Review of Economic Studies (2006) **73**, 31–53 0034-6527/06/00020031$02.00
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Minorities and Endogenous Segregation

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First version received January 2002; *final version accepted March* 2005 (*Eds.*)

A theoretical analysis is proposed of segregation as an equilibrium phenomenon in a random-matching model of the marriage market. Otherwise identical partners possess a pay-off-irrelevant characteristic, colour. We derive the set of colour-blind equilibria and show that they are generically constrained inefficient. Equilibrium segregation strategies are strategies that condition actions on the type of match. It is shown that distributions of types exist such that segregation equilibrium pay-offs Pareto dominate colour-blind pay-offs. For other distributions, segregation also generates conflict, where the majority unambiguously gains, while the minority group may lose. Giving preferential treatment, that is, minority bias, can increase overall welfare.

1. INTRODUCTION

Segregation based on race is widely prevalent in society. For example, in the 1990's only 5·5% of black men in the U.S. married white women. Eighty-five per cent of all marriages would be mixed race in the hypothetical experiment of randomly matching black men to all women, black and white.[1] This suggests there is a force at work generating such extreme segregation. In this paper, we analyse segregation in a random-matching model of the marriage market. Agents search to form a partnership in which they choose to cooperate or defect, and then they decide to continue or separate. People are identical except for a pay-off-irrelevant characteristic such as colour. The main feature of the current analysis is that the stochastically drawn colour of the new partner acts as a *public randomization device*. The set of equilibria is studied, and we identify the most efficient equilibria attainable. Colour-blind strategies typically are not constrained efficient. Strategies that condition actions on colour (*i.e.* on the public randomization device) can Pareto dominate the best colour-blind equilibria. These strategies typically lead to segregation.

Segregation as a result of discriminatory behaviour has, of course, received a lot of attention in the literature. Much like the model of statistical discrimination (Arrow, 1973; Coate and Loury, 1993), the segregation derived here is the result of *equilibrium behaviour* in society.[2] We identify a new channel of discrimination—namely, colour acting as a public randomization device—within a dynamic model. This channel generates predictions that significantly differ from those in the standard models of discrimination. For example, for certain distributions of types, segregation actually gives rise to Pareto dominant outcomes, while in most models of statistical discrimination, discrimination generates lower welfare. In further contrast, we find that the size of minorities matters. For certain asymmetric distributions, segregation can make smaller

1. See Wong (2003). Of course, in reality there is heterogeneity which leads to selection, that is, the matching is not uniformly random. Wong (2003) estimates that even when accounting for selection, the fraction of such mixed matches should be 64%, and attributes segregation to "mating taboo".

2. In Becker's (1957) model, discrimination derives from preferences.

minority groups worse off while making the majority better off. This is due to the fact that meeting probabilities are a function of the distribution of colour types in the economy. In the standard model of statistical discrimination, size does not matter.

The model is laid out in the next section. We use a random-matching model with infinite horizon. Upon meeting, two agents play a partnership game modelled as a prisoner's dilemma. After any transaction, each partner can choose either to remain matched or to terminate the partnership and randomly match with a new agent. The modelling choice of the prisoner's dilemma as the stage game is motivated by the idea of trust in repeated interactions, an aspect typical of marriage. For example, partners can either provide effort by performing household chores, working long hours to increase the economic well-being of the family, and so on, or alternatively, free ride on the partner. Our model is highly stylized, but it does have the key features of the standard marriage search models like those of Mortensen (1988), Burdett and Coles (1997), and Burdett, Imai and Wright (2004).[3]

In each period, there is an intertemporal trade-off between myopic gains from deviation and the continuation pay-off. Because trust and cooperative behaviour within a partnership are fragile and a partner can always break trust and search for a new partnership, full cooperation and hence efficiency cannot be an equilibrium. The best we can hope for is a constrained efficient equilibrium. An economy-wide equilibrium arrangement can make some cooperative behaviour in repeated partnerships incentive compatible, even though those arrangements do not necessarily guarantee second best efficiency. Several such social arrangements are possible, including arrangements without segregation, that is, colour-blind equilibrium. Incubation strategies, for example, are strategies that start a partnership cautiously, involving some initial phase of playing defect, or alternatively an initial phase where a mixed strategy is played. These arrangements turn into full cooperative behaviour as long as players stick to the arrangement. Once full cooperative behaviour is attained, deviation is costly, as new partnerships go through a costly initial phase. In other words, the equilibrium path of play must serve as its own punishment.

In Section 3, the *best* of the colour-blind equilibria is analysed. Proposition 1 establishes that, generically, it is not constrained efficient. The best colour-blind equilibrium very often involves partners using a mixed strategy in the first period, with the continuation of marriage being conditional on the realization of actions (whether your partner was cooperative). This, interestingly, fits the notion of *dating* well. When partners meet, they are indifferent between "putting in a lot of effort" or "putting in none". If both happen to have made the effort, the partnership continues. Alternatively, both get a new match and play the same game until a partnership is found that "works". The dating phase does not obtain full efficiency but is necessary to avoid deviation once a partnership turns into full cooperation. In general, not even constrained efficiency can be obtained, because even mixed strategies are not sufficient to correlate actions.

In Section 4, segregation strategies are introduced. In Proposition 2, it is proven that, generically, there exist distributions of pay-off-irrelevant characteristics, such that the best colour-blind equilibrium is Pareto dominated by the corresponding segregation equilibrium. In the dating example, segregation lets partners coordinate actions to make dating less costly: with a partner of a different colour, for example, you do not expect cooperation in a new match, thus avoiding miscoordination. Due to random matching, mixed matches always occur in equilibrium, but there are two separate sources of segregation: first, there may be less cooperation in mixed matches than in same-colour matches, and second, mixed matches may be of shorter duration.

3. Other papers closely related to Burdett and Coles (1997) include Smith (1997), Eeckhout (1999), Bloch and Ryder (2000), among others. For an overview of this literature, see Burdett and Coles (1999). With the model of Burdett *et al*. (2004), our model shares the notion of trust in repeated interactions. There, it is derived from the decision to search "on-the-job" while being married, whereas in our model, it is derived explicitly from a partnership game. For the special case of identical agents, Burdett *et al*. (2004) also find equilibrium to be inefficient, providing a benchmark for our result.

We also analyse simple strategies including cases with endogenous equilibrium distributions of singles of different types. They illustrate that in general, for asymmetric distributions, segregation strategies lead to inequality between different types. In particular, members of minorities may do worse under segregation than under the colour-blind equilibrium. Segregation, therefore, may generate *conflict*. Minorities tend to suffer as the gains from segregation are distributed unequally. Moreover, group size matters, and the smaller the minority, the more it suffers. This conflict is non-existent in models of statistical discrimination,[4] the implication being that discrimination against blacks does not affect the utility of whites. In our framework, minority members would clearly prefer an equilibrium without segregation. However, the majority benefits from segregation. Any change in equilibrium that eliminates segregation will make the majority worse off.

We perform an experiment labelled *minority bias*. It consists of a strategy giving preferential treatment to the minority in a mixed match, and in some economies, overall welfare even increases relative to segregation. However, the majority is worse off under minority bias than under segregation. Minority bias is an equilibrium, so no government action is required. Still, policy intervention may be needed in order to induce this strategy, for example, to move equilibrium in society from segregation into minority bias.[5]

In addition to the marriage literature, this paper is also related to a large literature on random matching and repeated games. The basic framework we use builds on work by Datta (1993), Ghosh and Ray (1996), Kranton (1996), Watson (1999), and Lindsey, Polak and Zeckhauser (1999), who derive cooperation using strategies with increasing levels of cooperation. We use a very stylized version of those models and show that a pay-off-irrelevant characteristic can improve efficiency. We also consider the set of strategies that involve mixing in the initial stages. Finally, since our objective is to study segregation, type-dependent strategies are a key component of this paper. Work by Mailath, Samuelson and Shaked (2000) considers strategies related to ours. Their analysis is performed in a different setting, a labour market model with heterogeneous workers. They find segregation equilibria driven by search externalities and study the effect of these externalities on investment in skills. The driving force behind segregation in our paper is quite distinct: the efficiency gains from coordination where colour acts as an endogenous public randomization device. Moreover, our result is derived in a context where agents are *homogeneous*.

Of course, it is immediate from the game theory literature that the type of coordination provided by the random arrival of different coloured partners can be mimicked by *any* public randomization device. In fact, sunspots, a coin toss before play in each match, or simply cheap talk can provide an equally useful device for efficiency improving coordination. However, while *exogenous* public randomization devices may be common, for example, in the case of traffic lights, they are far less common in other environments with decentralized social interaction. Here, the point is precisely that a randomization device is being used and that the one used is readily available from the composition of the population. We believe that this *endogenous* coordination device is appealing from a behavioural point of view. The strategies involved are simple and can easily be considered as rule of thumb.

2. THE MODEL

Consider a society with a continuum of infinitely lived agents. In each period a mass one of the agents is born. Time is discrete. At the beginning of each time period t, agents are either

4. The notable exception being that of Moro and Norman (2003), who consider statistical discrimination where inputs in production are complements. In addition, in their model as in ours, the size of the minority matters.

5. In a recent paper, Moro and Norman (2004) show that in a general equilibrium model of statistical discrimination, reverse discrimination (a policy not unlike minority bias) will offset the loss the minority suffers due to discrimination.

single or matched in pairs. The singles, including the new entrants, are matched in pairs.[6] All members of society are identical, except for a pay-off-irrelevant characteristic $c \in \mathcal{C}$, say colour. The characteristic is costlessly observable and cannot be hidden. Let $\pi(c)$ denote the measure of the population of colour c, and such that $\int_{c\in\mathcal{C}} \pi(c) = 1$. From Section 4.4 onwards, we will concentrate on a two-type distribution, $c \in \{b, w\}$, indicating black or white.

