



Incomplete markets, labor supply and capital accumulation[☆]

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Abstract

Endogenous labor supply decisions are introduced in an equilibrium model of limited insurance against idiosyncratic shocks. Unlike in the standard case with exogenous labor (e.g. [Aiyagari, S.R., 1994. Uninsured idiosyncratic risk and aggregate saving. *Quarterly Journal of Economics* 109, 659–684; Huggett, M., 1997. The one-sector growth model with idiosyncratic shocks: steady states and dynamics. *Journal of Monetary Economics* 39, 385–403]), labor supply is likely to be lower than under complete markets. This is due to an ex post wealth effect on labor supply (rich productive agents work fewer hours) that runs counter the precautionary savings motive. As a result, equilibrium savings and output may be lower under incomplete markets. It is also found that long-run savings remain finite even when the interest rate equals the inverse of the discount factor.

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1. Introduction

The conventional wisdom in the precautionary savings literature is that capital market imperfections and the presence of uninsured idiosyncratic risk lead agents to save more than they would if there were no uncertainty.¹ The literature typically compares the optimal behavior of agents with *inelastic* labor supply who receive random labor productivity (or, equivalently, who receive random endowments) with the outcome that would obtain were they to receive with certainty the implied mean productivity. It concludes, with Aiyagari (1994) and Huggett (1997), that in production economies uninsurable uncertainty leads to higher individual savings, and thus to a higher long-run capital-labor ratio.² Since labor supply is inelastic, this translates into higher output. We dub this positive effect of employment uncertainty on aggregate output the *Aiyagari–Huggett effect*.

The conclusion that an economy with less developed financial markets will achieve a higher output is somewhat paradoxical. The development literature often finds an effect of the opposite sign.³ In any case, for most calibrations, the effect predicted by the models is quite small. This may question the usefulness of equilibrium models with uncertainty in studying the interaction between financial market development and growth.

We show that the Aiyagari–Huggett effect need not hold anymore when labor supply is endogenous. The reason is that, if leisure is a normal good, incomplete markets introduce an ex post wealth effect which reduces labor supply. The mechanism is simple and general: agents who end up employed are ex post richer and, therefore, they work less under incomplete markets than under complete markets because they could not and did not buy insurance against unemployment. If the ex post wealth effect that shrinks labor supply is large enough at the aggregate level to overcome the Aiyagari–Huggett effect, then aggregate capital and output are lower under incomplete markets.⁴ Numerical methods are used to investigate which effect dominates for various parameter sets, and to show that it is possible to construct plausibly calibrated economies in which the ex post wealth effect dominates the Aiyagari–Huggett effect. The lower output occurs when the elasticity of hours worked is large relative to the elasticity of consumption. In fact, for some parameter values that have been used extensively in the literature (a relative risk aversion equal to five) the effect can be very large: completing the markets doubles output.

A theoretical finding of our paper is that, by contrast with the exogenous-labor case, the accumulation of capital remains bounded from above at the individual level even if the return to assets equals the rate of time preference. This occurs because, when leisure is a normal good, the incentive to work decreases with the level of assets. Past an upper bound

¹See among others Leland (1968), Sandmo (1970), and more recently, Kimball (1990), Deaton (1991), Carroll (1994), and Huggett (1993).

²This is the so-called “precautionary savings” effect, which refers here to increased capital formation under uncertainty rather than to the convexity of marginal utility. See Huggett and Ospina (2001) for a discussion.

³For example, using data from a number of countries, Levine (1997) reports “there is a strong positive relationship between (...) financial indicators and (...) long run real per capita growth rates, capital accumulation and productivity growth” (p. 706).

⁴Baxter and Crucini (1995) describe a similar wealth effect under incomplete markets. They use it to explain the low consumption correlation across countries. Wealth effects on labor supply have also been explored by Hansen (1985), Benhabib et al. (1991), Kydland (1995), and the related quantitative literature about real business cycles. See also Abowd and Card (1989), Ríos-Rull (1994), Flodén (1998), Krusell and Smith (1998), Low (2002), Castañeda et al. (2003), and Obiols-Homs (2003) among others.

on wealth, agents would stop working, so that they do not face uncertainty anymore and they can forever maintain a constant consumption flow with probability one. This demonstrates the ex post wealth effect and it also simplifies the analysis of the model.

The arguments presented in this paper have a wide range of applications. Any input that is subject to random productivity has a positive wealth elasticity and is complementary with capital is likely to display similar effects on output and savings. Were we to model, say, technology adoption, entrepreneurship or human capital, investment and output would in all likelihood shrink under incomplete markets for the very same reasons that they do in our paper.

The paper is organized as follows. Section 2 explores a partial equilibrium, static model, and develops the main intuition for the results. Section 3 analyzes a dynamic, multi-period, general equilibrium model with production, borrowing constraints, idiosyncratic shocks and incomplete insurance.⁵ The section establishes that the interest rate in steady state is smaller than the rate of time preference, so even with endogenous labor the capital-labor ratio is higher under incomplete markets. In Section 4 we report numerical results corresponding to several parameter values of interest. Section 5 concludes by outlining directions for further research.⁶

2. Wealth effects on labor supply in a static world

Does the unavailability of unemployment insurance lead people to work on average more or less than when unemployment insurance is available? This question is addressed here in the simplest possible environment: a static economy in which a fraction of the population might be randomly unemployed.

