# International Capital Flows with Dispersed Information: Theory and Evidence<sup>1</sup>

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#### Abstract

Existing models of international capital flows have an important shortcoming: they are representative agent models where all investors have the same information. This stands in sharp contrast to reality. Even within a country investors have widely different expectations about asset prices and macroeconomic fundamentals. Moreover, there is extensive evidence of the importance of information asymmetries across countries. In this paper we develop a general equilibrium model of international capital flows based on dispersed information across agents, with possible asymmetries in information across countries. We develop the implications for both gross and net international capital flows from a portfolio choice perspective. We show that dispersed information affects capital flows through asset prices, timevarying risk and differences between domestic and foreign investors in expected returns. We develop a series of testable implications of the model that are confronted to the data. This is the first paper that integrates dispersed information into a full DSGE model. In order to do so we develop a solution method that combines and extends elements from the noisy rational expectations literature and the recent literature on representative agent open economy DSGE models with portfolio choice.

JEL classification: F32, F36, F41 Keywords: international capital flows, information dispersion

### 1 Introduction

Existing theories of international capital flows are largely based on representative agent models where all investors have the same information. There is good reason to believe that this may not be a good benchmark to think about international capital flows. First, it implies that capital flows are entirely driven by observed macro fundamentals, in stark contrast to the well-known difficulty of explaining fluctuations in net capital flows.<sup>1</sup> Second, survey evidence has documented that investors have widely different views about the value of future macro fundamentals and future asset prices. This should affect capital flows through expectations of asset returns. Third, it is likely that determinants of relative asset prices are closely linked to determinants of capital flows. It is well known that especially over short to medium horizons asset price volatility is more closely linked to order flow than to observed macro fundamentals. This again suggests a key role for dispersed information.

The goal of this paper is to develop a theory of international capital flows based on dispersed information and to confront the empirical implications of the theory to the data. We develop a two-country general equilibrium model where agents make decisions about portfolio allocation, physical investment and saving. Information dispersion is introduced by adopting the two key elements of noisy rational expectations (NRE) models of asset pricing. First, agents have private information about future macro fundamentals. We allow for the possibility that agents have more precise information about domestic than foreign returns. Second, asset prices are prevented from revealing the private information through unobserved exogenous portfolio shifts across countries. Since our focus is on the impact of dispersed information, we keep the other elements as simple as possible in order to preserve the analytic tractability and transparency of the results. There is only one good in the world economy (no real exchange rate fluctuations), agents can purchase only two assets (called Home and Foreign equity), and we focus on a dynamic OLG structure where agents live only two periods.

<sup>&</sup>lt;sup>1</sup>For example, for the U.S. current account there are almost as many papers as explanations. Nason and Rogers (2006) write, "Current account fluctuations resist easy explanations. Large current account deficits have persisted in the U.S. through periods of large government budget deficits and surpluses, large and persistent real appreciations and depreciations of the dollar and all phases of the business cycle."

We adopt a portfolio perspective on capital flows, writing both capital inflows and outflows as a function of all the standard elements of portfolio allocation: changes in wealth (saving), changes in expected returns and changes in the riskcharacteristics of assets. Each of these is in turn solved as a function of the underlying shocks of the model in a full general equilibrium context. Information dispersion plays a role in three fundamentally different ways. First, we show that as a result on information dispersion relative asset prices depend on two unobservables: unobserved exogenous portfolio shifts across countries (the noise in the model) and unobserved relative future productivity shocks (which enter through private information). Relative asset prices in turn affect saving and investment, which affect capital flows through changes in wealth and changes in equilibrium expected excess returns. Second, time-variation in these two unobservables also leads to time-variation in second moments (risk) that affect capital flows. Third, differences in information about future fundamentals across countries leads to differences in expected returns that affect capital flows.

The model has various implications for capital flows that will be confronted to the data. First, both gross and net capital flows cannot be fully explained by observed macro fundamentals. Second, conditional on observed macro fundamentals, relative asset prices have explanatory power for both gross and net capital flows. Third, both gross and net capital flows have predictive power for future macro fundamentals (e.g. future world GDP growth and relative GDP growth across countries). Finally, controlling for relative saving and investment, the model implies that capital inflows and outflows are positively correlated. We will evaluate these implications based on asset price and capital flow data for industrialized countries.

The paper also makes an important methodological contribution. The model has four elements whose joint presence makes the solution challenging: (i) nonlinearity, (ii) general equilibrium, (iii) portfolio choice and (iv) information dispersion. Standard DSGE models only contain the first two elements while standard NRE models only contain the last two elements. Recently Devereux and Sutherland (2007) and Tille and van Wincoop (2008) have developed a solution method for open economy DSGE models with portfolio choice, which can therefore handle the first three of the elements described above. We further extend that method to handle information dispersion as well. This is not straightforward as standard NRE models that contain dispersed information are all linear partial equilibrium models. This stands in sharp contrast to the non-linear DSGE models that are standard fare in macroeconomics.

The paper is related to a small set of papers that have introduced NRE asset pricing features into open economy models. These include Albuquerque, Bauer and Schneider (2006), Bacchetta and van Wincoop (2004,2006), Brennan and Cao (1997), Gehrig (1993) and Veldkamp and van Nieuwerburgh (2007). These papers focus on a variety of issues, ranging from exchange rate puzzles to international portfolio home bias and the relationship between asset returns and portfolio flows. Together they show that information dispersion and information asymmetries can tell us a lot about a wide range of stylized facts related to international asset prices and portfolio allocation.

None of these papers have implications for aggregate capital inflows and outflows or even net capital flows. This is not just because the focus is on other questions but more fundamentally because these are not true general equilibrium models. In order to facilitate the solution these models always assume that there is a riskfree global asset with a constant return that is in infinite supply. The models only have implications for capital flows that do not involve this asset. Apart from the constant riskfree interest rate, these models inherit two additional features from the NRE asset pricing literature to facilitate the solution: constant absolute risk-aversion preferences and full linearity of the system of equations. These assumptions are rarely ever satisfied in general equilibrium macroeconomic models.

In this paper we will fully integrate information dispersion in a standard DSGE open economy model. We will not make any of the special assumptions that are commonly made in the NRE literature. Agents make decisions about production, saving, investment and portfolio allocation. Financial, goods and labor markets all need to clear. The model is highly non-linear. There is no riskfree asset that is in infinite supply and we adopt constant relative risk-aversion preferences.

The paper is organized as follows. Section 2 describes the model. The solution method is discussed in section 3. Section 4 derives the asset pricing implications of the model while section 5 derives expressions for capital inflows and outflows from a portfolio choice perspective. Section 6 discusses various implications for gross and net capital flows that will be confronted to the data in section 7. Section 8 concludes.

# 2 The Model

The key ingredient of the model is the dispersion of information across individual investors, both within and across countries. To focus on this aspect, the other elements of the model are designed to keep the model as simple and transparent as possible, while remaining rich enough to generate implications for both gross and net international capital flows.

The world consists of two countries of equal size, Home and Foreign. Both countries produce the same good using labor and capital. The good can be used for consumption or investment, the latter entailing an adjustment cost. We consider an overlapping generation setup with agents in each country living for two periods. In the first period of their life, young agents supply one unit of labor and earn a wage. Agents make decisions on the allocation of consumption over their lifetime, saving some of their wage to finance consumption when old. Saving is invested in claims on capital in both countries, which we refer to as Home and Foreign equity. Each agent allocates her portfolio across the two equities based on public information as well as private information on future equity returns. During the second period of life, when old, the agents consume the return on their investment.

### 2.1 Production and investment

The consumption good is taken as the numeraire. It is produced in both countries using a constant returns to scale technology in labor and capital:

$$Y_{i,t} = A_{i,t} K_{i,t}^{1-\omega} N_{i,t}^{\omega} \quad i = H, F$$

$$\tag{1}$$

where H and F denote the Home and Foreign country respectively.  $Y_i$  is the output in country i,  $A_i$  is a country-specific exogenous stochastic productivity term,  $K_i$ is the capital input and  $N_i$  the labor input that we normalize to unity. The log productivity follows an autoregressive process:

$$a_{i,t+1} = \rho a_{i,t} + \varepsilon_{i,t+1}$$

where  $\varepsilon_{i,t+1}$  has a  $N(0, \sigma_a^2)$  distribution and is uncorrelated across countries.

The dynamics of the capital stock reflects depreciation at a rate  $\delta$  and investment  $I_{i,t}$ :

$$K_{i,t+1} = (1 - \delta) K_{i,t} + I_{i,t} \quad i = H, F$$
(2)

A share  $\omega$  of output is paid to labor, with the remaining going to capital. The wage rate in country *i* is then

$$W_{i,t} = \omega A_{i,t} \left( K_{i,t} \right)^{1-\omega} \quad i = H, F \tag{3}$$

Capital is supplied by a competitive installment firm. In period t the installment firm produces  $I_{i,t}$  units of new capital and sells them at a price  $Q_{i,t}$  that the firm takes as given. The production of  $I_{i,t}$  units of capital good requires purchasing  $I_{i,t}$  units of the consumption good and incurring a quadratic adjustment cost, so the total cost in units of the consumption good is:

$$I_{i,t} + \frac{\xi}{2} \frac{(I_{i,t} - \delta K_{i,t})^2}{K_{i,t}}$$
(4)

The profit of installing  $I_{i,t}$  units of capital in country *i* is then  $Q_{i,t}I_{i,t}$  minus the cost (4). Profit maximization by the installment firm implies a standard Tobin's Q relation:

$$\frac{I_{i,t}}{K_{i,t}} = \delta + \frac{Q_{i,t} - 1}{\xi}$$

$$\tag{5}$$

### 2.2 Two assets: rates of return

There is a unit mass of atomistic investors in each country. A unit of Home equity is a claim on a unit of Home capital. Its value is then equal to the cost of purchasing one unit of capital from the installment firm,  $Q_{H,t}$ , which can be interpreted as the equity price. An investor purchasing a unit of Home equity at the end of period tgets a dividend of  $(1 - \omega)Y_{H,t+1}/K_{H,t+1}$  in period t + 1, and can sell the remaining  $1 - \delta$  units of equity at a price  $Q_{H,t+1}$ . The returns on Home and Foreign equity are then

$$R_{H,t+1} = \frac{(1-\omega) A_{H,t+1} (K_{H,t+1})^{-\omega} + (1-\delta) Q_{H,t+1}}{Q_{H,t}}$$
(6)

$$R_{F,t+1} = \frac{(1-\omega) A_{F,t+1} (K_{F,t+1})^{-\omega} + (1-\delta) Q_{F,t+1}}{Q_{F,t}}$$
(7)

Investing in equity abroad entails a cost, as in Tille and van Wincoop (2008). Specifically, a Home agent j investing in the Foreign country receives only the return (7) times an iceberg cost  $e^{-\tau_{Hj,t}} < 1$ . Similarly, a Foreign agent j investing in the Home country receives the return (6) times an iceberg cost  $e^{-\tau_{Fj,t}} < 1$ . The cost of investment abroad does not represent a loss in resources but is instead a fee paid to brokers from the investor's country. The costs  $\tau$  can vary across time, across countries, and across agents. We assume that each agent can only observe his own cost of investing abroad, but not the average cost.<sup>2</sup> The average cost across Home and Foreign investors is respectively  $\tau_{H,t}$  and  $\tau_{F,t}$ :

$$\tau_{i,t} = \tau \left( 1 + \epsilon_{i,t+1}^{\tau} \right) \qquad \qquad i = H, F$$

where  $\epsilon_{i,t+1}^{\tau}$  has a  $N(0, \theta \sigma_a^2)$  distribution and is uncorrelated across countries.  $\theta$  captures the relative variance of cost shocks relative to that of productivity shocks  $\sigma_a^2$ .

The cost of investing abroad is introduced for two reasons. First, it is a simple way to capture the hurdles of investing outside the domestic country, reflecting the cost of gathering information on an unfamiliar market for instance. Second, the time-variation in the average costs in both countries will play the role of "noise" in noisy-rational expectations models. In such models asset prices depend on future fundamentals through private information, while additional unobserved portfolio shifts prevent the asset price from fully revealing future fundamentals. In our model time-variation in the relative cost  $\tau_{H,t} - \tau_{F,t}$  leads to portfolio shifts across countries that prevents the relative asset price from revealing private information about future fundamentals. This is a convenient way of modeling the unobserved noise, although by no means the only possible way. In the literature the noise is usually simply introduced exogenously as noise trade or liquidity trade. Some papers have introduced it endogenously in various forms of hedge trade and liquidity trade.<sup>3</sup> For our purposes the existence of a source of noise is more important than the exact nature of it.

In period t a Home agent j invests a fraction  $z_{Hj,t}$  of her wealth in Home equity and a fraction  $1 - z_{Hj,t}$  in Foreign equity. The overall real return on her portfolio is then

$$R_{t+1}^{p,Hj} = z_{Hj,t} R_{H,t+1} + (1 - z_{Hj,t}) e^{-\tau_{Hj,t}} R_{F,t+1}$$
(8)

Similarly, the real return for a Foreign agent j is

$$R_{t+1}^{p,Fj} = z_{Fj,t} e^{-\tau_{Fj,t}} R_{H,t+1} + (1 - z_{Fj,t}) R_{F,t+1}$$
(9)

 $<sup>^{2}</sup>$ More precisely, we assume that the individual cost is an infinitely noisy signal of the average cost. This assumption can be relaxed but simplifies the analysis.

<sup>&</sup>lt;sup>3</sup>See for example Bacchetta and van Wincoop (2006), Dow and Gorton (1995), Spiegel and Subrahmanyam (1992) and Wang (1994).

The average portfolio shares of Home and Foreign investors are denoted  $z_{H,t} = \int_0^1 z_{Hj,t} dj$  and  $z_{F,t} = \int_0^1 z_{Hj,t} dj$ .

