Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt.*

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November 24, 2006

Abstract

This paper proposes a politico-economic theory of debt and government expenditure. Policies are chosen by governments that are voted through repeated elections. The resolution of an inter-generational political conflict over public goods, lead governments to adopt some fiscal discipline even if voters are imperfectly (or not at all) altruistic, and debt has no effect on the national interest rate. On the one hand, if tax does not distort labor supply, the economy progressively depletes its resources through a progressive debt accumulation that leaves future generations "enslaved". On the other hand, if tax distortions are sufficiently large, the economy converges to a stationary debt level which is bounded away from the endogenous debt limit. The model delivers empirical implications about the short- and long-run effect of fiscal shocks (a war) and political shifts affecting people's appreciation of public goods..

JEL No D72, E62, H41, H62, H63.
Keywords: Government debt, Fiscal policy, Fiscal discipline, Intergenerational conflict, Markov equilibrium, Political economy, Repeated voting.

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*VERY PRELIMINARY AND INCOMPLETE. An empirical section, appendix and references are missing. We would like to thank Andreas Müller for research assistance.

1 Introduction

There are large differences in government debt, government spending, and taxes across countries and across time. For instance, while the debt-GDP ratios of a large number of OECD countries ranges between 50-60%, those of Belgium, Italy and Japan range between 100% and 150%, and, on the opposite tail of the distribution, those of Australia, Korea and Ireland range between 15-30%. Even more extreme, the Norwegian government has a net financial wealth (negative debt) of about 130% of GDP. Budgetary policies raise year-after-year passionate political debates within each country. In spite of this, while a growing politico-economic literature has studied a variety of economic policy issues, there is still a limited theoretical understanding of the driving forces determining public debt.

This paper aims at filling this gap, by proposing a theory in which fiscal policies are chosen by governments that are voted through repeated elections and cannot bind the debt policies of future governments. A fundamental question for a politico-economic theory of debt is what prevents a process of never-ending debt accumulation, shifting the burden of financing public expenditure from current voters to future generations. Clearly, there is a limit for how much governments can expand their debt. First, markets must believe that government liabilities will not be repudiated. Second, a large government debt may put an upward pressure local interest rates. Yet, in most industrialized countries, public debt remains significantly below the level that would raise concerns about solvency. And the figures above, together with growing international capital market integration, suggest that many countries could expand or contract significantly their debt without major effects on their interest rate.

In this paper, we explore a complementary explanation. We argue that an inter-generational political conflict over public goods, combined with lack of commitment to future policies, can limit the desire of voters to expand public debt, even in a world where agents have no concern for future generations. To this end, we construct a politico-economic theory, where the dynamics and steady-state of fiscal policy are driven by a dynamic game over taxation, debt, and public-good provision. The framework is a small open economy populated by two-period-lived agents who work when young and consume a private and a government-provided public good both periods of life. The government can issue debt up to its natural borrowing constraint. Every period agents vote on public-good provision, distortive labor taxation on the young, and future government debt.

The intergenerational conflict plays out as follows. Old agents support the largest possible current government expenditure financed through deficit spending, in order to maximize the amount of public good that can be provided in the current period. Young agents, however, are more averse to running large deficits, because they care not only about the current level of public good provision, but also about the provision in the next period. In particular, they are concerned that a large current deficit will force the future government to cut back on public spending. The political process, that we model as a probabilistic-voting model a la Lindbeck and Weibull (1987) generates a compromise between these two desired policies.

The voting behavior of the young forward-looking agents is central for understanding our results. Clearly, they want to set the future debt strategically so as to manipulate future young voters to choose large public-good provision. In this sense, the voting behavior of the future young voters disciplines the current voters to limit debt accumulation. This fiscal discipline hinges on lack of commitment to future policies. In fact, when voters have commitment (so there is once-and-for-all voting on future debt and public goods), debt is systematically larger than under repeated voting.[1] The strength of such fiscal discipline depends crucially on the expectations of the young voters about the fiscal conduct of future governments. The government coming next has three options for dealing with a large inherited debt: it can increase taxes, reduce expenditure, or, expand the public debt further. If young voters expect a response consisting mainly of expenditure cuts, they will fear an old age without welfare state. This induces more fiscal discipline today. However, if they expect a response consisting mainly of higher taxes on the next generation of young, or even of a further debt expansion, they will be less opposed to run a large deficit today, and the fiscal discipline is lax.

In our model, expectations are built into the process of dynamic voting. In particular, we focus on Markov equilibria where the strategies of current voters can be conditioned only on pay-off-relevant state variables. In our model, the only such state variable is government debt, and this allows us a high tractability. A key result is that, along the equilibrium path, the expectations about the conduct of future governments depend crucially on the extent of tax distortions. Intuitively, the more distortionary taxation, the less future governments will be tempted to increase future taxes. Instead, when taxes are very distortionary, they will react to a larger debt by cutting public good provision. Therefore, the fiscal discipline becomes stronger the more distortionary taxes are, i.e., the more concave the Laffer curve.[2]

[1] Since the political outcome is always influenced by the forward-looking young voters, such fiscal discipline is a persistent force in the model.

[2] International tax competition provides a simple example. Suppose that at some level of taxation, labor supply became infinitely elastic due to international tax competition. Then, future governments could not increase taxes beyond that level, and any marginal adjustment to a larger debt must be in the form of a reduction in expenditure. This strengthens the fiscal discipline as the tax competition kicks in.

We show that, in the absence of labor supply distortions, the economy would progressively deplete its resources through a progressive debt accumulation that would "enslave" future generations. Namely, future generations would be forced to work to service the outstanding debt accumulated by previous generations, while their consumption, both private and public, would fall down to zero. Instead, if tax distortions are sufficiently large, the economy converges to an "interior" debt level which is bounded away from the endogenous debt limit. In this steady-state, both private consumption and public good provision are positive. In other words, labor market distortions provide future generations with a credible threat that prevent them from being abused by their rotten parents.

The model delivers a number of interesting empirical implications. The first class of predictions regards fiscal shocks, such as a war. When the economy is in steady-state, a war will result in larger debt, higher taxation, and lower public-good provision. Over time, debt, taxes, and public goods revert back to their steady state levels.

Such auto-regressive dynamics of debt stands in sharp contrast to the debt dynamics under commitment, analyzed in the seminal contribution of Barro (1979). He provided a normative benchmark emphasizing the notion of tax smoothing; If the distortionary costs of taxation are convex, governments should use debt to absorb fiscal shocks spreading their effects evenly over future periods. For instance, if government expenditures were unexpectedly increased by a war, the government should finance it through debt, and increase current and future taxes evenly so as to service the additional debt. If governments were to follow Barro's recommendations, debt should not be mean-reverting; after the war, there is no reason to reduce debt unless new shocks occur.

The data support the empirical prediction of our theory. Bohn (1998) shows that a short-lived increase in US government expenditures implies an increase in debt with a subsequent reversion in debt. In our empirical section we show that this stylized fact holds up for a panel data set of OECD countries [A DESCRIPTION OF THE EMPIRICAL ANALYSIS WILL BE ADDED HERE].[3]

The second class of empirical implications regards fiscal policy in response to political shifts. Following Persson and Svensson (1989), we identify political left (right) by a large (small) weight on the public good in the utility function. The key insight is that young right-wing voters are less scared of low future public-good provision, so their fiscal discipline is

[3]Our theory has also other testable implications which we have not yet confronted with data. To the best of our knowledge, all alternative models of debt (see review below) take government expenditure as exogenous, and are silent on non-military spending after wars. In our model non-military spending is autoregressive in response to war shock. Incidentally, Barro (1986) notes that non-military spending is crowded out during wars – exactly as our model predicts.

weaker than that of left-wing voters. Thus, by allowing cohort-specific shocks to the taste for public good, we get a number of sharp empirical predictions. In the short run, after a change in regime towards a right-wing government, the level of debt (taxes) should be increasing (falling). In the long run, i.e. after a long sequence of right-wing governments, the steady-state level of debt (public good) should be higher (lower).

To test these predictions, we construct a panel data set of fiscal variables and political color of the government, based on work by Franzese (2001, 2002) and Persson and Tabellini (2004). Our results confirm that right-wing governments have larger debt than left-wing governments.

[A DESCRIPTION OF THE EMPIRICAL ANALYSIS IS TO BE ADDED HERE]

Our paper is related to a number of contributions on the determination of government debt. The papers closest to us are Barro (1979), Ayiagari, Marcet, Seppälä, and Sargent (2002), and Krusell, Martin, and Rios-Rull (2005). These papers have, as in our model, distortionary taxation and non-state-contingent government debt (as opposed to Lucas and Stokey, 1983). Different from us, however, these papers focus on representative-agent economies, so there is no scope for political conflict. Moreover, these papers have no public-good consumption, so our disciplining effect is not present.

Interestingly, Ayiagari et al. (2002) find that government debt should be stationary, essentially for the same reason that individual wealth is stationary in an Aiyagari-Bewley-Huggett economy.[4] Hence, even with commitment government debt can be (weakly) auto-regressive. We view our mechanism as a complementary explanation for why government debt indeed is auto-regressive in response to surprising expenditure shocks. Moreover, we show that our mechanism gives rise to a quantitatively large reduction in mean reversion, compared to the mechanism in Ayiagari et al. (2002).

