



# Fiscal policy under loose commitment

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## Abstract

Due to time-inconsistency or political turnover, policymakers' promises are not always fulfilled. We analyze an optimal fiscal policy problem where the plans made by the benevolent government are periodically revised. In this loose commitment setting, the properties of labor and capital income taxes are significantly different than under the full-commitment and no-commitment assumptions. Because of the occasional re-optimizations, the average capital income tax is positive even in the long-run. Also, the autocorrelation of taxes is lower, their volatility with respect to output increases and the correlation between capital income taxes and output changes sign. Our method can be used to analyze the plausibility and the importance of commitment in a wide-class of dynamic problems.

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## 1. Introduction

In this paper, we analyze a classical fiscal policy problem, namely how a government should optimally distribute the taxation burden between labor and capital income. A common feature of this type of problems is that a planner influences households' expectations through its actions, and households' expectations influence the actions of the planner. Following the seminal papers by Kydland and Prescott [23] and Barro and Gordon [3], the literature has taken two dif-

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ferent approaches to tackle optimal policy problems – commitment and no-commitment. Both the approaches are to some extent unrealistic. The commitment approach does not match the observation that governments have defaulted on past promises. The no-commitment approach rules out the possibility that the government achieves the benefits of making and keeping a promise, even if there is an *a posteriori* incentive to renege on it.

Our study distinguishes from the existing literature in considering that some of the policymakers' promises are fulfilled, while others are not. There may be several interpretations for the *loose commitment* settings just described. A political economy interpretation is that governments fulfill their own promises, but it is possible that a new government is elected and the previous government's promises are not considered. Another interpretation is that a government commits to future plans, but defaulting on past promises becomes inevitable if particular events arise, e.g. induced political instability, pressures from the overall society, etc.<sup>1</sup> Another interpretation of these settings is that policymakers are often required to reevaluate policy. In this paper, we abstract from the specific source of reoptimizations. Our goal is instead to analyze how the optimal policy recommendation should take into account that policy will be revised in the future. To this end, we first consider a setting where current promises will be fulfilled with a given probability.<sup>2</sup> Lastly, we make the likelihood of reoptimizations a function of endogenous variables.

We characterize the properties of labor and capital income taxes in a loose commitment setting. The optimal policies are sensibly affected by the possibility of reoptimizations and do not necessarily lie in between the policies prevailing under full-commitment and no-commitment. On average, even if the probability of keeping promises is high, labor and capital income tax rates are close to the values prevailing under the no-commitment assumption. Chamley [6] and Judd [17] showed that with full-commitment it is optimal not to tax capital in the long-run. On the contrary, we find that under loose commitment the average capital tax rate is positive.

Other business cycle properties are also significantly different than in the full-commitment and no-commitment cases. Indeed, the occasional policy reoptimizations, peculiar to a loose commitment setting, act as an additional source of fluctuations. This changes dramatically volatilities and correlations. For example, the volatility of taxes with respect to output is higher than both under full-commitment and no-commitment. This is because, whenever a reoptimization occurs, policy instruments move more than the other variables. Also, capital income taxes and output move in the same direction, resulting in a positive correlation between the two variables. This is the opposite of what happens with full-commitment and no-commitment, where the correlation among variables is only determined by their co-movements in response to technology shocks. Finally, because of the occasional reoptimizations, the persistence of taxes is lower.

More generally, the main message of our analysis is that solving a model under either the full-commitment or the no-commitment assumption is not indicative of what would happen in a more realistic context where policy plans are occasionally revised. Solving a model under a loose commitment assumption fills this gap, and can therefore shed light on the plausibility and the importance of the commitment assumption. Interestingly, the loose commitment setting improves the empirical performance of the model along several dimensions.

We also characterize how the welfare gains change as a function of the probability to commit or the implied average time period before a reoptimization. Finally, in the endogenous probability

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<sup>1</sup> The implicit assumption is that there are exceptional events not contemplated in the policymaker's state-contingent plans. Flood and Isard [14] consider a central bank commitment to a rule with escape clauses. One can interpret that our probability of reoptimization is their probability of anomalous shocks triggering the use of the escape clauses.

<sup>2</sup> This setting can easily be extended to one where promises are only kept during a finite tenure.

model, we find that since the probability of commitment depends on endogenous state variables, the planner actively manipulates these state variables in order to enhance commitment.

This paper is related to several works analyzing the importance of commitment in optimal fiscal policy problems. Many works have tried to overcome the time-inconsistency of optimal plans through reputation mechanism, trigger strategies or the appropriate management of the debt maturity structure. Such strategies are typically quite intricate and raise enforcement and coordination issues.<sup>3</sup> Here we instead consider that policymakers do have the incentive to deviate from their announced plans. Klein and Ríos-Rull [21], Klein et al. [20] analyze policy problems with no-commitment, focusing on Markov perfect equilibria, where the policy plan is only a function of payoff-relevant state variables. Following this approach, Martin [25] studies the optimal long-run labor and capital income taxation. Building on the latter, we study both the long-run and the business cycle properties of the tax rates in a loose commitment setting.

Roberds [31] and Schaumburg and Tambalotti [33] also consider that promises may not always be kept. Our main contribution with respect to this literature is a methodological one. Our methods can indeed be used to analyze *loose commitment* problems in a wide class of non-linear models, as the fiscal policy problem considered here. On the contrary, the method proposed by Schaumburg and Tambalotti [33] could only be applied to linear-quadratic frameworks.<sup>4</sup> Our method is also more efficient, since it relies on the solution of only one fixed point problem. This is important when solving models with endogenous state variables, a feature of particular interest in commitment problems. Indeed, it is precisely through the choice of state variables that policymakers can affect strategically the actions of their successors.<sup>5</sup> Finally, we also show how to handle problems where the current policymaker can affect the likelihood of fulfilling promises.

The paper is organized as follows. Section 2 describes the model and Section 3 analyzes the optimal policy under loose commitment. In Sections 4 and 5 we calibrate our model and describe the main implications for the properties of capital and labor income taxes and for social welfare. Section 6 considers an extension with endogenous probabilities and Section 7 concludes.

## 2. The model

To facilitate comparisons with the earlier literature, we build on the model of Martin [25], who analyzed the classical choice between labor and capital income taxes both under the full-commitment and the no-commitment assumptions, focusing on Markov strategies.

A representative household derives utility from private consumption ( $c$ ), public consumption ( $g$ ) and leisure ( $1 - l$ ). The household has 1 unit of time each period that she can allocate between leisure and labor ( $l$ ). She rents capital ( $k$ ) and supplies labor ( $l$ ) to a firm. Labor and capital earnings are taxed at a rate ( $\tau^l$ ) and ( $\tau^k$ ) respectively.

<sup>3</sup> For example, Backus and Driffill [2] analyze a reputational mechanism where a planner of a certain type resembles another type. Barro and Gordon [4] and Persson et al. [28] analyze, respectively, equilibria with trigger strategies and management of the debt maturity structure. As shown by Rogers [32], when the provision of public good is endogenous as in our model, debt restructuring cannot enforce the commitment solution.

<sup>4</sup> At a first glance, it may be thought that linear quadratic methods can provide an accurate approximation of non-linear models. However, this procedure would often require to impose the so-called “timeless perspective” assumption, which is in sharp contrast with the *loose commitment* settings under consideration.

<sup>5</sup> For example, in the absence of endogenous state variables the optimal policy problem under no-commitment reduces to a static optimization problem.

Following Greenwood et al. [15], the household can also decide on the capital utilization rate  $v$ . Therefore, the amount of capital rented to firms will be  $vk$ . We are also going to assume that the depreciation rate of capital is a function of its utilization rate,  $(\delta(v_t))$ .

For given capital taxes  $(\tau_t^k)$ , labor taxes  $(\tau_t^l)$ , wages  $(w_t)$ , and interest rate  $(r_t)$ , the household problem is:

$$\begin{aligned} & \max_{\{k_{t+1}, c_t, l_t, v_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t, l_t), \\ \text{s.t. } & c_t + k_{t+1} = (1 - \tau_t^k)r_t v_t k_t + (1 - \tau_t^l)w_t l_t + [1 - \delta(v_t)]k_t \end{aligned} \tag{1}$$

where  $\beta$  denotes the discount factor. The households' first-order conditions (FOCs) are:

$$u_{c,t} - \beta E_t u_{c,t+1} ((1 - \tau_{t+1}^k)r_{t+1} v_{t+1} + 1 - \delta(v_{t+1})) = 0, \tag{2}$$

$$u_{c,t} (1 - \tau_t^l)w_t + u_{l,t} = 0, \tag{3}$$

$$(1 - \tau_t^k)r_t - \delta_{v,t} = 0. \tag{4}$$

In this problem, the source of time-inconsistency appears in Eq. (2). Today's household decisions depend on the expectation of future variables. In particular, the contemporaneous capital accumulation decision depends on future returns on capital. It is important to note that Eq. (4) is stating that the current capital tax is distortionary. If the planner would raise capital taxes, households could choose a lower capital utilization rate.<sup>6</sup> This feature is important for our model, because if the planner does not keep her past promises and the capital utilization is fixed, then the capital tax could be set at an extremely high and implausible value. Martin [25] showed that with fixed capital utilization, and for plausible calibrations, an equilibrium under no-commitment does not exist. There may be other reasons that inhibit the planner to choose an extremely high capital rate when it defaults on past promises. We chose this specification that guarantees the model to have a well-defined solution for both the commitment and the no-commitment cases.

Total output  $y_t$  is produced according to the function  $F(A_t, k_t, v_t, l_t) = A_t(k_t v_t)^\theta l_t^{1-\theta}$ , where  $A_t$  represents a productivity shock. Firms operate in perfectly competitive markets. Hence, wages and interest rates are given by

$$r_t = F_{kv,t},$$

$$w_t = F_{l,t}.$$

The planner provides the public good  $g$ , sets taxes  $\tau^k$  and  $\tau^l$ , satisfying the balanced budget constraint<sup>7</sup>:

$$g_t = \tau_t^k r_t v_t k_t + \tau_t^l w_t l_t. \tag{5}$$

<sup>6</sup> As discussed in Martin [25], it is important that at least some depreciation is not tax deductible, as assumed for instance in Greenwood et al. [15]. If this is not the case, the current capital tax is not viewed by the current government as distortionary and an equilibrium in such economy may not exist. In some developed economies, there is a tax allowance for accounting depreciation, which differs from the actual depreciation. If there is excess depreciation due to a high capacity utilization such depreciation would still not be tax deductible.

