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In search of a theory of debt management

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ABSTRACT

The complete market approach to government debt management argues that a portfolio of non-contingent bonds at different maturities should be chosen so that fluctuations in market value offset changes in expected future deficits. However, this approach recommends huge fluctuations in positions, enormous changes in portfolios for minor changes in maturities and no presumption it is always optimal to issue long and invest short term in a wide array of model specifications. These extreme, volatile and unstable features are undesirable for two reasons. Firstly fragility of portfolios to small changes in assumptions means that it is often better to follow a balanced budget rather than issue the optimal debt portfolio under some possibly misspecified model. Secondly for even miniscule transaction costs, governments prefer a balanced budget rather than the large positions complete markets recommends. The complete market recommendations conflict with a number of features we believe are integral to bond market incompleteness, e.g. transaction costs, liquidity effects, robustness, etc. and which need to be explicitly incorporated into the portfolio problem.

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1. Introduction

The complete market approach to debt management focuses on the idea that fiscal policy and debt structure should be *jointly* determined. In a seminal contribution, Angeletos (2002) shows that even if a Ramsey government only has access to non-contingent bonds it can still achieve the complete market outcome by exploiting fluctuations in the yield curve. Analyzing the case of government expenditure shocks only he shows how it is optimal for governments to issue long term debt and invest in short term assets. A similar recommendation is found in Barro (1999, 2003) and Nosbusch (2008). This paper argues that there are significant problems in using the complete market approach in the case where governments can only issue non-contingent bonds. The paper also finds that the qualitative implications to issue long and invest short are not robust. Whilst the complete market approach provides an important intuition about the role of long bonds in providing fiscal insurance it is an incomplete analysis as explicit consideration of small market frictions can make a big difference in the desirability of a certain policy.

As has already been documented by Buera and Nicolini (2004) the magnitude of positions derived from the complete market approach are large multiples of GDP, e.g. the government should hold 5 or 6 times GDP in privately issued short bonds and issue similar amounts of long bonds. No government in the real world conducts debt management this way, not even approximately. Logically this gap between the positions recommended by the models in Angeletos (2002) and Buera

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and Nicolini (2004) (henceforth ABN) and those observed in practice could be due to four different possible explanations. The first is that preferences and technology in reality are different from the simple endowment model of ABN and a richer structural model needs to be considered. The second is that market imperfections matter, especially financial market frictions. A third possible explanation is that governments do not know exactly the value of some parameters in the economy as they possess imperfect structural knowledge. Finally governments may pursue suboptimal policies or be subject to other constraints such as time consistency, etc. Angeletos (2002) suggests that the need for a richer structural model is a likely candidate for explaining away the gap[1] but that the qualitative implications of complete markets (to issue long term debt and buy short term bonds) are robust to variations in preferences and technology. Our view in contrast is that market imperfections and imperfect structural knowledge are more likely explanations for the discrepancy between the predictions of the complete market approach and the data.[2]

The paper begins by showing that the lack of a richer structural model is unlikely to be the reason for the discrepancy. It first reviews ABN (Section 2) and then introduce capital accumulation (Section 3). Introducing capital only exacerbates the problem—positions become even larger. Not only that, one nice feature of debt management under ABN was that debt positions were constant through time, leading to a "stable" recommendation for debt management, but when capital is introduced the optimal debt positions become extremely volatile. In an effort to reduce the size of these positions Section 4 introduces habits.[3] The size of the positions is lower than in the capital accumulation case, but positions are still very large and very volatile, they even frequently reverse sign from one period to the next.

This extreme sensitivity to the actual model and the value of state variables causes profound problems for the complete market approach. Firstly it makes the recommended positions very large but also counterfactually volatile across time. Secondly the sensitivity is so great that the qualitative implications stressed by Barro (1999, 2003), Angeletos (2002) and Nosbusch (2008) are not robust. Small variations even in the choice of maturities available to the government can easily reverse the issue-long-buy-short recommendation as can allowing for both productivity and expenditure shocks. Allowing for habits and capital accumulation introduces additional state variables that cause the same reversal even from period to period. This sensitivity is unlikely to disappear in even more sophisticated specifications of the model. It is due to the fact that yield premia are not very volatile, implying that bonds at different maturities offer similar returns and, therefore, are not good instruments for fiscal insurance.

Section 5 introduces specific reasons why markets may be incomplete and examines whether the large and volatile positions recommended by the complete market approach are costly in this setting. Firstly the analysis considers the case when governments misspecify various features of the economy (the imperfect structural knowledge model mentioned above). The sensitivity of debt positions to the specification of the model is such that even for small misperceptions following the complete market approach leads to significant welfare losses. These losses are so large that the government would frequently prefer to run a balanced budget and completely forego the advantages of tax smoothing in order to avoid the costs of incorrect debt management that would arise from following the optimal policy under a misspecified model. Further, so great is the sensitivity that no robust debt management policies emerge—which maturities perform best depends entirely on the misspecification. The importance of this example is that in misspecifying the economy the government is effectively in an incomplete market setting. In other words when one is explicit about the reasons for market incompleteness the complete market approach may lead to very costly mistakes. This point is reinforced by introducing another explicit form of market incompleteness—transaction costs. For minimal levels of transaction costs the government would once more prefer to operate a balanced budget.

We conclude that a theory of optimal debt management needs to supplement the focus of providing insurance against fiscal shocks with an explicit recognition of capital market imperfections—such as transaction costs, short selling constraints and liquidity effects.

2. Complete market approach to debt management

This section outlines the model and results of ABN.

2.1. The economy

The economy is the same as Lucas and Stokey (1983). There is a single non-storable good, the agent is endowed with one unit of time that it allocates between leisure and labour. Technology is given by

$$c_t + g_t \leq \theta_t(1 - x_t), \tag{1}$$

where x_t, c_t and g_t represent leisure, private consumption and government expenditure, respectively, and θ_t a productivity shock for all t. This model will be referred to as the endowment economy. The only sources of uncertainty are $h_t \equiv (g_t, \theta_t)$ which are stochastic and exogenous. Every period there is a finite number, N, of possible realizations of these shocks

[1] "However, this disturbing result [of debt holdings exploding to plus and minus infinity] is mostly an artefact of an economy without capital"

[2] We do not explore in this paper the possibility that it is non-Ramsey behaviour that explains the discrepancy, we think this may be a promising direction for future research.

[3] Wachter (2006) studies how introducing habits helps explain the volatility of the yield curve.

$\overline{h}_n \equiv (\overline{g}_n, \overline{\theta}_n)$, $n = 1, \ldots, N$. As usual, $h^t = (h_0, h_1, \ldots, h_t)$ represents the history of shocks up to and including period t. Government and consumers have full information, all variables dated t are restricted to be measurable with respect to h^t. Preferences are

$$E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) + V(x_t)], \tag{2}$$

where $0 < \beta < 1$. For simplicity U and V are assumed strictly increasing and strictly concave in their respective arguments.

The government finances its expenditure via a tax on labour or through debt. The case of complete markets using Arrow securities requires the government to issue N distinct contingent bonds at time t, each paying one unit of consumption contingent on $h_{t+1} = \overline{h}_n$ for $n = 1, \ldots, N$. The quantity $b_t(h^t, \overline{h}_n)$ denotes the amount of government bonds issued in period t for realization h^t that pay one unit of consumption in period $t+1$ if $h_{t+1} = \overline{h}_n$. The consumer's budget constraint is

$$\begin{aligned} & c_t(h^t) + \sum_{n=1}^{N} q_t(h^t, \overline{h}_n)\ b_t(h^t, \overline{h}_n) \\ & \quad \le (1 - \tau_t^x(h^t)) w_t(h^t)(1 - x_t(h^t)) + b_{t-1}(h^{t-1}, h_t), \end{aligned} \tag{3}$$

for all t and h^t, where $q_t(h^t, \overline{h}_n)$ is the price in terms of consumption of one bond $b_t(h^t, \overline{h}_n)$, $\tau_t^x(h^t)$ is the tax on labour and $w_t(h^t)$ is the wage earned by the consumer.

Finally, the government faces the constraint:

$$g_t(h^t) + b_{t-1}(h^{t-1}, h_t) \le \tau_t^x(h^t) w_t(h^t)(1 - x_t(h^t)) + \sum_{n=1}^{N} q_t(h^t, \overline{h}_n) b_t(h^t, \overline{h}_n). \tag{4}$$

Both the government and the consumer are subject to No-Madoff-game conditions. The government fully commits in the initial period to all future contingent taxes.