The economy consists of a continuum of ex ante identical consumers over the unit interval. Consumers have preferences $U(c, l)$ over consumption and leisure. The utility function U satisfies

A1. $U : R_+ \times [0, 1] \rightarrow R_+$, is continuous and differentiable.

A2. U is strictly increasing and strictly concave in each of its arguments, with $\lim_{c \rightarrow 0} U_c(c, l) = +\infty \forall l \in [0, 1]$, and $\lim_{l \rightarrow 0} U_l(c, l) = +\infty \forall c \geq 0$.

Labor supply is elastic with respect to the wages (work effort would otherwise be the same under complete and incomplete markets). In addition, it is required that *leisure be a normal good*.

All consumers are endowed with one unit of time, but they are subject to exogenous idiosyncratic employment shocks (or shocks of labor productivity, like health shocks): when a consumer wakes up in the morning, she is either employed or unemployed.⁷ Therefore, the employment state s is modeled as a random variable that takes the value 0 with probability $1 - \phi$, and the value 1 with probability ϕ . The employment probability ϕ is the same for all agents, but the ex post realization of the employment process is individual-specific. In the aggregate, given the continuum assumption, ϕ also measures the fraction of the population that is employed.

⁵Hernández (1991) is an early example of the effect of borrowing constraints in the neoclassical model of growth. See also Jappelli and Pagano (1994), Levine (1997), and Smith (2002) for a more recent treatment.

⁶An earlier version of the paper with some analytical examples and all the proofs of lemmas and propositions is available online at <http://www.philippeweil.com/research/MOW.pdf>.

⁷The assumption is that health cannot be influenced by the consumer's actions. Thus, moral hazard considerations are absent from our analysis.

Consumers also receive a non-produced endowment Ω of the consumption good. This endowment is non-random, and is identical across agents. Consumers can supplement their endowment of the consumption good by devoting some of their time to work: one unit of time produces 1 unit of the consumption good in the employment state, and zero otherwise. Obviously, only employed consumers will ever choose to sacrifice leisure to work. The equilibrium outcomes corresponding to two market arrangements is described next.

(A) *Complete markets*: Suppose that the consumers' employment state can be perfectly and costlessly monitored by third parties. In that setup, competitive insurance companies can offer unemployment insurance to the consumers at the actuarially fair price $p = 1 - \phi$. By paying p units of the consumption good to the unemployment insurance company before the shock is realized, consumers buy the right to get 1 unit of the consumption good if they end up unemployed at the end of the period, they get 0 otherwise. Letting Q denote the demand for insurance, a consumer with non-produced endowment Ω chooses c^e, c^u, l , and Q in order to $\max \phi U(c^e, l) + (1 - \phi)U(c^u, 1)$ subject to $c^e + pQ = \Omega + (1 - l)$, $c^u + pQ = \Omega + Q$, and $Q \geq 0, l \in [0, 1]$. In this formulation c^e and c^u denote consumption in the employment and unemployment states, and l denotes leisure in the employment state (in the unemployment state leisure equals its maximum possible value 1).⁸ One can readily verify that the implied consumption allocation coincides with the command optimum $\max_{c^e, c^u, l} U(c^e, c^u, l)$ subject to $\phi c^e + (1 - \phi)c^u = l + \Omega$. Hence, the competitive equilibrium described above is indeed a complete market equilibrium. A basic, and intuitive, property of the consumer's insurance decision is that:

Lemma 1. *If leisure is a normal good, then the demand for insurance is strictly positive ($Q > 0$) in an interior solution with $l < 1$.*

(B) *Incomplete markets*: An unemployed consumer under incomplete markets just consumes her endowment: $c^{u,IM} = \Omega$, where variables superscripted IM hereafter denote incomplete market magnitudes. When she is employed, our consumer chooses consumption and work effort $(c^{e,IM}, l^{IM})$ so as to maximize $U(c^{e,IM}, l^{IM})$ subject to $c^{e,IM} = \Omega + (1 - l^{IM}), 0 \leq c^{e,IM}, l^{IM} \in [0, 1]$.

The following proposition describes of the market arrangement on labor supply decisions.

Proposition 1. *If leisure is a normal good (and for interior solutions), labor supply is lower under incomplete markets than under complete markets (i.e., $l < l^{IM} < 1$).*

The intuition is clear: in the same way that you are better off ex post if your house does not burn and you have not bought fire insurance, consumers who do end up in the employed state are richer ex post under incomplete markets than under complete markets—because they did not pay an insurance premium for an unrealized contingency!—and they will work less under incomplete markets.⁹

⁸The Inada conditions guarantee that c and l are strictly positive at the optimum. To make sure that the constraint $l \leq 1$ is not binding, we need only impose that Ω be not too large (otherwise the constraint $l \leq 1$ binds and our consumer chooses never to work, regardless of whether she is employed or not).

⁹This statement is most definitely only a statement about ex post wealth, and not ex ante, wealth. But it is ex post wealth that matters for labor supply decisions under incomplete markets.