While Barro (1979) and Ayiagari et al. (2002) assume commitment, Krusell et al. (2005) focus on time-consistent policies without risk. Their main point is that, due to an incentive to manipulate interest rates, there exists time-consistent policies replicating the commitment solution of Barro (1979).

The paper is organized as follows. In section 2 we describe the model environment and derive the Generalized Euler Equation which is key to the characterization of the political equilibrium. Section 3 provides two examples that admit an analytical solution. Section 4 analyzes the general case. Section **??** discusses two applications (fiscal and political shocks).

[4]In these economies agents have a precautionary savings motive due to stochastic income. Wealth is bounded below by a borrowing constraint. Due to the equilibrium interest rate being smaller than the discount rate and that the absolute risk aversion is falling in consumption, the intertemporal incentive to reduce wealth will dominate when wealth becomes sufficiently large. Therefore, individual wealth will be stationary if the income process is stationary.

Section 6 concludes.

2 Model economy

The model economy is populated by overlapping generations of two-period lived agents who work in the first period and live off their savings in the second period of their lives. The population size is assumed to be constant. Agents earn utility from the consumption of two goods: a private good (c) and a public good (g) which is provided by the government.

Private goods can be produced via two technologies – market and household production. Market production is subject to constant returns, and agents earn a hourly wage w. The household production technology is represented by the following production function;

$$y_H = F\left(h - h_M\right), \; F'\left(\cdot\right) > 0, \; F''\left(\cdot\right) \leq 0,$$

where h is the total individual time endowment, h_M is the market labor supply, and $h - h_M \geq 0$ is the household activity. Since the government cannot tax household production, taxation distorts the share of time that agents devote to market activity. Agents choose the allocation of their time so as to maximize total labor income, which is denoted $A\left(\tau\right)$;

$$A\left(\tau\right) = \max_{h_M} \left\{ \left(1 - \tau_1\right) w h_M + F\left(h - h_M\right) \right\}. \tag{1}$$

This program defines the optimal market labor supply as a function of the tax rate, τ. In particular, we denote its solution as

$$h_M = h_M\left(\tau\right), \; h'_M\left(\cdot\right) \leq 0. \tag{2}$$

The preferences of a young agent i born in period one are represented by the following utility function;

$$U_Y^i\left(1\right) = \log\left(c_1^i\right) + \theta \log\left(g_1\right) + \beta\left(\log\left(c_2^i\right) + \theta \log\left(g_2\right) + \lambda U_Y^i\left(2\right)\right), \tag{3}$$

where Y stands for "young", superscripts denote individual-specific variables, and subscripts denote the age at which the agent consumes. β is the discount rate, θ is a parameter describing the intensity of preferences for public good consumption, and λ is the altruistic weight on the utility of the agent's child. We omit time subscripts when there is no source of confusion.

For the time being, we maintain that agents do not leave any monetary bequests to their children. It can be shown that this is indeed the optimal choice in equilibrium under some parametric restrictions [INCOMPLETE...]. This allows us to isolate the effect of altruism via political choices. Absent bequests, agents choose labor supply, h_M, according to (2), and the consumption sequence (c_1, c_2) so as to maximize utility (3) subject to their lifetime budget constraint given by

$$c_1^i + c_2^i/R = A(\tau_1), \tag{4}$$

where R is the gross interest rate. The solution yields

$$c_1^i = c_1 = \frac{A(\tau_1)}{1+\beta},\ c_2^i = c_2 = \frac{\beta R A(\tau_1)}{1+\beta}. \tag{5}$$

The fiscal policy is determined period-by-period through repeated elections. We model electoral competition as a two-candidate political model of probabilistic voting à la Lindbeck and Weibull (1987), which is extensively discussed in Persson and Tabellini (2000). In this model, agents cast their votes on one of two office-seeking candidates. Voters have different preferences not only over fiscal policy, but also over some policy dimension that is orthogonal to fiscal policy and about which the candidates cannot make binding commitments. In a probabilistic voting equilibrium, both candidates propose the same fiscal policy, which turns out to maximize a weighted sum of individual utilities, where the weights are the same for all agents of a given age but may differ between young and old agents. Thus, the equilibrium policy maximizes a "political objective function" which is a weighted average utility of all voters.

The elected government chooses the tax rate ($\tau \in [0,1]$), the public good provision ($g \geq 0$) and the debt (b') to be passed through to the following generation, subject to the following dynamic budget constraint

$$b' = g + Rb - \tau w h_M(\tau). \tag{6}$$

Both private agents and governments have access to an international capital market providing borrowing and lending at the gross rate $R > 1$. The government is committed not to repudiate the debt. This implies that debt cannot exceed the present discounted value of the maximum tax revenue that can be collected;

$$b \leq \frac{\max_\tau \{\tau w h_M(\tau)\}}{R-1} \equiv \bar{b}, \tag{7}$$

where $\bar{b}$ denotes the endogenous debt ceiling. This constraint rules out government Ponzi schemes.

Since agents vote twice in their life, first when they are young, and then when they are old, the first step to characterize the political equilibrium is to write the indirect utility of young and old agents. In the case of the young, substituting (1) and (5) into (8) yields:

$$U_Y(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) = (1+\beta)\log\left(\frac{(1+\beta R)A(\tau_1)}{1+\beta}\right) + \theta\log(g) + \beta\left(\theta\log(g') + \lambda U_Y(\boldsymbol{b}', \boldsymbol{\tau}', \boldsymbol{g}')\right), \tag{8}$$

where the primes denote next period's variables and boldface variables are vectors, defined as follows:

$$\boldsymbol{x} = \begin{bmatrix} x \\ x' \\ x'' \\ \ldots \end{bmatrix} = \begin{bmatrix} x \\ \boldsymbol{x}' \end{bmatrix}$$

Similarly, the indirect utility of old voters is given by[5]

$$U_O(\boldsymbol{b}, \tau, \boldsymbol{g}) = \log\left(\frac{(1+\beta R) A (1-\tau_{-1})}{1+\beta}\right) + \theta \log(g) + \lambda U_Y(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}), \tag{9}$$

where τ_{-1} denotes the tax rate in the period when the current old were young. Note that the old care about their children who are alive contemporarily with them. Thus, the children's utility, U_Y, is not discounted.

The equilibrium of a probabilistic voting model can then be represented as the choice over time of τ, g and $b\prime$ maximizing a weighted average indirect utility of young and households, given b. We denote the weights of the old and young as, respectively, ω and $1-\omega$. Then, the "political objective function" which is maximized by both political candidates is

$$U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) = (1-\omega) U_Y(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) + \omega U_O(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}), \tag{10}$$

subject to (6) and (7).

2.1 The commitment solution

A key feature of our model is that fiscal policy is not time consistent. The source of time inconsistency is different from those identified by other papers, and stems from the fact that agents vote repeatedly over their lifetime, and can condition the fiscal policy choice at different stages of their life.[6] To establish a benchmark, it is useful to characterize the fiscal policy sequence that would be chosen by the first generation of voters if they could commit the entire future path of fiscal policy.

Consider, first, a special case in which there is no time inconsistency. Namely, suppose that the first generation of old agents can dictate its preferred policy ($\omega = 1$). In this case $U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) = U_O(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})$. From (8) and (9), it follows immediately that the problem admits the following recursive formulation;

$$V_O^{comm}(b) = \max_{\{\tau, g, b'\}} v(\tau, g) + \beta\lambda V_O(b') \tag{11}$$

[5]With some abuse of notation, we write $U_O(\boldsymbol{b}, \tau, \boldsymbol{g})$ instead of $U_O(\boldsymbol{b}, \tau_{-1}, \tau, \boldsymbol{g})$ since τ_{-1} is not relevant for the political choice, due to the focus on Markov equilibrium and because prefereces are separable.

[6]For instance... (DISCUSS LUCAS-STOKEY AND THE LITERARTURE ON CAPITAL TAXATION).

subject to (6) and (7), where

$$v(\tau, g) \equiv (1+\lambda)\,\theta \log g + (1+\beta)\,\lambda \log A(\tau) \tag{12}$$

is the flow utility accruing to the initially old agents from the current public and private consumption, either directly or through their altruism for their children.

Note that (11) is a standard recursive problem. The solution is time consistent, and is the same irrespective of whether the entire sequence is dictated by the first generation of old voters or chosen period-by-period by subsequent generations of old voters.

