Money and the Size of Transactions

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Abstract

Consumers make transactions of different sizes over time. This paper shows that this fact can highlight many issues in monetary theory. First, it helps to explain the demand for money, and why should it be a store of value. Even if there are other stores of value, like capital, land, or bonds, with transaction costs, the model shows that there is still need for money for small transactions. The same approach is applied to commercial banks, as suppliers of a transaction technology that differs from cash in its cost structure. As a result consumers use cash for smaller transactions, and demand deposits for larger transactions. Finally, the paper shows that modeling banks as suppliers of liquidity leads to a better understanding of their role as financial intermediaries.

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Money and the Size of Transactions

1. Introduction

This paper claims that monetary theory can benefit significantly from incorporating the observation that transactions have different sizes. People purchase vegetables in the market on one day, purchase a CD on another day, and buy a car in the following month. Each purchase involves a different size of payment. Acknowledging this simple fact of life can lead to a better understanding of why we need money to begin with, and why money should not only be a medium of exchange, but also a store of value. Furthermore, it contributes to understanding additional issues of monetary economics, like precautionary demand for money, portfolio choice between bonds and money, the emergence of commercial banks, and why banks specialize in financial intermediation.

The paper presents a very simple framework of analysis to demonstrate how the different sizes of transaction, together with various transaction technologies, can shed new light on monetary issues. It assumes that people consume different amounts in each period of time, which also differ from the amounts they produce and sell, namely their income. That gives rise to a demand for money, as money is used to buy more than income in some periods and it is stored if purchases are smaller than income in other periods. Thus money must be a store of value, as it is used to transfer income over time. Money, therefore, gives us the flexibility to purchase different amounts in different periods of time. This flexibility is one aspect of what we usually call 'liquidity.' Note that the paper's approach to the role of money sits well in the tradition of money as way to overcome the lack of 'double coincidence of wants.' But here, the mismatch is not of the types of goods transacted, but of their sizes, or values.[1] Note that the type of money issued is not discussed, and is only assumed to be a durable and fully divisible asset.

Taste shocks are not known in advance. Hence, people leave aside some money in order to be able to perform large purchases in case there is a need. This is the precautionary demand for money. In this paper it is a result of future taste shocks and risk aversion. Hence, this framework can formalize the precautionary demand for money and examine how it is related to various parameters, like the distribution of taste shocks, risk aversion, inflation, etc.

The role of money in transferring income over time becomes less crucial when other stores of value like land, capital and bonds, with higher rates of return, exist. To cope with it we add to the model the well-known Baumol-Tobin assumption that the additional financial asset has a higher rate of return, but also transaction costs. Embedding this assumption in the framework of different sizes of transactions leads to the result that small purchases are paid by money, while large purchases are paid from savings, which are held in high return assets. This explains why consumers hold positive amounts of both assets. This Baumol-Tobin result therefore depends crucially on the having both small and large transactions. Otherwise only one of the two assets would be superior. Hence, this paper sheds new light on the Baumol-Tobin approach.

Next, the paper discusses the coexistence of cash and demand deposits and introduces commercial banks to the analysis. It does so by introducing an additional transaction technology, demand deposits. While transactions with cash are costless, holding cash is costly, due to possibility of theft, loss, etc. Deposits eliminate this cost,

[1] Double coincidence of wants by types of goods is an old idea, which has been formally modeled in a series of papers by Kiyotaki and Wright (1989, 1991 and 1993) and followed by many others.

but create a cost per transaction, either from going to the bank to get cash, or of processing a check in a clearing house. As a result people face two competing transaction technologies, with different cost structures. They use cash for small transactions, and demand deposits for larger transactions. Since they have both types of transactions they hold both cash and deposits. Hence, this approach can also shed light on the emergence of banks as innovators of a new transaction technology.

Banks hold money, or cash, that people have deposited. Some of this money can be lent and increase the bank's profits. This raises several issues, like optimal size of reserves, danger of runs on banks, etc. The paper briefly shows that its framework can deal with these issues analytically and even contribute to the understanding of why commercial banks are also leading financial intermediaries. The model shows that banks have a lower cost for their funds than other financial intermediaries, since they do not pay a rate of return on the deposits they use for lending. Hence, banks are successful financial intermediaries not because of superior information, but rather because they have cheaper funds, as they pay their depositors with liquidity.

The paper discusses all these issues in a series of models, where taste shocks affect demand for consumption. The models are quite similar, and differ only because each highlights a different issue. While the basic framework can be presented in a model with infinitely lived consumers, a simpler overlapping generation model is used in the paper, for reasons of tractability.[2]

The contribution of this paper to monetary theory can be described in the following way. It builds a general equilibrium model of money as a means of transaction and as a store of value, and shows that this model can be applicable to many issues in monetary economics, both general and applicable. The model draws from two main pervious traditions, but also sheds new light on them. The first tradition is that of Wallas (1980) on money as a store of value. Important contributions in this tradition are Bewley (1980, 1983), which discuss the role of money in the face of income and taste shocks. To this literature we add the tradition of Baumol (1952) and Tobin (1956), where assets differ by their cost structure, namely by their return and liquidity.[3] The marrying together of the two traditions is done by realizing that each of the assets is used for different size of transactions. Thus all assets are held in positive amounts. This combination of the two traditions enables this paper to highlight many monetary issues.

The structure of the paper is as follows. Section 2 presents the first model, which describes the demand for money and its role as a store of value. Section 3 formalizes the precautionary demand for money. Section 4 adds bonds with transaction costs and shows that the demand for money is still positive. Section 5 introduces a new transaction technology, through bank deposits. Section 6 discusses briefly bank loans and financial intermediation. Section 7 summarizes, Appendix A describes a Ramsey version of the model and Appendix B presents proofs to propositions.

2. Model I: The Demand for Money

This is the basic model in the paper, which shows how the role of money as a medium of exchange and its role as a store of value are closely related when transactions differ in size from one period to the other. This model shows how money can facilitate transactions and increase welfare so that demand for it is positive. We show it by using

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[2] See Appendix A for a Ramsey formulation of the basic model.

the simplest possible model, where people wish to consume different quantities than what they produce and thus need a medium of exchange, which also stores value over time.

Consider an economy with one physical good, which is used for consumption only. The consumption good is not durable. The economy consists of overlapping generations, each with a continuum of people of size 1. Each person lives 2 periods, works in both and produces a fixed amount y in each period. The individual consumes in the 2 periods, but consumption can differ over time, due to inter-temporal preferences. The utility function of an individual is:

(I.1) $$U = \theta \log c_1 + (1-\theta) \log c_2,$$

where c_1 is consumption in first period of life, c_2 is consumption in second period and θ is a taste shock, which realizes in first period of life. It is assumed that taste shocks differ across individuals and are uniformly distributed on [½ - σ, ½ + σ] in each generation.

If the consumption good is not durable each individual consumes exactly y in each period of life. That is true even if barter trade is taking place, if people cannot consume their own products and trade with others. Hence, barter does not raise utility. We therefore introduce an asset called money, which is durable. The type of money can differ widely. It can be metal like silver or gold, or fiat money, but the analysis is the same. Let the outstanding amount of money in the economy in period t be denoted M_t and let the price of the physical good in period t in terms of money be P_t.

