Q-Monetary Transmission*

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Abstract

We study the transmission of monetary policy to corporate investment through policy-induced changes in Tobin's q. We provide identification and empirical evidence of this channel, develop a model of the economic mechanism, evaluate the ability of the quantitative theory to match the evidence, and assess the aggregate relevance of the channel in monetary transmission to investment.

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1. Introduction

The chain of causal links that lie between monetary policy actions and their ultimate effects on the macroeconomic variables is broadly referred to as *the monetary transmission mechanism*. Since the immediate effect of these policy actions is to influence a wide array of interest rates and prices of financial and non-financial assets, it is difficult to imagine many economic decisions that are not affected by monetary policy. Consequently, textbook treatments of the effects of monetary policy contain extensive taxonomies of a myriad of transmission mechanisms.[1] The broadest classification typically consists of three main transmission channels: the (*direct* or *traditional*) *interest-rate channel*, the *asset-price channel*, and the *credit channel*.

The *interest-rate channel* is best described as a *user-cost* or *real discount-rate channel*: Suppose there is an unexpected increase in the nominal policy rate, and that (as is usually the case) some of the increase passes through to real rates. Then, since the real rate is a key component of the user cost of capital, and the user cost of capital is a key determinant of the demand for capital (e.g., as in Jorgenson (1963)), investment should fall as a result of the monetary policy action.[2] The *asset-price channel* is best described as a *Tobin's q channel*: Suppose an unexpected increase in the nominal policy rate causes stock prices to fall (as is well documented empirically, e.g., Bernanke and Kuttner (2005)). Then the conventional q-theory of investment (e.g., Hayashi (1982)) implies real corporate investment should fall as a result of the monetary policy action. The *credit channel*, which includes the well-known *balance-sheet channel*, is best described as an amplification mechanism associated to the other two channels: Suppose an unexpected increase in the nominal policy rate causes asset prices to fall (e.g., through either of the previous two channels), which in turn deteriorates borrowers' net worth. Then the resulting increase in external finance premia (Bernanke and Gertler (1989)) or tightening of borrowing constraints (Kiyotaki and Moore (1997)) imply credit-financed investment should fall as a result of the monetary policy action.

The user-cost channel is well-studied, widely taught, and present in most quantitative models used in policy analysis. The credit channel has received much attention in the past decade, and is now standard in theoretical and quantitative policy-oriented modelling. The asset-price channel of monetary transmission was one of the key mechanisms that Tobin (1969) sought to

[1]See, e.g., Mishkin (1995, 1996, 2001), or Boivin et al. (2010).

[2]Our focus here is on corporate investment, but all these channels have obvious household counterparts for spending in consumption of durables and real estate.

model by introducing the famous q; it is described in undergraduate textbooks, and discussed in policy circles. There seems to be, however, essentially no academic research on this channel, either empirical or theoretical. In this paper we study the transmission of monetary policy to corporate investment through an asset-price channel activated by policy-induced changes in Tobin's q. We refer to this mechanism as *q-monetary transmission*. We provide empirical evidence of this channel, develop a model of the economic mechanism, evaluate the ability of the quantitative theory to match the evidence, and assess the aggregate relevance of the channel in monetary transmission to investment.

The main challenge for estimating the q-monetary transmission mechanism is that monetary policy can potentially affect firms' investment decisions and stock prices through other channels as well as indirect general equilibrium effects. For example, if a monetary shock lowers demand for a firm's output and this decreases profit, both the firm's investment and stock price may fall, but this does not imply that investment falls *because* the stock price falls. Similarly, a contractionary money shock may lower investment directly through the traditional user-cost channel, and the (anticipated) reduction of investment may lead to a reduction in the firm's stock price. The stock price is also likely to fall simply because of higher discounting of future dividends. But again, in this case the fall in the stock price is not *causing* the fall in investment. In the first example, both investment and the stock price are endogenous outcomes responding to the reduction in demand. In the second example, the stock price is responding to the interest-rate induced reduction in investment, and both drop due to higher discounting. Thus, we cannot hope to expose the causal relationship between q and investment that is inherent in the q-monetary transmission mechanism by simply estimating the comovement of investment and q induced by monetary shocks.

We meet this empirical challenge by exploiting the cross-sectional variation in the responses of stock prices to monetary shocks. Lagos and Zhang (2020b) provide evidence that stock turnover is a strong predictor of monetary policy passthrough to stock prices in the cross-section of U.S. publicly traded firms. Therefore, stock turnover can be used as a measure of the cross-sectional differences in the exposure of stock prices to monetary policy shocks. Given this, our empirical strategy builds on the idea that, if the cross-sectional variation in stock turnover is uncorrelated with other other sources of response-heterogeneity at the time of a monetary policy shock, then identified money shocks combined with heterogeneity in cross-sectional stock turnover can be used as a source of exogenous cross-sectional variation in Tobin's q. We then use this cross-sectional variation in the responses of stock prices to money shocks for firms with different stock turnover, to identify the effects of stock-price changes on firms' investment decisions. Specifically, based on this logic, we construct an instrument for the cross-sectional variation in Tobin's q by interacting monetary policy shocks with firm-specific stock turnover (calculated in the quarter prior to the shock). Our main exercise consists of estimating whether such instrumented variation in Tobin's q has significant effects on firms' equity issuance and investment behavior. We find it does.

Our work contributes to four literatures. First, we contribute to the literature on monetary transmission by filling the empirical and theoretical void around the asset-price channel that operates through Tobin's q. Second, we contribute to the empirical literature on investment by proposing a novel instrument for Tobin's q that can address the usual concerns related to the endogeneity of q in the standard q regressions (see, e.g., Hayashi and Inoue (1991), Blundell et al. (1992)). As mentioned above, our innovation is to construct an instrument by exploiting a combination of identified monetary policy shocks and the cross-sectional variation in stock price responses to these shocks. Third, our theoretical and empirical results on the response of firms' equity issuance to fluctuations in stock prices (induced by monetary shocks) contribute to the corporate finance literature that studies the relationship between firms' capital structure and macroeconomic conditions in general, and stock prices in particular (e.g., Baker and Wurgler (2002), Baker et al. (2003), Korajczyk and Levy (2003), Hovakimian et al. (2004), Gilchrist et al. (2005)). Fourth, we contribute to the literature that builds micro-founded models of monetary exchange to uncover new channels through which money affects macroeconomic outcomes (e.g., Lagos (2011), Lagos and Zhang (2015, 2019, 2020a,b), Rocheteau et al. (2018)).

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 characterizes the main properties of the equilibrium. Section 4 considers a special case in which the equilibrium can be solved with paper and pencil to convey the main ideas. Section 5 contains the empirical analysis. In Section 6 we calibrate and simulate the model to assess the ability of the theory to fit the empirical evidence on the effects of monetary-policy induced changes in Tobin's q on equity issuance and investment. Finally, in Section 7 we employ our empirical estimates to provide a back-of-the-envelope evaluation of the aggregate relevance of the channel in monetary transmission to investment.

2. Model

Time is represented by a sequence of periods indexed by $t \in \{0, 1, ...\}$. Each time period is divided into two subperiods where different activities take place. There is a continuum of agents of three types: *investors*, each identified with a point in the set $\mathcal{I} = [0, 1]$, *brokers*, each identified with a point in the set $\mathcal{B} = [0, 1]$, and *entrepreneurs* (whom we will interchangeably refer to as *firms*), each identified with a point in the set $\mathcal{F} = [0, 1]$. Brokers and investors are infinitely lived. Entrepreneurs live for a random number of periods: a fraction $1 - \pi \in [0, 1]$ of the population of entrepreneurs who are alive at the beginning of the second subperiod of period t, dies (i.e., exits the economy) at the beginning of the second subperiod of period $t + 1$. The set of entrepreneurs who die is a uniform random draw from the population of entrepreneurs, and each is immediately replaced by a newly born entrepreneur.

There are three commodities at each date: two consumption goods, called *good 1* and *good 2*, and a *capital* good. The consumption goods are perishable: good 1 and good 2 can only be consumed in the first and second subperiods, respectively. Capital is storable, but depreciates at rate $\delta \in [0, 1]$ between periods. Upon entering the economy, an entrepreneur $i \in \mathcal{F}$ is endowed with $w_0^i \in \mathbb{R}_+$ units of good 2 and $k_0 \in \mathbb{R}_+$ units of capital. We use a cumulative distribution function Ω to describe the heterogeneity in the initial endowment of (claims to) good 2 relative to capital, $\omega_0^i \equiv w_0^i / k_0$, across entrepreneurs. In the second subperiod of every period, investors and brokers are endowed with a resource called *labor (effort)* that they can use to produce good 2 one-for-one. There are two other production technologies that can be managed only by entrepreneurs. The first, uses capital available at the beginning of period t to produce good 1 in the first subperiod of period t. Specifically the capital stock k_t operated by an entrepreneur delivers zk_t units of good 1 at the end of the first subperiod of t, where $z \in \mathbb{R}_{++}$. The second production technology can be operated by an entrepreneur in the second subperiod of period t, and uses good 2 and the capital the entrepreneur has in place at the beginning of period t to augment the capital that the entrepreneur will have in place to produce good 1 in period $t + 1$. Formally, this technology is represented by a cost function, $\mathrm{C}(x_t, k_t) \equiv x_t + \Psi(x_t/k_t) k_t$, interpreted as the cost (in terms of good 2) of producing and installing x_t units of capital for an entrepreneur whose current capital is k_t. We assume $0 < \Psi''$, and that there is a $\iota_0 \in \mathbb{R}_+$ such that $\Psi(\iota_0) = \Psi'(\iota_0) = 0$. It is convenient to define $\mathrm{c}(x_t/k_t) \equiv \mathrm{C}(x_t, k_t)/k_t$, i.e., the cost of investment per unit of installed capital. The assumptions on Ψ

imply $\text{C}(\iota_0) - \iota_0 = \text{C}'(\iota_0) - 1 = 0 < \text{C}''(\cdot)$. Once installed, capital is entrepreneur-specific, i.e., capital installed by entrepreneur i is only productive when operated by entrepreneur i.

The asset structure is as follows. In the second subperiod of every period, in order to finance the cost of investing in new capital, every entrepreneur can issue identical, durable, and perfectly divisible equity claims to the future returns from the newly created capital. (Entrepreneurs are also allowed to sell equity claims on any existing capital they currently own.) An equity share issued by an entrepreneur in the second subperiod of t represents ownership of 1 unit of capital along with the stream of *dividends* produced by that unit of capital. When an entrepreneur dies, the outstanding equity claims she had previously issued disappear, and the underlying capital plus any financial assets, physical capital, or claims owned by the entrepreneur are distributed uniformly (lump sum) to the cohort of newly born entrepreneurs. There are two other financial instruments: a one-period real pure-discount government *bond*, and *money.* A unit of the bond issued in the second subperiod of t represents a risk-free claim to one unit of good 2 in the second subperiod of $t+1$. The stock of bonds outstanding at time t is B_t, and all private agents take the sequence $\{B_t\}_{t=0}^{\infty}$ as given. Money is intrinsically useless (it is not an argument of any utility or production function, and unlike equity or bonds, money does not constitute a formal claim to any resources). The nominal money supply at the beginning of period t is denoted A_t^m, and we assume $A_{t+1}^m = \mu A_t^m$, with $\mu \in \mathbb{R}_{++}$ and $A_0^m \in \mathbb{R}_{++}$ given. The government injects or withdraws money via lump-sum transfers or taxes to investors in the second subperiod of every period. At the beginning of period $t = 0$, each investor is endowed with an equal portfolio of money. We assume brokers do not hold financial assets (money, bonds, or equity).[3]

The market structure is as follows. In the second subperiod, all agents can trade good 2, labor services, equity shares, bonds, and money, in a spot Walrasian market.[4] In the first subperiod, investors can trade equity shares and money in a random bilateral *over-the-counter (OTC) market* with brokers, while brokers can also trade equity shares and money with other

[3]This assumption allows us to abstract from the broker's portfolio problem in the first subperiod, which is not essential for the questions we study in this paper. See Lagos and Zhang (2015, 2020b) for a treatment of the broker's portfolio problem in a related model.

[4]Notice that equity shares (i.e., the claims on installed capital and its returns) can be traded freely, but the actual physical capital created and installed by a particular entrepreneur is assumed to be non tradable. The idea is that, once installed by an entrepreneur, physical capital becomes entrepreneur-specific and cannot be operated by another entrepreneur. An entrepreneur can, however, disinvest (which entails bearing the adjustment cost, Φ) to turn installed capital into good 2, which can then be traded freely in the Walrasian market. Similarly, when the entrepreneur dies, the quantity of good 2 obtained from unistalling the capital that the entrepreneur used to manage (net of adjustment costs) is distributed to newly born entrepreneurs.

brokers in a spot Walrasian *interbroker market*. We use $\alpha \in [0,1]$ to denote the probability that an individual investor is able to make contact with a broker in the OTC market. Once a broker and an investor have contacted each other, the pair negotiates the quantity of equity shares and money that the broker will trade in the interdealer market on behalf of the investor, and a fee for the broker's intermediation services. The terms of the trade between an investor and a broker in the OTC market are determined by Nash bargaining, where $\theta \in [0,1]$ is the investor's bargaining power. We assume the fee is negotiated in terms of good 2, and paid at the beginning of the following subperiod.[5] The timing is that the round of OTC trade takes place in the first subperiod and ends before equity pays out first-subperiod dividends.[6] Equity purchases in the OTC market cannot be financed by borrowing (e.g., due to anonymity and lack of commitment and enforcement). This assumption and the structure of preferences described below create the need for a medium of exchange in the OTC market.[7]

An individual broker's preferences are given by

$$\mathbb{E}_0^B \sum_{t=0}^{\infty} \beta^t (y_t - h_t),$$

where $\beta \in (0,1)$ is the discount factor, and y_t and h_t denote a broker's consumption of good 2, and utility cost from supplying h_t units of labor in the second subperiod of period t, respectively. The expectation operator, $\mathbb{E}_0^B$, is with respect to the probability measure induced by the random trading process in the OTC market. Dealers get no utility from good 1.[8] An individual investor's preferences are given by

$$\mathbb{E}_0^I \sum_{t=0}^{\infty} \beta^t \left(\varepsilon_t c_t + y_t - h_t\right),$$

where y_t and h_t denote an investor's consumption of good 2, and utility cost from supplying h_t units of labor in the second subperiod of period t, respectively, and c_t is the investor's consumption of good 1 at the end of the first subperiod of period t. The variable ε_t denotes the realization of an idiosyncratic valuation shock for good 1 that is distributed independently

[5] This is the specification used in Lagos and Zhang (2020b). Lagos and Zhang (2015) instead assume the investor must pay the intermediation fee to the broker on the spot (with money or equity). The timing convention in Lagos and Zhang (2020b) simplifies the exposition without affecting the mechanisms of interest.

[6] As in previous search models of OTC markets, e.g., Duffie et al. (2005) and Lagos and Rocheteau (2009), an investor must own the equity in order to consume the dividend flow of consumption good in the OTC round.

[7] See Lagos and Zhang (2020a, 2019) for a similar model where investors can buy equity *on margin*.

[8] This assumption implies that dealers have no direct consumption motive for holding the equity share. It is easy to relax, but we adopt it because it is the standard benchmark in the search-based OTC literature, e.g., see Duffie et al. (2005), Lagos and Rocheteau (2009), Lagos et al. (2011), and Weill (2007).

over time and across investors with a differentiable cumulative distribution function G with support $[\varepsilon_L, \varepsilon_H] \subseteq [0, \infty]$, and mean $\bar{\varepsilon} \equiv \int \varepsilon dG(\varepsilon)$. An investor learns the realization ε_t at the beginning of the first subperiod of period t, immediately before the OTC trading round. The expectation operator, $\mathbb{E}_0^I$, is with respect to the probability measure induced by the investor's valuation shocks, and the trading process in the OTC market.

The preferences of an entrepreneur born in the second subperiod of t are given by

$$\sum_{j=t}^{\infty} (\beta\pi)^{(j-t)} \left(y_j + \beta\hat{\varepsilon}c_{j+1}\right),$$

where y_j is the consumption of good 2 in the second subperiod of period j, and c_{j+1} is the entrepreneur's consumption of good 1 at the end of the first subperiod of period $j+1$, and $\hat{\varepsilon} \in \mathbb{R}_{++}$ is the entrepreneur's valuation of her own production of good 1.

3. Equilibrium

Consider the determination of the terms of trade in a bilateral meeting in the OTC round of period t between a broker and an investor with valuation ε and portfolio $\boldsymbol{a}_t = (a_t^b, a_t^m, a_t^s)$, where a_t^b denotes bond holdings, a_t^m money holdings, and a_t^s holdings of shares. Let $W_t(\boldsymbol{a}_t, \varpi_t)$ denote the maximum expected discounted payoff at the beginning of the second subperiod of period t of an investor who is holding portfolio $\boldsymbol{a}_t$ and has to pay a broker fee ϖ_t. Let $[\overline{\boldsymbol{a}}_t(\boldsymbol{a}_t, \varepsilon), \varpi_t(\boldsymbol{a}_t, \varepsilon)]$ represent the bargaining outcome in a bilateral trade at time t between a broker and an investor with portfolio $\boldsymbol{a}_t$ and valuation ε, where $\overline{\boldsymbol{a}}_t(\boldsymbol{a}_t, \varepsilon) \equiv \left(\overline{a}_t^b(\boldsymbol{a}_t, \varepsilon), \overline{a}_t^m(\boldsymbol{a}_t, \varepsilon), \overline{a}_t^s(\boldsymbol{a}_t, \varepsilon)\right)$ denotes the investor's post-trade portfolio. Then, $[\overline{\boldsymbol{a}}_t(\boldsymbol{a}_t, \varepsilon), \varpi_t(\boldsymbol{a}_t, \varepsilon)]$ solves

$$\max_{(\overline{\boldsymbol{a}}_t, \varpi_t) \in \mathbb{R}_+^4} \Gamma(\overline{\boldsymbol{a}}_t, \boldsymbol{a}_t, \varepsilon)^{\theta} \varpi_t^{1-\theta} \tag{1}$$

with $\overline{\boldsymbol{a}}_t \equiv \left(\overline{a}_t^b, \overline{a}_t^m, \overline{a}_t^s\right)$,

$$\begin{aligned} \Gamma(\overline{\boldsymbol{a}}_t, \boldsymbol{a}_t, \varepsilon) \equiv\ & \varepsilon z \overline{a}_t^s + W_t\left(\overline{a}_t^b, \overline{a}_t^m, \pi\overline{a}_t^s, \varpi_t\right) \\ & - \varepsilon z a_t^s - W_t(a_t^b, a_t^m, \pi a_t^s, 0), \end{aligned}$$

and subject to

$$\begin{aligned} \overline{a}_t^m + p_t\overline{a}_t^s &\leq a_t^m + p_t a_t^s \\ 0 &\leq \Gamma(\overline{\boldsymbol{a}}_t, \boldsymbol{a}_t, \varepsilon), \end{aligned}$$

and $\overline{a}_t^b = a_t^b$, where p_t denotes the dollar price of an equity share in the interbroker market of period t. The first and second constraints are the investor's budget, and participation constraints, respectively. The last constraint reflects the assumption that the real bond is illiquid in that it cannot be directly used as means of payment in stock-market trades.