<sup>7</sup> Dealing with debt and commitment issues is a topic that requires extensive treatment on itself and is beyond the scope of this paper. The reader is referred to Krusell et al. [22] and Debortoli and Nunes [12].

Combining the households and governments' budget constraint one obtains the feasibility constraint

$$y_t = c_t + g_t + k_{t+1} - (1 - \delta(v_t))k_t. \quad (6)$$

In order to simplify the expressions, using Eq. (4), the household's and the government budget constraints, we substitute  $v_t$ ,  $c_t$  and  $g_t$  into (2) and (3). Hence, the FOCs can be written in the more compact form

$$b_1(x_t(\omega^t), k_t(\omega^t)) + \beta E_t b_2(x_{t+1}(\omega^{t+1}), k_{t+1}(\omega^{t+1})) = 0. \quad (7)$$

The vector of functions  $b_1, b_2$  depends on several variables, where  $x_t \equiv (k_{t+1}, l_t, \tau_t^k, \tau_t^l)$  is the vector of contemporaneous control variables,  $k_t$  is the state variable and  $\omega^t$  is the history of events up to  $t$ .<sup>8</sup>

### 3. Optimal policy under loose commitment

We will consider a model where a planner is not sure whether his promises will be kept or not. As explained above, this uncertainty can be due to several factors. For simplicity, we assume that these events are exogenous. In other words, in any period the economy may experience a reoptimization with a given exogenous probability.<sup>9</sup> Since a "reoptimization" implies a "default" on past promises, we use the two terms interchangeably.<sup>10</sup>

To make matters simple, we abstract from any shock other than the random variable  $s_t$  describing Default ( $D$ ) or No-Default ( $ND$ ) in period  $t$ . It is a straightforward generalization to include other sources of uncertainty, but the notation would be harder to follow.<sup>11</sup> More formally, suppose the occurrence of Default or No-Default is driven by a Markov stochastic process  $\{s_t\}_{t=1}^{\infty}$  with possible realizations  $\bar{s}_t \in \Phi \equiv \{D, ND\}$ , and let  $\Omega^t$  be the set of possible histories up to time  $t$ :

$$\Omega^t \equiv \{\omega^t = \{D, \{\bar{s}_j\}_{j=1}^t\}: \bar{s}_j \in \Phi, \forall j = 1, \dots, t\}. \quad (8)$$

Notice that we only consider the histories  $\omega^t = \{D, \bar{s}_1, \bar{s}_2, \dots, \bar{s}_t\}$ , i.e. histories that start with a default on past promises. This is because in the initial period, when the current government has just been settled, there are no promises to be fulfilled.

#### 3.1. Individual agents and constraints

Before turning to the planner's problem, we describe the problem of individual agents. In Eq. (7) we wrote the households' FOCs. These equations depend on future variables and hence households need to form rational expectations using available information. Given our institutional setting, households believe the promises of the current planner, but consider that if a

<sup>8</sup> The class of models that our methodology is able to handle is fairly general and has the same requirements of Marcet and Marimon [24]. The separability in Eq. (7) is not necessary and the terms  $k_t, k_{t+1}, x_t, x_{t+1}$  can all interact in a multiplicative way. Our methodology is also able to handle participation constraints and other infinite horizon constraints, as also described in Marcet and Marimon [24].

<sup>9</sup> In Section 6, we relax this assumption.

<sup>10</sup> For the purposes of this paper it is indifferent whether the reoptimization is undertaken by the same planner or by a newly appointed one.

<sup>11</sup> In other words, for the derivations only, we are assuming that the productivity parameter  $A$  is constant. Productivity shocks are instead incorporated in the numerical analysis of Section 5.

reoptimization occurs then different policies will be implemented and past promises will not be kept. As it is common in the time-consistency literature, economic agents will take future controls that cannot be committed upon as functions of the state variable, i.e.  $x_{t+1}(\{\omega^t, D\}) = \Psi(k_{t+1}(\{\omega^t, D\}))$  where we use the short notation  $\{\omega^t, D\}$  to denote  $\{\omega^t, \bar{s}_{t+1} = D\}$ . The function  $\Psi(\cdot)$  denotes the vector of policy functions that rational agents anticipate to be implemented in future periods.<sup>12</sup> Therefore, the constraint in Eq. (7) becomes:

$$b_1(x_t(\omega^t), k_t(\omega^t)) + \beta \text{Prob}(\{\omega^t, ND\}|\omega^t)b_2(x_{t+1}(\{\omega^t, ND\}), k_{t+1}(\{\omega^t, ND\})) + \beta \text{Prob}(\{\omega^t, D\}|\omega^t)b_2(\Psi(k_{t+1}(\{\omega^t, D\})), k_{t+1}(\{\omega^t, D\})) = 0 \tag{9}$$

where we use the short notation  $\text{Prob}(\{\omega^t, ND\}|\omega^t)$  to denote  $\text{Prob}(\{s_j\}_{j=0}^{t+1} = \{\omega^t, ND\} | \{s_j\}_{j=0}^t = \omega^t)$ . Note that  $k$  is a state variable and hence it is understood that  $k_{t+1}(\{\omega^t, ND\}) = k_{t+1}(\{\omega^t, D\}), \forall \omega^t$ .

### 3.2. The planner

When a default occurs, a new plan is made from that point onwards. In order to be able to solve the problem, it is convenient to separate all histories  $\omega^t$  with respect to the first time when default occurs. This is because we want to know which histories correspond to which plan. We now define the subset of  $\Omega^t$  of histories where only commitment has occurred up to time  $t$  as:

$$\Omega_{ND}^t \equiv \{\omega^t = \{D, \{\bar{s}_j\}_{j=1}^t\} : \bar{s}_j = ND, \forall j = 1, \dots, t\} \tag{10}$$

and the subsets of histories where the first default occurs in period  $i$ ,

$$\Omega_{D,i}^t \equiv \{\omega^t = \{D, \{\bar{s}_j\}_{j=1}^t\} : (\bar{s}_i = D) \wedge (\bar{s}_j = ND), \forall j = 1, \dots, i - 1\}, \quad \text{if } i \leq t, \tag{11}$$

$$\Omega_{D,i}^t \equiv \emptyset, \quad \text{if } i > t. \tag{12}$$

By construction note that  $\{\Omega_{ND}^t, \Omega_{D,1}^t, \dots, \Omega_{D,t}^t\}$  is a partition of the set  $\Omega^t$ . Moreover, it can be seen that the sets  $\Omega_{ND}^t$  and  $\Omega_{D,i}^t$  are singletons.<sup>13</sup> Therefore, in order to avoid confusion between histories and sets of histories, we will refer to these singleton sets as  $\omega_{ND}^t$  and  $\omega_{D,i}^t$  respectively.

In Fig. 1, we show a more intuitive representation of the particular partition of histories specified above, where we use the name of the unique history ending in a given node to denote the node itself. White nodes indicate when past promises are disregarded (a new plan is made), while black nodes indicate the cases where promises are fulfilled (no default has occurred). We can see that in any period  $t$  there is only one history  $\omega_{ND}^t$  such that a default has never occurred in the past. Moreover, there is also only one history  $\omega_{D,i}^t = \{\omega_{ND}^{i-1}, D\}$ , meaning that the first default occurred in period  $i$ . In our institutional setting, a new plan is then made from the node  $\omega_{D,i}^t$  onward, specifying the planner’s actions over the possible histories passing through the node  $\omega_{D,i}^t$ , that is the sets  $\Omega_{D,i}^t, \forall t \geq i$ .

Given this partition of histories, the planner’s objective can be conveniently formulated as stated in the following proposition.

<sup>12</sup> For further discussions on this issue see Klein et al. [20].

<sup>13</sup>  $\Omega_{ND}^t$  only contains the history  $\{D, \bar{s}_1 = ND, \bar{s}_2 = ND, \dots, \bar{s}_t = ND\}$  and similarly the set  $\Omega_{D,i}^t$  only contains the history  $\{D, \bar{s}_1 = ND, \bar{s}_2 = ND, \dots, \bar{s}_{i-1} = ND, \bar{s}_i = D\}$ .

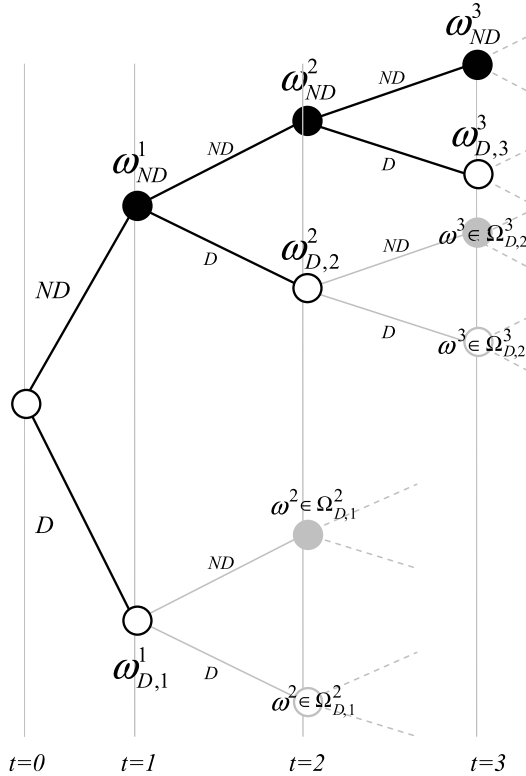


Fig. 1. Diagram of the possible histories.

**Proposition 1.** *The planners objective function in a loose commitment setting can be written as*

$$\begin{aligned}
 W(k_0) = & \max_{\{x_t, (\omega_{ND}^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left\{ \beta^t [Prob(\omega_{ND}^t) u(x_t(\omega_{ND}^t), k_t(\omega_{ND}^t))] \right. \\
 & \left. + \sum_{i=1}^{\infty} \beta^i Prob(\omega_{D,i}^i) \xi(k_i(\omega_{D,i}^i)) \right\}. \tag{13}
 \end{aligned}$$

**Proof.** See Appendix A. □

Notice that according to this formulation, the planner maximizes over the single history  $\{\omega^t: \omega^t \in \Omega_{ND}^t\} \equiv \omega_{ND}^t$ . This is because all the remaining histories  $\{\Omega_{D,1}^t, \dots, \Omega_{D,t}^t\}$  are incorporated in the value functions  $\xi(k_i)$ , summarizing the optimal plans made if a reoptimization occurs in a future period  $i$ . Also, since  $\Omega_{D,i}^t \cap \Omega_{D,j}^t = \emptyset$  for  $i \neq j$ , the future plans are independent between themselves.<sup>14</sup>

<sup>14</sup> In this formulation, we are assuming that the institutional setting faced when reoptimizations occur are all identical. Debortoli and Nunes [12] relax this assumption and focus on political disagreement issues.

