Security Design in Non-Exclusive Markets with Asymmetric Information

Vladimir Asriyan and Victoria Vanasco*

Preliminary Draft.

Abstract

We revisit the problem of a seller (e.g., bank) who is privately informed about her asset and needs to raise funds from uninformed buyers (e.g., investors) by issuing securities backed by her asset cash flows. In our setting, buyers post menus of contracts to screen the seller, but the seller cannot commit to accept contracts from only one buyer, i.e., markets are *non-exclusive*. Equilibrium existence is ensured by allowing the buyers to withdraw offered menus at a small cost after initial menu offers are observed. We show that non-exclusive markets behave very differently from exclusive ones in the presence of information asymmetries. In particular, when markets are non-exclusive: (i) separating contracts are never part of equilibrium; and (ii) equilibrium features semi-pooling for a wide range of parameters. Our model's predictions are consistent with empirical evidence on issuance and pricing of mortgage-backed securities, and we use the theory to evaluate some of the reforms aimed at enhancing transparency and exclusivity in financial markets.

JEL Codes: G14, G18, D47, D82, D86.

Keywords: Adverse Selection; Security Design; Non-Exclusivity; Tranching; Liquidity; Securitization; Transparency; Opacity; Complexity; Market Design; Regulation.

---

*Asriyan: CREi, UPF, Barcelona GSE and CEPR; vasriyan@crei.cat. Vanasco: CREi, UPF and Barcelona GSE; vvanasco@crei.cat. We thank Bruno Biais, Peter DeMarzo, Willie Fuchs, Piero Gottardi, Denis Gromb, Qi Li (discussant), Alberto Martin, Giorgia Piacentino, Jason Roderick Donaldson, and seminar/conference participants at Cambridge Corporate Finance Workshop 2020, Amsterdam Business School, CREI-UPF, SED Meetings in Mexico City and ESSFM in Gerzensee for helpful feedback and suggestions. We also thank Joon Sup Park and Gerard Martín Escofet for excellent research assistance.

1 Introduction

The question of how markets function in the presence of information asymmetries has been central to economics since the seminal work of Akerlof (1970), who showed that information asymmetries between sellers and buyers can give rise to equilibrium multiplicity and market failures. In the study of asset markets, a large literature followed to explore the role of security design, i.e. the ability of sellers to optimally design and issue securities backed by asset cash flows, in ameliorating such information frictions. From it, we learned that by retaining exposure to an asset's cash flows a seller may be able to credibly convey to buyers information about her asset quality (e.g. Leland and Pyle (1977); DeMarzo and Duffie (1999)); and that standard debt emerges as the optimal security design since it minimizes the distortions due to adverse selection (e.g. Nachman and Noe (1994); DeMarzo and Duffie (1999); Biais and Mariotti (2005); DeMarzo (2005); Daley, Green, and Vanasco (2020b)). This literature, however, has implicitly assumed that there is exclusivity in contracting, i.e. that a buyer of a security can ensure that his seller does not engage in financial trade with other agents in the market. In many settings of interest, however, exclusivity may be difficult to enforce.

Exclusive contracting effectively requires that the seller is able to commit to trade with only one buyer, even if gains from trade arise with other buyers; or that the buyers are able to observe and contract upon the entire set of the seller's trades. In the context of modern financial markets, however, these requirements are unlikely to be fulfilled: there is little information about agents' trades, and the complexity of certain financial products makes it difficult for potential buyers to understand a seller's overall asset positions and resulting risk-exposures.[1] This has been of particular concern to policymakers in the US and Europe, who are in the process of implementing policies aimed at enhancing exclusivity in markets for securitized products (see more details below).

Motivated by these observations, we revisit the classic problem of a seller who is privately informed about her asset and raises funds from uninformed buyers by issuing securities fully backed by her asset cash flows. We consider a screening game, where buyers post menus of contracts (securities and prices) to be accepted by the seller, but where the seller cannot commit to trade with only one buyer, i.e. markets are *non-exclusive*. We use our framework to study the implications of non-exclusivity for equilibrium allocations, as characterized by issued securities and their prices, which we show are consistent with recent empirical evidence from markets for mortgage-backed securities. We then investigate the theory's normative implications, which we relate to policy discussions and proposed reforms in the aftermath of the global financial crisis.

Our setup is as follows. There is a risk-neutral seller endowed with an asset that pays random

[1] This observation was first made and analyzed by Jaynes (1978) in the context of insurance markets.

cash flow, X, in the future; and, there is a large number of competitive, risk-neutral, deep-pocket buyers. Gains from trade arise because the seller is more impatient than the buyers. The seller, however, is privately informed about the quality of her asset, which could be *high*- or *low*-quality, and where higher cash flows are more likely to be obtained from a *high*-quality asset. Buyers compete by posting menus of bilateral contracts, where each contract is a security-price pair (F, p), where F maps cash flow X to a payment $F(X)$ to be received by the buyer in the future. We allow the buyers to withdraw their menus at an infinitesimal cost after observing initial menu offers from other buyers.[2] The seller can sell multiple asset backed securities but is subject to limited liability, i.e. all securities must be fully backed by her asset cash flows. As is standard in the literature on security design with asymmetric information, we restrict attention to securities for which the payoffs to the seller and the buyers are monotone in cash flows. Our equilibrium notion is perfect Bayesian.

Our setting features two frictions: the seller is privately informed about her asset quality and the securities market is non-exclusive. As a preliminary step, we consider two settings in which we shut down each of these frictions in turn. First, we study the full information benchmark, where the asset quality is public information. Here, we show that the first-best allocations are implemented, as all asset cash flows are transferred from the seller to the buyers: equity is the optimal security design and it is priced at its full information expected value.

Second, we study an *exclusive* market setting, where the seller is restricted to trade with at most one buyer. Here, we show that an equilibrium exists and it is unique, and can be separating or feature cross-subsidization. In it, the seller of a *high*-quality asset (*high*-type) issues standard debt whereas the seller of a *low*-quality asset (*low*-type) issues equity. When the buyers' prior belief is low, the equilibrium is separating and prices reflect the true underlying quality of the asset; that is, debt trades at *high*-type's valuation (*high*-valuation) whereas equity trades at *low*-valuation. The face value of the debt security is in turn chosen so that the *low*-type is indifferent between her equilibrium allocation and mimicking the *high*-type. The ability of buyers to withdraw loss-making menus after initial menus are posted allows us to support a cross-subsidizing equilibrium when buyer's beliefs are high, as "cream-skimming" deviations are rendered unprofitable. In this scenario, prices do not reflect the true underlying quality of the asset; that is, debt trades below *high*-valuation whereas equity trades above *low*-valuation. The face value of the debt security is in turn chosen to optimally trade-off the gains from issuing more cash flows with the costs from the increase in mispricing, both for the *high*-type.

In the presence of asymmetric information, non-exclusive markets behave very differently

[2] We follow Netzer and Scheuer (2014) and introduce the possibility of menu withdrawal at an infinitesimal cost to ensure existence and uniqueness of equilibrium both in the exclusive and non-exclusive screening games that we analyze.

from exclusive ones. In particular, we show that when markets are non-exclusive: (i) an equilibrium exists but may not be unique; (ii) separating contracts are never part of equilibrium; (iii) equilibria can be implemented with contracts that give the buyer zero expected profits, and (iv) equilibria feature semi-pooling for a wide range of parameters, in a sense that some *but not all* traded contracts are accepted by both seller types. In what follows, we discuss these findings and their implications in more detail.

In any equilibrium in non-exclusive markets there is cross-subsidization from the *high*- to the *low*-type seller, i.e. perfect separation is not possible.[3] One might intuitively think that, as cash flow retention is more costly for the *low*-type than for the *high*-type, it should be possible for buyers to separate the *high*-type seller by offering her a contract that requires enough cash flow retention to be unattractive for the *low*-type. When markets are non-exclusive, however, retention of cash flows cannot be enforced: there is always a profitable deviation for a buyer to offer to buy the retention implied by the contract of the *high*-type seller at a price slightly below *low*-valuation. Lack of exclusivity allows the *low*-type to then accept the contract meant for the *high*-type while also selling the resulting retention to the deviating-buyer; both the *low*-type and this buyer profit from such a deviation. Standard debt then emerges as the optimal security design for the *high*-type seller, as it minimizes the mispricing she faces due to adverse selection. In addition, in any equilibrium, the *low*-type seller issues all of her cash flows to fully exploit gains from trade.

Moreover, in non-exclusive markets, there is always an equilibrium where both seller types issue the same non-trivial debt security (senior tranche). The *high*-type optimally chooses to retain (i.e. not sell) her remaining cash flows (junior tranche), while the *low*-type issues both the debt security and the remaining cash flows but to distinct buyers. We refer to this as the *star* equilibrium, and note that it is the unique equilibrium among equilibria in which the seller can issue any set of cash flows at low valuation. In it, the senior tranche is *mispriced*, in the sense that its price reflects average rather than true asset quality; whereas the junior tranche is priced at its true, *low*-valuation. The face value of the debt security is in turn chosen so that it maximizes the payoff to the *high*-type seller, conditional on being priced at *average*-valuation.

An equilibrium of our screening game with non-exclusive markets always exists, but it requires that some buyers post *latent* contracts. These are contracts that are offered in some buyers' menus but that are not accepted by the seller in equilibrium. We show that in the absence of latent contracts there is always a profitable deviation for a buyer to, first, induce withdrawal of equilibrium menus by trying to cream-skim the *high*-type, and, second, to offer a cross-subsidizing contract to attract all seller types priced below valuation. In our setting, however,

[3] This result extends the finding in Attar, Mariotti, and Salanié (2011) that separation is not possible in non-exclusive markets for divisible goods to a setting in which securities are designed optimally.

buyers can deter such deviations by posting auxiliary, latent contracts that ensure that, if a buyer were to deviate to cream-skim the *high*-type, then the *low*-type would also find it optimal to accept the deviant contract together with a collection of latent contracts from other buyers, rendering the cream-skimming deviation unprofitable.[4] In addition to latent contracts, the ability of buyers to withdraw loss-making menus is essential in supporting equilibria in non-exclusive markets by ruling out deviations that aim to attract both seller types to separate contracts.

Our theory's predictions are consistent with evidence from markets where exclusivity is difficult to enforce. First, we provide a new rationale for the practice of tranching underlying cash flows that are sold separately in markets. Indeed, within the context of markets for commercial mortgage-backed securities (CMBS), where tranching is common practice, Ashcraft, Gooriah, and Kermani (2019) argue that complex products like collateralized debt obligations (CDOs) enabled informed parties in the securitization pipeline to reduce their cash flow retention in a way not observable to other market participants, suggesting that exclusivity is hard to enforce. Second, and in sharp contrast to conventional models, our theory predicts that the amount of cash flows retained by a seller *should not* predict differential pricing in the market for her senior tranches, but that it *should* predict differential quality of these tranches. This is consistent with findings in Ashcraft et al. (2019) that, in the CMBS market, initially retained cash flows sold into CDOs in the twelve months following a transaction are not correlated with the prices of the more senior tranches, though they do predict a higher probability of default of these tranches.

After the 2008-09 financial crisis, a number of reforms were discussed in the US, which would either directly or indirectly enhance exclusivity in contracting. For instance, the Dodd-Frank Act explicitly prohibits the sellers of asset-backed securities to engage in trades that have any material conflicts of interest with the investors of trades completed within the previous year.[5] A natural obstacle to the enforcement of such rules is the complexity of balance sheets of financial institutions and the opacity of markets in which they can trade. To address this, a number of complementary rules were implemented, primarily consisting of more stringent information disclosure requirements combined with the relocation of trading of certain securities from opaque over-the-counter markets to more transparent platforms. Our framework provides a natural laboratory within which one can evaluate the effects of such interventions.

[4] The role of latent contracts in supporting equilibria in non-exclusive markets was first analyzed in Arnott and Stiglitz (1991) and Attar et al. (2011) in environments with moral hazard and adverse selection, respectively.

[5] Statement at Open Meeting: Asset-Backed Securities Disclosure and Registration, by Commissioner Kara M. Stein on Aug. 27, 2014 states that "Section 621 prohibits an underwriter, placement agent, initial purchaser, sponsor, or any affiliate or subsidiary of any such entity, of an asset-backed security from engaging in any transaction that would involve or result in any material conflict of interest with respect to any investor in a transaction arising out of such activity for a period of one year after the date of the first closing of the sale of the asset-backed security."