A matched pair plays a partnership game. It is a symmetric prisoner's dilemma with action space $A = \{C, D\} \times \{C, D\}$ and stage game pay-off functions $\gamma : A \to \mathbb{R}^2$ as given:

	C	D
C	$1, 1$	$-l, 1+g$
D	$1+g, -l$	$0, 0$

where $g > 0$, $l > 0$. We assume that $g - l \leq 1$. Let $\mathcal{A}$ be the probability distribution over A. The game is common knowledge.[7] The unique Nash (and dominant strategy) equilibrium of the stage game is (D, D) with pay-offs $(0, 0)$, which also corresponds to the minmax value. The feasible (and individually rational) set of stage game pay-offs $\mathcal{F}$ is defined as

$$\mathcal{F} = \text{convex hull}\{v \mid \exists\, a \in A \text{ with } \gamma(a) = v\} \cap \mathbb{R}^2_+.$$

Matching is assumed to be anonymous and endogenous. The endogeneity implies that any member, after the realization of the outcome[8] of the partnership game, decides whether to continue being matched with the current partner or to terminate the current match. The match is dissolved if at least one of them *chooses* to terminate the match.[9] The assignment to new partners is anonymous, uniform,[10] and independent over time.

In general, players observe some realized actions at the end of each period. However, because we assume anonymity, this can only include the actions within the current match. Let $a^t = (a_1^t, a_2^t)$ be the actions in period t observed by a player in a particular match.[11] In addition to assuming anonymity, we concentrate on public strategies.[12] Then there is no relevant information about past actions beyond the current match. Therefore, the information any member possesses can be denoted by h^k, that is, we can "reset" histories in function of the lifetime of a match. For any match that is newly formed, the history is the null history $h^k = h^0$, which includes the time of entry in the market. For any $k > 0$, that is, a match that has survived k periods, both agents observe the sequence of realized actions at $t-1, \ldots, t-k+1$, so that $h^k = h^0 \cup \{a^{t-1}, \ldots, a^{t-k+1}\}$. Let $H^t = (A_1 \times A_2)^t$ be all possible period t histories. A strategy s_i for each player i will then be a sequence $\{s_i^t\}$ that maps histories $h^k \in H^t$ into mixed actions in $\mathcal{A}_i$ and termination decisions.

6. Even though agents are infinitely lived and there is a constant inflow of new agents, the measure of the market of singles will be bounded in equilibrium due to the outflow.

7. We assume there is no role identification: in this symmetric game, a player cannot distinguish whether she/he is the row or column player.

8. It is most natural to assume that players observe the pay-off realization. For pure strategies that assumption is without consequence. In the case of a mixing, however, this implies partners do not observe the mixing probability of the opponent.

9. In order to keep the exposition tractable, there is no exogenous separation. In an earlier version of this paper, there was exogenous separation, but results were not qualitatively affected by that assumption.

10. Because each single matches with probability 1, this corresponds to the standard assumption in marriage-matching models that the matching function is constant returns to scale. That is, an individual's probability of matching is independent of the number of singles in the market.

11. Past continuation/termination decisions of the match are trivially included. If an action a^t is observed at time t, then automatically, all decisions at earlier t in the current match have been continued.

12. A public strategy is a strategy where all players ignore their own private histories. As a result, a partner is not allowed to condition her/his action on some action she/he has taken in another partnership.

The pay-offs of the repeated matching game are given by v. Players discount the future at a common factor $\delta < 1$. The objective of a player is to maximize the normalized sum

$$v = E_s(1-\delta)\sum_{t=0}^{\infty}\delta^t\gamma(s^t(h)). \tag{1}$$

The normalization allows us to compare the repeated game pay-off directly with the stage game pay-off. E_s denotes the expectation over the infinite histories generated by the strategy profile s. Note that all possible histories in each of the matches are entirely represented by h^k.

A partnership-matching economy will be completely described by (g, l, δ). We will be looking for a strategy profile s that is a stationary perfect public equilibrium (PPE), that is, each individual strategy s_i is a public strategy, and for each date t and history h^t, the strategies yield a Nash equilibrium from that date on.

This model has the main features of a marriage model. In the next section, we analyse the benchmark case of colour-blind equilibrium. We then continue to analyse segregation.

3. THE BEST COLOUR-BLIND EQUILIBRIUM

In order to provide a benchmark for the segregation equilibrium of the partnership-matching economy, we first analyse equilibrium in which strategies are type independent. Those colour-blind strategies are symmetric, from the fact that there is no role identification. It follows immediately from the description of the model above that it is impossible to implement the strategies usually used to prove folks theorem for repeated games between a fixed set of players. Given anonymity, a deviator cannot be punished once she or he has separated from her or his partner. Still, cooperation is possible, albeit through strategies that involve costly "punishments" across different matches. In this random-matching environment, some degree of non-cooperation between agents (*i.e.* punishment) must occur in equilibrium. Were there no punishment in equilibrium, then any member could defect, get the pay-off $1 + g$, separate and rematch, and defect again. In this section we first give a simple example of a colour-blind strategy and then continue to present the set of colour-blind strategies. We then proceed by showing that the best colour-blind strategy is not constrained efficient and by characterizing the set of best colour-blind equilibria.

3.1. *A simple example: incubation strategies*

Consider first the simplest type of colour-blind strategy that generates some cooperation in equilibrium: the incubation strategy. Incubation strategies capture the notion that many observed relationships start out "cautiously". In the initial stage, partners do not cooperate fully but switch to full cooperation after the partners have gone through a sufficiently long period of low cooperation. Similar strategies have been considered in Datta (1993), Ghosh and Ray (1996), Kranton (1996), and Watson (1999), among others. In the context of our model, the incubation strategy for any player i implies that in a new partnership (*i.e.* for history h^0), (D, D) is played in the first period, and (C, C) is played from the second period on. If a partner deviates, the partnership is separated and both members return to the pool of singles. Let v^I denote the discounted pay-off from entering a new match (where the superscript I denotes incubation). Then, on the equilibrium path

$$v^I = 0 + \delta.$$

A new match implies both partners playing D with pay-off zero and switching to C thereafter. Consider first deviations in the first period of a match. Since the first-period pay-off is the

myopic Nash equilibrium pay-off of the stage game, no player can do better by choosing an action different from D. Also, immediate separation is dominated because that delays the pay-off v^I by one period. Next, we look at one-shot deviations in all later periods. These yield a continuation pay-off $v^D = (1+g)(1-\delta)+\delta v^I$. There is no deviation in future periods provided playing C yields a greater pay-off than deviating: $1 \geq v^D$, or

$$g \leq \delta. \tag{2}$$

When condition (2) is not satisfied, members will deviate when in a cooperative partnership if this incubation strategy is played. Of course, playing a k-incubation strategy increases punishment for deviators. This strategy amounts to playing (D, D) for k periods. For the remainder of the paper, we will concentrate on the case where $g < \delta$.[13]

3.2. *The formulation of colour-blind strategies*

There are obviously many possible strategies that can lead to some degree of cooperation. We now introduce additional notation to include the relevant colour-blind strategies (including incubation). Strategies will consist of the actions taken in the first period, and those taken in all later periods. The action taken in any of the later periods is to play C as long as there has not been a deviation from the equilibrium strategy in the earlier periods and to continue the match. In case of a deviation, separate the match. Given an infinite horizon, cooperation forever leads to a normalized continuation pay-off of 1. In what follows, we refer to this as the cooperation phase. In the first period, the strategy consists of a pair (σ, P), where $\sigma \in [0,1]$ is the probability with which a player plays C, and the vector $P = (P_1, P_2, P_3) \in [0,1]^3$ is the continuation probability in the second period, conditional on the outcome. After (C, C) is observed, a player chooses to continue with probability P_1; after (C, D) is observed, the player chooses to continue with probability P_2; and after (D, D) is observed, he or she continues with probability P_3. We will represent the full colour-blind strategy s by $\{\sigma, P\}$, keeping in mind that in the cooperation phase, players play C on the equilibrium path and separate in case of a deviation. For the sake of example, the incubation strategy is represented by $\{\sigma^I, P^I\} = \{0, (0,0,1)\}$. Let $v(s)$ denote the equilibrium pay-off obtained using a colour-blind strategy $s = \{\sigma, P\}$. Then in the cooperation phase, playing C is robust to no deviation, provided the pay-off from deviating does not exceed that from playing C, which has a normalized continuation pay-off of 1. The pay-off from a one-shot deviation in the cooperation phase is $v^D = (1+g)(1-\delta)+\delta v(s)$, where $v(s)$ is the pay-off of the candidate equilibrium. The no-deviation constraint requires $v^D \leq 1$ or

$$v(s) \leq 1 - g\frac{1-\delta}{\delta}. \tag{3}$$

If we let $\overline{V}$ denote the constrained efficient or second best pay-off, that is, the highest possible attainable pay-off that guarantees no deviation in the cooperation phase, then it is equal to $\overline{V} = 1 - g\frac{(1-\delta)}{\delta}$. Then, subject to the constraint that $v(s) \leq \overline{V}$, an equilibrium satisfies $s_i \in \arg\max_{s_i=\{\sigma_i, P_i\}} v(s_i, s_{-i})$, where i is the strategy chosen by any player i, and $-i$ is the strategy chosen by all other players. In the absence of role identification, the colour-blind equilibrium actions in the first period of a match are considered to be symmetric, and the subscripts $i, -i$ will be dropped.

13. It is easily verified that for k incubation strategies, such equilibrium can be sustained for a larger set of parameter values. The equivalent of (2) is $g \leq \delta\frac{(1-\delta^k)}{(1-\delta)}$. For example, when $k = 2$, cooperation can be sustained for $g \leq \delta(1+\delta)$, which includes values of g larger than δ.

We now proceed to study the colour-blind equilibrium strategy with the highest attainable pay-off. We will refer to this as the *best colour-blind* equilibrium strategy, denoted by s^* and with pay-off $v(s^*)$.

Proposition 1. *For a generic partnership-matching economy* (g, l, δ) *the best colour-blind equilibrium is constrained inefficient:* $v(s^*) < \overline{V}$.

