Identifying and Estimating the Effects of Unconventional Monetary Policy in the Data: How to Do It And What Have We Learned?

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Abstract: How should one identify monetary policy shocks in unconventional times? Are unconventional monetary policies as effective as conventional ones? And has the transmission mechanism of monetary policy changed in the zero-lower bound era? The recent financial crisis led Central banks to lower their interest rates in order to stimulate the economy, and interest rates in many advanced economies hit the zero lower bound. As a consequence, the traditional approach to the identification and the estimation of monetary policy faces new econometric challenges in unconventional times. This article aims at providing a broad overview of the recent literature on the identification of unconventional monetary policy shocks and the estimation of their effects on both financial as well as macroeconomic variables. Given that the prospects of slow recoveries and long periods of very low interest rates are becoming the norm, many economists believe that we are likely to face unconventional monetary policy measures often in the future. Hence, these are potentially very important issues in practice.

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1 Introduction

Are unconventional monetary policies as effective as conventional ones? And has the transmission mechanism of monetary policy changed in an era where interest rates are at the zero lower bound? This article aims at providing a broad overview of the recent literature on the identification of monetary policy shocks in unconventional times and the estimation of their effects on financial and macroeconomic variables.

The zero-lower bound (ZLB) refers to a situation where the monetary policy instrument (the short-term interest rate) is close to zero; hence, it cannot be lowered further in order to stimulate the economy. A decade ago the zero lower bound problem was considered an interesting but rare phenomenon, mainly associated with the Bank of Japan policy of keeping short-term interest rates close to zero for several years. However, the recent financial crisis in the US and Europe led Central banks to lower their interest rates in order to stimulate the economy, and interest rates in many advanced economies hit the zero lower bound. For example, in the US, the Federal Reserve Board kept interest rates close to zero between December 2008 and December 2015. Given that the prospects of slow recoveries and long periods of very low interest rates are becoming the norm, many economists believe that we are likely to face the zero lower bound problem often in the future. As Kocherlakota (2018) notes, there are two reasons why we should anticipate long stays at the zero lower bound: first, recent empirical estimates of the natural real rate of interest (that is, the real interest rate consistent with output equaling its natural rate and stable inflation) based on Laubach and Williams' (2003) methodology suggest that it has fallen steadily in the last ten years; thus, even small adverse shocks may push interest rates below the zero lower bound. Second, in the presence of another financial crisis, even if minor, pre-existing low real interest rate will imply that the Central bank will be unable to insulate the economy. As a result, aggregate output will decline substantially, thus contributing to another long recession and a prolonged stay at the zero lower bound.

Since traditional expansionary policies of lowering interest rates cannot be implemented at the zero lower bound, Central banks had to rely on alternative monetary policies to stimulate the economy. Monetary policy at the zero lower bound has been generally dealt with by using two kind of unconventional monetary policy interventions: forward guidance and large scale asset purchases. Large scale asset purchases ("LSAP") refer to purchases of

assets of private financial firms to inject liquidity and counteract the tightening in financial markets. Forward guidance refers to announcements that are intended to change the public belief about future Central banks' actions. For example, the Central bank could announce that it could keep interest rates "lower for longer" than the markets anticipated at the time, or it could say that interest rates would rise "more gradually for longer", meaning that interest rates would rise from their current low level more gradually than markets anticipated.[1] Quantitative Easing ("QE") instead is a term that refers to both changes in the size (LSAP) and the composition (changes in the maturity structure) of the Central bank balance sheet.

Clearly, in the presence of unconventional monetary policy, the traditional approach to the identification and estimation of monetary policy faces new econometric challenges. For example, the VAR cannot be estimated with an endogenous variable that is constant and equal to zero, and the short-term interest rate is zero at the ZLB. Furthermore, the sample is short and one would presumably want to use both pre- and post-ZLB data; however, the data are from potentially different regimes and care must be taken if the regimes are indeed different. Third, it is unclear which variables to include in a VAR to describe unconventional monetary policy. Hence, how to identify and estimate monetary policy shocks in unconventional times is a challenging issue in practice. This article discusses the econometric challenges faced by researchers when identifying monetary policy shocks in unconventional times as well as estimating their effects on the economy. It also provides insights on the empirical findings so far.

The focus in this article is on VAR models. Alternatively, one can estimate the effects of unconventional monetary policy shocks in structural DSGE models, in which case the shocks would be already identified by construction. The trade-off between the two approaches is that the first is more robust to misspecification (which is an important issue in the latest financial crisis, when DSGE models did not fare well).

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[1]See Eggertsson and Woodford (2003) for a theoretical justification of why forward guidance should work.

2 Econometric Approaches to the Identification of Monetary Policy Shocks

2.1 Traditional Approaches

Consider the following Structural VAR of order p model for the $(n \times 1)$ vector of variables X_t:

$$B(L) X_t = c + \varepsilon_t, \tag{1}$$

where $B(L) = B_0 - B_1 L - ... - B_p L^p$ is the lag polynomial, c is a vector of constants and ε_t is an $(n \times 1)$ vector of zero-mean, serially uncorrelated structural shocks of interest to the researcher, with identity covariance matrix: $E(\varepsilon_t \varepsilon_t') = I_n$.

Assuming invertibility and other standard assumptions, the Structural VAR can be rewritten as a Structural MA model (Watson, 1994):

$$X_t = k + \Theta_0 \varepsilon_t + \Theta_1 \varepsilon_{t-1} + ... + \Theta_{q-1} \varepsilon_{t-(q-1)} + \Theta_q \varepsilon_{t-q} + ... \tag{2}$$

Let's define the corresponding reduced-form VAR(p) as:

$$A(L) X_t = \mu + u_t, \tag{3}$$

where the lag polynomial $A(L) = I - A_1 L - ... - A_p L^p$ is such that $A_j = B_0^- B_j$, $\mu = B_0^- c$, $B_0^- \equiv B_0^{-1}$ and, in particular,

$$u_t = B_0^- \varepsilon_t. \tag{4}$$

In practice, the reduced-form VAR model is the model typically estimated as a linear system of equations with homoskedastic and serially uncorrelated errors and the same regressors in each equation. The model can be efficiently and conveniently estimated equation by equation by OLS (see Hayashi, 2001). This leads to estimates of A_1, A_2, ..., A_p, μ and the symmetric matrix $\Omega \equiv E(u_t u_t')$. The challenge is then to recover the structural parameters of interest in eq. (1) from the reduced-form VAR, eq. (3). The identification problem arises because the number of estimated parameters in (3) is $n^2 \times p + n + n(n+1)/2$ while the number of structural parameters is $n^2 \times (p+1) + n$ – see Watson (1994); hence, one needs to impose $n(n-1)/2$ identification restrictions on eq. (1).[2]

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[2] We assume that the VARs satisfy the usual covariance stationarity assumptions. We are already imposing that $E(\varepsilon_t \varepsilon_t') = I_n$.

A traditional approach to identification involves recursive identification (Sims, 1980). Note that $u_t = B_0^- \varepsilon_t$ and $E\left(\varepsilon_t \varepsilon_t'\right) = I_n$ imply $E\left(u_t u_t'\right) = B_0^- B_0^{-'}$; this, together with $\Omega \equiv E\left(u_t u_t'\right)$, implies that $B_0^- B_0^{-'} = \Omega$. This is a (nonlinear) system of equations that, in general, has exactly $n(n+1)/2$ over-identified parameters. Define the $(n \times n)$ lower-triangular matrix P such that

$$P'P = \Omega. \tag{5}$$

The $n(n-1)/2$ zero restrictions in the lower-triangular matrix P make the system of equations (5) just-identified. $P = B_0^-$ is known as the Cholesky factor of Ω.

