Falling Interest Rates and Credit Misallocation: Lessons from General Equilibrium

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Abstract

What is the effect of declining interest rates on the efficiency of resource allocation and overall economic activity? We study this question in a setting in which entrepreneurs with different productivities invest in capital, subject to financial frictions. We show that a fall in the interest rate has an ambiguous effect on aggregate output. In partial equilibrium, a lower interest rate raises aggregate investment both by relaxing financial constraints and by prompting relatively less productive entrepreneurs to invest. In general equilibrium, this higher demand for capital raises its price and crowds out investment by the more productive entrepreneurs. When this crowding-out effect is strong enough, a fall in the interest rate becomes contractionary. Moreover, in a dynamic setup, such reallocation effects among entrepreneurs can interact with the classic balance-sheet channel, giving

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rise to a *boom-bust* impulse response of output to a fall in the interest rate. We provide evidence in support of our mechanism using data from the US and Spain.

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1 Introduction

One distinctive feature of recent decades has been the sustained and significant decline in real interest rates across the globe. Although the conventional wisdom holds that declining interest rates should stimulate economic activity –at least over the short-term,– there are mounting concerns that declining rates may have undesirable side effects, such as endangering financial stability (Rajan, 2015; Martinez-Miera and Repullo, 2017; Coimbra and Rey, 2017; Brunnermeier and Koby, 2018; Bolton et al., 2021) or slowing-down the pace of technological innovation and long-term growth (Liu et al., 2019; Quadrini, 2020; Benigno et al., 2020).

Perhaps the most common worry among economists and policy makers is that declining rates may foster the proliferation of unproductive economic activities. This view is consistent with recent evidence, which suggests that periods of fast credit growth fueled by low interest rates have been characterized by low aggregate productivity growth (Reis, 2013; Gopinath et al., 2017; García-Santana et al., 2020). Despite its intuitive appeal, this view leaves many open questions. Can declining interest rates foster socially unproductive activities? If so, under which conditions? Can this effect be strong enough to hamper economic activity and growth?

In this paper, we develop a tractable framework to address these questions. We consider an economy populated by entrepreneurs who have the ability to invest in capital. We make two crucial assumptions. First, entrepreneurs are heterogeneous in their productivity, i.e., they differ in their effectiveness at using capital to produce consumption goods. Second, entrepreneurs face financial frictions, i.e., they cannot pledge the entire surplus from their activities to outsiders. To keep matters simple, we focus throughout on a small-open economy that takes the world interest rate as given. We then ask a simple question: how does a fall in the interest rate affect the allocation of capital across entrepreneurs, aggregate investment and output?[1]

The conventional view holds that lower interest rates stimulate economic activity by raising both entrepreneurs' willingness and ability to invest. Our framework challenges this view by highlighting that, in a world of heterogeneous productivity and financial frictions, lower interest rates may foster investment by the wrong mix of agents. In particular, declining interest rates make it attractive for relatively less productive entrepreneurs to invest. This raises the equilibrium price of capital and crowds-out investment by more productive (albeit financially constrained) entrepreneurs; that is, capital is reallocated from more to less productive entrepreneurs. We formalize this general-equilibrium *reallocation effect* of declining interest rates and show that it mitigates their stimulative effect on output. In fact, we show that this effect

[1]The main lessons from our analysis extend to a closed-economy setup, where interest rates are endogenous and interest rate changes are driven by economic fundamentals (see Section 3.1).

can be strong enough to make aggregate output decline in response to a fall in the interest rate!

Our mechanism relies on the two central assumptions highlighted above: heterogeneous productivity and financial frictions. When individual entrepreneurs invest, they do not internalize the effect of their investment on the price of capital. This gives rise to a pecuniary externality. In the absence of heterogeneity or financial frictions, the return to investment is equalized across entrepreneurs and this externality has no first-order effect on output or welfare. With heterogeneity and financial frictions, however, this is no longer the case. Simply put, less productive entrepreneurs invest too much because they do not internalize the crowding-out effect of their investment on that of more productive entrepreneurs.

We characterize the determinants of the reallocation effect, and show that its strength depends crucially on the elasticity of the capital supply and on the severity of the financial friction. In particular, when the supply of capital is sufficiently inelastic and the financial friction is severe enough, the reallocation effect is so strong that a fall in the interest rate may become contractionary. Crucially, this never happens if investment is dictated by a benevolent social planner, who is subject to the same financial constraints that limit entrepreneurial investment but who has the power to prevent certain entrepreneurs from investing altogether. By controlling the strength of the general-equilibrium reallocation effect, the social planner ensures that a fall in the interest rate is always expansionary.

Our findings extend to a dynamic setting in which entrepreneurial net worth is endogenous. This is important because the macro-finance literature has traditionally emphasized the balance sheet channel (e.g. Kiyotaki and Moore (1997)), by which rising asset prices are expansionary through their effect on entrepreneurial wealth. In particular, an increase in asset prices boosts the net worth of entrepreneurs with larger holdings of capital thereby relaxing their financial constraints. Since equilibrium capital holdings are positively correlated with productivity in dynamic settings like ours, one would intuitively expect the balance-sheet channel to amplify the stimulative effect of declining rates. We analyze the economy's response to a permanent decline in the interest rate and show that this intuition is only partially correct. The reason is that balance-sheet effects are driven by unexpected changes in the price of capital and are thus inherently transitory, whereas the general-equilibrium forces that drive reallocation are permanent. As a result, the response of output to a fall in the interest rate may feature a transitory boom, followed by a permanent bust: the balance-sheet channel temporarily boosts investment by more productive entrepreneurs, but its effect gradually wears-off as the contractionary reallocation effect begins to dominate.

Our theory is consistent with recent evidence on the macroeconomic effects of credit booms driven by low interest rates. In particular, it sheds light on the experience of several Southern European economies (e.g. Spain and Italy) during the early 2000s, when a reduction in interest rates coincided with local booms in credit and asset prices and, at the same time, with a decline in aggregate TFP and an increase in productivity dispersion across firms. Interpreted through the lens of our framework, this happened partly because – although they boosted investment and consumption – lower interest rates had an adverse effect on resource allocation by favoring the investment of unproductive firms. One implication of our theory is that this unproductive investment can be socially undesirable despite having a positive NPV from a private standpoint.

To test the theory more directly, we analyze the effects of interest rate changes across geographical regions, sectors, and firms in the United States and Spain. The key prediction is that the stimulative effect of declining interest rates should be weaker when the reallocation effect induced by the increase in the price of capital is stronger. Testing this in the data requires taking a stance on a specific type of capital that may be relevant in practice. Although the theory applies to any input of production that requires investment (i.e., that need to be partly paid off in advance), we focus throughout on real estate. The main advantage of doing so is that there are readily available measures of real-estate supply elasticities, which play a key role in our theory and thus in our empirical strategy.[2]

In particular, we measure the response of sectorial output at the local level (MSA for the case of the US, municipality for the case of Spain) to changes in the interest rate, and analyze how this response correlates with the local elasticity of real-estate supply and with the real-estate intensity of the sector. The data confirms the theory's core prediction, in the sense that the stimulative effect of a decline in interest rate is weaker for sectors that are real-estate intensive in regions where the supply of real estate is relatively inelastic. Of course, this could be driven by other mechanisms beyond the reallocation effect, e.g., real-estate intensive sectors may simply be unable to expand in regions where the supply of real estate is inelastic. Thus, we also use data from the Spanish firm census to analyze the differential effect of interest rates across firms of different productivities. The main result is in line with the reallocation channel: namely, within the sectors and regions identified above, it is low-productivity firms that expand relatively more in response to a decline in the interest rate.

In spirit, our paper is closely related to recent work that studies the dismal productivity performance that often characterizes credit booms in practice (Reis, 2013; Gopinath et al., 2017; Doerr, 2018; García-Santana et al., 2020; Gorton and Ordonez, 2020). Within this work, the papers closest to us are Reis (2013) and Gopinath et al. (2017). Like us, both papers show

[2]Moreover, real-estate price dynamics have been widely documented to affect macroeconomic outcomes (see, for instance, Iacoviello (2005), Liu et al. (2013) and Berger et al. (2018)).

that lower interest rates can lead to a decline in TFP in the presence of financial frictions, as some firms – not necessarily the most productive ones – take advantage of cheaper credit. Differently from them, however, our focus is on how general-equilibrium effects crowd-out the activity of the most productive firms and lead, not just to greater misallocation, but even to a fall in aggregate output.[3] We also show how this negative effect of lower interest rates on TFP interacts dynamically with traditional balance-sheet effects (e.g. Kiyotaki and Moore (1997)), according to which lower interest rates – by boosting asset prices – transfer resources towards the economy's most productive agents. A related line of work emphasizes "zombie" lending, broadly defined as relatively unproductive firms that remain active in low-interest rate environments (Caballero et al., 2008; Adalet McGowan et al., 2018; Tracey, 2019; Schivardi et al., 2020). In contrast, in our model, all investment has a positive net present value from a private standpoint, but some investments are nonetheless excessive from a social standpoint.

Our paper is also related to the growing literature on macroeconomics with heterogeneous agents. Much of this literature has studied how heterogeneity shapes an economy's response to monetary policy. Although this research focuses mostly on heterogeneity among households (e.g. Cloyne et al. (2020), Kaplan et al. (2018), Slacalek et al. (2020)), a growing body of work also analyzes heterogeneity among firms (e.g. Anderson and Cesa-Bianchi (2020), Cloyne et al. (2018), Manea (2020), Ottonello and Winberry (2020), Jeenas (2020)).[4] Relative to this work, whose main objective is to understand the transmission of monetary policy, we focus on the effects of interest rate changes on the allocation of resources between high- and low-productivity firms, and how these effects are shaped by general-equilibrium considerations.

Finally, from a more conceptual standpoint, our model is closely related to previous work that stresses inefficiencies in the allocation of factors of production due to financial frictions. One recurring theme in this work is that individual firms do not internalize the effect of their demand on factor prices, which may lead to inefficient outcomes in the presence of financial constraints. Biais and Mariotti (2009), Ventura and Voth (2015), Martin et al. (2018) and Asriyan et al. (2019) provide recent examples of this work. In a related vein, Coimbra and Rey (2017) study the allocation of risky capital across financial intermediaries that are subject

[3] Reis (2013) and Benigno and Fornaro (2014) show that financial integration with the rest of the world can induce a general-equilibrium reallocation of resources from tradable to non-tradable sector, a feature not present in our setting. Recently, Kiyotaki et al. (2021) develop a model in which – due to financial frictions – entrepreneurs borrow predominantly against their near-term incomes, which means that a fall in the interest rate may actually tighten financial constraints.

[4] Relatedly, Leahy and Thapar (2019) study how age-structure shapes the impact of monetary policy: they find that monetary policy is most potent in regions with a larger share of middle-aged due to their higher propensity for entrepreneurship. On the other hand, Caggese and Pérez-Orive (2020) show how lower interest rates may become less expansionary in economies where intangible investments become more important.

to financial constraints and are heterogenous in their tolerance for risk. They show how, by reallocating capital towards riskier intermediaries, a decline in the interest rate may increase financial instability.

The paper is organized as follows. Section 2 presents the model and the main equilibrium conditions. Section 3 characterizes the equilibrium effects of declining interest rates, as well as the normative properties of equilibria. Section 4 presents a dynamic extension of the baseline model. Section 5 provides supporting evidence, and Section 6 concludes.

2 Baseline model

Consider an economy that lasts for two periods, $t = 0, 1$. There are two goods: a perishable consumption good (c) and capital (k). There are two sets of risk-neutral agents, entrepreneurs and capitalists, each of unit mass.

Preferences. The preferences of all agents are given by:

$$U = E_0[c_1],$$

where c_1 is the consumption at $t = 1$ and $E_0[\cdot]$ is the expectations operator at $t = 0$.

Endowments. Each entrepreneur is endowed with w units of the consumption good at $t = 0$, while capitalists have no endowment.

Technology. Each capitalist has access to a production technology that converts $\chi(k)$ units of the consumption good into k units of capital at $t = 0$, where χ is quasi-convex and weakly increasing in k.[5] Capital can be used for production by entrepreneurs. Specifically, each entrepreneur has access to a production technology that converts one unit of capital at $t = 0$ into A units of the consumption good at $t = 1$. We refer to A as entrepreneurial productivity and assume that it is distributed independently across entrepreneurs, with distribution G that has an associated density g with full support on the interval $[0, 1]$.

Markets. The economy is small and open and there is an international financial market where agents can borrow and lend consumption goods at a world interest rate R. Here, we introduce a central friction of our analysis by supposing that an entrepreneur can always walk away with a fraction $1 - \lambda$ of her output at $t = 1$. This pledgeability friction will endogenously limit the borrowing and investment that each entrepreneur can undertake. There is also a competitive

[5] The case where capitalists are simply endowed with $\bar{K}$ units of capital is captured by the following cost function: $\chi(k) = 0$ for $k \leq \bar{K}$ and $\chi(k)$ is infinite thereafter.

capital market, where agents can trade capital at a unit price q in period $t = 0$. Note that, as the economy ends at $t = 1$, capital is no longer valuable after production at that date.

2.1 Optimization and equilibrium

Since agents can borrow and lend consumption goods in the international financial market at the interest rate R, only the clearing of the capital market is crucial for equilibrium. To characterize this market clearing condition, we analyze next the demand and supply of capital.

Capital demand. Let b_A and k_A respectively denote total borrowing and capital demand by an entrepreneur with productivity A. Given prices $\{q, R\}$, the entrepreneur makes her optimal borrowing and investment decisions to maximize her expected consumption:

$$\max_{\{b_A, k_A\}} \quad A \cdot k_A - R \cdot b_A, \tag{1}$$

subject to budget, borrowing and feasibility constraints:

$$q \cdot k_A \leq \; w + b_A, \tag{2}$$

$$R \cdot b_A \leq \; \lambda \cdot A \cdot k_A, \tag{3}$$

$$0 \leq k_A \; . \tag{4}$$

Optimization leads to the following capital demand:

$$k_A(q, R) \begin{cases} = 0 & \text{if } \frac{A}{q} < R \\ \in \left[0, \frac{1}{q - \frac{\lambda \cdot A}{R}} \cdot w\right] & \text{if } R = \frac{A}{q} \\ = \frac{1}{q - \frac{\lambda \cdot A}{R}} \cdot w & \text{if } \frac{\lambda \cdot A}{q} < R < \frac{A}{q} \\ = \infty & \text{if } R \leq \frac{\lambda \cdot A}{q} \end{cases}, \tag{5}$$

which has an associated level of borrowing:

$$b_A(q, R) = q \cdot k_A(q, R) - w. \tag{6}$$

Equation (5) states that and entrepreneur's demand of capital is decreasing in the interest rate, R. When the interest rate is smaller than an her return to capital, A/q, an entrepreneur demands finds it optimal to invest in financial markets and demands no capital. When both returns are equalized, the entrepreneur is indifferent between investing in capital and not doing so. When the interest rate is smaller than the return to capital but greater than its pledgeable return, the entrepreneur demands capital until her borrowing constraint binds.[6] Finally, when the interest rate is below the pledgeable return to capital, the entrepreneur's demand of capital is unbounded.

Equation (5) also implies that the demand function $k_A(q, R)$ is decreasing in q. For an entrepreneur who is unconstrained, i.e., $A \leq q \cdot R$, lower values of q raise the return to investing in capital. For an entrepreneur who is constrained, lower values of q relax the borrowing constraint and enable her to expand her borrowing and her purchases of capital. Finally, the demand function $k_A(q, R)$ is weakly increasing in λ, because a higher pledgeability of output enables constrained entrepreneurs to expand their borrowing and thus their purchases of capital.

We denote the aggregate demand for capital by entrepreneurs by:

$$K^D(q, R) \equiv \int_0^1 k_A(q, R) \cdot dG(A). \tag{7}$$

Capital supply. Given the price of capital, each capitalist chooses his supply of capital to maximize profits. Formally, we use $K^S(q)$ to denote a solution to:

$$\max_{k \geq 0} \; q \cdot k - \chi(k). \tag{8}$$

Since all capitalists are identical, $K^S(q)$ denotes both the individual and the aggregate supply of capital, which is weakly increasing in q.[7]

Market clearing. The price of capital, q, ensures that the capital market clears:

$$K^S(q) = K^D(q, R). \tag{9}$$

Aggregate domestic output is given by:

$$Y(q, R) = \int_0^1 A \cdot k_A(q, R) \cdot dG. \tag{10}$$

Throughout, our main objective is to characterize the effect of changes in the interest rate on aggregate output and, in particular, on the aggregate stock of capital and its allocation among entrepreneurs. As we shall see, this latter allocative effect will drive the changes in the

[6] Note that $q - \lambda \cdot A \cdot R^{-1}$ represents the "down payment" that is required to purchase a unit of capital: the price of each unit is q, but a part $\lambda \cdot A \cdot R^{-1}$ can be financed by borrowing against the unit's future output.

[7] E.g. if $\chi(\cdot)$ is increasing and convex with $\chi(0) = \chi'(0) = 0$, then $K^S(q) = \chi'^{-1}(q)$ and is increasing in q.

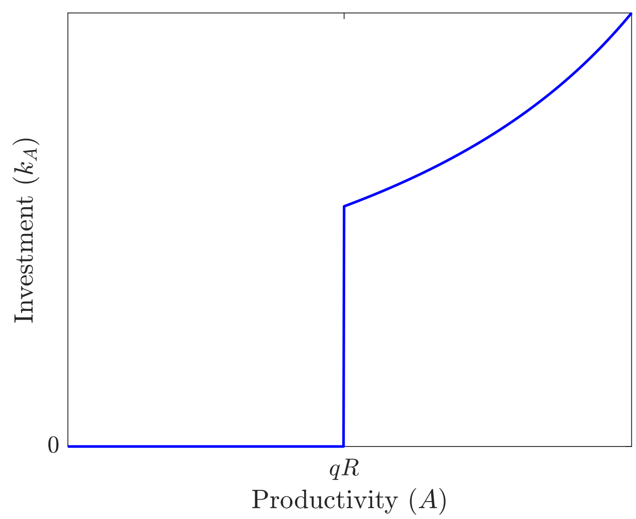


Figure 1: **Distribution of capital across entrepreneurs given prices $\{q, R\}$.**

aggregate productivity of capital, defined as the ratio $Y(q,R)/K^S(q)$.