Proof. In Appendix. ‖

To prove the proposition, a contradiction argument is used. Starting from the premise that a candidate equilibrium is constrained efficient (*i.e.* $v(s) = \overline{V}$), conditions on the parameters of the model are derived. It is shown that generically there is no solution that simultaneously satisfies those conditions. Even though at isolated points in the parameter space those conditions may well be satisfied thereby rendering the economy constrained efficient, slight perturbations to economies in the neighbourhood of those isolated points fail to satisfy the conditions for constrained efficiency. Generically therefore the partnership-matching economy is constrained inefficient.

This inefficiency result is quite surprising, because despite the possibility of mixed strategies that convexify the set of feasible outcomes, constrained efficiency is not guaranteed. The inefficiency derives from the fact that mixing implies pay-offs that are not Pareto efficient (in this case those as a result of playing (C, D)) are played with positive probability. This indicates that the result in Proposition 1 may extend to more general stage games beyond the Prisoner's Dilemma, for example, coordination games. Future work should analyse the class of stage games in this context that obtain generic constrained inefficiency. In an interesting recent paper, Lima-Fieler (2004) stresses the importance of the non-convexity of the set of uncorrelated stage game pay-offs for the inefficiency result in Proposition 1 to hold. In other words, it must be the case that not all pay-offs in the feasible set $\mathcal{F}$ are attainable in the absence of correlated actions. For public randomization devices to be efficiency improving, they must have a role: that of convexifying. In Section 3.3, we show that colour in a random-matching environment serves this purpose, which will lead to Pareto improvements.

3.3. *Characterization of the best colour-blind equilibrium*

In order to characterize the set of best colour-blind equilibrium strategies $s^* = \{\sigma^*, P^*\}$, we find the solution to the problem

$$s^* \in \arg \max_{s \in [0,1] \times \{0,1\}^3} v(s),$$
$$\text{s.t. } v(s) \leq \overline{V},$$

and in addition, indifference between the pay-off of playing C or D if $\sigma \in (0, 1)$. Given the inequality constraint, and given, from subgame perfection, that the continuation probabilities are restricted to lie in $\{0, 1\}^3$, the solution s^* is obtained by exhaustive comparison of each of the resulting cases. We describe an illustrative example in what follows. Let $g = 0{\cdot}5, \delta = 0{\cdot}8$. Parameterized by the value of $l \in (0, \infty)$, we compare the pay-off of all different strategies $s = \{\sigma, P\}$ (Table 1).

For this particular example, the incentive constraint upon cooperation implies that no pay-off be higher than $\overline{V} = 1 - g\frac{(1-\delta)}{\delta} = 0{\cdot}875$. Therefore, conditional on not exceeding $\overline{V}$, the best colour-blind equilibrium strategy $\{\sigma^*, P^*\}$ is the strategy that yields the highest pay-off $v(\sigma, P)$

TABLE 1

Equilibrium pay-offs for different strategies

$\{\sigma, P\}$	$v(\sigma, P)$
$\{0,(0,0,1)\}$	$0{\cdot}8$
$\{\sigma,(1,0,0)\}$	$0{\cdot}13l+0{\cdot}44+0{\cdot}06\sqrt{(4l^2-68l+49)}$
$\{\sigma,(1,1,0)\}$	$1{\cdot}0625-0{\cdot}125l-0{\cdot}0125\sqrt{(185+60l+100l^2)}$
$\{\sigma,(1,0,1)\}$	σ noninterior
$\{\sigma,(0,1,0)\}$	$0{\cdot}667-0{\cdot}167l+(0{\cdot}167l+0{\cdot}417)\frac{1}{8(2l+5)}\left(29+6l-\sqrt{(201+252l+100l^2)}\right)$
$\{\sigma,(0,0,1)\}$	σ noninterior
$\{\sigma,(0,1,1)\}$	σ noninterior

Parameter values: $g=0{\cdot}5, \delta=0{\cdot}8$.

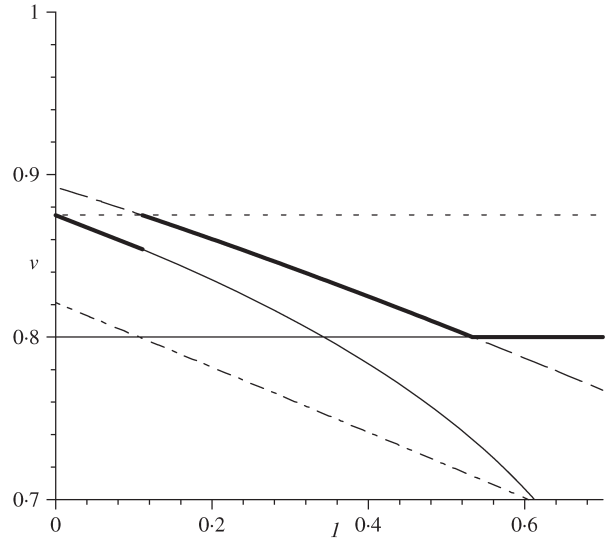

FIGURE 1

Characterization of the best colour-blind equilibrium pay-off $v(\sigma, P)$ given l ($g=0{\cdot}5, \delta=0{\cdot}8$)

given l and satisfies

$$\{\sigma^*, P^*\} = \begin{cases} (\sigma,(1,0,0)) & \text{if } l \in (0, 0{\cdot}11) \\ (\sigma,(1,1,0)) & \text{if } l \in (0{\cdot}11, 0{\cdot}53) \\ (0,(0,0,1)) & \text{if } l \in (0{\cdot}53, +\infty). \end{cases}$$

The characterization of the best colour-blind equilibrium pay-offs for these parameter values is illustrated in Figure 1. In the figure the downward sloping dashed line represents the colour-blind pay-off $v(\sigma,(1,1,0))$, the pay-off from the incubation strategy $v(0,(0,0,1)) = 0{\cdot}8$, the solid downward sloping line is $v(\sigma,(1,0,0))$, and the dot-dashed line is $v(\sigma,(0,1,0))$. The best constrained efficient pay-off is $\overline{V} = 0{\cdot}875$. The thick solid line is the upper envelope $v(\sigma^*, P^*)$ for different l.

Observe here the non-genericity of the constrained efficient colour-blind equilibrium. For the given parameters, it is the case that when $l = 0{\cdot}11$, $v(\sigma,(1,1,0)) = \overline{V}$. However, a minor change in l implies that $v(\sigma^*, P^*) < \overline{V}$. For l slightly larger than $0{\cdot}11$, $v(\sigma,(1,1,0))$ decreases. For l slightly smaller than $0{\cdot}11$, $v(\sigma,(1,1,0))$ increases, thereby violating the incentive constraint and rendering the best colour-blind equilibrium to be $v(\sigma^*, P^*) = v(\sigma,(1,0,0)) < \overline{V}$.

4. SEGREGATION

Consider the economy as defined above. From Section 3 it is clear that partners need some punishment in the first period in order to sustain cooperation in the later phase, but for colour-blind equilibria, the punishment is typically not fine-tuned. Playing the incubation strategy is too costly relative to the constrained efficient outcome. And while playing a mixed strategy combined with the adequate continuation probability may actually improve on the incubation strategy (*e.g.* for low enough l), it does not obtain the constrained efficient outcome for a generic partnership-matching economy. The reason is that the minmax condition making the player indifferent between playing C or D necessarily puts positive weight on the off-the-diagonal pay-off combination $(1+g,-l)$. That is inefficient given this pay-off is dominated by (C,C). Ideally, players would like to coordinate their actions on playing (D,D) with the "appropriate" probability to make the punishment sufficiently big to avoid deviation, while still obtaining the highest feasible pay-off. Mixing allows for some fine-tuning of the pay-off, but it cannot fully perform the role of correlation. In this section, we show that under random matching, partners can use the pay-off-irrelevant characteristic as a coordination device. In the remainder of this section, we first analyse the efficiency properties of segregation. We then consider some simple segregation strategies for populations with two types (black and white), and we analyse the impact of inequality in the distribution of types and the effect of minority bias where preferential treatment is given to the minority.

4.1. *Efficiency*

Starting from the best colour-blind equilibrium strategy s^*, we show in this section that there always exist distributions of types and segregation strategies s^S that Pareto dominate the best colour-blind equilibrium pay-off. This amounts to showing sufficient conditions under which this is the case. We will, therefore, restrict attention to environments where distributions are symmetric for the efficiency result. The symmetry (*i.e.* $\pi(c)=\pi$ for all c) simplifies the proof because the distribution of different types c within the pool of singles is identical to the distribution of types in the entire population. This holds even for agents playing the same but type-dependent segregation strategies. Below we also analyse economies where agents play type-dependent strategies in the absence of symmetric distributions of types, inducing different stationary distributions of singles.

The main efficiency result is that there always exist distributions such that segregation strategies Pareto dominate the best colour-blind equilibrium outcome. The basic idea is simple: depending on the match, partners will play C immediately (*e.g.* when both are of the same type), and they will continue to play the best colour-blind strategy s^* otherwise. For example, in the case where the best colour-blind strategy is the incubation strategy, you play C immediately when your partner is of the same type (which happens with probability π), and you stick to the incubation strategy in all other cases. This segregation strategy s^S is Pareto dominant because $v(s^S)=\pi\cdot 1+(1-\pi)v(s^*)>v(s^*)$. Moreover, for small enough π, this pay-off will not violate the incentive constraint $\overline{V}$.

The proof for any generic economy (g,l,δ) is similar, though somewhat more involved when the best colour-blind strategy involves an interior σ. This is due to the minmax condition, which implies that the exact same strategy s^* cannot be played conditional on a match (say, with a partner of another type). This, in turn, follows from the fact that when a match is separated, the continuation pay-off is not $v(s^*)$ but the pay-off of playing the segregation strategy $v(s^S)$. The main implication is that to find a sufficient condition for the segregation strategy to be Pareto dominant, the segregation strategy will change for different economies and it will typically depend on what the best colour-blind strategy is.