Let c denote the sequence of all consumptions $\{c_0, c_1, \ldots\}$, and similarly for other variables. A competitive equilibrium is a feasible allocation (c,x,g), a price system (w,q) and a government policy (g,τ^x,b) such that, given the price system and government policy, (c,x) solves the firm's and consumer's maximization problem and satisfies the sequence of government budget constraints (4). The optimal Ramsey problem chooses policy by selecting the competitive equilibrium that maximizes (2). As shown in Chari and Kehoe (1999), this is equivalent to choosing (c,x) that maximizes utility subject to (1) and the implementability constraint

$$E_0 \sum_{t=0}^{\infty} \beta^t [c_t U_{c,t} - (1 - x_t) V_{x,t}] = W_0 U_{c,0}, \tag{5}$$

where $W_0 = b_{-1}$ is the amount of liabilities inherited by the government in period 0, U_c is the marginal utility of consumption and V_x is the marginal utility of leisure.

2.2. The complete markets approach to debt management

Under complete markets it is always possible to back out the optimal bond holdings. For any given c,x that satisfy (5) define a sequence of random variables z such that

$$z_t(h^{t-1}, h_t) \equiv E\left(\sum_{s=0}^{\infty} \frac{\beta^s}{U_{c,t}} [c_{t+s}\ U_{c,t+s} - (1 - x_{t+s})\ V_{x,t+s}] | h^{t-1}, h_t \right). \tag{6}$$

It can be shown that all government budget constraints are satisfied with the given sequence c,x if the government issues in period $t-1$ an amount of debt/credit given by

$$b_{t-1}(h^{t-1}, \overline{h}_n) = z_t(h^{t-1}, \overline{h}_n). \tag{7}$$

ABN show how to build a model with non-contingent bonds of different maturities where the optimal allocations coincide with those in the equilibrium just described. Hereafter this will be termed the complete markets approach to debt management even though it is applied to the case of bonds with a non-contingent payoff. Assume that the number of maturities equals N (that is the number of possible realizations of the shocks), e.g. the government completes the markets. Let b_t^j denote the amount of bonds issued by the government that pay one unit of consumption with certainty in period $t+j$, and p_t^j denote the market price of this bond in terms of consumption in period t, both p_t^j and b_t^j are a function of h^t. Assume a bond matures for each $j = 1, \ldots, N$ and the government buys back each period the entire stock of debt. The budget constraint of the government at t is

$$g_t + \sum_{j=1}^{N} p_t^{j-1} b_{t-1}^j \le \tau_t^x w_t (1 - x_t) + \sum_{j=1}^{N} p_t^j\ b_t^j \tag{8}$$

for all t and h^t, and symmetrically for the consumer, where $p_t^0 \equiv 1$. Equilibrium prices satisfy

$$p_t^j = \beta^j \frac{E_t U_{c,t+j}}{U_{c,t}} \tag{9}$$

ABN prove that if bond prices are sufficiently variable one can choose each period a portfolio of maturities $(b_t^1, \ldots, b_t^N)$ such that

$$\sum_{j=1}^{N} p_t^{j-1} b_{t-1}^j (h^{t-1}) = z_t(h^t) \tag{10}$$

almost surely, for all t where z_t is found by plugging the optimal allocations in (6). It is possible to find such bonds because even though bonds issued in $t-1$ are not contingent on the realization of h_t, today's value of last period's debt $\sum_{j=1}^N p_t^{j-1}(h^t) b_{t-1}^j (h^{t-1})$ is state contingent through dependence of prices $p_t^{j-1}(h^t)$ on the current value of h_t.

Consider the case in which productivity is constant $\theta_t = \overline{\theta}$ and government expenditure follows a two step Markov process with values $\overline{g}_H > \overline{g}_L > 0$ and probabilities of remaining in the same state π_{HH} and π_{LL}. If $b_{-1}^j = 0$ for $j = 1,2$, then variables dated t in the Ramsey allocation depend only on the shock g_t so that in equilibrium consumption, prices, etc. each period can only take two values, one for each realization of the shock. Formally, $c_t(h^{t-1}, \overline{g}_i) = \overline{c}^i$, $p_t^1(h^{t-1}, \overline{g}_i) \equiv \overline{p}^i$ and so on for $i = H,L$ and for all t all h^{t-1}. Assuming in addition that $g_0 = \overline{g}_H$ it turns out $\overline{z}^H = 0 < \overline{z}^L$. Under these conditions, since $N = 2$, (10) becomes

$$b_{t-1}^1(h^{t-1}) + \overline{p}^i b_{t-1}^2 (h^{t-1}) = \overline{z}^i \quad \text{for } i = H,L \quad \forall t. \tag{11}$$

The necessary and sufficient condition for this equation to give a unique solution for b_{t-1}^j is $\overline{p}^L \neq \overline{p}^H$; so that

$$\begin{pmatrix} b_{t-1}^1(h^{t-1}) \\ b_{t-1}^2(h^{t-1}) \end{pmatrix} = \begin{pmatrix} 1 & \overline{p}^H \\ 1 & \overline{p}^L \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ \overline{z}^L \end{pmatrix} = \begin{pmatrix} \dfrac{\overline{p}^H \overline{z}^L}{\overline{p}^H - \overline{p}^L} \\ \dfrac{-\overline{z}^L}{\overline{p}^H - \overline{p}^L} \end{pmatrix} \equiv \begin{pmatrix} B^1 \\ B^2 \end{pmatrix} \tag{12}$$

for all t. Therefore in this case debt issuance at each maturity is time invariant. It is well known that for standard utility functions gives $\overline{p}^H < \overline{p}^L$ so that this equation gives $B^2 > 0$ and $B^1 < 0$.

This reproduces the two main messages from ABN: first, that the government should issue constant amounts of each bond, second, that it should hold short term assets and issue long term liabilities.

2.3. Simulations

As stressed by Buera and Nicolini (2004) the one-period ahead variability of long rates $(\overline{p}^H - \overline{p}^L)$ is not large so (12) implies large positions in B^2 are needed to achieve the complete market outcome and a matching but offsetting large position in B^1. To document this problem the model is calibrated to US data assuming the utility function:

$$\frac{c_t^{1-\gamma_1}}{1-\gamma_1} + \eta \frac{x_t^{1-\gamma_2}}{1-\gamma_2}$$

with $\beta = 0.98$, $\gamma_1 = 1$ and $\gamma_2 = 2$. The parameter η is calibrated so that the government's deficit equals zero in the non-stochastic steady state and use the steady state condition to fix the fraction of leisure at 30% of the time endowment. Initial debt b_{-1} is assumed to equal zero and government spending and technological processes are calibrated as in Chari et al. (1991). Assuming a two state symmetric Markov process for government expenditure gives $\overline{g}_i = g^*(1+\xi_i)$, $i = H,L$ and $\xi_H = 0.07 = -\xi_L$,g^* equals to 25% of GDP in the non-stochastic steady state and the transition probabilities are $\pi_{HH}^g = \pi_{LL}^g = 0.95$. The assumptions for technological process are $\overline{\theta}_i = \exp(\phi_i)$, $i = H,L$ and $\phi_H = 0.04 = -\phi_L$. The transition probabilities of the symmetric Markov process are $\pi_{HH}^\theta = \pi_{LL}^\theta = 0.91$. Also shown is the case where technology is more persistent than government expenditure ($\pi_{HH}^\theta = \pi_{LL}^\theta = 0.98$). Results are shown for transition probabilities $\left(\begin{smallmatrix} \pi_{HH} & \pi_{HL} \\ \pi_{LL} & \pi_{LH} \end{smallmatrix}\right) = \mu\Delta + (1-\mu)I$ where Δ are the above calibrated probabilities and $I = \left(\begin{smallmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{smallmatrix}\right)$. When $\mu = 1$ shocks have the persistence suggested by Chari et al. (1991), when $\mu = 0$ is shocks are i.i.d. and for $\mu \in (0,1)$ there is intermediate levels of persistence. Critical to the size of debt positions is the volatility of the yield curve so as the persistence of the shocks is changed the unconditional variance is maintained at the same calibrated level.

Table 1 reports results[4] and quotes the unconditional average of the ratio of the value of bond positions with total output (in other words 7.50 means a position of issuing 750% of GDP on average). As in Buera and Nicolini (2004) the issued maturities are chosen by minimizing the absolute value of the bond positions.

With only one source of uncertainty the results repeat the qualitative recommendations of Angeletos (2002)—governments issue long term debt and invest in short term assets. In the case of persistent government expenditure shocks optimal positions are large multiples of GDP (long term debt is more than 7 times GDP) because with persistent productivity shocks fluctuations

[4] The Appendix, available from the authors, provides detailed description of the computational methods used to produce the simulations. The moments in the table are computed as an average of 10 000 period simulations.