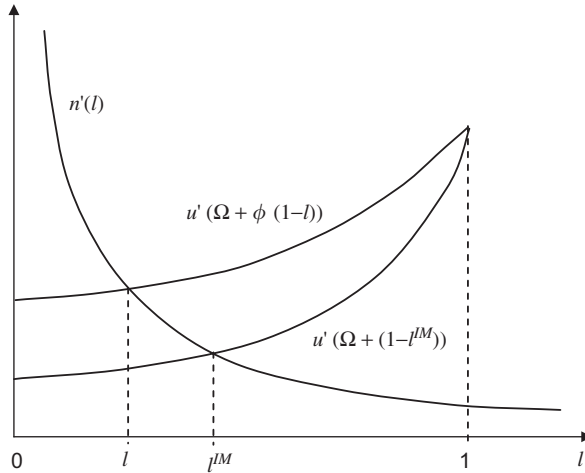


Fig. 1. Leisure with complete markets (l), and with incomplete markets (l^{IM} (wealth effect)).

As an example, consider a separable utility function $U(c, l) = u(c) + n(l)$. In this case it is straightforward to show that leisure under complete markets satisfies $n'(l) = u'[\Omega + \phi(1 - l)]$, and that under incomplete markets l^{IM} solves $n'(l^{IM}) = u'[\Omega + (1 - l^{IM})]$. Fig. 1 displays the optimal choices of an agent in the employment state, plotting both sides of the previous equations.¹⁰ It is clear that $l < l^{IM}$. Also, this figure suggests that the wealth effect will be big if u is very concave (so that u' is very steep) and/or if n is close to being linear (so that n' is nearly flat). Therefore, one would expect a large wealth effect when the curvature of n is much smaller than the curvature of u .

The conclusion is that when labor supply is elastic there is a fundamental economic mechanism—ex post wealth effects in labor supply—that tends to “shrink” the size of incomplete market economies relative to complete markets. This mechanism runs counter to the “enlarging” mechanisms (precautionary and/or buffer stock saving) at the heart of the Aiyagari–Huggett effect. It is therefore natural to ask how wealth effects in labor supply may interact with the precautionary motive for higher savings in an equilibrium multi-period model.

3. Dynamic equilibrium

Endogenous labor supply decisions are introduced in a version of Huggett’s (1997) model. We keep his notation, and draw on some of his results.

A continuum of agents uniformly distributed in the unit interval maximize the expected value of discounted utility $E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$, which depends on the infinite sequence of consumption and leisure, $\{c_t, l_t\}_{t=0}^{\infty}$. The operator E_0 denotes expectation with respect to the idiosyncratic shocks conditional on information available at time 0 (agents have

¹⁰As in the general case, interior solution is guaranteed, when $u(\cdot)$ and $n(\cdot)$ satisfy Inada conditions, if Ω is not too large: $u'(\Omega) > n'(1)$.

identical preferences and discount factor). In addition to A1–A2, we sometimes use assumption¹¹

A3. $U(c, l) = u(c) + n(l)$ is homogeneous of degree $\gamma \in (0, 1)$, with $u'(1) \neq 0$ and $n'(1) \neq 0$.

Each agent is endowed with both a unit of time and a labor productivity endowment (or employment shock) s_t that satisfies

A4. $s_t \in S \equiv \{1, 0\}$ with $\sum_{s'} \pi_{s'|s} = 1$ and $\pi_{s'|s} > 0$ for all $s, s' \in S$. Furthermore, $\pi_{1|1} \geq \pi_{1|0}$.

There is only one asset (capital) which may be used as a buffer to smooth out consumption in the unemployment state. Thus, the budget constraint of the consumer reads

$$c_t + k_t \leq (1 + r_t)k_{t-1} + w_t s_t (1 - l_t),$$

where $(1 + r_t)$ is the net interest factor from capital (i.e., the rental price of capital plus undepreciated capital) and w_t is the wage rate. The agent also faces the constraints $0 \leq l_t \leq 1$ and $c_t \geq 0$, together with a borrowing limit in capital holdings: $k_t \geq \underline{B}$.

Output in period t is given by an aggregate production function $f(K, H)$, assumed to satisfy

A5. f displays constant returns to scale, with $f_j \geq 0, f_{jj} < 0$.

There is a single firm that maximizes profits each period taking prices as given. This firm operates the technology for production, and rents capital and labor from the agents. In equilibrium, the first-order conditions of the firm are given by

$$f_{K,t} + 1 - d = (1 + r_t) \quad \text{and} \quad f_{H,t} = w_t, \quad \forall t \geq 0, \tag{1}$$

where $d \in (0, 1)$ is the depreciation rate of capital, and $f_{K,t}$ and $f_{H,t}$ stand, respectively, for the marginal productivity of capital and labor evaluated at the optimal time t input levels.

Letting $b(w_t) \equiv [n'(1)/(w_t u'(1))]^{1/(1-\gamma)}$, the first-order conditions for the leisure-labor choice of the consumer are

$$\text{either: } \frac{c_t}{l_t} = b(w_t)^{-1} s_t \quad \text{and} \quad l_t < 1 \tag{2}$$

$$\text{or: } c_t \geq b(w_t)^{-1} s_t \quad \text{and} \quad l_t = 1, \tag{3}$$

while the first-order conditions for capital in the consumer problem are

$$\text{either: } U_{c,t} = \beta(1 + r_{t+1})E_t(U_{c,t+1}) \quad \text{and} \quad k_t > \underline{B} \tag{4}$$

$$\text{or: } U_{c,t} \geq \beta(1 + r_{t+1})E_t(U_{c,t+1}) \quad \text{and} \quad k_t = \underline{B}. \tag{5}$$