The proof conjectures two different segregation strategies, denoted as s_1^S and s_2^S, and it shows that depending on the economy, one of these two strategies can always generate a Pareto improving equilibrium outcome for adequate distributions of types. The first segregation strategy prescribes playing C forever in a match with a partner of the same colour and playing the strategy corresponding to the best colour-blind strategy s^* in all other matches. The second segregation strategy prescribes playing D in a same-colour match and separating thereafter, and playing the strategy corresponding to the best colour-blind strategy s^* in all other matches.[14] Because the latter strategies (in mixed matches) differ from $s^* = (\sigma^*, P^*)$ in the mixing probability σ (in order to satisfy the minmax condition) but not in separation probability P, we denote these strategies by $s_1^* = (\sigma_1^*, P^*)$ and $s_2^* = (\sigma_2^*, P^*)$, respectively. First, we state the result (the proof is in the Appendix), then we illustrate using an example.

Proposition 2. *Consider the best colour-blind equilibrium strategy s^* of a generic partnership-matching economy (g, l, δ). Then there exist symmetric distributions $\pi(c)$ and a segregation strategy pay-off $v(s^S)$ that Pareto dominates the best colour-blind equilibrium outcome $v(s^S) > v(s^*)$.*

Proof. In Appendix. ‖

Because these segregation strategies rely on the play of the best colour-blind equilibrium, they imply that there is always cooperation and repeated interaction with positive probability between different types. These segregation strategies are, therefore, inclusive in the sense that they maintain the same interaction between different types as in a colour-blind equilibrium. While colour acts as a coordination device in order to enhance efficiency, it is also costly in terms of efficiency to separate matches. Therefore, these strategies maintain the same separation rate as in the case of colour-blind strategies. Below we elaborate on segregation strategies that are exclusive in the sense that in equilibrium there is neither cooperation nor repeated interaction between different types.

Revisiting the earlier example provides an insight into the result in Proposition 2. The l parameter space (the positive reals) was partitioned into three subsets, each corresponding to a different s^*. Consider one value in each of the partitions: $l = 0{\cdot}05, 0{\cdot}5, 1$ and the respective strategies s^*. The pay-offs are reported in Table 2, and values are calculated for $\pi = \frac{1}{2}$ and $\frac{1}{3}$.

1. Most straightforward is the case $l = 1$, where the strategy s^* corresponds to the incubation strategy. In that case, $s_1^* = s_2^* = s^*$, and $v(s_1^S)$ is simply the weighted sum of 1 and $v(s^*) : v(s_1^S) = \pi \cdot 1 + (1-\pi)v(s^*)$. Here s_1^S is clearly Pareto improving (whereas s_2^S is not, since $v(s_2^S) = [\pi\delta + (1-\pi)]v(s^*) < v(s^*)$), but there is an upper bound on the distribution of types imposed by the incentive constraint. This is illustrated in the third line of Table 2, where for $\pi = 1/2$ the incentive constraint under s_1^S is violated.

2. When $l = 0{\cdot}5$, it turns out that s_1^S is also the Pareto improving segregation strategy. Here, however, $v(s_1^*)$ is not equal to $v(s^*)$. To see this, observe that the pay-off from playing C is

$$v(s_1^*) = \sigma_1^* + (1-\sigma_1^*)[-l(1-\delta) + \delta],$$

which, since σ is interior, must be equal to the expected pay-off from playing D:

$$v(s_1^*) = \sigma_1^*[(1+g)(1-\delta) + \delta] + (1-\sigma_1^*\delta v(s_1^S)).$$

14. This strategy implies discrimination *against* the own type. Notice, however, that exactly the same pay-off would result if each type discriminates against exactly one other type (as long as there is an even number). For example, let $\pi = \frac{1}{4}$ with types c_1, c_2, c_3, c_4 and suppose type c_1 plays D when matched with c_2 (and vice versa), and c_3 and c_4 play D, and in all other matches s^* is played.

TABLE 2

Equilibrium pay-offs for segregation strategies

			$v(s_1^S)=\pi+(1-\pi)v(s_1^*)$		$v(s_2^S)=[\pi\delta+(1-\pi)]v(s_2^*)$	
l	s^*	$v(s^*)$	$\pi=1/2$	$\pi=1/3$	$\pi=1/2$	$\pi=1/3$
0·05	$\{\sigma,(1,0,0)\}$	0·8658	σ noninterior	0·8472	**0·8737**	**0·8716**
0·5	$\{\sigma,(1,1,0)\}$	0·8063	**0·8631**	**0·8419**	0·7596	0·7766
1	$\{0,(0,0,1)\}$	0·8	0·9	**0·8667**	0·72	0·6133

Parameter values: $g=0{\cdot}5, \delta=0{\cdot}8$; bold pay-offs are Pareto dominant *and* satisfy the incentive constraint.

The mixing probability σ_1^* and the pay-off $v(s_1^*)$ differ from σ^* and $v(s^*)$, respectively, because in the best colour-blind equilibrium, the pay-off from playing D satisfies $v(s^*)=\sigma^*[(1+g)(1-\delta)+\delta]+(1-\sigma^*)\delta v(s^*)$. The expected pay-off tomorrow after separation (*i.e.* after (D,D)) is exactly the same as the expected pay-off today. With this segregation strategy, however, the pay-off after (D,D) is $v(s_1^S)$, that is, with positive probability a partner of the same type will be met, therefore inducing cooperation and a different (higher) pay-off. The outside option of rematching has improved relative to that of the colour-blind strategy, thereby changing the mixing probability. Notice in Table 2 (second line) that $v(s_1^S)$ is increasing in π, and that s_2^S is not Pareto improving.

3. For $l=0{\cdot}05$, the segregation strategy s_2^S is Pareto improving, while s_1^S is not. This may seem counterintuitive, as conditioning the strategy on the partner's type by playing D and separating automatically imply delaying the pay-off from playing s^* (which itself is not constrained efficient). However, the minmax condition induces players to put more weight on the probability of playing C (*i.e.* σ_2^S increases), thereby increasing the pay-off $v(s_2^*)$. In other words, the threat of matching with someone of the same type induces sufficiently more cooperative behaviour in mixed matches, resulting in a higher pay-off of adhering to s_2^S. Again, as is apparent in the first line of Table 2, the pay-off increases with larger π.

It is important to note that the symmetry in the distribution, used to prove Proposition 2, is not a necessary requirement for a distribution and strategies to exist that Pareto dominate the best colour-blind equilibrium. The symmetry is a sufficient condition that simplifies the proof, as the distribution of types in the pool of singles is identical to the distribution in the entire population. Alternatively, consider an only slightly asymmetric distribution, in which case there remains an increase in the pay-offs.

4.1.1. Age-dependent strategies. Proposition 2 establishes that there always exists a segregation strategy that is Pareto improving over the best colour-blind segregation strategy. One feature of that result and the way it is established is that continuation pay-offs $v(s_i^*)$ are endogenous, implying in turn, that the Pareto improving segregation strategy differs depending on the parameters of the economy. We will now briefly show, however, that this need not be the case and that one simple segregation strategy is Pareto improving as long as one additional assumption is made: that agents can observe the age of the partner. If players can distinguish between newly entered singles and older ones, they can play the following segregation strategy: when two newly born singles of the same colour meet, they play C forever; in any other partnership, they play the best colour-blind equilibrium strategy s^*.

By restricting attention to strategies where actions are conditioned on whether a single is new or old, the continuation pay-offs are exactly the same as under colour-blind strategies $v(s^*)$. Now because strategies depend on the fraction of newly born singles, the measure of singles is endogenous to the strategy used. In the Appendix, we derive the stationary measure of singles when

the best colour-blind strategy is used, denoted by M, and similarly when segregation strategies are used, denoted by M^S.

Consider as a benchmark the best colour-blind strategy s^*. From Proposition 1, for any generic partnership economy, even the best colour-blind strategy is inefficient $v(s^*) < \overline{V}$. The expected pay-off from the segregation strategy described above is

$$v(s^S) = \pi' + (1-\pi')v(s^*),$$

where $\pi' = \pi/M^S$ and $1/M^S$ is the stationary fraction of new entrants in the market of singles, with M^S satisfying (A.9). Notice that $1/M^S$ depends on the continuation probabilities in the colour-blind equilibrium strategy. With probability π' a new entrant meets another new entrant of the same type, in which case both play C forever yielding a normalized pay-off of 1. With complementary probability, the new entrant meets someone of the other type or an old single, in which case both play the best colour-blind strategy s^*, with a (type-independent) pay-off of $v(s^*)$. Notice that because this segregation strategy prescribes type-dependent play only in the period of entry in the matching market, the expected continuation pay-off in the case of separation is $v(s^*)$ and independent of c. As before, for any positive π (and therefore π'), $v(s^S)$ Pareto dominates $v(s^*)$. Moreover, given that s^* is generically constrained inefficient, there exists a π small enough such that the segregation strategy satisfies the incentive constraint.

Return to the earlier example with the same parameter values ($g = 0{\cdot}5, \delta = 0{\cdot}8$), and let $l = 0{\cdot}5$. Then, as derived above, the best colour-blind equilibrium strategy is to mix in the first period and separate the match only if both players played D (continue in all other cases): $s^* = (\sigma, (1,1,0))$, where $\sigma = 0{\cdot}3545$. The equilibrium pay-off is $v(s^*) = 0{\cdot}8063$ and constrained inefficient. Now we can calculate the expected equilibrium pay-off from adhering to the segregation strategy described above, given a symmetric distribution of types: $v(s^S) = \pi' + (1-\pi')v(s^*)$, where $\pi' = \pi/M^S$ and M^S is derived from (A.9). The expected pay-off of this segregation strategy is always strictly larger for any π, and there is no incentive for deviation as long as $v(s^S) \leq \overline{V}$. It is immediately verified that this condition holds for any symmetric distributions with two types or more. For example, for a distribution with two types, the equilibrium pay-off $v(s^S) = 0{\cdot}8238$ is both incentive compatible and Pareto dominates the best colour-blind equilibrium pay-off.