### 2.3 Private information

The model allows for dispersed information across individual investors both within and across countries. Each agent receives private signals about next period's productivity innovations in both countries. The signals observed by Home investor jabout respectively the log of Home and Foreign productivity are:

$$v_{j,t}^{H,H} = \varepsilon_{H,t+1} + \epsilon_{j,t}^{H,H} \qquad \epsilon_{j,t}^{H,H} \sim N\left(0,\sigma_{HH}^2\right) \qquad (10)$$

$$v_{j,t}^{H,F} = \varepsilon_{F,t+1} + \epsilon_{j,t}^{H,F} \qquad \epsilon_{j,t}^{H,F} \sim N\left(0,\sigma_{HF}^2\right)$$
(11)

Each signal consists of the true innovation and a stochastic error. Similarly, agent j in the Foreign country observes the signals:

$$v_{j,t}^{F,H} = \varepsilon_{H,t+1} + \epsilon_{j,t}^{F,H} \qquad \epsilon_{j,t}^{F,H} \sim N\left(0,\sigma_{HF}^{2}\right)$$
(12)

$$v_{j,t}^{F,F} = \varepsilon_{F,t+1} + \epsilon_{j,t}^{F,F} \qquad \epsilon_{j,t}^{F,F} \sim N\left(0,\sigma_{HH}^2\right)$$
(13)

As is standard in noisy rational expectations models, we assume that the errors of the signals average to zero across investors in a given country  $(0 = \int_0^1 \epsilon_{j,t}^{H,H} dj = \int_0^1 \epsilon_{j,t}^{H,F} dj)$ .

For simplicity we assume that the variance of signals on domestic productivity is the same for agents in the two countries, as is the variance of signals on productivity abroad. We allow for an information asymmetry with agents receiving more precise signals about shocks in their own country than abroad:  $\sigma_{HH}^2 \leq \sigma_{HF}^2$ .

### 2.4 Consumption and Portfolio Choice

Our assumption of an overlapping generation structure simplifies the model in two ways. First, it removes the well-known pitfall in open economy models that temporary income shocks can have a permanent effect on the distribution of wealth across countries when agents have infinite lives. Second, investors have only a one period investment horizon and therefore do not face the issue of hedging against changes in future expected returns. A young Home agent j at time t chooses her consumption and portfolio to maximize

$$\frac{\left(C_{y,t}^{Hj}\right)^{1-\gamma}}{1-\gamma} + \beta E_t^{Hj} \frac{\left(C_{o,t+1}^{Hj}\right)^{1-\gamma}}{1-\gamma} \tag{14}$$

subject to the budget constraint:

$$C_{o,t+1}^{Hj} = (W_{H,t} - C_{y,t}^{Hj})R_{t+1}^{p,Hj}$$

and the portfolio return (8).  $C_{y,t}$  is consumption when young and  $C_{o,t+1}$  is consumption when old. Foreign agents face an analogous decision problem. While (14) is a standard time-separable expected utility, the expectation operator  $E_t^{Hj}$ is conditioned on the information by the specific agent. This reflects the fact that each young agent receives a private signal on productivity shocks in the second period of her life. The first-order conditions for consumption and portfolio choice are:

$$\left(C_{y,t}^{Hj}\right)^{-\gamma} = \beta \left(W_{H,t} - C_{y,t}^{Hj}\right)^{-\gamma} E_t^{Hj} \left(R_{t+1}^{p,Hj}\right)^{1-\gamma}$$
(15)

$$E_t^{Hj} \left( R_{t+1}^{p,Hj} \right)^{-\gamma} \left( R_{H,t+1} - R_{F,t+1} e^{-\tau_{Hj,t}} \right) = 0$$
(16)

The corresponding conditions for a Foreign investor j are:

$$\left(C_{y,t}^{Fj}\right)^{-\gamma} = \beta \left(W_{F,t} - C_{y,t}^{Fj}\right)^{-\gamma} E_t^{Fj} \left(R_{t+1}^{p,Fj}\right)^{1-\gamma}$$
(17)

$$E_t^{Fj} \left( R_{t+1}^{p,Fj} \right)^{-\gamma} \left( R_{H,t+1} e^{-\tau_{Fj,t}} - R_{F,t+1} \right) = 0$$
(18)

(15) and (17) are the standard consumption Euler equations. (16) and (18) show that the optimal portfolio allocation equates the expected discounted return (the expected product of the asset pricing kernel and asset returns) across assets. The asset pricing kernel is the marginal utility of future consumption, which is proportional to the return on the agent's portfolio.

### 2.5 Asset and Goods Market Clearing

We assume that the brokers who receive the fees on investment abroad fully consumer it. Also, we consider that the installment firms in both countries are owned by agents who consume all profits each period. The goods market equilibrium condition is

$$Y_{H,t+1} + Y_{F,t+1} = \int_0^1 C_{y,t+1}^{Hj} dj + \int_0^1 C_{y,t+1}^{Fj} dj + \int_0^1 (W_{H,t} - C_{y,t}^{Hj}) (z_{Hj,t} R_{H,t+1} + (1 - z_{Hj,t}) R_{F,t+1}) dj + \int_0^1 (W_{F,t} - C_{y,t}^{Fj}) (z_{Fj,t} R_{H,t+1} + (1 - z_{Fj,t}) R_{F,t+1}) dj + Q_{H,t+1} I_{H,t+1} + Q_{F,t+1} I_{F,t+1}$$

The left hand side is world output. The first two terms on the right hand side represent consumption by young agents. The next two terms represent consumption by old agents and the brokers.<sup>4</sup> The last two terms represent investment.<sup>5</sup>

Asset market clearing requires that the value of capital in a country is equal to the value of holdings of the country's equity by young agents. The financial assets of a young Home agent j at time t is given by  $W_{Ht} - C_{y,t}^{Hj}$ . Similarly, the assets of a young Foreign agent j are  $W_{Ft} - C_{y,t}^{Fj}$ . The asset market clearing conditions are then

$$Q_{H,t}K_{H,t+1} = \int_{0}^{1} (W_{Ht} - C_{y,t}^{Hj}) z_{Hj,t} dj + \int_{0}^{1} (W_{Ft} - C_{y,t}^{Fj}) z_{Fj,t} dj \qquad (19)$$

$$Q_{F,t}K_{F,t+1} = \int_{0}^{1} (W_{Ht} - C_{y,t}^{Hj}) (1 - z_{Hj,t}) dj + \int_{0}^{1} (W_{Ft} - C_{y,t}^{Fj}) (1 - z_{Fj,t}) dj \qquad (20)$$

# 3 Solution Method

The model has four features whose joint presence makes the solution challenging. These elements are (i) non-linearity, (ii) general equilibrium nature, (iii) portfolio choice and (iv) information dispersion. Noisy rational expectations models with dispersed information have the last two features, but not the first two. This significantly simplifies the solution. In particular, the linearity of these models leads to

<sup>&</sup>lt;sup>4</sup>The cost of investing abroad does not enter, as the income of the brokers exactly offsets the cost for old agents.

 $<sup>{}^{5}</sup>$ The installation cost does not enter. On the one hand it raises demand for the good (from the installation process itself). On the other hand it reduces profits, and therefore consumption, of the owners of installment firms.

simple linear signal extraction problems. On the other hand, most DSGE models in macro and open economy macro only have the first two features. The most common method for solving DSGE models is through a first or second-order local approximation around the deterministic steady state. However, apart from the absence of information dispersion, these methods have difficulty dealing with portfolio choice. The allocation around which the model is expanded cannot be the deterministic steady state as portfolio choice is undetermined in such an environment.

The solution method will adapt standard first and second-order local approximation methods to incorporate information dispersion and portfolio choice. As a result of space constraints we will only describe the solution method in broad terms, leaving all algebraic details to the Appendix and the Technical Appendix that is available on request.

### **3.1** Information Dispersion

We first discuss how the solution handles the issue of information dispersion, which is at the heart of this paper. Noisy rational expectation models (from hereon NRE models) are usually solved in three steps. The first step involves conjecturing an equilibrium asset price. It is assumed to depend linearly on a future asset payoff, which enters through private information, and on a noise variable. The second step involves a signal extraction problem in order to compute the expectation of the future asset payoff. This combines endogenous information from the asset price, private information and public information. The resulting expectation of the asset payoff is substituted in expressions for optimal portfolio shares. The final step involves imposing asset market equilibrium in order to make sure that the conjectured asset price equation is correct and to compute its parameters.

#### asset price conjecture

The approach we take here is related, but is obviously complicated by the fact that we have a non-linear DSGE model. As in the NRE literature, we first conjecture an equilibrium asset price equation. Only the relative asset price will be affected by private information. Due to the absence of non-equity assets, world equity demand will be entirely determined by world saving by the young, which will determine the average equity price. In what follows we will use lower case letters for

logs and use superscripts A and D to denote respectively the average and difference of a variable across the two countries  $(x^D = x_H - x_F, x^A = (x_H + x_F)/2)$ . We will conjecture that

$$q_t^D = f(S_t, x_t^D) \tag{21}$$

where

$$S_t = (a_t^D, a_t^A, k_t^D, k_t^A - k^A(0))$$
(22)

is the vector of observed state variables and

$$x_t^D = \varepsilon_{t+1}^D + \lambda \tau_t^D / \tau \tag{23}$$

is an unobserved state variable. We will verify that the solution for  $q_t^D$  indeed takes this conjectured form.

The logic behind this conjecture is as follows. First, as in any DSGE model the solution for control variables (including asset prices) will be a function of state variables. Usually these state variables are observed. In our model this is the case for the variables  $S_t$ . However, there are now also unobserved state variables, which are assumed to jointly affect the asset price through  $x_t^D$ . It is reasonable to conjecture that it is the difference  $\varepsilon_{t+1}^D$  in future productivity innovations that affects the difference in log asset prices. It is similarly reasonable to expect that the difference  $\tau_t^D = \tau_{H,t} - \tau_{F,t}$  in average financial frictions affects the difference in log asset prices. A rise in  $\tau_t^D$  leads to a portfolio shift from Foreign equity to Home equity that should raise the relative price of Home equity.

#### signal extraction

This conjecture significantly simplifies signal extraction. While the function f(.) will be non-linear in  $x_t^D$ , so that  $q_t^D$  depends in a non-linear way on  $\varepsilon_{t+1}^D$  and  $\tau_t^D$ , two aspects make simple linear signal extraction feasible. First, we have conjectured that the asset price depends on a variable  $x_t^D$  that is linear in the unknowns  $\varepsilon_{t+1}^D$  and  $\tau_t^D$ . Second, we will adopt a local approximation method. Locally  $q_t^D$  will depend on  $x_t^D$  with a positive slope. This means that we can extract  $x_t^D$  from knowledge of the relative asset price  $q_t^D$ , and the state space  $S_t$ . The asset price signal therefore translates into a signal that is linear in the future fundamental  $\epsilon_{t+1}^D$  and the "noise"  $\tau_t^D$ .

We then have three linear signals about next period's technology innovations: (i) the price signal, which tells us the level of  $\varepsilon_{t+1}^D + \lambda \tau_t^D / \tau$ , (ii) the private signals and (iii) the public signals that  $\varepsilon_{H,t+1}$  and  $\varepsilon_{F,t+1}$  are drawn from independent  $N(0, \sigma_a^2)$  distributions. We solve this signal extraction problem in Appendix B. It gives conditional normal distributions of  $\varepsilon_{H,t+1}$  and  $\varepsilon_{F,t+1}$  that vary across agents. The expectation of future productivity innovations by agent j in the Home country takes the form

$$E_t^{H,j} \begin{pmatrix} \varepsilon_{H,t+1} \\ \varepsilon_{F,t+1} \end{pmatrix} = \begin{pmatrix} \alpha_{x,H} x_t^D + \alpha_{HH} v_{j,t}^{H,H} + \alpha_{HF} v_{j,t}^{H,F} \\ -\alpha_{x,F} x_t^D + \alpha_{FH} v_{j,t}^{H,H} + \alpha_{FF} v_{j,t}^{H,F} \end{pmatrix}$$
(24)

All coefficients are positive and are defined in the Appendix. The average expectation across Home agents is then

$$\bar{E}_{t}^{H}\left(\begin{array}{c}\varepsilon_{H,t+1}\\\varepsilon_{F,t+1}\end{array}\right) = \left(\begin{array}{c}(\alpha_{x,H} + \alpha_{HH})\varepsilon_{H,t+1} + (\alpha_{HF} - \alpha_{x,H})\varepsilon_{F,t+1} + \alpha_{x,H}\lambda\tau_{t}^{D}/\tau\\(\alpha_{FH} - \alpha_{x,F})\varepsilon_{H,t+1} + (\alpha_{x,H} + \alpha_{FF})\varepsilon_{F,t+1} - \alpha_{x,F}\lambda\tau_{t}^{D}/\tau\end{array}\right)$$
(25)

Analogous results apply to Foreign agents. Average expectations about future productivity therefore depend on future productivity levels themselves and on the noise  $\tau_t^D$ . Through rational confusion an increases in  $\tau_t^D$  raises the expectation of  $\epsilon_{t+1}^D$ . This is because a rise in  $\tau_t^D$  leads to a higher relative price of Home equity, which agents use as a signal of future relative productivity.

### **3.2** Local Approximation

The final step in the solution of NRE models involves imposing asset market clearing conditions. In a DSGE model this step is more involved since we will need to invoke all model equations, including multiple asset market and goods market clearing conditions and Euler equations for portfolio choice and consumption. We will adopt a local approximation method that involves imposing model equations at various orders of approximation.

