The Impact of Government Debt Maturity on Inflation *

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Abstract

In the context of a sticky price DSGE model subject to government expenditure and preference shocks where governments issue only nominal non-contingent bonds we examine the implications for optimal inflation of varying the average maturity of goverment debt. We focus on two main channels through which inflation can be used to offset the impact of adverse government expenditure shocks. A real balance effect which reduces the value of nominal bonds that the government issues and an effective tax on corporate profits through squeezing profit margins. These two effects operate at different time horizons and have different effects depending on the sign and size of government debt. The profit tax effect works only in the initial period whilst the optimal scale of the real balance effect is limited when the government issues short term debt because of price stickiness. Issuing longer term debt enables greater use of the real balance effect. The result is that the persistence and volatility of inflation depends on the sign, size and maturity structure of government debt. We show analytically and numerically how inflation dynamics are determined by the interaction of maturity and these two channels. We find an important role for inflation with long term debt in reducing debt fluctuations but even low levels of price stickiness mean that adjusting prices is a costly way of stabilising debt. We find that markets remain signficantly incomplete even

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with long bonds and inflation and that the inflation channel provides a minor role in achieving debt sustainability.

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1 Introduction

Rising levels of government debt in the OECD in the wake of the financial crisis of 2007/8 are raising numerous concerns. Most obviously issues of fiscal sustainability are triggering a sovereign debt crisis, notably in the Euro area. Related to this concern is the notion that governments will use inflation in order to achieve fiscal solvency without resorting to distortionary labour taxes. For instance, Aizenman and Marion (2009) calculate that a persistent inflation rate of 5% will contribute significantly to stabilising US public finances. In contrast the evidence of Giannitsarou and Scott (2007) and Hall and Sargent (2010), using historical data, suggests that inflation has played a relatively minor role in achieving debt sustainability. Much discussion around the link between debt and inflation also includes a focus on the maturity structure of government debt with a widespread belief that the longer is the maturity of government debt the greater the incentive to use inflation. In this paper we examine optimal Ramsey policy for a government that controls labour taxes and inflation in an economy characterised by monopolistic competition and sticky prices. In doing so we follow the work of Schmitt-Grohe and Uribe (2004) and Siu (2004) who show how price adjustment costs limit the ability of inflation to make a substantial contribution to fiscal sustanability and Lustig, Sleet and Yeltekin (2008) who extend this work to consider long maturity bonds and show that governments use inflation more if they can issue long term bonds and that it is optimal to do so.

Our model is outlined in Section 2 and is broadly similar to those of Schmitt-Grohe and Uribe (2004), Siu (2004) and Lustig et al (2008). It focuses on a government which sets optimal policy under commitment in a monopolistically competitive environment characterised by sticky prices and incomplete bond markets in which governments cannot issue a complete set of contingent claims, instead they issue nominal non-state contingent bonds. As do Lustig et al (2008) we allow the maturity of government debt to be greater than a single period. Using the computational method of Faraglia, Marcet and Scott (2011) enables us to consider much longer maturities (in this case up to 20 periods) than previously analysed although unlike Lustig et al (2008) we do not focus on the composition of debt but simply focus on changes to average maturity. We extend previous work in this area by focusing not just on government expenditure shocks but also allow for preference shifts between consumption and leisure. Hall (1997) and Holland and Scott (1998) show how such preference shocks are necessary to produce plausible levels of volatility in employment in a basic DSGE model and we show theoretically how their addition can potentially remove the advantage of issuing long term debt found by Lustig et al (2008).

Because governments cannot issue fully contingent debt ours is an incomplete market model. Absent state contingent debt the government has three channels through which to offset fiscal fluctuations and satisfy the intertemporal budget constraint in a way that minimises fluctuations in distortionary taxes. The first is through inflation (Schmitt-Grohe and Uribe (2004) and Siu (2004)) which can be used to reduce the real value of government liabilities. The second is through the twisting of interest rates in a way which reduces government funding costs (Lucas and Stokey (1983) and Faraglia, Marcet and Scott (2011)) and the final way is through endogenous fluctuations in bond prices which covary appropriately with stochastic disturbances (Angeletos (2002)). Through consideration of a one period model (Section 3) as well as full stochastic simulations (Section 4) we investigate the relative role of each channel. Our analysis also helps identify two channels through which inflation can ease the government's fiscal burden. A real balance effect which reduces the real value of government's liabilities and an effective profit tax which can be used to offset adverse shifts in labour supply. We show how these two effects operate at different time horizons and have differing effects depending on the size and sign of government debt. When government is indebted the two effects work in opposite directions and this conflict can be avoided by issuing long term debt in a way that produces non-monotonic inflation dynamics. We show analytically and numerically how issuing long term debt increases the volatility and persistence of inflation and the importance of inflation in achieving fiscal sustainability. Outlining the different channels through which these effects operate, showing their role under both government expenditure shocks and preference shifts and showing how inflation dynamics vary depending on whether debt is close to zero or if the government is a debtor or creditor are a major contribution for this paper.

We also use our analysis to consider another issue in Section 5. Whilst Lustig et al (2008) show that issuing longer maturity debt enables governments to make greater use of inflation to stabilise the debt we also consider the relative importance of inflation in helping produce complete markets and stabilise debt fluctuations. Longer term bonds and inflation do significantly help reduce the persistence of government debt but it still displays more than unit root persistence (see Schmitt-Grohe and Uribe (2004)). In particular using the decomposition of the government budget constraint proposed by Hall and Sargent (2011) we find a relatively small impact from inflation on debt reduction, interest rate twisting has a bigger impact and most of the fiscal response is achieved through changes in the primary surplus - consistent with the empirical evidence of Gianitsarou and Scott (2007). Costs of price adjustment are such that distortionary taxes smoothed over time are preferable to even higher levels of inflation.

In Section 6 we further develop the model by including two extensions. The first is we relax the assumption (shared by Lustig et al (2008)) that governments cannot be a creditor with the private sector. Because the behaviour of inflation depends critically on the level of debt this not surprisingly has a significant effect on inflation - reducing its volatility and persistence as the government holds lower levels of debt. The other extension we consider is the empirically motivated case where governments issue debt but do not buy it back at the end of each period and reissue but buy it back only at redemption. We show how this leads to much more complex inflation dynamics including the possibility of inflation cycles.

A final section concludes.

2 Model

2.1 Agents

Preferences

We consider an infinite horizon economy populated by a large number of identical households. Each household has preferences defined over consumption and leisure given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, \xi_t) + v(h_t) \tag{2.1}$$

where c_t denotes consumption, h_t labor effort and β is the subjective discount factor. Variable ξ_t is a disturbance to the household's utility, a preference shifter (see for example Woodford (2003) Ch. 3). For each value of ξ_t, utility $u(c_t, \xi_t)$ is increasing and concave in consumption.

Firms

In every period t the economy produces a final good that we denote by Y_t and which is assumed to be a composite of a continuum of differentiated intermediate products. Each household is a monopolistic producer of one of these products and the aggregator function is of the Dixit Stiglitz form. Intermediate goods are produced with a linear technology whose sole input is labor and households hire labor in a perfectly competitive market. Demand for the intermediate good is given by $Y_t d(p_t)$ where p_t is the relative price of the good in terms of the composite final good. The demand function d satisfies additional assumptions that guarantee the existence of a symmetric equilibrium: $d(1) = 1$ and $d'(1) < -1$.

To introduce sticky prices in the economy we assume firms face a quadratic cost of adjusting their prices each period. Let $P_{i,t}$ be the price of a generic intermediate product i, then adjustment costs are given by $\frac{\theta}{2}(\frac{P_{i,t}}{P_{i,t-1}} - 1)^2$. The parameter θ governs the degree of price stickiness. The higher θ is, the higher the resource cost of adjusting prices. When $\theta = 0$ prices are fully flexible.

Intermediate good producers seek to maximize:

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{u_c(c_{t+j}, \xi_{t+j})}{u_c(c_t, \xi_t)} [\, \frac{P_{i,t+j}}{P_{t+j}} Y_{t+j} d(\frac{P_{i,t+j}}{P_{t+j}}) - w_{t+j} h_{i,t+j} - \frac{\theta}{2}(\frac{P_{i,t+j}}{P_{i,t+j-1}} - 1)^2] \tag{2.2}$$

subject to the constraint $h_{i,t+j} = Y_{t+j} d(\frac{P_{i,t+j}}{P_{t+j}})$. The quantity $\beta^j \frac{u_c(c_{t+j}, \xi_{t+j})}{u_c(c_t, \xi_t)}$ is the appropriate stochastic discount factor used to evaluate the stream of profits of the generic firm i and w_{t+j} is the wage rate in the competitive labor market. The first order condition with respect to P_t is given by:

$$\begin{aligned} \frac{1}{P_t} Y_t d(\frac{P_{i,t}}{P_t}) \quad &+ \quad \frac{P_{i,t}}{P_t^2} Y_t d'(\frac{P_{i,t}}{P_t}) - w_t Y_t d'(\frac{P_{i,t}}{P_t}) \frac{1}{P_t} - \theta(\frac{P_{i,t}}{P_{i,t-1}} - 1) \frac{1}{P_{i,t-1}} \\ &+ \quad \beta E_t \frac{u_c(t+1)}{u_c(t)} \theta(\frac{P_{i,t+1}}{P_{i,t}} - 1) \frac{P_{i,t+1}}{P_{i,t}^2} = 0 \end{aligned} \tag{2.3}$$

This equation is the "Phillips curve" which describes the inflation output tradeoff in our model. Letting $d'(1) = \eta$ and imposing a symmetric equilibrium such that all firms set the same price gives:

$$\frac{Y_t\eta}{\theta}(\frac{(1+\eta)}{\eta} - w_t) - (\pi_t - 1)\pi_t + \beta E_t \frac{u_c(c_{t+1}, \xi_{t+1})}{u_c(c_t, \xi_t)}(\pi_{t+1} - 1)\pi_{t+1} = 0$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ denotes gross inflation.

Government and Markets

The government engages in two activities. First it levies taxes τ_t on households' labor income and second it trades with the household sector in bond markets to finance a spending process $\{g_t\}_0^\infty$. We assume only one type of asset, a nominal bond of maturity N. We denote by B_t^N the quantity of this bond issued in period t and its price by q_t^N.

At the beginning of each period households enter the market with a portfolio B_{t-1}^N, the quantity of bonds issued by the government in the previous period. The prevailing market price is q_t^{N-1} i.e. the competitive price of a bond of maturity $N-1$. We assume each period the government buys back the entire stock of debt issued in the previous period. The government budget constraint is therefore :

$$q_t^N B_t^N = q_t^{N-1} B_{t-1}^N + P_t(g_t - \tau_t w_t h_t) \tag{2.4}$$

2.2 The Ramsey Problem

To solve for the optimal policy we follow the roote of substituting out of the planner's constraint taxes and bond prices. In a competitive equilibrium the tax rate satisfies $(1-\tau_t)w_t = -\frac{v_h(h_t)}{u_c(c_t,\xi_t)}$ and the price of an N period bond is $q_t^N = \beta^N E_t \frac{u_c(c_{t+N},\xi_{t+N})}{u_c(c_t,\xi_t)\Pi_{j=1}^N \pi_{t+j}}$ with the property $q_t^0 = 1$. With these substitutions the government budget constraint is given by the following expression:

$$\begin{aligned} \beta^N E_t \frac{u_c(c_{t+N}, \xi_{t+N})}{u_c(c_t, \xi_t)\Pi_{j=1}^N \pi_{t+j}} B_t^N &= \beta^{N-1} E_t \frac{u_c(c_{t+N-1}, \xi_{t+N-1})}{u_c(c_t, \xi_t)\Pi_{j=1}^{N-1} \pi_{t+j}} B_{t-1}^N \\ &+ P_t(g_t - (1 + \frac{v_h(h_t)}{u_c(c_t, \xi_t) w_t}) w_t h_t) \end{aligned} \tag{2.5}$$

Multiplying by the marginal utility of consumption in period t, dividing by the price level P_t and letting $b_t^N = \frac{B_t^N}{P_t}$ denote the real debt level, we can write the constraint as:

$$\begin{aligned} \beta^N E_t \frac{u_c(c_{t+N}, \xi_{t+N})}{\Pi_{j=1}^N \pi_{t+j}} b_t^N &= \beta^{N-1} E_t \frac{u_c(c_{t+N-1}, \xi_{t+N-1})}{\Pi_{j=1}^{N-1} \pi_{t+j}} \frac{b_{t-1}^N}{\pi_t} \\ &+ u_c(c_t, \xi_t)(g_t - (1 + \frac{v_h(h_t)}{u_c(c_t, \xi_t) w_t}) w_t h_t) \end{aligned} \tag{2.6}$$

In a competitive equilibrium aggregate output equals labor effort i.e $Y_t = h_t$ so that our Phillips curve becomes:

$$(\pi_t - 1)\pi_t = \frac{h_t\eta}{\theta}(\frac{(1+\eta)}{\eta} - w_t) + \beta E_t \frac{u_c(c_{t+1},\xi_{t+1})}{u_c(c_t,\xi_t)}(\pi_{t+1}-1)\pi_{t+1} \tag{2.7}$$