Let $V_t(\boldsymbol{a}_t, \varepsilon)$ denote the maximum expected discounted payoff of an investor with valuation ε and portfolio $\boldsymbol{a}_t$ at the beginning of the first subperiod of period t. In the second subperiod of period t, let $\boldsymbol{\phi}_t \equiv (\phi_t^b, \phi_t^m, \phi_t^s)$, where ϕ_t^b is the real price of a newly issued government bond , ϕ_t^m is the real price of a unit of money, and ϕ_t^s is the real price of an equity share (all in terms of the good 2). At the beginning of the second subperiod the investor solves

$$W_t(\boldsymbol{a}_t, \varpi_t) = \max_{(y_t, h_t, \boldsymbol{a}_{t+1}) \in \mathbb{R}_+^5} \left[y_t - h_t + \beta \int V_{t+1}(\boldsymbol{a}_{t+1}, \varepsilon) \, dG(\varepsilon) \right] \tag{2}$$

$$\text{s.t. } y_t + \boldsymbol{\phi}_t \boldsymbol{a}_{t+1} \leq \boldsymbol{\phi}_t' \boldsymbol{a}_t + h_t - \varpi_t + T_t,$$

where y_t is consumption of good 2, h_t is the disutility of labor, $\boldsymbol{a}_{t+1} \equiv (a_{t+1}^b, a_{t+1}^m, a_{t+1}^s)$, $\boldsymbol{\phi}_t' \equiv (1, \phi_t^m, \phi_t^s)$, and $T_t \in \mathbb{R}$ is the real value of the lump-sum monetary transfer. The value function of an investor who enters the first subperiod of t with portfolio $\boldsymbol{a}_t$ and valuation ε is

$$\begin{aligned} V_t(\boldsymbol{a}_t, \varepsilon) &= \alpha \left\{ \varepsilon z \overline{a}_t^s(\boldsymbol{a}_t, \varepsilon) + W_t\left[\overline{\boldsymbol{a}}_t'(\boldsymbol{a}_t, \varepsilon), \varpi_t(\boldsymbol{a}_t, \varepsilon)\right] \right\} \\ &\quad + (1-\alpha)\left[\varepsilon z a_t^s + W_t\left[\boldsymbol{a}_t'(\boldsymbol{a}_t), 0\right]\right], \end{aligned} \tag{3}$$

where $\overline{\boldsymbol{a}}_t'(\boldsymbol{a}_t, \varepsilon) \equiv (\overline{a}_t^b(\boldsymbol{a}_t, \varepsilon), \overline{a}_t^m(\boldsymbol{a}_t, \varepsilon), \pi \overline{a}_t^s(\boldsymbol{a}_t, \varepsilon))$ and $\boldsymbol{a}_t'(\boldsymbol{a}_t) \equiv (a_t^b, a_t^m, \pi a_t^s)$.

Let $J_t(\boldsymbol{b}_t)$ denote the maximum expected discounted payoff at the beginning of the second subperiod of period t, of an entrepreneur who currently has balance sheet $\boldsymbol{b}_t \equiv (a_t^b, k_t, s_t)$, composed of (claims to) a_t^b units of good 2, installed capital k_t, and s_t outstanding equity claims on installed capital. The value function satisfies

$$J_t(\boldsymbol{b}_t) = \max_{y_t, a_{t+1}^b, e_t, x_t} \left\{ y_t + \beta \left[\hat{\varepsilon} z (k_{t+1} - s_{t+1}) + \pi J_{t+1}(\boldsymbol{b}_{t+1}) \right] \right\} \tag{4}$$

$$\begin{aligned} \text{s.t. } y_t + \mathrm{C}(x_t/k_t) k_t + \phi_t^b a_{t+1}^b &\leq \phi_t^s e_t + a_t^b & (5) \\ k_{t+1} &= (1-\delta) k_t + x_t & (6) \\ s_{t+1} &= (1-\delta) s_t + e_t & (7) \\ 0 &\leq s_{t+1} \leq k_{t+1} & (8) \\ y_t, a_{t+1}^b &\in \mathbb{R}_+, & (9) \end{aligned}$$

where $\boldsymbol{b}_{t+1} \equiv \left(a_{t+1}^b, k_{t+1}, s_{t+1}\right)$, y_t denotes consumption of good 2, x_t is the quantity of good 2 invested to produce new capital (net of the installation cost), and e_t is the number of newly issued equity shares. Condition (5) is the entrepreneur's budget constraint (expressed in terms of good 2), while (6) and (7) are the laws of motion for the stock of installed capital and outstanding equity shares on the entrepreneur's installed capital, respectively. The first inequality in (8) states that an entrepreneur cannot buy claims on her own dividend of good 1 from others; or equivalently equity issuance must satisfy $-(1-\delta)\, s_t \leq e_t$, i.e., the entrepreneur is allowed to buy back her own equity shares, but cannot buy back more than the stock currently outstanding. The second inequality in (8) states that entrepreneurs cannot sell claims on capital that are not backed by capital owned by the entrepreneur, i.e., equity issuance must satisfy $e_t \leq x_t + (1-\delta)(k_t - s_t)$. The nonnegativity constraints in (9) rule out negative consumption and borrowing by shorting the government bond. Notice the formulation (4) assumes an entrepreneur does not hold cash.[9] Let the function $\boldsymbol{g}_t : \mathbb{R}_+^3 \to \mathbb{R}_+^2 \times \mathbb{R}^2$ denote the optimal decision rule corresponding to (4), i.e., $\boldsymbol{g}_t(\boldsymbol{b}_t) \equiv \left(g_t^y(\boldsymbol{b}_t), g_t^b(\boldsymbol{b}_t), g_t^e(\boldsymbol{b}_t), g_t^x(\boldsymbol{b}_t)\right)$ gives the entrepreneur's optimal choices of second-subperiod consumption, bond holdings, equity issuance, and investment, as a function of her initial balance sheet, $\boldsymbol{b}_t$. Then, conditional on survival, the optimal path for the entrepreneur's balance sheet is described by $\boldsymbol{b}_{t+1} = \bar{\boldsymbol{g}}_t(\boldsymbol{b}_t) \equiv \left(\bar{g}_t^b(\boldsymbol{b}_t), \bar{g}_t^k(\boldsymbol{b}_t), \bar{g}_t^s(\boldsymbol{b}_t)\right)$, with $\bar{g}_t^b(\boldsymbol{b}_t) \equiv g_t^b(\boldsymbol{b}_t)$, $\bar{g}_t^k(\boldsymbol{b}_t) \equiv (1-\delta)\, k_t + g_t^x(\boldsymbol{b}_t)$, and $\bar{g}_t^s(\boldsymbol{b}_t) \equiv (1-\delta)\, s_t + g_t^e(\boldsymbol{b}_t)$.

Let $j \in \{E, I\}$ denote the agent type, i.e., "E" for entrepreneurs and "I" for investors, and let $h \in \{b, m, s\}$ denote the type of financial asset, i.e., "b" for bonds, "m" for money, and "s" for equity shares. Then let A_{It}^h denote the quantity of financial asset h held by all investors at the beginning of period t. That is, $A_{It}^h = \int a_t^h dF_{It}(\boldsymbol{a}_t)$, where F_{It} is the cumulative distribution function over portfolios $\boldsymbol{a}_t = \left(a_t^b, a_t^m, a_t^s\right)$ held by investors at the beginning of period t. Similarly, let $\bar{F}_{Et}$ denote the joint cumulative distribution function over entrepreneur's balance sheets, $\boldsymbol{b}_t = \left(a_t^b, k_t, s_t\right)$, at the beginning of the second subperiod of period t. Let A_{Et}^b denote the quantity of bonds held by entrepreneurs at the beginning of period t. Let K_t and S_t denote the beginning-of-period t capital stock managed by all entrepreneurs, and outstanding equity claims on all installed capital, respectively. Then, we have the beginning-of-period t aggregates, $A_{Et}^b = \int a_t^b dF_{Et}(\boldsymbol{b}_t)$, $K_t = \int k_t dF_{Et}(\boldsymbol{b}_t)$, and $S_t = \int s_t dF_{Et}(\boldsymbol{b}_t)$. Let $\bar{A}_{It}^m$ and $\bar{A}_{It}^s$ denote the quantities of money and shares held after the first-subperiod round

[9] This assumption merely simplifies the exposition. In this environment, entrepreneurs are not involed in transactions for which cash is used as a medium of exchange, so we can anticipate that an entrepreneur will never choose to carry cash given she has the option to hold interest-bearing bonds.

of trade of period t by all the investors who are able to trade in the first subperiod. Then we have $\bar{A}^h_{It} = \alpha \int \bar{a}^h_t(\boldsymbol{a}_t, \varepsilon) dH_{It}(\boldsymbol{a}_t, \varepsilon)$ for $h \in \{m, s\}$, where H_{It} denotes the joint cumulative distribution of portfolios and valuation shocks across investors at the beginning of period t. We are now ready to define an equilibrium.

Definition 1. *An equilibrium is a sequence of prices, $\{\boldsymbol{\phi}_t\}_{t=0}^{\infty}$, terms of trade in the first-subperiod stock market, $\{\bar{\boldsymbol{a}}_t(\cdot), \varpi_t(\cdot)\}_{t=0}^{\infty}$, investor end-of-period portfolio choices, $\{\boldsymbol{a}_{t+1}\}_{t=0}^{\infty}$, and entrepreneur decision rules, $\{\boldsymbol{g}_t(\cdot)\}_{t=0}^{\infty}$, such that: (i) the terms of trade $\{\bar{\boldsymbol{a}}_t(\cdot), \varpi_t(\cdot)\}_{t=0}^{\infty}$ solve (1); (ii) taking prices and the bargaining protocol as given, the portfolios $\{\boldsymbol{a}_{t+1}\}_{t=0}^{\infty}$ solve the individual investor's optimization problem (2), and the decision rules $\{\boldsymbol{g}_t(\cdot)\}_{t=0}^{\infty}$ solve (4); and (iii) prices, $\{\boldsymbol{\phi}_t\}_{t=0}^{\infty}$, are such that all Walrasian markets clear, i.e., $A^b_{Et+1} + A^b_{It+1} = B_{t+1}$ (the end-of-period t Walrasian bond market clears), $A^m_{It+1} = A^m_{t+1}$ (the end-of-period t Walrasian market for money clears), $A^s_{It+1} = S_{t+1}$ (the end-of-period t Walrasian market for equity clears), $\bar{A}^m_{It} = \alpha A^m_t$ (the market for money in the first subperiod of t clears), and $\bar{A}^s_{It} = \alpha S_t$ (the market for equity in the first subperiod of t clears). An equilibrium is "monetary" if $\phi^m_t > 0$ for all t and "nonmonetary" otherwise.*

4. Theoretical results

In this section we assume $\pi = 0$, i.e., entrepreneurs live for one period, and focus on stationary equilibria, that is, equilibria in which the aggregate supply of equity, S, is constant over time, real equity prices are time-invariant linear functions of the aggregate dividend, i.e., $\phi^s_t = \phi^s \equiv \varphi^s z$, $p_t \phi^m_t = \bar{\varphi}^s z$, and aggregate real money balances are constant, i.e., $\phi^m_t A^m_t \equiv \mathrm{M}_t = \mathrm{M}$.

To characterize the equilibrium it is useful to define the valuation of the *marginal investor* in the stock market of the first subperiod of t, $\varepsilon^*_t \equiv p_t \phi^m_t / z$, and the *nominal interest rate* between period t and $t+1$,

$$r_{t+1} \equiv \frac{\phi_t}{\beta \phi_{t+1}} - 1. \tag{10}$$

The valuation ε^*_t is the one that makes an investor indifferent between holding equity or selling it for cash.[10] In a stationary equilibrium, $\varepsilon^*_t = \varepsilon^* \equiv \bar{\varphi}^s$. The nominal interest rate r_{t+1} is the

[10] To see the role that (20) will play in the equilibrium, consider an investor with valuation ε who, in the stock market of the first subperiod of period t, is deciding whether to keep an equity share or sell it for cash. If he keeps the share, his payoff is $\varepsilon z + \pi \phi^s_t$, namely his valuation of the period dividend, εz, plus the expected value (in terms of good 2) of the share in the following subperiod, $\pi \phi^s_t$. If he sells it for cash, he gets payoff $p_t \phi^m_t$ (i.e., sells the share for p_t dollars, worth ϕ^m_t units of good 2 in the following subperiod). Hence, the investor keeps the share if $\varepsilon z + \pi \phi^s_t > p_t \phi^m_t$, sells it for cash if $\varepsilon z + \pi \phi^s_t < p_t \phi^m_t$, and is indifferent if $\varepsilon = \varepsilon^*_t$.

nominal yield of a one-period risk-free nominal bond issued in the second subperiod of t and redeemed in the second subperiod of $t+1$ that is illiquid (in the sense that it cannot be used to purchase stocks in the first-subperiod of $t+1$). In a stationary equilibrium, $r_{t+1} = r \equiv (\mu - \beta)/\beta$, so we regard r as the *nominal policy rate*, which can be implemented by changing the growth rate in the money supply, μ.

For an entrepreneur who enters with initial conditions w and k in the context of a stationary equilibrium of an economy with $\pi = 0$, (4) specializes to

$$J(w,k,0) = \max_{x,y,s_{+1}} [y + \beta\hat{\varepsilon}z(k_{+1} - s_{+1})] \tag{11}$$

$$\begin{aligned} \text{s.t. } y + \text{C}(x/k)\,k &\leq \phi^s s_{+1} + w \\ k_{+1} &= (1-\delta)\,k + x \\ 0 &\leq s_{+1} \leq k_{+1} \\ 0 &\leq y. \end{aligned}$$

Let $g^x(w,k)$, $g^y(w,k)$, and $g^e(w,k)$ denote the investment, consumption, and equity issuance that solve (11). Define $x^* \equiv g^x(w,k)/k$, $y^* \equiv g^y(w,k)/k$, $s^*_{+1} \equiv g^e(w,k)/k$, $\omega \equiv w/k$, and $\hat{\phi}^s \equiv \beta\hat{\varepsilon}z$. The following result characterizes the solution (x^*, y^*, s^*_{+1}) to (11).

Lemma 1. *Let $\iota(\phi)$ denote the unique number, ι, that solves $\text{C}'(\iota) = \phi$ for any $\phi \in \mathbb{R}_+$. Assume $\delta - \iota_0 \leq 1 \leq \min(\phi^s, \hat{\phi}^s)$. (i) If $\hat{\phi}^s \leq \phi^s$,*

$$\begin{aligned} x^* &= \iota(\phi^s) \\ s^*_{+1} &= \begin{cases} 1-\delta+x^* & \text{if } \hat{\phi}^s < \phi^s \\ \left[\max\left\{0, \frac{\text{C}(x^*)-\omega}{\phi^s}\right\}, 1-\delta+x^*\right] & \text{if } \hat{\phi}^s = \phi^s \end{cases} \\ y^* &= \omega + \phi^s s^*_{+1} - \text{C}(x^*) \end{aligned}$$

(ii) If $\phi^s < \hat{\phi}^s$,

$$\begin{aligned} x^* &= \begin{cases} \iota(\hat{\phi}^s) & \text{if } \text{C}(\iota(\hat{\phi}^s)) \leq \omega \\ \text{C}^{-1}(\omega) & \text{if } \text{C}(\iota(\phi^s)) < \omega < \text{C}(\iota(\hat{\phi}^s)) \\ \iota(\phi^s) & \text{if } \omega \leq \text{C}(\iota(\phi^s)) \end{cases} \\ s^*_{+1} &= \begin{cases} 0 & \text{if } \text{C}(\iota(\hat{\phi}^s)) \leq \omega \\ 0 & \text{if } \text{C}(\iota(\phi^s)) < \omega < \text{C}(\iota(\hat{\phi}^s)) \\ \frac{\text{C}(\iota(\phi^s))-\omega}{\phi^s} & \text{if } \omega \leq \text{C}(\iota(\phi^s)) \end{cases} \\ y^* &= \begin{cases} \omega - \text{C}(\iota(\hat{\phi}^s)) & \text{if } \text{C}(\iota(\hat{\phi}^s)) \leq \omega \\ 0 & \text{if } \text{C}(\iota(\phi^s)) < \omega < \text{C}(\iota(\hat{\phi}^s)) \\ 0 & \text{if } \omega \leq \text{C}(\iota(\phi^s)). \end{cases} \end{aligned}$$

In Lemma 1, $\hat{\phi}^s$ is the entrepreneur's marginal private value of investing capital, while ϕ^s can be interpreted as the marginal value of capital to the outside investors who are pricing the entrepreneur's equity. Part (i) focuses on the case in which outside investors value the entrepreneur's marginal investment in capital more than the entrepreneur herself. In this case the entrepreneur chooses the investment rate, x^*, so that the marginal cost, $\text{C}'(x^*)$ equals the marginal value of the investment to the outside investor, ϕ^s. Moreover, because her valuation is lower than the market valuation, the entrepreneur issues equity shares on any capital she owns at the beginning of the period, and finances new investment entirely by equity issuance, i.e., she chooses $s^*_{+1}k = (1 - \delta + x^*)k$. (In the knife-edge case with $\hat{\phi}^s = \phi^s$, the entrepreneur is indifferent between financing by equity issuance or out of her own funds, ωk.)

