

# The Good, the Bad and the Complex: Product Design with Imperfect Information

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## Abstract

We study the joint determination of product quality and complexity in a rational setting. We introduce a novel notion of complexity, which affects how costly it is for an agent to acquire information about product quality. In our model, an agent can accept or reject a product proposed by a designer, who can affect the quality and the complexity of the product. Examples include banks that design financial products that they offer to retail investors, or policymakers who propose policies for approval by voters. We find that complexity is not necessarily a feature of low quality products. While an increase in alignment between the agent and the designer leads to more complex but better quality products, higher product demand or lower competition among designers leads to more complex and lower quality products. Our findings produce novel empirical implications on the relationship between quality and complexity, which we relate to evidence within the context of financial products and regulatory policies.

**Keywords:** complexity; information acquisition; signaling; regulation; financial products.

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# 1 Introduction

Rapidly increasing complexity has concerned policymakers and financial market participants alike. Complex financial products and regulations have been documented to contain jargon-rich descriptions, complicated explanations (C  lerier and Vall  e, 2017; Ghent, Torous, and Valkanov, 2019) or vague provisions (McMillan, 2014; Davis, 2017).<sup>1</sup> Products or policies with these features are difficult to understand and evaluate, leading to the concern that complexity may foster the production and proliferation of lower quality policies and products.

Central to the above concern is understanding how changes in *complexity* relate to changes in the *quality* of policies/products. A well-established literature has focused on the purposeful obfuscation of bad product attributes by firms that are faced with unsophisticated consumers (see Spiegler (2016) for a survey). In this view, unsophistication is essential, as otherwise consumers would eventually stop demanding products whose attributes they do not understand well (Milgrom, 1981). The recently documented proliferation of complex products and policies nonetheless begs the question of whether other factors beyond unsophistication may be at work, and if so, of whether complexity necessarily brings about bad quality policies/products. The goal of this paper is to shed light on these issues.

We develop a novel notion of complexity to study the joint endogenous determination of quality and complexity in a rational setting. Our starting point is that agents often receive proposals of uncertain quality which they have to evaluate before deciding whether to accept. In many situations, such proposals must contain all the information pertaining to them that is needed to make a proper evaluation. Agents have to process this information to make their acceptance decision. Some examples include financial products proposed to retail investors, or policy reforms proposed to voters. Proposal designers, in turn, not only affect the quality of their proposal, but they can also influence how costly it is for agents to process information by complexifying or simplifying their proposal. For example, a financial product can be complexified by adding unnecessary contingencies and complicated jargon, while a policy reform can be complexified by not putting effort to be concise and clear about its applicability. Motivated by this, we model complexity as a product feature that influences the agent’s cost of acquiring information about its quality. There are two distinguishing features of our approach: first, an agent can only acquire *imperfect* information about product quality; and second, the designer can *influence*, but not fully control, the agent’s information acquisition process. These features will matter both conceptually and in terms of applied implications.

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<sup>1</sup>E.g., an average sentence in the Basel Committee for Banking Supervision texts consists of 25.7 words, with the second sentence of the very first document spanning over 72 words. This is significantly longer than the average 21 words in a sentence of the British National Corpus, a collection of texts in modern British English. This analysis was done by the Swiss newspaper *Neue Z  rcher Zeitung* (Kolly and M  ller, 2017).

Our framework rationalizes the proliferation of complex products and policies by uncovering novel drivers of complexity. First, we break the link between bad product attributes and complexity. We show that when the information that an agent acquires is imperfect, the designer of a good product may choose to complexify it in order to reduce the agent’s reliance on noisy information. Alternatively, the designer of a bad product may choose to simplify it in order to gamble on the noise of the acquired information. Second, we show that complexity can result from high product demand by agents, low competition among designers, or high alignment (e.g., little conflicts of interest) between the agent and the designer. Such a rise in complexity, however, is only accompanied by worsening quality when driven by high demand or low competition. These results lead to new testable implications on the relationship between quality and complexity, which we relate to empirical findings.

We consider a setting where an agent (e.g., investor, median-voter) demands a product that is supplied by a designer. First, the designer takes private actions to affect the product’s quality and complexity, and then proposes the product to the agent. Next, the agent acquires information about the product’s quality and decides whether to accept the product or take her outside option. Whereas a product’s quality (good or bad) determines the direct payoff to the agent, a product’s complexity affects her costs of processing information about the product’s quality. While the objective of the designer is to get the agent to accept his product, the agent only wants to accept a good product. For example, a bank wants to convince a retail investor to accept a savings account, or a policymaker wants to get voter support for his policy proposal. To make the problem interesting, we suppose that the designer is misaligned with the agent, as he receives a higher payoff from having a bad product accepted. Such misalignment aims to capture conflicts of interest stemming from good products being more costly to produce, career concerns, or ideological preferences.

The designer in our setting takes actions to separately affect the product’s quality and complexity. For example, while the quality of a financial product can be interpreted as the net present value (NPV) that it generates to an investor, its complexity refers to features such as the number of contingencies which deliver that NPV or whether payments are linked to indices that an investor is unlikely to be familiar with. Similarly, while the quality of a policy can be interpreted as its effectiveness in addressing a particular inefficiency, the policy’s complexity refers to its length, clarity, and use of complicated or vague terminology.<sup>2</sup> By studying both notions separately, we gain a better understanding of the incentives to produce good/bad quality versus complex/simple products.

In practice, an agent’s ability to learn about a product depends on the product’s inherent

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<sup>2</sup>This strategic decision of policymakers is discussed in the literature on strategic ambiguity or noise by politicians (Alesina and Cukierman, 1990; Aragonés and Postlewaite, 2002; Espinosa and Ray, 2018).

complexity (e.g., a zero-coupon bond versus an asset-backed security, a flat rate tax versus tiered taxation with activity-specific exemptions), on her own ability to process relevant information (e.g., her opportunity cost of time, education), and on the actions towards simplification or complexification taken by the designer described above. For instance, even though a policymaker may take actions to simplify a proposed regulation, it may be costly for the median voter to learn about it if it addresses a topic that she finds complicated. We capture this by supposing that the agent’s cost of processing information depends on both the designer’s choice to complexify or simplify the product and on some component outside of the designer’s control. Importantly, we suppose that it is prohibitively costly for the agent to acquire perfect information about the product’s quality.<sup>3</sup>

A surprising result, at first, is that the designer of a good product may find it optimal to complexify it, or that the designer of a bad product may find it optimal to simplify it. Intuitively, when the agent would accept the product in the absence of new information –e.g., when her prior belief about quality being good or her demand for the product are sufficiently high,- the designer has an incentive to complexify the product to discourage the agent from acquiring noisy information. For example, a good policymaker who has enough support to pass a tax reform may not want to incur the risk of having the median voter reading a proposal that she could misinterpret. Analogously, when the agent would reject the product in the absence of new information, the designer has an incentive to simplify the product to encourage noisy information acquisition. For example, a bad policy maker with no support for tax reform can only obtain such support if the median-voter engages in reading and misinterprets a proposal.

When choosing his product quality, the designer anticipates the resulting probability with which his product will be accepted by the agent. He then faces a trade-off between producing a good product, which has a higher probability of acceptance, and producing a bad product, which has a higher payoff conditional on acceptance. We show that, when the agent’s prior belief about the product’s quality is sufficiently low, the designer obtains a higher payoff from producing good products. Conversely, when the agent’s prior belief is sufficiently high, the designer prefers to produce bad products. But, of course, the agent’s prior belief must be consistent with the equilibrium supply of good versus bad products. Indeed, we show that there is a unique equilibrium with positive trade, and in it the designer produces good products with interior probability.

We next explore how the equilibrium quality-complexity relationship changes with various features of the economic environment. First, we consider the effect of a decrease in the agent’s

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<sup>3</sup>As we show in Section 2.1, when product simplification ensures that the agent can acquire perfect information at no cost, there is a unique equilibrium where only good products are designed, and they are simplified and accepted with probability one.

relative outside option, which can be interpreted as an increase in the agent’s demand for the product. We find that this leads to lower product quality and higher product complexity. Intuitively, as the agent’s relative outside option falls, she is more likely to disregard information and have a looser acceptance strategy. This in turn encourages the designer to produce worse quality, more complex products. Second, we consider a reduction in the conflict of interest between the designer and the agent. We find that this leads to both higher product quality and higher product complexity. Intuitively, higher alignment between the designer and the agent increases the agent’s trust in the designer since product quality does indeed increase, which loosens the agent’s acceptance strategy. This in turn encourages the designer to produce more complex products. We show therefore that the relationship between quality and complexity depends crucially on the underlying drivers of heterogeneity.

In practice, not all designers may be misaligned with the agent. To address this, we extend our analysis by supposing that the agent may meet either an aligned or a misaligned designer. Surprisingly, the introduction of aligned designers only affects equilibrium outcomes when the probability of meeting one is sufficiently high, in which case both product quality and complexity increase. This is consistent with our previous finding on the effect of a reduction in conflicts of interest. Initially, as aligned designers enter, misaligned designers respond by producing bad products with higher probability, keeping average quality and complexity of products unchanged — until the misaligned designers produce only bad products.

Finally, we extend the analysis to study the effect of competition among designers. We consider a sequential search setting where, if the agent rejects a given product, she meets another designer with some probability, capturing the presence of search frictions. We show that as search frictions decrease, which we interpret as competition among designers intensifying, product quality increases and complexity decreases. This result is in contrast to the literature on obfuscation or price complexity, which typically finds that competition leads to more obfuscation.<sup>4</sup> In that literature, by obfuscating, firms reduce the consumer’s ability to uncover the attributes of competing products by effectively increasing the consumer’s search costs. Obfuscation in those settings counterbalances higher competition: the consumer searches fewer products, competition effectively declines, benefiting firm profits. Instead, in our model, the designer uses complexification to influence the information that an agent extracts about his own product. As a result, more intense competition incentivizes each designer to supply products that are better for the agent: those that are good and simple.

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<sup>4</sup>See, for instance, [Spiegler \(2006\)](#); [Ellison and Ellison \(2009\)](#); [Carlin \(2009\)](#); [Ellison and Wolitzky \(2012\)](#). Some papers in this literature also highlight that obfuscation gives rise to price dispersion, which in turn allows firms to price-discriminate among different consumer-types (e.g., fast versus slow searchers, sophisticated versus unsophisticated agents), a force that is present even in the absence of competition ([Salop, 1977](#)). This mechanism is not present in our model, where there is one type of rational agent.

We relate our model’s implications to two leading applied settings in which rising complexity has been at the forefront of policy debates: financial products and regulatory policy. For financial products, our model suggests that the proliferation of worse and more complex structured products documented by [C  lerier and Vall  e \(2017\)](#) could have been an optimal response of product designers to an increasing demand for relatively safe financial assets combined with an increasing trust in financial advisors. Within the context of regulation, our model’s predictions provide an additional channel for the evidence presented by [Gratton et al. \(2019\)](#) on the worsening quality and increasing complexity of laws proposed by Italian politicians in periods of low bureaucratic effectiveness (i.e., low outside options for policymakers, or equivalently, costly *status quo*).

Our findings complement the growing literature that examines the incentives of firms to shroud certain product attributes from agents ([Gabaix and Laibson, 2006](#); [Auster and Pavoni, 2018](#)) or to increase the opacity of their products in order to draw unsophisticated investors into the market ([Pagano and Volpin, 2012](#)). A crucial ingredient of these models is that there is a fraction of unsophisticated agents who make no inferences from the fact that they do not observe a certain product attribute. Complexity then allows sellers to extract rents from unsophisticated agents. This mechanism is not present in our setting, where there is a single type of rational agent.

Our model relates to the literature on strategic information transmission in games ([Milgrom, 1981](#); [Grossman, 1981](#); [Crawford and Sobel, 1982](#); [Kartik, 2009](#)) and the recent papers on the value of ignorance or opacity in incentive provision schemes ([Brocas and Carrillo, 2007](#); [Boleslavsky and Cotton, 2015](#); [Ederer, Holden, and Meyer, 2018](#)). Our contribution to this literature is the joint determination of product quality and complexity, which leads to previously unexamined feedback effects: the designer’s complexification strategy affects the quality of products produced in equilibrium, which in turn influences the designer’s incentives to complexify in the first place. Moreover, as in [Dewatripont and Tirole \(2005\)](#), information transmission in our model is imperfect: the designer tries to influence the information received by the agent, but he cannot control it fully, as the agent’s ability to process information depends also on external factors.

Our approach is influenced by the literature on costly information processing or rational inattention ([Sims, 2003](#); [Aragones, Gilboa, Postlewaite, and Schmeidler, 2005](#); [Wiederholt, 2010](#)), since by complexifying the designer makes it more costly for the agent to extract information, as in [Perez-Richet and Prady \(2011\)](#). Through this lens, our model also relates to [Roesler and Szentes \(2017\)](#), who study buyer-optimal learning, but where the seller cannot affect the buyers’ learning process, and to [Oehmke and Zawadowski \(2019\)](#), who study sellers’ incentives to complexify, but where complex products give more value to buyers. Our

approach, however, adds the aforementioned feedback effects generated by the joint determination of quality and complexity.

That more information may not always be desired has been pointed out in an early work by [Hirshleifer \(1978\)](#) and more recently by [Dang, Gorton, and Holmström \(2012\)](#) in the context of financial markets. This idea is also present in the literature on Bayesian persuasion ([Rayo and Segal, 2010](#); [Kamenica and Gentzkow, 2011](#)) and information design more broadly ([Bergemann and Morris, 2016](#); [Taneva, 2019](#)). The reason is that, from an ex-ante perspective, there may be benefits from committing to transmit imperfect information (e.g., through the design of noisy information structures or of assets that deter information acquisition) to allow worse types/states to be pooled with good ones. This is in contrast to our setting, where the decision to complexify is effectively undertaken ex-post, since the designer knows his product type. The gains from complexification in our setting are conceptually different, as “cross-subsidization” is not a driver of our results. In our model, the good (bad) product designer may complexify (simplify) in order to reduce (increase) the agent’s reliance on noisy information.<sup>5</sup>

Finally, our paper relates to the literature in industrial organization that studies pricing and marketing strategies jointly ([Saak, 2006](#); [Anand and Shachar, 2009](#); [Bar-Isaac, Caruana, and Cuñat, 2010](#)). In this literature, informative marketing strategies affect the dispersion of consumers’ valuations and, thus, in the language of [Johnson and Myatt \(2006\)](#), rotate a firm’s demand curve, which may increase profits when choosing the appropriate pricing strategy. In contrast, in our setting, the product designer faces one agent and he knows her valuation of the product. As the agent acquires imperfect information, however, the designer is exposed to risk, which is key for our results. Furthermore, to the best of our knowledge, our findings on the relation between quality and complexity are new to this literature.

The rest of the paper is organized as follows. In [Section 2](#), we setup our baseline model. In [Sections 3](#) and [4](#), we present our main results. We consider several robustness exercises in [Section 5](#), and we conclude in [Section 6](#). All proofs are relegated to the Appendix.

## 2 The Model

We consider the following interaction between a consumer and a product designer. The consumer needs a product, which only the designer can produce. The designer privately takes two actions  $\{y, \kappa\}$ , where  $y \in \{\mathbf{Good}, \mathbf{Bad}\}$  affects the product’s quality and  $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$  affects the product’s complexity. The designer then proposes the product to the consumer, who acquires costly information about the product’s quality and decides whether to accept it

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<sup>5</sup>Furthermore, our results would not arise if the designer could choose to transmit perfect information, as is typically the case in information design/Bayesian persuasion settings.



( $a = 1$ ) or take an outside option ( $a = 0$ ).

*A Product's Quality.* A product's quality,  $y$ , determines the agents' payoffs. The payoff to the consumer from accepting a product with quality  $y$  (which we refer to as a  $y$ -product) is  $w(y)$ , and her outside option if no product is accepted is  $w_0$ . The designer receives payoff  $v(y)$  from having a  $y$ -product accepted, and zero otherwise. We make the following assumptions:

**Assumption 1** *The payoffs satisfy the following properties:*

1.  $w(G) > w_0 > w(B)$ ,  $w_0 \geq 0$ .
2.  $v(B) > v(G) > 0$ .

The first assumption states that the consumer wants to accept a  $G$ -product but reject a  $B$ -product, making information about product quality relevant for the consumer's acceptance decision. The second assumption states that the designer prefers to have a  $B$ -product accepted, *misaligning* the designer's objective with that of the consumer.<sup>6</sup> Since in practice not all designers may be misaligned with the consumer, we extend our analysis and introduce aligned designers in Section 4.3. As we show in Section 5.3, for some applications our payoff structure can be rationalized by introducing prices and costs of production, whereby  $G$ -products are costlier to produce.

*A Product's Complexity.* A product's complexity, which we denote by  $\chi \in X \subset \mathbb{R}$ , determines the consumer's cost of acquiring information about the product's quality. We suppose that it depends on two components,  $\chi = \chi(\eta, \kappa)$ . The first component, denoted by  $\eta \in \mathbb{R}$ , captures the product's natural (inherent) level of complexity to the consumer, which is random and has an associated cdf  $H$ . The second component is the designer's action,  $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$ . The consumer observes  $\chi$ , i.e., she understands how costly it is for her to acquire information about the product's quality, but she does not observe  $\eta$  or  $\kappa$ , i.e., she does not know whether this is due to nature or the designer's action.<sup>7</sup> Let  $F(\cdot|\kappa)$  denote the cdf of  $\chi$  conditional on  $\kappa$ , which is induced by the cdf  $H$ . We assume that it has an associated pdf  $f(\cdot|\kappa)$  which has full support on  $X$  and satisfies MLRP: that is,  $\frac{f(\chi|\bar{\kappa})}{f(\chi|\underline{\kappa})}$  is increasing in  $\chi$ . Thus, we say that the

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<sup>6</sup>In the financial products industry, misalignment may arise due to financial advisors receiving higher fees for selling products that are not necessarily the best fit for their clients (i.e., fixed versus adjustable-rate mortgages). In the policy sphere, misalignment of policymakers vis-à-vis the public may arise due to ideological differences, lobbying, or career concerns.

<sup>7</sup>The imperfect link between the action  $\kappa$  and the product's complexity  $\chi$  allows us to obtain a unique equilibrium, which facilitates rich comparative statics, by ruling out equilibrium multiplicity arising from the freedom in specifying off-equilibrium beliefs (see [Matthews and Mirman \(1983\)](#) for a related modeling approach). Furthermore, it has the natural interpretation that there are features of the environment unknown to the consumer (e.g. a product's natural complexity) or the designer (e.g., the consumer's opportunity cost of time), that affect the consumer's cost of information acquisition.



designer *complexifies* the product when he increases the product's expected complexity, i.e., when he chooses  $\kappa = \bar{\kappa}$ . Otherwise, we say that the designer *simplifies* the product.

The consumer's information acquisition technology is as follows. After the designer proposes product  $(y, \kappa)$ , the consumer observes the complexity of the product,  $\chi$ , and acquires a signal  $S \in \{b, g\}$  about the product's quality with noise  $z$ , where:

$$z \equiv \mathbb{P}(S = b|y = G) = \mathbb{P}(S = g|y = B) \in \left[0, \frac{1}{2}\right]. \quad (1)$$

The consumer can reduce the noise of the signal by exerting effort with associated cost  $C(z, \chi)$ , which is weakly decreasing in noise,  $z$ , and weakly increasing in complexity,  $\chi$ , with  $C(\frac{1}{2}, \cdot) = 0$ . As a result, it is more costly for the consumer to acquire information about products that are either naturally more complex (high  $\eta$ ) or that have been purposefully complexified by the designer ( $\kappa = \bar{\kappa}$ ). Finally, we assume that acquiring a perfectly informative signal is prohibitively costly for the consumer:  $C(0, \chi(\eta, \kappa)) > \max\{w(G) - w_0, w_0 - w(B)\}$  with probability one for  $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$ .