To further simplify the problem, we assume that the random variable  $s_t$  is i.i.d. At any given point in time, promises are fulfilled with the probability  $\pi$ , while a new plane is made with probability  $1 - \pi$ . This implies that

$$Prob(\omega_{ND}^t) = \pi^t, \tag{14}$$

$$Prob(\omega_{D,t}^t) = \pi^{t-1}(1 - \pi). \tag{15}$$

The planner’s problem thus becomes

$$\begin{aligned} & \max_{\{x_t(\omega_{ND}^t)\}_{t=0}^\infty} \sum_{t=0}^\infty (\beta\pi)^t \{u(x_t(\omega_{ND}^t), k_t(\omega_{ND}^t)) + \beta(1 - \pi)\xi(k_{t+1}(\omega_{D,t+1}^{t+1}))\}, \\ \text{s.t. } & b_1(x_t(\omega_{ND}^t), k_t(\omega_{ND}^t)) + \beta\pi b_2(x_{t+1}(\omega_{ND}^{t+1}), k_{t+1}(\omega_{ND}^{t+1})) \\ & + \beta(1 - \pi)b_2(\Psi(k_{t+1}(\{\omega_{ND}^t, D\})), k_{t+1}(\{\omega_{ND}^t, D\})) = 0. \end{aligned} \tag{16}$$

We are now ready to show that this problem can be written recursively, and that the optimal policy functions are time-invariant and depend on a finite set of states. This is not straightforward. Due to the fact that we do have future controls in the constraints through the term  $\beta\pi b_2(x_{t+1}(\omega_{ND}^{t+1}), k_{t+1}(\omega_{ND}^{t+1}))$ , the usual Bellman equation is not satisfied.<sup>15</sup> Building on the results of Marcet and Marimon [24], we show that problems of this type can be rewritten as a saddle point functional equation (SPFE) that generalizes the usual Bellman equation. This result is summarized in Proposition 2.

**Proposition 2.** *Problem (16) can be written as saddle point functional equation as:*

$$\begin{aligned} W(k, \gamma) &= \min_{\lambda \geq 0} \max_x \{H(x, k, \lambda, \gamma) + \beta(1 - \pi)\xi(k') + \beta\pi W(k', \gamma')\}, \\ \text{s.t. } & \gamma' = \lambda, \quad \gamma_0 = 0, \end{aligned} \tag{17}$$

where

$$\begin{aligned} H(x, k, \lambda, \gamma) &= u(x, k) + \lambda g_1(x, k) + \gamma g_2(x, k), \\ g_1(x, k) &= b_1(x, k) + \beta(1 - \pi)b_2(\Psi(k'), k'), \\ g_2(x, k) &= b_2(x, k). \end{aligned}$$

**Proof.** See Appendix A.  $\square$

Proposition 2 states that the current planner maximizes utility of the representative agent subject to the constraints  $g_1(x, k) + \beta\pi g_2(x', k') = 0$ , where the latter is incorporated in  $H$ . If there is no commitment, the continuation of the problem is  $\xi(k')$ . If the current promises will be fulfilled, then the continuation of the problem is  $W(k', \gamma')$ , and promises are summarized in the co-state variable  $\gamma'$ . The co-state variable is not a physical variable and the policymaker always faces the temptation to set it to zero. Also note that in our problem only the first constraint contained in Eq. (7) contains future control variables. Therefore, only the first element of the vector  $\lambda$  needs to be included as a co-state variable. The optimal policy functions of such problem are time invariant and depend on a finite number of states, as Proposition 3 describes.<sup>16</sup>

<sup>15</sup> For details see Stokey et al. [34].

<sup>16</sup> As it is common in the time-consistent literature and also in the optimal taxation literature we do not prove that the optimal policy function is unique. Nevertheless, in our numerical exercises we found no evidence of multiple solutions.



**Proposition 3.** *The solution of problem (16) is a time invariant function with state variables  $(k_t, \gamma_t)$ , that is to say:*

$$\begin{aligned} \psi(k, \gamma) \in \arg \min_{\lambda \geq 0} \max_x \{ & H(x, k, \lambda, \gamma) + \beta(1 - \pi)\xi(k') + \beta\pi W(k', \gamma') \}, \\ \text{s.t. } \gamma' = \lambda, \quad \gamma_0 = 0. & \end{aligned} \quad (18)$$

**Proof.** See Appendix A.  $\square$

### 3.3. Equilibrium

In the previous section, we have assumed that the institutional setting faced when reoptimizations occur from period  $t = 1$  onward are all identical. From now on, we also assume that such settings are also the same as the one we specify in period 0. In other words, we set up the reoptimization problems in the same way as the problem of the planner in period 0. Thus we can use the following definition of equilibrium.

**Definition 1.** A Markov perfect equilibrium where each planner faces the same institutional setting must satisfy the following conditions.

1. Given  $\Psi(k)$  and  $\xi(k)$ , the sequence  $\{x_t\}$  solves problem (16).
2. The value function  $W(k, \gamma)$  is such that  $\xi(k) = W(k, 0) \equiv W(k)$ .
3. The policy functions  $\psi(k, \gamma)$  solving problem (16) are such that  $\Psi(k) = \psi(k, 0)$ .

The second part of the definition imposes directly that the planner's problem in period 0 is identical to the problem of a planner reoptimizing at a generic date  $t$ . When a planner comes to office, she has not previously made any promise. In our formulation, this means that co-state variable  $\gamma$  is reset to zero. While a planner is in office, she makes promises, and faces the temptation to deviate and reoptimize. In other words, the multiplier encoding the planner's promises is not a physical state variable and could always be put to zero. The third part of the definition imposes a consistency requirement in the constraints. More precisely, we require the policy functions  $\Psi(k)$  that agents expect to be implemented under default to be consistent with the optimal policy function. We refer to the notion of Markov perfect equilibrium because the function  $\Psi$  only depends on the natural state variables  $k$ . Also, in this equilibrium neither the planner nor individual agents desire to change behavior. Individual agents are maximizing and their beliefs are correct. The planner, taking as given  $\Psi$  and  $\xi = W$ , is also maximizing.

### 3.4. Solution strategy

The previous propositions showed that the problem can be written recursively. At first sight, solving this problem looks complex. Indeed our loose commitment setting raises both the difficulties of problems with commitment and those of problems with no-commitment. More precisely, as in the problems with commitment, making the problem recursive requires to include the Lagrange multiplier as a co-state variable. Moreover, as in the problems with no-commitment, the derivatives of the (unknown) policy functions  $\Psi(k)$  enter in the FOCs of the problem, and thus matter for the solution. Finally, the value function  $\xi(k)$  appears in the objective function (13), in

any term of the summation. In this section, we discuss a relatively inexpensive way to solve the problem, which relies on the solution of only one fixed point problem.

In particular, we use the FOCs of the Lagrangian associated to (13).<sup>17</sup> It is important to notice that the derivative of the value function  $\xi(\cdot)$  appears in the FOCs. This reflects the possibility for the current planner to influence the choices made when reoptimizations occur. This is done through the appropriate choice of the state variables left to future periods.<sup>18</sup> As described in Definition 1, we consider the case where  $\xi(k_t) = W(k_t)$  and hence  $\xi_{k,t+1} = W_{k,t+1}$ . Therefore, to obtain the derivative  $W_{k,t+1}$  we can use an envelope result, which is summarized in Result 1.<sup>19</sup>

**Result 1.** Using envelope results it follows that:

$$\frac{\partial W(k_t)}{\partial k_t} = \frac{\partial u(x_t(k_t), k_t)}{\partial k_t} + \lambda_t g_{1,k_t,t} \quad (19)$$

where all variables are evaluated using the optimal policy of a planner making her plan in period  $t$ , given the state  $k_t$ .

Using this envelope result to substitute  $\xi_{k,t+1} = W_{k,t+1}$ , the FOCs only depend on the functions  $\psi(k_t, \lambda_{t-1})$  and  $\Psi(k)$ . Moreover, using Definition 1 and Proposition 2 we know that  $\Psi(k) = \psi(k, 0)$ ,  $\forall k$ . Hence, we can solve the problem relying on only one fixed point on the policy functions  $\psi(\cdot)$ .

A possible alternative to our approach has been proposed by Schaumburg and Tambalotti [33], for a linear-quadratic optimal monetary policy problem. Our method constitute a contribution along several dimensions. First, it can be applied to a general class of microfounded non-linear problems, as the fiscal policy problem considered here. Such problems could not be addressed by taking a linear-quadratic approximation of the original non-linear problem, as proposed, e.g., in Benigno and Woodford [5]. Indeed, this would require to impose the “timeless-perspective” assumption, stating that policy reoptimizations never occur.<sup>20</sup> Hence, this would prevent analyzing a loose commitment setting, where the purpose is precisely to study the optimal policy problems where policymakers occasionally revise their plans. Second, Schaumburg and Tambalotti [33] only solved a model with no endogenous state variables, and suggest a procedure to handle endogenous state variables relying on three fixed points. Since our method relies on solving only one fixed point, it significantly simplifies the solution of models with endogenous state variables. This is of particular interest in a loose commitment setting. Indeed, it is precisely through the choice of a state variables that policymakers can affect strategically the actions of their successors.

Finally, we use a collocation method to solve for the optimal policy functions. Besides the arguments presented by Judd [18] and Judd [19], there are other specific reasons why global solution methods are more appropriate in our framework. For the solution to be accurate, one

<sup>17</sup> Details on the FOCs can be found in Appendix A.

<sup>18</sup> Note that, when default occurs, the Lagrange multiplier is set to zero and cannot be used to influence incoming planners.

<sup>19</sup> A proof of this envelope result is available upon request.

<sup>20</sup> This is shown in Curdia et al. [10], Debortoli and Nunes [11] and Benigno and Woodford [5]. In order to derive a correct linear-quadratic approximation, the “timeless-perspective” is a necessary assumption in all the optimal policy models where the first-best allocation cannot be attained, as in the presence of distortionary taxes.

Table 1  
Parameter values.