4.1.2. The most efficient symmetric distribution. When restricting attention to age-dependent strategies, the symmetric distribution that achieves the highest pay-off is that with the smallest number of groups provided the incentive constrained is not violated. If $\overline{\pi}$ is defined such that $v(s^S, \overline{\pi}) = \overline{V}$, where $v(s^S, \overline{\pi}) = \overline{\pi}/M^S + (1-\overline{\pi}/M^S)v(s^*)$, then it is immediate that the highest pay-off is achieved for a symmetric distribution with the smallest number of groups $\widehat{N}$ of different types where $\pi = 1/N$, $\widehat{N} = \{N \in \mathbb{N}_{++} : N \in \arg\max\{1/N\} \text{ and } 1/N \leq \overline{\pi}\}$. Because of the increased complexity of date-independent strategies, the answer there is not immediate. However, the same is true for those economies where the incubation strategy is the best colour-blind equilibrium strategy: the symmetric distribution achieving the highest pay-off is that with the smallest number of groups such that the incentive constraint is not violated.

4.2. *Minorities*

In this section, we analyse minorities in the framework of a partnership economy while concentrating on distributions with two types. With only two types the existence of a minority implies an asymmetric distribution. Let $c \in \{b, w\}$ indicate the colour of an agent, either "black" or "white". As before, the fraction $\pi(c)$ is the proportion of the population of players of colour c, such that $\pi(b) + \pi(w) = 1$. Without loss of generality, we assume that the black population is the minority: $\pi(w) \geq \frac{1}{2} \geq \pi(b)$. Because of the asymmetry in the distribution, in the presence of a

type-dependent strategy, the stationary distribution of singles differs from the stationary population distribution. Denote, therefore, by $\Pi(c)$ the proportion of types c that are single, given there is a proportion $\pi(c)$ in the population. Wherever appropriate we will use the shorthand notation $\pi(w) = \pi$ and $\Pi(w) = \Pi$ (and $\pi(b) = 1 - \pi$, $\Pi(b) = 1 - \Pi$).

With just two types, size matters, and in general we typically do not obtain Pareto dominance using segregation strategies. The segregation strategies shown in the former section generated Pareto dominant outcomes for small enough groups of each type. With only two types, the size of the majority may, in general, not be small enough. First, we address the issue of inequality in the distribution of types and its effect on efficiency when strategies are symmetric. Then, we consider asymmetric strategies and analyse their effect on welfare. Finally, for sufficiently asymmetric distributions and in the absence of Pareto improving segregation strategies, we show that exclusive segregation strategies make the majority better off while at the same time making the minority worse off.

4.2.1. Inequality and efficiency. Symmetry in the distribution and therefore equality between the different groups is good for efficiency because it leaves the possibility open for Pareto improving segregation equilibria to also be constrained efficient. As soon as there is some inequality, such constrained efficiency cannot be achieved with symmetric type-dependent strategies. These strategies are match specific, but conditional on the match (either same colour or mixed), they prescribe the same behaviour.

Proposition 3. *Whenever $\pi \neq \frac{1}{2}$, an equilibrium involving symmetric type-dependent strategies is constrained inefficient.*

Proof. In Appendix. ‖

Consider the class of economies with $l > l^*$, where the best colour-blind equilibrium strategy is the incubation strategy,[15] with $v(s^*) = \delta$. Then a strategy with cooperation in a same-type match and s^* in mixed matches yields the following pay-offs:

$$v_w = \pi + (1 - \pi)\delta$$
$$v_b = (1 - \pi) + \pi\delta$$

(because no matches are separated, $\pi = \Pi$). Since $\pi > \frac{1}{2}$, it follows that $v_w > v_b$. So even if $v_w = \overline{V}$, the equilibrium will never be efficient since $v_b < \overline{V}$. In other words, symmetric type-dependent strategies cannot achieve constrained efficiency and, therefore, relative size matters. We now turn to an asymmetric type-dependent strategy and show that it may be welfare improving relative to the symmetric strategies.

4.2.2. Minority bias. With asymmetric distributions, efficiency under symmetric type-dependent strategies is ruled out because the majority is treated differently from the minority, while both simultaneously have to satisfy the same incentive constraint. Clearly, when transfers can be made from the majority types to the minority types, there may exist ways to increase aggregate welfare while still satisfying the incentive constraint. Even though this is a game with non-transferable utility, there do exist means by which intertemporal transfers can be made between types. Here we show for a class of economies that asymmetric strategies with a bias towards the minority are welfare improving relative to the symmetric segregation strategy. Minority

15. It can be verified that for colour-blind equilibria with interior σ, the probability that C is played is decreasing in l, and that σ becomes negative for large enough l. There exists, therefore, an l^* such that for all $l > l^*$ the pay-off of the incubation strategy dominates any other strategy.

bias strategies are those strategies that give the black partner an advantage at the beginning of a mixed match. While minority bias is welfare improving, it does not Pareto dominate the symmetric segregation outcome, as the majority types are made worse off. On the other hand, minority bias does Pareto dominate the best colour-blind equilibrium outcome.

Minority bias can be achieved exploiting the fact that in mixed matches, different pay-offs in the feasible set can be reached.[16] In particular, we will consider the case where in mixed matches, in the first stage, the white player always plays C while the black player plays D. This yields the asymmetric pay-off pair $(-l, 1+g)$. In the second phase, play switches to (C, C). The pay-offs to whites and blacks, respectively, are

$$v_w = \pi + (1-\pi)(-l(1-\delta)+\delta) \tag{4}$$

$$v_b = (1-\pi)\delta + \pi[(1+g)(1-\delta)+\delta]. \tag{5}$$

Because we will concentrate on economies with $l > l^*$, for b types the pay-off is always strictly better in a mixed match, while the w types are always strictly worse off. These pay-offs must satisfy the incentive constraints, and for black–black matches, there will, therefore, be an incubation period: (D, D) is played in the first phase. Whites always play C with all partners. This strategy has whites "acting in good faith", even though they know they will suffer in a mixed match. This strategy will constitute an equilibrium in which the minority benefits from the cooperation in repeated matches, *and* it stops whites from deviating as the mixed matches provide a punishment. We can then establish the following result.

Proposition 4. *There are distributions and economies for which the minority bias equilibrium is welfare improving relative to the symmetric type-dependent segregation equilibrium.*

Proof. In Appendix. ‖

4.2.3. Minorities, exclusive segregation, and conflict. More equality in the size of each group can improve efficiency (Proposition 3). Moreover, economies with relatively equal distributions of types and where Pareto improving segregation strategies satisfy the incentive constraint do not necessarily have such an equilibrium when distributions are more unequal. When π increases, Pareto improving segregation strategies eventually violate the incentive constraint for the majority. Nonetheless, other segregation strategies do exist for sufficiently asymmetric distributions. Because those are not guaranteed to be Pareto improving, the minority may be made worse off.

Consider exclusive segregation strategies. They are symmetric type-dependent strategies in which there is neither cooperation nor repeated interaction in mixed matches: play C forever in a same-type match and play D and rematch in a mixed match. Then the expected pay-off is given by $v(s^S) = \Pi(c) + (1-\Pi(c))\delta v(s^S)$ or equivalently

$$v(s^S) = \frac{\Pi(c)}{1-(1-\Pi(c))\delta}.$$

The stationary distribution of singles is now endogenous with $\pi \geq \Pi \geq \frac{1}{2}$, where $\Pi = \frac{(-\pi+\sqrt{\pi(1-\pi)})}{(1-2\pi)}$ (see Appendix for the derivation). Note that Π is strictly increasing in π and

16. Observe that this strategy highlights a different function played by the pay-off-irrelevant characteristic. Here, in a mixed match, the particular match of characteristics can substitute for role identification. Even though by assumption players do not know whether they are the row or column player (trivially implying that first-period strategies of colour-blind equilibria are symmetric), in a mixed match actions can be conditioned on their type.

that the majority in a stationary equilibrium remains the majority among the population of new matches: that is, if $\Pi > \frac{1}{2}$ then $\pi > \frac{1}{2}$.

Compared with the inclusive segregation strategies employed to show Pareto improving outcomes, this exclusive equilibrium outcome is Pareto dominated. To see this, consider, for example, the majority. Conditional on the incentive constraint being met, exclusive segregation strategies can never achieve as high a pay-off. Observe that for the majority $v(s^S) < \pi + (1-\pi)\delta$, since $\Pi < \pi$ and $v(s^S) < 1$. In contrast, the pay-off from using a Pareto improving inclusive segregation strategy is at least as large as $\pi(c) + (1-\pi(c))\delta$.[17] As a result, provided the incentive constraint holds, the pay-off for the majority from the inclusive segregation strategy is always larger than that of the exclusive segregation strategy.

When π is large, the pay-off from the inclusive segregation strategy may not exist as the pay-off violates the incentive constraint. Exclusive segregation is still an equilibrium, but now for large π, it no longer Pareto dominates the colour-blind equilibrium strategy, and the minority is worse off under segregation than under colour-blind equilibrium.

When there is sufficient inequality in the distribution of types, the exclusive segregation equilibrium does not Pareto dominate the best colour-blind equilibrium. To see this, we need to verify the following. First, the incentive constraint holds for the majority types: $\frac{\Pi}{1-(1-\Pi)\delta} < \overline{V}$, which is satisfied for all $\Pi < \Pi_0 = 1 - \frac{g}{(1+g)\delta}$. Second, the pay-off of the minority types is less than that of colour-blind strategy equilibrium. The best colour-blind strategy has a pay-off of at least $v(s^*) \geq \delta$. Then the pay-off to the minority is less than that of the colour-blind equilibrium, provided $\frac{1-\Pi}{1-\Pi\delta} < \delta$, or $\Pi > \Pi_1 = \frac{1}{1+\delta}$. Therefore, for all $\Pi \in (\Pi_1, \Pi_0)$, the exclusive segregation strategy yields a lower pay-off for the minority than the colour-blind strategy. For a low enough g ($g < \frac{\delta^2}{1+\delta-\delta^2}$), there exists an open set of distributions for which the minority is worse off.