This approach requires that the researcher is able to identify zero restrictions that can be justified based on economic grounds. In fact, an area where the recursive approach has been used extensively is in the identification of monetary policy shocks – see Christiano et al. (1999) for an overview. To illustrate the approach, for simplicity, we will focus on the case where $n = 3$ and the variables used in the VAR are output (y_t), inflation (π_t) and the short-term interest (Federal Funds) rate $(r_t$, henceforth FFR), as in the benchmark VAR in Stock and Watson (2001). Let $X_t = (\pi_t, y_t, r_t)'$ and let $B_{s,ij}$ denote the *i-th* row and *j-th* column scalar value in the matrix B_s. Hence, the structural VAR we consider is:[3]

$$\begin{pmatrix} B_{0,11} & B_{0,12} & B_{0,13} \\ B_{0,21} & B_{0,22} & B_{0,23} \\ B_{0,31} & B_{0,32} & B_{0,33} \end{pmatrix} \begin{pmatrix} \pi_t \\ y_t \\ r_t \end{pmatrix} = c + B_1 X_{t-1} + ... + B_p X_{t-p} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix} \tag{6}$$

The traditional "backward-looking" Taylor rule for monetary policy is:

$$r_t = r^* + \phi_\pi \left(\pi_t - \pi^*\right) + \phi_y \left(y_t - y^*\right) + \varepsilon_{mp,t}, \tag{7}$$

where r^* is the desired interest rate, $(\pi_t - \pi^*)$ is the deviation of the inflation rate from its desired level and $(y_t - y^*)$ is the output gap.[4] The residual $\varepsilon_{mp,t}$ is the monetary policy shock. Note that the Taylor rule can be rewritten by redefining the constant term as: $r_t = \kappa + \phi_\pi \pi_t + \phi_y y_t + \varepsilon_{mp,t}$.

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[3]Differently from Stock and Watson (2001), our VAR features output instead of unemployment; however, empirical results are similar no matter which of the two is used (Christiano et al., 1999).

[4]For simplicity, we are ignoring the lagged value of the interest rate (or other lagged values).

The monetary policy rule in eq. (7) implies that output and inflation are pre-determined when the monetary authority sets interest rates – otherwise, if inflation or output could react to the interest rate set by the Central bank the coefficients ϕ_π and ϕ_y would not be structural coefficients and the structural analysis would be subject to the Lucas critique. This timing assumption implies two zero restrictions, namely that inflation and output cannot react contemporaneously to the FFR, i.e.

$$B_{0,13} = B_{0,23} = 0. \tag{8}$$

This implies

$$\begin{pmatrix} B_{0,11} & B_{0,12} & 0 \\ B_{0,21} & B_{0,22} & 0 \\ B_{0,31} & B_{0,32} & B_{0,33} \end{pmatrix} \begin{pmatrix} \pi_t \\ y_t \\ r_t \end{pmatrix} = c + B_1 X_{t-1} + ... + B_p X_{t-p} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix} \tag{9}$$

These restrictions are sufficient to eliminate the endogeneity problem for the purpose of identifying the monetary policy shock and estimating its effects on the economy. In fact, note that, by inverting B_0 after imposing the restrictions, one obtains a matrix B_0^- where $B_{0,13}^- = B_{0,23}^- = 0$. That is,

$$\begin{pmatrix} \pi_t \\ y_t \\ r_t \end{pmatrix} = \begin{pmatrix} B_{0,11}^- & B_{0,12}^- & 0 \\ B_{0,21}^- & B_{0,22}^- & 0 \\ B_{0,31}^- & B_{0,32}^- & B_{0,33}^- \end{pmatrix} \left\{ c + B_1 X_{t-1} + ... + B_p X_{t-p} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix} \right\} \tag{10}$$

Eq. (10) implies that both π_t and y_t are functions only of $\varepsilon_{1,t}$, $\varepsilon_{2,t}$ and past values of the endogenous variables, and not of $\varepsilon_{3,t}$. On the other hand, from eq. (9), r_t is a function of π_t, y_t, $\varepsilon_{3,t}$ and past values of the endogenous variables. That is, $r_t = -B_{0,33}^{-1} B_{0,31} \pi_t - B_{0,33}^{-1} B_{0,32} y_t + f(X_{t-1}, c) + B_{0,33}^{-1} \varepsilon_{3,t}$, where $f(X_{t-1}, c)$ denotes a linear function of past values of X and the constant. Note that the error term, $B_{0,33}^{-1} \varepsilon_{3,t}$, is such that $E(\varepsilon_{3,t} | \pi_t, y_t, X_{t-1}) = E(\varepsilon_{3,t} | \varepsilon_{1,t}, \varepsilon_{2,t}, X_{t-1}) = 0$, where the first equality follows from the fact that, under eq. (8), both π_t and y_t are functions only of $\varepsilon_{1,t}$, $\varepsilon_{2,t}$ and past values of the endogenous variables, and the last follows from the fact that shocks are unpredictable given past information and are mutually uncorrelated. Thus, OLS regressions of r_t on π_t, y_t and past values of the endogenous variables recover consistent estimates of the parameter estimates and of the monetary policy shock. Ignoring the presence of lagged variables, hence,

$\varepsilon_{mp,t} = B_{0,33}^{-}\varepsilon_{3,t}$. Once the coefficients in the last equation are consistently estimated, $\varepsilon_{3,t}$ can also be consistently estimated; hence, the impulse response coefficients can be estimated by directly regressing the macroeconomic variables on $\varepsilon_{3,t}$, $\varepsilon_{3,t-1}$, ...

However, note that the two zero restrictions imposed by eq. (8) are not sufficient to identify the other shocks in the Structural VAR, namely the output and the inflation shocks. In order to identify all the shocks and all the parameters, one needs one more restriction. A typical restriction is that $B_{0,12} = 0$, which implies that prices are pre-determined when firms make their production plans. This assumption is typically justified by price stickiness or menu costs. Clearly, under the restrictions $B_{0,13} = B_{0,23} = B_{0,12} = 0$, B_0 is lower-triangular and the remaining parameters in B_0 can be estimated as the inverse of the Cholesky factor of Ω.

2.2 Why Do Traditional Approaches Fail at the ZLB?

Clearly, the traditional recursive identification approach described in the previous sub-section cannot be used at the zero lower bound since the short-term FFR rate is zero, and the VAR cannot be estimated with an endogenous variable that is constant and equal to zero.

To overcome this problem, a researcher might consider a VAR with long-term interest rates instead of the short-term interest rate. That is, $X_t = \left(\pi_t, y_t, r_t^{long}\right)'$, where r_t^{long} is the long-term interest rate (e.g. the 2-year or the 5-year maturity rate). While this may avoid the problem of working with endogenous variables that equal zero, as the long-term interest rate is typically positive even at the ZLB, it is still possible that long-term rates might get close to zero in the future, which would invalidate this procedure as well. Furthermore, it is unclear which long-term maturity to use, and empirical results may depend on the choice. Either way, working with long-term rates will make comparisons across conventional and unconventional monetary policy regimes very difficult as monetary policy shocks are identified using different methodologies/variables in the two sub-samples.