The "order" of variables and equations is defined as follows. A variable  $x_t$  can always be written as the sum of its zero-order, first-order and higher-order components, namely:  $x_t = x(0) + x_t(1) + x_t(2) + \dots$  The zero-order component, x(0), is the value of  $x_t$  when the volatility of shocks in the model becomes arbitrarily small. The first-order component,  $x_t(1)$ , is proportional to model innovations or their standard deviation. The second-order component,  $x_t(2)$ , is proportional to the product of model innovations (or their variance), and so on. The order component of equations can be found by writing down a Taylor expansion of the

equation around x(0) and substituting  $x_t = x(0) + x_t(1) + x_t(2) + \dots$  For example, the second-order component of  $f(x_t)$  for a single variable  $x_t$  is  $f'(x(0))x_t(2) + 0.5f'(x(0))x_t(1)^2$ .

In solving the model we will make the following assumption about the order of some key model parameters.

**Assumption 1** The parameter  $\tau$  is second-order. The parameters  $\sigma_{HH}^2$  and  $\sigma_{HF}^2$  (variance of errors of private signals) are zero-order.

The logic behind these assumptions is as follows. Portfolio choice is driven by expected returns divided by risk (variance of returns). If the mean international financial friction  $\tau$  would be zero or first-order, expected excess returns would be large relative to risk, so that portfolios explode for low levels of risk. The second-order  $\tau$  leads to second-order expected return differences, which leads to zero-order portfolio home bias.

The assumption that  $\sigma_{HH}^2$  and  $\sigma_{HF}^2$  are zero-order is also meant to avoid an explosion of portfolios when risk becomes small. While errors in private signals are not "shocks" to the model, consider what would happen if we assumed them to be first-order like model innovations, so that  $\sigma_{HH}^2$  and  $\sigma_{HF}^2$  are second-order. In that case differences in expected returns across investors would be first-order, so that differences in portfolio shares explode for low levels of risk. When instead the errors in private signals are zero-order (and therefore  $\sigma_{HH}^2$  and  $\sigma_{HF}^2$  are zero-order as well), these errors will be large relative to the other signals. The weight given to private signals will then be small, of order two and higher. Differences in expected returns across investors well, so that differences in optimal portfolio shares are of order zero (depend on the zero-order errors of the private signals) and higher. There will then be a well-defined distribution of portfolio shares across agents that does not depend on the level of risk in the economy and therefore does not explode when risk becomes small.

### 3.3 Three-Step Solution

As discussed above, standard first and second-order local approximation methods cannot be applied with portfolio choice as the deterministic steady state is not well defined. Devereux and Sutherland (2007) and Tille and van Wincoop (2008), from hereon DS and TvW, have developed an extension of standard first and secondorder solution methods to incorporate portfolio choice. We will further adept it to incorporate information dispersion as well.

We will be brief here in describing the solution method developed by DS and TvW as these papers go into great detail. The solution uses standard first and second-order solution approximation methods for all equations except the optimality conditions for portfolio choice. These need to be approximated to higher orders. Intuitively, this reflects the fact that portfolio allocation is about risk, a dimension that is not captured by a zero and first-order approximation. Solving for portfolio choice in the allocation around which the model is expanded requires a secondorder expansion of optimality conditions for portfolio choice in order to capture the risk at the core of portfolio choice. In order to capture the time-variation of portfolio allocation a third-order expansion of the optimality conditions for portfolio choice is needed.

The complexity is particularly related to the difference across countries in portfolio allocation. The average across countries of portfolio allocation can be solved even from a deterministic steady state from the asset market clearing conditions. These tell us how much Home and Foreign equity agents will need to hold in equilibrium, but they do not tell us the allocation across investors from different countries. The solution method therefore starts by distinguishing between the difference across countries in portfolio shares and all "other variables" and between the difference across countries in optimality conditions for portfolio choice and all "other equations".

The solution involves two steps. The first step uses the first-order component of the "other equations" together with the second-order component of the difference across countries in portfolio Euler equations to jointly solve the zero-order component of the difference across countries in portfolio shares and the first-order component of the "other variables". The second step is the same but one order higher. It uses the second-order component of the "other equations" together with the third-order component of the difference across countries in portfolio Euler equations to jointly solve the first-order component of the difference across countries in portfolio shares and the second-order component of the "other variables".

We will leave all algebraic details associated with implementing these two steps of the solution method to the Appendices. One issue needs to be discussed though. The Euler equations for consumption and portfolio choice involve expectations of highly non-linear functions of future control and state variables. We need to compute these expectations before we can impose the order components of the equations. We proceed as follows. We start by conjecturing a solution for all control variables as a quadratic function of the observed and unobserved state variables  $S_t$  and  $x_t^D$ . This is sufficient for our purposes as the solution method will only compute first and second-order components of model variables. Specifically, we conjecture

$$q_{t}^{h} = \alpha_{h}S_{t} + S_{t}^{\prime}A_{h}S_{t} + \eta_{h}x_{t}^{D} + \phi_{h}S_{t}x_{t}^{D} + \mu_{h}\left(x_{t}^{D}\right)^{2} \quad h = D, A$$
(26)

$$c_{yt}^{h} = \alpha_{y,h}S_{t} + S_{t}'A_{y,h}S_{t} + \eta_{y,h}x_{t}^{D} + \phi_{y,h}S_{t}x_{t}^{D} + \mu_{y,h}\left(x_{t}^{D}\right)^{2} \quad h = D, A \quad (27)$$

$$k^{h} = \alpha_{t,t}S_{t} + S_{t}'A_{t,t}S_{t} + \eta_{t,h}x_{t}^{D} + \phi_{t,t}S_{t}x_{t}^{D} + \mu_{t,t}\left(x_{t}^{D}\right)^{2} \quad h = D, A \quad (27)$$

$$\kappa_{t+1} = \alpha_{k,h}S_t + S_tA_{k,h}S_t + \eta_{k,h}x_t + \phi_{k,h}S_tx_t + \mu_{k,h}(x_t) \qquad h = D, A$$

$$z_t^h = \alpha_{z,h}S_t + S_t'A_{z,h}S_t + \eta_{z,h}x_t^D + \phi_{z,h}S_tx_t^D + \mu_{z,h}(x_t^D)^2 \qquad h = D, A \quad (29)$$

These equations also imply that the state space accumulates according to

$$S_{t+1} = N_1 S_t + N_2 \epsilon_{t+1} + N_3 x_t^D + N_4 \left(x_t^D\right)^2 + N_5 S_t x_t^D + \begin{pmatrix} 0 \\ 0 \\ S_t' N_6 S_t \\ S_t' N_7 S_t \end{pmatrix}$$
(30)

where  $\epsilon_t = (\varepsilon_{H,t}, \varepsilon_{F,t})$  and the parameters in the matrices  $N_i$  follow directly from the parameters in (26)-(29).

In addition we will adopt Taylor expansions of the model equations. If we need to impose the first-order component of model equations it is sufficient to use a linear Taylor expansion. Quadratic and cubic Taylor expansions are sufficient when computing respectively second and third-order components of model equations. Model equations can then be written in the form of polynomials in  $S_t$ ,  $x_t^D$ ,  $x_{t+1}^D$  and  $\epsilon_{t+1}$ . Using the signal extraction results, which gives the conditional distribution of  $\epsilon_{t+1}$ , we can then compute expectations. After that we impose the various order components of the equations, following the two-step method described above. This gives us the zero and first-order components of the parameters  $\alpha$  and  $\eta$  (with various subscripts) in (26)-(29) and the zero-order component of all the other parameters.

A final step is needed to compute the parameter  $\lambda$  in the expression for  $x_t^D$  in (23). This step is specific to the presence of information dispersion in the model. The parameter  $\lambda$  captures the noise to signal ratio in the equilibrium

relative asset price. In NRE models this parameter is computed by imposing asset market equilibrium. Here we do the same, but we need to be careful about the order components of equations. The first-order component of  $z_t^A$  is determined from the first step of the solution method from the first-order component of the "other equations". This captures the share of Home equity in the world equity market from a supply perspective. We need to equate this to  $z_t^A(1)$  from the demand or portfolio choice perspective. This is done by using the third-order component of the average of the Euler equations for portfolio choice, which leads to an expression discussed in the next section. Leaving the algebraic details to the Appendix, equating  $z_t^A(1)$  from the supply side to that from the demand side of the model yields a solution for  $\lambda$ .

### 4 Relative Asset Prices

Before discussing the implications of the model for gross and net capital flows, some comments are in order about the solution for the relative asset price  $q_t^D$ . The first-order solution to the relative asset price is

$$q_t^D(1) = \alpha_D(0)S_t(1) + \eta_D(0)x_t^D(1) = \alpha_{D,1}(0)a_t^D + \alpha_{D,3}(0)k_t^D(1) + \eta_D(0)\varepsilon_{t+1}^D + \eta_D(0)\lambda\tau_t^D(3)/\tau$$
(31)

with all parameters positive. The relative asset price is therefore driven by both observable fundamentals,  $a_t^D$  and  $k_t^D$ , and by unobservables  $\varepsilon_{t+1}^D$  and  $\tau_t^D$ . Both of these unobservables generate a disconnect between asset prices and observed fundamentals that is widely documented.

In the absence of information dispersion the relative asset price would, to the first-order, be entirely determined by the observed fundamentals  $S_t$ . In that case a change in  $\tau_t^D$  still leads to first-order portfolio shifts. However, this would affect the relative asset price only to the third order. This is the standard portfolio balance effect. An asset supply or demand shock affects asset prices through the risk-premium channel. But changes in risk premia correspond to changes in second moments, which are third and higher order.

Another intuitive way to see this is as follows. A rise in  $\tau_t^D$  will lead to a firstorder portfolio shift towards the Home country. In order to clear asset markets a third-order drop in the expected excess return on Home equity is sufficient as this will lead to a first-order drop in demand for Home equity (optimal portfolio shares depend on expected returns divided by the variance of returns). A third-order drop in the expected excess return on Home equity requires only a third-order increase in the Home equity price.

How then is it possible that in the presence of information dispersion a change in  $\tau_t^D$  has a first-order effect on the relative asset price? The answer is that there is an amplification effect due to rational confusion. Agents do not know whether the increase in  $q_t^D$  is due to an increase in  $\tau_t^D$  or  $\varepsilon_{t+1}^D$ . They will give substantial weight to the latter as the relative price is a higher quality signal about  $\varepsilon_{t+1}^D$  than private signals (whose errors have a zero-order variance). The increased expectation about  $\varepsilon_{t+1}^D$  therefore further raises  $q_t^D$ . Order accounting shows that this amplification effect leads to a first-order impact of  $\tau_t^D$  on the relative asset price.<sup>6</sup>

### 5 International Capital Flows

We now turn to a discussion of the implications of the model for gross and net capital flows. We conduct the analysis of capital flows from a portfolio perspective. Analogous to Tille and van Wincoop (2008), we will relate capital flows to time-variation in expected returns and risk, the key elements that drive portfolio choice. A key difference is that information dispersion played no role in Tille and van Wincoop (2008).

#### Portfolio Growth and Portfolio Reallocation

After some straightforward balance of payments accounting presented in Appendix E we can write the first-order components of capital outflows and inflows as:

$$outflows_t = (1 - z_H(0)) s_t^H(1) - [\Delta z_{H,t}(1) - \Delta z_t^p(1)]$$
(32)

$$inflows_{t} = (1 - z_{H}(0)) s_{t}^{F}(1) + [\Delta z_{F,t}(1) - \Delta z_{t}^{p}(1)]$$
(33)

<sup>&</sup>lt;sup>6</sup>One might also wonder why  $\epsilon_{t+1}^{D}$  itself has a first-order effect on the asset price as the weight given to private signals about future productivity innovations is second-order. The reason is again that the relative price is a key coordination mechanism through which the impact of (weak) private information is amplified. While little weight is given to private signals when forming expectations about future productivity innovations, a high (zero-order) weight is given to the relative price that reflects the private information.

where  $s_t^i$  is net national saving in country *i* and  $z_t^p$  is the so-called "passive portfolio share" invested in Home equity. The latter represents the share invested in Home equity when the quantity of equity holdings is held at its steady-state level. It reflects changes in the share invested in Home equity due to relative asset price changes in the absence of any asset trade. Specifically, the passive portfolio share for all investors is  $z_t^p(1) = z_H(0)(1 - z_H(0))q_t^D(1)$ . Saving and capital flows in (32)-(33) are scaled by steady-state wealth.

The first term on the right hand side of (32)-(33) represents portfolio growth, namely the change in outflows and inflows when Home and Foreign saving are invested abroad at the steady state portfolio share  $1 - z_H(0)$ . The last terms on the right hand side of (32)-(33) represent portfolio reallocation. Only changes in portfolio shares in deviation from the passive portfolio share leads to asset trade and therefore capital flows. For example, an increase in the fraction  $z_{F,t}$  that Foreign investors invest in Home equity, leads to capital flows only to the extent that is differs from the passive portfolio share.

The portfolio growth component depends entirely on Home and Foreign saving, which are written as:

$$s_t^H(1) = \alpha_H \Delta S_t(1) - 0.5 z^D(0) \,\Delta q_t^D \tag{34}$$

$$s_t^F(1) = \alpha_F \Delta S_t(1) + 0.5 z^D(0) \,\Delta q_t^D \tag{35}$$

where  $\alpha_H$  and  $\alpha_F$  are zero-order vectors. Home and Foreign saving depend both on changes in observed state variables  $S_t(1)$  and changes in relative asset prices. The latter represent a wealth effect that affects the consumption of the old generation. When the relative price of Home equity rises, the old generation in the Home country will be relatively wealthy and will consume this additional wealth. This lowers Home saving.