To solve the program, note that the intra-temporal first-order condition linking g and τ in problem (12) is given by[7]

$$\frac{1+\beta}{\left(1+\frac{1}{\lambda}\right)\theta} g = A(\tau)\,(1-e(\tau)), \tag{13}$$

where $e(\tau) \equiv -\left(\partial h_M(\tau)/d\tau\right)\left(\tau / h_M(\tau)\right)$ is the elasticity of labor supply. The intertemporal first order condition leads then to the standard Euler equation for public consumption:

$$\frac{g'}{g} = \beta\lambda R. \tag{14}$$

If $\beta = R^{-1}$ and $\lambda = 1$ (perfect altruism), the solution features a stationary policy, whereby debt, taxes, and consumption are kept constant at their initial levels. An unexpected once-and-for all fiscal shock (e.g., a war) should be financed by a permanent increase in the debt level, to be financed through a time-invariant higher tax level in future (see Barro, 1979).[8]

Next, we move to the general case in which the young affect the fiscal outcome ($\omega < 1$). In this case, a standard recursive formulation of the problem does not exist. However, the program admits a "two-stage-recursive" formulation. This is formalized in the following lemma;

Lemma 1 *The "commitment" problem admits a "two-stage recursive" formulation where;*

(i) In the initial period, policies are set through

$$\{\tau_0, g_0, b_1\} = \arg \max_{\{\tau_0, g_0, b_1\}} v(\tau, g) - (1-\psi\lambda)\,\theta \log g + \beta\lambda V_O^{comm}(b_1),$$

[7]Two first order condtions with respect to τ and g are

$$\begin{aligned} \frac{(1+\beta)\,\lambda}{A(\tau)} \frac{\frac{\partial A(\tau)}{\partial \tau}}{w h_M(\tau) + \tau w \frac{\partial h_M(\tau)}{\partial \tau}} &= -\beta\lambda \hat{V}_O' \;\; b' \;\;, \\ -\frac{(1+\lambda)\,\theta}{g} &= -\beta\lambda \hat{V}_O' \;\; b' \;\;, \end{aligned}$$

The two FOCs, together with the fact that $A'(\tau) = -wh(\tau)$, lead to (13).

[8]We will return to the analysis of fiscal shocks in Section 5.1.

subject to (6) and (7), where the function $V_O(.)$ *is given by (11), and the constant* ψ *is*

$$\psi \equiv \frac{\omega}{1-\omega(1-\lambda)} \in \left(0, \frac{1}{\lambda}\right).$$

(ii) After the first period, the problem is equivalent to (11).

Proof in Appendix.

Lemma 1 implies that a set of rules applies to the first period, and another set of rules applies recursively to all future periods.[9] Namely, when the young have some political influence ($\omega < 1$), the solution is time inconsistent; the fiscal policy sequence chosen under commitment differs from the one resulting from repeated decisions. This is an important point to which we return after characterizing the political equilibrium without commitment.

In spite of the differences in the first period, the long-run properties of this model are observationally equivalent to Barro's solution. In particular, equation (14) governs the dynamics of public debt after the first period, and whether debt grows, fall or remain constant over time only depends on the term $\beta\lambda R$. The following Proposition follows immediately from Lemma 1:

Proposition 1 *The "commitment" solution is such that (i) if* $\beta\lambda R < 1$*, then* $\lim_{t\to\infty} b_t = \bar{b}$*, (ii) If* $\beta\lambda R > 1$*, then* $\lim_{t\to\infty} b_t = -\infty$*, (iii) if* $\beta\lambda R = 1$*,* $b_{t+1} = b_t$ *for* $t \geq 1$*.*

2.2 The political equilibrium

We now move to the main contribution of the paper, that is the characterization of the political equilibrium when fiscal policy is set through repeated elections whereby voters cannot commit future policies. In general, a dynamic game between the current and future voters arises, and the set of equilibria is potentially large. We restrict attention to Markov perfect equilibria where agents condition their choice only on pay-off relevant state variables. Subsequent periods are in principle linked by two state variables: the government debt, b, and the private wealth of the old. However, since preferences are separable between private consumption and public goods, the wealth of the old does not affect the preference of the old for public goods. Therefore, b is the only pay-off relevant state variable. Our Markov equilibria thus feature policy rules as functions of b only.

Definition 2 *A (Markov perfect) political equilibrium is defined as a 3-tuple of functions* $\langle B, G, T\rangle$*, where* $B : (-\infty, \bar{b}] \to [\underline{b}, \bar{b}]$ *is a debt rule,* $b' = B(b)$*,* $G : (-\infty, \bar{b}] \to R^+$ *is a*

[9] Note that when $\omega = 1$, $\psi\lambda = 1$, and there is no difference between the first-period problem and the continuation.

government expenditure rule, $g = G(b)$ *and* $T : (-\infty, \bar{b}] \rightarrow [0,1]$ *is a tax rule, such that the following functional equations hold:*

1. $\langle B(b), G(b), T(b) \rangle = \arg\max_{\{b' \leq \bar{b}, g \geq 0, \tau \in [0,1]\}} U(\mathbf{b}, \boldsymbol{\tau}, \mathbf{g})$, *subject to (6) and (7), where*

$$\boldsymbol{\tau} = \begin{bmatrix} \tau \\ T(b') \\ T(B(b')) \\ T(B(B(b'))) \\ \cdots \end{bmatrix}, \mathbf{g} = \begin{bmatrix} g \\ G(b') \\ G(B(b')) \\ G(B(B(b'))) \\ \cdots \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} b \\ b' \\ B(b') \\ B(B(b')) \\ \cdots \end{bmatrix}$$

   and $U(\mathbf{b}, \boldsymbol{\tau}, \mathbf{g})$ *is defined as in (10).*

2. $B(b) = G(b) + Rb - T(b) h_M (T(b)$.

In words, the government chooses the current fiscal policy (taxation, expenditure and debt level left to the next generation) subject to the budget constraint, and under the expectation that future fiscal policies will be conducted according to the equilibrium policies rules, $\langle B(b), G(b), T(b) \rangle$. Furthermore, the vector of policy functions must be a fixed point of the system of functional equations in part 1 and 2 of the definition (where part 2 requires that the equilibrium policy functions are consistent over time with the resource constraint).

The following Lemma (proof in the appendix) is a useful step to characterizing the Markov equilibrium.

Lemma 2 *The first functional equation in Definition 2 admits the following two-stage recursive formulation:*

$$\langle B(b), G(b), T(b) \rangle = \arg \max_{\{b' \leq \bar{b}, g \geq 0, \tau \in [0,1]\}} \left\{ v(\tau, g) - (1 - \psi\lambda)\theta \log g + \beta\lambda V_O(b') \right\}, \qquad (15)$$

where $v(.)$ *is defined as in 12, subject to (6) and (7), and where* V_O *satisfies the following functional equation*

$$V_O(b') = v(T(b'), G(b')) + \beta\lambda V_O(B(b')). \qquad (16)$$

The key difference between the commitment solution and the political equilibrium can be seen by comparing the expressions of V_O^{comm} in (11) and that of V_O in (16). In the political equilibrium, the first generation of voters cannot choose the whole sequence of future policies, but must take the mapping from the state variable into the (future) policy choices as given. For this reason, there is no max operator in the definition of V_O. However, the two programs are identical when $\omega = 1$ (only the old vote), and in this case fiscal policy is time consistent.

What is the source of time inconsistency? When $\omega < 1$, the young, who care directly (i.e., not only through their altruism) about next-period public expenditure, want more public savings than the old. Hence, the young are fiscally more disciplined than their parents. In the commitment solution, the effect of the conflict between "rotten parents" and "disciplined children" is limited to the first-period fiscal policy (as from the second period onwards, their preferences are perfectly aligned). In contrast, such effect is persistent in the political equilibrium since subsequent generations of young voters enter the stage in each new election. The result is less debt accumulation.

We characterize the political equilibrium as follows. First, similar to (13) in the commitment solution, the intra-temporal first-order condition linking g and τ in problem (15) is given by

$$\frac{1+\beta}{(1+\psi)\,\theta} g = A(\tau)\,(1-e(\tau))\,. \tag{17}$$

The only difference between (13) and (17) is in the denominator of the left hand-side term, where λ^{-1} is replaced by ψ.

Next, applying standard recursive methods to the First Order Conditions of (15)-(16), together with (17), leads to the following generalized Euler equation (GEE) describing the equilibrium dynamics of public good provision:[10]

$$\frac{G(B(b))}{G(b)} = \beta\lambda R - \underbrace{\beta\lambda G'(B(b))\left(\frac{1+\lambda^{-1}}{1+\psi}-1\right)}_{\text{the disciplining effect}}, \tag{18}$$

which is a key equation to characterize the political equilibrium. Note, first, that when the political power lies entirely in the old' hands ($\omega = 1$), then $\psi = \lambda^{-1}$, and the "disciplining effect" of the young disappears. In this case, the GEE coincides with the commitment solution in which $g'/g = \beta\lambda R$.

Suppose that a steady-state debt level exists, and denote such level by b^*. Since, in steady state, $G(B(b)) = G(b) = G(b^*)$, then (18) implies that

$$G'(b^*) = -\frac{(1+\psi)(1-\beta\lambda R)}{\beta(1-\lambda\psi)} \equiv \zeta < 0, \tag{19}$$

which is constant and independent of the value of b^*. Thus, $G'(.)$ is negative in the neighborhood of any steady state; higher debt is associated, as one might expect, with lower public spending. Plugging-in $G'(b^*)$ into (18) shows immediately that – in the neighborhood of a steady state – the growth rate of public spending without commitment is higher than with commitment, with the difference being proportional to ζ.