Equilibrium. An equilibrium consists of prices $\{q, R\}$ and allocations $\{\{k_A, b_A\}_A, K^S, Y\}$, such that, given prices, $\{k_A, b_A\}_A$ satisfy Equations (5) and (6), K^S satisfies Equation (8), the capital market clears according to Equation (9), and Y satisfies Equation (10).

3 Equilibrium effects of changes in interest rates

We want to understand the equilibrium effects of changes in the interest rate R. For now, we interpret changes in R as being induced by exogenous factors, such as changes in the world interest rate or in capital inflows. We later show that out results go through when R is an endogenous variable and its decline is driven by changes in model fundamentals.

Figure 1 depicts the distribution of capital across entrepreneurs for given prices $\{q, R\}$. To determine the aggregate effects of a change in R, we need to understand how this distribution of capital responds to a such change. In what follows, we refer to those entrepreneurs who find it optimal not to invest (i.e., $A < q \cdot R$) as "infra-marginal", to those that invest until their borrowing constraint binds (i.e., $A > q \cdot R$) as "supra-marginal", and to those that are indifferent (i.e., $A = q \cdot R$) as "marginal" entrepreneurs.

At first sight, the effect of a change in R on investment and output seems trivial. It follows immediately from Equation (5) that, for a given price q, all entrepreneurs raise their demand of capital when R falls. Supra-marginal entrepreneurs demand more capital because a lower value

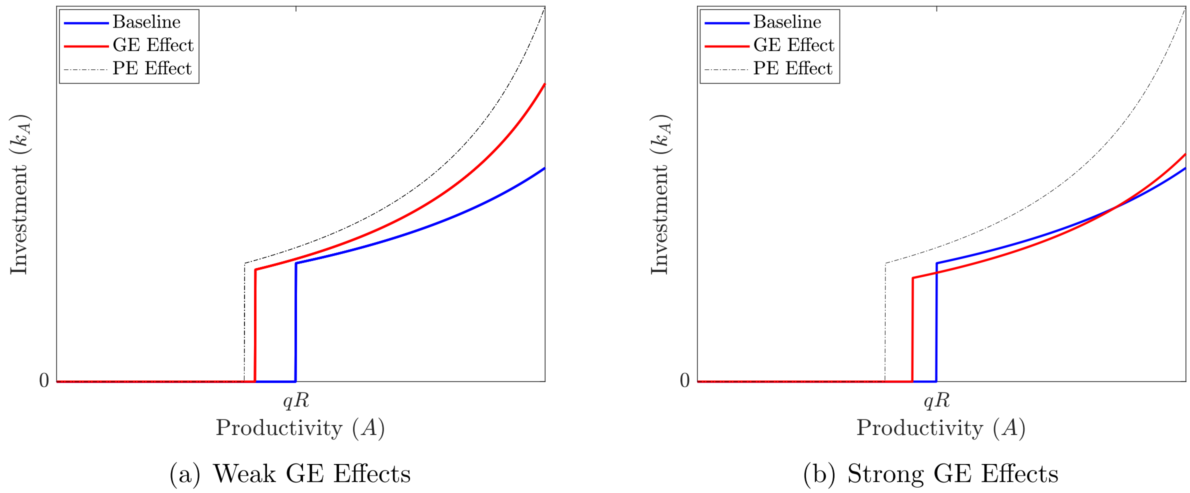


Figure 2: **Partial- and general-equilibrium effects of a fall in the interest rate on k_A.**

of R raises the present value of pledgeable output thus relaxing their borrowing constraints. Moreover, some infra-marginal entrepreneurs start investing because a lower value of R raises the present value of investment. This partial-equilibrium effect of a decline in R is depicted through a shift from the solid-blue to the dashed curve in panels (a) and (b) of Figure 2.

As long as the supply of capital is not perfectly elastic, however, the effects of a decline in R do not end here. There is also a general-equilibrium effect because the price of capital q must increase to insure capital-market clearing. This general-equilibrium effect makes capital less attractive and reduces investment along all margins, but it cannot be so strong as to raise the productivity of the marginal entrepreneur, $q \cdot R$: otherwise, all entrepreneurs would reduce their demand of capital, which is a contradiction. This implies that a decline in the interest rate must necessarily raise the investment of some infra-marginal entrepreneurs, although it may reduce the investment of some supra-marginal entrepreneurs.

Formally, the change in the investment of a supra-marginal entrepreneur with productivity A is given by:

$$\frac{dk_A}{dR} = \frac{\left|\frac{dq}{dR}\right| - \frac{\lambda \cdot A}{R^2}}{q - \frac{\lambda \cdot A}{R}} \cdot k_A. \tag{11}$$

Equation (11) shows that the change in investment induced by a change in the interest rate has a partial- and a general-equilibrium component. The second term in the numerator represents the partial-equilibrium effect, by which a decline in R increases the pledgeable return to capital and thus entrepreneurs' ability to invest. The first term in the numerator captures instead the general-equilibrium effect, by which a decline in R raises the price of capital thereby reducing entrepreneurial demand of capital.

Equation (11) suggests that the investment of supra-marginal entrepreneurs may decline when the interest rate falls, provided that the general-equilibrium effect is strong and the partial-equilibrium effect is weak enough. The strength of the general-equilibrium effects depend largely on the elasticity of capital supply, while the strength of the partial-equilibrium effect depends on the tightness of the financial friction. As $\lambda \to 0$, for instance, the partial-equilibrium effect disappears altogether. Panels (a) and (b) in Figure 2, respectively, depict the general-equilibrium effect of a fall in the interest rate on entrepreneurial investment, through a shift from the dashed to the solid-red curve. In panel (a), general-equilibrium effects are weak and all supra-marginal entrepreneurs invest more when the interest rate falls; in panel (b), instead, general-equilibrium effects are strong and some supra-marginal entrepreneurs invest less when the interest rate declines.

The effect of interest rates on output is shaped by the behavior of investment across entrepreneurs. Formally, we can combine Equations (9) and (10) to obtain:

$$\frac{dY}{dR} = \underbrace{\int_{q\cdot R}^{1} (A - q\cdot R)\cdot \frac{dk_A}{dR}\cdot dG(A)}_{\text{Capital-reallocation effect } \equiv \mathcal{R}} + \underbrace{q\cdot R\cdot \frac{dK^S(q)}{dR}}_{\text{Capital-supply effect } \equiv \mathcal{K}}, \tag{12}$$

where dk_A/dR is given by Equation (11). The first term in Equation (12), which we denote by $\mathcal{R}$, captures the capital-reallocation effect: the change in output driven by changes in the investment of supra-marginal entrepreneurs. As we have already noted, this effect can be positive or negative depending on the relative strength of the general- and partial- equilibrium effects. In what follows, we say that the capital-reallocation effect is *stronger* when $\mathcal{R}$ is more positive. The second term in Equation (12), which we denote by $\mathcal{K}$, captures instead the capital-supply effect: the change in output driven by adjustments in the aggregate capital stock. This effect is always (weakly) negative, since lower interest rates raise the demand for capital and thus the equilibrium stock of capital. In what follows, we say that the capital-supply effect is *stronger* when $\mathcal{K}$ is more negative.

Equation (12) illustrates the role of both, heterogeneity and financial frictions in shaping the aggregate response of output to changes in the interest rate. In the absence of heterogeneity, all entrepreneurs would have the same productivity; in the absence of financial frictions, only the most productive entrepreneur would invest. In either case, the capital-reallocation effect would disappear and the response of output to the interest rate would be negative and driven only by the capital-supply effect, i.e., on the economy's ability to adjust the capital supply to the shifting demand. With heterogeneous productivity and financial frictions, however, the response of output to changes in the interest rate depends not just on the behavior of aggregate investment but also on its reallocation across entrepreneurs. In fact, the capital reallocation effect can be so strong that falling interest rates may become contractionary!

To illustrate this, consider a simple example in which there is no borrowing and lending (i.e., $\lambda = 0$) and the capital stock is fixed (i.e., $K^S(q) = \bar{K}$). The lack of borrowing and lending means that the investment of supra-marginal entrepreneurs is equal to w/q, and thus independent of the interest rate: this maximizes the strength of the reallocation effect, $\mathcal{R}$. The fixed capital supply, in turn, eliminates the capital supply effect, $\mathcal{K}$, altogether. In this case, a decline of the interest rate must necessarily reduce aggregate output. By boosting the investment of infra-marginal entrepreneurs, a lower interest rate raises the equilibrium price of capital and thus reduces supra-marginal investment (which is more productive).

This is of course a stark example but, as the next proposition shows, the result is more general and does not rely on such extreme scenarios.

Proposition 1 *Consider two economies that have the same equilibrium allocations and are identical in all respects except the capital supply schedule. Let ε denote the elasticity of capital supply with respect to the price of capital q in equilibrium. Then, in the economy with lower ε:*

- *the capital-reallocation effect, $\mathcal{R}$, is stronger;*
- *the capital-supply effect, $\mathcal{K}$, is weaker;*
- *the response of output to a change in the interest rate, dY/dR, is less negative;*

moreover, for low enough ε, there is a threshold $\bar{\lambda}_\varepsilon > 0$ such that dY/dR is positive if $\lambda < \bar{\lambda}_\varepsilon$.

Proposition 1 states that the response of output to the interest rate is decreasing in the elasticity of the capital supply, ε, which governs the strength of the general-equilibrium effect.[8] This is illustrated in Figure 3, which plots dY/dR against ε for low and high values of λ, respectively. Both panels show that dY/dR increases as ε decreases. Lower values of ε weaken the capital-supply effect and, by reinforcing the general-equilibrium response of the price of capital, strengthen the capital-reallocation effect. When λ is low, moreover, the reallocation effect becomes positive and – for low values of ε – a fall in the interest rate leads to a decline in aggregate output (see panel (a)).

[8]The parameterization of the cost function $\chi(\cdot)$ used for Figure 3 is provided in the proof of Proposition 1.

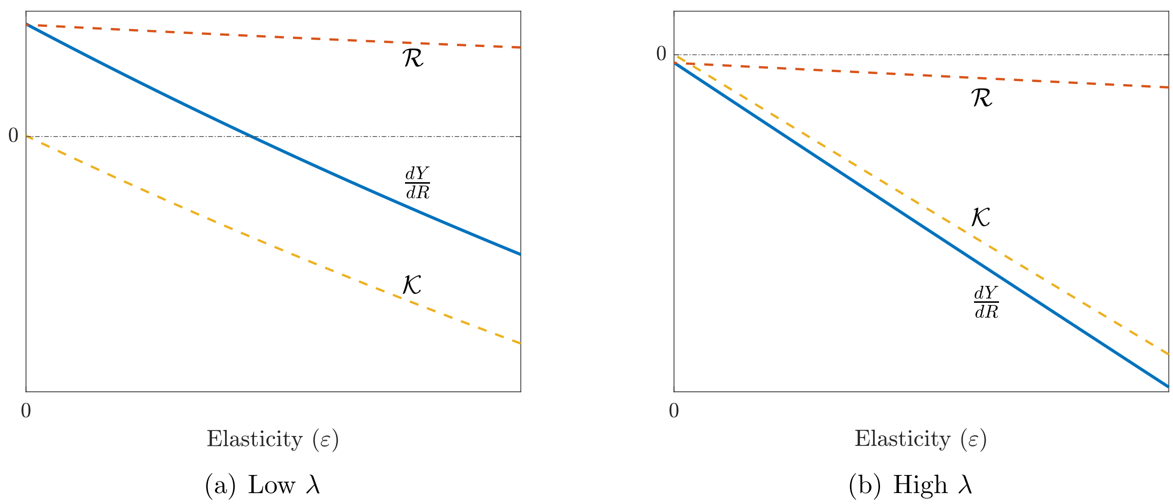


Figure 3: **Effect of the elasticity of capital supply on dY/dR.**

3.1 Robustness

Proposition 1 shows that the interaction of heterogeneous productivity and financial frictions gives rise to capital-reallocation effects in general equilibrium, which can mitigate or even overturn the expansionary effects of declining interest rates. This result has been derived in a fairly stylized environment. As we discuss next, however, it extends to more general settings.

Unconstrained firms. We have assumed throughout that all entrepreneurs are subject to financial constraints. Yet, one may wonder how our results change if some agents are unconstrained. This is in line with recent macroeconomic models that study the potentially heterogeneous reaction of constrained and unconstrained firms to monetary policy (e.g. Cloyne et al. (2018); Ottonello and Winberry (2020)). To this end, Appendix B.1 introduces unconstrained firms into our baseline setup and shows that nothing substantial changes as long as there are some constrained entrepreneurs in equilibrium. This requires that the unconstrained firms not be too productive; otherwise, they would absorb the entire capital stock and the economy would effectively be frictionless. When unconstrained firms are active, however, our results may even be strengthened: since the capital demand of these firms can adjust freely to changes in the interest rate, their presence strengthens the capital reallocation effect, thus raising dY/dR.

Diminishing returns at entrepreneur level. In our baseline model, entrepreneurs operate a linear production technology. As a result, entrepreneurial investment displays "bang-bang" behavior, i.e., an entrepreneur either finds it unattractive to invest in capital or she wants to invest as much as possible. Appendix B.2 shows that our results remain unchanged if, as is commonly assumed in the firm-dynamics literature (e.g. Hopenhayn (1992)), there are diminishing returns at the entrepreneur level. The key difference is that the set of marginal entrepreneurs now has positive measure and, if Inada conditions are satisfied, it includes all entrepreneurs below a certain threshold on productivity A. In equilibrium, these entrepreneurs are unconstrained because – given their lower marginal product of capital – they operate at a smaller scale than the constrained, more productive entrepreneurs.[9] Under similar conditions as in our baseline model, moreover, a fall in the interest rate generates reallocation of capital from more to less productive entrepreneurs, thus reducing output.

Closed economy and savings gluts. We have considered an economy that is small and open, which takes the world interest rate as given. One may wonder what would change in a closed economy, where the interest rate is determined endogenously. To this end, Appendix B.3 shows that our results remain unchanged if the interest rate is determined endogenously and its fall is prompted by an increased desire to save (i.e., a savings glut). In a closed-economy version of our baseline model, such an increase in savings is triggered either by a shift in preferences (higher patience) or in endowments (capitalists become richer). Thus, our findings are consistent with one of the most popular hypotheses to explain the sustained decline in interest rates over the past several decades (e.g. Bernanke et al. (2005); Caballero et al. (2008)).

Dynamics of wealth accumulation. Perhaps the main limitation of the baseline economy is that it is essentially static, as it lasts for only two periods and entrepreneurial wealth is exogenously specified. In a dynamic economy, however, entrepreneurial wealth would naturally be endogenous to (i) productivity, as more productive entrepreneurs may accumulate wealth faster; and (ii) asset prices, due to the well-known balance-sheet effects à la Kiyotaki and Moore (1997). Since this limitation may be perceived to be important given the pervasiveness of dynamics in macroeconomics, Section 4 extends our analysis to a dynamic economy and shows how our main result interacts with both wealth accumulation and balance sheet effects.

3.2 Normative properties

Before moving to the fully dynamic model, we turn to the normative properties of equilibrium. In particular, we analyze the extent to which the equilibrium level of output, as well as its response to the interest rate, are (in)efficient.

Consider the problem of a social planner whose objective is to maximize aggregate consump-

[9] Note that, since now there is a mass of marginal entrepreneurs who are unconstrained in equilibrium, this extension is conceptually similar to the previous one where some firms are assumed to be unconstrained.

tion at $t = 1$.[10] The planner is constrained to only choosing investment $\{k_A\}$ for entrepreneurs, but agents make all the other decisions on their own. This implies, in particular, that the planner must respect individual budget and financial constraints. To simplify the exposition, we assume throughout that $\chi(\cdot)$ is strictly convex.

Formally, the social planner solves the following maximization problem:

$$\max_{\{k_A\}} \int A \cdot k_A \cdot dG(A) - R \cdot [\chi(K^S) - w], \tag{13}$$

subject to:

$$R \cdot (q \cdot k_A - w) \leq \lambda \cdot A \cdot k_A \quad \text{and} \quad 0 \leq k_A \quad \forall A, \tag{14}$$

and to capitalists' optimization and market clearing:

$$\chi^{-1}(q) = K^S = \int k_A \cdot dG(A). \tag{15}$$

The objective function of the planner in (13) says that aggregate consumption at $t = 1$ is equal to aggregate output net of repayments to international lenders, which are given by R times the difference between the cost of capital production and aggregate endowment at $t = 0$. Equations (14) and (15) state that the planner must respect individual budget and financial constraints, feasibility, and market clearing. In particular, the planner is not able to make transfers so as to overcome financial frictions.

To understand the solution to the planner's problem, consider the (social) net present value (NPV_A^{SP}) of unit of investment, k_A, by entrepreneur with productivity A:

$$\text{NPV}_A^{SP} \equiv \frac{A}{R} - q - \left[\chi''\left(K^S\right) \cdot \int \gamma_{\widehat{A}} \cdot k_{\widehat{A}} \cdot dG(\widehat{A})\right] \tag{16}$$

where $\gamma_{\widehat{A}}$ denotes the multiplier on the borrowing constraints of entrepreneurs with productivity $\widehat{A}$. The first observation is that, since NPV_A^{SP} is linear and increasing in A, there exists a marginal entrepreneur $\widetilde{A}$ with $\text{NPV}_{\widetilde{A}}^{SP} = 0$, such that only entrepreneurs with productivity above $\widetilde{A}$ invest and they do so until their borrowing constraints bind. The second observation is that, since the term in brackets is positive, the planner perceives a higher social cost (or a lower social benefit) of investment than individual entrepreneurs, who only compare A/R to q. This is because the planner internalizes that each additional unit of investment by entrepreneurs with productivity A raises the equilibrium price of capital (as $\chi''(K^S) > 0$) and thus tightens

[10]Since preferences are linear, such an objective is equivalent to maximizing the equally-weighted aggregate welfare. We thus abstract from distributional considerations.

the borrowing constraints of all entrepreneurs with productivity above $\widetilde{A}$. Since the borrowing constraints bind for all such entrepreneurs (as $\gamma_A > 0 \; \forall A > \widetilde{A}$), this entails a first-order welfare loss. Consequently, the planner restricts investment by some entrepreneurs by setting $\widetilde{A} > q \cdot R$.

