Debt and deficit fluctuations and the structure of bond markets

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Abstract

We analyse the implications of optimal taxation for the stochastic behaviour of debt. We show that when a government pursues an optimal fiscal policy under complete markets, the value of debt has the same or less persistence than other variables in the economy and it *declines* in response to shocks that cause the deficit to increase. By contrast, under incomplete markets debt shows more persistence than other variables and it *increases* in response to shocks that cause a higher deficit. Data for US government debt reveals diametrically opposite results from those of complete markets and is much more supportive of bond market incompleteness.
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1. Introduction

As shown by Aiyagari, Marcet, Sargent and Seppälä [1] the structure of debt is a critical ingredient for determining the properties of optimal taxes. Optimal taxes under incomplete markets (where governments cannot achieve full insurance) display a near-unit root component that is

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absent under complete markets. However, it is far from obvious how to determine whether complete or incomplete markets offer the best paradigm for issues in public finance. Advocates of incomplete markets appeal to alleged moral hazard difficulties as well as the problems of limited commitment and transaction costs that government face. All of these features combine to reduce the range of contingent securities at the government's disposal. However, it is well known that when markets are incomplete some properties of the aggregate real allocations and asset prices are close to the first best.[1] Further, it has been shown that various policy instruments effectively complete the markets even when assets insuring for all contingencies do not exist.[2] These results suggest that it is not possible to discriminate between the relevance of complete or incomplete market models by looking at the range of securities governments can issue.

The aim of this paper is to consider the properties of debt under a variety of optimal tax models and use these results to propose tests for the empirical importance of complete versus incomplete markets. In particular we suggest two specific tests for market incompleteness based on comparing the properties of debt and deficit in the data with the behavior of these variables in the model. The first concerns the relative persistence of debt and deficits. That debt is highly persistent under incomplete markets has been commented on before (the analysis of Barro [3], Aiyagari et al. [1] suggests that debt contains a unit root). The idea of this paper is to test for market incompleteness by studying the *relative* persistence of debt compared to the primary deficit: under incomplete markets debt is much more persistent than deficits. By contrast, we show that deficit and debt tend to have a similar persistence under complete markets.

Our second test for incomplete markets has to do with the co-movement of debt and deficits and it builds on an insight that has, to our knowledge, not been discussed before. We argue that under complete markets, government debt should *fall* in response to shocks that cause the primary deficit to *increase*. This is because under complete markets the optimal debt portfolio held by the government to achieve smooth taxes involves an apparent "over-insurance," in the sense that the optimal portfolio pays much more than the income loss experienced in the period where a bad shock occurs. By contrast, the optimal policy under incomplete markets entails using debt as a buffer stock so that a bad shock brings about both a higher deficit and a higher debt.

Evaluating US post WW II data using these two criteria suggests strongly the importance of incomplete markets: the relative persistence of debt is very high and deft and deficits co-move in the same direction. We also show that other aspects of a simple incomplete markets model fit important aspects of the data.

Justifying and implementing these two tests is the core of our paper. Aside from the importance of the hypothesis being investigated the strength of these tests is their sharpness. Applying classical unit root tests to either debt or taxes is subject to well-known size and power problems, as well as the sensitivity of asymptotic distributions to small changes in the null hypothesis. Furthermore, strictly speaking, a model of complete markets with capital accumulation does have a near-unit root, so existence or not of a unit root is not a good way to discriminate between complete and incomplete markets.

The plan of the paper is as follows. Section 2 examines US post-1950 data and documents a number of facts on the stochastic properties of debt and deficits. Section 3 begins our theoretical analysis of the behaviour of debt when governments pursue an optimal taxation approach. It uses the canonical Lucas and Stokey [21] model and considers the dynamic behaviour of debt under

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[1] See, for example, Telmer [27], Heaton and Lucas [18], Krusell and Smith [20] and Marcet and Singleton [22].

[2] See Angeletos [2], Buera and Nicolini [5] and Chari, Christiano and Kehoe [9].

both complete and incomplete markets when the economy is subject to both productivity and expenditure shocks. Section 4 provides a broader characterization of debt dynamics across a class of standard macro models, and we focus on the case where capital accumulation is introduced. In Section 5 we turn to simulations to both confirm the relevance of our two tests but also to quantify the importance of these effects vis a vis the stylized facts discussed in Section 2. Section 6 considers a number of implications and extensions of our work while a final section concludes.

2. The behaviour of US government debt

The purpose of this section is to establish some stylized facts about debt in preparation for evaluating our competing theoretical models. The main theoretical variable of interest in our analysis will be the market value of outstanding government debt. However, the nearest published equivalent is the face value of outstanding government debt. As detailed in Appendix A we use the work and methods of Seater [25], Cox and Hirschorn [12] and Butkiewicz [6] to construct an annual series for the market value covering the period 1900–1999. However, in what follows we focus on the post-1950 period, although none of our results are seriously affected by using the longer sample. Our focus on the post-1950 data is to exclude the effects of war-inclusion of war time data should requires fitting an exogenous process for government expenditure that adequately capture large outliers. Failure to adequately deal with these outliers is well known to lead to positive biases in estimating persistence and so we avoid this potential complication by focusing on post-1950 data.

To summarize the properties of the data we first consider a minimal VAR which includes the primary deficit, GDP and the market value of government debt. In order to work with data that is stationary, we use the primary deficit/GDP ratio, the change in the logarithm of GDP and the debt/GDP ratio. We do not think of this VAR as a direct way of testing a particular structural model nor as providing a definitive characterization of how fiscal policy impacts on the economy[3] and because it is non-structural we do not wish to imbue the results as having any causal interpretation. Rather, we use the VAR as a convenient way to summarize the data and how debt evolves in response to deficit and GDP shocks. Our identification scheme for the VAR is to perform a Cholesky decomposition and we order the VAR by placing the deficit first, output growth next and finally our market value of government debt variable. By placing debt last we are able to detail how debt responds to variations in the prior listed variables. We sometimes follow the common practice of calling the first orthogonalized shock the 'deficit shock,' the second 'the output shock' and so on. However, we note that this nomenclature may mislead—for instance, in the models we consider the first shock in our estimated VAR (that is, the "deficit shock") contains a large part of the innovation to productivity, since this will influence tax revenues and, therefore, the deficit.[4]

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[3] See Blanchard and Perotti [4] or Burnside, Eichenbaum and Fisher [7] for such an analysis.

[4] The observant reader will note that in the introduction we discussed how government expenditure shocks affect debt while in our VAR we use the primary deficit. We do not use government spending data in the VAR for two reasons. The first is that our analysis covers a variety of shocks i.e. productivity as well as expenditure shocks, and our focus is how the optimal response of primary deficits to these shocks feeds into debt. The second is an econometric issue. Over our sample period there is evidence that government expenditure relative to GDP is non-stationary whereas the primary deficit relative to GDP is stationary. The consequence of using the primary deficit rather than expenditure is that our "deficit shocks" are not primitives of our true theoretical model. However, we overcome these problems by (i) estimating exactly the same VARs using our simulated data as we do with actual US data i.e. using deficits rather than
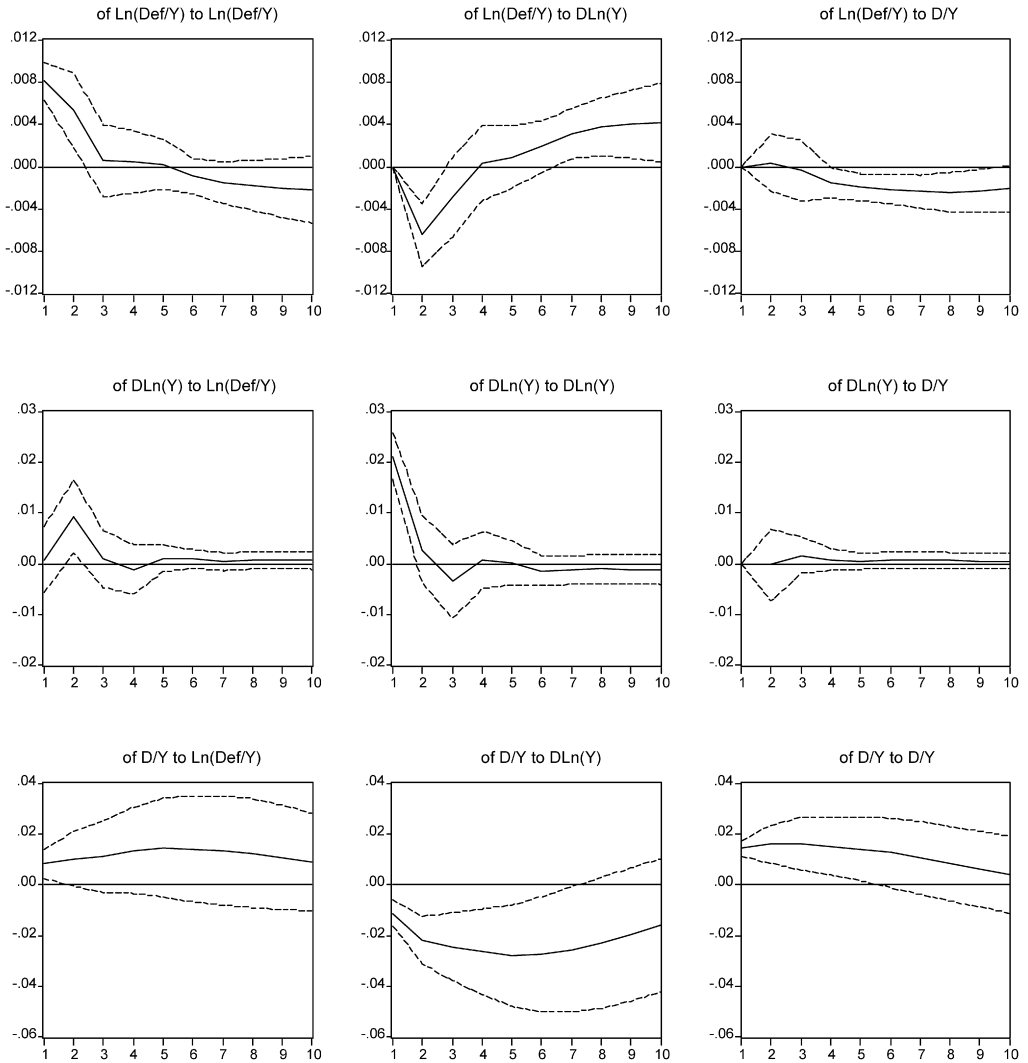
Response to Cholesky One S.D. Innovations ± 2 S.E.

Fig. 1. Estimated impulse response functions on US data 1950-1999.

Fig. 1 shows the estimated impulse response functions from our basic trivariate VAR specification.[5] The stochastic properties of debt and deficit can be summarized as follows:

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expenditure in the model also, and (ii) also for our theoretical models calculating the true impulse response functions of endogenous variables to the raw shocks i.e. expenditure shocks, so that the reader can easily map the relationship between an (exogenous) innovation to government spending and an (endogenous) innovation to deficit.

[5] Results are robust to a wide range of different VAR specifications and estimators.

Fact 1. **Relative Persistence.** Both the innovations to the deficit and GDP equations are followed by large and very persistent swings in the market value of debt. The swings in debt are far more persistent than those in any other variable of interest.

Fact 2. **Co-movement of Deficit and Debt.** Positive innovations to the primary deficit are followed by an *increase* in the market value of debt.

Fact 3. A higher deficit today signals a higher deficit over the next five or six years, that is, deficits show persistent fluctuations.