Part (ii) of Lemma 1 focuses on the case in which the entrepreneur values the marginal investment in capital more than outside investors, i.e., $\phi^s < \hat{\phi}^s$. In this case, the investment, financing, and consumption decisions of the entrepreneur depend not only on her valuation and the outside investors' valuation of investment, but also on the entrepreneur's own financial wealth, represented by ω. First, if the entrepreneur's wealth is high enough, i.e., $\text{C}(\iota(\hat{\phi}^s)) \leq \omega$, then the entrepreneur is financially unconstrained: she chooses her first-best investment rate, $\iota(\hat{\phi}^s)$ (the x^* that equates the marginal cost of investment, $\text{C}'(x^*)$, to her own marginal valuation, $\hat{\phi}^s$), finances it entirely with own funds, i.e., $s^*_{+1} = 0$ (issues no equity), and consumes the unspent financial wealth, i.e., sets $y^* = \omega -\text{C}(\iota(\hat{\phi}^s))$. On the opposite extreme, if the entrepreneur's own financial wealth is very low, specifically $\omega \leq \text{C}(\iota(\phi^s))$, i.e., lower than what would be needed to self-finance the level of investment that would be chosen based on outside investors' marginal valuation of investment, ϕ^s, then she chooses the investment rate $\iota(\phi^s)$ (the x^* that equates the marginal cost of investment, $\text{C}'(x^*)$, to the outside investors' marginal valuation, ϕ^s), uses up all of her own funds to finance investment (sets $y^* = 0$), and also resorts to equity issuance. Finally, if the entrepreneur's financial wealth is too low to self-finance her first-best investment rate but high enough to self-finance the investment rate that would be chosen based on outside investor's valuations, i.e., if $\text{C}(\iota(\phi^s)) < \omega < \text{C}(\iota(\hat{\phi}^s))$, then the entrepreneur invests the maximum that can be financed with her internal funds, i.e., the investment rate x^* that satisfies $\text{C}(x^*) = \omega$, sets $y^* = 0$, and issues no equity.

The following corollary of Lemma 1 shows that in every case the optimal investment rate, x^*, equates the marginal (technological) cost of investing, $\text{C}'(x^*)$, to *Tobin's q*, i.e., the marginal return to investing, which we donte by q.

Corollary 1. *If* $\hat{\phi}^s \le \phi^s$, $q = \phi^s$. *If* $\phi^s < \hat{\phi}^s$,

$$q = \begin{cases} \hat{\phi}^s & \text{if } \mathrm{C}(\iota(\hat{\phi}^s)) \le \omega \\ \mathrm{C}'(\mathrm{C}^{-1}(\omega)) & \text{if } \mathrm{C}(\iota(\phi^s)) < \omega < \mathrm{C}(\iota(\hat{\phi}^s)) \\ \phi^s & \text{if } \omega \le \mathrm{C}(\iota(\phi^s)) . \end{cases}$$

In every case, the optimal investment rate, x^*, *satisfies*

$$\mathrm{C}'(x^*) = q.$$

For what follows, let $x^*(\omega)$ and $s_{+1}^*(\omega)$ denote the optimal investment and equity issuance decisions taken by an entrepreneur who enters with a ratio of financial wealth to physical capital equal to ω, as characterized in Lemma 1. With this notation, we can write the aggregate investment chosen by all active entrepreneurs at the end of a period as

$$X^* = \int x^*(\omega)\, d\Omega(\omega), \tag{12}$$

and the aggregate stock of equity shares outstanding at the beginning of a period as

$$S^* = \int s_{+1}^*(\omega)\, d\Omega(\omega). \tag{13}$$

For the remainder of this section, we assume $\delta - \iota_0 \le 1 \le \min\{\underline{\phi}^s, \hat{\phi}^s\}$, where $\underline{\phi}^s \equiv \beta\bar{\varepsilon}z$. The following proposition characterizes the nonmonetary equilibrium.

Proposition 1. *A nonmonetary equilibrium exists for any parametrization. In the nonmonetary equilibrium, money has no value, i.e.,* $\mathrm{M} = 0$, *and* $\phi^s = \underline{\phi}^s$ *is the price of an equity share issued by entrepreneurs. Moreover: (i) If* $\hat{\phi}^s < \underline{\phi}^s$, *then* $X^* = \iota(\underline{\phi}^s)$, *and* $S^* = 1 - \delta + \iota(\underline{\phi}^s)$. *(ii) If* $\underline{\phi}^s < \hat{\phi}^s$, *then*

$$X^* = \Omega[\mathrm{C}(\iota(\underline{\phi}^s))]\iota(\underline{\phi}^s) + \int_{\mathrm{C}(\iota(\underline{\phi}^s))}^{\mathrm{C}(\iota(\hat{\phi}^s))} \mathrm{C}^{-1}(\omega) d\Omega(\omega) + \{1 - \Omega[\mathrm{C}(\iota(\hat{\phi}^s))]\}\iota(\hat{\phi}^s),$$

and

$$S^* = \frac{1}{\underline{\phi}^s} \int_0^{\mathrm{C}(\iota(\underline{\phi}^s))} [\mathrm{C}(\iota(\underline{\phi}^s)) - \omega] d\Omega(\omega).$$

The following proposition characterizes the monetary equilibrium. Before stating the result, it is convenient to define $\overline{\phi}^s \equiv \beta\left[\bar{\varepsilon} + \alpha\theta\left(\varepsilon_H - \bar{\varepsilon}\right)\right] z$ and $\bar{r} \equiv \alpha\theta\left(\bar{\varepsilon} - \varepsilon_L\right)/\varepsilon_L$.

Proposition 2. *Assume $r \in (0, \bar{r})$. (i) There exists a unique stationary monetary equilibrium. (ii) The equity price is*

$$\phi^s(r) = \beta \left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon) \right] z, \tag{14}$$

where $\varepsilon^* \in (\varepsilon_L, \varepsilon_H)$ is the unique solution to

$$\alpha\theta \int_{\varepsilon^*}^{\varepsilon_H} \frac{\varepsilon - \varepsilon^*}{\varepsilon^*} dG(\varepsilon) = r. \tag{15}$$

(iii) If $\hat{\phi}^s \in (\underline{\phi}^s, \overline{\phi}^s)$, let $\hat{r} \in (0, \bar{r})$ be defined by $\phi^s(\hat{r}) = \hat{\phi}^s$. Then: (a) If $r \in (0, \hat{r})$, then $X^* = \iota(\phi^s(r))$, and $S^* = 1 - \delta + \iota(\phi^s(r))$. (b) If $r \in (\hat{r}, \bar{r})$, then

$$X^* = \Omega[\mathrm{C}(\iota(\phi^s(r)))]\iota(\phi^s(r))) + \int_{\mathrm{C}(\iota(\phi^s(r)))}^{\mathrm{C}(\iota(\hat{\phi}^s))} \mathrm{C}^{-1}(\omega) d\Omega(\omega) + \{1 - \Omega[\mathrm{C}(\iota(\hat{\phi}^s))]\}\iota(\hat{\phi}^s),$$

and

$$S^* = \frac{1}{\phi^s(r)} \int_0^{\mathrm{C}(\iota(\phi^s(r)))} [\mathrm{C}(\iota(\phi^s(r))) - \omega] d\Omega(\omega).$$

(iv) If $\hat{\phi}^s < \underline{\phi}^s$, X^* and S^* are as in part (iii)(a). (v) If $\overline{\phi}^s < \hat{\phi}^s$, X^* and S^* are as in part (iii)(b). (vi) In every case, aggregate real money balances are given by $\mathrm{M} = \frac{G(\varepsilon^*)}{1 - G(\varepsilon^*)} S^*$.

The following corollary of Proposition 2 documents how asset prices and the investment rate respond to changes in the monetary policy rate, r.

Corollary 2. *In the stationary monetary equilibrium: (i) As $r \to \bar{r}$, $\mathrm{M} \to 0$, and $\phi^s \to \underline{\phi}^s$. (ii) As $r \to 0$, $\phi^s \to \overline{\phi}^s$. (iii) $d\varepsilon^*/dr < 0$ and $d\phi^s(r)/dr < 0$. (iv) $d\iota(\phi^s(r)))/dr < 0$, (v) $d^2\phi^s(r)/(drd\eta) < 0$, where $\eta \equiv \alpha\theta$.*

5. Empirical results

5.1. Data

We follow the literature that employs high-frequency price changes in financial markets around Federal Open Market Committee (FOMC) press releases to isolate unexpected components of monetary policy announcements from endogenous responses to macroeconomic conditions.[11] A conventional way to do this is by measuring changes in rates implied by federal funds futures

[11] Prominent early examples of such an event study based approach to the effects of monetary policy are Kuttner (2001), Cochrane and Piazzesi (2002), Bernanke and Kuttner (2005), Gürkaynak et al. (2005).

contracts in narrow 30-minute windows around policy announcement times. Given that futures contracts capture market participants' expectations about interest rates, these changes provide a proxy for exogenous policy rate shocks. The identification assumption is that in the narrow window around the press release there are no other, non-monetary shocks affecting futures rates.

Earlier work has pointed out that the unexpected component of monetary policy decisions as measured by the high-frequency movements in federal funds futures rates may nontheless contain additional information about the conduct of monetary policy, such as the implicit revelation of the monetary authority's information about economic fundamentals imperfectly observed by the private sector.[12] Treating these measures as purely exogenous changes in policy rates may thus lead to further imprecisions in inference. To address this issue, we also consider as a monetary shock proxy the series implied by the method of Jarociński and Karadi (2019).[13] Yet because the identification approach of Jarociński and Karadi (2019) relies on inference using sign restrictions, it only provides set identification and introduces further uncertainty in the shock proxy series. To abstract from potential complications introduced by generated instruments and model misspecification for our inference based on panel local projections, we construct our baseline shock proxy series based on the raw, high-frequency changes in 3-month ahead federal funds futures contracts at FOMC announcement times. And we consider alternative shock series following Jarociński and Karadi (2019) in robustness analysis.

Given that we work with quarterly firm-level data, we add up the high-frequency changes in the federal funds futures rate by quarter to arrive at our quarterly series of monetary policy shock proxies denoted as ε_t^m. While we acknowledge that ε_t^m is still very likely a noisy measure of true, fundamental monetary shocks and should be applied as an instrument in IV regressions (Stock and Watson, 2018), we will treat ε_t^m simply as a measure of monetary shocks in our reduced form specifications. In our main empirical IV specifications, we use ε_t^m in the construction of an instrument for stock prices. We use the convention that a positive ε_t^m stands for an unexpected increase in interest rates, and thus a contractionary monetary shock.

Our firm-level data comes from the Center for Research in Security Prices (CRSP) and

[12] See, for example, Nakamura and Steinsson (2018), Miranda-Agrippino and Ricco (2019), and Jarociński and Karadi (2019).

[13] Their proposed approach employs a structural vector autoregression (VAR) model identified using high-frequency changes in federal funds futures rates alongside sign restrictions imposing that conventional monetary policy shocks generate opposite-signed surprises in futures rates and returns in the S&P500 index. This purges the proxy series from informational components that generate positive high-frequency comovement between interest rates and stock returns.

Compustat databases. We use daily time series for all individual common stocks available in the CRSP database. As a measure of trade volume for each stock, we construct the daily *turnover rate* as the ratio of trade volume (total number of shares traded) to the number of outstanding shares, for any given day. After averaging each stock's daily turnover rates by quarter, we merge the resulting quarterly turnover series (denoted $\mathcal{T}_t^i$ for firm i quarter t below) with the corresponding firm fundamentals from the quarterly Compustat universe of publicly listed U.S. incorporated non-financial firms.

The key objects of interest from the Compustat database are measures of q, firms' equity issuances, and investment rates. As our measure of q, we employ the market-to-book ratio, i.e. an *average* q, computed as the book value of total assets plus the market value of common equity minus the book value of common equity, scaled by the book value of total assets, or $q = 1 + \frac{\text{MV equity} - \text{BV equity}}{\text{book total assets}}$. Moreover, as Eberly et al. (2012), we use the natural logarithm of q in our estimated regressions. Doing so provides a better fit, improving the precision of our estimates due to skewness in the firm-level data, while not affecting the qualitative implications. As the measure of *equity issuances* (denoted e_t^i), we employ the net measure of Compustat reported sales of equity minus purchases of equity in quarter t. We normalize the net issuances by the total balance sheet size at the beginning of quarter t (denoted b_t^i).[14] As the main measure of *investment*, we use the Compustat reported capital expenditures variable net of sales of property plant and equipment (denoted x_t^i). We normalize the investment measures with Compustat's net property plant and equipment at the beginning of the quarter (denoted k_t^i). In robustness analysis, we also employ measures of firms' *size*, *age*, *leverage* and *liquidity ratios* as additional controls.[15]

Our sample covers the period 1990Q1–2016Q4, for which high-frequency data on federal funds futures is available.[16] Because our regression specifications include simple firm fixed effects in a dynamic panel setting, we only include data from firms which are observed for at least 40 uninterrupted quarters at any point during the sample period. Appendix B.1 discusses sample selection and data construction in more detail.

[14] We measure the "beginning of quarter t" values of firms' stock variables, such as balance sheet size b_t^i or capital k_t^i, with the values reported in Compustat as of the end of quarter $t-1$.

[15] For constructing a measure of firm age, we follow the approach of Cloyne et al. (2018) and use data from Thomson Reuters' WorldScope database to infer time since the firm's incorporation.

[16] In constructing our various measures of ε_t^m, we employ the dataset used by Jarociński and Karadi (2019), in turn based on an updated version of the Gürkaynak et al. (2005) dataset.

5.2. Identification through turnover heterogeneity

In line with a model of monetary exchange in equity markets, analogous to the model presented above, Lagos and Zhang (2020b) provide evidence using daily data that stock turnover in the four weeks prior to a monetary policy announcement is a strong predictor of policy pass-through to stock prices in the cross-section of U.S. public firms. Stock turnover can thus be considered as a measure of firms' stock price exposure to monetary policy shocks.

If the variation in turnover were assigned randomly across firms, in a manner uncorrelated with any firm characteristics or fundamentals at the time of a monetary policy shock, then identified policy shocks combined with cross-sectional turnover heterogeneity can be used as a source of exogenous cross-sectional variation in stock prices and Tobin's q. And this cross-sectional variation in stock price responses for high- versus low-turnover firms can then in turn be applied to identify the effects of stock price fluctuations on firms' decisions.

The focus on cross-sectional variation is important because monetary policy shocks can affect the levels of all stock prices and firms' choices, potentially through intricate general equilibrium effects. For example, by lowering demand for the firm's production and decreasing profits, a contractionary monetary shock can cause stock prices and investment to fall. But it is not necessarily the case that investment falls *because* the stock price falls. Rather, both are endogenous outcomes responding to a worsening of demand. Thus, by simply observing the comovements of q and investment in the aftermath of monetary shocks, one cannot hope to make any causal statements.[17]

Based on the logic above, we construct an instrument for the cross-sectional variation in Tobin's q by interacting monetary policy shocks and individual stock turnover in the quarter prior to the shock. To isolate cross-sectional variation, we consider regression specifications in which industry-time fixed effects pick up any direct and general equilibrium effects that are common across all firms in an industry. And our exercise of main interest is to assess whether such instrumented variation in q predicts firms' equity issuance and investment behavior. In order for such IV estimates to have a causal interpretation in measuring the effect stock prices on firms' choices, the following identification assumption must hold: firms with different stock turnover respond to monetary policy shocks differently only because of differences in their stock price responses to the shocks.

[17] More formally, one can think of exploring such comovements by regressing investment on q, instrumenting the latter with identified monetary policy shocks.

If stock turnover was truly assigned randomly immediately prior to the realization of monetary shocks, the identification assumption for causality would be satisfied. The fact that this is unlikely to be the case in reality leads to potential concerns in the validity of a causal interpretation of our estimates. The randomness of variation in stock turnover could be violated in either of two main ways:

1. Certain firm characteristics cause their stocks to be traded relatively more or less. For example, the stocks of bigger firms may provide a more liquid market and invite higher trading activity, even relative to their potentially large market capitalizations.

2. For reasons not related to firm characteristics, stocks may experience heterogeneous trading activity. As suggested by the model, this can lead to differences in stock prices which in turn can affect firm behavior, for example their financing or portfolio decisions.

Either of these scenarios can introduce covariance between certain firm characteristics and stock turnover. If these characteristics are in turn predictive of firms' responsiveness to monetary policy shocks in their own right through other channels, as for example leverage could be (Anderson and Cesa-Bianchi, 2020), then any heterogeneous behavior predicted by turnover alone could instead be explained by reasons other than heterogeneous stock price changes, violating the identification assumption for causality. We address these concerns in robustness analysis by allowing various other firm-level controls to explain heterogeneous responsiveness of equity issuance and investment alongside turnover. To address point 1. above, we introduce measures of firm size and age, and regarding 2., we also include measures of leverage and liquid asset holdings.

For our approach in instrumenting Tobin's q to plausibly provide evidence of the effects of q on firms' choices, we should observe that firms with different stock turnover exhibit heterogeneous responsiveness to monetary shocks both in q and in their behavior. We first explore whether this is true by estimating 'reduced form' OLS regressions. The corresponding regression for q serves to illustrate the first stage of our main instrumental variables regression. And having established whether and when firms with higher turnover exhibit differential equity issuance and investment responses to identified monetary shocks, we move to the IV regressions of main interest.