First, we show that when the distribution of asset qualities in the market is exogenous (as in our baseline setting) non-exclusive markets increase welfare vis-à-vis exclusive markets if and only if they generate higher market liquidity (i.e. more trade), which is only the case when average asset quality is sufficiently high. This finding contrasts with the by-now conventional 'ignorance is bliss' view of Dang, Gorton, and Holmström (2010) and Dang, Gorton, Holmström, and Ordonez (2017), according to which market liquidity and efficiency are maximized through complexity of assets and opacity of issuers' balance sheets. Our model instead provides a more nuanced view: to the extent that complexity/opacity inhibits exclusive contracting, its effects on liquidity and efficiency will depend on the average quality of assets in the market.

Second, in some applications (e.g. loan origination), both the liquidity of markets and the manner by which claims are priced may impact efficiency by distorting incentives to improve asset quality. To address this, we consider a simple extension where we allow the seller (who is now also the asset originator) to exert costly, unobservable effort to increase the likelihood of having a high-quality asset. Here, we uncover a robust result: the average quality of originated assets is *always* lower with non-exclusive markets than with exclusive markets. Taking the above results together, we conclude that non-exclusive markets increase welfare vis-à-vis exclusive markets if and only if the potential (though not guaranteed) gains from increased market liquidity more than compensate for the (guaranteed) fall in asset quality. Thus, in contrast to Dang et al. (2010) and Dang et al. (2017), our results suggest that complexity/opacity is desirable only when efficiency gains are mostly driven by reallocation of assets in markets *and* originators need not be too incentivized to produce high-quality assets.

Our paper naturally relates to the literature that studies non-exclusive competition in markets plagued with adverse selection (e.g. Pauly Mark (1974); Jaynes (1978); Bisin and Gottardi (1999, 2003); Ales and Maziero (2009, 2016); Attar et al. (2011); Kurlat (2016)). We contribute to this literature by studying the implication of non-exclusivity for the optimal design and pricing of financial securities. Within this literature, the paper that is closest to ours is Attar et al. (2011), who study non-exclusive competition in the market for lemons. In their model the seller can only accept securities of the form $F(X) = q \cdot X$ for some $q \in [0, 1]$, so there is no room to study the role of security design. Furthermore, buyers in their setting cannot withdraw posted menus. There are two important differences in terms of results from Attar et al. (2011). First, due to optimal security design, we do not obtain Akerlof-like outcomes: there is always a non-trivial debt security that the *high*-type trades at average valuation and there is never a market collapse. Second, in our setting, the equilibrium demand for securities must generally be non-linear, in the sense that buyers must stand ready to buy one collection of securities at average or pooling valuation while another collection at low valuation, as in Jaynes (1978). This feature yields novel predictions regarding issuance and pricing of securities which are in line with empirical evidence.

To ensure existence of equilibria both in the exclusive and non-exclusive-market settings we follow Miyazaki (1977), Wilson (1977) and Netzer and Scheuer (2014) by introducing a stage after all initial menus are posted in which buyers can withdraw loss-making menus at an infinitesimal cost. The ability to withdraw loss-making menus prevents cream-skimming deviations, and therefore enlarges the set of equilibria in the competitive screening games we analyze. We contribute to this literature by analyzing the role of costly menu withdrawal in supporting equilibria in non-exclusive-market settings, where equilibria is hard to obtain, as shown by Attar et al. (2014).

Finally, on the normative front, we contribute to a growing literature that studies the costs and benefits of transparency in financial markets plagued with adverse selection (e.g. Chemla and Hennessy (2014); Dang et al. (2010); Dang et al. (2017); Asriyan, Fuchs, and Green (2017, 2019a); Daley, Green, and Vanasco (2020a)). This literature primarily focuses on how transparency affects the agents' ability to obtain additional information about the seller's asset quality (e.g. by observing signals). In contrast, we focus on the implications of transparency through its effects on exclusivity; that is, through the ability of an agent to observe *and* contract upon the set of trades that his counterparty enters into with other agents in the market.

Our paper is organized as follows. In Section 2, we present the setup of our model, and we establish two useful benchmarks against which to compare our results. In Section 3, we characterize the equilibrium of our model. We consider the model's normative implications, which we relate to policy discussions, in Section 4; and its positive predictions, which we relate to empirical facts, in Section 5.

2 The Model

There are two dates, indexed by $t \in \{1, 2\}$. There is an asset seller (e.g., bank) and a large number N of "deep pocket" buyers (e.g., investors). The seller's preferences are:

$$U^S = c_1^S + \delta \cdot c_2^S, \tag{1}$$

where $\delta \in (0, 1)$ and c_t^S is the cash flow she receives in period t. A buyer's preferences are:

$$U^B = c_1^B + c_2^B, \tag{2}$$

where c_t^B is the cash flow he receives in period t. Thus, gains from trade between the seller and the buyers arise due to heterogeneity in discount factors.[6]

The seller is endowed with an asset that delivers a random cash flow X in period $t = 2$. The asset can be of high- or low-quality, denoted by $\theta \in \{H, L\}$, and its cash flow is distributed according to cdf G_θ. We assume that G_θ has an associated pdf g_θ with full support on the interval $[0, \bar{X}]$ for some $\bar{X} > 0$. The pdfs are in turn related by the monotone likelihood ratio property (MLRP); that is, $\frac{g_H(x)}{g_L(x)}$ is increasing in x. In the spirit of Akerlof (1970), asymmetric information arises because the seller knows the quality θ of her asset, whereas the buyers are uninformed and have a prior belief $\mu_0 = \mathbb{P}(\theta = H) \in (0, 1)$.

To realize gains from trade, the seller raises funds at $t = 1$ by issuing securities fully backed by her asset cash flows to buyers. Formally, a security is a function $F : [0, \bar{X}] \to \mathbb{R}$ and its payoff is denoted by $F(x)$ when the realized cash flow is $X = x$. Let $\mathcal{F}$ be the collection of securities issued by the seller, then we say that this collection is *feasible* if:

1. (Limited Liability - LL) $\sum_{F \in \mathcal{F}} F(x) \leq x$ and $F(x) \geq 0$ for all $F \in \mathcal{F}$.
2. (Weak Monotonicity - WM) $F(x)$ and $x - \sum_{F \in \mathcal{F}} F(x)$ are weakly increasing in x.

We denote the set of all feasible (collections of) securities by Φ.

The securities market is non-exclusive in the sense that trade between the seller and the buyers is bilateral, and a buyer cannot exclude the seller from trading with other buyers. Formally, we study the following three-stage screening game:

- Stage 1: buyers simultaneously post menus of contracts, where a menu $\mathcal{M}^i$ posted by buyer i is a set of contracts or security-price pairs (F^i, p^i).
- Stage 2: buyers observe all posted menus and simultaneously decide whether to remain in the market or to become inactive by withdrawing their menu at infinitesimal cost $c > 0$.
- Stage 3: seller observes all active menus and accepts at most one contract from each, subject to the accepted collection of securities being feasible.[7]

Contracts are executed at the end of the game. Namely, if the seller has accepted contract (F^i, p^i) from buyer i at $t = 1$, then the buyer makes a transfer p^i to the seller at $t = 1$ and in exchange has a claim to $F^i(x)$ at $t = 2$ when the realized cash flow is $X = x$.

[6] The assumption that the seller is more impatient than the buyers is a common modeling device to rationalize gains from trade (e.g., DeMarzo and Duffie (1999); Biais and Mariotti (2005); DeMarzo (2005); Daley et al. (2020b,a)). In practice, there are many reasons why an asset owner might want to raise funds by selling asset cash flows. For example, a bank that is financially constrained and has new profitable investment opportunities may benefit from selling a fraction of its loans to finance these new opportunities. Alternatively, asset securitization may allow loan originators to share-risks with market investors.

[7] The restriction that the seller accepts at most one contract from each menu is without loss of generality.

Discussion. Our modeling approach of allowing a buyer to react to other buyers' menus is in the spirit of Wilson (1977) and Miyazaki (1977). In particular, to ensure that an equilibrium always exists, both in exclusive and non-exclusive markets, we follow Netzer and Scheuer (2014) and introduce the menu-withdrawal stage (i.e., Stage 2). As in their paper, introducing costly menu withdrawal refines the set of equilibria. Such withdrawal cost can be interpreted directly as communication/administrative cost or indirectly as the loss of reputation associated with withdrawal of contracts posted in a marketplace.[8]

For tractability, we restrict the seller to issue a feasible set of securities, as stated in conditions (LL) and (WM) above. These conditions are generalizations of the limited liability and the monotonicity constraints often used in the literature on asset-backed security design with asymmetric information (e.g., Nachman and Noe (1994); DeMarzo and Duffie (1999); Biais and Mariotti (2005)) to a setting with multiple securities. Importantly, the (LL) condition states that neither the seller nor the buyers can trade asset cash flows that they do not have. It is a generalization of the capacity constraint in Attar et al. (2011) to security design, and it helps us isolate the mechanism arising due to adverse selection in asset sales.[9]

In Appendix B, we provide a microfoundation that rationalizes our feasibility conditions in our non-exclusive- market setting. In a nutshell, we show that our setting is formally equivalent to one with lack of commitment and with spot markets in which the transfer of asset cash flows, defined as infinitesimal debt tranches, can be observed and verified only by the buyer involved in a given transaction. A natural example is the market for asset-backed securities, where bundles of tranches – fully backed by the cash flows of underlying assets (e.g., loans) – are traded over the counter. In these markets, a buyer can easily verify his own purchase, but he cannot readily observe what other transactions the seller enters into with other buyers.

Payoffs. At Stage 3, the θ-type seller decides which contracts to accept from the active menus. Let $\mathcal{C}_\theta$ denote the set of chosen contracts, where we set $\mathcal{C}_\theta = \{(0,0)\}$ if the seller chooses not to trade. Then, the seller's payoff at Stage 3 is:

$$u_\theta(\mathcal{C}_\theta) \equiv \sum_{(F,p)\in\mathcal{C}_\theta} p + \delta \cdot \mathbb{E}_\theta\left[X - F(X)\right], \tag{3}$$

where $\mathbb{E}_\theta[\cdot]$ is the expectations operator conditional on the seller's type being θ.

[8]We refer the reader to Netzer and Scheuer (2014) for a rich analysis of equilibria in screening games with asymmetric information in exclusive markets without a withdrawal stage or with costless withdrawal.

[9]A large literature has focused on the problem of dilution in financial markets, which arises when a firm is able to dilute existing claims by issuing new claims on the same set of cash flows (e.g., Parlour and Rajan (2001); Santos and Scheinkman (2001a,b); DeMarzo and He (2016); Admati, DeMarzo, Hellwig, and Pfleiderer (2018); Donaldson, Gromb, and Piacentino (2019)). Instead, we focus on asset sales by supposing that the transfer of asset cash flows in spot markets can be easily verified.

At Stage 2, after observing all posted menus $\cup_j \mathcal{M}^j$, buyer i decides whether to withdraw his own menu $\mathcal{M}^i$. Let $\mu_2(F,p)$ denote the buyer i's belief at this stage that the seller is an H-type if she were to accept contract $(F,p) \in \mathcal{M}^i$. Then, buyer i's payoff at Stage 2 is:

$$\max\left\{\sum_{(F,p)\in\mathcal{M}^i} \mathbb{P}\left(\text{seller accepts } (F,p) | \cup_j \mathcal{M}^j\right) \cdot \left(-p + \mathbb{E}_{\mu_2}[F(X)]\right),\ -c\right\}, \tag{4}$$

where $\mathbb{E}_\mu[F(X)] \equiv \mu(F,p) \cdot \mathbb{E}_H[F(X)] + (1 - \mu(F,p)) \cdot \mathbb{E}_L[F(X)]$ for any contract (F,p) that is accepted with positive probability.

At Stage 1, buyer i decides which contracts to post in his menu. From inspection of his payoff in (4) it is immediate that buyer i will not post a menu that he expects to withdraw at Stage 2, since he can always ensure a payoff of zero by posting only the trivial contract. Let $\mu_1(F,p)$ denote the buyer i's belief at this stage that the seller is an H-type if she were to accept contract $(F,p) \in \mathcal{M}^i$. Then, buyer i's payoff at State 1 is:

$$\sum_{(F,p)\in\mathcal{M}^i} \mathbb{P}\left(\text{seller accepts } (F,p)\right) \cdot \left(-p + \mathbb{E}_{\mu_1}[F(X)]\right). \tag{5}$$

We study pure strategy perfect Bayesian Nash equilibria of the above screening game, which has the following implications. First, the seller's acceptance strategy must be optimal, given the menus that remain active at Stage 2 (*Seller Optimality*). Second, a buyer's menu chosen at Stage 1 and his decision to withdraw at Stage 2 must be optimal given his belief at each stage (*Buyer Optimality*). Finally, a buyer's belief at Stages 1 and 2 about the seller's type who is likely to accept a contract from his menu must be consistent with other buyers' posting and withdrawal strategies, the seller's acceptance strategy and Bayes' rule (*Belief Consistency*).