Table 1
Simulation results—endowment economy.[a]

Shocks						Interest rates				
g							H	L		
		B^1	B^{30}			R^1	2.23	1.85		
	$\mu=1$	-7.04	7.16			R^{30}	2.10	1.98		
		B^1	B^{30}			R^1	3.95	0.13		
	$\mu=0$	-0.79	0.81			R^{30}	2.28	1.80		
$\boldsymbol{\theta}$							H	L		
		B^1	B^{30}			R^1	1.07	2.93		
	$\mu=1$	-0.85	0.90			R^{30}	1.85	2.21		
		B^1	B^{30}			R^1	-3.13	7.21		
	$\mu=0$	-0.17	0.18			R^{30}	1.86	2.21		
g,$\boldsymbol{\theta}$							HH	HL	LH	LL
		B^1	B^4	B^{13}	B^{30}	R^1	1.23	3.25	0.90	2.71
	$\mu=1$	-16.15	41.32	-86.71	57.66	R^{30}	1.92	2.28	1.79	2.15
$\pi^g_{HH}=0.95$		B^1	B^2	B^3	B^{29}	R^1	-5.75	7.21	-2.98	4.16
$\pi^\theta_{HH}=0.91$	$\mu=1/3$	-4.22	58.48	-161.22	106.37	R_{29}	1.92	2.28	1.79	2.14
		B^1	B^5	B^{18}	B^{30}	R^1	2.00	2.45	1.64	2.71
	$\mu=1$	63.82	-140.94	163.15	-75.64	R^{30}	1.97	2.22	1.85	2.09
$\pi^g_{HH}=0.95$		B^1	B^2	B^3	B^{29}	R^1	-3.34	6.96	-2.74	4.02
$\pi^\theta_{HH}=0.98$	$\mu=1/3$	5.77	-85.8	210.19	-129.51	R^{29}	1.91	2.28	1.79	2.14

[a] Table shows maturity structure and yield curve for endowment economy subject to various combinations of productivity and expenditure shocks.

in z are large and fluctuations in the long term interest rate are small. In the case of i.i.d. expenditure shocks or only productivity shocks (whether they are i.i.d. or persistent) the optimal debt positions are much smaller. It is with both shocks that the problems noted by Buera and Nicolini (2004) are clearly evident. Firstly, the required positions are enormous—governments need to issue debt at each maturity between 400% and 16 000% GDP. Secondly, although the model still recommends issuing long and investing short the maturity structure is complex and varies dramatically with small changes in maturity. In the case of intermediate persistence in shocks ($\mu=0.33$) the government should invest in one period bonds, issue 2 year bonds worth 5900% of GDP and invest in three year bonds worth 16 000% GDP.

The final rows of Table 1 show results when productivity is more persistent than government expenditure and shows two further areas in which the complete market recommendations are volatile and non-robust. Firstly the recommendation that governments issue long and invest short is reversed. Changing the persistence of shocks affects the slope of the yield curve and flips around the sign of the positions. Whilst interest rates still rise with adverse expenditure shocks the yield curve is now downward sloping, as short rates are more responsive to temporary shocks than long rates in rational expectations models. The second non-robustness occurs when the option that the government can change the maturities it issues is removed. In particular, in the case of $\mu=0.333$ the maturity structure that minimizes the absolute positions is 1,2,3 and 29 but if the government is restricted to issue maturities at 1,4,13 and 30 (the maturities that minimize the debt positions in the case of persistent shocks, $\mu=1$) then the matrix of returns becomes singular and the optimal positions tend to plus and minus infinity. Therefore, holding fixed maturity, small changes in model specification lead to huge changes in positions.

Therefore in the case of an endowment economy calibrated to US data the complete market approach to debt management (i) recommends positions that are large multiples of GDP (ii) the size of debt positions varies sharply with small changes in maturity and involves simultaneously both issuing and investing in bonds of adjacent maturities (iii) is extremely sensitive to small changes in parameter specifications with no presumption that it is always optimal for the government to issue long term debt and invest in short term bonds.

3. Introducing capital accumulation

The endowment economy is a useful workhorse model but the magnitude and sensitivity of the debt positions outlined in the previous section could be an artefact of its simplicity. Therefore in this section the complete market optimal tax model of Chari et al. (1994) is used to consider Angeletos' (2002) claim that capital mitigates these problems (see footnote 1).

3.1. Complete markets

Assume there are two factors of production: labour $(1-x)$ and capital k, with output produced through a Cobb Douglas function F such that

$$c_t+g_t+k_t-(1-\delta)k_{t-1} \le \theta_t k_{t-1}^{\alpha}(1-x_t)^{1-\alpha} = \theta_t F(k_{t-1},x_t) \tag{13}$$

where δ is the depreciation rate. As before, exogenous shocks are $h=(g,\theta)$. The government now has three policy instruments to finance g: taxes on labour τ^x, taxes on capital τ^k and debt. For this problem to be of interest capital taxes are restricted in two ways. First the initial period capital tax is bounded to prevent the planner from achieving the first best through a capital levy. This is done by adding the constraint $\tau_0^k \leq \overline{\tau}^k$ for a fixed constant $\overline{\tau}^k$. It is also assumed that capital taxes are decided one period in advance otherwise debt and taxes in equilibrium would be underdetermined and the role of debt management could be supplanted by state contingent capital taxation (see Chari and Kehoe, 1999). Note therefore that τ_t^k denotes the tax that is applied to capital income in period t even though it is set with information on h^{t-1}.

In the case where the government has full access to a complete set of contingent Arrow–Debreu securities the consumer's budget constraint is

$$c_t(h^t)+k_t(h^t)+\sum_{n=1}^{N} q_t(h^t,\overline{h}_n)\ b_t(h^t,\overline{h}_n) \leq [(1-\tau_t^k(h^{t-1}))r_t(h^t)+1-\delta]k_{t-1}(h^{t-1})$$
$$+(1-\tau_t^x(h^t))w_t(h^t)(1-x_t(h^t))+b_{t-1}(h^{t-1},h_t)$$

and the government's:

$$g_t(h^t)+b_{t-1}(h^{t-1},h_t) \leq \tau_t^k(h^{t-1})r_t(h^t)k_{t-1}(h^{t-1})$$
$$+\tau_t^x(h^t)w_t(h^t)(1-x_t(h^t))+\sum_{n=1}^{N} q_t(h^t,\overline{h}_n)\ b_t(h^t,\overline{h}_n),$$

where r_t denotes the rental price of capital.

The set of constraints in the Ramsey problem is now augmented with the consumer's Euler equation with respect to capital, viz.,

$$U_{c,t}=\beta E_t\{U_{c,t+1}[(1-\tau_{t+1}^k)r_{t+1}+1-\delta]\}. \tag{14}$$

Firms' maximization implies $r_t=F_{k,t}$, $w_t=F_{l,t}$. The results of Chari and Kehoe (1999) guarantee that the implementability constraint (5) taking $W_0=[r_0(1-\overline{\tau}^k)+1-\delta]k_{-1}+b_{-1}$ plus the feasibility constraint (13) are necessary and sufficient conditions for a competitive equilibrium. For sequences c,k,x that satisfy these conditions build the expected discounted sum of future government surpluses in each period z as

$$z_t^k(h^{t-1},h_t) \equiv E\left(\sum_{s=0}^{\infty}\beta^s\left[c_{t+s}\frac{U_{c,t+s}}{U_{c,t}}-(1-x_{t+s})\frac{V_{x,t+s}}{U_{c,t}}\right]\middle| h^t\right)$$
$$-[(1-\tau_t^k(h^{t-1}))F_{k,t}(h^t)+1-\delta]k_{t-1}(h^{t-1}). \tag{15}$$

Chari et al. (1994) show the Ramsey solution to this problem satisfies the recursive structure:

$$[k_t,c_t,x_t,\tau_t^x,\tau_{t+1}^k]'=G(h_t,k_{t-1})$$

for all $t \geq 1$ for some time-invariant function G. Using Proposition 1A in Marcet and Scott (2009) this implies the existence of a time-invariant function D such that

$$D(k_{t-1},\overline{h}_n)=z_t^k(h^{t-1},\overline{h}_n)$$

for all $t \geq 1$, all h^t all n and when k_t and the z_t^k are determined by the optimal solution. Therefore using (7) the Ramsey optimum for debt under complete markets is $b_{t-1}(h^{t-1},\overline{h}_n)=D(k_{t-1},\overline{h}_n)$. The result of adding capital is that the contingent bond positions that complete the market are no longer constant but are a function of the capital stock.

3.2. The complete market approach to debt management

Return now to the case of a government issuing non-contingent debt at N consecutive maturities. The government effectively completes the markets if it can find b_{t-1}^j such that

$$\sum_{j=1}^{N} p_t^{j-1}(h^{t-1},\overline{h}_n)\ b_{t-1}^j(h^{t-1})=D(k_{t-1}(h^{t-1}),\overline{h}_n) \tag{16}$$

for all t, h^{t-1} and n.