Parameter	Value	Description
$\beta$	0.96	Discount factor
$\phi_c$	0.285	Weight of consumption vs. leisure
$\phi_g$	0.119	Weight of public vs. private consumption
$\theta$	0.36	Capital income share
$\chi_0$	0.171	Depreciation function parameter
$\chi_1$	1.521	Depreciation function parameter
$\underline{A}$	0.976	Low productivity level
$\bar{A}$	1.024	High productivity level
$\Gamma$	0.946	Probability of remaining in the current state

needs to perform a good approximation of the policy functions both when promises are fulfilled and when a reoptimization occurs. These two states and the corresponding policy functions are not similar. This raises the challenge to find an appropriate value around which to approximate the solution, thus reducing the reliability of local methods in general.<sup>21</sup>

#### 4. Calibration

In order to facilitate comparisons with the previous literature analyzing optimal fiscal policy problems without commitment, and not to bias results in our favor, we calibrate the model choosing the same functional forms and parameters as in Martin [25]. In particular, we specify a period utility function<sup>22</sup>

$$u(c_t, g_t, l_t) = (1 - \phi_g)[\phi_c \log(c_t) + (1 - \phi_c) \log(1 - l_t)] + \phi_g \log(g_t),$$

the depreciation function

$$\delta(v_t) = \frac{\chi_0}{\chi_1} v_t^{\chi_1},$$

and the production function

$$F(A, k, l, v) = A(kv)^\theta l^{1-\theta}$$

where, following Chari et al. [7], we calibrate the technology shock as a symmetric two-state Markov chain, so that  $A \in \{\underline{A}, \bar{A}\}$  and  $E(A) = 1$ . As common in this literature, we also choose a year as the period length.<sup>23</sup>

Table 1 summarizes the values used for the parameters. The parameters  $\chi_0$  and  $\chi_1$  imply that in steady state the capital utilization rate ( $v$ ) is about 0.8, and the depreciation rate  $\delta(v)$  is about 0.08. The parameters related to the technology shocks, as in Klein and Ríos-Rull [21], are chosen to match the standard deviation (0.024) and autocorrelation (0.88) reported in Prescott [29] and consistent with the values we also obtain for the period 1963–2003.

<sup>21</sup> This is particularly evident when considering the behavior of the Lagrange multipliers, the co-state variables in the policy functions. When promises are fulfilled the (absolute) value of the Lagrange multipliers increase. On the contrary, in case of default the Lagrange multipliers are reset to zero.

<sup>22</sup> This utility function is also the same as in Klein et al. [20].

<sup>23</sup> This choice is mostly justified by the fact that tax rates are typically adjusted only once per year.

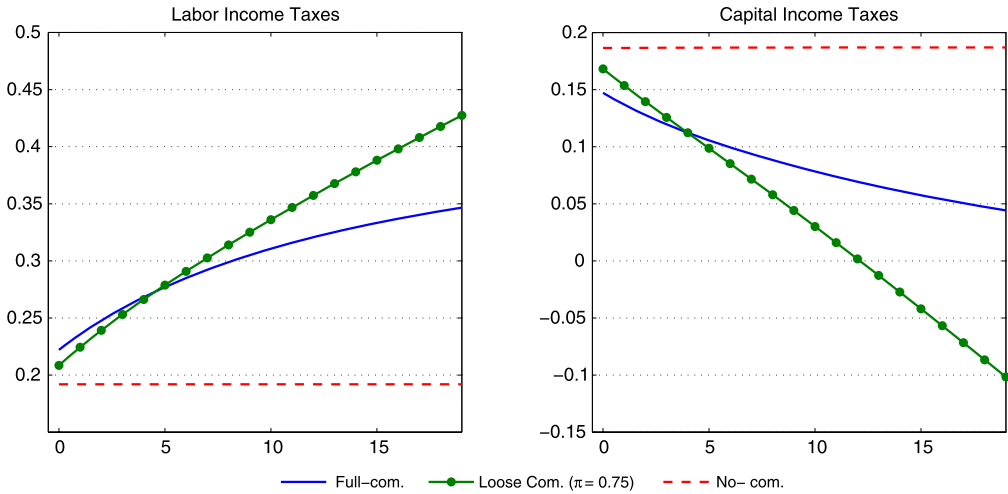


Fig. 2. Optimal labor and capital income taxes under different commitment settings, assuming that no reoptimization occurs. The stock of capital is initialized at the steady-state value under no-commitment.

A relevant parameter for our analysis is the probability of commitment  $\pi$ . Providing a definitive answer on the probability  $\pi$  or empirically estimating these models is beyond the scope of this paper. Nevertheless, we can provide some interpretations. First, a probability  $\pi$  implies an expected tenure of  $1/(1 - \pi)$  years. For example, a value of 0.75 implies that a planner is expected to be in power for 4 years on average. A calibration based on the political history of the US implies a value of 0.8, while the political history of Italy would imply a calibration around 0.3.<sup>24</sup>

Second,  $\pi$  is related to the proportion of promises fulfilled. Rallings [30] tried to obtain a measure of how many manifesto pledges were actually implemented. Some of these pledges often reveal political options, such as the composition of expenditures, and may not always be related to time-inconsistency issues. Nevertheless, the average number reported by the author is 0.63 and 0.71 for Britain and Canada respectively.

In our analysis, we consider  $\pi = 0.75$  as our benchmark. This implies a relative small departure from the full-commitment assumption. However, the specific value considered is not qualitatively important for our considerations. Moreover, in Section 6 we discuss a case where  $\pi$  is not constant over time but it endogenously depends on economic conditions.

## 5. Results

We now characterize the properties of the optimal policy under loose commitment, in comparison to the full-commitment and no-commitment cases. Fig. 2 plots the optimal plans announced by the policymakers, contingent on the fact that reoptimizations do not occur. In that picture, to highlight the role played by loose commitment, we are abstracting from other sources of variability, like transition dynamics to steady state and changes in productivity.

<sup>24</sup> In Italy, if we consider a change in government when the same prime minister is in power but the coalition changes, then this number drops to 0.

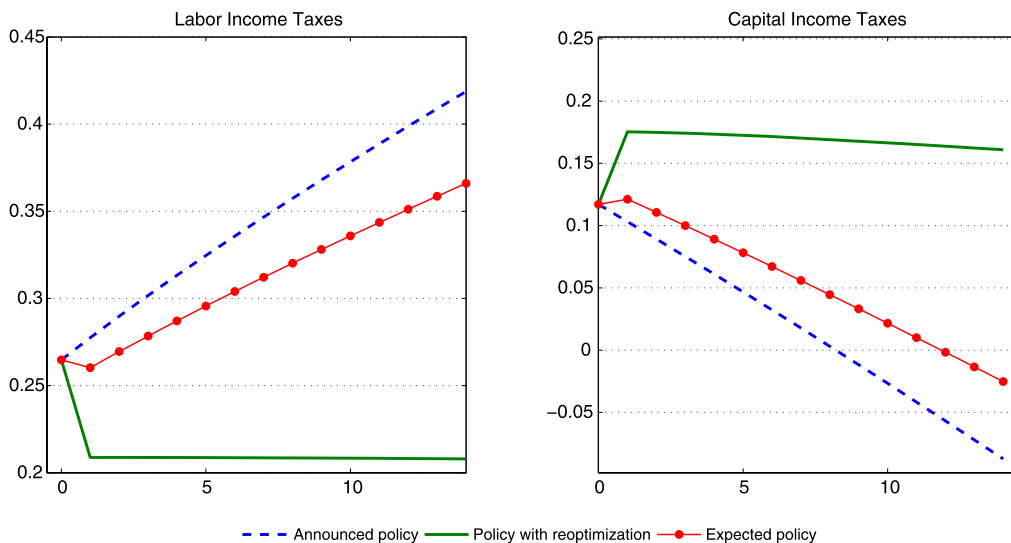


Fig. 3. The figure plots the optimal policy under loose commitment ( $\pi = 0.75$ ). The solid line represents the optimal policy prevailing when a reoptimization occurs in period  $t$ , given that no reoptimizations occurred before. The dashed line indicates the optimal policy announced in period 0 regarding period  $t$ , while the line with dots indicates the weighted average of the two.

Under full-commitment capital taxes are decreasing, and converge to zero in the long-run. In period 0, capital taxes are positive, since capital is a relatively inelastic tax base. As time passes by, the incentive of reducing capital taxes increases. This is because a lower capital tax influence capital accumulations in all the previous periods. Labor taxes are instead increasing over time, mainly to balance the budget. Under no-commitment, capital and labor income taxes are instead constant over time. In this case, the planner cannot make credible promises about future taxes and, for any given level of capital, it must set the same tax rates over time. This reflects the time-consistency of the plan.

A planner with loose commitment, in the beginning, sets capital taxes at a higher level than a fully committed policymaker. However, she also promises to rapidly reduce taxes in the future. For example, when  $t = 5$  promised capital taxes are lower than under full-commitment. The rapid reduction in capital taxes counteracts the effect of occasional reoptimizations, thus keeping expectations about capital taxes on track. This is also highlighting that announced policies under loose commitment do not necessarily lie in between the policies prevailing with full-commitment and no-commitment.

Importantly, under loose commitment the promised pattern of taxes is not necessarily implemented. Fig. 3 illustrates what happens in those cases. The solid line plots the tax rates that would be chosen by a policymaker reoptimizing in period  $t$ , given that no reoptimizations occurred until period  $t - 1$ . For comparison, the policy promised at period 0 and the expected policy (i.e. the weighted average between the promised policy and the policy prevailing if the first reoptimization occurs in period  $t$ ) are also reported.

In principle, the planner could choose equal tax rates whether reoptimizations occur or not. However, this does not turn out to be the optimal policy. What is relevant for the current policymaker are the expectations about future taxes. Agents expect that when a reoptimization occurs the capital tax rate will be set at an inefficiently high level. To keep the expected capital rate

low, the policymaker thus promises that, if a reoptimization does not occur, capital taxes will be reduced. When promising a certain tax rate for next period, the planner needs to equate the benefits of correcting the expected tax rate, and the costs of creating tax rate variability. This observation thus suggests that the optimal policy under loose commitment differs from the usual tax smoothing prescription.

Finally, notice that capital tax rates do not converge to zero, as in the full-commitment case. Indeed, after some periods where no reoptimizations occur, both the actual and the expected capital tax rates (conditional on no reoptimizations up to period  $t$ ) become negative.<sup>25</sup> This is because in many other states of nature the capital tax rate is positive, which leads to an under-accumulation of capital.