The following proposition summarizes the above discussion and also states its main implication for the response of output to changes in interest rates.

Proposition 2 *Let $\widetilde{A}$ denote the productivity of the marginal entrepreneur at the social planner allocation, and q^{CE} and q^{SP} respectively denote the price of capital in the competitive equilibrium and in the social planner's allocation. Then:*

$$\widetilde{A} > R \cdot q^{CE} > R \cdot q^{SP},$$

i.e., relative to the competitive equilibrium, the planner restricts investment by some supra-marginal entrepreneurs thereby depressing asset prices. Moreover, letting Y^{SP} denote output in the social planner allocation, it holds that:

$$\frac{dY^{SP}}{dR} < 0,$$

i.e., a fall in the interest rate is always expansionary in the social planner's allocation.

The first part of Proposition 2 follows directly from our previous discussion. It only illustrates why the planner forbids some entrepreneurs from investing altogether: by doing so, she reduces the price of capital relative to the competitive equilibrium thus allowing more productive entrepreneurs to expand their investment. The second part of Proposition 2 follows directly from the first. To see this, simply note that a fall in the interest rate can only reduce output if it reallocates capital from supra- to infra-marginal entrepreneurs (see Equation (12)). But the planner can always keep these reallocation effects under control by adjusting the productivity of the marginal entrepreneur, $\widetilde{A}$, when the interest rate changes.

This type of planner intervention, which prevents some entrepreneurs from investing altogether, may seem far-fetched and informationally demanding for the planner (i.e., she needs to know which entrepreneurs to exclude). However, it is straightforward to show that the planner allocations can be decentralized through a simple Pigouvian subsidy τ on savings, financed with lump-sum taxes on capitalists. By choosing the subsidy appropriately, the planner can ensure that all entrepreneurs with productivity lower than $\widetilde{A}$ prefer to save their endowments at the market interest rate and collect the subsidy rather than investing in capital.

These results are reminiscent of the literature on zombie firms, which emphasizes that low interest rates incentivize unproductive activities (Caballero et al., 2008; Adalet McGowan et al., 2018; Tracey, 2019; Schivardi et al., 2020). In much of that literature, however, the emphasis is on distorted incentives that make agents engage in unproductive activities (e.g. evergreening by banks). We show instead that investment can be socially excessive despite having a positive net present value from a private standpoint. The reason is that individual entrepreneurs do not internalize the crowding-out effect that they have on more productive investment. It is this externality that drives our results.

4 Dynamics of wealth accumulation

Our main results have been derived under the assumption that entrepreneurial wealth is exogenous. This raises the question of whether they extend to a dynamic economy in which entrepreneurial wealth evolves endogenously. In particular, productive entrepreneurs may stand to gain from a fall in the interest rate, as they benefit both through lower costs of borrowing and – potentially – through higher asset prices and their associated balance-sheet effects. We turn to this question next.

4.1 Dynamic economy

Suppose that time t is continuous. To simplify notation, we omit time subscripts whenever possible. As in the baseline of Section 3, the economy is populated by a continuum of entrepreneurs with mass one. There key difference relative to the baseline is that entrepreneurs differ not only in their productivity, which varies stochastically, but also in their wealth, which evolves endogenously.

Individual productivity A is stochastic and it fluctuates over time according to an idiosyncratic Poisson process with common arrival rate $\theta > 0$. If an individual entrepreneur is hit by the Poisson shock, she draws a new productivity from an aggregate distribution G. Otherwise, her productivity remains unchanged. Individual wealth w in turn evolves endogenously according to the individual return on wealth and individual consumption choices. Absent the Poisson shock, the law of motion of entrepreneurial wealth is

$$\dot{w} = y + \dot{q} \cdot k - r \cdot b - c, \tag{17}$$

where $y = A \cdot k$ is the output rate obtained from operating capital stock $k \geq 0$; $\dot{q}$ is the rate of change of the price of capital $q > 0$; $r > 0$ is the interest rate on debt/savings $b = q \cdot k - w$;

and $c \geq 0$ is the consumption rate.

As in the baseline economy, entrepreneurs may be financially constrained. To simplify the exposition, we now introduce financial constraints by assuming that an entrepreneur can walk away with a portion $1 - \lambda \in (0, 1)$ of her capital stock immediately after issuing debt. From this, we obtain the standard collateral constraint in the literature (e.g. Moll (2014)):

$$b \leq \lambda \cdot q \cdot k. \tag{18}$$

Entrepreneurs have logarithmic preferences for consumption and discount future consumption at rate $\rho > r$.[11] This simplifies their consumption choice and implies that entrepreneurs consume a fraction ρ of wealth at each instant. Also for simplicity, we assume throughout that the capital supply is fixed and equal to $\bar{K}$, which implies that capitalist optimization is irrelevant. These choices are without loss of generality, but they allow us to focus on our main objective, i.e., to show that the general-equilibrium reallocation effect extends to the dynamic economy, with minimal complications.

4.2 Equilibrium

In any period t, entrepreneurs choose their consumption c, their capital stock k, and their debt b, given the path of asset prices and the interest rate. The optimal choice of entrepreneurs is:

$$k = \begin{cases} \frac{1}{1-\lambda} \cdot \frac{w}{q} & \text{if } A + \dot{q} \geq r \cdot q \\ 0 & \text{otherwise} \end{cases}, \tag{19}$$

and

$$b = q \cdot k - \omega \tag{20}$$

$$c = \rho \cdot w. \tag{21}$$

Equation (19) says that, just as in the baseline model, there is a threshold entrepreneur who is indifferent between saving or investing in capital. The only difference is that this threshold is now given by $r \cdot q - \dot{q}$, as part of the return to capital accrues in the form of capital gains. Entrepreneurs above the threshold (i.e., supra-marginals) borrow and invest as much as possible whereas those below (i.e., infra-marginals) save at the interest rate r. Equation (21) says that,

[11] Entrepreneurs must be impatient relative to international lenders because otherwise their wealth would grow away from the collateral constraint.

due to logarithmic preferences, entrepreneurs consume a portion ρ of their wealth each period.

From substituting these optimal choices into law of motion (17), it follows that individual wealth w evolves according to:

$$\dot{w} = \begin{cases} \left[\left(\frac{A+\dot{q}}{q} - \lambda \cdot r\right) \cdot \frac{1}{1-\lambda} - \rho\right] \cdot w & \text{if } A + \dot{q} \geq r \cdot q \\ (r-\rho) \cdot w & \text{otherwise} \end{cases}. \tag{22}$$

Equation (22) captures the endogeneity of wealth accumulation in this dynamic economy, which depends both on individual productivity and on the interest rate. In particular, note that more productive entrepreneurs accumulate wealth at a faster pace, and that the interest rate has different effects on the wealth accumulation of supra-marginal and infra-marginal entrepreneurs.

To characterize the equilibrium, it is convenient to aggregate all entrepreneurs with the same productivity A. We thus define the aggregate wealth of entrepreneurs with productivity A as:

$$W_A \equiv \int w \cdot f(A, w) \cdot dw, \tag{23}$$

where $f(A, w)$ is the population share of entrepreneurs with productivity A and wealth w. It follows from Equation (22) that W_A evolves according to:

$$\dot{W}_A = \begin{cases} \left[\left(\frac{A+\dot{q}}{q} - \lambda \cdot r\right) \cdot \frac{1}{1-\lambda} - \rho - \theta\right] \cdot W_A + \theta \cdot g(A) \cdot W & \text{if } A + \dot{q} \geq r \cdot q \\ (r - \rho - \theta) \cdot W_A + \theta \cdot g(A) \cdot W & \text{otherwise} \end{cases} \tag{24}$$

where g is the density related to cumulative probability function G and $W > 0$ is aggregate wealth, that is:

$$W \equiv \int W_A \cdot dA. \tag{25}$$

The per-capita investment of entrepreneurs with productivity A is then given by:

$$k_A = \begin{cases} \frac{1}{1-\lambda} \cdot \frac{W_A}{q} \cdot \frac{1}{g(A)} & \text{if } A + \dot{q} \geq r \cdot q \\ 0 & \text{otherwise} \end{cases}, \tag{26}$$

so that market-clearing and aggregate output can respectively be expressed as:

$$\bar{K} = \int_{A \geq r \cdot q - \dot{q}} k_A \cdot dG(A), \tag{27}$$

and

$$Y = \int_{A \geq r \cdot q - \dot{q}} A \cdot k_A \cdot dG(A). \tag{28}$$

An equilibrium consists of paths of prices $\{q, r\}$ and of allocations $\{\{W_A, k_A\}_A, W, Y\}$ such that Equations (24)-(28) are satisfied in all periods.

4.3 Reallocation effects in steady state

We first analyze the steady-state effects of a fall in the interest rate r. The key feature of the steady state is that prices and aggregate variables are constant over time, i.e., $\dot{q} = 0$ and $\dot{W}_A = 0$ for all A. This implies, from Equation (24), that the steady-state wealth of entrepreneurs with productivity A is a constant share of aggregate wealth W:

$$W_A = \begin{cases} \frac{\theta}{\theta + \rho + \frac{1}{1-\lambda} \cdot \left(\lambda \cdot r - \frac{A}{q}\right)} \cdot g\left(A\right) \cdot W & \text{if } A \geq r \cdot q \\ \frac{\theta}{\theta + \rho - r} \cdot g\left(A\right) \cdot W & \text{otherwise} \end{cases}. \tag{29}$$

Equation (29) shows that, in the dynamic economy, there is a natural link between productivity and wealth. In particular, per-capita wealth $W_A / g(A)$ is strictly increasing in A for all supra-marginal entrepreneurs. The reason is that entrepreneurs with a higher productivity have higher return on wealth.

Together with the definition of aggregate wealth, Equation (29) implies that:

$$\int_{r \cdot q}^{1} \frac{\theta}{\theta + \rho + \frac{1}{1-\lambda} \cdot \left(\lambda \cdot r - \frac{A}{q}\right)} \cdot dG(A) = 1 - \frac{\theta}{\theta + \rho - r} \cdot [G(r \cdot q) - G(0)], \tag{30}$$

while the market clearing condition can be written as:

$$\bar{K} = \left[\int_{r \cdot q}^{1} \frac{\theta}{\theta + \rho + \frac{1}{1-\lambda} \cdot \left(\lambda \cdot r - \frac{A}{q}\right)} \cdot dG(A)\right] \cdot \frac{1}{1-\lambda} \cdot \frac{W}{q}. \tag{31}$$

Equations (29)-(31) determine $\{W_A\}$, W, and q, and jointly illustrate the steady-state effects of a fall in the interest rate, r. Equation (29) shows that, for given aggregate wealth W, a fall in r reduces the wealth of infra-marginal entrepreneurs and raises the wealth of supra-marginal entrepreneurs: in a nutshell, lower interest rates transfer wealth from creditors to debtors. This also implies that the productivity of the marginal entrepreneur, $r \cdot q$, falls as saving becomes relatively less attractive relative to investing in capital. Thus, if aggregate wealth W were

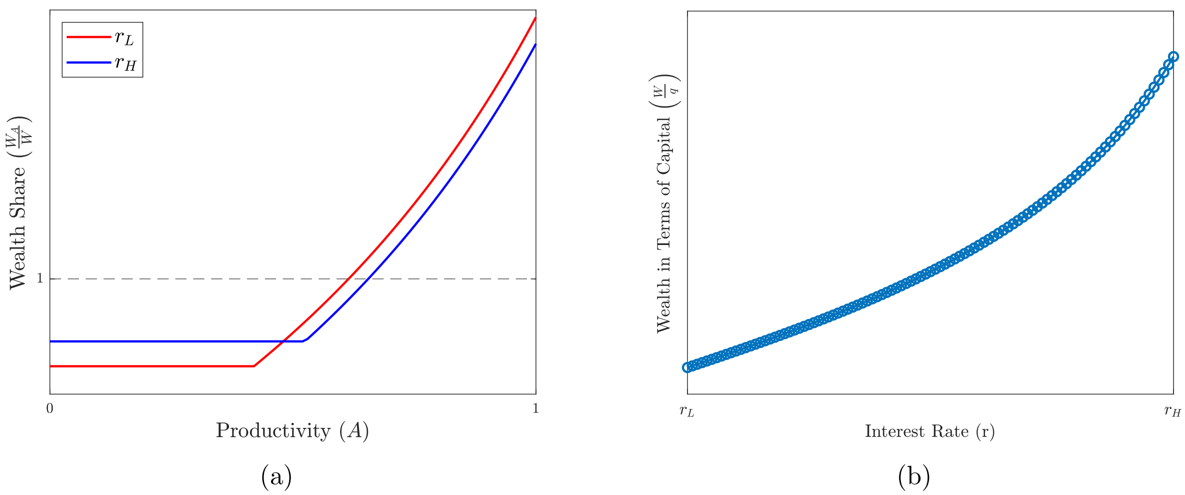


Figure 4: **Steady-state effect of a fall in the interest rate on wealth shares and total wealth in terms of capital.**

fixed, the price of capital would rise along with the increase in demand (see Equation (31)). But aggregate wealth is itself endogenous, and it may rise or fall when r declines. Intuitively, if the losses inflicted on infra-marginal entrepreneurs by a lower value of r are not compensated by the gains of supra-marginal entrepreneurs (e.g. if λ is small), then aggregate wealth will decline when r falls. As a result, the net effect on the price of capital, q, is ambiguous. What is unambiguous, however, is that – just as in the baseline model – aggregate wealth in terms of capital (W/q) must fall to ensure market clearing.

Figure 4 summarizes the above discussion. Panel (a) shows the steady-state shares of aggregate wealth held by entrepreneurs as a function of their productivity, for two levels of the interest rate, $r_H > r_L$. The figure shows how a fall in the interest rate from r_H to r_L raises the share of wealth of supra-marginal entrepreneurs and reduces the productivity of the marginal entrepreneur. Panel (b) instead shows how the steady-level of aggregate wealth in terms of capital, W/q, declines as the interest rate falls from r_H to r_L.

But how does all of this affect aggregate output? To address this question, we can combine Equations (27) and (28) to obtain:

$$\frac{dY}{dr} = \int_{r\cdot q}^{1} (A - r \cdot q) \cdot \frac{dk_A}{dr} \cdot dG(A). \tag{32}$$

Equation (32) is the equivalent of Equation (12) for the dynamic setup. Since the capital stock is fixed at $\bar{K}$, there is no capital-supply effect. There is, however, a capital-reallocation

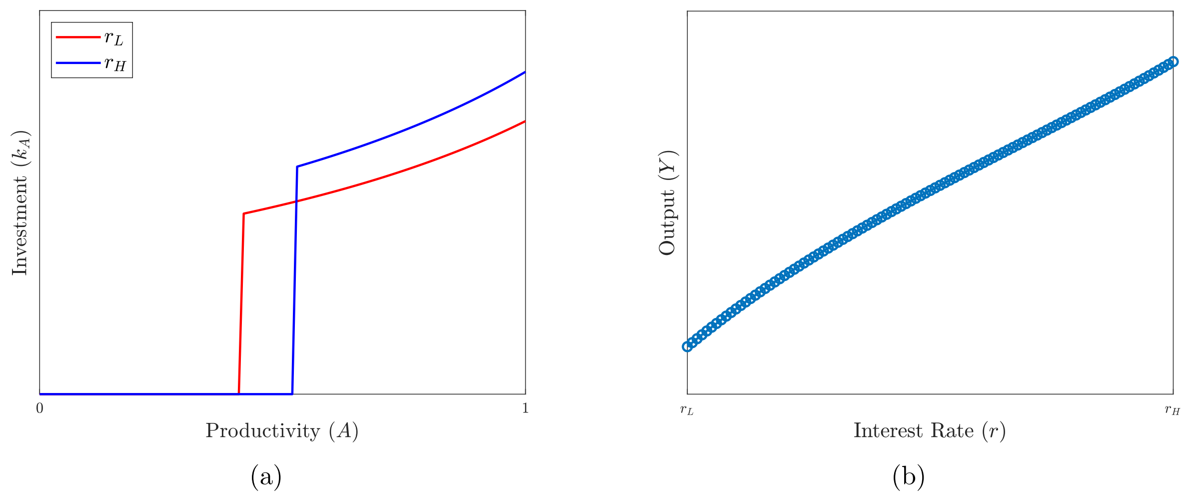


Figure 5: **Steady-state effect of a fall in the interest rate on entrepreneurial investment and aggregate output.**

effect, which depends on the change in the equilibrium level of investment of supra-marginal entrepreneurs. As in the baseline model, this effect can be positive or negative because it captures both the reallocation of capital from supra- to infra-marginal entrepreneurs (which reduces aggregate output) and the reallocation of capital among supra-marginal entrepreneurs (why may increase or reduce aggregate output).

Figure 5 illustrates the effect of a fall in r for a specific parametrization of the dynamic economy. Panel (a) shows the per-capita investment of entrepreneurs as a function of their productivity, for two levels of the interest rate, $r_H > r_L$. The figure shows how a fall in the interest rate from r_H to r_L reduces the investment of all supra-marginal entrepreneurs and raises that of infra-marginal entrepreneurs. Due to this reallocation, Panel (b) shows that the steady-level of output declines monotonically as the interest rate falls from r_H to r_L.

In the baseline model, Proposition 1 characterized conditions under which $dY/dr > 0$: the reason is that the reallocation among supra-marginals could be weakened arbitrarily by reducing λ. In the dynamic setup, however, this is no longer possible because the redistribution among supra-marginals operates through the dynamic accumulation of wealth (even if λ equals zero). Although we have been unable to provide general conditions under which $dY/dr > 0$ in the dynamic economy, we have been unable to find a parametrization where this does not hold.