In order to derive the equilibrium in the economy we begin with an individual born in period t with a taste shock θ. The individual sells output y, buys first period

[3] Recent formalizations of the Baumol-Tobin approach are Romer (1986), and Starr (2003).

consumption and is left with an amount of money equal to $m_t(\theta)$. This amount of money is chosen to maximize utility and is given by

$$\text{(I.2)} \qquad m_t(\theta) = \arg\max_{m \geq 0} \left\{ \theta \log\left(y - \frac{m}{P_t} \right) + (1-\theta) \log\left(y + \frac{m}{P_{t+1}} \right) \right\}.$$

Note that P_{t+1} is the expected price in the next period and due to rational expectations it is known in period *t*, since aggregates are deterministic in this economy. The constraint $m \geq 0$ is the liquidity constraint of the consumer in the first period of life.

We distinguish between two cases in maximization of utility. In the first case the individual has low consumption in first period of life and does not reach the liquidity constraint. In the second case the consumer wants to purchase a larger amount and the liquidity constraint is binding. The first case is described by point A in Figure 1, where the optimal amount of money is derived from the first order condition:

$$\text{(I.3)} \qquad \frac{m_t(\theta)}{P_t} = y\left(1 - \theta - \theta \frac{P_{t+1}}{P_t} \right).$$

Hence, case A holds when:

$$\text{(I.4)} \qquad \theta \leq \theta_t \equiv \left(1 + \frac{P_{t+1}}{P_t} \right)^{-1}.$$

Hence, the individual demand for real balances depends negatively on the expected rate of inflation, up to a rate of inflation where $\theta_t = 1/2 - \sigma$. Above it all taste shocks are higher than θ_t, case A is empty and the demand for real balances is zero. It can be shown that this threshold rate of inflation is

$$\text{(I.5)} \qquad \pi_{\max} = \frac{1/2 + \sigma}{1/2 - \sigma}.$$

[Insert Figure 1 here]

The second case, where $\theta \geq \theta_t$, is called B and is shown by point B in Figure 1. In this case the liquidity constraint is binding, no money is left and the individual consumes exactly y in each period of life. Clearly, the introduction of money increases utility of those who are in case A. Hence, this model can justify the introduction of money to the economy. Note that in this model money raises welfare only if it is durable, namely if it stores value.

We next turn to derive the monetary equilibrium. We sum up the demands for money by the young. These are the amounts they keep by the end of the period as the amount of money used for transactions within the period only changes hands and is not counted as net demand for money. Hence, the aggregate demand for real balances is

(I.6) $$\frac{M_t^d}{P_t} = \frac{1}{2\sigma P_t}\int_{1-\sigma}^{1+\sigma} m_t(\theta)d\theta = \frac{y}{2\sigma}\int_{1-\sigma}^{\theta_t}\left(1-\theta-\theta\frac{P_{t+1}}{P_t}\right)d\theta.$$

Calculation shows that the demand for real balances is equal to:

(I.7) $$\frac{M_t^d}{P_t} = \frac{y}{4\sigma}\left[\frac{1}{2}+\sigma-\left(\frac{1}{2}-\sigma\right)\frac{P_{t+1}}{P_t}\right]\left[\left(1+\frac{P_{t+1}}{P_t}\right)^{-1}-\frac{1}{2}+\sigma\right].$$

Hence, the demand for real balances depends negatively on the expected rate of inflation, P_{t+1}/P_t. If inflation is above the threshold level $\pi_{\max}$ = (½ + σ)/(½ - σ), the demand for money is equal to zero. Note that if σ = ½, the threshold is infinite and the demand for money is everywhere positive. If the gross rate of inflation is 1, namely at price stability, the demand for money is equal to $y\sigma/2$. Hence the demand for money depends positively on the variability of transactions. The demand for liquidity is shown in Figure 2.

[Insert Figure 2 here]

To complete the derivation of the monetary equilibrium we need to add a description of the dynamics of the quantity of money, namely of the monetary policy. We first take the simplest case, where the quantity of money is fixed: $M_t = M$ for all *t*. In this case the monetary policy is described by the horizontal curve at 1 in Figure 2. It is easy to show that the rational expectations equilibrium in this case is at point A, where the rate of inflation is 0 and the price is fixed and equal to:

(I.8) $$P_t = \frac{2M}{y\sigma}.$$

If monetary policy is expansionary and money increases either at a fixed rate μ, or in order to pay for a deficit of size *D*, prices are rising and the rate of inflation is positive. The equilibrium rate of inflation can be easily derived in each case.

This section therefore shows how money can be useful when people have different sizes of transactions over time. They use money to transfer income from one period to another if their consumption in first period is low enough. For money to function this way it must be a store of value. Hence, the roles of money as a means of transaction and as a store of value are closely related when transactions differ in size over time. Note that the demand for money depends on the variability of transactions and is zero if there is no variability.[4] Broadly speaking, this model of money also belongs to the tradition of 'mismatch of wants.' But instead of mismatch of types of goods to barter, here we have a mismatch of values of goods transacted, as the amount sold (income) is different from the amount bought.

[4] The demand for money can be positive even if σ=0, if there is deflation, due to the positive rate of return of money. But that is not the demand for money we usually try to explain.

3. Model II: The Precautionary Demand for Money

According to our basic approach people use money in transactions when the amount they wish to purchase is larger than the amount they earn as income. Hence, money enables them to use the opportunity to make a larger purchase. The more money they have available, the greater the chance that they will be able to purchase without being liquidity constrained. This leads to the precautionary demand for money. One of the reasons people hold money is to be able to make large unanticipated purchases. Our framework of analysis enables us to formalize this concept and quantify it. We demonstrate it in this section by use of a simple extension of Model I.

Consider a closed economy with only one physical good, which is used for consumption and is perishable. Population consists of overlapping generations, where each generation is a continuum of size 1. Each person lives 3 periods, produces in each period an amount y and consumes in the three periods according to the following utility function:

(II.1) $$U = \frac{1}{2}\log c_1 + \theta \log c_2 + (1-\theta)\log c_3 .$$

The taste parameter θ differs across people and is independent and uniformly distributed on [½ - σ, ½ + σ]. Furthermore, the taste parameter is stochastic, namely unknown to the individual in first period of life and is revealed only in the second period. Hence, in the first period of life all periods have the same expected weights. We assume that the only asset in the economy is money. Its quantity is M_t and the nominal price of the physical good is P_t. In the following analysis we assume for simplification that money is fixed: $M_t = M$ and hence we analyze the economy at the steady state equilibrium, where the price is fixed as well.