5.3. Reduced form regressions

Our empirical analysis builds on local projections in the spirit of Jordà (2005), applied in a panel setting. As mentioned, we first estimate what we refer to as 'reduced form' specifications. The main goal of these regressions is to verify whether in our sample, firms with different stock turnover, as measured prior to monetary policy shocks, exhibit differential responses in q, equity issuances, and investment.[18] Given that in our theoretical model of the turnover channel, stock prices affect investment because firms adjust their equity issuances to fluctuations in the former, we also analyze equity issuances to test whether there is evidence of such "market timing" behavior (Baker and Wurgler, 2002). As our baseline, we estimate panel regression specifications of the following form on our full sample of firm-level data:

$$y^i_{t+h} = f^i_h + d_{s,h,t+h} + \rho_h y^i_{t-1} + \beta_h \mathcal{T}^i_{t-1} + \gamma_h \mathcal{T}^i_{t-1}\varepsilon^m_t + u^i_{h,t+h} \tag{16}$$

$h = 0, 1, \ldots, H$ denotes the horizon at which the shock impact effects are being estimated. y^i_t refers to firm i's outcome variable of interest in quarter t. Based on the notation introduced above, y^i_t is one of $\log(q^i_t)$, e^i_t/b^i_t, or x^i_t/k^i_t.

f^i_h denotes firm i's fixed effect in the projection at horizon h. $d_{s,h,t+h}$ is shorthand for industry-quarter dummies at the SIC 2-digit level, given the h-quarter projection horizon and the outcome variable being measured in period $t+h$. ε^m_t is a measure of the quarterly monetary policy shock as discussed above. $u^i_{h,t+h}$ is the error term in the projection of the outcome variable in period $t+h$, given the h-quarter projection horizon. $\rho_h, \beta_h, \gamma_h$ are regression coefficients. The main object of interest is the estimate for γ_h which captures any heterogeneity in shock responsiveness predicted by stock turnover.

We lag firm controls to ensure they are unaffected by the realization of ε^m_t and can be thought of as measures of shock-exposure. As long as there is persistence in stock turnover from one quarter to another, the turnover measured in $t-1$ proxies for turnover immediately before the FOMC announcement in quarter t. As discussed above, our focus is on cross-sectional differences in how firms' stock turnover predicts their responsiveness to monetary policy shocks. Including a detailed industry-time dummy $d_{s,h,t+h}$ allows for a flexible way to isolate this cross-sectional variation. Thus, the identification of the mechanism of interest is driven by *within-industry between-firm* variation across time.

[18] In doing so, we are also providing a test for whether the main empirical findings of Lagos and Zhang (2020b) hold when measuring equity valuations based on q at a quarterly frequency, instead of using daily stock returns.

We multiply all the y_t^i considered by 100 for convenience, so the coefficients for changes in q can be interpreted in percentage terms and the issuance and investment ratios in percentage points. We standardize the turnover measure $\mathcal{T}_t^i$ by the standard deviation of turnover in the cross-section of firms, averaged across time over our sample. And we standardize the monetary shock measures ε_t^m by their standard deviation between 1990Q1–2016Q4 of approximately 9.66 bp, as measured by changes in federal funds futures rates.

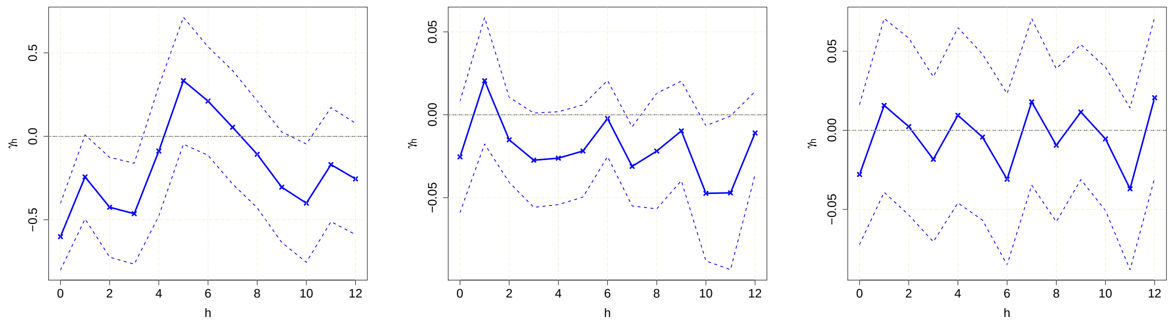
Figure 1 presents the point estimates and 95% confidence intervals for γ_h given the three outcome variables of interest. As one would expect based on financial markets incorporating the FOMC announcements virtually immediately, the heterogeneity in stock price responses predicted by turnover is strongest in the quarter of the monetary policy shock. The point estimate of approximately -0.5 says that an increase in stock turnover by 1 sd predicts a 0.5% stronger contraction in the firm's q in the quarter of a 1 sd contractionary monetary policy shock.[19] And the predicted differences between stock prices persist for about up to a year after the shock.

Heterogeneity in the responsiveness of equity issuances predicted by stock turnover appears about three quarters after the shock. Yet the estimated heterogeneity is just marginally statistically insignificant at the 95% confidence level. In terms of quantitative magnitudes, the coefficient of approximately -0.03 implies that a 1 sd increase in stock turnover predicts a 0.03 pp larger drop in net equity issuances relative to book assets three quarters after a 1 sd contractionary monetary shock. The negative predictive effect of turnover for quarterly issuances in the full sample of firm-quarters appears to be persistent and yields statistically significant estimates 7 and 10 quarters after the shock. Finally, the estimates in the last panel of Figure 1 imply that there is no significant evidence of turnover predicting stronger or weaker investment responses in the full sample of firm-quarters. Although, the point estimates suggest that there might be a weak negative relationship of higher turnover predicting stronger investment contractions after unexpected policy rate increases.

The theoretical model presented above provides a stark prediction about which firms' choices should be affected by the turnover-liquidity transmission mechanism. Firms which have few liquid resources available, relative to their size, are more likely to rely on external equity financing and expose themselves to fluctuations in stock prices. Firms that do not issue equity are isolated from these fluctuations. So even though among such firms stock prices respond

[19] More precisely, given specification (16), a negative γ_h only allows to infer a drop in q *relative* to other firms.

Figure 1: Heterogeneity in responses to monetary policy shock conditional on stock turnover



(a) $y_t^i = \log(q_t^i)$ (b) $y_t^i = e_t^i/b_t^i$ (c) $y_t^i = x_t^i/k_t^i$

Notes: Point estimates and 95% confidence intervals for γ_h from estimating specification (16). Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

to monetary policy shocks, and more so for those with high turnover, their choices of equity issuance and investment are unaffected by this. And therefore, no heterogeneous responses of issuances and investment conditional on turnover should be observed.

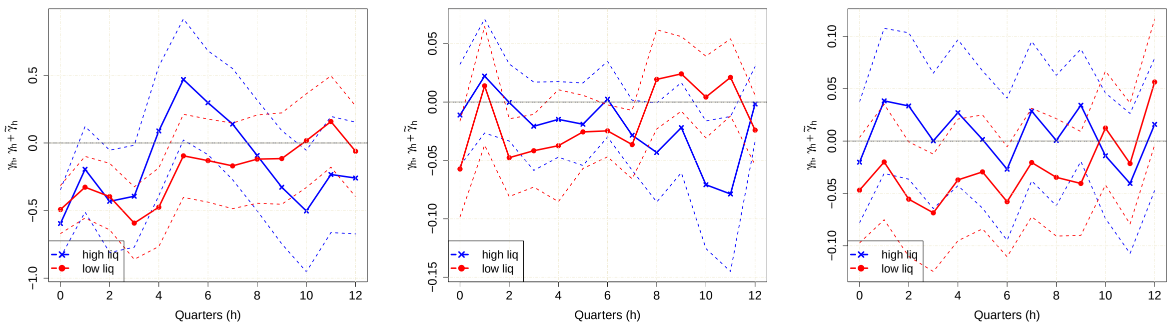
To test the empirical validity of these predictions and allow for differences in the strength of the turnover-liquidity channel across groups of firms, we define the indicator $\mathbb{I}_{L,t}^i$ which equals 1 if firm i belongs in the bottom half of the liquidity ratio distribution of the cross-section of firms in quarter t, and 0 otherwise. We define the *liquidity ratio* for firm i in quarter t as the ratio of Compustat reported *cash and short-term investments* to i's total assets in t, meant to capture the holdings of various assets that firms use to manage their liquidity and financial savings. And we estimate the following specification:

$$\begin{aligned} y_{t+h}^i =& f_h^i + d_{s,h,t+h} + \alpha_h \mathbb{I}_{L,t-1}^i + \left(\rho_h + \tilde{\rho}_h \mathbb{I}_{L,t-1}^i\right) y_{t-1}^i \\ &+ \left(\beta_h + \tilde{\beta}_h \mathbb{I}_{L,t-1}^i\right) \mathcal{T}_{t-1}^i + \left(\gamma_h + \tilde{\gamma}_h \mathbb{I}_{L,t-1}^i\right) \mathcal{T}_{i,t-1} \varepsilon_t^m + u_{h,t+h}^i \end{aligned} \tag{17}$$

In this case, γ_h measures the predictive power of turnover heterogeneity for firms with high liquidity ratios prior to the shock, and $\gamma_h + \tilde{\gamma}_h$ for those with low liquidity ratios. Figure 2 presents the point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ from the estimation of (17) for the three outcome variables of interest.

The first panel in Figure 2 indicates that the predictive power of turnover heterogeneity for stock price responses is similar across the two liquidity ratio groups. As predicted by the model, the turnover-liquidity channel is operative for all stocks, with the high-turnover ones responding relatively more in the quarter of a monetary shock, independently of the firms' liquid asset positions. The point estimates for γ_0 are close to the estimates of the full sample of firms in specification (16). While the differences in stock prices persist for about a year after the shock, the statistical significance of the estimates for the high liquidity group in quarters after the shock is weakened slightly.

Figure 2: Heterogeneity in responses to monetary policy shock conditional on stock turnover, across liquidity ratio groups



(a) $y_t^i = \log(q_t^i)$ (b) $y_t^i = e_t^i/b_t^i$ (c) $y_t^i = x_t^i/k_t^i$

Notes: Point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ from estimating specification (17). Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

The middle panel of Figure 2 shows that the relation between turnover and equity issuance responses to monetary policy shocks during the first two years is driven by firms with low liquid asset holdings. Among firms with below-median liquidity ratios prior to the shock, higher stock turnover predicts significantly stronger contractions in equity issuance both in the shock impact quarter and two and three quarters after a contractionary shock. Among such firms, an increase of 1 sd in turnover predicts an approximately 0.05 pp larger decrease in equity issuances, measured as a fraction of total assets, both immediately and two and three quarters after a 1 sd contractionary monetary shock. Also, among firms with high liquidity ratios, higher turnover predicts lower equity issuances in the aftermath of policy rate increases, although this relation is weaker over the two-year horizon, and becomes more pronounced at longer horizons. The extended version of our model with long-lived firms below provides a potential rationale for why firms with high liquidity ratios at the time of the shock may respond with a considerable delay. This happens whenever they draw down their liquid assets and engage in equity financing in subsequent quarters, while the effects of the shock on stock prices have not yet dissipated. Also, this empirical finding is not robust in our IV regressions when controlling for other firm-level covariates.

Finally, the estimates in the last panel of Figure 2 indicate that among firms with below median liquid asset holdings, those with higher turnover exhibit relatively lower investment rates after a contractionary monetary policy shock. For these firms, a 1 sd higher stock turnover predicts approximately 0.07 pp lower investment, relative to the capital stock, three quarters after a 1 sd contractionary monetary policy shock. As for equity issuances, the differences in investment rate responses predicted by turnover are persistent, with higher turnover predicting statistical differences in investment responses also six quarters after a monetary shock. Yet among firms with high liquidity ratios, heterogeneity in stock turnover does not predict any differential responses in investment.

5.4. IV regressions

We now turn to our main exercise of interest. We combine the cross-sectional heterogeneity in the monetary shock responses of Tobin's q, equity issuances and investment explained by stock turnover into an instrumental variables specification, in order to evaluate the effects of stock price fluctuations on equity issuances and investment. To do so, we construct the analogue of specification (16) by replacing the interaction term between turnover and the monetary shock $\mathcal{T}^i\varepsilon^m$ with the firm's measure of q, which is then instrumented with the $\mathcal{T}^i\varepsilon^m$-term.

As suggested by the OLS estimates for the reduced form specification, the heterogeneity in the monetary shock responses of q, equity issuances, and investment as explained by turnover heterogeneity can materialize at different horizons. Because of this, we consider allowing for the possibility that the variation in q instrumented by turnover and the monetary shock in period t is measured in period $t + h_q$, and the predicted effects on issuances and investment measured in period $t+h$, with $0 \leq h_q \leq h$. For example, if the effect of a monetary shock on stock prices required 1 quarter to fully materialize, yet the effects of stock price fluctuations on investment take 3 quarters to transmit, the main interest would be to study how heterogeneous variation in q in period $t + 1$ explains investment in $t + 4$ after a monetary policy shock in t. However, given that the heterogeneity in stock prices appears strongest in the impact quarter, as seen in Figures 1a and 2a, we focus the main estimations below on the case of $h_q = 0$.

Our baseline instrumental variable specification is as follows:

$$y^i_{t+h} = f^i_h + d_{s,h,t+h} + \rho_h y^i_{t-1} + \beta_h \mathcal{T}^i_{t-1} + \gamma_{h,h_q} \log\left(q^i_{t+h_q}\right) + u^i_{h,t+h} \tag{18}$$

where $\log\left(q^i_{t+h_q}\right)$ is instrumented with $\mathcal{T}_{i,t-1}\varepsilon^m_t$, and $0 \leq h_q \leq h$ for some h_q, with $h = 0, 1, \ldots, H$.

Figure 3 depicts the point estimates and 95% confidence intervals for γ_{h,h_q} from the estimation of (18) with 2SLS, given $h_q = 0$, for equity issuances and investment as dependent variables. The IV estimates are in line with what one would expect based on the reduced form OLS results in Section 5.3. The cross-sectional variation in q instrumented with turnover-based monetary shock exposure predicts higher equity issuances after increases in q caused by monetary shocks. Or, in light of the identification assumption stated in Section 5.2, this suggests that firms' equity issuances respond positively to exogenous increases in Tobin's q. The point estimates, although just barely statistically insignificant, indicate that a 1% increase in q leads to an approximately 0.02 pp increase in equity issuances relative to total assets three quarters later. The instrumented variation in q predicts statistically significantly higher issuances 7 and 10 quarters later, although these estimates are not robust to all specifications, as seen below. As expected based on the reduced form estimates, cross-sectional variation in q instrumented with monetary shocks and stock turnover does not predict significant heterogeneity in investment in the sample of all firm-quarters. Although, the point estimates suggest a tendency of higher q predicting higher investment, if anything.

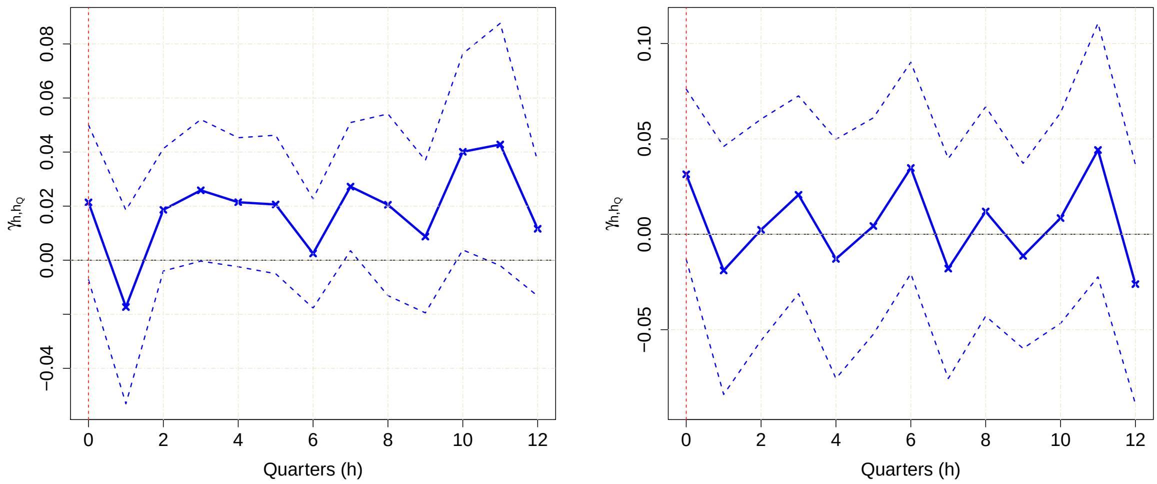
Following the predictions of our model and the evidence presented in Section 5.3, we finally turn to estimating the IV specification by allowing for differences in coefficient estimates for firms with high versus low liquid asset holdings. Employing the indicator $\mathbb{I}^i_{L,t}$ of having a below-median liquidity ratio in t, defined in Section 5.3, we consider the following specification:

$$\begin{aligned} y^i_{t+h} =& f^i_h + d_{s,h,t+h} + \alpha_h \mathbb{I}^i_{L,t-1} + \left(\rho_h + \tilde{\rho}_h \mathbb{I}^i_{L,t-1}\right) y^i_{t-1} \\ &+ \left(\beta_h + \tilde{\beta}_h \mathbb{I}^i_{L,t-1}\right) \mathcal{T}^i_{t-1} + \left(\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q} \mathbb{I}^i_{L,t-1}\right) \log\left(q^i_{t+h_q}\right) + u^i_{h,t+h} \end{aligned} \tag{19}$$

where the vector $\left[\log\left(q^i_{t+h_q}\right), \mathbb{I}^i_{L,t-1} \log\left(q^i_{t+h_q}\right)\right]$ is instrumented with $\left[\mathcal{T}^i_{t-1}\varepsilon^m_t, \mathbb{I}^i_{L,t-1}\mathcal{T}^i_{t-1}\varepsilon^m_t\right]$.