#### **Optimal Portfolio Shares**

In order to shed light on the portfolio reallocation component of capital flows we need expressions for optimal portfolio shares. It will be useful to write portfolio shares in terms of the average and difference in portfolio shares across countries:  $z_{H,t} = z_t^A + 0.5z_t^D$ ,  $z_{F,t} = z_t^A - 0.5z_t^D$ . Starting with the zero-order portfolio shares, the clearing of asset markets requires that the average share be equal to the relative size of asset markets:  $z^A(0) = 0.5$ . The difference in portfolio shares, computed from the second-order component of the difference in portfolio Euler equations, is driven by the mean level  $\tau$  of international financial frictions:

$$z^{D}(0) = \frac{2\tau}{\gamma[var_{t}(er_{t+1})](2)}$$
(36)

We obtain expressions for the first-order component of the average and difference in optimal portfolio shares from the third-order component of respectively the average and difference in portfolio Euler equations:

$$z_t^A(1) = \frac{\tau_t^D(3)}{\gamma[var_t(er_{t+1})](2)} + (1-\gamma)\frac{[var_t(r_{H,t+1})](3) - [var_t(r_{F,t+1})](3)}{\gamma[var_t(er_{t+1})](2)}$$
(37)

$$+ \frac{1}{2}(1-\gamma)^{2} \frac{\left[\bar{E}_{t}\left(r_{t+1}^{A}\right)^{2}er_{t+1}\right](3)}{\gamma\left[var_{t}(er_{t+1}](2)} + \frac{0.5\left(\left[\bar{E}_{H,t}er_{t+1}\right](3) + \left[\bar{E}_{F,t}er_{t+1}\right](3)\right)}{\gamma\left[var_{t}(er_{t+1})\right](2)} \right]$$

$$z_{t}^{D}(1) = \frac{2\tau_{t}^{A}(3)}{\gamma\left[var_{t}(er_{t+1})\right](2)} - \frac{1}{2}z^{D}(0)\frac{\left[var_{t}(er_{t+1})\right](3)}{\left[var_{t}(er_{t+1})\right](2)} + \frac{\left[\bar{E}_{H,t}er_{t+1}\right](3) - \left[\bar{E}_{F,t}er_{t+1}\right](3)}{\gamma\left[var_{t}(er_{t+1})\right](2)}$$

$$+ \frac{\left[\bar{E}_{H,t}er_{t+1}\right](3) - \left[\bar{E}_{F,t}er_{t+1}\right](3)}{\gamma\left[var_{t}(er_{t+1})\right](2)}$$

$$(38)$$

The first-order component  $z_t^A(1)$  is driven by four intuitive elements. First, a rise in  $\tau_t^D(3)$  leads to a portfolio shift towards Home equity as the cost of investment abroad rises for Home relative to Foreign investors. Second, a rise in the variance of the Home return relative to that of the Foreign equity return leads to a shift towards Foreign equity. As discussed in detail in Tille and van Wincoop (2008), changes in second moments are captured by the third-order component of these moments. Third, when the excess return on Home equity is expected to be high during periods of high global volatility  $((r_{t+1}^A)^2 \text{ high})$ , Home equity is a good hedge against such global risks and there is a shift towards Home equity. Finally, a higher average expected excess return on Home equity leads to a portfolio shift towards Home equity.

The second and third terms capture time-varying second moments. The first two steps of the solution imply that

$$[var_t(r_{H,t+1})](3) - [var_t(r_{F,t+1})](3) = \psi_1(x_t^D)^3 + \sigma_a^2 \psi_2 x_t^D + \sigma_a^2 \psi_3 S_t(1)$$
  
$$[\bar{E}_t (r_{t+1}^A)^2 er_{t+1}](3) = \psi_4(x_t^D)^3 + \sigma_a^2 \psi_5 x_t^D$$

where the parameters  $\psi_i$  are zero-order coefficients. Time-variation in second moments is therefore associated both with changes in observed state variables and unobserved state variables. The latter is specifically related to information dispersion in the model. The expression (38) for the difference  $z_t^D(1)$  in portfolio shares captures timevariation in portfolio home bias. It is driven by three factors. First, an increase in the average financial friction  $\tau_t^A$  leads to increased home bias. Second, an increase in the variance of the excess return leads to decreased home bias. Intuitively, an increase in the variance of the excess return leads to an increased incentive for diversification. This reduces home bias relative to its zero-order component (36). Finally, an increase in the expected excess return on Home equity by Home investors relative to Foreign investors will lead to increased home bias.

Using the results from the first two steps of the solution method we get:

$$[var_t(er_{t+1})](3) = \delta_1(x_t^D)^3 + \sigma_a^2 \delta_2 x_t^D + \sigma_a^2 \delta_3 S_t(1)$$
(39)

$$[\bar{E}_{H,t}er_{t+1}](3) - [\bar{E}_{F,t}er_{t+1}](3) = \delta_4 \sigma_a^2 \left[\frac{1}{\sigma_{HH}^2} - \frac{1}{\sigma_{HF}^2}\right] \varepsilon_{t+1}^A \tag{40}$$

where the parameters  $\delta_i$  are zero-order and follow from the first and second-order solutions of the "other variables". (39) implies that changes in the variance of the excess return over time are driven by both changes in observed and unobserved state variables. The latter again reflects the role of information dispersion. In order to give some intuition behind (40), assume that  $\sigma_{HH}^2 < \sigma_{HF}^2$ , so that agents have better quality signals about their domestic equity market. When productivity levels rise in both countries next period, agents from both countries will expect that productivity in their own country will rise more than that of the foreign country. This is because they have better quality information that their own productivity will rise. As a result they both expect the return on their own country's equity to rise relative to that of the other country, which leads to increased portfolio home bias ( $\delta_4 > 0$ ).

#### Equilibrium Expected Excess Returns

The first-order component of portfolio shares depends on expected returns and time-varying second moments. We have already provided expressions for the timevarying second moments and the difference across countries in expected returns. In order to complete the analysis of the determinants of capital flows we need to compute the determinants of average expected excess returns in the equilibrium of the model. The cross-country difference of the asset market clearing conditions implies

$$\Delta z_t^A(1) - \Delta z_t^p(1) = \frac{1}{4} \left[ i_t^D(1) - z^D(0) s_t^D(1) \right]$$
(41)

Intuitively, the average share invested in Home equity rises when the relative supply of Home equity goes up. This can take place either through an increase in the relative price of home equity, as reflected in  $\Delta z_t^p(1)$ , or through in increase in the relative size of the Home capital stock due to the investment differential  $i_t^D(1)$ . Finally, in the presence of portfolio home bias, an increase in Home saving relative to Foreign saving leads to an excess demand for Home equity when invested at steady state portfolio shares. A decrease in the share invested in Home equity is then necessary to clear equity markets.

Cross-country differences in saving and investment are equal to

$$s_t^D(1) = \Delta a_t^D(1) + (1 - \omega)\Delta k_t^D(1) - z^D(0)\Delta q_t^D(1)$$
(42)

$$i_t^D(1) = \frac{1}{\xi} q_t^D(1)$$
(43)

Relative asset prices affect relative saving through a wealth effect and relative investment through a standard Tobin's Q equation.

Substituting the optimal average portfolio share (37) into (41) gives an expression for changes in the average expected excess return  $\bar{E}_t er_{t+1} = 0.5(\bar{E}_t^H er_{t+1} + \bar{E}_t^F er_{t+1})$ :

$$\Delta \bar{E}_t er_{t+1}(3) = (\Delta \bar{E}_t er_{t+1}](3))^{\tau} + (\Delta [\bar{E}_t er_{t+1}](3))^{TVM} + (\Delta \bar{E}_t er_{t+1}(3))^{P} + (\Delta \bar{E}_t er_{t+1}(3))^{IS}$$
(44)

where

$$\begin{aligned} \Delta \bar{E}_t er_{t+1}(3)^{\tau} &= -\Delta \tau_t^D(3) \\ \Delta \bar{E}_t er_{t+1}(3)^{TVM} &= -\gamma \left( [var_t(r_{H,t+1})](3) - [var_t(r_{F,t+1})](3) \right) \\ &- \gamma \frac{(1-\gamma)^2}{2} \bar{E}_t \left( r_{t+1}^A \right)^2 er_{t+1} ](3) \\ \Delta \bar{E}_t er_{t+1}(3)^P &= \gamma [var_t(er_{t+1})](2) \Delta z_t^P(1) \\ \Delta \bar{E}_t er_{t+1}(3)^{IS} &= \frac{\gamma [var_t(er_{t+1})](2)}{4} \left[ i_t^D(1) - z^D(0) s_t^D(1) \right] \end{aligned}$$

Four factors drive changes in the equilibrium expected excess return. First, an increase in the relative friction  $\tau_t^D$  leads to a portfolio shift to Home equity. A drop in the expected excess return on Home equity is then needed to clear asset markets. Second, a change in second moments that increases the optimal average portfolio share invested in Home equity leads to a drop in the expected excess

return on Home equity in order to clear asset markets. Third, an increase in the relative price of Home equity raises the relative supply of Home equity. A higher expected excess return on Home equity is needed to induce agents to hold the higher passive portfolio share. The last term depends on relative investment and saving across countries. A rise in relative investment raises the relative supply of Home equity. A higher expected excess return on Home equity is then needed to clear asset markets. When Home saving is large relative to Foreign saving there will be an excess demand for Home equity due to portfolio home bias. A lower expected excess return on Home equity is then needed to clear asset markets.

#### **Determinants of Capital Flows**

We are now in a position to derive the determinants of capital flows. As a first step it is useful to substitute (44) into (37), which yields

$$\Delta z_t^A(1) - \Delta z_t^p(1) = \frac{\Delta \bar{E}_t er_{t+1}(3)^{IS}}{\gamma [var(er_{t+1})](2)}$$
(45)

Average portfolio reallocation towards Home equity is driven by only one element: the component of changes in the expected excess return due to relative saving and investment. The other components of the expected excess return have no effect on capital flows. Moreover, time-varying second moments do not affect average portfolio reallocation. This can be understood as follows. An increase in  $\tau_t^D$ leads to a portfolio shift towards Home equity, but this is offset by a decrease in the expected excess return on Home equity to clear asset markets. Time-varying second moments that lead to an increase in average portfolio demand for Home equity are also exactly offset by a corresponding decrease in the expected excess return. The third component of changes in expected excess return captures the need to induce agents to hold the passive portfolio when the relative asset price changes. This again does not lead to active portfolio reallocation as the passive portfolio involves no asset trade. Substituting (45) into (32)-(33), we have

$$outflows_{t} = (1 - z_{H}(0)) s_{t}^{H}(1) - \frac{\left(\Delta \left[\bar{E}_{t}er_{t+1}\right](3)\right)^{IS}}{\gamma [var_{t}(er_{t+1})](2)} - \frac{\Delta \tau_{t}^{A}(3)}{\gamma [var_{t}(er_{t+1})](2)} \\ + \frac{z^{D}(0)}{4} \frac{\Delta [var_{t}(er_{t+1})](3)}{[var_{t}(er_{t+1})](2)}$$

$$(46)$$

$$-\frac{1}{2} \frac{\Delta [\bar{E}_{H,t}er_{t+1}](3) - \Delta [\bar{E}_{F,t}er_{t+1}](3)}{\gamma (var_{t}(er_{t+1})](2)} \\ (46)$$

$$inflows_{t} = (1 - z_{H}(0)) s_{t}^{F}(1) + \frac{\left(\Delta \left[\bar{E}_{t}er_{t+1}\right](3)\right)^{IS}}{\gamma [var_{t}(er_{t+1})](2)} - \frac{\Delta \tau_{t}^{A}(3)}{\gamma [var_{t}(er_{t+1})](2)} \\ + \frac{z^{D}(0)}{4} \frac{\Delta [var_{t}(er_{t+1})](3)}{[var_{t}(er_{t+1})](2)}$$

$$(47)$$

$$-\frac{1}{2} \frac{\Delta [\bar{E}_{H,t}er_{t+1}](3) - \Delta [\bar{E}_{F,t}er_{t+1}](3)}{\gamma (var_{t}(er_{t+1})](2)}$$

Together with the explicit expressions provided earlier in this section for the terms in the numerator of these components, this gives a solution for capital inflows and outflows.

Capital outflows and inflows are broken into five intuitive components. The first is associated with portfolio growth. The remaining terms are due to portfolio reallocation. The second term is a result of changes in the average expected excess return. As discussed above, only changes in the expected excess return due to differences across countries in saving and investment lead to capital flows. The last three terms are associated with portfolio reallocation due to changes  $\Delta z_t^D(1)$  in portfolio home bias. An increase in the average international financial friction  $\tau_t^A$  leads to increased home bias, resulting in a drop in both capital inflows and outflows. The fourth term reflects changes in portfolio home bias due to changes in the variance of the excess return. An increase in the variance of the excess return makes portfolio diversification more attractive and therefore leads to increased capital inflows. The last term reflects differences across countries in the expectation of the excess return. When investors from both countries become more optimistic about the expected excess return on their domestic equity, capital outflows and inflows will both drop.

#### The role of information dispersion

Information dispersion affects capital flows in various ways. First, we have already seen that only in the presence of information dispersion do changes in the unobservables  $\tau_t^D$  and  $\varepsilon_{t+1}^D$  have a first-order impact on relative asset prices,  $q_t^D(1)$ . This takes placed through the unobserved state variable  $x_t^D$ . The impact of these unobservables on relative asset prices is transmitted to saving (34)-(35) and relative investment (43). This in turn affects capital flows through the portfolio growth component and the average expected excess return component.

Second, the unobservables affect the variance of the excess return (39) only in the presence of information dispersion. This affects the fourth component of capital flows in (46)-(47). Finally, information dispersion leads to differences in expected excess returns (40) across investors in different countries that affect the last component of capital flows (46)-(47). As shown in (40), this depends on the unobserved average future productivity  $\varepsilon_{t+1}^A$ .

To summarize, as a result of information dispersion a variety of unobserved macro fundamentals affect both gross and net capital flows. This takes place through a various channels that are shut down in the absence of information dispersion.