A person earns y in first period of life, consumes c_1 and keeps her savings in money, with real value $l_1 = y - c_1$. In period 2 she consumes $y + l_1 - l_2$, where l_2 is the real amount of money left in the end of period 2, and in period 3 she consumes $y + l_2$. We next analyze the optimal behavior of the individual recursively. In the second period of life she realizes what her taste shock θ is and maximizes utility for periods 2 and 3. The amount of money held in period 2 is

(II.2) $$l_2(\theta) = \arg\max_{l \geq 0} \{\theta \log(y + l_1 - l) + (1-\theta)\log(y + l)\}.$$

This maximization can lead to two possible outcomes. One, denoted A, occurs when the first order condition holds and the consumer has sufficient liquidity. The other, denoted B, holds when the consumer wants to purchase more in period 2 and is liquidity constrained. The two cases can be described in a diagram similar to Figure 1. In A the demand for money is determined by the FOC and is

(II.3) $$l_2(\theta) = (1 - 2\theta)y + (1-\theta)l_1.$$

The set A includes the sufficiently small taste shocks, formally:

(II.4) $$\theta \leq \theta_1 \equiv \frac{1 + l_1/y}{2 + l_1/y}.$$

Let us denote first period money relative to income by f_1, namely $f_1 = l_1 / y$. Then the borderline between sets A and B is $\theta_1 = (1 + f_1)/(2 + f_1)$. In set A consumption in period 2 is $\theta(2y + l_1)$ and consumption in period 3 is $(1-\theta)(2y + l_1)$. Hence, the lifetime utility in this set is described by

(II.5) $$U_A(\theta) = 1/2\log(y - l_1) + \theta\log\theta + (1-\theta)\log(1-\theta) + \log(2y + l_1).$$

In set B, where $\theta > \theta_1$, the consumer faces a liquidity constraint, leaves no money and consumption in periods 2 and 3 are $y + l_1$ and y respectively. In this set lifetime utility is

$$U_B(\theta) = 1/2\log(y - l_1) + \theta\log(y + l_1) + (1 - \theta)\log y. \tag{II.6}$$

Note that θ_1 is larger than ½, so it is possible that it is larger than ½ + σ as well. In that case the set B is empty and all consumers are in A. This happens if:

$$f_1 > \frac{2\sigma}{1/2 - \sigma}. \tag{II.7}$$

It is later shown that the optimal f_1 is smaller than this threshold, so B is never empty and there are always some consumers who are liquidity constrained.

Since in period 1 of life the individual does not know her future taste shock θ, the demand for money in period 1 is determined by maximizing expected utility. The expected utility is

$$EU = \frac{1}{2\sigma}\int_{1/2-\sigma}^{\theta_1} U_A(\theta)d\theta + \frac{1}{2\sigma}\int_{\theta_1}^{1/2+\sigma} U_B(\theta)d\theta, \tag{II.8}$$

if $f_1 \le 2\sigma/(1/2 - \sigma)$. If $f_1 > 2\sigma/(1/2 - \sigma)$, the expected utility is

$$EU = \frac{1}{2\sigma}\int_{1/2-\sigma}^{1/2+\sigma} U_A(\theta)d\theta. \tag{II.9}$$

The following proposition describes the results of the maximization of expected utility.

<u>Proposition 1:</u> There is a unique optimal amount of money l_1 that maximizes the expected utility (II.8) and (II.9). This amount of money is proportional to income, namely $l_1 = f_1 y$, and f_1 is independent of income. f_1 is positive and is smaller than $2\sigma/(½ - \sigma)$, so

that B is not empty. f_1 depends positively on the variability of taste shocks σ and when taste shocks have no variability people hold no money in first period of life.

Proof: In Appendix B.

We next demonstrate that the demand for money by the young, namely l_1, is the precautionary demand for money. In order to show that assume that in the second period of life individuals have an asset which is superior to money: it not only stores value, but also enables them to borrow. In other words, assume that they have a bond with the same interest rate as money, 0, but that can be used both to save for next period and to borrow in order to purchase in the present. In this case liquidity of grownups is never constrained. It can be shown that in this case the expected utility of a consumer is:

$$EU = \frac{1}{2\sigma}\int_{1/2-\sigma}^{1/2+\sigma}\left[\theta\log\theta - (1+\theta)\log(1+\theta)\right]d\theta + \frac{1}{2}\log(2y+l_1) + \frac{1}{2}\log(y-l_1).$$

In this case the optimal l_1 is zero. Hence, the amount of money held by the young in our model with money is purely a precautionary demand for money.

We next calculate the overall demand for real balances in the economy, where the demand of grownups is due to variable sizes of transactions, while the demand of the young is the precautionary demand for money. We get:

$$\frac{M_d}{P} = l_1 + \frac{1}{2\sigma}\int_{1/2-\sigma}^{\theta_1} l_2(\theta)d\theta = y\left[f_1 + \frac{1}{2\sigma}\int_{1/2-\sigma}^{\theta_1}[1+f_1-(2+f_1)\theta]d\theta\right] = \tag{II.10}$$

$$= y\left[f_1 + \frac{2+f_1}{2\sigma}\left(\frac{f_1}{2(2+f_1)}+\sigma\right)^2\right].$$

It can be shown that the demand for money depends positively on L_1 and positively on σ. Hence the demand for money depends positively on σ, namely on the variability of taste shocks. The monetary equilibrium in the steady state of zero monetary injections and zero inflation can be directly derived from equation (II.10) as well.

4. Model III: Money and Bonds

In this section we add one more asset to the economy, which can be described either as lending or as physical capital. For the sake of simplicity we assume that this asset has a constant positive real rate of return r, where $r > 0$. This asset has higher returns than money, but it is less liquid. It is costly to invest in this asset and to withdraw the investment for use. More specifically we assume that finding an investor and signing a contract has utility cost of size x, and withdrawing the loan after a period or two periods has a utility cost as well, though smaller, $z < x$. This assumption follows the Baumol-Tobin tradition, of a costly exchange between bonds and money. Below we discuss the difference between this model and previous models in this tradition.

The rest of the model is similar to the above models. Each person lives three periods, produces an amount y in the first period of life only, and consumes in the second and third periods of life.[5] The utility from consumption is

(III.1) $$U = \theta \log c_2 + (1-\theta)\log c_3 .$$

The taste shock θ is revealed only in the second period of life. We assume for simplicity that the taste shocks are uniformly distributed on [0, 1], namely that σ = ½. As in Section 3 we assume that the supply of money is fixed and equal to M, and hence we analyze only the steady state equilibrium with fixed prices. Extending the analysis to a dynamic environment is straightforward.

We first analyze the decisions of a representative consumer, who chooses not only consumption levels but also the composition of her portfolio between money and bonds. In the first period of life the individual saves all income and divides it between money and lending:

(III.2) $$y = \frac{m_1}{P} + b_1 = l_1 + b_1,$$

where l denotes real balances of money.

In the second period of life there are three cases, which depend on the taste shock θ. In the first case A, the transaction size in period 2 is smaller than money, so that some money is left after the transaction. It is not invested in bonds as we assume that x is large enough, so it is kept as money for third period of life. In the second case B, the size of transaction is larger, so that all the money is used. In the third case C, the transaction is even larger, so that it is paid for by all the money and by adding some savings. This is done by going to the investor and getting some of the loan back.

In case A the consumer leaves money l_2 to third period of life and maximizes the following utility:

$$\theta \log(l_1 - l_2) + (1-\theta)\log[b_1(1+r)^2 + l_2] - x - z.$$

The optimal amount of money left is:

(III.3) $$l_2(\theta) = l_1 - \theta[l_1 + b_1(1+r)^2] = l_1 - \theta[y(1+r)^2 - (2r + r^2)l_1].$$

Clearly, case A occurs as long as l_2 is non-negative, namely if the taste shock satisfies:

(III.4) $$\theta \le \theta_1 \equiv \frac{l_1}{y(1+r)^2 - (2r+r^2)l_1}.$$

[5] This assumption is made for simplicity only. Since the main focus of this model is not saving and consumption, but portfolio decisions, the main results of the model are not affected by this assumption.

Calculating the optimal levels of consumption in periods 2 and 3 we get that case A utility is equal to

(III.5) $$U_A(\theta) = N(\theta) + \log[y(1+r)^2 - (2r + r^2)l_1] - x - z.$$

where the function N denotes: $N(\theta) = \theta \log \theta + (1-\theta)\log(1-\theta)$.