Alternative identification schemes that are suitable at the zero lower bound include using a shadow rate instead of the FFR at the ZLB (Wu and Xia, 2016; Krippner, 2013; among others), heteroskedasticity-based identification (Wright, 2012), High-Frequency identification (HFI, Kuttner, 2001), External Instruments (Gertler and Karadi, 2015; Montiel-Olea et al., 2012;

Stock-Watson, 2018), Functional VARs (Inoue and Rossi, 2018). We review each of these identification schemes in the following sub-sections.

2.3   Shadow Rates

A shadow rate model assumes the existence of a "shadow" yield curve, that is a yield curve which is linear in Gaussian factors and hence can become negative at short maturities, even though the actual short-term rate is the maximum of the shadow rate and zero. The shadow rate is the shortest maturity rate from the estimated shadow yield curve. For example, it will take negative values in unconventional monetary policy environments to signal an unconventional monetary policy that is more accommodative than a zero short-term (policy) rate by taking into account the effects that unconventional policies have on longer-maturity rates. When the zero lower bound is not binding, the shadow rate equals the short-term interest rate. Since the shadow rate is the same as the short-term interest rate outside the ZLB, one can easily compare conventional and unconventional sub-samples. Hence, an advantage of shadow rates is that they are an intuitive and convenient indicator of the stance of monetary policy in both conventional and unconventional periods.

However, note that since the shadow rate is determined from an estimated yield curve, in practice the estimate may depend on the model used to fit the yield curve. In addition, the shadow rate is a theoretical concept and does not correspond to interest rate values used in actual transactions.

Krippner (2013, 2016), Christensen and Rudebusch (2014), Wu and Xia (2016) and Bauer and Rudebusch (2016) have proposed shadow-rate measures of interest rates to quantify the stance of US monetary policy in unconventional times. Lemke and Vladu (2017), Kortela (2016) and Wu and Xia (2018) focus on Euro data. In particular, Wu and Xia (2018) propose a shadow rate for the Euro-area where interest rates have reached negative levels. Kim and Singleton (2012) and Ichiue and Ueno (2013) develop term structure models at the ZLB for Japan. Lombardi and Zhu (2014) propose an alternative model-free shadow rate based on a factor model, extracting information from a large dataset of variables linked to Central Banks' monetary policies.

An alternative way to summarize the stance of monetary policy in an indicator is to use the "Expected Time to Zero" (for this, as well as alternative measures, see Krippner, 2016). This indicator is the horizon at which the

short-term rate will reach zero, conditional on the shadow rate being negative at the current time. It is less convenient than the shadow rate since the time series is available only when the shadow rate is negative.

2.4 Heteroskedasticity-based Identification

The heteroskedasticity-based approach to identification exploits additional restrictions deriving from the variance of the shocks changing over time. Assume that the variance of the structural errors changes at a given point in time (due to particular events) from $E\left(\varepsilon_t\varepsilon_t'\right) = \Lambda_A$ to $E\left(\varepsilon_t\varepsilon_t'\right) = \Lambda_B$, where Λ_A and Λ_B are diagonal matrices with elements $\lambda_i^{(A)}$ and $\lambda_i^{(B)}$, respectively. In our context, in order to identify a monetary policy shock, it is reasonable to assume that, on days of a monetary policy announcement, the variance of the monetary policy shock is bigger than the variance of the monetary policy shock on any other day. Hence, monetary policy announcement days identify the two sub-samples.

If the researcher aims at identifying all the shocks using the heteroskedasticity-based method, he/she needs to assume that the variance of all structural shocks is different in the relevant sub-samples. On the other hand, if the researcher only wants to identify the monetary policy shock, it is sufficient to assume that the variance of the monetary policy shock is different in the relevant sub-samples.

In the former case where the researcher's goal is to identify all the shocks, let the variance of the reduced-form shocks in the two sub-samples be denoted by Ω_A and Ω_B. Thus, following Rigobon (2003), from equations (4), $E\left(\varepsilon_t\varepsilon_t'\right) = \Lambda_A$ and $E\left(\varepsilon_t\varepsilon_t'\right) = \Lambda_B$, we have:

$$\Omega_A = B_0^{-1}\Lambda_A B_0^{-1\prime} \text{ and } \Omega_B = B_0^{-1}\Lambda_B B_0^{-1\prime}. \tag{11}$$

Eq. (11) is sufficient to identify the structural parameters of interest after normalizing either Λ_A or Λ_B to the identity matrix (as Λ_A, $B_0^{-1\prime}$ and Λ_B are not separately identifiable). In fact, eq. (11) is a system of $n\left(n+1\right)$ equations[5] in $n^2 + n$ unknowns.[6]

In the latter case where the researcher's goal is to identify only the monetary policy shock, it is sufficient to assume that only the variance of the

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[5]The number of equations in each of the two equations is $n\left(n+1\right)/2$, as the $(n \times n)$ matrices Ω_A and Ω_B are symmetric.

[6]There are n^2 unknown parameters in B_0 and n unknown parameters in the diagonal matrix Λ_B – assuming Λ_A has been normalized to the identity matrix.

monetary policy shock changes across sub-samples. This assumption provides one identification restriction that identifies the monetary policy shock as follows. Following Wright (2012), note that $u_t = B_0^{-1}\varepsilon_t$ implies that

$$u_t = \Sigma_{i=1}^n b_{0,i}^{(-)} \varepsilon_{i,t}, \tag{12}$$

where $\varepsilon_{i,t}$ is the i-th structural shock and $b_{0,i}^{(-)}$ is the i-th $(n \times 1)$ column vector of B_0^{-1}. Let the monetary policy shock be the first shock, $\varepsilon_{1,t}$. The identification restriction is that the variance of the monetary policy shock, i.e. $var(\varepsilon_{1,t}) \equiv \Omega_{\varepsilon_1}$, changes at time t from $\Omega_{\varepsilon_1,A}$ to $\Omega_{\varepsilon_1,B}$, while the variance of all the other shocks remains constant: $\Omega_{\varepsilon_j,A} = \Omega_{\varepsilon_j,B}$ for every $j \neq 1$. From eq. (12), calculating the variance of the reduced form shocks in the two sub-samples, we have $\Omega_{u,A} = \Sigma_{i=1}^n b_{0,i}^{(-)} \lambda_i^{(A)} b_{0,i}^{(-)\prime}$ and $\Omega_{u,B} = \Sigma_{i=1}^n b_{0,i}^{(-)} \lambda_i^{(B)} b_{0,i}^{(-)\prime}$, a system of equations that identifies the structural parameters of interest. In fact, $\Omega_{u,A} - \Omega_{u,B} = b_{0,1}^{(-)} \left(\lambda_1^{(A)} - \lambda_1^{(B)} \right) b_{0,1}^{(-)\prime} = \left(\lambda_1^{(A)} - \lambda_1^{(B)} \right) b_{0,1}^{(-)} b_{0,1}^{(-)\prime}$. Normalizing $\lambda_1^{(A)} - \lambda_1^{(B)} = 1$, and letting $\widehat{\Omega}_{u,A}$ denote the estimate of $\Omega_{u,A}$ (and similarly for $\widehat{\Omega}_{u,B}$), one can then estimate $b_{0,1}^{(-)}$ in a minimum distance problem by choosing $b_{0,1}^{(-)}$ in order to minimize the distance $d\left(b_{0,1}^{(-)} \right) \equiv vech\left(\widehat{\Omega}_{u,A} - \widehat{\Omega}_{u,B} \right) - vech\left(b_{0,1}^{(-)} b_{0,1}^{(-)\prime} \right)$:

$$\underset{b_{0,1}^{(-)}}{argmin} \ d\left(b_{0,1}^{(-)} \right)' \left[V_{vech(\Omega_A)} + V_{vech(\Omega_B)} \right]^{-1} d\left(b_{0,1}^{(-)} \right),$$

where $V_{vech(\Omega_A)}$ and $V_{vech(\Omega_A)}$ are the variance of $\widehat{\Omega}_{u,A}$ and $\widehat{\Omega}_{u,B}$, respectively.