4.4 Balance-sheet effects and transition dynamics

We have thus far shown that a fall in r can reduce output in steady state due to the capital-reallocation effect. This may come as a surprise to readers that are familiar with the macro-finance literature, which has traditionally emphasized the expansionary role of rising asset prices through balance-sheet effects. As we now show, however, these affects are also present in our economy but they are transitory.

To see this, suppose that the economy is in steady state at time t_0 when it experiences an unexpected, permanent fall in the interest rate from r_H to r_L. For concreteness, we focus throughout on the parametrization in which the steady-state value of q rises in response to the fall in r. As is standard in the literature (e.g. Kiyotaki and Moore (1997)), changes in q give rise to balance-sheet effects provided that financial contracts are non-contingent, which we have assumed. In our environment, as a result, changes in q will affect the net worth of supra-marginal entrepreneurs, proportionally to their capital holdings.[12]

To see this, note that on impact a fall in r leads to an instantaneous increase in the price of capital from q_{t_0} to $q_{t_0^+}$, where the subindexes t_0 and t_0^+ respectively denote the instants before and after the shock.[13] As a result, the wealth of supra-marginal entrepreneurs jumps in proportion to their capital holdings, while the wealth of infra-marginals is unaffected:

$$W_{A,t_0^+} = \begin{cases} \left(\frac{q_{t_0^+}}{q_{t_0}} - \lambda\right) \cdot \frac{1}{1-\lambda} \cdot W_{A,t_0} & \text{if } A \geq r \cdot q_{t_0} \\ W_{A,t_0} & \text{otherwise} \end{cases}. \tag{33}$$

From then on, the evolution of aggregate variables $\{\{W_A, k_A\}_A, W, q, Y\}$ is characterized as before by Equations (24)-(28).

The transition of the economy to its new steady state is depicted in Figures 6 and 7. Panel (a) of Figure 6 shows the evolution of the price of capital q. As the figure shows, q rises on impact and then gradually declines to its new, higher steady-state value. Consistent with Equation (33), panel (b) shows that, on impact, this rise in q boosts the wealth of all supra-marginal entrepreneurs (see shift from the solid blue curve to the dashed curve). Figure 7 then depicts the resulting non-monotonic evolution of output. On impact, output increases due to balance-

[12]Note that the conditions necessary for the existence of balance-sheet effects are more stringent than those necessary for reallocation effects. Besides heterogeneity and financial frictions, balance-sheet effects require that entrepreneurs retain exposure to the price of capital on their balance sheets (see, e.g., Krishnamurthy (2003); Di Tella (2017); Asriyan (2021)). Indeed, these effects do not arise if entrepreneurs can hedge this exposure through the use of derivatives, or if they can avoid it altogether by renting capital instead of buying it.

[13]If the price of capital q did not rise, there would an excess demand of capital since the permanent fall in r would induce infra-marginal entrepreneurs to invest.

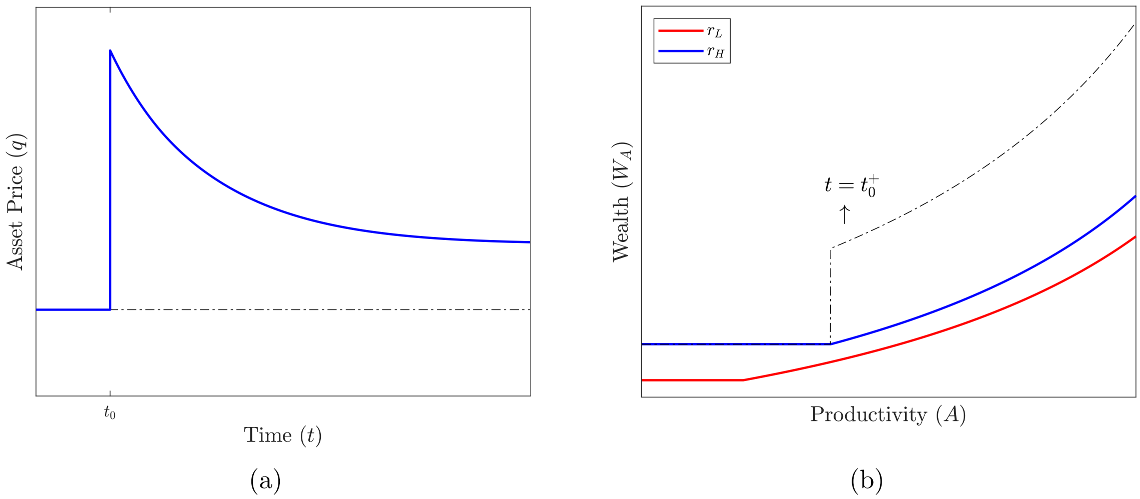


Figure 6: **Balance-sheet effects induced by a fall in the interest rate.**

sheet effects, which benefit the most productive supra-marginal entrepreneurs and enable a reallocation of capital in their favor. From then on, output declines monotonically to its new, lower steady-state value. Thus, balance-sheet effects are a one-time transfer to supra-marginal entrepreneurs, which materialize in the instant that r falls to its new level. The reallocation effects that we have emphasized throughout are instead permanent, just as the fall in r, and thus become dominant eventually.[14]

This discussion suggests that the duration of interest-rate changes must be crucial to understand their effects on investment and output. To illustrate this, Figure 8 depicts the equilibrium dynamics of output for interest rate declines of different durations. The dashed red line depicts the case of a permanent fall in the interest rate, just as in Figures 6 and 7; the solid-black line depicts the case of a temporary fall in the interest rate, but which is long-lived; lastly, the solid-blue line depicts the case of a temporary fall that is short-lived. As the figure shows, transitory falls in the interest rate have a smaller effect on asset prices and thus smaller balance-sheet and reallocation effects. Moreover, once the fall in the interest rate is sufficiently short-lived, the balance-sheet effect dominates throughout and the traditional expansionary effect of interest-rate declines resurfaces.

[14]For the interested reader, in Appendix B.4 we show that the insight – i.e., that balance-sheet effects mask the reallocation effects induced by lower interest rate only temporarily – can also be obtained in the classic setting of Kiyotaki and Moore (1997).

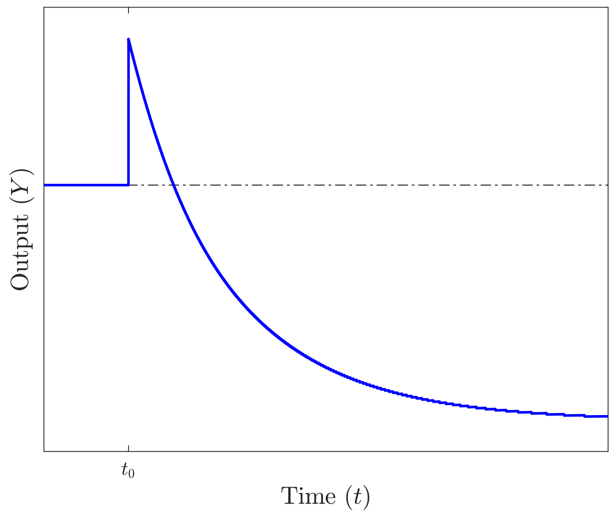


Figure 7: **Evolution of output after a fall in the interest rate.**

5 Supporting evidence

The main insight of our theory is that, in the presence of financial frictions, the expansionary effect of a decline in the interest rate is weakened (or even overturned!) by general-equilibrium reallocation effects. In particular, the expansion of relatively unproductive investment tends to raise the price of capital thereby crowding-out more productive investment.

In this section, we test the key implications of our theory using US and Spanish data. Doing so requires taking a stance on the empirical counterpart of capital in the model, which could in principle be any input of production that requires an investment. We use real estate for such a counterpart. The advantage of doing so is that there are readily available measures of real-estate supply elasticities, which play a key role in our theory (see Proposition 1) and thus in our empirical strategy.

We test the theory's main predictions on region-, sector- and firm-level data. Using US data, we assess whether the expansionary effect of a fall in the interest rate is weaker (i) in regions where the supply of real estate is less elastic, and (ii) for sectors that use real estate more intensively as an input. To show that this is indeed due to reallocation effects, we use Spanish firm-level data to assess whether interest rate declines are associated to the relative expansion of low-productivity firms in these regions and sectors.

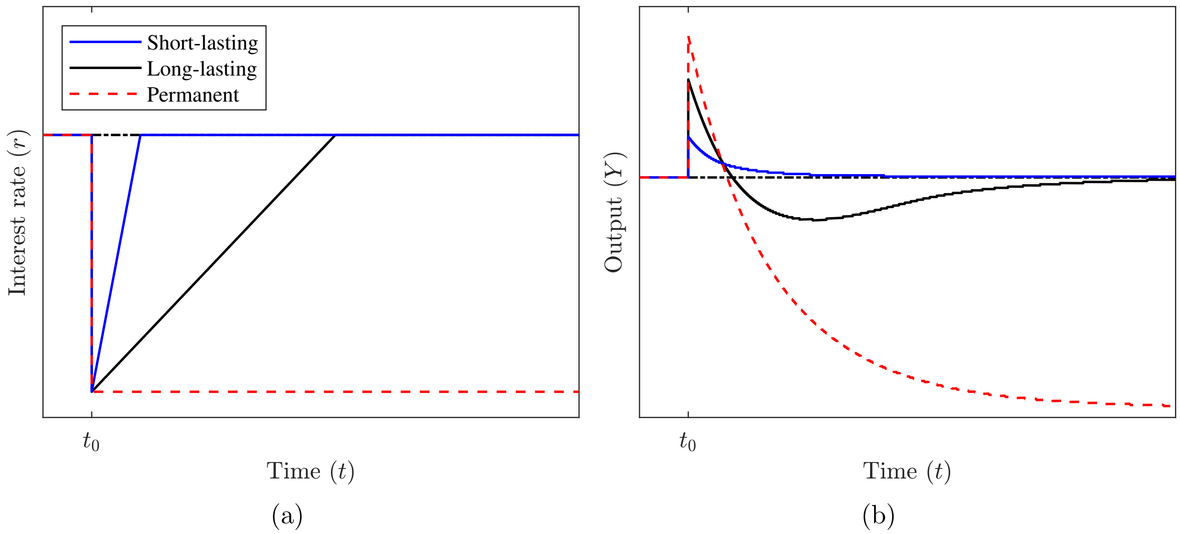


Figure 8: **Effects of the duration of fall in the interest rate on output dynamics.**

5.1 Regional- and sector-level evidence

Testing the regional- and sector-level predictions of changes in interest rates requires overcoming three obstacles. First, we need to identify changes in the price of real estate that are orthogonal to changes in its demand. The literature has dealt with this problem by measuring the local elasticity of real-estate supply based on constraints on land availability (Saiz, 2010).

Second, we need to classify sectors according to the intensity of their use of real estate as a productive input. We follow Vom Lehn and Winberry (2019) and use sectorial data from the US Bureau of Economic Analysis (BEA) Fixed Assets tables, which provides information on real estate and other fixed assets. We follow their approach to construct a sector-level measure of real-estate intensity, computed as the share of real estate in total fixed assets.

Third, we need a measure of changes in the interest rate that are orthogonal to local economic conditions. We follow the literature and use various measures of monetary policy shocks to proxy for such changes. Our baseline measure is the high-frequency monetary shock from Jarociński and Karadi (2020), who identify monetary surprises based on high-frequency data around monetary policy announcements, combined with sign restrictions to take out the component of the announcement that reflects the economic outlook. We aggregate their shocks over annual periods to construct yearly measures of changes in monetary policy. The Appendix provides a detailed explanation of this variable, and of two other measures of interest rate changes that we use for robustness.

To ensure that the results reflect the dynamics of the overall economy, we require data covering all firms in the US. To this end, we use data on the economic activity of US firms by region and sector from the US Bureau of Economic Analysis (BEA) Regional Accounts. To the best of our knowledge, these accounts provide the widest coverage of disaggregated data on firm output. We relegate a detailed description of the dataset to Appendix C.

5.1.1 Empirical strategy

To test the prediction that the expansionary effects of real interest rates on economic activity is weaker in regions with a lower elasticity of real estate supply, we estimate – for sector i in region j at date t – the following equation:

$$\Delta y_{ijt} = \alpha_{ij} + \alpha_{it} + \delta \cdot y_{ijt-1} + \beta_1 \cdot r_t \cdot H_j + \varepsilon_{ijt} \tag{34}$$

where Δy_{ijt} is the real GDP growth rate in sector i in Metropolitan Statistical Area (MSA) j in year t, α_{ij} is a sector i by region j fixed effect, α_{it} is a sector i by year t fixed effect, y_{ijt-1} is lagged GDP in sector i in MSA j, r_t is the annual monetary policy shock in year t, and H_j is the elasticity of land supply in MSA j. Δy_{ijt} and y_{ijt-1} are respectively computed as $ln(Y_{ijt}/Y_{ijt-1})$ and $ln(Y_{ijt-1})$, where Y is output expressed in constant (chained 2012) US dollars. The sector-region fixed effects capture permanent differences in output dynamics across sectors and regions, and the sector-year fixed effects control for sectorial differences in exposure to aggregate shocks. We include the lagged level of real GDP to capture growth convergence effects, although doing so does not affect our main results.

The theory predicts that $\beta_1 < 0$ and significant. In other words, the effect of a monetary expansion on economic activity is weaker in regions with a lower elasticity of real-estate supply. Of course, there may be multiple channels through which real-estate prices affect local economic activity (e.g. they may boost local demand), which may confound the reallocation effects identified in the theory.

Thus, we test whether the differential effect of interest rates across regions is stronger for sectors that are more real-estate intensive. Specifically, we estimate variations of the following equation:

$$\Delta y_{ijt} = \alpha_{ij} + \alpha_{it} + \delta \cdot y_{ijt-1} + \beta_1 \cdot r_t \cdot H_j + \beta_2 \cdot r_t \cdot H_j \cdot RE_{it-1} + \Gamma' Z_{ijt-1} + \varepsilon_{ijt} \tag{35}$$

where $RE_{i,t-1}$ is the average real estate intensity of sector i in year $t-1$ (one year lagged) and Z_{ijt-1} is a vector of controls. As described in the Appendix, real-estate intensity is computed as (i) the share of real-estate related fixed assets in total fixed assets or (ii) the share of real-estate related investment in total investment.

The theory predicts that $\beta_2 < 0$ and significant, implying that the differential effect of interest rates across regions should be stronger for sectors that are more real-estate intensive.

5.1.2 Empirical results

Our main results on sector and regional effects are reported in Table 1. Column 1 presents estimates of Equation (34).[15] The estimate of β_1 is not statistically significant, i.e., we do not find that the expansionary effects of real interest rates to be weaker in regions with a lower elasticity of real estate supply. As we anticipated, however, this is a weak test of the theory as there may be multiple mechanisms at work (Mian and Sufi, 2011).

Column 2 reports estimates of Equation (35). The estimated value of β_2 is negative and statistically significant, which is consistent with a strong reallocation effect. The estimated effect is economically substantial, moreover: the semi-elasticity of GDP growth to monetary easing is reduced by 0.035 units when the elasticity of real estate and the real-estate intensitiy of the sector are respectively reduced and raised by one standard deviation relative to the average in the sample. This is a meaningful effect, taking into account that the average growth rate of real GDP of 0.018.

Column 3 includes region-year fixed effects. The estimated value of β_2 now increases to -0.041, and it is statistically significant at the 1% level. Column 4 in turn drops both the sector-year and region-year fixed effects, which makes it possible to estimate the average effect of monetary shocks. The average semi-elasticity of a monetary shock is -0.019, which is comparable to our differential effect of interest, even if it is estimated with large error. This implies that the differential effect we have identified is economically meaningful.

Table 5 in the Appendix shows that our main results are robust to the use of alternative definitions of the monetary policy shocks, to the use of the flow measure of real-estate intensity, and to the inclusion of financial leverage in the regression as a way to control for balance-sheet effects. The Appendix also shows that the result reported in Table 1 is persistent and remains significant up to five years after the monetary policy shock (see Figure 10).

Taken together, these findings are consistent with the presence of strong reallocation effects: namely, the expansionary effects of declining interest rates are weaker in regions with a lower elasticity of real-estate supply and for sectors that are more real-estate intensive. But they

[15] For ease of interpretation, we invert the sign of r_t and H_j such that higher values of r_t denote an easing of interest rates and higher values of H_j denote a more inelastic land supply. All explanatory variables except for the monetary policy shock are standarized. In all regressions, we cluster standard errors two-ways, by sector-region and year. This increases the standard errors compared to not clustering and should therefore be seen as a conservative estimate.

Table 1: **Monetary transmission, real estate supply, and real estate intensity**

VARIABLES	(1) ΔGDP	(2) ΔGDP	(3) ΔGDP	(4) ΔGDP
mp_med_t				-0.019
				(0.064)
$mp\_med_t * H_j$	0.011	0.017		0.025*
	(1.010)	(0.010)		(0.010)
$mp\_med_t * H_j * RE_{i,t-1}$		-0.035**	-0.041***	-0.032***
		(0.009)	(0.009)	(0.008)
$mp\_med_t * RE_{i,t-1}$				0.011
				(0.026)
$ln(GDP)_{i,j,t-1}$	-0.225***	-0.225***	-0.237***	-0.192***
	(0.026)	(0.026)	(0.027)	(0.027)
Sector-Year FE	Yes	Yes	Yes	No
Sector-Region FE	Yes	Yes	Yes	Yes
Region-Year FE	No	No	Yes	No
Observations	17756	17756	17752	17756
R-squared	0.276	0.277	0.345	0.168
p>F	0.000	0.000	0.000	0.000
Clustering	Yes	Yes	Yes	Yes

Notes: Regression results from estimating the following specification: $\Delta y_{ijt} = \alpha_{ij} + \alpha_{it} + \delta \cdot y_{ijt-1} + \beta_1 \cdot r_t \cdot H_j + \beta_2 \cdot r_t \cdot H_j \cdot RE_{it-1} + \varepsilon_{it}$. Δy_{ijt} is the real GDP growth rate at the sector-MSA level, and y_{ijt-1} is the one-period lag of log of real GDP. GDP is expressed in chained 2012 US dollars and growth rates are constructed using log changes. mp_med_t is the high-frequency monetary shock from Jarociński and Karadi (2020), obtained with median rotation sign restrictions, aggregated over each year. H_j is the elasticity of real-estate supply at the MSA level. We invert the sign of the mp_med_t and H_j variables to ease interpretation, so that higher values of mp_med_t denote an easing of monetary policy and higher values of H_j denote a more inelastic housing supply. RE_{it} is the ratio of real estate assets to total fixed assets of the sector. All explanatory variables except the monetary shock are standardized. All variables are as defined in Table 1. Standard errors are clustered two-ways by sector-region and year.

could also be driven by alternative explanations: when interest rates fall, for instance, real-estate intensive sectors may simply find it hard to expand in regions where the supply of real estate is inelastic. We thus turn to firm-level data to look for traces of reallocation effects.