This gives N equations to solve for the unknowns $(b_{t-1}^1(h^{t-1}),\ldots,b_{t-1}^N(h^{t-1}))$ in each period. Since the recursive structure of the Ramsey solution implies $P^n(k_{t-1}(h^{t-1}),\overline{h}_j) \equiv p_t^n(h^{t-1},\overline{h}_j)$ for N time-invariant functions P^n, for all $t \geq 1$, all h^{t-1} and all $n,j=1,\ldots,N$, this gives a recursive solution for the optimal bond portfolio. More precisely, letting $\Pi: R_+ \to R^{N\times N}$

be defined as

$$\Pi(k) \equiv \begin{bmatrix} 1 & P^1(k,\overline{h}_1) & \dots & P^{N-1}(k,\overline{h}_1) \\ \vdots & & & \vdots \\ 1 & P^1(k,\overline{h}_N) & \dots & P^{N-1}(k,\overline{h}_N) \end{bmatrix} \tag{17}$$

and assuming $\Pi(k_t)$ is an invertible matrix with probability one,[5] then the time-invariant function $B: R_+ \to R^N$ defined by

$$\begin{bmatrix} b^1_{t-1} \\ \vdots \\ b^N_{t-1} \end{bmatrix} = [\Pi(k_{t-1})]^{-1} \begin{bmatrix} D(k_{t-1},\overline{h}^1) \\ \vdots \\ D(k_{t-1},\overline{h}_N) \end{bmatrix} \equiv B(k_{t-1}) \tag{18}$$

gives the portfolio that effectively completes the markets for all $t \geq 1$, all h^t.

Formula (18) strongly hints that the resulting bond positions are likely to be very volatile. As can be seen from the definition of $\Pi(k)$ in (17), each row of $\Pi(k)$ contains the yield curve conditional on each realization of the shock, hence the yield curve moves through time from one row of $\Pi(k)$ to another row of $\Pi(k)$. As is well known, both in the model and in the real world, the yield curve does not change much from one period to the next, therefore for any realistic calibration the rows of $\Pi(k)$ are quite similar and so $\Pi(k)$ is likely to be nearly singular. As is well known $\partial A^{-1}/\partial A$ is very large when a matrix A is nearly singular, therefore small changes in k will bring about large changes in $[\Pi(k)]^{-1}$.

3.3. Simulations

Table 2 summarizes the results for the model with capital accumulation. Setting $\alpha = 0.4$, the depreciation rate $\delta = 0.05$, assume the initial value of government debt is zero and set initial capital equal to its deterministic steady state Ramsey value. As the bond holdings issued in each period are no longer constant the average structure of the value of debt and also the average of the 5% lowest and 5% highest positions for each maturity are reported as $E_{\pm 5\%}$ in the table.[6] Functions G and D are found by standard Parameterized Expectation Algorithm (PEA) combining long and short simulations. Given G, P is found by approximating the expectations of future marginal utilities in one step and construct Π. See Appendix for details.

The results show that adding capital exacerbates the magnitude of the positions. Allowing for capital accumulation introduces another margin through which agents can smooth consumption and so interest rates and bond prices are less volatile requiring larger positions to achieve the complete market outcome. As explained above capital accumulation also makes the debt positions time varying and Table 2 shows the variation to be substantial. For instance, in the case of persistent productivity and expenditure shocks although on average the government issues long term debt worth 3344% of GDP in 5% of the periods it issues long term debt worth on average around 1594% GDP and at the other extreme in 5% of periods issuance averages around 6629%.

The qualitative recommendations of the complete markets approach are undermined in yet another dimension. Return to the case where $g_t = \overline{g}$ for all t and assume the government issues a one-period bond and a j-period bond, $j = 2, \dots, 30$. The (average) optimal bond positions are shown in Fig. 1 for many j's. For $j < 18$ the government should issue short term debt and invest in long term bonds. However, for $j \geq 18$ the result flips around and now issuing long and investing short becomes optimal. Further evidence of sensitivity to small changes in specification and how bond positions change over time is shown in Fig. 2 which plots the policy function for the j=16 case. Fig. 2 shows that there is a value of capital k^* (near 1291) such that, if $k_t < k^*$ the government should issue short term debt and invest in long term assets, and these signs are reversed for $k_t > k^*$. Furthermore, short bonds converge to plus (minus) infinity as $k_t \nearrow k^*$ ($k_t \searrow k^*$), the opposite signs for long bonds. The reason that this policy function displays an asymptote is that the matrix of bond returns $\Pi(\cdot)$ is non-invertible at k^*. More precisely, from (18) if the shock takes only two values then $b^{16}_{t-1} \equiv B^N(k_{t-1}) = (D(k_{t-1},\theta^L) - D(k_{t-1},\theta^H))/(P^{15}(k_{t-1},\overline{\theta}^L) - P^{15}(k_{t-1},\overline{\theta}^H))$. It turns out that the denominator in this expression is zero at $k_{t-1} = k^*$, negative (positive) for lower (higher) k_{t-1} whilst the numerator is never close to zero. This singularity is the reason for the change in sign and the asymptotes in Fig. 2. In the simulations k^* is found to be very close to the median of the steady state distribution, hence a switch from one side to the other of the singularity is bound to occur.

In summary, far from confirming the qualitative insights of Angeletos (2002) the addition of capital accumulation significantly undermines the recommendation to always issue long and buy short and to issue constant bond positions.

4. Habits and term structure volatility

In this section habits are introduced into the utility function. Habits have been widely used as a means of matching asset market puzzles in the literature e.g. Constantinides (1990), Campbell and Cochrane (1999) and Wachter (2006).

[5] The "probability" statement is with respect to the distribution on k_t induced by the Ramsey solution. If $\Pi(k_t)$ is singular with positive probability then, quite simply, the complete markets approach cannot be implemented with N maturities.

[6] Same comments as in footnote 4 apply to this table.

Table 2
Simulation results—capital accumulation.[a]

Shocks						Interest rates				
g			B^1		B^{30}			H		L
	$\mu=1$		−14.49		12.36	R^1		2.08		1.98
	$E_{+5\%}$		−18.29		9.41	R^{30}		2.07		2.00
	$E_{-5\%}$		−11.65		16.3					
			B^1		B^{30}					
	$\mu=0$		−9.23		7.19	R^1		2.06		1.99
	$E_{+5\%}$		−9.50		6.90	R^{30}		2.04		2.03
	$E_{-5\%}$		−8.94		7.46					
θ			B^1		B^{30}			H		L
	$\mu=1$		−8.49		6.26	R^1		2.26		1.85
	$E_{+5\%}$		−12.5		3.56	R^{30}		2.01		2.07
	$E_{-5\%}$		−5.62		10.10					
			B^1		B^{30}					
	$\mu=0$		−3.49		1.47	R^1		2.01		2.07
	$E_{+5\%}$		−3.93		1.19	R^{30}		2.02		2.06
	$E_{-5\%}$		−3.12		1.82					
g,θ							HH	HL	LH	LL
		B^1	B^4	B^{16}	B^{30}					
	$\mu=1$	−30.10	42.54	−48.18	33.44	R^1	2.46	1.67	2.26	1.48
	$E_{+5\%}$	−34.33	26.14	−97.58	15.94	R^{30}	2.03	2.07	1.94	1.98
	$E_{-5\%}$	−26.30	63.28	−16.46	66.29					
$\pi_H^g=0.95$		B^1	B^9	B^{13}	B^{29}					
$\pi_H^\theta=0.91$	$\mu=1/3$	−14.38	32.62	−30.74	10.42	R^1	2.04	1.97	2.00	1.92
	$E_{+5\%}$	−18.80	26.24	−36.75	8.16	R^{29}	2.01	2.05	2.00	2.03
	$E_{-5\%}$	−11.00	41.44	−25.37	11.91					
		B^1	B^5	B^{18}	B^{30}					
	$\mu=1$	−77.85	153.10	−207.77	130.19	R^1	2.55	1.63	2.42	1.50
	$E_{+5\%}$	−109.15	138.74	−226.37	106.12	R^{30}	2.09	2.05	2.02	1.99
	$E_{-5\%}$	−55.63	167.34	−189.63	161.17					
$\pi_H^g=0.95$		B^1	B^9	B^{14}	B^{29}					
$\pi_H^\theta=0.98$	$\mu=1/3$	−12.58	21.44	−23.13	12.20	R^1	2.07	1.94	2.03	1.90
	$E_{+5\%}$	−34.93	13.46	−54.90	8.63	R^{29}	2.03	2.00	2.05	2.01
	$E_{-5\%}$	−5.48	70.24	−18.56	17.44					

[a] Table shows maturity structure and yield curve for simulations of an economy with capital subject to productivity and expenditure shocks.