*The Consumer's Problem.* The consumer's problem can be expressed in two steps, backwards. Given her information set, as summarized by the posterior belief  $\mu(s, z, \chi) \equiv \mathbb{P}(y = G|s, z, \chi)$ , the consumer makes an optimal acceptance decision:

$$W(s, z, \chi) \equiv \max_{a \in \{0,1\}} a [\mu(s, z, \chi) w(G) + (1 - \mu(s, z, \chi)) w(B)] + (1 - a) w_0. \quad (2)$$

Next, in anticipation of her optimal acceptance decision and given her interim belief  $\mu(\chi) \equiv \mathbb{P}(y = G|\chi)$ , which incorporates the potential information contained in the product's complexity,  $\chi$ , the consumer makes an optimal information acquisition decision:

$$\max_{z \in [0, \frac{1}{2}]} \sum_{s \in \{b, g\}} \mathbb{P}(S = s|z, \chi) W(s, z, \chi) - C(z, \chi), \quad (3)$$

where  $\mathbb{P}(S = s|z, \chi) = \mathbb{P}(S = s|y = G)\mu(\chi) + \mathbb{P}(S = s|y = B)(1 - \mu(\chi))$  and where  $\mathbb{P}(S = s|y)$  is given by (1). We denote the consumer's strategy by  $\{z(\chi), a(s, \chi)\}_{s, \chi}$ .<sup>8</sup>

*The Designer's Problem.* The designer's expected payoff is given by:

$$V(y, \kappa) \equiv \mathbb{P}(a = 1|y, \kappa) \cdot v(y) \quad (4)$$

where  $\mathbb{P}(a = 1|y, \kappa)$  denotes the probability that product  $\{y, \kappa\}$  is accepted by the consumer. The designer chooses  $y \in \{G, B\}$  and  $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$  to maximize (4). We denote the designer's

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<sup>8</sup>As it will become clear soon, focusing on pure strategies for the consumer is without loss of generality.

strategy by  $\{m, \sigma_G, \sigma_B\}$ , where  $m = \mathbb{P}(y = G)$  is the probability that the designer chooses a  $G$ -product and  $\sigma_y = \mathbb{P}(\kappa = \bar{\kappa}|y)$  is the probability that he chooses to complexify a  $y$ -product.

*Equilibrium Concept.* We use Perfect Bayesian Equilibrium (PBE) as our equilibrium concept. This has the following implications. First, given her belief, the consumer's strategy must maximize her expected payoff (*Consumer Optimality*). Second, the designer's strategy must maximize his expected payoff, given the consumer's strategy (*Designer Optimality*). Finally, the consumer's belief must be consistent with the designer's strategy and updated using Bayes' rule when possible (*Belief Consistency*).

## 2.1 Benchmark with Perfect Information

Before we proceed to equilibrium analysis, it is useful to establish a benchmark against which our results can be compared. To highlight the role of imperfect information, we consider a benchmark where (only) by simplifying the product the designer is able to reduce the cost of information acquisition of the consumer to zero. The following proposition states that in this scenario the efficient outcome is attained.

**Proposition 1** *Suppose that the consumer's cost of information acquisition is zero with probability one if and only if the designer chooses  $\kappa = \underline{\kappa}$ .<sup>9</sup> Then, in equilibrium, only the  $G$ -product is produced, it is always simplified and accepted with probability one.*

Intuitively, the designer of a  $G$ -product never wants to expose himself to the noise of imperfect information; thus, he chooses to simplify his product by choosing  $\underline{\kappa}$ .<sup>10</sup> As a result, the designer of a  $B$ -product cannot exploit the noise of imperfect information to get his product accepted, since the consumer rationally infers that the designer has chosen a  $B$ -product if she faces a positive cost of information acquisition, and thus rejects all products that are costly to understand. Therefore, in equilibrium, only  $G$ -products are produced and they are accepted with probability one, i.e., the efficient outcome is attained.

It follows that our results will be driven by the fact that the designer cannot ensure that the consumer acquires perfect information, i.e., the consumer's information set is imperfect.

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<sup>9</sup>Formally, we assume that  $C(0, \chi(\eta, \underline{\kappa})) = 0$  with probability one, but we maintain that  $C(0, \chi(\eta, \bar{\kappa})) > \max\{w(G) - w_0, w_0 - w(B)\}$  with probability one.

<sup>10</sup>We assume that if the consumer's cost of information acquisition is zero, then she can effectively observe the product quality.

### 3 Equilibrium

In this section, we characterize the equilibria of our game. First, we consider the consumer's optimal strategy, given her belief about the product proposed by the designer (Section 3.1). Second, we analyze the designer's strategy: his optimal choice of complexification (Section 3.2) and of quality (Section 3.3), given the consumer's acceptance strategy. Finally, we impose belief consistency to obtain the model's equilibria (Section 3.4).

By inspection, it is immediate that there is always a trivial equilibrium with zero trade in which (i) the consumer correctly believes that the designer has chosen a  $B$ -product; thus she acquires no information and rejects it with probability one, and (ii) the designer indeed chooses a  $B$ -product with probability one as he is indifferent to producing  $G$ - versus a  $B$ -products (both yield a zero expected payoff). In what follows, we focus on the more interesting equilibria with positive trade, where the designer chooses a  $G$ -product with positive probability.

In the main analysis, in order to facilitate analytical tractability, we make the following assumption on the cost of information acquisition.

**Assumption 2** *The consumer's information cost is given by:*

$$C(z, \chi) = \begin{cases} 0 & \text{if } z \geq \chi \\ \bar{C} & \text{if } z < \chi \end{cases} \quad (5)$$

where  $\bar{C} > \max\{w(G) - w_0, w_0 - w(B)\}$  and  $X = [0, \frac{1}{2}]$ .

This information cost implies that it is free for the consumer to reduce the noise of the signal to  $\chi$ , but prohibitively costly to reduce it further. Though this formulation is very convenient for obtaining sharp analytical results, we will extend our results to a broader class of information costs in Section 5.

#### 3.1 The Consumer's Strategy

Observe that Assumption 2 has the immediate implication that the consumer's optimal information acquisition strategy that solves problem (3) is:

$$z(\chi) = \chi, \quad (6)$$

and the consumer never finds it optimal to incur the cost  $\bar{C}$ . Thus, when the product is more complex, the consumer optimally acquires a noisier signal. Since the consumer sets  $z(\chi) = \chi \forall \chi$ , for brevity of notation we will write the posterior belief as:  $\mu(s, \chi) \equiv \mu(s, \chi, \chi)$ .

Given this, it follows that the consumer's optimal acceptance strategy that solves problem (2) is a threshold strategy: the consumer accepts the product,  $a(s, \chi) = 1$ , if and only if her posterior belief about the product having good quality is sufficiently high,

$$\mu(s, \chi) \geq \frac{w_0 - w(B)}{w(G) - w(B)} \equiv \omega, \quad (7)$$

where  $\omega$  captures the relative value of the consumer's outside option.<sup>11</sup>

To understand the consumer's optimal acceptance strategy, we need to analyze the determinants of her posterior belief. Let  $\mu \equiv \mathbb{P}(y = G)$  denote the consumer's prior belief, which must be positive as we are looking at equilibria with positive trade.<sup>12</sup> After the designer proposes his product, the consumer observes the product complexity  $\chi$ . Since complexity is informative about the designer's action  $\kappa$ , it may contain information about quality  $y$ . The consumer's interim belief upon observing  $\chi$  is:

$$\mu(\chi) = \frac{\mu}{\mu + (1 - \mu)\ell(\chi)}, \quad (8)$$

where  $\ell(\chi) \equiv \frac{\mathbb{P}(\chi|y=B)}{\mathbb{P}(\chi|y=G)}$ . Note that, in equilibrium, the likelihood ratio  $\ell(\chi)$  will depend on the designer's complexification strategy  $\{\sigma_y\}$  and the primitive likelihood ratio  $\frac{f(\chi|\bar{\kappa})}{f(\chi|\underline{\kappa})}$ . Given the interim belief in (8), the consumer optimally acquires information and observes signal  $s$  with noise  $\chi$ . As a result, the consumer's posterior belief is given by:

$$\mu(s, \chi) = \frac{\mathbb{P}(S = s|y = G) \cdot \mu(\chi)}{\mathbb{P}(S = s|y = G) \cdot \mu(\chi) + \mathbb{P}(S = s|y = B) \cdot (1 - \mu(\chi))}. \quad (9)$$

The consumer's acceptance strategy is contingent on the acquired information whenever she accepts the product after observing a good signal,  $S = g$ , but rejects it after observing a bad signal,  $S = b$ . For this to be an optimal acceptance strategy, the signal has to be informative enough so that:

$$\mu(b, \chi) < \omega \leq \mu(g, \chi). \quad (10)$$

Otherwise, if the acquired information is not sufficiently precise, the consumer chooses to disregard this information. In this case, she either always accepts the product (if  $\mu(s, \chi) \geq \omega \forall s$ ), or always rejects it (if  $\mu(s, \chi) < \omega \forall s$ ). To ensure that, for each  $s$ , the posterior  $\mu(s, \chi)$  is monotonic in  $\chi$  (see Lemma A.1), we impose the following regularity condition:

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<sup>11</sup>If the consumer is indifferent, we assume without loss of generality that she accepts the product. Since such an indifference will arise with probability zero, what happens in that event is inconsequential.

<sup>12</sup>We will slightly abuse notation by referring to  $\mu$  as a prior belief,  $\mu(\chi)$  as an interim belief following observation of  $\chi$ , and  $\mu(s, \chi)$  as a posterior belief following observation of both  $\chi$  and  $s$ .

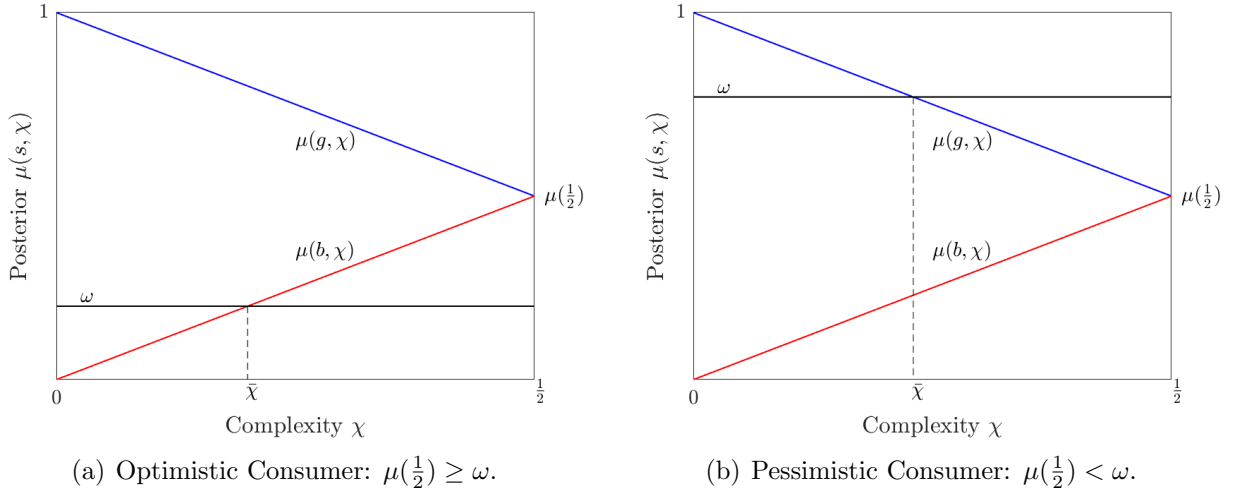


Figure 1: Illustrates the behavior of posterior belief  $\mu(s, \chi)$  as it depends on the signal,  $s$ , and complexity,  $\chi$ .

**Condition 3.1**  $\frac{f(\chi|\underline{k})}{f(\chi|\bar{k})} \cdot \frac{\chi}{1-\chi}$  is monotonic in  $\chi$ .

Given a prior belief  $\mu \in (0, 1)$ , the posterior beliefs satisfy  $\mu(g, 0) = 1$ ,  $\mu(b, 0) = 0$ , and  $\mu(g, \frac{1}{2}) = \mu(b, \frac{1}{2}) = \mu(\frac{1}{2}) \in (0, 1)$ . That is, the signal perfectly reveals quality when complexity is zero,  $\chi = 0$ , and it is uninformative when complexity is maximal,  $\chi = \frac{1}{2}$ . As  $\chi$  increases from 0 to  $\frac{1}{2}$ , MLRP guarantees that the posterior is monotonic when the designer's equilibrium strategy satisfies  $\sigma_G = \sigma_B$ , whereas MLRP combined with Condition 3.1 guarantees that this is also the case when  $\sigma_G \neq \sigma_B$ . Intuitively, Condition 3.1 ensures that the information content of the signal,  $s$ , is greater than the information content of the complexity,  $\chi$ . These features of the posterior beliefs are depicted in Figure 1, where note that the consumer optimally chooses to disregard (or, equivalently, not acquire) information about sufficiently complex products.

The following definition will be useful in characterizing the consumer's optimal strategy.

**Definition 1** We say that the consumer is **optimistic** if her interim belief satisfies  $\mu(\frac{1}{2}) \geq \omega$ , whereas we say that she is **pessimistic** if  $\mu(\frac{1}{2}) < \omega$ .

The consumer is optimistic when, upon receiving an uninformative signal, she chooses to accept the product. This will happen if her interim belief after observing a product with the highest possible complexity (i.e.,  $\chi = \frac{1}{2}$ ) is higher than her relative outside option. This case is depicted in Figure 1(a), where the consumer only rejects the product after observing a sufficiently informative negative signal, i.e.,  $\mu(S, \chi) < \omega$  iff  $S = b$  and  $\chi \leq \bar{\chi}$ . In contrast, the consumer is pessimistic when, upon receiving an uninformative signal, she chooses to reject the product. This will happen if her interim belief after observing a product with the

highest possible complexity (i.e.,  $\chi = \frac{1}{2}$ ) is lower than her relative outside option. This case is depicted in Figure 1(b), where the consumer only accepts the product after observing a sufficiently informative positive signal, i.e.,  $\mu(S, \chi) \geq \omega$  iff  $S = g$  and  $\chi \leq \bar{\chi}$ . We formalize this discussion in the following lemma.

**Lemma 1** *When the consumer is optimistic, her acceptance strategy is:*

$$a(s, \chi) = \begin{cases} \mathcal{I}_{\{S=g\}} & \text{if } \chi \leq \bar{\chi} \\ 1 & \text{if } \chi > \bar{\chi} \end{cases}, \quad (11)$$

where  $\mu(b, \bar{\chi}) = \omega$  and  $\mathcal{I}_{\{S=g\}}$  is the indicator equal to one when the signal is good. When the consumer is pessimistic, her acceptance strategy is:

$$a(s, \chi) = \begin{cases} \mathcal{I}_{\{S=g\}} & \text{if } \chi \leq \bar{\chi} \\ 0 & \text{if } \chi > \bar{\chi} \end{cases}, \quad (12)$$

where  $\mu(g, \bar{\chi}) = \omega$ .<sup>13</sup>

Therefore, an optimistic consumer will accept products that are sufficiently complex ( $\chi > \bar{\chi}$ ), whereas a pessimistic consumer will reject such products. This will be essential for understanding the designer's incentives to complexify or simplify his product.

Finally, note that both the threshold level of complexity  $\bar{\chi}$  and whether the consumer is optimistic or pessimistic are endogenous to equilibrium, since the prior belief  $\mu$  and the likelihood ratio  $\ell(\chi)$ , which determine the consumer's beliefs  $\mu(\chi)$  and  $\mu(s, \chi)$ , will need to be consistent with the equilibrium strategy of the designer and Bayes' rule.

### 3.2 The Designer's Complexification Strategy

Given the consumer's acceptance strategy described in the previous section, we next consider the designer's choice of  $\kappa$  for a  $y$ -product. From the designer's objective in (4), it follows that a designer who chooses a  $y$ -product also (weakly) prefers to simplify,  $\underline{\kappa}$ , whenever

$$\mathbb{P}(a = 1|y, \underline{\kappa}) \geq \mathbb{P}(a = 1|y, \bar{\kappa}). \quad (13)$$

Otherwise, the designer prefers to complexify,  $\bar{\kappa}$ . From Lemma 1, we can compute the probability of acceptance of a  $y$ -product conditional on the product's complexity  $\chi$ .

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<sup>13</sup>When  $\mu = 1$  or  $\mu = 0$ , we set without loss of generality  $\bar{\chi} = 0$ . As we will see, however, in any positive trade equilibrium  $\mu \in (0, 1)$ .

When the consumer is optimistic,

$$\mathbb{P}(a = 1|G, \chi) = \begin{cases} 1 - \chi & \text{if } \chi < \bar{\chi} \\ 1 & \text{if } \chi \geq \bar{\chi} \end{cases} \quad \text{and} \quad \mathbb{P}(a = 1|B, \chi) = \begin{cases} \chi & \text{if } \chi < \bar{\chi} \\ 1 & \text{if } \chi \geq \bar{\chi} \end{cases}. \quad (14)$$

Instead, when the consumer is pessimistic,

$$\mathbb{P}(a = 1|G, \chi) = \begin{cases} 1 - \chi & \text{if } \chi \leq \bar{\chi} \\ 0 & \text{if } \chi > \bar{\chi} \end{cases} \quad \text{and} \quad \mathbb{P}(a = 1|B, \chi) = \begin{cases} \chi & \text{if } \chi \leq \bar{\chi} \\ 0 & \text{if } \chi > \bar{\chi} \end{cases}. \quad (15)$$

Hence, a designer who proposes a  $(y, \kappa)$  product to the consumer expects it to be accepted with probability:

$$\mathbb{P}(a = 1|y, \kappa) = \int_0^{\frac{1}{2}} \mathbb{P}(a = 1|y, \chi) \cdot f(\chi|\kappa) d\chi. \quad (16)$$

The following proposition characterizes the optimal complexification strategy of a designer who has produced a  $y$ -product.

**Proposition 2** *Let  $\hat{\chi} \in (0, \frac{1}{2})$  denote the unique solution to  $\int_0^{\hat{\chi}} \chi \cdot f(\chi|\underline{\kappa}) d\chi = \int_0^{\hat{\chi}} \chi \cdot f(\chi|\bar{\kappa}) d\chi$ . Then, when the consumer is optimistic,*

$$\sigma_B = 1 \quad \text{and} \quad \sigma_G \begin{cases} = 1 & \text{if } \bar{\chi} < \hat{\chi} \\ \in [0, 1] & \text{if } \bar{\chi} = \hat{\chi}, \\ = 0 & \text{if } \bar{\chi} > \hat{\chi} \end{cases}, \quad (17)$$

whereas, when the consumer is pessimistic,

$$\sigma_B \begin{cases} = 0 & \text{if } \bar{\chi} < \hat{\chi} \\ \in [0, 1] & \text{if } \bar{\chi} = \hat{\chi} \\ = 1 & \text{if } \bar{\chi} > \hat{\chi} \end{cases} \quad \text{and} \quad \sigma_G = 0. \quad (18)$$

This result says that when the consumer is optimistic, the designer has a tendency to complexify his product; conversely, when the consumer is pessimistic, the designer has a tendency to simplify it. The intuition for this result can be obtained from Figure 2, which illustrates the acceptance probability,  $\mathbb{P}(a = 1|y, \chi)$ , as it depends on the product's quality,  $y$ , and complexity,  $\chi$ . An optimistic consumer does not acquire (or disregards) information and



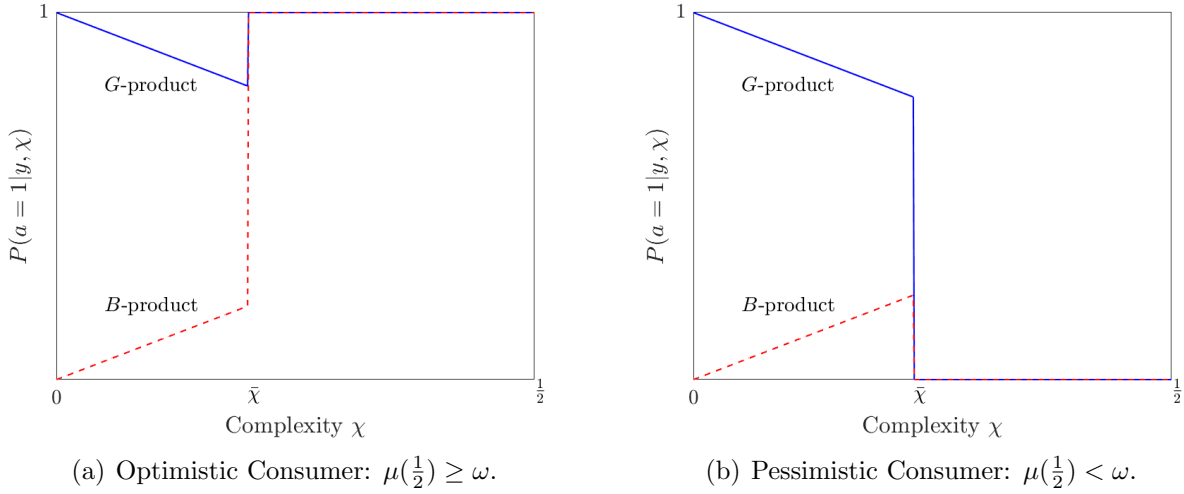


Figure 2: Illustrates the probability of acceptance of a  $y$ -product as a function of complexity  $\chi$ . Panel (a) shows the case of an optimistic consumer. Panel (b) shows the case of a pessimistic consumer.

accepts with probability one when a product is sufficiently complex. Thus, even the producer of a  $G$ -product may benefit from complexifying if the consumer is sufficiently optimistic, i.e.,  $\bar{\chi}$  is sufficiently low (see Figure 2(a)). Conversely, a pessimistic consumer does not acquire (or disregards) information and rejects with probability one when a product is sufficiently complex. Thus, even the producer of a  $B$ -product may benefit from simplifying if the consumer is sufficiently pessimistic, i.e.,  $\bar{\chi}$  is sufficiently low (see Figure 2(b)).