5. DISCUSSION

5.1. *On the informational assumption*

One might argue that it is innocuous to assume that agents have access to a public randomization device. There are many environments where correlation devices have been extremely successful (the use of traffic lights at road crossings being the most prominent example) and flipping a coin between two agents is costless.

The point in this paper is precisely that a correlation device is used, specifically one that is endogenous in the model (*i.e.* derived from the population distribution). Moreover, it is used in such a way that strategies are simple and nearly rule of thumb. In a decentralized trading environment, it seems plausible to rely on such a device rather than on a centralized (*i.e.* provided by a planner) correlation device (*e.g.* traffic lights).

The most prominent criticism to our endogenous public randomization device then comes from cheap talk extensive form extensions of the stage game. Forges (1990) shows that any solution that can be achieved by an arbitrary communication mechanism is a correlated equilibrium pay-off of the game, a principle that applies to our game as well. However, the result holds in general for games of four players or more. While with mediated communication (*e.g.* through a planner) the same result holds for even a two-player partnership game like ours, it seems natural to assume that communication in this decentralized environment is unmeditated.

For the sake of completeness, we derive the constrained efficient equilibrium outcome when partners dispose of an exogenous public randomization device prior to choosing actions. The public randomization device is observed by both partners within a match and in each period t. Let

17. The best colour-blind strategy yields a pay-off that is no smaller than that of the incubation strategy $v(s^*) \geq \delta$.

q be a random variable distributed uniformly over $[0, 1]$. Then, consider the following strategy where both players play C only if q is below a threshold level $\overline{q}$. Otherwise, they play D and terminate the match. Before being matched, the expected continuation pay-off is $v = \overline{q} + (1 - \overline{q})\delta v$, implying

$$v = \frac{\overline{q}}{1 - (1 - \overline{q})\delta}.$$

Conditional on expecting the other player to play C, the expected continuation pay-off given this strategy profile is 1. This strategy is robust to no deviation provided $v \leq \overline{V} = 1 - g\frac{1-\delta}{\delta}$, implying

$$\overline{q} \leq 1 - \frac{g}{(1+g)\delta}.$$

The highest possible pay-off that can be achieved using this strategy is where this condition holds with equality.

5.2. *Efficiency as a positive theory?*

One of the main objectives of this paper is to provide a theoretical analysis of the marriage market in light of segregation. The centrepiece of the analysis is the study of the efficiency properties of both colour-blind and segregation equilibria. An open question remains whether efficiency of equilibrium necessarily provides a positive theory of decentralized behaviour and norms in society. There are three relevant aspects related to this debate. First, the analysis here focuses on non-cooperative equilibria, that is, each of the equilibrium strategies is robust to unilateral deviations. While there may be reasonable grounds on which to consider cooperative equilibria (equilibrium strategies that are robust to deviations by coalitions) as well, this is beyond the scope of the current paper. Here we take the view that as far as a non-cooperative equilibrium exists, it is a robust prediction of the behaviour of rational agents. Second, when multiple equilibria exist, economic theory in general (and game theory in particular) has frustratingly little predictive power. From our first point, any equilibrium provides a robust prediction of behaviour, but in the presence of multiple equilibria, non-cooperative equilibrium theory does not provide us with guidance on the selection of any one of the equilibria. Of course, mapping the set of equilibria and their efficiency properties, which is achieved in this paper, is of immediate theoretical importance. The efficiency properties are also of positive importance, even without a prediction of behaviour in a decentralized world, because they allow us to understand the properties of different observed outcomes. Third, the efficiency properties are important from a policy perspective. While decentralized behaviour may lead to any of the multiple equilibrium outcomes, policies (that either affect pay-offs directly or that intend to coordinate equilibrium beliefs) may be adopted that achieve more efficient outcomes. Before contemplating policy intervention, knowledge of the efficiency properties is not only desired but necessary to justify the intervention. In particular, in the case of the minority bias equilibrium studied in Section 4.2.2, achieving this outcome increases efficiency relative to another segregation equilibrium outcome. Imagine an economy in which the latter segregation equilibrium is observed. In such a scenario, there is a role for a benevolent government in achieving the minority bias equilibrium. On the other hand, in the absence of such a benevolent government, majority voting may help explain why such minority bias equilibrium is not obtained. The majority is worse off under minority bias, and the majority of self-interested voters will vote against adopting policies that lead to minority bias. A political economy model may, therefore, help shed further light on the predictive power of this theory.

One way to increase the predictive power of behaviour in the presence of multiple equilibria is to consider equilibrium refinements. For example, renegotiation proofness is a refinement that typically aims to select those equilibria in which the continuation path outcomes Pareto dominate.

In our paper, equilibrium strategies require a punishment involving termination of the partnership if either party deviates. In the current set-up, renegotiation will always break down equilibrium. After any action, renegotiation rules out all $P_i = 0$ and only admits $P_i = 1, i \in \{1, 2, 3\}$, which does not induce cooperation as a best response. The same applies if the equilibrium concept were cooperative, in the sense that deviations by coalitions of individuals are also considered. In particular, any partnership would always gain if it were to coordinate its actions in order to start cooperation immediately (an individual partnership has zero mass in the population). However, there is an existence problem: equilibrium with pairwise deviations does not exist, since all partnerships will start cooperating immediately, making cooperation in the later stages no longer incentive compatible. Ghosh and Ray (1996) manage to construct a renegotiation proof equilibrium in a similar but slightly altered environment. They endow a fraction of the population with myopic preferences and show that when players have incomplete information about their partners, equilibrium can be shown to be renegotiation proof.

6. CONCLUDING REMARKS

This paper provides a theoretical analysis of the role of segregation in a marriage-matching model. First, we have characterized the set of colour-blind equilibria as a benchmark. Then, we have considered strategies that depend on a pay-off-irrelevant characteristic like colour. In a dynamic random-matching environment, colour acts as an endogenous public randomization device, and the correlated equilibrium may achieve more efficient outcomes than an equilibrium without segregation.

For given distributions and simple segregation strategies, there may also be a conflict of interest between the majority and the minority. The majority unambiguously gains from the increased coordination, but the minority may be worse off. A policy of minority bias can both increase the welfare of the minority and improve overall efficiency. However, the majority is worse off under this policy. Both the conflict of interest and the Pareto dominance of segregation equilibria are absent in the standard statistical discrimination model (Arrow, 1973). This paper, therefore, sheds new light on this phenomenon in the marriage market, where segregation is prevalent.

Several assumptions made in this model deserve to be extended or reconsidered. First, it is assumed that meetings are random. While this is certainly an important part of the story, it is also the case that in reality agents *choose* which pool of potential partners to meet through directed search. Such search behaviour may generate segregation in addition to the segregation that arises from repeated matching: whites may search for social interaction only with other whites. This type of directed search is certainly a promising path for future research. Of course, as long as the search process involves some randomness, the post-match segregation will remain. Second, one may want to consider a more sophisticated model of the marriage market. In particular, for empirical purposes, it is desirable to introduce heterogeneity as in Burdett and Coles (1997) in order to account for selection.

APPENDIX

A.1. *Proof of Proposition* 1

In order to prove Proposition 1, we will introduce the following notation. Given a strategy s, denote the pay-off of playing C by $F(s)$ and the pay-off from playing D by $G(s)$. Then the expected continuation pay-off for a newly matched partner at the start of the game is

$$v(s) = \sigma F(s) + (1-\sigma)G(s), \tag{A.1}$$

where

$$F(s) = \sigma[1-\delta+\delta(P_1+(1-P_1)v(s))]+(1-\sigma)[-l(1-\delta)+\delta(P_2+(1-P_2)v(s))] \tag{A.2}$$

$$G(s) = \sigma[(1+g)(1-\delta)+\delta(P_2+(1-P_2)v(s))]+(1-\sigma)\delta(P_3+(1-P_3)v(s)). \tag{A.3}$$

First, in Lemma A1 and A2, we establish two results that apply to colour-blind equilibria and that are used in the proof of Proposition 1 that continuation probabilities are either 0 or 1 and never interior and that there is no colour-blind equilibrium where C is played with certainty.

Lemma A1. *In equilibrium,* $P \in \{0,1\}^3$.

Proof. Consider any candidate equilibrium strategy s with expected pay-off $v(s) \leq \overline{V}$. After any history (C,C), (C,D), (D,D), a player's choice of P_j either leads to the cooperation phase with continuation pay-off of 1 or to the pay-off from rematching $v(s)$. Denote P_j^i the continuation probability, $j \in \{1,2,3\}$ of player i, and P_j^{-i} of the opponent. Then the expected continuation pay-off is $P_j^i(P_j^{-i} + (1-P_j^{-i})v(s)) + (1-P_j^i)v(s)$. That is, only if both choose $P_j = 1$ (with probability $P_j^i \cdot P_j^{-i}$) will a player get to the cooperation phase with pay-off 1. In all other cases the pay-off is $v(s)$. Subgame perfection requires that the choice of (P_j^i, P_j^{-i}) constitutes a Nash equilibrium in this subgame. The Nash equilibria of this subgame are either $(P^i, P^{-i}) = (0,0)$ or $(1,1)$ (there is a "third" Nash equilibrium in mixed strategies with degenerate mixing probabilities which coincides with the pure strategy equilibrium $(0,0)$). ‖

In a subgame after the actions of the partner in the partnership game are observed, players simultaneously choose continuation probabilities. Lemma A1 establishes that if a player believes the other will continue the partnership, then the best response is to continue, and hence $(P_j^i, P_j^{-i}) = (1,1)$ is a Nash equilibrium in the subgame. Equally, if a player believes the opponent will terminate, then a player will get $v(s)$ whatever she/he chooses. Therefore, it is an equilibrium if both choose to terminate, hence $(P_j^i, P_j^{-i}) = (0,0)$. Notice that if any player will continue with positive probability, then the best response is to continue with probability 1, since continuing always obtains a higher utility than terminating. This also explains why no interior mixed strategy equilibrium exists.