In case B the consumer is mildly liquidity constrained. She uses all her money, but is not sufficiently constrained to call off a loan. In this case consumption levels are $c_2 = l_1$ and $c_3 = b_1(1+r)^2$ in the second and third periods respectively and utility is

(III.6) $$U_B(\theta) = \theta \log(l_1) + (1-\theta)\log[(y - l_1)(1+r)^2] - x - z.$$

In case C the taste shock θ is larger and so is consumption. The liquidity constraint is so binding that the consumer goes to the investors and redeems part of the loan, of size d. The consumer maximizes

$$\theta \log(l_1 + d) + (1-\theta)\log\{[b_1(1+r) - d](1+r)\} - x - 2z.$$

The amount of loan called off d is therefore equal to:

(III.7) $$d(\theta) = \theta[y(1+r) - rl_1] - l_1.$$

Hence, utility in this case is

(III.8) $$U_C(\theta) = N(\theta) + (1-\theta)\log(1+r) + \log[y(1+r) - rl_1] - x - 2z.$$

We next examine who find themselves in case B and who in case C. A consumer decides to stay in B and not open her savings if $U_B(\theta) \geq U_C(\theta)$. Hence, the threshold taste shock between cases B and C, which is denoted θ_2, is determined by the condition: $U_B(\theta_2) = U_C(\theta_2)$. This condition boils down to

(III.9) $$\theta_2 \log l_1 + (1-\theta_2)\log[(y - l_1)(1+r)] = N(\theta_2) + \log[y(1+r) - rl_1] - z.$$

This threshold preference level θ_2 is binding if it is smaller than 1. If U_B is everywhere higher than U_C then θ_2 is equal to 1. It is easy to see that this does not happen if l_1 is low.

We can next calculate the expected utility of the individual in period 1 of life, which is when portfolio decisions are made:

$$EU = \int_0^{\theta_1} U_A(\theta) d\theta + \int_{\theta_1}^{\theta_2} U_B(\theta) d\theta + \int_{\theta_2}^{1} U_C(\theta) d\theta. \tag{III.10}$$

The individual determines the amount of money to hold by equating the marginal utility of money to zero or by equating the cost and the benefits of holding money. Increasing l_1 is costly as it reduces the future income from bonds. But it is also beneficial, as it reduces the probability of selling bonds and thus reduces the probability of suffering the utility cost of $-z$. In other words, increasing the amount of money reduces the probability of hitting the liquidity constraint. Proposition 2 shows that the individual always prefers to hold a positive amount of money, despite its lower rate of return.

Proposition 2: The amount of money held by the young is positive, and proportional to income: $l_1 = yf(r)$, where $f'(r) < 0$. The demand for money is positively related to z. If the transaction cost x is high, the consumer holds money only and $l_1 = y$.

Proof: In Appendix B.

In this specific model the overall demand for money is by the young and by the grownup of case A, who do not use all their money in second period of life. Hence the equilibrium in the money market is determined by the following equilibrium condition:

$$\frac{M}{P} = l_1 + \int_0^{\theta_1} l_2(\theta) d\theta = yf(r)\left\{1 + \frac{1}{2}\frac{f(r)}{1 + (2r + r^2)[1 - f(r)]}\right\}. \tag{III.11}$$

This condition determines the price level in this economy. Clearly the right hand side, which is the demand for real balances, depends negatively on the rate of return *r*.

In this paper money is used to transfer income over time, in order to have different size of transactions over time. In this model we introduce a new asset – bonds – that can also do the same job, transfer income over time. This asset has a higher rate of return, but it also has transaction costs. In other words, the two assets have different cost structures. If all transactions were of the same size, then they would all be performed either by money or by bonds, and the two assets would not co-exist. Here the assumption that transactions come in different sizes becomes crucial, as some transactions are better financed by money and some by savings. Thus, there are positive demands for the two assets, money and bonds. Hence, this model follows the Baumol-Tobin tradition, but embeds it in a framework of different sizes of transactions, which that clarifies why the Baumol-Tobin assumption works. It is interesting to note that in the original Baumol-Tobin model all purchases are the same, but they are not equal to income, as consumption is done every period while income arrives only once in a while. While this seems to be an explicit assumption in the original model, in this paper it becomes an explicit assumption and its role is fully emphasized. This model also clarifies how money and savings are used for different types of purchases, where money pays for smaller transactions, while for a larger transaction a loan is called off and used for purchase.

5. Model IV: Commercial Banks – Cash and Demand Deposits

This section introduces commercial banks as a technological innovation in the area of transactions, which offers a solution to the risk involved with carrying large amounts of money. Holding large amounts of cash creates risk of loss or theft and banks offer to hold the amount of money in safe demand deposits. Hence banks create an additional transaction technology, demand deposits. The way demand deposits are used by consumers, namely going to the bank to draw cash for each purchase or using checks, is less crucial for our argument, since in any case there is a cost involved in the transaction itself. Cash transactions are not costly, but the holding of cash is costly due to risk of loss or theft. Therefore the two transaction technologies, or the two assets cash and demand deposits, have different cost structures, similar to money and bonds. While using cash is not costly, holding it usually is, due to possible theft or loss. Similarly to money and bonds, the two forms of money – cash and demand deposits – will be used, one for small size transactions and the other for large size transactions. Hence, banks emerge as a way to reduce the cost of holding cash, by offering a different transaction technology, but since this technology is not completely costless and has a different cost structure, banks do not completely crowd out the use of cash, and the two forms of money coexist.

To demonstrate this idea, consider a similar economy to those in previous sections. The economy has overlapping generations, each of size 1, and each person lives three periods, produces y in the first period of life, and consumes in the second and third periods of life where utility is

(IV.1) $$U = \theta \log c_2 + (1-\theta)\log c_3 .$$

The taste shock θ is random, it is revealed only in the second period of life, and θ is uniformly distributed on $[0, \tau]$, where $\tau < 1$. This is assumed for simplicity only. For further simplicity assume that there are no bonds but money only. People can hold cash, which faces risk of theft or loss, or they can deposit money in banks, which are safe.

Going to the bank to deposit or withdraw money has utility cost of size *v*.[6] Holding cash is costly, but there are many ways to model this cost. We assume that if the amount of cash held too large it is stolen. Since it is reasonable that this threshold of theft is relative to income, we assume that it is equal to $d\,y$, where $d > 0$. Hence, *d* measures danger of theft in the economy. [7] We assume that $d < 1 - \tau$. Furthermore, assume that the supply of cash is fixed at *M*, so that only the steady state equilibrium with fixed prices is analyzed.

A person earns income *y* in first period of life and keeps it in two forms of money. A real amount l_c is held in cash and a real amount l_b is deposited in the bank, so that $l_c + l_b = y$. The demand deposit pays no interest. In the second period of life the person realizes her θ and four cases are possible. In case A, a small amount is purchased with some of the cash, and some cash is left for third period: $l_c - c_2$. In case B the transaction is larger and all the cash l_c is used in period 2, but no bank deposit is used to avoid the cost of going to the bank. In the third case C the consumer wishes to consume a larger amount in period 2 and both cash and some of the demand deposit are used, but some of the demand deposit is still left in the bank. This is done because the anticipated purchase in period 3 is too large to be paid by cash only. In case D all the money in the bank is withdrawn, some is used for purchase and some remains as cash. Since $1 - \theta$ is larger than *d* the consumer is constrained by the threat of theft and $c_3 = l_c + l_b - c_2 = dy$. The following proposition describes the main characteristics of the equilibrium.