Our analysis thus highlights the central trade-off faced by the designer when choosing whether to complexify his product. The designer trades off the benefit of relying on the consumer's prior belief about the product's quality against the benefit of making the consumer acquire and react to information. While the latter is higher for the designer of a good product, the former does not depend on the product quality chosen by the designer. For example, when the consumer's prior is sufficiently high (so that the consumer is optimistic and  $\bar{\chi} < \hat{\chi}$ ), all products are complexified to reduce the consumer's reliance on information that is imperfect. Conversely, when the consumer's prior is sufficiently low (so that the consumer is pessimistic and  $\bar{\chi} < \hat{\chi}$ ), all products are simplified to reduce the consumer's reliance on information that is imperfect. Finally, for intermediate beliefs, the designer of a good product wants the consumer to rely on information (and therefore simplifies), whereas the designer of a bad product wants the consumer to ignore information and rely on her prior (and therefore complexifies).

### 3.3 The Designer's Quality Strategy

When choosing the product's quality, the designer faces a trade-off between increasing the product's acceptance probability (by choosing  $y = G$ ) or increasing his payoff conditional on acceptance (by choosing  $y = B$ ). Given the consumer's acceptance strategy, the net expected payoff to the designer from choosing the  $G$ -product over the  $B$ -product is:

$$\gamma \equiv \max_{\kappa} \mathbb{P}(a = 1|G, \kappa) \cdot v(G) - \max_{\kappa} \mathbb{P}(a = 1|B, \kappa) \cdot v(B). \quad (19)$$

The first term is the expected payoff from choosing the  $G$ -product given the corresponding optimal choice of  $\kappa$ , as characterized in Proposition 2. The second term is the expected payoff from choosing the  $B$ -product given the corresponding optimal choice of  $\kappa$ . The probabilities in each scenario are computed as in (16). Note that these probabilities, and as a result the payoff  $\gamma$ , depend on the consumer's belief  $\mu$  and the likelihood ratio  $\ell(\cdot)$ , as the latter determine  $\bar{\chi}$ . The next result then follows immediately.

**Proposition 3** *Given the consumer's acceptance strategy, the designer chooses the  $G$ -product with probability:*

$$m \begin{cases} = 1 & \text{if } \gamma > 0 \\ \in [0, 1] & \text{if } \gamma = 0, \\ = 0 & \text{if } \gamma < 0 \end{cases} \quad (20)$$

where  $\gamma$  is given by (19).

### 3.4 Characterization of Equilibria

In Section 3.1, we characterized the consumer's strategy given her belief. In Sections 3.2 and 3.3, we characterized the designer's quality and complexification strategy given the consumer's acceptance strategy. To characterize the equilibria of our model, we now require that the consumer's belief be consistent with the designer's strategy and Bayes' rule. We find it instructive to proceed in two steps.

*Equilibrium Complexity.* In the first step, we take the consumer's prior belief  $\mu$  as given and find the designer's equilibrium complexification strategy  $\{\sigma_y\}$  by requiring that the consumer's interim belief,  $\mu(\chi)$ , be consistent with it and Bayes' rule.

**Proposition 4** *Suppose that in equilibrium the consumer's prior belief is  $\mu \in (0, 1)$ , then there exist thresholds  $0 < \mu_1 < \mu_2 < \mu_3 < \mu_4 < 1$  such that:*

1. *If  $\mu \in (0, \mu_1]$ , all products are simplified,  $\sigma_G = \sigma_B = 0$ .*

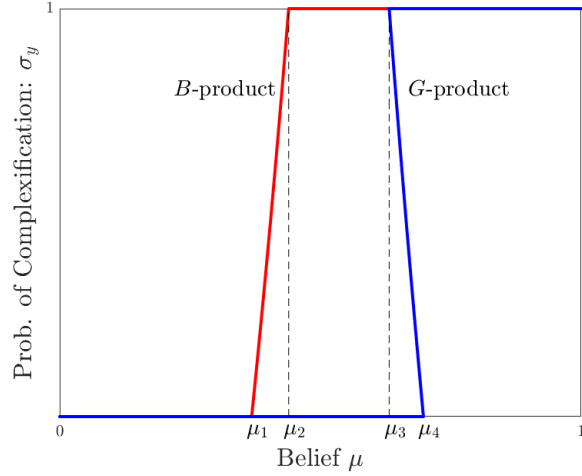


Figure 3: Illustrates how the complexification strategy of the designer who chooses a  $y$ -product varies with the consumer's belief  $\mu$ .

2. If  $\mu \in (\mu_1, \mu_2]$ ,  $G$ -products are simplified,  $\sigma_G = 0$ , whereas  $B$ -products are complexified with probability

$$\sigma_B = \left( \frac{f(\hat{\chi}|\bar{\kappa})}{f(\hat{\chi}|\underline{\kappa})} - 1 \right)^{-1} \left( \frac{1 - \hat{\chi}}{\hat{\chi}} \frac{\mu}{1 - \mu} \frac{1 - \omega}{\omega} - 1 \right).$$

3. If  $\mu \in (\mu_2, \mu_3]$ ,  $G$ -products are simplified,  $\sigma_G = 0$ , whereas  $B$ -products are complexified,  $\sigma_B = 1$ .

4. If  $\mu \in (\mu_3, \mu_4)$ ,  $G$ -products are complexified with probability

$$\sigma_G \in \left\{ 0, 1 - \left( 1 - \frac{f(\hat{\chi}|\underline{\kappa})}{f(\hat{\chi}|\bar{\kappa})} \right)^{-1} \left( 1 - \frac{1 - \hat{\chi}}{\hat{\chi}} \frac{1 - \mu}{\mu} \frac{\omega}{1 - \omega} \right), 1 \right\},$$

whereas  $B$ -products are complexified,  $\sigma_B = 1$ .

5. If  $\mu \in [\mu_4, 1)$ , all products are complexified,  $\sigma_G = \sigma_B = 1$ .

This result is illustrated in Figure 3. As we can see, all products will be complexified (simplified) when the consumer's prior belief is sufficiently high (low), and  $G$ -products will be simpler than  $B$ -products for intermediate values of  $\mu$ . In what follows, we provide an intuition for this result by sketching the proof of the proposition.

First, let us ask whether an equilibrium in which all products are simplified, i.e.,  $\sigma_G = \sigma_B = 0$ , exists. By Proposition 2, this can only happen if the consumer is pessimistic, i.e.,

$\mu(\frac{1}{2}) = \mu < \omega$ .<sup>14</sup> In this case,  $G$ -products are always simplified, whereas  $B$ -products are simplified only if  $\bar{\chi} \leq \hat{\chi}$ . Since the threshold  $\bar{\chi}$  is given by  $\mu(g, \bar{\chi}) = \omega$  (see Lemma 1) and  $\mu(g, \cdot)$  is decreasing, we have that  $\bar{\chi} \leq \hat{\chi}$  if and only if  $\mu(g, \hat{\chi}) \leq \omega$ , which is equivalent to:

$$\mu \leq \frac{\omega \cdot \frac{\hat{\chi}}{1-\hat{\chi}}}{\omega \cdot \frac{\hat{\chi}}{1-\hat{\chi}} + 1 - \omega} \equiv \mu_1. \quad (21)$$

Since  $\hat{\chi} \in (0, \frac{1}{2})$ , also  $\mu_1 < \omega$ . Thus, an equilibrium in which all products are simplified exists if  $\mu \in (0, \mu_1]$ ; that is, if the consumer is ‘sufficiently pessimistic’ about the product’s quality.

Second, let us ask whether an equilibrium in which all products are complexified, i.e.,  $\sigma_G = \sigma_B = 1$ , exists. By Proposition 2, this can only happen if the consumer is optimistic, i.e.,  $\mu(\frac{1}{2}) = \mu \geq \omega$ . In this case,  $B$ -products are always complexified, whereas  $G$ -products are complexified only if  $\bar{\chi} \leq \hat{\chi}$ . Since now the threshold  $\bar{\chi}$  is given by  $\mu(b, \bar{\chi}) = \omega$  (see Lemma 1) and  $\mu(b, \cdot)$  is increasing, we have that  $\bar{\chi} \leq \hat{\chi}$  if and only if  $\mu(b, \hat{\chi}) \geq \omega$ , which is equivalent to:

$$\mu \geq \frac{\omega \cdot \frac{1-\hat{\chi}}{\hat{\chi}}}{\omega \cdot \frac{1-\hat{\chi}}{\hat{\chi}} + 1 - \omega} \equiv \mu_3. \quad (22)$$

Since  $\hat{\chi} \in (0, \frac{1}{2})$ , also  $\mu_3 > \omega$ . Thus, an equilibrium in which all products are complexified exists if  $\mu \in [\mu_3, 1)$ ; that is, if the consumer is ‘sufficiently optimistic’ about the product’s quality.

Third, let us ask whether an equilibrium in which  $G$ -products are simplified and  $B$ -products are complexified, i.e.,  $\sigma_G = 0$  and  $\sigma_B = 1$ , exists. Note that, in this case, the consumer makes an inference about the product’s quality upon observing complexity,  $\chi$ , since more complex products are more likely to be  $B$ -products. By Proposition 2, there are two cases to consider, depending on whether the consumer is optimistic or pessimistic. If the consumer is pessimistic, i.e.,  $\mu(\frac{1}{2}) < \omega$ , then  $G$ -products are always simplified whereas  $B$ -products are complexified only if  $\bar{\chi} \geq \hat{\chi}$ . But, both conditions hold if and only if:

$$\mu_2 \equiv \frac{\omega \cdot \ell(\hat{\chi}) \cdot \frac{\hat{\chi}}{1-\hat{\chi}}}{\omega \cdot \ell(\hat{\chi}) \cdot \frac{\hat{\chi}}{1-\hat{\chi}} + 1 - \omega} \leq \mu \leq \frac{\omega \cdot \ell(\frac{1}{2})}{\omega \cdot \ell(\frac{1}{2}) + 1 - \omega}, \quad (23)$$

where  $\ell(\cdot) = \frac{f(\cdot|\bar{\kappa})}{f(\cdot|\underline{\kappa})}$ . Instead, if the consumer is optimistic, i.e.,  $\mu(\frac{1}{2}) \geq \omega$ , then  $B$ -products are always complexified whereas  $G$ -products are simplified only if  $\bar{\chi} \geq \hat{\chi}$ . But, both conditions

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<sup>14</sup>Note that, when all products are simplified (or all products are complexified), the consumer does not make inferences upon observation of complexity and, thus,  $\mu(\chi) = \mu \forall \chi$ .

hold if and only if:

$$\frac{\omega \cdot \ell\left(\frac{1}{2}\right)}{\omega \cdot \ell\left(\frac{1}{2}\right) + 1 - \omega} < \mu \leq \frac{\omega \cdot \ell(\widehat{\chi}) \cdot \frac{1-\widehat{\chi}}{\widehat{\chi}}}{\omega \cdot \ell(\widehat{\chi}) \cdot \frac{1-\widehat{\chi}}{\widehat{\chi}} + 1 - \omega} \equiv \mu_4. \quad (24)$$

Thus, an equilibrium in which  $G$ -products are simplified and  $B$ -products are complexified exists provided that  $\mu \in [\mu_2, \mu_4]$ ; that is, the consumer is neither too optimistic nor too pessimistic about the product's quality.

Lastly, because the designer's incentive to complexify is always greater for  $B$ - than for  $G$ -products, there cannot be an equilibrium in which  $G$ -products are complexified and  $B$ -products are simplified (see Proposition 2). Moreover, note that the ranking of the belief thresholds follows by inspection of (21)-(24), since  $\widehat{\chi} \in (0, \frac{1}{2})$  and MLRP implies that  $\frac{f(\widehat{\chi}|\kappa)}{f(\widehat{\chi}|\underline{\kappa})} > 1$ . Such ranking implies that for  $\mu \in (\mu_1, \mu_2)$  there cannot exist equilibria that involve either pooling or separation on  $\kappa$ , whereas for  $\mu \in (\mu_3, \mu_4)$  both pooling and separation on  $\kappa$  is possible. In these regions, we can construct equilibria that involve mixing on  $\kappa$ , as illustrated in Figure 3. For a detailed construction of these equilibria, see the proof of Proposition 4.

We have thus characterized the designer's equilibrium complexification strategy,  $\{\sigma_y\}_\mu$ , that is consistent with a given prior belief  $\mu$ , where it is now convenient to make the dependence of the strategy on  $\mu$  explicit.

*Equilibrium Quality.* In the second step, we pin down the equilibrium prior belief, which we will denote by  $\mu^*$ . That is, we need to show that the complexification strategies and designer payoffs implied by  $\mu^*$  are consistent with the designer producing  $G$ -products with probability  $m = \mu^*$ . To this end, we use equation (19) to compute the designer's net payoff  $\gamma(\mu, \{\sigma_y\}_\mu)$  from producing  $G$ - versus  $B$ -products, as a function of the prior belief,  $\mu$ , and the corresponding complexification strategy,  $\{\sigma_y\}_\mu$ , from Proposition 4.

The following result establishes existence and uniqueness of the positive trade equilibrium.

**Proposition 5** *There is generically a unique equilibrium with positive trade, and in it  $m = \mu^* = \psi \in (0, 1)$ , where  $\psi$  and  $\{\sigma_y\}_\psi$  are solutions to:*

$$\gamma(\psi, \{\sigma_y\}_\psi) = 0. \quad (25)$$

First, it is easy to rule out an equilibrium with  $\mu^* = 1$ , since in that case the consumer would accept the product with probability one, making it optimal for the designer to only produce a  $B$ -product, contradicting belief consistency. As discussed before, an equilibrium with  $\mu^* = 0$  always exists, but in it products are rejected with probability one, and thus there is no trade. Therefore, in any equilibrium with positive trade, it must be that  $\mu^* \in (0, 1)$ .

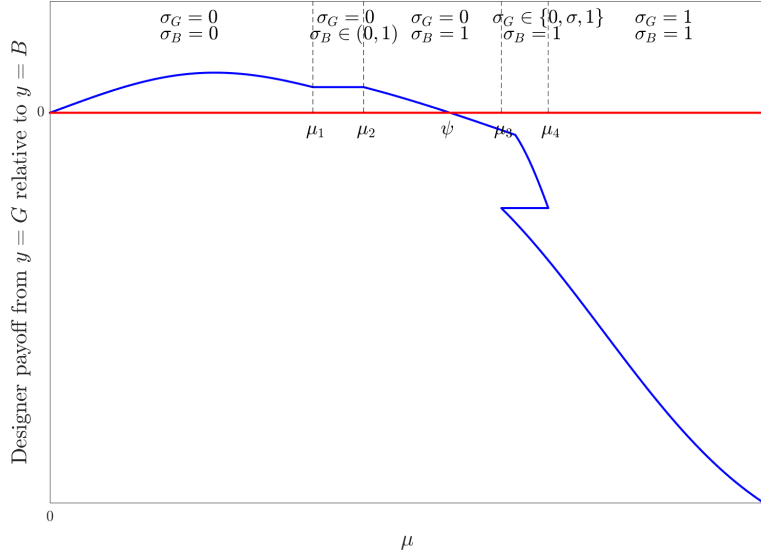


Figure 4: Illustrates how the designer’s net payoff from choosing the  $G$ -product varies with the consumer’s belief  $\mu$ .

Such an equilibrium exists if there is belief,  $\psi$ , and corresponding complexification strategy  $\{\sigma_y\}_\psi$ , that make the designer indifferent between producing  $G$ - versus  $B$ -products.

Figure 4 illustrates the generic behavior of the designer’s net payoff from producing  $G$ - versus  $B$ -products, as it depends on  $\mu$ . First, it is positive for  $\mu$  small and becomes negative for  $\mu$  large. Intuitively, when  $\mu$  is small, the consumer is pessimistic and, thus, accepts products with low probability. The designer then expects a higher payoff from producing a  $G$ -product, given its higher probability of acceptance. Second, as  $\mu$  increases, the difference between the probabilities of acceptance for  $G$ - versus  $B$ -products increases to further favor producing the  $G$ -product, since the consumer becomes more likely to rely on information. Third, as  $\mu$  increases further, the consumer becomes sufficiently optimistic, which reduces her reliance on information, and the gap between the probabilities of acceptance of the two products as a result begins to shrink. Finally, as  $\mu$  becomes sufficiently large, the probabilities of acceptance become large enough so that the designer obtains a higher expected payoff from  $B$ -products. See proof of Proposition 5 for formal details. Using continuity arguments, we can then show that an intersection  $\psi$  exists, and that it can generically lie in anywhere outside of the interval  $(\mu_1, \mu_2)$ , where the designer’s payoff is independent of  $\mu$ .

We have thus provided a full characterization of the equilibrium of our signaling game. We note that uniqueness of equilibrium is obtained due to two distinguishing features of our model. First, the designer can influence but not fully control the product complexity

for the consumer. This is because in our model a product’s complexity depends on both the designer’s choice to complexify or simplify it (action  $\kappa$ ) and on some component outside of the designer’s control ( $\eta$ ), which the consumer cannot tell apart. This feature ensures that multiplicity due to the freedom of specifying off-equilibrium beliefs does not arise in our setting. This formulation is not only analytically convenient, but it is also reasonable within our applications where a number of factors can determine the consumer’s ability to process information about a product. Second, the endogeneity of product quality rules out multiplicity arising from several complexification strategies being consistent with a given prior belief (as stated in Proposition 4 for  $\mu \in (\mu_3, \mu_4)$ ). In particular, there is generically a single value of  $\psi$  and a single complexification strategy,  $\{\sigma_y\}_\psi$  such that both the designer’s strategy is optimal and the consumer’s belief is consistent.

Next, we exploit the uniqueness of our equilibrium to obtain sharp comparative statics and to discuss how our results lead to applied implications for the relationship between product quality and complexity.

## 4 The Quality-Complexity Relationship

Our model generates sharp predictions about the relationship between quality and complexity of products, and how this relationship varies with features of the environment. We now explore these predictions through comparative statics and several model extensions. In doing so, we say that a product’s expected quality increases when the probability that a product has good quality,  $\mu = \mathbb{P}(y = G)$ , increases; and that a product’s expected complexity increases when the probability that a product is complexified,  $\mathbb{P}(\kappa = \bar{\kappa}) = \mu\sigma_G + (1 - \mu)\sigma_B$ , increases.

### 4.1 Relative Outside Option

We begin by considering the effect of a decrease in  $\omega$ , which reflects an increase in the relative payoff of the product to the consumer. Such an increase could result from an increase in the consumer’s direct payoff from the product, i.e.,  $w(G)$  or  $w(B)$ , which we interpret as higher product demand, or a fall in the consumer’s outside option,  $w_0$ .

**Proposition 6** *As  $\omega$  decreases,  $\mu$  decreases while  $\sigma_G$  and  $\sigma_B$  do not change. As a result, expected quality falls while expected complexity weakly increases.*

Figure 5 illustrates the effect of a change in the consumer’s relative outside option on



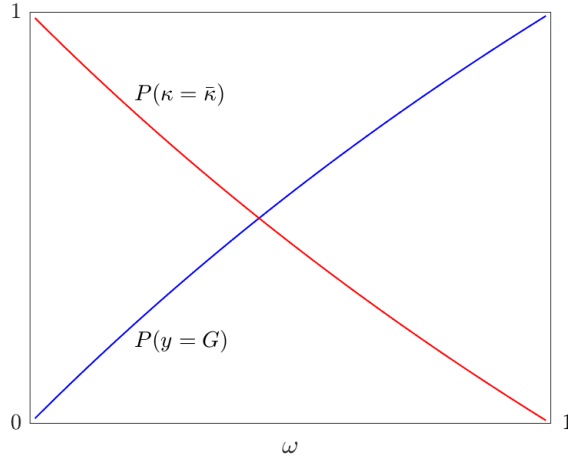


Figure 5: Illustrates how expected product quality and complexity vary with the consumer’s relative outside option,  $\omega$ .

expected product quality and complexity.<sup>15</sup> Intuitively, as the consumer’s relative outside option falls, she is more likely to accept the product as her acceptance decision becomes less strict. This makes the  $B$ -product more likely to be accepted, increasing the designer’s incentives to produce a  $B$ -product, lowering expected quality. Interestingly, the designer’s equilibrium complexification strategy is independent of  $\omega$ , as all of the adjustment happens through changes in average quality, leaving the consumer’s acceptance strategy unchanged. Finally, as complexification is always weakly higher for a  $B$ - than a  $G$ -product, a decrease in expected quality results in an increase in expected complexity.