### 6 Empirical Implications

The results for capital flows in the previous section lead to a number of implications that can be brought to the data. We will first discuss the implications for net capital flows and then move on to gross capital flows.

#### Net Capital Flows

Taking the difference between (46) and (47), we have

$$outflows - inflows = (1 - z_H(0)) \left( s_t^H(1) - s_t^F(1) \right) - 2 \frac{\Delta \bar{E}_t er_{t+1}(3)^{IS}}{\gamma [var(er_{t+1})](2)}$$

Net capital flows are driven by two factors. The first is the difference in portfolio growth associated with capital outflows and inflows, which depends on the difference in saving across the two countries. The second is changes in the average expected excess return that result from differences in the level of saving and investment across countries. Therefore net capital flows are determined by differences in saving and investment across the two countries, both through portfolio growth and through portfolio reallocation. This is not surprising as this is also implied by a standard saving minus investment view of the current account:

$$outflows - inflows = s_t^H(1) - i_t^H(1) = \frac{1}{2} \left[ s_t^D(1) - i_t^D(1) \right]$$

The first implication of this is

#### **Implication 1** Observed macro fundamentals cannot fully explain net capital flows.

We have seen from the (42)-(43) that differences in saving and investment across countries depend on differences in asset prices, which in turn depend on the unobservables  $\tau_t^D$  and  $\varepsilon_{t+1}^D$  through the unobserved state variable  $x_t^D$ . Therefore both observed macro fundamentals, captured by the vector  $S_t$  of observed state variables, and unobserved macro fundamentals, affect net capital flows. As discussed in section 4, information dispersion plays a key role here. In the absence of information dispersion the relative asset price would to the first-order only be driven by observed macro fundamentals. The following implication naturally follows as well:

**Implication 2** Conditional on observed macro fundamentals, both the level and change in the level of relative asset prices have significant explanatory power for net capital flows.

Information dispersion again plays a key role here. In the absence of information dispersion the relative asset price would be driven entirely by observed fundamentals  $S_t$ , so that conditional on these observed fundamentals relative asset prices have no explanatory power for net capital flows.

One of the unobserved fundamentals is relative future productivity  $\varepsilon_{t+1}^D$ . It positively affects the relative price of Home equity. This lowers relative saving (42) and raises relative investment (43), leading to net capital inflows. It follows that

**Implication 3** After conditioning on observed macro fundamentals, net capital flows negatively predict future relative productivity, relative GDP growth and relative profits.

#### Gross Capital Flows

Capital inflows and outflows can always be written as a function of net capital flows and the sum of inflows and outflows. For example, outflows = 0.5(outflows - inflows) + 0.5(outflows + inflows). Since we have already discussed implications for net capital flows, we now turn to the sum of capital inflows and outflows. Taking the sum of (46) and (47), we have

$$outflows_{t} + inflows_{t} = (1 - z_{H}(0)) \left(s_{t}^{H}(1) + s_{t}^{F}(1)\right) - \frac{2\Delta\tau_{t}^{A}(3)}{\gamma[var_{t}(er_{t+1})](2)} + \frac{z^{D}(0)}{2} \frac{\Delta[var_{t}(er_{t+1})](3)}{[var_{t}(er_{t+1})](2)} - \frac{\Delta[\bar{E}_{H,t}er_{t+1}](3) - \Delta[\bar{E}_{F,t}er_{t+1}](3)}{\gamma(var_{t}(er_{t+1})](2)}$$
(48)

The sum of the portfolio growth components depends on the sum of saving across the countries, which from (34)-(35) depends only on observed state variables  $S_t$ . Nonetheless, just like net capital flows, the sum of capital inflows and outflows depends on unobserved macro fundamentals through various channels. First, the unobserved state variable  $x_t^D$  affects the variance of the excess return (39), which affects capital inflows and outflows in the same direction. Second, the difference across the countries in the expected excess return (40) depends positively on the unobserved  $\varepsilon_{t+1}^A$ . Information dispersion is key in driving this disconnect from observed macro fundamentals. Finally, a rise in the unobserved average financial friction  $\tau_t^A$  lowers both capital inflows and outflows. This would be the case though even in the absence of information dispersion. To summarize, analogous to Implication 1 for net capital flows, we have

**Implication 4** Observed macro fundamentals cannot fully explain the sum of capital inflows and outflows.

We also have an analogous result to Implication 2 for net capital flows:

**Implication 5** Conditional on observed macro fundamentals, relative asset prices have explanatory power for the sum of capital inflows and outflows.

The only channel through which this last implication holds is time-variation in the variance of the excess return. We know from (39) that the variance of the excess return depends on  $x_t^D$  and  $S_t$ . In turn the relative asset price  $q_t^D$  is also a function of  $x_t^D$  and  $S_t$ . We can therefore write  $[var(er_{t+1})](3)$  as a function of  $S_t$ and  $q_t^D$ . The sum of capital inflows and outflows also has predictive power for future fundamentals. This happens through two channels. First, the variance of the excess return (39) depends on  $\varepsilon_{t+1}^D$  through  $x_t^D$ . Second, the difference in the expected excess return across countries (40) depends positively on  $\varepsilon_{t+1}^A$ . We therefore have

**Implication 6** After conditioning on observed macro fundamentals, the sum of capital inflows and outflows (i) positively predicts future world productivity, world GDP growth and world profits, (ii) predicts future relative productivity, relative GDP growth and relative profits.

One final implication for gross capital inflows and outflows relates to their correlation. (46)-(47) imply that three factors unambiguously contribute to a positive correlation between capital inflows and outflows: (i) time-variation in average financial frictions  $\tau_t^A(3)$ , (ii) time-variation in the variance of the excess return (39) and (iii) time-variation in the difference of the expected excess return across countries. One element unambiguously leads to a negative co-movement between capital inflows and outflows: time-variation in the average expected excess return due to differences in saving and investment across countries (45). The last determinant of capital flows, portfolio growth, can generate either a positive or negative co-movement between capital inflows and outflows, dependent on the co-movement between saving across countries. This implies:

**Implication 7** Controlling for differences in saving and investment across countries, the model implies a positive co-movement between capital inflows and outflows.

# 7 Empirical Results

# 8 Conclusion

To be written

# Appendix

# A Equations of the model

The various equations of the model can be written in terms of the logs of the various variables, denoted by lower-case letters. We denote the worldwide average of log equity prices by  $q_t^A = 0.5 (q_{H,t} + q_{F,t})$ , and the cross-country difference in log equity prices by  $q_t^D = q_{H,t} - q_{F,t}$ . We define similar variables for the capital stock  $(k_{t+1}^A, k_{t+1}^D)$ , productivity  $(a_t^A, a_t^D)$  and asset returns  $(r_{t+1}^A, r_{t+1}^D = er_{t+1})$ .

The Tobin's Q (5) in Home and Foreign are:

$$e^{k_{t+1}^A + \frac{1}{2}k_{t+1}^D} = \left(1 + \frac{e^{q_t^A + \frac{1}{2}q_t^D} - 1}{\xi}\right)e^{k_t^A + \frac{1}{2}k_t^D}$$
(49)

$$e^{k_{t+1}^A - \frac{1}{2}k_{t+1}^D} = \left(1 + \frac{e^{q_t^A - \frac{1}{2}q_t^D} - 1}{\xi}\right)e^{k_t^A - \frac{1}{2}k_t^D}$$
(50)

The consumption Euler equations (15) and (17) are:

$$\left(\omega e^{a_t^A + \frac{1}{2}a_t^D + (1-\omega)\left(k_t^A + \frac{1}{2}k_t^D\right) - c_{yt}^{H_j}} - 1\right)^{\gamma} = \beta E_t^{H_j} e^{(1-\gamma)r_{t+1}^{p,H_j}}$$
(51)

$$\left(\omega e^{a_t^A - \frac{1}{2}a_t^D + (1-\omega)\left(k_t^A - \frac{1}{2}k_t^D\right) - c_{yt}^{Fj}} - 1\right)^{\gamma} = \beta E_t^{Fj} e^{(1-\gamma)r_{t+1}^{p,Fj}}$$
(52)

The portfolio Euler equations for individual investors (16) and (18) are:

$$0 = E_t^{Hj} \left( e^{-\gamma r_{t+1}^{p,Hj} + r_{t+1}^A + \frac{1}{2}er_{t+1}} - e^{-\gamma r_{t+1}^{p,Hj} - \tau_{Hj,t} + r_{t+1}^A - \frac{1}{2}er_{t+1}} \right)$$
(53)

$$0 = E_t^{Fj} \left( e^{-\gamma r_{t+1}^{p,Fj} - \tau_{Fj,t} + r_{t+1}^A + \frac{1}{2}er_{t+1}} - e^{-\gamma r_{t+1}^{p,Fj} + r_{t+1}^A - \frac{1}{2}er_{t+1}} \right)$$
(54)

The asset market clearing conditions (19)-(20) are:

$$e^{k_{t+1}^{A} + \frac{1}{2}k_{t+1}^{D} + q_{t}^{A} + \frac{1}{2}q_{t}^{D}} = \int \left( \omega e^{a_{t}^{A} + \frac{1}{2}a_{t}^{D} + (1-\omega)\left(k_{t}^{A} + \frac{1}{2}k_{t}^{D}\right)} - e^{c_{yt}^{Hj}} \right) z_{Hj,t} dj \qquad (55)$$

$$+ \int \left( \omega e^{a_{t}^{A} - \frac{1}{2}a_{t}^{D} + (1-\omega)\left(k_{t}^{A} - \frac{1}{2}k_{t}^{D}\right)} - e^{c_{yt}^{Fj}} \right) z_{Fj,t} dj$$

$$+ \int \left( \omega e^{a_{t}^{A} + \frac{1}{2}a_{t}^{D} + (1-\omega)\left(k_{t}^{A} + \frac{1}{2}k_{t}^{D}\right)} - e^{c_{yt}^{Hj}} \right) (1 - z_{Hj,t}) dj \qquad (56)$$

$$+ \int \left( \omega e^{a_{t}^{A} - \frac{1}{2}a_{t}^{D} + (1-\omega)\left(k_{t}^{A} - \frac{1}{2}k_{t}^{D}\right)} - e^{c_{yt}^{Fj}} \right) (1 - z_{Fj,t}) dj$$

The rates of returns on Home and Foreign equity (6)-(7) are given by:

$$e^{r_{t+1}^{A} + \frac{1}{2}er_{t+1}} = (1 - \omega) e^{a_{t+1}^{A} + \frac{1}{2}a_{t+1}^{D} - \omega\left(k_{t+1}^{A} + \frac{1}{2}k_{t+1}^{D}\right) - q_{t}^{A} - \frac{1}{2}q_{t}^{D}} + (1 - \delta) e^{q_{t+1}^{A} + \frac{1}{2}q_{t+1}^{D} - q_{t}^{A} - \frac{1}{2}q_{t}^{D}}$$
(57)

$$e^{r_{t+1}^{A} - \frac{1}{2}er_{t+1}} = (1 - \omega) e^{a_{t+1}^{A} - \frac{1}{2}a_{t+1}^{D} - \omega\left(k_{t+1}^{A} - \frac{1}{2}k_{t+1}^{D}\right) - q_{t}^{A} + \frac{1}{2}q_{t}^{D}} + (1 - \delta) e^{q_{t+1}^{A} - \frac{1}{2}q_{t+1}^{D} - q_{t}^{A} + \frac{1}{2}q_{t}^{D}}$$
(58)

The portfolio returns of individual investors (8)-(9) are:

$$e^{r_{t+1}^{p,H_j}} = z_{Hj,t}e^{r_{t+1}^A + \frac{1}{2}er_{t+1}} + (1 - z_{Hj,t})e^{r_{t+1}^A - \frac{1}{2}er_{t+1} - \tau_{Hj,t}}$$
(59)

$$e^{r_{t+1}^{p,Fj}} = z_{Fj,t}e^{r_{t+1}^A + \frac{1}{2}er_{t+1} - \tau_{Fj,t}} + (1 - z_{Fj,t})e^{r_{t+1}^A - \frac{1}{2}er_{t+1}}$$
(60)

The zero order components of the variables are:

$$a(0) = q(0) = 0$$
  

$$e^{r(0)} = (1-\omega)e^{-\omega k(0)} + (1-\delta)$$
  

$$e^{c_y(0)} = \omega e^{(1-\omega)k(0)} - e^{k(0)}$$

where k(0) solves:

$$(\omega e^{-\omega k(0)} - 1)^{-\gamma} = \beta [(1 - \omega) e^{-\omega k(0)} + (1 - \delta)]^{1-\gamma}$$

The ratio of young consumption to the wage is:

$$\bar{c} = \frac{1}{\omega} e^{c_y(0) - (1-\omega)k(0)} = 1 - \frac{e^{\omega k(0)}}{\omega}$$

The average portfolio share is computed from the asset market clearing (55):

$$z^{A}(0) = \frac{z_{H}(0) + z_{F}(0)}{2} = \frac{1}{2}$$

### **B** Signal extraction

#### General approach

We focus on the signal extraction of a Home investors. The inferences of a Foreign investors are computed along similar lines.

A Home investor observes the component  $x_t^D$  of the equity price differential that is not attributable to the state variables, as well as her private signals on Home and Foreign future productivity shocks,  $v_{j,t}^{H,H}$  and  $v_{j,t}^{H,F}$ . From (62),  $x_t^D$  only has a first order component. The Home investor infers the Home and Foreign future productivity shocks,  $\varepsilon_{t+1}^H$  and  $\varepsilon_{t+1}^F$  from these signals.