Lemma A2. *There exists no colour-blind equilibrium with* $\sigma = 1$.

Proof. Observe that in case $\sigma = 1$, the pay-off from playing C is $v(s) = F(s) = 1-\delta+\delta(P_1 + (1-P_1)v(s))$ or

$$v(s) = \frac{1-\delta+\delta P_1}{1-\delta(1-P_1)} = 1$$

for all P_1. Since $\overline{V} = 1 - g\frac{1-\delta}{\delta} < 1$ for all $\delta < 1$, it follows that $\sigma = 1$ is never robust to deviations in the cooperation phase. ‖

This result is fairly intuitive. If all players will play C from the start, then in the presence of anonymous matches, there is no punishment for deviators. Once someone has deviated (and reaps the short run gain of $1+g$), he or she can match immediately with a new partner, who according to this candidate equilibrium plays C from the start. As a result a dominant strategy is to deviate and rematch. We can now proceed to prove Proposition 1.

Proposition 1. *For a generic partnership-matching economy* (g,l,δ)*, the best colour-blind equilibrium is constrained inefficient:* $v(s^*) < \overline{V}$.

Proof. Consider first the case where σ is not interior. When $\sigma = 0$, then the best equilibrium strategy involves $P_3 = 1$, so that $v(0,P) = G(0,P) = \delta$ ($v = 0$ when $P_3 = 0$). This is an equilibrium strategy, as a deviation entails a strictly lower pay-off $F(0,P) < G(0,P)$. Then, given $g < \delta$, $v(0,P) = \delta < \overline{V} = 1 - g\frac{1-\delta}{\delta}$ for all $\delta < 1$. When $\sigma = 1$, from Lemma A2, we know that no colour-blind equilibrium exists.

Next, suppose $\sigma \in (0,1)$ is interior and that the contrary is true, that is, that an equilibrium exists that is constrained efficient. Then that candidate equilibrium must satisfy

1. Constrained efficiency: $v(s) = \overline{V}$
2. Indifference: $F(s) = G(s)$
3. No deviation: $\overline{V} = 1 - g\frac{1-\delta}{\delta}$.

Recall that the latter equation is derived from $v^D = (1+g)(1-\delta)+\delta v \leq 1$. In the event that $v = \overline{V}$, this holds with equality, and as a result, $(1+g)(1-\delta) = 1-\delta\overline{V}$.

Using constrained efficiency and indifference, we get that

$$\begin{aligned}&\sigma[1-\delta+\delta(P_1+(1-P_1)\overline{V})] + (1-\sigma)[-l(1-\delta)+\delta(P_2+(1-P_2)\overline{V})]\\ &= \sigma[(1+g)(1-\delta)+\delta(P_2+(1-P_2)\overline{V})] + (1-\sigma)\delta(P_3+(1-P_3)\overline{V}),\end{aligned}$$

or solving for σ

$$\sigma = \frac{-l + \frac{\delta}{1-\delta}(P_2 - P_3)(1-\overline{V})}{g - l + \frac{\delta}{1-\delta}(2P_2 - P_1 - P_3)(1-\overline{V})}.$$

Now we can use the fact that

$$\begin{aligned}\overline{V} &= F(s) \\ &= \sigma\left[1 + l + \frac{\delta}{1-\delta}(P_1 - P_2)(1-\overline{V})\right] - l + \frac{\delta}{1-\delta}(P_2 + (1-P_2)\overline{V})\end{aligned} \tag{A.4}$$

and equation (A.2). We substitute for σ and the no-deviation condition in equation (A.4):

$$1 - g\frac{1-\delta}{\delta} = \frac{-l + (P_2 - P_3)g}{g - l + (2P_2 - P_1 - P_3)g}[1 + l + (P_1 - P_2)g] - l + \frac{\delta}{1-\delta} - (1-P_2)g.$$

Rearranging, we obtain a quadratic form in g, l, δ:

$$Ag^2 - gl - Bg^2\frac{\delta}{1-\delta} + Cgl\frac{\delta}{1-\delta} + Ag\left(\frac{\delta}{1-\delta}\right)^2 - l\left(\frac{\delta}{1-\delta}\right)^2 + Dg\frac{\delta}{1-\delta} = 0, \tag{A.5}$$

where $A = 1 + 2P_2 - P_1 - P_3$, $B = 1 + P_2 - P_2^2 + P_1P_3 - P_1 - P_3$, $C = -P_2$, and $D = P_1 - P_2 - 1$ are constants and $A, B, C, D \in \{-2, -1, 0, 1, 2, 3\}$.

Equation (A.5) is a necessary condition for an efficient equilibrium. Generically, an economy (g, l, δ) does not have an efficient equilibrium, that is, there exists an open and dense subset of the space of economies such that equation (A.5) is not satisfied, or alternatively, the solutions to (A.5) are contained in the complement of an open and dense set of (g, l, δ). If in each solution of (A.5) the implicit function theorem applies (the set of solutions for which the implicit function theorem does not apply has a complement that is open and dense), then the set of solutions is a hyperplane in $\mathbb{R}^3$ and therefore its complement is open and dense. ‖

A.2. *Proof of Proposition* 2

Proposition 2. *Consider the best colour-blind equilibrium strategy s^* of a generic partnership-matching economy (g, l, δ). Then, there exist symmetric distributions $\pi(c)$ and a segregation strategy pay-off $v(s^S)$ that Pareto dominates the best colour-blind equilibrium outcome $v(s^S) > v(s^*)$.*

Proof. Consider any generic economy (g, l, δ) and let $s^* = (\sigma, (P_1, P_2, P_3))$ be the best colour-blind strategy. In order to show that a Pareto dominant segregation strategy exists, it is sufficient to provide one segregation strategy s^S, such that the expected pay-off $v(s^S) > v(s^*)$. We conjecture a segregation strategy s^S, and from the necessary conditions, we derive the pay-off $v(s^S; \pi)$ given the symmetric distribution π. For some π we verify Pareto dominance $(v(s^S; \pi) \geq v(s^S; 0) = v(s^*))$ and the feasibility of the incentive constraint $(v(s^S; \pi) \leq \overline{V})$.

Consider first those best colour-blind strategies where σ is not interior. From Lemma A2, we can concentrate attention on the incubation strategy, that is, where $s^* = (0, (0, 0, 1))$. Then $v(s^*) = \delta$. Now consider a segregation strategy s^S where upon a match with a partner of the same type, both play C forever, and in all other matches, they play s^*. Then the expected pay-off of this segregation strategy is $v(s^S) = \pi + (1-\pi)\delta$. First, for any π, pay-offs under segregation are higher $v(s^S) \geq v(s^*)$, therefore satisfying Pareto dominance. Second, for $\pi = 0$, the pay-off is generically inefficient, and there exists π small enough such that $v(s^S) \leq \overline{V}$, thereby satisfying the incentive constraint.

For those best colour-blind strategies with σ interior, we conjecture two segregation strategies s_1^S and s_2^S and show that one of the two will be Pareto improving while satisfying the incentive constraint. Consider the first segregation strategy s_1^S : when in a match with a partner of the same type, play C forever; in all other cases, play the strategy corresponding to the best colour-blind strategy $s^* = (\sigma, (P_1, P_2, P_3))$. Because in equilibrium σ will differ, denote this strategy by s_1^*, with σ_1^*.

For all interior solutions of σ, two necessary conditions are

$$\begin{aligned}v(s_1^*) &= \sigma_1^*[1 - \delta + \delta(P_1 + (1-P_1)v(s_1^S))] + (1-\sigma_1^*)[-l(1-\delta) + \delta(P_2 + (1-P_2)v(s_1^S))] \\ v(s_1^*) &= \sigma_1^*[(1+g)(1-\delta) + \delta(P_2 + (1-P_2)v(s_1^S))] + (1-\sigma_1^*)\delta(P_3 + (1-P_3)v(s_1^S)),\end{aligned}$$

the pay-offs from playing C and D, respectively. These conditions imply:

$$\sigma_1^* = \frac{-l + \frac{\delta}{1-\delta}(P_2 - P_3)(1 - v(s_1^S))}{g - l + \frac{\delta}{1-\delta}(2P_2 - P_1 - P_3)(1 - v(s_1^S))},$$

and

$$v(s_1^*) = \sigma_1^*[(1+g)(1-\delta) + \delta(P_2 - P_3)(1 - v(s_1^S))] + \delta(P_3 + (1-P_3)v(s_1^S)).$$

Following the segregation strategy s_1^S, the expected pay-off $v(s_1^S) = \pi + (1-\pi)v(s_1^*)$, which gives rise to the implicit function $\Phi_1(v(s_1^S), \pi) = 0$, given by

$$-v(s_1^S) + \pi + (1-\pi)\{\sigma_1^*[(1+g)(1-\delta) + \delta(P_2 - P_3)(1 - v(s_1^S))] + \delta(P_3 + (1-P_3)v(s_1^S))\} = 0.$$

From the implicit function theorem, we derive

$$\frac{dv(s_1^S)}{d\pi} = -\frac{\partial\Phi_1/\partial\pi}{\partial\Phi_1/\partial v(s_1^S)} = -\frac{1 - v(s^*)}{-1 + (1-\pi)\dfrac{\partial v(s_1^*)}{\partial v(s_1^S)}}, \tag{A.6}$$

where the numerator $\partial\Phi_1/\partial\pi$ is always positive, and the denominator $\partial\Phi_1/\partial v(s_1^S)$ generically has an ambiguous sign.