## 4.2 Conflicts of Interest

We next analyze the effect of an increase in the designer’s alignment with the consumer given by an increase in the designer’s payoff from producing a  $G$ -product,  $v(G)$ .<sup>16</sup>

**Proposition 7** *As  $v(G)$  increases,  $\mu$  increases for large changes in  $v(G)$ , though it may be non-monotonic locally, while  $\sigma_G$  and  $\sigma_B$  increase. Thus, after a large increase in  $v(G)$ , both expected quality and complexity increase.*

Figure 6(a) illustrates the effect of a change in the designer’s alignment with the consumer on expected product quality and complexity. Intuitively, an increase in  $v(G)$  increases the net payoff to the designer from producing a  $G$ -product. Therefore, unsurprisingly, expected

<sup>15</sup>Throughout this section, unless stated otherwise, the figures are produced under the following parametrization:  $\chi \sim \text{Truncated Normal}(\mu_{\kappa}, \sigma, 0, 0.5)$  with  $\mu_{\kappa} = 0.2$ ,  $\mu_{\bar{\kappa}} = 0.3$ ,  $\sigma = 0.2$ ,  $\omega = 0.5$ ,  $v(G) = 0.3$  and  $v(B) = 1$ . None of the qualitative results discussed depend on these parametric restrictions.

<sup>16</sup>Note that the effect of an increase in  $v(G)$  is the same as the effect of a decrease in  $v(B)$ .

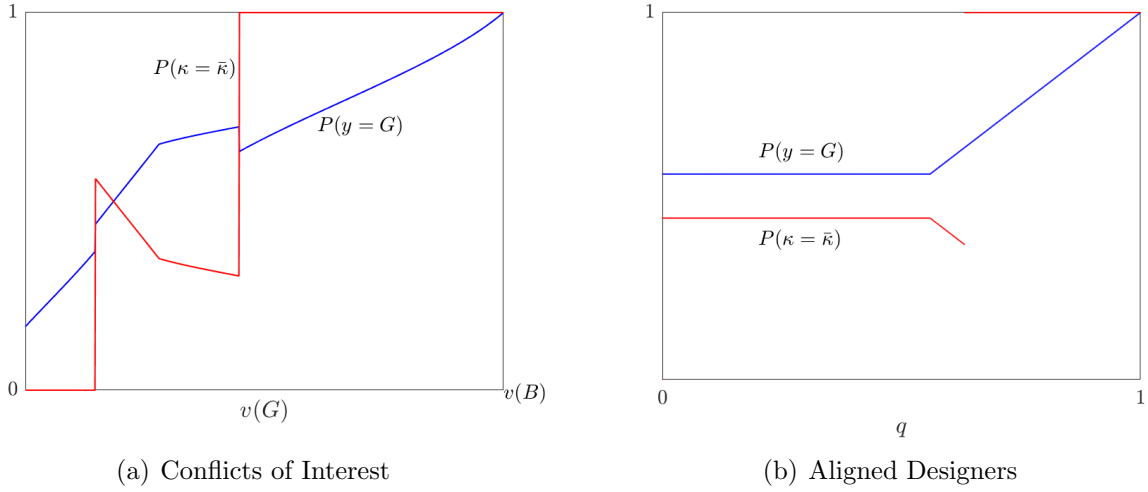


Figure 6: illustrates how expected product quality and complexity vary with the consumer-designer alignment. Panel (a) shows the comparative statics on the designer’s payoff from producing a  $G$ -product,  $v(G)$ . Panel (b) shows the comparative statics on the probability of finding an aligned designer,  $q$ .

product quality increases. Since in equilibrium the consumer’s prior belief  $\mu$  must increase as well, she becomes less selective in accepting products, which increases the designer’s incentive to complexify.

More precisely, an increase in  $v(G)$  generates an upward shift of the correspondence  $\gamma$ , depicted in Figure 4, while leaving the thresholds  $\mu_1 - \mu_4$  unchanged. Generically, this leads to an increase in  $\psi$ , and thus to an improvement in overall product quality. There are, however, two values of  $v(G)$  at which  $\psi$  jumps, generating the two kinks in Figure 6(a). First, as  $v(G)$  increases,  $\psi$  gradually approaches  $\mu_1$  and then jumps up to  $\mu_2$ : the reason is that the equilibrium switches from pooling at simplification to separation (i.e.,  $\sigma_G = 0$  and  $\sigma_B = 1$ ), generating a discontinuous change in the consumer’s interim belief as she begins inferring product quality from observed complexity, which incentivizes the supply of good products. Second, as  $v(G)$  increases further,  $\mu$  gradually approaches  $\mu_4$  and then jumps down to  $\mu_3$ : the reason is that the equilibrium switches from separation to pooling at complexification, generating a discontinuous change in the consumer’s interim belief as she stops inferring product quality from observed complexity, which incentivizes the supply of bad products. As we show next, this result is consistent with other mechanisms that reduce the misalignment between the designer and the consumer.

### 4.3 Aligned Designers

We next study the implications of introducing a designer whose payoffs are *aligned* with the consumer's. We suppose that with probability  $q \in [0, 1]$  the consumer encounters an aligned designer, who obtains a higher payoff from having a  $G$ -product being accepted,  $\bar{v}(G) > \bar{v}(B) > 0$ . With probability  $1 - q$ , however, the consumer meets the misaligned designer as in the baseline model. Finally, whether the designer is aligned or misaligned is not observable to the consumer. Observe that our baseline model corresponds to the case of  $q = 0$ .

An aligned designer takes private actions  $\{y, \kappa\}$  to maximize his expected payoff,  $\mathbb{P}(a = 1|y, \kappa) \cdot \bar{v}(y)$ . As both his probability of acceptance and payoff conditional on acceptance are higher for a  $G$ -product, the aligned designer always produces a  $G$ -product. As in our baseline model, the complexification strategy for the  $G$ -product is given by Proposition 2. Thus, even an aligned designer chooses to complexify his  $G$ -product when the consumer is sufficiently optimistic.

The presence of an aligned designer only affects the equilibrium analyzed in Section 3 through the belief that a  $G$ -product is offered, which is now given by  $\mu = q + (1 - q) \cdot m$ . As before,  $m$  is the probability with which the misaligned designer produces a  $G$ -product. The following proposition characterizes the main effects of introducing an aligned designer.

**Proposition 8** *The presence of an aligned designer need not change the equilibrium outcome unless  $q$  is sufficiently large, in which case  $\mu$ ,  $\sigma_G$  and  $\sigma_B$  increase. Thus, after a large increase in  $q$ , both expected quality and complexity increase.*

When the probability of meeting an aligned designer is sufficiently small ( $q < \psi$ ), there is no effect on equilibrium outcomes, as the presence of an aligned designer is fully offset by an increase in the misaligned designer's incentives to produce a  $B$ -product. As a result, expected product quality and complexity are as in the baseline model, with  $\mu = \psi$ . When  $q$  is sufficiently large, however, the misaligned designer only produces a  $B$ -product,  $m = 0$ , and further increases in  $q$  lead to higher product quality and, thus, to more complexification (see Proposition 4). These effects are depicted in Figure 6(b). The upward jump in expected complexity illustrated in the figure arises due to an equilibrium switch from separation to pooling at complexification.<sup>17</sup>

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<sup>17</sup>Since for  $q > \psi$  the expected quality is effectively exogenous, there may be multiple equilibria whenever  $q \in (\mu_3, \mu_4)$ , due to multiple complexification strategies being consistent with belief  $\mu = q$  (see Proposition 4). Figure 6(b) is produced for the equilibrium where  $\sigma_G = 0$  in this region, so that the increase in expected complexity occurs at  $q = \mu_4$  rather than sooner.

## 4.4 Sequential Search and Competition

We now study the effects of competition among designers. In particular, we suppose that if the consumer rejects a product, she searches for a new designer whom she finds with probability  $\beta \in (0, 1)$ . The new designer proposes a product to the consumer, and the game repeats until the consumer accepts an offered product. In this setting, higher  $\beta$  corresponds to lower search frictions and, hence, more intense competition among designers.

In a stationary equilibrium, in which  $U$  denotes the consumer’s equilibrium value, we have:

$$U = \mathbb{E} \left[ \max_{a \in \{0,1\}} \{a \cdot [\mu(s, \chi) \cdot w(G) + (1 - \mu(s, \chi)) \cdot w(B)] + (1 - a) \cdot \beta U\} \right]. \quad (26)$$

Note that for  $w_0 = \beta U$ , the equilibrium is fully characterized in Section 3.4. Thus, the main difference with our baseline model is that the consumer’s outside option is now endogenous. Further, we assume that  $w(B) < 0$  (but maintain that  $w(G) > 0$ ), which ensures that the consumer prefers to search rather than accept a bad product; this rules out an uninteresting equilibrium where  $B$ -products are produced and accepted with probability one.<sup>18</sup>

**Proposition 9** *An equilibrium exists, and in it  $\beta U \in (0, w(G))$ . Furthermore,  $\beta U$  is increasing in  $\beta$ . Thus, an increase in competition among designers increases expected quality and decreases expected complexity.*

As  $\beta U$  is the consumer’s outside option in our search setting, comparative statics with respect to  $\beta$  are as those with respect to the relative outside option,  $\omega$ , in Proposition 6. We find that competition has the desirable effect of increasing incentives to design products that the consumer wants: those that are good and simple. This prediction is in contrast to the literature on obfuscation and price complexity (Spiegler, 2006; Ellison and Ellison, 2009; Carlin, 2009; Ellison and Wolitzky, 2012), which finds that higher competition leads to more obfuscation, as obfuscation effectively increases the producer’s market power by making it harder for the consumer to observe the attributes of competing products, e.g., by effectively increasing search costs. The consumer searches fewer products, competition effectively declines, which benefits firm profits. This channel is not present in our setting. Instead, here, complexification influences the information the consumer extracts about the product. More intense competition effectively increases the consumer’s outside option. This, in turn, incentivizes the designer to offer products that are more likely to be accepted by the consumer: those that are good and simple.<sup>19</sup>

<sup>18</sup>Note that, in such an equilibrium, the consumer’s outside option would be  $w_0 = \beta U = \beta w(B) < w(B)$ .

<sup>19</sup>A related result, though in a Bayesian persuasion setting, is found in Au and Kawai (2020), who study competition in Bayesian persuasion where senders disclose information about their respective qualities. They

## 4.5 Empirical implications

In this section, we summarize the main empirical implications, and discuss them within the context of our two leading applications.

**Remark 1** *The model generates the following testable implications:*

1. *As product demand by the consumer increases, products become more complex and of worse quality (Proposition 6).*
2. *As the consumer-designer alignment increases, products become more complex and of better quality (Propositions 7 and 8.)*
3. *As competition among the designers increases, products become less complex and of better quality (Proposition 9).*

We now discuss these predictions within the context of our two leading applications: financial products and regulatory design.

*Financial Products.* We interpret the product designer as a financial advisor, and the consumer as a retail investor. Financial advisors design or select financial products to offer to investors, such as investment funds, credit cards, and securitized products. Investors then decide whether they are willing to accept the offer or not. Within this context, our model suggests that financial products are more likely to be complex and of low quality when demand for these products is high. This suggests that the proliferation of low quality and complex structured financial products documented by [Jaffee et al. \(2009\)](#) and [C  l  rier and Vall  e \(2017\)](#) could have been driven by an increase in the demand for such products.<sup>20</sup>

Our model also highlights the importance of financial advisors' compensation structures or career concerns, as they determine the level of alignment between financial advisors and retail investors. In particular, our model suggests that when the trust in financial advisors is high due to perceived alignment, we should observe better quality and more complex products being offered. This is consistent with the observation that the proliferation of complex products in recent decades has been accompanied by a period of increased trust in financial advisors, which may have culminated with the financial crisis. Even though in this scenario, and unlike

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find that competition (i.e., a higher number of senders) induces each sender to disclose information more aggressively.

<sup>20</sup>It is by now conventional wisdom that the last two decades have witnessed an unprecedented increase in the demand for stores of value produced by the US, due to the so-called "global savings glut" ([Bernanke, 2005](#)). Structured financial products were perceived as a class of safer assets that could satisfy this demand.

in our model, the perceived alignment may have been unjustified, and accompanied by the proliferation of bad quality products.<sup>21</sup>

Recent empirical work has examined the effects of policies aimed at reducing conflicts of interest between financial advisors and clients. [Bhattacharya, Illanes, and Padi \(2019\)](#) exploit state-level variation in fiduciary duty laws in the United States and find that broker-dealers bound by fiduciary duty propose higher quality products. Our model provides an additional prediction, so far untested: although such policies can be effective in improving product quality, they may have the side effect of increasing product complexity. This insight is relevant given current debates to expand fiduciary duty in the financial advice industry.<sup>22</sup>

*Regulatory Policies.* In the regulatory sphere, we interpret the designer as a policymaker, and the consumer as the median voter that has to accept a policy proposal. Within this context, we interpret a “good” (“bad”) policy proposal as one that is ideologically aligned (misaligned) with the median voter’s preferences, and  $q$  as the fraction of policymakers who are aligned with the median voter (as in Section 4.3). Finally, a lower median voter outside option corresponds to a costlier status-quo, or equivalently, to a greater urgency to pass a policy.

In a study of Italian legislative proposals, [Gratton et al. \(2019\)](#) show that worse quality and more complex legislation is proposed by politicians when the bureaucracy is less effective and the expected duration of the legislative sessions is shorter. The authors argue that these conditions reduce the voters’ ability to gather information on the competence of politicians. This in turn incentivizes bad politicians to pool with good politicians in producing legislation, which results in many low quality legislative proposals. Our model suggests an additional channel for these outcomes. Bureaucratic ineffectiveness and short legislative sessions could map to a costly status-quo and urgency to pass policies, given substantial pressure for reforms in a short time interval. The results of Proposition 6 suggest that laws passed under such time pressure would indeed be more complex and of lower quality.

In the legislative context, alignment between voters and policymakers is an important factor, and it is likely to arise due to similar ideological or political preferences. Such alignment between policymakers and the median voter is reflected in public opinion data, which provides politicians with real time information about voters’ support. Thus, our results suggest that

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<sup>21</sup>For example, in 2011, the Federal Housing Finance Agency filed lawsuits against some of the largest US financial institutions, involving allegations of securities law violations and fraud in the packaging and sale of mortgage-backed-securities. For a detailed description, see <https://www.fhfa.gov/SupervisionRegulation/LegalDocuments/Pages/Litigation.aspx>.

<sup>22</sup>For instance, the Securities and Exchange Commission (SEC) has proposed to forbid the use of the term “financial advisor” for those managing brokerage accounts (particularly retirement funds) unless the broker has formally accepted a fiduciary duty to act in the investor’s best interest. See “*Fiduciary Rule*” *Poised for Second Life Under Trump Administration*, article by Dave Michaels on the Wall Street Journal, January 10th, 2018: <https://www.wsj.com/articles/fiduciary-rule-poised-for-second-life-under-trump-administration-1515580200>.

policymakers who face high public opinion are more likely to propose policies that are complex but aligned with the median voter’s preferences, and vice-versa. Indeed, legal scholars have argued that in policy domains where public opinion is low (e.g., financial services or pharmaceuticals), policy proposals from the US Congress tend to be simpler, leaving it to federal agencies to draft additional rules (Stiglitz, 2017). Our results provide a theoretical basis for further exploring this empirical observation.

## 5 Robustness of our modeling approach

In our baseline model, we assumed a stark functional form for the cost of information acquisition to isolate the key mechanisms driving our results, and we also took the payoffs to the agents as given. In this section, we show that our results are robust to considering alternative information acquisition problems, i.e., convex costs and rational inattention, and to endogenizing the agents’ payoffs by introducing prices and production costs. To streamline the exposition, we relegate formal derivations to the Appendix.

### 5.1 Convex Costs

In this section, we suppose that the consumer’s cost of information acquisition is:

$$C(z, \chi) = \chi \cdot h\left(\frac{1}{2} - z\right), \quad (27)$$

where  $\chi = \chi(\eta, \kappa) \in (0, \infty)$  and  $h(\cdot)$  is continuously differentiable, increasing and convex, with  $h'(0) = 0$  and  $\lim_{x \rightarrow \frac{1}{2}} h'(x) = \infty$ . These properties imply that the consumer’s choice of information acquisition,  $z(\chi)$ , will be positive (i.e., information is always imperfect) and increasing in complexity,  $\chi$ .

If the consumer chooses to acquire information, i.e.,  $z(\chi) < \frac{1}{2}$ , it is because she will make her acceptance decision contingent on the received information: she will accept the product after observing signal  $g$  and reject it after observing signal  $b$ . Given this, the payoff from acquiring information is:

$$W^I(\chi, z) \equiv \max_{z \in [0, \frac{1}{2}]} w_0 + \mu(\chi) \cdot (1 - z) \cdot (w(G) - w_0) + (1 - \mu(\chi)) \cdot z \cdot (w(B) - w_0) - C(z, \chi). \quad (28)$$

We can already see the main difference between this and our baseline model: the mapping between a product’s complexity and the noise of the acquired information given by the solution to (28),  $z(\chi)$ , now depends on the consumer’s prior belief,  $\mu$ . To highlight the main



mechanisms of our paper, in the baseline model we chose a formulation for the cost function that eliminated this dependence. As we show next, though this dependence complicates the analysis, it does not change our main qualitative results.

If the consumer chooses not to acquire information,  $z(\chi) = \frac{1}{2}$ , her payoff is:

$$W^U(\chi) \equiv \max \{ \mu(\chi) \cdot w(G) + (1 - \mu(\chi)) \cdot w(B), w_0 \}. \quad (29)$$

It follows that the consumer acquires information and makes her decision contingent on the received information whenever  $W^I(\chi, z(\chi)) > W^U(\chi)$ . After some algebra, it follows that the consumer makes her decision conditional on information when:

$$z(\chi) < \begin{cases} \frac{(1-\mu(\chi)) \cdot \omega}{(1-\mu(\chi)) \cdot \omega + \mu(\chi) \cdot (1-\omega)} - \frac{h(\frac{1}{2}-z(\chi))}{h'(\frac{1}{2}-z(\chi))} & \text{if } \mu(\chi) \geq \omega \\ \frac{\mu(\chi) \cdot (1-\omega)}{(1-\mu(\chi)) \cdot \omega + \mu(\chi) \cdot (1-\omega)} - \frac{h(\frac{1}{2}-z(\chi))}{h'(\frac{1}{2}-z(\chi))} & \text{if } \mu(\chi) < \omega \end{cases} \quad (30)$$

In contrast to our baseline model, the consumer now takes into account the cost of information acquisition when deciding whether to make her decision contingent on information. This is captured by the new term  $\frac{h(\frac{1}{2}-z(\chi))}{h'(\frac{1}{2}-z(\chi))}$  on the right hand side of (30). As in our baseline analysis, we impose a regularity condition on the likelihood ratio  $\frac{f(\cdot|\bar{\kappa})}{f(\cdot|\underline{\kappa})}$  to ensure that there is a unique threshold  $\bar{\chi}$  such that the consumer makes her decision contingent on information if and only if  $\chi < \bar{\chi}$ . With this, we are able to prove the analogue of Lemma 1.

Given the consumer's optimal strategy, we proceed to the designer's problem. For this, we first compute the probability of having a product with attributes  $(y, \kappa)$  accepted, which is the same as in the baseline model, given by (14)-(16), except that now the noise of the signal is given by  $z(\chi)$  rather than  $\chi$ . We can then study the designer's complexification strategy. To do this, it is straightforward to prove Proposition 2, where  $\hat{\chi}$  is now defined by:

$$\int_0^{\hat{\chi}} z(\chi) \cdot (f(\chi|\underline{\kappa}) - f(\chi|\bar{\kappa})) \cdot d\chi = 0. \quad (31)$$

Finally, it is also clear that the designer's optimal quality strategy continues to be characterized by Proposition 3. We have thus shown that the optimal strategies of the consumer and of the designer qualitatively coincide with those in the baseline model. As a final step, we show that an equilibrium with positive trade exists, and that it shares the same broad features as our baseline equilibrium. To do so, however, we need to impose a regularity condition on the cost of information acquisition, which we explain below.