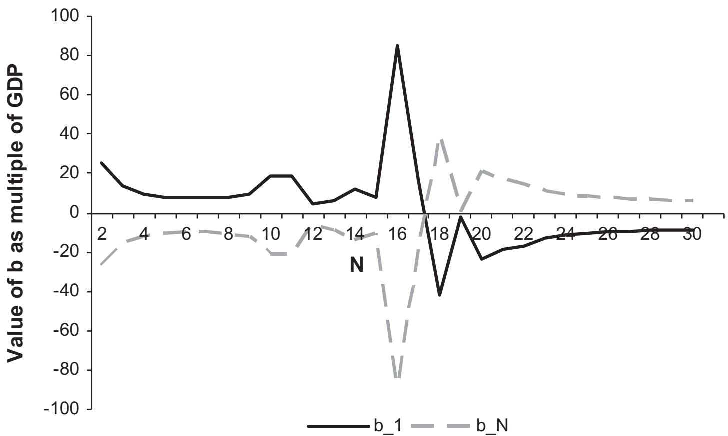

Fig. 1. Sensitivity of portfolio structure—capital accumulation and persistent technology shocks. Note: When long bond is less than 17 period maturity then invest long, for longer maturity then issue long.

In essence, with habits interest rates are a function of consumption growth and so the slope of the yield curve depends on the rate of change of consumption growth, increasing its volatility. This is important because a potential criticism to our findings in Section 3 of extreme, volatile and unstable positions is that they are driven by a counterfactually low volatility in the yield curve.

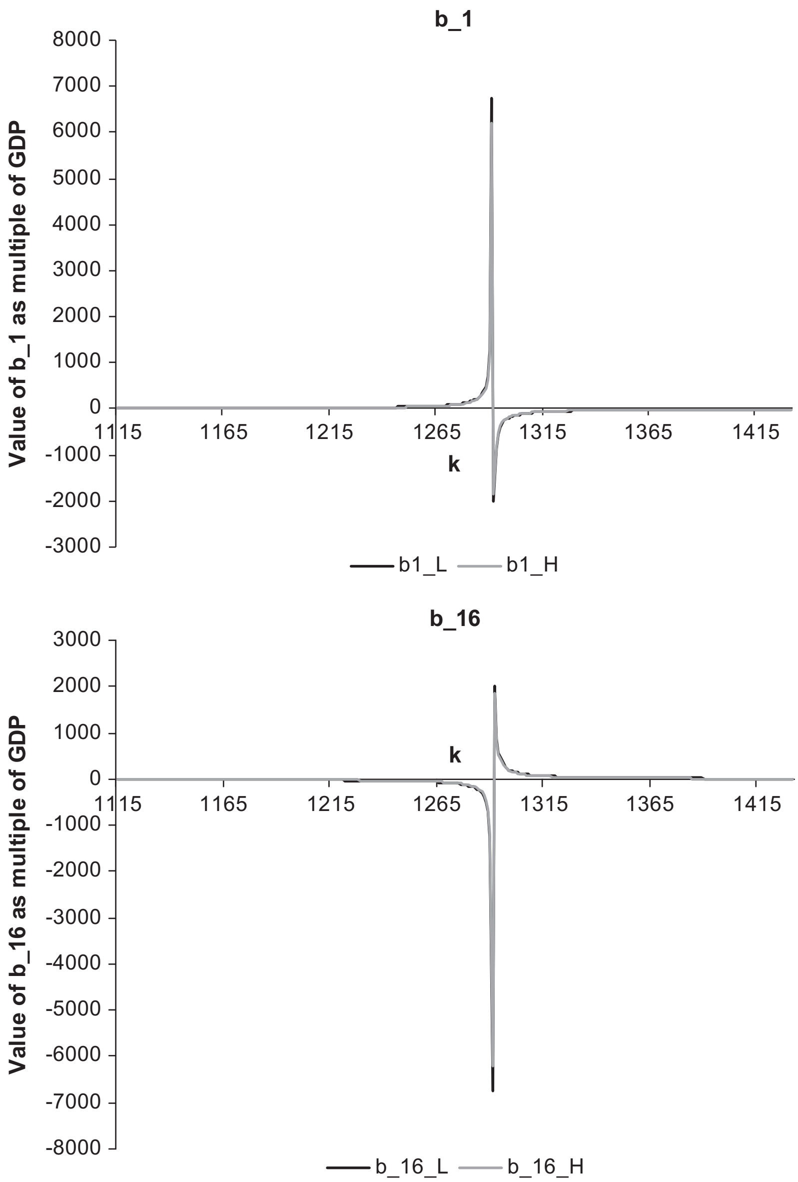

Fig. 2. Policy functions for debt issuance—capital accumulation and persistent technology shocks. Note: Sign of optimal positions changes dramatically for small fluctuations in capital stock around its median.

The following discussion highlights the importance of the volatility of the yield curve, and tries to bring this issue to the data. Consider again the model of Section 2 when g can take two possible values $\overline{g}_H,\overline{g}_L$, θ is a constant, and assume the government issues a short bond that matures in S periods and a long bond maturing at date $M>S$. To effectively complete the markets a portfolio b_{t-1}^S,b_{t-1}^M needs to be issued satisfying $\overline{p}_t^{S-1,i}b_{t-1}^S+\overline{p}_t^{M-1,i}b_{t-1}^M=\overline{z}^i$ for $i=H,L$ for all h^{t-1} where $\overline{p}_t^{S-1,H}$ is shorthand for $\overline{p}_t^{S-1}(h^{t-1},\overline{g}_H)$ and so on. Using the complete market methodology gives the following optimal long position:

$$b_{t-1}^M=\frac{\overline{p}_t^{S-1,H}\overline{z}^L-\overline{p}_t^{S-1,L}\overline{z}^H}{\overline{p}_t^{S-1,H}\overline{p}_t^{M-1,L}-\overline{p}_t^{S-1,L}\overline{p}_t^{M-1,H}} \tag{19}$$

The closer to zero is the denominator then, ceteris paribus, the larger is the absolute value of b_{t-1}^M. Now, let $y_t\equiv(S-1)spr_t+(M-S)r_t^{M-1}$ where $spr_t\equiv r_t^{M-1}-r_t^{S-1}$ is the interest rate spread between long and short bonds, and r is the annualized net interest rate at each maturity. Log-linearizing the denominator in (19) around 1 and rearranging gives that this denominator is approximately equal to $y_t(h^{t-1},\overline{g}^H)-y_t(h^{t-1},\overline{g}^L)$. This is the difference of the two possible realizations of y_t conditional on a past history h^{t-1}, so that the denominator in (19) is proportional to the one-period ahead conditional variance of y. Hence greater *conditional* volatility of y means lower optimal position b_{t-1}^M. Indeed one could generate bond positions that are huge by constructing a model with a volatility of y that is unrealistically low. This suggests that $\phi \equiv \sqrt{var_{t-1}(y_t)}/Er_t^1$ is the relevant aspect of the yield curve volatility that needs to be matched for the issue at hand.

We estimate ϕ with US data over the period 1949–2004 and set $M=11$ and $S=2$ years, so that the spread involved is between 10 and 1-year bonds. The term $var_{t-1}(y_t)$ is found by building a prediction mode for y. Applying standard model selection criteria on a VAR using a set of related macroeconomic variables (e.g. GDP growth, interest rates, primary deficit, inflation π) yields an equation of the form

$$y_t = \alpha_1 + \alpha_2 y_{t-1} + \alpha_3 \pi_t + \alpha_4 \pi_{t-1} + \varepsilon_t$$

as the best predictive model. The estimated variance of ε is our measure of $var_{t-1}(y_t)$ which leads to an estimate $\hat{\phi}=3.38$. Comparing this to the simulations of Sections 2 and 3 confirms how poorly those models reproduce volatility in the yield curve. In the model without capital (Section 2) this leads to $\phi=0.052$ for the model with government spending shocks, $\phi=0.317$ for the model with technology shocks and only $\phi=1.172$ even allowing for both shocks.

We now try to match ϕ in a model with habits in consumption with utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t,c_{t-1}) + V(x_t)]. \tag{20}$$

The resource, budget constraints, market equilibrium concept and government policy are as in Section 2. Equilibrium prices are given as before by (9) where marginal utility of consumption is now

$$U_{c,t} \equiv \frac{\partial U(c_t,c_{t-1})}{\partial c_t} + \beta E_t\left[\frac{\partial U(c_{t+1},c_t)}{\partial c_t}\right]. \tag{21}$$

The implementability condition is (5) but with $U_{c,t}$ given by the above formula. The presence of future variables in $U_{c,t}$ introduces some technical difficulties and non-standard aspects in the optimal policy. Unlike the case of Section 2 the first order conditions of the Ramsey policy include intertemporal terms since c_t now appears in $U_{c,t}$, $U_{c,t-1}$ and $U_{c,t+1}$. Further in order to write recursively the problem it is necessary to operate on the implementability constraint until the Lagrangian is expressed as a recursive sum from period 1 onwards.