[6] When people pay with checks this is usually an income loss rather than utility cost, as checks cost money. The cost is different, but the two costs structures are similar and the results of the model are similar as well.
[7] An alternative modeling strategy is to assume that cash depreciates at some rate. In this case the model is fully equivalent to model III of money and bonds.

Proposition 3: The amount of cash held by each young consumer is $d\,y$. Both demands for cash and for demand deposits are proportional to income y. The demand for cash depends positively on the average size of taste shocks τ and the demand deposits depend negatively on it. The overall demand for money depends negatively on τ:

$$L = y\left[1 + d + \frac{(1-d)^2}{2\tau}\right].$$

Proof: In Appendix B.

The monetary equilibrium in this economy with cash and demand deposits is therefore described by the following equation:

$$\text{(IV.2)} \qquad \frac{M}{P} = y\left[1 + d + \frac{(1-d)^2}{2\tau}\right].$$

Note that the outstanding amount of money must be equal to the total amount of cash and demand deposits since banks do not lend in this model without bonds, so they hold a reserve ratio of 100%. In the next section we add lending to the model and examine its effects on money and on financial intermediation.

6. Banks and Financial Intermediation

The situation described in model IV, where banks have large stocks of cash in their safes, tempts banks to begin lending some of the money in order to earn interest on it. This leads us to the issues of bank lending, optimal size of reserves, and how these are related to the monetary equilibrium. This section presents a preliminary analysis of these issues, using a model similar to the previous ones. Let us assume that there is a class of investors, who have projects that require investment of 1 in one period. These projects yield 1 + R in the next period if the investor is good and 0 if bad. It is possible to perfectly monitor and screen investors before giving them resources, but this monitoring has a cost of e per project selected. This gives rise to financial intermediaries as delegated monitors as in Diamond (1984), and the resulting net rate of return for consumers who lend to these financial intermediaries is: $r = R - e$.

We next describe consumers in the model. There are overlapping generations, each of size 1, and each person lives three periods, produces y in the first period of life, and consumes in the second and third periods of life where utility is

(V.1) $$U = \theta \log c_2 + (1-\theta) \log c_3.$$

The taste shock θ is random, it is revealed only in the second period of life, and θ is uniformly distributed on $[0, \tau]$, where $\tau < 1$. There is a utility cost of going to the financial intermediary and create a loan contract or withdraw resources from the contract. This utility cost is x.[8] In addition to loans or bonds people can hold cash, which faces risk of theft or loss, or they can deposit money in banks, which are safe. Going to the bank to deposit or withdraw money has utility cost of size v. We assume realistically that $v < x$. Similar to model IV assume that if the amount of cash held is larger than $d\,y$, it is stolen. Assume that the supply of cash is fixed at M, so that only the steady state equilibrium with fixed prices is analyzed.

We can now sketch the equilibrium of this economy. In first period of life consumers build a portfolio of the three assets: cash, deposits, and bonds, in the amounts l_{1c}, l_{1b}, and b_1 respectively. In second period of life they realize θ, consume, and keep cash, deposits and bonds in the amounts $l_{2c}(\theta), l_{2b}(\theta)$ and $b_2(\theta)$ respectively. Summing up over all individuals we get the overall deposits in the banks in each period: $L_{1b} + L_{2b}$, and the overall withdrawals from banks: $L_{1b} + L_{2b}$, which are equal. Hence, if banks are fully diversified in the population and the distribution of θ among their clients is the same as in the population, then they can lend all their deposits and keep reserves at size zero. What is clear from this description is that banks can lend funds to investors and monitor them, just like other financial intermediaries, but the funds they use are less expensive, as they don't have to pay the interest rate of lenders. They pay zero interest to depositors and only supply them with liquidity. Hence, banks have an advantage in competition with other financial intermediaries. To illustrate it assume that the bank lends all the demand deposits $L = L_{1b} + L_{2b}$ and also intermediate an amount B of bonds. Clearly the bank can pay savers a higher rate of return:

(V.2) $$r_b = r\frac{L+B}{B} > r.$$

This gives the commercial bank an edge in competing with other financial intermediaries. Note, that this edge is not due to any comparative advantage in information processing over other institutions, but only because banks pay less for funds, for which they pay with the transaction services to depositors.

Another extension of the model can model reserves more realistically and have positive reserves. For that we need to add some uncertainty to money holding, or some uncertainty to the distribution of risk. The best way to model it is to assume that taste shocks have an aggregate component, namely that τ_t is random. Hence banks need to

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[8] We can also assume as in model III that the cost of writing a contract is not the same as withdrawing the money. The results are similar.

keep some reserves in order to ensure that they do not risk becoming illiquid. We do not fully model banking decision making here, but the way to do it seems quite obvious.

Focusing on liquidity shocks and reserves can highlight another issue in monetary economics, which is how central banks affect real activity. Note that central banks lend to banks who are hit by liquidity shocks, in this model τ_t. The interest rate of this lending is lower than the market interest rate. Hence, this can be viewed as a subsidy to financial intermediation by banks. By changing the interest rate of lending to banks the central bank affects the cost of funds to banks and thus affects the interest rate they charge for their loans. Thus such a policy has real effects and not only nominal effects. The intuitive reason for that is that such a change in the interest rate by the central bank changes not only the stock of money, but the flow of income from the bank to the economy, or to the commercial banks. In a way, this approach views monetary policy as part of fiscal policy, where the central bank directly subsidizes the banking sector and through it investors in the economy. Note, that this view on the effect of monetary policy does not require any assumption of price rigidity. Monetary policy affects the economy as it directs real resources to some sectors in the economy.

7. Summary

This paper develops a model of money as a store of value that enables consumers to transfer income over time, as their consumption differs from one period to the other, and it differs from their income as well. Taking this simple and intuitive intuition to the Baumol-Tobin approach, according to which assets differ by their cost structure, namely by their return on one side and by their cost of transaction on the other side, sheds light on the role of liquidity in equilibrium. It shows that the more liquid assets are used for the smaller transactions, and the less liquid assets, with higher returns, are used for the larger transactions. Since people face all sizes of transactions over time, they need to hold a portfolio that has assets with different degrees of liquidity: cash, demand deposits, bonds, etc. This is the main insight of this paper.

The paper shows that this approach can be helpful in understanding many issues in monetary economics, like precautionary demand for money, financial intermediation by banks, bank reserves, and even the operation of the central bank. The paper does not touch upon many other issues that have been dealt with in other studies. It does not discuss the issue of commodity versus fiat money. It does simply assume that money is a durable asset which is fully divisible. It does not fully model banks in the case of uncertain liquidity and does not discuss issues of bank stability, like in Dybvig-Diamond (1983). But it is our belief that the framework of analysis this paper offers can be useful to analyze these issues and many others.

Appendix A: The Infinite Horizon Model

Consider an economy with a single physical good. Time is discrete. There is a continuous mass of size 1 of infinite horizon individuals in the economy. Each one produces a constant amount y in each period of time. Individuals derive utility from consumption, but they have taste shocks in each period, that affect their utility. The utility from consumption in period t of individual j is:

(A.1) $$E_t \sum_{s=0}^{\infty} \beta^s u[c_{t+s}(j), \theta_{t+s}(j)].$$

The utility function is concave in consumption and satisfies the Inada conditions. The taste shocks θ are independent and identically distributed over time and across individuals. Each shock is revealed in its period, but future taste shocks are unknown in advance. Assume that output is produced by labor only and there is no capital.