Finally the economy wide resource contraint sets output equal to the sum of consumption, government spending and the costs of adjusting inflation such that :

$$h_t = c_t + g_t + \frac{\theta}{2}(\pi_t - 1)^2 \tag{2.8}$$

Our Ramsey planner seeks to maximize (2.1) subject to (2.6),(2.7) and (2.8). We attach a multiplier $\lambda_{s,t}$ to the budget constraint, $\lambda_{p,t}$ to the Phillips curve and $\lambda_{f,t}$ to the resource constraint. The Lagrangian for the planner's program is then given by :

$$\begin{aligned}\mathcal{L} = E_0 \sum_t \beta^t (u(c_t,\xi_t) + v(h_t) \quad &+ \quad \lambda_{f,t}(h_t - c_t - g_t - \frac{\theta}{2}(\pi_t-1)^2) \\ +(\lambda_{s,t-N} - \lambda_{s,t-N+1})\frac{u_c(c_t,\xi_t)}{\Pi_{t-N+1}^t \pi_j} b_{t-N}^N \quad &- \quad \lambda_{s,t}(g_t u_c(c_t,\xi_t) - (w_t u_c(c_t,\xi_t)h_t + v_h(h_t)h_t)) \\ +(\lambda_{p,t-1} - \lambda_{p,t})u_c(c_t,\xi_t)\pi_t(\pi_t-1) \quad &+ \quad \lambda_{p,t}\frac{\eta}{\theta}h_t u_c(c_t,\xi_t)(\frac{1+\eta}{\eta} - w_t))\end{aligned} \tag{2.9}$$

given $\lambda_{s,-N},...\lambda_{s,-1}$ and $\lambda_{p,-N},...\lambda_{p,-1}$. The first order conditions for this problem with respect to c_t, b_t^N, h_t, w_t and π_t are :

$$\begin{aligned}u_c(c_t,\xi_t) - \lambda_{f,t} + \lambda_{s,t}u_{cc}(c_t,\xi_t)(w_t h_t - g_t) \quad &+ \quad (\lambda_{s,t-N} - \lambda_{s,t-N+1})b_{t-N}^N \frac{u_{cc}(c_t,\xi_t)}{\Pi_{t-N+1}^t \pi_j} = \\ -\lambda_{p,t}\frac{\eta h_t u_{cc}(c_t,\xi_t)}{\theta}(\frac{1+\eta}{\eta} - w_t) + (\lambda_{p,t} \quad &- \quad \lambda_{p,t-1})u_{cc}(c_t,\xi_t)\pi_t(\pi_t-1)\end{aligned} \tag{2.10}$$

$$\begin{aligned}v_h(h_t) + \lambda_{f,t} \quad &+ \quad \lambda_{s,t}(w_t u_c(c_t,\xi_t) + v_{hh}(h_t)h_t + v_h(h_t)) \\ &+ \quad \lambda_{p,t}\frac{\eta}{\theta}u_c(c_t,\xi_t)(\frac{1+\eta}{\eta} - w_t) = 0\end{aligned} \tag{2.11}$$

$$\lambda_{s,t} - \frac{\eta}{\theta}\lambda_{p,t} = 0 \tag{2.12}$$

$$\begin{aligned}-\theta\lambda_{f,t}(\pi_t-1) \quad &- \quad \sum_{k=1}^N (\lambda_{s,t-k} - \lambda_{s,t-k+1})\beta^{N-k} E_t \frac{u_c(c_{t+N-k},\xi_{t+N-k})b_{t-k}^N}{\pi_t \, \Pi_{j=t-k+1}^{j=t+N-k}\pi_j} \\ &- \quad (\lambda_{p,t} - \lambda_{p,t-1})u_c(c_t,\xi_t)(2\pi_t - 1) = 0\end{aligned} \tag{2.13}$$

$$E_t\lambda_{s,t}\frac{u_c(c_{t+N},\xi_{t+N})}{\Pi_{t+1}^{t+N}\pi_j} - E_t\lambda_{s,t+1}\frac{u_c(c_{t+N},\xi_{t+N})}{\Pi_{t+1}^{t+N}\pi_j} = 0 \tag{2.14}$$

Equations (2.10) and (2.11) are the planner's first order conditions for consumption and hours respectively whilst (2.13) determines the optimal inflation level. The first term in (2.13) captures the marginal impact of higher inflation on the resource costs associated with price changes. The second term measures the effect of higher inflation on the inherited liability of the government in period t and the last term represents the intertemporal effects of a current change in inflation via the Phillips curve. In (2.12) the planner changes wages (marginal costs) so as to balance the benefits of higher wages in financing the deficit with the costs in terms of higher inflation in period t.

Finally, (2.14) is the Euler equation for the optimal choice of b_t^N and following the argument in Aiyagari et al (2002) the multiplier $\lambda_{s,t}$ behaves as a risk adjusted random walk. As discussed in Aiyagari et al (2002) and Marcet and Scott (2009), since $\lambda_{s,t}$ is a state variable to the model, the optimal policy possesses a propagation mechanism which yields different serial correlation properties for taxes, debt and deficits than for the exogenous process for government spending and the preference shift. Typically the persistence of these variables increases relative to the fundamental disturbances in the economy. By contrast if markets provided a full set of contingent bonds or goods prices were fully flexible then the excess burden of taxation, $\lambda_{s,t}$, would remain constant over time and therefore the model's endogenous variables would inherit the stochastic properties of the spending process and the preference shocks (see Scott (2007)).

2.3 Calibration

Each period represents one calendar year. The discount factor β is set to .96. We assume in period 0 the economy is in steady state with constant prices, consumption, government spending and hours. The values for these variables are such that they solve the system of first order conditions for the Ramsey policy problem. The optimal inflation rate is set equal to zero, the level of government expenditure is set to 25% of value added and the market value of debt to 60% of output. Since in the steady state the market value of debt is given by $\beta^N b^N$, as we vary the maturity N we have to vary the quantity of bonds in the steady state to keep the market value constant. When we simulate the economy we choose initial conditions for b_t^N, $\lambda_{s,t}$ and $\lambda_{p,t}$ for $t = \{-N, ... -1\}$ equal to the steady state values for these objects.

Household preferences are of the form $u(c_t, h_t) = \xi_t \log(c_t) + \zeta \log(1 - h_t)$. We choose ζ so that households spend 20% of their unitary time endowment working which gives $\zeta = 3.0417$. For price adjustment costs θ we follow SGU (2004) and set $\theta =$ 4.375 which gives a linearized version of the Phillips curve consistent with the empirical estimates of Sbordone (2004). Taking log deviations of wages (marginal costs) and inflation around the zero inflation rate steady state the Phillips curve in our model is:

$$\hat{\pi}_t = \beta E_t \, \hat{\pi}_{t+1} - \frac{\overline{h}(1+\eta)}{\theta} \hat{w}_t \tag{2.15}$$

where circumflexes denote log devations from the steady state. To calibrate the elasticity η we follow SGU (2004) and choose a value of -6. Moreover $\overline{h} = .2$ is hours in the steady state in the model. Sbordone's empirical estimates imply a value of 17.5 for the fraction $\frac{\overline{h}(1+\eta)}{\theta}$. This gives a value for θ equal 17.5 but since the horizon in the model is annual we divide this value by a factor of 4.

We assume the following stochastic processes for government spending and preferences :

$$\begin{aligned} \ln g_t &= (1-\rho_g)\ln \overline{g} + \rho_g \ln g_{t-1} + \varepsilon_{g,t}, \varepsilon_{g,t} \sim N(0,\sigma_g^2) \\ \ln \xi_t &= (1-\rho_\xi)\ln \overline{\xi} + \rho_\xi \ln \xi_{t-1} + \varepsilon_{\xi,t}, \varepsilon_{\xi,t} \sim N(0,\sigma_\xi^2) \end{aligned}$$

Our principle in calibrating the parameters ρ_i and σ_i for $i \in \{\xi, g\}$ is to make the log-linearized version of the model consistent with estimates of the IS equation in Ireland (2004).[1] In particular if our model had a monetary authority setting short term interest rates, the demand side would be described by :

$$\beta(1+i_t)E_t \frac{c_t}{c_{t+1}} \frac{\xi_{t+1}}{\xi_t} \frac{1}{\pi_{t+1}} = 1$$

Log linearising this equation and imposing that in equilibrium $y_t = c_t + g_t + \frac{\theta}{2}(\pi_t - 1)^2$ yields :

$$\widehat{y}_t = E_t \widehat{y}_{t+1} - \frac{\overline{c}}{\overline{y}}(\widehat{\imath}_t - E_t \widehat{\pi}_{t+1}) + \frac{\overline{g}}{\overline{y}}\widehat{g}_t + \frac{\overline{c}}{\overline{y}}\widehat{\xi}_t - E_t \frac{\overline{g}}{\overline{y}}\widehat{g}_{t+1} + \frac{\overline{c}}{\overline{y}}\widehat{\xi}_{t+1}$$

where $\widehat{x}$ denotes the log deviation of variable x from its steady state value.

Ireland (2004) estimates a version of this IS equation (along with a Phillips curve and an interest rate rule) using quarterly data for the US economy. In his analysis he leaves out government spending shocks and instead estimates the IS equation with only one disturbance. His estimates of the process for this unique shock map into our vector of disturbances with the linear combination $\frac{\overline{g}}{\overline{y}}\hat{g}_t + \frac{\overline{c}}{\overline{y}}\hat{\xi}_t$. In order to retrieve the implied process of ξ_t we assume, following SGU(2004), that spending shocks have a standard deviation $\sigma_g = .03$ in annual data and a first order autocorrelation $\rho_g = .9$. Ireland (2003) on the other hand obtains a point estimate for the autocorrelation of $\frac{\overline{g}}{\overline{y}}\hat{g}_t + \frac{\overline{c}}{\overline{y}}\hat{\xi}_t$ equal to .947. The value of .974 that would give us a yearly autocorrelation of .9 is within the range of his estimates. We therefore set $\rho_g = \rho_\xi = .9$. Finally the standard deviation of the preference shock is obtained by mapping the yearly aggregated process into the quarterly estimates of Ireland (2004) for this statistic. We retrieve a value of .0804 for this parameter.

[1] Since he uses log consumption without government spending shocks the loading of the term $(\hat{\imath}_t - E_t\ \hat{\pi}_{t+1})$ is unity rather than $\frac{\overline{c}}{\overline{y}}$. Nevertheless his model seems to be the closest to ours in the empriical New Keynesian literature.

3 Model with First Period Uncertainty

In this section we consider a variant of our economy where an unanticipated shock in either government spending or preferences occurs only in period one and for all subsequent periods there is no uncertainty. In this simplified environment we can solve the model exactly and use the first order conditions to point to properties of optimal inflation. In a later section we use full stochastic simulations to study the properties of inflation but the aim of this section is to understand the channels through which governments use inflation to achieve fiscal sustainability.

In a world of non-contingent bonds governments will seek to structure debt and use other policy instruments to minimise the distortionary costs of volatile taxes. In the context of our model there are three channels through which they can achieve such fiscal insurance a) through variations in inflation (Schmitt-Grohe and Uribe (2004) and Siu (2004)) b) interest rate twisting (Lucas and Stokey (1983), Lustig, Sleet and Yeltekin (2008) and Faraglia, Marcet and Scott (2011)) c) endogenous variations in bond prices (Angeletos (2002) and Buera and Nicoloni (2004)). We consider in this section how the relative importance of each of these channels varies as you change the maturity of government debt. A particular focus of this paper is the role of inflation and how this is influenced by variations in the maturity structure. In our model the inflation effect operates through two channels. The first channel arises from quadratic costs of adjustment. Long bonds allow governments to use inflation more by spreading any price increase over longer time periods. A second channel arises from what is effectively a profit tax whereby the government uses inflation to tax monopoly profits. In some parts of the state space we show how these two channels may be in conflict especially if bonds are of short maturity. Issuing longer bonds has the advantage of reducing the conflict between these two aims.

3.1 Fiscal Insurance Against Expenditure Shocks

We first consider the case of a shock to government expenditure. Under perfect foresight the government's intertemporal budget constraint is:

$$-\sum_{1} \beta^{t-1}(g_t u_c(c_t,\bar{\xi}) - w_t u_c(c_t,\bar{\xi})h_t - v_h(h_t)h_t) = \beta^{N-1}u_c(c_N,\bar{\xi})\frac{b_0^N P_0}{P_N} \quad (3.1)$$

In (3.1) we have replaced the product of inflation rates from period one to period N with the ratio of initial and end prices. We also evaluate the budget constraint setting the preference shock to its steady state value, $\bar{\xi}$, so as to concentrate solely on the effects of changes in g. The government's intertemporal budget constraint equates the present discounted value of government supluses to the initial liability inherited by the government. After an unexpected change in g_1, the government can either adjust tax revenue $(w_t u_c(c_t,\xi_t)h_t + v_h(h_t)h_t)$ or adjust the right hand side of (3.1) to ensure that it holds. We refer to changes in the right hand side as fiscal insurance following Faraglia, Marcet and Scott (2008) and Lustig, Sleet and Yeltekin (2011).