2.5 High-Frequency Identification and Event-Study Approaches

In a seminal paper, Kuttner (2001) proposed to identify monetary policy shocks as the changes in financial market's expectations on a short window of time around a monetary policy announcement. In particular, Kuttner (2001) measured monetary policy shocks based on the change in the daily federal funds futures rate around FOMC announcements.[7] Bernanke and

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[7]In this approach, an implicit but realistic assumption is that risk premia do not change over such a short time interval; otherwise the monetary policy shock would also capture changes in risk premia.

Kuttner (2005) study the effects of conventional monetary policy on stock markets. Gürkaynak et al. (2005) have shown that, based on this identification, monetary policy announcements contain valuable information above and beyond the actual changes in the Federal Funds rate.

The HFI is implemented in a simple regression:

$$(\Delta X_{i,t} \cdot d_t) = \alpha + \beta \left(\Delta r_t \cdot d_t \right) + \gamma W_t + \varepsilon_{mp,t},$$

where $\Delta r_t \cdot d_t$ is the surprise component of the policy rate change due to monetary policy (that is, the change in the policy rate in the time period identified by the dummy variable d_t, which equals to one at a time of a monetary policy announcement), $\Delta X_{i,t} \cdot d_t$ is the change in a variable over the same time interval (hence, the terminology "high-frequency" identification) and W_t is a set of control variables. Clearly, changes in interest rates may not necessarily be always due to monetary policy; therefore, key to this identification strategy is that the change in financial markets' expectations be measured in a short window of time around the announcement, to avoid the measure to be contaminated by other shocks that might happen within the same time period. Hence, this methodology is also sometimes referred to as an "event-study", due to the fact that the identification really relies on selecting specific episodes that allow researchers to extract the exogenous component of monetary policy.

A potential issue faced by this type of identification is that the identified shocks may not be pure monetary policy shocks, since they might be contaminated by either (i) other shocks (e.g. news about the state of the economy) or (ii) information that the Central bank is releasing about the future state of the economy in their announcements. Regarding (i), one could include regressors that control for information that jointly affects $\Delta X_{i,t}$ and Δr_t among the control variables W_t, such as the release of economic news. Regarding (ii), there is a debate on whether informational effects are empirically important. On the one hand, there are methodologies that allow researchers to clean the shocks from informational effects by regressing them on Central bank's own forecasts (e.g. Miranda-Agrippino and Ricco, 2018; Jarocinski and Karadi, 2018). On the other hand, informational effects are a concern if Central banks' forecasts are more accurate than the private sector's; however, the forecasting advantage of Central banks relative to the private sector was significant in the past (Romer and Romer, 2000) but has disappeared in the most recent period (Rossi and Sekhposyan, 2016).

2.6 External Instruments and the Local Projection-IV Approach

An alternative to identifying unconventional monetary policy shocks from VARs is to use external sources of information – that is, external instruments (Montiel-Olea et al., 2012). External instruments are variables that are correlated with the shock of interest but not with the other shocks. External instruments are not necessarily the shock of interest, as they might contain some measurement error, but, as long as they are uncorrelated with the other shocks in the system, one can use them to identify the shock of interest; however, they need to be exogenous. For example, an instrument to identify the conventional monetary policy shock could be the Romer and Romer (2004) narrative measure, which may contain measurement error. Alternatively, another instrument could be the change in the Federal Funds future rates around FOMC announcements (Gertler and Karadi, 2015), as such changes are uncorrelated with any other shock in a short window of time around the announcements.

2.6.1 VARs with External Instruments

In the VAR-based approach, one needs a valid measure of a policy rate at the ZLB to include in the VAR as an endogenous variable. Clearly, the short-term interest rate cannot be used, as it is zero at the ZLB. One could rely on an interest rate at a longer maturity, although it is not clear which maturity to use. For example, in the US, some argue that the horizon of the Central bank is two years while Gertler and Karadi (2015) use the one-year government bond rate since the two year rate turns out to be a weak instrument; it is also not clear which maturity to use when considering other countries. Note that using a rate that is not the same as the rate commonly used in conventional times makes the exercise of comparing conventional and unconventional monetary policies difficult.

Assume that r_t^l is a medium- or long-term rate that can be considered a valid measure of the policy rate in unconventional times. Gertler and Karadi (2015) consider a VAR where $X_t = (y_t, P_t, r_t^l)'$, where P_t is the log of the price level. They estimate the responses to the monetary policy shock, ε_t^{mp}, by considering the portion of the Structural VAR related to the monetary policy shock: from eqs. (3-4), the latter is: $X_t = \sum_{j=1}^{p} A_j X_{t-j} + b_0^- \varepsilon_t^{mp}$, where b_0^-

is the column of B_0^- that multiplies the monetary policy shock. They assume that there is an instrument Z_t such that $E\left(Z_t \varepsilon_t^{mp}\right) \neq 0$ while $E\left(Z_t \varepsilon_{j,t}\right) = 0$ for every shock $\varepsilon_{j,t}$ that is not a monetary policy shock. Such instruments come from the HFI literature and are surprises in Fed Funds and Eurodollar futures in a short window of time around FOMC announcements. Then b_0^- can be estimated by 2SLS. In the first step, regress $u_{3,t}$ (the reduced form residual ordered last in the VAR, and associated with the policy indicator r_t^l) on the instrument Z_t to obtain the fitted value $\widehat{u}_{3,t}$, whose variation is only due to exogenous movements in Z_t. Thus, regressing the remaining reduced form residuals $[u_{1,t}, u_{2,t}]'$ on $\widehat{u}_{3,t}$ one obtains a consistent estimate of b_0^- up to a constant of proportionality.