3.1. Characterization of decision rules with constant prices

Consider the case where an agent faces a deterministic sequence of constant prices, so that $r_t = r$ and $w_t = w$ for all t . As the utility function is unbounded above, an upper bound \bar{B} on capital holdings is needed to guarantee existence of the Bellman equation in the usual way. The agent's problem is studied using standard dynamic programming techniques. The position at a point in time of an agent is described by $x = (k, s)$ that belong

¹¹Separability and homogeneity of A3 are sufficient to guarantee that the utility function is bounded from below, and that in equilibrium consumption and leisure are proportional (see the interior first-order condition below). These assumptions simplify the results based on dynamic programming, but the results are likely to go through for any utility function in which consumption and leisure are both normal goods. A more general class of preferences is considered in the numerical simulations in Section 4.

to the state space $X = [\underline{B}, \bar{B}] \times S$. Letting v be the value function, the problem of an agent can be written recursively as

$$v(x; w, r) = \sup_{(c,l,k') \in \Gamma(x;w,r)} \{U(c, l) + \beta E[v(x'; w, r)|x]\}, \tag{6}$$

where

$$\Gamma(x; w, r) = \{(c, l, k') : c + k' \leq ws(1 - l) + (1 + r)k; \\ c, l \geq 0; l \leq 1, \text{ and } \underline{B} \leq k' \leq \bar{B}\}.$$

The following results are an extension of Huggett (1993, 1997), and characterize some features of the value function and optimal decision rules (note that since $(w, 1 + r)$ are constant by assumption, we simplify notation and remove them from the value function and decision rules):

Remark. Assume A1–A5, $(w, 1 + r) > 0$ and $\beta(1 + r) \leq 1$. Then:

R1. $v(x)$ is strictly increasing and strictly concave in k , and $c(x)$, $l(x)$ and $k(x)$ are continuous in k .

R2. $c(x)$, $l(x)$ and $k(x)$ are strictly positive, $c(k, s)$ is strictly increasing in k and $k(k, s)$ and $l(k, 1)$ are increasing in k .

R3. For all $k \in [\underline{B}, \bar{B}]$, $k(k, 0) \leq k$ (with strict inequality if $\underline{B} < k < \bar{B}$ and $\beta(1 + r) < 1$).

Let us define $\bar{k} \equiv (b(w)r)^{-1}$. It is easy to check that at this level of savings, for both values of the shock s , the budget constraint of the consumer is satisfied if consumption equals $b(w)^{-1}$, capital stays constant and hours of work equal zero. Furthermore, the first-order condition with respect to labor (3) is satisfied at these choices for both values of the shock. In the case that $\beta(1 + r) = 1$, (4) is satisfied since consumption is constant. Therefore, if $\beta(1 + r) = 1$ the optimal choices are

$$c(\bar{k}, s) = b(w)^{-1}, \\ l(\bar{k}, s) = 1, \\ k(\bar{k}, s) = \bar{k}$$

for $s = 1, 0$. Therefore, if the agent ever reaches this level of savings, she will maintain a consumption stream equal to $b(w)^{-1}$ without ever working. For lower interest rates, constant consumption does not satisfy the first-order condition for capital, and the consumer never reaches \bar{k} .

The following proposition makes these statements more precise.

Proposition 2. Assume A1–A5, assume that $\bar{B} > \bar{k}$ and $w > 0$. Then:

- (a) If $\beta(1 + r) \leq 1$, for any $k \leq \bar{k}$ then $k(k, s) \leq \bar{k}$.
- (b) If $\beta(1 + r) = 1$, for any $k \geq \bar{k}$ we have $k(k, s) = k$, $l(k, s) = 1$ and $c(k, s) = kr$.
- (c) If $\beta(1 + r) = 1$ and $k_{-1} \leq \bar{k}$, then $k_t \rightarrow \bar{k}$, $c_t \rightarrow b(w)^{-1}$ and $l_t \rightarrow 1$ a.s.

Parts (a) and (b) are displayed in Fig. 2, showing the decision rule for capital accumulation. In the case $\beta(1 + r) < 1$ we have a similar picture as in the exogenous labor case: any capital stock can be reached from any initial capital stock, so that a stationary wealth distribution arises in the long run. In the first graph of Fig. 2, the bold line in the horizontal axis denotes the support of the stationary distribution of wealth.¹² But in the

¹²For a similar picture in the standard case, see Figure I in Aiyagari (1994).