Figure 4 presents the point estimates and 95% confidence intervals for γ_{h,h_q} and $\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q}$ from the estimation of (19) given $h_q = 0$, for equity issuances and investment as dependent variables. Again, the IV estimates confirm the findings from the reduced form regressions. Among firms with low liquid asset holdings, the cross-sectional variation in q instrumented

Figure 3: Issuances and investment predicted by instrumented q



(a) $y_t^i = e_t^i / b_t^i$ (b) $y_{i,t} = x_t^i / k_t^i$

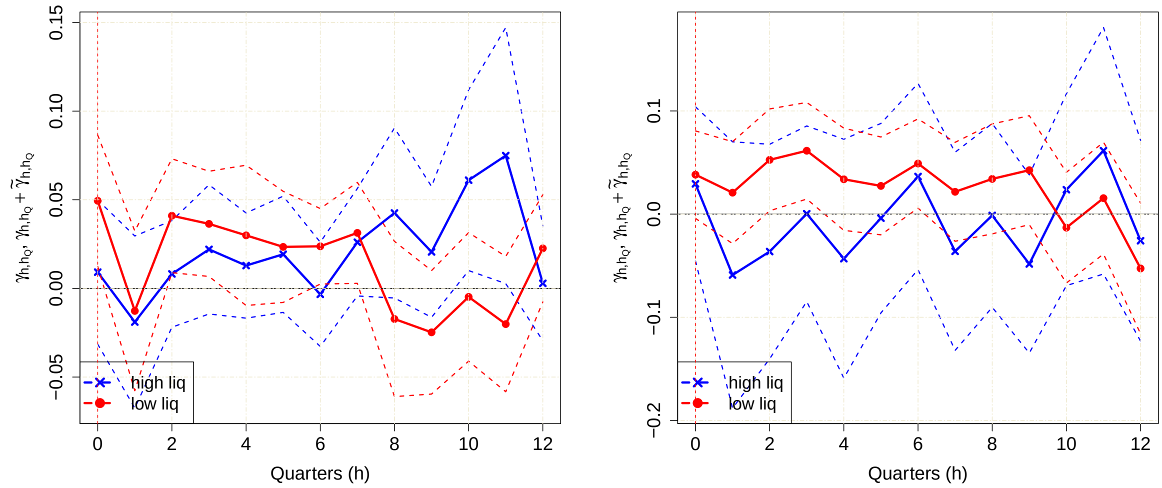
Notes: Point estimates and 95% confidence intervals for γ_{h,h_q} from estimating specification (18). Vertical red dashed line marks the value of $h_q = 0$. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

with turnover and identified monetary policy shocks predicts significant heterogeneity in equity issuances. A 1% increase in q leads to an approximately 0.05 pp increase in equity issuances relative to total assets in the same quarter. Similar, statistically significant point estimates are implied for equity issuances also two and three quarters later. For firms with high liquidity ratios, the positive relation between instrumented variation in q and equity issuances is weaker at the two year horizon, but becomes more evident later on.

Finally, increases in instrumented q predict higher investment for firms with low liquid asset holdings. For these firms, a 1% increase in q implies an approximately 0.06 pp higher investment rate three quarters later. For firms with liquidity ratios above the median, instrumented variation in q does not predict heterogeneity in investment.

Robustness. In Appendix B.2, Figure 6 we include additional firm-level controls interacted with the monetary shock measure in specification (19) to verify that the predicted heterogeneity in issuance and investment responsiveness is not in fact explained by other firm-level covariates. Based on the discussion in Section 5.2, we consider as the main controls measures of size, leverage and liquidity ratios. Comparing the results in Figures 4 and 6, it is clear that while

Figure 4: Issuances and investment predicted by instrumented q, across liquidity ratio groups



(a) $y_t^i = e_t^i / b_t^i$ (b) $y_t^i = x_t^i / k_t^i$

Notes: Point estimates and 95% confidence intervals for γ_{h,h_q} and $\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q}$ from estimating specification (19). Vertical red dashed line marks the value of $h_q = 0$. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

the confidence intervals on the estimates of γ_{h,h_q} and $\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q}$ widen due to cross-sectional correlation between stock turnover and the various firm-level controls, the point estimates are in large part unchanged and our main results hold. In Figure 7 in Appendix B.2, we further add firm age as a control. Because of worse coverage of the age variable, we lose almost a fifth of the firm-quarter observations from the full sample behind the results in Figure 4, so the estimates are again slightly less precise. But the main finding remains. An increase in firms' q, as instrumented by stock turnover and monetary policy shocks, leads to significantly higher investment among low-liquidity firms, and this finding cannot be explained by the other firm-level covariates predicting heterogeneous responsiveness to monetary shocks.

In Figure 8 in Appendix B.2, we present the main OLS and IV coefficient estimates for an alternative ε_t^m series identified based on the 'poor man's sign restrictions' by Jarociński and Karadi (2019).[20] As seen from the figures, our main findings hold when potential informational

[20] We focus on the 'poor man's sign restrictions' series by Jarociński and Karadi (2019) since their benchmark identification approach relies on (set-)identification with a linear model which can lead to further imprecisions during the financial crisis and zero lower bound periods after 2008 during which nonlinear dynamics most likely played a central role in the economy.

effects of policy announcements are purged from the monetary shock series.

6. Quantitative analysis

In this section we use a quantitative version of the model presented in Section 2 to assess the ability of the theory to match the dynamic responses of investment documented in Section 5. The theory consists of two building blocks: an asset-pricing block that determines equity prices given monetary policy, and an investment block that determines the capital structure and investment decisions of firms.

We extend the theory of Section 2 and introduce monetary policy shocks in the form of an unexpected change in the path of the nominal interest rate (10). The shock we consider is an unexpected increase of ε^m in r_{t+1} in period $t = 0$, after which the nominal rate follows an autoregressive path back to its steady state value according to: $r_{t+1} = \bar{r} + \rho_n (r_t - \bar{r})$ where $\bar{r}$ is the steady state net nominal rate. We choose ε^m so as to generate a 1% increase in firms' stock prices at the time of the announcement of the shock, conditional on other parameter values.

We extend the problem of the entrepreneurs seen in Section 2 in two ways. First, we introduce stochastic fixed equity issuance costs. More specifically, we assume that for an entrepreneur with capital stock k_t^i to issue new equity in period t, i.e. choose $e_t^i > 0$ in the second subperiod of t, he must exert effort and suffer a disutility of $\xi_t^i k_t^i$. The stochastic cost ξ_t^i is i.i.d. across entrepreneurs and time, distributed uniformly $\xi_t^i \sim U\left[0, \bar{\xi}\right], \forall (i,t)$, and is drawn at the beginning of the second subperiod of t by each entrepreneur.

Second, we assume that a unit of capital available at the beginning of period t also delivers $a \in \mathbb{R}_+$ units of good 2 in the second subperiod of t, in addition to producing z units of good 1 at the end of the first subperiod. A share issued by entrepreneur i in the second subperiod of t now represents ownership of 1 unit of capital along with the stream of *dividends* of both good 1 and good 2. That is, holding 1 share in period t constitutes the right to receive z units of good 1 in subperiod 1, and a units of good 2 in subperiod 2. We make an additional simplifying assumption by imposing that instead of receiving the a units of good 2, an investor who owns a share of entrepreneur i's capital at the beginning of subperiod 2 is instead given new shares $\tilde{e}_t^i$ in i's capital stock. And the size of this equity distribution is such that the market value of the new shares is equal to the market value of the dividends, i.e. $\psi_t^i \tilde{e}_t^i = a$, and thus the investor is indifferent. This means that we are implicitly imposing that the returns of capital in subperiod 2 remain within the firm and shareholders are compensated by an increase in the value of their shares they own, i.e. the firm simply retains all earnings in subperiod 2.[21] Since this operation constitutes a firm growing by retaining earnings, the distribution of $\tilde{e}_t^i > 0$ is not subject to fixed equity issuance costs.

As illustrated by the analytical example in Section 4, the introduction of the equity issuance costs and capital's ability to produce good 2 in subperiod 2 are in no way necessary to produce our main qualitative results. Rather, their main purpose to, in a straighforward manner, improve the quantitative characteristics and flexibility of the model. The equity issuance cost allows the model to yield a more realistic fraction of firms issuing equity at any given point in time and a nontrivial stationary distribution of liquid asset holdings. With the productivity $a > 0$, firms in the quantitative model can invest and grow not only using new equity issuances, but also using retained earnings, thus allowing the model to yield realistic average investment rates while matching the empirical frequency and size of equity issuances by public firms.

As for the remaining functional forms in the model, we assume quadratic capital adjustment costs $\Psi(x/k) = \frac{\kappa}{2}\left(\frac{x}{k}\right)^2$, and a lognormal distribution G for $\varepsilon \sim \log\mathcal{N}(\mu_\varepsilon, \sigma_\varepsilon)$. For the entrepreneur's problem, the relevant idiosyncratic state variable is the ratio of bond holdings to its capital stock.[22] For the exercises relevant below, because of this, the characteristics of newly born entrepreneurs simply need to be specified in terms of the distribution of initial endowment of good 2 to capital. As *ex post* heterogeneity of entrepreneurs is generated by the stochastic equity issuance costs and the occurrence of death, we simply assume that all entrepreneurs are born with a given $\omega_0 \equiv w_0/k_0 \in \mathbb{R}_{++}$, dictating the liquidity of the entrepreneurs' balance sheets at birth.

As the main exercise, we compare the impulse responses of investment rates for firms with high and low liquidity ratios in the stationary distribution of our model to the estimates from the data. The empirical IV coefficients from Section 5.4 estimate how much investment rates respond to a 1% increase in q generated by a monetary policy shock. In our stylized model, the *only* channel of monetary transmission from nominal rates to stock prices and investment is the turnover-liquidity channel. Therefore, we can simply study the impulse responses of *ex ante* identical firms (who also have identical stock turnover), with *ex post* heterogeneous liquidity positions. In contrast, in the data, it was necessary to employ cross-sectional variation in the

[21]It can be shown that in the stationary equilibrium of the model, retaining earnings and distributing equity is strictly preferred by all entrepreneurs over paying out any of the dividends a to outside investors.

[22]The entrepreneur's problem is homogeneous of degree 1 in k_t^i. So choices of investment rates, equity issuance rates, and bond holdings relative to capital are independent of the incoming capital stock.

monetary shock responses of firms with different stock turnover in order to identify and isolate the turnover-liquidity channel.

The parameter values we currently use for the quantitative exercise are chosen based on one time period being a quarter: $\beta = 0.995$, $\delta = 0.025$, $1 - \pi = 0.017$ (exit rate targeted by Begenau and Salomao (2019)), $\alpha = \theta = 1$, $\rho_n = 0.5$, $\bar{r} = 0.04/4$. As Lagos and Zhang (2020b), we consider a baseline calibration $\theta = 1$ to abstract from micro-level pricing frictions induced by bargaining. Also, given that the current quantitative exercise does not rely on turnover heterogeneity among firms in the model, we consider a baseline $\alpha = 1$. We calibrate σ_ε in the distribution of ε so that the stock price sensitivity of the firms (with $\alpha = 1$) in the model matches the impact effects of monetary policy shocks on the prices of the 10% highest turnover stocks in our empirical work.[23] And we normalize $\mu_\varepsilon = -\frac{\sigma_\varepsilon^2}{2}$. We use $\omega_0 = 2/3$ which is consistent with the approximate average cash-to-assets ratio of 0.40 for firms "entering" Compustat, i.e. engaging in an IPO and entering our sample of public firms during the period that we study, following Begenau and Palazzo (2020).

We calibrate the values of the remaining parameters $\hat{\varepsilon}$, z, a, $\bar{\xi}$, and κ to match moments yielded by the stationary equilibrium of our model to the sample of Compustat firms used in our empirical analysis of Section 5. More specifically, we target: the median liquidity (cash-to-assets) ratio, the average investment rates for firms with below-median and above-median liquidity, the unconditional frequency of equity issuance across firms and time, and the average ratio of equity issuance relative to total assets conditional on positive equity issuance.[24] Table 1 provides an overview of the employed structural parameter values and calibration targets.

Figure 5 depicts the model impulse responses of investment rates alongside the corresponding point estimates and confidence intervals already presented in panel (b) of Figure 4 in Section 5.4. Low-liquidity firms increase their investment rates by about 0.05 pp in response to a monetary shock that increases q by 1%. And both qualitatively and quantitatively, the whole

[23]Given that in this simple model, the only real effect of nominal rate shocks works through their effect on firms' stock prices and we normalize the monetary shock size in our main exercise so that stock prices respond by 1%, this choice simply governs the size of the required nominal rate shock in the background.

[24]We follow convention in the corporate finance literature and define the incidence of a firm issuing equity in our sample as the net equity issuance to asset ratio e_t^i/b_t^i exceeding some chosen cutoff. For example Leary and Roberts (2005) use a cutoff of 5% when working with annual Compustat data. For our analysis with quarterly data, we consider the cutoff of 1%. One reason for employing such a cutoff rule is that, as McKeon (2015) points out, the proceeds from sales of stock reported on firms' statements of cash flows often come from employees excercising options, rather than a managerial decision to sell stock as we are interested in identifying. Since firm-initiated issuances tend to be large and infrequent, he shows that using relative issuance size can with high reliability identify equity issuance proceeds that contain a firm-initiated component.

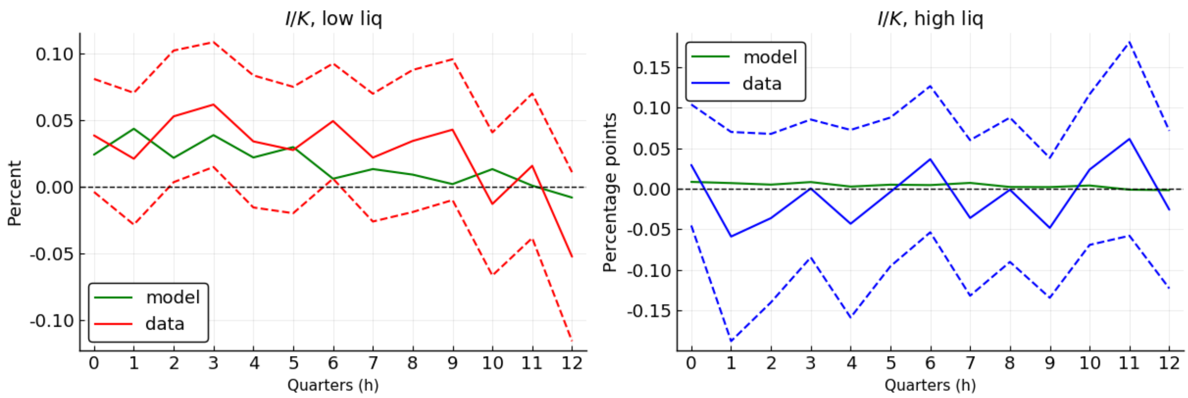
Table 1: Calibrated parameter values and calibration targets

Parameter	Value	Target / Source
Externally calibrated		
β	0.995	2% annual real rate
$\bar{r}$	0.04/4	4% annualized nominal rate
δ	0.025	Conventional
$1-\pi$	0.017	Compustat exit (Begenau and Salomao, 2019)
σ_ε	2.56	Top 10% turnover ψ_t^i response to MP
$(\alpha, \theta, \mu_\varepsilon)$	$\left(1, 1, -\frac{\sigma_\varepsilon^2}{2}\right)$	Normalization (Lagos and Zhang, 2020b)
ω_0	2/3	Average cash-to-assets at IPO (Begenau and Palazzo, 2020)
Internally calibrated		
z	0.031	$\text{med}\left(\frac{\text{cash}^i}{\text{assets}^i}\right) = 7.96\%$ (**model: 7.81%**)
a	0.038	$\text{avg}(x^i/k^i)\|_{\mathbb{I}_{L,t-1}=1} = 2.74\%$ (**2.80%**)
$\hat{\varepsilon}$	4.21	$\text{avg}(x^i/k^i)\|_{\mathbb{I}_{L,t-1}=0} = 3.69\%$ (**3.68%**)
$\bar{\xi}$	0.244	$\text{freq}(e^i/b^i > 0.01) = 0.0714$ (**0.0719**)
κ	27.80	$\text{avg}(e^i/b^i)\|_{e^i/b^i>0.01} = 9.57\%$ (**9.73%**)

path of the average response of investment rates is very similar in the model and the data.

For high-liquidity firms, the average investment rate response in the model is considerably smaller, consistent with no evidence of the turnover channel affecting such firms' investment in the data. Although, their investment response in the model is not exactly zero. This happens for two main reasons. First, in any given period, some firms designated as "high-liquidity" may get low enough draws of the equity issuance cost ξ_t^i and take advantage of the beneficial circumstances to issue equity. Although, they are significantly less likely to issue equity than the low-liquidity firms. But if they happen to do so exactly at the time of the monetary shock, their investment will respond to the change in the price of equity and is thus not "isolated" from the shock. Second, whenever not issuing equity, high-liquidity firms draw down their liquid assets, slowly becoming "low-liquidity", and experience an increase in their probability of issuing equity over time. If the monetary shock is persistent and its effect on stock prices lasts for several periods, the direct effect on these firms simply appears with a lag and to a smaller extent, depending on the shock's persistence. Moreover, because the high-liquidity firms anticipate this immediately when the shock is revealed, and they want to smooth investment due to the convex adjustment costs, they respond already at shock impact. To do so, they invest more out of their liquid asset holdings, even though they are not yet accessing the equity market.

Figure 5: Comparison of investment rate responses from model and data estimates



(a) Low liquidity ratio firms

(b) High liquidity ratio firms

Notes: Data refers to point estimates and 95% confidence intervals for γ_{h,h_q} and $\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q}$ from estimating specification (19) with $y_t^i = x_t^i / k_t^i$ as the outcome variable. *Model* response is computed as the average firm-level impulse response of investment rates, averaged over a large panel of firms drawn from the stationary distribution of the model. High and low liquidity ratios are defined as above or below the cross-sectional median cash-to-assets ratio in both model and the data.

7. Aggregate relevance in monetary transmission

Having established that our empirical estimates of the q-monetary (turnover) channel are both qualitatively and quantitatively consistent with the calibrated theoretical model, we proceed to provide a back-of-the-envelope assessment of the importance of the channel for monetary transmission to aggregate investment. We do so by directly employing our empirical regression estimates from Section 5, instead of relying on the calibrated structural model.