We now turn to illustrate how some basic statistical properties of our variables are affected by the loose commitment assumption. Even though the goal of our analysis is not to fit the business cycle statistics, we also compare the properties of the model with their counterparts in the data. The main statistics are summarized in Table 2.

### 5.0.1. Long-run averages

In general, average allocations under loose commitment are close to those under no-commitment. This warns us that a relatively small departure from the full-commitment assumption changes dramatically the average optimal tax rates and other allocations.

In particular, as it is common in the literature, under full-commitment capital taxes are zero in the long-run.<sup>26</sup> On the contrary, with no-commitment capital income taxes are positive (19%, under our calibration).<sup>27</sup> In our loose commitment setting, when reoptimizations occur on average only every 4 years ( $\pi = 0.75$ ), optimal capital taxation is also positive, at about 13%, which is close to the no-commitment value.

This relatively small departure from the full-commitment assumption is therefore enough to account for an important property of the data, where capital income taxes are positive, with an average of 40% over the period 1976–2003. Moreover, given the balanced budget assumption, labor income taxes under full-commitment are higher (38%) than in the data (28%). In that case, the entire burden of taxation is indeed borne by labor taxes. This is no longer the case departing from the full-commitment assumption. Under no-commitment, however, labor income taxes are too low (19%). Interestingly, in our loose commitment setting labor income taxes are closer to the data, at a level of 25%. Given these effects on tax rates, we also have that capital increases with the level of commitment, while hours worked decrease.<sup>28</sup>

### 5.0.2. Volatilities

As reported in Table 3, and as shown in Klein and Ríos-Rull [21], volatilities are higher under full-commitment than under no-commitment. With loose commitment, the volatilities of taxes are not between the two extremes, but are in general higher. This is because the occasional reop-

<sup>25</sup> This is however, a very unlikely event. Expected capital tax rates are indeed negative only after 13 periods with no reoptimizations. Given that  $\pi = 0.75$ , the probability of this event is less than 2.5%.

<sup>26</sup> The robustness of this result across several model specifications is documented, e.g., in the survey by Chari and Kehoe [8].

<sup>27</sup> This result has been shown, e.g., by Klein and Ríos-Rull [21], Klein et al. [20] and Martin [25], restricting their analysis on Markov perfect equilibria, as we do here.

<sup>28</sup> The latter statistics are not reported for brevity, but available from the authors upon request.

Table 2  
Properties of taxes – data vs. model.

	US data (1967–2003)	Model		
		Full-com.	Loose com. ( $\pi = 0.75$ )	No-com.
<i>Labor income taxes</i>				
Mean	0.28	0.38	0.25	0.19
Std. dev. (w.r.t. output)	0.63	0.104	0.77	0.01
Autocorrelation	0.90	0.92	0.72	0.92
Cross correlations:				
with output	−0.28	−0.22	−0.38	0.85
with tech. shocks	−0.55	−0.34	−0.004	0.91
<i>Capital income taxes</i>				
Mean	0.40	0.00	0.13	0.19
Std. Dev. (w.r.t. output)	1.24	0.26	0.92	0.097
Autocorrelation	0.93	0.77	0.75	0.75
Cross correlations:				
with output	0.26	−0.50	0.27	−0.81
with tech. shocks	0.15	−0.35	−0.08	−0.70
with labor taxes	−0.66	−0.69	−0.98	−0.45
<i>Government expenditure</i>				
Mean (w.r.t. output)	0.16	0.25	0.21	0.19
Std. dev. (w.r.t. output)	1.099	0.76	0.87	0.88
Autocorrelation	0.89	0.86	0.81	0.87
Cross correlations:				
with output	0.45	0.99	0.62	0.99
with tech. shocks	0.66	0.99	0.88	0.99
with labor taxes	−0.40	−0.27	0.43	0.90
with capital taxes	−0.07	−0.41	−0.49	−0.73
<i>Output</i>				
Mean (normalized)	1.00	1.00	0.97	0.95
Std. dev.	0.03	0.057	0.06	0.054
Autocorrelation	0.80	0.84	0.81	0.83

*Note:* Marginal tax rates are from Chen et al. [9], who use the methodology of Joines [16] and McGrattan [26]. Results are robust considering average tax rates, as estimated by Mendoza et al. [27]. The other series are from the United States National Income and Product Accounts (NIPA). Standard deviations and correlations of taxes, (the log of) GDP and (the log of) public expenditure have been computed after applying a linear trend. Our conclusions do not change using the HP filter. The series of technology shocks have been obtained as the Solow residual from the production function, assuming constant capacity utilization. Results are robust if we consider variable capacity utilization. Statistics implied by the models are computed across 100 simulations of 500 observations each.

timizations, peculiar to our loose commitment setting, act as a separate source of fluctuations. In particular, there are two different forces affecting volatilities. First, the frequency of reoptimizations, which is decreasing in  $\pi$ . Second, the magnitude of the responses in case of reoptimization, which is instead increasing in  $\pi$ . The relative strength of these two forces determines at which degree of commitment volatilities are higher.

In a model with loose commitment, technology shocks account for a significantly smaller proportion of aggregate volatility. This can be seen in Table 3, where only 58% of the volatility of output can be attributed to technology shocks, with the remaining part being associated to loose commitment. Remarkably, this proportion is only 2% for capital and labor income taxes.

Table 3

	Full-com.	Loose com.			No-com.
		$\pi = 0.75$	$\pi = 0.5$	$\pi = 0.25$	
<i>Coefficient of Variation</i>					
Labor income taxes	0.016	0.18	0.13	0.06	0.00
Capital income taxes	–	0.44	0.21	0.09	0.03
Public expenditure	0.018	0.021	0.019	0.018	0.018
Output	0.0585	0.06	0.0582	0.054	0.053
<i>Proportion due to technology shocks</i>					
Labor income taxes	1.00	0.02	0.00	0.00	1.00
Capital income taxes	1.00	0.02	0.03	0.06	1.00
Public expenditure	1.00	0.53	0.73	0.88	1.00
Output	1.00	0.58	0.66	0.82	1.00

*Note:* The table reports the coefficient of variation (upper panel) and the proportion of variance due to technology shocks (lower panel) depending on the commitment setting considered. Output and Public expenditure are considered in logs.

This exercise warns us of the risks of overestimating the importance of technology shocks in the business cycle in empirical estimates that do not take into account the possibility of policy reoptimizations or related political factors.

Loose commitment also increases the relative volatility of taxes with respect to output. As can be seen in Table 2, this improves the performance of the model, since the (relative) volatility of capital and labor income taxes under full-commitment and no-commitment are much lower than in the data. In this respect, our analysis can be related to the literature on political cycles, as described for instance in Drazen [13]. This literature has argued the political cycle affects substantially the policy instruments, whereas output and consumption are less affected. Our model has the same prediction. Labor and capital income taxes are the policy instruments and these face greater variations in relative terms. This also implies that governments do not succeed in affecting consumption and output before elections, even though policy instruments may aim at such outcomes. It is not our goal to match political cycles, because electoral competition is absent from the model. But, our simple model does not contradict the empirical evidence found in that literature.

### 5.0.3. Autocorrelations

In a loose commitment setting, the autocorrelation statistics are lower than both under full-commitment and no-commitment. First, this is because when a reoptimization occurs the dependence of the variables on past-promises vanishes, thus reducing their degree of history-dependence. This explains why autocorrelations are lower than under full-commitment.

Second, allocations are less persistent than under no-commitment. For a given level of capital and technology shock, the optimal policy under no-commitment prescribes to keep taxes constant over time. On the contrary, under loose commitment, the ability to (partially) commit makes it optimal to set taxes at different rates over time, thus exploiting the fact that when a reoptimization occurs capital is a relatively inelastic tax base. This, in turn implies a lower persistence than under no-commitment. Overall, independently on the assumption about commitment, the autocorrelations of both capital and labor income taxes implied by the model are high, as in the data.



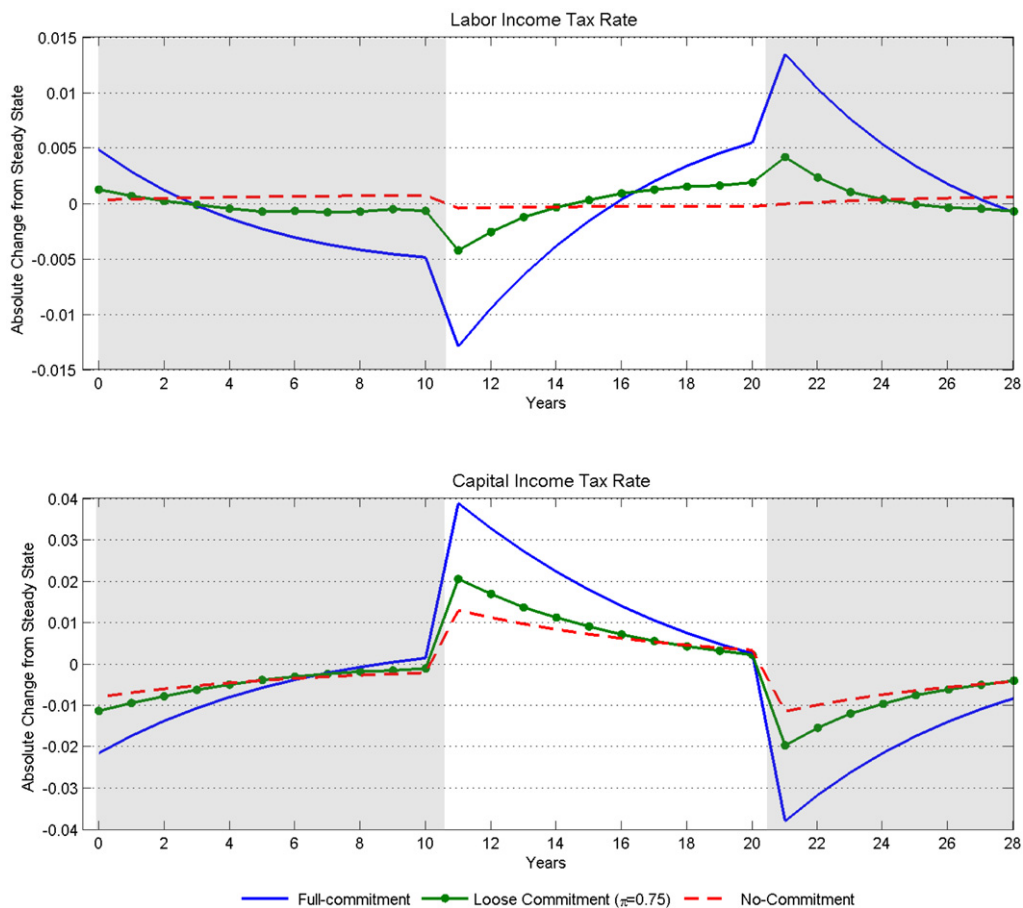


Fig. 4. Absolute change from steady state values of labor income (upper panel) and capital income (lower panel) tax rates in the presence of productivity shocks. The shaded areas indicate periods with high productivity, while the white areas periods with low productivity. The figures under loose commitment are averages across 100 simulated histories of default vs. no default shocks.