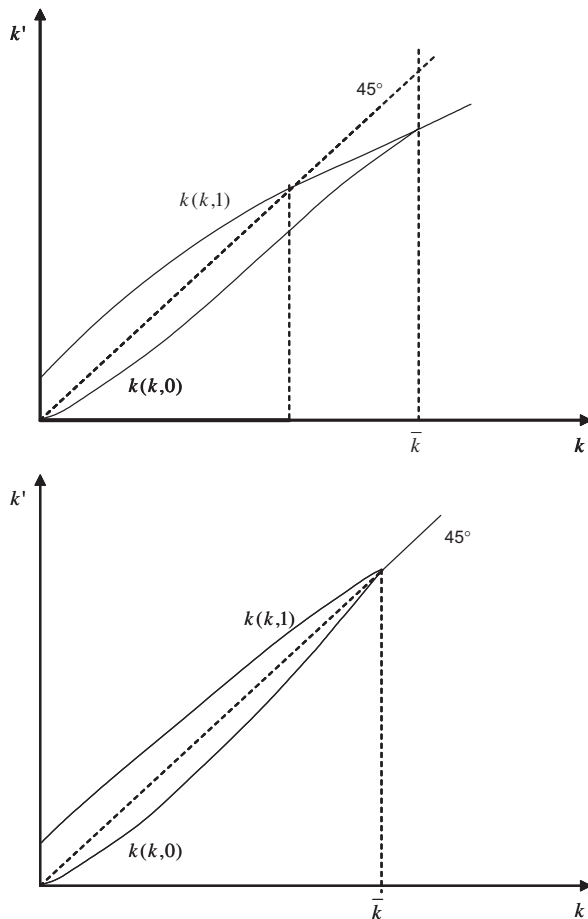


Fig. 2. Optimal decision rules for capital with endogenous labor supply when $\beta(1+r) < 1$ (top) and when $\beta(1+r) = 1$ (bottom). Notice that with $\beta(1+r) < 1$, the support of the equilibrium distribution (the thick segment on the k axis) does not include \bar{k} .

case $\beta(1+r) = 1$ we have that \bar{k} is an absorbing state, once capital reaches \bar{k} it stays there. This is what leads to part (c) of the proposition, saying that if $\beta(1+r) = 1$, then capital accumulation in the long run is bounded and it converges to \bar{k} , unlike the case of exogenous labor where long run savings is unbounded. This exhibits the wealth effect on labor supply in a partial equilibrium version of the model and it shows that in the mapping from interest rates to long run savings there is no singularity at $\beta(1+r) = 1$.¹³

The following remark follows immediately from part (a) of Proposition 2:

R4. If we choose $\bar{B} > \bar{k}$ and if $k_{-1} \leq \bar{k}$, then the upper limit on capital is never binding.

This implies that the upper bound on capital that was introduced to obtain existence and uniqueness of the value function is, in fact, not binding under the conditions of the remark.

¹³For example, this singularity is displayed for the standard case in Figure II, Aiyagari (1994).

3.2. Stationary equilibrium

Let ψ be the probability measure (time-invariant in a stationary equilibrium) describing the mass of agents in each level of wealth and value of s , defined on the sigma algebra \mathcal{X} , containing all Borel subsets of X . This measure is the aggregate state in the economy. Call $P(x, M)$ the transition function giving the probability that a worker in individual state x at time t will have an individual state that lies in the set $M \in \mathcal{X}$ next period.¹⁴ Finally, let $K(\psi)$ and $H(\psi)$ denote the aggregate stock of capital and labor as a function of the aggregate state ψ . A standard notion of stationary competitive equilibrium for the economy is as follows:

Definition. A stationary recursive competitive equilibrium with incomplete markets is a list of functions (v, c, l, k, ψ, K, H) and a pair of prices (w, r) such that:

- (1) v satisfies the functional equation in (6) and $c(x)$, $l(x)$ and $k(x)$ are the associated optimal decision rules given (w, r) .
- (2) $\{w_t, r_t\} = \{w, r\}$ satisfy Eq. (1) for all $t \geq 0$ for the levels of input $K(\psi)$, $H(\psi)$.
- (3) Aggregate factor inputs are generated by decision rules of the agents:
 - (i) $\int_X k(x) d\psi = K(\psi)$,
 - (ii) $\int_X s(1 - l(x)) d\psi = H(\psi)$.
- (4) ψ is a stationary distribution in the law of motion determined by the transition P implied by the decision rules. Formally,

$$\psi(M) = \int_X P(x, M) d\psi \quad \text{for all } M \in \mathcal{X}.$$

The following result extends Theorem 1 in Huggett (1997), and establishes the relationship between the rate of time preference and the interest rate in a stationary equilibrium. K^{IM} and K^{CM} denote the value of aggregate capital in the stationary recursive competitive equilibrium under incomplete and complete markets. Similarly, for hours we use H^{IM} and H^{CM} .

Proposition 3. Assume A1–A5. In a stationary recursive competitive equilibrium with positive capital and labor $\beta(1 + r) < 1$. Therefore, $K^{\text{IM}}/H^{\text{IM}} > K^{\text{CM}}/H^{\text{CM}}$.

The intuition for this proposition is the following. From part (c) of Proposition 2, we know that if $\beta(1 + r) = 1$ then capital goes to \bar{k} , labor goes to zero, so that the capital labor ratio goes to infinity. If this happened, we would observe $f_{K,t} \rightarrow 0$ and (1) cannot be satisfied together with $\beta(1 + r) = 1$. Therefore, a lower interest rate is needed for markets to clear. At any steady state the distribution of wealth is uniquely determined.¹⁵ Since under complete markets $\beta(1 + r) = 1$, the interest rate is lower and the capital labor ratio is larger under incomplete markets.

This reproduces, in terms of the capital/labor ratio, the standard precautionary savings result. But total output depends both on the capital/labor ratio and on total hours. Since the ex post wealth effect we have described at length in this paper decreases total labor, this

¹⁴For a construction of the transition function, see Theorem 9.13 in Stokey and Lucas (1989, p. 284).