Fact 4. Positive innovations to GDP are followed by reductions in future deficits and debt.[6]

As we shall see later, Facts 1 and 2 form the core of our test for discriminating between complete and incomplete markets. Fact 3 will also be important in order to provide a relevant calibration of the exogenous processes. Fact 4 will also be brought out by the incomplete markets model.

Studying impulse response functions provides a highly detailed picture of interactions between endogenous variables but is sensitive to the ordering of variables and the dimension of the system. Therefore as an additional more robust measure of persistence we utilize the k-variance ratio defined as[7]

$$P_y^k = \frac{Var(y_t - y_{t-k})}{k Var(y_t - y_{t-1})}.$$

In the case of a stationary and ergodic variable we have $Var(y_t - y_{t-k}) \to 2\,var(y_t)$ and, therefore, $P_y^k \to 0$ as $k \to \infty$. For instance, in the case of an i.i.d. process $P_y^k = 1/k$. By contrast in the case of a pure unit root $P_y^k = 1$ for all k. Roughly speaking, the more persistent is variable y, the longer it takes for P_y^k to go to zero as k grows.

Estimates of P_y^k for a selection of key variables are shown in Fig. 2. Since the k-variance ratios for debt are far higher than those for other variables, our earlier finding that debt shows greater persistence than other variables is repeated. The persistence of debt is marked even after 10 years and is still rising. Because $P_y^k = 1 + \sum_{j=1}^{k-1} \frac{k-j}{k} \rho_j$ where ρ_j is the correlation of Δy_t with Δy_{t-j} the rising profile of P_{Debt}^k in Fig. 2 is consistent with a unit root for debt and with positively serially correlated deficits, providing further insight into the co-movement between deficits and debt.

Having documented empirically the dynamic behaviour of debt we now turn out attention to characterize theoretically its behaviour under complete and incomplete markets.

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[6] Notice that innovations to the debt ratio are significantly different from zero for the first few lags of the Debt/GDP equation and persistent. It is not clear why this occurs, in our model the government budget constraint states an identity between debt, deficits and interest rates. We have tested if this shock captures interest rate effects, but when we include interest rates in the VAR the size and effect of these "debt shocks" remains essentially unchanged. Various possible explanations for the existence of these "shocks" are: (i) measurement error—over a long sample there are many changes of definition of debt, revaluation effects and asset sales which will the cause the identity not to hold exactly, (ii) approximation error—a linear VAR may not fully capture the non-linear nature of the government's budget constraint and of the Debt/output ratio, (iii) identification problems; it may be that the current identification leaves some of the innovations in the other variables to appear as "debt shocks."

[7] Cochrane [11] uses this statistic in the macroeconomics literature to measure persistence in US GDP data.
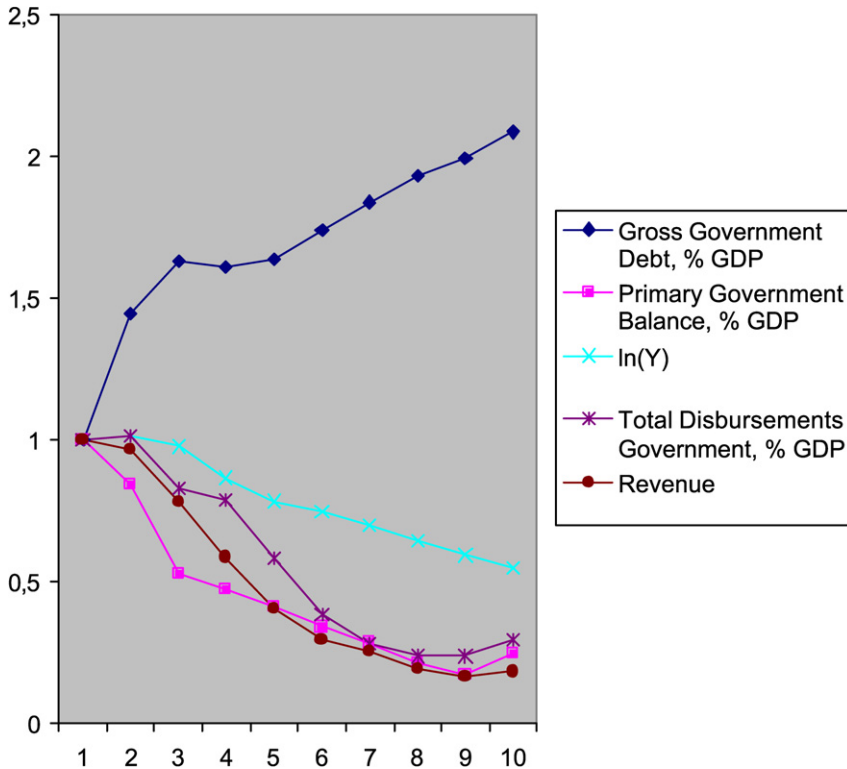
Fig. 2. k-variance statistic for US data 1950–1999.

3. A model without capital

In this section we offer a characterization of debt dynamics in a canonical model of optimal taxation. The complete markets version of the model is very close to Lucas and Stokey [21], the incomplete markets version is very close to Aiyagari et al. [1]. We show how the relative persistence of debt and the co-movement between deficit and debt is very different under complete and incomplete markets and that the latter fits the data much better. Many of the results will be derived analytically, and where analytic results cannot be found we provide a simple intuition.

We augment the model of Lucas and Stokey [21] to include a productivity shock θ_t so that output is given by $y_t = \theta_t(1 - l_t)$ where l_t denotes leisure.[8] The resource constraint is $y_t = c_t + g_t$ where c denotes private consumption and g government expenditure. Using the notation $s_t \equiv (g_t, \theta_t)$, the stochastic process $\{s_t\}$ is assumed exogenous and Markov. The representative consumer has utility function $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$. A representative firm maximizes profits. Consumers and firms are competitive, they take prices and taxes as given, hence wage is equal to θ_t. The consumer pays a proportional tax rate τ_t on labor income. Consumer, firms and government observe all shocks up to the current period.

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[8] Angeletos [2], Buera and Nicolini [5], Chari, Christiano and Kehoe [9], Farhi [16], Gorostiaga [17], Scott [24], Zhu [28] and a few others also study optimal fiscal policy in models with productivity shocks.

We look for a Ramsey equilibrium, where government chooses taxes and debt in order to maximize the consumer's utility, government satisfies its budget constraint, and it knows the mapping between taxes and competitive equilibrium allocations. The government has full commitment to implement the best sequence of (possibly time inconsistent) taxes and government debt.

3.1. Complete markets

Under complete markets (the case studied by Lucas and Stokey) the government faces the budget constraint

$$b_{t-1}^g(s_t) = g_t - \tau_t \theta_t (1 - l_t) + \int b_t^g(\bar{s}) p_t^b(\bar{s}) \, d\bar{s} \tag{1}$$

where $\bar{s}$ is a possible realization of s_t and $b_t^g(\bar{s})$ denotes the amount of one-period real bonds held by the government paying one unit of consumption in the next period in the event that $s_{t+1} = \bar{s}$. For all other realizations the bond will pay nothing. The price of this bond is $p_t^b(\bar{s})$. As usual, $u_{c,t}$ denotes the marginal utility of consumption in period t.

It is well known that all equilibrium conditions can be summarized by one implementability condition holding at time $t = 0$.[9] Using the notation for the primary deficit $\omega_t^g \equiv g_t - \tau_t \theta_t (1 - l_t)$, results in Lucas and Stokey insure that the optimal allocation satisfies

$$\left(\tau_t, c_t, l_t, y_t, \omega_t^g \right) = G(s_t) \tag{2}$$

for all $t > 0$, and a time-invariant function G. That is, all variables at time t depend only on the shocks of the current period.

The value of the bond portfolio held by the government in period t is

$$vb_t^g \equiv \int b_t^g(\bar{s}) p_t^b(\bar{s}) \, d\bar{s}.$$

Correspondingly, $-vb_t^g$ is the value of government debt.

From (1) it can be seen that there is a potential ambiguity about whether it is $-vb_t^g$ or $-b_{t-1}^g(s_t)$ that should be taken as the quantity in the model representing the value of debt.

3.1.1. Persistence

The budget constraint (1) shows the payoff of past period's bond portfolio $b_{t-1}^g(s_t)$ in the left side of the equation, suggesting that the current value of debt depends on past outstanding debt. Therefore, a quick look at this equation would suggest that today's value of debt depends on past shocks and that debt is likely to be more persistence than other variables in the economy. For example, it would seem that if s_t is i.i.d. the value of debt is likely to be serially correlated. What follows is an argument that under complete markets this intuition is misleading.

It can be shown that,

$$vb_t^g = E_t \sum_{j=1}^{\infty} \beta^j \frac{u_{c,t+j}}{u_{c,t}} \omega_{t+j}^g \tag{3}$$

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[9] More precisely, a feasible allocation for c, l in this model is a competitive equilibrium if and only if the implementability condition $E_0 \sum_{t=0}^{\infty} \beta^t \frac{u_{c,t}}{u_{c,0}} (g_t - \tau_t \theta_t (1 - l_t)) = b_{-1}^g$ holds.

for all t and all realizations, and that this equation combined with (2) gives[10]

$$vb_t^g = \mathrm{V}^g(s_t) \tag{4}$$

for all $t > 0$, and a time-invariant function V^g. This says that there is no direct impact of past shocks and past debt on today's level of debt. This shows that debt is likely to have the *same* persistence as output and deficit, more precisely, if g_t and θ_t each have the same serial correlation then, up to a linear approximation to the functions V^g and G, output, debt and deficit all have the same serial correlation as s_t. This provides a strict proof that, up to linear approximations, complete markets is at odds with Fact 1.[11]

The intuition for this result is the following. Tax smoothing means that, to a large extent, the government wants to keep similar level of taxes over time. So, the optimal portfolio insulates government's discounted wealth from shocks, government wealth is constant (on average) over time and in this way a given average level of taxes can be maintained forever.

3.1.2. Co-movement of deficit and debt

It is well known that under complete markets and a Ramsey planner labor taxes are essentially constant.[12] Consider the case where g_t is positively serially correlated and consider a realization where g_t is unexpectedly high. Constant taxes and high g mean that the deficit ω_t^g is likely to increase, and in the case that g is persistent this means all future deficits are likely to be higher. Therefore, vb_t^g is likely to increase, since according to (3) it is roughly an expectation of future deficits. Therefore, debt (equal to $-vb_t^g$) is likely to fall in response to persistent and adverse expenditure shocks while deficits go up. Hence, in general we would expect the co-movement of debt and deficit to be negative under complete markets. The reason for this odd behavior of debt is that in a reasonable calibration of the model the optimal choice for debt government will apparently "over-insure" bad shocks.

To be precise let us specialize the setting. Let G^c, G^ω denote the coordinates of the G function in (2) corresponding to consumption and deficit, let $G^{c\prime}$, $G^{\omega\prime}$ be the partial derivatives with respect to g, assume

(Ai) θ is constant and $g_t = a + \rho g_{t-1} + \varepsilon_t^g$ for ε_t^g i.i.d. mean zero, and $\rho > 0$.

(Aii) G^c, G^ω are differentiable, and their derivatives satisfy $G^{\prime c} < 0$ and $G^{\prime \omega} > 0$. Also assume u is separable in consumption and leisure.

The first assumption is for simplicity and because $\rho > 0$ is required for the model is consistent with Fact 3. The second assumption requires that optimal policy involves increasing the deficit and lowering consumption when government spending is higher; it is an assumption on the equilibrium laws of motion that holds in most applications found in the literature in the case of complete markets.[13]

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[10] These two results are established formally in part (B) of Proposition 1 in Section 4.