5.2 Firm-level evidence

If the results uncovered in the previous section are driven by reallocation effects, we would expect them to be accompanied by some form of rising misallocation. In particular, it should be firms with a relatively low marginal product of capital that expand their investment the most in response to a decline in the interest rate. This differential firm-level effect should be observable in sectors with a higher real-estate intensity and in regions with a relatively inelastic supply of real estate.

To test this, we need a firm-level measure of productivity. We use throughout the marginal (revenue) product of capital. Since this measure is not available for a broad cross-section of US firms, we turn instead to Spanish firm-level data from the Bureau Van Dijk Orbis database. This data has previously been used by Gopinath et al. (2017) to study the evolution of firm-level productivity during the Spanish credit boom of the early 2000's. Spain has the additional advantage of having readily available measures of real-estate elasticities at the regional level, which have been computed by Basco et al. (2018).

Specifically, we follow following Gopinath et al. (2017) and compute the marginal revenue product of capital of firm f in sector i in region j at time t, as

$$MRPK_{fijt} = \kappa \cdot \left(\frac{VA_{fijt}}{k_{fijt}} \right),$$

where κ is a constant, VA_{fijt} is firm real value added, and k_{fijt} is the real capital stock at the firm level.[16] We follow Gopinath et al. (2017) and set $\kappa = 0,23$. We then take the log of $MRPK_{fijt}$ and label this $mprk_{fijt}$. To reduce the influence of outliers, we winsorize $mrpk_{fijt}$ at the 2% and 98% levels. We relegate a detailed description of the dataset and its construction to Appendix C.

[16] In particular, $\kappa = \frac{\alpha}{\mu}$, where α represents the capital share of income and μ captures the average mark-up in the economy; VA_{fijt} is computed as the difference between gross output and materials divided by an output price deflator, and; k_{fijt} is computed as gross fixed assets divided by a gross fixed assets formation price deflator. Since we do not have prices at the firm level, we use the sector-specific gross output price deflator for Spain from the Eurostat KLEMS database at the two-digit NACE industry level to deflate value added, and the gross fixed capital formation price deflator for Spain at the country level from the World Bank's World Development Indicators database to deflate the capital stock.

5.2.1 Empirical strategy

We want to establish whether changes in interest rates have differential effects across firms depending on their productivity. In particular, we test whether firms with a low marginal revenue product of capital grow relatively more than firms with a high marginal revenue product of capital after a fall in the interest rate, and whether this differential effect is stronger in sectors with a higher real-estate intensity and in regions with a lower elasticity of real-estate supply. To this end, we extend the specification in Equation (35) as follows:

$$\Delta y_{fijt} = \alpha_f + \alpha_{it} + \alpha_{jt} + \beta' X_{fij,t-1} + \gamma' Z_{ij,t-1} + \varepsilon_{fijt} \tag{36}$$

where Δy_{fijt} denotes log output growth of firm f, α_f, α_{it} and α_{jt} respectively represent firm, sector-year, and province-year fixed effects, $X_{fij,t-1}$ is a vector containing r_t, H_j, $RE_{US,i,t-1}$ and $mrpk_{fij,t-1}$ and all of their possible interactions, and Z_{ijt-1} is a vector of controls.

According to the theory, we would expect the estimated coefficient on the quadruple interaction $r_t \cdot H_j \cdot RE_{US,i,t-1} \cdot mrpk_{fij,t-1}$ to be negative and significant, implying that the stimulative effect of interest-rate reduction on the output of a specific firm should be decreasing in the real-estate intensity of the firm's sector, increasing in the elasticity of real-estate in the region where the firm operates, and decreasing in the productivity of the firm.

5.2.2 Empirical results

The regression results are presented in Table 2, which only reports the interactions of interest. Column 1 includes only the variables r_t, $mrpk_{fij,t-1}$ and their interaction term $r_t \cdot mrpk_{fij,t-1}$ to test whether – on average – firms with a low marginal revenue product of capital grow relatively more after a fall in the interest rate. The estimated coefficient on the interaction term is positive albeit not statistically significant, which implies that the reallocation effect identified in the theory is not strong for the average sector.

To test whether this differential effect across firms is stronger in areas with a more inelastic housing supply, Column 2 adds the measure of housing supply elasticity at the provincial level, H_j, and its interactions with the variables r_t, $mrpk_{fij,t-1}$ and the interaction term $r_t \cdot mrpk_{fij,t-1}$. We only report the estimated coefficients on variables $r_t \cdot mrpk_{fij,t-1}$ and $r_t \cdot H_j \cdot mrpk_{fij,t-1}$, neither of which is statistically significant. The implication is that the reallocation effect does not appear to be strong for the average sector regardless of regional characteristics.

Column 3 further enriches the model by including our proxy for real-estate intensity at the sector level, $RE_{US,i,t-1}$, and its interactions with the variables r_t, H_j, $mrpk_{fij,t-1}$, along with all possible interaction terms between these four variables. The key finding is that the estimated coefficient on the on the interaction term $r_t \cdot H_j \cdot RE_{US,i,t-1} \cdot mrpk_{fij,t-1}$ is negative and significant. This implies that the reallocation effect appears to be strong precisely among firms that operate in relatively real-estate intensive sectors and in regions with a relatively inelastic supply of real estate. The estimated effect is economically meaningful, moreover: the semi-elasticity of output growth to monetary easing is lowered by 0.013 units when the real-estate intensity of the sector and the elasticity of real-estate in the region are respectively increased and reduced by one standard deviation relative to the sample average. This is a meaningful effect compared to the sample average output growth of 0.021.

Table 2: **Monetary transmission and misallocation of capital in the Spanish sample**

VARIABLES	(1) $\Delta Output$	(2) $\Delta Output$	(3) $\Delta Output$	(4) $\Delta Output$
$r_t * mrpk_{fij,t-1}$	0.001	-0.001	-0.003	0.019
	(0.024)	(0.024)	(0.024)	(0.028)
$r_t * H_j * mrpk_{fij,t-1}$		0.001	0.002	0.000
		(0.004)	(0.004)	(0.005)
$r_t * H_j * RE_{US,i,t-1} * mrpk_{fij,t-1}$			-0.013***	-0.012**
			(0.004)	(0.004)
Controls	No	No	No	Yes
Individual Fixed Effects	Yes	Yes	Yes	Yes
Industry-Year Fixed Effects	Yes	Yes	Yes	Yes
Region-Year Fixed Effects	Yes	Yes	Yes	Yes
Observations	1,007,883	936,268	936,056	936,056
R-squared	0.348	0.349	0.349	0.378
p > F	0.000	0.000	0.000	0.000
Clustering	Yes	Yes	Yes	Yes

Notes: This table contains the regression results for $\Delta Output_{fijt} = \alpha_f + \alpha_{it} + \alpha_{jt} + \beta' X_{fij,t-1} + \gamma' Z_{ij,t-1} + \varepsilon_{fijt}$, where $X_{fij,t-1}$ is a vector containing all possible interactions between a subset of the four variables r_t, H_j, $RE_{US,i,t-1}$, and $mrpk_{fij,t-1}$, and where $Z_{fij,t-1}$ is a vector of control variables containing all interactions between r_t, H_j, $s_{fij,t-1}$, and $mrpk_{fij,t-1}$ to control for size effects. Only coefficient estimates of interest are reported. The clustering of errors is performed twoways at the firm and year level. All regressors except monetary shocks have been standardized. t-statistics in parenthesis.

Finally, Column 4 controls for firm size and its interactions with r_t, H_j, and $mrpk_{fij,t-1}$. This is a way to control for balance sheet effects that may be correlated with firm size, as pointed out by Gopinath et al. (2017). Our results are robust to this inclusion.

6 Conclusions

What is the effect of declining interest rates on the efficiency of resource allocation and overall economic activity? We study this question in a setting where entrepreneurs with different productivities invest in capital, subject to financial frictions. We show that a fall in the interest rate has an ambiguous effect on aggregate output. In partial equilibrium, a lower interest rate raises aggregate investment both by relaxing borrowing constraints and by prompting relatively unproductive entrepreneurs to increase investment. In general equilibrium, however, this higher demand for capital raises its price. This crowds out investment by relatively more productive entrepreneurs, reallocating it to less productive ones. Contrary to the conventional wisdom, we show that a fall in interest rates can be contractionary if this general equilibrium is strong enough, which occurs when: (i) the supply of capital is sufficiently inelastic and (ii) the financial frictions are severe enough. We show that such contractionary effects arise because of an externality, whereby less productive entrepreneurs fail to internalize the effect of their capital demand on its price. We also show that such these effects can be temporarily masked by balance sheet effects resulting from the effect of interest rates on the price of capital. Empirical evidence from the US and Spain supports our model mechanism.

References

Adalet McGowan, M., D. Andrews, and V. Millot (2018). The walking dead? zombie firms and productivity performance in oecd countries. *Economic Policy 33*(96), 685–736.

Anderson, G. and A. Cesa-Bianchi (2020). Crossing the credit channel: credit spreads and firm heterogeneity.

Asriyan, V. (2021). Balance sheet channel with information-trading frictions in secondary markets. *The Review of Economic Studies 88*(1), 44–90.

Asriyan, V., L. Laeven, and A. Martin (2019). Collateral booms and information depletion.

Basco, S., D. Lopez-Rodriguez, and E. Moral-Benito (2018). Housing bubbles and misallocation: evidence from spain. Technical report, mimeo.

Benigno, G. and L. Fornaro (2014). The financial resource curse. *The Scandinavian Journal of Economics 116*(1), 58–86.

Benigno, G., L. Fornaro, and M. Wolf (2020). The global financial resource curse. *FRB of New York Staff Report* (915).

Berger, D., V. Guerrieri, G. Lorenzoni, and J. Vavra (2018). House prices and consumer spending. *The Review of Economic Studies 85*(3), 1502–1542.

Bernanke, B. S. et al. (2005). The global saving glut and the us current account deficit. Technical report.

Biais, B. and T. Mariotti (2009). Credit, wages, and bankruptcy laws. *Journal of the European Economic Association 7*(5), 939–973.

Bolton, P., T. Santos, and J. A. Scheinkman (2021). Savings gluts and financial fragility. *The Review of Financial Studies 34*(3), 1408–1444.

Brunnermeier, M. K. and Y. Koby (2018). The reversal interest rate. Technical report, National Bureau of Economic Research.

Caballero, R. J., E. Farhi, and P.-O. Gourinchas (2008). An equilibrium model of" global imbalances" and low interest rates. *American economic review 98*(1), 358–93.

Caballero, R. J., T. Hoshi, and A. K. Kashyap (2008). Zombie lending and depressed restructuring in japan. *American economic review 98*(5), 1943–77.

Caggese, A. and A. Pérez-Orive (2020). How stimulative are low real interest rates for intangible capital? Technical report, UPF working paper.

Cloyne, J., C. Ferreira, M. Froemel, and P. Surico (2018). Monetary policy, corporate finance and investment. Technical report, National Bureau of Economic Research.

Cloyne, J., C. Ferreira, and P. Surico (2020). Monetary policy when households have debt: new evidence on the transmission mechanism. *The Review of Economic Studies 87*(1), 102–129.

Coimbra, N. and H. Rey (2017). Financial cycles with heterogeneous intermediaries. Technical report, National Bureau of Economic Research.

Di Tella, S. (2017). Uncertainty shocks and balance sheet recessions. *Journal of Political Economy 125*(6), 2038–2081.

Doerr, S. (2018). Collateral, reallocation, and aggregate productivity: Evidence from the us housing boom. Unpublished paper.

García-Santana, M., E. Moral-Benito, J. Pijoan-Mas, and R. Ramos (2020). Growing like spain: 1995-2007. *International Economic Review 61*, 383–416.

Gopinath, G., Ş. Kalemli-Özcan, L. Karabarbounis, and C. Villegas-Sanchez (2017). Capital allocation and productivity in south europe. *Quarterly Journal of Economics 132*(4), 1915–1967.

Gorton, G. and G. Ordonez (2020). Good booms, bad booms. *Journal of the European Economic Association 18*(2), 618–665.

Hopenhayn, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica: Journal of the Econometric Society*, 1127–1150.

Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American economic review 95*(3), 739–764.

Jarociński, M. and P. Karadi (2020). Deconstructing monetary policy surprises—the role of information shocks. *American Economic Journal: Macroeconomics 12*(2), 1–43.

Jeenas, P. (2020). Firm balance sheet liquidity, monetary policy shocks, and investment dynamics. *Working Paper*.

Jordà, Ò. (2005). Estimation and inference of impulse responses by local projections. *American economic review 95*(1), 161–182.

Kaplan, G., B. Moll, and G. L. Violante (2018). Monetary policy according to hank. *American Economic Review 108*(3), 697–743.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of political economy 105*(2), 211–248.

Kiyotaki, N., J. Moore, and S. Zhang (2021). Credit horizons. Technical report, National Bureau of Economic Research.

Krishnamurthy, A. (2003). Collateral constraints and the amplification mechanism. *Journal of Economic Theory 111*(2), 277–292.

Leahy, J. V. and A. Thapar (2019). Demographic effects on the impact of monetary policy. Technical report, National Bureau of Economic Research.

Liu, E., A. Mian, and A. Sufi (2019). Low interest rates, market power, and productivity growth. Technical report, National Bureau of Economic Research.

Liu, Z., P. Wang, and T. Zha (2013). Land-price dynamics and macroeconomic fluctuations. *Econometrica 81*(3), 1147–1184.

Manea, C. (2020). Monetary policy with financially-constrained and unconstrained firms.

Martin, A., E. Moral-Benito, and T. Schmitz (2018). The financial transmission of housing bubbles: evidence from spain.

Martinez-Miera, D. and R. Repullo (2017). Search for yield. *Econometrica 85*(2), 351–378.

Mian, A. and A. Sufi (2011). House prices, home equity-based borrowing, and the us household leverage crisis. *American Economic Review 101*(5), 2132–56.

Moll, B. (2014). Productivity losses from financial frictions: Can self-financing undo capital misallocation? *American Economic Review 104*(10), 3186–3221.

Nakamura, E. and J. Steinsson (2018). High-frequency identification of monetary non-neutrality: the information effect. *The Quarterly Journal of Economics 133*(3), 1283–1330.

Ottonello, P. and T. Winberry (2020). Financial heterogeneity and the investment channel of monetary policy. *Econometrica 88*(6), 2473–2502.

Quadrini, V. (2020). The impact of industrialized countries' monetary policy on emerging economies. *IMF Economic Review 68*, 550–583.

Rajan, R. (2015). Competitive monetary easing: is it yesterday once more? *Macroeconomics and Finance in Emerging Market Economies 8*(1-2), 5–16.

Reis, R. (2013). The portuguese slump and crash and the euro crisis. *Brookings Papers on Economic Activity*, 143.

Saiz, A. (2010). The geographic determinants of housing supply. *The Quarterly Journal of Economics 125*(3), 1253–1296.

Schivardi, F., E. Sette, and G. Tabellini (2020). Identifying the real effects of zombie lending. *The Review of Corporate Finance Studies 9*(3), 569–592.

Slacalek, J., O. Tristani, and G. L. Violante (2020). Household balance sheet channels of monetary policy: A back of the envelope calculation for the euro area. *Journal of Economic Dynamics and Control 115*, 103879.

Tracey, B. (2019). The real effects of zombie lending in europe.

Ventura, J. and H.-J. Voth (2015). Debt into growth: how sovereign debt accelerated the first industrial revolution. Technical report, National Bureau of Economic Research.

Vom Lehn, C. and T. Winberry (2019). The investment network, sectoral comovement, and the changing us business cycle. Technical report, National Bureau of Economic Research.

A Proofs

Proof of Proposition 1. The capital market clearing condition is:

$$K^{S}(q) = \int_{q \cdot R}^{1} \frac{R}{q \cdot R - \lambda \cdot A} \cdot g(A) \cdot dA \cdot w, \tag{37}$$

and aggregate output, Y, is given by:

$$Y = \int_{q \cdot R}^{1} A \cdot \frac{R}{q \cdot R - \lambda \cdot A} \cdot g(A) \cdot dA \cdot w. \tag{38}$$

The derivative of aggregate output, Y, with respect to the interest rate, R, is:

$$\begin{aligned} \frac{dY}{dR} = & \int_{q \cdot R}^{1} \frac{A}{(q \cdot R - \lambda \cdot A)^2} \cdot \left(\left| \frac{dq}{dR} \right| - \frac{\lambda \cdot A}{R^2} \right) \cdot g(A) \cdot dA \cdot w \\ & + \frac{R}{1-\lambda} \cdot g(q \cdot R) \cdot w \cdot \left(R \cdot \left| \frac{dq}{dR} \right| - q \right), \end{aligned} \tag{39}$$

which, all else equal, is increasing in $|dq/dR|$. To arrive at Equation (12), we totally differentiate the capital market clearing condition and replace the last term in Equation (39).