¹⁵The arguments in Huggett (1993), based on Theorem 2 in Hopenhayn and Prescott (1992), can be readily applied to the endogenous labor case considered in this paper.

means that total capital and total output may be higher or lower under incomplete markets depending on which effect (the ex post wealth effect or the precautionary motive) dominates. Numerical methods are used in the following section to investigate this issue in several examples.¹⁶

4. Simulations

The functional form for technology used in all numerical simulations is Cobb–Douglas: $f(K, H) = TK^\alpha H^{1-\alpha}$, where T is a scale parameter and α measures the capital share of total income. Preferences are also standard (a special case of the separable preferences studied at the end of Section 2):

$$U(c, l) = c^{1-\gamma_c}/(1-\gamma_c) + Al^{1-\gamma_l}/(1-\gamma_l).$$

The benchmark calibration sets the following parameters $\alpha = 0.36$, $\beta = 0.99$, $A = 2$, and $d = 0.025$ to roughly match quarterly observations of the US economy on capital income share, interest rates, hours worked, and the capital output ratio (see, for instance, Hansen, 1985). The probabilities of transition for the idiosyncratic endowment are given by $\pi_{1|1} = 0.94$ and $\pi_{1|0} = 0.91$. These values are similar to the ones in Imrohoroglu (1989) and Krusell and Smith (1998) and approximately match the 93% average employment rate (after normalizing with the participation rate) and the 13-week average duration of unemployment observed in the US economy since the Korean War. Finally, we fix $T = 0.5$ and $\underline{B} = 2.6$. These values help us to obtain high accuracy in our simulations by reducing the size of the state space.¹⁷ Panel I in Table 1 reports the equilibrium outcome for aggregate capital, hours worked, output, interest rate and saving rate (respectively K , H , Y , r , and sr) under both market arrangements and for several combinations of (γ_c, γ_l) .¹⁸ Consider the case of $\gamma_c = \gamma_l = 1$ corresponding to the log–log utility case. The ex post wealth effect causes labor supply to be smaller under incomplete markets than in the complete markets setting. Nevertheless, the effect is too small to overcome the precautionary savings, so that both K and Y are larger in the incomplete markets arrangement. But many authors have argued that in order to match observed fluctuations in consumption and hours the elasticity of hours worked should be much higher than the elasticity of consumption. Given the intuition provided in Section 2, it is precisely in this case that the drop in hours worked is large and that the ex post wealth effect might dominate. To explore this possibility, in panel I the curvature of U with respect to c is increased (i.e., increasing γ_c) and the curvature with respect to l is decreased (i.e., decreasing γ_l). Output is smaller than under complete markets for all parameter values (except the already mentioned $\gamma_c = \gamma_l = 1$), so that the ex post wealth effect tends to dominate the Aiyagari–Huggett effect when labor is more elastic than in the log–log case.

¹⁶The working paper version of this paper contains an analytical example with uncertainty only in the first period, and we show that the ex post wealth effect can easily dominate precautionary savings and the Aiyagari–Huggett effect.

¹⁷Our solution method is the same one as in Huggett (1993): approximation of the derivative of the value function by piecewise linear functions on a grid of points. The usual grid contains 3,000 points and a distance between them between 0.0005 and 0.005. See the working paper version for a detailed accounting of issues involved in the computations, accuracy, and an extensive sensitivity analysis.

¹⁸The complete markets allocation we report corresponds to the usual planner's solution with a representative agent.

Table 1
Equilibrium allocations

	$\gamma_c = 1$		$\gamma_c = 2$		$\gamma_c = 5$	
	CM	IM	CM	IM	CM	IM
<i>I. Endogenous labor supply</i>						
$\gamma_l = 1$						
<i>K</i>	3.794	3.801	6.128	4.029	8.830	4.980
<i>H</i>	0.295	0.294	0.476	0.303	0.686	0.318
<i>r</i>	3.510	3.505	3.510	3.438	3.510	3.099
<i>Y</i>	0.369	0.370	0.597	0.384	0.861	0.428
<i>sr</i>	0.256	0.256	0.256	0.261	0.256	0.290
$\gamma_l = 0.7$						
<i>K</i>	4.128	3.819	6.615	4.078	9.333	5.098
<i>H</i>	0.320	0.296	0.514	0.305	0.725	0.321
<i>r</i>	3.510	3.503	3.510	3.424	3.510	3.068
<i>Y</i>	0.402	0.371	0.645	0.387	0.910	0.434
<i>sr</i>	0.256	0.256	0.256	0.262	0.256	0.293
$\gamma_l = 0.5$						
<i>K</i>	4.409	3.836	7.024	4.118	9.744	5.195
<i>H</i>	0.342	0.297	0.546	0.306	0.757	0.323
<i>r</i>	3.510	3.500	3.510	3.414	3.510	3.042
<i>Y</i>	0.429	0.373	0.684	0.390	0.950	0.439
<i>sr</i>	0.256	0.257	0.256	0.263	0.256	0.295
<i>II. Exogenous labor supply</i>						
<i>K</i>	3.781	3.802	3.897	3.930	4.090	4.162
<i>H</i>	0.294	0.294	0.303	0.303	0.318	0.318
<i>r</i>	3.510	3.497	3.510	3.490	3.510	3.471
<i>Y</i>	0.368	0.369	0.379	0.381	0.398	0.401
<i>sr</i>	0.256	0.257	0.256	0.257	0.256	0.259

Notice that in all these examples, as predicted by Proposition 3, the capital labor ratio is larger under incomplete markets, but the lower labor supply reduces output. In these examples, therefore, production and consumption under incomplete markets is “too small”, and a better developed financial intermediation sector would not only increase welfare, but also savings and output.