If there is no money either, consumers cannot shift income over time to meet their different taste shocks and must consume y in each period. The utility of consumer j in period t is therefore equal to:

(A.2) $$E_t \sum_{s=0}^{\infty} \beta^s u[y, \theta_{t+s}(j)].$$

Next assume that there is money in the economy and people can use it to trade with. Assume that money is durable, namely it is coins or fiat money. Hence, it can be used to transfer income over time. Assume that the overall amount of money in the economy in period t is M_t. Let us denote the amount of money held by individual j by the end of period t by $m_t(j)$. Then the budget constraint in period t is described by:

(A.3) $$c_t(j) = \frac{m_{t-1}(j)}{P_t} + y - \frac{m_t(j)}{P_t}.$$

The consumer maximizes utility (A.1) given the budget constraints (A.3) and the non-negativity of money constraint:

(A.4) $$m_t(j) \geq 0.$$

Clearly, the optimal utility with money is higher than utility without money (A.2). Hence, the role of money in this model to enable consumers to adjust better to the taste shocks, namely to make transactions of different sizes over time.

We next turn to describe equilibrium in this economy. The equilibrium prices are the sequence $(P_t, P_{t+1}, ...)$ of the nominal prices of the physical good in terms of money. As shown below these prices are fully known in advance, namely prices are deterministic. The consumer reacts to these prices as they determine the rates of return of money. Hence, the individual maximizes (1), given the budget constraints (3) and (4) and taking the price levels as given.

It can be shown that the optimal amount of money held by end of period t is described by a function of initial money, taste shock, and expected rates of inflation:

(A.5) $$\frac{m_t(j)}{P_t} = l\left(\frac{m_{t-1}(j)}{P_t}, \theta_t(j), \pi_t\right),$$

where π_t is the vector of all future rates of inflation:

(A.6) $$\pi_t = \left(\frac{P_t}{P_{t+1}}, \frac{P_{t+1}}{P_{t+2}}, ...\right).$$

Equation (A.5) describes the individual demand for money. Note that this demand for money can be positive, if the constraint (A.4) is not binding, namely if the consumer does not consume much in period t and can leave money to future consumption. But if the taste shock raises the marginal utility of consumption in t the constraint (A.4) becomes binding and the consumer does not leave money for future periods.

Equation (A.5) also describes how the distribution of money across individuals evolves over time. Note that since the taste shock is independent across individuals it is also independent of the amount of money from period t-1. Hence, the distribution of money in period t is determined uniquely by the distribution in t-1, namely it evolves deterministically over time.

The aggregate demand and the aggregate supply of money determine the equilibrium in the market for money, which also determines the price of goods P_t:

(A.7) $$\frac{M_t}{P_t} = \int_0^1 l\left(\frac{m_{t-1}(j)}{P_t}, \theta(j), \pi_t\right) dj.$$

Since the taste shock is independent of the initial distribution of money from period t-1, it follows that the equilibrium price level is fully determined by the distribution of money in t-1 and is therefore not stochastic. Hence, the prices in the present and the future are fully determined by the initial distribution of money, and by the future anticipated amounts of money.

In order to complete the proof of existence and uniqueness of equilibrium (which is only sketched here) note that the price levels in all periods depend on the path of future prices as shown by equations (A.5) and (A.7). A fixed-point argument shows the existence and uniqueness of an equilibrium price path. Note, that the equilibrium price is finite in each period and does not equal to infinity, namely the price of money is positive, despite the fact that it does not produce any utility in itself. If for example there are no taste shocks the price of money will be equal to zero and it has no value. To see this note that in this case all those who hold money in period t want to consume it since they have preference to the present (as β<1). As a result the demand for goods exceeds y, but since the equilibrium amount of consumption must be y in this exchange economy, the equilibrium real balances become 0, namely money has no value. Hence, money has value in this economy only because transactions have different sizes.

The model can be extended by adding an asset that has higher return than money but is less liquid, in a similar way to what is done in the Baumol-Tobin approach. Consider the possibility of bonds in the economy that pay a fixed real rate of interest r, but each sale of bonds has a fixed cost of real size x. Note that although bonds have a higher rate of return than money, people hold money none the less, for making small purchases. If the purchase is big enough, people might be willing to bear the cost x and not lose utility as a result of not making the full purchase. But if the purchase is small they prefer to pay with money, despite the loss of interest, in order to avoid spending x. Note that the demand for money is a result of having transactions of many sizes, since otherwise one type of asset would be preferred over the other.

Clearly the main ideas of the paper can be developed within this rather general framework, but with less tractability. Hence, we turn to a much simpler version of the model, where life cycles are short and the model can be solved fully. This will enable us to analyze all the mechanisms that we have outlined above.

Appendix B: Proofs

Proof of Proposition 1:

Derivation of (II.8) with respect to l_1 yields

$$\frac{\partial EU}{\partial l_1}=\frac{1}{2\sigma}\frac{\partial \theta_1}{\partial l_1}\left[U_A(\theta_1)-U_B(\theta_1)\right]+\frac{1}{2\sigma}\int_{1/2-\sigma}^{\theta_1}\frac{\partial U_A(\theta)}{\partial l_1}d\theta+\frac{1}{2\sigma}\int_{\theta_1}^{1/2+\sigma}\frac{\partial U_B(\theta)}{\partial l_1}d\theta.$$

Since the first item in the RHS is zero, we get that the marginal utility of first period money for $f_1 \le 2\sigma/(1/2-\sigma)$ is

(B.1)
$$\frac{\partial EU}{\partial l_1}=-\frac{1}{2}\frac{1}{y-l_1}+\frac{1}{2\sigma}\int_{1/2-\sigma}^{\theta_1}\frac{1}{2y+l_1}d\theta+\frac{1}{2\sigma}\int_{\theta_1}^{1/2+\sigma}\frac{\theta}{y+l_1}d\theta=$$
$$=\frac{1}{2y}\left[-\frac{1}{1-f_1}+\frac{1}{\sigma}\int_{1/2-\sigma}^{\theta_1}\frac{1}{2+f_1}d\theta+\frac{1}{\sigma}\int_{\theta_1}^{1/2+\sigma}\frac{\theta}{1+f_1}d\theta\right].$$

The marginal utility if $f_1 > 2\sigma/(1/2-\sigma)$ is:

(B.2)
$$\frac{\partial EU}{\partial l_1}=-\frac{1}{2}\frac{1}{y-l_1}+\frac{1}{\sigma}\int_{1/2-\sigma}^{1/2+\sigma}\frac{1}{2y+l_1}d\theta=\frac{1}{2y}\left[-\frac{1}{1-f_1}+\frac{1}{\sigma}\int_{1/2-\sigma}^{1/2+\sigma}\frac{1}{2+f_1}d\theta\right].$$

The expected utility can also be written in a general form in the following way:

(B.3)
$$\frac{\partial EU}{\partial l_1}==\frac{1}{2y}\left[-\frac{1}{1-f_1}+\frac{1}{\sigma}\int_{1/2-\sigma}^{1/2+\sigma}\max\left(\frac{1}{2+f_1},\frac{\theta}{1+f_1}\right)d\theta\right].$$

From (B.3) it is clear that marginal expected utility is decreasing with liquidity f_1. Furthermore, if liquidity is equal to zero, $f_1=0$, the marginal expected utility is equal to $\sigma/4y$. As f_1 approaches 1, the marginal expected utility goes to -∞. Hence, there exists a positive f_1 where marginal expected utility is equal to zero. This is the optimal ratio between money and income. Hence, the optimal l_1 is proportional to y. Note also that (B.2) is everywhere negative, so optimal f_1 must be smaller than $2\sigma/(1/2-\sigma)$, and there are always liquidity constrained consumers.