[10]The notion of GEE was first introduced in the literature by XXX (add some discussion)

The disciplining effect introduces a discrepancy between the commitment solution and the political equilibrium that can lead to qualitatively different dynamics. Note that it continues to be possible that the GEE admits a linear equilibrium solution, i.e., one of the type $G(b) = \alpha_{0G} + \alpha_{1G} b$ and $B(b) = \alpha_{0B} + \alpha_{1B} b$ (in this case, $G'(.)$ is a constant). Indeed , we will see a special case featuring a linear equilibrium. However, the presence of $G'(.)$ on the right hand-side opens up the possibility that the equilibrium dynamics of public expenditure and debt be non-linear, and possibly feature multiple steady states.

Another interesting observation is that if an "interior" steady-state, $b^* < \bar{b}$ exists, and b converges monotonically to b^* in a neighborhood of b^*, then $G(b)$ must be concave around b^*.[11] Intuitively, when debt is above the steady state, there must exist a stronger disciplining effect to tighten the public consumption, for debt to fall and go back to the steady state. Conversely, when debt is lower than the steady state, there must exist a weaker disciplining effect leading to an increasing debt towards the steady state.

3 Two examples

In the rest of the paper we parameterize the household production technology as follows:

$$F(h - h_M) = X(h - h_M)^{\xi},$$

where h is the total individual time endowment, $h - h_M \geq 0$ denotes household activity, X is a parameter and $\xi \in ([0, 1]$. To rule out trivial solutions where $h_M = 0$, we assume that $X < w$. An analytical solution of the political equilibrium cannot be obtained in general. However, the model can be solved analytically in some special cases.

In the first case, we set $\xi = 0$, implying that agents cannot substitute market hours with household activity. Due to the log-utility function, labor taxation does not distort labor supply. We will see that in this case, a linear equilibrium exists, and the dynamics of debt resemble qualitatively the commitment solution.

In the second case, we set $\xi = 1$. This implies that market hours are supplied inelastically as long as $\tau \leq \bar{\tau} \equiv 1 - X/w$. However, if taxation exceeds $\bar{\tau}$ market labor supply and tax revenues fall to zero. In this case, the equilibrium expenditure function, $G(.)$, is concave, and a stable interior steady state with positive public good provision can be attained.

[11] Consider a small perturbation of debt from the steady state; $\tilde{b} = b^* + \varepsilon$, $\varepsilon > 0$. The monotone convergence implies that $B \;\; \tilde{b} \;\; \in \;\; b^*, \tilde{b}$. Due to the negative slope of $G(b)$ around b^*, $G \;\; B \;\; \tilde{b} \;\;\;\; > G \;\; \tilde{b}$, which implies that $G' \;\; B \;\; \tilde{b} \;\;\;\; < \zeta$ according to (18). Since $B \;\; \tilde{b} \;\; > b^*$, this establishes that $G'(b) < \zeta$ for $b > b^*$. A similar argument establishes that $G'(b) > \zeta$ for $b > b^*$, by letting $\varepsilon < 0$. So, $G(b)$ must be concave around b^*.

3.1 Example I: $\xi = 0$

As in this example the market labor supply is constant at h, we have that $A(\tau) = (1-\tau)\,wh$ and $e(\tau) = 0$. Furthermore, as the top of the Laffer Curve is attained at $\tau = 1$, the maximum debt is $\bar{b} = wh/(R-1)$. The FOC, (17), can be written as

$$1-\tau = \frac{1+\beta}{(1+\psi)\,\theta wh} g. \tag{20}$$

Plugging-in this solution into the government budget constraint yields

$$b' = \left(1 + \frac{1+\beta}{\theta\,(1+\psi)}\right) g + Rb - wh. \tag{21}$$

Next, we guess G to be linear, $G(b) = \gamma\left(\bar{b} - b\right)$. Then, the GEE, (18), yields:

$$\frac{\gamma\left(\bar{b} - B(b)\right)}{\gamma\left(\bar{b} - b\right)} = \beta\lambda R - \beta\lambda\gamma\left(\frac{1+\lambda^{-1}}{1+\psi} - 1\right). \tag{22}$$

Then, using (22), the budget constraint, (21), the equilibrium condition that $b' = B(b)$, and the expression of $\bar{b}$ given above leads to the following solution for γ;

$$\gamma = \frac{(1-\beta\lambda)\theta(1+\psi)R}{(1+\theta)(1+\beta) + (1-\beta\lambda)\theta\psi}.$$

Finally, substituting g by its equilibrium expression, $g = \gamma\left(\bar{b} - b\right)$, into (20) and (21), leads to a complete analytical characterization. This is summarized in the following Proposition (proof in the text).[12]

Proposition 3 *Assume that $\xi = 0$. Then, the time-consistent equilibrium is given by the following policy functions*

$$\tau = T(b) = 1 - \frac{1}{wh}\frac{(1-\beta\lambda)(1+\beta)R}{(1+\theta)(1+\beta) + (1-\beta\lambda)\theta\psi}\left(\bar{b} - b\right), \tag{23}$$

$$g = G(b) = \frac{(1-\beta\lambda)\theta(1+\psi)R}{(1+\theta)(1+\beta) + (1-\beta\lambda)\theta\psi}\left(\bar{b} - b\right), \tag{24}$$

$$b' = B(b) = \bar{b} - \frac{\theta + \lambda(1+\beta+\theta)}{(1+\theta)(1+\beta) + (1-\beta\lambda)\theta\psi}\beta R\left(\bar{b} - b\right), \tag{25}$$

where $\bar{b} \equiv wh/(R-1)$.

It is interesting to note that $G'(.) = -\gamma < 0$, implying that the disciplining effect in (18) increases the growth rate of public spending, as in the general discussion above.

[12] The results of Proposition 3 extend to economies with population growth and technical change. The analysis of this extension is presented in appendix **??**.

Due to the linearity of $G(.)$, however, the disciplining effect is a constant for any debt level. For this reason, the dynamics do not lead to any stable interior steady state. If $(\theta + \lambda(1 + \beta + \theta)) \cdot ((1 + \theta)(1 + \beta) + (1 - \beta\lambda)\theta\psi)^{-1} \cdot \beta R < 1$, the economy converges asymptotically to the maximum debt level $\bar{b}$, while if this inequality is reverted the debt approaches minus infinity in the long-run. Although the long-run properties of the debt dynamics are qualitatively identical to those of the commitment solution, two differences should be noted. First, there exists a range of parameters such that, under commitments, the economy would accumulate debt till the maximum level ($b \to \bar{b}$), while the political equilibrium leads to an ever growing surplus ($b \to -\infty$). Namely, the political empowerment of future generations is beneficial to them. Second, if we take an economy converging to $\bar{b}$ under both regimes, the slope of the debt function, B(b), is always steeper in the political equilibrium. In other words, public debt grows more slowly in the political equilibrium than under commitment.

Figure 1 provides a geometric representation of an equilibrium converging to $\bar{b}$.[13] Panel *a* shows the equilibrium tax policy: the tax rates increases linearly with the debt level. Panel *b* shows the equilibrium expenditure: public good provision declines linearly with the debt level. Panel *c*, finally, shows the upward sloping equilibrium debt dynamics.

FIGURE 1 (THREE PANELS) HERE

In this example, the economy depletes its resources over time: generation after generation, agents find their private and public consumption progressively crowded out by debt repayment to foreign lenders. This occurs gradually, even in a model without any altruism (i.e., if we set $\lambda = 0$, which would give a standard OLG model). In this case, in the commitment solution the debt converges to $\bar{b}$ in only two periods. In contrast, the political equilibrium features

$$\bar{b} - b' = \bar{b} - B(b) = \frac{\theta}{(1 + \theta)(1 + \beta) + \theta\psi}\beta R(\bar{b} - b),$$

where $\psi = \omega / (1 - \omega)$. As the expression above shows, in spite of the lack of concern for future generations, voters do not support a "big party" which would consume the present value of the entire future income stream. In fact, the old would always support a big party, but young voters disagree because they care about what public expenditure will be when they become old. If debt were set to its maximum level right away, the young would suffer from their public consumption falling to zero in their old age. To see how crucial the concern for *public*

[13]Paramter values are given in Table X-1 below except for wh, which is adjusted to make $\bar{b}$ equal to its counterparts in the following cases.

consumption is, observe that, if $\theta = 0$, then the initial young and old voters would agree to set $b = \bar{b}$, and the young would secure private consumption in old age through savings. Thus, the key assumption is that private savings cannot buy public goods. The concern for public consumption in old age becomes then a partial substitute for lack of altruism towards future generations.

As it is young voters who discipline fiscal policy, increasing the political influence of the old (i.e., increasing ω) leads to higher debt, higher taxes, and an increase in current public good provision. Changing ω does not affect the steady state, but a larger ω implies a faster depletion of both private and public consumption. If the young have no influence on the political process ($\omega = 1$), the maximum debt is attained in the first period. Conversely, if the old have no political representation ($\omega = 0$) the debt dynamics converge to $\bar{b}$ at the slowest rate.