We first provide a brief discussion on how our empirical estimates based on between-firm variation can allow us to take the extra step and give an assessment of the overall effect of monetary transmission working through the turnover channel. To fix notation, let us use $\frac{dy}{dx}\Big|^{\text{TC}}$ to denote the effect of variable x on y *through the turnover channel.* This is in contrast to $\frac{dy}{dx}$ by which we mean the effect of x on y *through all possible transmission channels.* For concreteness, let us first focus the discussion on the effects that monetary policy shocks have on q. The estimates of γ_h from the reduced form OLS regressions of Section 5.3 provide an estimate of $\frac{d^2 \log\left(q_{t+h}^i\right)}{d\varepsilon_t^m d\mathcal{T}_{t-1}^i}$. That is, $\widehat{\gamma}_h$ captures how the estimated effect of ε_t^m on $\log\left(q_{t+h}^i\right)$ differs conditional on past turnover $\mathcal{T}_{t-1}^i$. By the identifying assumption that differences in firms' responses, as predicted by turnover, appear *only* because of the turnover channel, we can attribute these differences in full to the turnover channel, i.e. $\frac{d^2 \log\left(q_{t+h}^i\right)}{d\varepsilon_t^m d\mathcal{T}_{t-1}^i} = \frac{d^2 \log\left(q_{t+h}^i\right)}{d\varepsilon_t^m d\mathcal{T}_{t-1}^i}\Big|^{\text{TC}}$, estimated by γ_h.

However, without further moment restrictions or identifying assumptions, regressions relying on between-firm differences in responses only allow to identify these cross-derivatives: they tell us how the monetary shock affects the q of firms with different turnover differently (through the turnover channel). Yet the ultimate goal is to evaluate how the monetary shock affects firms' q through the turnover channel. That is, we would like to identify the first derivative $\frac{d\log\left(q_{t+h}^i\right)}{d\varepsilon_t^m}\Big|^{\text{TC}}$. Continuing with imposing linearity, one could "integrate out" $\mathcal{T}_{t-1}^i$ from the cross-derivatives and write:

$$\frac{d\log\left(q_{t+h}^i\right)}{d\varepsilon_t^m}\Bigg|^{\text{TC}} = \bar{\gamma}_h^i + \gamma_h \mathcal{T}_{t-1}^i$$

where $\bar{\gamma}_h^i$ could be thought of as a "missing intercept" (Wolf, 2019), referring to the (potentially firm specific) effect of monetary shocks on q_{t+h}^i through the turnover channel that is not explained by variation in turnover. $\bar{\gamma}_h^i$ cannot be identified solely based on our empirical regressions. But it can be identified based on our theoretical model of the turnover channel: for all stocks with zero turnover, the turnover channel is inactive. So the effect of monetary shocks on the corresponding firms through the channel must be zero. And we can pin down the missing empirical "intercept" as:

$$\frac{d\log\left(q_{t+h}^i\right)}{d\varepsilon_t^m}\Bigg|^{\text{TC}}_{\mathcal{T}_{t-1}^i=0} = 0 \Longrightarrow \bar{\gamma}_h^i = 0 \;\text{ and }\; \frac{d\log\left(q_{t+h}^i\right)}{d\varepsilon_t^m}\Bigg|^{\text{TC}} = \gamma_h \mathcal{T}_{t-1}^i$$

Note, importantly, that due to the stylized nature of our theoretical model, there exist no general equilibrium effects through which responsive firms can affect market prices, which in turn influence firms with zero stock turnover. So based on our model, this additional moment restriction is precise. In reality, its validity depends on whether such general equilibrium feedback effects are negligible or not.

Having established that $\frac{d\log\left(q_{t+h}^i\right)}{d\varepsilon_t^m}\Big|^{\text{TC}}$ can be gauged using $\widehat{\gamma}_h \mathcal{T}_{t-1}^i$, we compute that in our Compustat sample, across time and firms, the average effect of a 25 bp contractionary shock in ε_t^m, as measured by quarterly aggregated 3m Federal funds futures rate changes, is to decrease q_t^i by 1.65% at impact through the turnover channel.

Next, we can use the IV estimates from Section 5.4 to evaluate the effects of monetary shocks through the turnover channel on firms' investment. For illustration, let us first consider the specification that does not split the sample into firm-quarters with high and low liquidity ratios. Based on our identification assumption, the IV coefficient $\gamma_{h,0}$ in specification (18) for $y_t^i = x_t^i/k_t^i$ provides an estimate of $\frac{d\left(x_{t+h}^i/k_{t+h}^i\right)}{d\log\left(q_t^i\right)}\Big|^{\text{TC}}$. By the chain rule, we can therefore write:

$$\frac{d\left(x_{t+h}^i/k_{t+h}^i\right)}{d\varepsilon_t^m}\Big|^{\text{TC}} = \frac{d\left(x_{t+h}^i/k_{t+h}^i\right)}{d\log\left(q_t^i\right)}\Big|^{\text{TC}} \cdot \frac{d\log\left(q_t^i\right)}{d\varepsilon_t^m}\Big|^{\text{TC}} = \gamma_{h,0}\cdot\gamma_0\mathcal{T}_{t-1}^i$$

where $\gamma_{h,0}$ refers to the coefficient on the instrumented $\log\left(q_{t+h_q}^i\right)$ in specification (18) for $y_t^i = x_t^i/k_t^i$, with $h_q = 0$. And γ_0 refers to the coefficient on $\mathcal{T}_{t-1}^i\varepsilon_t^m$ in specification (16) for $y_t^i = \log\left(q_t^i\right)$. More precisely, since our estimates indicate that monetary shocks transmit to investment through the turnover channel only for firms with low liquid asset holdings, we use the following calculation to condition on the liquidity positions:

$$\frac{d\left(x_{t+h}^i/k_{t+h}^i\right)}{d\varepsilon_t^m}\Big|^{\text{TC}} = \left[(1-\mathbb{I}_{L,t-1}^i)\cdot\gamma_{h,0}\cdot\gamma_0 + \mathbb{I}_{L,t-1}^i\cdot(\gamma_{h,0}+\tilde{\gamma}_{h,0})\cdot(\gamma_0+\tilde{\gamma}_0)\right]\cdot\mathcal{T}_{t-1}^i$$

where $\gamma_{h,0}$ and $\tilde{\gamma}_{h,0}$ are estimated in specification (19), for $y_t^i = x_t^i/k_t^i$ with $h_q = 0$. And γ_0 and $\tilde{\gamma}_0$ come from estimating specification (17) for $y_t^i = \log\left(q_t^i\right)$. Based on this, we compute that in our Compustat sample, across firms and time, the average effect of a 25 bp contractionary shock in ε_t^m is to decrease x_{t+3}^i/k_{t+3}^i, i.e. the investment rate three quarters after the shock, by 0.037 pp through the turnover channel.

Finally, to assess the relevance of the turnover channel in monetary transmission to aggregate investment, we compute the implied semi-elasticity of firm i's quarterly investment x_{t+3}^i with respect to ε_t^m based on the estimates of $\frac{d\left(x_{t+3}^i/k_{t+3}^i\right)}{d\varepsilon_t^m}\Big|^{\text{TC}}$ relative to x_{t-1}^i/k_{t-1}^i. For each quarter t, we compute the cross-sectional average of these semi-elasticities, weighted by firms' capital expenditures x_{t-1}^i, in our Compustat panel, to get an estimate of the semi-elasticity of aggregate public firm investment in quarter $t+3$ with respect to a monetary shock in t. Taking the average of these aggregate semi-elasticities across time and adjusting for the share of approximately 46.5% of US aggregate nonresidential investment being done by public firms (Asker et al., 2011), we find that in response to a 25 bp unexpected increase in the Federal funds rate, aggregate investment drops by 0.25% three quarters later due to the q-monetary channel. For comparison, the corresponding peak effect on aggregate investment estimated by Christiano et al. (2005) is approximately 0.45%. We can thus conclude that the effects of monetary policy shocks on public firms' investment due to equity price responses have the potential to explain a nonnegligible fraction of overall monetary transmission to aggregate investment in the US.

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A. Appendix: proofs

A.1. Investor's portfolio and bargaining problems

Lemma 2. *Let*

$$\varepsilon_t^* \equiv \frac{p_t \phi_t^m - \pi \phi_t^s}{z} \tag{20}$$

and define the correspondence $\chi : \mathbb{R}^2 \rightrightarrows [0,1]$ *as*

$$\chi(\varepsilon_t^*, \varepsilon) \begin{cases} = 1 & \text{if } \varepsilon_t^* < \varepsilon \\ \in [0,1] & \text{if } \varepsilon_t^* = \varepsilon \\ = 0 & \text{if } \varepsilon < \varepsilon_t^*. \end{cases}$$

Consider a bilateral meeting in the first subperiod of period t between a dealer and an investor with portfolio $\boldsymbol{a}_t$ and valuation ε. The investor's post-trade portfolio, $\bar{\boldsymbol{a}}(\boldsymbol{a}_t, \varepsilon) \equiv (\overline{a}_t^b(\boldsymbol{a}_t, \varepsilon), \overline{a}_t^m(\boldsymbol{a}_t, \varepsilon), \overline{a}_t^s(\boldsymbol{a}_t, \varepsilon))$, is given by

$$\begin{aligned} \overline{a}_t^b(\boldsymbol{a}_t, \varepsilon) &= a_t^b \\ \overline{a}_t^m(\boldsymbol{a}_t, \varepsilon) &= [1 - \chi(\varepsilon_t^*, \varepsilon)](a_t^m + p_t a_t^s) \\ \overline{a}_t^s(\boldsymbol{a}_t, \varepsilon) &= a_t^s + \frac{1}{p_t}[a_t^m - \overline{a}_t^m(\boldsymbol{a}_t, \varepsilon)], \end{aligned}$$

and the intermediation fee charged by the dealer is

$$\varpi_t(\boldsymbol{a}_t, \varepsilon) = (1-\theta)(\varepsilon_t^* - \varepsilon) z \frac{1}{p_t}[\overline{a}_t^m(\boldsymbol{a}_t, \varepsilon) - a_t^m].$$

Proof. The value function (2) can be written as

$$\begin{aligned} W_t(\boldsymbol{a}_t, \varpi_t) &= \boldsymbol{\phi}_t' \boldsymbol{a}_t - \varpi_t + \bar{W}_t \\ &= a_t^b + \phi_t^m a_t^m + \phi_t^s a_t^s - \varpi_t + \bar{W}_t, \end{aligned} \tag{21}$$

where

$$\bar{W}_t \equiv T_t + \max_{\boldsymbol{a}_{t+1} \in \mathbb{R}_+^3} \left[-\boldsymbol{\phi}_t \boldsymbol{a}_{t+1} + \beta \int V_{t+1}(\boldsymbol{a}_{t+1}, \varepsilon) \, dG(\varepsilon) \right]. \tag{22}$$

With (21) we can write

$$\begin{aligned} \Gamma(\overline{\boldsymbol{a}}_t, \boldsymbol{a}_t, \varepsilon) &= \overline{a}_t^b + \phi_t^m \overline{a}_t^m + (\varepsilon z + \pi \phi_t^s) \overline{a}_t^s \\ &\quad - \left[a_t^b + \phi_t^m a_t^m + (\varepsilon z + \phi_t^s \pi) a_t^s \right] - \varpi_t, \end{aligned}$$

so the solution to (1) is

$$\begin{aligned}
\overline{a}_t^b(\boldsymbol{a}_t,\varepsilon) &= a_t^b \\
\overline{a}_t^s(\boldsymbol{a}_t,\varepsilon) &= a_t^s + \frac{1}{p_t}[a_t^m - \overline{a}_t^m(\boldsymbol{a}_t,\varepsilon)] \\
\varpi_t(\boldsymbol{a}_t,\varepsilon) &= (1-\theta)(\varepsilon_t^* - \varepsilon) z \frac{1}{p_t}[\overline{a}_t^m(\boldsymbol{a}_t,\varepsilon) - a_t^m] \\
\overline{a}_t^m(\boldsymbol{a}_t,\varepsilon) &= \arg\max_{0\le \overline{a}_t^m \le p_t a_t^s + a_t^m}\left[(\varepsilon_t^* - \varepsilon) z \frac{1}{p_t}(\overline{a}_t^m - a_t^m)\right].
\end{aligned}$$

This concludes the proof. ■

Lemma 3. *Let* $\left(a_{t+1}^b, a_{t+1}^m, a_{t+1}^s\right)$ *denote the portfolio chosen by an investor in the second subperiod of period* t*. This portfolio must satisfy the following first-order necessary and sufficient conditions:*

$$\phi_t^b \ge \beta, \text{ with "} = \text{" if } a_{t+1}^b > 0 \tag{23}$$

$$\phi_t^m \ge \beta\left[\phi_{t+1}^m + \alpha\theta\int_{\varepsilon_{t+1}^*}^{\varepsilon_H}\left(\varepsilon - \varepsilon_{t+1}^*\right) z dG(\varepsilon)\frac{1}{p_{t+1}}\right], \text{ with "} = \text{" if } a_{t+1}^m > 0 \tag{24}$$

$$\phi_t^s \ge \beta\left[\bar{\varepsilon} z + \pi\phi_{t+1}^s + \alpha\theta\int_{\varepsilon_L}^{\varepsilon_{t+1}^*}\left(\varepsilon_{t+1}^* - \varepsilon\right) z dG(\varepsilon)\right], \text{ with "} = \text{" if } a_{t+1}^s > 0. \tag{25}$$

Proof. With (21) and the bargaining outcome described in the statement of Lemma 2, (3) can be written as

$$\begin{aligned}
V_t(\boldsymbol{a}_t,\varepsilon) = {} & a_t^b + (\varepsilon z + \pi\phi_t^s) a_t^s + \phi_t^m a_t^m + \bar{W}_t \\
& + \alpha\theta(\varepsilon - \varepsilon_t^*) z \frac{1}{p_t}[a_t^m - \overline{a}_t^m(\boldsymbol{a}_t,\varepsilon)].
\end{aligned}$$

Hence, using the expression for $\overline{a}_{t+1}^m(\boldsymbol{a}_{t+1},\varepsilon)$ from Lemma 2,

$$\begin{aligned}
\int V_{t+1}(\boldsymbol{a}_{t+1},\varepsilon)\, dG(\varepsilon) = {} & \bar{W}_{t+1} + a_{t+1}^b + \left[\bar{\varepsilon} z + \pi\phi_{t+1}^s + \alpha\theta\int_{\varepsilon_L}^{\varepsilon_{t+1}^*}\left(\varepsilon_{t+1}^* - \varepsilon\right) z dG(\varepsilon)\right] a_{t+1}^s \\
& + \left[\phi_{t+1}^m + \alpha\theta\frac{1}{p_{t+1}}\int_{\varepsilon_{t+1}^*}^{\varepsilon_H}\left(\varepsilon - \varepsilon_{t+1}^*\right) z dG(\varepsilon)\right] a_{t+1}^m.
\end{aligned}$$

Thus, the necessary and sufficient first-order conditions corresponding to the maximization problem in (22) are as in the statement of the lemma. ■

A.2. Stock-market clearing

Lemma 4. *In period t, the first-subperiod market-clearing condition for equity is*

$$[1 - G(\varepsilon_t^*)] \frac{1}{p_t} A_t^m = G(\varepsilon_t^*) S_t. \tag{26}$$

Proof. Recall that $\bar{A}_{It}^s = \alpha \int \bar{a}_t^s(\boldsymbol{a}_t, \varepsilon) dH_{It}(\boldsymbol{a}_t, \varepsilon)$, so using the bargaining outcomes in Lemma 2, we have

$$\bar{A}_{It}^s = \alpha [1 - G(\varepsilon_t^*)] \left(S_t + \frac{1}{p_t} A_t^m \right).$$

With this expression, the market-clearing condition for equity in the first subperiod of period t, i.e., $\bar{A}_{It}^s = \alpha S_t$, can be written as (26). ■

A.3. Equilibrium characterization: stock prices and real money balances

The following result characterizes the equilibrium paths $\{\mathrm{M}_t\}_{t=0}^{\infty}$ and $\{\phi_t^s\}_{t=0}^{\infty}$ taking as given the path for the outstanding aggregate quantity of stocks, $\{S_t\}_{t=0}^{\infty}$.

Corollary 3. *In equilibrium, aggregate real money balances, $\{\mathrm{M}_t\}_{t=0}^{\infty}$, and the real price of equity shares, $\{\phi_t^s\}_{t=0}^{\infty}$, satisfy the following conditions:*

$$\mathrm{M}_t \geq \frac{\beta}{\mu} \left[1 + \alpha\theta \int_{\varepsilon_{t+1}^*}^{\varepsilon_H} \left(\varepsilon - \varepsilon_{t+1}^* \right) z dG(\varepsilon) \frac{1}{\varepsilon_{t+1}^* z + \pi \phi_{t+1}^s} \right] \mathrm{M}_{t+1}, \text{ with "} = \text{" if } \mathrm{M}_{t+1} > 0 \tag{27}$$

$$\phi_t^s = \beta \left[\bar{\varepsilon} z + \pi \phi_{t+1}^s + \alpha\theta \int_{\varepsilon_L}^{\varepsilon_{t+1}^*} \left(\varepsilon_{t+1}^* - \varepsilon \right) z dG(\varepsilon) \right], \tag{28}$$

where for all $t \geq 0$, ε_t^* satisfies

$$[1 - G(\varepsilon_t^*)] \mathrm{M}_t = G(\varepsilon_t^*) S_t. \tag{29}$$

Proof. Conditions (27), (28), and (29) follow from (24), (25), and (26), respectively, using $\mathrm{M}_t \equiv \phi_t^m A_t^m$, $A_{t+1}^m / A_t^m = \mu$, and (20). ■

The following result characterizes the equilibrium paths $\{\mathrm{M}_t\}_{t=0}^{\infty}$ and $\{\phi_t^s\}_{t=0}^{\infty}$ taking as given the path for the outstanding aggregate quantity of stocks, $\{S_t\}_{t=0}^{\infty}$—in the context of a stationary equilibrium.

Corollary 4. *In a stationary equilibrium, $S_t = S$, $\varepsilon_t^* = \varepsilon^*$, $\phi_t^s = \varphi^s z$, and $\mathrm{M}_t = \mathrm{M}$ for all t, and $(\varepsilon^*, \varphi^s, \mathrm{M})$ satisfy the following conditions:*

$$r \geq \alpha\theta \int_{\varepsilon^*}^{\varepsilon_H} \frac{\varepsilon - \varepsilon^*}{\varepsilon^* + \pi\varphi^s} dG(\varepsilon), \text{ with "} = \text{" if } \mathrm{M} > 0 \tag{30}$$

$$\varphi^s = \frac{\beta}{1-\beta\pi}\left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon)\right], \tag{31}$$

where ε^* satisfies

$$[1 - G(\varepsilon^*)]\,\mathrm{M} = G(\varepsilon^*)\, S. \tag{32}$$

Proof. Conditions (27)-(29) follow immediately from (30)-(32) imposing the stationarity conditions described in the statement. ■

Lemma 5. *Let $S > 0$ be given. Then:*

(i) There always exists a solution to (30)-(32) in which money is not valued, i.e., $\mathrm{M} = 0$, $\varepsilon^* = \varepsilon_L$, and $\varphi^s = \frac{\beta}{1-\beta\pi}\bar{\varepsilon}$.

