = U^{1-\sigma}$ for $0 < \sigma < 1$ and $g(U) = U^{\sigma-1}$ for $\sigma > 1$. We thank the editor for suggesting replacing U with $g(U)$ in our definition of utility, (1).

1. The elasticity of the relative demand for two different goods with respect to a monotonic transformation $g(\cdot)$ of utility is constant, i.e.,

$$\frac{\partial \log\left(C_i/C_j\right)}{\partial \log g(U)} = (1-\sigma)\left(\varepsilon_i - \varepsilon_j\right), \qquad \forall i, j \in \mathcal{I}. \tag{4}$$

2. The elasticity of substitution between goods of different sectors is constant[12]

$$\frac{\partial \log\left(C_i/C_j\right)}{\partial \log\left(p_j/p_i\right)} = \sigma, \qquad \forall i, j \in \mathcal{I}. \tag{5}$$

The first property ensures that the nonhomothetic features of these preferences do not systematically diminish as income (and therefore utility) rises. This property helps us account for the empirical patterns discussed in the Introduction that consumption across sectors shows non-vanishing, stable and heterogeneous income elasticities, both at the macro and micro levels. The second property ensures that different goods have a constant elasticity of substitution and price elasticity regardless of the level of income.[13] It is because of this property that we refer to these preferences as nonhomothetic *CES*.[14]

The demand system implied by nonhomothetic CES for the *relative* consumption expenditures of goods transparently summarizes the two properties above. The Hicksian demand for any pair of expenditure shares ω_i and ω_j, and for all $i, j \in \mathcal{I}$, satisfies

$$\log\left(\frac{\omega_i}{\omega_j}\right) = (1-\sigma)\log\left(\frac{p_i}{p_j}\right) + (1-\sigma)\left(\varepsilon_i - \varepsilon_j\right)\log g(U) + \log\left(\frac{\Upsilon_i}{\Upsilon_j}\right). \tag{6}$$

Equation (6) highlights one of the key features of the nonhomothetic CES demand system, which is the separation of the price and the income effects. The first term on the right hand side shows the price effects characterized by a constant elasticity of substitution σ, and the

[12]Note that for preferences defined over I goods when $I > 2$, alternative definitions for elasticity of substitution do not necessarily coincide. In particular, Equation (5) defines the so-called Morishima elasticity of substitution, which in general is not symmetric. This definition may be contrasted from the Allen (or Allen-Uzawa) elasticity of substitution defined as $\frac{E \cdot \partial C_i/\partial p_j}{C_i C_j}$, where E is the corresponding value of expenditure. Blackorby and Russell (1981) prove that the only preferences for which the Morishima elasticities of substitution between any two goods are symmetric, constant, and identical to Allen-Uzawa elasticities have the form of Equation (1), albeit with a more general dependence of weights on U.

[13]Nonhomothetic CES preferences inherit this property because they belong to the class of implicitly additively separable preferences (Hanoch, 1975). In contrast, any preferences that are explicitly additively separable in sectoral goods (e.g., Stone-Geary, price independent generalized linear or constant relative income elasticity preferences) imply parametric links between income and substitution elasticities. This result is known as Pigou's Law (Snow and Warren, 2015). For a discussion of specific examples, see Appendix A.

[14]Alternatively, if we assume that consumer preferences satisfy the two properties (4) and (5) for given parameter values $(\sigma, \varepsilon_1, \cdots, \varepsilon_I)$, the preferences correspond to the nonhomothetic CES preferences given by Equation (1). More specifically, imposing condition (5) defines a *general* class of nonhomothetic CES preferences, defined in Equation (A.1) in the appendix. Further imposing condition (4) yields the definition in Equation (1). See Appendix A for more details.

second term on the right hand side shows the change in relative sectoral demand as consumers move across indifference curves. Equation (6) implies a log-linear form for relative demand of different goods in terms of prices and our utility aggregator, but the latter is not readily observed in the data. Below, we show how to use the Marshallian demand to rewrite our utility aggregator in terms of observables and obtain a log-linear demand equation in terms of observables that can be readily estimated.

Marshallian Demand The expenditure function in Equation (3) is monotonically increasing in $g(U)$ (and therefore utility U) under the parametric restrictions we imposed. Define the indirect utility function $V(\cdot;\boldsymbol{p})$ as the inverse of function $E(\cdot;\boldsymbol{p})$, such that $V(E(U;\boldsymbol{p});\boldsymbol{p}) = g(U)$ for all U. Substituting $V(E;\boldsymbol{p})$ for $g(U)$ in Equation (2), gives the expression for the Marshallian demand expressed in terms of prices $\boldsymbol{p}$ and expenditure E.

Implicit Marshallian Demand We derive an alternative expression for the Marshallian demand that expresses the expenditure shares of $I-1$ goods in terms of prices $\boldsymbol{p}$, total expenditure E, and the expenditure share of a base good $b \in \mathcal{I}$.[15] We first use the expression for the demand of the base good in Equation (2) to write the function $g(\cdot)$ of utility in terms of the price and expenditure of the base good b, as well as the total consumption expenditure

$$\varepsilon_b \log g(U) + \frac{1}{1-\sigma} \log \Upsilon_b = \log\left(\frac{E}{p_b}\right) + \frac{1}{1-\sigma} \log \omega_b. \tag{7}$$

Substituting for $g(U)$ from expression (7) in the demand for other goods $i \in \mathcal{I}_{-b} \equiv \mathcal{I}\backslash\{b\}$, we find that the consumption expenditure shares satisfy

$$\log \omega_i = (1-\sigma)\log\left(\frac{p_i}{p_b}\right) + (1-\sigma)\left(\frac{\varepsilon_i}{\varepsilon_b} - 1\right)\log\left(\frac{E}{p_b}\right) + \frac{\varepsilon_i}{\varepsilon_b}\log\omega_b + \log\left(\frac{\Upsilon_i}{\Upsilon_b^{\varepsilon_i/\varepsilon_b}}\right). \tag{8}$$

Equation (8) gives us a collection of $I-1$ constraints implied by the nonhomothetic CES preferences that together fully characterize the Marshallian demand.

Sectoral Income Elasticities The income (expenditure) elasticity of demand for sectoral good i is given by

$$\eta_i \equiv \frac{\partial \log C_i}{\partial \log E} = \sigma + (1-\sigma)\frac{\varepsilon_i}{\bar{\varepsilon}}, \tag{9}$$

where E is the consumer's total consumption expenditure, and we have defined the expenditure-weighted average of nonhomotheticity parameters, $\bar{\varepsilon} \equiv \sum_{i=1}^{I} \omega_i \varepsilon_i$ with ω_i denoting the expen-

[15] Our approach is similar in spirit to the idea of *implicit Marshallian demand* introduced by Lewbel and Pendakur (2009) and *M-demands* introduced by Browning (1999), both of which also rely on substituting for the utility in the Hicksian demand as a function of expenditure shares and prices.

diture share in sector i as defined above.[16] Equation (9) implies that $\eta_i > \eta_j$ if and only if $\varepsilon_i > \varepsilon_j$ for $0 < \sigma < 1$, and $\varepsilon_i < \varepsilon_j$ for $\sigma > 1$. As Engel aggregation requires, the income elasticities average to 1 when sectoral weights are given by expenditure shares, $\sum_{i=1}^{I} \omega_i \eta_i = 1$. For $0 < \sigma < 1$, if good i has a nonhomotheticity parameter ε_i that exceeds (is less than) the consumer's average nonhomotheticity parameter $\bar{\varepsilon}$, then good i is a luxury (necessity) good, in the sense that it has an expenditure elasticity greater (smaller) than 1 at that point in time. The converse holds for $\sigma > 1$. This implies that being a luxury or a necessity good is not an intrinsic characteristic of a good, but rather depends on the consumer's current composition of consumption expenditures and, ultimately, income.

2.1.2 Identifiability of Nonhomothetic CES

There are classes of functions $F(\boldsymbol{C})$ introduced in Equation (1) that can be written as monotonic transformations of one another. To see this, note that the expressions for the implicit Marshallian demand in Equation (8) only depend on the ratios $\left(\varepsilon_i/\varepsilon_b, \Upsilon_i/\Upsilon_b^{\varepsilon_i/\varepsilon_b}\right)_{i\in\mathcal{I}_{-b}}$. Therefore, all uniform scalings of parameters $\boldsymbol{\varepsilon}$ and $\boldsymbol{\Upsilon}$ imply the same patterns of observable choice behavior. As another example, if we let $\varepsilon_i = \varepsilon_b > 0$ for all i and some $b \in \mathcal{I}$, the function $g(U)$ satisfies:

$$\Upsilon_b^{\frac{1}{1-\sigma}} g(U)^{\varepsilon_b} = \left(\sum_{i=1}^{I} \left(\frac{\Upsilon_i}{\Upsilon_b}\right)^{\frac{1}{\sigma}} C_i^{\frac{1-\sigma}{\sigma}}\right)^{\frac{\sigma}{1-\sigma}}.$$

Therefore, the definition above for different values of ε_b and Υ_b corresponds to different isoelastic transformations of the standard homothetic CES utility, and are observationally equivalent with it.

Motivated by these observations, we introduce the reparameterization $\boldsymbol{\epsilon} \equiv (\epsilon_i)_{i=1}^{I}$ and $\boldsymbol{\Omega} \equiv (\Omega_i)_{i=1}^{I}$ of the nonhomothetic CES preferences corresponding to a given base good b:

$$\epsilon_i \equiv \frac{\varepsilon_i}{\varepsilon_b}, \qquad \Omega_i \equiv \frac{\Upsilon_i}{\Upsilon_b^{\varepsilon_i/\varepsilon_b}}. \tag{10}$$

Note that, by definition, $\epsilon_b = \Omega_b = 1$ for the base good. We emphasize that the construction that follows holds and leads to identical implications for observed behavior irrespective of the specific choice of base good $b \in \mathcal{I}$.[17]

We now show that all our variables of interest only depend on the identifiable parameters $\boldsymbol{\epsilon}$ and $\boldsymbol{\Omega}$. For this exercise, it is helpful to consider a sequence of prices $(\boldsymbol{p}_t)_{t=1}^{T}$, expenditures $(E_t)_{t=1}^{T}$, consumption expenditure shares $(\boldsymbol{\omega}_t)_{t=1}^{T}$, and utility $(U_t)_{t=1}^{T}$ that all maximize the

[16] The expenditure elasticity of relative demand is $\partial \log (C_i/C_j) / \partial \log E = (1-\sigma)(\varepsilon_i - \varepsilon_j)/\bar{\varepsilon}$. Note the difference with Equation (4) that expresses the elasticity instead in terms of the function $g(\cdot)$ of utility.

[17] Appendix A.2.2 generalizes this construction to bases made from arbitrary convex combinations of all goods.

utility function $U_t = F(\boldsymbol{C}_t)$ defined in Equation (1) under the budget constraint $\sum_{i=1}^{I} p_{it} C_{it} \leq E_t$ for all $1 \leq t \leq T$.

Identifying Nonhomothetic CES Demand From Equation (8), the implicit Marshallian demand can be written solely in terms of the reparameterization. That is, the sequence of expenditures, expenditure shares of goods, and prices satisfies

$$\log \omega_{it} = (1-\sigma) \log \left(\frac{p_{it}}{p_{bt}}\right) + (1-\sigma)(\epsilon_i - 1) \log \left(\frac{E_t}{p_{bt}}\right) + \epsilon_i \log \omega_{bt} + \log \Omega_i, \tag{11}$$

for all $i \neq b$ and all t. As we further discuss in the next section, the implicit demand Equations (11) can be used to identify parameters $\boldsymbol{\epsilon}$ and $\boldsymbol{\Omega}$. The next lemma shows that the Marshallian demand can also be fully characterized in terms of the same parameters, $\boldsymbol{\epsilon}$ and $\boldsymbol{\Omega}$.

Lemma 1. *The Marshallian demand corresponding to the nonhomothetic CES preferences given by Equation (1) can be expressed as $\boldsymbol{\omega}(\boldsymbol{p}_t, E_t; \sigma, \boldsymbol{\epsilon}, \boldsymbol{\Omega})$, where parameters $(\boldsymbol{\epsilon}, \boldsymbol{\Omega})$ are defined in Equation (10) for given base good b. Similarly, the vector of sectoral income elasticities defined in Equation (9) can also be written in terms of the same set of parameters as $\boldsymbol{\eta}(\boldsymbol{p}_t, E_t; \sigma, \boldsymbol{\epsilon}, \boldsymbol{\Omega})$.*

Proof. See Section B. □

CES Index of Real Consumption The discussions above motivate a natural cardinalization for nonhomothetic CES preferences (following the terminology of Deaton and Muellbauer 1980, page 42). For any function $g(\cdot)$ and the corresponding nonhomothetic CES utility function $U = F(\cdot)$ introduced in Equation (1), we define the aggregator function $C \equiv G(\boldsymbol{C})$ as a monotonically increasing transformation of U given by

$$\log C \equiv \varepsilon_b \log g(U) + \frac{1}{1-\sigma} \log \Upsilon_b, \tag{12}$$

as the *nonhomothetic CES index of real consumption* for base good $b \in \mathcal{I}$. Note that the definition above corresponds to the left hand side of Equation (7), implying that the index C_t at time t can be *expressed as a function of observables and the parameter* σ according to

$$\log C_t = \log \left(\frac{E}{p_{bt}}\right) + \frac{1}{1-\sigma} \log \omega_{bt}. \tag{13}$$

Using this definition, we can rewrite Equation (1) in terms of the index C, parameters $\boldsymbol{\epsilon}$ and $\boldsymbol{\Omega}$ as

$$\sum_{i=1}^{I} \Omega_i^{\frac{1}{\sigma}} \left(\frac{C_i}{C^{\epsilon_i}}\right)^{\frac{\sigma-1}{\sigma}} = 1. \tag{14}$$

The key property of aggregator $C \equiv G(\boldsymbol{C})$ is that the elasticity of relative demand with respect to the value of this aggregator is constant everywhere. This is readily verified by writing Equation (6) in terms of C,

$$\log\left(\frac{\omega_i}{\omega_j}\right) \quad = \quad (1-\sigma)\log\left(\frac{p_i}{p_j}\right) + (1-\sigma)\left(\epsilon_i - \epsilon_j\right)\log C + \log\left(\frac{\Omega_i}{\Omega_j}\right). \tag{15}$$

We refer to index C as the index of real consumption as it generalizes the standard real consumption aggregators typically assumed under homothetic CES preferences. In particular, for the case of $\varepsilon_i = \varepsilon_b > 0$ for all i discussed in the beginning of the section, we find the standard definition for homothetic CES, $C = \left(\sum_{i=1}^{I} \Omega_i^{1/\sigma} C_i^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$. The definition also implies a corresponding average price index defined as $P = E/C$.[18]

Separating the Roles of Relative Prices and Income Effects Given parameters σ, $\boldsymbol{\epsilon}$, and $\boldsymbol{\Omega}$, along with observed data on expenditures and prices, we can decompose the time variations in relative expenditures into the component driven by variation in relative prices and that driven by changes in utility. To see this, note that Equation (6) and the definition (7) together imply

$$\log\left(\frac{\omega_{it}}{\omega_{jt}}\right) = (1-\sigma)\log\left(\frac{p_{it}}{p_{jt}}\right) + (1-\sigma)\left(\epsilon_i - \epsilon_j\right)\underbrace{\left[\log\left(\frac{E_t}{p_{bt}}\right) + \frac{1}{1-\sigma}\log\omega_{bt}\right]}_{\equiv \log C_t} + \log\left(\frac{\Omega_i}{\Omega_j}\right), \tag{16}$$

where the last term on the right hand side is a constant, the first term corresponds to the contribution of the variations in prices, and the second term to the variations in the nonhomothetic CES index of real consumption $\log C_t$, following Equation (13). Importantly, the second term that accounts for the role of variations in utility can be written *only in terms of observables and identifiable model parameters (σ, $\boldsymbol{\epsilon}$).* This expression constitutes the foundation of our approach to the empirical decomposition of the variations in expenditure shares in Section 4. In particular, it allows us to distinguish the roles of relative prices and nonhomotheticity in a simple log-linear specification.

2.2 Multi-sector Growth with Nonhomothetic CES

We now integrate the nonhomothetic CES preferences into a general-equilibrium growth model to study the effect of the demand forces documented in the Introduction on shaping the long-run patterns of structural change. On the supply side, the model combines two distinct

[18]The homothetic CES has the property that sectoral consumption C_i grows linearly in the index of real consumption C, when the price of good i relative to the corresponding price index p_i/P remains constant. The nonhomothetic CES index of real consumption for given base good b preserves this property only for the base good, that is, $C_b = (p_b/P)^{-\sigma} C$.

potential drivers of sectoral reallocation previously highlighted in the literature: heterogeneous rates of sectoral technological progress (Ngai and Pissarides, 2007) and heterogeneous capital-intensity across sectors (Acemoglu and Guerrieri, 2008).

Households A unit mass of homogeneous households has preferences over an infinite stream of consumption bundles $\{\boldsymbol{C}_t\}_{t=0}^{\infty}$ defined by utility function

$$\mathcal{U}\left(\{\boldsymbol{C}_t\}_{t=0}^{\infty}\right) \equiv \sum_{t=0}^{\infty} \beta^t v\left(F\left(\boldsymbol{C}_t\right)\right), \tag{17}$$

where the function $F(\cdot)$ is defined by Equation (1) and $\beta \in (0,1)$ is the discount factor. To complete the characterization of the household behavior, we assume that each household inelastically supplies one unit of perfectly divisible labor, and starts at period 0 with a homogeneous initial endowment $\mathcal{A}_0$ of assets.

Firms Firms in each consumption sector produce sectoral output under perfect competition. In addition, firms in a perfectly competitive investment sector produce investment good, Y_{0t}, that is used in the process of capital accumulation. We assume constant-returns-to-scale Cobb-Douglas production functions with time-varying Hicks-neutral sector-specific productivities,

$$Y_{it} \quad = \quad A_{it} K_{it}^{\alpha_i} L_{it}^{1-\alpha_i}, \qquad i \in \{0\} \cup \mathcal{I},$$

where K_{it} and L_{it} are capital and labor used in the production of output Y_{it} in sector i at time t (we have identified the sector producing investment good as $i = 0$) and $\alpha_i \in (0,1)$ denotes sector-specific capital intensity. The aggregate capital stock of the economy, K_t, accumulates using investment goods and depreciates at rate δ, $Y_{0t} = K_{t+1} - (1-\delta) K_t$.

2.2.1 Competitive Equilibrium

Given an initial stock of capital K_0 and a sequence of sectoral productivities $\left\{(A_{it})_{i=0}^{I}\right\}_{t\geq 0}$, a competitive equilibrium is defined as a sequence of allocations $\left\{K_{t+1}, (Y_{it}, C_{it}, K_{it}, L_{it})_{i=0}^{I}\right\}_{t\geq 0}$ and a sequence of prices $\left\{w_t, R_t, (p_{it})_{i=0}^{I}\right\}_{t\geq 0}$ such that *(i)* agents maximize the present discounted value of their utility given their budget constraint, *(ii)* firms maximize profits and *(iii)* markets clear. In this section we focus on characterizing the features of the competitive equilibrium of this economy that motivate our empirical specifications.

Household Problem Households take the sequence of wages, real interest rates, and prices of goods $\{w_t, r_t, \boldsymbol{p}_t\}_{t=0}^{\infty}$ as given, and choose a sequence of asset stocks $\{\mathcal{A}_{t+1}\}_{t=0}^{\infty}$ and consumption bundles $\{\boldsymbol{C}_t\}_{t=0}^{\infty}$ to maximize their utility defined in Equations (1) and Equations (17), subject to the per-period budget constraint

$$\mathcal{A}_{t+1}+\sum_{i=1}^{I}p_{it}C_{it}\leq w_t+\left(1+r_t\right)\mathcal{A}_t, \tag{18}$$

where we have normalized the price of assets to 1. The next lemma provides the solution to the household problem, and shows that it can be characterized in terms of identifiable parameters $(\sigma,\, \boldsymbol{\Omega},\, \boldsymbol{\epsilon})$ for any base good $b\in\{1,\cdots,I\}$.

Lemma 2. *(Household Behavior) Consider a household with preferences as described by Equations (1) and (17) with monotonically increasing* $g(\cdot)$*,* $\sigma\in(0,1)\cup(1,\infty)$ *and* $\boldsymbol{\Upsilon},\boldsymbol{\varepsilon}\geq 0$*. Define the function*

$$u\left(C\right)\equiv\left(v\circ g^{-1}\right)\left(\Upsilon_b^{\frac{1}{(\sigma-1)\varepsilon_b}}C^{\frac{1}{\varepsilon_b}}\right),$$

for any base good $b\in\{1,\cdots,I\}$*, as the felicity function expressed in terms of the corresponding nonhomothetic CES index of real consumption. Assume that the function* $u\left(\cdot\right)$ *is differentiable, monotonically increasing, concave, and satisfies* $\eta_{u'}\left(C\right)\equiv Cu''\left(C\right)/u'\left(C\right)\leq-\underline{\theta}$ *for some* $\underline{\theta}>0$ *such that*

$$\underline{\theta}>\begin{cases}1-\epsilon_{min}, & 0<\sigma<1,\\ 1-\epsilon_{min}\left[1-\frac{1}{4}\left(\sigma-1\right)\left(\frac{\epsilon_{max}}{\epsilon_{min}}-1\right)^2\right], & 1<\sigma,\end{cases} \tag{19}$$

where the vector of parameters $\boldsymbol{\epsilon}$ *and* $\boldsymbol{\Omega}$ *satisfy Equation (10) for the same base good* b*. Given a sequence of prices* $\{w_t,r_t,\boldsymbol{p}_t\}_{t=0}^{\infty}$ *and an initial stock of assets* $\mathcal{A}_0$*, the utility maximization problem of households subject to the budget constraint (18) and the No-Ponzi condition* $\lim_{t\to\infty}\mathcal{A}_t\left(\prod_{t'=1}^{t-1}\frac{1}{1+r_{t'}}\right)=0$*, has a unique solution, fully characterized by the following conditions.*

1. *The intratemporal allocations of consumption goods* $\left\{\boldsymbol{C}_t,E_t=\sum_{i=1}^{I}p_{it}C_{it}\right\}_{t=0}^{\infty}$ *satisfy* $C_{it}=\Omega_i\left(p_{it}/E_t\right)^{-\sigma}\ C_t^{(1-\sigma)\epsilon_i}$ *as well as Equation (11), where the index of real consumption* C_t *at time* t *is implicitly given by* $E_t=\left(\sum_{i=1}^{I}\Omega_i\left(C_t^{\epsilon_i}p_{it}\right)^{1-\sigma}\right)^{1/(1-\sigma)}$*.*

2. *The intertemporal allocation of expenditures, the nonhomothetic CES indices of real aggregate consumption, and assets* $\{E_t,C_t,\mathcal{A}_{t+1}\}_{t=0}^{\infty}$ *satisfy the Euler equation*

$$\frac{C_{t+1}u'\left(C_{t+1}\right)}{C_tu'\left(C_t\right)}=\frac{1}{\beta\left(1+r_{t+1}\right)}\frac{\bar{\epsilon}_{t+1}}{\bar{\epsilon}_t}\frac{E_{t+1}}{E_t}, \tag{20}$$

and the transversality condition

$$\lim_{t\to\infty}\beta^t\left(1+r_t\right)\frac{\mathcal{A}_tu'\left(C_t\right)}{\bar{\epsilon}_tP_t}=0, \tag{21}$$

where $\bar{\epsilon}_{it} \equiv \sum_{i=1}^{I} \omega_{it}\epsilon_i$ denotes the expenditure share weighted average of the vector of parameters $\boldsymbol{\epsilon}$.

Proof. See Section B. $\square$

The first insight from Lemma 2 is that the household problem can be decomposed into two sub-problems: one involving the allocation of consumption expenditures and savings over time, and one involving the within-period allocation of consumption across sectors conditional on the total expenditure allocated for a given period. This is an application of two-stage budgeting that applies to all explicitly additive preferences, in our case over time (see Blackorby et al., 1978).

The first part of the lemma characterizes the *intratemporal* problem of allocating consumption across different goods based on the sectoral demand implied by the nonhomothetic CES preferences. The lemma establishes that the sectoral allocations and the nonhomothetic CES index of real consumption (for any base good b) in every period satisfy the same constraints as those imposed by the demand in the static case. In particular, a corollary of the lemma is that for any base good b, the sequences of expenditure shares $\boldsymbol{\omega}_t$ satisfy the $I-1$ constraints in Equation (11) for all $i \neq b$ at all times t. We rely on these constraints to derive a Generalized Method of Moments (GMM) estimator for the parameters of the model in the next section.