Finally, it is worth noting that the political equilibrium and the commitment solution are identical in the first period (proof available upon request). This equivalence implies that the disciplining effect in the political solution is of the same size as in the first period of the commitment solution. This is due to the log-preference assumption over public goods, and that future public goods are linear in $\left(\bar{b} - b\right)$. These two features imply the cancellation of two opposing effects; if public funds were to be spent more lavishly in the future, then current decision makers might be expected to leave less for the future. On the other hand, if future governments were to spend more lavishly, they would be driven into public poverty earlier, which might be expected to induce the current policy decision makers to increase public savings.[14]

3.2 Example II: $\xi = 1$

Next, we turn to the second tractable case, where we assume constant returns to labor in the household production technology, i.e., $\xi = 1$. In this case, taxation does not distort labor supply as long as $\tau \leq \bar{\tau} \equiv 1 - X/w$, namely, agents only work in the market. If $\tau > \bar{\tau}$, however, agents switch all their time endowment into household production, and the tax revenue falls to zero. Thus, $\bar{\tau}$ is the top of the Laffer curve. Since rational voters would never choose a tax rate inducing no public good provision, the political equilibrium necessarily features $\tau \leq \bar{\tau}$. Thus, the model is observationally equivalent to one in which the government is committed not to tax income over the upper bound rate $\bar{\tau}$.

[14]To see this result technically, note that whenever the policy rule is on the following form $G(b) = \gamma \ \bar{b} - b$ for some γ, the cross derivative $\frac{\partial^2 V_Y(b)}{\partial b \partial \gamma}$ is always equal to zero. This means that the future lavishness, i.e. γ, will not impact on current political decisions.

Three sub-cases can be distinguished. First, when the interest rate is sufficiently low, the economy behaves similarly to the linear equilibrium case: debt converges asymptotically to its maximum level, which is now $\bar{b} = \bar{\tau}wh/(R-1)$, and the economy features public poverty in the long run, i.e. $\lim_{t\to\infty} g_t = 0$. However, since taxes are bounded from above by $\bar{\tau}$, private consumption does not fall to zero, but converges to $(1-\bar{\tau})\, wh > 0$. Since the equilibrium dynamics resembles those of the benchmark model, we omit the analysis of this case (details available upon request). Second, when the interest rate is sufficiently high, the economy accumulate a perpetual surplus, and again there are no novel aspects.

The third case, which corresponds to an intermediate range of R, is more interesting. Here, the equilibrium is qualitatively different; an economy starting from low initial debt converges in finite time to a steady-state equilibrium such that $\tau = \bar{\tau}$, but debt is strictly lower than $\bar{b}$. In a neighborhood of the steady state, the equilibrium dynamics of the fiscal variables are given by steady-state debt level is given by[15]

$$b' = B(b) = b_0^* \equiv \bar{b}\left(1 - \frac{\theta(1+\psi)(1-\bar{\tau})}{\bar{\tau}(1+\beta)}\right) \tag{26}$$

$$\tau = T(b) = \bar{\tau} - \frac{R(1+\beta)}{wh(1+\beta+\theta(1+\psi))}(b_0^* - b) \tag{27}$$

$$g = G(b) = \frac{wh\theta(1+\psi)(1-\bar{\tau})}{1+\beta} + \frac{\theta(1+\psi)R}{1+\beta+\theta(1+\psi)}(b_0^* - b) \tag{28}$$

Figure 2 provides a geometric representation of the equilibrium. Panel *a* shows the equilibrium tax policy: taxes increase linearly with the debt level as long as $b \le b_0^*$. Thereafter, T is flat at $\tau = \bar{\tau}$. Panel *b* shows the equilibrium expenditure: public good provision declines linearly with the debt level as $b \le b_0^*$. To the right of b_0^*, the government loses the ability to adjust taxes, and thus the government expenditure function becomes steeper. Panel *c*, finally, shows that the policy is flat around b_0^*. Therefore, if the debt level starts sufficiently close to b_0^*, it converges to b_0^* in one period and remains at b_0^* thereafter. In other words, debt is strongly mean-reverting after a shock. The figure also shows that the debt and expenditure policy function feature discontinuous dynamics for high initial debt levels. Moreover, there are multiple steady states. However, these are fragile features of this particular example which disappear when one consider smooth labor supply distortion. Instead, as the next section will show, the existence of a locally stable steady-state debt level lower than $\bar{b}$ with an associated tax level lower than one and positive public good provision are robust features that carry on to the more general case.

[15] A formal Proposition with a complete characterization of the equilibrium and its proof are provided in the appendix.

What is the intuition for the dynamics around b_0^*? Imagine, to make the case sharper, that voters are not altruistic ($\lambda = 0$). Yet, young voters care about public good provision one-period ahead. In the linear equilibrium of example I, this concern for the near future did not prevent the debt from increasing in every period, progressively impoverishing the future generations. Why? Because future generations could not threaten credibly current voters to cut public good provision drastically should they inherit a large debt. Voters would anticipate that the next generation would make part of the adjustment to a larger debt in the form of higher taxes and debt. Although government expenditure would also fall, these adjustments mitigate the expenditure-cutting effect. As a result, each generation of voters "passes the bill" to the next and only suffers a partial sacrifice of public consumption.

Passing the bill to future generations becomes harder, however, when taxation is increasingly distortionary. In example II, this is particularly stark; the tax rate cannot exceed $\bar{\tau}$. As the debt approaches b_0^* (and taxes approach $\bar{\tau}$), Voters anticipate that future generations will not be able to contain the reduction of public expenditure by increasing taxes over $\bar{\tau}$. Hence, the disciplining effect becomes very strong. Note that $G(.)$ is (piece-wise-linear-) concave around the steady state b_0^*. To the right of b_0^*, the disciplining effect is so strong that debt falls and reverts to b_0^* in just one period. In contrast, to the left to b_0^*, $G(b)$ is less steep, implying a smaller disciplining effect. In fact, voters support an increasing debt, and b_0^* is a steady state.[16]

FIGURE 2 (THREE PANELS) HERE

4 The General Case: $\xi \in (0,1)$

The intuition behind the result of example II carries over to the general case with $\xi \in (0,1)$, with smooth labor supply distortions. In this case, however, the equilibrium policy function are non-linear (nor piecewise linear), and the model does not admit an analytical solution. We must therefore resort to numerical analysis.[17]

[16] A related intuition explains why there is no internal steady state when the interest rate is low? The reason is that G' is bounded from below by ζ. Since the function G is continuous, the GEE (18) therefore implies an ever-decreasing sequence of public goods. Hence, with a low interest rate, the disciplining effect is not strong enough to generate falling debt for any $b \leq \bar{b}$, so $b \to \bar{b}$, irrespectively of the initial b.

[17] We adopt a standard projection method with Chebyshev collocation (Judd, 1992) to approximate T and G, according to the First Order Conditions (17) and (18). The basic idea of the projection method is to approximate some unknown functions on a basis of functional space.

We calibrate parameters as follows. First we think of one period a corresponding to thirty years. Thus, we set $\beta = 0.98^{30}$ and $R = 1.025^{30}$. implying a 2% annual discount rate and a 2.5% annual interest rate. This value of β is standard in the macroeconomics literature, and the value of R is consistent with the average real long-term U.S. government bond yields (2.5%) between 1960 and 1990. There are few quantitative clues for ω and λ. So we simply set $\omega = 0.5$ (equal political weights on the young and old) and $\lambda = 0.75$.[18] We use the results from our example II to calibrate the two parameters, θ and $\bar{\tau}$. In particular, we choose parameters so as to match, in the example, the average debt-GDP ratio (0.30) and the government expenditure-GDP ratio (0.18) in the U.S. from 1960 to 1990. The calibration yields $\bar{\tau} = 0.51$ and $\theta = 0.37$.[19]. Finally, we normalize wh to unity in this tractable case. Table X-1 summarizes the parameters.

Table X-1

$\beta = 0.98^{30}$	$R = 1.025^{30}$	$\omega = 0.50$	$\lambda = 0.75$	$\theta = 0.37$	$\bar{\tau} = 0.51$	$wh = 1$

In addition, we must assign values to w, X and ξ. To this aim, we normalize $w = 1$ in the tractable case with $\xi = 1$ and we let $h = 1$ in all cases. Then, to make it easier to compare the simulated economy with the tractable case in which $\xi = 1$, we set w and X in a sequence of economies with different ξ according to the following two conditions. First, the top of the Laffer curve is constant across experiments at $\tau = \bar{\tau}$, and second, the tax revenue at the top of the Laffer curve is also constant and equal to the one in the tractable case with $\xi = 1$. The details are given in the appendix.

Figure 3 describes the equilibrium dynamics of two simulated economies, with respectively $\xi = 0.90$ and $\xi = 0.50$.[20] In both cases, the tax policy function is increasing in b (panel a) while the public expenditure function is a decreasing in b (panel b). The debt policy is an increasing convex function of b which crosses the 45-degree twice: first at an interior steady-state level, and then at the maximum debt. Interestingly, only the interior steady-state is stable. Namely,

[18] We must also assume $\lambda \in (\lambda_{\min}, \lambda_{\max})$, where $\lambda_{\min}$ and $\lambda_{\max}$ are implied by the conditions $R > 1+(1+\psi)/\zeta$ and $\beta\lambda R < 1$. Given the parameter values of β, R and ω, $\lambda_{\min}$ and $\lambda_{\max}$ are equal to 0.68 and 0.87, respectively.