(ii) Let

$$\bar{r} \equiv \frac{\alpha\theta(\bar{\varepsilon} - \varepsilon_L)}{\varepsilon_L + \frac{\beta\pi}{1-\beta\pi}\bar{\varepsilon}}.$$

If $r \in (0, \bar{r})$ there exists a unique solution to (30)-(32) with $\mathrm{M} > 0$, i.e.,

$$\mathrm{M} = \frac{G(\varepsilon^*)}{1 - G(\varepsilon^*)} S \tag{33}$$

$$\varphi^s = \frac{\beta}{1-\beta\pi}\left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon)\right], \tag{34}$$

where $\varepsilon^* \in (\varepsilon_L, \varepsilon_H]$ is the unique solution to

$$\frac{\alpha\theta \int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^*)\, dG(\varepsilon)}{\varepsilon^* + \frac{\beta\pi}{1-\beta\pi}\left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon)\right]} = r. \tag{35}$$

Moreover:

(a) As $r \to \bar{r}$, $\varepsilon^* \to \varepsilon_L$, $\mathrm{M} \to 0$, and $\varphi^s \to \frac{\beta}{1-\beta\pi}\bar{\varepsilon}$.

(b) As $r \to 0$, $\varepsilon^* \to \varepsilon_H$ and $\varphi^s \to \frac{\beta}{1-\beta\pi}[\bar{\varepsilon} + \alpha\theta(\varepsilon_H - \bar{\varepsilon})]$.

(c) $\frac{\partial \varepsilon^*}{\partial r} < 0$, $\frac{\partial \mathrm{M}}{\partial r} < 0$, and $\frac{\partial \varphi^s}{\partial r} < 0$.

Proof. To establish part (i), simply set $\mathrm{M} = 0$ in (30)-(32). To establish part (ii), proceed as follows. Assume $\mathrm{M} > 0$; then (30) holds with equality, and using (31) to substitute φ^s from (30) gives $T(\varepsilon^*; r) = 0$, where

$$T(\varepsilon^*; r) \equiv \frac{\alpha\theta \int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^*)\, dG(\varepsilon)}{\varepsilon^* + \frac{\beta\pi}{1-\beta\pi}\left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon)\right]} - r.$$

First, notice that

$$\frac{\partial T(\varepsilon^*; r)}{\partial \varepsilon^*} = -\frac{[1-G(\varepsilon^*)]\left\{\varepsilon^* + \pi \frac{\beta}{1-\beta\pi}\left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) dG(\varepsilon)\right]\right\} + \left[\int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^*) dG(\varepsilon)\right]\left[1 + \pi \frac{\beta}{1-\beta\pi} \alpha\theta G(\varepsilon^*)\right]}{\frac{1}{\alpha\theta}\left\{\varepsilon^* + \frac{\beta\pi}{1-\beta\pi}\left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) dG(\varepsilon)\right]\right\}^2} < 0.$$

Assume $r \in (0, \bar{r})$. Then

$$T(\varepsilon_H; r) = -r < 0 < T(\varepsilon_L; r) = \bar{r} - r. \tag{36}$$

Since T is a continuous function of ε^*, $\partial T(\varepsilon^*; r)/\partial \varepsilon^* < 0$ and (36) imply that for any $r \in (0, \bar{r})$ there exists a unique ε^* that solves $T(\varepsilon^*; r) = 0$ on the interval $(\varepsilon_L, \varepsilon_H)$. Given the ε^* that solves $T(\varepsilon^*; r) = 0$, M and ϕ_t^s are given by (33) and (34), respectively.

Part $(ii)(a)$ is immediate from (33) and (34), and the observation that $T(\varepsilon_L; \bar{r}) = 0$. Part $(ii)(b)$ is immediate from (34), and the observation that $T(\varepsilon_H; 0) = 0$. Part $(ii)(c)$, follows from

$$\begin{aligned} \frac{\partial \mathrm{M}}{\partial r} &= \frac{G'(\varepsilon^*)}{[1 - G(\varepsilon^*)]^2} S \frac{\partial \varepsilon^*}{\partial r} + \frac{G(\varepsilon^*)}{1 - G(\varepsilon^*)} \frac{\partial S}{\partial r} \\ \frac{\partial \varphi^s}{\partial r} &= \alpha\theta \frac{\beta}{1 - \beta\pi} G(\varepsilon^*) \frac{\partial \varepsilon^*}{\partial r} \end{aligned}$$

together with the fact that

$$\frac{\partial \varepsilon^*}{\partial r} = \frac{1}{\frac{\partial T(\varepsilon^*; r)}{\partial \varepsilon^*}}$$

and $\partial T(\varepsilon^*; r)/\partial \varepsilon^* < 0$. ■

A.4. Economy with $\pi = 0$

A.4.1. Entrepreneur's choice of investment and capital structure

Proof of Lemma 1. The Lagrangian for the optimization problem of the one-period-lived entrepreneur at entry, i.e., (11), is

$$\begin{aligned} \mathcal{L} &= y + \hat{\phi}^s [(1 - \delta) k + x - s_{+1}] \\ &\quad + \xi [\phi^s s_{+1} + w - y - \mathrm{c}(x/k) k] \\ &\quad + \zeta_L^e s_{+1} + \zeta_H^e [(1 - \delta) k + x - s_{+1}] + \zeta_L^c y, \end{aligned}$$

where ξ, ζ_L^e, ζ_H^e, and ζ_L^c are the Lagrange multipliers on the entrepreneur's budget constraint, nonnegativity constraint on equity issuance, upper bound on equity issuance, and nonnegativity constraint on consumption, respectively.

The first-order conditions are

$$\begin{aligned}
0 &= 1 - \xi + \zeta_L^c && (37)\\
0 &= \hat{\phi}^s - \xi \mathrm{C}'(x/k) + \zeta_H^e && (38)\\
0 &= -\hat{\phi}^s + \xi \phi^s + \zeta_L^e - \zeta_H^e && (39)\\
0 &= \xi \left[\phi^s s_{+1} + w - y - \mathrm{C}(x/k)\, k\right] && (40)\\
0 &= \zeta_L^c y && (41)\\
0 &= \zeta_L^e s_{+1} && (42)\\
0 &= \zeta_H^e \left[(1-\delta)\, k + x - s_{+1}\right]. && (43)
\end{aligned}$$

Conditions (37)-(39) are the first-order conditions with respect to y, x, and s_{+1}, respectively. Condition (37) implies $\xi = 1 + \zeta_L^c > 0$, so (40) implies

$$0 = \phi^s s_{+1} + w - y - \mathrm{C}(x/k)\, k. \tag{44}$$

There are potentially eight cases depending on whether the multipliers $(\zeta_L^c, \zeta_L^e, \zeta_H^e)$ are positive or equal to zero. We consider each in turn. Recall ι_0 is the investment rate that satisfies $\mathrm{C}'(\iota_0) = 1$, so $\mathrm{C}'' > 0$ and the assumption in the statement of the lemma imply

$$\delta - 1 \leq \iota_0 \leq \min\{\iota(\phi^s), \iota(\hat{\phi}^s)\}. \tag{45}$$

Case 1: $\zeta_L^e = \zeta_H^e = 0 < \zeta_L^c$. In this case, condition (41) implies

$$y = 0,$$

condition (44) implies

$$\phi^s s_{+1} = \mathrm{C}(x/k)\, k - w, \tag{46}$$

and conditions (38) and (39) imply

$$\mathrm{C}'(x/k) = \phi^s.$$

For this case to be a solution we need three conditions to be satisfied. First, $0 < \zeta_L^c$, which by (37) is equivalent to $\xi > 1$, which by (39) is equivalent to

$$\phi^s < \hat{\phi}^s.$$

Second, since the solution must satisfy the constraints $0 \leq s_{+1} \leq (1-\delta)\,k + x$, (46) implies we must have

$$\Xi\left(\iota\left(\phi^{s}\right)\right) \leq \omega \leq \mathrm{C}\left(\iota\left(\phi^{s}\right)\right),$$

where

$$\Xi\left(\iota\right) \equiv \mathrm{C}\left(\iota\right) - \mathrm{C}'\left(\iota\right)\left(1-\delta+\iota\right). \tag{47}$$

Notice $\Xi\left(\iota_0\right) = \delta - 1 \leq 0$ and $\Xi'\left(\iota\right) = -\,\mathrm{C}''\left(\iota\right)\left(1-\delta+\iota\right) \leq 0$ for all $\iota \geq \iota_0$, so (45) implies the condition $\Xi\left(\iota\left(\phi^{s}\right)\right) \leq \omega$ is satisfied for any $\omega \geq 0$.

Case 2: $\zeta_L^c = \zeta_H^e = 0 < \zeta_L^e$. In this case (37) implies $\xi = 1$, (38) implies

$$\mathrm{C}'\left(x/k\right) = \hat{\phi}^{s},$$

(39) implies

$$\zeta_L^e = \hat{\phi}^{s} - \phi^{s}, \tag{48}$$

(42) implies

$$s_{+1} = 0,$$

and (44) implies

$$y = w - \mathrm{C}(\iota(\hat{\phi}^{s}))k. \tag{49}$$

For this case to be a solution we need three conditions to be satisfied. First, $0 < \zeta_L^e$, which by (48) is equivalent to

$$\phi^{s} < \hat{\phi}^{s}.$$

Second, $0 \leq y$, which by (49) is equivalent to

$$\mathrm{C}(\iota(\hat{\phi}^{s}))k \leq w.$$

Third, $0 \leq k_{+1} - s_{+1}$, is equivalent to

$$0 \leq 1 - \delta + \iota(\hat{\phi}^{s}).$$

This condition is implied by (45).

Case 3: $\zeta_L^c = \zeta_L^e = 0 < \zeta_H^e$. In this case (37) implies $\xi = 1$, (39) implies

$$\zeta_H^e = \phi^{s} - \hat{\phi}^{s}, \tag{50}$$

and this together with (38) implies

$$\mathrm{C}'(x/k) = \phi^s.$$

Then condition (43) implies

$$s_{+1} = [1 - \delta + \iota(\phi^s)]\, k \tag{51}$$

and (44) implies

$$y = \{\phi^s [1 - \delta + \iota(\phi^s)] + \omega - \mathrm{C}(\iota(\phi^s))\}\, k. \tag{52}$$

For this case to be a solution we need three conditions to be satisfied. First, $0 < \zeta_H^e$, which by (50) is equivalent to

$$\hat{\phi}^s < \phi^s.$$

Second, $0 \leq s_{+1}$, which by (51) is equivalent to

$$0 \leq 1 - \delta + \iota(\phi^s).$$

This condition is implied by (45). Third, $0 \leq y$, which by (52) is equivalent to

$$\Xi(\iota(\phi^s)) \leq \omega, \tag{53}$$

where $\Xi(\cdot)$ is as defined in (47). Notice $\Xi(\iota_0) = \delta - 1 \leq 0$ and $\Xi'(\iota) = -\mathrm{C}''(\iota)(1 - \delta + \iota) \leq 0$ for all $\iota \geq \iota_0$, so (45) implies (53) is satisfied for any $\omega \geq 0$.

Case 4: $\zeta_H^e = 0 < \min(\zeta_L^c, \zeta_L^e)$. In this case (41) implies

$$y = 0,$$

(42) implies

$$s_{+1} = 0,$$

and hence (44) implies

$$x/k = \mathrm{C}^{-1}(\omega).$$

Conditions (37) and (38) imply

$$\zeta_L^c = \frac{\hat{\phi}^s - \mathrm{C}'(\mathrm{C}^{-1}(\omega))}{\mathrm{C}'(\mathrm{C}^{-1}(\omega))}, \tag{54}$$

and conditions (38) and (39) imply

$$\zeta_L^e = \frac{\mathrm{C}'(\mathrm{C}^{-1}(\omega)) - \phi^s}{\mathrm{C}'(\mathrm{C}^{-1}(\omega))}\hat{\phi}^s. \tag{55}$$

For this case to be a solution we need three conditions to be satisfied. First, $0 < \zeta_L^c$, which by (54) is equivalent to

$$\mathrm{C}'(\mathrm{C}^{-1}(\omega)) < \hat{\phi}^s \Leftrightarrow \mathrm{C}^{-1}(\omega) < \iota(\hat{\phi}^s) \tag{56}$$

Second, $0 < \zeta_L^e$, which by (55) is equivalent to

$$\phi^s < \mathrm{C}'(\mathrm{C}^{-1}(\omega)) \Leftrightarrow \iota(\phi^s) < \mathrm{C}^{-1}(\omega). \tag{57}$$

Notice that conditions (56) and (57) can both be satisfied only if

$$\phi^s < \hat{\phi}^s.$$

The third condition that needs to be satisfied for this case to be a solution is $0 \leq k_{+1} - s_{+1}$, which by (43) is equivalent to

$$0 \leq 1 - \delta + \mathrm{C}^{-1}(\omega). \tag{58}$$

From (57), we know that $\mathrm{C}(\iota(\phi^s)) < \omega$, which together with (45) implies

$$\iota_0 = \mathrm{C}(\iota_0) \leq \mathrm{C}(\iota(\phi^s)) < \omega.$$

Hence, $\iota_0 < \mathrm{C}^{-1}(\omega)$, which implies condition (58) is satisfied.

Case 5: $\zeta_L^e = 0 < \min(\zeta_L^c, \zeta_H^e)$. In this case (41) implies

$$y = 0,$$

and conditions (38) and (39) imply

$$\mathrm{C}'(x/k) = \phi^s.$$

Then (43) implies

$$s_{+1} = [1 - \delta + \iota(\phi^s)]\, k. \tag{59}$$

For this case to be a solution we need four conditions to be satisfied. First, $0 \leq s_{+1}$, which with (59) is equivalent to

$$0 \leq 1 - \delta + \iota(\phi^s).$$

This condition is implied by (45). Second, (40) and (43) require that

$$\omega = \Xi(\iota(\phi^s)) \tag{60}$$

with $\Xi(\cdot)$ as defined in (47). As argued in Case 3, the assumptions in the statement of the lemma imply $\Xi(\iota(\phi^s)) \leq 0$. Since $\omega \geq 0$, (60) implies this case is only possible if $\omega = 0$. Third,

$0 < \zeta_L^c$ requires that $1 < \xi$. Fourth, $0 < \zeta_H^e$ requires that $\zeta_H^e = \xi\phi^s - \hat{\phi}^s > 0$. There exist values of ξ that satisfy both these conditions.

Case 6: $\zeta_L^c = 0 < \min(\zeta_L^e, \zeta_H^e)$. In this case (42) implies

$$s_{+1} = 0$$

and then (43) implies

$$x/k = \delta - 1,$$

and condition (40) implies

$$y = [\omega - \text{C}(\delta - 1)]\, k. \tag{61}$$

Conditions (38) and (39) imply

$$\zeta_L^e = \text{C}'(\delta - 1) - \phi^s \tag{62}$$

$$\zeta_H^e = \text{C}'(\delta - 1) - \hat{\phi}^s. \tag{63}$$

For this case to be a solution, we need three conditions to hold. First, $0 \leq y$, which by (61) is equivalent to

$$\text{C}(\delta - 1) \leq \omega.$$

And $0 < \min(\zeta_L^e, \zeta_H^e)$, which by (62) and (63) are equivalent to

$$\max(\phi^s, \hat{\phi}^s) < \text{C}'(\delta - 1). \tag{64}$$

Notice (45) implies

$$\text{C}'(\delta - 1) \leq \text{C}'(\iota_0) \leq \text{C}'(\min\{\iota(\phi^s), \iota(\hat{\phi}^s)\}) = \min(\phi^s, \hat{\phi}^s), \tag{65}$$

which contradicts (64), so this case cannot be a solution.

Case 7: $0 < \min(\zeta_L^c, \zeta_L^e, \zeta_H^e)$. In this case, (**??**)-(**??**) imply

$$y = 0$$

$$s_{+1} = 0$$

$$x/k = \delta - 1.$$

For this to be a solution, we need the following conditions to hold

$$\begin{aligned} w &= \text{C}\left(\delta-1\right)k \\ 1 &< \xi \\ \zeta_L^e &= \xi\left[\text{C}'\left(\delta-1\right)-\phi^s\right]>0 \qquad (66)\\ \zeta_H^e &= \xi\left[\text{C}'\left(\delta-1\right)\right]-\hat{\phi}^s>0. \qquad (67)\end{aligned}$$

The first is implied by (40), the second by the condition $0<\zeta_L^c$, and the third and fourth by the conditions (38) and (39), and the requirement that $0<\min\left(\zeta_L^e,\zeta_H^e\right)$. Notice (45) implies (65), which contradicts (66) and (67), so this case cannot be a solution.