2.6.2 LP-IV

Another convenient way to estimate impulse responses in the external instrument approach is via Local Projection Instrumental Variable (LP-IV) regressions (Stock and Watson, 2018). The LP regression (Jorda', 2005) follows from the Structural MA representation in eq. (**??**):

$$X_{t+h} = k + \Theta_h X_t + u_{t+1,t+h}, \tag{13}$$

where $u_{t+1,t+h}$ is a function of $\varepsilon_{t+h}, \varepsilon_{t+h-1}, ..., \varepsilon_{t+1}, \varepsilon_{t-1}, \varepsilon_{t-2},...$ The LP-IV approach estimates directly the responses from eq. (13) using an instrument.[8] Suppose the researcher is interested in the response of the j-th variable in eq. (13), $X_{j,t}$, to the monetary policy shock. Let $\Theta_h^{(j)}$ denote the j-th row in Θ_h and $\Theta_h^{(j,j)}$ denote the $j-th$ row and $j-th$ column element of Θ_h. Let the exogenous instrument for the monetary policy shock $\varepsilon_{mp,t}$ be denoted by the scalar Z_t – that is, $E\left(Z_t \varepsilon_{3,t}\right) \neq 0$ but $E\left(Z_t \varepsilon_{i,t}\right) = 0$ for every $i \neq 3$.[9] Then, the LP-IV estimator is:

$$\frac{E\left(X_{j,t+h} Z_t\right)}{E\left(X_{j,t} Z_t\right)} = \frac{E\left(\Theta_h^{(j)} X_t Z_t\right)}{E\left(\Theta_0^{(j)} \varepsilon_t Z_t\right)} = \frac{E\left(\sum_{s=1}^n \Theta_h^{(j,s)} \varepsilon_{j,t} Z_t\right)}{E\left(\Theta_0^{(j,3)} \varepsilon_{3,t} Z_t\right)} = \frac{\Theta_h^{(j,3)} E\left(\varepsilon_{3,t} Z_t\right)}{\Theta_0^{(3,3)} E\left(\varepsilon_{3,t} Z_t\right)}, \tag{14}$$

which recovers the parameter of interest, $\Theta_h^{(j,3)}$, up to scale.

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[8] Note that external instruments are used as instruments and are not included among the variables X_t in the VAR.

[9] Future shocks are assumed to be uncorrelated with past values of X as well as past values of Z.

2.6.3 A note on Instruments' Relevance

An important condition required in this approach to identification is that the instruments be relevant (i.e. $E\left(X_t Z_t\right) \neq 0$), otherwise the instrument is weak. The presence of weak instruments can be detected by weak instrument tests. In the VAR-based approach, a standard first-stage F-statistic can be used (see Stock, Wright and Yogo, 2002). In the LP-IV approach, the errors in the LP-IV regression ($u_{t+1,t+h}$) are possibly serially correlated by construction. Thus, the usual tests for weak instruments that impose absence of serial correlation cannot be used. However, Montiel-Olea and Pflueger (2013) discuss a generalization of the first-stage F-test in the presence of serial correlation that can be used to judge instrument strength and Ganics, Inoue and Rossi (2018) discuss confidence intervals for the structural estimates in eq. (14) robust to both the presence of weak instruments and serial correlation.

2.7 VARs with Functional Shocks

An alternative approach to identify unconventional monetary policy shocks is via a "VAR with functional shocks" (Inoue and Rossi, 2018), where the shock is the exogenous shift in the yield curve due to monetary policy. Thus, the shock itself is a function, and it is referred to as a "functional shock". In detail, Inoue and Rossi (2018) approximate the yield curve function using the Nelson and Siegel (1987) and Diebold and Li (2006) model:

$$r_t\left(\tau\right) = \beta_{1,t} + \beta_{2,t}\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3,t}\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right), \qquad (15)$$

where $r_t\left(\tau\right)$ is the yield as a function of the maturity τ. The functional monetary policy shocks are then defined as:

$$\varepsilon_{1,t}\left(\tau\right) \equiv \Delta r_t\left(\tau\right) \cdot d_t, \qquad (16)$$

where d_t is a dummy variable equal to one if there is a monetary policy shock at time t and Δ denotes time differences: $\Delta r_t\left(\tau\right) \equiv r_t\left(\tau\right) - r_{t-1}\left(\tau\right)$. To capture the exogenous component of monetary policy, the change in the yield curve is calculated in a short interval of time around monetary policy announcements.

The main difference between this identification approach and the previous approaches is that the shock itself is a function, while in the previous approaches the shock is a scalar. Inoue and Rossi (2018) show that the functional shock provides a more comprehensive measure of monetary policy shocks. Also, the shocks have different shapes in the conventional and unconventional periods. The idea to define the shock as a multi-dimensional object and then derive an impulse response function to it differentiates their work from other papers, which either include in the VAR interest rates at longer maturities or some factors describing the yield curve, and separately calculate the responses to each maturity or each factor. The functional shock has the advantage of being defined in the same way no matter whether one considers the conventional or the unconventional monetary policy regime and hence can be used to study both conventional and unconventional monetary policy in a unified manner.

Inoue and Rossi (2018) show how to trace out the effects of monetary policy shocks in the economy via VARs using a procedure called "*VARs with functional shocks*". They consider a reduced-form VAR model with an economic variable X_t and a function $f_t(\cdot)$:[10]

$$X_t = c_1 + \phi_{1,1}X_{t-1} + \phi_{1,2}\int w(\tau)f_{t-1}(\tau;\lambda)d\tau + u_{X,t}, \qquad (17)$$

$$f_t(\tau;\lambda) = c_2(\tau) + \phi_{2,1}(\tau)X_{t-1} + \phi_{2,2}f_{t-1}(\tau;\lambda) + u_{f,t}(\tau;\lambda), \qquad (18)$$

where the function is a linear combination of q time-varying factors ($\beta_{j,t}$, where t denotes time) with coefficients that are functions of the maturity τ and tuning parameters λ:

$$f_t(\tau;\lambda) = \sum_{j=1}^{q} \beta_{j,t}g_j(\tau;\lambda). \qquad (19)$$

They show that the model (17)-(19) can be estimated by a VAR that includes X_t and the time-varying factors $\beta_{j,t}$. The response of the macroeconomic variables (X_{t+h}) to the monetary policy shock ($\varepsilon_{mp,t}(.)$) is a combination of the changes in each of the time-varying components that drive the functional shock:

$$\frac{\partial X_{t+h}}{\partial \varepsilon_{1,t}(.)} = \sum_j \Lambda_{j,h}\left(\Delta\beta_{j,t}d_t\right), \qquad (20)$$

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[10]In the monetary policy case, the function $f_t(\cdot)$ is $r_t(\tau)$.

where $\Lambda_{j,h} \equiv \frac{\partial X_{t+h}}{\partial \Delta \beta'_{j,t}}$ is estimated in the VAR, while $\Delta \beta_{j,t} d_t$ are estimated by the change in the term structure in a short window of time around the monetary policy announcement.

2.8 Sign-Restrictions

Sign-restrictions is an identification approach that has been used to identify monetary policy shocks in conventional times (Faust, 1998; Canova and De Nicolo, 2002; Uhlig, 2005). In order to identify the monetary policy shock, one needs to impose that the responses of some variables to specific structural shocks have a certain sign. Typically, one randomly generates a large number of uncorrelated shocks[11] and then keeps only the responses that have the desired sign. The difficulty in imposing sign restrictions when identifying unconventional monetary policy shocks is that it is unclear what the response of monetary policy in unconventional times should be. If so, either it is unclear which restrictions should be imposed or, by imposing certain sign restrictions, one may obtain results that are, by construction, those imposed by assumption.

3 Measuring the Effects of Unconventional Monetary Policy in the Data: What Have We Learned?

To measure how quantitative easing affects macroeconomic variables, one can focus on either high-frequency financial variables, such as asset prices, or more low-frequency macroeconomic variables, such as output and inflation. We consider the former in the next section and the latter in Section 3.2. One can also analyze international variables, reviewed in Section 3.3, and on agents' expectations via surveys, reviewed in Section 3.4.

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[11]See Lutkepohl and Kilian (2017) for specific algorithms.