The second part of the lemma characterizes the *intertemporal* consumption-savings problem. The proof of the lemma shows that the household solves for the sequence of $\{\mathcal{A}_{t+1}, C_t\}_{t=0}^{\infty}$ that maximizes $\sum_t \beta^t u\left(C_t\right)$, subject to the constraint

$$\mathcal{A}_{t+1} + E\left(g^{-1}\left(\Upsilon_b^{\frac{1}{(\sigma-1)\varepsilon_b}} C_t^{\frac{1}{\varepsilon_b}}\right); \boldsymbol{p}_t\right) \leq w_t + \mathcal{A}_t\left(1+r_t\right), \tag{22}$$

where $E\left(\cdot; \boldsymbol{p}_t\right)$ is the expenditure function for the nonhomothetic CES preferences, defined in Equation (3). Conditions (19) are sufficient to ensure that the instantaneous utility term corresponding to this dynamic consumption/savings problem is concave in per-period expenditure everywhere. A simple example that ensures all these conditions are satisfied is given by the choice of $g\left(U\right) = U$ and $v\left(U\right) = \left(U^{1-\theta}-1\right)/\left(1-\theta\right)$ with $\sigma \in (0,1)$ and $\theta > 1$, which we use in the calibration exercise in Section 7.[19]

The Euler equation (20) illustrates the consequences of nonhomotheticity for the optimal savings behavior of the households. Because of nonhomotheticity, consumption expenditure rises nonlinearly in the index of real consumption C_t to reflect changes in the sectoral compo-

[19]We used this benchmark setup in the working paper version of this paper Comin et al. (2018), where we imposed an alternative set of sufficient conditions to ensure the concavity: $\sigma \in (0,1)$, $\theta > 0$, and additional constraints on the nonhomotheticity parameters $\epsilon_i \geq 1$ for all $i \in \mathcal{I}$. The proof of the lemma in Section B also provides the reasoning behind this alternative sufficient condition.

sition of consumption as income grows. The Euler equation (20) shows that this nonlinearity creates a wedge $\bar{\epsilon}_{t+1}/\bar{\epsilon}_t$ between the growth of marginal utility of consumption and the growth in the relative price (cost-of-living) index $P_t \equiv E_t/C_t$, compared to the benchmark case with homothetic CES ($\epsilon_i \equiv 1$ for all $i \in \mathcal{I}$).[20] The size of this wedge depends on the growth in the average income elasticities across sectors $\bar{\epsilon}_t$, and varies over time.

Firm Problem Firm profit maximization and equalization of the prices of labor and capital across sectors pin down prices of sectoral consumption goods,

$$p_{it} = \frac{p_{it}}{p_{0t}} = \frac{\alpha_0^{\alpha_0}\left(1-\alpha_0\right)^{1-\alpha_0}}{\alpha_i^{\alpha_i}\left(1-\alpha_i\right)^{1-\alpha_i}}\left(\frac{w_t}{R_t}\right)^{\alpha_0-\alpha_i}\frac{A_{0t}}{A_{it}}, \tag{23}$$

where, since the units of investment good and capital are the same, we normalize the price of investment good, $p_{0t} \equiv 1$. Equation (23) shows that price effects capture both supply-side drivers of sectoral reallocation: heterogeneity in productivity growth rates and heterogeneity in capital intensities.

General Equilibrium Goods market clearing ensures that household sectoral consumption expenditure equals the value of sectoral production output, $\omega_{it}E_t = P_{it}Y_{it}$.[21] Competitive goods markets and profit maximization together imply that a constant share of sectoral output is spent on the wage bill,

$$L_{it} = (1-\alpha_i)\omega_{it}\frac{E_t}{w_t}, \tag{24}$$

where ω_{it} is the share of sector i in household consumption expenditure.

The main prediction of the theory that we take to the data in the next section is the intratemporal consumption decision (Equation 11 and its empirical counterpart, 36). It provides a log-linear relationship between relative sectoral demand, relative sectoral prices, and total expenditure that has to hold at every period. From the market-clearing Equation (24) note that

$$\frac{L_{it}}{L_{jt}} = \frac{1-\alpha_i}{1-\alpha_j}\frac{\omega_{it}}{\omega_{jt}}, \qquad i,j \in \mathcal{I}. \tag{25}$$

This implies that relative sectoral employment is proportional to relative expenditure shares. Thus, relative sectoral employment also follows the same log-linear relationship with relative prices and total expenditure. Equation (23) suggests that relative prices capture the effect of supply-side forces in the form of differential rates of productivity growth and heterogeneous capital intensities in the presence of capital deepening. Therefore, given the parameters of the nonhomothetic CES preferences, we can rely on Equation (16) to separate out the impact

[20] Note that the dynamic implications of the model only depend on the curvature of function $u(\cdot)$ that, in turn, depends on the relative curvatures of the two functions $g(\cdot)$ and $v(\cdot)$.

[21] In our empirical applications, we additionally account for sectoral trade flows.

of demand and supply-side forces in shaping long-run patterns of structural change.

For the case in which there are three sectors, agriculture, manufacturing, and services, Equation (15) also makes transparent how nonhomothetic CES can generate a steady decline in agricultural consumption (real and nominal), a hump-shaped pattern in manufacturing consumption and a steady increase in services. Suppose that relative prices are constant. In this case, the evolution of the vector of expenditure shares $\boldsymbol{\omega}_t$ and sectoral consumption $\boldsymbol{C}_t$ depend only on the evolution of the index of real consumption C_t (and correspondingly utility U_t) as well as the relative ranking of nonhomotheticity parameters. If nonhomotheticity parameters satisfy $\epsilon_a < \epsilon_m < \epsilon_s$ and sectors are gross complements, as the index C_t grows, the relative consumption of manufacturing to agriculture and of services to manufacturing steadily grow. Thus, the share of consumption raises monotonically for services and declines monotonically for agriculture. For manufacturing, it is clear that it asymptotically has to decline too. But, it is also easy to see that the share of manufacturing can temporarily rise and generate an inverted U-pattern if the initial share of agricultural consumption is sufficiently high.[22]

Finally, we note that Equation (6) also shows how our model can generate a positive correlation between relative sectoral consumption in real and nominal terms, as it is observed in the data (Herrendorf et al., 2014). In the empirically relevant case of gross complementarity ($\sigma < 1$), the price effect implies that relative sectoral consumption should negatively correlate with relative sectoral prices, as is the case for homothetic preferences with gross complementarity.[23] However, our demand system has an additional force: income effects. The nonhomothetic effect of aggregate consumption affects both series in the same way and thus is a force that makes both time series co-move. Thus, if income effects are sufficiently strong, both time series can be positively correlated. We revisit this result in Section 6.2, where we show that this is indeed the case empirically.

2.2.2 Constant Growth Path

We characterize the asymptotic dynamics of the economy when total factor productivities at the sectoral level grow at heterogeneous but constant rates. To this end, let us assume that

[22] Under the assumption that relative prices remain constant, Equation (6) implies that the relative growth rate of sector i to sector j is $(\epsilon_i - \epsilon_j)\gamma^*$, where γ^* denotes the growth rate of C_t. Using the fact that shares add up to one, we can write the growth rate of the manufacturing sector expenditure share as $g_m = \left((\epsilon_m - \epsilon_a)\omega_a - (\epsilon_s - \epsilon_m)\omega_s\right)\gamma^*$. Thus, the sign of g_m depends on whether $(\varepsilon_m - \varepsilon_a)\omega_a \lesseqgtr (\epsilon_s - \epsilon_m)\omega_s$. Since $\epsilon_a < \epsilon_m < \epsilon_s$, this depends on whether $\omega_a \lesseqgtr \frac{\epsilon_s - \epsilon_m}{\epsilon_m - \epsilon_a}\omega_s$. If the initial expenditure share in agriculture is sufficiently large to satisfy the previous inequality, then the evolution of manufacturing will be hump-shaped. Since ω_a decreases monotonically and ω_s increases monotonically over time, g_m changes sign at most once.

[23] To see why, note that relative real consumption is decreasing in relative prices with an elasticity of $-\sigma$, while relative nominal expenditure is increasing with an elasticity of $1 - \sigma$. Thus, with CES aggregators and gross complementarity, real and nominal variables are negatively correlated–a counterfactual prediction.

the function $u(\cdot)$ is such that for some $\theta > 0$, we have

$$\lim_{C \to \infty} \frac{C\, u''(C)}{u'(C)} = -\theta. \tag{26}$$

Moreover, we assume that sectoral productivity growth is given by

$$\frac{A_{it+1}}{A_{it}} = 1 + \gamma_i, \qquad i \in \{0\} \cup \mathcal{I}. \tag{27}$$

Under these assumptions, the competitive equilibrium of the economy converges to a path of constant per-capita consumption growth. Along this path, consumption expenditure, investment, and the stock of capital all grow at a rate dictated by the rate of growth of the investment sector γ_0. Denoting the rate of growth of the index of real consumption C_t by γ^*, the share of each sector i in consumption expenditure also exhibits constant growth along a constant growth path, characterized by constants

$$1 + \xi_i \equiv \lim_{t \to \infty} \frac{\omega_{it+1}}{\omega_{it}} = \left[\frac{(1+\gamma^*)^{\epsilon_i}}{(1+\gamma_0)^{\frac{\alpha_i}{1-\alpha_0}} (1+\gamma_i)} \right]^{1-\sigma}. \tag{28}$$

Given the fact that expenditures shares have to be positive and sum to 1, Equation (28) allows us to find the rate of growth of real consumption as a function of nonhomotheticity parameters, factor intensities, and the rates of technical growth. The next proposition characterizes the asymptotic dynamics of the competitive equilibrium.

Proposition 1. *Let γ^* be defined as*

$$\gamma^* = \begin{cases} \min_{i \in \mathcal{I}} \left[(1+\gamma_0)^{\frac{\alpha_i}{1-\alpha_0}} (1+\gamma_i) \right]^{\frac{1}{\epsilon_i}} - 1, & 0 < \sigma < 1, \\ \max_{i \in \mathcal{I}} \left[(1+\gamma_0)^{\frac{\alpha_i}{1-\alpha_0}} (1+\gamma_i) \right]^{\frac{1}{\epsilon_i}} - 1, & 1 < \sigma. \end{cases} \tag{29}$$

Assume that conditions (26) and (27) hold, and that γ^* satisfies the following condition

$$(1+\gamma_0)^{-\frac{\alpha_0}{1-\alpha_0}} < \beta (1+\gamma^*)^{1-\theta} < \min \left\{ \frac{(1+\gamma_0)^{-\frac{\alpha_0}{1-\alpha_0}}}{\alpha_0 + (1-\alpha_0)(1+\gamma_0)^{-\frac{1}{1-\alpha_0}} (1-\delta)}, 1 \right\}. \tag{30}$$

Then, for any initial level of capital stock, K_0, there exists a unique competitive equilibrium *along which the index of real consumption asymptotically grows at rate* γ^**,*[24]

$$\lim_{t\to\infty} \frac{C_{t+1}}{C_t} = 1 + \gamma^*. \tag{31}$$

Along the constant growth path, (i) the real interest rate is constant, $r^* \equiv (1+\gamma_0)^{1/(1-\alpha_0)}/\beta(1+\gamma^*)^{1-\theta} - 1$*, (ii) consumption expenditure, total nominal output, and the stock of capital grow at rate* $(1+\gamma_0)^{\frac{1}{1-\alpha_0}}$*, and (iii) only the subset of sectors* $\mathcal{I}^*$ *that achieve the minimum in Equation (29) employ a non-negligible fraction of workers.*

Proof. See Section B. $\square$

Equation (29) shows how the long-run growth rate of the nonhomothetic CES index of real consumption is affected by nonhomotheticity parameters, ϵ_i, rates of technological progress, γ_i, and sectoral capital intensities, α_i. To build intuition, consider the case in which all sectors have the same capital intensity and preferences are homothetic. In the empirically relevant case of $\sigma \in (0,1)$, Equation (29) implies that the long-run growth rate of the real consumption index is pinned down by the sectors with the lowest technological progress, as in Ngai and Pissarides (2007). Consider now the case in which there is also heterogeneity in income elasticities. In this case, sectors with higher income elasticity and faster technological progress can co-exist in the long-run with sectors with low income elasticity and slow technological progress. The intuition is that the agents shift their consumption expenditure towards income-elastic goods as they become richer, and away from goods that are becoming cheaper due to technical progress. Finally, the role of heterogeneity in capital shares in shaping the long-run rate of consumption growth is analogous to the role of technological progress, as they both ultimately shape the evolution of prices.

Which sectors survive in the long-run? At all points in time, all sectors produce a positive amount of goods, and their production grows over time. In relative terms, however, only the subset of sectors $\mathcal{I}^*$ satisfying Equation (29) will comprise a non-negligible share of total consumption expenditure in the long-run. Indeed, if the initial number of sectors is finite, generically only one sector survives in the long-run.

2.2.3 Transitional Dynamics

To study the transitional dynamics of the economy, we focus on the special case where all sectors have a common capital intensity, $\alpha_i = \alpha$ for all $i \in \mathcal{I}$, and the felicity function in terms of the index of real consumption is isoelastic, i.e., $u(C) = \left(C^{1-\theta} - 1\right)/(1-\theta)$, where θ

[24]Here we follow the terminology of Acemoglu and Guerrieri (2008) in referring to our equilibrium path as a constant growth path. Kongsamut et al. (2001) refer to this concept as generalized balanced growth path. As with these papers, we normalize the investment sector price. See Duernecker et al. (2017a) for a discussion on the connection between this price normalization and chained-price indexing of real consumption.

controls the elasticity of intertemporal substitution.[25] Let us normalize each of the aggregate variables by their respective rates of growth, introducing normalized consumption expenditure $\tilde{E}_t \equiv (1+\gamma_0)^{-\frac{t}{1-\alpha}} E_t$, per-capita stock of capital $\tilde{k}_t \equiv (1+\gamma_0)^{-\frac{t}{1-\alpha}} K_t$, and real per-capita consumption $\tilde{C}_t \equiv (1+\gamma^*)^{-t} C_t$. Using the assets market clearing condition, we can translate Equations (20) and (22) into equations that characterize the evolution of the normalized aggregate variables

$$\tilde{k}_{t+1} = (1+\gamma_0)^{-\frac{1}{1-\alpha}} \left[\tilde{k}_t^{\alpha} + \tilde{k}_t (1-\delta) - \tilde{E}_t\right], \tag{32}$$

$$\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{\theta-1} \frac{\bar{\epsilon}_{t+1}}{\bar{\epsilon}_t} \frac{\tilde{E}_{t+1}}{\tilde{E}_t} = \frac{1+\alpha\tilde{k}_{t+1}^{\alpha-1}-\delta}{1+r^*}, \tag{33}$$

where the normalized consumption expenditure $\tilde{E}_t$ is a function of $\tilde{C}_t$ and the two functions of the growth in $\tilde{C}_t$, that is, $\tilde{C}_{t+1}/\tilde{C}_t$, as[26]

$$\left(\frac{\tilde{E}_{t+1}}{\tilde{E}_t}\right)^{1-\sigma} = \sum_{i=1}^{I} \omega_{it} \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{\epsilon_i(1-\sigma)} (1+\xi_i)^t, \tag{34}$$

$$\frac{\bar{\epsilon}_{t+1}}{\bar{\epsilon}_t} = \left(\frac{\tilde{E}_t}{\tilde{E}_{t+1}}\right)^{1-\sigma} \sum_{i=1}^{I} \omega_{it} \left(\frac{\epsilon_i}{\bar{\epsilon}_t}\right) \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{\epsilon_i(1-\sigma)} (1+\xi_i)^t. \tag{35}$$

Starting from any initial levels of normalized per-capita consumption $\tilde{C}_0$ and stock of capital $\tilde{k}_0$, we can find that period's allocation of expenditure shares $\boldsymbol{\omega}_t$ using Equations (2) and (3), and compute the normalized per-capita consumption and stock of capital of the next period using Equations (32) and (33). Proposition 1 establishes that the equilibrium path exists, is unique, and is therefore fully characterized by the dynamic equations above.

At the aggregate level, the transitional dynamics of this economy deviates from that of the standard neoclassical growth model since the household's elasticity of intertemporal substitution (EIS) varies with income.[27] Goods with lower income elasticity are less intertemporally substitutable. Since the relative shares of high and low income-elastic goods in the consumption expenditure of households vary over time, the effective elasticity of intertemporal substitution of households correspondingly adjusts. Typically, as income rises, low income-elastic goods constitute a smaller share of the households' expenditure and therefore the

[25]The online appendix characterizes the dynamics along an equilibrium path in the more general case with heterogeneous capital intensities α_i in a continuous-time rendition of the current model.

[26]See Section B.2 for the derivation of this result.

[27]For general multi-good consumption/savings problems, Crossley and Low (2011) show that the standard assumption of constant EIS imposes the strong restriction on *within period* allocation of consumption that the shapes of Engel curves have to be of at most rank 2. More specifically, they show that the only nonhomothetic preferences satisfying this restriction is the PIGL and PIGLOG preferences of Muellbauer (1975) and Muellbauer (1976) (see also Muellbauer, 1987). As with our findings in the next section, Crossley and Low (2011) find that the data clearly rejects this restriction on the shapes of Engel curves.

effective elasticity of intertemporal substitution rises over time. When the economy begins with a normalized stock of capital $\tilde{k}_t$ below its long-run level $\tilde{k}^*$, the interest rate along the transitional path exceeds its long-run level. With a rising elasticity of intertemporal substitution, households respond increasingly more strongly to these high interest rates. Therefore, the accumulation of capital and the fall in the interest rate both accelerate over time.[28]

In general, the transitional dynamics of the economy can generate a rich set of different patterns of structural transformation depending on nonhomotheticity parameters and the rates of productivity growth of different sectors $\{\epsilon_i, \gamma_i\}_{i=1}^{I}$. In Section 3, we estimate the demand-side parameters of the model using both micro and macro level data. We then use these parameters to calibrate the model in Section 7 and study the implications for the evolution of sectoral shares as well as the paths of interest rate and savings.

3 Model Estimation

In this section, we bring our model to the data with two goals in mind. Our first goal is to show that nonhomothetic CES preferences, despite their parametric parsimony, provide a reasonable account of the relation between the sectoral composition of the economy, relative prices, and total expenditure, both at the household and and at the aggregate levels. Our second goal is to provide estimates of parameters of the nonhomothetic CES preferences (i.e., nonhomotheticity parameters, $\{\epsilon_i\}_{i\in\mathcal{I}}$, and the elasticity of substitution, σ) that can be used to calibrate our model and study its transitional dynamics in Section 7. In both household and aggregate data, our estimating strategy relies on the construction of Section 2.1.2 and particularly the intratemporal specification (16) for the allocations of consumption across three sectors: $a$griculture, $m$anufacturing, and $s$ervices.

To study relative sectoral demand at the household level, we use the U.S. Consumer Expenditure Survey (CEX) that reports the composition of household consumption expenditures on different final goods. For the aggregate data, we use a panel of 39 countries over the post-war period. The sample of countries covers a wide range of growth experiences, including developing countries (e.g., Botswana and India), miracle economies (e.g., South Korea and Taiwan) and developed economies (e.g., the U.S. and Japan). The aggregate data contains sectoral employment, value-added output, sectoral prices and consumption per capita. We briefly discuss each of these datasets below, and present further details on the data sources in Section D of the online appendix.

[28] King and Rebelo (1993) discuss this mechanism in the context of a neoclassical growth model with Stone-Geary preferences. In the current model, if the rate of productivity growth in high income-elastic sectors is large enough, the share of these sectors may in fact fall over time, and the effective elasticity of intertemporal substitution of households may correspondingly fall. However, as we will see in the calibration of the model in Section 7, the empirically relevant case is one in which the share of more income-elastic goods rises as the economy grows.

3.1 Data Description

U.S. Household Expenditure Data We use U.S. household quarterly consumption data for the period 1999-2010 from the Consumption Expenditure Survey (CEX). In the CEX, each household is interviewed about their expenditures for up to four consecutive quarters. Our data construction is based on Aguiar and Bils (2015), who in turn follow very closely Heathcote et al. (2010) and Krueger and Perri (2006). As these authors, we focus on a sample of urban households with a present household head aged between 25 and 64. We also use the same total income measure (net of taxes) and household controls as Aguiar and Bils (2015). These controls are demographic dummies based on age range of the household head (25-37, 38-50, 51-64), household size dummies ($\leq$2, 3-4, 5+) and dummies for the number of household earners $(1, 2+)$.

The key difference from Aguiar and Bils (2015) is that we construct our consumption categories to match expenditure in agriculture, manufacturing and services. We follow Herrendorf et al. (2013) to construct these three categories. The agricultural sector is composed of food-at-home expenditures. The main expenditure categories for the manufacturing sector are vehicles, housing equipment, other durables, clothing, shoes and personal care items. For services, these are housing, utilities, health, food away from home, television subscriptions, and other entertainment fees.

We combine the CEX data with disaggregated regional quarterly price series from the BLS's urban CPI (CPI-U). Similar to Hobijn and Lagakos (2005) and Hobijn et al. (2009), we construct the price for each sector faced by a household by taking the household expenditure-weighted average of the log-price of each of the expenditure categories belonging to the sector. Since expenditure weights are household-specific, this allows us to, albeit imperfectly, account for the fact that the effective price for each sector may be different across households.

Aggregate Data Our aggregate data comes from two sources. The sectoral data comes from Groningen's 10-Sector Database (Vries et al., 2014). The 10-Sector Database provides a long-run internationally comparable dataset on sectoral measures for 10 countries in Asia, 9 in Europe, 9 in Latin America, 10 in Africa and the United States. The variables covered in the data set are annual series of production value added (nominal and real) and employment for 10 broad sectors starting in 1947. In our baseline exercise, we aggregate the ten sectors into agriculture, manufacturing and services following Herrendorf et al. (2013). In Section 6, we estimate our model for 10 sectors. Our consumption expenditure per capita data comes from the ninth version of the Penn World Tables, (Feenstra et al., 2015). Combining these two datasets gives us a final panel of 39 countries with an average number of observations of 42 years per country. As we have discussed, the countries in our sample span very different growth trajectories. For example, the ratio of the 90th to the 10th percentile of consumption per capita in year 2000 is 18.2.[29]

3.2 Household-Level Results

Empirical Strategy and Identification Following the construction of Section 2.1.2, we assume that the sectoral composition of the consumption of each household n across sectoral goods follows nonhomothetic CES demand for a set of identifiable parameters $(\sigma, \boldsymbol{\epsilon}, \boldsymbol{\Omega}^n)$, where we have allowed the taste shifter $\boldsymbol{\Omega}^n$ to potentially vary at the household level. We consider the manufacturing sector m as the base. We can then write an empirical counterpart of Equations (16) for each sector $i \in \mathcal{I}_{-m} = \{a, s\}$ as

$$\log\left(\frac{\omega_{it}^n}{\omega_{mt}^n}\right) = (1-\sigma)\log\left(\frac{p_{it}^n}{p_{mt}^n}\right) + (1-\sigma)\left(\epsilon_i - 1\right)\log\left(\frac{E_t^n}{p_{mt}^n}\right) + \left(\epsilon_i - 1\right)\log \omega_{mt}^n + \zeta_i^n + \nu_{it}^n \quad (36)$$

where ω_{it}^n and p_{it}^n denote the share of consumption and the price of sector-i goods of household n at time t, E_t^n denotes their total expenditure, $\zeta_i^n \equiv \log(\Omega_i^n/\Omega_m^n)$ accounts for relative taste parameters, and ν_{it}^n for the error terms.

Furthermore, we impose the additional assumptions that the household level taste-shocks are linear functions of observables $\zeta_i^n \equiv \beta_i' X^n + \delta_{ir}$, and that the error term may contain common sector-time fixed effects across households $\nu_{it}^n \equiv \delta_{it} + \tilde{\nu}_{it}^n$. The first assumption imposes the constraint that the cross-household heterogeneity in time-invariant taste parameters can be fully explained as a linear function of the vector X^n of household characteristics discussed above (age, household size, and number of earners dummies) and sector-region (ir) fixed effects. The second assumption allows for a dyad of sector-time (it) fixed effects to absorb potential aggregate consumption shocks. This specification identifies income elasticities based on the within-region covariation between expenditure shares and total household expenditures, controlling for household characteristics.