The signal extraction problem therefore consists of inferring a vector  $\xi_{t+1} = [\varepsilon_{H,t+1}, \varepsilon_{F,t+1}]'$  conditional on a vector of signals  $Y_t^{jh} = \left[x_t^D, v_{j,t}^{H,H}, v_{j,t}^{H,F}\right]'$  which are linked as follows:

$$Y_t^{jh} = H^{h\prime}\xi_{t+1} + w_t^{jh}$$

where  $w_t^{jh} = \left[\lambda \frac{\tau_t^D}{\tau}, \epsilon_{j,t}^{H,H}, \epsilon_{j,t}^{H,F}\right]'$  are shocks and  $H^{h'}$  is a 3 by 2 matrix:

$$H^{h\prime} = \begin{vmatrix} 1 & -1 \\ 1 & 0 \\ 0 & 1 \end{vmatrix}$$

The variances of the productivity and the signals are:

$$\tilde{P} = var_t \left(\xi_{t+1}\right) = \begin{vmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_a^2 \end{vmatrix} \qquad R^h = var_t \left(w_t^{jh}\right) = \begin{vmatrix} 2\lambda^2 \theta \sigma_a^2 & 0 & 0 \\ 0 & \sigma_{H,H}^2 & 0 \\ 0 & 0 & \sigma_{H,F}^2 \end{vmatrix}$$

Based on her information, the Home agent's assessment of the expected productivity shocks and their variance are:

$$E_t^{jh}\left(\xi_{t+1}\right) = M^h Y_t^{jh} \qquad \qquad Var_t^{jh}\left(\xi_{t+1}\right) = \tilde{P} - M^h H^{h\prime} \tilde{P}$$

where  $M^h$  is a 2 by 3 matrix:

$$M^{h} = \tilde{P}H^{h} \left[ H^{h\prime} \tilde{P}H^{h} + R^{h} \right]^{-1}$$

#### Expected productivity shocks

The expected values of future Home and Foreign productivities are:

$$E_t^{jh} \varepsilon_{H,t+1} = \alpha_x^{h,H} x_t^D + \alpha_{vH}^{h,H} v_{j,t}^{H,H} + \alpha_{vF}^{h,H} v_{j,t}^{H,F}$$
$$E_t^{jh} \varepsilon_{F,t+1} = \alpha_x^{h,F} x_t^D + \alpha_{vH}^{h,F} v_{j,t}^{H,H} + \alpha_{vF}^{h,F} v_{j,t}^{H,F}$$

where:

$$\begin{split} \alpha_{x}^{h,H} &= \frac{\sigma_{H,H}^{2} \left[ \sigma_{a}^{2} + \sigma_{H,F}^{2} \right]}{V} \\ \alpha_{vH}^{h,H} &= \sigma_{a}^{2} \frac{2\lambda^{2}\theta \left[ \sigma_{a}^{2} + \sigma_{H,F}^{2} \right] + \sigma_{H,F}^{2}}{V} \\ \alpha_{vF}^{h,H} &= \frac{\sigma_{a}^{2}\sigma_{H,H}^{2}}{V} \\ \alpha_{x}^{h,F} &= -\frac{\sigma_{H,F}^{2} \left[ \sigma_{a}^{2} + \sigma_{H,H}^{2} \right]}{V} \\ \alpha_{vH}^{h,F} &= \frac{\sigma_{a}^{2}\sigma_{H,F}^{2}}{V} \\ \alpha_{vF}^{h,F} &= \sigma_{a}^{2} \frac{2\lambda^{2}\theta \left[ \sigma_{a}^{2} + \sigma_{H,H}^{2} \right] + \sigma_{H,H}^{2}}{V} \\ V &= 2 \left[ 1 + \lambda^{2}\theta \right] \left[ \sigma_{a}^{2} + \sigma_{H,H}^{2} \right] \left[ \sigma_{a}^{2} + \sigma_{H,F}^{2} \right] \\ -\sigma_{a}^{2} \left[ \sigma_{a}^{2} + \sigma_{H,H}^{2} \right] - \sigma_{a}^{2} \left[ \sigma_{a}^{2} + \sigma_{H,F}^{2} \right] \end{split}$$

While these coefficients are complex functions, we can distinguish between their various orders. We consider components up to order two. The coefficients on  $x_t^D$   $(\alpha_x^{h,H} \text{ and } \alpha_x^{h,F})$  only have components of order zero and two:

$$\begin{aligned} \alpha_{x}^{h,H}\left(0\right) &= -\alpha_{x}^{h,F}\left(0\right) = \frac{1}{2\left[1+\lambda^{2}\theta\right]} \\ \alpha_{x}^{h,H}\left(2\right) &= \frac{\sigma_{H,H}^{2} - \left[1+2\lambda^{2}\theta\right]\sigma_{H,F}^{2}}{4\left[1+\lambda^{2}\theta\right]^{2}\sigma_{H,H}^{2}\sigma_{H,F}^{2}}\sigma_{a}^{2} \\ \alpha_{x}^{h,F}\left(2\right) &= \frac{\left[1+2\lambda^{2}\theta\right]\sigma_{H,H}^{2} - \sigma_{H,F}^{2}}{4\left[1+\lambda^{2}\theta\right]^{2}\sigma_{H,H}^{2}\sigma_{H,F}^{2}}\sigma_{a}^{2} \end{aligned}$$

The coefficients on the private signals only have components of order two:

$$\alpha_{vH}^{h,H}(2) = \frac{1+2\lambda^{2}\theta}{2\left[1+\lambda^{2}\theta\right]\sigma_{H,H}^{2}}\sigma_{a}^{2} \quad , \quad \alpha_{vF}^{h,H}(2) = \frac{1}{2\left[1+\lambda^{2}\theta\right]\sigma_{H,F}^{2}}\sigma_{a}^{2}$$

$$\alpha_{vH}^{h,F}(2) = \frac{1}{2\left[1+\lambda^{2}\theta\right]\sigma_{H,H}^{2}}\sigma_{a}^{2} \quad , \quad \alpha_{vF}^{h,F}(2) = \frac{1+2\lambda^{2}\theta}{2\left[1+\lambda^{2}\theta\right]\sigma_{H,F}^{2}}\sigma_{a}^{2}$$

The various orders of the Home agent's expectations of future Home produc-

tivity are then:

$$\begin{bmatrix} E_{t}^{jh} \varepsilon_{H,t+1} \end{bmatrix} (1) = - \begin{bmatrix} E_{t}^{ih} \varepsilon_{F,t+1} \end{bmatrix} (1) = \alpha_{x}^{h,H} (0) x_{t}^{D} (1)$$

$$\begin{bmatrix} E_{t}^{jh} \varepsilon_{H,t+1} \end{bmatrix} (2) = \alpha_{vH}^{h,H} (2) v_{j,t}^{H,H} (0) + \alpha_{vF}^{h,H} (2) v_{j,t}^{H,F} (0)$$

$$= \alpha_{vH}^{h,H} (2) \epsilon_{j,t}^{H,H} + \alpha_{vF}^{h,H} (2) \epsilon_{j,t}^{H,F}$$

$$\begin{bmatrix} E_{t}^{jh} \varepsilon_{F,t+1} \end{bmatrix} (2) = \alpha_{vH}^{h,F} (2) \epsilon_{j,t}^{H,H} + \alpha_{vF}^{h,F} (2) \epsilon_{j,t}^{H,F}$$

A useful result if the third-order component of the expected productivity difference:

$$\begin{bmatrix} E_{t}^{jh} \left(\varepsilon_{H,t+1} - \varepsilon_{F,t+1}\right) \end{bmatrix} (3)$$

$$= \left[ \alpha_{x}^{h,H} \left(2\right) - \alpha_{x}^{h,F} \left(2\right) \right] x_{t}^{D} \left(1\right) + \left[ \alpha_{vH}^{h,H} \left(2\right) - \alpha_{vH}^{h,F} \left(2\right) \right] v_{j,t}^{H,H} \left(1\right) + \left[ \alpha_{vF}^{h,H} \left(2\right) - \alpha_{vF}^{h,F} \left(2\right) \right] v_{j,t}^{H,F} \left(1\right)$$

$$= \frac{-\lambda^{2} \theta \sigma_{a}^{2}}{2 \left[ 1 + \lambda^{2} \theta \right]^{2}} \frac{\sigma_{H,H}^{2} + \sigma_{H,F}^{2}}{\sigma_{H,H}^{2} \sigma_{H,F}^{2}} x_{t}^{D} \left(1\right) + \frac{\lambda^{2} \theta \sigma_{a}^{2}}{1 + \lambda^{2} \theta} \left( \frac{\varepsilon_{H,t+1}}{\sigma_{H,H}^{2}} - \frac{\varepsilon_{F,t+1}}{\sigma_{H,F}^{2}} \right)$$

$$\tag{61}$$

### Variance of productivity shocks

The Home agent also infers the variances and covariances of the productivities shocks:

$$\begin{aligned} Var_t^{jh}\left(\varepsilon_{H,t+1}\right) &= \frac{\sigma_a^2 \sigma_{H,H}^2}{V} \left[ 2\lambda^2 \theta \left[ \sigma_a^2 + \sigma_{H,F}^2 \right] + \sigma_{H,F}^2 \right] \\ Var_t^{jh}\left(\varepsilon_{F,t+1}\right) &= \frac{\sigma_a^2 \sigma_{H,F}^2}{V} \left[ 2\lambda^2 \theta \left[ \sigma_a^2 + \sigma_{H,H}^2 \right] + \sigma_{H,H}^2 \right] \\ Covar_t^{jh}\left(\varepsilon\varepsilon_{H,t+1},\varepsilon_{F,t+1}\right) &= \frac{\sigma_a^2 \sigma_{H,H}^2 \sigma_{H,F}^2}{V} \end{aligned}$$

These terms only have second-order components:

$$Var_{t}^{jh}\left(\varepsilon_{H,t+1}\right)\left(2\right) = Var_{t}^{h}\left(\varepsilon_{F,t+1}\right)\left(2\right) = \frac{1+2\lambda^{2}\theta}{2\left[1+\lambda^{2}\theta\right]}\sigma_{a}^{2}$$
$$Covar_{t}^{jh}\left(\varepsilon_{H,t+1},\varepsilon_{F,t+1}\right)\left(2\right) = \frac{1}{2\left[1+\lambda^{2}\theta\right]}\sigma_{a}^{2}$$

The expected values of squared and cubic shocks are computed as:

$$E_{t}^{jh} (\varepsilon_{H,t+1})^{2} = \left(E_{t}^{jh} \varepsilon_{H,t+1}\right)^{2} + Var_{t}^{jh} (\varepsilon_{H,t+1})$$

$$E_{t}^{jh} (\varepsilon_{H,t+1})^{3} = \left(E_{t}^{jh} \varepsilon_{H,t+1}\right)^{3} + 3\left(E_{t}^{jh} \varepsilon_{H,t+1}\right)\left(Var_{t}^{jh} (\varepsilon_{H,t+1})\right)$$

$$E_{t}^{jh} (\varepsilon_{F,t+1})^{2} = \left(E_{t}^{jh} \varepsilon_{F,t+1}\right)^{2} + Var_{t}^{jh} (\varepsilon_{F,t+1})$$

$$E_{t}^{jh} (\varepsilon_{F,t+1})^{3} = \left(E_{t}^{jh} \varepsilon_{F,t+1}\right)^{3} + 3\left(E_{t}^{jh} \varepsilon_{F,t+1}\right)\left(Var_{t}^{jh} (\varepsilon_{F,t+1})\right)$$

# C First order solution

To a first order, the variables are linear functions of the state space:

$$q_{t}^{D}(1) = \alpha(0) S_{t}(1) + \alpha_{5}(0) x_{t}^{D}(1)$$

$$c_{yt}^{A}(1) = \alpha_{y}(0) S_{t}(1) + \alpha_{5y}(0) x_{t}^{D}(1)$$

$$c_{yt}^{D}(1) = \alpha_{yD}(0) S_{t}(1) + \alpha_{5,yD}(0) x_{t}^{D}(1)$$

$$q_{t}^{A}(1) = \alpha_{qA}(0) S_{t}(1) + \alpha_{5,qA}(0) x_{t}^{D}(1)$$

$$k_{t+1}^{A}(1) = \alpha_{kA}(0) S_{t}(1) + \alpha_{5,kA}(0) x_{t}^{D}(1)$$

$$k_{t+1}^{D}(1) = \alpha_{kD}(0) S_{t}(1) + \alpha_{5,kD}(0) x_{t}^{D}(1)$$

$$z_{t+1}^{A}(1) = \alpha_{zA}(0) S_{t}(1) + \alpha_{5,zA}(0) x_{t}^{D}(1)$$

where:

$$S_{t}(1) = [a_{t}^{D}(1), a_{t}^{A}(1), k_{t}^{D}(1), k_{t}^{A}(1)]'$$
  

$$x_{t}^{D}(1) = \varepsilon_{t+1}^{D} + \lambda (\tau_{t}^{D}/\tau)$$
(62)