Our environment features two frictions: (i) the seller is privately informed about θ, and (ii) the securities market is non-exclusive. Before we proceed to the equilibrium analysis, it is useful to consider two benchmarks in which we shut down each of these frictions in turn.

2.1 Benchmark without Asymmetric Information

We first consider the allocations that would be attained in a setting without asymmetric information; that is, we assume that the seller's asset quality, θ, is observable to the buyers.

Proposition 1 *Suppose that buyers observe asset quality θ before posting their menus. Then, the aggregate cash flows issued by the θ-type seller are $F_\theta(X) = X$, which are priced at their full information valuation $p_\theta = \mathbb{E}_\theta[X]$.*

In the absence of asymmetric information, first-best allocations are obtained, as all gains from trade between the seller and the buyers are realized. Moreover, due to competition, the seller's cash flows are priced at their expected value, conditional on the true quality of the seller's asset. As we can see, in this setting, the fact that the securities market is non-exclusive has no bite. Therefore, all our novel findings will be due to the interaction of non-exclusivity with asymmetric information.

2.2 Benchmark with Exclusive Markets

We next consider the allocations that would be attained in a setting where the securities market is exclusive; that is, the seller can only trade with one buyer. Consider the following optimization program, which yields the security design version of the Miyazaki-Wilson allocations characterized in the insurance literature:

$$\max_{\{(F_\theta, p_\theta)\}\in\Phi} \quad p_H + \delta \cdot \mathbb{E}_H[X - F_H] \tag{P1}$$

subject to the following set of constraints:

$$p_L + \delta \cdot \mathbb{E}_L[X - F_L] \geq p_H + \delta \cdot \mathbb{E}_L[X - F_H], \tag{6}$$

$$p_H + \delta \cdot \mathbb{E}_H[X - F_H] \geq p_L + \delta \cdot \mathbb{E}_H[X - F_L], \tag{7}$$

$$\mu_0 \cdot (\mathbb{E}_H[F_H] - p_H) + (1 - \mu_0) \cdot (\mathbb{E}_L[F_L] - p_L) \geq 0, \tag{8}$$

$$p_L \geq \mathbb{E}_L[F_L]. \tag{9}$$

The solution to program P1, which we denote by $\mathcal{C}^{P1}$, maximizes the H-type's payoff subject to: (i) the seller's incentive compatibility constraints (6) and (7), which implicitly assume that accepting allocation (F, p) implies retaining (i.e., not transferring to buyers) the remaining cash flows; (ii) the buyers' participation constraint (8), which ensures that buyers do not make losses in expectation; and (iii) the constraint (9), which ensures that the L-type receives at least her full information payoff. The following lemma describes the solution to P1, which we will shortly use to characterize equilibria in exclusive markets.

Lemma 1 *The unique solution to P1 is as follows. There exists $\widetilde{\mu} \in (0, 1)$ s.t.*

1. *If $\mu_0 \leq \widetilde{\mu}$, the solution features perfect separation: there exists $d^S \in (0, \bar{X})$ such that*
   - *(i) $F_H(X) = \min\{d^E, X\}$ and $p_H = \mathbb{E}_H[\min\{d^E, X\}]$;*
   - *(ii) $F_L(X) = X$ and $p_L = \mathbb{E}_L[X]$.*

2. *If $\mu_0 > \widetilde{\mu}$, the solution features cross-subsidization: there exists $d^C(\mu_0) \in (d^S, \bar{X}]$ s.t.*

   (i) *$F_H(X) = \min\{d^C(\mu_0), X\}$ and*

$$p_H = E_{\mu_0}[F_H] + (1-\mu_0)(1-\delta)E_L[X - F_H] < \mathbb{E}_H[F_H(X)];$$

   (ii) *$F_L(X) = X$ and*

$$p_L = E_{\mu_0}[F_H] + [(1-\mu_0)(1-\delta)+\delta]E_L[X - F_H] > \mathbb{E}_L[X].$$

The first result in Lemma 1 states that when the buyers' prior belief is low the solution to P1 is separating; i.e., constraint (9) binds. The seller's type is screened through two distinct contracts: (F_H, p_H), which offers to buy less cash flows, but at high valuation; and (F_L, p_L), which offers to buy all cash flows, but at low valuation. Because cash flow retention is more costly for the L-type seller, the security F_H is designed so that the L-type is indifferent between issuing all of her cash flows at low valuation or mimicking the H-type by accepting contract (F_H, p_H). Consistent with this, debt is the optimal design as it relaxes the incentive compatibility constraint of the L-type relative to other feasible securities. Furthermore, d^S is exactly chosen to ensure that the incentive compatibility constraint of the L-type binds:

$$E_{\mu_0}\left[\min\{d^S, X\}\right] + [(1-\mu_0)(1-\delta)+\delta]E_L\left[X - \min\{d^S, X\}\right] = \mathbb{E}_L[X]. \quad (10)$$

The second result in Lemma 1 states that when the buyers' prior belief is high the solution features cross-subsidization from the H- to the L-type seller; i.e., constraint (9) is slack. In this scenario, the H-type is better off by selling more cash flows, as $d^C(\mu_0) > d^S$, even though this comes at the expense of receiving less than her valuation. Debt continues to be the optimal security design, as it minimizes the subsidy that the H-type has to give to the L-type by being the feasible security for which the difference in valuations between the two types is the smallest. Furthermore, for a given μ_0, d^C is exactly chosen to optimally trade off the marginal benefit for the H-type of selling more cash flows with the marginal cost of subsidizing the L-type:

$$d^C(\mu_0) = \arg\max_{d \in [0, \bar{X}]} \mathbb{E}_{\mu_0}[\min\{d, X\}] - \delta \cdot \mathbb{E}_H[\min\{d, X\}] - (1-\mu_0)\cdot(1-\delta)\cdot E_L[\min\{d, X\}]. \quad (11)$$

Moreover, $d^C(\mu_0) \leq \bar{X}$ where the last inequality is strict iff $\mu_0 \leq \delta \cdot \frac{\lim_{x\to\bar{X}} \frac{g_H(x)}{g_L(x)} - 1}{\lim_{x\to\bar{X}} \frac{g_H(x)}{g_L(x)} - \delta}$.

Proposition 2 (Equilibrium in Exclusive Markets) *Suppose that the seller can accept con-*

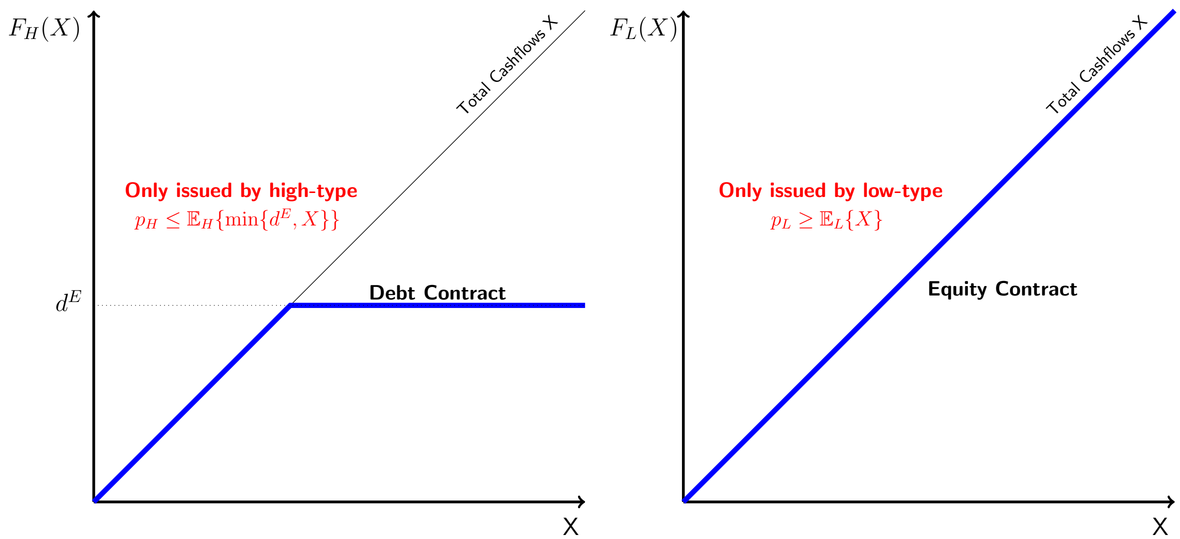

Figure 1: **Security Design in Exclusive Markets.** The left-panel depicts the security issued by the H-type seller, which is priced at or below high valuation; whereas the right-panel depicts the security issued by the L-type seller, which is priced at or above low valuation.

tracts from only one buyer. Then, an equilibrium always exists and it is unique.[10] *In it, the H-type seller accepts contract (F_H, p_H), while the L-type seller accepts contract (F_L, p_L), given by the solution to P1 in Lemma 1.*

The proof of Proposition 2 consists of, first, showing that any equilibrium allocation must solve program P1. As markets are exclusive, any cash flow that is not transferred to the buyer must be retained by the seller, as stated by the incentive compatibility constraints (6)-(7).[11] With this, we show that if equilibrium allocations are such that one of the constraints in P1 is violated or if the H-type's payoff is not maximized, there is a profitable deviation for a buyer.

Second, we show that the solution to P1 can be supported as a PBE, i.e., there are no profitable deviations for the buyers nor for the seller. Let u_θ^E denote the payoff to the θ-type seller in this equilibrium. First, note that by construction, there are no profitable deviations for a seller as the allocations maximize the H-type's payoff subject to incentive compatibility constraints. Second, as buyers make zero profits (remember their participation constraint binds) no buyer benefits from withdrawing equilibrium menus at Stage 2, as doing so incurs a cost. Thus, it suffices to rule out buyer deviations at Stage 1.

When the buyers' prior belief is low, i.e., $\mu \leq \widetilde{\mu}$, the equilibrium is separating. Its alloca-

[10]As there are many buyers, a given buyer's equilibrium menu is not pinned down. We say an equilibrium is unique when the allocation of transfer and cash flows between each seller type and the buyers is uniquely pinned down.

[11]We highlight this, as understanding what is the appropriate incentive compatibility constraint will prove to be essential in the study of non-exclusive markets.

tions coincide with those of the least-costly separating equilibrium (LCSE) in signaling games (DeMarzo, 2005; Daley et al., 2020b), which obtains the highest possible payoff to the H-type seller by implementing the minimum cash flow retention needed for separation. As a result, it is clear that there are no profitable buyer-deviations that can attract the H-type to a separating contract. In addition, there are also no profitable buyer-deviations to offer a cross-subsidizing contract since, when $\mu \leq \widetilde{\mu}$, the LCSE allocations give the H-type a higher payoff than any cross-subsidizing allocations, as shown in the solution to P1.

When the buyers' prior belief is high, i.e., $\mu > \widetilde{\mu}$, the equilibrium features cross-subsidization from the H-to the L-type. In this scenario, given the equilibrium menus, a profitable "cream-skimming" contract that gives the H-type a payoff above u_H^E and the L-type a payoff below u_L^E always exists. Such a contract destroys pooling equilibria in standard screening games, as shown in the context of insurance markets by Rothschild and Stiglitz (1978). In our setting, however, the ability of buyers to withdraw loss-making menus renders such a deviation unprofitable, as was first pointed out by Wilson (1977) and Miyazaki (1977).

Next, we show that non-exclusive markets behave very differently from exclusive ones. In particular, when markets are non-exclusive: (i) equilibrium always exists but need not be unique, (ii) perfectly separating allocations are never part of equilibrium, (iii) equilibrium allocations can always be implemented with non-cross subsidizing contracts, and (iv) all equilibria are semi-pooling in a sense that some *but not all* contracts are accepted by both seller types. We will then argue that these findings have both important positive and normative implications.

3 Equilibrium

We are now ready to characterize equilibria of our model, where the seller is able to accept contracts from multiple menus. As a first important result, we show that in any equilibrium in non-exclusive markets, there must be cross-subsidization from the H-type to the L-type seller.