To deal with potential measurement error and endogeneity issues, we use instruments for the observed measures of household expenditures and relative prices. First, we follow Aguiar and Bils (2015) and instrument household expenditures (total and on the reference good) in a given quarter with the annual household income after taxes and the income quintile of the household. The instruments capture the permanent household income and are therefore correlated with household expenditures without being affected by transitory measurement

[29] The countries in our sample are Argentina, Bolivia, Botswana, Brazil, Chile, Colombia, Costa Rica, Denmark, Ethiopia, France, Germany, Ghana, Hong Kong, India, Indonesia, Italy, Japan, Kenya, Korea, Malawi, Malaysia, Mauritius, Mexico, Netherlands, Nigeria, Perú, Philippines, Senegal, Singapore, South Africa, Spain, Sweden, Taiwan, Tanzania, Thailand, United Kingdom, United States of America, Venezuela, and Zambia. In a previous version of this paper (Comin et al., 2015), we used the Barro-Ursua dataset instead of the PWTv9.0. The main advantage of the PWTv9.0 is that it allows us to estimate our model for all countries in the 10-sector database (39) relative to Barro-Ursua, for which we only had data for 25 countries. We find very similar results using both data sets.

error in total expenditures.[30] Second, we instrument household relative prices with a "Hausman" relative-price instrument. Each of the prices used in the relative-price instrument is constructed in two steps. First, for each sub-component of a sector, we compute the average price across regions excluding the own region. Then, the sectoral price for a region is constructed using the average region expenditure shares in each sub-component as weights.[31] These price instruments capture the common trend in U.S. prices while alleviating endogeneity concerns due to regional shocks (and measurement error of expenditure).[32]

Equation (36) defines a system of log-linear equations for $i \in \mathcal{I}_{-m}$ with constraints in its coefficients. First, there is the constraint that σ is the same across equations. Second, for each equation, the product of the coefficient on relative prices, $(1-\sigma)$, and expenditure share on manufacturing, $(\epsilon_i - 1)$, has to be equal to the coefficient on expenditure, $(1-\sigma)(\epsilon_i - 1)$. We estimate the parameters $\{\sigma, \epsilon_i, \zeta_i^n\}_{i \in \mathcal{I}_{-m}}$ of this system of equations (imposing these constraints) via the generalized method of moments (GMM).[33]

We present our estimation results under two alternative weighting schemes. We use the household weights provided in the CEX data to make the household sample representative of the entire U.S. population. Additionally, we re-weight households by their total level of expenditure to bridge the gap with the estimates with aggregate-level data.[34] Comparing the alternative weighting schemes allows us to examine the stability of the estimated parameters across income groups.

Estimation Results Table 1 reports our estimation results. Columns (1) and (2) report the estimates when we control only for household characteristics X^n but we do not include any time or region fixed effects. Column (1) corresponds to the weighting scheme that replicates the U.S. population, while column (2) corresponds to the expenditure re-weighted estimates. In both cases we find very similar estimates. The estimates show that the nonhomotheticity parameter is lower for agriculture relative to manufacturing ($\epsilon_a - 1 = -0.80$ in column 1) and higher for services relative to manufacturing ($\epsilon_s - 1 = 0.65$ in column 1). The price

[30]The measure of total household income corresponds to a separate question in the CEX and is not constructed adding household expenditures over the year. Boppart (2014a) also instruments quarterly expenditure levels by household income.

[31]Formally, we instrument relative prices $\log(p_{it}^r/p_{mt}^r)$ with $\log(p_{it}^{-r}/p_{mt}^{-r})$, where $\log p_{it}^{-r}$ for $i \in \{a, m, s\}$ is constructed as follows. Suppose that for sector j we have information on the price of subcomponents $k \in \{1, \ldots, K\}$, then $\log p_{it}^{-r} = \sum_{k=1}^{K} \hat{\omega}_{kt}^r \log \hat{p}_{kt}^{-r}$ where $\hat{\omega}_{kt}^r$ denotes the average expenditure share of k in region r and $\hat{p}_{kt}^{-r}$ denotes the log of the average price in the U.S. excluding region r. We have verified that constructing the instrument using the price in the own region or the average national price delivers similar results.

[32]Using the average price in the U.S. excluding the own region addresses the concern of regional shocks, while capturing the common component of prices across regions. Using average expenditures in the region addresses the concern of mismeasurement of household expenditure shares in that region to the extent that the mismeasurement averages out in the aggregate.

[33]We also note that our estimation strategy is different from the one proposed in Hanoch (1975). Hanoch proposes an estimation based on double differences that can only identify $I-2$ nonhomotheticity parameters.

[34]We thank the editor for this suggestion.

Table 1: Estimates, CEX Final Good Expenditure

	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.26	0.28	0.28	0.20	0.31	0.33
	(0.04)	(0.04)	(0.03)	(0.05)	(0.04)	(0.05)
$\epsilon_a - 1$	-0.80	-0.83	-0.81	-0.70	-0.95	-0.97
	(0.06)	(0.07)	(0.06)	(0.07)	(0.09)	(0.10)
$\epsilon_s - 1$	0.65	0.68	0.75	0.67	0.82	0.85
	(0.07)	(0.07)	(0.06)	(0.07)	(0.09)	(0.10)
Expenditure Re-Weighted	N	Y	N	Y	N	Y
Region FE	N	N	Y	Y	Y	Y
Year $\times$ Quarter FE	N	N	N	N	Y	Y

Notes: All regressions include household controls (described in the text). Standard errors clustered at the household level shown in parenthesis. The number of observations is 60,925 in all regressions.

elasticity estimates are less than one ($\sigma = 0.26$ in the first column) suggesting that agriculture, manufacturing and services are gross complements in household preferences. Using Equation (9), we find that the expenditure elasticities for the average household in our sample are 0.37, 0.83, and 1.20 for agriculture, manufacturing, and services, respectively. This implies that for the average U.S. household agricultural and manufacturing goods are necessities, while services are luxury goods.

We subsequently add region and time fixed effects in columns (3) to (6). We find very similar coefficients to those in columns (1) and (2). An important observation from Table 1 is that our estimates of relative income elasticities do not change significantly between the specifications with U.S. population weights (odd columns) and those with expenditure weights (even columns). This finding suggests that the assumption in our model of nonhomotheticity parameters $\{\epsilon_i\}_{i \in \mathcal{I}}$ being constant across income groups provides a good description of the data.[35]

Table 2 explores the stability of the slope of the relative demand in expenditure across different subsamples of the data. First, we split households in two groups: above and below the annual median income in the sample. Columns (1) and (2) report the estimates of specification (36) when we estimate it separately for each subsample. We find that the estimated elasticities are not significantly different from each other. We also study the stability of the estimates over time and estimate our baseline regression in the pre- and post-2005 sub-samples. Columns (3) and (4) report the estimates, where we again find estimates that are close in magnitude.

A key prediction of nonhomothetic CES is that relative expenditure shares are log-linear in the real consumption index. We use the estimate of the elasticity of substitution $\hat{\sigma}$ from

[35] Table E.1 in the online appendix reports the regression of our instruments on aggregate expenditure and prices, which would correspond to the "first-stage" in a 2SLS setting, and show that the coefficients have the expected sign and are significant at conventional levels.

Table 2: Sample Splits, CEX Final Good Expenditure

	< P50	> P50	Pre '05	Post '05
	(1)	(2)	(3)	(4)
σ	0.35	0.31	0.33	0.25
	(0.07)	(0.05)	(0.07)	(0.05)
$\epsilon_a - 1$	-0.89	-0.99	-0.98	-0.92
	(0.17)	(0.12)	(0.15)	(0.08)
$\epsilon_s - 1$	0.75	0.59	0.74	0.65
	(0.19)	(0.16)	(0.14)	(0.10)

Notes: Regressions estimated using CEX-replicate weights. Households controls included in all regressions (as described in the main text). All regressions include Region and Year $\times$ Quarter fixed effects. Standard errors clustered at the household level. The estimations in columns (2) and (3) are performed imposing the constraint $\epsilon_a \geq 0$ (by estimating an exponential transformation of the variable). The corresponding standard errors are computed using the delta method.

Table 1 in Equation (13) to obtain our measure of the nonhomothetic CES index of real consumption

$$\log C_t^n = \log\left(\frac{E_t^n}{p_{mt}^n}\right) + \frac{1}{1-\hat{\sigma}} \log \omega_{mt}^n. \tag{37}$$

Our theory (e.g., Equation 6) implies that log-relative expenditure shares are a linear function of log-relative prices and the log real consumption index. Figure 2 plots the (binned) residuals after all controls and relative prices have been partialled-out from the instrumented real consumption measure and relative expenditure shares. As implied by our model, we find that residual variation in relative shares is well approximated by a log-linear function of residual consumption, both for agriculture relative to manufacturing (2a) and services relative to manufacturing (2b).

3.3 Cross-Country Aggregate-Level Results

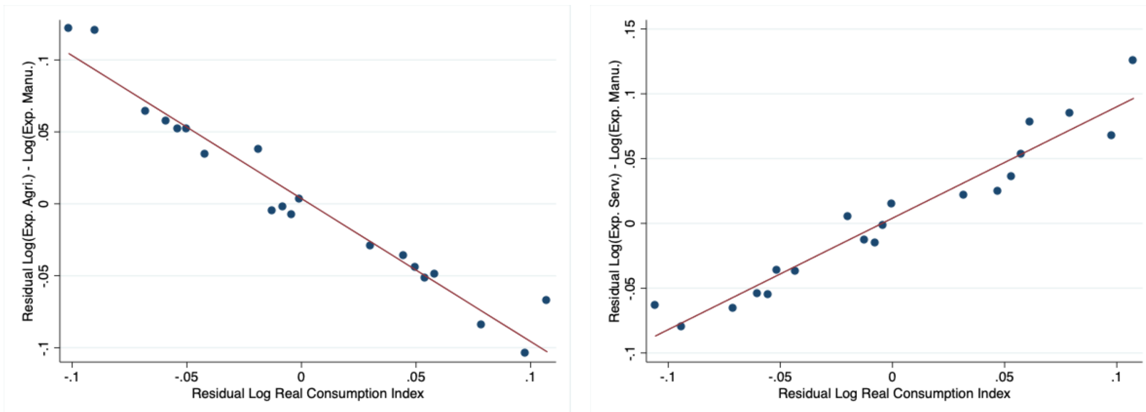
After estimating the model with household data, we explore the ability of these preferences preferences to account for the broad patterns of structural transformations observed across countries during the post-war period.

Empirical Strategy and Identification We employ a strategy similar to the one we used for micro data to estimate the preferences with aggregate data. Recall that our model assumes that each country is inhabited by homogeneous households.[36] Hence, the specifications

[36]The growth model developed in Section 2.2 abstracts from within-country dispersion of income and assumes all households are identical. In Section B of the online appendix, we derive approximate expressions for aggregate sectoral demand in an environment featuring within-country heterogeneity in income. In particular, we show that the equations characterizing household-level and aggregate-level allocation of expenditure are

Figure 2: Partial Correlation of the Real Consumption Index and Relative Log Expenditure Shares, CEX

(a) Agriculture relative to Manufacturing (b) Services Relative to Manufacturing



Notes: These plots depict the (binned) residuals corresponding to the average value of 20 equal-sized bins of the data. The red line depicts the linear regression between the residualized variables.

discussed in Section 2.1.2 and, in particular, Equations (11) and (16) apply to aggregate data for a set of identifiable parameters $(\sigma, \boldsymbol{\epsilon}, \boldsymbol{\Omega}^n)$, where the taste shifters $\boldsymbol{\Omega}^n$ can vary at the level of each country n. Once again, we use the manufacturing sector $b = m$ as the base.[37]

In our baseline exercise, we estimate our model from the patterns of structural change in employment. In particular, Equation (25) implies that relative sectoral consumption expenditures are proportional to relative sectoral employment shares, yielding

$$\log\left(\frac{L_{it}^n}{L_{mt}^n}\right) = (1-\sigma)\log\left(\frac{p_{it}^n}{p_{mt}^n}\right) + (1-\sigma)(\epsilon_i - 1)\log\left(\frac{E_t^n}{p_{mt}^n}\right) + (\epsilon_i - 1)\log\omega_{mt}^n + \zeta_i^n + \nu_{it}^n. \tag{38}$$

The term ζ_i^n denotes a country-sector fixed effect.[38] In addition, we include controls for log-sectoral exports and imports in Equation (38) to account for the fact that some goods can be traded, thus affecting the sectoral composition of employment.[39] Using employment

identical up to first order of approximation in the standard deviation of the logarithm of consumption expenditure, if the latter has a symmetric distribution such as the log-normal distribution (see Battistin et al., 2009, for evidence for the log-normality of the distribution of total consumption expenditure across households).

[37] Section 5 shows estimates with alternative bases.

[38] In this case ζ_i^n, in addition to constant taste parameters, $\log\Omega_i^n$, also absorbs country-specific heterogeneity in sectoral capital intensity, α_i^n's.

[39] We note also that our sectoral price measures have embedded the effect of traded intermediate inputs and that total expenditures embed the effect of trade on income. We use the "trade detail" data from the PWT to construct sectoral exports and imports. Agricultural trade flows correspond to trade in food and beverages. Manufacturing trade flows correspond to trade in industrial supplies, fuels and lubricants, capital goods, transport equipment, and consumer goods. Our baseline specifications directly include log-sectoral exports and imports as controls. Alternatively, we can rely on a model with exogenous trade flows to derive

Table 3: Cross-Country Estimates, $\epsilon_m = 1$

	World		OECD		Non-OECD	
	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.57	0.50	0.25	0.35	0.63	0.48
	[0.32,0.69]	[0.26,0.71]	[0.20,0.66]	[0.03,0.55]	[0.06,0.74]	[0.34,0.75]
$\epsilon_a - 1$	-0.98	-0.89	-0.99	-0.99	-0.91	-0.80
	[-1.13,-0.41]	[-1.14,-0.46]	[-1.00,-0.38]	[-1.00,-0.66]	[-1.15,-0.58]	[-1.14,-0.40]
$\epsilon_s - 1$	0.17	0.21	0.27	0.25	0.18	0.37
	[0.07,0.60]	[0.03,0.67]	[0.03,0.55]	[0.09,1.95]	[0.11,2.08]	[0.03,0.67]
Country × Sector FE	Y	Y	Y	Y	Y	Y
Trade Controls	N	Y	N	Y	N	Y
Observations	1626	1626	492	492	1134	1134

Notes: Bootstrapped 95% confidence intervals clustering at the country level shown in square brackets (computed through bootstrapping 50 samples with replacement). The estimations in columns (3) and (4) are performed by imposing the constraint that $\epsilon_a \geq 0$ (by estimating an exponential transformation of the variable).[40]

rather than value-added shares in Equation (38) is our favored specification for investigating the cross-country data because it does not use the price data (an explanatory variable) to construct the dependent variables (Section 6 shows that we find similar estimates if we use value-added shares as dependent variables). All other steps of the estimation procedure are identical to those used in the case of household-level data.

Our cross-country estimation relies on the within-country variation of employment shares, expenditure, and relative prices to identify the price and income elasticities. The identification assumption to obtain consistent estimates is that, for each country, the shocks to relative prices and income are uncorrelated with the relative demand shocks ν_{it}^n. This assumption would be violated if, for example, sectoral taste shocks (which are part of ν_{it}^n) are correlated with aggregate demand or relative price shocks. To alleviate these endogeneity concerns, we estimate our model separately for OECD and Non-OECD countries and show that the estimates do not change significantly across sub-samples. While the estimates could in principle be biased in both cases, this would require sectoral taste shocks (or any other omitted variable) to be correlated with aggregate demand or relative price shocks in the same way across sub-samples, which we deem less likely.

less flexible estimation equations that control for trade flows and are consistent with the model. In this case, we need to assume that factor intensities are identical in the production function of the same sector across different countries approach. We can then use the accounting identity $p_{it}^n C_{it}^n = p_{it}^n Y_{it}^n - NX_{it}^n$, where NX_{it}^n denotes the nominal value of net exports in sector i, time t and country n. It follows that the expressions for sectoral employment in sector i should be adjusted by terms involving the observed values of $NX_{it}^n / p_{it}^n Y_{it}^n$. Using these alternative model-driven controls for trade flows, we have found results very similar to what is presented here.

Estimation Results Table 3 reports the results obtained from estimating (38) for the full sample of 39 countries and separately for OECD and Non-OECD sub-samples. Columns (1) and (2) report the estimates for our entire sample with and without trade controls, respectively. The estimated nonhomotheticity parameter is lower for agriculture relative to manufacturing ($\epsilon_a < 1$) and larger for services compared to manufacturing ($\epsilon_s > 1$). The price elasticity is also less than unity ($\sigma = 0.57$). Introducing trade controls hardly changes our estimates, as shown in column (2).[41] Using Equation (9), we find that the implied expenditure elasticities for the average country-year in our sample are 0.56, 1.03 and 1.14 for agriculture, manufacturing and services, respectively. This implies that agriculture is a subsistence good, while manufacturing (marginally) and services are luxury goods.

As we discuss in Section 2.1, whether good i is a luxury or a necessity is not an intrinsic characteristic of the good. Rather, it depends on the composition of consumer expenditures and the relative ranking of ϵ_i. To illustrate this point, we compute the expenditure elasticities for the country at the 10th decile of income per capita in our sample (Tanzania) and at the 90th decile (the Netherlands) in year 2000. For Tanzania, we find that the expenditure elasticites are 0.57, 1.15 and 1.29 for agriculture, manufacturing and services, respectively. Thus, manufacturing was a luxury good from the perspective of Tanzania's representative consumer in year 2000. In contrast, for the Netherlands, the expenditure elasticities are 0.55, 0.95 and 1.04, implying that manufacturing was a necessity good from the standpoint of a Dutch consumer.

Columns (3) and (4) report the estimated elasticities for OECD countries and columns (5) and (6) report the estimates for the Non-OECD sample. The estimates are similar for the two sub-samples. In fact, we cannot reject the null that the estimates for the nonhomotheticity parameters are the same for both sub-samples at conventional levels. We find that our estimates of nonhomotheticity parameters appear to be stable across countries of different levels of income. The point estimates of σ vary more across subsamples. We find values between 0.25 and 0.63. However, they always remain less than unity in all specifications. The estimates appear more stable when controlling for sectoral trade. Moreover, the estimate of any specification falls within the confidence interval of the estimates in the other sub-samples. Overall, the similarity of the estimates across sub-samples is reassuring, as we deem less likely that unobserved relative demand shocks may be correlated with relative prices and income in the same way across two such different groupings of countries.

[40] We report bootstrapped standard errors because the weighting matrix in the second step of the GMM estimation when we allow for clustering at the country-level becomes almost singular, as we include a large number of fixed effects.

[41] In a previous version of the paper (Comin et al., 2015), we used the Barro-Ursua measures of real consumption. In that case, we only had data for 25 countries. Almost all of the differences from our current sample come from the fact that we now have more Non-OECD countries. We find similar estimates using either sample. Also, we can reject the null hypothesis that $\log(E_t)$ and $\log(E_t/p_{mt})$ have unit roots in our sample. Thus, the variables in our regression are not cointegrated.

4 Accounting for Structural Change

Next, we use our estimated model to formally evaluate the fit of the model and quantify how much of the variation in employment shares within countries is accounted for by changes in relative prices and the real consumption index.

To conduct this exercise, we rely on the estimated parameters of the nonhomothetic CES preferences based on the aggregate data in Section 3.3, and the theory developed in Section 2.1. In particular, Equation (16) implies that the within-country variation in log-relative employment shares can be decomposed into the contribution of income and price effects using knowledge of the demand parameters $\{\sigma, \epsilon_i\}_{i\in\mathcal{I}}$, relative prices, and the nonhomothetic CES index of real consumption C (defined in terms of observables in Equation 12). Denoting with a "hat" the estimated parameter values, $\{\hat{\sigma}, \hat{\epsilon}_i, \hat{\zeta}_i^n\}$, the predicted values of our estimation Equations (38) for log-relative employment shares are

$$\widehat{\log\left(\frac{L_{it}^n}{L_{mt}^n}\right)} = \underbrace{(1-\hat{\sigma})\log\left(\frac{p_{it}^n}{p_{mt}^n}\right)}_{\text{Price Effect}} + \underbrace{(1-\hat{\sigma})\left(\hat{\epsilon}_i - 1\right)\log\left(\frac{E_t^n}{p_{mt}^n}\right) + \left(\hat{\epsilon}_i - 1\right)\log\omega_{mt}^n}_{\text{Income Effect}} + \hat{\zeta}_i^n \quad (39)$$

for $i = \{a, s\}$. Using the definition of the nonhomothetic CES index of real consumption (12), note that the term corresponding to income effects in the estimating Equations (39) is

$$(1-\hat{\sigma})(\hat{\epsilon}_i - 1)\widehat{\log C_t^n} \equiv (1-\hat{\sigma})(\hat{\epsilon}_i - 1)\left(\log\left(\frac{E_t^n}{p_{mt}^n}\right) + \frac{1}{1-\hat{\sigma}}\log\omega_{mt}^n\right). \quad (40)$$

To assess the overall fit of our model, we report in Table 4 the ratio of the variance of predicted and actual log-employment shares, $\mathrm{Var}\left[\widehat{\log\left(\frac{L_{it}^n}{L_{mt}^n}\right)}\right] \Big/ \mathrm{Var}\left[\log\left(\frac{L_{it}^n}{L_{mt}^n}\right)\right]$ for $i = \{a, s\}$. We find that the model explains most of the variation in data, as the two values are 0.97 for agriculture to manufacturing employment shares and 0.57 for services relative to manufacturing.[42] Next, we compute the fraction of the predicted within-country variation in employment shares accounted for by the price and income effects. The second row of Table 4 reports the contribution of income and price effects to the total predicted variation in employment shares,

$$\frac{\mathrm{Var}\left[(1-\hat{\sigma})\log\left(\frac{p_{it}^n}{p_{mt}^n}\right) + (1-\hat{\sigma})\left(\hat{\epsilon}_i - 1\right)\widehat{\log C_t^n}\right]}{\mathrm{Var}\left[\widehat{\log\left(\frac{L_{it}^n}{L_{mt}^n}\right)}\right]}. \quad (41)$$

We find that this ratio is 46% and 61% in the equations for agriculture and services, respectively. The remainder of the predicted variance is accounted for by the country-sector

[42] Figure F.1 in the online appendix depicts the predicted fit against the data.

Table 4: Accounting for Structural Change, Baseline Estimates

	$\log\left(\frac{\text{Agriculture}}{\text{Manufacturing}}\right)$	$\log\left(\frac{\text{Services}}{\text{Manufacturing}}\right)$
Explained over Total Variance	0.97	0.57
Within over Explained Variance	0.46	0.61
	Within-Country Variance Decomposition	
Price Effects	0.02	0.27
Income Effects	0.98	0.84
Both Effects	1.00	1.00

Notes: Explained over Total Variance is computed as $\text{Var}\left[\widehat{\log\left(\frac{L_{it}^n}{L_{mt}^n}\right)}\right]/\text{Var}\left[\log\left(\frac{L_{it}^n}{L_{mt}^n}\right)\right]$. Within over Explained Variance is computed as $\text{Var}\left[(1-\hat{\sigma})\log\left(\frac{p_{it}^n}{p_{mt}^n}\right)+(1-\hat{\sigma})\left(\hat{\epsilon}_i-1\right)\widehat{\log C_t^n}\right]/\text{Var}\left[\widehat{\log\left(\frac{L_{it}^n}{L_{mt}^n}\right)}\right]$.

fixed effects $\hat{\zeta}_i^n$ and their covariance with the within-country terms. Thus, the within-country evolution of sectoral employment shares, which is our object of interest in studying structural change, accounts for a substantial part of the total variation at the estimated parameter values.[43]

Next, we analyze the drivers of structural change within countries. We quantify the contribution of price and income effects to the predicted within-country evolution of log-relative employment shares by calculating

$$\frac{\text{Var}\left[(1-\hat{\sigma})\log\left(\frac{p_{it}^n}{p_{bt}^n}\right)\right]}{\text{Var}\left[\widehat{\log\left(\frac{L_{it}^n}{L_{mt}^n}\right)}-\hat{\zeta}_i^n\right]} \quad \text{and} \quad \frac{\text{Var}\left[(1-\hat{\sigma})(\hat{\epsilon}_i-1)\widehat{\log C_t^n}\right]}{\text{Var}\left[\widehat{\log\left(\frac{L_{it}^n}{L_{mt}^n}\right)}-\hat{\zeta}_i^n\right]}. \tag{42}$$

for $i=\{a,s\}$, where we have used the fact that $\widehat{\log\left(\frac{L_{it}^n}{L_{mt}^n}\right)}-\hat{\zeta}_i^n=(1-\hat{\sigma})\log\left(\frac{p_{it}^n}{p_{mt}^n}\right)+(1-\hat{\sigma})\left(\hat{\epsilon}_i-1\right)\widehat{\log C_t^n}$ from Equation (39). The last three rows in Table 4 report this variance decomposition exercise. They report the contribution that we obtain for price and income effects in our two estimating equations. The last row reminds the reader that, by construction, the variance of the sum of both terms accounts for 100% of the within-country predicted variation.