Recall that now the consumer's choice of information acquisition,  $z(\chi)$ , varies with prior belief  $\mu$ . This adds an additional consideration that was absent in our baseline model; namely,

that now not only the consumer's threshold,  $\bar{\chi}$ , but also the designer's threshold,  $\hat{\chi}$ , given by (31), change with  $\mu$ . Recall from the discussion following Proposition 4 that understanding how the ranking between  $\bar{\chi}$  and  $\hat{\chi}$  depends on the prior belief  $\mu$  is essential for characterizing the designer's complexification strategy that is consistent with an equilibrium belief  $\mu$ . In the baseline model, monotonicity of  $\bar{\chi} - \hat{\chi}$  was ensured because  $\bar{\chi}$  was monotonic in  $\mu$  and  $\hat{\chi}$  was independent of it. This monotonicity property does not come for free in the current setting, but we recover it by imposing a regularity condition on the cost function so that  $\bar{\chi}$  is more sensitive to changes in  $\mu$  than  $\hat{\chi}$ . With this, we establish the following result.

**Proposition 10** *An equilibrium with positive trade exists. In it, the designer produces a G-product with probability  $\mu^* \in (0, 1)$  and there exist thresholds  $0 < \tilde{\mu}_1 < \tilde{\mu}_2 < \tilde{\mu}_3 < \tilde{\mu}_4 < 1$  such that:*

1. *If  $\mu^* \in (0, \tilde{\mu}_1]$ , all products are simplified,  $\sigma_G = \sigma_B = 0$ .*
2. *If  $\mu^* \in (\tilde{\mu}_1, \tilde{\mu}_2]$ , G-products are simplified,  $\sigma_G = 0$ , and B-products complexified with probability  $\sigma_B \in (0, 1)$ .*
3. *If  $\mu^* \in (\tilde{\mu}_2, \tilde{\mu}_3]$ , G-products are simplified,  $\sigma_G = 0$ , and B-products complexified,  $\sigma_B = 1$ .*
4. *If  $\mu^* \in (\tilde{\mu}_3, \tilde{\mu}_4)$ , G-products are complexified with probability  $\sigma_G \in \{0, \tilde{\sigma}, 1\}$  for some  $\tilde{\sigma} \in (0, 1)$ , and B-products complexified,  $\sigma_B = 1$ .*
5. *If  $\mu^* \in [\tilde{\mu}_4, 1)$ , all products are complexified,  $\sigma_G = \sigma_B = 1$ .*

This result states that the structure of the equilibrium of the model with convex costs of information is effectively the same as that in our baseline model, as summarized by Propositions 4 and 5. The main difference is that we can no longer ensure uniqueness of equilibrium, which was useful for obtaining sharp comparative statics results (Section 4).

## 5.2 Rational Inattention

We also consider an even more general information acquisition problem, by supposing that the consumer can choose how much uncertainty about the product quality to reduce, subject to an entropy-reduction cost, where entropy measures the consumer's uncertainty (Sims, 2003). Thus, a product is more complex if it has a higher entropy-reduction cost. Although this approach allows for a more flexible information acquisition technology, it has the drawback that we can no longer obtain as sharp of an equilibrium characterization as in our baseline specification or as in the previous section. Nevertheless, we argue next that the model's main mechanisms remain robust to this alternative specification.

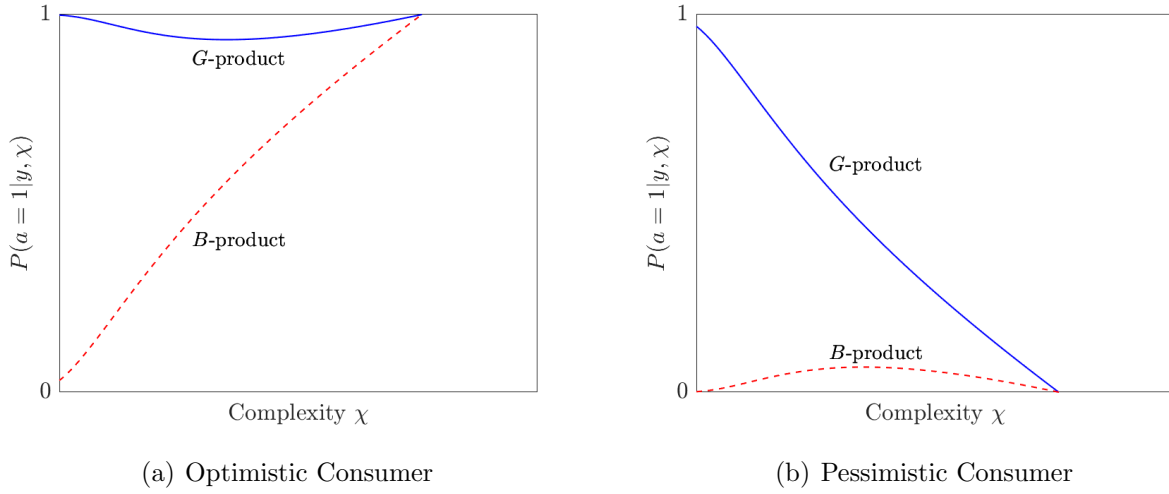


Figure 7: Illustrates the probability of acceptance of a  $y$ -product as a function of the product’s complexity,  $\chi$ .

Since the consumer’s action is binary, i.e., accept or reject, it is without loss of generality to focus on binary signals  $S \in \{b, g\}$  (Woodford, 2009; Yang, 2015), where the consumer accepts the product if and only if she receives a  $g$  signal. Thus, the main difference from the analysis in Section 5.1 is that now the consumer’s signal need not be symmetric, as the consumer may allocate “precision” optimally between the  $g$  and the  $b$  signals, trading off the costs of rejecting a  $G$ -product (type I error) with the costs of accepting a  $B$ -product (type II error).

Equipped with the optimal information structure, we compute the probability of acceptance of a  $y$ -product, as it depends on the product’s complexity,  $\chi$ . These probabilities are depicted in Figure 7, which we can see closely resemble those in our baseline model (see Figure 2). When the product’s complexity is low, the consumer extracts an informative signal and makes her decision contingent on its realization. Otherwise, the consumer accepts the product with probability one if she is optimistic, and she rejects it with probability one if she is pessimistic.

Although a full analytical characterization of the equilibrium set is difficult to obtain, we check (numerically) that it resembles closely that of our baseline model. Figure 8(a) depicts the complexification strategy  $\{\sigma_y\}$  of the designer that is consistent with an equilibrium prior belief  $\mu$ . And, Figure 8(b) depicts the designer’s net payoff from producing a  $G$ - versus a  $B$ -product as it depends on  $\mu$ . Thus, and in line with the results in Proposition 10, the right panel determines the expected product quality,  $\mu = \psi$ , whereas the left panel determines the equilibrium complexification of a  $y$ -product, given that the consumer’s prior belief is  $\mu = \psi$ .

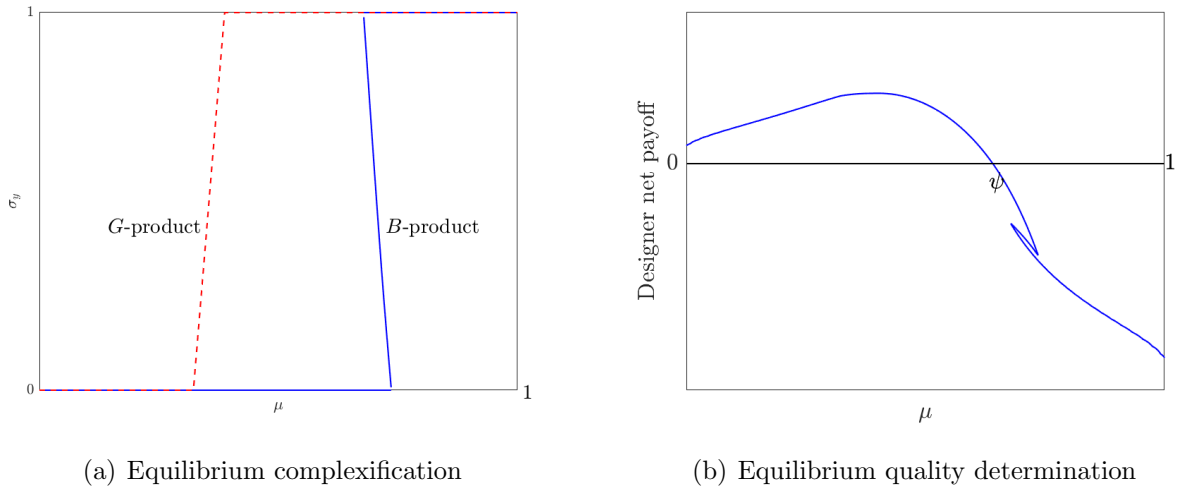


Figure 8: The left panel illustrates how the complexification strategy of the designer who produces a  $y$ -product varies with equilibrium belief  $\mu$ . The right panel illustrates the designer’s net payoff from choosing the  $G$ -product, given belief  $\mu$ . Note that equilibrium quality  $\psi$  is set so that the net payoff is equal to zero.

### 5.3 Prices and Production Costs

For some applications, it is natural to assume that a designer not only proposes a product to the consumer but that he also sets a price that is observable to the consumer. To analyze the role of such transfers in our environment, we modify the agents’ payoff as follows. If a product is accepted, the designer’s payoff is given by the price he charges the consumer minus the cost of production,  $p - c(y)$ .<sup>23</sup> In turn, the consumer’s payoff from accepting a  $y$ -product is given by her valuation minus the price she pays,  $\tilde{w}(y) - p$ . As before, the consumer’s outside option is given by  $w_0$ . The following assumption replaces Assumption 1.

**Assumption 3** *The payoffs satisfy the following properties:*

1.  $\tilde{w}(G) - c(G) > w_0 > \tilde{w}(B) - c(B)$ , with  $w_0 \geq 0$ .
2.  $c(G) > c(B) \geq 0$ .

The first assumption states that  $G$ -products are efficient to produce, whereas  $B$ -products are not. The second assumption states that  $G$ -products are costlier to produce than  $B$ -products.

As prices are set by the designer, the consumer makes inferences not only from the product’s complexity,  $\chi$ , but also from its price,  $p$ ; so the consumer’s posterior belief is now denoted by  $\mu(s, \chi, p)$ . It is easy to see that the consumer will accept the product if and only if her

<sup>23</sup>We assume that the cost of production is incurred upon product acceptance in order to stay close to the payoff structure of our baseline model.

posterior belief is greater than a price-adjusted relative outside option:

$$\mu(s, \chi, p) \geq \frac{\tilde{w}_0 - \tilde{w}(B) + p}{\tilde{w}(G) - \tilde{w}(B)}. \quad (32)$$

Given the consumer's acceptance strategy, the designer chooses  $\{y, \kappa, p\}$  to maximize his expected payoff:

$$\mathbb{P}(a = 1|y, \kappa, p) \cdot (p - c(y)). \quad (33)$$

For simplicity, we focus on equilibria in which the designer has a pure strategy over the price. The following proposition summarizes the main results of this section.

**Proposition 11** *In any positive trade equilibrium, the price set by the designer is independent of the product's quality. Moreover, any price  $p^* \in (c(G), \tilde{w}(G) - w_0)$  is consistent with equilibrium. The expected product quality and complexity are determined as in the baseline model with payoffs given by  $w(y) \equiv \tilde{w}(y) - p^*$  and  $v(y) \equiv p^* - c(y)$ .*

The result that separation through prices is not possible is intuitive. As a  $B$ -product is cheaper to produce, the designer of such a product is willing to set any price the  $G$ -product designer is willing to set. As a result, the  $B$ -product designer always mimics the pricing strategy of a  $G$ -product designer in order to avoid being identified. Due to the freedom in specifying off equilibrium beliefs, multiple prices can be supported as an equilibrium. The bounds on possible prices are due to the fact that in any positive trade equilibrium a  $G$ -product designer will not post a price below his cost of production, or a price high enough for the product to be rejected with probability one.

## 6 Concluding Remarks

In this paper, we explore the incentives of product designers to produce complex products, and the resulting implications for overall product quality. Our novel framework examines the joint determination of a product's quality and its complexity in a setting with only rational agents. We view our approach and our results as complementary to those studied by the literature on obfuscation and shrouded attributes.

We find that product complexity is not necessarily a feature of low quality products. In particular, complexification or simplification may be used strategically by designers of both high and low quality products. Exploring the model's implications, we highlight the importance of understanding the underlying drivers of product heterogeneity for deriving empirical predictions regarding the relationship between product quality and complexity.

We focus our model's implications on two domains in which increasing complexity has received close scrutiny: financial products and regulatory policies. We provide a new rationale for the observed proliferation of complex financial products and regulatory policies. In the context of the financial products' industry, we argue that high demand for safe assets may have been an important driver of the increasing complexity and worsening quality of structured products. In the context of regulatory design, we argue that increased complexity and worsening quality of regulatory proposals may be driven by high urgency of passing regulatory reform or high cost of inaction (of maintaining the status quo).

Finally, our model contributes to the policy debate on whether policymakers should be concerned by rising complexity and whether they should act towards reducing it. If the policymakers' worries are about the effect of complexity on the quality of products offered to consumers, then our results help isolate which features of the environment should be monitored for signs that rising complexity will lead to lower quality.

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## A Proofs for Sections 2-4

**Lemma A.1** *Suppose that Condition 3.1 holds. Then, in equilibrium, (i) the posterior belief  $\mu(g, \chi)$  is increasing in  $\chi$ , (ii) the posterior belief  $\mu(b, \chi)$  is increasing in  $\chi$ , and (iii) the threshold  $\bar{\chi}$  defined in Definition 1 is unique.*

**Proof.** From (9), the posterior belief  $\mu(b, \chi)$  is increasing in  $\chi$  if and only if the likelihood ratio  $\frac{\sigma_G f(\chi|\bar{\kappa}) + (1 - \sigma_G) f(\chi|\underline{\kappa})}{\sigma_B f(\chi|\bar{\kappa}) + (1 - \sigma_B) f(\chi|\underline{\kappa})} \cdot \frac{\chi}{1 - \chi}$  is increasing in  $\chi$ . But the latter follows from MLRP and Condition 3.1. Analogously, the posterior belief  $\mu(g, \chi)$  is decreasing in  $\chi$  if and only if the likelihood ratio  $\frac{\sigma_B f(\chi|\bar{\kappa}) + (1 - \sigma_B) f(\chi|\underline{\kappa})}{\sigma_G f(\chi|\bar{\kappa}) + (1 - \sigma_G) f(\chi|\underline{\kappa})} \cdot \frac{\chi}{1 - \chi}$  is increasing in  $\chi$ . But the latter also follows from MLRP and Condition 3.1. Finally, the uniqueness of the threshold  $\bar{\chi}$  follows from the monotonicity of the posteriors combined with the facts that  $\mu(g, 0) = 1$ ,  $\mu(b, 0) = 0$ , and  $\mu(g, \frac{1}{2}) = \mu(b, \frac{1}{2}) = \mu(\frac{1}{2}) \in (0, 1)$ . ■

We will use the result in Lemma A.1 in the proofs that follow.

**Proof of Proposition 1.** Observe that in any equilibrium a  $G$ -product must be accepted with probability 1, since the  $G$ -product designer can always choose  $\underline{\kappa}$  and effectively reveal the product's quality to the consumer, and the consumer would accept it since  $w(G) > w_0$ .

Next, let  $\kappa_y$  denote the complexification choice of the  $y$ -product designer, and suppose for contradiction that in equilibrium the designer of a  $G$ -product chooses  $\kappa_G = \bar{\kappa}$  with positive probability. Since the product has to be accepted with probability 1, it must be that the consumer accepts it independently of her signal; thus, the consumer will not acquire costly information. But then, the designer of a  $B$ -product can also set  $\kappa_B = \bar{\kappa}$  and get his product accepted with probability one. If in equilibrium, however, both  $G$ - and  $B$ -products were accepted with probability 1, then the designer would only produce a  $B$ -product, since he gets a higher payoff with that product,  $v(B) > v(G)$ . But then, the consumer would reject all products with probability one since  $w_0 > w(B)$ , a contradiction.

Therefore, in any equilibrium, the designer of a  $G$ -product must choose  $\kappa_G = \underline{\kappa}$  and perfectly reveal his product quality to the consumer. Hence, there does not exist an equilibrium in which the designer produces a  $B$ -product with positive probability, since then the consumer would find this out and would reject such a product with probability one. ■

**Proof of Lemma 1.** See text. ■

**Proof of Proposition 2.** We begin by studying the designer's optimal choice of  $\kappa$  in the case when the consumer is optimistic (see Definition 1).

*Case 1 (consumer is optimistic).* In this case, the designer's product is accepted with probability 1 when complexity is high enough,  $\chi \geq \bar{\chi}$ . So, his optimal choice of  $\kappa$  solves:

$$\max_{\kappa \in \{\underline{\kappa}, \bar{\kappa}\}} \int_0^{\bar{\chi}} P(S = g|y) \cdot f(\chi|\kappa) d\chi + \int_{\bar{\chi}}^{1/2} f(\chi|\kappa) d\chi. \quad (34)$$

Thus, it is optimal for the designer of  $B$ -product to choose  $\bar{\kappa}$  if

$$\int_0^{\bar{\chi}} \chi \cdot f(\chi|\bar{\kappa}) d\chi + \int_{\bar{\chi}}^{1/2} f(\chi|\bar{\kappa}) d\chi \geq \int_0^{\bar{\chi}} \chi \cdot f(\chi|\underline{\kappa}) d\chi + \int_{\bar{\chi}}^{1/2} f(\chi|\underline{\kappa}) d\chi, \quad (35)$$

and it is uniquely optimal if the inequality is strict. This is equivalent to:

$$\int_0^{\bar{\chi}} (1 - \chi) \cdot (f(\chi|\underline{\kappa}) - f(\chi|\bar{\kappa}))d\chi \geq 0. \quad (36)$$

But, note that we have:

$$\int_0^{\bar{\chi}} (1 - \chi) \cdot (f(\chi|\underline{\kappa}) - f(\chi|\bar{\kappa}))d\chi > (1 - \bar{\chi})(F(\bar{\chi}|\underline{\kappa}) - F(\bar{\chi}|\bar{\kappa})) > 0 \quad (37)$$

for  $\bar{\chi} > 0$ , as will be the case in any equilibrium. Thus, condition (35) is satisfied with strict inequality, and it is uniquely optimal for the designer of the  $B$ -product to choose  $\bar{\kappa}$ .

On the other hand, it is optimal for the designer of  $G$ -product to choose  $\bar{\kappa}$  if

$$\int_0^{\bar{\chi}} (1 - \chi) \cdot f(\chi|\bar{\kappa})d\chi + \int_{\bar{\chi}}^{1/2} f(\chi|\bar{\kappa})d\chi \geq \int_0^{\bar{\chi}} (1 - \chi) \cdot f(\chi|\underline{\kappa})d\chi + \int_{\bar{\chi}}^{1/2} f(\chi|\underline{\kappa})d\chi, \quad (38)$$

and it is uniquely optimal if the inequality is strict. This is equivalent to:

$$\int_0^{\bar{\chi}} \chi \cdot (f(\chi|\underline{\kappa}) - f(\chi|\bar{\kappa}))d\chi \geq 0. \quad (39)$$

Condition (38) is satisfied if  $\bar{\chi} \leq \hat{\chi}$ , and holds with strict inequality if  $\bar{\chi} < \hat{\chi}$ . Thus, if  $\bar{\chi} < \hat{\chi}$ , it is uniquely optimal for the designer of  $G$ -product to choose  $\bar{\kappa}$ . Otherwise, if  $\bar{\chi} = \hat{\chi}$ , the designer is indifferent to the choice of  $\kappa$ , and if  $\bar{\chi} > \hat{\chi}$ , it is uniquely optimal to choose  $\underline{\kappa}$ .

Next, we study the designer's choice of  $\kappa$  in the case when the consumer is pessimistic.