Proposition 3 *In any equilibrium in non-exclusive markets, there must be some level of cross-subsidization from the H-type to the L-type seller, i.e., perfect separation is not possible.*

Proposition 3 highlights the importance of exclusivity for separation. Intuitively, suppose that the contracts (F_H, p_H) and (F_L, p_L) described in Proposition 2 were offered by buyer i and accepted by the H- and the L-type respectively. Consider then a deviation by buyer $j \neq i$, which allows the L-type to mimic the H-type seller: he offers to purchase any feasible security at a price slightly below low valuation, including $\widetilde{F}(X) = X - F_H(X)$. In addition, he offers a cross-subsidizing contract where the security is priced slightly below average valuation. If no menu is withdrawn after such a deviation, buyer j attracts the L-type: as markets are non-exclusive, she can now accept both (F_H, p_H) and $(\widetilde{F}, \widetilde{p})$ and obtain a strictly higher payoff than her full information payoff. In this scenario, however, it is possible that menus with contract (F_H, p_H), for example, are withdrawn, as they generate losses to buyers. After all loss-making menus are withdrawn, however, we show that buyer j has either attracted both seller types to his cross-subsidizing contract, or only the L-type to one of his low valuation contracts. In either scenario, buyer j makes profits.[12]

Next, we describe the solution to an optimization program, which will be helpful in characterizing equilibrium with non-exclusive markets:

$$\max_{\{(F_\theta, p_\theta)\}\in\mathcal{F}} \quad p_H + \delta \cdot \mathbb{E}_H[X - F_H] \tag{P2}$$

subject to,

$$p_L + \mathbb{E}_L[X - F_L] \geq p_H + \mathbb{E}_L[X - F_H] \tag{12}$$

$$p_H + \delta \cdot \mathbb{E}_H[X - F_H] \geq p_L + \delta \cdot \mathbb{E}_H[X - F_L] \tag{13}$$

$$\mu_0 \cdot (\mathbb{E}_H[F_H] - p_H) + (1 - \mu_0) \cdot (\mathbb{E}_L[F_L] - p_L) \geq 0 \tag{14}$$

$$p_L \geq \mathbb{E}_L[F_L] \tag{15}$$

The solution to program P2, which we denote by $\mathcal{C}^{P2}$, maximizes the H-type's payoff subject to the same constraints as in program P1 with the exception of the L-type's incentive compatibility (12). In contrast to P1, we now suppose that if the L-type were to mimic the H-type she would be able to issue her remaining cash flows at low-valuation. The following proposition fully characterizes the solution to P2.

Lemma 2 *The unique solution to P2 is as follows. There exists $d^{NE} \in (0, \bar{X}]$ such that*

(i) $F_H(X) = \min\{d^{NE}, X\}$ and $p_H = \mathbb{E}_{\mu_0}[F_H(X)]$;

(ii) $F_L(X) = X$ and $p_L = \mathbb{E}_{\mu_0}[F_H(X)] + \mathbb{E}_L[X - F_H(X)]$.

By contrasting the results in Lemma 2 with those obtained in Lemma 1, it follows that when the L-type is able to sell her remaining cash flows at low valuation, the solution is never separating: there is always cross-subsidization from the H- to the L-type seller. Furthermore, the subsidy is such that the H-type seller issues all of her cash flows at average valuation. As

[12] Observe that such a deviation would not be profitable if markets were exclusive, as the L-type could not jointly accept $(\widetilde{F}, \widetilde{p})$ and (F_H, p_H); additionally, such a deviation would still be profitable in the absence of a withdrawal stage, making the result in Proposition 3 robust to the absence of Stage 2.

in the solution to program P1 in the presence of cross-subsidies, the optimal security issued by the H-type is debt, as it is the design that maximizes the H-type's payoff by minimizing the subsidy given to the L-type among all feasible securities. Consistent with this, the debt level is given by

$$d^{NE} = \arg\max_{d\in[0,\bar{X}]} \mathbb{E}_{\mu_0}[\min\{d, X\}] - \delta \cdot \mathbb{E}_H[\min\{d, X\}]. \tag{16}$$

Moreover, $0 < d^{NE} \leq \bar{X}$ where the last inequality is strict iff $\mu_0 \leq \frac{\lim_{x\to\bar{X}} \delta\cdot\frac{g_H(x)}{g_L(x)}-1}{\lim_{x\to\bar{X}} \frac{g_H(x)}{g_L(x)}-1}$.

The allocations given by the solution to P2 can be implemented through non-cross-subsidizing contracts, i.e. contracts in a menu that give zero expected profits to the buyer, as follows:

(i) A *senior tranche*, $F_S(X) = \min\{d^{NE}, X\}$, priced at average valuation, $p_S = \mathbb{E}_{\mu_0}[F_S(X)]$, and accepted by all seller types.

(ii) A *junior tranche*, $F_J(X) = \max\{X - d^{NE}, 0\}$, priced at low valuation, $p_J = \mathbb{E}_L[F_J(X)]$, and accepted only by the L-type seller.

In the remainder of the paper, we use contracts $\mathcal{C}^* \equiv \{(F_S, p_S), (F_J, p_J)\}$ to represent the set of contracts whose resulting allocations solve P2.[13]

In what follows we show that the allocations that solve P2 can always be supported as an equilibrium, and that they provide a lower bound on the payoff of other possible equilibria in non-exclusive markets.

Proposition 4 (Equilibrium in Non-Exclusive Markets) *Suppose that the seller can accept contracts from multiple buyers. Then, there exists an equilibrium in which all seller types accept contract* (F_S, p_S)*, and in addition the* L*-type seller accepts contract* (F_J, p_J).

Proposition 4 states that there always exists an equilibrium in which both seller types issue the same, non-trivial, debt security. Moreover, if buyers' belief about the underlying asset quality is sufficiently high, the seller issues a claim to all of her cash flows, i.e., $F_S(X) = X$. In addition to accepting the same contract as the H-type, the L-type seller issues her remaining cash flows $F_J(X) = X - F_S(X)$ at low valuation to a separate buyer, in order to further exploit gains from trade.

The equilibrium in Proposition 4 is supported by the presence of menus that offer to purchase all feasible securities at low valuation. Many of these contracts, often referred to as *latent*

[13]Focusing on contracts $\mathcal{C}^*$ is without loss as the solution to P2 has been shown to be unique in terms of allocations for each seller type and from sellers to buyers. We choose these contracts as we believe they capture best the nature of the solution to P2.

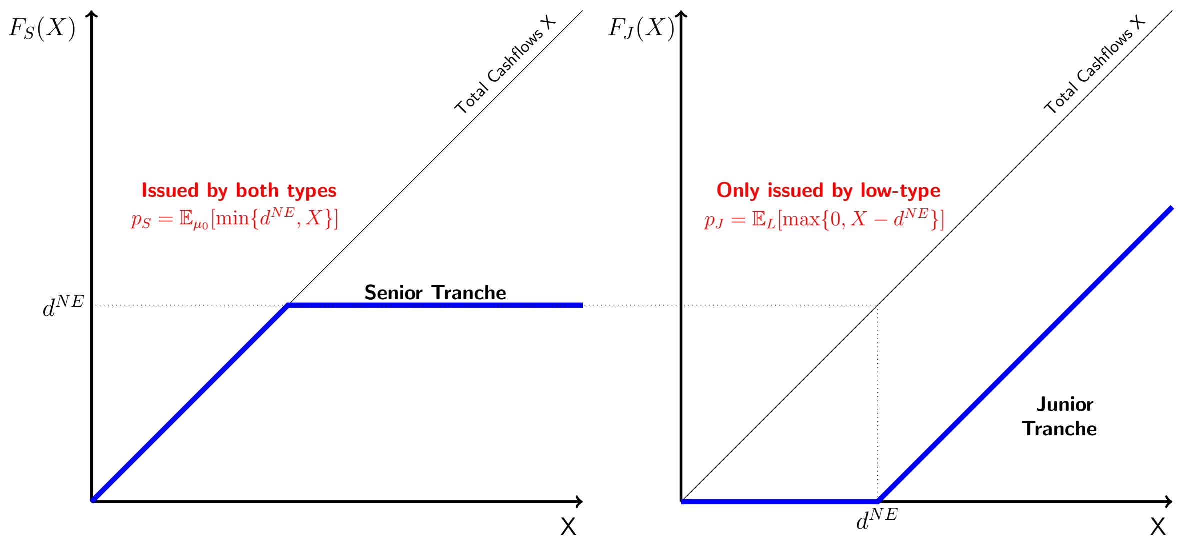

Figure 2: **Security Design in Non-Exclusive Markets.** The left-panel depicts the security issued by both seller types, which is priced at average valuation; whereas the right-panel depicts the security issued by the L-type seller only, which is priced at low valuation.

contracts, are not accepted on equilibrium path but they deter Stage 1 deviations by buyers, as we explain in what follows. First, deviations to offer alternative pooling contracts cannot attract the H-type, as (F_S, p_S) is the pooling contract that provides her with maximal payoff. Second, deviations to offer contracts that attempt to separate seller types cannot be profitable either. To see this, first note that if a deviating menu were to attract the H-type, then all equilibrium menus containing contract (F_S, p_S) are withdrawn at Stage 2. But then, we show that the L-type accepts the deviating contract meant for the H-type together with a low-valuation latent contract from a remaining active menu, generating losses for the deviating buyer.

We have characterized the essential properties of what we refer to as our *star* equilibrium. First, asset cash flows are split into a senior (F_S) and a junior (F_J) tranche. While the L-type seller issues both tranches, each to a separate buyer, the H-type seller only issues the senior tranche but retains the junior one, as she is not willing to sell these cash flows at average valuation. Second, the L-type seller is effectively subsidized by the H-type seller in the market for the senior tranche, but she receives her full information valuation in the market for the junior tranche. Figure 2 provides a graphical illustration of these findings.

Proposition 5 *The* star equilibrium *is unique among equilibria in which the seller is able to sell any cash flows at low valuation.*

Proposition 5 highlights the robustness of the *star equilibrium*, as it is the unique equilibrium that can be supported when the seller is able to issue any set of feasible cash flows at low valuation, which we view as the case that approximates financial markets best. The proof of this proposition is straightforward, as when the seller can always issue any cash flows at low valuation, contracts $\mathcal{C}^*$ maximize the H-type's payoff subject to incentive compatibility and participation constraints (as they implement the allocations implied by the solution to P2). It can be shown, however, that other equilibria can be supported if buyers coordinate on not posting certain securities at low valuation.[14] We show, however, that our *star equilibrium* provides a lower bound on seller payoffs on all possible equilibria.

Proposition 6 *The* star equilibrium *payoffs provide a lower bound for the payoff of each seller in any equilibrium.*

The proof of Proposition 6 consists of showing that if contracts $\mathcal{C}^*$ are not offered on some equilibrium menu, then there is a profitable deviation for a buyer. Thus, if other equilibria exist, they must Pareto dominate the *star* equilibrium. An immediate implication is that when $d^{NE} = \bar{X}$, i.e. all seller types sell a full claim to their cash flows, the *star equilibrium* is the unique equilibrium, as its allocations are on the Pareto frontier.

In what follows we discuss the welfare properties of equilibria in exclusive vs. non-exclusive markets. In doing so, when needed, we focus the analysis of non-exclusive markets on the *star* equilibrium.

4 Costs and Benefits of Non-Exclusivity

As we already discussed in the introduction, after the global financial crisis, a number of transparency-enhancing financial market reforms were discussed in the US and Europe, which would either directly or indirectly enhance exclusivity in contracting. Despite of these efforts of policymakers and regulators, there is surprisingly little theoretical work on the policy implications of non-exclusivity in markets with asymmetric information. Motivated by this, in this section we consider the normative implications of our theory by studying the potential costs and benefits of non-exclusivity. First, we study the welfare properties of our baseline model, where the distribution of asset qualities is exogenous. Second, we consider a simple extension of the model that endogenizes asset quality.

4.1 Non-Exclusivity and Market Liquidity

We begin by introducing the notion of efficiency/welfare in our setting. Since buyers always break-even, efficiency is determined by the ex-ante expected payoff to the seller:

$$W(\mu_0) \equiv \mu_0 \cdot u_H(\mu_0) + (1 - \mu_0) \cdot u_L(\mu_0). \tag{17}$$

[14] We are at the moment working on obtaining a characterization of all equilibria in non-exclusive markets.

where $u_\theta(\mu_0)$ denotes the equilibrium payoff of a θ-type seller when the buyers' prior belief that the seller is H-type is μ_0.[15] In what follows, we will sometimes use superscripts to indicate the outcomes in the first-best (FB), exclusive (E) or non-exclusive (NE) market settings.

In the presence of asymmetric information, the equilibrium allocations may be distorted away from first-best for two reasons. First, due to retention of cash flows by the H-type seller, some gains from trade remain unrealized. We say that the market is more *liquid* when more gains from trade between the seller and the buyers are realized, i.e. when cash flow retention is lower. Second, because the prices of claims need not reflect true underlying asset quality θ, the H-type seller may effectively subsidize the L-type seller. When this occurs, we say that there is *mispricing*. To illustrate the effects of these distortions, the payoff of a θ-type seller can be expressed as follows:

$$u_L(\mu_0) = \underbrace{\mathbb{E}_L[X]}_{=u_L^{FB}} \quad + \quad \underbrace{\Delta(\mu_0)}_{\text{Mispricing Subsidy}} \quad , \tag{18}$$

$$u_H(\mu_0) = \underbrace{\mathbb{E}_H[X]}_{=u_H^{FB}} - \underbrace{(1-\delta)\cdot\mathbb{E}_H[X - F_H(X)]}_{\text{Cost of Retention}} - \underbrace{\frac{1-\mu_0}{\mu_0}\cdot\Delta(\mu_0)}_{\text{Mispricing Tax}}. \tag{19}$$

where F_H denotes the cash flows issued by the H-type seller in equilibrium.