Let us first focus on the decomposition of within-country variation in log-relative agriculture to manufacturing employment, which corresponds to the first column in Table 4. We find quite a dramatic result: price effects account for only 2% of the within-country varia-

[43]Nevertheless, the fixed effects indeed account for a substantial share of the variation as well. This is not surprising since countries differ widely in their relative employment shares, and country-sector fixed effects capture the average differences across countries. These fixed effects also absorb potential differences in technological parameters, α_i^n.

tion, while income effects alone account for 98%. For log-relative services to manufacturing employment, we find that price effects alone account for 27% of the overall within-country variation. Income effects alone account for 84%. These individual contributions add up to more than one because they are not independent from each other.

Overall, Table 4 is consistent with the view that the nonhomotheticity of demand plays a dominant role in accounting for within-country structural change in our panel of countries. If we attribute all the covariation in prices and consumption to prices in our full sample, we find that the within-country variation accounted for by real consumption is 98% for the log-relative agriculture to manufacturing equation, and 73% (=100%-27%) for log-relative services to manufacturing.[44] Thus, we conclude that nonhomotheticities account for over 73% of the structural change in our sample.[45]

5 Additional Robustness Analyses

In this section, we present two important sets of additional empirical analyses that showcase the robustness and generality of our results. In Section 5.1, we show that our results are robust to using services, agriculture, or a linear combination of three sectors as base sectors instead of manufacturing. In Section 5.2, we analyze household expenditure data from the Indian National Sample Survey (NSS) to show that our results generalize to a developing country with average household incomes far below that of the US. We show evidence that the estimates of relative income elasticities based on NSS are closely in line with those presented in Section 3.2 based on the CEX data.

[44] If we break down our analysis between OECD and Non-OECD countries, a similar picture emerges (see Tables F.7 and F.8 in the online appendix). For OECD countries, we find that the contribution of price effects alone is somewhat larger, 13% and 36%. This may partially reflect better measurement by statistical agencies. Even in this case, income effects play a more substantial role.

[45] This conclusion differs from Boppart (2014a) who studies the evolution of services relative to the rest of the economy in the U.S. during the postwar period. He finds that the contribution of price and income effects are roughly of equal sizes. First, the differences in the results are partly due to the differences in the level of sectoral aggregation. If we confine our analysis to the U.S. and lump together agriculture and manufacturing into one sector, we find that price effects account for 26% of the variation. Second, our specification of demand is different from Boppart (2014a) because in our specification the price elasticity is constant. In contrast, Boppart's demand system implies that the price elasticity of services relative to the rest of consumption is declining as the economy grows. As noted by Buera and Kaboski (2009), since the relative expenditure and value added of services grows at a faster rate than services relative price, a declining price elasticity automatically increases the explanatory power of relative prices. We have checked that a declining variable elasticity is quantitatively important for the decomposition exercise. We have generated a synthetic panel of countries with two sectors (agriculture plus manufacturing, and services) with preferences given by nonhomothetic CES calibrated to capture the key features of our true cross-country panel. We then do two decomposition exercises with these data: one estimating a nonhomothetic CES demand, and another estimating a PIGL demand. We find that the within variation accounted for by prices is four times larger with PIGL than with nonhomothetic CES.

Table 5: Variance Decomposition using Alternative Bases

	Estimates under Alternative Bases (see Table 9)			
	Agri. as base	Manu. as base	Serv. as base	3 sec. as base
σ	0.33	0.57	0.33	0.40
	(0.03)	(0.07)	(0.03)	(0.04)
$\varepsilon_a/\varepsilon_m - 1$	-0.67	-0.98	-0.86	-0.68
	(0.06)	(0.06)	(0.07)	(0.04)
$\varepsilon_s/\varepsilon_m - 1$	0.26	0.17	0.23	0.31
	(0.05)	(0.04)	(0.05)	(0.06)

	Variance Decomposition using Alternative Bases							
	$\log\left(\frac{\text{Agriculture}}{\text{Manufacturing}}\right)$				$\log\left(\frac{\text{Services}}{\text{Manufacturing}}\right)$			
	A-base	M-base	S-base	3-base	A-base	M-base	S-base	3-base
Price Effects	0.03	0.02	0.02	0.03	0.10	0.27	0.16	0.08
Income Effects	0.94	0.98	0.94	0.94	0.95	0.84	1.00	1.00

Notes: Column A-base reports the result of performing the variance decomposition in Equation (42) using the estimates obtained with agriculture as the base sector (reported in the first three rows of column "Agri. as base" in this table). Analogously, M-base, S-base and 3-base denote the variance decomposition results using the estimates obtained with manufacturing, services and the linear combination of the 3 sectors as bases. Robust standard errors shown in parenthesis.

5.1 Estimation with Alternative Base Sectors

Our baseline empirical specification used manufacturing as a base sector in the estimating Equations (36). Here, we show that our findings are robust to using services, agriculture, or even more general bases as reference sectors in our empirical estimation. Section A.2.2 of the appendix generalizes the construction of Section 2.1.2 for the identification and the definition of the nonhomothetic CES index of real consumption to one in which we can use a convex combination of the different sectors as base. Table 5 compares the key results of Sections 3 and 4 using three different choices of base: manufacturing, agriculture, service, and a uniform convex combination of all three sectors.[46]

The first three rows of Table 5 provide the estimated parameters (σ, $\varepsilon_a/\varepsilon_m$, $\varepsilon_s/\varepsilon_m$) for each choice of base. Let ϵ_i^b denote the nonhomotheticity parameter for sector i corresponding to base b following Equation (10) (or Equation A.14 in the appendix). It is easy to see that the ratios of the parameters $\boldsymbol{\varepsilon}$, which have to be invariant to the choice of the base b, can be

[46]In Comin et al. (2018), we also show results of simultaneously estimating the system of moment conditions that combines all three sets of specifications corresponding to three different sectors as bases. The results of that exercise is similar and in line with those reported here.

written in terms of the estimated parameters $\boldsymbol{\epsilon}^b$. In particular, we can write for any base b:

$$\frac{\varepsilon_i}{\varepsilon_m} - 1 = \frac{\varepsilon_b \cdot \epsilon_i^b}{\varepsilon_b \cdot \epsilon_m^b} - 1 = \frac{\epsilon_i^b}{\epsilon_m^b} - 1, \qquad i \in \{a, s\}.$$

The first column of the table reports this value under our benchmark of manufacturing as base $b = m$ and $\epsilon_m^m = 1$, which also equals $\epsilon_i - 1$ in Table 3. We find that $\varepsilon_s/\varepsilon_m - 1$ ranges between 0.17 and 0.33 across our specifications, meaning that services are more income elastic than manufacturing, and $\varepsilon_a/\varepsilon_m - 1$ ranges between -0.99 and and -0.60, implying that agriculture is less income elastic than manufacturing. The estimated values for the price elasticity are between 0.3 and 0.6, implying that the three sectors are estimated to be gross complements in all specifications.[47]

Finally, the last two rows of Table 5 repeat our variance decomposition for the within-country variation in relative employment shares in Section 4 for the same four choices of bases. We find that our decomposition results are robust to using these alternative choices for the base. Income effects still account for the bulk of the within-country variation–in fact, using manufacturing as base gives the lowest average value for the income effects.

5.2 Evidence from Indian Household Expenditure Data

One key challenge in revisiting the robustness of our household-level estimation in the case of NSS data is the absence of reliable price information. To circumvent this problem, we first present an alternative econometric specification to partially estimate our demand system when price data are missing. This estimation strategy allows us to recover all nonhomotheticity parameters up to a scaling constant and it can thus be used to identify the rank-ordering and relative magnitudes of the nonhomotheticity parameters. We apply this estimation approach to the Indian NSS household data and to the CEX (as a robustness check). We find very similar nonhomotheticity parameters in both samples. We also show that the estimates are consistent with our baseline specification.[48]

Inference without Price Data Consider the following log-linear demand specification for $i \in \{a, s\}$:

$$\log\left(\frac{\omega_{it}^n}{\omega_{mt}^n}\right) = \sum_{j \in \{a,m,s\}} \varsigma_{ij} \log p_{jt}^n + (\tilde{\epsilon}_i - \tilde{\epsilon}_m) \log E_t^n + \zeta_i^n + \nu_{it}^n \tag{43}$$

[47] Table 9 in the appendix presents the full set of parameter estimates for all bases.

[48] Online Appendix I elaborates on this estimation strategy and introduces two additional estimation strategies: *(i)* a non-linear specification that directly incorporates the average cost index and *(ii)* an iterative linear least squares approach that uses a second-order approximation of the real consumption index. Online Appendix J compares estimates of different econometric specifications using synthetic data.

where, as before, n stands for a household. On first sight, this equation appears to be an ad hoc and naive approach to estimating demand. However, Lemma 6 in Appendix C shows that this specification indeed identifies $\epsilon_i - \epsilon_m$ for any base b up to a scaling factor, i.e., $\tilde{\epsilon}_i - \tilde{\epsilon}_m = \tilde{\lambda}(\epsilon_i - \epsilon_m)$ for some $\tilde{\lambda} > 0$. In other words, the ratio of estimates $(\tilde{\epsilon}_s - \tilde{\epsilon}_m)/(\tilde{\epsilon}_a - \tilde{\epsilon}_m)$ is a consistent estimator of $(\epsilon_s - \epsilon_m)/(\epsilon_a - \epsilon_m)$ for any base. More generally, this approach identifies $I-2$ nonhomotheticity parameters and allows us to find the rank-ordering of nonhomotheticity parameters with a simple log-linear regression.[49,50]

In some instances, especially when dealing with household survey data, price data may not be available and it may not be possible to fully estimate Equation (43) or our baseline specification, Equation (36). This is the case for our Indian household data. Building on Equation (43), we propose an approach that approximately retrieves the nonhomotheticity parameters of our demand system, up to a scaling factor. We estimate a model where we substitute prices faced by households in (43) with a full set of interactions between region r, time t, sector i, and household income quintile q fixed effects. Formally, we estimate the following system of equations for $i \in \mathcal{I}_{-m}$

$$\log\left(\frac{\omega_{it}^n}{\omega_{mt}^n}\right) = (\tilde{\epsilon}_i - \tilde{\epsilon}_m)\log E_t^n + \pi_{it}^{rq} + \zeta_i^n + \nu_{it}^n, \tag{44}$$

where π_{it}^{rq} denotes the $t \times r \times q$ fixed effects. This approach allows us to capture the effect of prices in a non-parametric way through π_{it}^{rq}. It imposes the assumption that households in the same income quintile, region and time should face the same prices and choose the same consumption bundles up to the heterogeneity that we allow in household characteristics through ζ_i^n.

Estimation Results from Indian Household Expenditure Data We next present the estimation results based on Equation (44) for a household survey in India, and also compare it to what we would obtain using the U.S. household expenditure data. We use data from rounds 64, 66 and 68 of the India National Sample Survey (NSS), which span the years 2007 to 2012. The NSS is a representative survey of household expenditure that collects repeated cross-sections of expenditures incurred by households in goods and services. We construct total expenditure in agriculture, manufactures and services following the same classification as

[49] We note that that this specification closely corresponds to the exercise underlying Figure 1 presented in the Introduction. We report in Tables E.2 and F.3 of the online appendix the results of the estimating Equation (43) on household and aggregate data, respectively. In the last two rows of each table, we show that the ratios $(\tilde{\epsilon}_s - \tilde{\epsilon}_m)/(\tilde{\epsilon}_a - \tilde{\epsilon}_m)$ are close to the corresponding ratios $(\epsilon_s - 1)/(\epsilon_a - 1)$ found in our baseline specifications in Tables 1 and 3.

[50] Lemma 6 also implies that there exists an alternative scaling of ϵ_m such that our baseline estimation results in Tables 1 and 3 would coincide with the estimates $\tilde{\epsilon}_i - \tilde{\epsilon}_m$ in Tables E.2 and F.3 of the online appendix. Conversely, it is possible to estimate the full demand system in two stages. In the first step, we can estimate Equation (43) (or Equation (44) below) to obtain $\tilde{\epsilon}_s - \tilde{\epsilon}_m$. In the second step, we substitute in our baseline specification $\epsilon_i - 1 = \tilde{\lambda}^{-1}(\tilde{\epsilon}_s - \tilde{\epsilon}_m)$ and estimate σ and $\tilde{\lambda}$.

Table 6: Baseline Regression for India, NSS Expenditure

			< P50	> P50	Only Urban		U.S.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\tilde{\epsilon}_a - \tilde{\epsilon}_m$	-0.63	-0.55	-0.62	-0.69	-0.57	-0.52	-0.61
	(0.05)	(0.07)	(0.05)	(0.29)	(0.05)	(0.07)	(0.09)
$\tilde{\epsilon}_s - \tilde{\epsilon}_m$	0.49	0.42	0.69	0.51	0.56	0.45	0.49
	(0.07)	(0.10)	(0.53)	(0.08)	(0.08)	(0.11)	(0.09)
	Comparison to US Baseline						
$(\tilde{\epsilon}_s - \tilde{\epsilon}_m)/(\tilde{\epsilon}_a - \tilde{\epsilon}_m)$	-0.78	-0.76	-1.11	-0.74	-0.98	-0.87	-0.80
$(\epsilon_s - 1)/(\epsilon_a - 1)$, baseline estimates	-0.81	-0.82	-0.84	-0.60	-0.81	-0.81	-0.81
Expenditure Re-Weighted	N	Y	N	N	N	Y	N
Time $\times$ Region $\times$ Inc. Quintile FE	Y	Y	Y	Y	Y	Y	Y
T. $\times$ Reg. $\times$ Inc. Quint. $\times$ Rural FE	Y	Y	Y	Y	N	N	N

Notes: Standard errors clustered at the year×state×district shown in parenthesis. All regressions include household controls (discussed in the main text). Observations for the full sample are 293,007. Urban observations are 118,681. Time fixed effects are the interaction of year×month. Region fixed effects are the interactions of state×district. The ratios for the baseline estimates corresponding to columns 1, 2 and 5 to 7 are computed from columns 1 and 2 of Table 1. The ratios for columns 3 and 4 are computed from Table 2.

for the CEX data. Household total income is constructed from an earnings measure that averages (potential) different sources of income within the household from different occupations, including received benefits (net of taxes).

We construct the controls in an analogous way to the U.S., with the only difference that the requirement of a prime age household is between ages of 18 and 60. In contrast to the U.S., we do not discard rural population as it represents more than half of the Indian population (around 55%). We instead show results for the entire sample and the subsample of urban households. The composition of expenditure in India is vastly different from the CEX. The average expenditure share in food and agricultural in the sample is 52% (versus 12% in the CEX). Expenditure shares in manufactures and services in the NSS represent, on average, 27% and 21% of total expenditure (versus 27% and 61% in the CEX).

Table 6 reports our estimation results from estimating equation (44).[51] Columns (1) and (2) report our baseline estimates using the full sample for the same two weighting schemes we used for the CEX. The first makes the estimates representative of the Indian population and the second re-weights households according to their total expenditure. We find that

[51] We use total household annual income as an instrument for household quarterly expenditure. The first stage includes all controls used in the second stage. The coefficient on household annual income is positive and significant in all first-stage regressions. We note also that for columns (1) to (4) we augment specification (44) interacting the income-quintile×time×region with a dummy that indicates whether the household is classified as rural to account for potential constant difference between rural and urban households.

the relative income elasticities between agriculture and manufacturing, $\tilde{\epsilon}_a - \tilde{\epsilon}_m$, are negative, and between services and manufacturing, $\tilde{\epsilon}_s - \tilde{\epsilon}_m$, are positive. Likewise, comparing the point estimates in Columns (1) and (2) we see that they again remain stable across the two weighting schemes. We further explore the stability of the parameter estimates by applying the same specification separately to the subset of households above and below the median income level. Columns (3) and (4) show that we find very similar estimates of income elasticities in the two sub-samples. We show in Columns (5) and (6) that when we restrict our attention to urban households, we also obtain very similar estimates regardless of the weighting scheme used.

Column (7) shows the coefficient we would obtain if we run the same regression for the U.S. CEX data. Despite the vast differences in the level of development between the US and India, we find that the US estimates, $\tilde{\epsilon}_a - \tilde{\epsilon}_m = -0.61$ and $\tilde{\epsilon}_s - \tilde{\epsilon}_m = 0.49$, are very similar in magnitude to the estimates for India. The last two rows of the table compare the ratios $(\tilde{\epsilon}_s - \tilde{\epsilon}_m)/(\tilde{\epsilon}_a - \tilde{\epsilon}_m)$ obtained in the without-price specification, Equation (44), for India and the US with the ratios $(\epsilon_s - 1)/(\epsilon_a - 1)$ from the baseline US CEX estimation. As implied by Lemma 6, we find very similar ratios. For the US, the ratio of the estimates in column (7) of Table 6 is -0.80. The corresponding number from the baseline estimates in Table 1 is -0.81. For India, the ratio in column (1) is also very close to US estimates, -0.78, and fairly stable across specifications. We take these results as evidence of nonhomothetic CES being able to capture parsimoniously with the same nonhomotheticity parameters demand conditions from very different stages of development.

6 Comparison to Alternative Models and Extensions

In this section, we provide comparisons to alternative models and discuss several extensions and robustness checks of our empirical results for both micro and macro data. In Section 6.1 we compare the model fit of nonhomothetic CES to alternative specifications of within-period utility that have been used previously in the literature: generalized Stone-Geary and PIGL. In Section 6.2, we shift the focus of our analysis of the patterns of structural change from the evolution of employment to sectoral value added in production (which has been the focus of some recent work, e.g., Herrendorf et al. 2013). We also show that our estimated model accounts for structural change in both real and nominal value added. In addition, we revisit our results based on household-level CEX data when we specify consumption expenditures in terms of sectoral value added rather than expenditure in final goods. In Section 6.3, we show that the growth rate of the model-implied nonhomothetic CES real consumption index strongly correlates with the growth rate of off-the-shelf indices of real consumption. Finally, in Section 6.4, we extend our estimation to more than three sectors.

6.1 Fit Comparison with Stone-Geary and PIGL preferences

We compare the cross-country fit of our model to alternative specifications where we replace the nonhomothetic CES aggregator with Stone-Geary (Herrendorf et al., 2014) and PIGL preferences (augmented to three sectors as described in Boppart, 2014b). Appendix D introduces these two demand systems and the estimation procedure. Here, we highlight two similarities between these demand specifications and ours that allow us to perform this comparison. First, the number of parameters to be estimated in these two demand systems is the same as that in nonhomothetic CES. Second, as with nonhomothetic CES, there exist sets of parameters for these two demand systems such that the expenditure shares are constant for each country-sector (they correspond to Cobb-Douglas with expenditure shares equal to country-sector averages). Thus, we benchmark the fit of these three demand systems relative to only using the country-sector average as a prediction for each sector. This amounts to computing the R^2 for agriculture, manufacturing and services shares after subtracting country-sector means for each sector.[52]

We find that the within-R^2 for Stone-Geary is 0.14, meaning that 14% of the residual variation in agricultural, manufacturing and service shares after we partial out country-sector averages is accounted for by the Stone-Geary demand system. The corresponding number for nonhomothetic CES is more than two-times larger, 0.29. The intuition for the worse fit of Stone-Geary is that income effects are very low for rich countries, since for high levels of income, the subsistence levels responsible for introducing the nonhomotheticity become negligible (see, also, Dennis and Iscan, 2009).[53] For the PIGL demand system we find an R^2 of 0.13, which is very similar to that of Stone-Geary. PIGL preferences track the trends in services more accurately than Stone-Geary due to the fact that they feature a non-vanishing nonhomotheticity of the service sector. However, they under-perform relative to nonhomothetic CES mostly because they assume a homothetic composite between agriculture and manufacturing, while nonhomothetic CES allows for sector-specific nonhomotheticities. Figure 3 illustrates the fit for the case of Taiwan for the three demand systems.[54]

[52]The R^2 compares the sum of squared errors of the model fit to the sum of squared errors obtained by using the country-sector average as a prediction. Formally, $R^2 = 1 - \frac{1}{I}\sum_{i=1}^{I}\left(\sum_{t=1}^{N}(y_{it}^n - \hat{y}_{it}^n)^2 / \sum_{t=1}^{N}(y_{it}^n - \bar{y}_i^n)^2\right)$ where N denotes the total number of observations per sector, I, the number of sectors, y_{it}^n, observed employment shares in sector i and country c, $\hat{y}_{it}^n$, predicted employment shares, $\bar{y}_i^n$ the sample average of y_{it} for country c in sector i, and $i \in \mathcal{I} = \{a, m, s\}$. We also note that the estimates used to compute the within-R^2 for nonhomothetic CES correspond to the structural estimates in column (1) of Table 3. Finally, note that in this exercise we are computing the R^2 on employment shares (and not relative log-shares). The reason is that the estimation of the three demand systems is based on different left-hand-side variables (e.g., Stone-Geary is not log-linear and it is estimated on shares directly). We chose to benchmark the fit of the three demand systems based on the level of employment shares as it is arguably the most basic object of interest.

[53]For the U.S., the value of the nonhomothetic terms $p_{it}\bar{c}_i$ relative to total expenditure is never higher (in absolute terms) than 0.1%, which suggests that nonhomotheticity are insignificant. The highest values of the nonhomotheticities in the sample are 37% for agriculture and 18% for services.

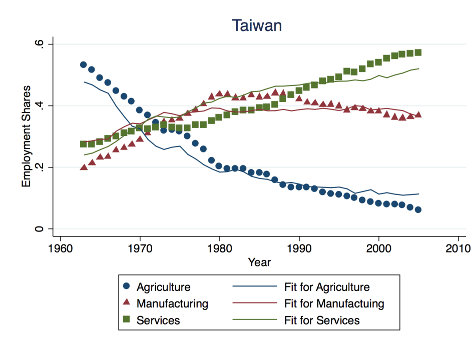
[54]We report the fit of each country for nonhomothetic CES in Section F and for Stone-Geary and PIGL in

Figure 3: Comparison of Demand Systems and Fit for Taiwan

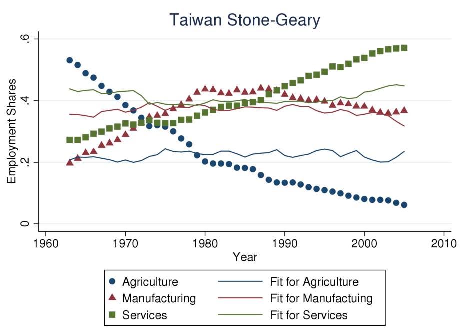
(a) Overall Fit Comparison

Model:	Within-R^2
Stone-Geary	0.14
PIGL	0.13
Nonhom.-CES	0.29

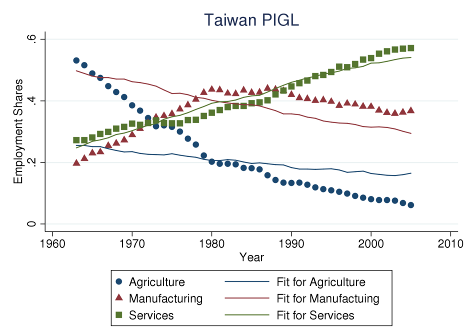
(b) Nonhomothetic CES Fit for Taiwan



(c) Stone-Geary Fit for Taiwan



(d) PIGL Fit for Taiwan



6.2 Value-Added Estimation and Accounting of Structural Change

Estimation in Aggregate Data with Value-Added Shares We investigate whether we find similar estimates to the baseline cross-country results when we use value-added output shares as dependent variable (instead of employment shares). Table F.2 in the online appendix reports the estimation results using sectoral output value added shares as dependent variables in our baseline estimation, Equation (36). The estimates of the nonhomotheticity parameters appear with the expected signs and overall similar magnitudes as in the baseline regression. The price elasticity appears to be somewhat smaller, between 0.3 and 0.4.

Structural Change in Real and Nominal Value Added A salient feature of the patterns of structural transformation observed in the data on sectoral value added is that they appear regardless of whether we document them in nominal or real terms (Herrendorf et al., 2014). To investigate our model's ability to account for this fact, we combine our structural cross-country estimates from Table 3 and the sectoral demands from our theory to generate the predicted evolution of nominal and real sectoral demands.

Section H of the online appendix.

Table 7: Correlation of Nominal and Real Value Added

	Data	Model
Agriculture/Manufacturing	0.96	0.94
Services/Manufacturing	0.87	0.71

Note: Model-predicted values constructed using the structural estimates from column (2) in Table 3.

Table 7 reports the correlation between nominal and real shares both in our estimated model and in the data. We find that the model is able to generate correlation similar to the data. In particular, the correlation between the nominal and real relative demand of agricultural goods to manufactured goods is 0.94 in our model, while in the data it is 0.96. For services, the model generates a correlation of 0.71 while in the observed correlation in the data is 0.87.