If bond markets are fully contingent (Chari and Kehoe (1999)) or prices fully flexible (Siu (2004)) then given the optimal allocation it is always possible to construct a sequence of real liabilities that satisfy (3.1) for every contingency. Under either of these cases the planner's maximisation problem involves a single implementability constraint in period zero. However, with sticky prices and non-contingent bonds then (3.1) will not be satisfied for every g_1 unless it is imposed as an additional constraint. The result is that the excess burden of taxation is not constant over time and the economy does not reach the complete market outcome. By varying inflation (and so affecting P_0/P_N) and using taxes to influence interest rates or exploiting endogenous fluctuations in bond prices $(u_c(.))$ the maturity structure of government debt governments can achieve fiscal insurance - reducing the volatility in tax rates so that the optimal sequence is closer to the complete market outcome.

In order to determine the optimal inflation path in the case of a first period shock to government expenditure we assume that the government enters into period one after a long commitment so that $\lambda_{s,t}$ and $\lambda_{p,t}$ have settled at constant values. From the martingale property of $\lambda_{s,t}$ we have $\lambda_{s,0} = E_0 \frac{u_c(c_N,\bar{\xi})\lambda_{s,1}}{P_N} / E_0 \frac{u_c(c_N,\bar{\xi})}{P_N}$ and from the first order condition with respect to wages $\lambda_{s,0} = \lambda_{p,0}\frac{\eta}{\theta}$. We assume the analogous values for the multipliers in periods $t = -N, -N+1, -1$ are equal to these period zero values. Since all uncertainty is removed after the shock to g_1 from (2.14) it follows $\lambda_{s,t} = \lambda_{s,t+1}\ \forall\, t \geq 1$ and similarly from (2.12) $\lambda_{p,t} = \lambda_{p,t+1}$. As a result the first order condition for inflation is:

$$\begin{aligned} -\theta\lambda_{f,t}(\pi_t - 1) \quad &- \quad (\lambda_{s,0} - \lambda_{s,1})\beta^{N-t}\frac{u_c(c_N,\bar{\xi})b_0^N}{\pi_t\, P_N}\,\mathcal{I}(t \leq N) \\ &- \quad (\lambda_{p,1} - \lambda_{p,0})u_c(c_t,\bar{\xi})(2\pi_t - 1)\,\mathcal{I}(t = 1) = 0 \qquad (3.2) \end{aligned}$$

where $\mathcal{I}(t = k)$ is an indicator function that takes the value one in period k and zero otherwise.

This optimality condition is nonlinear and cannot be solved analytically but we can note some properties of the solution. First in (3.2) inflation responds to the g shock for a maximum of N periods. In period $N+1$ gross inflation satisfies $-\theta\lambda_{f,t}(\pi_t - 1) = 0$ so that the inflation rate equals zero $(\pi_t = 1)$ as in the case of sticky prices $\theta > 0$ and $\lambda_{f,t} \neq 0$. Clearly it is optimal for the planner to frontload changes in prices for the duration of outstanding government debt. If instead the optimal policy was to commit to generate inflation in some period $T > N$ then current prices would also be affected through the Phillips curve as current inflation responds to expectations of future inflation. Such a policy would entail non-zero resource costs for all periods between $N+1$ and T that add nothing to improve the governments debt position in period 1.

From periods 1 to N the term $(\lambda_{s,1} - \lambda_{s,0})\beta^{N-t-1}\frac{u_c(c_N,\bar{\xi})b_0^N}{\pi_t\, P_N}$ determines optimal inflation as a function of the governments inherited liability and in particular the sign of the inflation response depends on whether goverments are creditors or debtors. When $b_0^N > 0$, the government wants to increase prices in response to a rise in g_1 in order to decrease the liability and absorb part of the shock. The converse holds if $b_0^N < 0$. The higher is debt the higher is the level of optimal inflation. Therefore both the level and maturity of government debt matters.

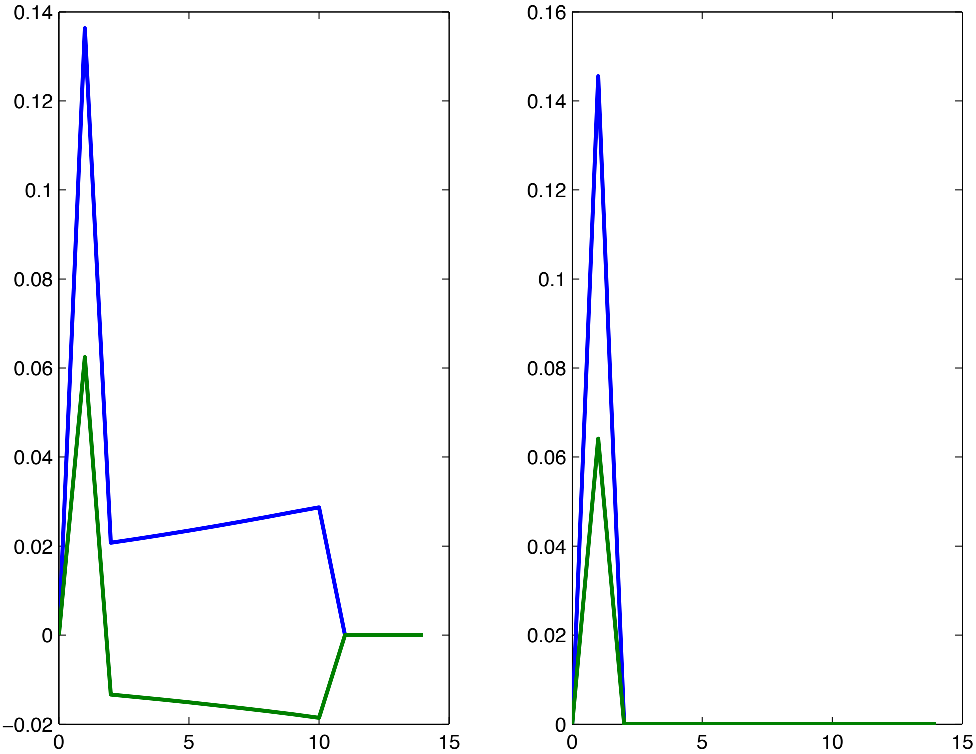
An additional term $(\lambda_{p,1} - \lambda_{p,0}) u_c(c_1, \overline{\xi})(2\pi_1 - 1)$ operates through the Phillips curve that only operates in period 1. This term captures the effect of changes in wages and inflation on firm monopoly profits. In our model there are no profit taxes but the government can use inflation to effectively levy a tax on firms, and since profits have a negative wealth effect on the supply of labor, the planner would like to tax them precisely in those states where a rise in effort is needed (high g_1 states). To do so the planner can either have the real marginal costs of the firm rise or through an adjustment in prices, increase the real resource costs facing firms from inflation.

These two forces - a real balance effect and an effective profit tax - determine inflation dynamics. The impact of these two forces depends on the sign of the shock g_1 as well as whether the government is a creditor or debtor. For example assume a positive innovation to government spending and $b_0^N > 0$. In this case a rise in wages and inflation in period one helps the government reduce its debt and encourages labour supply by mitigating the wealth effect from profits. The two channels therefore work in the same direction in this case. However if $b_0^N < 0$ the planner would like to reduce inflation to increase the value of savings but also increase wages and thus inflation to head off the wealth effect through the implicit profit tax. Therefore when $b_0^N > 0$ both forces serve to raise inflation but when $b_0^N < 0$ the two forces push in different directions. If the government can issue longer term bonds then the tradeoff between these two forces is less. Because the effective tax term applies only in period 1 if N=1 these two factors are in direct conflict. If N>1 then the government can commit to lower inflation between periods 2 and N to meet both objectives.

In Figure 1 we show the optimal inflation path in our economy in response to a rise in spending (a one standard deviation shock) in the case of one period bonds (right panel) and for a long bond where N=10 (left panel). The initial shock is propagated through the first order autoregressive process so spending differs from $\overline{g}$ for several periods before it eventually converges back its steady state value. The blue lines on both panels show the responses of inflation when the governments inherited liabilty is 60% of the steady state level of output whereas the green lines represent an initial position of -60% of output.[2]

When $b_0^N > 0$ (blue lines) the government commits to increase inflation for N periods in response to the shock. In period one the response of prices is larger because the real balance effect and the implicit profit tax effect point in the same direction. After period 1, in the case of the long bond, optimal inflation

[2]In (3.2) the optimal inflation path is influenced by the behavior of the multipliers which here evolve as pure random walks. This property is preserved independent of the first order autocorrelation coefficient of the process of government spending. As a result the optimal inflation path would be qualitatively very similar in the case of i.i.d inovations to g_1. We verified this numerically but for the sake of brevity do not report these results.



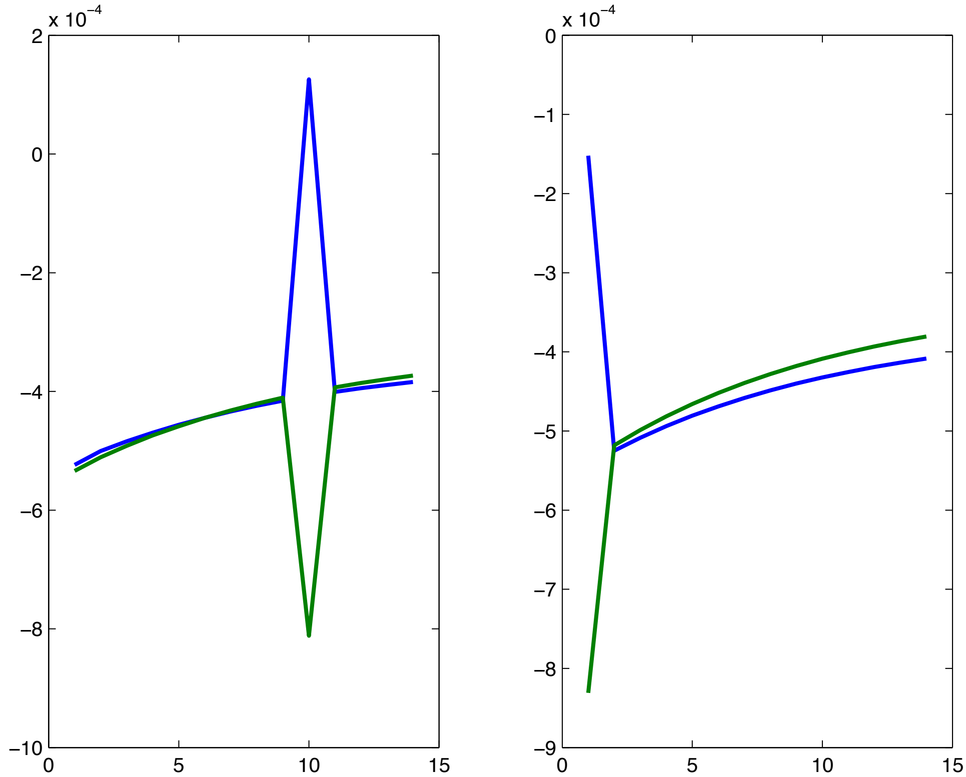
The left panel shows the optimal inflation path (in percentage terms) for the long bond economy ($N = 10$). The blue line represents the inflation path when the governments inherited liability b_0^N is 60% of the steady state GDP. The green line shows the analogous path starting from initial position of - 60% of GDP. The right panel shows the responses for short maturities ($N = 1$).

Figure 1: **Reponses of inflation to government spending shocks**

adjusts downwards, to reflect only the real balance effect. The optimal path entails a rise in inflation from periods 2 to N due to discounting. The govenent must commit to increase wages in period N to engineer this rise in inflation. The wage path (not shown) has a spike in period one and a spike in period N , but in all other periods wages are approximately equal to steady state value $\frac{1+\eta}{\eta}$. Notice that given that the government has more periods to adjust inflation in the long bond economy it can can smooth price increases across a longer period.

When $b_0^N < 0$ (green lines) prices increase with both maturities in period 1 because a rise in wages helps boost hours and helps offset the impact of the increase in government expenditure. In our baseline calibration therefore, the implicit profit tax effect dominates the real balance effect in period 1. Long bonds however permit the government to commit to set gross inflation below unity for every subsequent period until period N and so boost the value of assets. To engineer this path, wages must be set below the steady state value in period N. The price level increases by 0.045 % after the shock in the short bond economy where as in the long bond model it drops by 0.0521%.

In Figure 2 we show the responses of consumption to a one standard deviation shock in government spending. As discussed earlier the government will commit to change consumption in period N in order to adjust the interest payment on its inherited debt (see Faraglia Marcet and Scott (2011)). In the long bond economy this shows as a 'blip' in the consumption path in period 10, whereas in the short bond economy the 'blip' occurs in period one. The sign of these 'blips' depends on the sign of the liability. When $b_0^N > 0$ the government will increase consumption in period N in order to lower the marginal utility $u_c(c_N, \bar{\xi})$ and reduce the interest payment on its debt. The converse holds if $b_0^N < 0$. To engineer the change in consumption the planner must commit to change the tax schedule in period N.



The left panel shows the optimal consumption path for the long bond econonmy ($N = 10$). The blue line represents the consumption path when the governments inherited liability b_0^N is 60% of the steady state GDP. The green line shows the analogous path starting from initial position of - 60% of GDP. The right panel shows the responses for short maturities ($N = 1$).