#### 5.0.4. Correlations

In general, correlations under loose commitment look very different than under no-commitment and full-commitment. To understand the underlying mechanism, we separately analyze the effects due to technology shocks and occasional reoptimizations.

Fig. 4 illustrates the pattern of tax rates in response to changes in the technology levels. When productivity increases, capital income taxes are decreased on impact in all the economies. This is mainly related to the persistence of the productivity shock. Since productivity is expected to be high also in future periods, it is beneficial to lower capital income taxes to foster capital accumulation. This explains why capital income taxes are negatively correlated with technology shocks in all the commitment settings. Regarding labor income taxes, the picture is more variegated. On impact, lower capital income taxes require slightly higher labor income taxes to balance the budget. As the boom persists, labor income taxes are decreased both under full-commitment and loose commitment, but slightly increased under no-commitment. In this respect, loose com-

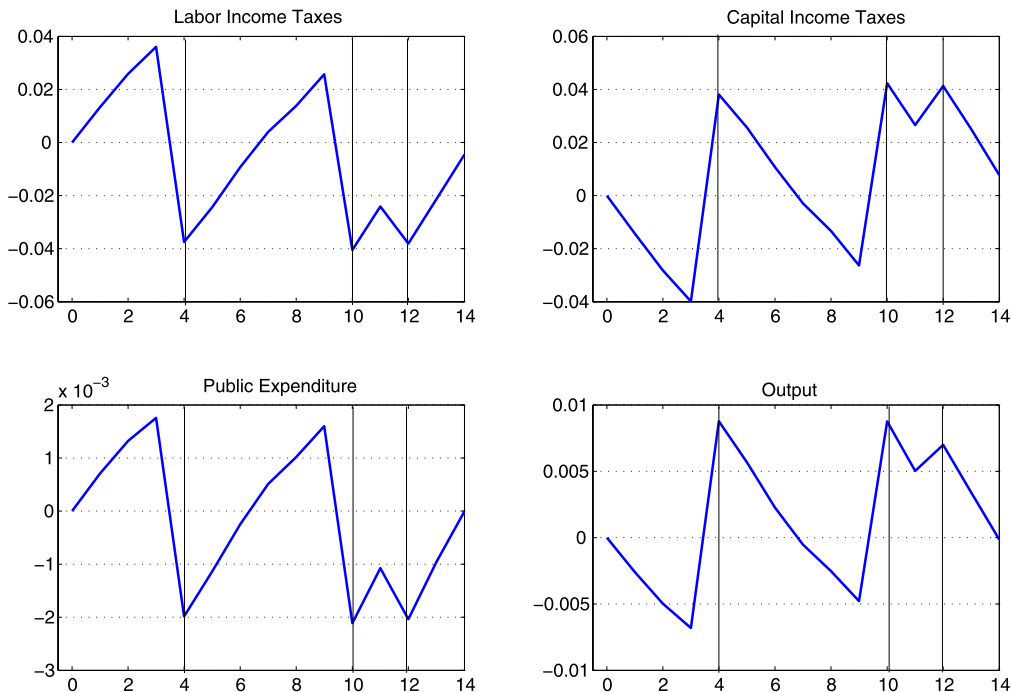


Fig. 5. Absolute change from steady state values over a particular history of default vs. no default shocks. The vertical lines indicate the periods when a reoptimization occurs.

mitment looks therefore more similar to full-commitment than to no-commitment. Interestingly, as in the data, both under full-commitment and loose commitment labor income taxes are negatively correlated with technology shocks. The correlation has instead the wrong sign under no-commitment.<sup>29</sup>

In terms of magnitude, the response of taxes to technology shocks is significantly weaker than under full-commitment. Indeed, in the latter case, capital is less elastic with respect to current taxes. For example, in response to a decrease (increase) in productivity, a tax increase (cut) has smaller implications on capital accumulation, as long as it is accompanied by the promise to lower (increase) taxes in the future. Moreover, as discussed above, under loose commitment the absolute volatility of all the variables is higher, because of the presence of an additional source of fluctuations. As a result, the correlation between taxes and technology shocks under loose commitment is lower than in the other cases. Interestingly, the statistic implied by a model with loose commitment is also closer to the data, where the correlation between capital income taxes and technology shocks is mild (but positive).

The second source of fluctuations, peculiar to our loose commitment setting, is the occurrence of policy reoptimizations. This not only increases aggregate volatilities, but also drives variables in opposite directions than implied by technology shocks, as shown in Fig. 5. When the government reoptimizes, it increases capital income taxes, it reduces labor income taxes and, as a result, output increases. Therefore, under loose commitment, capital income taxes are positively corre-

<sup>29</sup> The latter feature is also present in Klein and Ríos-Rull [21].

lated with output. This result is consistent with what is observed in the data. On the contrary, this feature cannot be accounted for in the other commitment settings considered. In those cases, since technology shocks are the only source of fluctuations, capital income taxes are negatively correlated with output.<sup>30</sup>

Finally, loose commitment reduces the correlation of government expenditure with output, which becomes closer to the data. The correlation among tax rates under loose commitment is instead higher than in the data, but with the correct sign. This is because, whenever a reoptimization occurs, government expenditure decreases and output decreases, while capital and labor income taxes move in opposite directions.

To summarize, the properties of taxes under loose commitment look different than both under commitment and no-commitment. Average allocations are similar to the no-commitment case, even when the degree of commitment is relatively high. Volatilities and correlations change dramatically, mainly because of the additional source of fluctuations not present in the alternative settings. Interestingly, without making a formal econometric test, loose commitment helps explaining some properties of the data, not captured assuming either full-commitment or no-commitment. Altogether, these results emphasize why it may be important to solve a model with loose commitment, as opposed to consider only more extreme commitment assumptions.

### 5.1. Welfare

In this section, we turn to measure the welfare implications of building commitment. In our framework, this means considering the welfare gains of changing the value of  $\pi$ . The next proposition states that an economy facing a better commitment technology achieves a higher welfare.

**Proposition 4.** *Denote the welfare of an economy with  $\pi = \pi_1$  as  $V(K, \pi_1)$ . Then,  $\pi_1 \leq \pi_2$  if and only if  $V(K, \pi_1) \leq V(K, \pi_2)$ .*

**Proof.** See Appendix A.  $\square$

It was already known in the literature that a planner with full-commitment can achieve a higher welfare than a planner with no-commitment. Proposition 4 generalizes this result. The reasoning is that a planner with a better commitment technology could always mimic the allocations of a planner with a worse commitment technology. Note that *a priori* Proposition 4 is not straightforward. First, as it is well known, in the presence of several distortions (partially) correcting one distortion does not necessarily improve welfare. Second, loose commitment can be considered as an additional shock to the economy which, as discussed above, increases the volatility of our variables. This is typically associated with a decrease in welfare. This does not happen in this context, mainly because increasing commitment leads to more efficient allocation levels. It follows that any loose commitment technology with  $0 < \pi \leq 1$  allows the planner to improve welfare with respect to the no-commitment case.

In Table 4 we report the welfare changes, measured in consumption equivalent variation, implied by passing from an economy with no-commitment to an economy with a degree of

<sup>30</sup> In a different model, where taxes are set one period in advance and capacity utilization is constant, Klein and Ríos-Rull [21] find that capital income taxes are positively correlated with output both under full-commitment and no-commitment.

Table 4  
Welfare gains from commitment.

	Full-com.	Loose com.			No-com.
		$\pi = 0.75$	$\pi = 0.5$	$\pi = 0.25$	
Welfare gains (% CEV)	3.60	1.88	0.95	0.37	0
Relative to total gains	1.00	0.52	0.26	0.10	0

*Note:* The table reports the welfare gains of passing from no-commitment to a degree of commitment  $\pi$ . The first line reports the consumption equivalent variation (CEV), while the second line reports the relative gain with respect to passing from no-commitment to full-commitment.

commitment  $\pi$ .<sup>31</sup> According to our calibration, the welfare improvement from no-commitment to commitment is equivalent to an increase in private consumption of 3.60%.

As implied by Proposition 4, welfare is (weakly) increasing in  $\pi$ .<sup>32</sup> When the degree of commitment is  $\pi = 0.75$ , the welfare gains with respect to no-commitment are 1.88% of consumption, which amounts to roughly 50% of the total gains of passing from no-commitment to full-commitment. This suggests that increasing  $\pi$  from low to intermediate levels results in relative small welfare gains. In other words, most of the gains from enhancing commitment can only be achieved when  $\pi$  is already high.<sup>33</sup>

Finally, the benefits of building commitment are much larger than the costs associated with the presence of technology shocks. The latter, in our model, are equivalent to less than 0.2% of consumption.

In a related work on optimal monetary policy, Schaumburg and Tambalotti [33] found qualitatively different results. At low levels of commitment, the relative welfare gains in their model are higher. The previous result is independent on the metric used to measure the degree of commitment.<sup>34</sup> Also, the difference from their result is not due to specific modeling choices like the presence of endogenous state variables (absent in [33]) or the model time period (annual vs. quarterly).

## 6. Endogenous probability model

We finally consider an extension where the probability of commitment depends on the states of the economy. In this context, our purpose is to see whether the planner acts strategically to influence the probability of commitment. For simplicity, and without loss of generality, we abstract from the presence of productivity shocks. Therefore, the only shock affecting the economy is whether there are reoptimizations or not.<sup>35</sup>

Since capital is the only natural state variable in the economy, we assume that the probability of commitment depends on the capital stock. More formally, the planner and households consider

<sup>31</sup> Details about the calculations are provided in Appendix A.

<sup>32</sup> More precisely, according to Proposition 4 welfare is increasing in any metric  $f(\pi)$  with  $f' > 0$  for  $0 \leq \pi \leq 1$ . The expected time before a reoptimization  $1/(1 - \pi)$ , also satisfies this property.