In the standard case, as in Section 2, it is well known that the policy function is time-invariant for all periods (including $t=0$) assuming zero initial government debt. In the case with habits even assuming zero initial debt leads to a different function giving optimal policy in period 0, therefore in the equilibrium $c_t = G(h_t,c_{t-1})$ for a time-invariant G only for $t \geq 1$. Using a similar argument as in the last section gives

$$[b_t^1, \ldots, b_t^N]' = B(c_t) \tag{22}$$

for some time-invariant function B and all h^t for all $t \geq 1$. Thus the level and composition of debt that effectively complete the markets now varies with consumption. The formula for B is obtained, analogous to the model for capital, by constructing a matrix $\Pi(c_t)$ with the yield curve for each realization in each row and inverting this matrix for each c_t.

In the simulations it is assumed

$$U(c_t,c_{t-1}) + V(x_t) \equiv \frac{(c_t - \chi c_{t-1})^{1-\gamma_1}}{1-\gamma_1} + \eta \frac{x_t^{1-\gamma_2}}{1-\gamma_2},$$

where χ is chosen to match ϕ.

In the case of habits, no capital and productivity shocks alone the model is able to match exactly ϕ with $\chi=0.273$. Unfortunately, for the case of expenditure shocks only or both expenditure and productivity shocks no value of χ can be found to match ϕ. In these cases the conditional volatility of spr_t[7] is instead matched. For the case of expenditure and productivity shocks $var_{t-1}(spr_t)$ is matched for $\chi=0.25$. Unfortunately, no value for χ matches $var_{t-1}(spr_t)$ either for the case of just expenditure shocks, so this case is excluded from the analysis.

Table 3 considers only persistent shocks. Consistent with our motivation, the higher volatility of the interest rate spread now lowers the magnitude of the positions relative to Sections 2 and 3. But these remain large (for instance with both shocks the government issues 22 year bonds worth 11.48 times GDP and invests in 10 year bonds worth 18.23 times GDP). Furthermore, allowing for habits reduces the size of the positions but creates a substantial additional problem, the volatility of positions is now huge. For instance, focusing on the higher 5% realizations of the long bond issuance they are on average 99.10 times GDP while for the lowest 5% realizations the government invests in 22 year bonds to the value of 62.69 GDP on average. Once again, the simple qualitative recommendation of issuing long and investing short is easily overturned since the government invests heavily in long maturities in many periods.

The reason behind these results is that the policy functions are similar to those of Fig. 2. The spike is now at the level of consumption $c^* = 52.42$, at this level of consumption the matrix of returns is non-invertible and the sign of the bond

[7] The model selection procedure for the spread gives the empirical model

$$spr_t = \alpha_1 + \alpha_2 spr_{t-1} + \alpha_3 \frac{def_{t-2}}{gdp_{t-2}} + \alpha_4 r_{t-2}^1 + u_t,$$

where def_t/gdp_t is the primary deficit/GDP ratio. This leads to an estimate of $\sqrt{var_{t-1}(u_t)}/Er_t$ equal to 0.341.

Table 3
Simulation results—consumption habits.[a]

Habits						**Interest rates**				
θ **shock**										
							H	L		
$\chi=0$		B^1	B10			R^1	1.07	2.93		
	$\mu=1$	−1.03	1.07			R^{10}	1.58	2.47		
$\chi=0.273$		B^1	B^{10}							
	$\mu=1$	−0.63	0.62			R^1	−0.58	5.10		
	$E_{+5\%}$	−0.68	0.59			R^{10}	1.37	2.73		
	$E_{-5\%}$	−0.58	0.66							
g,θ **shocks with** $\pi^g_{HH}=0.95$ **and** $\pi^\theta_{HH}=0.91$										
							HH	HL	LH	LL
$\chi=0$		B^1	B^{10}	B^{16}	B^{30}	R^1	1.23	3.15	0.90	2.71
	$\mu=1$	−4.60	71.74	−159.02	101.39	R^{30}	1.92	2.28	1.79	2.15
$\chi=0.25$		B^1	B10	B15	B22					
	$\mu=1$	−0.48	−18.23	7.01	11.48	R^1	0.09	5.39	−0.77	3.50
	$E_{+5\%}$	−0.50	−27.45	−91.14	−62.69	R^{22}	1.82	2.44	1.62	2.21
	$E_{-5\%}$	−0.45	−7.24	90.36	99.10					

[a] Table shows maturity structure and yield curve for simulations using a model with habits subject to productivity and expenditure shocks.

holding switches. This reversal of optimal debt positions occurs despite the fact that interest rates in the model do rise in response to adverse expenditure shocks—a combination that Angeletos (2002) and others stress as important for making it optimal for governments to issue long term debt.

5. Robustness

The paper has so far shown how the complete market approach leads to large positions which in the presence of habits or capital accumulation show high volatility and how variations in model calibration lead to substantial changes in optimal portfolio structure such that there is no presumption it is optimal to issue long and invest short. However, within the context of the framework used the size, sensitivity and volatility of the positions cannot be used as a criticism of the complete market approach—given the planner's knowledge of the environment and the absence of transaction costs these are the optimal positions. Pointing to extreme magnitudes or volatility cannot be a justified criticism unless these positions come with some cost. As a result the analysis of this section considers how robust these portfolios are to alternative specifications. Firstly the focus is on the case where the government incorrectly specifies the nature of the economy and show how relatively small misspecifications lead to large welfare costs in pursuing the complete market recommendations such that governments are better pursuing a balanced budget. Secondly the size of transaction costs necessary to offset the insurance benefits of the complete market approach are considered and it is shown how even *de minimus* transaction costs make the complete market approach inferior to a balanced budget. In other words once market incompleteness is introduced explicitly into the model the complete market approach is far from optimal and not even an approximate guide for policy.

5.1. Government misperceptions

Key to the size of the positions that the complete market approach recommends is the persistence of the shocks. Errors in perceiving persistence of shocks will therefore translate into sub-optimal portfolio positions. To evaluate the welfare costs of these errors the following measure is used:

$$R(\rho,\rho^*)=\frac{W_X-W_{BB}}{W_{CM}-W_{BB}}$$

where W_i denotes the welfare level obtained for policy regimes i=CM,BB, X and ρ denotes the true vector of parameters of the economy . Here, under CM the government implements the complete market fiscal policy when it correctly knows ρ. Under BB the government runs a balanced budget every period. Policy regime X occurs when the government implements the optimal debt policy as if the primitives of the economy were given by vector ρ^* possibly different ρ. Obviously, $W_X=W_{CM}$ when $\rho=\rho^*$. In all cases agents have rational expectations.

The combination of debt and tax policy is standard in the cases CM and BB, where only ρ plays a role. Under the misspecified policy X it is clear that under rational expectations for the agents the government will not be able to implement both the debt and tax policy that would be optimal under complete markets for the parameter ρ^*: since consumers do not behave as the government conjectures, the complete markets debt and tax policies are not both feasible.

Our assumption in this section is that the government chooses the *debt* positions that are optimal for ρ^* but that taxes then adjust so as to be consistent with a competitive equilibrium when parameters are in fact ρ. Formally, the model solved is one where positions of short and long bonds in all periods $B^1_{\rho^*}, B^M_{\rho^*}$ are determined by the optimal policy under complete markets if the true parameters are ρ^*, as in Section 2. Then tax policy $\tau(\rho,\rho^*)$ is a sequence of contingent taxes that satisfies the budget constraint of the government each period when bonds are fixed at $B^1_{\rho^*}, B^M_{\rho^*}$, and the sequences c,x are competitive equilibrium allocations when consumers have rational expectations, given this tax policy.

The ratio $R(\cdot,\cdot)$ captures the proportion of the gains of optimal debt issuance that are preserved when the government misperceives the economy. The denominator measures the maximal welfare gains that come from issuing debt and so $R(\cdot,\cdot)$ is bounded from above by 1. For values of $R(\cdot,\cdot)$ between 0 and 1 the misspecification of the primitives reduces the welfare gains from debt management but still leads to an improvement over the balanced budget case. In the case when $R(\cdot,\cdot)<0$ then attempts at optimal debt management produce worse outcomes than a balanced budget.

5.1.1. *Misperceiving persistence*

Consider the earlier model of an economy without capital and subject only to government expenditure shocks. Government expenditure can take only two values, high and low and, as in Section 2 π_{HH} and π_{LL} denote the probabilities of staying in each state. Assume as in Section 2 that $\pi_{HH}=\pi_{LL}=0.95$ but that the government has beliefs $\pi^*_{HH}=\pi^*_{LL}\neq 0.95$. Assuming the government can issue only one year and 30 year bonds Fig. 3 shows how $R(\pi_{HH},\pi^*_{HH})$ varies as beliefs alter with respect to reality. The horizontal axis shows π^*_{HH}, denoted *prob** in the figure. So long as $\pi^*_{HH}<\pi_{HH}$ then $R(\cdot,\cdot)$ is always positive even if less than 1. However, in the case that the government overestimates persistence (and so takes larger positions) then $R(\cdot,\cdot)$ drops rapidly and quickly turns negative. Governments are better off following a balanced budget than operating complete market policies when they overestimate the persistence of expenditure shocks.