Figure 2: **Reponses of consumption to government spending shocks**

3.2 Fiscal Insurance Against Preference Shocks

We now consider the effects of a positive innovation in preferences, specifically an increase ξ_t relative to the steady state value $\bar{\xi}$. This shock will increase the households appetite to consume. In order to finance higher consumption, hours must increase, but since the shock alters the marginal rate of substitution between consumption and leisure, households will find it optimal to increase the labor effort. From the point of view of the government, tax revenue will rise and given the level of expenditures in the economy this increase in revenue will decrease the deficit of the government (or increase the surplus).

To illustrate the effects of this shock we rewrite the government's intertemporal constraint as:

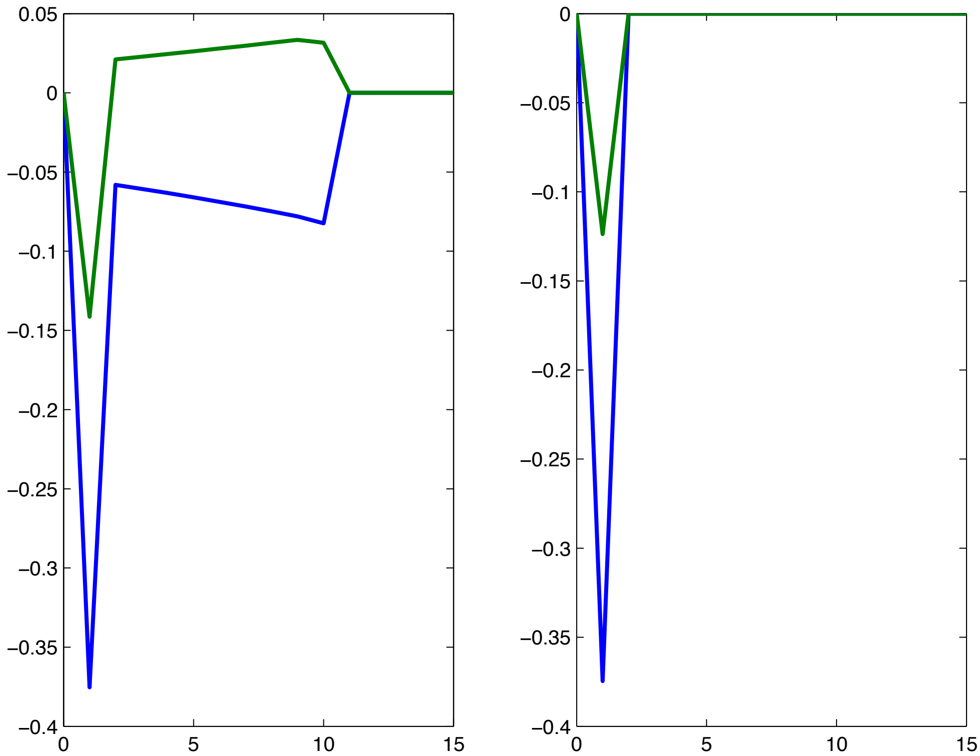
$$-\sum_{1}\beta^{t-1}(\overline{g}u_c(c_t,\xi_t)-w_tu_c(c_t,\xi_t)h_t-v_h(h_t)h_t)=\beta^{N-1}u_c(c_N,\xi_N)\ b_0^N\frac{P_0}{P_N} \quad (3.3)$$

Consider first the case $b_0^N > 0$. A rise in ξ_1 relative to the steady state means that the LHS of equation 3.3 will increase, for the reasons just explained. As before movements in inflation and consumption can help the government smooth any required change in taxes. A positive innovation to ξ_1 will force the government to reduce prices P_N or reduce c_N by rearranging the tax schedule, in order to provide fiscal insurance and insulate tax rates from preference shocks. If, in contrast, $b_0^N < 0$, i.e the government holds savings, then in response to an increase in the value of ξ the planner must now adjust the liability by increasing prices, to reduce the market value of savings, and change c_N accordingly. In both of these cases the government still has the incentive to vary the effective profit tax through varying wages and inflation. Once more in the case where $b_0^N < 0$ and the government issues short bonds then the real balance effect and the effective profit tax conflict.

Focusing on preference shocks also reveals a third channel through which varying the maturity of government debt influences the degree of fiscal insurance. Preference shocks impact on asset prices and hence change the tradeoffs facing the government when it wishes to hedge preference shocks using long maturities rather than short bonds. To illustrate this we divide the RHS of (3.3) by the marginal utility of consumption in period 1 to express the government's intertemporal budget constraint as:

$$-\sum_{t}\beta^{t-1}\frac{u_c(c_t,\xi_t)}{u_c(c_1,\xi_1)}(g_t-w_th_t\tau_t)=\beta^{N-1}\frac{u_c(c_N,\xi_N)}{u_c(c_1,\xi_1)}\frac{P_0}{P_N}b_0^N \quad (3.4)$$

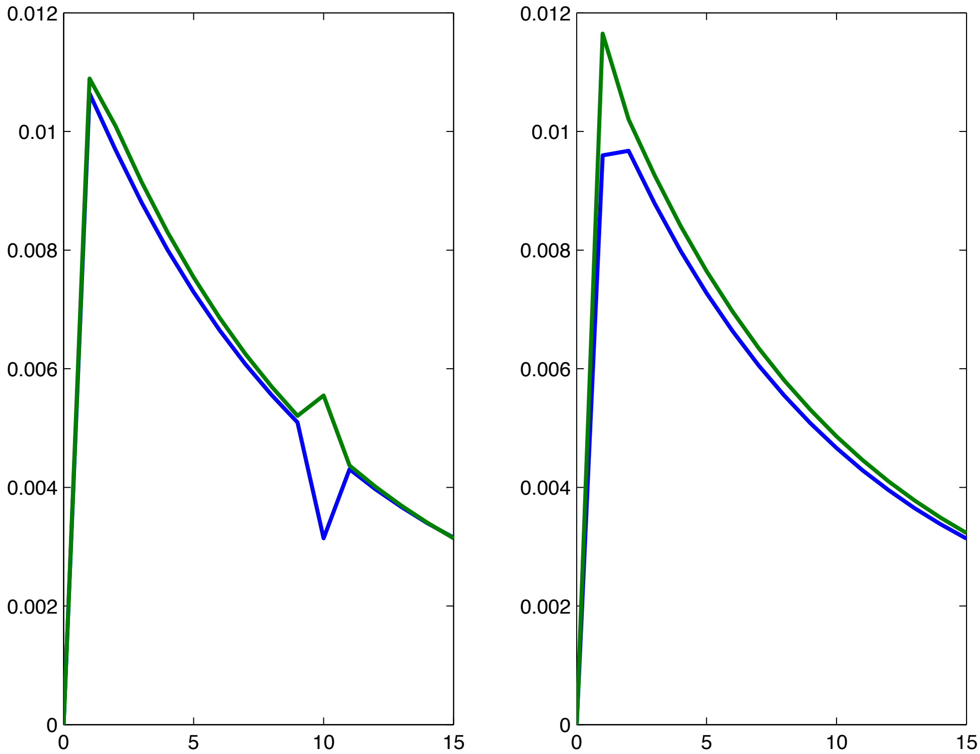
Notice that if $\xi_1 > \xi_N$ the change in the marginal utilities of consumption in periods 1 and N may push the government's inherited liability in the opposite direction than what is needed to insure against the shock. When $b_0^N > 0$ the planner would like to increase the value of debt; if the shock is mean reverting, as we assumed, and relative movements in consumption are not large enough to compensate, then the ratio $\frac{u_c(c_N,\xi_N)}{u_c(c_1,\xi_1)}$ will fall and hence send the liability in the opposite direction. The longer is the maturity N, the more vulnerable the government is to this adverse effect. The only type of bond that is completely insulated from this effect is the one period bond, because its price is always equal to one independent of the shocks and consumption. Whether shorter maturities help the government smooth the burden of taxation better than longer ones ultimately depends on how the importance of being able to use inflation more readily under long N fares against of the importance of these movement in the real returns. Moreover it is crucial to stress that this apparent advantage of short maturities over longer ones can only hold when the government has positive debt. When $b_0 < 0$ the government will want to reduce the value of its savings in response to the shock and movements in asset prices of long maturities help accomplish precisely that.



The left panel shows the optimal inflation path for the long bond economy ($N = 10$). The blue line represents the path when the governments inherited liability b_0^N is 60% of the steady state GDP. The green line shows the analogous path starting from initial position of - 60% of GDP. The right panel shows the responses for short maturities ($N = 1$).

Figure 3: **Reponses of inflation to preference shocks**

In Figure 3 we show the responses of inflation to the preference shock. The left panel shows the optimal path in the long bond economy. The blue line represents the path when the government has an initial position with positive debt. The green line shows an initial position with savings. The left panel plots the analogous quantities in the short bond economy. Inflation follows a similar path as in the case of spending shocks. Of course here prices must decrease to help the government absorb the shock though the liability. As in the case of spending shocks with long bonds the planner can smooth price changes and mitigate the undesirable fall in prices in period 1, due to the corporate tax, when $b_0^N < 0$. In Figure 4 we plot the consumption paths. A positive shock to preferences increases household consumption. The government changes the path in period N to affect the real returns and move the liability in the desired direction. Given the magnitude of the responses we can verify that a positive innovation to ξ_1 reduces asset prices for 10 period debt. Therefore as discussed above, long bonds present the government with a less favorable tradeoff when $N = 10$. We leave it to the next section, where we present the analysis based on the stochastic version of the model, to investigate whether the asset pricing channel of preference shocks implies that governments can better control their debt when the debt maturity is short.



The left panel shows the optimal consumption path for the long bond econonmy ($N = 10$). The blue line represents the path when the governments inherited liability b_0^N is 60% of the steady state GDP. The green line shows the analogous path starting from initial position of - 60% of GDP. The right panel shows the responses for short maturities ($N = 1$).

Figure 4: **Reponses of consumption to preference shocks**

4 Stochastic Simulations

The previous section articulated the key channels which influenced optimal inflation using a model of one period uncertainty only. In this section we turn to full stochastic simulations to see how varying debt maturity influences the behaviour of inflation. We simulate the economy using random draws of government spending and preference shocks and study the cyclical behavior of optimal inflation, taxes, deficits and debt. As we discussed in the previous section, these optimal paths will differ depending on the maturity structure and sign of government debt. Longer maturities make changes in inflation optimally run for several periods in response to shocks, and they yield different properties for tax rates, consumption and hence deficits and debt. We therefore compare the outcomes for our model economy as we vary the maturity of debt.

The previous analysis suggested that the response of inflation to expenditure or preference shocks depends on the existing level of government debt. Therefore the range of positions that the government is allowed to take in the bond market will affect the cyclical properties of the variables of interest. To limit this range we assume that bond issuances are subject to ad hoc constraints of the form $\beta^N b_t^N \in \{\underline{M}_N, \ \overline{M}_N\}$. Notice that since β^N is the steady statemarket price of an N period bond, these constraints are expressed in terms of the market value of debt. The main results presented in this section derive from a model where $\frac{\underline{M}_N}{\beta^N} = 0$, meaning that the government can only borrow from the private sector. In Lustig et al (2008) this constraint is imposed to ensure that the government cannot take extreme positions that would effectively complete the market as in Angeletos (2002). In our case we use this constraint because, as we have explained optimal policy is very different and also because its has been shown that allowing the government to go short and long leads in equilibrium to large accumulation of precautionary savings by the government (Faraglia et al (2011)). In all cases we consider we assume that the upper bound $\overline{M}_N$ is loose enough so as to bind very infrequently in the simulations. In the appendix we explain how to extend the planners program to allow for these bounds and in a later section we allow for the case of$\underline{M}_N$<0.

Our numerical approach to solve for the equilibium is to approximate the conditional expectations in the first order conditions with polynomials of the appropriate state variables. Full details are contained in the Appendix but we essentially borrow from the work of Faraglia, Marcet and Scott (2011) who show how to model long maturity bonds in a manner which avoids using the full state space. In our application the state vector is a high dimensional object; it includes the value of government spending and the preference shock in period t, the history of debt obligations b_{t-j}^N, past values of the multipliers $\lambda_{s,t-j}$ and past values of gross inflation π_{t-j} for $j = 1, ...N$. To give an idea of its size note that if the maturity of government bonds is 10 periods then the state space consists of 31 variables. Faraglia, Marcet and Scott (2011) show that in order to make the computation of models with large N manageable, one has to reduce the number of states in the approximating polynomials. Their approach is to partition the state space into variables that are of primary importance and variables of secondary importance. The latter are introduced in the approximating functions as sucessive linear combinations. The authors provide a procedure to test for the number of linear combinations that are necessary to get accurate approximations

Model	$N=1$	$N=5$	$N=10$	$N=20$
	No Lending $\frac{M}{\beta^N}=0$			
$\overline{b}^N$	.0660	.0639	.0980	.1255
$\overline{MV}^N$	.0635	.0524	.066	.058
σ_π	.429%	.462%	.470%	.557%
corr (π_t, π_{t-1})	.136	.27	.338	.534
σ_Y	.0279	.0283	.0282	.0283
σ_τ	.0333	.0354	.0354	.0352

Notes: σ_x is the standard deviation of variable x; corr(x,y) denotes the correlation between variables x and y. Upper bars denote sample means. The market value of debt is constructed using the formula $MV_t = \beta^N E_t \frac{u_c(c_{t+N},\xi_{t+N})}{\Pi_{t+1}^{t+N}\pi_j u_c(c_t,\xi_t)} b_t^N$.