Since the mispricing of claims generates a transfer from the H-type to the L-type seller, equilibrium welfare is distorted away from first-best only due to inefficient cash flow retention:

$$W(\mu_0) = \underbrace{\mu_0\cdot\mathbb{E}_H[X] + (1-\mu_0)\cdot\mathbb{E}_L[X]}_{=W^{FB}(\mu_0)} - \underbrace{\mu_0\cdot(1-\delta)\cdot\mathbb{E}_H[X - F_H(X)]}_{\text{Expected Cost of Retention}}. \tag{20}$$

It follows that, when asset quality is exogenous, a more liquid market is also more efficient. Therefore, non-exclusive markets are more efficient than exclusive markets if and only if they implement higher liquidity, e.g. when $d^{NE}(\mu_0) > d^E(\mu_0)$. The next proposition characterizes when this is the case.

Proposition 7 *There exists $\underline{\mu} < \bar{\mu}$ both $\in (0,1)$, such that the welfare with exclusive markets is strictly greater than in non-exclusive markets for $\mu_0 < \underline{\mu}$, strictly lower than in non-exclusive markets for $\mu_0 \in (\underline{\mu}, \bar{\mu})$, and equal for $\mu_0 \geq \bar{\mu}$ as in both markets the seller issues a full claim to her cash flows.*

Figure 3 illustrates this result graphically. The proof is intuitive. Let us focus on the *star equilibrium* for concreteness. First, we have shown that $d^E(\mu_0) = \max\{d^S, d^C(\mu_0)\}$, as the

[15] We are using the fact that the average quality of assets and the buyers' prior belief are the same.

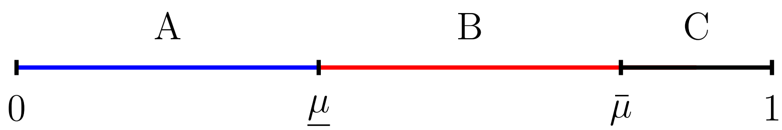

Figure 3: **Which market structure is more efficient for an exogenously given average quality μ_0?** Region A depicts the values of μ_0 for which *exclusive* markets dominate *non-exclusive* markets. Region B depicts the values for which *non-exclusive* markets dominate *exclusive* markets. Region C depicts the values for which both market structures obtain the same welfare, as $W^E(\mu_0) = W^{NE}(\mu_0) = \mathbb{E}_{\mu_0}[X]$.

equilibrium in exclusive markets is separating for $\mu_0 \leq \widetilde{\mu}$, and features cross-subsidization for $\mu_0 > \tilde{\mu}$, with $d^S = d^C(\tilde{\mu})$ and with $d^C(\mu_0)$ continuous and monotonically increasing in μ_0. Second, in the *star* equilibrium, $d^{NE}(\mu_0)$ is continuous and monotonically increasing in μ_0. Moreover, it is straightforward to show that $d^{NE}(\mu_0) \geq d^C(\mu_0)$ with the inequality being strict for $\mu_0 \in (0, \bar{\mu})$. With this, it is straightforward to show that d^{NE} is smaller than d^E when μ_0 is sufficiently small but both go to $\bar{X}$ as μ_0 goes to 1. The first threshold is given by $d^S = d^{NE}(\underline{\mu})$, while the second threshold is given by $d^C(\bar{\mu}) = d^{NE}(\bar{\mu}) = \bar{X}$. As the *star* equilibrium allocations represent a lower bound on the seller's equilibrium payoffs in non-exclusive markets, we note that the results in Proposition 7 hold for any equilibrium in non-exclusive markets.

These findings contrast with the by-now conventional 'ignorance is bliss' view of Dang et al. (2010) and Dang et al. (2017), according to which market liquidity and efficiency are maximized through complexity of assets and opacity of issuers' balance sheets. Our results instead suggest that to the extent that complexity/opacity inhibits exclusive contracting, it actually reduces market liquidity and efficiency whenever the underlying asset quality is low.

4.2 Non-Exclusivity and Origination Incentives

In the previous section, we showed that non-exclusive markets may either increase or decrease market liquidity, and therefore efficiency, depending on the average quality of assets. Recall, however, that non-exclusive markets always induce a larger mispricing of claims than exclusive markets. In our baseline setting, such mispricing was irrelevant for efficiency, as the distribution of asset quality was exogenous. In many applications, however, such mispricing may impact efficiency by distorting agent's decisions, e.g. distorting incentives of loan origination. To address this, we now explore how non-exclusivity, through its effects on market liquidity and the pricing of claims, affects incentives to originate high quality assets.

We consider a simple extension of our baseline setting, where we now allow the seller (who is now also an asset originator) to exert costly, unobservable effort $c(q)$ to ensure that her asset is of high quality with probability $q \in [0, 1]$. For interior solution, we suppose that $c(0) = c'(0) = 0$,

$c'(q) > 0$ and $c''(q) > 0$ for $q \in (0,1)$, and $\lim_{x\to 1} c'(x) = \infty$.[16] Given the buyers' prior belief μ_0, efficiency is given by the ex-ante payoff of the seller, now given by:

$$W(\mu_0) = \max_q \; q \cdot u_H(\mu_0) + (1-q) \cdot u_L(\mu_0) - c(q), \tag{21}$$

where $u_\theta(\mu_0)$ denotes the payoff of a θ-type seller in the equilibrium of the *trading stage* as defined in Section 2, for given belief μ_0. The solution q^* to problem (21) exists, is unique, and satisfies:

$$c'(q^*) = \underbrace{\mathbb{E}_H[X] - \mathbb{E}_L[X] - (1-\delta) \cdot \mathbb{E}_H[X - F_H(X)] - \frac{\Delta(\mu_0)}{\mu_0}}_{= u_H(\mu_0) - u_L(\mu_0)}. \tag{22}$$

An equilibrium of the entire game now in addition requires that (i) given the seller's payoffs $\{u_\theta(\mu_0)\}_\theta$ at the trading stage, her effort choice is optimal, i.e. solves (22); and (ii) the buyers' prior belief is consistent with the seller's optimal effort choice, i.e. $\mu_0 = q^*$. It is straightforward to show that an equilibrium exists both in exclusive and non-exclusive markets.

From equation (22), we see that both market liquidity (as captured by the cash flows sold by the H-type at the *trading stage*, F_H, which note varies with μ_0) and the extent to which the claims are mispriced (as captured by $\Delta(\mu_0)$) are relevant for determining the originator's effort incentives. Moreover, even though market liquidity may be higher or lower in non-exclusive markets (see Proposition 7), the mispricing of claims is always larger in non-exclusive markets, which, as we show next, is crucial for understanding origination incentives.

Proposition 8 *When markets are non-exclusive, the average quality of originated assets is always lower than when markets are exclusive:* $0 < \mu_0^{NE} < \mu_0^E < \mu_0^{FB} < 1$.

Proposition 8 establishes a very strong result: non-exclusive markets always implement lower equilibrium asset quality than exclusive markets. The reason behind it is intuitive, and it is driven by fundamental differences between exclusive and non-exclusive markets. Recall that the originator's incentive to exert effort increases with the payoff gap between seller types, $u_H(\mu_0) - u_L(\mu_0)$ (see equation (22)). The proof then consists of showing that this gap is always smaller in non-exclusive markets.

By combining the results of Propositions 7 and 8, we conclude that non-exclusive markets can only be more efficient if the potential (though not guaranteed) gains from increased market

[16] This formulation is standard and has been employed in several papers that study the effect of secondary market liquidity on origination incentives (e.g. Chemla and Hennessy (2014); Vanasco (2017); Caramp (2017); Neuhann (2017); Daley, Green, and Vanasco (2020a); Fukui (2018); Asriyan, Fuchs, and Green (2019b)).

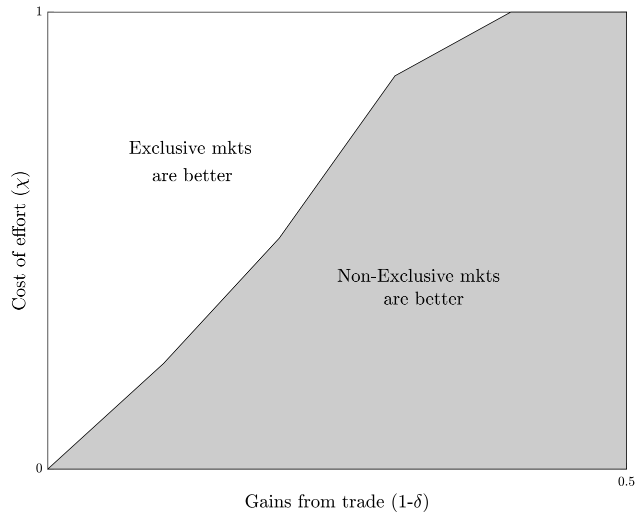

Figure 4: **Which market structure is more efficient when average quality is endogenous?** The unshaded region depicts parameter values for which *exclusive* markets dominate non-*exclusive* ones. The shaded region depicts parameter values for which *non-exclusive* markets dominate *exclusive* ones.

liquidity more than compensate for the (guaranteed) fall in asset quality. Figure 4 illustrates that this happens when the gains from market trade (as captured by $1 - \delta$) are large relative to the cost of exerting effort (as captured by χ), where we use a simple parameterization, $c(q) = \chi \cdot \frac{q^2}{1-q}$. It is important to highlight that when asset quality is endogenous, there is never an equilibrium in which all seller types sell a full claim to their cash flows: there is always retention in equilibrium. Otherwise, $u_H - u_L = 0$, resulting in very low average quality, which is inconsistent with the H-type being willing to sell all of her asset cash flows.

These findings provide a further contrast to Dang et al. (2010) and Dang et al. (2017), by showing that complexity/opacity is desirable only in environments where efficiency gains are mostly driven by reallocation of assets in markets *and* at the same time the originators need not be too incentivized to produce high-quality assets.

5 Empirical Implications

Our model has important implications for markets in which exclusivity is difficult to enforce. This is likely to be the case in markets where sellers' (e.g. firms, banks) risk exposures or trades are either not observable or hard to understand by other market participants (e.g. investors, regulators, courts). Understanding the implications of non-exclusivity is particularly relevant for the study of modern financial markets, where the increasing complexity of assets and balance sheets of financial intermediaries combined with the opacity of markets where these assets are traded makes it virtually impossible for outsiders to ensure that a seller retains a particular risk-exposure. In what follows, we present the novel empirical implications of our model and relate them to empirical evidence in the market for mortgage-backed securities. When doing so, we focus on the *star* equilibrium in non-exclusive markets.

Prediction 1. As exclusivity becomes harder to enforce, the practice of splitting asset cash flows into different tranches that are sold separately in markets is more likely to occur.

Indeed, in recent decades, the expansion of securitization and of the practice of tranching loan cash flows coincided with an increase in the complexity of financial intermediaries' balance sheets, whose risk-exposures became harder to understand and contract upon. As argued in a recent paper by Ashcraft et al. (2019), the complexity of collateralized debt obligations (CDOs) "enabled informed parties in the commercial mortgage-backed securitization pipeline to reduce their skin-in-the-game [retention] in a way not observable to other market participants."

Prediction 2. In non-exclusive markets, the amount of cash flows retained **should not** *predict differential pricing of securities in the market for senior tranches.*

This prediction follows from Proposition 4, which states that the senior tranche, issued by all seller types in equilibrium is priced at average valuation. As a result, whether the seller retains (H-type) or sells (L-type) her junior tranche does not affect its pricing. This result is consistent with findings in Ashcraft et al. (2019), who study cash flow retention and its relation to security performance in the conduit segment of the commercial mortgage-backed securities market.[17] They find that the fraction of initially retained cash flows sold into CDOs in the twelve months following a transaction, i.e., not observed at the time of the transaction, is not correlated with the prices of the more senior tranche.

Prediction 3. In non-exclusive markets, the amount of cash flows retained **should** *predict differential quality of the senior tranches.*

This prediction also follows from Proposition 4, which states that while the H-type seller only issues a senior tranche, and thus retains a junior tranche, the L-type seller issues both tranches to distinct buyers, and thus does not retain cash flows in equilibrium. This result is consistent with evidence in Ashcraft et al. (2019), who find that a higher fraction of initial cash flow retention sold into CDOs predicts a higher probability of default of the more senior tranches, even after controlling for all information available at issuance.