*Case 2 (consumer is pessimistic).* In this case, the designer's product is rejected if complexity is too high,  $\chi > \bar{\chi}$ . So, his optimal choice of  $\kappa$  solves:

$$\max_{\kappa \in \{\underline{\kappa}, \bar{\kappa}\}} \int_0^{\bar{\chi}} P(S = g|y) \cdot f(\chi|\kappa)d\chi. \quad (40)$$

Thus, it is optimal for the designer of  $B$ -product to choose  $\underline{\kappa}$  if

$$\int_0^{\bar{\chi}} \chi \cdot f(\chi|\bar{\kappa})d\chi \leq \int_0^{\bar{\chi}} \chi \cdot f(\chi|\underline{\kappa})d\chi, \quad (41)$$

and it is uniquely optimal if the inequality is strict. This is equivalent to:

$$\int_0^{\bar{\chi}} \chi \cdot (f(\chi|\underline{\kappa}) - f(\chi|\bar{\kappa}))d\chi \geq 0. \quad (42)$$

Condition (41) is satisfied if  $\bar{\chi} \leq \hat{\chi}$ , and holds with strict inequality if  $\bar{\chi} < \hat{\chi}$ . Thus, if  $\bar{\chi} < \hat{\chi}$ , it is uniquely optimal for the designer of  $B$ -product to choose  $\underline{\kappa}$ . Otherwise, if  $\bar{\chi} = \hat{\chi}$ , the designer is indifferent to the choice of  $\kappa$ , and if  $\bar{\chi} > \hat{\chi}$ , it is uniquely optimal to choose  $\bar{\kappa}$ .

On the other hand, it is optimal for the designer of the  $G$ -product to choose  $\underline{\kappa}$  if

$$\int_0^{\bar{\chi}} (1 - \chi) \cdot f(\chi|\bar{\kappa})d\chi \leq \int_0^{\bar{\chi}} (1 - \chi) \cdot f(\chi|\underline{\kappa})d\chi, \quad (43)$$

and it is uniquely optimal if the inequality is strict. This is equivalent to:

$$\int_0^{\bar{\chi}} (1 - \chi) \cdot (f(\chi|\underline{\kappa}) - f(\chi|\bar{\kappa}))d\chi \geq 0. \quad (44)$$

Re-writing the above condition, we have

$$\int_0^{\bar{\chi}} (f(\chi|\underline{\kappa}) - f(\chi|\bar{\kappa}))d\chi > \int_0^{\bar{\chi}} \chi \cdot (f(\chi|\underline{\kappa}) - f(\chi|\bar{\kappa}))d\chi, \quad (45)$$

which immediately implies that condition (43) is satisfied for all  $\bar{\chi} > 0$ , as will be the case in any equilibrium, and it is uniquely optimal for the designer of the  $G$ -product to choose  $\underline{\kappa}$ . ■

**Proof of Proposition 3.** The proof is straightforward. ■

**Proof of Proposition 4.** Suppose that, in equilibrium, the consumer's belief that the designer has produced a  $G$ -product is  $\mu \in (0, 1)$ .

*Pooling on  $\kappa$ .* Consider first the candidate equilibrium in which  $\sigma_B = \sigma_G = 0$ . By Proposition 2, this requires that  $\mu \leq \omega$  and  $\bar{\chi} \leq \hat{\chi}$ . On equilibrium path, the consumer does not update upon observing  $\chi$  and, thus, threshold  $\bar{\chi}$  is given by  $\mu(g, \bar{\chi}) = \omega$ , which is equivalent to:

$$\bar{\chi} = \frac{(1 - \omega) \cdot \mu}{(1 - \omega) \cdot \mu + \omega \cdot (1 - \mu)}. \quad (46)$$

This is an equilibrium if and only if  $\bar{\chi} \leq \hat{\chi}$ , which is equivalent to:

$$\mu \leq \frac{\omega \cdot \frac{\hat{\chi}}{1 - \hat{\chi}}}{\omega \cdot \frac{\hat{\chi}}{1 - \hat{\chi}} + 1 - \omega} \equiv \mu_1. \quad (47)$$

Consider next the candidate equilibrium in which  $\sigma_B = \sigma_G = 1$ . By Proposition 2, this requires that  $\mu \geq \omega$  and  $\bar{\chi} \leq \hat{\chi}$ . On equilibrium path, the consumer does not update upon observing  $\chi$  and, thus, threshold  $\bar{\chi}$  is given by  $\mu(b, \bar{\chi}) = \omega$ , which is equivalent to:

$$\bar{\chi} = \frac{\omega \cdot (1 - \mu)}{(1 - \omega) \cdot \mu + \omega \cdot (1 - \mu)}. \quad (48)$$

This is an equilibrium if and only if  $\bar{\chi} \leq \hat{\chi}$ , which is equivalent to:

$$\mu \geq \frac{\omega \cdot \frac{1 - \hat{\chi}}{\hat{\chi}}}{\omega \cdot \frac{1 - \hat{\chi}}{\hat{\chi}} + 1 - \omega} \equiv \mu_3. \quad (49)$$

Therefore,  $\sigma_B = \sigma_G = 0$  is an equilibrium if and only if  $\mu \in (0, \mu_1]$ , whereas  $\sigma_B = \sigma_G = 1$  is an equilibrium if and only if  $\mu \in [\mu_3, 1)$ .

*Separation on  $\kappa$ .* Consider the candidate equilibrium in which  $\sigma_B = 1$  and  $\sigma_G = 0$ . There are two cases to consider, depending on whether the consumer is optimistic or pessimistic.

First, suppose that

$$\mu \left( g, \frac{1}{2} \right) = \mu \left( b, \frac{1}{2} \right) = \frac{\mu}{\mu + (1 - \mu) \cdot \ell \left( \frac{1}{2} \right)} \leq \omega, \quad (50)$$

where  $\ell(\cdot) = \frac{f(\cdot|\bar{\kappa})}{f(\cdot|\underline{\kappa})}$ . Then, the consumer is pessimistic (see Definition 1). On equilibrium path, the consumer updates upon observing  $\chi$ , and thus threshold  $\bar{\chi}$  is given by

$$\mu(g, \bar{\chi}) = \frac{\mu}{\mu + (1 - \mu) \cdot \ell(\bar{\chi}) \cdot \frac{\bar{\chi}}{1 - \bar{\chi}}} = \omega. \quad (51)$$

This is an equilibrium if and only if  $\bar{\chi} \geq \hat{\chi}$ , i.e.

$$\mu_2 \equiv \frac{\omega \cdot \ell(\hat{\chi}) \cdot \frac{\hat{\chi}}{1 - \hat{\chi}}}{\omega \cdot \ell(\hat{\chi}) \cdot \frac{\hat{\chi}}{1 - \hat{\chi}} + 1 - \omega} \leq \mu \leq \frac{\omega \cdot \ell \left( \frac{1}{2} \right)}{\omega \cdot \ell \left( \frac{1}{2} \right) + 1 - \omega} \equiv \tilde{\mu}. \quad (52)$$

Second, suppose that

$$\mu \left( g, \frac{1}{2} \right) = \mu \left( b, \frac{1}{2} \right) = \frac{\mu}{\mu + (1 - \mu) \cdot \ell \left( \frac{1}{2} \right)} > \omega, \quad (53)$$

Then, the consumer is optimistic. The threshold  $\bar{\chi}$  is now given by

$$\mu(b, \bar{\chi}) = \frac{\mu}{\mu + (1 - \mu) \cdot \ell(\bar{\chi}) \cdot \frac{1 - \bar{\chi}}{\bar{\chi}}} = \omega. \quad (54)$$

This is an equilibrium if and only if  $\bar{\chi} \geq \hat{\chi}$ , i.e.

$$\tilde{\mu} = \frac{\omega \cdot \ell \left( \frac{1}{2} \right)}{\omega \cdot \ell \left( \frac{1}{2} \right) + 1 - \omega} < \mu \leq \frac{\omega \cdot \ell(\hat{\chi}) \cdot \frac{1 - \hat{\chi}}{\hat{\chi}}}{\omega \cdot \ell(\hat{\chi}) \cdot \frac{1 - \hat{\chi}}{\hat{\chi}} + 1 - \omega} \equiv \mu_4. \quad (55)$$

Therefore,  $\sigma_B = 0$  and  $\sigma_G = 1$  is an equilibrium if and only if  $\mu \in [\mu_2, \mu_4]$ .

*Semi-separation on  $\kappa$ .* Consider the candidate equilibrium, in which  $\sigma_B \in (0, 1)$ . By Proposition 2, such an equilibrium requires that the consumer be pessimistic and so  $\sigma_G = 0$ . On equilibrium path, there is updating from observing  $\chi$ , and threshold  $\bar{\chi}$  must equal  $\hat{\chi}$  so that the designer of  $B$ -product is indifferent to the choice of  $\kappa$  (Proposition 2) and is willing to mix:

$$\mu(g, \hat{\chi}) = \frac{\mu}{\mu + (1 - \mu) \cdot (\sigma_B \cdot \ell(\hat{\chi}) + 1 - \sigma_B) \cdot \frac{\hat{\chi}}{1 - \hat{\chi}}} = \omega, \quad (56)$$

which in turn implies that:

$$\sigma_B = \frac{\frac{1 - \hat{\chi}}{\hat{\chi}} \cdot \frac{\mu}{1 - \mu} \cdot \frac{1 - \omega}{\omega} - 1}{\frac{f(\hat{\chi}|\bar{\kappa})}{f(\hat{\chi}|\underline{\kappa})} - 1}. \quad (57)$$

Since the posterior belief  $\mu(g, \hat{\chi})$  is continuous and decreasing in  $\sigma_B$  (MLRP implies that  $\ell(\hat{\chi}) > 1$ ), this equilibrium exists if and only if:

$$\mu(g, \hat{\chi})|_{\sigma_B=1} < \omega < \mu(g, \hat{\chi})|_{\sigma_B=0}, \quad (58)$$

which is equivalent to:

$$\mu_1 = \frac{\omega \cdot \frac{\hat{\chi}}{1-\hat{\chi}}}{\omega \cdot \frac{\hat{\chi}}{1-\hat{\chi}} + 1 - \omega} < \mu < \frac{\omega \cdot \ell(\hat{\chi}) \cdot \frac{\hat{\chi}}{1-\hat{\chi}}}{\omega \cdot \ell(\hat{\chi}) \cdot \frac{\hat{\chi}}{1-\hat{\chi}} + 1 - \omega} = \mu_2. \quad (59)$$

Therefore,  $\sigma_G = 0$  and  $\sigma_B \in (0, 1)$  is an equilibrium if and only if  $\mu \in (\mu_1, \mu_2)$ .

Consider the candidate equilibrium in which  $\sigma_G \in (0, 1)$ . By Proposition 2, such an equilibrium requires that the consumer be optimistic and so  $\sigma_B = 1$ . On equilibrium path, there is updating from observing  $\chi$ , and threshold  $\bar{\chi}$  must equal  $\hat{\chi}$  so that the designer of  $G$ -product is indifferent to the choice of  $\kappa$  and is willing to mix:

$$\mu(b, \hat{\chi}) = \frac{\mu}{\mu + (1 - \mu) \cdot \frac{1-\hat{\chi}}{\hat{\chi}} \cdot \frac{1}{\sigma_G + (1-\sigma_G) \cdot \ell(\hat{\chi})^{-1}}} = \omega. \quad (60)$$

which in turn implies that

$$\sigma_G = 1 - \frac{1 - \frac{1-\hat{\chi}}{\hat{\chi}} \cdot \frac{1-\mu}{\mu} \cdot \frac{\omega}{1-\omega}}{1 - \frac{f(\hat{\chi}|\bar{\kappa})}{f(\hat{\chi}|\bar{\kappa})}}. \quad (61)$$

Since the posterior belief  $\mu(b, \hat{\chi})$  is continuous and increasing in  $\sigma_G$ , this equilibrium exists if and only if:

$$\mu(b, \hat{\chi})|_{\sigma_G=0} < \omega < \mu(b, \hat{\chi})|_{\sigma_G=1}, \quad (62)$$

which is equivalent to:

$$\mu_3 = \frac{\omega \cdot \frac{1-\hat{\chi}}{\hat{\chi}}}{\omega \cdot \frac{1-\hat{\chi}}{\hat{\chi}} + 1 - \omega} < \mu < \frac{\omega \cdot \ell(\hat{\chi}) \cdot \frac{1-\hat{\chi}}{\hat{\chi}}}{\omega \cdot \ell(\hat{\chi}) \cdot \frac{1-\hat{\chi}}{\hat{\chi}} + 1 - \omega} = \mu_4. \quad (63)$$

Therefore,  $\sigma_G \in (0, 1)$  and  $\sigma_B = 1$  is an equilibrium if and only if  $\mu \in (\mu_3, \mu_4)$ .

We have thus characterized all the possible equilibrium  $\{\sigma_y\}$ , as a function of belief  $\mu$ :

1. If  $\mu \in (0, \mu_1]$ , then  $\sigma_G = \sigma_B = 0$ .
2. If  $\mu \in (\mu_1, \mu_2)$ , then  $\sigma_G = 0$  and  $\sigma_B = \frac{\frac{1-\hat{\chi}}{\hat{\chi}} \cdot \frac{1-\mu}{\mu} \cdot \frac{1-\omega}{\omega} - 1}{\frac{f(\hat{\chi}|\bar{\kappa})}{f(\hat{\chi}|\bar{\kappa})} - 1}$ .
3. If  $\mu \in [\mu_2, \mu_3]$ , then  $\sigma_G = 0$  and  $\sigma_B = 1$ .
4. If  $\mu \in (\mu_3, \mu_4)$ , then  $\sigma_G \in \left\{ 0, 1 - \frac{1 - \frac{1-\hat{\chi}}{\hat{\chi}} \cdot \frac{1-\mu}{\mu} \cdot \frac{\omega}{1-\omega}}{1 - \frac{f(\hat{\chi}|\bar{\kappa})}{f(\hat{\chi}|\bar{\kappa})}} \right\}$  and  $\sigma_B = 1$ .
5. If  $\mu \in [\mu_4, 1]$ , then  $\sigma_G = \sigma_B = 1$ .

This establishes the stated result. ■

**Proof of Proposition 5.** The designer's net expected payoff from choosing a  $G$ -product relative to a  $B$ -product is defined in (19). Define the correspondence  $\Gamma : [0, 1] \rightarrow 2^{\mathbb{R}}$ , where  $\Gamma(\mu)$  is the set of designer net payoffs  $\gamma(\mu)$  implied by all the possible  $\{\sigma_y\}$ , given in Proposition 4, which are consistent with the consumer's prior belief  $\mu$ , where we now make explicit the dependence of the net payoff on the consumer's belief  $\mu$ .

First, note that  $0 \in \Gamma(0)$ , and that  $\Gamma(0)$  is a singleton, since the product is rejected with probability one when the consumer's belief is  $\mu = 0$ . Second, consider  $\mu \in (0, 1]$ . Note that  $\Gamma(\mu)$  is a singleton for  $\mu \notin (\mu_3, \mu_4)$ , since  $\{\sigma_y\}$  corresponding to such  $\mu$  are unique. On the other hand,  $\Gamma(\mu)$  consists of three elements if  $\mu \in (\mu_3, \mu_4)$ , since then either (i)  $\sigma_G = 0$  and  $\sigma_B = 1$ , (ii)  $\sigma_G \in (0, 1)$  and  $\sigma_B = 1$ , or (iii)  $\sigma_G = 1$  and  $\sigma_B = 1$ . We consider each case next.

*Case  $\mu \in (0, \mu_1]$ .* By Proposition 4, equilibrium must have  $\sigma_G = \sigma_B = 0$ , and it must be that the consumer is pessimistic, since  $\mu_1 < \tilde{\mu}$ . Furthermore,  $\Gamma(\mu)$  is a singleton with:

$$\gamma(\mu) = v(G) \cdot \int_0^{\bar{\chi}(\mu)} (1 - \chi) f(\chi|\underline{\kappa}) d\chi - v(B) \cdot \int_0^{\bar{\chi}(\mu)} \chi f(\chi|\underline{\kappa}) d\chi, \quad (64)$$

since the product is rejected whenever  $\chi > \bar{\chi}(\mu)$ . Therefore:

$$\gamma'(\mu) = [v(G) - (v(G) + v(B)) \cdot \bar{\chi}(\mu)] \cdot f(\chi|\underline{\kappa}) \cdot \frac{d\bar{\chi}(\mu)}{d\mu}. \quad (65)$$

where  $\bar{\chi}(\mu)$  is given by (46) and, thus, satisfies  $\frac{d\bar{\chi}(\mu)}{d\mu} > 0$ ,  $\bar{\chi}(0) = 0$ , and  $\bar{\chi}(\mu_1) = \hat{\chi}$ . As a result, for  $\mu$  sufficiently small,  $\gamma'(\mu) > 0$  and  $\gamma(\mu) > 0$ . Next, consider the value  $\mu_v$  such that:

$$\bar{\chi}(\mu_v) = \frac{v(G)}{v(G) + v(B)} \implies \mu_v \equiv \frac{v(G) \cdot \omega}{v(G) \cdot \omega + v(B) \cdot (1 - \omega)}. \quad (66)$$

If  $\mu_v > \mu_1$ , then  $\gamma'(\mu) > 0 \forall \mu \in (0, \mu_1)$ . Otherwise,  $\gamma'(\mu) > 0$  for  $\mu \in (0, \mu_v)$  and  $\gamma'(\mu) < 0$  for  $\mu \in (\mu_v, \mu_1)$ .

*Case  $\mu \in (\mu_1, \mu_2]$ .* Equilibrium must have  $\sigma_G = 0$  and  $\sigma_B \in (0, 1)$ , and it must be that the consumer is pessimistic, since  $\mu_2 < \tilde{\mu}$ . In this case,  $\bar{\chi}(\mu) = \hat{\chi}$  and  $\Gamma(\mu)$  is singleton with:

$$\gamma(\mu) = v(G) \cdot \int_0^{\hat{\chi}} (1 - \chi) f(\chi|\underline{\kappa}) d\chi - v(B) \cdot \int_0^{\hat{\chi}} \chi f(\chi|\underline{\kappa}) d\chi, \quad (67)$$

since the product is rejected whenever  $\chi > \bar{\chi}(\mu) = \hat{\chi}$ . It therefore follows that  $\gamma(\mu)$  is constant on interval  $(\mu_1, \mu_2)$  and equal to  $\gamma(\mu_1)$ .

*Case  $\mu \in (\mu_2, \mu_3]$ .* Equilibrium must have  $\sigma_G = 0$  and  $\sigma_B = 1$ . The consumer is pessimistic if  $\mu < \tilde{\mu}$ , and she is optimistic otherwise. Here, again  $\Gamma(\mu)$  is a singleton.

Suppose that  $\mu < \tilde{\mu}$ . Then, the consumer is still pessimistic, and we have:

$$\gamma(\mu) = v(G) \cdot \int_0^{\bar{\chi}(\mu)} (1 - \chi) f(\chi|\underline{\kappa}) d\chi - v(B) \cdot \int_0^{\bar{\chi}(\mu)} \chi f(\chi|\bar{\kappa}) d\chi, \quad (68)$$



since the product is rejected whenever  $\chi > \bar{\chi}(\mu)$ . Therefore:

$$\gamma'(\mu) = [v(G) \cdot (1 - \bar{\chi}(\mu)) \cdot f(\bar{\chi}(\mu)|\underline{\kappa}) - v(B) \cdot \bar{\chi}(\mu) \cdot f(\bar{\chi}(\mu)|\bar{\kappa})] \cdot \frac{d\bar{\chi}}{d\mu}. \quad (69)$$

where  $\bar{\chi}(\mu)$  is now given by (51). Since  $\bar{\chi}(\mu_2) = \hat{\chi}$ ,  $\gamma(\mu)$  is continuous at  $\mu_2$ . Furthermore,  $\gamma'(\mu) \geq 0$  iff

$$\bar{\chi}(\mu) \leq \frac{v(G)}{v(G) + v(B) \cdot \ell(\bar{\chi}(\mu))} \iff \frac{(1 - \mu) \cdot \omega}{\mu \cdot (1 - \omega)} \geq \frac{v(B)}{v(G)} \iff \mu \leq \mu_v, \quad (70)$$

with strict inequalities iff  $\mu < \mu_v$ . And, since  $\frac{(1 - \mu) \cdot \omega}{\mu \cdot (1 - \omega)} \geq \frac{v(B)}{v(G)}$  is decreasing in  $\mu$  and equal to  $\ell(\frac{1}{2}) < 1$  when  $\mu = \tilde{\mu}$ , it follows that  $\mu_v < \tilde{\mu}$  and thus  $\gamma'(\tilde{\mu}) < 0$ .