Table 1: **Moments: Long samples**

of the conditional expectation terms.

4.1 Simulation Results - Long Sample

The model is run with one long sample of 100,000 observations and in calculating moments for the endogneous variables we drop the first 5,000 observations so as to get rid of the influcence of initial conditions. In Table 1 below we show sample moments for four maturities - one, five, ten and twenty period bonds. The first two rows of Table 1 report the mean values of bonds and the market value of debt in the samples. Increasing the maturity seems to have a small impact on the market value of debt. For example, with bonds of one period maturity the government on average issues debt equal to roughly 30% of average GDP in the model. We get similar magnitudes for longer maturities.

Note that near the zero debt region the optimal response of inflation should be nearly independent of the maturity of debt. If debt is equal to zero, the models are identical because inflation changes do not affect the government's position, and thus do not help the government smooth the excess burden of taxation. Under these circumstances the only concern for the planner is to use inflation to manipulate the wealth effect of profits on labor supply over the business cycle and therefore, independent of the level of maturity, the response of inflation to shocks would only be instantaneous.

Even though on average debt is low, the models still differ in terms of the first order autocorrelation properties of inflation. In row 4 we report this quantity. As the maturity increases from one year to twenty years the correlation increases from .136 to .534. This convinces us that under long maturities its still optimal to use price changes for several periods, in spite of the fact that the average level of debt in the economy is only 30% of GDP.

Notice that, contrary to the predictions of Section 3 , inflation is positively autocorrelated even under one year maturity. This property reflects the impact of the bounds on the government's optimal plan. For example near the no lending constraint the multiplier $\lambda_{s,t}$ ceases to behave as a risk adjusted random walk - there is strong mean reversion in its response to shocks. Evidently this adds to the persistence of the inflation process at all horizons.

Finally, in the third row of the table we report the standard deviation of inflation in our sample. This quantity increases with maturity from roughly 0.43% in the one year bond economy to 0.56% in the 20 year maturity model. Note that the quantity reported is the unconditional standard deviation of inflation, a measure of the resource costs of the economy due to price adjustments, so these differences reflect differences in the peristence of the process. Relative to the volatility of output, which is similar across models (row 5), price changes are extremely smooth because the resource costs are too large given our calibration of the Phillips curve.

4.2 Short Samples

As discussed above the properties of inflation, taxes and the market value of debt are affected by the magnitude of the position of the government in bond markets. In the stochastic model with one long sample the government issued on average little debt in terms of market value. In our model when debt is equal to zero optimal policy is the same in long and short bond economies. We therefore want to study the behavior of the economy in short samples where we can condition on different levels of debt to better understand the impact on inflation. The results, summarized in Table 2 represent the moments from 1000 samples of 100 observations each run with different initial conditions, uniformly distributed across the range $\{0, \frac{\overline{M}}{\beta^N}\}$. We report the properties of inflation when the initial condition is 5, 55 and 95 percent of average GDP. To conserve on space we only consider the case of one period and 10 period bonds.

The last column of Table 2 reports the average of the market value of debt in our samples. Although initial conditions exert an influence the differences in the means are considerably smaller than differences in initial conditions. For example when we use an initial condition of 95% of GDP in market value we get an average of 0.0955 (nearly 50% lower) under short bonds and 0.078 (nearly 60% lower) under ten year maturity. This suggests that the planner can very effectively use their policy instruments to manage the debt, an implication that is consistent with the sharp rise in inflation variability in these high initial debt cases. With one year bonds the standard deviation of inflation increases from

		σ_π	corr (π_t, π_{t-1})	$\overline{\text{MV}}$
One period bonds				
b_0^N	5%	.43%	.19	.0424
	55%	.43%	.15	.0633
	95%	.54%	.08	.0955
Ten period bonds				
b_0^N	5%	.46%	.25	.0469
	55%	.45%	.28	.0628
	95%	.64%	.27	.0768

Table 2: **Moments: Short samples**

0.43% to 0.54% and with 10 year maturity it rises from 0.46% to 0.64%. showing that governments use the real balance effect to greater effect. The reason for this considerable increase is that the influence of the real balance effect is greater when debt is high. In response to fiscal shocks the government will move prices more forcefully to adjust its liability, adding to the volatility of inflation under optimal policy.

4.3 Decomposing the result

In this subsection we investigate what happens in the economy if we only allow for spending shocks and what happens if we only allow for preference shocks. The results are summarized in Table 3. The first two columns of the table report outcomes in the model with g shocks only and the last two columns show the moments in the model with only preference shocks. We note that the economy with spending shocks shows substantially less volatility in inflation and taxes as well as output than the economy with preference shocks. The implication is to be anticipated given the results of Holland and Scott (1998). Since changes in ξ_t cause larger fluctuations in hours, they exert a larger influence on the governments finances. Tax rates have to respond more to finance deficits and also larger increases and decreases of inflation are required to adjust the liability in response to shocks. The volatility of inflation is around 0.15% when only spending shocks are included in the model and above 0.40% under the model with preference shocks. Finally note that the first order autocorrelation of inflation is independent of the source of disturbances in the economy. Since in equilibrium the government issues similar amounts of debt the properties of inflation are also similar.

4.4 Varying the Degree of price stickiness

Model	$N = 1$	$N = 10$	$N = 1$	$N = 10$
	Spending Shocks		**Preference Shocks**	
$\bar{b}^N$	.061	.102	.070	.100
$\overline{MV}^N$	.058	.067	.067	.067
σ_π	.146%	.153%	.402%	.440%
$\text{corr}(\pi, \pi_{-1})$	.11	.27	.13	.34
σ_τ	.0181	.0187	.0305	.0333

Notes: The first two columns summarize the moments from a model with only g shocks. Columns three and four present the analogous moments with preference shocks only.

Table 3: **Moments**

In this section we consider the impact of varying the degree of price stickiness. We simulate the model using a value of θ equal to a third of the baseline value. In terms of the Phillips curve this choice implies approximately one quarter of sticky prices. As a consequence, inflation persistence changes considerably with long debt - it increases in the case of 10 period bonds from 0.34 to 0.74. Further, and not surprisingly, inflation volatility increases considerably especially for long maturities. In the case of 10 period bonds we get a standard deviation of inflation equal to 0.827% , almost double the baseline model (0.43), whereas with one period debt we get a more modest increase to 0.56%.

To understand these findings notice that when prices are less sticky the government's incentive to manipulate wage costs and mitigate the wealth effect on labor supply becomes weaker. Firms can pass on higher costs to product prices thus insulating their profits and thus the government now puts a larger weight on the real balance effect of inflation. With lower price stickiness the real balance effect can also be used to greater effect. In terms of the impulse response analysis presented in Section 3, the first period surge in prices is smoothed and inflation increases spread across later periods. With one period debt inflation volatility doesn't rise by much because the more impoartant real balance effect exerts an influence on prices for only the first period and is partly offset by the weakened corporate tax effect.

5 The Effectiveness of Inflation

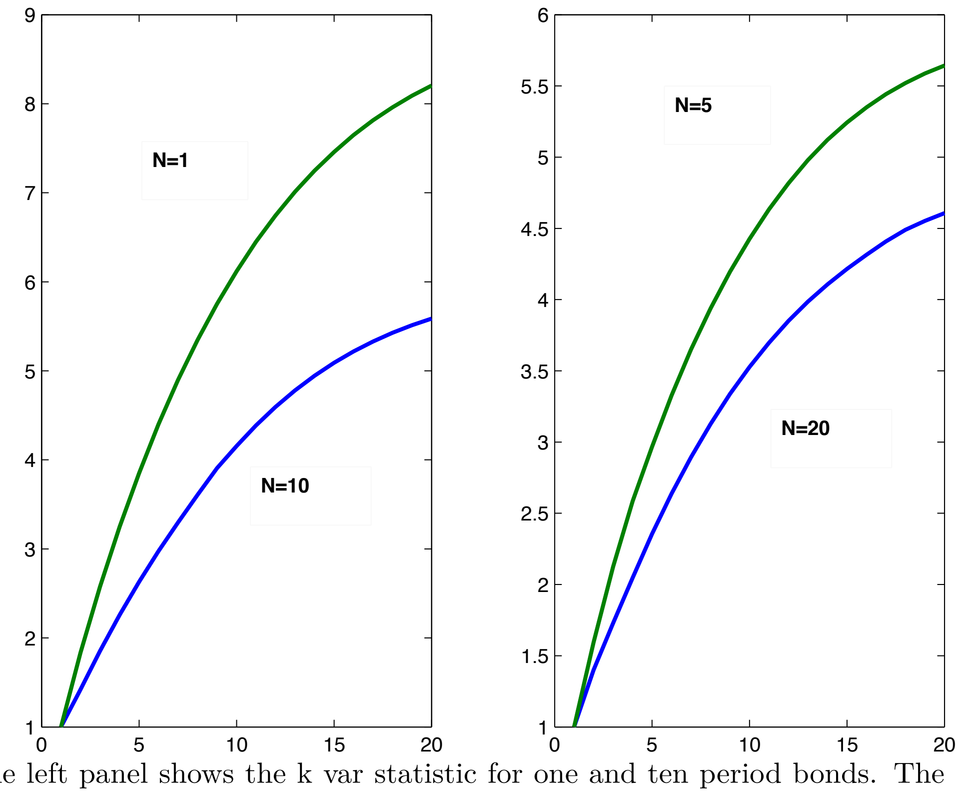
Marcet and Scott (2008) show how under complete markets and persistent shocks the market value of debt shows less persistence than other endogenous variables whereas when markekts are incomplete debt shows relatively greater persistence. Faraglia, Marcet and Scott (2009) show that Cochrane's (1988) measure of persistence is a good indicator with which to assess the effectiveness of debt management in achieving fiscal insurance. Previous sections have shown how issuing long bonds in the presence of sticky prices enables governments to make greater use of inflation to achieve fiscal insurance. In this section we investigate the role of inflation in shifting closer to the complete market outcome.

5.1 The role of inflation in achieving complete market outcome

We compute for our model economy the following measure of persistence :

$$P_{MV}^{k} = \frac{\text{Var}\left(\text{MV}_{t} - \text{MV}_{t-k}\right)}{k\ \text{Var}\left(\text{MV}_{t} - \text{MV}_{t-1}\right)} \tag{5.1}$$

Figure 5 plots our k-variance measure of persistence for government debt.



The left panel shows the k var statistic for one and ten period bonds. The green line is the short bond economy and the blue line represents the long bond economy. The right panel depicts the analogous objects for maturities of 5 (green line) and 20 years (blue line)

Figure 5: **Measure of Persistence of the market value of debt**

In the right panel the blue line shows the case $N = 10$ and the green line $N = 1$. On the left panel the green line is the five year maturity and the blue the twenty year maturity. Notice that there is a sizeable difference between the one and the ten period bond and also a large difference between the five and twenty period bond. The larger the maturity the lower the persistence. At $k = 20$, under twenty period maturity the value of P_{MV}^{k} is roughly 4.5. Under one year debt the analogous value is roughly 8. Since Faraglia Marcet and Scott (2011) show that with real debt long bonds are not that different than short bonds, we interpret these differences in (5.1) as reflecting the ability of governments to control debt via inflation. Notice that though in Figure 5 very short bonds perform much worse than long bonds, there is relatively little difference between five and ten period maturities. Under ten year debt P_{MV}^{k} equal 5.58 at $k = 20$ whereas under five year debt we get a value of 5.69. In the case of five period bonds the government issues less debt and so the real balance effect is smaller. However the asset pricing effect from preference shocks, which operates in a different direction, is also muted so the net result is little difference between the five and ten period case. Going from ten to twenty periods does produce a substantially reduction in persistence suggesting that the real balance effect dominates the asset price effect. Given the highly autocorrelated nature of the the shocks the shifts in asset prices at longer maturities are relatively not as large as in the case for i.i.d shocks which would also raise the relative importance of the real balance effect. The results therefore support the analysis of Lustig et al (2008) who show in the context of a monetary economy with government spending shocks and sticky prices that long bonds lead to outcomes closer to complete markets. We show that this holds for even longer maturities than they consider and in the case of preference as well as government expenditure shocks.

We close this section by briefly commenting on how P_{MV}^{k} behaves in some of the alternative environments that we considered in section 4. First when we run the model with short samples our measure of peristence plummets when the initial condition sets debt to a high level. This convinces us that the ability of the government to control its debt via inflation, crucially depends on the size of the government debt; when debt is high inflation is more useful to the government in order to manage its liabilities. Second, when we callibrate our model to have a lower degree of price stickiness we find that there is no difference in the peristence measure in the models, except for a slight drop under the longest, 20 year, maturity. As discussed previously lower persistence means more use of the real balance effect, but weakens considerably the corporate tax channel and hence the governments ability to influence hours in response to shocks. According to our results the ability of the government to tax firm profits indirectly by varying wages and inflation is more important when the maturity is short. In the case of one year debt we find that the measure of peristence increases slightly when we lower the resource costs of inflation.