<sup>33</sup> Clearly, the welfare gains may be a concave function in a different metric. This is the case, for example, if we measure the welfare gains with respect to the expected time before reoptimization  $1/(1 - \pi)$ , which is a convex function of  $\pi$ .

<sup>34</sup> In other words, whether we measure commitment in the metric  $\pi$  or  $1/(1 - \pi)$ , the relative welfare gains achieved at low levels of commitment are higher in the Schaumburg and Tambalotti model.

<sup>35</sup> Nevertheless, our setup is also valid in the presence of other aggregate shocks. Indeed, also in that case it is possible to separate the histories of events, depending both on the occurrence of reoptimizations and the realizations of productivity shocks. The formal derivations are available from the authors upon request.

that the probability of commitment in the next period is given by a function  $P(k_{t+1})$ , instead of  $\pi$ . Given the assumptions at hand, the probabilities of events are

$$\begin{aligned}
 Prob(\omega_{ND}^t) &= \frac{\prod_{j=0}^t (P(k_j))}{P(k_0)}, \\
 Prob(\omega_{D,i}^t) &= \frac{\prod_{j=0}^{t-1} (P(k_j))}{P(k_0)} (1 - P(k_t)).
 \end{aligned}$$

As in Section 3, we need to start from the planner’s original objective function and obtain an expression similar to (13). To do so, notice that the partition of events considered in Fig. 1 still applies. Then, we can still define  $\Omega_{ND}^t$  and  $\Omega_{D,i}^t$  as in (10)–(12), where by construction  $\{\Omega_{ND}^t, \Omega_{D,1}^t, \dots, \Omega_{D,t}^t\}$  is a partition of the set  $\Omega^t$ . Hence, following the same steps as in (A.1)–(A.3), and plugging the probability of events described above, we obtain

$$\begin{aligned}
 W(k_0) &= \max_{\{x_t(\omega_{ND}^t)\}_{t=0}^\infty} \left\{ \sum_{t=0}^\infty \beta^t \left[ \frac{\prod_{j=0}^t (P(k_j))}{P(k_0)} u(x_t(\omega_{ND}^t), k_t(\omega_{ND}^t)) \right] \right. \\
 &\quad \left. + \sum_{i=1}^\infty \beta^i \frac{\prod_{j=0}^{i-1} (P(k_j))}{P(k_0)} (1 - P(k_i)) \xi(k_i(\omega_{D,i}^i)) \right\}. \tag{20}
 \end{aligned}$$

Rearranging this expression and considering  $\xi = W$ , the objective function of the planner becomes

$$\sum_{t=0}^\infty \beta^t \frac{\prod_{j=0}^t (P(k_j))}{P(k_0)} (u(x_t, k_t) + \beta(1 - P(k_{t+1}))W(k_{t+1})). \tag{21}$$

As it seems reasonable, we also assume that private agents take the probability of commitment as given, thus not internalizing the effects of aggregate capital on the commitment technology.<sup>36</sup> Hence, the constraints that the planner is facing are

$$\begin{aligned}
 b_1(x_t(\omega_{ND}^t), k_t(\omega_{ND}^t)) + \beta(1 - P(k_{t+1}))b_2(\Psi(k_{t+1}(\{\omega_{ND}^t, D\})), k_{t+1}(\{\omega_{ND}^t, D\})) \\
 + \beta P(k_{t+1})b_2(x_{t+1}(\omega_{ND}^{t+1}), k_{t+1}(\omega_{ND}^{t+1})) = 0. \tag{22}
 \end{aligned}$$

We need to prove that this setting can also be written as an SPFE. This is done in the following proposition.

**Proposition 5.** *The problem of a planner maximizing Eq. (21) subject to Eq. (22) can be written as saddle point functional equation.*

**Proof.** See Appendix A. □

With Proposition 5 at hand, it then follows that the solution to the problem is a time-invariant function.

We now turn to characterize the solution of this problem. With respect to the exogenous probability case, the margins determining the optimal level of capital are now different. Indeed, capital

<sup>36</sup> More formally, one could model a continuum of agents on a real interval between 0 and 1. All agents would be equal, and therefore their decisions would be equivalent to a representative agent, who takes aggregate capital as given.

Table 5

Endogenous probability – average values.

	Exog. Prob. ( $\pi = 0.5$ )	Endog. Prob.
Labor income taxes	0.22	0.27
Capital income taxes	0.16	0.15
Consumption (w.r.t. output)	0.60	0.57
Gov't. exp. (w.r.t. output)	0.20	0.22
Capital (w.r.t. output)	2.46	2.55
Probability of commitment	0.5	0.74

has two additional effects, reflecting its influence on the probability of commitment.<sup>37</sup> First, capital affects agents expectations, as can be seen by the appearance of  $P(k_{t+1})$  in the constraints of households (22). Second, it changes the expected utility, induced by the change in the commitment probability. This is captured by the term  $P_{k_{t+1}}\beta(W(k_{t+1}, \lambda_t) - W(k_{t+1}, 0))$ . If capital is increased, the commitment probability is increased by  $P_{k_{t+1}}$ . This increases the chances of the current planner to obtain the continuation value implied by following the announced plan  $W(k_{t+1}, \lambda_t)$ . Nevertheless, it decreases the chances of obtaining tomorrow's continuation value implied by a reoptimization  $W(k_{t+1}, 0)$ .<sup>38</sup>

In what follows, we consider a case where higher capital leads to a higher probability of commitment. This assumption could be justified on political economy grounds. More capital implies more output and a higher probability of reelection. We will consider the following probability function:

$$P(k_t) = 1 - \frac{1}{\left(\frac{k_t}{\tilde{k}}\right)^\rho + 1}. \quad (23)$$

The parameter  $\tilde{k}$  is a normalization such that  $P(\tilde{k}) = 0.5$ . The higher is  $\rho$ , the easier it is for the planner to influence its probability of commitment. When  $\rho = 0$  the probability is constant, as in the cases considered in the previous sections. We chose  $\rho = 30$  and  $\tilde{k}$  to be equal to the average capital allocation when  $\pi = 0.5$ . This normalization allows us to directly compare the results with the case with (constant) exogenous probability considered before.

Results are presented in Table 5. In the endogenous probability model, given the higher marginal benefit of accumulating capital, the capital stock is higher. To foster households' capital accumulation, the planner has to lower capital income taxes. Consequently, labor income taxes increase. At the same time, private consumption decreases and public consumption increases.<sup>39</sup>

Finally, the welfare gains with respect to the no-commitment case is equivalent to 2.6% of consumption, which is higher than in the benchmark case (as shown in Table 4). This is for two reasons. First, the average probability of commitment is higher (74%). Second, the welfare gains function is convex in  $\pi$ . Thus, a varying probability around the mean may induce some additional welfare gains.

<sup>37</sup> This can also be seen by comparing the first-order conditions of the planner's problem, as shown in Appendix A.

<sup>38</sup> The presence of the level of the value function in the FOCs raises an extra difficulty in terms of computational work. This is because an envelope result would not suffice to eliminate the value function from the problem. Thus, also such value function needs to be approximated.

<sup>39</sup> All the other allocations remain roughly identical.

In a political economy interpretation, our model would suggest that governments accumulate more capital to increase the probability of being reelected. This policy improves social welfare, since it reduces political turnover thus increasing the commitment probability.

## **7. Concluding remarks**

In this paper, we analyzed the relevance of policymakers commitment. We showed how to address problems where policymakers occasionally revise their plans. Optimal policy problems are typically analyzed either assuming that policymakers are able to fully commit or that they always reoptimize. With respect to these two extremes, in a loose commitment setting the properties of capital and labor income tax rate change significantly, in terms of averages, volatilities and correlations. Given its simplicity, the purpose of the paper is not to propose a model which is able to match all the business cycle statistics. However, we believe that our loose commitment setting constitutes a necessary step towards testing different alternatives regarding the policymakers' ability to commit.

There are many aspects deserving further exploration. Throughout the paper, we have abstracted from divergences in the political views of governments, which would obviously affect the behavior of policy instruments when plans are revised. In this respect, our loose commitment setting can be a useful tool to analyze political economy problems, where the presence of political turnover constitute a natural limitation of the policymakers' commitment horizon. Due to technical reasons that this paper overcomes, such literature (e.g. Alesina and Tabellini [1]) had always assumed a no-commitment approach or avoided time-inconsistency issues. For example, in a companion paper we study how political disagreement and lack of commitment influence the behavior of government debt. Also, our setting describes what happens in the event of a reoptimization, rather than explicitly modeling why policymakers occasionally renege on their promises. Modeling such a choice is an interesting topic, which may help understand what is the optimal degree of policymakers' commitment.

Finally, the applications of the methodology proposed here are not restricted to optimal policy problems. Indeed, it can be used in other interesting dynamic problems where commitment plays an important role, like in the relationship between firms and its customers and shareholders, or in other principle-agent type of problems.

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**Appendix A**

*A.1. Exogenous probability model*

**Proof of Proposition 1.** We need to show how to derive the formulation of the planner’s objective function (13). Given the partition of histories detailed in Section 3, the planner’s objective is

$$\begin{aligned}
 W(k_0) = & \max_{\substack{\{x_t(\omega^t)\}_{t=0}^\infty \\ \omega^t \in \Omega^t}} \left[ \sum_{t=0}^\infty \sum_{\omega^t \in \Omega_{ND}^t} \beta^t \{ \text{Prob}(\omega^t) u(x_t(\omega^t), k_t(\omega^t)) \} \right. \\
 & + \max_{\substack{\{x_t(\omega^t)\}_{t=1}^\infty \\ \omega^t \in \Omega_{D,1}^t}} \left\{ \sum_{t=1}^\infty \sum_{\omega^t \in \Omega_{D,1}^t} \beta^t \{ \text{Prob}(\omega^t) u(x_t(\omega^t), k_t(\omega^t)) \} \right\} \\
 & \left. + \max_{\substack{\{x_t(\omega^t)\}_{t=2}^\infty \\ \omega^t \in \Omega_{D,2}^t}} \left\{ \sum_{t=2}^\infty \sum_{\omega^t \in \Omega_{D,2}^t} \beta^t \{ \text{Prob}(\omega^t) u(x_t(\omega^t), k_t(\omega^t)) \} \right\} + \dots \right]. \tag{A.1}
 \end{aligned}$$