[19] More precisely, we use the steady-state expressions of g and b in (28)-(26), each divided by wh, as proxies for the debt-GDP ratio. Thus, we set

$$\begin{aligned} \frac{\theta(1+\psi)(1-\bar{\tau})}{(1+\beta)} &= 0.30, \\ \frac{1}{R-1} \quad \bar{\tau} - \frac{\theta(1+\psi)(1-\bar{\tau})}{1+\beta} &= 0.18. \end{aligned}$$

Given the other parametrs, these two equations identify $\bar{\tau}$ and θ.

[20] In the internal steady state of the two simulated economies with $\xi = 0.90$ and $\xi = 0.50$, the elasticities of market labor supply with respect to w, denoted by $\chi(h_M^*) = \frac{\partial h_M^*/h_M^*}{\partial w/w}$, are equal to 0.2309 and 0.4878, respectively. (ADD SOME DISCUSSION ON THE ESTIMATED VALUE OF THE ELASTICITY).

as long as the economy starts at $b < \bar{b}$, it converges to the internal steady state with no public poverty.[21]

FIGURE 3 (THREE PANELS) HERE

To earn intuition, it is useful to go back to the analytical examples. In all cases, the tax function is non-decreasing and concave (strictly concave if $\xi > 0$), while the expenditure function is decreasing and concave (strictly concave if $\xi > 0$). In example II, the policy functions are piece-wise linear with a kink at the steady state. This is because taxation is non-distortionary to the left of $\bar{\tau}$ and infinitely distortionary to the right of it. In the general case of $\xi \in (0,1)$, the tax function flattens as b increases, since larger b implies requires higher taxes to be financed, and tax-collection becomes increasingly ineffective. At high debt levels, governments tend to react to further debt increases by cutting expenditure more than by increasing taxes. This shows up in the concave shape of the and T functions. In example II, the slope of the G function changes discontinuously, whereas in the numerical examples the derivative of G falls smoothly. In example I ($\xi = 0$), taxation is not distortionary. Thus, a larger debt is matched by a proportional increase in taxation and cut in expenditure.

Table X-2 reports steady state values of variables of interests under different values of ξ.

Table X-2

	$\chi(h_M^*)$	τ^*	g^*	b^*	g^*/wh_M^*	b^*/wh_M^*
$\xi = 1.000$	0.0000	0.5100	0.1843	0.2967	0.1843	0.2967
$\xi = 0.975$	0.0488	0.4724	0.1935	0.2639	0.1892	0.2580
$\xi = 0.950$	0.1150	0.4562	0.1938	0.2566	0.1859	0.2463
$\xi = 0.900$	0.2309	0.4388	0.1901	0.2551	0.1775	0.2381
$\xi = 0.700$	0.4373	0.4032	0.2139	0.2235	0.1878	0.1963
$\xi = 0.500$	0.4878	0.3696	0.2998	0.1277	0.2519	0.1073

5 Three Applications

5.1 War Finance

In this section, we introduce uncertainty and fiscal shocks. In particular, we assume that at occasions the government is forced to "fight wars", whose financing requires an exogenous

[21]Clearly, simulations do not establish that these equilibria are unique. However, we run many simulations and never found any qualitatively different equilibrium from those display in the figure.

spending of Z units per war period. The metaphor of wars is intended to capture more generally fiscal shocks increasing the marginal value of government spending. During a war, the government's budget constraint (6) changes to

$$b' = g + Rb - \tau w h_M(\tau) + Z, \tag{29}$$

while during peace it reverts to (6).

To earn some intuition, it is instructive to start by analyzing two simple examples where the war is an unanticipated transitory shock (formally, war is a zero-probability event). In this case, a one-period war is identical to a temporary increase in the debt, from b to $b + Z/R$. The local dynamics around the steady state determine how the economy reacts to the shock. For instance, in the tractable case of example II, analyzed in Section 3.2 ($\xi = 1$), if the size of the war is relatively small (so that the local analysis applies), the war shifts the real debt from b_0^* to $b_0^* + Z/R$. Clearly, since the tax constraint ($\tau \leq \bar{\tau}$) was binding even before the war, an additional exogenous spending need will only make the constraint more binding and τ remains constant at $\bar{\tau}$. Moreover, the government sets $b' = b_0^*$. Consequently, the war is financed entirely by a reduction in spending:

$$G(b_0 | z_W) = G(b_0 | z_P) - X.$$

In this case, there is no (non-war) expenditure smoothing, and the debt level returns to the pre-war level in just one period after the war ends. The dotted lines of Figure 4 shows the post-war expenditure and debt dynamics of this simple case (clearly, taxes do not move as the constraint is binding).

In the general case where $\xi < 1$ (see the solid line in Figure 4), supposing that the economy was in a stable interior steady state before the war, the tax rate shoots up and public expenditure shoots down. Public debt first increases to finance the war, and then returns smoothly (as opposed to the case of $\xi = 1$, when this happened in just one period) to its steady-state level. Noticeably, debt is used to finance the war respectively.

FIGURE 4 (THREE PANELS) HERE

We can now move now to the case in which the probability distribution of fiscal shocks is non-degenerate. Since there is a positive probability that the country experiences a perpetual war, and the government must be solvent in all states of nature, the maximum debt level now becomes

$$b \leq \frac{\max_\tau \{\tau w h_M(\tau)\} - Z}{R-1} \equiv \bar{b}. \tag{30}$$

We denote by z^i the state of the economy, where $i \in \{P, W\}$ stands for peace and war, respectively. The state of the economy is assumed to evolve following a first-order Markov process, with transition probability matrix Π, whose elements we denote by p_{ij} (where $p_{i,i} + p_{j,i} = 1$, and $j \neq i$).

The political equilibrium is characterized formally by the following fixed-point problem;

$$\left\langle \begin{array}{c} B\left(b|z^i\right), \\ G\left(b|z^i\right), \\ T\left(b|z^i\right) \end{array} \right\rangle = \arg \max_{\left\{b' \leq \bar{b}, g \geq 0, \tau \in [0,1]\right\}} \left\{ \begin{array}{c} (1+\psi)\,\theta \log g + (1+\beta) \log A(\tau) \\ +\beta\left(p_{i,i} V_O\left(b'|z^i\right) + p_{j,i} V_O\left(b'|z^j\right)\right) \end{array} \right\},$$

subject to either (6) or (29), and (30). $V_O\left(b|z^i\right)$, denoting the utility of the old $V_O\left(b|z^i\right)$, is given by the following functional equation

$$\begin{aligned} V_O\left(b|z^i\right) &= (1+\lambda)\,\theta \log\left(G(b|z)\right) + (1+\beta)\,\lambda \log A\left(T(b|z)\right) \\ &\quad +\beta\lambda\left(p_{z,z} V_O\left(b'|z\right) + p_{z',z} V_O\left(b'|z'\right)\right). \end{aligned} \tag{31}$$

The analysis leads to the following generalization of the GEE to a stochastic environment (see appendix for the derivation);[22]

$$E\left(\left.\frac{G(B(b))}{G(b)}\right| z = z^i\right) = \beta\lambda R - \beta\lambda E\left(G'(B(b))\right| z = z^i\left)\left(\frac{1+\lambda^{-1}}{1+\psi} - 1\right). \tag{32}$$

Figure 5 show the dynamics of a simulated economy where we assume the following transition Markov matrix

$$\Pi = \begin{bmatrix} p_{PP} = 0.9 & p_{PW} = 0.1 \\ p_{WW} = 0.75 & p_{WW} = 0.25 \end{bmatrix}.$$

This implies that war is less likely than peace, and that the state is characterized by some persistence.

FIGURE 5 (six panels) HERE

The first three panels represent, respectively, g, τ and b' as function of b and z.[23] Continuous (dotted) lines represent the level of the policy conditional on war (peace). The first panel shows that taxes are increasing in b and larger in war than in peace times. The second panel shows

[22]Note that the left hand-side of (32)is the conditional expectation of the marginal rate of substitution of public consumption between time t and t+1, given the state of nature (war or peace) at t.

[23]We assume that a war costs 10% of maximum tax revenues.

that expenditure (excluding war expenditure) is decreasing in b and larger in peace than in war times. Finally, the third panel show the dynamics of debt. In all panels, the continuous (dotted) line can be interpreted as the decisions rule associated with one particular history, namely when the economy experiences an infinite sequence of war (peace) times. The stationary distribution of debt is between the upper and lower steady states.

Panel 4-6 plot the evolution of policy policies. The results are qualitatively similar to those solid lines of Figure 4. The main differences are that in this case the anticipation of the possibility of future wars induces an additional precautionary motive for public savings in times of peace.