5.2 The relative role of inflation and interest rates

Ours is a model of incomplete bond markets where the Ramsey planner uses inflation and interest rate twisting to achieve fiscal insurance. We have shown that when the government has access to longer maturity bonds then inflation can be utilised more and the economy shifts closer to the complete market outcome. In this section we examine the relative importance of inflation and interest rates in driving the behaviour of debt. To answer this question we use the government budget constraint of Hall and Sargent (2010) and decompose the change in the market value of debt, following a shock to either spending or preferences, into a component that is due to inflation and a component due to the movement in interest rates. The formula that we derive allows us to trace the effect of these changes over the many periods.

Using taxes and asset prices the budget contraint can be written as:

$$q_t^N b_t^N = q_{t-1}^N b_{t-1}^N + g_t - w_t h_t \tau_t = \frac{q_t^{N-1}}{q_{t-1}^N} q_{t-1}^N b_{t-1}^N + g_t - w_t h_t \tau_t \tag{5.2}$$

Notice that since $\mathrm{MV}_t = q_t^N b_t^N$ we can rewrite (5.2) in the following form:

$$\mathrm{MV}_t = \frac{q_t^{N-1}}{q_{t-1}^N} \frac{\mathrm{MV}_{t-1}}{\pi_t} + g_t - w_t h_t \tau_t \tag{5.3}$$

Let $(1+i_{t,j})^j$ be the inverse of the price of a bond of maturity j in period t. Then the law of motion of the market value of debt becomes:

$$\mathrm{MV}_t = \frac{(1+i_{t-1,N})^N}{(1+i_{t,N-1})^{N-1}} \frac{\mathrm{MV}_{t-1}}{\pi_t} + g_t - w_t h_t \tau_t \tag{5.4}$$

Linearizing (5.4) we arrive at the following formula:

$$\mathrm{MV}_t = (1 - i_{t,N-1}(N-1) + i_{t-1,N}N - \hat{\pi}_t)\mathrm{MV}_{t-1} + g_t - w_t h_t \tau_t \tag{5.5}$$

where $\hat{\pi}_t$ denotes net inflation in period t.

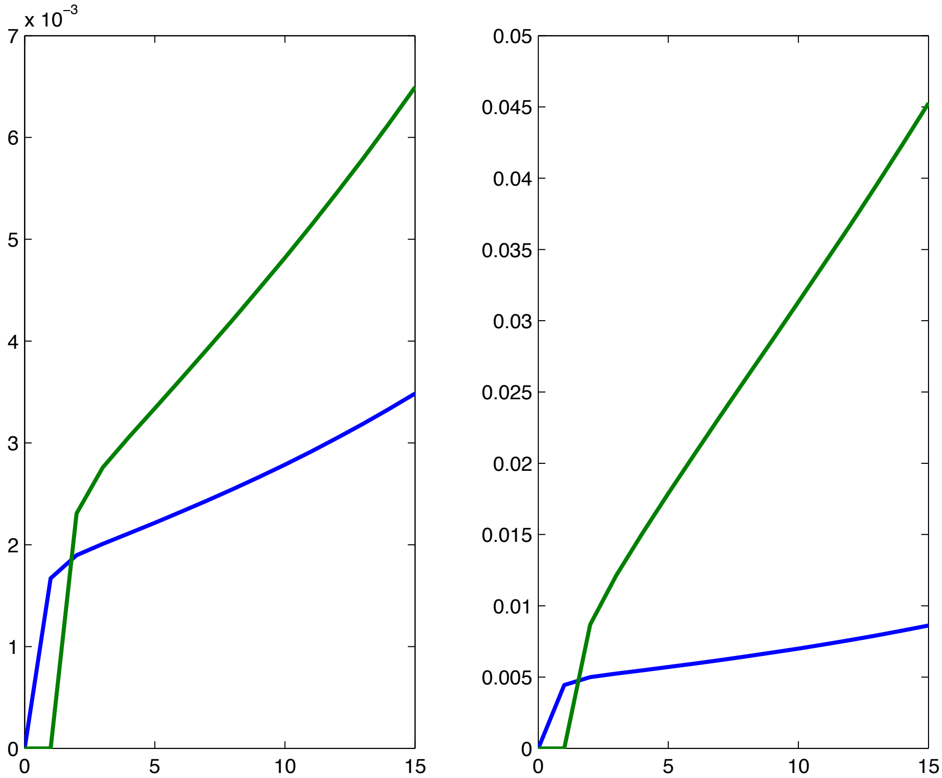
In order to quantify the effect of interest rates and inflation we create two series. The first sets $\hat{\pi}_t$ equal to zero. The second sets $i_{t,N-1}$ and $i_{t-1,N}$ equal to their steady state values. Our accouting exercise is to look at percentage differences between these series and the market value of debt in (5.5). We plot the results in Figures 6 and 7.

Consider first the case $N = 1$. Assume that a spending shock hits the economy at $t = 1$; the interest rate $i_{0,1}$ in (5.5) is clearly irrelevant because it involves the conditonal expectation in period 0. What matters is the time path of interest rates after period 1 and the level of inflation. For the latter the analysis of Section 3 revealed that under short term debt there is only an immediate response of prices. We find the same pattern in the stochastic model of this section. Inflation only increases immediately after the shock and therefore mitigates the rise of the market value. The path of the nominal interest rates is as follows: Because the planner would like to reduce the debt burden initially they will use changes in the real rate, and therefore changes in consumption in the period of the shock - the interest rate twisting effect of Faraglia, Marcet and Scott (2011). As a result, assuming positive debt, there is a rise in consumption that lowers marginal utility and decreases the interest rate in period t. For every subsequent period interest rates will rise relative to what they would have been in the absence of the shock (similar to Lustig et al (2008)). Since hours typically increase in response to a spending shock, interest rates are countercyclical in the first period and procyclical along the remainder of the path.

In the left panel of Figure 6 the blue line traces the effect of inflation. It plots the percentage increase in the market value of debt that would occur, had the inflation rate been held at its steady state value after the shock. Accoding to the graph the market value would be 0.2% higher immediately after the shock; since our formula compounds the effect of inflation after 15 periods the rise in the market value would be larger (nearly 0.35%). The green line represents the effects of interest rates. A fall in the interest rate in period 1 helps to stabilize the debt. As a result the market of debt would have been 0.25% higher had this change in the return not occured. After period 1 the short term interest rate increases but that rise is not nearly as large so as to revert the positive effect of the initial fall in $i_{1,1}$. In effect after 15 model periods the market value of debt would have been 0.75% larger had the entire path of interest rates been set equal to its steady state value. The interest twisting effect therefore is larger than the inflation effect although both exert a relatively small impact on debt.

The right panel repeats this analysis for the case of a preference shock. In order to facilitate the exposition we consider an innovation that lowers the value of ξ_t and hence lowers hours worked and the tax revenue of the government. In response to the shock the planner will increase prices instantaneously to adjust the liability and also will increase consumption in the period of the shock to change the real returns. The difference here is that the term $\frac{\xi_2}{\xi_1}$ also has an influence. Because shocks are mean reverting we generally expect $\xi_{t+1} > \xi_t$ and therefore interest rates will stay low for several periods after the shock and this helps the planner stabilize the debt. The figure shows that setting inflation equal to zero, would affect the debt dynamics on a rather modest scale, 0.5% initially and 0.9% after 15 periods. As with government expenditure shocks the bigger impact comes from interest rate twisting - eliminating the variability of interest rates would entail much larger debt swings 1% initially and 4.5% in the longer term.

The case of $N = 10$ is shown in Figure 7. Again the blue line represents an inflation rate equal to the steady state and the green line rules out interest rate twisting. In response to both spending and preference shocks the planner increases prices to lower the governmnet inherited liability. According to our decomposition the market value of debt would be nearly 0.2% higher when the expenditure shock occurs if it were not for this price adjustment and nearly 0.5% after 10 years. For the preference shock the effects are roughly 0.5% instantaneously and 1.5% in period 10. Notice that this decomposition underestimates the impact of inflation as a significant component of the inflation effect operates through bond prices. For example the term $i_{1,N-1}(N-1)$ is influenced by future anticipated inflation



The left panel shows the response of the market value of debt if inflation (blue line) or the interest rate (green line) are set to their steady state values. The right panel shows the analogous objects for the case of a preference shock. The economy has one period debt.

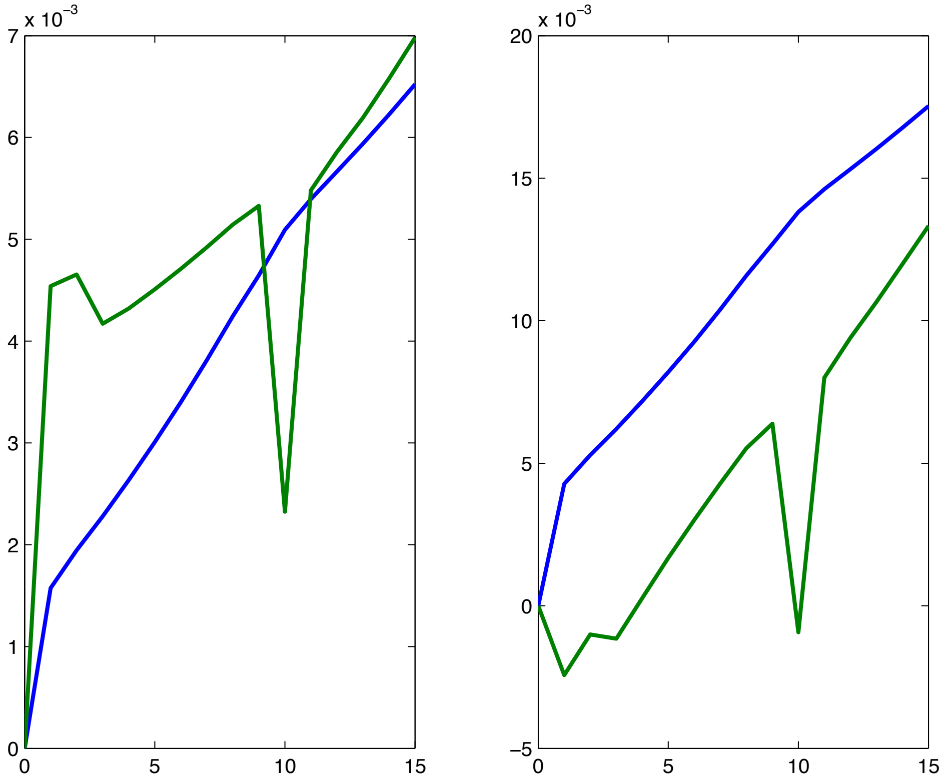
Figure 6: **Responses of the Market Value of Debt**

and this will feature all the inflation rates from periods 2 to 10. Therefore the blue line underestimates the effect of setting inflation equal to the steady state value at early periods. In subsequent periods it removes this effect though the adjustment in interest rate. In period 2 for example the term $i_{2,N-1}(N-1) + i_{1,N}N$ would up to a first order add the term $\hat{\pi}_2$ to the slope in 5.5. In period three it would add $\hat{\pi}_3$ and so on. That is to say that the effect of adding the inflation rates $\hat{\pi}_2$ to $\hat{\pi}_{10}$ in period 1 will be gradually removed by interest rate differences at later periods.

The adjustment of debt due to interest rates, represented by the green lines in Figure 7, is even more noteworthy in the case of long bonds. Notice that because in response to spending shocks the planner would like to change the real rate and reduce the debt burden, the term $i_{1,N-1}(N-1)$, adds to the variability of debt initially. The next kink occurs in period N in which case the path of the marginal utility of consumption increases the short term nominal interest rate. The green line that removes the changes in interest rates lowers the market value of debt in that period.

Even more interesting is the case of preference shocks on the right panel. We said earlier that in response to a preference shock that lowers the tax revenue and increases the deficit the planner would like to reduce the inherited liability $\beta^N \frac{u_c(c_N,\xi_N)}{u_c(c_1,\xi_1)} \frac{P_0}{P_N}$ but that due to the effect of the preference shock to the marginal rate of subsitution this adjustment becomes difficult. When the shock reduces the value of ξ_1 the term $\frac{\xi_N}{\xi_1}$ is in expectation greater than one and so in effect bond prices are pushed up and interest rates fall in spite of the adjustment in inflation or consumption. This difficulty facing the planner shows up in Figure 7 as a drop in the green line on the right panel. Notice that stated in terms of (5.5) the difference between long bonds and short bonds is that in the latter the term $(N-1)i_{t,N-1}$ is equal to zero. This is precisely the term that summarizes the adverse effect of changes in asset prices on government finances. In a nutshell the pattern that emerges in Figure 7, indicates why insurance against preference shocks becomes more difficult under long maturities even though, as suggested by the evolution of the market value of debt shows that the adverse asset price effect is rather short lived short lived. We also used the Hall and Sargent decomposition for low costs of price adjustment case considered above. Naturally the impact of inflation on debt fluctuations was larger - around twice the size - but still relatively small.

These results of the relative importance of inflation echo those of Schmitt-Grohe and Uribe (2004) and Siu (2004) that in the presence of price adjustment costs inflation plays only a limited role in achieving fiscal insurance. As Schmitt-Grohe and Uribe (2004) note even allowing for inflation still leaves debt following a unit root process that reflects the importance of market incompleteness. We show that this result still holds even when the government can more fully exploit inflation by issuing long run bonds. It also accords with the empirical work of Giannitsarou and Scott (2007) and Hall and Sargent (2011) that in practice inflation has provided only a relatively small role in achieving fiscal sustainability.