Assume $\chi(\cdot)$ is locally twice differentiable, so that the capital supply elasticity is given by:

$$\varepsilon = \frac{q}{\chi''\left(\chi'^{-1}(q)\right) \cdot \chi'^{-1}(q)}. \tag{40}$$

From the capital market clearing condition (37), therefore, we have:

$$\left| \frac{dq}{dR} \right| = \frac{q \cdot \int_{q \cdot R}^{1} \frac{1}{(q \cdot R - \lambda \cdot A)^2} \cdot \frac{\lambda \cdot A}{R^2} \cdot g(A) \cdot dA \cdot w + \frac{q \cdot R}{1-\lambda} \cdot g(q \cdot R) \cdot w}{\varepsilon \cdot K^{S}(q) + q \cdot \int_{q \cdot R}^{1} \frac{1}{(q \cdot R - \lambda \cdot A)^2} \cdot g(A) \cdot dA \cdot w + \frac{R}{1-\lambda} \cdot g(q \cdot R) \cdot w}. \tag{41}$$

Thus, observe that, all else equal, $\left|\frac{dq}{dR}\right|$ is decreasing in the capital supply elasticity, ε, in equilibrium. It follows that dY/dR is decreasing in ε. Below, we will verify that it is indeed possible to change ε (by adjusting $\chi(\cdot)$) without affecting the equilibrium allocations.

Next, dY/dR is continuous in λ and, when $\lambda = 0$, aggregate output is given by:

$$Y = \left(\int_{q \cdot R}^{1} A \cdot \frac{g(A)}{1 - G(q \cdot R)} \cdot dA \right) \cdot K^{S}(q), \tag{42}$$

where:

$$K^S(q) = \frac{w}{q} \cdot (1 - G(q \cdot R)). \tag{43}$$

From the capital market clearing condition, it is clear that $q \cdot R$ must rise with R. Hence, it follows that:

$$\frac{d\left(\int_{q \cdot R}^{1} A \cdot \frac{g(A)}{1 - G(q \cdot R)} \cdot dA\right)}{dR} > 0, \tag{44}$$

that is, aggregate TFP is increasing in R. Moreover, since:

$$\frac{dY}{dR} = \left(\int_{q \cdot R}^{1} A \cdot \frac{g(A)}{1 - G(q \cdot R)} \cdot dA\right) \cdot K^S(q) + \left(\int_{q \cdot R}^{1} A \cdot \frac{g(A)}{1 - G(q \cdot R)} \cdot dA\right) \cdot \frac{dK^S(q)}{dR}, \tag{45}$$

we have that $dY/dR > 0$ provided that the capital supply elasticity, ε, is small enough, i.e., ε is below some $\bar{\varepsilon} > 0$. Fix such an $\varepsilon < \bar{\varepsilon}$. By continuity, there exists a threshold $\bar{\lambda}_\varepsilon$ such that $dY/dR > 0$ for all $\lambda < \bar{\lambda}_\varepsilon$.

Finally, to produce Figure 3, we consider the following parameterization of the capital supply schedule (which is equivalent to parametrizing the cost of capital production). Suppose that the interest rate is equal to R and let γ be defined as follows:

$$\bar{K} = \int_{\gamma \cdot R}^{1} \frac{1}{\gamma - \frac{\lambda \cdot A}{R}} \cdot w \cdot dG(A). \tag{46}$$

Consider the following family of capital supply schedules:

$$K^S(q; \varepsilon) = \max\left\{0,\ \bar{K} \cdot \left(1 + \varepsilon \cdot \frac{q - \gamma}{\gamma}\right)\right\},\ \varepsilon \in [0, \infty). \tag{47}$$

Note that, at the interest rate R, the equilibrium allocations are independent of ε, the elasticity of the capital supply. In particular, $K^S = \bar{K}$ and $q = \gamma$. But, as the interest rate changes, the equilibrium allocations will change since γ is a parameter and q will no longer equal γ. ■

Proof of Proposition 2. Consider the problem of the social planner:

$$\max_{\{k_A\}} \int A \cdot k_A \cdot dG(A) - R \cdot \left(\chi\left(\int k_A \cdot dG(A)\right) - w\right)$$

subject to:

$$R \cdot \left(\chi'\left(\int k_A \cdot dG(A)\right) \cdot k - w\right) \leq \lambda \cdot A \cdot k_A \quad (\gamma_A \cdot g(A)),$$

$$0 \leq k_A \quad (\omega_A \cdot g(A)).$$

In parentheses, we denote the multipliers on the constraints. We also suppose that the cost of capita production, $\chi(\cdot)$, is strictly convex (see below for the case of inelastic capital supply).

The first-order condition to the planner's problem are given by:

$$\frac{A}{R} - q - \chi''\left(K^S\right) \cdot \int \gamma_{\widehat{A}} \cdot k_{\widehat{A}} \cdot dG\left(\widehat{A}\right) = \gamma_A \cdot \left(q - \frac{\lambda \cdot A}{R}\right) - \omega_A \quad \forall A, \tag{48}$$

which together with the Kuhn-Tucker conditions characterize the solution to the problem. Since at the planner's allocation it must be that $q = \chi'\left(K^S\right) \geq \frac{\lambda}{R}$, it follows that there exists $0 < \widetilde{A} \leq 1$ such that $\omega_A = 0$ for all $A < \widetilde{A}$. There are thus two possibilities.

Case 1: $\widetilde{A} = 1$. In this case, $\omega_A > 0 = \gamma_A$ for $A < 1$ and, therefore, $\widetilde{A} > q^{CE} \cdot R$. Further, market clearing requires that $q^{SP} = \frac{\lambda}{R}$ and the capital stock and output are given by:

$$Y = K^{SP} = \chi^{-1}\left(\frac{\lambda}{R}\right),$$

and note that $q^{SP} < q^{CE}$.

Case 2: $\widetilde{A} < 1$. In this case, $\gamma_A > 0 = \omega_A$ for all $A > \widetilde{A}$ and after some algebra we have:

$$\widetilde{A} = q^{SP} \cdot R + \frac{\chi''\left(K^S\right) \cdot \int_{\widetilde{A}}^1 \left(A - q^{SP} \cdot R\right) \cdot \frac{1}{\left(q^{SP} - \frac{\lambda \cdot A}{R}\right)^2} \cdot w \cdot dG\left(A\right)}{1 + \chi''\left(K^S\right) \cdot \int_{\widetilde{A}}^1 \frac{1}{\left(q^{SP} - \frac{\lambda \cdot A}{R}\right)^2} \cdot w \cdot dG\left(A\right)}, \tag{49}$$

where

$$q^{SP} = \chi'\left(K^S\right) = \chi'\left(\int_{\widetilde{A}}^1 \frac{1}{q^{SP} - \frac{\lambda \cdot A}{R}} \cdot w \cdot dG\left(A\right)\right). \tag{50}$$

Clearly, again, $\widetilde{A} > q^{SP} \cdot R$ since the RHS in Equation (49) is positive. Note that, from Equation (50), the capital price q^{SP} is depressed below q^{CE} since the entrepreneurs who invest do so until their financial constraints bind, but there are fewer such entrepreneurs.

Next, note that when R falls, the planner can always ensure the same equilibrium allocations (with unchanged equilibrium price of capital, q^{SP}), in which case all supra-marginal entrepreneurs' financial constraints become slack. Moreover, it is clear that the planner would never want to reduce both $K^S(q^{SP})$ and Y^{SP} in response to a fall in R, since the aggregate productivity of capital (TFP) is higher than R times the marginal cost of producing a unit of capital, χ'^{S}), both before and after the fall in R. Finally, it follows that the decline in R must be expansionary. For instance, the planner can always increase $\widetilde{A}$ and k_A for $A > \widetilde{A}$ so that K^S is unchanged but TFP increases. ■

B Complementary Appendix

B.1 Unconstrained firms

Consider a variation of our baseline economy where, in addition to entrepreneurs, there is a mass of firms that do not face financial constraints, i.e., they are unconstrained. For concreteness, we assume that these firms are owned and operated by the capitalists. In particular, assume that these firms have a production technology that can converts k units of capital in $t = 0$ into $f(k)$ units of the consumption good in $t = 1$, where f is twice differentiable with $f(0) = 0$, $f'(\cdot) > 0$ and $f''(\cdot) < 0$.

For simplicity, suppose that the capital supply is inelastic and entrepreneurs cannot pledge their output to creditors, i.e., $\lambda = 0$. Lastly, to ensure that some entrepreneurs operate side-by-side with these firms, we assume that $f'(\bar{K}) < 1$; that is, the marginal product of capital in these firms – if they were to employ the entire stock of capital – is lower than that of the most productive entrepreneur.

In this economy, there are two types of equilibria depending on whether the interest rate, R, is above or below the threshold $\widetilde{R}$.[17] If $R > \tilde{R}$, then the unconstrained firms are inactive, $R \cdot q > f'(0)$, and the analysis of the previous section applies. If $R < \tilde{R}$, the unconstrained firms are active and $R \cdot q = f'(K^T(q, R)))$, where $K^T(q, R) > 0$ denotes the aggregate units of capital employed by these firms; it only depends on (and is decreasing in) $R \cdot q$. In this case, market clearing requires that:

$$\bar{K} = K^T(q, R) + \int_{q \cdot R}^{1} \frac{w}{q} \cdot dG\left(A\right), \tag{51}$$

It follows that in equilibrium a fall in R raises the price of capital q (and weakly reduces $R \cdot q$). The response of aggregate output to changes in the interest rate is given by:

$$\frac{dY}{dR} = \int_{R \cdot q}^{1} (A - R \cdot q) \cdot \frac{d}{dR}\left(\frac{w}{q}\right) \cdot dG(A), \tag{52}$$

which note only features the capital-reallocation effect (as capital supply is fixed). Moreover, since w/q decreases in response to a fall in R, we have that $dY/dR > 0$.

[17] If $\lim_{k\to 0} f'(k) = \infty$, then $\widetilde{R} = \infty$. Otherwise, $\tilde{R}$ is implicitly defined by:

$$\int_{\widetilde{R} \cdot q}^{\bar{A}} \frac{w}{q} \cdot dG\left(A\right) \equiv \bar{K},$$

with $q = f'(0) \cdot \widetilde{R}^{-1}$. At this interest rate, the entrepreneurs are just able to absorb the entire capital stock $\bar{K}$.

What is going on? When the unconstrained firms are active, the price of capital is determined by their marginal product of capital, since $R \cdot q = f'(K^T)$. Thus, any decline in the interest rate must be compensated by an increase in the price of capital, which implies that the demand of capital by supra-marginal entrepreneurs must fall. Ultimately, a decline in the interest rate simply redistributes capital from supra-marginal entrepreneurs to the less productive unconstrained firms, thereby reducing output. Although this example with fixed capital stock and no credit whatsoever is stark, it is straightforward to prove the analogue of Proposition 1 for this setting: namely, as the capital supply becomes less elastic, dY/dR increases and becomes positive provided that λ is below a threshold.

B.2 Diminishing returns at entrepreneur level

Consider a variation of our baseline economy in which entrepreneurial technology has diminishing returns. In particular, suppose that an entrepreneur with productivity A can convert k of capital at $t = 0$ into $A \cdot h(k)$ units of the consumption good at $t = 1$, where $h(\cdot)$ satisfies $h'(\cdot) > 0$, $h''(\cdot) < 0$ and $\lim_{k \to 0} h'(k) \to \infty$. As before, A is distributed according to the cdf G on interval $[0, 1]$ that has an associated density g. For simplicity, assume that the capital supply is inelastic and entrepreneurs cannot pledge their output to creditors, i.e., $\lambda = 0$.

Optimal investment of entrepreneur with productivity A is given by:

$$k_A(q, R) = \min \left\{ h'^{-1}\left(\frac{R \cdot q}{A}\right), \frac{w}{q} \right\}. \tag{53}$$

Therefore, it follows that there is a cut-off productivity give by:

$$\widetilde{A} = \frac{R \cdot q}{h'\left(\frac{w}{q}\right)},$$

such that all entrepreneurs with productivity $A > \widetilde{A}$ are constrained by their resources while the rest are unconstrained.

In this economy, the market clearing condition for capital is given by:

$$\bar{K} = \int_0^{\widetilde{A}} h'^{-1}\left(\frac{R \cdot q}{A}\right) \cdot g(A) \cdot dA + \frac{w}{q} \cdot (1 - G(\widetilde{A})), \tag{54}$$

and aggregate output is:

$$Y = \int_0^{\widetilde{A}} A \cdot h'^{-1}\left(\frac{R \cdot q}{A}\right) \cdot g(A) \cdot dA + \int_{\widetilde{A}}^1 A \cdot \frac{w}{q} \cdot g(A) \cdot dA. \tag{55}$$

Combining Equations (54) and (55), we have:

$$\frac{dY}{dR} = \int_0^{\widetilde{A}} \left(A - \int_{\widetilde{A}}^1 x \cdot \frac{g(x)}{1 - G(x)} \cdot dx\right) \cdot \frac{d}{dR}\left(h'^{-1}\left(\frac{R \cdot q}{A}\right)\right) \cdot dA. \tag{56}$$

From Equation (53) and the market clearing condition, we have that $d\left(h'^{-1}\left(R \cdot q/A\right)\right)/dR < 0$ since h is concave and $R \cdot q$ is increasing in R. Moreover, it is clear that $A < \int_{\widetilde{A}}^1 x \cdot \frac{g(x)}{1-G(x)} \cdot dx$ for $A \leq \widetilde{A}$. Hence, it follows that dY/dR is positive.

What is the intuition? In partial equilibrium, as R falls the investment of unconstrained entrepreneurs expands while the investment of constrained entrepreneurs is unchanged. This implies that, in general equilibrium, the price of capital must rise. Ultimately, exactly as in our baseline economy with linear technology, the decline in R simply reallocates capital from entrepreneurs with $A > \widetilde{A}$ to entrepreneurs with $A < \widetilde{A}$, thereby reducing output. Again, although this example with fixed capital stock and no credit whatsoever is stark, it is straightforward to prove the analogue of Proposition 1 for this setting: namely, as the capital supply becomes less elastic, dY/dR increases and becomes positive provided that λ is below a threshold.

B.3 Closed economy: endogenous interest rates and savings gluts

Throughout our main analysis, we considered a small open economy that experienced an exogenous fall in the world interest rate. In this appendix, we show that none of our main insights would change if the economy were closed, but where the fall in the interest rate instead were the result of a savings glut, i.e., an increase in the economy's desired savings.

Suppose now that the economy is closed, that the agents preferences are given by:

$$U = E_0\{c_0 + \beta \cdot c_1\} \tag{57}$$

for some $\beta \in (0, 1)$, and that the capitalists have an endowment $w^C > 0$ of the consumption good at $t = 0$. Given these adjustments, we next show that the main results from our baseline setting can be obtained by raising the desired savings in this economy.

Proposition 3 *The effects of a fall in the interest rate, R, as described in Proposition 1 are isomorphic to those of an increase in w^C and/or β.*

In what follows, we illustrate the proof of this result. First, note that the equilibrium interest rate, R, must be greater than β^{-1}. Otherwise, there would be a positive credit demand but no savings, as all agents who do not invest in capital would want to consume; hence, the credit market would not clear.

Second, observe that, given prices $\{q, R\}$, the aggregate savings of the savers (i.e., the capitalists and entrepreneurs with productivity $A < q \cdot R$ are given by:

$$S(q,R) \begin{cases} = w^C + q \cdot K^S(q) - \chi(K^S(q)) + w \cdot G(q \cdot R) & \text{if } R > \beta^{-1}, \\ \in [0,\ w^C + q \cdot K^S(q) - \chi(K^S(q)) + w \cdot G(q \cdot R)] & \text{if } R = \beta^{-1} \end{cases}. \tag{58}$$

Equation (58) states that if $R > \beta^{-1}$, then the savers save all their resources, which are given by their endowments of the consumption good, the market value of capital, less the costs of producing the capital. If $R = \beta^{-1}$, then the savers are indifferent between saving and consuming these resources. As a result, the credit market clearing condition is given by:

$$S(q,R) = \int_{q \cdot R}^{1} b_A(q,R) \cdot dG(A), \tag{59}$$

which together with Equations (5), (6), (8), (9) and (10), characterizes the equilibrium.

Lastly, observe that the aggregate credit demand can be expressed as:

$$\int_{q \cdot R}^{1} b_A(q,R) \cdot dG(A) = q \cdot K^S(q) - w \cdot (1 - G(q \cdot R)), \tag{60}$$

since the entrepreneurs, who invest in capital, use all of their endowment plus borrowing to finance purchases of capital.

Therefore, we can immediately see that there are two possibilities in equilibrium.

Case 1. Consider a candidate equilibrium where the interest rate, R, is equal to β^{-1}. For this to be an equilibrium, it must be that:

$$w^C + q \cdot K^S(q) - \chi(K^S(q)) + w \cdot G(q \cdot \beta^{-1}) \geq q \cdot K^S(q) - w \cdot (1 - G(q \cdot R)), \tag{61}$$

which holds if and only if:

$$w^C + w \geq \chi(K^S(q)), \tag{62}$$

where the equilibrium price of capital, q, satisfies clears the capital market:

$$K^S(q) = \int_{q\cdot R}^{1} k_A(q, \beta^{-1}) \cdot dG(A). \tag{63}$$

It is therefore immediate that in this case the effects of an increase β on the aggregate capital and output are isomorphic to those of a fall in R analyzed in Section 3. Moreover, observe that this candidate is an equilibrium if w^C and/or β are large enough.

Case 2. Consider a candidate equilibrium where the interest rate, R, is above β^{-1}. This candidate is in turn equilibrium if at $R = \beta^{-1}$, the inequality (62) is violated, i.e., if w^C and/or β are small. Hence, in this case, the equilibrium prices $\{q, R\}$ are such that:

$$w^C + w = \chi(K^S(q)), \tag{64}$$

and

$$K^S(q) = \int_{q\cdot R}^{1} k_A(q, R) \cdot dG(A). \tag{65}$$

Here, a rise in w^C raises the capital price (as $\chi(K^S(q))$ is increasing in q) and lowers the interest rate (to offset the effect of a higher q that depresses capital demand). Hence, the effects of an increase in w^C on the aggregate capital and output are isomorphic to those of a fall in R analyzed in Section 3.