Suppose that  $\mu > \tilde{\mu}$ . Now, the consumer is optimistic and  $\bar{\chi}(\mu) \geq \hat{\chi}$  is given by (54), with  $\frac{d\bar{\chi}(\mu)}{d\mu} < 0$ . Therefore, we have:

$$\gamma(\mu) = v(G) \cdot \left[ \int_0^{\bar{\chi}(\mu)} (1 - \chi) f(\chi|\underline{\kappa}) d\chi + 1 - F(\bar{\chi}|\underline{\kappa}) \right] - v(B) \cdot \left[ \int_0^{\bar{\chi}(\mu)} \chi f(\chi|\bar{\kappa}) d\chi + 1 - F(\bar{\chi}|\bar{\kappa}) \right], \quad (71)$$

since the product is now accepted whenever  $\chi > \bar{\chi}(\mu)$ . Thus:

$$\gamma'(\mu) = [v(B) \cdot (1 - \bar{\chi}(\mu)) \cdot f(\bar{\chi}(\mu)|\bar{\kappa}) - v(G) \cdot \bar{\chi}(\mu) \cdot f(\bar{\chi}(\mu)|\underline{\kappa})] \cdot \frac{d\bar{\chi}}{d\mu} < 0, \quad (72)$$

where the inequality follows from the observation that  $\ell(\bar{\chi}(\mu)) \geq \ell(\hat{\chi}) > 1$ . Recall that  $\tilde{\mu}$  is the threshold between the region where the consumer is pessimistic and the region where she is optimistic. Since  $\bar{\chi}(\tilde{\mu}) = \frac{1}{2}$  and thus  $F(\bar{\chi}|\kappa) = 1$  for  $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$ , it is easy to check that  $\gamma(\mu)$  is continuous at  $\tilde{\mu}$ .

*Case  $\mu \in [\mu_4, 1]$ .* Equilibrium must have  $\sigma_G = \sigma_B = 1$ , and it must be that the consumer is optimistic, since  $\mu_4 > \tilde{\mu}$ . Here, again  $\Gamma(\mu)$  is a singleton. Moreover,  $\bar{\chi}(\mu)$  given by (48),  $\frac{d\bar{\chi}(\mu)}{d\mu} < 0$ , and:

$$\gamma(\mu) = v(G) \cdot \left[ \int_0^{\bar{\chi}(\mu)} (1 - \chi) f(\chi|\bar{\kappa}) d\chi + 1 - F(\bar{\chi}|\bar{\kappa}) \right] - v(B) \cdot \left[ \int_0^{\bar{\chi}(\mu)} \chi f(\chi|\bar{\kappa}) d\chi + 1 - F(\bar{\chi}|\bar{\kappa}) \right], \quad (73)$$

since the product is accepted whenever  $\chi > \bar{\chi}(\mu)$ . It follows that  $\gamma(\mu)$  is decreasing in  $\mu$  since:

$$\gamma'(\mu) = [v(B) \cdot (1 - \bar{\chi}) - v(G) \cdot \bar{\chi}] \cdot f(\chi|\bar{\kappa}) \cdot \frac{d\bar{\chi}}{d\mu} < 0. \quad (74)$$

Finally, note that  $\gamma(1) = v(G) - v(B) < 0$ .

*Case  $\mu \in (\mu_3, \mu_4)$ .* By Proposition 4, equilibrium must have either: (i)  $\sigma_G = 0$  and  $\sigma_B = 1$ ; (ii)  $\sigma_G \in (0, 1)$  and  $\sigma_B = 1$ ; and (iii)  $\sigma_G = \sigma_B = 1$ ; and it must be that the consumer is optimistic, since  $\mu_3 > \tilde{\mu}$ . Thus,  $\Gamma(\mu)$  consists of three elements, and we let  $\gamma^j(\mu) \in \Gamma(\mu)$  for

$j \in \{1, 2, 3\}$  denote the net expected payoff to the  $L$ -type of choosing  $G$ -product, when the equilibrium  $\{\sigma_y\}$  is in region (i), (ii), and (iii) respectively.

We have already shown that the functions  $\gamma^1(\mu)$  and  $\gamma^3(\mu)$  are decreasing in  $\mu$  (see Case  $\mu \in [\mu_2, \mu_3)$  when  $\mu > \tilde{\mu}$ , and Case  $\mu \in [\mu_4, 1]$ ). Let us consider  $\gamma^2(\mu)$ , which is given by:

$$\gamma^2(\mu) = v(G) - v(G) \cdot \int_0^{\hat{\chi}} \chi f(\chi|\bar{\kappa}) d\chi - v(B) \cdot \int_0^{\hat{\chi}} \chi f(\chi|\bar{\kappa}) d\chi - v(B) \cdot (1 - F(\hat{\chi}|\bar{\kappa})), \quad (75)$$

and is thus constant on the interval  $(\mu_3, \mu_4)$ . Furthermore, it is easy to check that  $\lim_{\mu \downarrow \mu_3} \gamma^1(\mu) = \gamma(\mu_3)$  where  $\gamma(\mu_3)$  is defined in Case  $\mu \in (\mu_2, \mu_3]$ ,  $\lim_{\mu \uparrow \mu_4} \gamma^1(\mu) = \lim_{\mu \uparrow \mu_4} \gamma^2(\mu) = \lim_{\mu \downarrow \mu_3} \gamma^2(\mu) = \lim_{\mu \downarrow \mu_3} \gamma^3(\mu)$ , and  $\lim_{\mu \uparrow \mu_4} \gamma^3(\mu) = \gamma(\mu_4)$  where  $\gamma(\mu_4)$  is defined in Case  $\mu \in [\mu_4, 1]$ .

Therefore, we have shown that (i)  $\Gamma(\mu)$  is a singleton for  $\mu \in [0, \mu_3]$ , with  $\lim_{\mu \rightarrow 0} \gamma(\mu) = \gamma(0) = 0$  where  $\gamma(\mu)$  is continuous, and increasing on  $[0, \mu_v]$ , but decreasing on  $[\mu_v, \mu_3]$ ; (ii)  $\Gamma(\mu)$  is a singleton, where  $\gamma(\mu)$  continuous and decreasing on  $[\mu_4, 1]$ , with  $\gamma(1) < 1$ ; (iii) finally,  $\Gamma(\mu)$  has three elements on  $(\mu_3, \mu_4)$ , where  $\{\gamma^j(\mu)\}$  are continuous and (weakly) decreasing, with  $\lim_{\mu \downarrow \mu_3} \gamma^1(\mu) = \gamma(\mu_3)$ ,  $\lim_{\mu \uparrow \mu_4} \gamma^1(\mu) = \lim_{\mu \uparrow \mu_4} \gamma^2(\mu)$ ,  $\lim_{\mu \downarrow \mu_3} \gamma^2(\mu) = \lim_{\mu \downarrow \mu_3} \gamma^3(\mu)$ ,  $\lim_{\mu \uparrow \mu_4} \gamma^3(\mu) = \gamma(\mu_4)$ , and where  $\gamma^2(\mu)$  is constant (see Figure 4 for illustration). Hence, it must be that (generically) there is a unique  $\mu$  on  $(0, 1)$ , denoted by  $\psi$ , such that  $0 \in \Gamma(\psi)$ . We conclude that there is (generically) a unique positive trade equilibrium. In it, the designer produces the  $G$ -product with probability  $\psi \in (0, 1)$  and his complexification strategy  $\{\sigma_y\}$  is given by Proposition 4, where the consumer's prior belief is  $\mu = \psi$ . ■

**Proof of Proposition 6.** By inspection of the designer's net payoff  $\gamma$  from producing a  $G$ -product relative to a  $B$ -product, we see that  $\mu$  and  $\omega$  only affect it through their effect on threshold complexity  $\bar{\chi}$  (see proof of Proposition 5), which determines whether the consumer's acceptance decision is contingent on the signal or not (see Lemma 1). As a result, any change in  $\omega$  must be fully offset by a corresponding change in  $\mu = \psi$  so as to keep the designer indifferent between producing a  $G$ - vs. a  $B$ -product. It is easy to check that an increase in  $\omega$  increases the thresholds  $\mu_1 - \mu_4$ . Therefore, in equilibrium, it must be that  $\psi$  increases in  $\omega$ , but the designer's complexification strategy  $\{\sigma_y\}$  does not change. As a result, an increase in  $\omega$  increases expected quality, and it decreases expected complexity since  $\sigma_G \leq \sigma_B$ . ■

**Proof of Proposition 7.** An increase in  $v(G)$  affects the equilibrium complexification strategy  $\{\sigma_y\}$  only to the extent that it affects the consumer's equilibrium belief  $\mu$  (see proof of Proposition 4 and note that the thresholds  $\mu_1 - \mu_4$  are independent of  $v(G)$ ). Now, consider the designer's net payoff  $\gamma$  from producing a  $G$ -product relative to producing a  $B$ -product, as given by:

$$\gamma = \max_{\kappa} \mathbb{P}(a = 1|G, \kappa; \{\sigma_y\}, \mu) \cdot v(G) - \max_{\kappa} \mathbb{P}(a = 1|B, \kappa; \{\sigma_y\}, \mu) \cdot v(B), \quad (76)$$

where we now make explicit that the equilibrium probability of acceptance of a  $y$ -product,  $\mathbb{P}(a = 1|y, \kappa; \{\sigma_y\}, \mu)$ , will depend on the equilibrium belief  $\mu$  and complexification strategy  $\{\sigma_y\}$ . By our previous argument, for a given belief  $\mu$ ,  $\max_{\kappa} \mathbb{P}(a = 1|y, \kappa; \{\sigma_y\}, \mu)$  is independent of  $v(G)$  and therefore  $\gamma$  must be increasing in  $v(G)$ . Thus, if equilibrium features  $\mu = \psi \notin [\mu_3, \mu_4]$ , it must be that  $\psi$  increases with  $v(G)$  (see proof of Proposition 5 and Figure

4). The same holds if  $\psi \in [\mu_3, \mu_4]$  and the change in  $v(G)$  is large enough so that the new equilibrium  $\psi$  is greater than  $\mu_4$ . In particular, it is easy to check that  $\mu = \psi$  goes to 1 as  $v(G)$  goes to  $v(B)$ . However, when  $\psi \in (\mu_3, \mu_4)$ , it is possible that an increase in  $v(G)$  implies that  $\psi$  falls as the equilibrium jumps from separation on  $\kappa$  to pooling at  $\kappa = \bar{\kappa}$  (see Figure 6). From Proposition 4, since the thresholds  $\mu_1 - \mu_4$  are unchanged, as  $\mu = \psi$  increases,  $\{\sigma_y\}$  increase. As a result, a large enough change in  $v(G)$  increases both expected quality and expected complexity, though locally the effect may be non-monotonic. ■

**Proof of Proposition 8.** Note that the aligned designer's net benefit from choosing the  $G$ -product is

$$\gamma = \max_{\kappa} \mathbb{P}(a = 1|G, \kappa) \cdot \bar{v}(G) - \max_{\kappa} \mathbb{P}(a = 1|B, \kappa) \cdot \bar{v}(B). \quad (77)$$

Since the probability of acceptance is always higher for a  $G$ -product, i.e.  $\mathbb{P}(a = 1|G, \kappa) \geq \mathbb{P}(a = 1|B, \kappa) > 0$  for all  $\kappa$  and  $\bar{v}(G) > \bar{v}(B)$ , the aligned designer produces a  $G$ -product with probability one. In turn, this designer's complexification strategy is given by  $\sigma_G$ , as characterized in Proposition 2. Thus, the presence of an aligned designer will affect the equilibrium outcomes only by affecting the probability of a  $G$ -product being produced, captured by the fact that belief consistency now requires that  $\mu = q + (1 - q) \cdot m$ .

Consider the case of  $q > \psi$ . First, note that in equilibrium  $\mu \geq q$ , since  $m \geq 0$ . Second, note that it must be that  $m = 0$ , since the misaligned designer's net payoff from producing a  $G$ -product relative to a  $B$ -product is always negative for  $\mu > \psi$  (see proof of Proposition 5). Thus, we have that in equilibrium  $\mu = q \geq \psi$  and, thus, expected quality is higher in the presence of an aligned designer. It follows from Proposition 4 that  $\{\sigma_y\}$  are higher as well, since the presence of an aligned designer simply increases  $\mu$ .

Next, consider the case of  $q \leq \psi$ . We now show that the equilibrium  $\mu$  and  $\{\sigma_y\}$  need not change in the presence of an aligned designer; their presence is simply offset by the misaligned designer producing a  $G$ -product with smaller probability. If  $q < \mu_3$ , then the misaligned designer's payoff from producing a  $G$ -product is strictly positive if equilibrium had  $\mu = q$  (see proof of Proposition 5 and Figure 4), which is inconsistent with an equilibrium; thus, it must be that the misaligned designer produces a  $G$ -product with positive probability  $m = \frac{\psi - q}{1 - q}$  so that the equilibrium belief is  $\mu = \psi$  and he is indifferent to producing a  $G$ - vs.  $B$ -product. Now, suppose that  $q \in [\mu_3, \psi]$ . If  $\min\{\gamma : \gamma \in \Gamma(\mu_3)\} > 0$ , then the equilibrium is as the one described above since the misaligned designer's net payoff from producing a  $G$ -product is still strictly positive if equilibrium had  $\mu = q$ , which cannot be consistent with equilibrium. If, instead,  $\min\{\gamma : \gamma \in \Gamma(\mu_3)\} \leq 0$ , then multiple equilibria exist. In particular, the equilibrium where the misaligned designer produces a  $G$ -product with probability  $m = \frac{\psi - q}{1 - q}$  (so that  $\mu = \psi$ ) still exists, since  $0 \in \Gamma(\psi)$ . However, there is also an equilibrium where  $\mu = q \in [\mu_3, \psi]$ , since there exists a  $\gamma < 0$  such that  $\gamma \in \Gamma(q)$ . ■

**Proof of Proposition 9.** For each  $U \in [0, w(G)]$ , consider map  $T_\beta : U \mapsto \mathbb{R}$  defined by:

$$T_\beta(U) = \mathbb{E} \left\{ \max_{a \in \{0,1\}} \{a \cdot (\mu(s, \chi) \cdot w(G) + (1 - \mu(s, \chi)) \cdot w(B)) + (1 - a) \cdot \beta U\} \right\}, \quad (78)$$

where recall  $\mu(s, \chi)$  is the consumer's equilibrium belief that the proposed product has quality  $G$ , given signal  $s$  and complexity  $\chi$ . For an exogenously given value of  $U$ , which pins down the

consumer's outside option  $w_0 = \beta U$ , this map gives us the consumer's ex-ante value  $T_\beta(U)$ . An equilibrium is a fixed point of this map, and we denote it by  $U^*$ . Clearly,  $w(B) < T_\beta(U) \leq w(G)$ , which implies that in equilibrium  $w_0 = \beta U^* \in (w(B), w(G))$ , satisfying Assumption 1. As in our baseline model, we focus on positive trade equilibria, in which good products are produced with positive probability. And, for the same reason as in the baseline model, bad products must be produced with positive probability. Therefore, to show that an equilibrium exists, it suffices to show that  $T_\beta(\cdot)$  is increasing. But note that an increase in the outside option increases the consumer's ex-ante welfare directly and indirectly through its effects on equilibrium  $\mu$  and  $\{\sigma_y\}$ . The latter follows from Proposition 6, where we have shown that  $\mu = \psi$  increases in the outside option, whereas  $\{\sigma_y\}$  are independent of it.

For comparative statics, note that, for a given  $U$ , an increase from  $\beta$  to some  $\beta'$  is equivalent to an increase in the consumer's outside option. Thus, it must be that  $T_\beta(U) < T_{\beta'}(U)$ . The fixed point must therefore be higher at  $\beta'$  than at  $\beta$ , since  $T_\beta(\cdot)$  is increasing. If there are multiple fixed points, then the statement holds locally and for the maximal one. ■

## B Derivations and Proofs for Section 5

### B.1 Convex Costs

In this Appendix, we provide derivations for Section 5.1. Let us begin with the consumer's optimal strategy. Recall that upon observing  $\chi$ , and updating her belief to  $\mu(\chi)$ , the consumer decides whether to acquire information or not. Given the cost that the consumer must pay whenever information is acquired, it follows immediately that the consumer will acquire information only if her acceptance decision is made contingent on this information. If the consumer were to acquire information, the optimal noise would be:

$$z(\chi) = \arg \max_{z \in [0, \frac{1}{2}]} (1-z) \cdot \mu(\chi) \cdot (w(G) - w_0) + z \cdot (1 - \mu(\chi)) \cdot (w(B) - w_0) + w_0 - \chi \cdot h \left( \frac{1}{2} - z \right), \quad (79)$$

or equivalently:

$$z(\chi) = \frac{1}{2} - h'^{-1} \left( \frac{\mu(\chi) \cdot (w(G) - w_0) - (1 - \mu(\chi)) \cdot (w(B) - w_0)}{\chi} \right), \quad (80)$$

where  $h'^{-1}(x)$  is increasing in  $x$ . Thus, in that case, the consumer accepts a Good product with probability  $1 - z(\chi)$  and a Bad product with probability  $z(\chi)$ . This implies a payoff:

$$W^I(\chi) = w_0 + (1 - z(\chi)) \cdot \mu(\chi) \cdot (w(G) - w_0) + z(\chi) \cdot (1 - \mu(\chi)) \cdot (w(B) - w_0) - \chi \cdot h \left( \frac{1}{2} - z(\chi) \right). \quad (81)$$

Instead, if the consumer does not acquire information, she makes her optimal acceptance decision based on her interim belief alone, implying a payoff:

$$W^U(\chi) = \max \{ \mu(\chi) \cdot w(G) + (1 - \mu(\chi)) \cdot w(B), w_0 \}. \quad (82)$$

Thus, the consumer acquires information if  $W^I(\chi) > W^U(\chi)$ . And, there are two cases to consider. First, if  $\mu(\chi) > \omega$ , the condition for acquiring information reduces to:

$$z(\chi) + \frac{\chi \cdot h\left(\frac{1}{2} - z(\chi)\right)}{(1 - \mu(\chi)) \cdot \omega + \mu(\chi) \cdot (1 - \omega)} < \frac{(1 - \mu(\chi)) \cdot \omega}{(1 - \mu(\chi)) \cdot \omega + \mu(\chi) \cdot (1 - \omega)}. \quad (83)$$

It is straightforward to show that if the likelihood ratio  $\frac{f(\cdot|\bar{\kappa})}{f(\cdot|\underline{\kappa})}$  is not too steep, then (i)  $z(\chi)$  defined by (80) is monotonically increasing in  $\chi$ , i.e., the consumer's information gets noisier as the product gets more complex, and (ii) there is a unique threshold value for complexity, denoted by  $\bar{\chi}^o$ , such that inequality (83) holds if and only if  $\chi < \bar{\chi}^o$ . We assume this is the case from now on, which is similar to assuming Condition 3.1 holds in our baseline model.

Second, consider the case where  $\mu(\chi) < \omega$ . Then, the condition for acquiring information reduces to:

$$z(\chi) + \frac{\chi \cdot h\left(\frac{1}{2} - z(\chi)\right)}{[(1 - \mu(\chi)) \cdot \omega + \mu(\chi) \cdot (1 - \omega)]} < \frac{\mu(\chi) \cdot (1 - \omega)}{[(1 - \mu(\chi)) \cdot \omega + \mu(\chi) \cdot (1 - \omega)]}. \quad (84)$$

Here, again we can show that there is a unique threshold level of complexity, denoted by  $\bar{\chi}^p$ , such that inequality (84) holds if and only if  $\chi < \bar{\chi}^p$ .<sup>24</sup> Given the above observations, the following lemma then follows immediately.

**Lemma B.1** *When the consumer is optimistic, i.e.,  $\lim_{\chi \rightarrow \infty} \mu(\chi) \geq \omega$ , her acceptance strategy is:*

$$a(s, \chi) = \begin{cases} \mathcal{I}_{\{S=g\}} & \text{if } \chi \leq \bar{\chi}; \\ 1 & \text{if } \chi > \bar{\chi}; \end{cases} \quad (85)$$

*instead, when the consumer is pessimistic, i.e.,  $\lim_{\chi \rightarrow \infty} \mu(\chi) < \omega$ , her acceptance strategy is:*

$$a(s, \chi) = \begin{cases} \mathcal{I}_{\{S=g\}} & \text{if } \chi \leq \bar{\chi}; \\ 0 & \text{if } \chi > \bar{\chi}; \end{cases} \quad (86)$$

where  $\bar{\chi} = \begin{cases} \bar{\chi}^o & \text{if } \lim_{\chi \rightarrow \infty} \mu(\chi) > \omega \\ \infty & \text{if } \lim_{\chi \rightarrow \infty} \mu(\chi) = \omega. \end{cases}$ <sup>25</sup>

Note that Lemma B.1 is the counterpart of Lemma 1 in our baseline model. It shows that the consumer's optimal acceptance decision depends crucially on whether she is optimistic or pessimistic, i.e., what she does when the signal that she would receive, conditional on acquiring information, becomes uninformative, which occurs as  $\chi \rightarrow \infty$ .