The left panel shows the response of the market value of debt if inflation (blue line) or the interest rate (green line) are set to their steady state values. The right panel shows the analogous objects for the case of a preference shock. The economy has 10 year debt.

Figure 7: **Responses of the Market Value of Debt**

6 Extensions

In this final section we extend our model in two directions. Earlier we assumed that governments could only issue debt but could not lend to the private sector. Although small in number there are some countries e.g Norway who have positive assets and Sovereign Wealth Funds are growing in importance as a means for governments to manage their positive asset holdings. The picture is further complicated when consideration is made of uncollected tax liabilities on the part of the private sector, clearly an asset for the government, although this may be offset by future capital allowances governments may have granted. Therefore to see the robustness of our results we consider the case where we do not impose a zero lower bound on government debt.

The other consideration of this section is relaxing the assumption that every period the government buys back the entire existing stock of debt and then reissues anew. In the context of a one period bond this assumption is obviously innocuous but as stressed by Faraglia, Marcet and Scott (2011) when long bonds are introduced the assumption of what to do with outstanding government debt at the end of each period is a significant one. Above we followed the literature and assume each period all debt is bought in and then reissued but in this section we consider the consequences of holding debt until redemption - the case of no buy back. As documented by Marceshi (2004) it is clear that governments in practice do not buy back debt but tend to hold to redemption.

6.1 Government as Creditor

When we remove the constraint that governments cannot lend to the private sector it is well known that the government will accumulate large amounts of precautionary savings to buffer shocks to its budget and this is not surprisingly precisely what we find in our simulations. In an economy with one period debt the market value of debt is on average -0.1437 in our sample and its -0.1558 under the ten year maturity model. Hence on average the government holds savings equal to 75% of average steady state GDP.

With private sector assets being used as a precautionary balance the properties of inflation change significantly. First we find the standard deviation of inflation drops to 0.22% under $N = 1$ and 0.27% under $N = 10$ and the first order autocorrelation of inflation is 0.16 and 0.48 under short and long bonds respectively. To understand these implications note that when a shock hits, the inflation response is now considerably smaller as the real balance and corporate tax effects now work in opposite directions. This implies that with short maturities a spending shock or a preference shock may produce no inflation response at all, while for a long maturity economy the change in inflation must come from later periods. The second implication is that, under long term debt, the autocorrelation of inflation increases relative to the no lending model. This change comes about through the same force that mitigates the volatilty of inflation. When savings are high enough the government will reduce inflation in response to a shock that increases the deficit (thus leaning against the corporate tax channel). In effect prices will fall in the period of the shock and continue to fall in subsequent periods. This brings about a larger first order autocorrelation than in the no lending model because as we showed earlier in our simulations government debt was too low, which made policy responses under long debt closer to those under short term debt. Given how we displayed earlier how the response of inflation depends on the level of debt it is not surprising that removing the no lending constraint has an impact on inflation properties.

6.2 Hold to Redemption

This section contains a brief discussion of the model with no buyback and it highlights the challenges faced in modelling this more empirically motivated assumption. With outstanding debt that isn't bought out each period the government is not exposed to reissue risk and so may have a greater incentive to use inflation. We use the version of our model where an unexpected shock hits in period one and thereafter there is no uncertainty in the economy. We show that under no buy back it is not possible to summarize the planner′s program in a single implementability constraint in period one. Instead there are N different constraints that relate to N different values of initial debt and each one of these constraints has generally a different value for the multiplier (λ_s) attached to it. Although the first order conditions are very similar in a model with and without buyback the fact that the multiplier $\lambda_{s,t}$is not constant after the shock, gives rise to an optimal policy response that features oscillations even in the long run. Similar paths are followed by other endogenous variables such as debt and consumption.

Under no buy back we can write the governments budget constraint as follows:

$$E_t \frac{u_c(c_{t+N}, \xi_{t+N})}{u_c(c_t, \xi_t)} \beta^N \frac{b_t^N}{\Pi_{t+1}^N \pi_j} = \frac{b_{t-N}^N}{\Pi_{t-N+1}^t \pi_j} + g_t - (1 + \frac{v_h(h_t)}{u_c(c_t, \xi_t) w_t}) w_t h_t \tag{6.1}$$

Iterating forward on equation 6.1 for periods 1 to period N we get the following expressions:

$$\begin{aligned} - \sum_{t \in \ j, j+N, j+2N, \ldots} \beta^{t-1} (g_t u_c(c_t, \xi_t) \ &- \ w_t u_c(c_t, \xi_t) h_t - v_h(h_t) h_t) \\ &= \ \beta^{N-j} u_c(c_{N-j+1}, \xi_{N-j+1}) \frac{b_{-j+1}^N P_{-j}}{P_{N-j+1}} \ \text{ for } \ j = 1, 2, \ldots N \end{aligned} \tag{6.2}$$

Moreover, merging the intertemporal constraints into one we get:

$$- \sum_1 \beta^{t-1} (g_t u_c(c_t, \xi_t) - w_t u_c(c_t, \xi_t) h_t - v_h(h_t) h_t) = \sum_{j=1}^{N} \beta^{N-j} u_c(c_{N-j+1}, \xi_{N-j+1}) \frac{b_{-j+1}^N P_{-j}}{P_{N-j+1}} \tag{6.3}$$

Notice that the term $\sum_{j=1}^N \beta^{N-j} u_c(c_{N-j+1}, \xi_{N-j+1}) \frac{b_{-j+1}^N P_{-j}}{P_{N-j+1}}$ represents the inherited liability of the government, the market value of debt outstanding in period one. When expenditure changes in that period this term has to adjust to adsorb the shock. How can this adjustment take place? Again prices and consumption will be used to influence the governments debt obligation. Notice however that the government can extert control over its debt by adjusting consumption in period $N-1$ and prices P_{N-1} to change the part of the liability that involves b_{-1}^N, by adjusting c_{N-2} and P_{N-2} to influence the term mutliplied by b_{-2}^N and so on. Changes in inflation that occur at early periods are much more effective in managing the debt. To see this note consider the case where $\{b_0^N = b_{-2}^N = \ldots b_{-N+1}^N > 0\}$ and consider the effects of a rise in π_1, π_2 , ... and π_N. It is obvious that given the assumed distribution of debt obligations increases in π_1 increase uniformly the price level in periods 1 to N and hence can absorb a bigger part of the shock. Ultimately the optimal policy will depend on the distribution of liabilities of the government across maturities.

Assume that the economy enters in period 1 with a positive innovation to g_1. Since under no buy back, a bond that is issued in period t has to be redeemed in period $t+N$ the planner will use the asset to balnce the marginal contribution of an increase in b_t^N to welfare with a cost of paying back the loan in period $t+N$. The multipliers that have to be equated now are $\lambda_{s,t}$ and $\lambda_{s,t+N}$. This gives that N distinct values $\lambda_{s,1}, \lambda_{s,2}, ..., \lambda_{s,N}$ that now have to be pinned down in order to solve for the optimal path. How are these objects determined? We can show that their values must be such that they balance the N corresponding intertemporal budget constraints in 6.2.

The optimal allocation here will involve an inflation path that is different from one in perpetuity. To show this we can use the first order condition for inflation in the model, which is given by the following equation:

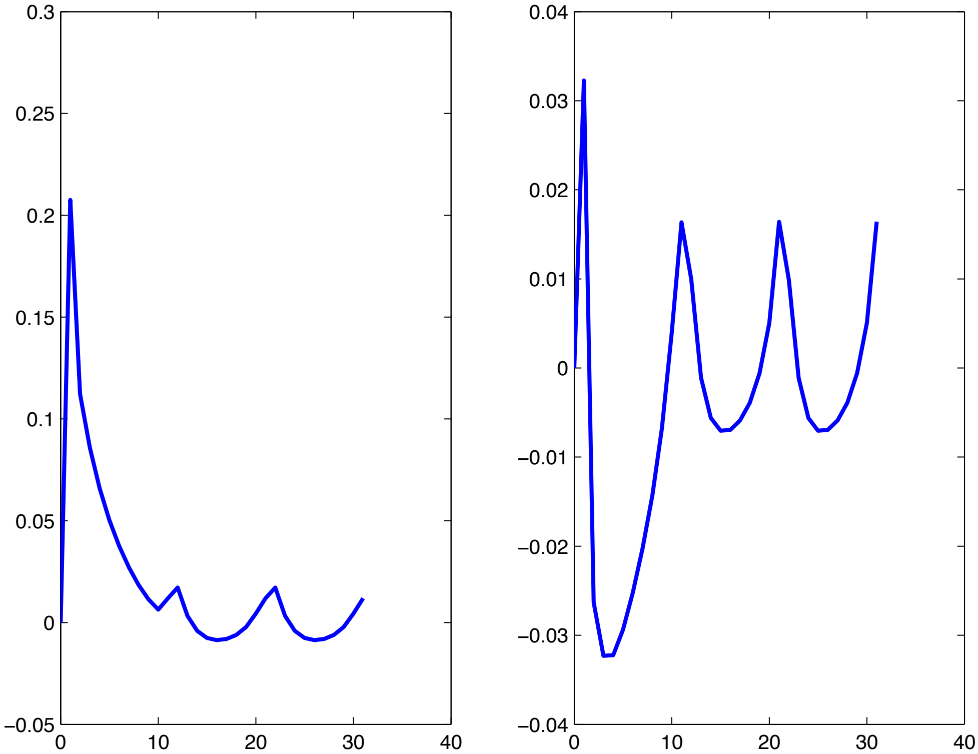
$$\begin{aligned} -\theta\lambda_{f,t}(\pi_t - 1) \quad &- \quad \sum_{k=1}^{N}(\lambda_{s,t-k} - \lambda_{s,t+N-k})\beta^{N-k}\frac{u_c(c_{t+N-k}, \xi_{t+N-k})b_{t-k}^N}{\pi_t \prod_{j=t-k+1}^{j=t+N-k}\pi_j}\mathcal{I}(t \leq N) \\ &- \quad (\lambda_{p,t} - \lambda_{p,t-1})u_c(c_t, \xi_t)(2\pi_t - 1) = 0 \end{aligned} \tag{6.4}$$

The second term becomes zero in period $N+1$ and hence the optimal inflation path is given by $-\theta\lambda_{f,t}(\pi_t - 1)(\lambda_{p,t} - \lambda_{p,t-1})u_c(t)(2\pi_t - 1) = 0$. Given that the mupltiplier $\lambda_{s,t}$ follows an N period cycle the term $(\lambda_{p,t} - \lambda_{p,t-1})$ is different from zero for each t and therefore inflation will be different from one even in the long run steady state.

In Figure 8 we plot the optimal inflation path in the equilibrium with no buy back. The left panel shows the response of inflation starting from an initial position with positive market value of government debt (again 60% of steady state GDP) and the right panel shows the analogous path when the initial position is negative. Both plots are constructed assuming a uniform disitrbution of liabilities and a shock of one standard deviation to government expenditure (similar results obtain when we consider preference shocks).

Once again the reponse of inflation in the first N periods is dominated by the presense of the standard real balance and profit tax concerns that the planner faces. When there is debt the planner raises prices aggressively in the first period and then decreases inflation. When the liability is negative the planner increases inflation in period 1 but attempts to reduce prices by keeping net inflation negative for several periods. Price changes continue after period N even when the economy converges to the new steady state.

We think that this behavior to commit to change prices forever derives from two properties of the no buy back model. The first is obvious; under sticky prices the planner cannot rearrange the inflation rates in such a way so as to satisfy the implementability constraints simultaneously consistent with a unique value of the multiplier λ_s after period 1. In contrast under flexible prices (complete markets) whether the government finances its debt with a buy back bond or a no buy back bond is immaterial for allocations, because in that case because in that case inflation can be as volatile as necessary. The second property is that since under incomplete markets the timing of debt payments matters, the planner will use changes in inflation, as well as consumption and taxes to alleviate the debt burden. But because optimal policy is framed here in a full commitment model these choices affect the behavior of the economy even in the long run.



Optimal inflation response in the no buy back model with $N = 10$. The left panel plots the inflation path when the governments inherited market value of debt is 60% of the steady state GDP. The right panel shows the analogous path starting from initial position of - 60% of GDP.

Figure 8: **Reponses of inflation to government spending shocks: No buy back model**

What can we do to control this erratic behavior of inflation? It seems obvious that if there were two types of bonds in the economy, a buy back and a non buy back bond then on the optimal path the condition $\lambda_{s,t} = \lambda_{s,t+1}$ would hold. For example if we gave the government the option to issue short term debt in the economy then oscillations in inflation and other variables would never occur as part of the optimal policy. Therefore whilst in reality governments hold debt to maturity, and obviously analyzing no buy back is important to understand this aspect of policy making, it seems reasonable to introduce more than one maturity to the model. In Faraglia et al (2012) we take up to this task.