[11] Simulations in Section 6 will confirm that even taking into account the non-linearity of V^g and G the complete market model does not agree with Fact 1.

[12] This is discussed in detail, for example, in Zhu [28] and Scott [24]. Also, see Chari and Kehoe [10] for a review.

[13] One can find utility functions that violate (Aii). For example, if consumption is an inferior good consumption may go up with higher g. To our knowledge nobody has argued that this may be an important element in the analysis of fiscal policy.

Ignoring interest rate effects (that is, assuming $u_{c,t} = 1$) the formula[14] for $\frac{\partial vb_t^g}{\partial \varepsilon_t^g}$ shows unambiguously that $\frac{\partial vb_t^g}{\partial \varepsilon_t^g} > 0$. Therefore, in this case the value of debt goes down when ε_t^g goes up. Since deficit increases at the same time, this shows that in this case the co-movement of debt and deficit is negative. Furthermore, for general u in order for $\frac{\partial vb_t^g}{\partial \varepsilon_t^g}$ to be negative three elements are needed: a high elasticity of interest rates to g, a positive amount of government savings, *and* low serial correlation of g. Since the data does not seem to show any of these properties, we would claim a reasonable calibration of the model will give that current debt should go down in response to a high ε_t^g.

The intuition for this odd behavior of debt is that in an economy where persistent shocks cause the deficit to go up the optimal insurance policy appears to provide "over-insurance." The optimal portfolio is such that when an adverse shock occurs, the bond portfolio pays out more than today's high deficit, in anticipation of the following high deficits that are likely to come. Under complete markets contingent debt provides insurance equal to the net present value of the shock, in the case of persistent shocks this is more than the current value and so the market value of debt actually declines.

3.2. Incomplete markets

We now assume a particular form of market incompleteness. Agents can only issue one-period debt which pays a risk free rate of return. In this case the government budget constraint is[15]

$$b_{t-1}^g = g_t - \tau_t \theta_t (1 - l_t) + b_t^g p_t^b \tag{5}$$

where b_t^g denotes the number of bonds the government holds at time t, each bond pays one unit of consumption good next period with certainty. The price of each bond is p_t^b. Analogously to the complete markets, the value of government debt is $-vb_t^g \equiv -p_t^b b_t^g$.

In this case the payoff of the bond is the same across realizations and it is impossible to construct a portfolio that allows the government to smooth taxes in the same way as in the complete markets model. This introduces additional constraints in the allocations that are available to the government, in particular, now it is not possible to summarize all equilibrium constraints in one period zero budget constraint, and the budget constraints in each period matter. The optimal allocation satisfies

$$(\tau_t, c_t, l_t, y_t, \lambda_t) = G_{IM}\big(s_t, \lambda_{t-1}, b_{t-1}^g\big) \tag{6}$$

where λ_t is the Lagrange multiplier associated with the implementability constraint in period t. Relative to complete markets, the policy function includes two new state variables λ_{t-1}, b_{t-1}^g. The multiplier λ_t contains a near-martingale component.[16]

3.2.1. Persistence

It is the presence of these new state variables that breaks the argument in Section 3.1.1, in which we proved that debt, deficit and output have the same persistence under complete markets. That result followed directly from (4), and this equation was obtained by combining (3) with (2).

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[14] See Appendix B in the fuller version of our paper available from our websites.

[15] This assumption has been used in models of optimal policy by Aiyagari et al. [1], Scott [24], Farhi [16].

[16] See Aiyagari et al. for a detailed discussion of these facts.

Under incomplete markets, it turns out that an equation such as (3) is still valid,[17] but the presence of the new state variables $(\lambda_{t-1}, b^g_{t-1})$ means that now we can only infer from (3) that $vb^g_t = V^g(s_t; \lambda_{t-1}, b^g_{t-1})$. This simply confirms that, under incomplete markets, current debt depends on past debt just as it would be inferred from a quick look at the budget constraint. Furthermore, the new and highly persistent martingale process $\{\lambda_t\}$ could influence differently the dynamics of debt compared to other variables leading to the possibility that debt shows greater persistence.

This shows the argument of equal persistence for debt and other variables breaks down but it does not yet prove that debt has a larger relative persistence under incomplete markets. To understand why this may be the case consider the following general intuition.

Under complete markets, a high g_t implied a high ω^g_t. The government could sustain this policy by constructing a portfolio that insured these shocks. For example, in the case that g is i.i.d. the value of debt vb^g_t is roughly constant, and it is the payoff of last period's debt $b^g_{t-1}(g_t)$ that makes the adjustment. By contrast, under incomplete markets the payoff b^g_{t-1} is predetermined and it cannot adjust. Therefore, for the government to sustain a higher ω^g_t the value of debt vb^g_t has to increase. Once increased it stays high for several periods and so shows greater persistence than other variables.

This is just a special case of a point that has been made in the incomplete markets literature: agents who cannot fully insure are likely to use debt as a buffer stock to smooth fluctuations, increasing debt in bad times and reducing it in good times. The government in our model is such an agent: it would like to smooth taxes but since it does not have access to contingent debt it cannot construct a portfolio whose payoff insulates its wealth from shocks to the economy. Using debt as a buffer stock means that the effect of today's bad shock is spread over time, since the higher debt will cause future taxes to be higher, in order to service higher future interest payments. The fact that the higher debt is not repaid immediately leads to additional persistence of debt compared to the behaviour of deficits and output.

Whilst the intuition behind this argument is strong it is not possible at this level of generality to prove that the relative persistence of debt is high under incomplete markets. In Section 5 we will establish this claim in a commonly used calibrated model via simulation. Now we prove analytically this result in a special case.

An analytic example for persistence   Consider the above model. Assume productivity is constant so that $y_t = 1 - l_t$. Assume $\{g_t\}$ is stochastic only at time $t = 1$, in particular, assume $P(g_1 = g^H) = P(g_1 = g^L) = 0.5$ for two values $g^H > g^L > 0$, but g is constant in all other periods: $g_0 = g_t = \bar{g} \equiv \frac{g^H + g^L}{2}$ for all $t \geqslant 2$. Utility is given by $u(c, l) = c + H(l)$ and further assume that initial debt is zero: $b^g_{-1} = 0$.

Denote with superscripts H and L the values of all variables in all periods under each realization of g_1. The following result shows equilibrium sequences under complete or incomplete markets.

Result
Under regularity assumptions (*stated in Appendix B*)

- *Under* **complete markets:**
  Deficit is influenced by the shock only in period $t = 1$:

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[17] More precisely, that equation holds with b^g_{t-1} in the left side. This is shown in Proposition 1 of Aiyagari et al.

$$\omega_0^g = \omega_2^g = \omega_3^g = \cdots = 0 \quad \text{both realizations}$$
$$\omega_1^{g,H} > 0$$

Debt does not respond to a deficit shock[18]:

$$vb_t^g = 0 \quad \text{for all } t = 0, 1, \ldots \quad \text{both realizations.}$$

- *Under **incomplete markets:***
  Deficit is higher at $t = 1$ and lower for all $t > 1$ after a high shock:

$$\omega_0^g < \omega_1^{g,H} \quad and \quad \omega_0^g > \omega_2^{g,H} = \omega_3^{g,H} = \cdots \tag{7}$$

  Debt increases permanently after a high shock

$$vb_0 > vb_1^H = vb_2^H = \cdots \tag{8}$$

  All inequalities reversed (and all equalities hold) in the event $g_1 = g^L$.

See proof in Appendix B.

Therefore in this stylized example debt under incomplete markets shows a permanent response to a temporary fiscal shock. The deficit increases as a result of a one time shock, then it decreases in all future periods but debt is permanently higher forever, thus displaying a persistence greater than any other variable.

3.2.2. Debt/deficit co-movement

In Section 3.2.1 we gave an intuitive argument about why optimal policy under incomplete markets is likely to use debt as a buffer stock. This implies that and adverse shock (high g or low θ) leads to high deficit and higher debt. Therefore, debt and deficit co-move positively under incomplete markets, a result we confirm by simulations in Section 5.

It is instructive to reconsider the argument we gave for the negative co-movement under complete markets and explore where it breaks down under incomplete markets. A key element in the argument of Section 3.1.2 was that a high g caused *all* future deficits to increase. Under incomplete markets a bad shock increases today's deficit but it decreases deficits sufficiently far in the future in order to service the additional interest.[19] These lower future primary deficits are the reason why the discounted sum in (3) goes down under incomplete markets.

4. What generalizes and what does not?

In the special model of Section 3 we could analytically establish many of our results. In this section we discuss how some of these results might generalize to a broader class of models.

The buffer stock behavior of debt, described intuitively in Section 3.2.1, is a very general feature of incomplete market models. The tax-smoothing effect of Barro [3] and Aiyagari et al. [1] imparts a unit root component to taxes, this has been shown to extend to models with

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[18] Since the deficit is not serially correlated, this example does not display the co-movement of deficit and debt under complete markets that was discussed in Section 3.1.2.

[19] To be precise, if g is highly serially correlated, it is only in the long run that future deficits are lower. The serial correlation of g may induce a high deficit in the near future. See impulse responses in Section 5 for incomplete markets serially correlated shocks.

capital by Farhi [16] and Scott [24], and it implies that debt is optimally used as a buffer stock. In general, this will imply high persistence of debt and positive co-movement of debt and deficit. Simulations of Section 5 confirm that this is the case for a model with capital accumulation.

As for the generalization of the complete markets results, we show a proposition for a general setup that is useful in arguing that debt is equally persistent as output under complete markets in a large class of models. Of independent interest is that this proposition was already used to derive (4) in Section 3, it is also useful in solving for debt under complete markets for the capital accumulation model of Section 5, and it has been used in Faraglia, Marcet and Scott [15] to solve for the portfolio of maturities in a diversity of complete market models.

Let us generalize the setup. Consider an economy consisting of exogenous shocks (s_t) and endogenous variables (x_t). The process $\{s_t\}$ is Markov and, for simplicity, assume that the distribution of the exogenous shocks conditional on the past has a density. Without loss of generality let us partition s_t into two subvectors $s_t = (s_t^1, s_t^2)$ with s_t^2 including those shocks known one period ahead and s_t^1 containing the rest.[20] There are I agents and each agent at time t chooses consumption and obtains net income from several sources, ω_t^i denotes the value of agent i's deficit (i.e. expenditure minus income) in period t in units of the numéraire consumption good.

Under complete markets there exists a spot market for claims contingent on all possible values of s_{t+1}^1. A bond contingent on a value $\bar{s}^1 \in S^1$ will pay one unit of the consumption numéraire if $s_{t+1}^1 = \bar{s}^1$ occurs, and zero otherwise. Here $S^1 \equiv \bigcup_{t=1}^{\infty} \{\text{support of } s_t^1\}$. $b_t^i(\bar{s}^1)$ denotes the quantity of bonds purchased by agent i at time t contingent on the occurrence of $\bar{s}^1$. Hence, agent i has to choose a function $b_t^i : S^1 \to R$ in each period, this function may depend on the period and the realization. The payoff of this portfolio next period is $b_t^i(s_{t+1}^1)$ and the budget constraint of each agent $i = 1, \ldots, I$ satisfies

$$b_{t-1}^i(s_t^1) = \omega_t^i + \int_{S^1} b_t^i(\bar{s}^1) p_t^b(\bar{s}^1) \, d\bar{s}^1 \tag{9}$$

for all agents, periods and realizations. Here $p_t^b(\bar{s}^1)$ is the price at time t of a bond contingent on $s_{t+1}^1 = \bar{s}^1$. All agents are prevented from defaulting and from running Ponzi schemes. The market value of the bond portfolio held by agent i is

$$vb_t^i \equiv \int_{S^1} b_t^i(\bar{s}^1) p_t^b(\bar{s}^1) \, d\bar{s}^1. \tag{10}$$

Obviously, the market value of debt held by agent i is $-vb_t^i$.