The success in generating a correlation between nominal and real measures of the same magnitude as in the data is important. Note that this is an out-of-sample test of the predictions of our model, since our baseline estimation has not targeted the evolution of sectoral shares of real or nominal value added. As discussed in Section 2, if we had used a homothetic CES framework, the correlation generated by the model would have been negative because the price elasticity of substitution is smaller than one. This implies that real and nominal variables can not have positive co-movement with homothetic CES.[55] Of course, any specification of preferences that asymptotically converges to a homothetic CES (e.g., Stone-Geary) would face a similar problem in explaining the nominal-real co-movement. This also holds for homothetic demand systems with variable elasticity of substitution (since agriculture, manufacturing and services are gross complements). In contrast, in our framework, there is a second force that makes the positive co-movement possible: income effects. The nonhomothetic effect of real consumption affects real and nominal variables in an identical way (this term is $C^{(1-\sigma)(\epsilon_i-\epsilon_m)}$, see Equation 2). At the estimated parameter values, the implied income effects are sufficiently strong to overcome the relative price effect and make both time series co-move positively. Therefore, we argue that the ability to simultaneously account for the evolution of real and nominal sectoral shares is a key feature of our specification of nonhomotheticity.

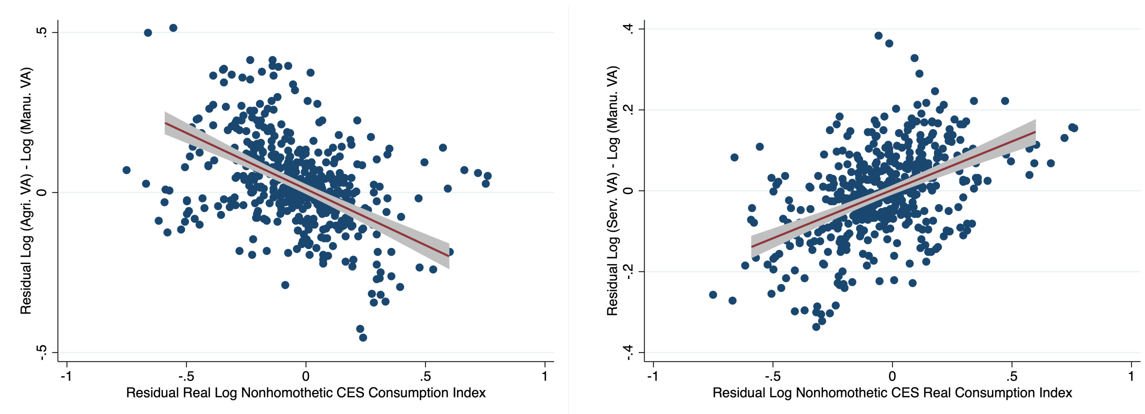
Partial Correlations of Relative Value Added and the Real Consumption Index We motivated our focus on the role of nonhomotheticity in the Introduction by highlighting the strong partial correlations between log sectoral expenditure shares and log aggregate income per capita in Figure 1. We revisit this relationship through the lens of our model to

[55] To see that, note that the relative trend in nominal values $\omega_{i,t}/\omega_{j,t}$ would be proportional to $(p_{i,t}/p_{j,t})^{1-\sigma}$. For real values, $c_{i,t}/c_{j,t}$, would be proportional to $(p_{i,t}/p_{j,t})^{-\sigma}$. As $0 < \sigma < 1$, both trends would move in opposite directions.

Figure 4: Partial Correlations of Sectoral Value Added and the Real Consumption Index

(a) Agriculture relative to Manufacturing

(b) Services Relative to Manufacturing



Notes: Data for OECD Countries, 1970-2005. Each variable has been residualized by partialling out country fixed effects and relative prices. Each point represents a country-year observation. The red line depicts the OLS fit, the shaded regions, the 95% confidence interval.

draw some intuitions about why the model predicts such a strong role for income effects in driving the evolution of relative employment shares. Figure 4 shows the partial correlation between log value-added shares and the log nonhomothetic CES real consumption index (after partialling out country-sector fixed effects and relative prices) for the same set of countries and time period as Figure 1.[56] We see that for both log-relative agriculture to manufacturing and log-relative services to manufacturing employment a substantial part of the variation is well captured by a simple log-linear relationship, similar to that in Figure 1. We take this result as additional supporting evidence for nonhomothetic CES being able to capture salient patterns in the data and nonhomotheticities playing an important role at all levels of development.

CEX Value-Added Demand Formulation So far, we have estimated household demand defined over households' final-good expenditure. Previous work has shown that the patterns of structural transformation are qualitatively similar whether we measure sectoral economic activity in terms of value-added or final-good expenditure shares (Herrendorf et al., 2013). We estimate our model defining household utility over the value added provided by each sector (rather than final good expenditure) and show that we obtain similar results. To do this, we follow Buera et al. (2015) and assign each consumption expenditure category to an industry of the U.S. input-output. We then compute the value added from each sector embedded in the final good expenditure of a given CEX category to express household demand over value

[56] Korea is excluded from the figure because it has substantially more variation in the partialled-out variables. Plotting it with the other countries would prevent seeing the rest of the within-country variation. Figure F.2 in the online appendix reproduces the same plot including Korea.

added (see the online appendix for details). Table 10 in the appendix presents the estimates of our baseline specification where we use as dependent variable household expenditure shares measured in value added (instead of final good expenditure). We find that the estimates of ϵ_a are less than one, while the estimates of ϵ_s are above one. The estimates do not vary once we re-weight by expenditure, suggesting that estimates are stable across the income distribution. The point estimates we obtain for the price elasticity are in the 0.3 to 0.5 range. The magnitudes of all estimated elasticities appear to be overall very comparable to the expenditure formulation. The major difference is that the elasticity of agriculture ϵ_a appears to be somewhat smaller in the value added formulation (specially in the specification without time fixed effects, where we find values of ϵ_a between 0.02 and 0.06, for values around 0.2 in the expenditure formulation).

6.3 Connection to Off-the-Shelf Measures of Real Consumption

Nonhomothetic CES Index vs. Törnqvist Index We next investigate if the model-implied real consumption index behaves in a similar way as the "standard" measures of real consumption. These measures deflate nominal expenditures with off-the-shelf price indeces. We establish a result connecting the changes in the nonhomothetic CES index of real consumption to the changes in off-the-shelf measures of real consumption. Let Δ denote the time difference operator between $t+1$ and t. In the last part of Appendix B, we show that up to second-order approximation

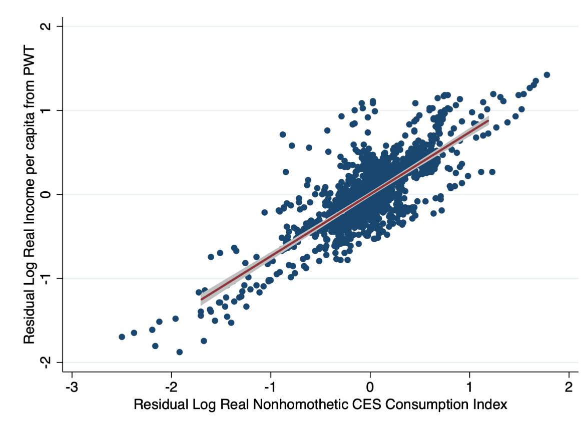
$$\Delta \log C_t^n \equiv \Delta \log \left(\frac{E_t^n}{P_t^n} \right) = \frac{1-\sigma}{\overline{\mathcal{E}}_t^n} \left(\Delta \log E_t^n - \Delta \log \mathcal{P}_t^n \right), \tag{45}$$

where $\mathcal{P}_t^n$ is the chained Törnqvist price index of consumption unit n at time t, $\Delta \log \mathcal{P}_t^n \equiv \frac{1}{2} \sum_{i=1}^{I} \left(\omega_{it}^n + \omega_{it+1}^n \right) \Delta \log p_{it}^n$ and $\overline{\mathcal{E}}_t^n$ is a correspondingly chained average of nonhomotheticity parameters ϵ_i's, $\overline{\mathcal{E}}_t^n \equiv \frac{1}{2} \sum_i \left(\omega_{it}^n + \omega_{it+1}^n \right) \epsilon_i$.[57]

Equation (45) sheds light on why the relationship between relative log-value-added shares and consumption expenditure deflated by standard price indices is well approximated by a log-linear relationship as suggested in Figure 1 in the Introduction. The relationship would be exactly log-linear if the term in the denominator $\overline{\mathcal{E}}^n$ was constant. Indeed, this term is not constant by construction because, as total income grows, expenditure shares change and $\overline{\mathcal{E}_t^n}$ changes accordingly. However, $\overline{\mathcal{E}_t^n}$ is slow-moving. Consider for example the case of the United States. The ratio of nominal consumption deflated by a chained Törnqvist price in year 2000 relative to 1950 is 3.20. During the same period, we have that the ratio $\overline{\mathcal{E}_{2000}^{US}}/\overline{\mathcal{E}_{1950}^{US}} = 1.09$,

[57] In Online Appendix I we show how to use this result to obtain an alternative estimating equation based on this approximation. We show that the resulting estimating equation belongs to a class of demand systems called "conditionally linear demand systems." Blundell and Robin (1999) show that this type of demand systems can be estimated using iterated OLS.

Figure 5: Real Income per capita from PWT and Nonhomothetic CES Index



Note: Both measures are residualized by partialling out country fixed effects.

implying a smaller increase of the true consumption index.[58] If we ignored the correction coming from the growth of $\overline{\mathcal{E}}$, we would expect a 3.2-fold increase in real consumption, while the true increase of the real consumption index is $3.2 \times \frac{1.01}{1.10} = 2.94$. This simple exercise also illustrates the broad fact that the growth rate of the real consumption index is smaller than the one implied by deflating expenditures by a chained Törnqvist index because $\overline{\mathcal{E}}_t^n$ grows as income grows.

Correlation with the Real Income Measure in the Penn World Tables The previous insights hold more broadly across countries. Figure 5 plots the log-real nonhomothetic CES index (constructed according to Equation 13) and the real income per capita measure reported in the Penn World Tables after partialling out country fixed effects to both measures. The figure shows that the these two measures are very well-approximated by a log-linear relationship.

6.4 Structural Change with More than Three Sectors

Jorgenson and Timmer (2011) have pointed out that in order to understand how structural transformation progresses in rich countries, it is important to zoom in on the service sector, as it represents the majority of rich economies' consumption shares (see also Buera and Kaboski, 2012b). Our framework lends itself to this purpose, as it can accommodate an arbitrary number of sectors. We estimate our demand system to more than three sectors for both micro and macro data. In the CEX data, the largest broad expenditure category is housing

[58] From 1950 to 2000, value added shares in agriculture, manufacturing and services went from 0.08, 0.41 and 0.51 to 0.02, 0.28, and 0.70, respectively. Normalizing $\epsilon_m = 1$ and taking the baseline estimates from the cross-country regression, this implies that $\overline{\mathcal{E}}_{1963} = 0.08 \times 0.02 + 0.41 \times 1 + 0.51 \times 1.17 = 1.01$ and, $\overline{\mathcal{E}}_{2000} = 0.02 \times 0.02 + 0.28 \times 1 + 0.70 \times 1.17 = 1.10$. Note that this ratio is invariant to re-scaling ϵ_i's.

services. We extend our analysis by separating housing from the rest of services and re-estimate our model with four sectors.[59] Table 11 in the appendix reports our findings. The price elasticity σ and the relative elasticity of agriculture to manufacturing, and services to manufacturing (excluding housing), remain very similar to our baseline estimates in Table 1. We find that the nonhomotheticity parameter of housing to manufacturing $\epsilon_{\text{housing}} - 1$ is around 0.9, and thus somewhat larger than for the rest of services (albeit not statistically different at conventional significance levels).

For the macro data, we extend our estimation to the original sectors in Groningen's data: (1) agriculture, forestry and fishing, (2) mining and quarrying, (3) manufacturing, (4) public utilities, (5) construction, (6) wholesale and retail trade, hotels and restaurants, (7) transport, storage and communication, (8) finance, insurance, real state, (9) community, social and personal services.[60] Table 12 in the appendix reports the results estimating the demand system in an analogous manner to our baseline estimation (38) with additional sectors. We find that the smallest income elasticities correspond to agriculture and mining, while the highest correspond to the finance, insurance and real state category. Columns (2) and (3) show that the ranking of sectors in terms of nonhomotheticity parameters is very similar when we estimate OECD and Non-OECD countries separately.[61]

7 Calibration Exercise

So far, we have only focused on the predictions of the model regarding the intratemporal allocation of consumption expenditures. In this section, we rely on a simple calibration exercise to study the dynamic, intertemporal predictions of the model implied by the Euler equation. As we discussed in Section 2.2.3, the qualitative properties of the transitional dynamics of the model heavily depend on the relationship between sectoral income elasticity and rates of productivity growth. In this section, we study the dynamics of the economy calibrated for the set of parameters estimated for the nonhomothetic CES preferences in Section 3. We then compare our model with simpler versions where we strip off different drivers of structural change. Relative to the Neoclassical Growth Model benchmark, we find that including any drivers of structural change in the model generates a slow-down of the convergence toward the long-run value.

[59] We define housing as expenditure in dwellings plus utilities. We use the same set of instruments plus a price instrument for housing constructed in an analogous way to the other price instruments.

[60] We exclude government services since it is missing for a third of the observations. The data set also contains information on dwellings that are not constructed within the period, but this information is very sparse and we abstract from them. Note that in this case, the manufacturing sector is more narrowly defined than in the baseline estimation as it excludes mining and construction.

[61] In working paper version Comin et al. (2015), we exploit that the nesting properties of nonhomothetic CES are analogous to homothetic CES and we also report the estimation results from a nested CES structure where we estimate the demand for each of the sectors that belong to services or manufacturing separately.

Table 8: Model Parameters for the Calibration Exercise

Parameter	γ_a	$\gamma_m = \gamma_0$	γ_s	σ	ϵ_a	ϵ_m	ϵ_s	α	θ	β	δ
Value	0.029	0.013	0.011	0.50	0.05	1.00	1.20	0.33	2.20	0.96	0.10

Model Calibration For the preferences, we rely on the values estimated for the sectoral nonhomotheticity parameters ϵ_i's and the elasticity of substitution σ using the macro data in Section 3 and set $\Omega_i \equiv 1$ for all $i \in \{a, m, s\}$. We assume that capital intensity is the same across sectors and choose the standard value $\alpha = 0.33$ and a rate of depreciation of $\delta = 0.1$ for capital. For the sectoral rates of productivity growth, we assume that the rates of productivity growth in the investment sector and manufacturing are the same $\gamma_m = \gamma_0$, and calibrate them to the rate of growth of labor productivity observed in the in the postwar period in the US.[62] We then use the rates of decline in relative sectoral prices within the same period to calibrate the rates of growth of sectoral productivity for agriculture and services. Finally, we choose the value of the parameter θ such that the asymptotic value of the elasticity of intertemporal substitution matches 0.5, a number within the range of various estimates provided in the literature (e.g., Guvenen, 2006; Havránek, 2015). Table 8 presents the set of model parameters used for the calibration.[63]

Dynamics of Capital Accumulation First, we study how the presence of nonhomothetic CES demand changes the dynamics of the process of capital accumulation and the real interest rate. For this exercise, we compare the transitional dynamics of the calibrated model, in which both the nonhomotheticity parameters and the rates of productivity growth vary across sectors, with the following three different increasingly simpler models: 1) a model where the rates of productivity growth are homogeneous across sectors and the evolution of sectoral allocations is *exclusively* driven by nonhomotheticity in demand, 2) a model with homothetic preferences where nonhomotheticity parameters are identical and, following Ngai and Pissarides (2007), the evolution of sectoral allocations is *exclusively* driven by the heterogeneity in sectoral rates of productivity growth, and 3) a standard neoclassical growth model (NGM) with homothetic CES preferences and homogeneous rates of productivity growth across sectors. We choose the parameters such that all models asymptotically converge to the same steady state as that of the calibrated model.[64]

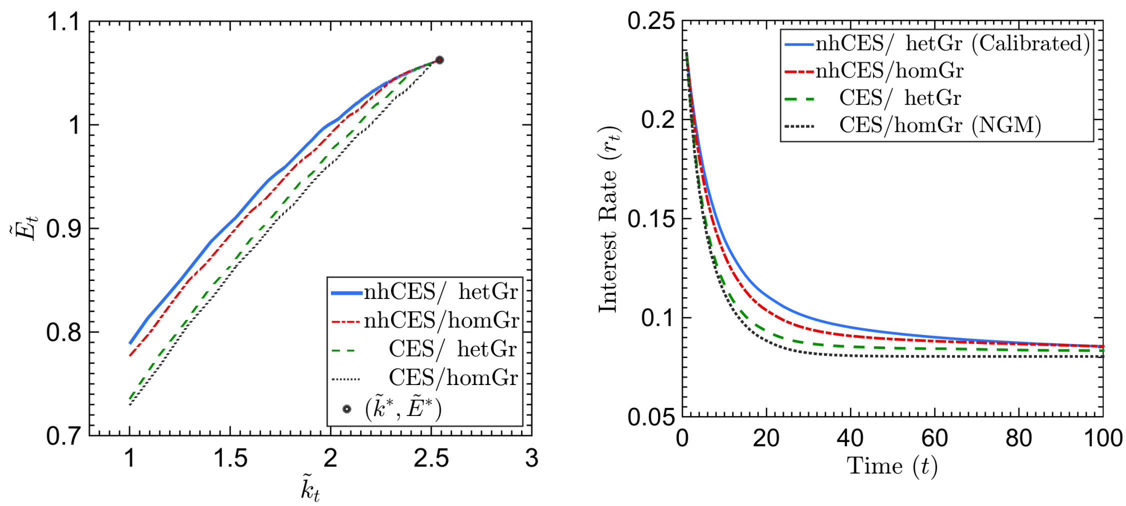
[62] Note that based on the model the rate of growth of labor productivity growth exceeds the rate of growth of multi-factor productivity (TFP) γ_m by a factor $\alpha/(1-\alpha)$.

[63] Section C in the appendix discusses the details of the method used for solving Equations (32) and (33) to derive the transitional dynamics of the model under these model parameters.

[64] Given the calibrated model parameters, the share of the service sector in consumption and employment converges to 1. Therefore, asymptotically all four models behave identical to a single-sector Neoclassical Growth Model where the instantaneous utility is defined as $C_t = C_{st}^{\epsilon_s}$ and the productivity in the final good sector grows at rate γ_s.

Figure 6: Transitional Dynamics: Comparison with Neoclassical Growth Model (NGM)

(a) Path in $(\tilde{k}, \tilde{E})$–space (b) Evolution of the Real Interest Rate



The evolution of the economy starting from initial per-capita stock of capital of $\tilde{k}_0 = 1 < \tilde{k}^* = 2.50$. The parameters for the Calibrated Model are given in Table 8. The nhCES/hetGr corresponds to the calibrated model with nonhomothetic CES and heterogeneous rates of sectoral productivity growth. The nhCES/homGr model corresponds to the case with nonhomothetic CES preferences and homogeneous rates of sectoral productivity growth, $\gamma_i = 0.011$ for $i \in \{a, m, s\}$. The CES/hetGr model corresponds to the case with homothetic CES preferences, $\epsilon_i = 1.20$ for $i \in \{a, m, s\}$. The CES/homGr corresponds to the case of the Neoclassical Growth Model (NGM) where both the rates of productivity growth and the nonhomotheticity parameters are homogeneous across sectors. $\tilde{k}^*$ and $\tilde{E}^*$ denote the asymptotic normalized per-capita stock of capital and total consumption expenditure, respectively.

Beginning at an initially low level of per-capita stock of capital of $\tilde{k}_0 = 1 < \tilde{k}^* = 2.5$, Figure 6a shows the path of the economy from this initial condition toward its steady state in the space of the normalized per capita stock of capital and per-capita consumption expenditure. The figure compares these paths for all four models. All three models featuring structural change have higher values of total consumption expenditure relative to the NGM at all levels of per-capita stock of capital along the transitional path. As a result, we conclude based on this calibration that the presence of structural change implies a slower process of capital accumulation compared to the NGM, whether it is driven through price or income effects.

The slowdown in capital accumulation relative to the NGM benchmark is driven by the same two forces that shape the evolution of sectoral shares, namely, the inter-sectoral heterogeneity in the elasticities of income and the rates of productivity growth. In Section 2.2.3, we explained the mechanism behind the former force: the elasticity of intertemporal substitution gradually rises as consumption shifts toward more income elastic goods that are also more intertemporally substitutable. The latter force is present in the benchmark theory of Ngai and Pissarides (2007): over time, household consumption shifts toward the sectors with the slower rates of productivity growth, lowering the rate of fall in the price of consumption. If household consumption is intertemporally inelastic (in the sense that $\theta > 1$), conditional on a given level of interest rate, the slowdown in the rate of decline of prices results in faster growth of consumption expenditure.[65] As the figure shows, these two forces, as well as their potential interactions, contribute to the slowdown in the accumulation of capital in the calibrated model, although nonhomotheticity plays a larger role.

Dynamics of Interest Rate Figure 6b compares the implications of all four models above for the evolution of the real interest rate. The slower process of capital accumulation implies that the real interest rate also converges toward its steady state more slowly in all three models featuring structural change, relative to the NGM benchmark. Once again, the model that solely features nonhomotheticity grows more slowly compared to the one solely featuring heterogeneous sectoral rates of productivity growth. Nevertheless, the overall difference between the evolution of the real interest rate between the calibrated model and the corresponding NGM is relatively small: the time it takes for the real interest rate to go from 200% to 150% of its steady state level (half-life) is 9.1 years in the former relative to 4.4 years in the latter.[66]

8 Conclusion

This paper presents a tractable model of structural transformation that accommodates both long-run demand and supply drivers of structural change. Our main contributions are to introduce the nonhomothetic CES utility function to growth theory, show its empirical relevance and use its structure to decompose the overall observed structural change into the contribution of income and price effects. These preferences generate nonhomothetic Engel curves at any level of development, which are in line with the evidence that we have from both rich and developing countries. Moreover, for this class of preferences, price elasticities are independent from nonhomotheticity parameters, and they can be used for an arbitrary number of sectors. We argue that these are desirable theoretical and empirical properties.

We estimate these preferences using household-level data for the U.S. and India, and aggregate data for a panel of 39 countries during the post-war period. We argue that nonhomothetic CES preferences provide a good fit of the data despite their parsimony. Armed with the estimated price and nonhomotheticity parameters, we then use the demand structure to decompose the broad patterns of reallocation observed in our cross-country data into the

[65] To better see this point, consider a constant nonhomotheticity parameter across sectors, $\epsilon_i = \epsilon$, and log-linearize the Euler Equation (20) to find $\theta \Delta \log E_t \approx (1-\theta)\left(\gamma_0 - \overline{\gamma}_t\right) + r_{t+1} - (1-\beta)$, where $\overline{\gamma}_t \equiv \sum_i \omega_{it}\gamma_i$ is the consumption-weighted average of the sectoral rates of productivity growth (see also Equation 22 in Ngai and Pissarides, 2007). When $\sigma < 1$, over time $\overline{\gamma}_t$ falls and therefore $\Delta \log E_t$ grows if $\theta > 1$.

[66] The corresponding numbers in the model with nonhomotheticity and in the model with differential rates of productivity growth are 7.2 and 5.0, respectively.

contribution of nonhomotheticities and changes in relative prices. We find that the majority of the within-country variation is accounted for by nonhomotheticities in demand.

To conclude, we believe that the proposed preferences provide a tractable departure from homothetic preferences. They can be used in other applied general equilibrium settings that currently use homothetic CES and monopolistic competition as their workhorse model, such as international trade. These preferences can be nested in the same manner as homothetic CES. Also, as we discuss in Appendix A, it is possible to generalize nonhomothetic CES to generate variable nonhomotheticity parameters. These properties may be useful in some applications. Even in this case, nonhomothetic CES remains a local approximation (with constant nonhomotheticity parameters) and can be used to guide how the varying elasticities should be parameterized, e.g., by estimating nonhomothetic CES across sub-samples.

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Appendix Click Here for the Online Appendix

A General Nonhomothetic CES Preferences

In this section of the appendix, we provide an overview of the properties of the general family of nonhomothetic Constant Elasticity of Substitution (CES) preferences. We first introduce the general family and then specialize them to the case of isoelastic nonhomothetic CES functions of Section 2.1.