Finally, as σ increases, the marginal expected utility rises, since the averaging in equation (B.3) is done over higher θ. Hence, the optimal f_1 increases as well. Namely, the higher the variability of tastes, the larger the amount of money held by consumers in first period of life. If there are no taste shocks and $\sigma = 0$, the optimal amount of money in first period of life is 0. Q.E.D.

Proof of Proposition 2:

The marginal expected utility with respect to first period money satisfies:

$$\frac{\partial EU}{\partial l_1} = \frac{\partial \theta_1}{\partial l_1}\left[U_A(\theta_1) - U_B(\theta_1)\right] + \frac{\partial \theta_2}{\partial l_1}\left[U_B(\theta_2) - U_C(\theta_2)\right] +$$
$$+ \int_0^{\theta_1} \frac{\partial U_A(\theta)}{\partial l_1} d\theta + \int_{\theta_1}^{\theta_2} \frac{\partial U_B(\theta)}{\partial l_1} d\theta + \int_{\theta_2}^{1} \frac{\partial U_C(\theta)}{\partial l_1} d\theta =$$
$$= \int_0^{\theta_1} \frac{\partial U_A(\theta)}{\partial l_1} d\theta + \int_{\theta_1}^{\theta_2} \frac{\partial U_B(\theta)}{\partial l_1} d\theta + \int_{\theta_2}^{1} \frac{\partial U_C(\theta)}{\partial l_1} d\theta.$$

Hence, the marginal expected utility is equal to:

(B.4)
$$\frac{\partial EU}{\partial l_1} = \theta_1 \frac{-2r - r^2}{y(1+r)^2 - (2r + r^2)l_1} - \frac{(1-\theta_1)^2 - (1-\theta_2)^2}{2} \frac{1}{y - l_1} +$$
$$+ \frac{\theta_2^2 - \theta_1^2}{2} \frac{1}{l_1} - (1-\theta_2) \frac{r}{y(1+r) - rl_1}.$$

Marginal expected utility is decreasing in l_1. We next show that the marginal expected utility at $l_1 = 0$ is infinite, and hence the optimal amount of money, where the marginal expected utility is zero, is positive. To show this note that as $l_1 \to 0$ both θ_1 and θ_2 go to zero as well. As a result the first and second items on the RHS of (B.4) go to zero as well. The fourth item in the RHS of (B.4) converges to $-r/[y(1+r)]$. We next show that the third item in the RHS of (B.4) goes to ∞. From the definition of θ_2 in (III.9) we get that as l_1 and θ_2 converge to zero we get:

$$\theta_2 \log l_1 \xrightarrow[l_1 \to 0]{} -z.$$

Hence, for l_1 close enough to 0 :

$$\theta_2 \geq -\frac{z}{2\log l_2}.$$

Hence:

$$\frac{\theta_2^2}{l_1} \geq \frac{z^2}{4(\log l_1)^2 l_1} \xrightarrow[l_1 \to 0]{} \infty.$$

Since from (III.4) we get:

$$\frac{\theta_1^2}{l_1} \leq \frac{l_1}{(y-l_1)^2(1+r)^4} \xrightarrow[l_1 \to 0]{} 0,$$

the third item in (B.4) goes to infinity and so does marginal expected utility as money diminishes to zero. Hence the optimal amount of money is positive.

From (B.4) and from the definitions of θ_1 and θ_2 we can see that l_1 is proportional to income y and hence: $y = f(r)$. We next show that the demand for money by the young depends negatively on r, namely $f'(r) < 0$. To see this note that expected utility can be written as:

(B.5) $$EU = \int_0^1 \max \begin{Bmatrix} N(\theta) + \log[y(1+r)^2 - (2r + r^2)l_1], \\ \theta \log l_1 + (1-\theta)\log(y - l_1) + 2(1-\theta)\log(1+r), \\ N(\theta) + \log[y(1+r) - rl_1] + (1-\theta)\log(1+r) - z \end{Bmatrix} d\theta - z - x.$$

Clearly a rise in the rate of return r increases the losses from holding l_1 and hence it reduces the optimal l_1.

Since optimal l_1 is proportional to y, it follows from (B.5) that the optimal expected utility can be written as:

(B.6) $$\max EU = \log y + \phi(r, z) - x.$$

Note that if the cost of finding an investor and signing a contract is high, the consumer might prefer not to hold bonds at all. In that case utility is equal to:

(B.7) $$\int_0^1 N(\theta)d\theta + \log y.$$

Hence, if $x > \phi(r,z) - \int_0^1 N(\theta)d\theta$, the consumer prefers to hold money only and does not go to the bonds market. QED.

Proof of Proposition 3:

Consider the four cases of money and consumption in second period of life. In case A the consumer has small consumption and purchases with part of her cash. Utility is:

$$\theta \log c_2 + (1-\theta)\log(l_c + l_b - c_2) - 2v.$$

Optimization leads to: $c_2 = \theta y$, $c_3 = (1-\theta)y$, and cash held for next period is: $l_c - \theta y$. Utility is:

$$U_A(\theta) = N(\theta) + \log y - 2v.$$

In case B consumption is larger so all the cash is consumed and all demand deposits are kept for next period. Hence:

$$U_B(\theta) = \theta \log l_c + (1-\theta)\log(y - l_c) - 2v.$$

In case C consumption is financed by all the cash and by some of the demand deposits, while some of the demand deposit is kept in the bank. The consumer maximizes:

$$\theta \log c_2 + (1-\theta)\log(l_c + l_b - c_2) - 3v.$$

Hence consumption in second period is $c_2 = \theta y$, and in third period is $c_3 = (1-\theta)y$, and utility is:

$$U_C(\theta) = N(\theta) + \log y - 3v.$$

In case D the consumer leaves only cash for third period but since it is bounded by $d\,y$, we get: $c_2 = (1-d)y$ and $c_3 = dy$. Hence utility in this case is:

$$U_D(\theta) = \theta \log(1-d) + (1-\theta)\log d + \log y - 2v.$$

Note that the borderline between cases A and B is $\theta_1 = l_c / y$. Denote the borderline between B and C (or D if C is empty) by θ_2. Note that the utility for C and D does not depend on l_c. Hence the marginal expected utility is equal to:

$$\frac{1}{\tau}\int_0^{\theta_1} \frac{\partial U_A(\theta)}{\partial l_c} d\theta + \frac{1}{\tau}\int_{\theta_1}^{\theta_2} \frac{\partial U_B(\theta)}{\partial l_c} d\theta = \frac{1}{\tau}\int_{\theta_1}^{\theta_2}\left(\frac{\theta}{l_c} - \frac{1-\theta}{y-l_c}\right)d\theta.$$

Note that for $\theta \geq \theta_1 = l_c / y$, the integral is positive. Hence, the young consumer raises the amount of cash as much as possible, so that $l_c = dy$.