Case 8: $\zeta_L^c=\zeta_L^e=\zeta_H^e=0$. In this case, conditions (38) and (39) imply

$$\text{C}'\left(x/k\right)=\hat{\phi}^s=\phi^s,$$

condition (44) implies

$$y=\phi^s s_{+1}+\left[\omega-\text{C}\left(\iota\left(\phi^s\right)\right)\right]k,$$

and s_{+1} is any number that satisfies that satisfies

$$\max\left\{0,\frac{\text{C}\left(\iota\left(\phi^s\right)\right)-\omega}{\phi^s}k\right\}\leq s_{+1}\leq\left[1-\delta+\iota\left(\phi^s\right)\right]k.$$

Cases 1, 2, and 4, are summarized in part (*ii*) of the statement of the lemma, while part (*i*) summarizes cases 3, 5, and 8. This concludes the proof. ■

Proof of Corollary . The Lagrangian for (11) can be written as

$$\begin{aligned}\mathcal{L} &= y+\hat{\phi}^s(k_{+1}-s_{+1}) \\ &\quad +\hat{q}\left[\left(1-\delta\right)k+x-k_{+1}\right] \\ &\quad +\xi\left[\phi^s s_{+1}+w-y-\text{C}\left(x/k\right)k\right] \\ &\quad +\zeta_L^e s_{+1}+\zeta_H^e\left(k_{+1}-s_{+1}\right)+\zeta_L^c y,\end{aligned}$$

where ξ, ζ_L^e, ζ_H^e, and ζ_L^c are the Lagrange multipliers on the entrepreneur's budget constraint, nonnegativity constraint on equity issuance, upper bound on equity issuance, and nonnegativity constraint on consumption, respectively. The Lagrange multiplier $\hat{q}$ is associated to the law of motion of the capital stock, and is interpreted as the shadow price of a margial unit of capital to the entrepreneur. The first-order conditions with respect to y, x, s_{+1}, and k_{+1} are, respectively,

$$
\begin{aligned}
0 &= 1 - \xi + \zeta_L^c && (68) \\
0 &= \hat{q} - \xi \mathrm{C}'(x/k) && (69) \\
0 &= -\hat{\phi}^s + \xi \phi^s + \zeta_L^e - \zeta_H^e && (70) \\
0 &= \hat{\phi}^s - \hat{q} + \zeta_H^e. && (71)
\end{aligned}
$$

Condition (71) implies the shadow price of capital to the entrepreneur, $\hat{q}$, is at least as large as the discounted value that she assigns to the return on capital, $\hat{\phi}^s$, but could exceed it if the entrepreneur is facing a binding financing constraint, i.e., in the form of a binding upper bound on equity issuance ($0 < \zeta_H^e$). If we use (71) to substitute $\hat{q}$ in (69), then (68)-(70) become identical to (37)-(39) in the proof of Lemma 1. For what follows, it is convenient to define

$$q \equiv \frac{\hat{q}}{\xi}. \tag{72}$$

Intuitively, ξ is the shadow price to the entrepreneur of a unit of good 2 (in terms of second-subperiod marginal utility). Since the entrepreneur's utility for good 2 is linear, this shadow price equals 1 in an interior solution. But it will exceed 1 if the entrepreneur is financially constrained in the sense that it would like to be able to borrow good 2 to invest but is unable to do so. This "binding financial constraint" manisfests itself with $0 < \zeta_L^c$, i.e., a situation in which the nonnegativity constraint on consumption binds. In sum, the q defined in (72) is the *return* (gross of adjustment costs) to the entrepreneur from investing an additional unit good 2 into capital. When investing an additional unit of good 2, the entrepreneur pays utility cost ξ to get payoff $\hat{q}$. Condition (69) then says that at an optimum, $\mathrm{C}'(x/k) = q$, i.e., the marginal (technological) cost of investing, $\mathrm{C}'(x/k)$, must equal the marginal return to investing, q. Next, we derive the value of q corresponding to every case in Lemma 1.

Case 1. This case corresponds to the lowest endowment range (i.e., $\omega \leq \mathrm{C}(\iota(\phi^s))$) in part (*ii*) of Lemma 1. In this case the Lagrange multipliers are:

$$
\begin{aligned}
\zeta_L^e &= \zeta_H^e = 0 < \zeta_L^c = \frac{\hat{\phi}^s}{\phi^s} - 1 = \xi - 1 \\
\hat{q} &= \hat{\phi}^s \\
q &= \phi^s,
\end{aligned}
$$

and the optimal investment rate, x^*, satisfies

$$\text{C}'(x^*) = \phi^s.$$

Case 2. This case corresponds to the highest endowment range (i.e., $\text{C}(\iota(\hat{\phi}^s)) \le \omega$) in part (ii) of Lemma 1. In this case the Lagrange multipliers are:

$$\begin{aligned} \zeta_L^c &= \zeta_H^e = 0 = \xi - 1 < \hat{\phi}^s - \phi^s = \zeta_L^e \\ \hat{q} &= \hat{\phi}^s \\ q &= \hat{\phi}^s, \end{aligned}$$

and the optimal investment rate, x^*, satisfies

$$\text{C}'(x^*) = \hat{\phi}^s.$$

Case 4. This case corresponds to the intermediate endowment range (i.e., $\text{C}(\iota(\phi^s)) < \omega < \text{C}(\iota(\hat{\phi}^s)))$ in part (ii) of Lemma 1. In this case the Lagrange multipliers are:

$$\begin{aligned} 0 &= \zeta_H^e \\ 0 &< \zeta_L^c = \xi - 1 = \frac{\hat{\phi}^s}{\text{C}'(x^*)} - 1 \\ 0 &< \zeta_L^e = \left[1 - \frac{\phi^s}{\text{C}'(x^*)}\right]\hat{\phi}^s \\ \hat{q} &= \hat{\phi}^s \\ q &= \text{C}'(x^*), \end{aligned}$$

and the optimal investment rate, x^*, satisfies

$$\text{C}(x^*) = \omega.$$

Case 3. This case corresponds to the case with $\hat{\phi}^s < \phi^s$ in part (i) of Lemma 1. In this case the Lagrange multipliers are:

$$\begin{aligned} \zeta_L^c &= \zeta_L^e = 0 < \zeta_H^e = \phi^s - \hat{\phi}^s \\ \xi &= 1 \\ q &= \hat{q} = \phi^s, \end{aligned}$$

and the optimal investment rate, x^*, satisfies

$$\mathrm{C}'(x^*) = \phi^s.$$

Case 5. This case corresponds to the case with $y^* = 0 < \phi^s - \hat{\phi}^s$ in part (i) of Lemma 1. In this case the Lagrange multipliers are:

$$\begin{aligned} 0 &< \xi - 1 = \zeta_L^c \\ 0 &< \hat{q} - \hat{\phi}^s = \zeta_H^e \\ q &= \phi^s, \end{aligned}$$

and the optimal investment rate, x^*, satisfies

$$\mathrm{C}'(x^*) = \phi^s.$$

Case 8. This case corresponds to the case with $\phi^s = \hat{\phi}^s$ in part (i) of Lemma 1. In this case the Lagrange multipliers are:

$$\begin{aligned} 0 &= \zeta_L^c = \zeta_L^e = \zeta_H^e = \xi - 1 \\ q &= \hat{q} = \phi^s = \hat{\phi}^s, \end{aligned}$$

and the optimal investment rate, x^*, satisfies

$$\mathrm{C}'(x^*) = \phi^s.$$

By collecting all cases we obtain the expressions in the statement. ■

Corollary 5. *The value function (11) can be written as*

$$J(w, k, 0) = [y^* + \hat{\phi}^s(1 - \delta + x^* - s_{+1}^*)]k,$$

with (x^*, y^*, s_{+1}^*) as given in Lemma 1.

(i) If $\hat{\phi}^s \leq \phi^s$,

$$\frac{J(w, k, 0)}{k} = \phi^s [1 - \delta + \iota(\phi^s)] + \omega - \mathrm{C}(\iota(\phi^s)).$$

(ii) If $\phi^s < \hat{\phi}^s$,

$$\frac{J(w, k, 0)}{k} = \begin{cases} \hat{\phi}^s[1 - \delta + \iota(\hat{\phi}^s)] + \omega - \mathrm{C}(\iota(\hat{\phi}^s)) & \text{if } \mathrm{C}(\iota(\hat{\phi}^s)) \leq \omega \\ \hat{\phi}^s[1 - \delta + \mathrm{C}^{-1}(\omega)] & \text{if } \mathrm{C}(\iota(\phi^s)) < \omega < \mathrm{C}(\iota(\hat{\phi}^s)) \\ \hat{\phi}^s[1 - \delta + \iota(\phi^s) - \frac{\mathrm{C}(\iota(\phi^s)) - \omega}{\phi^s}] & \text{if } \omega \leq \mathrm{C}(\iota(\phi^s)). \end{cases}$$

In every case, the value function can be written as $\mathcal{J}(\omega)k$, where $\mathcal{J}(\omega) \equiv J(\omega k, k, 0)/k$.

A.4.2. Equilibrium characterization

Proof of Proposition 1. In a stationary nonmonetary equilibrium, we know from Lemma 5 that $\mathrm{M} = 0$, $\varepsilon^* = \varepsilon_L$, and $\phi^s = \varphi^s z$, with $\varphi^s = \frac{\beta}{1-\beta\pi}\bar{\varepsilon}$. In this case $\pi = 0$, so $\phi^s = \beta\bar{\varepsilon}z \equiv \underline{\phi}^s$. The expressions for X^* and S^* in parts (i) and (ii) follow from (12) and (13), and the expressions in parts (i) and (ii) of Lemma 1. ■

Proof of Proposition 2. The existence and uniqueness claim in part (i) follows from the fact that there exists a unique ε^* that satisfies (15), as established in Lemma 5. Parts (ii) and (vi) also follow from Lemma 5. To establish parts (iii), (iv), and (v) we again rely on Lemma 5, which shows that $\varphi^s(r)$ is continuous, with $\frac{\partial\varphi^s(r)}{\partial r} < 0$, $\phi^s(0) = \overline{\phi}^s$, and $\phi^s(\bar{r}) = \underline{\phi}^s$. From this it follows that for every $\hat{\phi}^s \in (\underline{\phi}^s, \overline{\phi}^s)$ there exists a unique $\hat{r} \in (0, \bar{r})$ that satisfies $\phi^s(\hat{r}) = \hat{\phi}^s$, with $\phi^s(r) > \hat{\phi}^s$ for all $r \in (0, \hat{r})$, and $\phi^s(r) < \hat{\phi}^s$ for all $r \in (\hat{r}, \bar{r})$. Given this, the expressions for X^* and S^* then follow from (12), (13), and Lemma 1. ■

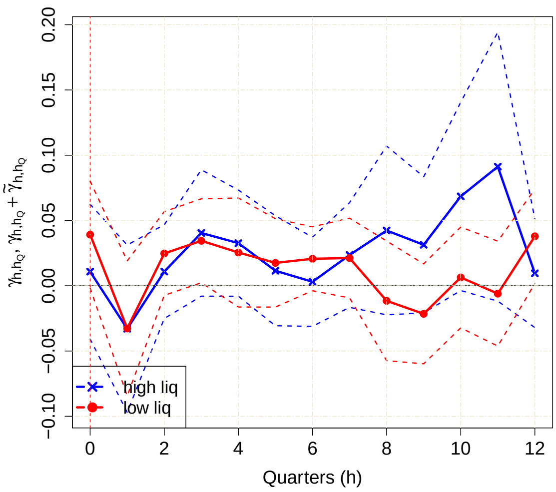
B. Appendix: data, additional regressions, and robustness

B.1. Data

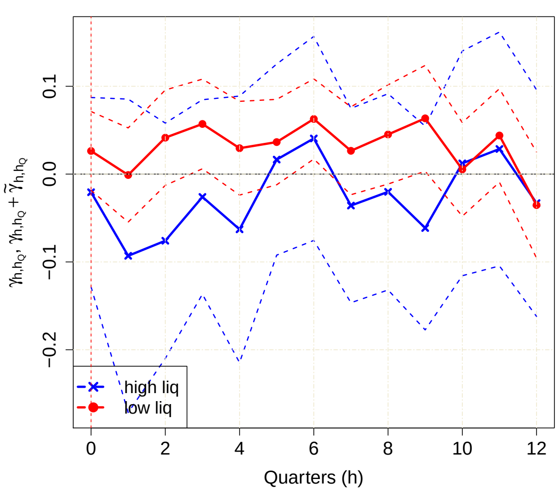
To be added ...

B.2. Robustness of regression estimates

Figure 6: Issuances and investment predicted by instrumented q, across liquidity ratio groups, with additional firm controls



(a) $y_t^i = e_t^i / b_t^i$



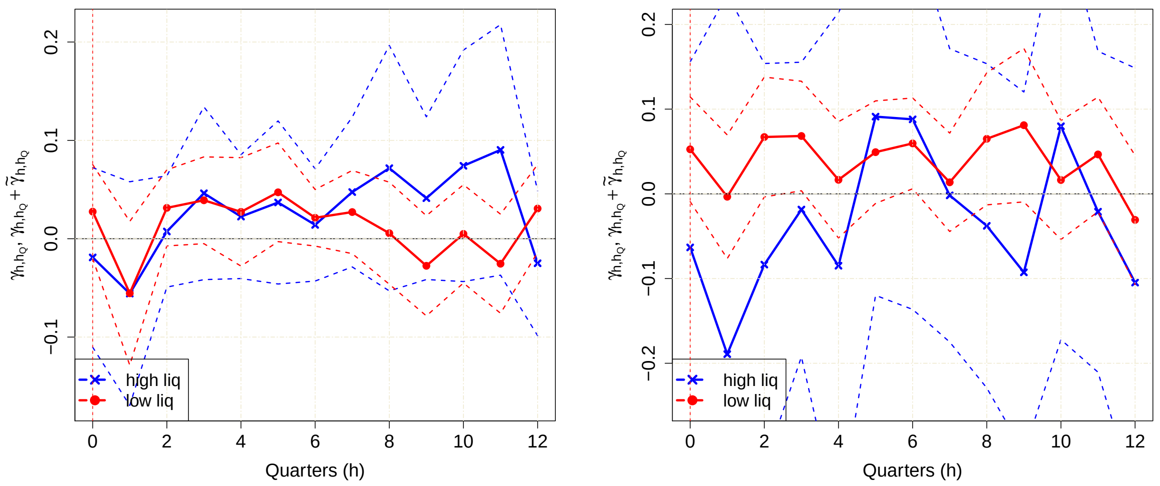
(b) $y_t^i = x_t^i / k_t^i$

Notes: Point estimates and 95% confidence intervals for γ_{h,h_q} and $\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q}$ from estimating specification

$$\begin{aligned} y_{t+h}^i =& f_h^i + d_{s,h,t+h} + \alpha_h \mathbb{I}_{L,t}^i + \left(\rho_h + \tilde{\rho}_h \mathbb{I}_{L,t-1}^i\right) y_{t-1}^i + \left(\Lambda_h + \tilde{\Lambda}_h \mathbb{I}_{L,t-1}^i\right) Z_{t-1}^i + \left(\Psi_h + \tilde{\Psi}_h \mathbb{I}_{L,t-1}^i\right) Z_{t-1}^i \varepsilon_t^m \\ &+ \left(\beta_h + \tilde{\beta}_h \mathbb{I}_{L,t-1}^i\right) \mathcal{T}_{t-1}^i + \left(\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q} \mathbb{I}_{L,t-1}^i\right) \log\left(q_{t+h_q}^i\right) + u_{h,t+h}^i \end{aligned}$$

where Z_t^i is a vector containing the firm's liquidity ratio, log total assets as a measure of firm size, and $\frac{\text{total debt}_t^i}{\text{total assets}_t^i}$ as a measure of leverage. $\log\left(q_{t+h_q}^i\right)$ is instrumented with $\mathcal{T}_{t-1}^i \varepsilon_t^m$. Vertical red dashed line marks the value of $h_q = 0$. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

Figure 7: Issuances and investment predicted by instrumented q, across liquidity ratio groups, with additional firm controls including age



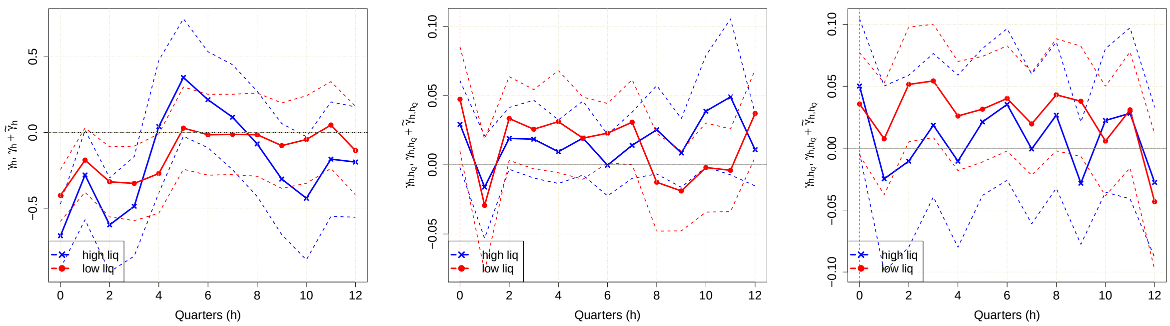
(a) $y_t^i = e_t^i / b_t^i$ (b) $y_t^i = x_t^i / k_t^i$

Notes: Point estimates and 95% confidence intervals for γ_{h,h_q} and $\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q}$ from estimating specification

$$y_{t+h}^i = f_h^i + d_{s,h,t+h} + \alpha_h \mathbb{I}_{L,t}^i + \left(\rho_h + \tilde{\rho}_h \mathbb{I}_{L,t-1}^i\right) y_{t-1}^i + \left(\Lambda_h + \tilde{\Lambda}_h \mathbb{I}_{L,t-1}^i\right) Z_{t-1}^i + \left(\Psi_h + \tilde{\Psi}_h \mathbb{I}_{L,t-1}^i\right) Z_{t-1}^i \varepsilon_t^m + \left(\beta_h + \tilde{\beta}_h \mathbb{I}_{L,t-1}^i\right) \mathcal{T}_{t-1}^i + \left(\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q} \mathbb{I}_{L,t-1}^i\right) \log\left(q_{t+h_q}^i\right) + u_{h,t+h}^i$$

where Z_t^i is a vector containing the firm's liquidity ratio, log total assets as a measure of firm size, $\frac{\text{total debt}_t^i}{\text{total assets}_t^i}$ as a measure of leverage, and time since incorporation as a measure of age. $\log\left(q_{t+h_q}^i\right)$ is instrumented with $\mathcal{T}_{t-1}^i \varepsilon_t^m$. Vertical red dashed line marks the value of $h_q = 0$. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

Figure 8: OLS and IV regression estimates, across liquidity ratio groups, given Jarociński and Karadi (2019) 'poor man's sign restrictions'



(a) $y_t^i = \log(q_t^i)$ (b) $y_t^i = e_t^i / b_t^i$ (c) $y_t^i = x_t^i / k_t^i$

Notes: Point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ from estimating specification (17) in panel (a), and specification (19) in panels (b) and (c) with y_{t+h}^i as dependent variable. ε_t^m is the shock series inferred based on the 'poor man's sign restrictions' of Jarociński and Karadi (2019), for 1990Q1–2016Q4. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.