Prior Work Sato (1975) derived a general family of CES functions as the solution to a partial differential equation that imposes the constancy of elasticity of substitution. This family includes standard homothetic CES functions as well as two classes of separable and non-separable nonhomothetic functions. Hanoch (1975) showed that additivity of the direct or indirect utility (or production) function results in price and income effects that are non-trivially dependent on each other. He then introduced *implicit additivity* and derived a family of functions where the income elasticity of demand is not fully dependent on the elasticity of substitution. Our nonhomothetic CES functions correspond to the non-separable class of functions in the sense of Sato (1975), which also satisfy the condition of implicit additivity in the sense of Hanoch (1975). Finally, Blackorby and Russell (1981) have proved an additional property that is unique to this class of functions. In general, different generalizations of the elasticity of substitution to cases involving more than two variables, e.g., the Allen-Uzawa definition or the Morishima definition, are distinct from each other. However, for the class of nonhomothetic CES functions, they become identical and the elasticity of substitution can be uniquely defined similarly to the case of two-variable functions.

A.1 General Definition

Consider now preferences over a bundle $\boldsymbol{C}$ of I goods defined through an implicit utility function:

$$\sum_{i=1}^{I} \Upsilon_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)}\right)^{\frac{\sigma-1}{\sigma}} = 1, \tag{A.1}$$

where functions g_i's are differentiable in U with $\sigma \neq 1$ and $\sigma > 0$. Theorem 2 in Blackorby and Russell (1981) implies that property (5) holds if and only if the preferences can be written as equation (A.1). In this sense, the definition above corresponds to the most general class of nonhomothetic CES preferences. Standard CES preferences are a specific example of Equation (A.1) with $g_i(U) = U$ for all i's.

These preferences were first introduced, seemingly independently, by Sato (1975) and Hanoch (1975) who each characterize different properties of these functions. Here, we state and briefly prove some of the relevant results to provide a self-contained exposition of our theory in this paper.

Lemma 3. *If $\sigma > 0$ and functions $g_i(\cdot)$ are positive and monotonically increasing for all i, the function $U(\boldsymbol{C})$ defined in Equation (A.1) is monotonically increasing and quasi-concave for all $\boldsymbol{C} \gg 0$.*

Proof. Establishing monotonicity is straightforward. To establish quasi-concavity, assume to the contrary that there exists two bundles of $\boldsymbol{C}'$ and $\boldsymbol{C}''$ and their corresponding utility values U' and U'', such that $U \equiv U(\alpha \boldsymbol{C}' + (1-\alpha)\boldsymbol{C}'')$ is strictly smaller than both U' and U''. We then have for the case $\sigma > 1$

$$\begin{aligned} 1 &= \sum_i \Upsilon_i^{1/\sigma} \left(\alpha \frac{C_i'}{g_i(U)} + (1-\alpha) \frac{C_i''}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}}, \\ &> \sum_i \Upsilon_i^{1/\sigma} \left(\alpha \frac{C_i'}{g_i(U')} + (1-\alpha) \frac{C_i''}{g_i(U'')} \right)^{\frac{\sigma-1}{\sigma}}, \\ &\geq \alpha \sum_i \Upsilon_i^{1/\sigma} \left(\frac{C_i'}{g_i(U')} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \sum_i \Upsilon_i^{1/\sigma} \left(\frac{C_i''}{g_i(U'')} \right)^{\frac{\sigma-1}{\sigma}}, \end{aligned}$$

where in the first inequality we have used monotonicity of the g_i's and in the second we have used Jensen's inequality and the assumption that $\infty > \sigma > 1$. Since the last line equals 1 from the definition of the nonhomothetic CES functions valued at U' and U'', we arrive at a contradiction. For the case that $0 < \sigma < 1$, we can proceed analogously. In this case, the inequality signs are reversed in both lines and we also reach a contradiction. □

Demand Function Henceforth, we assume the conditions in Lemma 3 are satisfied. The next lemma characterizes the demand for general nonhomothetic CES preferences and provides the solution to the expenditure minimization problem.

Lemma 4. *Consider any bundle of goods that maximizes the utility function defined in Equation (A.1) subject to the budget constraint $\sum_i p_i C_i \leq E$. For each good i, the real consumption satisfies:*

$$C_i = \Upsilon_i \left(\frac{p_i}{E} \right)^{-\sigma} g_i(U)^{1-\sigma}, \tag{A.2}$$

where U satisfies

$$E = \left[\sum_{i=1}^{I} \Upsilon_i \left(g_i(U) \, p_i \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{A.3}$$

and the share in consumption expenditure is given by

$$\omega_i \equiv \frac{p_i C_i}{E} = \Upsilon_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}} = \Upsilon_i \left[g_i(U) \left(\frac{p_i}{E} \right) \right]^{1-\sigma}. \tag{A.4}$$

Proof. Let λ and ρ denote the Lagrange multipliers on the budget constraint and constraint (A.1), respectively:

$$\mathcal{L} = U + \rho \left(1 - \sum_i \Upsilon_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}} \right) + \lambda \left(E - \sum_i p_i C_i \right).$$

The FOCs with respect to C_i yields:

$$\rho \frac{1-\sigma}{\sigma} \frac{\omega_i}{C_i} = \lambda p_i, \tag{A.5}$$

where we have defined $\omega_i \equiv \Upsilon_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}}$. Equation (A.5) shows that expenditure $p_i C_i$ on good i is proportional to ω_i. Since the latter sums to one from constraint (A.1), it follows that ω_i is the expenditure share of good i, and we have: $E = \sum_{i=1}^{I} p_i C_i = \frac{1-\sigma}{\sigma} \frac{\rho}{\lambda}$. We can now substitute the definition of ω_i in expression (A.5) and use the budget constraint above to find Equation (A.2), and (A.4). $\square$

Elasticities of Demand Lemma 4 implies that the equation defining the expenditure function (and implicitly the indirect utility function) for general nonhomothetic CES preferences is given by Equation (A.3). The expenditure function is continuous in prices p_i's and utility U, and homogeneous of degree 1, increasing, and concave in prices. The elasticity of the expenditure function with respect to utility is

$$\eta_E^U \equiv \frac{U\partial E}{E\partial U} = \sum_i \omega_i\, \eta_{g_i}^U \equiv \overline{\eta_{g_i}^U}, \tag{A.6}$$

which ensures that the expenditure function is increasing in utility if all g_i's are monotonically increasing. It is straightforward to also show that the elasticity of the utility function (A.1) with respect to consumption of good i is also given by

$$\eta_U^{C_i} \equiv \frac{C_i\partial U}{U\partial C_i} = \frac{\omega_i}{\overline{\eta_{g_i}^U}}, \tag{A.7}$$

where, again, ω_i is the ratio $\omega_i \equiv \Upsilon_i^{\frac{1}{\sigma}}\left(C_i/g_i\left(U\right)\right)^{\frac{\sigma-1}{\sigma}}$.

Examining demand for good i from Equation (A.2) along indifference curves, we can derive the main properties of nonhomothetic CES preferences. As expected, *on a given indifference curve*, the elasticity of substitution is constant

$$\eta_{C_i/C_j}^{p_i/p_j} \equiv \left.\frac{\partial \log\left(C_i/C_j\right)}{\partial \log\left(p_i/p_j\right)}\right|_{U=\text{const.}} = \sigma. \tag{A.8}$$

More interestingly, the elasticity of relative demand with respect to utility, in constant prices, is in general different from unity:[67]

$$\eta_{C_i/C_j}^U \equiv \left.\frac{\partial \log\left(C_i/C_j\right)}{\partial \log U}\right|_{\boldsymbol{p}=\text{const.}} = (1-\sigma)\frac{\partial \log\left(g_i/g_j\right)}{\partial \log U}. \tag{A.9}$$

Since utility has a monotonic relationship with expenditure, it then follows that the expenditure elasticity of demand for different goods are different. More specifically, we can use (A.6) to find the expenditure elasticity of demand:

$$\eta_{C_i}^E \equiv \frac{\partial \log C_i}{\partial \log E} = \sigma + (1-\sigma)\,\frac{\eta_{g_i}^U}{\overline{\eta_{g_i}^U}}. \tag{A.10}$$

[67]Preferences defined by Equation (A.1) belong to the general class of preferences with *Direct Implicit Additivity*. Hanoch (1975) shows that the latter family of preferences have the nice property that is illustrated by Equations (A.8) and (A.9): the separability of the income and substitution elasticities of the Hicksian demand. This is in contrast to the stronger requirement of *Explicit Additivity* commonly assumed in nonhomothetic preferences, whereby the utility is explicitly defined as a function $U = F\left(\sum_i f_i(C_i)\right)$. In Section G.1 of the Online Appendix, we will show examples of how substitution and income elasticities of Hicksian demand are *not* separable for preferences with explicitly additivity in direct utility, e.g., generalized Stone-Geary preferences (Kongsamut et al., 2001), or indirect utility, e.g., PIGL preferences (Boppart, 2014a).

Convexity of the Expenditure Function in Utility First, we express the second derivative of the expenditure function in terms of elasticities,

$$\frac{\partial^2 E}{\partial U^2} = \frac{E}{U^2}\eta_E^U\left(\eta_E^U + \eta_{\eta_E^U}^U - 1\right), \tag{A.11}$$

where $\eta_{\eta_E^U}^U$ is the second order elasticity of expenditure with respect to utility. We can compute this second order elasticity as follows:

$$\begin{aligned}
\eta_{\eta_E^U}^U &= U\frac{\partial}{\partial U}\log\sum_i \eta_{g_i}(U)\left(g_i(U)\,p_i\right)^{1-\sigma} - (1-\sigma)\frac{\partial \log E}{\partial \log U}, \\
&= \frac{\sum_i \eta_{\eta_{g_i}}\cdot\eta_{g_i}\left(g_i(U)\,p_i\right)^{1-\sigma} + (1-\sigma)\sum_i \eta_{g_i}^2\left(g_i(U)\,p_i\right)^{1-\sigma}}{\sum_i \eta_{g_i}\left(g_i(U)\,p_i\right)^{1-\sigma}} - (1-\sigma)\,\overline{\eta_{g_i}}, \\
&= \overline{\eta_{g_i}}\left[\frac{\overline{\eta_{\eta_{g_i}}\cdot\eta_{g_i}}}{\left(\overline{\eta_{g_i}}\right)^2} + (1-\sigma)\,Var\left(\frac{\eta_{g_i}}{\overline{\eta_{g_i}}}\right)\right],
\end{aligned} \tag{A.12}$$

where $\overline{X_i}$ and $Var\left(X_i\right)$ denote the expected value and variance of variable X_i across sectors with weights given by expenditure shares ω_i for prices $\boldsymbol{p}$ and utility U.

A.2 Isoelastic Nonhomothetic CES Preferences

A.2.1 Elasticities

We discussed that a class of preferences satisfies equation (5) if and only if we can write it as (A.1). If we further impose condition (4), it follows that there is some function $g\left(\cdot\right)$ such that $\log g_i(U) = \varepsilon_i \log g\left(U\right)$, where both $g\left(\cdot\right)$ is an increasing function and $\varepsilon_i > 0$ for all i. This gives us the definition of our basic model in Section 2. We then have $\eta_{g_i}^U = \eta_g\,\varepsilon_i$ where $\eta_g \equiv d\log g/d\log U$, and $\eta_{\eta_{g_i}} = \eta_{\eta_g}$ where $\eta_{\eta_g} \equiv d\log\eta_g/d\log U$. Equations (2) and (3) follow by substituting for g_i in the results of Lemma 4 above. From (A.6), the utility elasticity of the expenditure function is: $\eta_E^U \equiv \frac{U}{E}\frac{\partial E}{\partial U} = \eta_g\overline{\varepsilon}$, where $\overline{\varepsilon} = \sum_i \omega_i\varepsilon_i$. Therefore, a sufficient condition for the function $E\left(U;\boldsymbol{p}\right)$ to be a one-to-one mapping for all positive prices is that all sectors have an income elasticity larger than the elasticity of substitution. This directly follows from Lemma 3. Combining Equations(A.11) and (A.12), we find

$$\frac{\partial^2 E}{\partial U^2} = \frac{E}{U^2}\eta_g\overline{\varepsilon}\left[\eta_g\overline{\varepsilon}\left(1 + (1-\sigma)\,Var\left(\frac{\varepsilon_i}{\overline{\varepsilon}}\right)\right) + \eta_{\eta_g}\right]. \tag{A.13}$$

where we have substituted in Equation (A.12) to find $\eta_{\eta_E^U}^U = \eta_{\eta_g} + \eta_g\overline{\varepsilon}\left(1-\sigma\right)Var\left(\varepsilon/\overline{\varepsilon}\right)$. In the special case of $g\left(U\right) = U$, we find $\eta_E^U = \overline{\varepsilon}$ and $\partial^2 E/\partial U^2 = E/U^2 \times \overline{\varepsilon}^2\left(1 + (1-\sigma)\,Var\left(\varepsilon_i/\overline{\varepsilon}\right)\right)$.

A.2.2 Generalized Indices of Real Consumption and Demand Estimation

Let vector $\boldsymbol{\psi} \in \mathbb{R}_+^I$ denote a vector of weights defined over the I goods such that $\psi_i \geq 0$ for all i and $\sum_{i=1}^I \psi_i = 1$. For the nonhomothetic CES preferences of Equation (1), define a $\boldsymbol{\psi}$-index of nonhomothetic CES real consumption $C^{\boldsymbol{\psi}}$ as the following monotonic transformation of $F(\boldsymbol{C})$:

$$C^{\boldsymbol{\psi}}=\left(\prod_{i=1}^{I}\Upsilon_i^{\psi_i}\right)^{\frac{1}{1-\sigma}}g\left(F\left(\boldsymbol{C}\right)\right)^{\sum_{i=1}^{I}\psi_i\varepsilon_i}.$$

Substituting this expression in Equation (1), we find

$$\sum_{i=1}^{I}\left(\frac{\Upsilon_i}{\left(\prod_{j=1}^{I}\Upsilon_j^{\psi_j}\right)^{\frac{\varepsilon_i}{\sum_{j=1}^{I}\psi_j\varepsilon_j}}}\right)^{\frac{1}{\sigma}}\left(\frac{C_i}{\left(C^{\boldsymbol{\psi}}\right)^{\frac{\varepsilon_i}{\sum_{j=1}^{I}\psi_j\varepsilon_j}}}\right)^{\frac{\sigma}{\sigma-1}}=1,$$

which suggests defining the following set of parameters $\left(\sigma,\boldsymbol{\epsilon}^{\boldsymbol{\psi}},\boldsymbol{\Omega}^{\boldsymbol{\psi}}\right)$:

$$\epsilon_i^{\boldsymbol{\psi}}\equiv\frac{\varepsilon_i}{\sum_{j=1}^{I}\psi_j\varepsilon_j},\qquad \Omega_i^{\boldsymbol{\psi}}\equiv\frac{\Upsilon_i}{\left(\prod_{j=1}^{I}\Upsilon_j^{\psi_j}\right)^{\frac{\varepsilon_i}{\sum_{j=1}^{I}\psi_j\varepsilon_j}}},\qquad 1\le i\le I, \tag{A.14}$$

satisfying $\sum_{i=1}^{I}\psi_i\epsilon_i^{\boldsymbol{\psi}}=1$ and $\prod_{i=1}^{I}\left(\Omega_i^{\boldsymbol{\psi}}\right)^{\psi_i}=1$. Accordingly, we define the nonhomothetic CES preferences for an aggregator $C^{\boldsymbol{\psi}}$ satisfying $\sum_{i=1}^{I}\left(\Omega_i^{\boldsymbol{\psi}}\right)^{\frac{1}{\sigma}}\left(\frac{C_i}{\left(C^{\boldsymbol{\psi}}\right)^{\epsilon_i^{\boldsymbol{\psi}}}}\right)^{\frac{\sigma-1}{\sigma}}=1$, in line with Equation (14).

The derivation above suggests that there exists a family of indices of real consumption and the corresponding parameterizations indexed by vectors $\boldsymbol{\psi}$ that all characterize the same preferences. In particular, our definition of the index of real consumption and parameters $(\boldsymbol{\epsilon},\boldsymbol{\Omega})$ for a base good $b\in\{1,\cdots,I\}$ in Section 2.1.2 is a special case of the construction above for a vector $\boldsymbol{\psi}$ such that $\psi_b=1$ and $\psi_i=0$ for $i\neq b$.

To simplify the expressions, and in line with the convention in Section 2.1.2 , let us drop the superscript $\boldsymbol{\psi}$ and denote the index of real consumption and the parameters by C and $(\sigma,\boldsymbol{\epsilon},\boldsymbol{\Omega})$, bearing in mind that both correspond to a given vector $\boldsymbol{\psi}$. Solving for the Hicksian demand in terms of the $\boldsymbol{\psi}$-index of real consumption, we find $\omega_i=\Omega_i\left(p_iC^{\epsilon_i}/E\right)^{1-\sigma}$. Using the constraints $\sum_{i=1}^{I}\psi_i\epsilon_i=1$, $\prod_{i=1}^{I}\Omega_i^{\psi_i}=1$, and $\sum_i\psi_i=1$, we find that

$$\prod_{i=1}^{I}\omega_i^{\psi_i}=\left[\frac{\left(\prod_{i=1}^{I}p_i^{\psi_i}\right)}{E}C\right]^{1-\sigma}.$$

This allows us to write the $\boldsymbol{\psi}$-index of real consumption in terms of observed data as

$$\log C=\log E-\sum_{i=1}^{I}\psi_i\log p_i+\frac{1}{1-\sigma}\sum_{i=1}^{I}\psi_i\log\omega_i, \tag{A.15}$$

which generalizes the expression in Equations (7) and (12). Substituting this expression in the Hicksian demand, we find the following I constraints that together define the implicit Marshallian demand

$$\log\omega_i = (1-\sigma)\left[\log p_i - \sum_{j=1}^{I}\psi_j \log p_j\right] + (\epsilon_i - 1)(1-\sigma)\left[\log E - \sum_{j=1}^{I}\psi_j \log p_j\right] + \epsilon_i \sum_{j=1}^{I}\psi_j \log\omega_j + \log\Omega_i, \tag{A.16}$$

generalizing Equation (11). Note that, as before, the set of equations defined in (A.16) imply $I-1$ independent constraints on the expenditure shares.[68]

Finally, the following lemma shows that the knowledge of the parameters for a given base $\boldsymbol{\psi}$ is sufficient to be able to recover the parameters for any other base $\boldsymbol{\psi}'$.

Lemma 5. *Consider two vectors $\boldsymbol{\psi}$ and $\boldsymbol{\psi}'$, and the corresponding indices and parameters $(C, \boldsymbol{\epsilon}, \boldsymbol{\Omega})$ and $(C', \boldsymbol{\epsilon}', \boldsymbol{\Omega}')$. We can show that one can transform the parameters from one index to the other according to:*

$$\epsilon_i' = \frac{\epsilon_i}{\sum_j \psi_j' \epsilon_j},$$
$$\Omega_i' = \frac{\Omega_i}{\left(\prod_{j=1}^{I} \Omega_j^{\psi_j'}\right)^{\frac{\epsilon_i}{\sum_j \psi_j' \epsilon_j}}}.$$

Proof. The results follow from a simple substitution of the expressions implied by Equation (A.14) for $(\boldsymbol{\epsilon}, \boldsymbol{\Omega})$ and $(\boldsymbol{\epsilon}', \boldsymbol{\Omega}')$. We can recover $\boldsymbol{\epsilon}'$ through:

$$\frac{\epsilon_i}{\sum_j \psi_j' \epsilon_j} = \frac{\frac{\varepsilon_i}{\sum_{j'} \psi_{j'} \varepsilon_{j'}}}{\sum_j \psi_j' \left(\frac{\varepsilon_j}{\sum_{j''} \psi_{j''} \varepsilon_{j''}}\right)} = \frac{\varepsilon_i}{\sum_j \psi_j' \varepsilon_j} = \epsilon_i'.$$

[68] We also want to briefly make a complementary comment. After obtaining the estimates of the nonhomotheticity parameters, it is possible to refine the construction of the real consumption index and convexify the solution in a similar way to what we have discussed above. Let $(\hat{\Omega}_i, \hat{\epsilon}_i, \hat{\sigma})_i$ denote the set of estimates. We can invert the Hicksian demand (2) for *each sector* i

$$\log\omega_i = \log\hat{\Omega}_i + (1-\hat{\sigma})\log\left(\frac{p_i}{E}\right) + (1-\hat{\sigma})\,\hat{\epsilon}_i \log C, \qquad \forall i \in \mathcal{I}, \tag{A.17}$$

to obtain an estimate of the real consumption index, $\widehat{\log C^i}$, where i denotes the estimate for sector i. If the demand system was exactly a nonhomothetic CES observed without noise, we would recover the same real consumption index for all the equations. In general, however, this is not the case and we end up with a vector of $(\widehat{\log C^i})_{i\in\mathcal{I}}$ where each entry is different to each other. If we stick to the construction in Section 2, we would be only using the base sector to recover the real consumption index, i.e., we would be selecting $i = b$. Despite being perfectly logically consistent, this would disregard the information contained in the other entries of the vector. Thus, if the goal is to find the "best" measure of the real consumption index, one could try to find a set of weights for each entry of the vector that maximizes a given objective function. While a formal analysis of this is beyond the scope of the paper, we note that in Comin et al. (2018), we used this approach and selected as weights the average sectoral share of each country. We found that the model fit improved marginally.

Similarly, we can recover $\boldsymbol{\Omega}'$ through

$$\frac{\Omega_i}{\left(\prod_{j=1}^{I}\Omega_j^{\psi_j'}\right)^{\frac{\epsilon_i}{\sum_j \psi_j'\epsilon_j}}} = \frac{\frac{\Upsilon_i}{\left(\prod_{j'}\Upsilon_{j'}^{\psi_{j'}}\right)^{\epsilon_i}}}{\left(\frac{\prod_j \Upsilon_j^{\psi_j'}}{\left(\prod_j \Upsilon_j^{\psi_j}\right)^{\sum_{j''}\psi_{j''}'\epsilon_{j''}}}\right)^{\frac{\epsilon_i}{\sum_j \psi_j'\epsilon_j}}},$$

$$= \frac{\Upsilon_i}{\left(\prod_j \Upsilon_j^{\psi_j'}\right)^{\frac{\varepsilon_i}{\sum_j \psi_j'\varepsilon_j}}},$$

$$= \Omega_i',$$

where in the last equality we have used the fact that $\epsilon_i' = \epsilon_i / \sum_j \psi_j' \epsilon_j$. □

B Proofs

B.1 Proofs of Propositions and Lemmas

Proof of Lemma 1. As we discussed in Section 2.1, the expression for the nonhomothetic CES Marshallian demand is given by

$$C_i = \Upsilon_i \left(\frac{p_i}{E}\right)^{-\sigma} V(E;\boldsymbol{p})^{(1-\sigma)\varepsilon_i}, \qquad 1 \le i \le I, \tag{B.1}$$

where the indirect utility function $V(\cdot,\boldsymbol{p})$ satisfies:

$$E = \left[\sum_{i=1}^{I} \Upsilon_i V(E;\,\boldsymbol{p})^{(1-\sigma)\varepsilon_i} p_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{B.2}$$

In line with the definition of the nonhomothetic CES index of real consumption, define a monotonic transformation of the indirect utility function as $\mathcal{C}(E;\,\boldsymbol{p}) \equiv \Upsilon_b^{\frac{1}{1-\sigma}} V(E;\,\boldsymbol{p})^{\varepsilon_b}$. Simple algebra shows that we can rewrite Equations (B.1) and (B.2) solely in terms of parameters $\boldsymbol{\epsilon}$ and $\boldsymbol{\Omega}$ defined in Equation (10) as:

$$C_i = \left(\frac{\Upsilon_i}{\Upsilon_b^{\varepsilon_i/\varepsilon_b}}\right)\left(\frac{p_i}{E}\right)^{-\sigma} \mathcal{C}(E;\boldsymbol{p})^{(1-\sigma)(\varepsilon_i/\varepsilon_b)}, \qquad 1 \le i \le I, \tag{B.3}$$

where the function $\mathcal{C}(\cdot,\boldsymbol{p})$ satisfies:

$$E = \left[\sum_{i=1}^{I}\left(\frac{\Upsilon_i}{\Upsilon_b^{\varepsilon_i/\varepsilon_b}}\right)\mathcal{C}(E;\,\boldsymbol{p})^{(1-\sigma)(\varepsilon_i/\varepsilon_b)} p_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{B.4}$$

Similarly, the observable income (expenditure) elasticities may also be written in terms of $\boldsymbol{\epsilon}$ and $\boldsymbol{\Omega}$. Since we can express the Marshallian demand in terms of $\boldsymbol{\epsilon}$ and $\boldsymbol{\Omega}$, we can also write the expenditure-share-weighted average at time t as $\bar{\varepsilon}_t \equiv \varepsilon_b \times \bar{\epsilon}_t$, also in terms of these parameters. The expression for the income elasticity in Equation (9) then implies that the income elasticity of sector i at time t

satisfies $\eta_{it} = \sigma + (1-\sigma)\epsilon_i/\bar{\epsilon}_t$. $\square$