Eq. (A.1) makes it explicit that inside the maximization problem of the current government, there are many histories in which other maximizations (reoptimizations) occur.<sup>40</sup> Given that  $\{\Omega_{ND}^t, \Omega_{D,1}^t, \dots, \Omega_{D,t}^t\}$  is a partition of the set  $\Omega^t$ , all the histories are contemplated. Since  $\forall t > i, \Omega_{D,i}^t = \{\omega_{D,i}^t, \{s_j\}_{j=i}^t\}$ , we can rewrite the probabilities for  $\omega^t \in \Omega_{D,i}^t$  in the following way:

$$\text{Prob}(\omega^t) = \text{Prob}(\omega_{D,i}^t \wedge \omega^t) = \text{Prob}(\omega^t | \omega_{D,i}^t) \text{Prob}(\omega_{D,i}^t), \quad \forall \omega^t \in \Omega_{D,i}^t, t \geq i.$$

Substituting for these expressions into Eq. (A.1) and collecting the common term in the summation, we obtain:

$$\begin{aligned}
 W(k_0) = & \max_{\substack{\{x_t(\omega^t)\}_{t=0}^\infty \\ \omega^t \in \Omega^t}} \left\{ \sum_{t=0}^\infty \sum_{\omega^t \in \Omega_{ND}^t} \beta^t \{ \text{Prob}(\omega^t) u(x_t(\omega^t), k_t(\omega^t)) \} + \sum_{i=1}^\infty \beta^i \text{Prob}(\omega_{D,i}^i) \right. \\
 & \left. \times \left[ \max_{\substack{\{x_t(\omega^t)\}_{t=i}^\infty \\ \omega^t \in \Omega_{D,i}^t}} \sum_{t=i}^\infty \sum_{\omega^t \in \Omega_{D,i}^t} \beta^{t-i} \{ \text{Prob}(\omega^t | \omega_{D,i}^i) u(x_t(\omega^t), k_t(\omega^t)) \} \right] \right\}. \tag{A.2}
 \end{aligned}$$

Finally, we can define the value functions

$$\xi_i(k_i(\omega_{D,i}^i)) \equiv \max_{\substack{\{x_t(\omega^t)\}_{t=i}^\infty \\ \omega^t \in \Omega_{D,i}^t}} \sum_{t=i}^\infty \sum_{\omega^t \in \Omega_{D,i}^t} \beta^{t-i} \{ \text{Prob}(\omega^t | \omega_{D,i}^i) u(x_t(\omega^t), k_t(\omega^t)) \} \tag{A.3}$$

summarizing the happenings after the node  $\omega_{D,i}^i$ . Substituting this expression into (A.2) Eq. (13) is obtained.  $\square$

<sup>40</sup> We are using the short notation  $\text{Prob}(\omega^t) = \text{Prob}(\{s_j\}_{j=0}^t = \omega^t)$ .



**Proof of Proposition 2.** Drop history dependence and define

$$r(x_t, k_t) \equiv u(x_t, k_t) + \beta(1 - \pi)\xi(k_{t+1}), \tag{A.4}$$

$$g_1(x_t, k_t) \equiv b_1(x_t, k_t) + \beta(1 - \pi)b_2(\Psi(k_{t+1}), k_{t+1}), \tag{A.5}$$

$$g_2(x_{t+1}, k_{t+1}) \equiv b_2(x_{t+1}, k_{t+1}). \tag{A.6}$$

The planner’s problem is thus

$$\begin{aligned} & \max_{\{x_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta\pi)^t r(x_t, k_t), \\ \text{s.t. } & g_1(x_t, k_t) + \beta\pi g_2(x_{t+1}, k_{t+1}) = 0, \\ & k_0 \text{ given.} \end{aligned}$$

After writing the associated Lagrangian, and following Marcet and Marimon [24], solving the above problem is equivalent to solve the saddle point functional equation (SPFE)

$$\begin{aligned} W(k, \gamma) &= \min_{\lambda \geq 0} \max_x \{H(x, k, \lambda, \gamma) + \beta(1 - \pi)\xi(k') + \beta\pi W(k', \gamma')\}, \\ \text{s.t. } & \gamma' = \lambda, \quad \gamma_0 = 0, \end{aligned}$$

where

$$H(x, k, \lambda, \gamma) \equiv u(x, k) + \lambda g_1(x, k) + \gamma g_2(x, k). \quad \square$$

**Proof of Proposition 3.** Using Proposition 2, this proof follows trivially from the results of Marcet and Marimon [24].  $\square$

*A.1.1. First-order conditions*

The FOCs of the planner’s problem are<sup>41</sup>

$$\frac{\partial \mathcal{L}}{\partial z_t}: u_{z_t,t} + \lambda_t g_{1,z_t,t} + \lambda_{t-1} g_{2,z_t,t} = 0, \tag{A.7}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t}: g_1(x_t, k_t) + \beta\pi g_2(x_{t+1}, k_{t+1}) = 0, \tag{A.8}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial k_{t+1}}: & u_{k_{t+1},t} + \beta(1 - \pi)W_{k_{t+1},t+1} + \lambda_t(g_{1,k_{t+1},t} + \beta\pi g_{2,k_{t+1},t+1}) \\ & + \beta\pi(u_{k_{t+1},t+1} + \lambda_{t+1}g_{1,k_{t+1},t+1}) - \lambda_{t-1}g_{2,k_{t+1},t} = 0, \end{aligned} \tag{A.9}$$

$$\forall t = 0, \dots, \infty, \quad \lambda_{-1} = 0,$$

where according to the formulation of the problem provided in Section 2, we have  $z_t \equiv (l_t, \tau_t^k, \tau_t^l)$  and

$$\begin{aligned} g_{1,z_t,t} &= b_{1,z_t,t}, \\ g_{1,k_{t+1},t} &= b_{1,k_{t+1},t} + \beta(1 - \pi)(b_{2,x_{t+1},t+1}\Psi_{k_{t+1}} + b_{2,k_{t+1},t+1}), \\ g_{2,z_t,t} &= b_{2,x,t}, \end{aligned}$$

<sup>41</sup> The symbol  $f_{x_t,i}$  denotes the partial derivative of the function  $f(m_i)$  with respect to  $x_t$ . We suppressed the arguments of the functions for readability purposes.

$$g_{1,k_t,t} = b_{1,k_t,t},$$

$$g_{2,k_t,t} = b_{2,k_t,t},$$

$$g_{2,k_{t+1},t} = b_{2,k_{t+1},t}.$$

*A.2. Welfare analysis*

**Proof of Proposition 4.** The planner facing a commitment technology with  $\pi_2$  can in commitment states randomize its policy. With probability  $0 \leq 1 - p \leq 1$  the planner chooses to implement the same policy occurring in default periods. While with probability  $p$  the planner chooses not to do so. Define  $p$  such that  $\pi_2 p = \pi_1$ . It follows that the optimal allocation with  $\pi_1$  is a feasible allocation for the planner facing  $\pi_2$ . Since the planner maximizes utility it follows that  $V(K, \pi_1) \leq V(K, \pi_2)$ .  $\square$

*A.2.1. Welfare calculations*

Consider two regimes, a benchmark regime (B) and an alternative regime (A). The life-time utility  $W$  of the representative agent in both regimes is given by

$$W^i(k_{-1}, 0) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i, l_t^i, g_t^i), \quad i = A, B,$$

where  $\{c_t^i, l_t^i, g_t^i\}_{t=0}^{\infty}$  is the optimal allocation sequence in regime  $i$  and where expectations are taken with respect to the shock driving commitment and default and the technology shock. We define  $\varpi$  as the increase in private consumption in the benchmark regime that makes households indifferent between the benchmark and an alternative regime. More formally  $\varpi$  is implicitly defined as:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^A, l_t^A, g_t^A) = E_0 \sum_{t=0}^{\infty} \beta^t u((1 + \varpi)c_t^B, l_t^B, g_t^B).$$

For our calculations, we considered the benchmark regime to be the no-commitment case and we initialized capital at the steady state prevailing when  $\pi = 0.5$ .<sup>42</sup>

*A.3. Endogenous probability model*

**Proof of Proposition 5.** First, define an additional variable  $\eta$ , whose law of motion is

$$\eta_{t+1} = \eta_t P(k_{t+1}), \tag{A.10}$$

with  $\eta_0 = 1$ .

The planner’s problem is to maximize (21) subject to (22). Using the redefinitions (A.4)–(A.6), and writing the associated Lagrangian, we obtain

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<sup>42</sup> We also considered initializing capital at other steady states or expressing  $\varpi$  in consumption units of the alternative regime. The results remain unchanged.

$$\begin{aligned} \min_{\{\lambda_t, \varphi_t\}_{t=0}^{\infty}} \max_{\{x_t\}_{t=0}^{\infty}} \mathcal{L} = & \sum_{t=0}^{\infty} \beta^t \eta_t (u(x_t, k_t) + \beta(1 - P(k_{t+1}))\xi(k_{t+1})) \\ & + \lambda_t (g_1(x_t, k_t) + \beta P(k_{t+1})g_2(x_{t+1}, k_{t+1})) \\ & + \varphi_t (\eta_{t+1} - \eta_t P(k_{t+1})). \end{aligned} \quad (\text{A.11})$$

Finally, using the same reasoning as in the proof of Proposition 2, the result follows.<sup>43</sup>  $\square$

### A.3.1. First-order conditions of the planner's problem

In this case, the first-order condition of the planner's problem with respect to capital becomes

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial k_{t+1}} : & u_{k_{t+1}, t} + \beta(1 - P(k_{t+1}))W_{k_{t+1}, t+1} + \lambda_t [g_{1, k_{t+1}, t} + \beta P(k_{t+1})g_{2, k_{t+1}, t+1}] \\ & - \lambda_{t-1} g_{2, k_{t+1}, t} + \beta P(k_{t+1})[u_{k_{t+1}, t+1} + \lambda_{t+1} g_{1, k_{t+1}, t+1}] \\ & + \beta P_{k_{t+1}} [W(k_{t+1}, \lambda_t) - W(k_{t+1})] = 0 \\ \forall t = 0, \dots, \infty, \quad & \lambda_{-1} = 0, \end{aligned} \quad (\text{A.12})$$

where we have substituted for  $\eta$  and  $\varphi$  using the corresponding first-order conditions. As emphasized in the main text, the last two terms of (A.12) constitute the only difference with respect to the exogenous probability case.

## Supplementary material

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<sup>43</sup> For the purpose of this proof one has to include  $\eta_t$  as a state variable. This is only convenient for this proof and not necessary for the numerical solution.

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