3.1 The Effects on Long-Term Yields and Other Asset Prices

Overall, the literature agrees that the empirical effects of unconventional monetary policy are sizeable on long-term yields and other asset prices. These results hold across a variety of identification procedures. Using a heteroskedasticity-based identification, Wright (2012) finds that unconventional monetary policy shocks have a large effect on 10-year Treasury yields and long-maturity corporate yields, while the effect on two-year yields is very small. Thus, the long-term effects on yields were not only on government yields but were transmitted to the private sector as well. He also finds that short-term Treasury Inflation Protected Securities (TIPS) rates rose but long-term ones fell.

The high-frequency identification of monetary policy shocks uncovers similarly substantial responses and has generated a large literature. Gagnon et al. (2011), Krishnamurthy and Vissing-Jorgensen (2011), D'Amico and King (2013), Hamilton and Wu (2012) find that LSAP were successful at flattening the yield curve in the US; this means that, since short-term yields were constant at the ZLB, long-term yields decreased. For example, Gagnon et al. (2011) find that LSAP appear to reduce ten-year term premia somewhere between 30 and 100 basis points. Krishnamurthy and Vissing-Jorgensen (2011) find that the magnitudes of the effects depend on the episodes; furthermore, TIPS data show that expected inflation increased as a result of the first two QE episodes in the US, implying larger reductions in real than in nominal rates. Gilchrist, Lopez-Salido and Zakrajsek (2015) estimate the effects on real borrowing costs. By comparing conventional and unconventional monetary policy regimes, they find that conventional monetary policy steepens the yield curve (a 10 basis point reduction in the 2-year yield induces a decline of 4 basis points in the 10-year yield) while unconventional policy flattens it (the same reduction in the 2 year yield leads to a 16 basis point decline in the 10 year yield). They also find that most of the movement in nominal yields is reflected in real rates and hence, monetary policy actions are transmitted fully to real business borrowing costs and significantly to households' real borrowing costs in mortgage markets.

As announcements often contain information on both LSAP and forward guidance, typically high-frequency identification will estimate the joint effects of both. Rogers et al. (2014) attempt at disentangling the two by assuming that a monetary policy surprise that decreases all the yields is an expansion-

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ary LSAP shock while a shock that rotates the yield curve by pushing short rates down and long rates up is a forward guidance shock.

While it is clear that forward guidance decreases long-term rates by affecting agents' expectations, it is not clear what is the mechanism by which LSAP decrease long-term rates: it could be either a decrease in term premia (since the Central bank decreases the amount of long-term bonds in the private sector's portfolio, a "portfolio balance channel") or a decrease in the expected term structure (since markets expect a decrease in future short-term rates, a "signaling channel"). Disentangling the two channels generally requires a theoretical model to separate the risk-neutral expectation of the term structure and the term premium – see Bauer and Rudebusch (2014) and Gagnon et al. (2011) for attempts that depend on different models – although, under some assumptions (e.g. market segmentation of certain assets) it is possible to separate the two as well (Bauer and Rudebusch, 2014). Krishnamurthy and Vissing-Jorgensen (2011) find that the channels through which LSAP worked depend on the episodes; they detect a signaling effect in the first two QE episodes in the US, which drives down all yields, as well as a safety channel that drives down long-term bond yields and an inflation channel that increased inflation expectations; the first QE episode is also associated with a decrease in risk.

3.2 The Effects on Output and Inflation: The US Evidence

Using their shadow rate in a factor-augmented VAR model, Wu and Xia (2016) estimate that between July 2009 and December 2013, US unconventional monetary policy succeeded in decreasing the unemployment rate by 1 percent (which, they found, is 0.13% more stimulative than an historical average based on the Taylor rule). While Christensen and Rudebusch (2014) and Bauer and Rudebusch (2016) find that estimates of shadow rates differ depending on the model, Wu and Xia (2016) find that the effects on macroeconomic variables are, however, very similar across different measures of the shadow rate. Gertler and Karadi (2015) use an external instruments approach and find that, by taking into account financial variables' information, monetary policy substantially affects credit costs. Using the "Functional VAR approach", Inoue and Rossi (2018) find that the effects of unconventional monetary policy shocks on output and inflation are similar to those of

conventional shocks: an expansionary shock increases both output and infla-
tion; the response is typically hump-shaped, and peaks a few quarters after
the initial shock. However, in their approach, the responses are heterogenous
and differ depending on the way monetary policy affects the term structure
of interest rates.

As the effects on lower frequency variables, such as macroeconomic vari-
ables, require a longer span of data, researchers wishing to use traditional
approaches such as sign restrictions or VARs often need to pool information
from both conventional and unconventional monetary policy regimes. Hence,
taking into account time-variation becomes an important issue. Baumeister
and Benati (2013), Kulish, Morley and Robinson (2017) and Wu and Zhang
(2017) estimate monetary policy effects using structural DSGE models or
time-varying VARs during both periods of conventional monetary policy and
the ZLB. Kulish, Morley and Robinson (2017) find that an exogenous change
in the expected duration has significant effects on the real economy. In addi-
tion, there is considerable variation in the expected duration of the ZLB over
time; for example, it increased in 2011 as the US monetary authority moved
to calendar-based forward guidance. Using sign restrictions in a TVP-VAR,
Baumeister and Benati (2013) find that in conventional times, an increase
of 25 basis points in the FFR decreases inflation between -0.3 and -0.4 in
1970-1990's but the effect is larger in magnitude in 2000s, reaching estimates
between -1 and -1.5. A similar finding holds for output growth: the effect is
around -1 before 2000 and becomes -2 or -3 after then.

3.3   Exchange Rates and the International Evidence

Similar results have been found for countries other than the US. Gagnon et
al. (2011) find that, in Japan and the UK, LSAP had similar effects on asset
prices as in the US, and Joyce et al. (2012) note that LSAP led to a decrease
in long-term yields in the UK as well. In Europe, the survey by Hartmann
and Smets (2018) provides an overview of monetary policy since the start of
the ECB, and describes the unconventional monetary policies that the ECB
followed during the financial crisis.[12] Wang and Mayes (2012) analyze the
effects of monetary policy shocks on stock prices in New Zealand, Australia,
the UK and Europe.

Using a high-frequency approach, Rogers, Scotti and Wright (2014) iden-

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[12]See also Altavilla et al. (2016).

tify monetary policy shocks in two principal components estimated from a cross-section of yields on bond yields, stock prices and exchange rates in a short window of time around monetary policy announcements in the US, UK, Euro-area and Japan. They find that unconventional policies have been effective in improving financial conditions by lowering government bond yields and reducing risk premia. The pass-through from bond yields to other asset prices has been bigger for the US than for other countries. They also find evidence of spillovers across countries, where the US affected more the rest of the world (UK, Europe and Japan) than vice-versa.

Conventional monetary policy typically appreciates the currency of the country implementing an expansionary move (Clarida and Gali, 1994; Eichenbaum and Evans, 1995). Using a VAR with external instruments, Rogers, Scotti and Wright (2016) estimate the effects of unconventional monetary policy on exchange rates, bond yields and foreign risk premia; they find that a monetary policy easing lowers domestic and foreign bond term premia and appreciates the domestic currency. Similarly, using the Functional VAR, Inoue and Rossi (2019) show that a monetary policy easing leads to a depreciation of the country's spot nominal exchange rate in both conventional and unconventional periods; however, there is substantial heterogeneity in monetary policy shocks over time and their effects depend on the way they affect agents' expectations. Finally, monetary policy operates by affecting real interest rates, not just inflation expectations.