#### Worldwide averages

The solution in terms for the worldwide averages of consumption, equity prices and capital dynamics is computed by taking first-order expansions of the equations (49)-(60), and take worldwide averages of the relations for the Home and Foreign country. The complete solution is given by:

$$c_{yt}^{A}(1) = \lambda_1 a_t^{A}(1) + \lambda_2 k_t^{A}(1)$$
 (63)

$$q_t^A(1) = \frac{\xi}{1+\xi} \frac{1-\bar{c}\lambda_1}{1-\bar{c}} a_t^A(1) - \frac{\xi}{1+\xi} \frac{\bar{c}(\lambda_2-1)+\omega}{1-\bar{c}} k_t^A(1)$$
(64)

$$k_{t+1}^{A}(1) = \frac{1}{1+\xi} \frac{1-\bar{c}\lambda_{1}}{1-\bar{c}} a_{t}^{A}(1) + \left(1 - \frac{1}{1+\xi} \frac{\bar{c}(\lambda_{2}-1)+\omega}{1-\bar{c}}\right) k_{t}^{A}(1) \quad (65)$$

where:

$$r_q = \frac{1-\delta}{(1-\omega)\,e^{-\omega k(0)} + (1-\delta)}$$

 $\lambda_1$  is given by:

$$\lambda_{1} \left[ 1 + \bar{c} \frac{1 - \gamma}{\gamma} \left[ \frac{\xi}{1 + \xi} \left( 1 - r_{q} \rho \right) + \left[ r_{q} \frac{\xi}{1 + \xi} \frac{\bar{c} \left( \lambda_{2} - 1 \right) + \omega}{1 - \bar{c}} + (1 - r_{q}) \omega \right] \frac{1}{1 + \xi} \right] \right]$$

$$= 1 + \frac{1 - \gamma}{\gamma} \left[ \frac{\xi}{1 + \xi} \left( 1 - r_{q} \rho \right) + \left[ r_{q} \frac{\xi}{1 + \xi} \frac{\bar{c} \left( \lambda_{2} - 1 \right) + \omega}{1 - \bar{c}} + (1 - r_{q}) \omega \right] \frac{1}{1 + \xi} \right]$$

$$- \frac{(1 - \gamma)(1 - \bar{c})}{\gamma} (1 - r_{q}) \rho$$

 $\lambda_2$  is given by:

$$\lambda_2 = 1 + \frac{(1+\xi)(1-\bar{c})\Phi - \omega}{\bar{c}}$$

where  $\Phi \in [0, 1]$  is the coefficient on  $k_t^A(1)$  in (65) and is the root of the polynomial:

$$0 = \Phi \left[ 1 + \xi + \bar{c} \frac{1 - \gamma}{\gamma} (1 - r_q) \left(\xi + \omega\right) \right] - \frac{1}{\gamma} \left[ \gamma \left[ 1 - \bar{c} (1 - r_q) \right] + \bar{c} (1 - r_q) \right] \omega + \bar{c} \frac{1 - \gamma}{\gamma} r_q \xi \left(\Phi\right)^2$$

#### **Cross-country** differences

The solution is relies on taking first-order expansions of the equations (49)-(60), and express them in terms of cross-country differences. The results are:

$$q_t^D(1) = \alpha_1(0) a_t^D(1) + \alpha_3(0) k_t^D(1) + \alpha_5(0) x_t^D(1)$$
(66)

$$c_{yt}^{D}(1) = a_{t}^{D}(1) + (1 - \omega)k_{t}^{D}(1)$$
(67)

$$k_{t+1}^{D}(1) = \frac{\alpha_{1}(0)}{\xi}a_{t}^{D}(1) + \left(1 + \frac{\alpha_{3}(0)}{\xi}\right)k_{t}^{D}(1) + \frac{\alpha_{5}(0)}{\xi}x_{t}^{D}(1)$$
(68)

$$4z_t^A(1) = \left(\frac{1+\xi}{\xi}\alpha_1(0) - z^D(0)\right)a_t^D(1)$$
(69)

+ 
$$\left(1 + \frac{1+\xi}{\xi}\alpha_3(0) - (1-\omega)z^D(0)\right)k_t^D(1) + \frac{1+\xi}{\xi}\alpha_5(0)x_t^D(1)$$

where:

$$\alpha_{3}(0) = \frac{1}{2r_{q}} \left[ (1 - r_{q})(\xi + \omega) - ((1 - r_{q})^{2} (\xi + \omega)^{2} + 4r_{q}\omega(1 - r_{q})\xi)^{0.5} \right]$$
  

$$\alpha_{1}(0) = \frac{(1 - r_{q})\rho}{1 + [\omega (1 - r_{q}) - r_{q}\alpha_{3} (0)] \frac{1}{\xi} - r_{q}\rho}$$
  

$$\alpha_{5}(0) = \frac{1 - r_{q} + r_{q}\alpha_{1} (0)}{1 + [\omega (1 - r_{q}) - r_{q}\alpha_{3} (0)] \frac{1}{\xi}} \frac{1}{1 + \lambda^{2}\theta}$$

and  $\theta$  is the ratio between the variance of liquidity and productivity shocks:  $\sigma_{\tau}^2 = \theta \sigma_a^2$ . The coefficient  $\alpha_5(0)$  in (66)-(69) is defined conditional on the term  $\lambda$  in (62). In the absence of signal extraction  $\alpha_5(0) = 0$  and the first-order cross-country solution is given by (66)-(69).

To solve for  $\lambda$ , we first take the third-order component of the optimal portfolio

condition for a Home investor (53) which can be written as:

$$\gamma z_{Hj,t} (1) \left[ E_t^{Hj} (er_{t+1})^2 \right] (2)$$

$$= \left[ E_t^{Hj} er_{t+1} \right] (3) + \tau_{Hj,t} (3) + (1 - \gamma) \left[ E_t^{Hj} er_{t+1} r_{t+1}^A \right] (3)$$

$$+ (1 - \gamma) \tau_{Hj,t} (2) \left[ E_t^{Hj} r_{t+1}^A \right] (1)$$

$$- \left[ \frac{1 - \gamma}{2} + \gamma \left( 2z_{Hj} \left( 0 \right) - 1 \right) \right] \tau_{Hj,t} (2) \left[ E_t^{Hj} er_{t+1} \right] (1)$$

$$- \gamma \frac{2z_{Hj} \left( 0 \right) - 1}{2} \left[ E_t^{Hj} \left( er_{t+1} \right)^2 \right] (3)$$

$$+ \left[ - \frac{\gamma \left( 1 + \gamma \right)}{2} z_{Hj} \left( 0 \right) \left( 1 - z_{Hj} \left( 0 \right) \right) + \frac{1}{6} - \frac{1 - \gamma}{2} \frac{1 + \gamma}{4} \right] \left[ E_t^{Hj} \left( er_{t+1} \right)^3 \right] (3)$$

$$+ \frac{(1 - \gamma)^2}{2} \left[ E_t^{Hj} \left[ r_{t+1}^A \right]^2 er_{t+1} \right] (3)$$

$$- \gamma \left( 1 - \gamma \right) \frac{2z_{Hj} \left( 0 \right) - 1}{2} \left[ E_t^{Hj} r_{t+1}^A \left( er_{t+1} \right)^2 \right] (3)$$

We can undertake similar steps for the optimal portfolio condition for a Foreign investor (54). We then sum across investors to get a relation in terms of per capita variables in each country. Taking the average of these relations in the Home and the Foreign country, we get:

$$2\gamma z_{t}^{A}(1) \left[E_{t} \left(er_{t+1}\right)^{2}\right](2)$$

$$= \int \left[E_{t}^{Hj} er_{t+1}\right](3) dj + \int \left[E_{t}^{Fj} er_{t+1}\right](3) dj + \tau_{t}^{D}(3)$$

$$+ (1 - \gamma) \left(\int \left[E_{t}^{Hj} er_{t+1}r_{t+1}^{A}\right](3) dj + \int \left[E_{t}^{Fj} er_{t+1}r_{t+1}^{A}\right](3) dj\right)$$

$$+ \frac{(1 - \gamma)^{2}}{2} \left(\int \left[E_{t}^{Hj} \left[r_{t+1}^{A}\right]^{2} er_{t+1}\right](3) dj + \int \left[E_{t}^{Fj} \left[r_{t+1}^{A}\right]^{2} er_{t+1}\right](3) dj\right) \quad (70)$$

$$+ (1 - \gamma) \tau(2) \left[\int \left(E_{t}^{Hj} r_{t+1}^{A}\right)(1) dj - \int \left(E_{t}^{Hj} r_{t+1}^{A}\right)(1) dj\right]$$

$$-\gamma \left(1 - \gamma\right) \frac{z^{D}(0)}{2} \left[\int \left(E_{t}^{Hj} r_{t+1}^{A}(er_{t+1})^{2}\right)(3) dj - \int \left(E_{t}^{Fj} r_{t+1}^{A}(er_{t+1})^{2}\right)(3) dj\right]$$

$$-\gamma \frac{z^{D}(0)}{2} \left[\int \left[E_{t}^{Hj} \left(er_{t+1}\right)^{2}\right](3) dj - \int \left[E_{t}^{Fj} \left(er_{t+1}\right)^{2}\right](3) dj\right]$$

We can infer  $\lambda$  from using (69) to substitute for  $z_t^A(1)$  in (70).  $x_t^D(1)$  enter several components of (70), and.  $\tau_t^D(3)$  enters the second row of (70). Because agents do not observe the components of  $x_t^D(1)$  separately, the model requires that  $\varepsilon_{t+1}^{D}$  also enters (70) and that it does so in a way that when combined with  $\tau_{t}^{D}(3)$  is enters as  $x_{t}^{D}(1)$ .

 $\varepsilon_{t+1}^D$  does not enter through terms that are expectations of cross-products (as in lines 3 and following), as such terms would only lead to variances of shocks, or the expectation of  $\varepsilon_{t+1}^D$ . Instead  $\varepsilon_{t+1}^D$  only enters (70) independently through the first-order component of the private signals, as this component are the actual shocks to future productivity. The signal extraction section above shows that the coefficients on private signals only have second-order components. The product of these coefficients and  $\varepsilon_{t+1}^D$  is then of order three.  $\varepsilon_{t+1}^D$  can then only enter (70) through a linear term, with the only such terms being:

$$\int \left[ E_t^{Hj} er_{t+1} \right] (3) \, dj + \int \left[ E_t^{Fj} er_{t+1} \right] (3) \, dj$$

To assess these terms, we can focus on a linear approximation of (57)-(60):

$$er_{t+1} = -q_t^D + r_q q_{t+1}^D + (1 - r_q) \left( a_{t+1}^D - \omega k_{t+1}^D \right)$$

We can show that the only relevant terms in the expectation of  $er_{t+1}$  for a Home investor are:

$$\left[E_t^{Hj} e r_{t+1}\right](3) = \left[r_q \alpha_1(0) + (1 - r_q)\right] \left[E_t^{ih} \left(\varepsilon_{H,t+1} - \varepsilon_{F,t+1}\right)\right](3)$$

where  $\left[E_t^{ih}\left(\varepsilon_{H,t+1} - \varepsilon_{F,t+1}\right)\right]$  (3) is given by (61). We can undertake similar steps for a Foreign investors. Aggregating across individual investors, we obtain:

$$\int \left[ E_t^{Hj} er_{t+1} \right] (3) \, dj + \int \left[ E_t^{Fj} er_{t+1} \right] (3) \, dj$$
$$= \left[ r_q \alpha_1 \left( 0 \right) + \left( 1 - r_q \right) \right] \frac{\lambda^2 \theta \sigma_a^2}{1 + \lambda^2 \theta} \left( \frac{1}{\sigma_{H,H}^2} + \frac{1}{\sigma_{H,F}^2} \right) \varepsilon_{t+1}^D$$

Focusing on the terms of interest, (70) becomes:

$$0 = \int \left[ E_t^{Hj} er_{t+1} \right] (3) \, dj + \int \left[ E_t^{Fj} er_{t+1} \right] (3) \, dj + \tau_t^D (3)$$
$$= \left[ r_q \alpha_1 \left( 0 \right) + (1 - r_q) \right] \sigma_a^2 \frac{\lambda^2 \theta}{1 + \lambda^2 \theta} \left( \frac{1}{\sigma_{H,H}^2} + \frac{1}{\sigma_{H,F}^2} \right) \varepsilon_{t+1}^D + \tau_t^D (3)$$

The ratio between the coefficient on  $\varepsilon_{t+1}^D$  and the coefficient on  $\tau_t^D(3)$  must be the same as in  $x_t^D(1)$ , implying

$$\left[r_q \alpha_1 \left(0\right) + \left(1 - r_q\right)\right] \left(\frac{1}{\sigma_{H,H}^2} + \frac{1}{\sigma_{H,F}^2}\right) \lambda = \frac{1 + \lambda^2 \theta}{\lambda^2 \theta} \frac{\tau \left(2\right)}{\sigma_a^2} \tag{71}$$

The left-hand side of (71) is an increasing linear function of  $\lambda$  which is flatter the higher the variance of private signal. The right-hand side of (71) is decreasing function of  $\lambda$  that is infinite when  $\lambda \to 0$  and converges to  $\tau$  (2)  $/\sigma_a^2$  when  $\lambda \to \infty$ . (71) therefore gives an implicit solution for  $\lambda$ . Combining it with (66)-(69) gives the first-order solution for the model.