It is also useful to analyze the commitment solution in this stochastic environment. A simple generalization of Lemma 1 holds.[24] The analysis of the First Order Conditions leads to a stochastic version of the Euler equation under commitment, (14);

$$E\left(\left.\frac{g'}{g}\right| z = z^i\right) = \beta\lambda R, \qquad i \in \{P, W\}. \tag{35}$$

FIGURE 6 (3 panels) HERE

Figure 6 is the analogue of Figure 5. In particular, the third panel shows that an economy experiencing perpetual war converges to the debt limit, while an economy experiencing perpetual peace (but perceiving a positive probability that a war starts) settles down below the maximum debt. Note that an even under commitment there is some scope for the government to reduce debt in times of peace. However, such scope is limited to a precautionary motive: agents anticipate that some future generation may suffer war, and wish to limit the extent to which future government consumption must be cut. This effect is significantly smaller than in the politico-economic model. In Table X-3, we denote by b_P^* and b_R^* the steady-state debt levels with perpetual peace in the political equilibrium and the Ramsey allocation, respectively, for two different values of ξ. In all cases b_P^* is substantially lower than b_R^*, and is in fact rather

[24] In particular, after the first period, the problem can be expressed by the following recursive programme;

$$V_O \quad b|z^i \quad = \max_{\{\tau,g,b'\}} v(\tau, g) + \beta\lambda \quad p_{i,i} V_O \quad b'|z^i \quad + p_{j,i} V_O \quad b'|z^j \tag{33}$$

subject to (6) or (29) and (30). The functional equation (33) is the stochastic analogue of (11).

The analysis of the First Order Conditions leads to the following generalization, state-by-state, of equation (13);

$$\frac{1+\beta}{1+\frac{1}{\lambda} \quad \theta} g^i = A \quad \tau^i \qquad 1 - e \quad \tau^i \qquad , \tag{34}$$

See the appendix for the details of the analysis.

close to $\bar{b}$, showing that the precautionary motive can only induce a limited amount of public saving.

Table X-3

	b_P^*	b_R^*
$\xi = 0.90$	0.2295	0.4047
$\xi = 0.70$	0.1883	0.4047
$\xi = 0.50$	0.0932	0.4046

5.2 Political shocks

In this section, we introduce time-varying preference in the form of cohort-specific ideological shocks affecting agents' appreciation for public-good consumption. For simplicity, we assume that the realization of the shock is identical across all voters of a given age. In particular, we now let $\theta_Y \in \{\theta_r, \theta_l\}$ and $\theta_O \in \{\theta_r, \theta_l\}$ to denote the preference of the young and of the old, respectively, where $\theta_r < \theta_l$ (R stands for "right-wing" and L stands for "left-wing"). The late 1960's can be regarded as a leftist wave, where for no particular reason, agents' faith and taste for the size of governments increased. The neo-cons revolution of the 1980's is an example of a right-wing wave.

The realization of preference shocks is assumed to follow a first-order Markov process. More specifically, we denote by $p_{l,r}$ the probability that, conditional on the current young generation being rightist, a leftist young generation materializes in the next period. Equivalently, $p_{l,r}$ is the probability that, conditional on the current young voters being rightist, next period's voting population will consist of rightist old and leftist young agents. We define $p_{l,l}, p_{r,l}$ and $p_{r,r}$ in a similar fashion. By these definition,

$$p_{l,l} + p_{r,l} = p_{l,r} + p_{r,r} = 1.$$

We impose no restriction on the persistence of political shocks.

The equilibrium definition must in this case be amended to allow for heterogenous preferences of young and old over public good provision (formally, we have additional state variables). Thus, the equilibrium policy functions will be denoted by $T(b|\theta_Y, \theta_O), G(b|\theta_Y, \theta_O)$ and $B(b|\theta_Y, \theta_O)$, where θ_Y, θ_O denotes the state of preferences of the current voters, young and old.

For reasons that will be clarified later, we focus our main discussion on a version of the model without altruism ($\lambda = 0$), and then discuss separately the effect of altruism. Also, we start from the tractable case where $\xi = 0$ (the analogue of example I). In this case, a linear equilibrium exists, which the following Proposition characterizes (proof in appendix).

Proposition 4 *Assume that $\xi = 0$ and $\lambda = 0$. Then, the equilibrium with political uncertainty is given by the following policy functions.*

$$T(b|\theta_Y, \theta_O) = 1 - \frac{(1-\omega)R(1+\beta)}{wh((1-\omega)(1+\theta_Y)(1+\beta)+\omega\theta_O)}\left(\bar{b} - b\right),$$

$$G(b|\theta_Y, \theta_O) = \frac{((1-\omega)\theta_Y + \omega\theta_O)R}{\omega\theta_O + (1-\omega)(1+\theta_Y)(1+\beta)}\left(\bar{b} - b\right),$$

$$B(b|\theta_Y, \theta_O) = \bar{b} - \frac{(1-\omega)\theta_Y\beta R}{\omega\theta_O + (1-\omega)(1+\theta_Y)(1+\beta)}\left(\bar{b} - b\right),$$

where $\bar{b} \equiv wh/(R-1)$, and $\theta_Y \in \{\theta_r, \theta_l\}$ and $\theta_O \in \{\theta_r, \theta_l\}$ denotes the political preferences of the young and old voters, respectively.

A first interesting observation is that neither the variance nor the persistence of shocks have any effect on the equilibrium. A permanent change in political preferences, for instance, has the same effect as a temporary one, and more generally, the probabilities $p_{j,i}$ do not enter the equilibrium functions $T(.)$, $G(.)$ and $B(.)$ – they only depend on the state of debt and on the current distribution of political preferences –. This surprising result depends on the cancellation of two opposite effects, an income and a substitution effect. To understand this point, suppose both the young and the old to be initially (say, at t) leftist, but they anticipate that the next generation (born at $t+1$) will be rightist, i.e., has a low appreciation for government expenditure. Clearly, this has no influence on old voters at t. Consider the young at t. Since the next generation will spend a small share of $\bar{b}-b'$ into public good provision, a "responsible" fiscal policy has a lower return in terms of future public good consumption. The "substitution effect" calls for an increase in current debt. But the "income effect" goes in the opposite direction: precisely because the next generation will not deliver much public good, it is important that it inherits a low public debt. So, the expectation of a shift to the right strengthen (from the income effect standpoint) the fiscal policy of the leftist young. In the log-specification (and under no altruism).

This result is of independent interest. Persson and Svensson (1989)(Persson & Svensson 1989a) argued in an influential article that strategic considerations affect the debt policy of governments with heterogenous preferences for public consumption when there is a positive probability of non-reelection. For instance, a right-wing government with a low taste for public consumption may issue more debt when it knows that it will be replaced by a left-wing government with a stronger taste for expenditure. Their result is derived in a two-period model. Our generalization to an infinite horizon shows that in general strategic effects can go either way, due to the concomitant presence of income and substitution effects. This may explain why the empirical literature has found mixed results (see ...).

Second, the policy functions in Proposition 4 provide some interesting comparative statics. First, as expected, both T and G are increasing in both θ_O and θ_Y (taxes and public good provision are increasing with the appreciation for public consumption). However, θ_O and θ_Y have opposite effects on the debt policy, B. An increase in θ_O increases B whereas an increase in θ_Y decreases B. As the old become more eager to consume public goods, they push for more debt. In contrast, more appreciation for public consumption make the young wary of debt. Since they care about next-period public consumption, the more they care for public consumption the more they are debt averse.

An interesting experiment is that of a two-period transition from a society where all agents have a high preference for public consumption ($\theta_Y = \theta_O = \theta_L$) to one where all agents have a low preference for public consumption ($\theta_Y = \theta_O = \theta_R$). In the first period, θ_Y falls and θ_O does not change, whereas in the second period θ_O falls and θ_Y remains low. Figure 7 shows the effects of such sequence of policy shocks on the equilibrium policy function.[25] The tax policy, T, shifts down in the first period, and then further down. The policy function G shifts down in the first period, and then up in the second. Finally, the policy function B shifts up in the first period, and then down in the second. However, the net effects of the ideological shift on G and B are unambiguous. In particular, $G(b|\theta_r, \theta_r) < G(b|\theta_l, \theta_l)$, and $B(b|\theta_r, \theta_r) > B(b|\theta_l, \theta_l)$, namely a shift to the right leads unambiguously to more debt accumulation.

Similar results obtain in the case of elastic labor supply. While in the linear case political shocks have only transitory effects (as in the long run the economy falls in all cases into immiseration), in the case of elastic labor supply political shocks have both short-term and long-term effects. We calibrate the model as in Table X-1, with two exceptions. First, since we have set $\lambda = 0$, we must reparametrize R that we set equal to 1.06^{30} in order to have interior steady states under different political regimes. Second, we set $\omega = 0$, i.e., we assume that all political power is in the hands of the young. This simplification is introduced for purely expositional purposes, as it allows us to analyze changes in political preferences in the form of one-period shocks, since when $\omega = 0$, θ_O has no effect on the equilibrium (see the expressions in Proposition 4 for the linear case).[26] Finally, we assume that θ_L and θ_R are 10% above and below θ, respectively. We consider two alternative cases: in the first $p_{l,l} = p_{r,r} = 1$ (unanticipated shokcs), and in the second $p_{l,l} = p_{r,r} = 0.5$ (i.i.d. shocks). The values of

[25] The parameter values are given in Table X-4. See the discussion below for the parameterization.