Assume that in equilibrium deficits and interest rates can be formulated recursively in the sense that they are given by a time-invariant function of some state variables z_t. Formally

$$z_t = h(z_{t-1}, s_t), \tag{11}$$

$$\omega_t^i = f^i(z_{t-1}, s_t), \tag{12}$$

$$p_t^b(\bar{s}^1) = p(\bar{s}^1, z_{t-1}, s_t) \mu(\bar{s}^1, s_t) \tag{13}$$

for all $i, t, \bar{s}^1$, where μ is the density of s_{t+1}^1 given s_t.

A large variety of models satisfy these assumptions, including models with multiple agents, public goods, distorting taxation, time-non-separable utility function, externalities, monopolistic power, market frictions, non-rational expectations or credit constraints. Most models in the

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[20] More precisely, s_t^2 contains the elements of s_t that are measurable with respect to s_{t-1} and s_t^1 those that are not.

literature guarantee that a version of (13) holds with p being the intertemporal marginal rate of substitution of consumption for some agent.

Proposition 1. *Under the above assumptions and given functions* h, f, p

(A) *the equilibrium portfolio of bonds for agent i that satisfies the budget constraint of agent i satisfies*

$$b_t^i(\cdot) = D^i\left(\cdot, s_{t+1}^2, z_t\right) \quad a.s. \tag{14}$$

for all t, for the time invariant function D^i

$$D^i(s_t, z_{t-1}) \equiv \omega_t^i + E_t \sum_{j=1}^{\infty} \omega_{t+j}^i \prod_{\tau=0}^{j-1} p\left(s_{t+\tau+1}^1, z_{t+\tau-1}, s_{t+\tau}\right). \tag{15}$$

(B) *For each agent i, the value of the equilibrium portfolio of contingent bonds vb_t^i is given by a (time-invariant) function V^i such that*

$$vb_t^i = V^i(z_t, s_t) \equiv E_t\left[D^i\left(z_t, s_{t+1}^1, s_{t+1}^2\right) p\left(s_{t+1}^1, z_t, s_t\right)\right] \quad a.s.$$

for all t.

Proof. See Appendix B. □

   This proposition is a very general result that simply relates b_t^i and vb_t^i to current values of stochastic shocks and the state variables under complete markets. The first part of this proposition states that the portfolio of bonds issued at time t is independent of the realization of s_t^1—in other words, the structure of bonds issued in each period does not respond to current period unexpected shocks. It does however depend on shocks predictable one period ahead and on the state variables z_t. The second part of the proposition says the market value of government debt $(-vb_t)$ responds to *both* current period unexpected shocks *and* the state variables. The dependence of the market value on current unexpected shocks arises from their impact on bond prices.

   It has been shown that, under complete markets, equilibrium variables such as deficit and interest rates often do not depend on past debt.[21] This is because for many of these models all budget constraints can be substituted by one implementability constraint such as the one in footnote 10. Combining this knowledge with the above proposition allows us to conclude that in many complete markets models debt is likely to be independent of past debt; instead, debt is likely to be a function of real variables and, therefore, debt will have no more persistence than real variables. For example, in the model of Section 3, since we know from Eq. (2) that there are no state variables the above proposition implies (4), which allowed us to conclude that debt and deficit had the same persistence in Section 3.1.1.

   It is important to emphasize that Proposition 1(B) per se does not allow to discriminate between complete and incomplete markets. Since Eq. (3) also holds in some incomplete markets models, strictly speaking, part (B) above would also hold in those cases as well. But in an incomplete markets model the state vector z_t will typically include past debt and/or a Lagrange

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[21] See Chari and Kehoe [10] for a review.

multiplier, so that part (B) under incomplete markets only confirms that current debt is a function of past debt and it opens the door for debt to be more persistent than real variables.

Using an argument analogous to the one we used in Section 3.1.1, the proposition implies that any extension of the model in Section 3.1 (with any number of agents, any number of shocks, any public good or externality, any preferences of the government, . . .) where optimal deficit and interest rates are a function only of *exogenous* state variables (that is, z_t is empty) and where all the elements of s_t share the same serial correlation, debt and deficit will have the same persistence and the model will violate Fact 1.

In general, though, the complete markets solution will include some real variables in z_t (say, past capital or past consumption). In these cases the proposition shows that debt will not have more persistence than the most persistent real variable. This suggests that debt and deficit are likely to have similar persistence but, unfortunately, it falls short of proving this point. To see this let us extend the model to allow for capital accumulation so that the resource constraint is given by[22]

$$ c_t + k_t - (1-\delta)k_{t-1} + g_t = \theta_t k_{t-1}^\alpha (1-l_t)^{1-\alpha} = y_t $$

where δ is the depreciation rate of capital. For simplicity, we assume only labor income is taxed at a rate τ_t.[23]

For the case of complete markets this model fits in the general framework by having $x_t = (\tau_t, l_t, k_t, c_t)$, exogenous shocks $s_t = s_t^1 = (g_t, \theta_t)$ and endogenous state variable $z_t = k_t$. The solution is a time-invariant function of (g_t, θ_t, k_{t-1}) for $t > 0$. Therefore, part (A) of our proposition says

$$ b_t^g(\bar{g}, \bar{\theta}) = D^g(\bar{g}, \bar{\theta}, k_t) $$

so that the portfolio of debt depends only on today's value of the capital stock. According to part (B) the market value of bonds is given by

$$ vb_t^g = V_{CM}^g(g_t, \theta_t, k_t) \tag{16} $$

for a time invariant function V_{CM}^g. Again, past shocks, and the influence they possibly had on government debt, bear no influence on today's equilibrium value of government debt, over and above the effect that past shocks have on current capital, and so once more vb_t^g does not depend on past debt.

This proves that debt cannot be more persistent than capital, but it falls short of proving that debt is equally persistent as deficit. Debt could be more persistent than deficit if k (a very persistent variable) influences debt more strongly than output. This would seem a priori unlikely, but it can only be resolved in this specific model by simulation, as we do in the next section.

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[22] This model under incomplete markets has also been analyzed by Farhi [16] and Scott [24].

[23] Relative to papers studying optimal capital taxes we simplify by setting capital taxes to zero for all time periods and all states of nature. Allowing for stochastic capital taxes would potentially enable the complete market outcome to be achieved even in the absence of state contingent debt, so the incomplete markets model we analyze later would not be different from the complete markets one. Farhi [16] introduces capital taxation and assumes that governments have to commit one period ahead to the tax rate. This prevents the complete market outcome from being achieved and allows for incomplete markets and capital taxes.

5. Simulations

In this section we resort to simulations to further justify the validity of these tests. The simulations will show the quantitative effects we have discussed, and they will resolve a number of issues that could not be dealt with analytically in a model with capital. We do not set out to match US data as closely as possible, rather our interest is in establishing differences between complete and incomplete market outcomes. As such we choose canonical parameter values and functional forms rather than seek to estimate key parameters in order to mimic US data. However, it should be stressed that our main qualitative results are robust to alternative parameterization.

5.1. Model 1—No capital accumulation

In the model of Section 3 we assume the utility function

$$u(c_t, l_t) = \frac{c_t^{1-\gamma_1}}{1-\gamma_1} + B\frac{l_t^{1-\gamma_2}}{1-\gamma_2}$$

and set $\beta = 0.98$ and $\gamma_1 = 1$, $\gamma_2 = 2$. We set B so that the share of leisure in the time endowment equals 30% on average.

In keeping with the literature, we assume g follows a truncated AR(1), and θ_t a log AR(1) process

$$g_t = \begin{cases} \bar{g} & \text{if } (1-\rho^g)g^* + \rho^g g_{t-1} + \varepsilon_t^g > \bar{g}, \\ \underline{g} & \text{if } (1-\rho^g)g^* + \rho^g g_{t-1} + \varepsilon_t^g < \underline{g}, \\ (1-\rho^g)g^* + \rho^g g_{t-1} + \varepsilon_t^g & \text{otherwise}, \end{cases}$$
$$\log \theta_t = \rho^\theta \log \theta_{t-1} + \varepsilon_t^\theta$$

for $\varepsilon_t^g, \varepsilon_t^\theta$ i.i.d., mean zero and mutually independent. We assume $\varepsilon_t^\theta \sim N(0, 0.007^2)$, $\varepsilon_t^g \sim N(0, 1.44^2)$, $g^* = 17.5$, with $\bar{g} = 35\%$ and $\underline{g} = 15\%$ of average GDP. These values are chosen so that in the non-stochastic steady state of Model 1, government expenditure amounts to 25 percent of GDP and fluctuates within the range 15 and 35% of output. We consider two different assumptions regarding the persistence of the shocks: (a) both sequences are i.i.d. ($\rho^g = \rho^\theta = 0$) and, (b) strongly positively serially correlated shocks $\rho^g = \rho^\theta = 0.95$.[24] Assuming such high levels of persistence means in this respect our model matches US data closely.

We use both impulse-response functions (IRF) and the k-variance ratio to measure persistence. In using IRFs we start by studying the response of each variable to the fundamental shocks in the model economy (g_t, θ_t). We call this the "true IRF." We obtain the ith coefficient of the true IRF by computing numerically how each variable would change i periods ahead if, starting from the steady state mean, the innovation to each shock had a realization equal to $(\sigma_{\varepsilon^j}, 0, 0, \ldots)$, $j = g, \theta$. When the true IRF coefficients at low frequencies are large relative to the coefficients at high frequencies we will say that the variable has high persistence.

The *dashed lines* of Figs. 3 to 4 show the true IRFs and Fig. 6 shows the k-variance ratio for various models under both complete and incomplete markets. Fig. 3 shows Model 1 under i.i.d.