Concentrating on the numbers in boldface, for $(\gamma^c, \gamma^l) = (1, 0.5)$ shows that output would increase by almost 20% by completing the markets, and that for $\gamma^c = 5$ (and any γ^l) output would nearly double!. These parameter values fall within the range of parameters that have been used in the literature. To give a benchmark of how important this effect is compared to the precautionary motive discussed previously in the literature, panel II provides the equilibrium outcome in this model when labor supply is exogenously given.¹⁹ Panel II reproduces the standard result that precautionary savings lead to higher capital and higher output, but the effects are very small. Comparing panels I and II suggests that abstracting from endogenous labor supply decisions may seriously bias our estimates of the costs of

¹⁹To allow comparison, for each γ^c , the (exogenous) hours worked are set equal to the corresponding value for $\gamma^l = 1$ in the incomplete market equilibrium of panel I.

incomplete markets and that the ex post wealth effect is more important than the precautionary motive.

We have computed many other examples, not reported here for reasons of space, and our conclusion is that the wealth effect often (but not always) dominates precautionary savings and the Aiyagari–Huggett effect, and that this occurs more often as consumption becomes less elastic relative to hours of labor.^{20,21}

5. Concluding remarks

We have shown that poor financial development introduces an ex post wealth effect that can reduce output and savings when labor decisions are endogenous. With endogenous labor, the standard precautionary motive translates into a higher capital/labor ratio under incomplete markets, but total hours worked are lower due to the wealth effect, so that the total effect on output is, a priori, undetermined. For particular parameter values, when elasticity of consumption is low relative to the elasticity of labor (as in the calibration of many real business cycles papers) our numerical results suggest that the lack of complete insurance markets reduces output. For some reasonable parameter values, completing the markets doubles equilibrium output.

We find that, in contrast with the exogenous labor case, wealth in the long run is bounded even when the interest rate equals the inverse discount factor. The fact that the distribution of wealth does not go to infinity in this case removes a singularity in the mapping from interest rates to long run wealth, and facilitates the analysis of models with idiosyncratic risk.²²

Overall, our results suggest that equilibrium dynamic stochastic models can and should be used to understand the relationship between uncertainty, financial markets and the wealth of nations.

It is still an open question whether this decrease in output is empirically relevant. We stayed as close as possible to the standard models with incomplete markets with idiosyncratic uncertainty. These models are the heirs of the RBC literature that became standard in aggregate macroeconomics research during the 80s and 90s, but they are possibly not the best models to calibrate the wealth elasticity of total labor supply. It would be interesting to explore the effect we mention in models with an extensive margin for labor supply.

The results here have implications beyond the simple introduction of labor in models with idiosyncratic risk. The same effect is likely to appear in the supply of other inputs (such as innovation, technology acquisition, human capital, managerial skills, etc.). The effect on output is likely to be much more important if these inputs are considered. There may be other frictions that matter for savings and that have been left out of the model. All in all, it is quite likely that the drop in output is understated in our model relative to setups taking these factors into account.

²⁰See the working paper version of this paper for robustness analysis.

²¹It is worth mentioning that in fact, the empirical evidence regarding precautionary saving is far from clear. See for instance [Carroll et al. \(1999\)](#) and the references therein.

²²For example, [Marcet and Obiols-Homs \(2006\)](#) exploit this feature of the model with endogenous labor to prove polarization in the distribution of wealth when labor productivity depends on consumption or on wealth.

Finally, Aiyagari (1995) argued that the precautionary savings effect of incomplete markets would justify taxing capital income. Our results suggest that Aiyagari’s arguments should be reevaluated.

Appendix

Proof of Lemma 1. Consider the problem $\max_{c,l} U(c,l)$ subject to $c = \omega + (1 - l)$. The first-order condition for an interior solution is $U_l(c,l) = U_c(c,l)$. Manipulation of this first-order condition establishes that leisure is a normal good if and only if $\partial l / \partial \omega = (U_{cc} - U_{cl}) / (U_{cc} - 2U_{cl} + U_{ll}) > 0$. Since the second-order condition for a maximum imposes that the denominator of this expression be negative, normality of leisure requires that $U_{cc} - U_{cl} < 0$. Combining these results with the first-order condition with respect to Q we have

$$U_c[\Omega + \phi Q, 1] = U_c[\Omega - (1 - \phi)Q + (1 - l), l] < U_c[\Omega - (1 - \phi)Q, 1],$$

Now U_{cc} is strictly negative by assumption. Therefore the previous equation implies that $Q > 0$. \square

Proof of Proposition 1. Labor supply under complete markets satisfies

$$U_l[\Omega - (1 - \phi)Q + (1 - l), l] = U_c[\Omega - (1 - \phi)Q + (1 - l), l], \tag{7}$$

while labor supply under incomplete markets solves

$$U_l[\Omega + (1 - l^{IM}), l^{IM}] = U_c[\Omega + (1 - l^{IM}), l^{IM}]. \tag{8}$$