Glick and Leduc (2015) distinguish between shocks to three different assets around monetary policy announcements: the Fed funds rate, the one-year ahead euro-dollar future rate; and the first principal component from a set of long-term Treasury rate futures. They find that monetary policy is effective in both conventional and unconventional times; however, the U.S. dollar depreciates only in response to the first shock in unconventional times while it depreciates also in response to the other shocks in unconventional times. The result that the effects of monetary policy on exchange rates are similar in the conventional and unconventional periods seems very robust – see also Neely's (2015) and Bhattarai and Neely (2017) literature reviews.

3.4 The Effects on Survey Expectations

In Section 3.1, we reviewed the literature that studies how unconventional monetary policy shocks affect asset prices via influencing financial market expectations. More generally, it is interesting to study how monetary policy

affects very kind of expectations. When measuring the effects of monetary policy on survey expectations, one encounters a series of serious problems. In particular, survey expectations are measured at points in time that are not necessarily the days in which a monetary policy announcement is made; hence, changes in survey expectations may be affected by other shocks, thus rendering the identification difficult. To mitigate the problem, one can include control variables in the regressions to control for changes in expectations that are due to other news and shocks, although it might be difficult to include all the necessary control variables. Based on a high-frequency identification, Campbell et al. (2012) and Del Negro et al. (2015) find that forward guidance increases survey expectations of both inflation and output.[13] Since this is contrary to what we should expect in terms of how the economy should react to monetary policy shocks, this finding is often interpreted as empirical evidence that announcements reveal bad news about the state of the economy (under the assumption that the Central bank has an informational advantage on the private sector, or that the private sector believes that the Central bank has such informational advantage) rather than being monetary policy shocks. Altavilla and Giannone (2016) find that forecasters' bond yields drop significantly for at least one year after an easing.

4 The Future of Forward Guidance and Central Banks' Communication: Measures of Forecast Uncertainty and Predictive Densities

As mentioned, forward guidance refers to Central banks' announcements that are intended to change public's view on their future actions. Central banks around the world have communicated their monetary policy and their predictions about the state of the economy as well as their monetary policy in terms of "point forecasts"; that is, their statements refer to the expected,

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[13]Another finding in Del Negro et al. (2015) is that DSGE models overestimate the impact of forward guidance, and refer to this problem as the "forward guidance puzzle". Similarly, Gali' (2018) finds that financial markets' expectations of interest rate differentials in the near (distant) future have empirically larger (smaller) effects than implied by theory, an empirical finding that he refers to as "the forward guidance exchange rate puzzle".

average value of future interest rates or macroeconomic variables.

However, the uncertainty around these "point forecasts" is essential as well. In fact, in a highly uncertain environment, Central banks' forecasts of future macroeconomic variables or their conditional forecasts of future monetary policy (like any other forecast, be it financial or private sector's) may be far away from their target and could be highly unreliable. Measures of uncertainty around these forecasts – such as confidence intervals, quantiles or density forecasts – are more informative and, at the same time, would provide hedging in an uncertain world.

Several Central banks already routinely communicate measures of "confidence intervals" around their predictions for inflation and output via fan charts to communicate uncertainty around point forecasts. Fan charts provide percentiles of the forecast distribution of macroeconomic variables over a sequence of forecast horizons. In general, Central banks' fan charts are the result of convoluted methodologies that involve a variety of models and subjective assessments, although fan charts can be based on specific models as well (e.g., the Bank of England Inflation Report, the Economic Bulletin of the Bank of Italy, and the publications by the Bank of Canada, the Reserve Bank of Australia and the European Central Bank). In their Summary of Economic Projections, the US Federal Open Market Committee provides measures of predictive uncertainty based on the RMSFE of historical forecasts calculated in rolling windows over the previous twenty years.

But, how good are these measures of uncertainty? And can they be improved? Given that forward guidance is expected to be used more and more often in the future, the development of methodologies to improve predictive densities as well as to evaluate them is an econometric aspect that will likely play an important role in the future.

4.1   Methodologies to Convey Forecast Uncertainty

As mentioned, fan charts measure the uncertainty around a sequence of forecasts across horizons. Thus, they can be obtained from a sequence of forecast densities available over forecast horizons or, alternatively, as quantiles or confidence intervals for the forecasts.

4.1.1 Predictive Densities

Fan charts can be easily be obtained from predictive densities (or forecast densities). A predictive density is the conditional distribution of the target variable, say X_{t+h}, given a conditioning set of variables, say W_t, and will be denoted by $\Pr(X_{t+h}|W_t)$. Predictive densities can be parametric or non-parametric.

Predictive densities can be obtained from parametric models or non-parametrically (e.g. survey density forecasts). For a basic introduction to density forecasts in economics and policymaking, see Rossi (2014b).

4.1.2 Predictive Densities from Parametric Models

Predictive densities can be easily obtained from parametric models after making assumptions on the distribution of the forecast errors. For example, assume that $y_{t+h} = \alpha + \beta x_t + \varepsilon_{t,t+h}$, where $\varepsilon_{t,t+h}$ is unpredictable $(E(\varepsilon_{t,t+h}|\mathcal{I}_t) = 0)$ and with variance $E(\varepsilon_{t,t+h}^2|\mathcal{I}_t) = \sigma_h^2$, where $\mathcal{I}_t$ is the information set at time t. Then, the conditional predictive density can be obtained by assuming a parametric distribution for the error term. Suppose that the errors are Gaussian, that is $\varepsilon_{t,t+h}|\mathcal{I}_t \sim N(0, \sigma_h^2)$. Thus, $y_{t+h}|\mathcal{I}_t \sim N(\alpha + \beta x_t, \sigma_h^2)$ is the predictive density. For a technical introduction to density forecasts from parametric models, see Elliott and Timmermann (2016, Chp. 13).

In the simple example above, the variance of the forecast errors (σ_h^2) is constant over time. However, there is widespread empirical evidence that the mean square forecast errors (MSFE) are time-varying – e.g. Stock and Watson (2003), Rossi (2006, 2014a).

Changes in macroeconomic volatility are particularly important when producing density forecasts. Unlike point forecasts, where the misspecification of the volatility may result in inefficient estimates, misspecifying changes in volatilities may result in misspecification of the predictive density, and hence, a wrong assessment of the uncertainty around point forecasts. Recently, researchers have developed methodologies aimed at better fitting predictive densities in the presence of changes in volatility. For a recent survey of instabilities in density forecasts, see Rossi (2014a, section 2.3.4).

One way to guard against instabilities is to model it directly; the estimation of these models is typically computationally intense and researchers often rely on Bayesian methods. Clark (2011) considers density forecasts

from Bayesian VARs with stochastic volatility and show that they improve relative to models with constant volatility. An alternative way of guarding against instabilities in density forecasts is to combine densities from a set of models; this can be done via forecast combinations (Jore, Mitchell and Vahey, 2010), Bayesian model averaging (BMA), large-dimensional Bayesian VARs etc. Rossi and Sekhposyan (2014) compare the predictive performance of parametric models for predictive densities (e.g. BMA, forecast combinations, factor models, Bayesian VARs) and conclude that, empirically, density forecasts from equal-weight combinations are among the best forecasting models for macroeconomic data.