Proof of Lemma 2. To connect the household problem to the static problem solved in Section A, we explicitly add expenditure to the problem and state it as

$$\max_{(\boldsymbol{C}_t, E_t, \mathcal{A}_t)_t} \sum_{t=0}^{\infty} \beta^t v\left(F\left(\boldsymbol{C}_t\right)\right)$$
$$\text{s.t. } w_t + (1+r_t)\,\mathcal{A}_t - \mathcal{A}_{t+1} - E_t \geq 0,$$
$$E_t - \sum_{i\in\mathcal{I}} p_{it} C_{it} \geq 0,$$

along with the no-Ponzi constraint. Given the definition of the function $u(\cdot)$, it is straightforward to show that $v(F(\boldsymbol{C}_t)) = u(G(\boldsymbol{C}_t))$, with function $G(\cdot)$ defined in Equation (12). Therefore, letting $C_t \equiv G(\boldsymbol{C}_t)$, we can set up the intermediate Lagrangian problem:

$$\max_{(\boldsymbol{C}_t, E_t, C_t \mathcal{A}_t)_t} \sum_{t=0}^{\infty} \beta^t \left[u(C_t) + \rho_t \left(-1 + \sum_i \left(\Omega_i C_t^{(1-\sigma)\epsilon_i} \right)^{\frac{1}{\sigma}} C_{it}^{\frac{\sigma-1}{\sigma}} \right) + \lambda_t \left(E_t - \boldsymbol{p}_t \cdot \boldsymbol{C}_t \right) \right],$$
$$\text{s.t. } w_t + (1+r_t)\,\mathcal{A}_t - \mathcal{A}_{t+1} - E_t \geq 0.$$

The first order conditions for each of the elements in $\boldsymbol{C}_t$, (i.e., C_{it}'s) are given by

$$\rho_t \frac{\sigma-1}{\sigma} \frac{\omega_{it}}{C_{it}} = \lambda_t p_{it}, \tag{B.5}$$

where we have defined $\omega_{it} \equiv \Omega_i^{\frac{1}{\sigma}} \left(C_t^{-\epsilon_i} C_{it} \right)^{\frac{\sigma-1}{\sigma}}$. Note that this condition corresponds exactly to what we found in the static problem in Equation (A.5). Substituting this condition in $E_t = \boldsymbol{p}_t \cdot \boldsymbol{C}_t$ and using the definition of ω_{it}, we find $E_t = \frac{\sigma-1}{\sigma}\frac{\rho_t}{\lambda_t}$ and $C_{it} = \Omega_i \left(\frac{p_{it}}{E_t}\right)^{-\sigma} C_t^{\epsilon_i(1-\sigma)}$. Substituting for C_{it} in $E_t = \sum_{i=1}^{I} p_{it} C_{it}$ yields the expression relating E_t and C_t, and Equation (11) follows along the same line as the derivations in the main text.[69]

The derivation above shows that along any optimal path, the streams of indices of real consumption $(C_t)_t$ and total expenditures $(E_t)_t$ satisfy $E_t^{1-\sigma} = \sum_{i=1}^{I} \Omega_i \left(C_t^{\epsilon_i} p_{it}\right)^{1-\sigma}$. Given the parameter restrictions, we know that we can define a function C_t as a monotonically increasing and invertible function of E_t as $C_t = \mathcal{C}\left(E_t; \boldsymbol{p}_t\right)$. This function is the indirect utility function corresponding to the cardinalization of utility implied by the nonhomothetic CES index of real consumption. Therefore, we have decomposed the problem into two independent parts. The intratemporal problem involves allocating the expenditure E_t across I goods so as to maximize instantaneous utility U_t defined by Equation (1). The solution is given by Equations (2) and (3). The intertemporal one, which we will characterize next, involves allocating the stream of expenditures $(E_t)_t$ and assets $(\mathcal{A}_t)_t$ over time given the paths of good prices and initial assets.

[69] An alternative strategy for deriving the two-stage budgeting nature of the optimal solution is to substitute for one base good $C_{bt} = \left[w_t + (1+r_t)\,\mathcal{A}_t - \mathcal{A}_{t+1} - \sum_{i\neq b} p_{it} C_{it} \right] / p_{bt}$ and note that the choice of control variables $\boldsymbol{C}_{-bt} \equiv (C_{it})_{i\neq b}$ does not have any dynamic implications. The optimality conditions for these control variables coincide with Equation (B.5).

Substituting for the expenditure E_t from the period budget constraint, we can write the intertemporal problem as that of finding the sequence of assets $\{\mathcal{A}_{t+1}\}_{t=0}^{\infty}$ such that

$$\max_{(\mathcal{A}_t)_t} \sum_{t=0}^{\infty} \beta^t \, u\left(\mathcal{C}\left(w_t + \left(1 + r_t\right) \mathcal{A}_t - \mathcal{A}_{t+1}; \boldsymbol{p}_t\right)\right). \tag{B.6}$$

Next, we provide conditions that ensure that the function $u\left(C\left(\cdot; \boldsymbol{p}_t\right)\right)$ is monotonically increasing and strictly concave for all prices. Then, we invoke standard results from discrete dynamic programming (e.g., see Acemoglu, 2008, Chapter 6, Theorem 6.12) to conclude that the Euler equation

$$u'\left(C_t\right) \frac{\partial \mathcal{C}_t}{\partial E_t} \quad = \quad \beta \left(1 + r_{t+1}\right) \, u'\left(C_{t+1}\right) \frac{\partial \mathcal{C}_{t+1}}{\partial E_{t+1}},$$

and the transversality condition

$$\lim_{t \to \infty} \beta^t \left(1 + r_t\right) \mathcal{A}_t \, u'\left(C_t\right) \frac{\partial \mathcal{C}_t}{\partial E_t} = 0, \tag{B.7}$$

provide necessary and sufficient conditions for a sequence $\{\mathcal{A}_{t+1}\}_{t=0}^{\infty}$ to characterize the solution.

To ensure the concavity of $u\left(C\left(\cdot; \boldsymbol{p}_t\right)\right)$, let us compute its second derivative:

$$\begin{aligned} \frac{\partial^2}{\partial E_t^2} u\left(\mathcal{C}\left(E_t; \boldsymbol{p}_t\right)\right) &= u''\left(C_t\right) \cdot \left(\frac{\partial \mathcal{C}_t}{\partial E_t}\right)^2 + u'\left(C_t\right) \frac{\partial^2 \mathcal{C}_t}{\partial E_t^2}, \\ &= u'\left(C_t\right) \cdot \left(\frac{\partial \mathcal{C}_t}{\partial E_t}\right)^2 \cdot \frac{1}{C_t} \cdot \left[\frac{C_t u''\left(C_t\right)}{u'\left(C_t\right)} + \frac{C_t \partial^2 \mathcal{C}_t / \partial E_t^2}{\left(\partial \mathcal{C}_t / \partial E_t\right)^2}\right]. \end{aligned} \tag{B.8}$$

Since $u\left(\cdot\right)$ is increasing and $u'\left(\cdot\right) > 0$, to ensure that the second derivative of $u\left(C\left(\cdot; \boldsymbol{p}_t\right)\right)$ is negative, it is sufficient to ensure that the expression in the rightmost square brackets above is negative everywhere.

To derive a sufficient condition that ensures this, we use the results of the previous section, and in particular Equation (A.13) (under the choice of $g(U) = U$, $\boldsymbol{\Upsilon} = \boldsymbol{\Omega}$, and $\boldsymbol{\varepsilon} = \boldsymbol{\epsilon}$). Let $\mathcal{E}\left(\cdot; \boldsymbol{p}_t\right)$ be the corresponding expenditure function in terms of the nonhomothetic CES index of real consumption. Equation (A.13) implies that

$$\frac{\partial^2 \mathcal{E}\left(C_t; \boldsymbol{p}_t\right)}{\partial C_t^2} = \frac{\mathcal{E}_t}{C_t^2} \bar{\epsilon}_t^2 \left[1 + \left(1 - \sigma\right) Var\left(\frac{\epsilon_i}{\bar{\epsilon}_t}\right)\right].$$

Now, note that $\mathcal{E}\left(\mathcal{C}\left(E_t; \boldsymbol{p}_t\right)\right) = E_t$, which implies:

$$\frac{\partial \mathcal{E}_t}{\partial C_t} \times \frac{\partial \mathcal{C}_t}{\partial E_t} = 1, \qquad \frac{\partial \mathcal{E}_t}{\partial C_t} \times \frac{\partial^2 \mathcal{C}_t}{\partial E_t^2} + \frac{\partial^2 \mathcal{E}_t}{\partial C_t^2} \times \left(\frac{\partial \mathcal{C}_t}{\partial E_t}\right)^2 = 0$$

These results allow us to rewrite the expression within the square bracket in Equation (B.8) as follows.

$$
\begin{aligned}
\frac{C_t u''(C_t)}{u'(C_t)} + \frac{C_t \partial^2 \mathcal{C}_t / \partial E_t^2}{(\partial \mathcal{C}_t / \partial E_t)^2} &= \frac{C_t u''(C_t)}{u'(C_t)} - \frac{C_t \left(\partial^2 \mathcal{E}_t / \partial C_t^2\right)}{\partial \mathcal{E}_t / \partial C_t}, \\
&= \frac{C_t u''(C_t)}{u'(C_t)} - \frac{C_t^2}{\mathcal{E}_t \eta_{\mathcal{E}_t}^{C_t}} \times \frac{\partial^2 \mathcal{E}_t}{\partial C_t^2}, \\
&< -\underline{\theta} - \overline{\epsilon}_t \left(1 + (1-\sigma) Var_t \left(\frac{\epsilon_i}{\overline{\epsilon}}\right)\right) + 1,
\end{aligned}
$$

where in the first equality, we have used the expressions relating the partial derivatives of $\mathcal{C}_t$ with respect to E_t as those of $\mathcal{E}_t$ with respect to C_t, and in the last inequality we have used the expression for the second derivative of $\mathcal{E}(\cdot; \boldsymbol{p}_t)$ stated above, as well as the bound on the elasticity of $u'(\cdot)$.

We now have two cases. If $0 < \sigma < 1$, a sufficient condition for the upper bound in the expression above to be negative is that $\underline{\theta} > 1 - \epsilon_{min}$.[70] If $1 < \sigma$, we rely on the fact that $Var(\epsilon_i) \leq \frac{1}{4}(\epsilon_{max} - \epsilon_{min})^2$ to find Equation (19) to ensure the concavity of $u(\mathcal{C}(\cdot; \boldsymbol{p}_t))$.

Having established the concavity of the instantaneous utility function in terms of $\mathcal{A}_t$ and $\mathcal{A}_{t+1}$, it follows that the Euler equation

$$
u'(C_t) \frac{\partial \mathcal{C}_t}{\partial E_t} = \beta (1 + r_{t+1}) u'(C_{t+1}) \frac{\partial \mathcal{C}_{t+1}}{\partial E_{t+1}}
$$

and the transversality condition

$$
\lim_{t \to \infty} \beta^t (1 + r_t) \mathcal{A}_t u'(C_t) \frac{\partial \mathcal{C}_t}{\partial E_t} = 0, \tag{B.9}
$$

provide necessary and sufficient condition for a sequence $\{\mathcal{A}_{t+1}\}_{t=0}^{\infty}$ to characterize the solution.

Using the results of Section A, we can simplify each side of the Euler equation as

$$
u'(C_t) \frac{\partial \mathcal{C}_t}{\partial E_t} = u'(C_t) \frac{C_t}{E_t} \frac{1}{\eta_{\mathcal{E}_t}^{C_t}} = u'(C_t) \frac{C_t}{E_t} \frac{1}{\overline{\epsilon}_t},
$$

where in the first equality we have used the definition of the utility elasticity of the expenditure function $\eta_{\mathcal{E}}^{C}$, in the second inequality we have substituted $\eta_{\mathcal{E}}^{C} = \overline{\epsilon}$. We can now write the Euler equation as stated in Equation (20). Moreover, using the same results, we can also rewrite the transversality condition (B.9) as

$$
\lim_{t \to \infty} \beta^t (1 + r_t) u'(C_t) \frac{\mathcal{A}_t C_t}{\overline{\epsilon}_t E_t} = 0.
$$

□

Proof of Proposition 1. Our proof for the proposition involves two steps. First, we use the second Welfare Theorem and consider the equivalent centralized allocation by a social planner. Due to the concavity of the per-period indirect utility function $v(V(\cdot; \boldsymbol{p}_t)) \equiv u(\mathcal{C}(\cdot; \boldsymbol{p}_t))$, which is ensured by the conditions in Lemma 2, we can use standard arguments to establish the uniqueness of the equilibrium allocations (see Stokey et al., 1989, p. 291). Next, we construct a unique constant growth path (steady

[70] In Lemma 1 of the working paper draft of this paper (Comin et al., 2018), we have imposed the alternative set of assumptions $\epsilon_i \geq 1$ and $\underline{\theta} > 0$.

state) that satisfies the equilibrium conditions. It then follows that the equilibrium converges to the constructed Constant Growth Path (CGP).

Consider an equilibrium path along which consumption expenditure E_t, aggregate stock of capital K_t, and the capital allocated to the investment sector K_{0t} all asymptotically grow at rate $(1+\gamma_0)^{\frac{1}{1-\alpha_0}}$, and the labor employed in the investment sector asymptotically converges to $L_0^* \in (0,1)$. Henceforth, we use the tilde variables to denote normalization by $A_{0t}^{-\frac{1}{1-\alpha_0}}$, for instance, $\tilde{K}_t \equiv A_{0t}^{-\frac{1}{1-\alpha_0}} K_t$. Accordingly, we can write the law of evolution of aggregate stock of capital as

$$\tilde{K}_{t+1} = \frac{1-\delta}{(1+\gamma_0)^{1/(1-\alpha_0)}} \tilde{K}_t + \frac{1}{(1+\gamma_0)^{1/(1-\alpha_0)}} \tilde{K}_{0t}^{\alpha_0} L_{0t}^{1-\alpha_0}, \tag{B.10}$$

and the interest rate and wages as

$$r_t = R_t - \delta \quad = \quad \alpha_0 \left(\frac{\tilde{K}_{0t}}{L_{0t}} \right)^{\alpha_0 - 1} - \delta, \tag{B.11}$$

$$\tilde{w}_t \quad = \quad (1-\alpha_0)\, A_{0t}^{\frac{1}{1-\alpha_0}} \tilde{K}_{0t}^{\alpha_0} L_{0t}^{-\alpha_0}. \tag{B.12}$$

From the assumptions above, it follows that $\tilde{K}_{0t}/L_{0t}$ asymptotically converges to a constant, which from Equation (B.11) implies that the rate of interest also converges to a constant r^*.

We first derive an expression for the asymptotic growth of nominal consumption expenditure shares (and sectoral employment shares) of different sectors:

$$\begin{aligned} 1+\xi_i \equiv \lim_{t\to\infty} \frac{\omega_{it+1}}{\omega_{it}} &= \lim_{t\to\infty} \left(\frac{E_t}{E_{t+1}} \right)^{1-\sigma} \left(\frac{p_{it+1}}{p_{it}} \left(\frac{C_{t+1}}{C_t} \right)^{\epsilon_i} \right)^{1-\sigma}, \\ &= \left(\frac{1}{1+\gamma_0} \right)^{\frac{1-\sigma}{1-\alpha_0}} \left(\frac{(1+\gamma_0)^{\frac{1-\alpha_i}{1-\alpha_0}}}{1+\gamma_i} (1+\gamma^*)^{\epsilon_i} \right)^{(1-\sigma)}, \\ &= \left[\frac{(1+\gamma^*)^{\epsilon_i}}{(1+\gamma_0)^{\frac{\alpha_i}{1-\alpha_0}} (1+\gamma_i)} \right]^{1-\sigma}, \end{aligned} \tag{B.13}$$

where in the second line we have used the definition of the constant growth path as well as the fact that from Equations (B.11) and (B.12), the relative labor-capital price grows at rate $(1+\gamma_0)^{\frac{1}{1-\alpha_0}}$ and therefore from Equation (23) we have

$$\lim_{t\to\infty} \frac{p_{it+1}}{p_{it}} = \frac{1+\gamma_0}{1+\gamma_i} (1+\gamma_0)^{\frac{\alpha_0-\alpha_i}{1-\alpha_0}}. \tag{B.14}$$

Equation (B.13) shows that the expenditure shares asymptotically grow (or diminish) monotonically. Since the shares belong to the compact $I-1$ dimensional simplex, they asymptotically converge to a time-constant set of shares.

Since shares have to add up to 1, we need to have that $\xi_i \leq 0$ for all i. Moreover, this inequality has to be satisfied with equality at least for one non-vanishing sector. Now, consider the expression defined in (29) for the growth rate of the nonhomothetic CES index of real consumption. For sectors $i \in \mathcal{I}^*$ that achieve the minimum (maximum) for $0 < \sigma < 1$ $(1 < \sigma)$, the growth of nominal expenditure share becomes zero, and their shares converge to constant values ω_i^*. For sectors $i \notin \mathcal{I}^*$, we find that the rates of growth in nominal sectoral shares ξ_i in Equation (B.13) become negative.

Asymptotically, the expenditure-weighted average income elasticity and expenditure-weighted capital intensity in the consumption sector both converge to constants $\bar{\epsilon}^* \equiv \lim_{t\to\infty} \sum_{i=1}^{I} \epsilon_i \omega_{it} = \sum_{i\in\mathcal{I}^*} \epsilon_i \omega_i^*$ and $\bar{\alpha}^* \equiv \lim_{t\to\infty} \sum_{i=1}^{I} \alpha_i \omega_{it} = \sum_{i\in\mathcal{I}^*} \alpha_i \omega_i^*$. Henceforth, with slight abuse of notation, we use tilde to also indicate variables normalized by their corresponding asymptotic rate of growth (or decline) along our proposed constant growth path. For instance, we let $\tilde{p}_{it} \equiv p_{it}(1+\gamma_0)^{-\frac{1-\alpha_i}{1-\alpha_0}t}(1+\gamma_i)^t$ and $\tilde{C}_t \equiv C_t(1+\gamma^*)^{-t}$. Furthermore, we define starred notation to indicate the asymptotic value of each variable along the constant growth path, for example, we let $p_i^* \equiv \lim_{t\to\infty} \tilde{p}_{it}$ and $\tilde{C}^* \equiv \lim_{t\to\infty} \tilde{C}_t$.

We now show that a constant growth path exists and is characterized by γ^* as defined by equation (29). We also show the existence of the asymptotic values $\{\tilde{K}^*, \tilde{U}^*, \tilde{K}_0^*, L_0^*\}$. First, note that the left hand side in the Euler equation (20) asymptotically converges to $(C_{t+1}/C_t)^{1-\theta}$ due to assumption (26) as $C_t \to \infty$. Noting that $P_t = E_t/C_t$, the Euler equation then implies

$$(1+\gamma^*)^{1-\theta} = \frac{(1+\gamma_0)^{\frac{1}{1-\alpha_0}}}{\beta\,(1+r^*)}, \tag{B.15}$$

which pins down r^*, the asymptotic real interest rate in terms of γ^* given by Equation (29). Then from Equation (B.11), we find the asymptotic capital-labor ratio in the investment sector in terms of the asymptotic real interest rate

$$\kappa \equiv \frac{\tilde{K_0}^*}{L_0^*} = \left(\frac{\alpha_0}{r^*+\delta}\right)^{\frac{1}{1-\alpha_0}}. \tag{B.16}$$

This gives us the asymptotic relative labor-capital price from Equations (B.11) and (B.12) as

$$\frac{\tilde{w}^*}{R^*} = A_{00}^{\frac{1}{1-\alpha_0}} \frac{1-\alpha_0}{\alpha_0} \frac{\tilde{K}^*}{L_0^*} = A_{00}^{\frac{1}{1-\alpha_0}} \frac{1-\alpha_0}{\alpha_0} \left(\frac{\alpha_0}{r^*+\delta}\right)^{\frac{1}{1-\alpha_0}}. \tag{B.17}$$

From Equation (23), we find

$$\tilde{p}_i^* = \frac{\alpha_0^{\alpha_0}\,(1-\alpha_0)^{1-\alpha_0}}{\alpha_i^{\alpha_i}\,(1-\alpha_i)^{1-\alpha_i}} \left(\frac{\tilde{w}^*}{R^*}\right)^{\alpha_0-\alpha_i} \frac{A_{00}}{A_{i0}}, \tag{B.18}$$

where $\tilde{w}^*/R^*$ is given by Equations (B.17) and (B.15) and A_{i0} denotes the initial state of technology in sector i and $A_{00} \equiv 1$. Given asymptotic prices, we have that

$$\tilde{E}^* = \left[\sum_{i\in\mathcal{I}^*} \Omega_i \left(\left(\tilde{C}^*\right)^{\epsilon_i} \tilde{p}_i^*\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{B.19}$$

and

$$\omega_i^* = \Omega_i \left(\left(\tilde{C}^*\right)^{\epsilon_i} \frac{\tilde{p}_i^*}{\tilde{E}^*}\right)^{1-\sigma}. \tag{B.20}$$

Next, we combine the equation for accumulation of capital (B.10), the household budget constraint (22), and the market clearing condition of consumption goods to establish that there exists a unique $\{\tilde{K}^*, \tilde{C}^*, \tilde{K}_0^*, L_0^*\}$ satisfying the asymptotic equilibrium conditions and a $\kappa = \tilde{K}_0^*/L_0^*$ where κ is given by Equation (B.16). From market clearing, the sum of payments to labor in the consumption sector is $\sum_{i=1}^{I}(1-\alpha_i)\omega_{it}E_i$, which implies $(1-\bar{\alpha}_t)\,\tilde{E}_t = \tilde{w}_t\,(1-L_{0t})$. Asymptotically, we find that

$$(1-\bar{\alpha}^*)\,\tilde{E}^* = (1-\alpha_0)\,\kappa^{\alpha_0}\,(1-L_0^*)\,. \tag{B.21}$$

Similarly, from Equation (B.10) it follows that $\left[(1+\gamma_0)^{\frac{1}{1-\alpha_0}}-(1-\delta)\right]\tilde{K}^* = \kappa^{\alpha_0}L_0^*$. Defining the expression within the square brackets at a positive constant ϑ, we write the asymptotic employment in the investment sector in terms of the aggregate stock of capital as

$$L_0^* = \vartheta\kappa^{-\alpha_0}\tilde{K}^*. \tag{B.22}$$

Finally, using the market clearing condition in the assets market $\mathcal{A}_t = K_t$ and Equation (22), we find that $\tilde{E}_t = \tilde{w}_t + R_t\tilde{K}_t - \left(\frac{\tilde{K}_{0t}}{L_{0t}}\right)^{\alpha_0}L_{0t}$ for all t. Taking the limit, it follows that

$$\tilde{E}^* = (1-\alpha_0)\,\kappa^{\alpha_0} + \alpha_0\kappa^{\alpha_0-1}\tilde{K}^* - \kappa^{\alpha_0}L_0^*. \tag{B.23}$$

Substituting from Equation (B.22) into Equations (B.21) and (B.23) yields,

$$\bar{\alpha}^*\tilde{E}^* = \alpha_0\left(\kappa^{\alpha_0-1}-\vartheta\right)\tilde{K}^*. \tag{B.24}$$

We can show that the left hand side of this equation is a monotonically increasing function of $\tilde{C}^*$ with a given κ.[71] From condition (30), we have that $\kappa^{\alpha_0-1}-\vartheta>0$ and therefore the right hand side is a linear increasing function of $\tilde{K}^*$. Therefore, Equation (B.24) defines $\tilde{C}^*$, and correspondingly $\tilde{E}^*$, as an increasing function of $\tilde{K}^*$. Finally, substituting this function and Equation (B.22) in Equation (B.23), we find

$$\tilde{E}^* + \left(\vartheta-\alpha_0\kappa^{\alpha_0-1}\right)\tilde{K}^* = (1-\alpha_0)\,\kappa^{\alpha_0}. \tag{B.25}$$

From condition (30), we know that the left hand side is a monotonically increasing function of $\tilde{K}^*$ for constant κ. This function is 0 when $\tilde{K}^*$ is zero, and limits to infinity as the latter goes to infinity. Therefore, Equation (B.25) uniquely pins down $\tilde{K}^*$ as a function of κ, which in turn is given by Equation (B.16). Condition (30) also ensures that the transversality condition (21) is satisfied. Finally, we verify that $L_0^*\in(0,1)$. Combining equations (B.22), (B.21) and (B.23) we obtain that

$$L_0^* = \frac{\bar{\alpha}^*}{\left[\frac{1-\bar{\alpha}}{1-\alpha_0}\left(\alpha_0\kappa^{\alpha_0-1}\vartheta^{-1}-1\right)+1\right]} \tag{B.26}$$

Assuming that the term in square brackets is positive, we have that $L_0^*\in(0,1)$ if and only if $\vartheta<\kappa^{\alpha_0-1}$, which in terms of fundamental parameters requires that $\beta(1+\gamma^*)^{1-\theta} < \frac{(1+\gamma_0)^{-\frac{\alpha_0}{1-\alpha_0}}}{\alpha_0+(1-\alpha_0)(1+\gamma_0)^{-\frac{1}{1-\alpha_0}}(1-\delta)}$ which is the condition stated in (30). Also, it is readily verified that as long as $\vartheta<\kappa^{\alpha_0-1}$, L_0^* cannot be negative.