# D Portfolio difference

#### Zero order solution

We solve for  $z^{D}(0) = z_{H}(0) - z_{F}(0)$  by taking the second-order component of the optimal portfolio condition for a Home investor (53) which can be written as:

$$z_{Hj}(0) = \frac{1}{2} + \frac{\left[E_t^{Hj}er_{t+1}\right](2) + \tau_{Hj,t}(2)}{\gamma \left[E_t^{Hj}\left(er_{t+1}\right)^2\right](2)} + \frac{1-\gamma}{\gamma} \frac{\left[E_t^{Hj}er_{t+1}r_{t+1}^A\right](2)}{\left[E_t^{Hj}\left(er_{t+1}\right)^2\right](2)}$$

We can undertake similar steps for the optimal portfolio condition for a Foreign investor (54). We then sum across investors to get a relation in terms of per capita variables in each country. Taking the difference between these relations in the Home and the Foreign country, we get:

$$z^{D}(0) = \frac{2\tau(2)}{\gamma \left[E_{t}(er_{t+1})^{2}\right](2)} + \frac{\int \left[E_{t}^{Hj}er_{t+1}\right](2) dj - \int \left[E_{t}^{Fj}er_{t+1}\right](2) dj}{\gamma \left[E_{t}(er_{t+1})^{2}\right](2)} + (1-\gamma) \frac{\int \left[E_{t}^{Hj}er_{t+1}r_{t+1}^{A}\right](2) dj - \int \left[E_{t}^{Fj}er_{t+1}r_{t+1}^{A}\right](2) dj}{\gamma \left[E_{t}(er_{t+1})^{2}\right](2)}$$

We can show that  $\left[E_t^{Hj}er_{t+1}r_{t+1}^A\right](2) = \left[E_t^{Fj}er_{t+1}r_{t+1}^A\right](2) = 0$  and  $\left[E_t^{Hj}er_{t+1}\right](2) = \left[E_t^{Fj}er_{t+1}\right](2)$ . In addition:

$$\begin{bmatrix} E_t^{H_j} (er_{t+1})^2 \end{bmatrix} (2) = 2 \begin{bmatrix} (1 - r_q + r_q \alpha_1(0))^2 \frac{\lambda^2 \theta}{1 + \lambda^2 \theta} + (r_q \alpha_5(0))^2 \begin{bmatrix} 1 + \lambda^2 \theta \end{bmatrix} \sigma_a^2 \\ = 2\sigma_a^2 (1 - r_q + r_q \alpha_1(0))^2 \Gamma$$

where  $\Gamma \in [0, 1]$  is an increasing function of  $\lambda$  that converges to one when private signals are infinitely noisy  $(\lambda \to \infty)$ :

$$\Gamma = 1 - \left(1 - \left(\frac{r_q}{1 + [\omega (1 - r_q) - r_q \alpha_3 (0)] \frac{1}{\xi}}\right)^2\right) \frac{1}{1 + \lambda^2 \theta}$$

The zero-order portfolio difference is then::

$$z^{D}(0) = \frac{2\tau(2)}{\gamma \left[ E_{t} \left( er_{t+1} \right)^{2} \right](2)} = \frac{\tau(2)}{\gamma \sigma_{a}^{2}} \frac{1}{(1 - r_{q} + r_{q} \alpha_{1}(0))^{2}} \frac{1}{\Gamma}$$
(72)

#### **First-order** solution

The first-order component of the difference in portfolio shares is solved by taking the third-order component of the optimal portfolio condition for a Home investor (53), and aggregating across Home investors to obtain a per capita average for the Home country. We follow similar steps with the third-order component of the optimal portfolio condition for a Foreign investor (54). Taking the difference between the Home and Foreign per-capita relations we write:

$$\begin{split} \gamma z_t^D \left(1\right) \left[ E_t \left(er_{t+1}\right)^2 \right] (2) \\ &= \int \left[ E_t^{Hj} er_{t+1} \right] (3) \, dj - \int \left[ E_t^{Fj} er_{t+1} \right] (3) \, dj + 2\tau_t^A \left(3\right) \\ &+ \left(1 - \gamma\right) \left( \int \left[ E_t^{Hj} er_{t+1} r_{t+1}^A \right] (3) dj - \int \left[ E_t^{Fj} er_{t+1} r_{t+1}^A \right] (3) dj \right) \\ &+ \left(1 - \gamma\right) \tau \left(2\right) \left[ \int \left( E_t^{Hj} r_{t+1}^A \right) (1) \, dj + \int \left( E_t^{Hj} r_{t+1}^A \right) (1) \, dj \right] \\ &- \gamma \frac{2z_H \left(0\right) - 1}{2} \left( \int \left[ E_t^{Hj} \left(er_{t+1}\right)^2 \right] (3) \, dj + \int \left[ E_t^{Fj} \left(er_{t+1}\right)^2 \right] (3) \, dj \right) \\ &+ \frac{\left(1 - \gamma\right)^2}{2} \left( \int \left[ E_t^{Hj} \left[ r_{t+1}^A \right]^2 er_{t+1} \right] (3) dj - \int \left[ E_t^{Fj} \left[ r_{t+1}^A \right]^2 er_{t+1} \right] (3) dj \right) \\ &- \gamma \left(1 - \gamma\right) \frac{2z_H \left(0\right) - 1}{2} \left( \int \left( E_t^{Hj} r_{t+1}^A \left(er_{t+1}\right)^2 \right) (3) dj + \int \left( E_t^{Fj} r_{t+1}^A \left(er_{t+1}\right)^2 \right) (3) dj \right) \\ &- \gamma \left(1 - \gamma\right) \frac{2z_H \left(0\right) - 1}{2} \left( \int \left( E_t^{Hj} r_{t+1}^A \left(er_{t+1}\right)^2 \right) (3) dj + \int \left( E_t^{Fj} r_{t+1}^A \left(er_{t+1}\right)^2 \right) (3) dj \right) \\ &- \gamma \left(1 - \gamma\right) \frac{2z_H \left(0\right) - 1}{2} \left( \int \left( E_t^{Hj} r_{t+1}^A \left(er_{t+1}\right)^2 \right) (3) dj + \int \left( E_t^{Fj} r_{t+1}^A \left(er_{t+1}\right)^2 \right) (3) dj \right) \\ &- \gamma \left(1 - \gamma\right) \frac{2z_H \left(0\right) - 1}{2} \left( \int \left( E_t^{Hj} r_{t+1}^A \left(er_{t+1}\right)^2 \right) (3) dj + \int \left( E_t^{Fj} r_{t+1}^A \left(er_{t+1}\right)^2 \right) (3) dj \right) \\ &- \gamma \left(1 - \gamma\right) \frac{2z_H \left(0\right) - 1}{2} \left( \int \left( E_t^{Hj} r_{t+1}^A \left(er_{t+1}\right)^2 \right) (3) dj + \int \left( E_t^{Fj} r_{t+1}^A \left(er_{t+1}\right)^2 \right) (3) dj \right) \\ &- \gamma \left(1 - \gamma\right) \frac{2z_H \left(0\right) - 1}{2} \left( \int \left( E_t^{Hj} r_{t+1}^A \left(er_{t+1}\right)^2 \right) \left( E_t^{Hj} r_{t+1}^A \left(er_{t+1}\right)^2 \right) (3) dj + \int \left( E_t^{Fj} r_{t+1}^A \left(er_{t+1}\right)^2 \right) \left( E_t^{Fj} r_{t+1}^A \left(er_{t+$$

The various terms can be computed using the first- and second-order components of the solution. The detailed steps are complex and the solution takes the form:

$$z_{t}^{D}(1) = \frac{z^{D}(0)}{2\lambda} \frac{\sigma_{H,F}^{2} - \sigma_{H,H}^{2}}{\sigma_{H,H}^{2} + \sigma_{H,F}^{2}} \varepsilon_{t+1}^{A}(1) + z^{D}(0) \frac{\tau_{t}^{A}(3)}{\tau(2)} + \Omega_{S}S_{t}(1) + \frac{f\left(\sigma_{a}^{2}x_{t}^{D}(1), \left[x_{t}^{D}(1)\right]^{3}\right)}{\left[E_{t}\left(er_{t+1}\right)^{2}\right](2)}$$

where  $\Omega_S$  is a zero-order parameter and f is complex function.

# **E** Balance of Payments Accounting

#### Saving and investment

Saving is equal to income minus consumption. In line with national accounts, we consider savings net of the amount required to offset the depreciation of capital. National saving in the Home and Foreign countries are:

$$S_{t}^{H} = \int \left( w_{H,t} - C_{y,t}^{jH} \right) dj$$
  

$$- \int \left[ z_{Hj,t-1} \frac{Q_{H,t}}{Q_{H,t-1}} + (1 - z_{Hj,t-1}) \frac{Q_{F,t}}{Q_{F,t-1}} \right] \left( w_{H,t-1} - C_{y,t-1}^{jH} \right) dj$$
  

$$S_{t}^{F} = \int \left( w_{F,t} - C_{y,t}^{jF} \right) dj$$
  

$$- \int \left[ z_{Fj,t-1} \frac{Q_{H,t}}{Q_{H,t-1}} + (1 - z_{Fj,t-1}) \frac{Q_{F,t}}{Q_{F,t-1}} \right] \left( w_{F,t-1} - C_{y,t-1}^{jF} \right) dj$$

The first-order components of savings are:

$$s_{t}^{H}(1) = \frac{1}{1-\bar{c}} \left[ \Delta a_{H,t}(1) + (1-\omega) \Delta k_{H,t}(1) \right] -\frac{\bar{c}}{1-\bar{c}} \Delta c_{y,t}^{H}(1) - \Delta q_{t}^{A}(1) - \frac{z^{D}(0)}{2} \Delta q_{t}^{D}(1) s_{t}^{F}(1) = \frac{1}{1-\bar{c}} \left[ \Delta a_{F,t}(1) + (1-\omega) \Delta k_{F,t}(1) \right] -\frac{\bar{c}}{1-\bar{c}} \Delta c_{y,t}^{F}(1) - \Delta q_{t}^{A}(1) + \frac{z^{D}(0)}{2} \Delta q_{t}^{D}(1) \right]$$

where  $s_t^i(1)$  is the first-order component of savings, scaled by the steady state wealth:  $s_t^i(1) = S_t^i(1) / (W(0)(1-\bar{c}))$ . In addition for a variable  $g: \Delta g_t(1)=g_t(1)-g_{t-1}(1)$ . The first-order consumption in a country can be split between the worldwide average and the cross-country difference. Using (63) and (67), consumption in a specific country reflects only the observed state variables:

$$\Delta c_{y,t}^{H}(1) = \Delta c_{y,t}^{A}(1) + \frac{1}{2} \Delta c_{y,t}^{D}(1)$$
  
=  $\lambda_{1} \Delta a_{t}^{A}(1) + \lambda_{2} \Delta k_{t}^{A}(1) + \frac{1}{2} \left[ \Delta a_{t}^{D}(1) + (1-\omega) \Delta k_{t}^{D}(1) \right]$ 

Saving in a specific country are then affected by the information dispersion only

through relative equity prices:

$$s_{t}^{H}(1) = \alpha_{H} \Delta S_{t}(1) - \frac{z^{D}(0)}{2} \Delta q_{t}^{D}(1)$$
  

$$s_{t}^{F}(1) = \alpha_{F} \Delta S_{t}(1) - \frac{z^{D}(0)}{2} \Delta q_{t}^{D}(1)$$
  

$$s_{t}^{D}(1) = \Delta a_{t}^{D}(1) + (1 - \omega) \Delta k_{t}^{D}(1) - z^{D}(0) \Delta q_{t}^{D}(1)$$

where  $s_{t}^{D}(1) = s_{t}^{H}(1) - s_{t}^{F}(1)$ .

Investment is also defined net of depreciation:

$$I_{i,t}^{net} = I_{i,t} - \delta K_{i,t-1} = K_{i,t} - K_{i,t-1} \qquad i = H, F$$

The first-order component of investment, scaled by steady-state wealth, is then:

$$i_t^{D,net}\left(1\right) = \frac{I_t^{H,net}\left(1\right) - I_t^{F,net}\left(1\right)}{e^{w(0)}\left(1 - \bar{c}\right)} = \Delta k_{t+1}^D\left(1\right) = \frac{1}{\xi} q_t^D\left(1\right)$$

where we used (5).

#### Capital flows

The passive portfolio share combines the steady-state holdings of quantities of assets with the actual asset prices. For Home investors, we write:

$$z_{H,t}^{p} = \frac{z_{H}(0) e^{q_{H,t}}}{z_{H}(0) e^{q_{H,t}} + (1 - z_{H}(0)) e^{q_{F,t}}}$$

The first-order passive portfolio share is the same for all investors:

$$z_t^p(1) = z_H(0) (1 - z_H(0)) q_t^D(1)$$

Using the difference between the first-order components of (55) and (56) we get:

$$\Delta z_t^A(1) - \Delta z_t^p(1) = \frac{1}{4} \left[ i_t^{D,net}(1) - z^D(0) s_t^D(1) \right]$$

Gross capital outflows and inflows reflect the changes in the value of crossborder asset holdings:

$$OUTFLOWS_{t} = \int (1 - z_{Hj,t}) \left( w_{H,t} - C_{y,t}^{jH} \right) dj$$
  
$$- \frac{Q_{F,t}}{Q_{F,t-1}} \int (1 - z_{Hj,t-1}) \left( w_{H,t-1} - C_{y,t-1}^{jH} \right) dj$$
  
$$INFLOWS_{t} = \int z_{Fj,t} \left( w_{F,t} - C_{y,t}^{jF} \right) dj$$
  
$$- \frac{Q_{H,t}}{Q_{H,t-1}} \int z_{Fj,t-1} \left( w_{F,t-1} - C_{y,t-1}^{jF} \right) dj$$

The first-order components, scaled by steady-state wealth  $W\left(0\right)\left(1-\bar{c}\right)$ , are:

$$outflows_{t} = -(1 - z_{H}(0)) \Delta q_{F,t}(1) - \Delta z_{H,t}(1) + \frac{1 - z_{H}(0)}{1 - \bar{c}} \left[ \Delta a_{H,t}(1) + (1 - \omega) \Delta k_{H,t}(1) - \bar{c} \Delta c_{y,t}^{H}(1) \right] = (1 - z_{H}(0)) s_{t}^{H}(1) - \left[ \Delta z_{H,t}(1) - \Delta z_{t}^{p}(1) \right] inflows_{t} = (1 - z_{H}(0)) s_{t}^{F}(1) + \left[ \Delta z_{F,t}(1) - \Delta z_{t}^{p}(1) \right]$$

In terms of net capital flows, we write:

$$net_{t} = outflows_{t} - inflows_{t}$$
  
=  $(1 - z_{H}(0)) s_{t}^{D}(1) - 2 \left[\Delta z_{t}^{A}(1) - \Delta z_{t}^{p}(1)\right]$   
=  $\frac{1}{2} \left[s_{t}^{D}(1) - i_{t}^{D,net}(1)\right]$ 

The sum of gross capital flows is:

$$outflows_t + inflows_t = \frac{1 - z_H(0)}{2} s_t^A(1) - \Delta z_t^D(1)$$

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