As the positions recommended by complete markets are very sensitive to the choice of maturities so too will be the welfare losses. Table 4 investigates this by calculating $R(\cdot,\cdot)$ across all combinations of one-year bonds with bonds of up to 30 years and reporting the highest and lowest value for $R(\cdot,\cdot)$ in the case where $\pi_{HH}=0.95$ but government beliefs differ. Column R reports the same numbers as the line in Fig. 3, column max$R$ reports the maximum R that was found across all maturities up to 30 years and the analogous holds for min$R$. The table suggests that the choice of 1 and 30 year bonds in Fig. 3 was flattering to the complete market approach. Other maturities frequently lead to worse outcomes than the balance budget case when the persistence of expenditure shocks is underestimated and the losses are even greater in this direction than overestimating the persistence. Although issuing 30 year bonds is rarely the way to maximize $R(\cdot,\cdot)$ in the case of misperceptions it does seem that issuing such long bonds is a more robust way of minimizing the losses from underestimating the persistence of shocks. It does not, however, help against the costs of overestimating persistence.

To better understand the robustness of the complete market approach to model misspecification the following exercise is performed. Consider the optimal portfolios of 1 and 30 period debt when the government thinks persistence is either 0.65, 0.75, 0.85 or 0.95, i.e. four different portfolios. In the case where the government believes persistence is 0.95 the optimal portfolio is to issue 30 period debt worth 701% of output and go short by 689% in one year bonds. The absolute size of the positions is declining in the perceived persistence such that when the government thinks the persistence parameter is 0.65 the positions are 112% and −110%, respectively. Welfare is then calculated for all four portfolios but where the true persistence in the economy takes values between 0.1 and 0.9. The results are shown in Fig. 4. The results suggest that the balanced budget outcome is always worse than implementing complete markets under the mistaken belief that $\pi^*_{HH}=0.65$. By contrast when beliefs are that $\pi^*_{HH}=0.95$ then a balanced budget dominates nearly everywhere. The implication is that it is the size of the positions that leads to welfare losses from misspecification. Given these results, robustness considerations would suggest reducing the magnitude of positions advocated by complete markets.

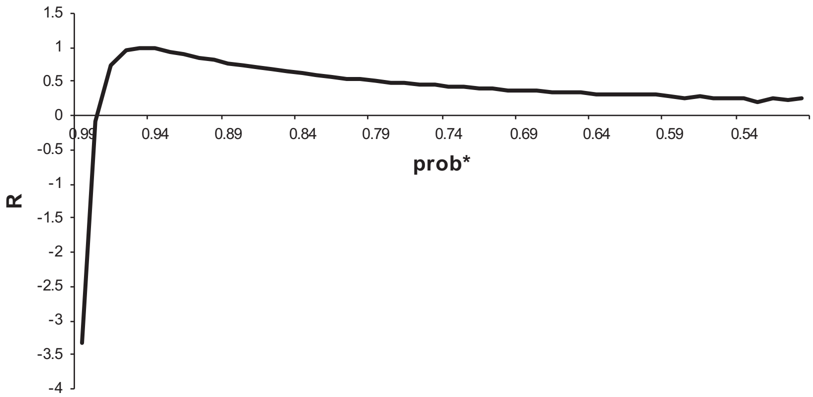

Fig. 3. Uncertainty in the transition probabilities: 2 state economy $-\pi_{HH}=0.95$. Note: When government overestimates shock persistence then balanced budget dominates. Substantial welfare loss for large underestimates.

Table 4
Robustness misperceptions across maturities.[a]

π^*_{HH}	max*R*	**R**	min*R*
0.99	−1.384	−3.319	−8.290
0.98	0.377	−0.067	−2.740
0.97	0.841	0.750	−0.026
0.96	0.974	0.962	0.818
0.95	1.000	1.000	1.000
0.94	0.987	0.982	0.727
0.93	0.959	0.944	−0.434
0.92	0.927	0.901	−3.269
0.91	0.894	0.857	−8.840
0.90	0.863	0.815	−17.998
0.89	0.834	0.776	−30.565
0.88	0.807	0.739	−44.899
0.87	0.782	0.705	−58.576
0.86	0.760	0.674	−69.745
0.85	0.739	0.645	−77.849
0.84	0.720	0.618	−83.321
0.83	0.703	0.593	−86.930
0.82	0.687	0.570	−89.365
0.81	0.672	0.548	−91.104

[a] Welfare loss across maturities when government misspecifies persistence of shocks in an endowment model with expenditure shocks ($R(\cdot)=1$ no welfare loss, $R(\cdot)<0$ balanced budget dominates).

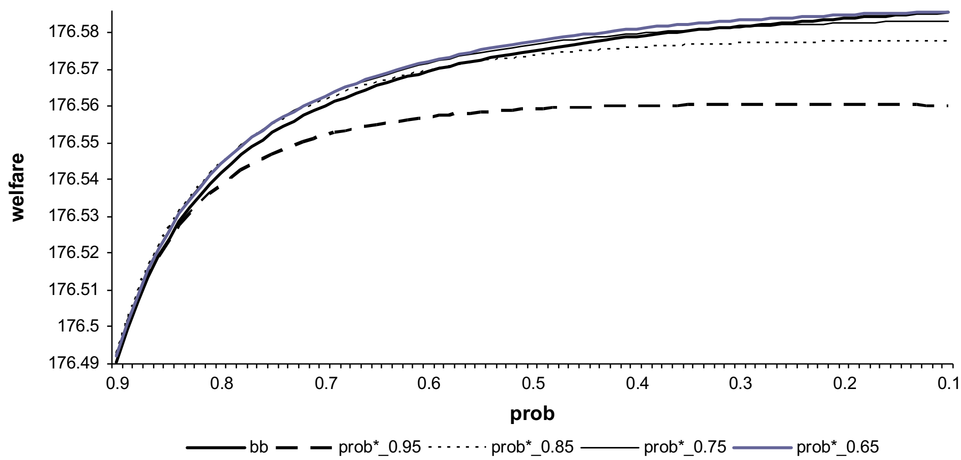

Fig. 4. Horse race: balanced budget (BB) against ABN. Note: Welfare comparison of balanced budget against optimal debt management with misspecified structural information.

5.1.2. *Misperceiving states of the world*

The previous subsection focused on a minor deviation from complete markets. The government still issued enough securities (2—the number of states of the world) to achieve the complete market outcome but because of misperceptions failed to do so. In this subsection a more serious failure is considered—the government continues to issue 2 securities but there exist three states of the world. As well as government expenditure taking on a high and a low value it can also with small probability take on a very large value, g^W(as would be the case with a war). Specifically the economy is characterized by a transition matrix

$$\begin{pmatrix} \pi & 1-\pi & 0 \\ 1-\pi & \pi-\pi^W & \pi^W \\ 0.05 & 0.9 & 0.05 \end{pmatrix}$$

but the government perceives only a transition matrix between two states as in Section 5.1.1. With initial $g_0=\overline{g}_H$, if $\pi^W=0$ the government is correct in believing that wars cannot occur. Fig. 5 shows the value of $R(\cdot,\cdot)$ as π^W varies from 0 to 0.05. Even for very small values of π^W there is a sharp fall in welfare such that it is often optimal to follow a balanced budget rather than the complete market outcome.

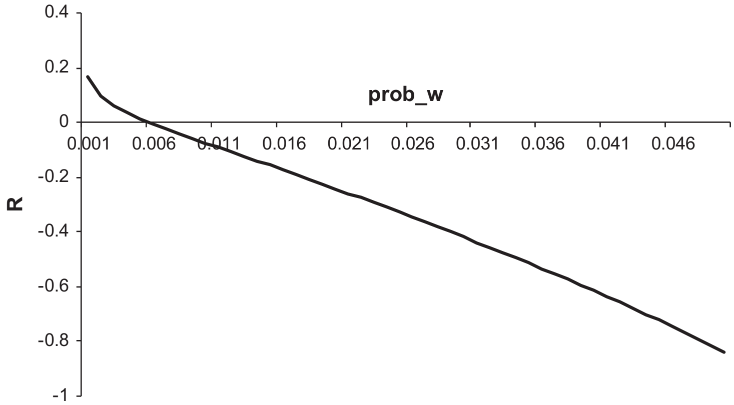

Fig. 5. Hidden state—3 state economy but 2 bonds. Note: Balanced budget dominates when probability of ignored state exceeds 0.6%.