[17]They study the retention of B-piece investors, who are buyers that perform due-diligence and re-underwrite all of the loans in a given pool, indicating there is no asymmetric information between the actual seller and the B-piece investors. Importantly, even though the size of the B-piece is disclosed to other (uninformed) buyers, how much the B-piece buyer actually retains over time is not transparent to these buyers. See Ashcraft et al. (2019) for a more detailed description of the environment and empirical strategy.

Prediction 4. As exclusivity becomes harder to enforce, the quality of originated assets declines.

This prediction is effectively a re-statement of Proposition 8. Though we are not aware of a formal test of this prediction, it is broadly consistent with the well-known stylized fact that the US credit boom of the early 2000s, fueled by securitization and financial engineering of complex assets traded in opaque markets, has been associated with falling lending standards and a decline in the quality of originated assets (e.g. Mian and Sufi (2009); Keys et al. (2010); Dell'Ariccia et al. (2012)). This is commonly attributed to the observed decline in the originators' cash flow retention (i.e. less skin-in-the-game) that the securitization process had apparently enabled (Parlour and Plantin (2008); Chemla and Hennessy (2014); Vanasco (2017)). As we showed in Section 4, however, the manner by which secondary markets price claims is also an essential determinant of the originators' incentives, above and beyond overall cash flow retention.

6 Concluding Remarks

We revisit the classic problem of a seller who is privately informed about her asset and needs to raise funds from uninformed buyers by issuing securities backed by her asset cash flows. We depart from the traditional literature by positing that the securities market is non-exclusive; that is, the seller cannot commit to trade with only one buyer. We show that non-exclusive markets behave very differently from exclusive ones in the presence of information asymmetries. In particular, when markets are non-exclusive: (i) separating contracts are never part of equilibrium; and (ii) equilibrium features semi-pooling for a wide range of parameters. Our model's predictions are consistent with empirical evidence on the issuance and pricing of mortgage-backed securities, and we use the theory to evaluate some of the reforms, recently proposed and implemented, aimed at enhancing transparency and exclusivity in financial markets.

References

Admati, A. R., P. M. DeMarzo, M. F. Hellwig, and P. Pfleiderer (2018): "The leverage ratchet effect," *The Journal of Finance*, 73, 145–198.

Akerlof, G. (1970): "The Market for Lemons: Quality Uncertainty and the Market Mechanism," *Quaterly Journal for Economics*, 84.

Ales, L. and P. Maziero (2009): "Non-Exclusive Dynamic Contracts," *Competition, and the Limits of Insurance, kworking paper.*

——— (2016): "Non-exclusive dynamic contracts, competition, and the limits of insurance," *Journal of Economic Theory*, 166, 362–395.

Arnott, R. and J. E. Stiglitz (1991): "Moral hazard and nonmarket institutions: Dysfunctional crowding out of peer monitoring?" *The American Economic Review*, 179–190.

Ashcraft, A. B., K. Gooriah, and A. Kermani (2019): "Does skin-in-the-game affect security performance?" *Journal of Financial Economics.*

Asriyan, V., W. Fuchs, and B. Green (2017): "Information spillovers in asset markets with correlated values," *American Economic Review*, 107, 2007–40.

——— (2019a): "Aggregation and design of information in asset markets with adverse selection," *Available at SSRN 2959043.*

——— (2019b): "Liquidity sentiments," *American Economic Review*, 109, 3813–48.

Attar, A., T. Mariotti, and F. Salanié (2011): "Nonexclusive competition in the market for lemons," *Econometrica*, 79, 1869–1918.

——— (2014): "Nonexclusive competition under adverse selection," *Theoretical Economics*, 9, 1–40.

Biais, B. and T. Mariotti (2005): "Strategic liquidity supply and security design," *Review of Economic Studies*, 72, 615–649.

Bisin, A. and P. Gottardi (1999): "Competitive equilibria with asymmetric information," *Journal of Economic Theory*, 87, 1–48.

——— (2003): "Competitive markets for non-exclusive contracts with adverse selection: The role of entry fees," *Review of Economic Dynamics*, 6, 313–338.

Caramp, N. (2017): "Sowing the seeds of financial crises: Endogenous asset creation and adverse selection," *Available at SSRN 3009977.*

Chemla, G. and C. Hennessy (2014): "Skin in the game and moral hazard," *Journal of Finance.*

Daley, B., B. Green, and V. Vanasco (2020a): "Securitization, ratings, and credit supply," *The Journal of Finance*, 75, 1037–1082.

Daley, B., B. S. Green, and V. Vanasco (2020b): "Security design with ratings," *Available at SSRN 2940791.*

Dang, T. V., G. Gorton, and B. Holmström (2010): "Financial Crises and the Optimality of Debt for Liquidity," *Working Paper*.

Dang, T. V., G. Gorton, B. Holmström, and G. Ordonez (2017): "Banks as secret keepers," *American Economic Review*, 107, 1005–29.

Dell'Ariccia, G., D. Igan, and L. U. Laeven (2012): "Credit booms and lending standards: Evidence from the subprime mortgage market," *Journal of Money, Credit and Banking*, 44, 367–384.

DeMarzo, P. and Z. He (2016): "Leverage dynamics without commitment," Tech. rep., National Bureau of Economic Research.

DeMarzo, P. M. (2005): "The pooling and tranching of securities: A model of informed intermediation," *Review of Financial Studies*, 18, 1–35.

DeMarzo, P. M. and D. Duffie (1999): "A Liquidity-Based Model of Security Design," *Econometrica*, 67, 65–99.

Donaldson, J. R., D. Gromb, and G. Piacentino (2019): "The paradox of pledgeability," *Journal of Financial Economics.*

Fukui, M. (2018): "Asset quality cycles," *Journal of Monetary Economics*, 95, 97–108.

Hellwig, M. F. (1988): "A note on the specification of interfirm communication in insurance markets with adverse selection," *Journal of Economic Theory*, 46, 154–163.

Jaynes, G. D. (1978): "Equilibria in monopolistically competitive insurance markets," *Journal of Economic Theory*, 19, 394–422.

Keys, B. J., T. Mukherjee, A. Seru, and V. Vikrant (2010): "Did securitization lead to lax screening? Evidence from Subprime Loans," *Quarterly Journal of Economics.*

Kurlat, P. (2016): "Asset markets with heterogeneous information," *Econometrica*, 84, 33–85.

Leland, H. E. and D. H. Pyle (1977): "Informational asymmetries, financial structure, and financial intermediation," *Journal of Finance*, 32, 371–387.

Mian, A. and A. Sufi (2009): "The consequences of mortgage credit expansion: Evidence from the US mortgage default crisis," *The Quarterly Journal of Economics*, 124, 1449–1496.

Miyazaki, H. (1977): "The rat race and internal labor markets," *The Bell Journal of Economics*, 394–418.

Nachman, D. and T. Noe (1994): "Optimal Design of Securities under Asymmetric Information," *Review of Financial Studies*, 7, 1–44.

Netzer, N. and F. Scheuer (2014): "A game theoretic foundation of competitive equilibria with adverse selection," *International Economic Review*, 55, 399–422.

NEUHANN, D. (2017): "Macroeconomic effects of secondary market trading," *Available at SSRN 3095730*.

PARLOUR, C. A. AND G. PLANTIN (2008): "Loan sales and relationship banking," *The Journal of Finance*, 63, 1291–1314.

PARLOUR, C. A. AND U. RAJAN (2001): "Competition in loan contracts," *American Economic Review*, 91, 1311–1328.

PAULY MARK, V. (1974): "Overinsurance and Public Provision of Insurance: the Roles of Moral Hazard and Adverse Selection," *Quarterly Journal of Economics*, 88, 44–62.

ROTHSCHILD, M. AND J. STIGLITZ (1978): "Equilibrium in competitive insurance markets: An essay on the economics of imperfect information," in *Uncertainty in economics*, Elsevier, 257–280.

SANTOS, T. AND J. A. SCHEINKMAN (2001a): "Competition among exchanges," *The Quarterly Journal of Economics*, 116, 1027–1061.

——— (2001b): "Financial intermediation without exclusivity," *American Economic Review*, 91, 436–439.

VANASCO, V. (2017): "The downside of asset screening for market liquidity," *The Journal of Finance*, 72, 1937–1982.

WILSON, C. (1977): "A model of insurance markets with incomplete information," *Journal of Economic theory*, 16, 167–207.

A Proofs

Proof of Proposition 1. The proof is straightforward. ■

Proof of Lemma 1. We prove Lemma 1 by establishing a set of intermediate lemmas. We begin by guessing that the incentive compatibility constraint (7) in program P1 is slack at the optimum and, thus, dropping it from the program. We will then verify that this is indeed the case.

The first lemma shows that the L-type transfers all of her cash flows to the buyers.

Lemma 3 *In any solution to* (P1), $F_L(x) = x \; \forall x$.

Proof. Suppose to the contrary that the solution to P1 consists of allocations $\{(F_\theta, p_\theta)\}$ with $F_L(x) < x$ for some $x \in [0, \bar{X}]$. Consider next the contracts $\{(F'_\theta, p'_\theta)\}$ where: (i) $F'_H(x) = F_H(x) \; \forall x$ and $p'_H = p_H + \frac{1-\mu_0}{\mu_0} \cdot \varepsilon$; and (ii) $F'_L(x) = x \; \forall x$ and $p'_L = p_L + E_L[X - F_L(X)] - \varepsilon$. For $\varepsilon > 0$ sufficiently small, with the new contracts $\{(F'_\theta, p'_\theta)\}$, the constraints (6), (8), and (9) are satisfied, but the objective of the program has increased, a contradiction. ■

The second lemma shows that at the optimum the incentive compatibility constraint of the L-type, given by (6), and the buyers' participation constraint, given by (8), bind.

Lemma 4 *In any solution to* (P1)*, the constraints* (6) *and* (8) *are satisfied with equality.*

Proof. Suppose not. If the constraint (8) were slack, then the objective of the program could be increased by raising p_H and p_L by a small amount $\varepsilon > 0$, which leaves the constraints (6) and (9) satisfied. If instead the constraint (6) were slack, then there are two possibilities. If also the constraint (9) is slack, then decreasing p_L by ε and increasing p_H by $\frac{1-\mu_0}{\mu_0} \cdot \varepsilon$ increases the objective while, for $\varepsilon > 0$ sufficiently small, still satisfying the constraints (6), (8) and (9), a contradiction. On the other hand, if the constraint (9) binds and thus $p_L = E_L[X]$, it must be that $F_H(x) < x$ for some x; otherwise, since the constraint (8) binds, the constraint (6) would have to be violated. But then, replacing contract (F_H, p_H) with contract (F'_H, p'_H), where $F'_H(x) = \min\{F_H(x) + \epsilon, x\} \; \forall x$ and $p'_H = E_H[F'_H(x)]$ and $\epsilon > 0$ is small, increases the objective of the program while still satisfying the constraints (6), (8) and (9). ■

Next, let us define $d^C : [0,1] \to [0, \bar{X}]$ as follows:

$$d^C(\mu_0) = \arg \max_{d \in [0, \bar{X}]} \int_0^{\bar{X}} \min\{d, x\} \cdot ((\mu_0 - \delta) \cdot g_H(x) + (1 - \mu_0) \cdot \delta \cdot g_L(x)) \cdot dx.$$

Also, define $d^S \in (0, \bar{X})$ as follows:

$$(1 - \delta) \cdot E_L[X] = E_H\left[\min\left\{d^S, X\right\}\right] - \delta \cdot E_L\left[\min\left\{d^S, X\right\}\right].$$

We can now prove our final lemma:

Lemma 5 $F_H(x) = \min\{d, x\}$ *where:*

$$d^E = \begin{cases} d^S & \text{if } \mu_0 \leq \tilde{\mu} \\ d^C(\mu_0) & \text{if } \mu_0 > \tilde{\mu} \end{cases}.$$

Proof. First, we show that $F_H(x) = \min\{d, x\}$ for all x and some $d \in [0, \bar{X}]$. Suppose to the contrary that F_H is not a debt contract, and let d' be such that $\mathbb{E}_H[F_H(X)] = \mathbb{E}_H[\min\{d', X\}]$. Since $F_H(\cdot)$ is monotonic, there exists $\widehat{x} \in [0, \bar{X}]$ such that $F_H(x) > d'$ if and only if $x > \widehat{x}$. By MLRP, it must then be that $\mathbb{E}_L[X - F_H(X)] > \mathbb{E}_L[X - \min\{d', X\}]$, since both securities have the same high valuation:

$$\mathbb{E}_L[F_H(X) - \min\{d', X\}] = \mathbb{E}_H\left[(F_H(X) - \min\{d', X\}) \cdot \frac{g_L(X)}{g_H(X)}\right]$$
$$< \mathbb{E}_H[F_H(X) - \min\{d', X\}] \cdot \frac{g_L(\widehat{x})}{g_H(\widehat{x})} = 0. \tag{23}$$

Thus, whereas the high-type is indifferent between contracts (F_H, p_H) and (F', p_H) with $F'(X) = \min\{d', X\}$, the low-type strictly prefers the equity contract to contract (F', p_H). Consider a deviation for a buyer to include contract (F'', p'') with $F''(X) = \min\{d'', X\}$, $p'' = \mathbb{E}_H[F''(X)] - \varepsilon$, and $d'' > d'$ in his menu. If d'' is close to d', then the low-type still prefers to accept the equity contract to contract (F'', p'') for any $\varepsilon \geq 0$. But, since $\mathbb{E}_H[F''(X)] > \mathbb{E}_H[F_H(X)]$, for ε small enough the Htype must prefer contract (F'', p'') to contract (F_H, p_H). Hence, such a deviation must be profitable.