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[24] All models are solved using the Parameterised Expectations Algorithm described in den Haan and Marcet [13].
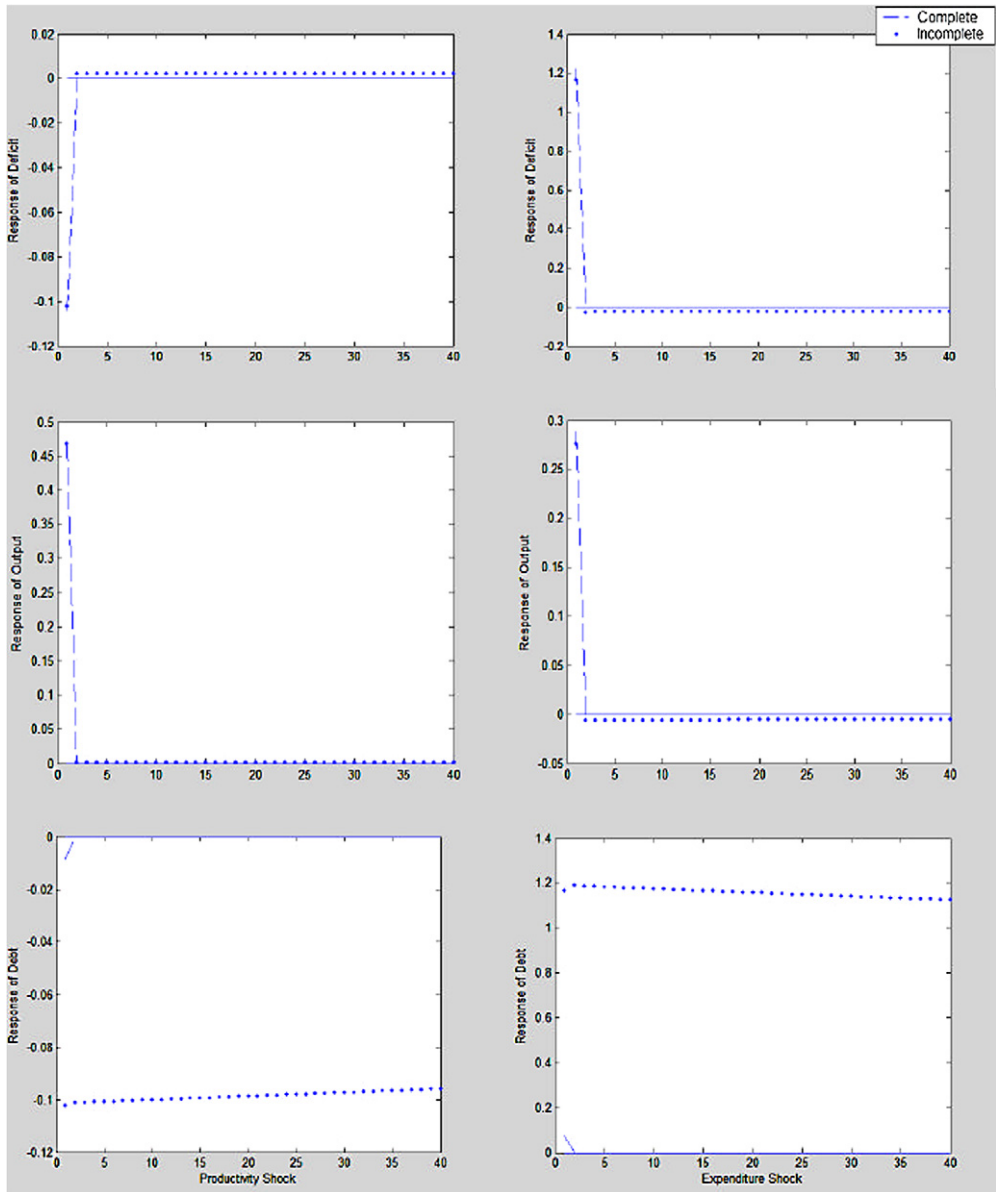
Fig. 3. True IRF, Model 1 (i.i.d. shocks).

shocks.[25] Not surprisingly, given our discussion in Section 3.1.1, all variables including debt are i.i.d., all have zero persistence. The first panel in the first column of Fig. 6 shows the k-variance ratio for the same variables and shows an identical degree of persistence across them all. By contrast, in the incomplete market case (shown by the dotted lines) the response of debt to both

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[25] The vertical axis in Figs. 3 to 5 are in units of the variable under consideration.
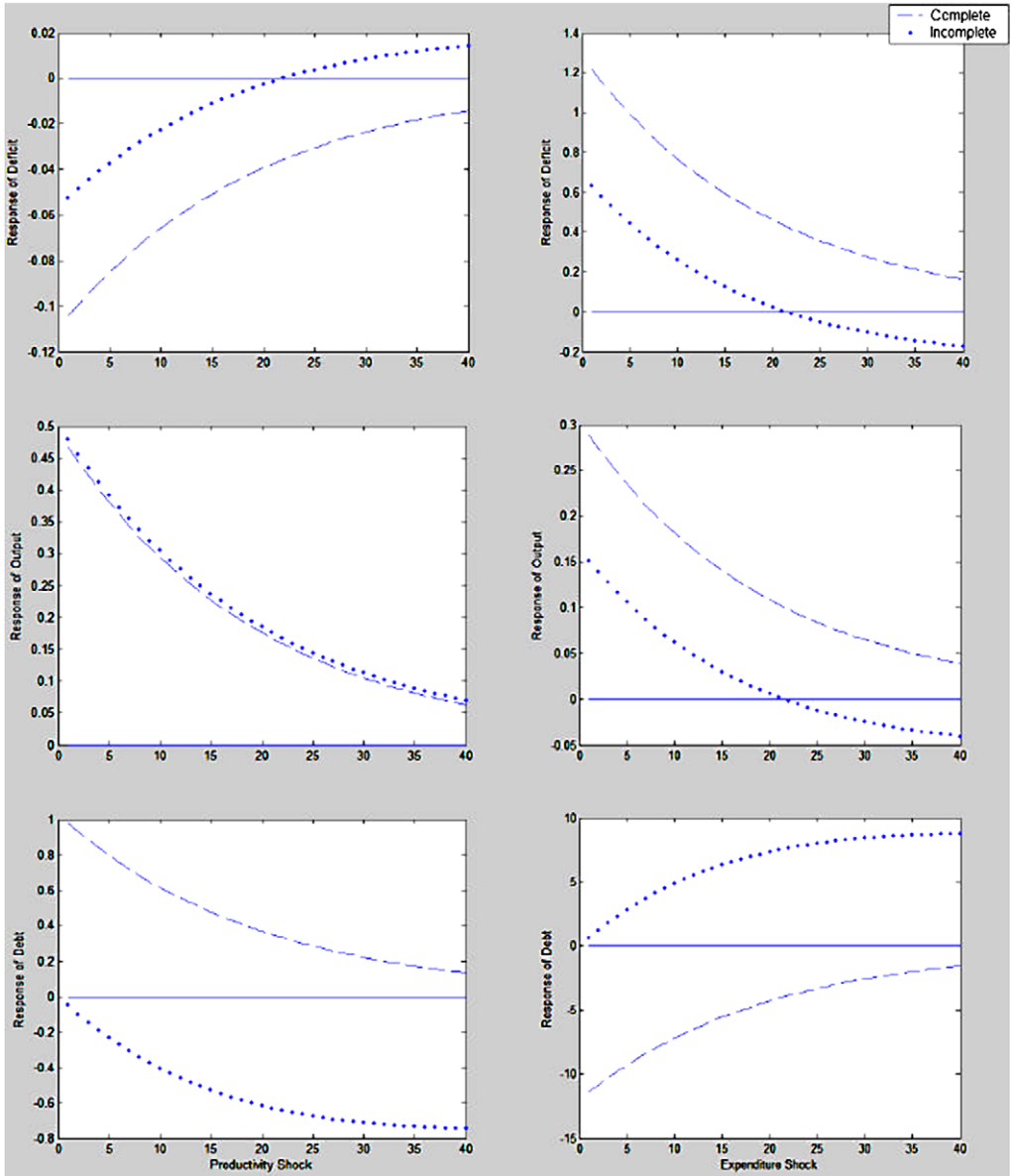
Fig. 4. True IRF, Model 1 (persistent shocks).

shocks is highly persistent, confirming that debt is used as a buffer stock to smooth fluctuations. This is confirmed by the k-variance ratio in Fig. 6 (top right panel) while most of the response of the other variables is on impact, consistent with Fact 1. The responses of the deficit and debt to each shock in the first few periods have the same sign, agreeing with Fact 2.[26]

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[26] Recall that for our argument in Section 3.1.2 that debt and deficit co-moved in opposite ways we needed serially correlated shocks. This is why this feature is not present in the i.i.d. case.
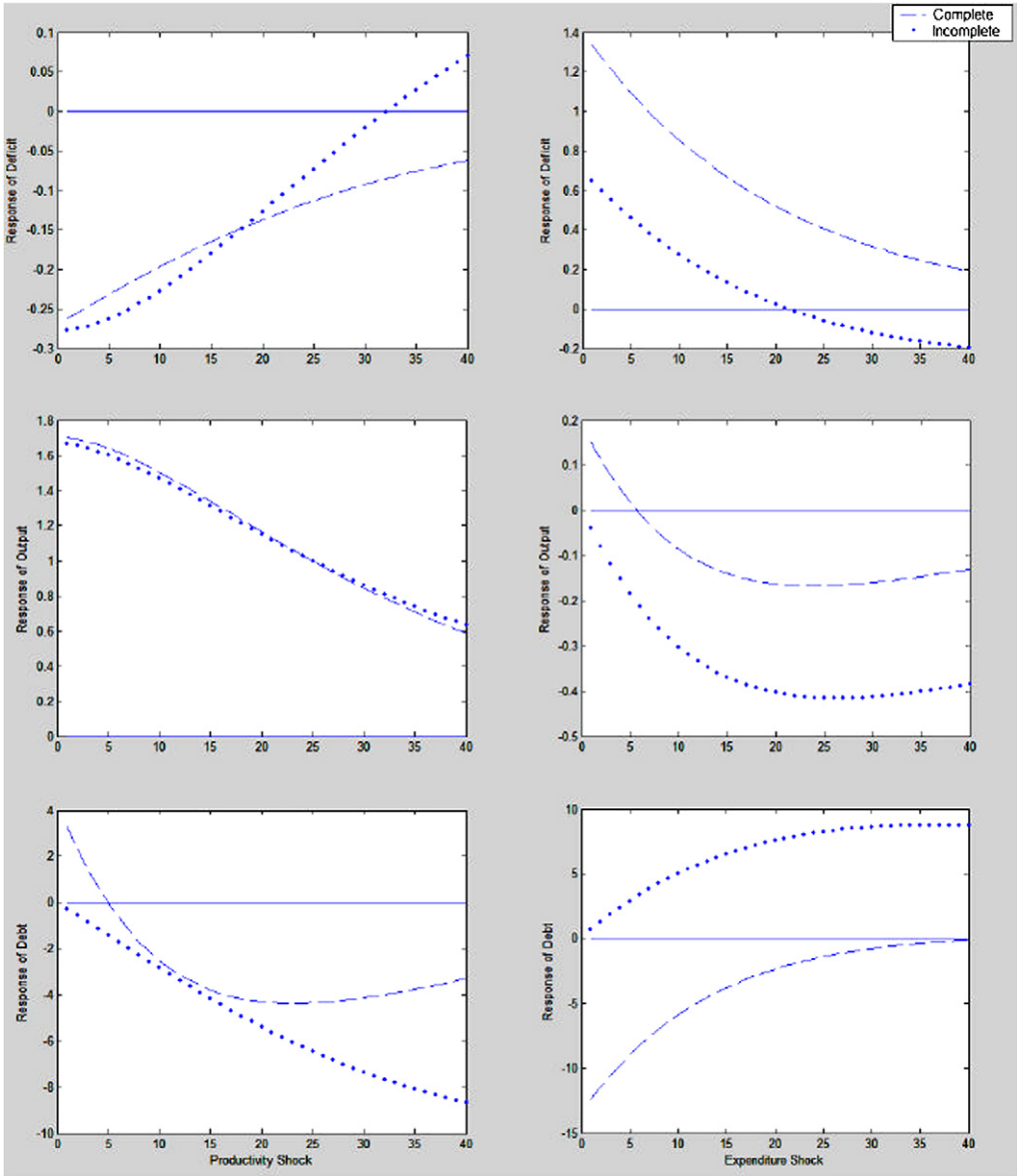
Fig. 5. True IRF, Model 2 (capital accumulation, persistent shocks).