Consider the second segregation strategy s_2^* : when in a match with a partner of the same type, play D, separate the match, and rematch in the next period; in all other cases, play the strategy corresponding to the best colour-blind strategy $s^* = (\sigma, (P_1, P_2, P_3))$. Denote this strategy by s_1^*, with σ_1^*. The expected pay-off of this segregation strategy s_2^S implies $v(s_2^S) = [\pi\delta + (1-\pi)]v(s_2^*)$, which when substituting for σ_2^* and the necessary conditions gives an implicit function $\Phi_2(v(s_2^S), \pi) = 0$, given by

$$-v(s_2^S) + [\pi\delta + (1-\pi)]\{\sigma_2^*[(1+g)(1-\delta) + \delta(P_2 - P_3)(1 - v(s_2^S))] + \delta(P_3 + (1-P_3)v(s_2^S))\} = 0.$$

From the implicit function theorem

$$\frac{dv(s_2^S)}{d\pi} = -\frac{\partial\Phi_2/\partial\pi}{\partial\Phi_2/\partial v(s_2^S)} = -\frac{(\delta - 1)v(s_2^*)}{-1 + [\pi\delta + (1-\pi)]\dfrac{\partial v(s_2^*)}{\partial v(s_2^S)}}, \tag{A.7}$$

where the numerator $\partial\Phi_2/\partial\pi$ is always negative, and the denominator $\partial\Phi_2/\partial v(s_2^S)$ generically has an ambiguous sign.

First, observe that

$$\frac{\partial v(s_1^*)}{\partial v(s_1^S)} = \frac{\partial\sigma_1^*}{\partial v(s_1^S)}[(1+g)(1-\delta) + \delta(P_2 - P_3)(1 - v(s_1^S))] - \sigma_1^*\delta(P_2 - P_3) + \delta(1 - P_3)$$

and that it is equal to $\dfrac{\partial v(s_2^*)}{\partial v(s_2^S)}$ (both $v(s_1^*)$ and $v(s_2^*)$ are essentially derived from the same necessary conditions).

Now consider the case where $\pi = 0$: observe that the pay-off $v(s_i^*, \pi)$ satisfies $v(s_1^*, 0) = v(s_2^*, 0) = v(s^*)$. Given $\dfrac{\partial v(s_1^*)}{\partial v(s_1^S)} = \dfrac{\partial v(s_2^*)}{\partial v(s_2^S)}$, at $\pi = 0$ the denominators in equations (A.6) and (A.7) are the same and equal to $\dfrac{\partial v(s^*)}{\partial v(s^S)} - 1$. For small enough π, the sign of the denominators is the same. Now observe that $\partial\Phi_1/\partial\pi > 0$ and $\partial\Phi_2/\partial\pi < 0$, that is, the numerators always have the opposite sign. Therefore, for small enough π, the sign of $\dfrac{dv(s_1^S)}{d\pi}$ is the opposite of the sign of $\dfrac{dv(s_2^S)}{d\pi}$. As a result, there is always one of the two strategies that is Pareto improving (and the other strategy will always be Pareto worsening).

Finally, we need to verify that the incentive constraint is not violated, that is, that $v(s^S) \leq \overline{V}$. From the definition of each of the segregation strategies, $v(s_1^S) = \pi + (1-\pi)v(s_1^*)$ and $v(s_2^S) = [\pi\delta + (1-\pi)]v(s_2^*)$, it follows that for small π, $v(s_1^S)$ and $v(s_2^S)$ are approximately equal to $v(s^*)$. Since generically there is constrained inefficiency for the best colour-blind strategy, $v(s^*) < \overline{V}$, there will always exist a small enough π such that $v(s_1^S)$ and $v(s_2^S)$ are no larger than $\overline{V}$. ‖

A.3. *The endogenous measure of singles under age-dependent strategies*

In the case of colour-blind strategies, the measure of singles M solves

$$1 + M[\sigma^2(1-2P_1) + 2\sigma(1-\sigma)(1-2P_2) + (1-\sigma)^2(1-2P_3)] = 0. \tag{A.8}$$

The inflow of singles is a measure 1 of newly born agents and out of the measure M of last period's singles, and $\sigma^2(1-P_1)$ is the probability that both partners played C and then remain single. Likewise for the other histories (C, D) and (D, D). The outflow is determined by $\sigma^2 P_1$, which is the probability that two agents leave the pool after both playing

C, and similarly for the other histories. The stationary measure M of singles solves equation (A.8) which is obtained after equating the inflow with the outflow.

Now consider the age-dependent segregation strategy s^S in which each player plays the best colour-blind strategy s^* when in a mixed match (*i.e.* $c_i \neq c_{-i}$) and plays C with certainty and continue the match when in a same-colour match ($c_i = c_{-i}$). In the case of the segregation strategy just described, the measure of singles M^S solves

$$1+\{-\pi/M^S+(M^S-\pi/M^S)[\sigma^2(1-2P_1)+2\sigma(1-\sigma)(1-2P_2)+(1-\sigma)^2(1-2P_3)]\}=0. \tag{A.9}$$

There are $1/M^S$ matches between two newly born singles and a fraction π of them is between types of the same colour. Compared to the colour-blind strategy, there is the same exit rate for mixed matches and those with "old" partners, while all the matches of new singles of the same type leave the pool of singles faster, that is, with probability 1.

A.4. *Proof of Proposition* 3

Proposition 3. *Whenever $\pi \neq \frac{1}{2}$, an equilibrium that involves symmetric type-dependent strategies is constrained inefficient.*

Proof. Symmetric type-dependent strategies imply pay-offs

$$\begin{aligned} v_w &= \Pi v(s_i)+(1-\Pi)v(s_{-i}) \text{ and} \\ v_b &= (1-\Pi)v(s_i)+\Pi v(s_{-i}), \end{aligned}$$

where s_i denotes the strategy when in a same-colour match (s_{-i} in a mixed match). For the equilibrium to be constrained efficient, it must be the case that $v_w = v_b = \overline{V}$ (if not, then either at least one pay-off is larger than $\overline{V}$, therefore violating the incentive constraint, or both are lower and the equilibrium is not constrained efficient). Using the expressions for the pay-offs, this implies $(1-2\Pi)[v(s_i)-v(s_{-i})]=0$, which requires that either $\Pi=\frac{1}{2}$ or $v(s_i)=v(s_{-i})$, or both. Now, if $v(s_i)=v(s_{-i})$, then strategies are colour-blind and by Proposition 1, generically the equilibrium is constrained inefficient. ‖

A.5. *Proof of Proposition* 4

Proposition 4. *There are distributions and economies for which the minority bias equilibrium is welfare improving relative to the symmetric type-dependent segregation equilibrium.*

Proof. Focus on economies where the colour-blind equilibrium strategy is the incubation strategy. Denote the aggregate pay-off from the colour-blind symmetric segregation strategy by W and that of the minority bias strategy by W^m. Then

$$\begin{aligned} W^m &= \pi v_w+(1-\pi)v_b \\ &= W+(1-\pi)(1-\delta)[\pi(2+g-l)-1] \end{aligned}$$

from equations (4) and (5), and where $W=\pi(\pi+(1-\pi)\delta)+(1-\pi)((1-\pi)+\pi\delta)$. The minority bias equilibrium is welfare improving ($W^m > W$) provided $\pi > 1/(2+g-l)$.

Deviation by the whites can occur either when matched to another white or when in a mixed match. Deviation in a mixed match in the first phase yields a continuation pay-off of $0+\delta v_w$. Compare this to the pay-off from conforming to the strategy when in a mixed match, which generates a continuation pay-off of $-l(1-\delta)+\delta$. No deviation requires that $\delta v_w \leq -l(1-\delta)+\delta$. After substitution for v_w from equation (4), we get

$$\pi \leq 1-\frac{l}{(1+l)\delta}. \tag{A.10}$$

No deviation in the continuation phase (and for a white also when matched to a white in the initial phase) requires that both v_w and v_b be no larger than $\overline{V}=1-g\frac{1-\delta}{\delta}$. Since $v_b > v_w$, the binding constraint is $v_b \leq \overline{V}$ and after substitution for v_b from equation (5), it follows that

$$\pi \leq 1-\frac{g(1+\delta)}{(1+g)\delta}. \tag{A.11}$$

The range of distributions π that satisfy these constraints is

$$\pi \in \left(\max\left\{\frac{1}{2},\frac{1}{2+g-l}\right\},\ \min\left\{1-\frac{l}{(1+l)\delta},1-\frac{g(1+\delta)}{(1+g)\delta}\right\}\right),$$

which guarantees that the equilibrium is welfare improving $W^m > W$, and that there is no deviation by any individual. ‖

A.6. *Derivation of the endogenous distribution of singles under exclusive segregation strategies*

Let $n_w \leq \pi$ be the measure of w-types that are single. Then, the number of whites that are single in the next period n'_w is given by (denote the measure of new agents entering the market of the λ; recall that $\lambda = 1$)

$$n'_w = n_w(1-\Pi) + \lambda\pi.$$

Those w who meet a b, which happens with probability $1-\Pi$ will be single again next period, in addition to the new entrants. A stationary PPE implies strategies are time invariant, and as a result $n'_w = n_w$. Likewise for n_b, in a PPE we get

$$n_b = n_b\Pi + \lambda(1-\pi).$$

We can solve for n_w, n_b:

$$n_w = \frac{\lambda\pi}{\Pi} \text{ and}$$
$$n_b = \frac{\lambda(1-\pi)}{1-\Pi}.$$

Now note that

$$\Pi = \frac{n_w}{n_w + n_b}.$$

Then substituting for n_w and n_b in Π we get

$$\Pi^2(1-2\pi) + \Pi 2\pi - \pi = 0.$$

Solving the quadratic, we get two roots: one is negative and hence infeasible, and the other is positive and as stated in the text.

Acknowledgements. I would like to thank Melvyn Coles, Julio Dávila, Leonardo Felli, Patrick Legros, George Mailath, Eric Maskin, Steve Matthews, Andrea Moro, Peter Norman, Ichiro Obara, Nicola Persico, Andy Postlewaite, Jeroen Swinkels, Randall Wright, the editor, two referees, and seminar participants for insightful discussions and comments.

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