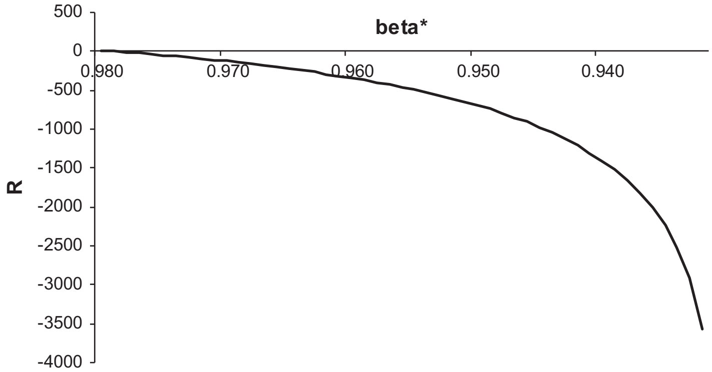

Fig. 6. Uncertainty in the discount factor: 2 state economy. Note: Underestimating the discount rate leads to substantial welfare losses and preference for balanced budget.

5.1.3. Misperceiving the discount rate

In this section we show how errors in estimating the discount rate again lead to it being better to use a balanced budget rather than issue debt. Consider the case where the agents discount rate is $\beta = 0.98$ but the government has beliefs in the range (0.93,0.98). Fig. 6 shows the welfare performance across the various combinations. For the case of issuing a 1 and 30 period bonds any incorrect beliefs over the discount factor lead to a worse outcome than a balanced budget. This example also illustrates another non-robustness problem. We documented in Section 5.1.1 that when governments made mistakes about the persistence of shocks there was some evidence that issuing long bonds was the most robust policy. However, in the case of errors in the discount rate issuing long bonds is usually worse than the balanced budget.

5.2. Transaction costs

In this subsection once more the case in which the government has perfect knowledge is considered but now allowing for transaction costs. Assuming the government pursues the complete market approach to debt management even when markets are transaction costs this section calculates the level of transaction costs that would make the government indifferent between pursuing this approach or a balanced budget. Specifically the level of transaction costs *TC* is calculated such that if government spending is actually $g_t + TC$ and government follows complete markets optimal debt management the welfare achieved is the same as under balanced budget (and, therefore, zero transaction costs).

In the case of only government expenditure shocks in Table 1 $\mu = 1$ a miniscule level of transaction costs equal to 0.016% of steady state government expenditure (equivalent to 0.003% of the absolute value of debt issued) are sufficient for the government to prefer a balanced budget. In the case of both productivity and expenditure shocks the level of transaction costs required to be indifferent with a balanced budget is 0.02% of steady state expenditure and 0.002% of the absolute value of debt issued. Arguably actual transaction costs for issuing actual debt are much larger than 0.002%, specially if the spread between borrowing and lending rates is supposed to reflect the presence of transaction costs.

This section has shown through a series of examples how explicitly introducing market incompleteness often produces outcomes where governments would rather avoid the insights of the complete market approach. This result echoes Siu (2004) who analyses the role of unexpected inflation as a means of varying the ex post real return on debt in a Ramsey model characterized by non-state contingent nominal debt (see also Lustig et al., 2009 for the role of debt management in a nominal setting). Key to Siu's model is the trade off between fluctuations in unanticipated inflation aimed at achieving the complete market outcome and the welfare costs such inflation induces because of sticky prices and the resulting incorrect relative prices and resource misallocation. Through his specification of sticky prices Siu is deriving explicitly an endogenous cost structure over the government's effective liability position. He finds that for government expenditure volatility similar to that experienced by the US since 1945 (as in our calibration) the costs exceed the benefits and making use of fluctuations in the real return of debt is sub-optimal. Only if government expenditure is characterized by large war time spikes is it worth using this margin to reduce tax volatility and incur the misallocation costs. Siu's focus on nominal issues makes his model very different from ours as does his focus on product rather than bond market imperfections. However, his conclusion is analogous to ours: for government expenditure shocks calibrated to post war volatility governments prefer to avoid using the complete market approach to debt management so as to avoid costs due to market imperfections.

6. Conclusion

Macroeconomists have become increasingly interested in trying to embed debt management into theories of optimal fiscal policy. This literature has produced an appealing theory—the complete market approach to debt management. By exploiting variations in the yield curve the government can structure its non-contingent debt so as to minimize the distortionary costs to taxation. A number of authors have argued this approach offers a robust qualitative recommendation to debt managers—governments should issue long term debt and invest in short term bonds.

In this paper we have extensively reviewed the insights and implications of this complete market approach to debt management and identified a number of areas where this methodology is problematic:

(i) As in Buera and Nicolini (2004) the magnitude of the debt positions the government is required to hold are implausibly large multiples of GDP. Buera and Nicolini's results are extended by calibrating the model to US data and considering a range of extensions including capital accumulation and habits. Although the magnitude of the positions does change substantially across these model specifications they remain throughout extremely large compared with observed practice.

(ii) Extending the model to allow for capital accumulation and habits identifies an additional problem. The required positions also show large volatility. In some cases this volatility is so large that optimal positions for long term debt fluctuate between large negative and positive positions from one period to the next. It is certainly not a robust qualitative recommendation of complete markets that governments should issue long term debt and invest in short bonds.

(iii) The complete market approach is very sensitive to small variations in parameters. Both the size and sign of positions can change dramatically with small changes in relative persistence of shocks or slight changes in the maturity of bonds governments can issue.

(iv) We show that by introducing varying degrees of market incompleteness these large volatile and unstable debt positions lead to sub-optimal outcomes. In particular, allowing for possible model misspecification or transaction costs it is shown that the government would prefer to follow a balanced budget rather than implement the optimal portfolio structure recommended by the complete market approach.

The fundamental problem of the complete market approach is that the limited volatility of the yield curve makes maturities a poor substitute for state contingent debt. Therefore in order to exploit the maturity structure of debt the complete market approach requires large positions. If governments were to try and implement these policy recommendations they would have to buy and sell enormous amounts of bonds each period. This would entail all kinds of transaction costs, refinancing risks, and it would force some private agents in the economy to hold the opposite of the huge positions the government decided to take, possibly facing credit constraints. The government would have to hold very large amounts of private debt which could be defaulted upon. By explicitly ignoring these features of market incompleteness we believe the complete market approach is potentially misleading. The great strength of the complete market approach is it recognizes the importance of debt management in providing insurance against fiscal shocks. However, the weakness with the complete market approach is it only focuses on fiscal insurance and abstracts from fundamental features of market incompleteness. A successful theory of debt management will need to balance the insights of fiscal insurance with the constraints that incomplete markets provide. We remain in search of a plausible theory of debt management.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.jmoneco.2010.08.005.

References

Angeletos, G.-M., 2002. Fiscal policy with non-contingent debt and optimal maturity structure. Quarterly Journal of Economics 27, 1105–1131.
Barro, R.J., 1999. Notes on optimal debt management. Journal of Applied Economics 2, 281–289.
Barro, R.J., 2003. Optimal management of indexed and nominal debt. Annals of Economics and Finance 4, 1–15.
Buera, F., Nicolini, J.P., 2004. Optimal maturity of government debt with incomplete markets. Journal of Monetary Economics 51, 531–554.
Campbell, J.Y., Cochrane, J., 1999. By force of habit: a consumption based explanation of aggregate stock market behaviour. Journal of Political Economy 107, 205–251.
Chari, V.V., Christiano, L.J., Kehoe, P.J., 1991. Optimal fiscal and monetary policy: some recent results. Journal of Money Credit and Banking 23, 519–539.
Chari, V.V., Christiano, L.J., Kehoe, P.J., 1994. Optimal fiscal policy in a business cycle model. Journal of Political Economy 102, 617–652.
Chari, V.V., Kehoe, P.J., 1999. Optimal fiscal and monetary policy. In: Taylor, J., Woodford, M. (Eds.), Handbook of Macroeconomics. North-Holland, Amsterdam.
Constantinides, G.M., 1990. Habit formation: a resolution of the equity premium puzzle. Journal of Political Economy 98, 519–543.
Lucas, R.E., Stokey, N.L., 1983. Optimal fiscal and monetary policy in an economy without capital. Journal of Monetary Economics 12, 55–93.
Lustig, H., Sleet, C., Yeltekin, S., 2009. Fiscal hedging with nominal assets. Journal of Monetary Economics 55, 710–727.
Marcet, A., Scott, A., 2009. Debt and deficit fluctuations and the structure of bond markets. Journal of Economic Theory 144 (2), 473–501.
Nosbusch, Y., 2008. Interest costs and the optimal maturity structure of government debt. Economic Journal 118, 477–498.
Siu, H., 2004. Optimal fiscal and monetary policy with sticky prices. Journal of Monetary Economics 51, 575–607.
Wachter, J., 2006. A consumption-based model of the term structure of interest rates. Journal of Finance 79, 365–399.