Fig. 4 shows the true IRFs from the same model but now assuming persistent shocks $\rho^g = \rho^\theta = 0.95$. As expected from our discussion in Section 3.1.2, the persistence in the shocks is passed through to all the endogenous variables under complete markets. In all cases the response declines roughly geometrically, at a rate of decay approximately equal to 0.95. The middle panel of the left column in Fig. 6 shows that all variables have approximately equal persistence.[27]

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[27] The persistence is not exactly equal in all variables due to non-linearities of the model.
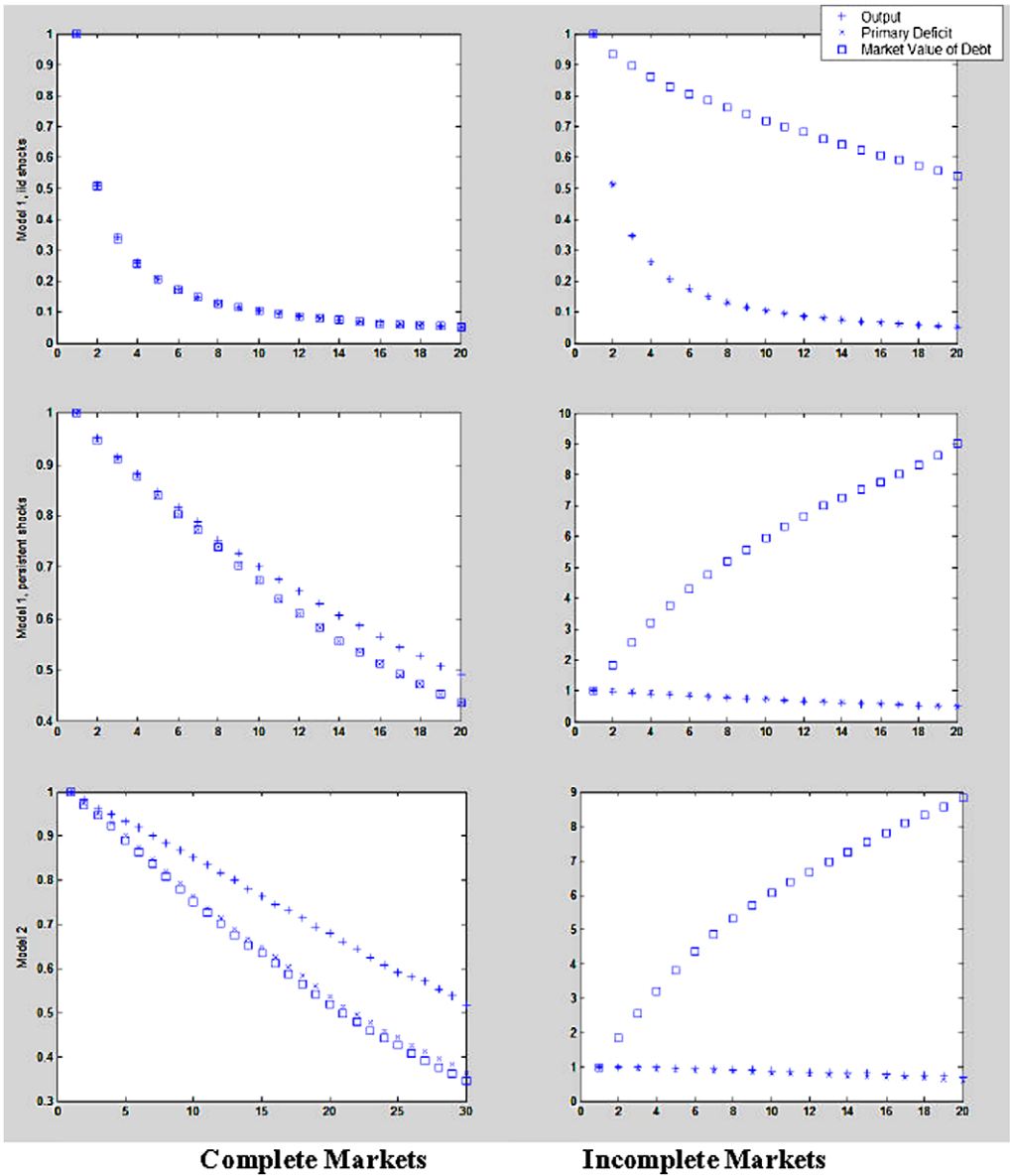
Fig. 6. *k*-variance ratios (persistence measures) for all models.

Fig. 4 also confirms our results in Section 3.1.2 and extends them to the productivity shock: under complete markets debt goes down (up) in response to a higher g_t (θ_t). The sign of the response of the primary deficit is now the opposite from the sign shown by the debt response. Therefore, debt and deficit co-move in opposite directions.

The incomplete market case in Fig. 4 for persistent shocks reveals a more complex picture. The response of debt to both shocks is increasing (in absolute value) for many periods and the effect is shifted to low frequencies. This combination causes the *k*-variance ratios to reveal sub-

stantially more persistence of debt in the case of incomplete markets, and much more persistence than the other variables—compare the middle panels of Fig. 6. Compared to our US findings in Fig. 2 the model seems to overshoot, since now the k-variance ratios for debt reach a value larger than 5 after 10 years compared with a k-variance ratio of 2 in the data.

Fig. 4 also shows that under incomplete markets the response of debt and deficit has the same sign, at least in the short and medium run. This confirms our discussion in Section 3.2 that the government uses debt as a buffer stock, increasing debt as a response to a bad shock, hence the co-movement of debt and deficit is as in the data. Notice also that under incomplete markets the response of deficits changes sign in the long run. This is because primary deficits eventually have to decrease in response to a bad shock, in order to service the additional interest payments caused by higher debt. This feature of the deficit response played an important role in our discussion in Section 3.2.2 and it agrees with the point estimates of the empirical impulse response of Fig. 1.

5.2. Model 2—Capital accumulation

We now study simulations of the capital accumulation model introduced at the end of Section 4. Notice that this model has a near unit root in debt even under complete markets, because capital imparts a near unit root behavior, showing that unit root tests are not the correct way to discriminate between complete and incomplete markets.

To perform the simulations we extend our calibration and set the depreciation rate $\delta = 0.05$, $\alpha = 0.4$ and to focus on the most persistent case we consider only the case $\rho^g = \rho^\theta = 0.95$.

The true IRFs are shown in Fig. 5. For the case of complete markets most responses decay to zero from an initially high level, the main exception being the response of output to government spending, which shows a relatively large long run negative effect after an initially positive response. Also notice that the responses to the productivity shock are longer-lived. None of the responses are humped-shaped, and certainly not the response of debt, suggesting that persistence is about the same in all variables. To give a final answer about persistence, the last panel of the left column in Fig. 6 shows that according to the k-variance ratio output and deficit are more persistent than debt under complete markets; so, the relative persistence of debt is now the opposite as in the data. The co-movement of deficit and debt, it is also unlike the data, since debt and deficit respond in opposite ways to each shock. In other words, the result of introducing capital is to take the complete markets model even further away from the data.

Fig. 5 also shows that the case of incomplete markets is once more diametrically opposed to the complete market outcome and much more consistent with the data.[28] It is interesting to note that both output and debt have a roughly equally persistent response to an innovation in g, while the response of debt to an innovation in θ is much more persistent than that of output. Debt seems to display the greatest persistence amongst the endogenous variables since it shows clearly humped shaped IRF. The lower panels of Fig. 5 again show that under incomplete markets the deficits and debt move in the same direction in response to each shock, consistent with US data. Once more in the long run the IRF of deficit has to reverse sign in order to service the additional interest payments. Fig. 6 confirms that under incomplete markets debt displays greater persistence than other variables although as with Model 1 there is too much persistence.

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[28] We have checked that this sort of behavior of debt under incomplete markets is present under most parameter values and under various models that we have explored. For example, the same sort of behavior occurs in the model of Gorostiaga [17], which introduces frictions in the labor market and endogenous government spending.

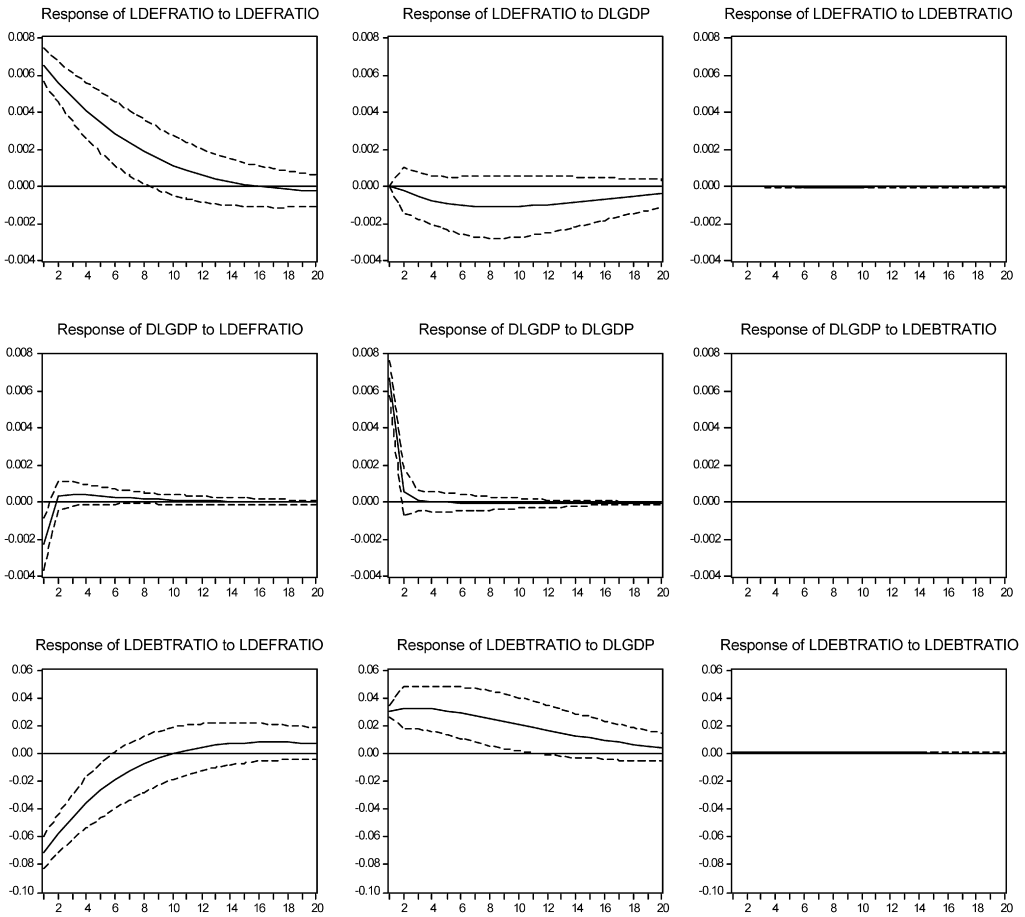Response to One S.D. Innovations ± 2 S.E.

Fig. 7. Estimated IRF, Model 2 (persistent shocks, complete markets).

The results above on true IRFs have been based around identifying the true theoretical innovations to productivity and government expenditure. These shocks are hard to identify in the data, this is why we used a VAR with a Cholesky decomposition to generate Fig. 1. To ensure consistency between theoretical and empirical results we also compute the IRF in the model by estimating an identical VAR as in Section 2 but using simulated data. We call this the "estimated IRF." The true IRF helps us understand the properties of our model, while the estimated IRF enable us to compare directly the model with the data and the validity of our tests.

The estimated IRF is shown in Figs. 7 and 8. Confirm the validity of our tests in this broader setting—complete markets (Fig. 7) shows debt is less persistence and falls in response to adverse "deficit" shocks (identified in the above method) whereas incomplete markets (Fig. 8) generates IRFs that display higher persistence of debt and the appropriate co-movement of debt and deficit.