Comparing Eq. (8) with (7), we observe the incomplete and complete market labor supply decisions differ only in that consumers who do work are *richer* under incomplete markets than under complete markets, since $Q > 0$, and thus, $\Omega > \Omega - (1 - \phi)Q$. Hence we conclude that, if leisure is a normal good, labor supply is lower under incomplete markets than under complete markets. \square

Proof of Remark. The assumptions on $U(c)$ and $n(l)$ imply that the utility function is bounded below by 0. Since capital is bounded above by \bar{B} , by standard arguments there exists a unique function $v(x)$ in the space of bounded continuous functions on X satisfying the functional (6), and there also exist the corresponding optimal decision rules (see, respectively, Theorem 9.6 in Stokey and Lucas, 1989 and Corollary 2 in Denardo, 1967). The proofs for R1 to R3 use these facts and follow from the same arguments as in Huggett (1993, pp. 964–968, 1997, pp. 399–400). \square

Proof of Proposition 2. To prove (a), first consider the case $r > 0$. Following the proof of Theorem 1 in Huggett (1993), under A1–A4 and $k \in [\underline{B}, \bar{B}]$ it can be shown by induction that $v'(k, 1) \leq v'(k, 0)$, i.e., $c(k, 0) \leq c(k, 1)$. For an agent in the unemployment state, R3 and the budget constraint imply that $c(\bar{k}, 0) \geq b(w)^{-1}$. Since $c(\bar{k}, 0) \leq c(\bar{k}, 1)$, then the FONC with respect to leisure implies $l(\bar{k}, 1) = 1$. Therefore $k(\bar{k}, 1) \leq \bar{k}$.

$r \leq 0$: Take $k_1 < k_2$, thus $c(k_1, 1) < c(k_2, 1)$. The budget constraint of an employed agent implies that $w(1 - l(k_1, 1)) + (1 + r)k_1 - k(k_1, 1) < w(1 - l(k_2, 1)) + (1 + r)k_2 - k(k_2, 1)$, thus $k(k_2, 1) - k(k_1, 1) < (1 + r)(k_2 - k_1) + w(l(k_1, 1) - l(k_2, 1))$. Since leisure is also strictly increasing in the level of capital, it follows that $(k(k_2, 1) - k(k_1, 1)) / (k_2 - k_1) < 1$.

To prove (b), consider the optimal choice of $\{c_t, l_t, k_t\}$ in the consumer problem when the initial condition satisfies $k_{-1} \geq \bar{k}$. It is immediate to check that the allocation $\{\tilde{c}_t, \tilde{l}_t, \tilde{k}_t\} = \{k_{-1}r, 1, k_{-1}\}$ satisfies all first-order conditions for any s . The FOC for labor are satisfied because $\tilde{c}_t = k_{-1}r \geq \bar{k}r = b(w)^{-1}$ and the FOC for capital are also satisfied because $U_{c,t}$ is constant. Since the problem is concave the first-order conditions are sufficient for an optimum. Since the policy function gives the optimum, we have that for $k_{-1} \geq \bar{k}$ it must hold that $c(k_{-1}, s) = \tilde{c}_0 = k_{-1}r$, $l(k_{-1}, s) = \tilde{l}_0 = 1$, and $k(k_{-1}, s) = \tilde{k}_0 = k_{-1}$. To prove (c), notice first that part (a) implies that $k_t \leq \bar{k}$ for all t , and R2 together with part (b) imply that $c_t \leq c(\bar{k}, s) = \bar{k}r$ so that consumption is bounded a.s. Notice that the FOC for capital (4) and (5) imply that $u_{c,t} \geq E_t(u_{c,t+1})$ a.s., so that $u_{c,t}$ is a super-martingale. Since $u_{c,t}$ is bounded below by $u'(\bar{k}r)$ the martingale convergence theorem applies and it implies that $u_{c,t}$ converges a.s. to a random variable. Assume, towards a contradiction, that $u_{c,t}$ converged to a value strictly larger than $u'(\bar{k}r)$, consumption would converge to a point $c^l < b(w)^{-1}$ so that we would have an interior solution for the consumption–leisure decision of employed agents and (2) applies if $s_t = 1$. Under these circumstances labor income would converge to $w(1 - b(w)c^l)s_t$, which is a non-degenerate i.i.d. random variable with positive variance. The arguments in Chamberlain and Wilson (2000) imply that the lower or upper bounds on capital will be violated with positive probability in this case, so that it is impossible for $u_{c,t}$ to converge to a value strictly larger than $u'(\bar{k}r)$. Therefore, the only possibility is that $u_{c,t}$ converges to $u'(\bar{k}r)$ and, since u' is invertible, consumption converges to $b(w)^{-1}$. The budget constraint implies that k_t must converge to \bar{k} . \square

Proof of Proposition 3. By the usual arguments (e.g., Aiyagari, 1994; Chamberlain and Wilson, 2000), if $\beta(1+r) > 1$ consumption diverges to infinity which is unfeasible. Proposition 3 (c) implies that if $\beta(1+r) = 1$ then capital is bounded but labor goes to zero for each agent. Therefore, the aggregate capital/labor ratio goes to infinity for each initial distribution of wealth, which is incompatible with equilibrium and (1). Therefore, the only possibility for existence of a stationary equilibrium under incomplete markets is that $\beta(1+r) < 1$. Since $\beta(1+r) = 1$ under complete markets, then we have that the capital labor ratio must be larger for incomplete markets if (1) is to be satisfied. \square

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