Given the initial demand for cash we can calculate the amounts of cash in the second period of any individual. To simplify the calculation assume that v is large enough so that the set C is empty. Hence, θ_2 is determined by the intersection of U_B and U_D:

$$\theta_2 \log d + (1-\theta_2)\log(1-d) + \log y - 2v = \theta_2 \log(1-d) + (1-\theta_2)\log d + \log y - 2v.$$

Clearly the solution to this equation is: $\theta_2 = ½$.

We can now calculate the overall demand for cash:

$$L_c = y\left[d + \frac{1}{\tau}\int_0^d (d-\theta)d\theta + \frac{\tau - 1/2}{\tau} d\right] = yd\frac{4\tau + d - 1}{2\tau}.$$

Hence, the demand for cash is rising with τ.

A similar calculation shows that the overall amount of demand deposits is:

$$L_b = y\left[1 - d + \frac{1}{2\tau}(1-d)\right] = y\frac{(1-d)(2\tau+1)}{2\tau}.$$

Hence the overall demand for money in the economy is:

$$L = y\left[1 + d + \frac{(1-d)^2}{2\tau}\right].$$

Hence the demand for money is falling with τ, the size of taste shocks. QED.

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Figures

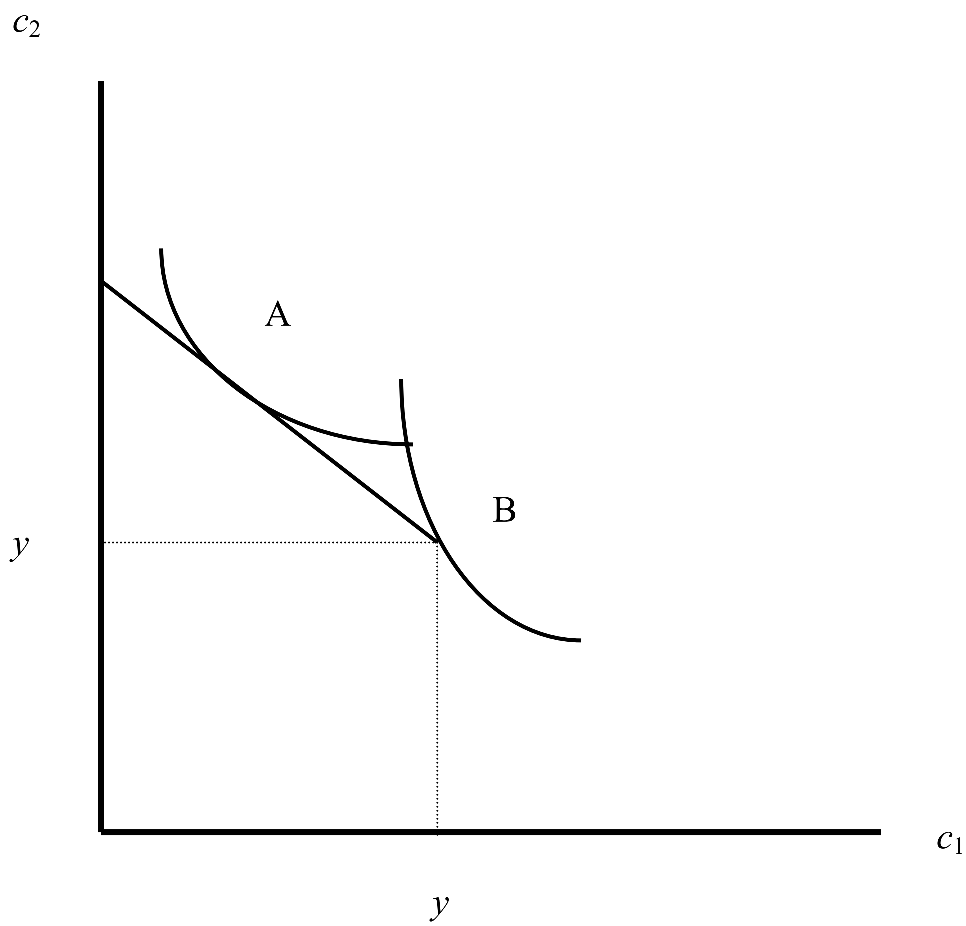

Figure 1

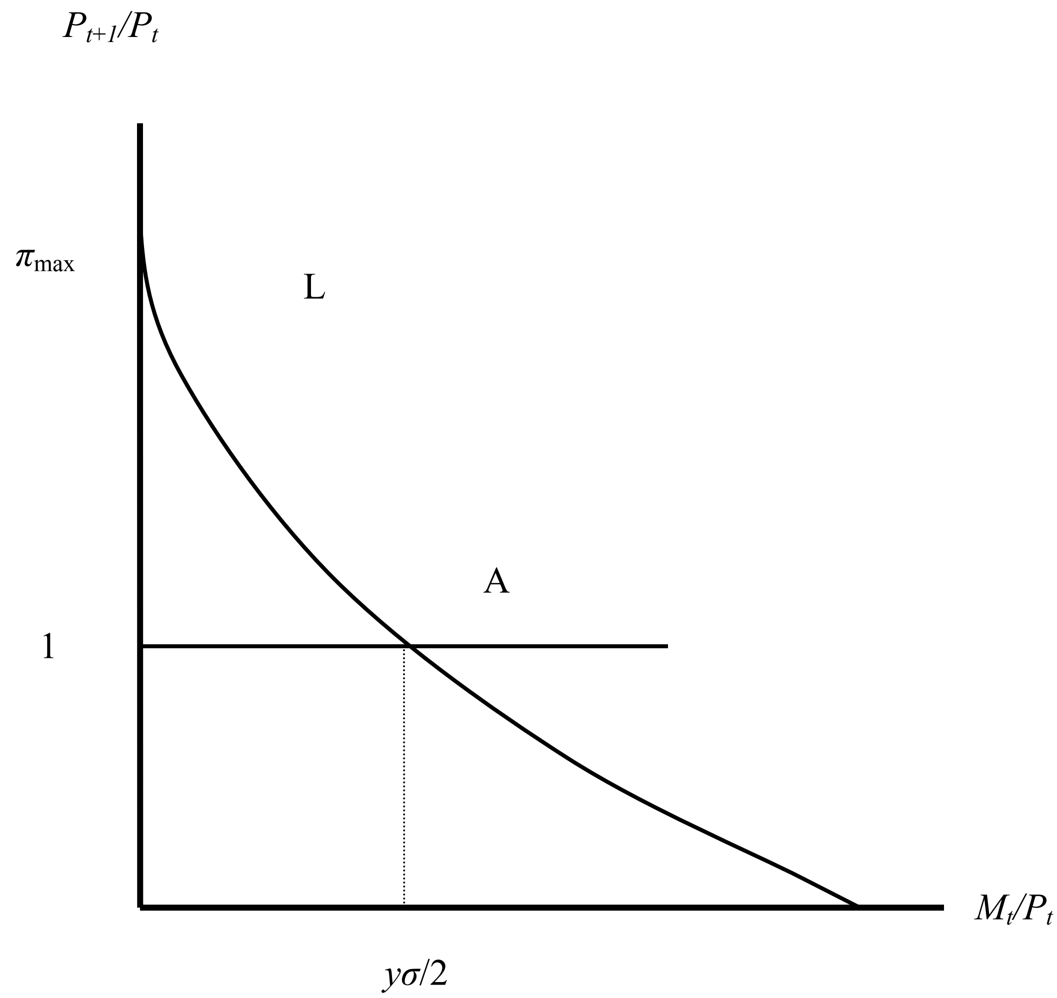

Figure 2