To summarize the results in this section: the complete market analysis is inconsistent with our findings for US data, particularly Facts 1 and 2, while introducing incomplete markets model

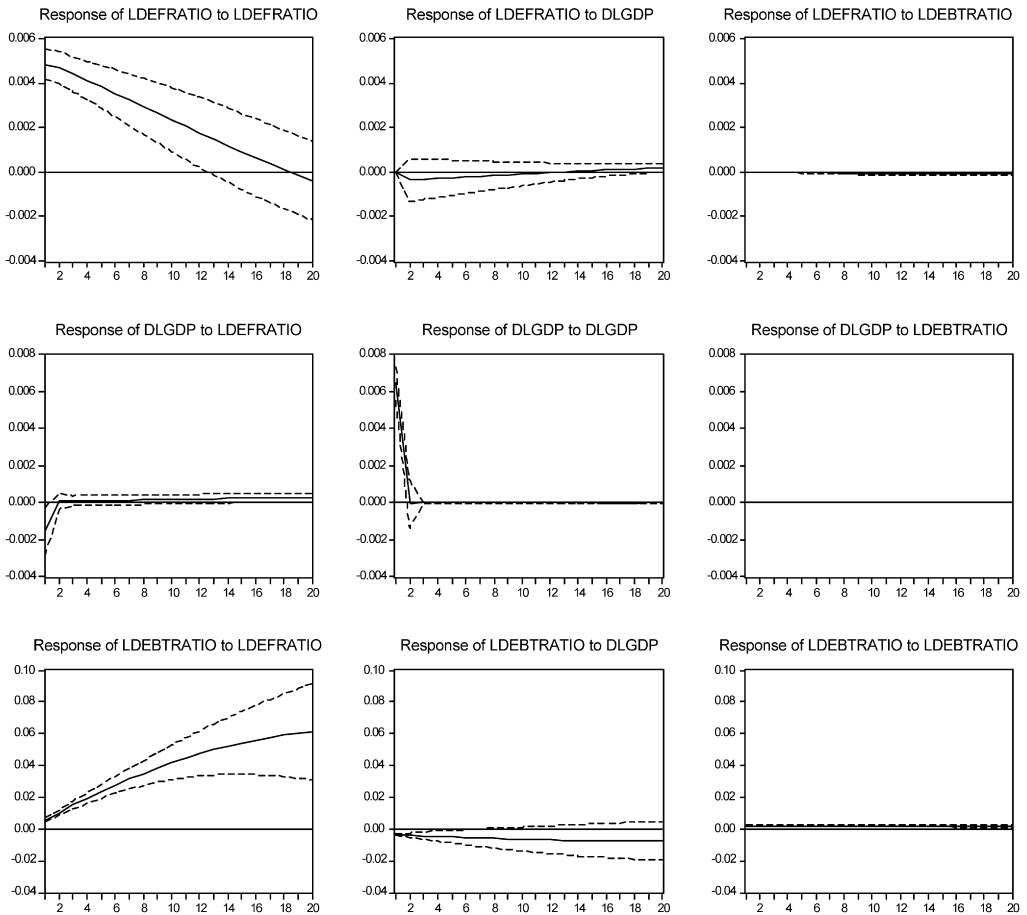Response to One S.D. Innovations ± 2 S.E.

Fig. 8. Estimated IRF, Model 2 (persistent shocks, incomplete markets).

provides a very large improvement. These results suggest that governments use variations in the level of debt as a buffer stock for fiscal shocks rather than using the insurance role that bond interest payments play in the case of complete markets.

6. Implications and extensions

The aim of our analysis has to been to assess the relative importance of complete or incomplete bond markets in affecting the behaviour of debt and government's fiscal policy. Our results here for the US, and those of Faraglia, Marcet and Scott [14] for the OECD, all suggest that bond markets are incomplete and that this incompleteness directly impacts on fiscal behaviour. However we have not sought in this paper to calibrate the degree of market incompleteness that exists. Indeed by focusing on the case of one period risk free bonds we find that our model now generates *too* much persistence in government debt. Presumably allowing for governments to gain some insurance against particular contingencies would give an intermediate position between the

two cases we consider and better match quantitatively the persistence of debt. An important research agenda is therefore gauging the degree of incompleteness and in particular which shocks governments are unable to obtain insurance for.[29]

The interpretation of our results we have stressed is both that bond markets are incomplete and that this matters for fiscal policy. However the conclusion that bond markets are incomplete rests on rejecting a *joint* hypothesis of a Ramsey government and complete markets. It could equally be argued that the real cause of this rejection is due to the Ramsey assumption rather than market incompleteness. Perhaps governments behave suboptimally due, for example, to political economy issues. Potentially this would be an interesting issue to analyse. It should be emphasized, however, that we have provided a model (based on market incompleteness) that improves the fit with the data very substantially, it is still to be shown if a model of political economy (and complete markets) can achieve such good results. One difficulty in pursuing this line of reasoning is the absence, to date, of a canonical political economy model and so we leave for future work an examination of this issue.

A number of papers in the optimal taxation literature accept that bond markets may be imperfect but that this need not constrain governments who can replicate optimal fiscal policy under complete markets by alternative policy instruments. To cite some examples within the literature on optimal policy, Angeletos [2] and Buera and Nicolini [5] show how to use *debt management*, Chari, Christiano and Kehoe [9] state contingent *capital taxation*, related to this, Farhi [16] investigates the possibility of using government ownership of capital to achieve the same outcome and Chari, Christiano and Kehoe [8] *monetary policy* in order to effectively complete markets.

Because our main results are on the behaviour of debt itself rather than the type of assets the government issues or the portfolio composition of debt, our results do show that the models in the above papers are not good descriptions of the data. Governments have not, for whatever reason, used these other policy instruments to achieve the complete market outcome. In other words, if we accept that bond markets are incomplete then as a matter of practice this has constrained fiscal policy as governments have not utilised these other channels to achieve the complete market outcome.

Another interpretation of the papers cited above would be that market incompleteness cannot be an important element in explaining the data, since in fact governments have many ways of achieving the complete markets outcome. This could be taken as indirect evidence that in fact governments have behaved suboptimally and the role of the papers mentioned above would be to tell governments how debt management, capital taxes or inflation policy should be run.

We dispute this interpretation because these alternative policies all have substantial costs attached to them which make the optimal policies difficult to implement. For instance, Faraglia, Marcet and Scott [15] show that, in many environments, completing the markets with debt management a la Angeletos [2] would require portfolio positions of several multiples of GDP at each maturity and that the optimal portfolio structure is both potentially volatile over time and extremely sensitive, both quantitatively and qualitatively, to small changes in the model.[30] Chari et al. [9] suggest using fluctuations in ex post capital taxes to achieve the complete market outcome but their own simulation results suggest this requires a standard deviation of around 40% in capital tax rates (see their Table 2). Even accepting the argument in Judd [19] that conventional

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[29] In this vein, Sleet [26] investigates the case where governments have access to private information so that rates of return offer limited state contingency but markets are still incomplete. He does find that debt is still persistent but less so than in the models of the present paper.

[30] This point was already made in a simple model by Buera and Nicolini [5].

measures of capital taxes understate volatility by ignoring corporate allowances it is hard to think that capital taxes in the real economy could adjust ex-post to this degree. Chari et al. [8] show that the inflation policy required to complete markets is also extremely volatile, inflation should have a standard deviation of between 20 or 30%.

It is possible that the implied volatility in bond positions, capital taxes or inflation are sufficiently costly or distortionary (for a variety of reasons—political economy factors, transaction costs in bond markets or real/nominal rigidities, difficulties in reaching the rational expectations outcome if agents use learning schemes) that governments do not wish to use them to achieve the complete market outcome. For example, Farhi [16] outlines reasons why an extremely volatile tax rate on capital might be very distortionary, and Schmidt-Grohe and Uribe [23] show that if mild costs of adjusting prices are introduced the benefits of completing the markets by adjusting inflation are lost. This would suggest that the Ramsey model with complete markets should not be used literally as a template to make recommendations about debt management, inflation or capital taxes. These instruments may play a role in partly insuring governments risks, and considering them may drive the behavior of debt closer to the data than our one period risk free assumption. However in our opinion it is important that enough constraints are kept in the model so that the Ramsey complete markets outcome cannot be attained.

7. Conclusion

The aim of this paper has been to revisit the topic of Barro's [3] paper—the determination of public debt—and to use the implications of optimal taxation to provide a test for discriminating between complete and incomplete markets. This refocusing on debt enables us to arrive at a simple but sharp test for the importance of incomplete markets which is robust to a range of assumptions and has broad applicability.

When markets are complete government debt shows the same or less persistence than other variables in the economy; furthermore, debt and deficit co-move in opposite directions. US data is inconsistent with both of these implications of complete markets: in fact, debt is more persistent than deficit and output, and debt moves in the same direction as deficit. We show that both puzzles are solved by the introduction of incomplete markets. This observation allows us to, first, reject complete markets, second, accept incomplete markets.

Our simulation results suggest that the way we model market incompleteness may be too extreme. A fruitful area of research could be to identify the precise nature of the causes of incompleteness and to allow governments to use some instruments to insure against shocks.

Our analysis is of a joint hypothesis of complete or incomplete markets in conjunction with a Ramsey planner. Therefore our findings against complete markets could alternatively be interpreted as a rejection of the Ramsey assumption and as evidence that the US government has been behaving suboptimally. But in order to prove this point one should provide a model that explains why governments chose the policies they did. Until a tractable and widely accepted alternative to the Ramsey planner exists (which it does not) it is not possible to know how robust our two tests are to alternative assumptions on government behaviour. It will be interesting to see if a model of, say, political economy, is able to match Facts 1 and 2 under complete markets as well as our off-the-shelf Ramsey models with incomplete markets.

While our analysis has focused exclusively on government fiscal policy we stress that our general results should extend to the behavior of consumer, firms or open economy debt. It seems that the behavior of debt offers the best chance to distinguish complete versus incomplete markets.

Acknowledgments

Appendix A. Data

We use annual US data. Data for the total deficit, government expenditure and tax revenue from 1900 to 1999 was from the Economic Report of the President and downloaded from www.ibert.org/deficit.html. The BEA website provided current and constant price GDP (available from 1900 and 1929 respectively). Interest payments were available on a fiscal year basis from 1940 onwards at http://w3.access.gpo.gov/usbudget. These were converted into a calendar year basis by multiplying adjoining years by 0.25 and 0.75 e.g. 1941 interest payments were calculated as 0.75 of 1940/1941 fiscal year payments plus 0.25 of 1941/1942 fiscal payments. The resulting series was then removed from the total deficit series to arrive at a primary deficit series from 1940 onwards.

The main complication was in finding a consistent series for the market value of US government debt. Seater [25] provides an annual series for 1919 to 1975. Cox and Hirschorn [12] provide a monthly series between 1945 and 1980. As the latter shows these two studies arrive at highly consistent results. Butkiewicz [6] outlines a more time efficient way of calculating the series and again reveals discrepancies to be small between the alternative approaches. We therefore construct a market value series from 1919 to 1999 using (a) Seater for 1919 to 1975, (b) Cox and Hirschorn 1976–1980 and (c) extend the series to 1999 using Butkiewicz formula and OECD data, Central Government Debt Statistics. The full data is available upon request.

Although we have been careful to do our study with the market value of debt, which is the appropriate counterpart of debt in the model, the results in Section 2 stay almost the same for a measure of the total outstanding debt from OECD.

Appendix B. Technical appendix

B.1. Assumptions and proof of Result 1 in Section 3.2.1

The first order conditions of the consumer and of the Ramsey optimizer under both complete and incomplete markets imply[31]

$$H'(l_t) = 1 - \tau_t, \tag{17}$$

$$H'(l_t) + \lambda_t \left[1 - H'(l_t) + H''(l_t)(1 - l_t) \right] = 0, \tag{18}$$

$$\lambda_t = E_t(\lambda_{t+1}), \tag{19}$$

where λ_t is the Lagrange multiplier of the government budget constraint.