Second, we determine the optimal debt level as follows. The derivative of the expression on the right-hand side of definition of $d^C(\mu_0)$ w.r.t. d is given by:

$$\Phi(d, \mu_0) = (\mu_0 - \delta) \cdot (1 - G_H(d)) + (1 - \mu_0) \cdot \delta \cdot (1 - G_L(d)),$$

and it satisfies the following properties: (i) $\Phi(0, \cdot) > 0$ for $\mu_0 > 0$, (ii) $\Phi(\cdot, 0) = 0$, (iii) $\Phi(\bar{X}, \cdot) = 0$, (iv) $\Phi_1 < 0 < \Phi_2$, and (v) there exists a unique $d \in [0, \bar{X})$ such that $\Phi(d, \mu_0) = 0$ if and only if $\mu_0 < \delta \cdot \lim_{x \to \bar{X}} \frac{\frac{g_H(x)}{g_L(x)} - 1}{\frac{g_H(x)}{g_L(x)} - \delta} \equiv \bar{\mu}$. Thus, $d^C(0) = 0$, $d^C(\cdot)$ is continuous and increasing on $[0, \bar{\mu}]$ and $d^C(\cdot) = \bar{X}$ for $\mu_0 \geq \bar{\mu}$. Finally, we have $d^C(0) < d^S$, since otherwise (6) would be slack. ■

The result stated in the proposition follows from Lemmas 3-5 together with the following verification that the constraint (7) is always slack. The latter holds if:

$$p_H + \delta \cdot E_H\left[X - \min\left\{d^E, X\right\}\right] \geq p_L = p_H + \delta \cdot E_L\left[X - \min\left\{d^E, X\right\}\right]$$

$$\Longleftrightarrow$$

$$E_H\left[X - \min\left\{d^E, X\right\}\right] \geq E_L\left[X - \min\left\{d^E, X\right\}\right].$$

The last inequality follows from MLRP, as $x - \min\left\{d^E, x\right\}$ is monotonically increasing in x. ■

Proof of Proposition 2. Note that on equilibrium path the menus posted at Stage 1 are not withdrawn at Stage 2, as this would yield a withdrawing buyer a sure payoff of $-c$ whereas he can guarantee a payoff of zero by posting a menu that contains the trivial contract only.

We next show that any equilibrium must consist of the allocations that solve program P1. To this end, consider an equilibrium with allocations $\{(F_\theta, p_\theta)\}$; that is, the θ-type transfers cash flows $F_\theta(x)$ to the buyers at $t = 2$ when $X = x$ for all x, and the buyers transfer p_θ to the seller at $t = 1$.

(i) In any equilibrium, each buyer must make zero expected profits. Suppose to the contrary that the buyers' aggregate profits are positive, and let $\{(F_\theta, p_\theta)\}$ denote the equilibrium allocations. Suppose that buyer i who were earning less than half of the aggregate profits were to deviate and add the contracts $\{(F_\theta, p_\theta + \varepsilon)\}$ to his menu. Clearly, the seller would pick these contracts instead of the equilibrium allocation $\{(F_\theta, p_\theta)\}$, independently of whether the other buyers withdraw their menus or not. Moreover, for ε small enough, such a deviation is profitable as buyer i effectively captures all of the aggregate expected profits, a contradiction.

(ii) It is also clear that the assumption that the equilibrium allocations satisfy the incentive compatibility constraints (6) and (7) is without loss of generality.

(iii) It must also be that $p_L \geq E_L[X]$. Suppose to the contrary that $p_L < E_L[X]$, and consider a buyer who were to deviate and add the following contract in his menu: (F', p') with $F'(X) = X$ and $p' = E_L[X] - \varepsilon$. For ε small, the low-type prefers this contract to his equilibrium allocation and, thus, would pick it up. Moreover, this contract makes strictly positive profits even if it is also picked up by the H-type. Thus, independently of whether other buyers withdraw their menus or not, such a deviation is profitable; a contradiction.

(iv) It must also be that the equilibrium allocations maximize H-type's payoff. Suppose to the contrary, and let $\left\{\left(F_\theta^{P1}, p_\theta^{P1}\right)\right\}$ denote the unique solution to program P1, which note cannot be offered on the proposed equilibrium menus. Suppose that these allocations are picked up from buyers i (and possibly j), and consider a deviation by buyer $k \neq i, j$ to post a menu that consists of the contracts $\left\{\left(F_\theta^{P1}, p_\theta^{P1} - \varepsilon\right)\right\}$ in addition to the trivial contract. For $\varepsilon > 0$ small enough, such a deviation attracts the H-type seller to contract $\left(F_H^{P1}, p_H^{P1} - \varepsilon\right)$, and the deviation is profitable whether or not the L-type is attracted to the contract $\left(F_L^{P1}, p_L^{P1} - \varepsilon\right)$, a contradiction.

Finally, we show that an equilibrium exists for c small enough. That the equilibrium allocations are unique will follow from the uniqueness of the solution to P1. Let buyers 1 and 2 offer the contracts $\left\{\left(F_\theta^{P1}, p_\theta^{P1}\right)\right\}$, whereas all the remaining buyers post the trivial contract. There are three types of deviations to consider.

(i) Consider a deviation that only attracts the L-type. Such a deviation clearly cannot be profitable as it would need to offer the L-type $p_L > E_L\{X\}$.

(ii) Consider deviation that attracts both types. Let $\{(F_\theta, p_\theta)\}$ denote the allocations of the seller when she accepts a contract from the deviating menu. For it to be profitable, $\{(F_\theta, p_\theta)\}$ must satisfy the constraints of P1. Additionally, in order to attract the H-type, it would have to offer her a higher payoff than the solution to P1, which is not possible.

(iii) Consider a deviation that only attracts the H-type. There are two cases to consider.

Case 1. Suppose that $\mu_0 \leq \tilde{\mu}$, so that at the solution to P1 is separating, i.e. $p_L^{P1} = E_L\{X\}$. A deviation that attracts only the H-type to some contract $\{(F_H, p_H)\}$ would have to satisfy the constraints:

$$p_H + \delta \cdot E_H\{X - F_H\} \geq E_L\{X\},$$

$$E_L\{X\} \geq p_H + \delta E_L\{X - F_H\},$$

$$p_H \leq E_H\{F_H\},$$

since the equilibrium menus do not make losses and are therefore not withdrawn. But note that these are exactly the constraints in program P1. Therefore, it is impossible that such a deviation yields the H-type a higher payoff than the equilibrium allocation, a contradiction.

Case 2. Suppose that $\mu_0 > \tilde{\mu}$, so that at the solution to P1 involves cross-subsidization, $p_L^{P1} > E_L\{X\}$. Consider a deviation by a buyer to attract the H-type to an allocation (F_H, p_H). If c is small enough (i.e., $c < \frac{1}{2} \cdot (1-\mu_0) \cdot (p_L^{P1} - E_L\{X\})$), buyers 1 and 2 will withdraw their menus as these are now loss-making. But then, the L-type must also be picking a non-trivial contract from the deviating menu, a contradiction. ∎

Proof of Proposition 2. Note that the only difference between P2 and P1 is given by the incentive compatibility constraint of the L-type seller as there is a δ discount in the retained cash flows in the incentive compatibility constraint of the L-type in P1, but no discount for such cash flows in P2, see constraint (6) vs. constraint (12). As a result, the proof is isomorphic to the proof of Lemma 1, with the different debt levels and transfers being solely driven by the absence of the discount on retained cash flows for the L-type. ∎

Proof of Proposition 3. Let $\{u_H, u_L\}$ denote the equilibrium allocations for each seller type that are implemented through cash flows and transfers $\{F_\theta, p_\theta\}$. Suppose now that the equilibrium can be implemented without cross-subsidizing contracts, i.e., $p_\theta = E_\theta[F_\theta]$, which must be the case if the equilibrium features no cross-subsidization across seller types. As in the exclusive-markets setting, it is straightforward to show that, in any equilibrium, buyers cannot expect to make positive profits, and that the L-type must issue all of her cash flows. With this, equilibrium payoffs should be given by $u_L = E_L[X]$ and $u_H = (1-\delta) \cdot E_H[F_H] + \delta \cdot E_H[X]$. In what follows, we find a profitable menu for the deviating buyer through an algorithm.

Let $\mathcal{M}_0^j$ be the starting menu of this algorithm. Begin with $d_0 \in (0, d^{NE}]$ and define:

$$\mathcal{M}_0^j = \{(\min\{d_0, X\}, E_{\mu_0}\{\min\{d_0, X\}\} - \epsilon), \{(F, E_L[F] - \epsilon)\} : \forall F \in \mathcal{F}\},$$

for $\epsilon > 0$ very small. There are two cases to consider:

1. If menu $\mathcal{M}_0^j$, when posted, attracts the H-type (and maybe the L-type) to the contract $(\min\{d_0, X\}, E_{\mu_0}\{\min\{d_0, X\}\} - \epsilon)$, or if it attracts the L-type to any contract other than this, then the buyer makes profits. The deviating menu is $\mathcal{M}_0^j$ and we stop.

2. If not, then the menu $\mathcal{M}_0^j$, when posted, must attract only the L-type to the contract $(\min\{d_0, X\}, E_{\mu_0}\{X\} - \epsilon)$. Note that not attracting any seller type to $\mathcal{M}_0^j$ is not possible, as the L-type gets payoff above her equilibrium payoff by accepting combined-contract $(X, E_{\mu_0}\{X\} + E_L[X - \min\{d_0, X\}] - 2\epsilon))$. But then, there must be another contract (F_H^0, p_H^0) in menu that remains active at Stage 2 such that:

$$p_H^0 - \delta E_H\left[F_H^0\right] \geq E_{\mu_0}[\min\{d_0, X\}] - \delta E_H[\min\{d_0, X\}].$$

It is straightforward to show that $p_H^0 > E_L[F_H]$.

With this, consider the second point of our algorithm defined as follows:

$$\mathcal{M}_1^j = \{(\min\{d_1, X\}, E_{\mu_0}\{\min\{d_1, X\}\} - \epsilon), \{(F, E_L[F] - \epsilon)\} : \forall F \in \mathcal{F}\},$$

where d_1 is such that:

$$p_H^0 + E_L\left[X - F_H^0\right] = E_{\mu_0}[\min\{d_1, X\}] + E_L[X - \min\{d_1, X\}].$$

Since the contract $\min\{d_0, X\}$ priced at average valuation minus ϵ attracted the L-type and since $p_H^0 > E_L[F_H^0]$, we must have $d_1 \in (0, d_0] \subset (0, d^{NE}]$.

Note that, the menus that are withdrawn after menu $\mathcal{M}_0^j$ is posted would continue to be withdrawn when menu $\mathcal{M}_1^j$ is posted. If no additional menu is withdrawn (perhaps because withdrawal is costly), then $\mathcal{M}_1^j$ is the deviating menu, as it only attracts the L-type to junior tranches. If not, we move to the second step of the algorithm with menu $\mathcal{M}_1^j$ defined by $d_1 \in (0, d^{NE}]$. Continuing in this manner, after at most $N-1$ iterations, we arrive at a menu:

$$\mathcal{M}^j = \{(\min\{d^*, X\}, E_{\mu_0}\{\min\{d^*, X\}\} - \epsilon), \{(F, E_L[F] - \epsilon)\} : \forall F \in \mathcal{F}\},$$

where $d^* \in (0, d^{NE}]$, such that both seller types are attracted to the contract:

$$(\min\{d^*, X\}, E_{\mu_0}\{\min\{d^*, X\}\} - \epsilon)$$

and, in addition, the L-type accepts some junior tranches. This is the deviating menu, and it is profitable to the buyer. ■

Proof of Proposition 4. Consider the following strategies:

- Buyers 1 and 2 post contract (F_S, p_S).
- The rest of the buyers price all contracts at low valuation: $\{(F, p) : F \in \mathcal{F}, p = E_L[F]\}$.
- Both types accepts contract (F_S, p_S) from buyers 1 or 2. In addition, the L-type accepts contract (F_J, p_J) from any of the remaining buyers.