More sophisticated methods to combine predictive densities are developed in Billio et al. (2013) via multivariate time-varying weights, where the weight dynamics is driven by the past performance of the predictive densities using learning mechanisms. Carriero, Clark and Marcellino (2016) consider large-dimensional VARs where the volatilities are driven by a single common factor and Koop and Korobilis (2013) propose methods to estimate large dimensional VARs with time-varying parameters (including time-varying volatilities), where the models' dimension can change over time. Clark and Ravazzolo (2015) empirically compare alternative models of time-varying volatility (Bayesian VARs and VARs with time-varying volatility such as GARCH, mixture models, stochastic volatility, etc.) and find that ARs and VARs with conventional stochastic volatility are among the best models.[14]

4.1.3   Non-parametric Predictive Densities

Survey-based density forecasts are among the most used non-parametric predictive densities. For example, in the US, the Philadelphia Fed maintains the Survey of Professional Forecasters (SPF); in Europe, the ECB maintains a European SPF. Survey forecasts provide both aggregate predictive densities, from which one can obtain actual measures of aggregate forecast uncertainty, and individual forecasters' predictions, from which one can obtain measures of disagreement that are sometimes interpreted as uncertainty. For a distinction between aggregate forecast uncertainty and disagreement across individual forecasters as measures of uncertainty, see Lahiri and Sheng (2010).

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[14]Additional issues include using disaggregate (Ravazzolo and Vahey, 2014) and/or mixed frequency data (Aastveit et al, 2017).

Surveys are often conducted for "fixed-events" (see e.g. the SPF). For example, in each quarter panelists are asked to forecast GDP growth and inflation for the current calendar year and the next, implying that the forecast horizon shrinks over time as one approaches the end of the year. The fixed-event nature limits the usefulness of survey density predictions for policymakers and market participants, who often wish to characterize uncertainty a fixed number of periods ahead ("fixed-horizon"). Ganics, Rossi and Sekhposyan (2018) develop fixed-horizon density forecasts from combining fixed-event probabilistic predictions available from surveys.

4.1.4   MSFE Confidence Intervals

Several Central banks use the historical forecast errors to quantify forecast uncertainty; for example, the FOMC SEP includes fan charts with uncertainty bands computed using the MSFEs of past historical forecasts, assuming uncertainty is constant within a certain rolling window of past data. Clark, McCracken and Mertens (2017) improve such estimates by explicitly modeling the time variation in the forecast error variances. Their model includes the forecast error from the previous quarter and forecast updates for subsequent quarters to summarize the information in the set of forecast errors at all horizons. As a model specification for the SPF, they use a multiple-horizon stochastic volatility model (estimated with Bayesian methods).

Note that confidence intervals are summary statistics of a distribution, hence they contain less information than a predictive density. Only in special cases they are as informative as a predictive density: for example, when the predictive density is Gaussian, knowing a confidence interval implies knowledge of the mean (which corresponds to the center of the confidence interval) and the variance of the distribution (as the extremes of a, say, 95% confidence interval equals the mean plus/minus 1.96 times the standard deviation), and, hence, knowledge of the whole predictive density. An alternative approach is to directly model quantiles of the distribution – see Adrian et al. (2017).

4.2   Methodologies to Evaluate Fan Charts and Predictive Densities

Clearly, when attempting to improve density forecasts, an important issue is how to evaluate whether they are correctly specified. Well-calibrated predictive densities would improve the public's confidence in Central banks'

announcements.

Testing for the correct specification of a predictive density means understanding whether the description of uncertainty provided by the forecast model is accurate (e.g. ex-post realizations in certain quantiles of the predictive density should be observed more often in the quantiles of the predictive density where the probability is the highest). Diebold et al. (1998, 1999), Corradi and Swanson (2006a,b) and Rossi and Sekhposyan (2018) develop methodologies to test whether empirical predictive densities match the true but unobserved data generating distribution based on the probability integral transform (PIT). Their approach tests for properties of the PITs, such as independence and uniformity, which imply the correct specification of the predictive density.[15] The papers differ depending on whether and how parameter estimation error is taken into account when evaluating the forecasts: in the pioneering approach by Diebold, Gunther and Tay (1998, 1999) parameter estimation error is ignored, while it is taken into account in Corradi and Swanson (2006a,b) and Rossi and Sekhposyan (2018), who use expanding and fixed parameter estimation windows, respectively. Alternative approaches include tests on raw moments of the distribution, such as Berkowitz (2001), based on a likelihood ratio test, and Knueppel (2015), who instead takes a GMM approach. The trade-offs between the PIT-based tests and the raw-moments-based tests are that the former jointly test for the correct specification of the whole predictive density (hence, all the moments at the same time), while the latter focus on a selected subset of the moments – if the researcher does not select all the relevant moments, the latter suffer from misspecification; however the latter are more powerful than the former if the correct subset of moments is selected.

Clearly, environments where monetary policy could occasionally or more systematically be at the ZLB would raise the issue that the data might be non-stationary, as the environment changes over time. Since, the non-stationarity is potentially problematic in all the tests above, Rossi and Sekhposyan (2013) develop PIT-based tests that are robust to the presence of instabilities while Rossi and Sekhposyan (2016) develop tests for the correct specification of point forecasts.

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[15] For example, uniformity follows from the fact that, in the 5th quantile of the density, one should observe about 5% of the realizations, ex-post; hence, the distribution of the cumulative distribution function of the predictive density evaluated at the realization should be a Uniform.

5 Conclusions

Economists predict that the ZLB will occur more and more often in the future. Traditional econometric techniques that identify and estimate the effects of monetary policy face the problem of merging datasets from periods when conventional identification was possible with periods at the ZLB, where conventional approaches fail. This gives rise to the need for alternative identification schemes, which we have reviewed and which are part of a promising and currently active area of research in econometrics.

Given the prominent role played by unconventional monetary policy, such as forward guidance, Central banks at the ZLB may face challenges on how to communicate their policies to the public. Tools that communicate not only the intended (average) policies, but the uncertainty that surrounds them as well, will likely become essential in Central banks' communication toolkits. Developing better measures of forecast uncertainty/predictive densities that Central banks can use to communicate uncertainty to the public is another exciting area of research.

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Tables

Table 1. **Selected Empirical Studies on the Effects of Unconventional Monetary Policy Shocks**
Panel A. **High Frequency Financial Variables**

Outcome Variable	Shadow Rate	Heterosk.	HFI/ Event-study	External-IV	Functional VAR	Sign Restrictions
Yields & other asset p.		Wright (2012)	Gagnon et al. (2011)			
			Krishnamurthy et al. (2011)			
			D'Amico & King (2012)			
			Hamilton & Wu (2012)			
			Gilchrist et al. (2013)			
International asset p.			Joyce et al. (2011)	Rogers et al. (2016)	Inoue & Rossi (2019)	
			Rogers et al. (2014)			
			Glick & Leduc (2015)			
			Hartmann et al. (2018)			

Table 1B. **Selected Empirical Studies on the Effects of Unconventional Monetary Policy Shocks**
Panel B. **Low Frequency Macroeconomic Variables**

Outcome Variable	Shadow Rate	HFI/ Event-study	External-IV	Functional VAR	Sign Restrictions
GDP/ Inflation	Wu & Xia (2016)		Gertler & Karadi (2015)	Inoue & Rossi (2018)	Baumeister & Benati (2013)
	Christensen & Rudebusch (2014)				
	Krippner (2013)				
	Bauer & Rudebusch (2016)				
Survey Expect.		Campbell et al. (2012)			
		Del Negro et al. (2015)			
		Altavilla et al. (2017)			

Notes to the table. The table lists selected empirical studies on unconventional monetary policy.