7 Conclusions

With currently high levels of government debt there is increasing interest and concern that governments may resort to inflation to achieve fiscal sustainability and that they are more likely to do so when they issue long maturity bonds. We consider the impact of debt maturity on inflation in a model of monopolistic competition and sticky prices. We build on the work of Schmitt-Grohe and Uribe (2004), Siu 2004) and Lustig, Sleet and Yeltekin (2008) by extending the model to consider additional shocks and longer maturities using the computational method of Faraglia, Marcet and Scott (2011). We also extend our analysis to consider the case where governments can lend to the private sector and where they hold bonds until redemption. Fiscal insurance in our model is affected by inflation, interest rate twisting and endogenous variations in bond prices. We identify two channels through which governments wish to use inflation in order to achieve fiscal insurance - a real balance effect and an implicit profit tax. We show how the real balance effect and the implicit profit tax effect depend on the level, sign and maturity of government debt. In the case of short term debt the two effects are in conflict with one another. Issuing long term debt can help overcome this conflict - as noted by Schmitt-Grohe and Uribe (2004). However in the presence of preference shocks the volaitlity of bond prices offsets the advantages of long term debt implying theoretically that governments may be better off issuing short term debt.

Using a model calibrated to US data we however confirm that even with preference shocks the finding of Lustig, Sleet and Yeltekin (2008) that long bonds dominate over short bonds still holds. In fact we show that the result holds for even longer maturities than they consider with significant advantages emerging from issuing 20 rather than 10 period debt. Issuing long term debt enables governments to overcome price stickiness and use inflation more and for longer. The result is that inflation is more persistent and more volatile when governments can issue long term bonds. The precise behaviour of inflation though is very sensitive to the initial level of debt as well as the maturity structure of government debt. We also find that the inflation channel tends to be less important than the interest rate twisting effect.[3] We also find that although longer maturity debt encourages the government to use inflation more there still exists signficiant market incompleteness and the contribution of inflation to debt sustainability is at best modest. Costs of adjustment in prices mean that shifts in taxation and the primary surplus are the dominant way in which debt sustainability is achieved.

The conclusion is that issuing long term debt does enable governments to use inflation more to achieve fiscal sustainability. The longer is the maturity of debt the more volatile and persistence is inflation. However the relative impact on inflation is modest and the relative importance of inflation in achieving fiscal sustainability is modest whatever the length of maturity. A more substantial contribution to debt stabilisation comes from twisting interest rates.

[3]It is interesting in the current environment to note much discussion of "financial repression" (see Reinhert and Rogoff (2011)) as a means of lowering the cost of government debt as well as Quantitative Easing which achieves the same aim.

8 Appendix

8.1 Numerical Procedures

One period uncertainty model. We briefly describe the numerical algorithm that we use to solve the oone period uncertainty model of section 3 and the no buy back model of section 6.2. Our approach is to find a value for $\lambda_{s,1}$ for every contingency g_1 and a value $\lambda_{s,0}$ that satisfies the euler equation (first order condition for bonds in period zero) such that after the shock the model economy converges to a long run steady state.

Step 1. Given initial conditions for $\lambda_{s,t}$ and $\lambda_{p,t}$ for $t = -N, -N+1, ...0$, the initial value of the government real liability b_0^N, and a value of the spending process in period one g_1, we pick an initial value for $\lambda_{s,1}$ and a simulation length T. We solve the system of optimality conditions to determine all of the endogenous variables from period 1 to period T. We assume that in period T the economy converges to a new long run steady state. Notice that since the first order conditions involve both lags and expectations of endogenous variables an inner loop is necessary to guarantee that the values of these variables converge.

Step 2. With the sequence of endogenous variables we construct the present value of the government's surplus in period one for each contingency g_1 as follows:

$$-\sum_1^{\infty} \beta^T (g_t u_c(c_t, \xi_t) \quad - \quad w_t u_c(c_t, \xi_t) h_t - v_h(h_t) h_t)$$
$$+\frac{\beta^{T+1}}{1-\beta}(g_T u_c(c_T, \xi_T) \quad - \quad w_T u_c(c_T, \xi_T) h_T - v_h(h_T) h_T) \tag{8.1}$$

Convergence obtains when the suplus in 8.1 close enough to the initial liability $\beta^{N-1} u_c(c_N, \xi_N) \frac{b_0^N P_0}{P_N}$. Otherwise we need to update the value of $\lambda_{s,1}$. We repeat steps 1 and 2 for every contingency g_1.

Step 3. We compute a new value for $\lambda_{s,0}$. From the first order condition of b_0^N the multiplier in period zero satisfies $\lambda_{s,0} = E_0 \frac{u_c(c_N,\xi_N)\lambda_{s,1}}{\Pi_1^N \pi_j} / \; E_0 \frac{u_c(c_N,\xi_N)}{\Pi_1^N \pi_j}$. To produce the figures in the main text we force the history of multipliers $\lambda_{s,t}$ and $\lambda_{p,t}$ for $t = \{-N, ..., -1\}$ to be equal to $\lambda_{s,0}$ and $\lambda_{p,0}$ respectively. We do this because we want to avoid having the endogenous variables in the model be affected by the initial conditions. [4] With the new updated values of $\lambda_{s,0}$, $\lambda_{p,0}$ and b_0^N we repeat steps 1 to 3. The algorithm converges when successive updates of the date zero endogenous variables are not far apart.

In the no buy back model of section 6.2 instead of one we have to find the values of 10 multiplies $\lambda_{s,1}$ to $\lambda_{s,N}$. These values are such that the following

[4] Otherwise the system of first order condions in the one period uncertainty model would include the terms $\lambda_{s,-1} - \lambda_{s,0}$ and $\lambda_{p,-1} - \lambda_{p,0}$.

constraints hold with equality:

$$\sum_{t \in j, N+j, 2N+j, \ldots}^{\infty} \beta^t (g_t u_c(c_t, \xi_t) \quad - \quad (w_t u_c(c_t, \xi_t) h_t + v_h(h_t) h_t))$$
$$+ \quad \frac{u_c(c_t, \xi_t)}{\Pi_{k=-N+j}^{j} \pi_k} b_{j-N}^N \quad j = 1, \ldots N$$

i.e. such that implementability constraints for periods 1 to N hold. We solve this problem by non-linear least squares. Since $N = 10$ we have to find ten values that minimize the errors of the constraints.

Finally notice that in **Step 3.** the goverment chooses consumption, debt, inflation and wages to maximize household utility. In general the values for these objects can be different from the assumed initial conditions. For example the value of debt b_0^N can differ from b_{-1}^N, if inflation is different from one, though we find that this difference is small. The impulse responses that are plotted in the main text are constructed as the difference between the optimal path after a positive shock to g_1 and the path that sets g_1 equal to its steady state value.

Stochastic Simulations Algorithm. We briefly the numerical procedure that we use to solve for the equilibrium in the model of section 4. Our algorithm is standard Paramterized expectationsH algorithm as in Den Haan and Marcet (1994) . Our approach is to approximate the condtional expectations in the first order conditions 2.14 to 2.13, the government budget constraint and the Phillips curve with polynomials of the states. For example we approximate the terms $E_t \frac{u_c(c_{t+k}, \xi_{t+k})}{\Pi_1^k \pi_{t+j}}$ for $k = 1, \ldots N$, with a functional form $\Phi(X_t, \delta^k)$ where X_t denotes the state vector in period t, and δ^k is the vector of coefficients attached to these polynomials.

The algorithm proceeeds as follows. First we pick an order of the polynomial and initial values for the coefficients δ_0. We use these objects to solve the system of first order conditons of the Ramsey problem. We store the simulated series for consumption, bonds, inflation and the multipliers. With the simulated paths we create the integrands in the conditional expectation terms (for example the term $\frac{u_c(c_{t+k}, \xi_{t+k})}{\Pi_1^k \pi_{t+j}}$). To update the coefficients δ we regress these expressions on the state variables in X_t. This gives us a new set of coefficients δ_1. We iterate on this procedure until we obtain convergence in the coefficients.

We mentioned in the main text that our is a large scale application with many of state variables. To give an idea of the size of X_t note that if $N = 10$ there are 31 state variables in the model. To reduce the number of state variables used in the approximating functions we apply the methodolgy of Faraglia Marcet and Scott (2011) The formal description of the algortihm and an application to an economy with real debt is contained in that paper.

Finally when we introduce bounds in the stochastic simulations we treat them as follows: The conditions $b_t^N \in \{M, \overline{M}\}$ implies that the euler equation of government debt will not be satisfied with equality when the constraint binds.

In order to solve for endogenous variables we replace $b_t = \overline{M}$ or $b_t^N = \underline{M}$ and use the remaining first order conditions along with the Phillips curve the budget constraint and the resource constraint.

Aiyagari, R., Marcet, A., Sargent, T.J. and Seppala, J. (2002) ”Optimal Taxation without State-Contingent Debt” *Journal of Political Economy*, 110, 1220-1254

Aizenman. J. and Marion. N (2009) ”Using Inflation to Erode the US Public Debt” *NBER Working Paper* No 15562

Angeletos, G-M (2002) ”Fiscal policy with non-contingent debt and optimal maturity structure”, *Quarterly Journal of Economics*, 27, 1105-1131

Calvo, G. and Guidotti, P. (1992) ”Optimal Maturity of Nominal Government Debt: An Infinite-Horizon Model,” *International Economic Review*, 33(4), 895-919

Calvo, G. Guidotti, P. and Leiderman, L (1991) ”Optimal maturity of nominal government debt : The first tests,” *Economics Letters*, 35(4), 415-421,

Christiano, L., Eichenbaum, M. and Rebelo, S.L (2011) ”When is the Government Spending Multiplier Large,” *Journal of Political Economy*, 119 (1), 78-121,

Cochrane, J. (1988) ” How Big is the Random Walk in GNP?” *Journal of Political Economy* 96, 893-920.

Den Haan, W. and Marcet, A. (1990) ”Solving the stochastic growth model by parameterizing expectations” *Journal of Business and Economic Statistics*, 8, 31-34.

Eggertsson, G. and Woodford, M. (2003) ”The Zero Bound on Interest Rates and Optimal Monetary Policy” *Brookings Papers on Economic Activity*, 34 (1), 139-235.

Eggertsson, G. and Woodford, M. (2004) ”Policy Options in a Liquidity Trap” *American Economic Review*, 94 (2), 76-79.

Faraglia, E, Marcet, A and Scott,A (2008). "Fiscal Insurance and Debt Management in OECD Economies," Economic Journal, Royal Economic Society, vol. 118(527), pages 363-386, 03

Faraglia, E., Marcet, A. and Scott. A (2011) ”Dealing with Maturity: Optimal Fiscal Policy in the Case of Long Bonds”, *mimeo LBS*

Giannitsarou. C. and Scott. A (2007) ”Inflation Implications of Rising Government Debt” , *NBER Working Paper* No. 12654.

Hall. G. J. and Sargent. T (2010) ”Interest Rate Risk and Other Determinants of Post-WWII U.S. Government Debt/GDP Dynamics”, *NBER Working Paper* No. 15702

Hall, Robert E, 1997. "Macroeconomic Fluctuations and the Allocation of Time," Journal of Labor Economics, University of Chicago Press, vol. 15(1), pages S223-50, January

Holland, A and Scott, A (1998) "The Determinants of UK Business Cycles," Economic Journal, Royal Economic Society, vol. 108(449), pages 1067-92, July

Ireland. P (2004) ”Technology Shocks in the New Keynesian Model ”, *Review of Economics and Statistics* 86, (4), 923-936

Jeanne. O. and Guscina. A (2006) ’Government Debt in Emerging Market Countries: A New Data Set,’ IMF Working Paper No. 06/98

Lustig. H., Christopher Sleet. C., and Yeltekin. S (2008) ’Fiscal Hedging with Nominal Assets’, *Journal of Monetary Economics* 55, (4), 710-727

Lustig. H., Christopher Sleet. C., and Yeltekin. S (2011) ” How does the US Government Finance Fiscal Shocks”, *American Economic Journal: Macroeconomics* 4, (1), 69-104

Marcet, A and Scott. A (2009) ”Debt and Deficit Fluctuations and the Structure of Bond Markets” *Journal of Economic Theory* 144, 473 -501

Marchesi (2004) ”Buybacks of Domestic Debt in Public Debt Management”, University of Sienna mimeo.

Mehl. A. and Reynaud. J (2005), ”The Determinants of Domestic Original Sin in Emerging Market Economies” ECB Working Paper, No. 560

Nosbuch, Y. (2008) ”Interest Costs and the Optimal Maturity Structure of Debt” *Economic Journal*, 118, 477-498

Rabanal, R. and Rubio-Ramirez, J. (2008) ” Comparing New Keynesian Models of the Business Cycle: A Bayesian Approach ”, *Journal of Monetary Economics*, 52, 1151-1166

Reinhart, C and Rogoff, K (2011) "This Time Is Different: Eight Centuries of Financial Folly" Princeton University Press, Princeton, NJ

Schmitt-Grohe. S. and Uribe. M (2004) ”Optimal Fiscal and Monetary Policy Under Sticky Prices”, *Journal of Economic Theory*, 114 198-230

Scott, A. (2007) ”Optimal Taxation and OECD Labor Taxes” *Journal of Monetary Economics*, 54 (3), 925-944

Siu. H (2004) ”Optimal fiscal and monetary policy with sticky prices” *Journal of Monetary Economics*, 51, 575-607