Lastly, note that if the equilibrium is initially in Case 2, then an increase in w^C eventually moves the equilibrium into Case 1.

B.4 Our mechanism in the Kiyotaki-Moore model

In this Appendix, we show that the capital-reallocation effects induced by falling interest rates that we emphasized through the main text are also present in the class macro-finance model of Kiyotaki and Moore (1997). As in our dynamic setup of Section 4, however, in that model as well these effects will be temporarily masked by the balance-sheet channel.

Time is now infinite, $t = 0, 1, ...$ Assume, for simplicity, that all entrepreneurs in the modern sector have the same productivity $A \in (0, 1)$, and that the capital stock is fixed at $\bar{K} > 0$. Thus, aggregate output in any period t depends solely on the allocation of capital between the modern and traditional sectors:

$$Y_t = A \cdot K_t + a \cdot f(\bar{K} - K_t), \tag{66}$$

where K_t denotes the aggregate stock of capital employed in the modern sector at time t.

We focus on equilibria in which the traditional sector is active in all periods and, hence, its demand for capital is given by:

$$\frac{a \cdot f'\left(\bar{K} - K_{t+1}\right) + q_{t+1}}{q_t} = R, \tag{67}$$

i.e., the return to capital within the traditional sector must equal the interest rate.

As in the static model, we introduce a financial friction by assuming that – in any period – an entrepreneur can walk away with a fraction $1 - \lambda$ of her resources, which now include her output and the market value of her capital. It thus follows that entrepreneurs face the following borrowing constraint:

$$R \cdot B_t \leq \lambda \cdot (A + q_{t+1}) \cdot K_{t+1}, \tag{68}$$

where B_t and K_{t+1} respectively denote entrepreneurial borrowing and investment in period t.[18] Note that, since all entrepreneurs are identical, B_t and K_{t+1} also represent aggregate borrowing and investment in the modern sector.

In any period t, the net worth of entrepreneurs equals the sum of their output and the market value of their capital minus repayments to creditors: $A \cdot K_t + q_t \cdot K_t - R \cdot B_{t-1}$. We assume that entrepreneurs consume a fraction $1 - \rho$ of this net worth in every period, where $\rho \cdot R < 1$.[19] This ensures, in the spirit of Kiyotaki and Moore (1997), that the financial constraint holds with equality in all periods. As a result, the modern-sector demand for capital is given by:

$$K_{t+1} = \frac{1}{q_t - \lambda \cdot \frac{A + q_{t+1}}{R}} \cdot \rho \cdot (1 - \lambda) \cdot (A + q_t) \cdot K_t, \tag{69}$$

where we make parametric assumptions to ensure that both sectors are active in a neighborhood of the steady state.[20]

Thus, given an initial value for $K_0 > 0$ and a no bubbles condition on the price of capital,

[18] In Kiyotaki and Moore (1997), the output of investment is not pledgeable but the resale value of capital is fully pledgeable. Although our results would also go through under that specification, we have chosen the current specification in order to preserve symmetry with the baseline model of Section 2.

[19] E.g., it is sufficient to assume that entrepreneurs have log-preferences, i.e., $U^E = \sum_{t=0}^{\infty} \rho^t \cdot log(c_t)$. Note that the preferences of other agents (i.e., capitalists and traditional investors) are irrelevant for the evolution of q_t, K_t and Y_t.

[20] In particular, if K_0 is close to steady state, this requires that:

$$\frac{a \cdot f'(0)}{R - 1} > \frac{R \cdot \rho \cdot (1 - \lambda) + \lambda}{R - R \cdot \rho \cdot (1 - \lambda) - \lambda} \cdot A > \frac{a \cdot f'(\bar{K})}{R - 1}.$$

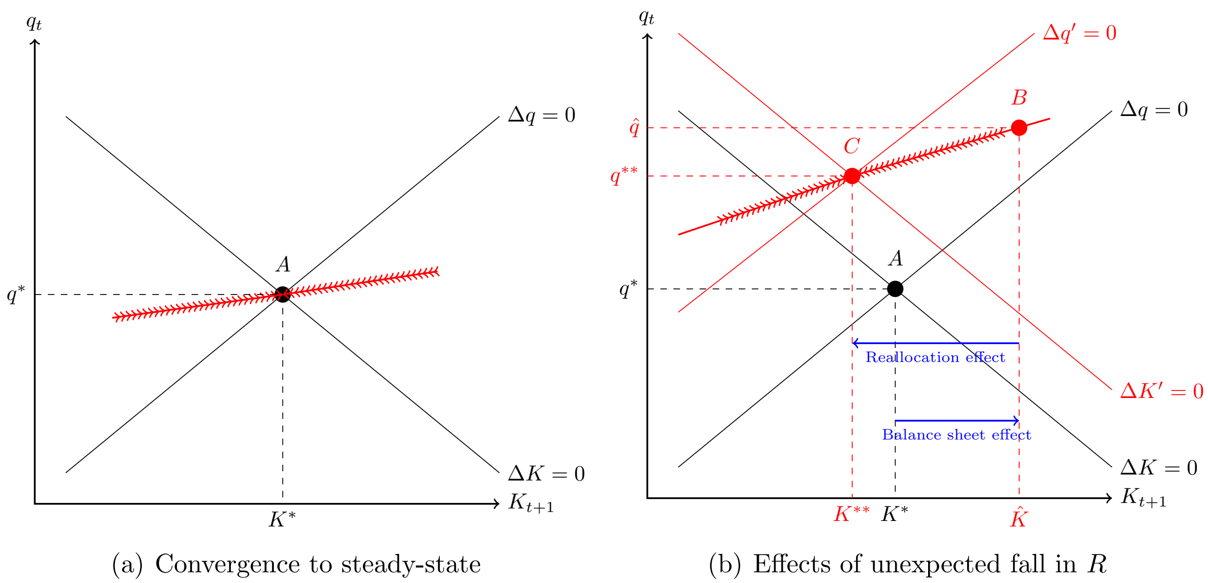


Figure 9: **Equilibrium dynamics and balance sheet effects.** The figure illustrates a phase diagram for the joint evolution of the price of capital and the stock of capital in the modern sector. The saddle path of the system is depicted by a red curve with arrows pointing to the steady state: the left panel depicts the dynamics before the unexpected decline in the interest rate, whereas the right panel depicts the dynamics after it.

Equations (67) and (69) fully characterize the equilibrium of this economy. Panel (a) of Figure 9 portrays the equilibrium dynamics with the help of a phase diagram in the (K_{t+1}, q_t)-space. The $\Delta q = 0$ locus depicts all the combinations of K_{t+1} and q_t for which Equation (67) is satisfied with $q_t = q_{t+1}$. The locus is upward sloping because a higher level of modern-sector investment, K_{t+1}, is associated with a higher productivity of capital in the traditional sector and – since capital is priced by this sector – with a higher level of q_t. The $\Delta K = 0$ locus depicts instead all the combinations of K_{t+1} and q_t for which Equation (67) is satisfied with $K_t = K_{t+1}$. The locus is downward sloping because a higher level of modern-sector investment, K_{t+1}, is only affordable to constrained entrepreneurs if the equilibrium price of capital, q_t, is lower. As the figure shows, the system displays saddle-path dynamics. From an initial condition $K_0 < K^*$, both K and q increase monotonically as the economy transitions to the steady state and modern-sector entrepreneurs accumulate net worth. The opposite dynamics follow from an initial condition $K_0 > K^*$.

The right-hand panel of Figure 9 portrays the response to a permanent and unanticipated decline in R in a given period t_0. In response to a lower R, both loci shift upwards. The $\Delta q = 0$ locus shifts up because the traditional sector's willingness to pay for capital increases alongside the net present value of dividends; the $\Delta K = 0$ also shifts up because entrepreneurs' ability to pay for capital increases as lower interest rates relax their borrowing constraint. The presence of financial frictions, however, mitigates the shift in the $\Delta K = 0$ locus. Thus, as the figure shows, a decline in R triggers an increase in the steady-state price of capital to q^{**}, and a reduction in the capital employed in the modern sector to K^{**}. Hence, a reduction in the interest rate leads to a fall in the steady-state level of output despite the presence of dynamics.

This does not mean, however, that balance sheet effects do not play a role. Indeed, on impact, in response to a decline in the interest rate, the value of capital increases from $q^* \cdot K^*$ to $q_{t_0} \cdot K^*$ while entrepreneurial debt payments - which are pre-determined - remain unaffected and equal to $R \cdot B^*$.[21] Therefore:

$$K_{t+1} = \begin{cases} \frac{1}{q_t - \lambda \cdot \frac{A+q_{t+1}}{R}} \cdot \rho \cdot ((1-\lambda) \cdot (A+q^*) + q_t - q^*) \cdot K^* & \text{if } t = t_0 \\ \frac{1}{q_t - \lambda \cdot \frac{A+q_{t+1}}{R}} \cdot \rho \cdot (1-\lambda) \cdot (A+q_t) \cdot K_t & \text{if } t > t_0 \end{cases}. \tag{70}$$

The evolution of q_t is still given by Equation (67). This means that the adjustment of K to the new steady-state is not monotonic. As the right-hand panel of Figure 9 shows, K_{t+1} rises to $\widehat{K}$ on impact: this, as stated in the figure, is the balance sheet effect. The expansion of the modern sector is short-lived, though, since from that period onwards the economy evolves along the saddle-path towards its new steady state, which features a higher price of capital but a lower capital stock in the modern sector and thus a lower level of output. This decline from $\widehat{K}$ to K^{**} is, as stated in the figure, due to the reallocation effect: the higher demand of capital by the traditional sector keeps capital prices high, and these slowly erode the net worth of modern-sector entrepreneurs. As a result, the dynamic behavior of aggregate output in this economy resembles closely that of the dynamic economy in Section 4, illustrated in Figure 7.

The key takeaway is that the same reallocation forces that we analyzed in our baseline model of Section 2 are also at work in a dynamic environment. Moreover, these forces are persistent in response to a permanent decline in the interest rate, while the balance-sheet effects that are often highlighted in the literature are transitory. To be sure, an unexpected decline in the interest rate does have an initial balance-sheet effect that benefits productive entrepreneurs and reallocates capital towards them, raising average productivity and output. But this effect is by nature temporary: the reason is that it represents a one-time shock to the level of entrepreneurial net worth, but it does not affect the dynamic evolution of net worth thereafter.

[21] As in Kiyotaki and Moore (1997), these balance sheet effects require that entrepreneurs' debt payments are not indexed to the price of capital.

C Empirical Appendix

This Appendix describes the datasets and definitions of the variables used in the empirical analysis in Section 5. The description is in two parts: the US dataset and the Spanish dataset.

C.1 US dataset

C.1.1 Data and variables

Our analysis uses US firm-level data across regions and sectors from the US Bureau of Economic Analysis (BEA), combined with information on the real-estate component of fixed asset holdings and investments at the sectorial level from the BEA. We supplement this data using information on monetary policy changes and real-estate supply elasticities at the MSA level. Given our focus on the use of real estate in production, we exclude the financial services industries, real-estate agents sector and the construction sector. The sample period is 2001 to 2019. We define industry groupings based on the BEA's industry classification, which follows the most recent 2016 North American Industry Classification System (NAICS) of industry codes, which covers a total of 17 sectors that are a combination of two digit and three digit level NAICS codes. For a detailed description of the construction of the dataset and definitions of the control variables, see Appendix C.1.2.

The growth rate of real GDP, Δy_{ijt}, is computed at the sector-MSA level using the US Bureau of Economic Analysis (BEA)'s Regional Accounts. GDP is expressed in chained 2012 US dollars and growth rates are constructed using log changes.

We compute two sectorial measures of real estate intensity. The first measure, RE_{it}, is constructed on a stock basis, as the ratio of real estate assets to total fixed assets. The second measure, $REinv_{it}$, is constructed on a flow basis, as the ratio of real estate investment to total fixed investments. Both measures are computed using the BEA Fixed Assets tables. Total fixed assets is computed as the sum of all non-residential fixed assets, including equipment, machinery, land, buildings, and intellectual property. The stock-based measure is our baseline measure of real estate intensity.

We use three alternative measures of monetary policy changes. First, the high-frequency monetary shock from Jarociński and Karadi (2020) obtained with median rotation sign restrictions, mp_med_t, computed by aggregating daily shocks during each year. This is our baseline measure. Second, the high-frequency based monetary shock from Jarociński and Karadi (2020) obtained with poor man's sign restrictions, mp_pm_t, also computed by aggregating daily shocks during each year. Both of these shocks extract the first principal component of surprises in interest rate futures with maturities from one month to one year, within a short window of 30 minutes around the times of the Federal Reserve's monetary policy announcements. They extend the approach in Nakamura and Steinsson (2018) by stripping out information shocks from policy shocks using a VAR model with sign restrictions. Both of these measures are true shocks in the sense that they are exogenous to current economic activity. Third, the annual change in the Federal Funds rate, Δr_t, is obtained from the Federal Reserve Bank of St. Louis' FRED database. This variable proxies for the change in nominal short-term interest rates.

Finally, the elasticity of housing supply at the MSA level, denoted H_j, is obtained from Saiz (2010). To capture balance-sheet effects related to financial leverage, we compute a sector-level measure of the leverage ratio, $Leverage_{i,t}$, computed as the median of the ratio of short and long term debt to total assets using firm-level data from the COMPUSTAT database.

Table 3 reports the descriptive statistics of our main variables. In our sample, real estate is a sizeable fraction of the fixed assets that corporations hold on their balance sheet. For the median sector in the sample, real estate represents 62 percent of total fixed assets. Real-estate supply is quite elastic, with an elasticity of 2 for the average MSA. The interest rate (shocks) data indicate that our sample period includes both periods of monetary expansions and contractions. Annual changes in short-term interest rates ranged from -4.1 to $+2$ percentage points, with a median of 0.

C.1.2 Dataset construction and definition of the control variables

The US dataset is created at the sector-region-year level primarily using data from the US BEA. We use MSA as unit for regions (using the latest MSA classification available from US Census or BEA) and we use the BEA sector classification, which is based on a combination of various NAICS sectors. This includes a total of 17 sectors, at either the two-digit or three-digit NAICS level: Agriculture, forestry, fishing and hunting (NAICS 11), Mining, quarrying, and oil and gas extraction (NAICS 21), Utilities (NAICS 22), Durable goods manufacturing (NAICS 321,327-339), Nondurable goods manufacturing (NAICS 311-316, 322-326), Wholesale trade (NAICS 42), Retail trade (NAICS 44-45), Transportation and warehousing (NAICS 48-49), Information (NAICS 51), Professional, scientific, and technical services (NAICS 54), Management of companies and enterprises (NAICS 55), Administrative and support and waste management and remediation services (NAICS 56), Educational services (NAICS 61), Health care and social assistance (NAICS 62), Arts, entertainment, and recreation (NAICS 71), Accommodation and food services (NAICS 72), Other services except government and government

Table 3: **Summary statistics of US sample**

VARIABLES	Mean	Median	St. Dev.	Min	Max	N. Obs.
$\Delta GDP_{i,j,t}$	.018	.021	.139	-2.314	2.927	17,816
mp_med_t	.019	.022	.109	-.159	.181	18
mp_pm_t	.043	.055	.128	-.321	.198	18
Δr_t	-.000	.000	.013	-.041	.020	18
H_j	2.011	1.795	1.015	.670	5.450	74
$RE_{i,t}$	.598	.621	.202	.258	.928	324
$RE\_inv_{i,t}$	.265	.206	.205	.049	.898	324
$Leverage_{i,t}$	.241	.229	.099	.025	.562	306

Notes: ΔGDP_{ijt} is the real GDP growth rate at the sector-MSA level for a given year from the US Bureau of Economic Analysis (BEA)'s Regional Accounts. GDP is expressed in chained 2012 US dollars and growth rates are constructed using log changes. mp_med_t is the high-frequency monetary shock from Jarociński and Karadi (2020), obtained with median rotation sign restrictions, aggregated over each year. H_j is the elasticity of real-estate supply at the MSA level from Saiz (2010). mp_pm_t is the high-frequency monetary shock from Jarociński and Karadi (2020), obtained with poor man's sign restrictions, aggregated over each year. δr_t is the change in the Federal Funds rate over the year. H_j is the elasticity of housing supply at the MSA level from Saiz (2010). RE_{it} is the ratio of real estate assets to total fixed assets of the sector in a given year from the BEA fixed assets tables. RE_inv_{it} is the ratio of real estate investment to total fixed investment of the sector in a given year from the BEA fixed assets tables. $Leverage_{it}$ is the leverage ratio of the sector, computed at the sector-year level as the median across firms of the ratio of short and long term debt over total assets.

enterprises (NAICS 81). We exclude firms operating in the construction (NAICS 23), finance and insurance (NAICS 52), and real estate (NAICS 53) sectors. We limit the sample to those 95 MSAs for which we have information on the housing supply elasticity. This leaves a sample of 1,512 region-year observations and 107,390 sector-region-year observations for the period 2001 to 2019.

Economic activity. We measure the level of real GDP and the growth rate of real GDP at the sector-MSA level using the Local Area Gross Domestic Product from the US BEA Regional Economic Accounts, available at `https://www.bea.gov/data/economic-accounts/regional`. We use the Real GDP by county and metropolitan area (CAGDP9) data files contained in the CAGDP9.zip file for the period 2001-2019. GDP is expressed in chained 2012 US dollars and growth rates are constructed using log changes, using annual data.

Real-estate intensity. We compute real-estate intensity at the NAICS sector-level using the Nonresidential detailed estimates from the BEA Nonresidential Industry Fixed Assets tables. These can be downloaded at: `https://www.bea.gov/data/investment-fixed-assets/industry`. We construct two alternative variables based on fixed assets or investments, respectively: (i) the ratio of real estate assets to total fixed assets, and (ii) the ratio of real estate investment to total fixed investments. These are computed as (i) the share of nonresidential real estate in net fixed assets (at fixed cost) from the net stocks at fixed cost table available at `https://apps.bea.gov/national/FA2004/Details/xls/detailnonres_stk2.xlsx` and (ii) the share of nonresidential real estate in net fixed investment (at fixed cost) from the investment at fixed cost table, available at `https://apps.bea.gov/national/FA2004/Details/xls/detailnonres_inv2.xlsx`.