Therefore, we constructed a unique constant growth path that asymptotically satisfies the equilibrium conditions whenever the parameters of the economy satisfy Equation (30). Together with the

[71] We have that $\frac{\partial\left(\bar{\alpha}^*\tilde{E}^*\right)}{\partial\tilde{C}^*} = \frac{\bar{\alpha}^*\tilde{E}^*}{\tilde{C}^*}\frac{\bar{\epsilon}}{1-\sigma}\left[1+(1-\sigma)\,\rho_{\epsilon_i,\alpha_i}\right]$ where $\rho_{\epsilon_i,\alpha_i}$ is the correlation coefficient between ϵ_i and α_i under a distribution implied by expenditure shares (see the online appendix for details of the derivation). Therefore, the derivative is always positive and the function is a monotonic function of $\tilde{C}^*$.

uniqueness of the competitive equilibrium, this completes the proof.

□

B.2 Other Derivations

Derivations for the Results in Section 2.2.3 We first characterize the dynamics of the state variable, the normalized per-capita stock of capital $\tilde{k}_t \equiv \tilde{K}_t/L$. Substituting in $K_{t+1} = A_{0t}K_{0t}^{\alpha}L_{0t}^{1-\alpha} + K_t(1-\delta)$ and noting the equality of per-capita stock of capital across sectors, we find

$$(1+\gamma_o)^{\frac{1}{1-\alpha}}\,\tilde{k}_{t+1} = \tilde{k}_t^{\alpha} l_{0,t} + \tilde{k}_t\,(1-\delta)\,,$$

where $l_{0,t} \equiv L_{0,t}/L$ is the share of labor employed in the investment sector. We can show that this share is given by $l_{0,t} = 1 - \tilde{E}_t/\tilde{k}_t^{\alpha}$ (see the online appendix), therefore establishing Equation (32).

For the evolution of per-capita consumption, we need to write C_{t+1}/C_t in terms of variables known at time t. Rewriting the Euler Equation (20) as $\left(C_{t+1}/C_t\right)^{1-\theta}\beta\,(1+r_{t+1}) = (E_{t+1}/E_t)\,\overline{\epsilon}_{t+1}/\overline{\epsilon}_t$, first note that the interest rate is given from Equation (B.11) as $r_t = \alpha\tilde{k}_t^{\alpha-1} - \delta$. Substituting for the normalized variables, we find

$$\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{1-\theta}\frac{(1+\gamma^*)^{1-\theta}}{(1+\gamma_0)^{\frac{1}{1-\alpha}}}\beta\,(1+r_{t+1}) = \left(\frac{\tilde{E}_{t+1}}{\tilde{E}_t}\right)\frac{\overline{\epsilon}_{t+1}}{\overline{\epsilon}_t}.$$

Using the expression for the asymptotic rate of interest r^* from (B.15) then gives us Equation (33).

Next, we can write the growth in per-capita consumption expenditure as

$$\begin{aligned}\left(\frac{E_{t+1}}{E_t}\right)^{1-\sigma} &= \sum_{i=1}^{I}\Omega_i\left(C_t^{\epsilon_i}\frac{p_{it}}{E_t}\right)^{1-\sigma}\left(\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i}\frac{p_{it+1}}{p_{it}}\right)^{1-\sigma},\\ &= \sum_{i=1}^{I}\omega_{it}\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i(1-\sigma)}\left(\frac{1+\gamma_0}{1+\gamma_i}\right)^{(1-\sigma)t},\end{aligned}$$

where we have used Equation (3), Equation (23), and the expression for expenditure shares $\omega_{it} = \Omega_i(p_{it}/E_t)^{1-\sigma}C_t^{\epsilon_i(1-\sigma)}$ under the assumption of $\alpha_i \equiv \alpha$. Substituting for the normalized variables $\tilde{E}_t$ and $\tilde{C}_t$ in the expression above gives Equation (34).

Finally, we use the same idea to rewrite the term $\overline{\epsilon}_{t+1}$ as follows

$$\begin{aligned}\frac{\overline{\epsilon}_{t+1}}{\overline{\epsilon}_t} &= \sum_{i=1}^{I}\Omega_i\left(C_{t+1}^{\epsilon_i}\frac{p_{it+1}}{E_{t+1}}\right)^{1-\sigma}\frac{\epsilon_i}{\overline{\epsilon}_t},\\ &= \left(\frac{E_t}{E_{t+1}}\right)^{1-\sigma}\sum_{i=1}^{I}\Omega_i\left(C_t^{\epsilon_i}\frac{p_{it}}{E_t}\right)^{1-\sigma}\left(\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i}\frac{p_{it+1}}{p_{it}}\right)^{1-\sigma}\left(\frac{\epsilon_i}{\overline{\epsilon}_t}\right),\\ &= \frac{\sum_{i=1}^{I}\omega_{it}\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i(1-\sigma)}\left(\frac{1+\gamma_0}{1+\gamma_i}\right)^{(1-\sigma)t}\left(\frac{\epsilon_i}{\overline{\epsilon}_t}\right)}{\sum_{i=1}^{I}\omega_{it}\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i(1-\sigma)}\left(\frac{1+\gamma_0}{1+\gamma_i}\right)^{(1-\sigma)t}}.\end{aligned}$$

Multiplying both the numerator and the denominator by $(1+\gamma_0)^{-\frac{t}{1-\alpha_0}}$ and substituting again for the normalized variables $\tilde{E}_t$ and $\tilde{C}_t$ gives us Equation (35).

Proof of Equation (45). From the definition of the expenditure function in Equation (3), we have

$$\begin{aligned}
\left(\frac{E_{t+1}}{E_t}\right)^{1-\sigma} &= \frac{\sum_i \Omega_i C_{t+1}^{\epsilon_i(1-\sigma)}\, p_{it+1}^{1-\sigma}}{\sum_i \Omega_i C_t^{\epsilon_i(1-\sigma)}\, p_{it}^{1-\sigma}} \\
&= \frac{\sum_i \Omega_i C_t^{\epsilon_i(1-\sigma)}\, p_{it}^{1-\sigma}\cdot\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i(1-\sigma)}\left(\frac{p_{it+1}}{p_{it}}\right)^{1-\sigma}}{\sum_i \Omega_i C_{t+1}^{\epsilon_i(1-\sigma)}\, p_{it+1}^{1-\sigma}\cdot\left(\frac{C_{t+1}}{C_t}\right)^{-\epsilon_i(1-\sigma)}\left(\frac{p_{it+1}}{p_{it}}\right)^{-(1-\sigma)}} \\
&= \left(\frac{E_{t+1}}{E_t}\right)^{-(1-\sigma)} \frac{\sum_i \omega_{it}\cdot\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i(1-\sigma)}\left(\frac{p_{it+1}}{p_{it}}\right)^{1-\sigma}}{\sum_i \omega_{it+1}\cdot\left(\frac{C_{t+1}}{C_t}\right)^{-\epsilon_i(1-\sigma)}\left(\frac{p_{it+1}}{p_{it}}\right)^{-(1-\sigma)}}.
\end{aligned}$$

Assuming that $\Delta\log E_t = \log\left(E_{t+1}/E_t\right) \ll 1$ and $\Delta\log p_{it} = \log\left(p_{it+1}/p_{it}\right) \ll 1$ for all i, we can rewrite the expression above up to the second order in $\Delta\log E_t$, $\Delta\log C_t$, and $\Delta\log p_{it}$ as

$$\begin{aligned}
\log\frac{E_{t+1}}{E_t} &\approx \frac{1}{2\left(1-\sigma\right)}\sum_i\left(\omega_{it}+\omega_{it+1}\right)\left(1-\sigma\right)\left(\log\frac{p_{it+1}}{p_{it}}+\epsilon_i\log\frac{C_{t+1}}{C_t}\right), \\
&= \underbrace{\left[\frac{1}{2}\sum_i\left(\omega_{it}+\omega_{it+1}\right)\log\frac{p_{it+1}}{p_{it}}\right]}_{\equiv\Delta\log\mathcal{P}_{it}} + \underbrace{\left[\frac{1}{2}\sum_i\left(\omega_{it}+\omega_{it+1}\right)\epsilon_i\right]}_{\equiv\overline{\mathcal{E}}_t}\cdot\log\frac{C_{t+1}}{C_t},
\end{aligned}$$

from which Equation (45) follows. □

C Discussion of the Estimation Strategy without Exact Price Index

To simplify the exposition of the derivations, we define the following notation, only to be used within this section of the Appendix: let $Y_{it}^n \equiv \log\left(\omega_{it}^n/\omega_{mt}^n\right)$, $P_{it}^n \equiv \log\left(p_{it}^n/p_{mt}^n\right)$, $X_t^n \equiv \log C_t^n$, and $Z_{it}^n \equiv \log\left(E_t^n/\mathcal{P}_t^n\right)$ for all $i\in\mathcal{I}_{-m}=\mathcal{I}\backslash\{m\}$. We can then rewrite Equation (6) as

$$Y_{it}^n = \left(1-\sigma\right)P_{it}^n + \left(\epsilon_i-\epsilon_m\right)X_t^n + \zeta_i^n + \nu_{it}^n,\ i\in\mathcal{I}_{-m}. \tag{C.1}$$

Henceforth, we assume i is always within set $\mathcal{I}_{-m}$ and drop the reference to the set.

Throughout, we maintain the following assumptions.[72]

Assumption 1. *Relative prices and income are orthogonal to the errors, that is,* $\mathbb{E}\left[P_{jt}^n\nu_{it}^n\right] = \mathbb{E}\left[X_t^n\nu_{it}^n\right] = 0$ *for all* i,j*. Moreover, relative prices are not perfectly correlated with either the real income index* X_t^n *or the proxy* Z_t^n*, that is,* $\left|\mathbb{E}\left[X_t^n P_{it}^n\right]\right| < \left(\mathbb{E}\left[X_t^n\right]\mathbb{E}\left[P_{it}^n\right]\right)^{1/2}$ *and* $\left|\mathbb{E}\left[Z_t^n P_{it}^n\right]\right| < \left(\mathbb{E}\left[Z_t^n\right]\mathbb{E}\left[P_{it}^n\right]\right)^{1/2}$.

[72] In the case of household-level data, instead of assuming the orthogonality of the covariates and the error, we use instruments for both relative prices and income, which would slightly complicate the derivations that follows. However, the main insights will remain intact whether we assume the orthogonality of the covariates or the existence of instruments for them.

The different approaches discussed in Section 5 and Online Appendix I involve replacing the unobserved index of real consumption X_t^n by a *proxy* variable, for example, the consumption expenditure or consumption expenditure deflated by a standard price index Z_t^n. For any population-level distribution of relative prices and income, the indirect utility function $X_t^n(Z_t^n, P_{1t}^n, \cdots, P_{It}^n)$ can be log-linearized around the population mean to yield

$$X_t^n = \sum_i \eta_i P_{it}^n + \gamma Z_t^n + \iota^n + u_t^n, \tag{C.2}$$

such that $\mathbb{E}[u_t^n] = \mathbb{E}[P_{it}^n u_t^n] = \mathbb{E}[Z_t^n u_t^n] = 0$ for all $i \in \mathcal{I}_{-m}$ (this corresponds to running an OLS regression if we were to observe X_t^n). It follows that we can write

$$\begin{aligned} Y_{it}^n = & \left(1 - \sigma + \eta_i \left(\epsilon_i - \epsilon_m\right)\right) P_{it}^n + \sum_{j \neq i} \eta_j \left(\epsilon_i - \epsilon_m\right) P_{jt}^n \\ & + \left(\epsilon_i - \epsilon_m\right) \gamma Z_t^n + \zeta_i^n + \left(\epsilon_i - \epsilon_m\right) \left(\iota^n + u_t^n\right) + \nu_{it}^n. \end{aligned} \tag{C.3}$$

The lemma below establishes that we can identify the model's nonhomotheticity parameters up to a constant factor using a system OLS estimate or a feasible GLS estimate of log relative shares on log relative prices and log real consumption expenditure, of the form[73]

$$Y_{it}^n = \sum_j \alpha_{ij} P_{jt}^n + \beta_i Z_t^n + \tilde{\zeta}_i^n + \tilde{\nu}_{it}^n. \tag{C.4}$$

Lemma 6. *Assume that the model in Equation (C.1) is well-specified, Assumption 1 holds, and that $\gamma \neq 0$, i.e., the real consumption index of nonhomothetic CES, X_t^n, and our proxy variable, Z_t^n, e.g., the real income calculated based on standard price indices, are correlated after controlling for relative prices. Let $\widehat{\beta}_i$ denote the coefficients on the real consumption expenditure based on estimating the system of Equations C.4. Then, the coefficients on the proxy variable Z_t^n satisfy plim $\widehat{\beta}_i / \widehat{\beta}_j = \left(\epsilon_i - \epsilon_m\right) / \left(\epsilon_j - \epsilon_m\right)$.*

D Comparison with Stone-Geary and PIGL prefrences

We compare the cross-country fit of our model to alternative specifications where we replace the nonhomothetic CES aggregator with Stone-Geary and PIGL preferences. A brief discussion of these preferences and estimation is given here. We relegate a detailed discussion to Online Appendix G.

We start considering a generalized Stone-Geary formulation (Herrendorf et al., 2014). These preferences define the intra-period consumption aggregator as

$$C_t^c = \left[\Omega_a^c \left(C_{at}^c + \bar{c}_a\right)^{\frac{\sigma-1}{\sigma}} + \Omega_m^c \left(C_{mt}^c\right)^{\frac{\sigma-1}{\sigma}} + \Omega_c^c \left(C_{st}^c + \bar{c}_s\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{D.1}$$

where C_t^c denotes aggregate consumption of country c at time t, $\Omega_i^c > 0$ are constant preference parameters that are country specific, $C_{i,t}^c$ denotes consumption in sector $i = \{a, m, s\}$, $\bar{c}_a$ and $\bar{c}_s$ are

[73] This result is fairly general. It can be applied to all cases where one unobserved covariate appears on the right-hand-side of more than one equation within a system of equations, and a proxy variable exists that is correlated with the unobserved covariate and is orthogonal to the error.

constants that govern the nonhomotheticity of these preferences, and σ is a parameter that tends to the price elasticity of substitution as $C_{it}^c \gg \max\{\bar{c}_a, \bar{c}_s\}$.[74] We use the first-order conditions of the intra-period problem to estimate the model. As with nonhomothetic CES preferences, we estimate three parameters that are common across countries $\{\sigma, \bar{c}_a, \bar{c}_s\}$ that govern the price and income elasticities and country-specific taste parameters $\{\Omega_i^c\}_{i \in \mathcal{I}, c \in C}$. Our estimation results (reported in Table G.1 of the online appendix) imply that the three sectors are gross complements, that nonhomotheticities are significantly different from zero, and of the expected sign, $\bar{c}_a < 0$ and $\bar{c}_s > 0$.

Next, we study the cross-country fit of PIGL preferences as specified in Boppart (2014b). This preference structure features a homothetic CES aggregator between agriculture and manufacturing with price elasticity σ and a nonhomothetic aggregator between services and the agriculture-manufacturing composite. The within-period indirect utility V of a household with total expenditure E^c in country c is

$$V = \frac{1}{\varepsilon}\left(\frac{E^c}{p_{st}^c}\right)^{\varepsilon} - \frac{\Omega_s^c}{\gamma} \cdot \frac{\left(\Omega_a^c \cdot (p_{at}^c)^{1-\sigma} + \Omega_m^c \cdot (p_{mt}^c)^{1-\sigma}\right)^{\frac{\gamma}{1-\sigma}}}{(p_{st}^c)^{\gamma}} - \frac{1}{\varepsilon} + \frac{\Omega_s^c}{\gamma}, \tag{D.2}$$

with $0 \leq \varepsilon \leq \gamma < 1$ and $\Omega_i^c > 0$ for $i \in \{a, m, s\}$. The nonhomotheticity and price elasticity between services and the agriculture-manufacturing CES composite are governed by two parameters, ε and γ. The nonhomotheticity is not vanishing as income grows, and the price elasticity grows with income but is bounded above by 1.[75]

We use the demand implied by these preferences to estimate the demand parameters. As with nonhomothetic CES and Stone-Geary, we estimate three elasticities that are common across countries $\{\varepsilon, \gamma, \sigma\}$ and we allow for country-specific constant taste parameters, $\{\Omega_i^c\}_{i \in \mathcal{I}, c \in C}$. We find that, at our estimated parameter values, manufacturing and agriculture are gross complements and nonhomotheticities are significantly different from zero. In fact, the nonhomotheticity parameter that we estimate is similar in magnitude to the U.S. estimate reported in Boppart (2014a) (see table G.3).

E Additional Tables and Figures

[74] Since these preferences are not implicitly additive, the price and income elasticities are not independent. In Appendix G.1 we show that the elasticity of substitution between i and j is $\sigma_{ij} = \sigma \eta_i \eta_j$, where η's denote income elasticities.

[75] The parameter ε governs the nonhomotheticity of preferences between services and the composite of agricultural and manufacturing goods. If $\varepsilon > 0$, the expenditure elasticity is larger than one for services and less than one for agricultural and manufacturing goods (and identical for both). The price elasticity of substitution between services and the agriculture-manufacturing composite never exceeds one, it is increasing with the level of income and it asymptotes to $1-\gamma$. The baseline model in Boppart contains only two sectors. Here we follow the extension proposed in Appendix B.3.3 (Boppart, 2014b) to account for three sectors such that there can be a hump-shape in manufacturing. We have generalized the demand to allow for constant taste parameters that are heterogeneous across countries and not symmetric between agricultural and manufacturing goods. We have also experimented with another proposed extension in which the expenditure share in the manufacturing sector constant (Appendix B.3.2), obtaining a worse fit.

Table 9: Estimation of Equation (38) under Different Base Sectors

	Manuf. as base, $\varepsilon_m = 1$		Serv. as base, $\varepsilon_s = 1$		Agri. as base, $\varepsilon_a = 1$		Linear Comb. Three sectors	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
σ	0.57	0.50	0.33	0.46	0.33	0.30	0.40	0.41
	(0.03)	(0.03)	(0.07)	(0.06)	(0.03)	(0.04)	(0.04)	(0.03)
ε_a	0.02	0.11	0.11	0.01	1	1	0.32	0.16
	(0.06)	(0.05)	(0.06)	(0.06)	-	-	(0.09)	(0.08)
ε_m	1	1	0.81	0.76	3.01	2.46	1	1
	-	-	(0.02)	(0.03)	(0.53)	(0.28)	-	-
ε_s	1.17	1.21	1	1	3.78	3.21	1.31	1.33
	(0.04)	(0.03)	-	-	(0.71)	(0.45)	(0.04)	(0.04)
Country × Sector FE	Y	Y	Y	Y	Y	Y	Y	Y
Trade Controls	N	Y	N	Y	N	Y	N	Y
	Implied Differences in Relative ε's							
$\varepsilon_a/\varepsilon_m - 1$	-0.98	-0.89	-0.86	-0.99	-0.67	-0.60	-0.68	-0.84
	(0.06)	(0.05)	(0.07)	(0.08)	(0.06)	(0.05)	(0.04)	(0.04)
$\varepsilon_s/\varepsilon_m - 1$	0.17	0.21	0.24	0.32	0.26	0.31	0.31	0.33
	(0.04)	(0.03)	(0.04)	(0.05)	(0.05)	(0.14)	(0.06)	(0.07)
	R^2 Measures of Fit							
Eq. 1	0.95	0.94	0.94	0.94	0.66	0.77	0.93	0.92
Eq. 2	0.37	0.33	0.45	0.49	0.50	0.61	0.37	0.41

Notes: Robust standard errors are shown in parenthesis. $\varepsilon_a/\varepsilon_m - 1$ and $\varepsilon_s/\varepsilon_m - 1$ in columns (3) to (6) are computed such that the results are comparable to our baseline exercise where we use manufacturing as base (i.e., $\epsilon_a - 1$ and $\epsilon_m - 1$). Standard errors for these variables are computed using the delta method. Columns (7) and (8) report the estimation where $\log C$ is a linear combination with equal weights across sectors as described in Equation (A.15). Since the predicted values of our model do not come from ordinary least squares regression, the R^2 measures are computed as the squared of the Pearson coefficient of correlation between true and predicted log-employment shares. This is equivalent to computing the R^2 of the regression of the true data on the predicted values plus a constant. For columns (1), (2), (7) and (8) Eqs. 1 and 2 corresponds to $\log\left(L^n_{at}/L^n_{mt}\right)$ and $\log\left(L^n_{st}/L^n_{mt}\right)$. For columns (3) and (4), Eqs. 1 and 2 correspond to $\log\left(L^n_{at}/L^n_{st}\right)$ and $\log\left(L^n_{mt}/L^n_{st}\right)$. For Columns (5) and (6), Eqs. 1 and 2 correspond to $\log\left(L^n_{mt}/L^n_{at}\right)$ and $\log\left(L^n_{st}/L^n_{at}\right)$. All regressions have 1626 observations.

Table 10: Value-Added Household Estimation, CEX

	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.34	0.34	0.31	0.52	0.30	0.33
	(0.02)	(0.02)	(0.02)	(0.04)	(0.03)	(0.03)
$\epsilon_a - 1$	-0.94	-0.98	-0.97	-0.99	-1.00	-0.99
	(0.08)	(0.09)	(0.08)	(0.47)	(0.11)	(0.10)
$\epsilon_s - 1$	0.64	0.62	0.66	0.90	0.88	0.94
	(0.08)	(0.09)	(0.08)	(0.58)	(0.11)	(0.11)
Household Controls	Y	Y	Y	Y	Y	Y
Region FE	N	N	Y	Y	Y	Y
Year $\times$ Quarter	N	N	N	N	Y	Y

Notes: Standard errors clustered at the household level. The number of observations is 60925 in all regressions.

Table 11: Housing as a Separate Group, CEX Expenditure, $\epsilon_m = 1$

	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.17	0.19	0.20	0.17	0.19	0.23
	(0.04)	(0.04)	(0.03)	(0.04)	(0.03)	(0.03)
$\epsilon_a - 1$	-0.71	-0.74	-0.73	-0.72	-0.81	-0.85
	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)	(0.06)
$\epsilon_{\text{services (excl. housing)}} - 1$	0.75	0.77	0.77	0.67	0.65	0.65
	(0.07)	(0.07)	(0.07)	(0.06)	(0.06)	(0.07)
$\epsilon_{\text{housing}} - 1$	0.94	1.00	0.96	0.89	0.78	0.82
	(0.09)	(0.10)	(0.09)	(0.09)	(0.07)	(0.08)
Expenditure Re-Weighted	N	Y	N	Y	N	Y
Region FE	N	N	Y	Y	Y	Y
Year $\times$ Quarter FE	N	N	N	N	Y	Y

Notes: Standard errors clustered at the household level shown in parentheses. The number of observations is 60925 in all regressions.

Table 12: 10-Sector Regression, $\epsilon_m = 1$

	World	OECD	Non-OECD
Price Elasticity σ	0.10	0.13	0.07
	(0.03)	(0.03)	(0.04)
Sector i Nonhomotheticity Parameter ϵ_i			
Agriculture	0.32	0.00	0.38
	(0.05)	(0.04)	(0.06)
Mining	0.41	0.01	0.67
	(0.06)	(0.04)	(0.05)
Public Utilities	1.59	1.32	1.61
	(0.05)	(0.03)	(0.05)
Transp., Storage, Communications	1.44	1.36	1.41
	(0.03)	(0.04)	(0.03)
Construction	1.03	0.72	1.09
	(0.02)	(0.02)	(0.02)
Community, Social and Personal Serv.	1.18	0.85	1.21
	(0.03)	(0.05)	(0.03)
Wholesale and Retail	1.62	1.59	1.58
	(0.04)	(0.05)	(0.04)
Finance, Insurance, Real State	2.17	2.36	2.04
	(0.07)	(0.11)	(0.07)
Observations	1596	492	1104

Notes: Standard errors clustered at the country level. All regressions include a sector-country fixed effect. For the OECD regression, we constrain the agriculture and mining parameters to be non-negative (by estimating the exponent of ϵ_i, standard errors are adjusted using the delta method).