Using Lemma B.1, we can compute the probability that a product  $(y, \kappa)$  proposed by the

<sup>24</sup>Since in equilibrium  $\mu(\chi)$  is weakly decreasing in  $\chi$ , we do not need to impose additional conditions on the likelihood ratio  $\frac{f(\cdot|\bar{\kappa})}{f(\cdot|\underline{\kappa})}$  to obtain this result.

<sup>25</sup>When  $\lim_{\chi \rightarrow \infty} \mu(\chi) = \omega$ , the consumer acquires information for any  $\chi \in (0, \infty)$ , so we set  $\bar{\chi} = \infty$ .

designer is accepted by the consumer:

$$\mathbb{P}(a = 1|G, \kappa) = \int_0^{\bar{\chi}} (1 - z(\chi)) \cdot f(\chi|\kappa) d\chi + \mathcal{I}_{\{\lim_{\chi \rightarrow \infty} \mu(\chi) \geq \omega\}} \cdot (1 - F(\bar{\chi}|\bar{\kappa})), \quad (87)$$

and

$$\mathbb{P}(a = 1|B, \kappa) = \int_0^{\bar{\chi}} z(\chi) \cdot f(\chi|\kappa) d\chi + \mathcal{I}_{\{\lim_{\chi \rightarrow \infty} \mu(\chi) \geq \omega\}} \cdot (1 - F(\bar{\chi}|\bar{\kappa})). \quad (88)$$

Since  $z(\chi)$  is increasing in  $\chi$ , by the same reasoning as in the proof of Proposition 2, we obtain that the crucial determinant of the designer's optimal choice of  $\kappa$  is whether the consumer is optimistic or pessimistic, and the threshold  $\bar{\chi}$ .

**Lemma B.2** *Fix  $\mu \in (0, 1)$ , and let  $\hat{\chi}$  denote the unique solution to  $\int_0^{\hat{\chi}} z(\chi) \cdot f(\chi|\underline{\kappa}) d\chi = \int_0^{\hat{\chi}} z(\chi) \cdot f(\chi|\bar{\kappa}) d\chi$ . When the consumer is optimistic,*

$$\sigma_B = 1, \text{ and } \sigma_G = \begin{cases} 0 & \bar{\chi} > \hat{\chi} \\ [0, 1] & \bar{\chi} = \hat{\chi} \\ 1 & \bar{\chi} < \hat{\chi} \end{cases}, \quad (89)$$

whereas when the consumer is pessimistic,

$$\sigma_B = \begin{cases} 0 & \bar{\chi} > \hat{\chi} \\ [0, 1] & \bar{\chi} = \hat{\chi} \\ 1 & \bar{\chi} < \hat{\chi} \end{cases}, \text{ and } \sigma_G = 0. \quad (90)$$

This result is essentially the same as Proposition 2, except that note that now the threshold  $\hat{\chi}$ , which controls the designer's preference between complexification and simplification, also depends on the prior belief  $\mu$ , as the latter affects the optimal information choice  $z(\chi)$ ; we will come back to this dependence shortly. Finally, it should be clear the designer's optimal quality choice is as before given by Proposition 3.

We have thus shown that both the consumer's and the designer's optimal strategies remain qualitatively unchanged from our baseline model; we are therefore left to solve for the equilibrium. We again proceed in two steps. We first take the consumer's prior belief  $\mu$  as given and find the designer's equilibrium complexification strategy by requiring that the consumer's interim belief,  $\mu(\chi)$  be consistent with the designer's strategy and Bayes' rule.

**Lemma B.3** *Suppose that in equilibrium the consumer's prior belief is  $\mu \in (0, 1)$ , then there exist thresholds  $0 < \tilde{\mu}_1 < \tilde{\mu}_2 < \tilde{\mu}_3 < \tilde{\mu}_4 < 1$  such that:*

1. *If  $\mu \in (0, \tilde{\mu}_1]$ , all products are simplified,  $\sigma_G = \sigma_B = 0$ .*
2. *If  $\mu \in (\tilde{\mu}_1, \tilde{\mu}_2]$ ,  $G$ -products are simplified,  $\sigma_G = 0$ , and  $B$ -products complexified with probability  $\sigma_B \in (0, 1)$ .*
3. *If  $\mu \in (\tilde{\mu}_2, \tilde{\mu}_3]$ ,  $G$ -products are simplified,  $\sigma_G = 0$ , and  $B$ -products complexified,  $\sigma_B = 1$ .*

4. If  $\mu \in (\tilde{\mu}_3, \tilde{\mu}_4)$ ,  $G$ -products are complexified with probability  $\sigma_G \in \{0, \tilde{\sigma}, 1\}$  for some  $\tilde{\sigma} \in (0, 1)$ , and  $B$ -products complexified,  $\sigma_B = 1$ .

5. If  $\mu \in [\tilde{\mu}_4, 1)$ , all products are complexified,  $\sigma_G = \sigma_B = 1$ .

**Proof.** Consider first the candidate equilibrium with  $\sigma_G = \sigma_B = 0$ . In this case, the consumer does not update upon observing complexity  $\chi$ , i.e.,  $\mu(\chi) = \mu$  for all  $\chi$ . For this to be an equilibrium, it must be that  $\bar{\chi}^p(\mu) \leq \hat{\chi}(\mu)$ , where  $\bar{\chi}^p(\mu)$  is given by the solution to:

$$z(\bar{\chi}^p) = \frac{\mu \cdot (1 - \omega) - \bar{\chi}^p \cdot h\left(\frac{1}{2} - z(\bar{\chi}^p)\right)}{(1 - \mu) \cdot \omega + \mu \cdot (1 - \omega)}, \quad (91)$$

and where  $z(\cdot)$  is given by (80). Thus, this equilibrium exists if and only if  $\mu$  belongs to the set  $\mathcal{M}_{P,\bar{\kappa}} \equiv \{\mu \in (0, 1) : \bar{\chi}^p(\mu) \leq \hat{\chi}(\mu)\}$ , which is non-empty since  $\bar{\chi}^p(\mu) \downarrow 0$  as  $\mu \downarrow 0$ , whereas  $\hat{\chi}(\mu)$  is bounded away from zero.

Next, consider the candidate equilibrium with  $\sigma_G = \sigma_B = 1$ . In this case, also, the consumer does not update upon observing complexity  $\chi$ , i.e.,  $\mu(\chi) = \mu$  for all  $\chi$ . For this to be an equilibrium, it must be that  $\bar{\chi}^o(\mu) \geq \hat{\chi}(\mu)$ , where  $\bar{\chi}^o(\mu)$  is given by the solution to:

$$z(\bar{\chi}^o) = \frac{(1 - \mu) \cdot \omega - \bar{\chi}^o \cdot h\left(\frac{1}{2} - z(\bar{\chi}^o)\right)}{(1 - \mu) \cdot \omega + \mu \cdot (1 - \omega)}, \quad (92)$$

and where  $z(\cdot)$  is given by (80). Thus, this equilibrium exists if and only if  $\mu$  belongs to the set  $\mathcal{M}_{P,\bar{\kappa}} \equiv \{\mu \in (0, 1) : \bar{\chi}^o(\mu) \geq \hat{\chi}(\mu)\}$ , which is non-empty since  $\bar{\chi}^o(\mu) \uparrow \infty$  as  $\mu \uparrow 1$ , whereas  $\hat{\chi}(\mu)$  is bounded above.

Next, consider the candidate equilibrium with  $\sigma_G = 0$  and  $\sigma_B = 1$ . In this case, the consumer does update upon observing complexity  $\chi$ , i.e.,  $\mu(\chi) = \frac{\mu}{\mu + (1 - \mu) \frac{f(\chi|\kappa)}{f(\chi|\bar{\kappa})}}$  for all  $\chi$ . There are two possibilities here, depending on whether  $\lim_{\chi \rightarrow \infty} \mu(\chi)$  is greater or smaller than  $\omega$ , which is equivalent to asking whether  $\mu$  is greater than or smaller than  $\tilde{\mu} \equiv \frac{\omega}{\omega + (1 - \mu) \lim_{\chi \rightarrow \infty} \frac{f(\chi|\kappa)}{f(\chi|\bar{\kappa})}}$ .

If  $\mu < \tilde{\mu}$ , then for this to be an equilibrium, it must be that  $\bar{\chi}^p(\mu) \leq \hat{\chi}(\mu)$ , where  $\bar{\chi}^p(\mu)$  is given by the solution to:

$$z(\bar{\chi}^p) = \frac{\mu(\bar{\chi}^p) \cdot (1 - \omega) - \bar{\chi}^p \cdot h\left(\frac{1}{2} - z(\bar{\chi}^p)\right)}{(1 - \mu(\bar{\chi}^p)) \cdot \omega + \mu(\bar{\chi}^p) \cdot (1 - \omega)}, \quad (93)$$

and where  $z(\cdot)$  is given by (80). Here, such an equilibrium exists if and only if  $\mu$  belong to the set  $\mathcal{M}_{S,a} \equiv \{\mu \in (0, \tilde{\mu}) : \bar{\chi}^p(\mu) \leq \hat{\chi}(\mu)\}$ , which is non-empty since  $\bar{\chi}^p(\mu) \downarrow 0$  as  $\mu \downarrow 0$ , whereas  $\hat{\chi}(\mu)$  is bounded away from zero. Instead, if  $\mu \geq \tilde{\mu}$ , then for this to be an equilibrium, it must be that  $\bar{\chi}^o(\mu) \geq \hat{\chi}(\mu)$ , where  $\bar{\chi}^o(\mu)$  is given by the solution to:

$$z(\bar{\chi}^o) = \frac{(1 - \mu(\bar{\chi}^o)) \cdot \omega - \bar{\chi}^o \cdot h\left(\frac{1}{2} - z(\bar{\chi}^o)\right)}{(1 - \mu(\bar{\chi}^o)) \cdot \omega + \mu(\bar{\chi}^o) \cdot (1 - \omega)}, \quad (94)$$

and where  $z(\cdot)$  is given by (80). Here, such an equilibrium exists if and only if  $\mu$  belongs to the set  $\mathcal{M}_{S,b} \equiv \{\mu \in (\tilde{\mu}, 1) : \bar{\chi}^o(\mu) \geq \hat{\chi}(\mu)\}$ , which is non-empty since  $\bar{\chi}^o(\mu) \uparrow \infty$  as  $\mu \uparrow 1$ ,



whereas  $\widehat{\chi}(\mu)$  is bounded above. We therefore conclude that an equilibrium with  $\sigma_G = 0$  and  $\sigma_B = 1$  exists if and only if  $\mu \in \mathcal{M}_S \equiv \mathcal{M}_{S,a} \cup \mathcal{M}_{S,b}$ .

Next, suppose that (i)  $\bar{\chi}^p(\mu) - \widehat{\chi}(\mu)$  is increasing in  $\mu$  in an equilibrium with  $\sigma_G = \sigma_B = 0$ , or with  $\sigma_G = 0, \sigma_B = 1$  when  $\mu < \tilde{\mu}$ ; and (ii)  $\bar{\chi}^o(\mu) - \widehat{\chi}(\mu)$  is decreasing in  $\mu$  in an equilibrium with  $\sigma_G = \sigma_B = 1$ , or with  $\sigma_G = 0, \sigma_B = 1$  when  $\mu \geq \tilde{\mu}$ . These two conditions will hold, for example, if the function  $h(\cdot)$  is convex enough so that  $z(\cdot)$  is not too sensitive to changes in  $\mu$ . We will assume this in what follows, in which case the three equilibrium regions can be represented as:  $\mathcal{M}_{P,\kappa} = (0, \tilde{\mu}_1)$ ,  $\mathcal{M}_{P,\kappa} = (\tilde{\mu}_2, \tilde{\mu}_4)$ , and  $\mathcal{M}_{P,\bar{\kappa}} = (\tilde{\mu}_3, 1)$  for some  $0 < \tilde{\mu}_1 < \tilde{\mu}_2 < \tilde{\mu}_3 < \tilde{\mu}_4 < 1$ ; moreover, following similar arguments as in the proof of Proposition 4, we can construct the mixed strategy equilibria for  $\mu$  in the intervals  $(\tilde{\mu}_1, \tilde{\mu}_2)$  and  $(\tilde{\mu}_3, \tilde{\mu}_4)$ . ■

Finally, we are left to pin down the equilibrium prior belief  $\mu^*$ . As before, in any positive trade equilibrium, the designer must be indifferent between producing either of the two products. With the results from Lemma B.2 it is easy to show that the correspondence  $\Gamma(\mu)$  (defined just as in proof of Proposition 5), which consists of the designer's net payoffs  $\gamma$  from producing  $G$ - vs.  $B$ -products, is well defined and both upper and lower hemicontinuous. But then, it is straightforward to use continuity arguments and find a  $\mu^* \in (0, 1)$  at which the designer's net payoff from producing  $G$ - vs.  $B$ -product is equal to zero (i.e.,  $\gamma(\mu^*, \{\sigma_y\}_{\mu^*}) = 0$ ).

## B.2 Rational Inattention

In this Appendix, we provide derivations for Section 5.2. We adjust our baseline setting to allow the consumer to optimally reduce her uncertainty about the product's quality, subject to an entropy-reduction cost, as in the literature on rational inattention (Sims, 2003). Within this framework, a more complex product is one that is associated with a high entropy-reduction cost for the consumer.

The uncertainty faced by the consumer with belief  $\tilde{\mu} = P(y = G)$  is measured by the entropy function:

$$H(\tilde{\mu}) = -(\tilde{\mu} \cdot \log(\tilde{\mu}) + (1 - \tilde{\mu}) \cdot \log(1 - \tilde{\mu})), \quad (95)$$

which reaches a minimum of zero at  $\tilde{\mu} \in \{0, 1\}$  and a maximum of  $-\log(\frac{1}{2})$  at  $\tilde{\mu} = \frac{1}{2}$ . As before, we let  $S$  denote the signal observed by the consumer and  $s$  denote its realization. The signal has a distribution conditional on the product's quality,  $\pi(s|y) \equiv \mathbb{P}(S = s|y)$ , which determines the consumer's posterior belief:

$$\tilde{\mu}(s) \equiv P(y = G|s) = \frac{\pi(s|G) \cdot \tilde{\mu}}{\pi(s|G) \cdot \tilde{\mu} + \pi(s|B) \cdot (1 - \tilde{\mu})}. \quad (96)$$

The entropy associated with the posterior belief is  $H(\tilde{\mu}(s))$ .

We measure the amount of information that the consumer obtains from a particular information structure  $\pi$  as the expected reduction in entropy:

$$I(\pi) = H(\tilde{\mu}) - \int_s H(\tilde{\mu}(s)) \cdot \pi(s) \cdot ds, \quad (97)$$

and we assume that the consumer faces a cost  $\chi \cdot I(\pi)$  of entropy-reduction, where  $\chi \in (0, \infty)$



depends on the two components  $\eta$  and  $\kappa$ , with a conditional pdf  $f(\chi|\kappa)$  that has full support and satisfies MLRP. Thus, when complexity is minimal,  $\chi \rightarrow 0$ , it is essentially costless for the consumer to find out the product's quality; instead, when complexity is maximal,  $\chi \rightarrow \infty$ , extracting any information about the product's quality becomes prohibitively costly.

Since the consumer's action is binary, i.e., she chooses to accept or reject the product, it is without loss of generality to restrict attention to information structures that consist of binary signals  $S \in \{b, g\}$  such that the consumer accepts the product if and only if  $S = g$  (Woodford, 2009; Yang, 2015). Let  $\pi_y$  denote the probability that the consumer accepts the product, conditional on the designer producing a  $y$ -product. Let  $\mu(\chi)$  be the consumer's interim belief after observing the product's complexity  $\chi$ . For a given  $\chi$ , the consumer's problem is then reduced to choosing  $\pi_G$  and  $\pi_B$  in order to maximize her expected payoff:

$$\mu(\chi) \cdot \pi_G \cdot (w(G) - w_0) + (1 - \mu(\chi)) \cdot \pi_B \cdot (w(B) - w_0) - \chi \cdot I(\pi) \quad (98)$$

where

$$I(\pi) = H(\mu(\chi) \cdot \pi_G + (1 - \mu(\chi)) \cdot \pi_B) - \mu(\chi) \cdot H(\pi_G) - (1 - \mu(\chi)) \cdot H(\pi_B). \quad (99)$$

Figure 7 illustrates the solution to this problem for a given prior belief  $\mu \in (0, 1)$ , for the case where the consumer's interim belief satisfies  $\mu(\chi) = \mu$ , i.e., when the equilibrium features pooling on complexification. Under a regularity condition on the likelihood ratio  $f(\cdot|\bar{\kappa})/f(\cdot|\underline{\kappa})$  (akin to Condition 3.1), there is a unique threshold value of complexity,  $\bar{\chi}$ , such that the consumer extracts an informative signal and makes her decision contingent on its realization if and only if  $\chi < \bar{\chi}$ . Otherwise, when complexity is high, the consumer makes her decision solely based on her interim belief. Finally, observe that when complexity is high enough, then the consumer either accepts the product with probability one or she rejects it with probability one. As in our baseline model, which of the two scenarios arises depends on whether the consumer is optimistic or pessimistic; that is, what she would do in the absence of an informative signal.<sup>26</sup>

Naturally, an equilibrium requires that the consumer's prior belief  $\mu$  and her interim belief  $\mu(\chi)$  be consistent with the designer's strategy  $\{m, \sigma_G, \sigma_B\}$  and Bayes' rule. Although a full analytical characterization of the equilibrium set is now difficult to obtain, we are able to check numerically that the equilibrium set of the model with optimal information extraction resembles closely that of our baseline model. As we discussed in the text, Figure 8 is the analogue of the Figures 3 and 4. And, an equilibrium is found by requiring that the belief  $\mu$  equals  $\psi$ , so that the designer is indifferent to producing a  $G$ - or a  $B$ -product, and then reading off the equilibrium complexification strategy of the designer from the left panel, given that the consumer's prior belief is  $\mu = \psi$ .

### B.3 Prices and Production Costs

**Proof of Proposition 11.** Consider an equilibrium in which the  $y$ -product has price  $p_y$ , with  $p_G \neq p_B$ . A the designer of a bad product would only trade at price  $p_B \geq c(B)$ , but

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<sup>26</sup>As with convex costs of information acquisition in Section 5.1, the consumer is optimistic if  $\lim_{\chi \rightarrow \infty} \mu(\chi) \geq \omega$ , and she is pessimistic otherwise.

then  $\tilde{w}(B) - p_B < 0$  and the consumer would reject all products with price  $p_B$ . Since also  $p_G \geq c(G) > c(B)$ , in any positive trade equilibrium, the  $B$ -product designer would expect to make profits by deviating to price  $p_G$ . Thus, in equilibrium, different quality products cannot be offered at different prices. Consider a candidate equilibrium in which  $p^*$  is the price set by the designer, which can be supported for example by an off-equilibrium belief that the designer has produced a  $B$ -product if he sets any other price. For any price  $p^* \in (c(G), \tilde{w}(G) - w_0)$ , define payoffs  $w(y) \equiv \tilde{w}(y) - p^*$  and  $v(y) \equiv p^* - c(y)$ , and note that they satisfy Assumption 1. That such an equilibrium exists follows by Proposition 5, and its characterization is the same as that of our baseline model.

We also note that  $p^* \leq c(G)$  cannot be part of a positive trade equilibrium, since then only  $B$ -products would be produced (if any) and rejected with probability one. Similarly,  $p^* > \tilde{w}(G) - w_0$  would induce a rejection with probability one by the consumer, as the product would generate losses to the consumer. An equilibrium with  $p^* = \tilde{w}(G) - w_0$  may exist, but it would require that the (indifferent) consumer accepts the product randomly, and in a manner that is correlated with the signal she acquires; moreover, such an equilibrium would unravel if we were to introduce an arbitrarily small cost of information acquisition in the region where information is costless in our baseline model. ■