We now prove that the above strategies constitute an equilibrium. In any SPE, the contracts posted on the first stage are not withdrawn on the second stage. Otherwise, there exists a buyer i with $\pi_i(\mathcal{M}_i|\mathcal{M}) = -c < 0$, who has a profitable deviation to $\mathcal{M}_{\text{i}} = (0, 0)$, i.e. to offering the trivial contract.

Thus, it is sufficient to rule out deviations by buyers on Stage 1. First, there does not exist a profitable deviation that attracts only the L-type. Such a deviation would need to pay the L-type seller above her valuation for the traded cash flows, which would incur losses for the deviating buyer. Thus, we only need to think of deviations that attract the H-type. There are two types of such deviations. First, there are deviations after which the H-type continues to accept contract (F_S, p_S) and, second, there are deviations after which she no longer accepts contract (F_S, p_S). Let us start with the latter.

Second, there does not exist a profitable deviation that attracts only the H-type. To see this, note that the menu of any buyer who posted contract (F_S, p_S) at stage 1 would only attract the L-typ and make losses and, thus, be withdrawn in stage 2. Consistent with this argument, any active menu that attracts the L-type to a contract priced above low valuation should be withdrawn, as the H-type is accepting a contract from the deviating menu. But if all such menus are withdrawn, the deviating buyer would also attract the L-type, a contradiction.

Third, there does not exist a profitable deviation that attracts both types. Suppose to the contrary that such a deviation existed and let $\{(F_\theta, p_\theta)\}$ be the contracts of the deviating buyer. Note that such a deviation must satisfy the constraints (12), (13) and (15) of program P2, since the L-typ has access to the menus of buyers $j \neq 1, 2$, which are not withdrawn at stage 2. But then, such a deviation cannot at the same time attract the H-type and be profitable for the buyers.

Finally, consider a deviation that attracts the H-type but that she still continues to issue contract (F_S, p_S). Such a deviation cannot only attract the H-type, since the L-type would mimic the H-type and issue the residual cash flows. Thus, suppose that the deviation attracts both types, and let $\{(F_\theta, p_\theta)\}$ be the contracts of the deviating buyer. But then again such contracts must satisfy the constraints (12), (13) and (15) of program P2, which therefore cannot attract the H-type and be profitable at the same time. ■

Proof of Proposition 6. Suppose not. First, assume that the H-type's payoff were below $u_H^{NE}(\mathcal{C}^*)$. Then there would be a profitable deviation for a buyer to enter and offer contract $(F_S, p_S - \varepsilon)$, which would attract the H-type and be profitable for $\varepsilon > 0$ small enough. Second, assume that the L-type's payoff were below $u_L^{NE}(\mathcal{C}^*)$ but the H-type's payoff is weakly above $u_H^{NE}(\mathcal{C}^*)$. We now construct a deviating menu using an algorithm analogous to that in the proof of Proposition 3.

Let $\mathcal{M}_0^j$ be the starting menu of this algorithm. Begin with $d_0 \in (0, d^{NE}]$ and define:

$$\mathcal{M}_0^j = \{(\min\{d_0, X\}, E_{\mu_0}\{\min\{d_0, X\}\} - \epsilon), \{(F, E_L[F] - \epsilon)\} : \forall F \in \mathcal{F}\},$$

for $\epsilon > 0$ very small. There are two cases to consider:

1. If menu $\mathcal{M}_0^j$, when posted, attracts the H-type (and maybe the L-type) to the contract $(\min\{d_0, X\}, E_{\mu_0}\{\min\{d_0, X\}\} - \epsilon)$, or if it attracts the L-type to any contract other than this, then the buyer makes profits. The deviating menu is $\mathcal{M}_0^j$ and we stop.

2. If not, then the menu $\mathcal{M}_0^j$, when posted, must attract only the L-type to the contract $(\min\{d_0, X\}, E_{\mu_0}\{X\} - \epsilon)$. Note that not attracting any seller type to $\mathcal{M}_0^j$ is not possible, as the L-type gets payoff above her equilibrium payoff by accepting combined-contract $(X, E_{\mu_0}\{X\} + E_L[X - \min\{d_0, X\}] - 2\epsilon))$. But then, there must be another contract (F_H^0, p_H^0) in menu that remains active at Stage 2 such that:

$$p_H^0 - \delta E_H[F_H^0] \geq E_{\mu_0}[\min\{d_0, X\}] - \delta E_H[\min\{d_0, X\}].$$

It is straightforward to show that $p_H^0 > E_L[F_H]$.

With this, consider the second point of our algorithm defined as follows:

$$\mathcal{M}_1^j = \{(\min\{d_1, X\}, E_{\mu_0}\{\min\{d_1, X\}\} - \epsilon), \{(F, E_L[F] - \epsilon)\} : \forall F \in \mathcal{F}\},$$

where d_1 is such that:

$$p_H^0 + E_L[X - F_H^0] = E_{\mu_0}[\min\{d_1, X\}] + E_L[X - \min\{d_1, X\}].$$

Since the contract $\min\{d_0, X\}$ priced at average valuation minus ϵ attracted the L-typ and since $p_H^0 > E_L[F_H^0]$, we must have $d_1 \in (0, d_0] \subset (0, d^*]$.

Note that, the menus that are withdrawn after menu $\mathcal{M}_0^j$ is posted would continue to be withdrawn when menu $\mathcal{M}_1^j$ is posted. If no additional menu is withdrawn (perhaps because withdrawal is costly), then $\mathcal{M}_1^j$ is the deviating menu, as it only attracts the L-typ to junior tranches. If not, we move to the second step of the algorithm with menu $\mathcal{M}_1^j$ defined by $d_1 \in (0, d^{NE}]$. Continuing in this manner, after at most $N - 1$ iterations, we arrive at a menu:

$$\mathcal{M}^j = \{(\min\{d^*, X\}, E_{\mu_0}\{\min\{d^*, X\}\} - \epsilon), \{(F, E_L[F] - \epsilon)\} : \forall F \in \mathcal{F}\},$$

where $d^* \in (0, d^{NE}]$, such that both seller types are attracted to the contract

$$(\min\{d^*, X\}, E_{\mu_0}\{\min\{d^*, X\}\} - \epsilon)$$

and, in addition, the L-type accepts some junior tranches. This is the deviating menu, and it is profitable to the buyer. ■

Proof of Proposition 7. Under both exclusive and non-exclusive market structures, the buyers break even. So the entire trading surplus accrues to the seller. In what follows, we suppose that non-exclusive markets implement the allocations of contract $\mathcal{C}^*$ on equilibrium. We later extend the result to all possible equilibria of non-exclusive markets.

Given buyers prior belief μ_0, when markets are non-exclusive, the expected trading surplus (i.e., the expected welfare of the seller) is given by:

$$W^{NE}(\mu_0) \equiv \mathbb{E}_{\mu_0}\{X\} - \mu_0 \cdot (1-\delta) \cdot \mathbb{E}_H\left\{X - \min\left\{d^{NE}(\mu_0), X\right\}\right\}, \tag{24}$$

where $d^{NE}(\mu_0) < \bar{X}$ if and only if $\mu_0 < \bar{\mu} \equiv \frac{\lim_{x\to\bar{X}} \delta \cdot \frac{g_H(x)}{g_L(x)} - 1}{\lim_{x\to\bar{X}} \frac{g_H(x)}{g_L(x)} - 1} \in (0,1)$, in which case $d^{NE}(\mu_0)$ is defined by the first order condition:

$$1 - \mu_0 \cdot G_H\left(d^{NE}\right) - (1-\mu_0) \cdot G_L\left(d^{NE}\right) = \delta \cdot \left(1 - G_H\left(d^{NE}\right)\right). \tag{25}$$

Thus, $d^{NE}(\cdot)$ is continuous and increasing, with the property that $d^{NE}(0) > 0$ and it goes to $\bar{X}$ as μ_0 goes to $\bar{\mu}$.

Instead, the expected trading surplus with exclusive markets is given by:

$$W^{E}(\mu_0) \equiv \mathbb{E}_{\mu_0}\{X\} - \mu_0 \cdot (1-\delta) \cdot \mathbb{E}_H\left\{X - \min\left\{d^{E}, X\right\}\right\}, \tag{26}$$

where $d^E \in \left(0, \bar{X}\right)$ is independent of μ_0.

We now show that there exists a $\underline{\mu} \in (0,1)$ and $\bar{\mu} \in (\underline{\mu}, 1]$ such that the expected trading surplus with non-exclusive markets is greater than with exclusive markets if $\mu_0 \in \left(\underline{\mu}, \bar{\mu}\right)$, the two coincide if $\mu_0 \geq \bar{\mu}$, and the expected trading surplus with exclusive markets is greater if $\mu_0 \in [0, \underline{\mu})$. To this end, note that:

$$W^{NE}(\mu_0) - W^{E}(\mu_0) = \mu_0 \cdot (1-\delta) \cdot \left(\mathbb{E}_H\left\{\min\left\{d^{NE}(\mu_0), X\right\}\right\} - \mathbb{E}_H\left\{\min\left\{d^{E}(\mu_0), X\right\}\right\}\right), \tag{27}$$

which is continuous and increasing in μ_0 on $[0, \bar{X}]$. Next, in non-exclusive markets, when μ_0 goes to zero, the H-type issues the security $d^{NE}(\mu_0)$ essentially at low valuation. However, in exclusive markets, she strictly prefers not to issue any additional cash flows (i.e. more than d^E) at low valuation. By continuity, this implies that $d^{NE}(\mu_0) < d^E(\mu_0)$ and, thus, $W^{NE}(\mu_0) < W^{E}(\mu_0)$ if μ_0 is sufficiently small. Finally, as in both market structures both seller types issue a full claim to their cash flows when buyers' beliefs are sufficiently high, the result follows.

Now consider other equilibria in non-exclusive markets. As shown in Proposition 6, it must be that in any equilibrium both the H- and the L- type obtain a payoff as high as one obtained in the *star* equilibrium. Thus, the welfare analysis conducted for the lower bound clearly extends to other possible equilibria in non-exclusive markets. ■

Proof of Proposition 8. This result follows from Proposition 6 and the observation that any equilibrium allocation with non-exclusive markets satisfies the constraints (6) and (8) with $F_L(x) = x \ \forall x$ and $p_L = u_L^{NE}$. Since for any μ_0 we have $u_L^{NE} \geq u_L^{NE}(\mathcal{C}^*) \geq u_L^E$, it follows that $u_H^{NE} \leq u_L^E$ and thus $0 \leq u_H^{NE} - u_L^{NE} \leq u_H^E - u_L^E$, with strict inequalities whenever $\mu_0 < \bar{\mu}$. ■

B A microfoundation for feasibility constraints

Suppose that the seller is endowed with an asset whose cash flows are defined as infinitesimal debt tranches indexed by $a \in [0, \bar{X}]$, and whose payoffs are given by:

$$a(x) = \begin{cases} 0 & \text{if } x < a \\ dx & \text{if } x \geq a \end{cases}, \tag{28}$$

and where trading the entire asset is equivalent to trading all of its cash flows: $X = \int_{a\in[0,\bar{X}]} a(x)\, da$. In this context, security design allows the seller to sell any bundle of asset cash flows that she desires, e.g., $F(X) = \int_{a\in A} a(X)da$ for some measurable subset A in $[0, \bar{X}]$.

Our feasibility conditions hold naturally as the seller is simply selling bundles of cash flows on spot markets where transactions are verified. In particular,

1. (LL) If the seller transfers a bundle of cash flows to a buyer, she cannot transfer those cash flows to another buyer.

2. (WM) The seller transfers a bundle of cash flows, $F(X) \equiv \int_{a\in A} a(X)\, da$ for some $A \subset \left[0, \bar{X}\right]$, where F could be any weakly increasing function of X and where her remaining payoff is weakly increasing in X.

One way to microfound that the seller can only transfer asset cash flows, as opposed to writing arbitrary contracts upon them, is that neither the seller nor the buyer have commitment to fulfill promises at $t = 1$. As a result, the only possible contracts are transfers of assets on the spot at $t = 0$.

Finally, the assumption of non-exclusivity is simply equivalent to a notion of market anonymity or opacity: each buyer observes the security/cash flows that the seller transfers to him, but she cannot observe other cash flows/securities that the seller transfers to other buyers.