Monetary shocks. We measure short-term rates using the effective Federal Funds rate from the Federal Reserve's FRED database. The data can be downloaded at: `https://fred.stlouisfed.org/series/FEDFUNDS`. We use the change in the Federal funds rate over the year as one measure of monetary shocks. We measure inflation rates using the CPI inflation rate from the Federal Reserve's FRED database available from `https://fred.stlouisfed.org/series/FPCPITOTLZGUSA`. We use the annual change in the Federal funds rate adjusted for inflation as a proxy for the change in the real short-term rate. Our alternative proxies for monetary shocks use cumulative high-frequency monetary shocks from Jarociński and Karadi (2020). The original dataset called jarocinski-karadi.zip is available fro the AEJ Macro website and provides data for the period 1990 to 2016. We are grateful to Peter Karadi for providing us with an updated version of the dataset that extends the monetary shocks until mid-2019. The dataset provides quarterly monetary shocks. We transform these quarterly shocks into annual shocks by aggregating the quarterly shocks for each year.

Land supply elasticity. Local real-estate supply elasticities for a total of 95 MSAs are obtained from Table VI in Saiz (2010), which is available from `https://academic.oup.com/qje/article/125/3/1253/1903664?login=true`. These elasticities capture the amount of local land that can be developed and are estimated using satellite-generated images of the terrain. We transform this dataset into the latest available MSA codes, as of March 2020, as defined by the United States Office of Management and Budget (OMB), by averaging elasticities across merged MSAs in those few cases where MSA definitions have changed because of the merging of several regions.

Control variables. Financial leverage is computed using data from COMPUSTAT (downloaded from WRDS) and defined as the ratio of short-term and long-term debt (DLC+DLTT) divided by the book value of total assets (AT). We construct a sectorial measure of financial leverage by taking the median of financial leverage across firms by sector. We compute this variable by matching the COMPUSTAT NAICS codes to the BEA sector codes using the concordance table available from the BEA website.

C.2 Spanish dataset

C.2.1 Data and variables

We start with the universe of Spanish firms from the Orbis database. There are a total of $290,752$ distinct firms in the dataset. We collect information on the industry, i, in which the firm operates at the two-digit NACE level. There are a total of 80 industries in the dataset. We exclude sectors that produce real estate (the construction and real estate sectors) and financial intermediaries (the financial services sector), which leaves 73 industries. We also collect information on the province, j, where the firm is located. There are a total of 52 provinces in the dataset (50 proper provinces plus the autonomous cities of Ceuta and Melilla). The dataset covers a total of 18 years, t, for the period 2000-2017. (This period differs slightly from the US data which covers the period 2001-2019. The reason is that we need 2000 data to compute growth rates for 2001, the 2019 data is sparsely covered in the Orbis database, and the sector-level deflator for gross value added from KLEMS is not available post-2017.)

In addition to $mrpk_{fijt}$ as outlined above, we compute the following variables: $\Delta Output_{fijt}$ is gross output growth, computed as $ln(Output_{fijt}/Output_{fij,t-1})$, where $Output_{fijt}$ is the operating revenue of firm f. Any sectorial level price effects of output growth will be absorbed by the inclusion of sector-year fixed effects. As monetary shock, r_t, we use the monetary policy shock for the euro area from Jarociński and Karadi (2020) with median rotation sign restrictions. As before with the US monetary shocks, we aggregate daily shocks over the year to construct an annual time series of monetary shocks. Since monetary policy in the euro area is implemented by the ECB with respect to overall euro area economic conditions, this monetary shock variable is truly exogenous to Spain.

We measure the elasticity of housing supply in province j, H_j, by averaging the housing-elasticity measure of Basco et al. (2018), which is computed at the municipal-level for the year 1995, across all municipalities in the province. To avoid large outliers, we winsorize this variable at the 5%-95% levels. Since the Orbis database does not offer any information on the real estate assets of firms, we use the US-based measure of real-estate intensity at the sector level used in Table 1, computed as the share of real-estate-related fixed assets in total fixed assets. We denote this variable $RE1_{US,i,t}$.[22]

Gopinath et al. (2017) find that misallocation during the housing boom period in Spain was associated with firm size, with effects being more pronounced for small firms that are more likely to face borrowing constraints. To account for size effects, we construct a size measure, $s_{f,i,j,t}$, computed as log of a firm's total assets.

Table 4 displays the summary statistics of the main variables in the Spanish sample. Compared to the US sample, the regional variation in housing elasticity is somewhat smaller though still substantial. The dispersion in output growth at the firm level is somewhat larger in the Spanish sample compared to the dispersion in GDP growth across sectors and regions in the US sample. Importantly, there is substantial variation in the log of the marginal revenue product of capital across Spanish firms, with a standard deviation of 1.3.

C.2.2 Dataset construction

Our main dataset for Spain combines firm-level data from the Bureau Van Dijk's Orbis database with measures of annual monetary policy shocks and regional housing-supply elasticity.

Firm-level data. We follow Gopinath et al. (2017) in compiling firm-level data from the Orbis database. Our sample contains annual financial data for 290,752 distinct firms during the period 2000-2017. Firms are classified into different two-digit economic activities following the EU NACE Rev. 2 standardized classification, and the firm postal code allows us to map the location of each firm into each one of the Spanish provinces [23] We index firms by f, industries by

[22] We match this variable to the Orbis dataset at the 2-digit NACE level using the concordance tables for the correspondence between 2002 NAICS codes and NACE Rev. 1.1 codes available from the US Census (2020).

[23] We consider the 50 proper provinces in Spain plus the two autonomous cities of Ceuta and Melilla.

Table 4: **Summary statistics of Spanish sample**

	mean	median	st. dev	min	max	num. obs.
$\Delta Output_{fijt}$	0.021	0.018	0.314	-4.410	4.041	1,052,222
r_t	0.031	0.002	0.098	-0.105	0.267	18
H_j	0.712	0.818	0.240	0.264	0.981	43
$RE_{US,it}$	0.524	0.490	0.187	0.275	0.928	306
$mrpk_{fijt}$	-0.899	-0.948	1.267	-8.773	6.501	1,452,887
$s_{f,i,j,t}$	13.555	13.478	1.430	8.063	19.291	1,101,585

Notes: $\Delta Output_{fijt}$ is output (gross revenues) growth at the firm level; r_t is the monetary policy shock for the euro area from Jarociński and Karadi (2020) with median rotation sign restrictions, aggregated over the year; H_j is the average real-estate elasticity from Basco et al. (2018) at the province level in the year 1995; $RE1_{US,j,t}$ is the share of real-estate related fixed assets in total fixed assets of the sector based on US data and computed at the two-digit NACE level; $mprk_{fijt}$ is the log of the marginal revenue product of capital computed following Gopinath et al. (2017); and $s_{f,i,j,t}$ is the log of the firm's total assets.

i, regions by j, and time by t. Our main variable of interest is gross output growth $\Delta output_{fijt}$, constructed as the log-change in operating revenues. In order to proxy for how productive a firm is, we follow Gopinath et al. (2017) in computing the (log) marginal revenue product of capital as the log of $MRPK_{fijt} = \left(\frac{\alpha}{\mu}\right)\left(\frac{VA_{fijt}}{k_{fijt}}\right)$, where VA_{fijt} is real value added, computed as the difference between gross output (operating revenues) and materials divided by an output price deflator, and k_{fijt} is the real capital stock, computed as gross fixed assets divided by a gross fixed assets formation price deflator. Since we do not have prices at the firm level, we use the sector-specific gross output price deflator for Spain from the Eurostat KLEMS database at the two-digit NACE industry level as output price deflator to deflate value added, and the gross fixed capital formation price deflator for Spain at the country level from the World Bank's World Development Indicators database to deflate the capital stock. We follow Gopinath et al. (2017) and set $\alpha = 0.35$ and $\mu = 1.5$. We then take the log of $MRPK_{fijt}$ and label this $mprk_{fijt}$. To reduce the influence of outliers, we winsorize $mrpk_{fijt}$ at the 2% and 98% levels.

In terms of firm-level controls, we consider firm size, s_{fijt}, computed as the log of total assets using firm-level data from the Orbis database. We exclude the firms classified in the real-estate agents (NACE 68) and construction sectors (NACE 41-43), since these industries produce (and not use as input) real-estate. We also exclude the financial services industry (NACE 64-66). We winsorize the (log) marginal revenue product of capital measure at the 2% and 98% percentiles to avoid having some large outliers in the sample.

Monetary policy shocks. We borrow ECB monetary policy shocks from Jarociński and Karadi (2020). These authors separate interest rates shocks into either *information shocks* or *monetary policy shocks* by distinguishing interest rate unexpected increases (decreases) associated to stock market unexpected revaluations (devaluations) during a narrow window after the unexpected interest-rate change announcement. We use their preferred shock, mp_med_t, which is obtained following the estimation of a VAR model that implements sign restrictions to identify both the central bank information shock and the monetary policy shock. We aggregate the daily shocks from Jarociński and Karadi (2020) over the year in order to merge the series to our panel of annual data.

Real-estate supply elasticity. Basco et al. (2018) compute Real-estate supply elasticities for each Spanish municipality as the ratio of buildable urban land over urban land with already built structures. This approach is similar to the one taken by Saiz (2010) for the US. Based on the data from Basco et al. (2018) for year 1995, we construct a province-level real-estate supply elasticity proxy as the simple average of buildable urban land ratios across municipalities in a given province. Finally, we winsorize the variable at the 5% and 95% percentiles to avoid having some large outliers in the sample.

Real-estate intensity. Since we do not have data on the real estate exposure of Spanish firms, we proxy for the real-estate intensity of the sector in which the firm operates by using $RE_{US,j,t}$, computed as the ratio of real estate assets to total fixed assets using US sectorial data from the BEA Fixed Assets tables, manually matched to the NACE Rev 2 industries using concordance tables from the US Census and Eurostat.

C.3 Robustness checks

Table 5 presents several robustness checks on our main results. Column 1 shows results when replacing the interest rate variable with the cumulative monetary shock from Jarociński and Karadi (2020), obtained with poor man sign restrictions. Results are broadly unchanged. We now obtain a statistically negative coefficient on the interaction variable of interest of -0.030. This implies a semi-elasticity of 0.03 units for a one standard deviation increase in both housing inelasticity and real estate intensity.

Column 2 presents results when using the change in the Federal Funds rate as measure of the change in monetary policy. We have reversed the sign on this variable such that higher values denote easing. Results are qualitatively similar. The regression coefficient of interest increases in size but is estimated less precisely compared to our baseline estimate.

Column 3 shows that our main results are robust to using a measure of real estate intensity based on investments in real estate instead of the stock of real estate.

Previous research has provided strong evidence of the existence of a balance sheet effect,

Table 5: **Monetary transmission, real estate supply, and real estate intensity: robustness**

VARIABLES	(1) ΔGDP	(2) ΔGDP	(3) ΔGDP	(4) ΔGDP
$mp\_pm_t * H_j * RE_{i,t-1}$	-0.030**			
	(0.009)			
$\Delta r_t * H_j * RE_{i,t-1}$		-0.173*		
		(0.062)		
$mp\_med_t * H_j * REinv_{i,t-1}$			-0.034**	
			(0.010)	
$mp\_med_t * H_j * RE_{i,t-1}$				-0.039***
				(0.007)
$mp\_med_t * H_j * Leverage_{i,t-1}$				0.0025
				(0.012)
$ln(GDP)_{i,j,t-1}$	-0.237***	-0.237***	-0.237***	-0.239***
	(0.027)	(0.027)	(0.027)	(0.027)
Sector-Year FE	Yes	Yes	Yes	Yes
Sector-Region FE	Yes	Yes	Yes	Yes
Region-Year FE	Yes	Yes	Yes	Yes
Observations	17752	17752	17752	17096
R-squared	0.345	0.344	0.345	0.347
p>F	0.000	0.000	0.000	0.000
Clustering	Yes	Yes	Yes	Yes

Notes: Regression results from estimating the following specification: $\Delta y_{ijt} = \alpha_{ij} + \alpha_{it} + \alpha_{jt} + \delta \cdot y_{ijt-1} + \beta_1 \cdot r_t \cdot H_j + \beta_2 \cdot r_t \cdot H_j \cdot RE_{it-1} + \gamma L_{ijt-1} + \varepsilon_{it}$. Δy_{ijt} is real GDP growth rate at the sector-MSA level, and y_{ijt-1} is the one-period lag of the log of real GDP. mp_pm_t is the high-frequency monetary shock from Jarociński and Karadi (2020), obtained with poor man's sign restrictions, aggregated over the year. Δr_t is the annual change in the Federal Funds rate. mp_med_t is the high-frequency monetary shock from Jarociński and Karadi (2020), obtained with median rotation sign restrictions, aggregated over the year. H_j is the elasticity of housing supply at the MSA level. We invert the sign of the monetary shock and H_j variables to ease interpretation, so that higher values of the monetary shock denote an easing of monetary policy and higher values of H_j denote a more inelastic housing supply. RE_{it} is the ratio of real estate assets to total fixed assets of the sector. $REinv_{it}$ is the ratio of real estate investments to total fixed investments of the sector. All explanatory variables except the monetary shock are standardized. $Leverage_{it}$ is the median across firms in the sector of the ratio of short and long term debt over total assets. All variables are as defined in Table 1. Standard errors are clustered two-ways by sector-region and year.

whereby the transmission of monetary shocks to firm output is affected by financial constraints. In Column 4, we control for balance sheet effects through the inclusion of financial leverage. We compute financial leverage using balance sheet data from COMPUSTAT. We do not find evidence of a balance sheet effect related to financial leverage. Our main result is not affected when controlling for financial leverage.

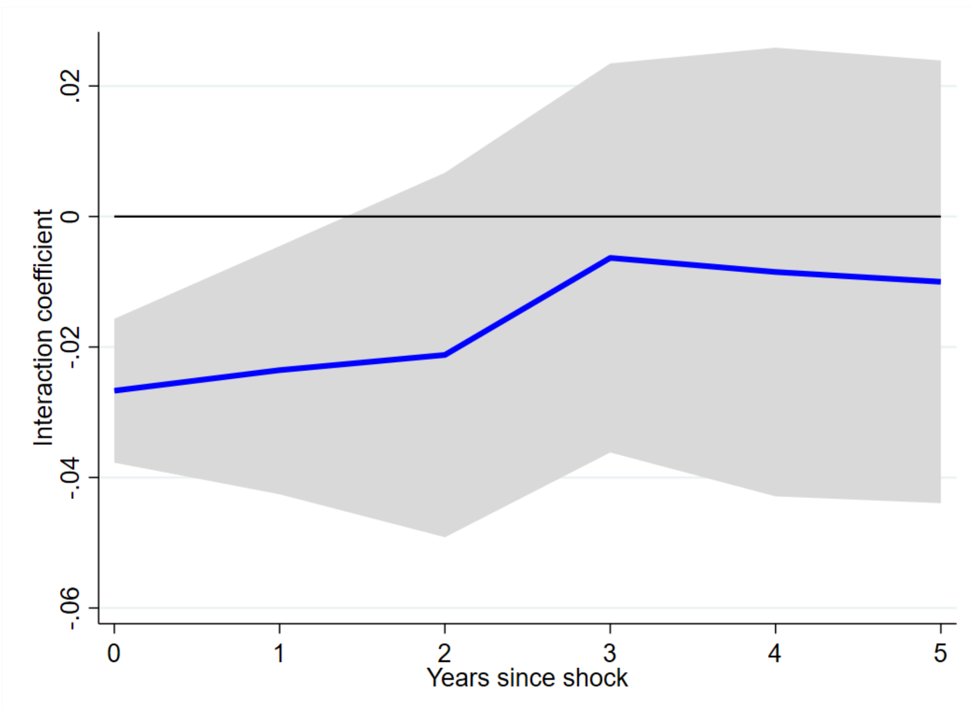
To analyze the dynamic impact of interest rate changes over time, we also estimate a Jordà (2005) type of local projection of Equation (35) as follows:

$$y_{ij,t+h} - y_{ij,t-1} = \alpha_{ijh} + \alpha_{jth} + \alpha_{ith} + \delta_h \cdot y_{ijt-1} + \beta_h \cdot r_t \cdot H_j \cdot RE_{i,t-1} + \varepsilon_{ijth} \tag{71}$$

for $h \geq 0$. The local projection shows the dynamics of the response of GDP to monetary shocks differentiated by housing elasticity H_j and real estate intensity $RE_{i,t-1}$. Specifically, the regression coefficients β_h indicate how the cumulative response of GDP over the period t to $t+h$ to a monetary shock in year t depend on the interaction between H_j and $RE_{i,t-1}$ in year $t-1$.

As before, all explanatory variables other than the monetary shock are standardized, and the signs of the monetary shock and housing elasticity are reversed to ease interpretation. The results of the local projection exercise are depicted in Figure 10. We use our baseline measure, mp_med_t, for the monetary shock. The results indicate that the result reported in Table 1 is persistent. Namely, the output of sectors that are more real-estate intensive and are located in regions where housing supply is more inelastic shows a consistently weaker response to expansionary monetary policy shocks for up to 5 years, even if this effect is estimated with large standard errors as shown by the 90% confidence intervals.

Figure 10: **Dynamics of differential response of GDP to monetary shocks**



Notes: The figure displays the regression coefficients β_h (the blue line) obtained when estimating the following specification: $ln(GDP_{ij,t+h}/GDP_{ij,t-1}) = \alpha_{ijh} + \alpha_{jth} + \alpha_{ith} + \delta_h \cdot y_{ijt-1} + \beta_h mp_m ed_t \cdot H_j \cdot RE_{i,t-1} + \varepsilon_{ijth}$ for $h = 0, 1, 2, 3, 4, 5$. The grey area depicts the 90% confidence interval. All variables are as defined in Table 1. Standard errors are clustered twoways by sector-region and year.