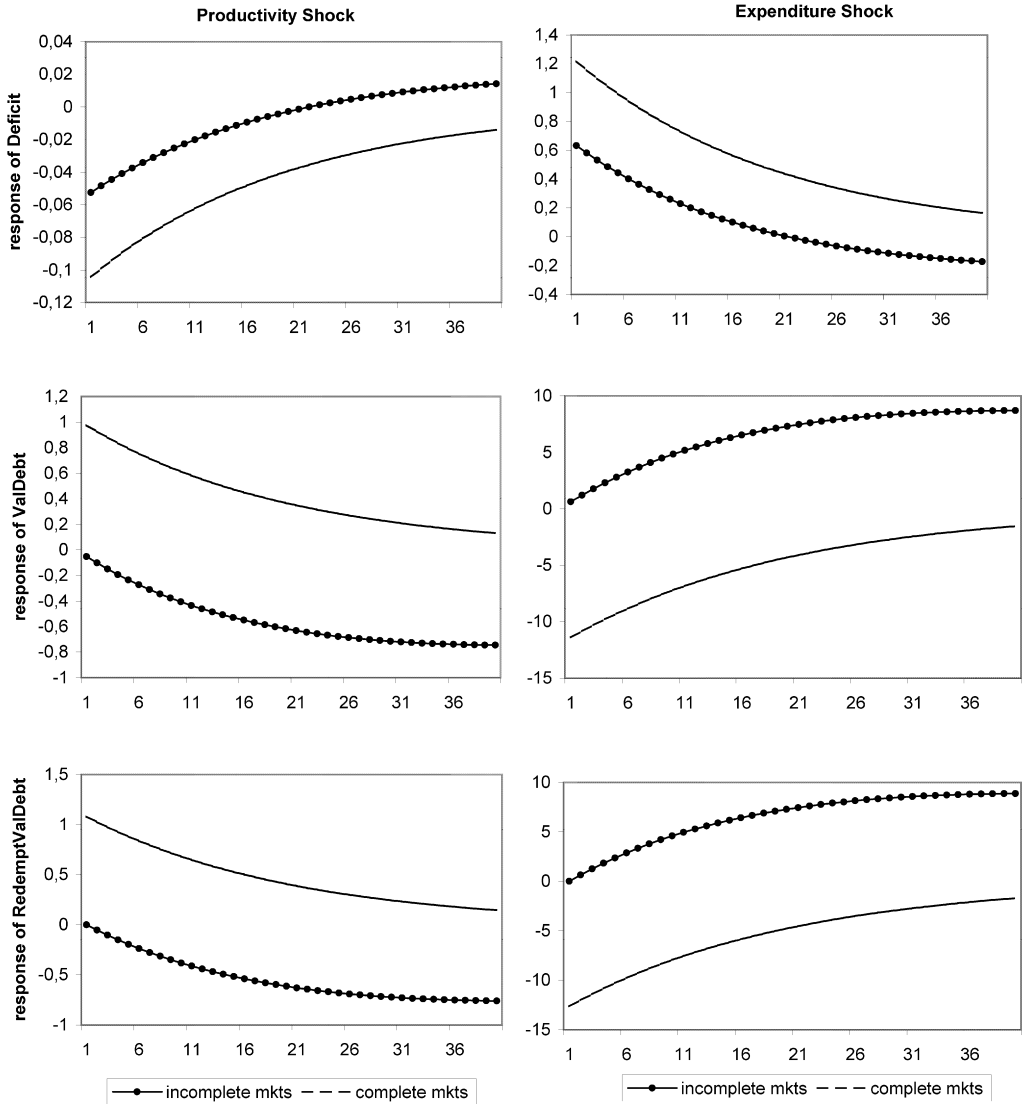[31] See, for example, Aiyagari et al. [1].

Fig. A.1.

We need to define the function $\Lambda(\cdot)$ mapping feasible revenue values into the corresponding multiplier λ guaranteeing that (18) and (17) hold. Formally, for any feasible $\overline{R}$, Λ is defined as

$$\Lambda(\overline{R}) \equiv -\frac{1 - H'(\check{l}) + H''(\check{l})(1 - \check{l})}{H'(\check{l})}$$

where $\check{l}$ is the corresponding labor satisfying $\overline{R} \equiv (1 - H'(\check{l}))(1 - \check{l})$.

We assume H is such that Λ is strictly monotone[32] and that there is an interior solution with probability one in all periods for both the consumer and the government.

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[32] Sufficient conditions on H for this result are found in footnote 14 in [1].

Finally we assume $\beta > 0.5$.

Under these conditions the result stated in Section 3.2.1 holds according to the following

Proof. For the government spending process above we have

$$E_t(g_{t+j}) = \bar{g} \quad \text{for all } t \geqslant 0 \text{ and for all } j > 0. \tag{20}$$

Let us consider first the complete markets part. Following Lucas and Stokey [21], (18) is satisfied with $\lambda_t = \lambda$ for all t. This, together with (17) implies that leisure and taxes are constant, say $l^{cm} = l_t$, $\tau^{cm} = \tau_t$.

From the budget constraint of the government at period zero we have

$$0 = E_0 \sum_{t=0}^{\infty} \beta^t \omega_t^g = \sum_{t=0}^{\infty} \beta^t \left[E_0(g_t) - \tau^{cm} \left(1 - l^{cm} \right) \right] = \frac{\bar{g} - \tau^{cm}(1 - l^{cm})}{1 - \beta}$$

where the first equality uses $u_c' = 1$, the second equality uses constancy of taxes and leisure, and the third equality uses (20). Therefore,

$$\omega_t^g = g_t - \tau^{cm} \left(1 - l^{cm} \right) = \bar{g} - \tau^{cm} \left(1 - l^{cm} \right) = 0, \quad t = 0, 2, 3, \ldots,$$
$$\omega_t^{g,H} = g^H - \tau^{cm} \left(1 - l^{cm} \right) = g^H - \bar{g} = \eta > 0, \quad t = 1.$$

This proves the statements about the deficit under complete markets.

The value of debt satisfies

$$vb_t^g = \sum_{j=1}^{\infty} \beta^j \left(\tau^{cm} \left(1 - l^{cm} \right) - E_t(g_{t+j}) \right) = 0$$

for all t, where the first equality follows from (3) and constancy of taxes and hours, and the second equality follows from (20) and our analysis of deficit. This shows our statements about debt under complete markets.

We now prove the statements about incomplete markets. Since there is no uncertainty after period 1, (19) implies that $E_t(\lambda_{t+1}) = \lambda_{t+1}$ for all $t \geqslant 1$, so that $\lambda_t = \lambda_1$ for all $t \geqslant 1$. Then (18) implies $l_1 = l_t$ and (17) $\tau_1 = \tau_t$ for all $t \geqslant 1$ and all realizations, and since g is also constant after period 2 we have $\omega_2^g = \omega_t^g$ for all $t > 2$ and all realizations, proving the equalities of (7).

From Proposition 1 in [1] and $u_{c,t} = 1$ we have

$$b_{t-1}^g = E_t \sum_{j=0}^{\infty} \beta^j \omega_{t+j}^g \quad \text{for all } t. \tag{21}$$

This and the fact that deficit is constant for all $t \geqslant 2$ imply

$$vb_t^g = \beta b_t^g = \beta \frac{\bar{g} - \tau_1(1 - l_1)}{1 - \beta} \quad \text{for all } t \geqslant 1$$

which proves the equalities in (8).

The remainder of this proof shows that $\omega_1^{g,H} > \omega_0^g$. Since (21) and the equalities in (7) imply that $b_1^g = \frac{\tau_1(1-l_1)-\bar{g}}{1-\beta}$, and by assumption $b_{-1}^g = 0$, the budget constraint of the government at periods 0 and 1 imply

$$\beta^{-1} \left(\bar{g} - \tau_0(1 - l_0) \right) = -b_0^g = \tau_1(1 - l_1) - g_1 + \beta \frac{\tau_1(1 - l_1) - \bar{g}}{1 - \beta}. \tag{22}$$

Let us denote the revenue by $R_t \equiv \tau_t(1 - l_t)$. Since the last equation holds for both realizations of g_1 and b_0^g is not random, we have

$$-b_0^g = R_1^H - g^H + \beta \frac{R_1^H - \bar{g}}{1 - \beta} = R_1^L - g^L + \beta \frac{R_1^L - \bar{g}}{1 - \beta} \tag{23}$$

and, using $g^H - g^L = 2\eta$ this implies

$$R_1^H - R_1^L = (1 - \beta)2\eta > 0. \tag{24}$$

Applying (19) at $t = 0$ gives

$$\Lambda(R_0) = \lambda_0 = E_0(\lambda_1) = \frac{\Lambda(R_1^H) + \Lambda(R_1^L)}{2} \tag{25}$$

and, since $R_1^H > R_1^L$ and Λ is strictly monotone we have

$$R_1^H > R_0 > R_1^L. \tag{26}$$

The equality in (24) and the assumption $\beta > 0.5$ imply

$$R_1^H - R_1^L < \eta$$

and together with (26) we have $R_1^H - R_0 < \eta$. Since $g_1^H - g_0 = \eta$, we have that $\omega_1^{g,H} > \omega_0^g$.   □

B.2. No-Ponzi-Scheme condition, Section 4

The no-Ponzi-Scheme condition we alluded to in Section 4 requires prices and bonds to satisfy

$$\lim_{T \to \infty} E_t \left(b_{t+T}\big(s_{t+T+1}^1\big) \prod_{j=1}^{T} \mathrm{p}\big(s_{t+j+1}^1, z_{t+j-1}, s_{t+j}\big) \right) = 0 \quad \text{a.s.} \tag{27}$$

for all t. This is satisfied in most models, where b is bounded and p is the intertemporal elasticity of consumption.

B.3. Proof of Proposition 1

To show part (A) we first show that a sequence of bond holdings that satisfies the budget constraints satisfies

$$
\begin{aligned}
b_{t-1}^i\big(s_t^1\big) &= \omega_t^i + \int_{S^1} b_t^i\big(\bar{s}^1\big) \mathrm{p}\big(\bar{s}^1, z_{t-1}, s_t\big) \mu\big(\bar{s}^1, s_t\big) \, d\bar{s}^1 \\
&= \omega_t^i + E_t\big(b_t^i\big(s_{t+1}^1\big) \mathrm{p}\big(s_{t+1}^1, z_{t-1}, s_t\big)\big) \\
&= \omega_t^i + E_t \sum_{j=1}^{\infty} \omega_{t+j}^i \prod_{\tau=0}^{j-1} \mathrm{p}\big(s_{t+\tau+1}^1, z_{t+\tau-1}, s_{t+\tau}\big) \quad \text{a.s.}
\end{aligned} \tag{28}
$$

The first equality uses (9) and (13), the second equality uses the definition of conditional expectation, and the third equality is obtained by recursive forward substitution of the random variable $b_t^i(s_{t+1}^1)$ and (27). Furthermore, the Markov assumption about s_t together with (11), (12), (13) imply that the conditional expectations of the discounted sum in (28) is a function only

of (s_t, z_{t-1}). This means that a time-invariant function D^i satisfying (15) exists and together with (28) it implies (14).

To establish part (B) note that plugging (14) in the budget constraint and using (13) imply

$$\int_{S^1} b_t^i(\bar{s}^1) p_t^b(\bar{s}^1)\, d\bar{s}^1 = \int_{S^1} \mathrm{D}^i\big(\bar{s}^1, s_{t+1}^2, z_t\big) \mathrm{p}\big(\bar{s}^1, z_t, s_t\big) \mu\big(\bar{s}^1, s_t\big)\, d\bar{s}^1$$

so that the right side of this function is a time invariant function of z_t, s_t.

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