[26] The reason why θ_O and θ_Y have different effects lies in the mechanics of the probabilistic voting model. Old voters only care about public good provision. Moreover, they support the maximum feasible budget deficit. Reducing θ_O is identical to reducing ω, since the model is sensitive to the intensity of voters preferences Therefore, lowering θ_O will lead to lower taxes, lower spending and lower debt due to a loss of political influence of the old.

Details of simulations with $\omega = 0.5$ are available upon request.

parameters are summarized in Table X-4.

Table X-4

$\beta = 0.98^{30}$	$R = 1.06^{30}$	$\omega = 0$	$\lambda = 0$	$\bar{\tau} = 0.51$
$\theta_L = 0.40$	$\theta_R = 0.33$	$p_{l,l} = 0.5/1$	$p_{r,r} = 0.5/1$	$wh = 1$

Figure 7 plots the equilibrium policy rules under the two different political regimes for the case in which the political change is both permanent and totally unanticipated, $p_{l,l} = p_{r,r} = 1$. Dotted lines are for the left-wing regime ($\theta_Y = \theta_l$), while solid lines for the left-wing regime ($\theta_Y = \theta_r$). The arrows in the first panel also shows the dynamic adjustment of an economy starting as in the steady state of the left-wing regime and moving to the right-wing regime. As one can see, though the political regime switch leads to less public good provision and more public debt over time, the evolution of tax rates can be non-monotonic. This is because more public debt leaves heavier financial burden on the government budget, which forces the subsequent rightists to raise tax rates. Figure 8 plots the time-series dynamics of g, τ and b under the political regime shift. The solid lines show the case described in Figure 7 ($p_{l,l} = p_{r,r} = 1$). The dashed lines give the results with persistent politcal regimes ($p_{l,l} = p_{r,r} = 0.9$) and the dotted lines show the i.i.d. case ($p_{l,l} = p_{r,r} = 0.5$). Two remarks are in order. First, public policies in all cases feature the same dynamic features. Public spending decreases and public debt increases over time, while the tax rate goes down in the first period, and then increases for financing larger public debt. Second, there is a tendency of policy convergence between the left-wing and the right-wing as political regimes being less persistent. [**To be explained...**]

FIGURE 7 (Three Panels) HERE

FIGURE 8 (Three Panels each with two cases) HERE

Finally, we discuss altruism. Introducing altruism in a model in which preferences change raises non-trivial issues. In particular, do parents value public consumption according to their own taste ("paternalistic preferences"), or do they respect the (unknown) preference of their children and grand-children. If we assume paternalistic preferences, the introduction of altruism has no major effect on the result discussed above. With non-paternalistic preferences, however, new effects arise. For instance, leftist parents may choose to leave high debt when they expect that their children will be, with high probability, rightist. In other words, non-paternalistic altruism introduces an intertemporal substitution motive in public consumption inducing dynasties to consume more (and issue more debt) in times of high preferences for public consumption (i.e., in leftist periods). When political shocks are highly persistent, this effect is dominated by the mechanism discussed above. For instance, consider again the calibrated example with $\lambda = 0.75$. Suppose that the unconditional probability of θ_L and θ_R is the same, but the realization of the shocks are highly persistent, ($p_{r,r} = p_{l,l} = 0.9$). Then, the dynamics are similar to those discussed earlier on, as shown in Figure 8.[27] However, in economies characterized by a combination of high (non-paternalistic) altruism, and a low persistence of ideological shock the results can even be reverted (i.e., leftist governments issue more debt). For the reasons just described, we do not regard the mechanism behind this result to be realistic, and we conclude therefore that the theory delivers its natural prediction in the case with no altruism or with paternalistic altruism.

FIGURE 9 (Three Panels) HERE

6 Conclusion

TO BE WRITTEN

[27] Similar results hold woth this parameterization for $p_{l,l} = p_{r,r} < 0.75$.

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Figure 1: equilibrium policy rules when ξ=0

Figure 1-1

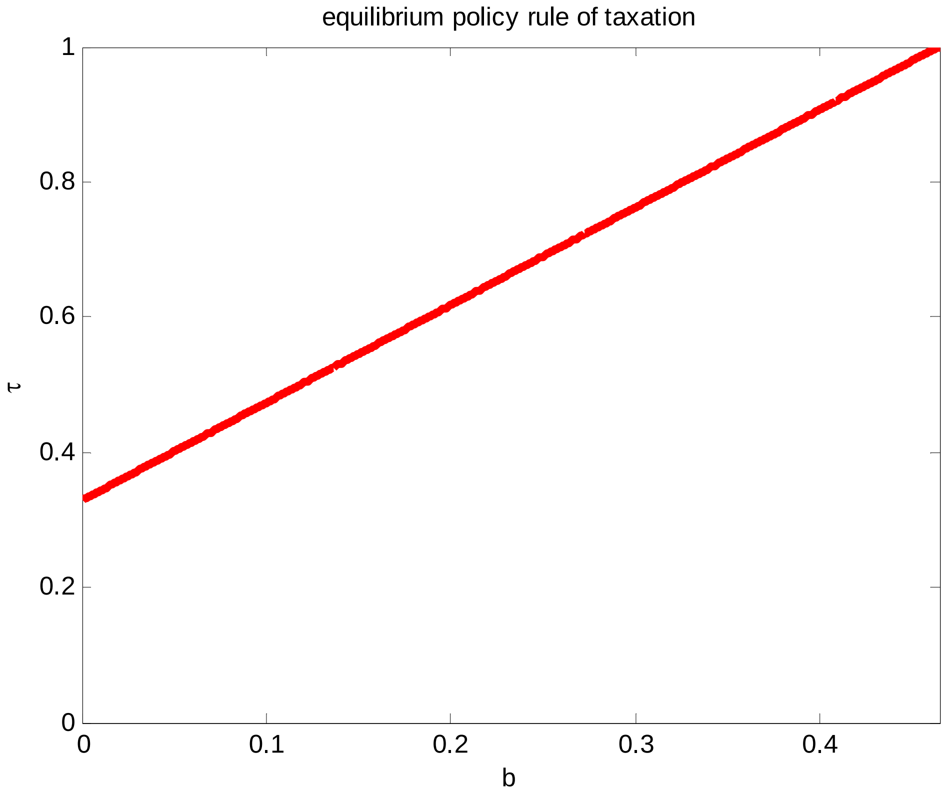


Figure 1-2

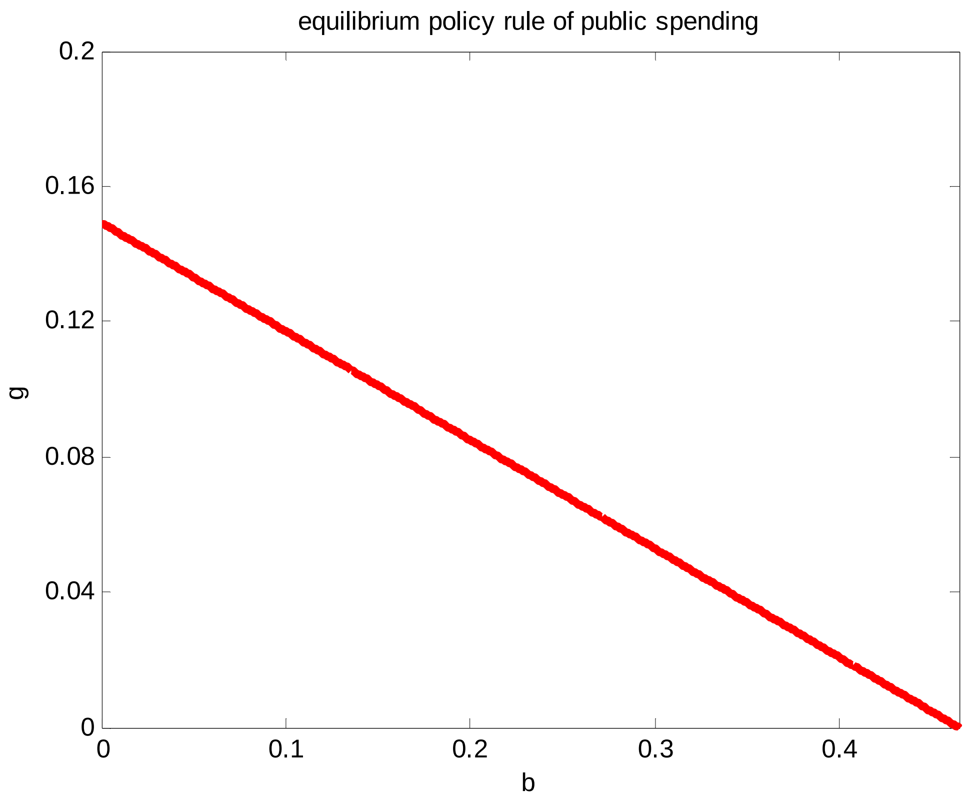

equilibrium policy rule of public spending
0.2
0.16
0.12
0.08
0.04
0
g
0
0.1
0.2
0.3
0.4
b

Figure 1-3

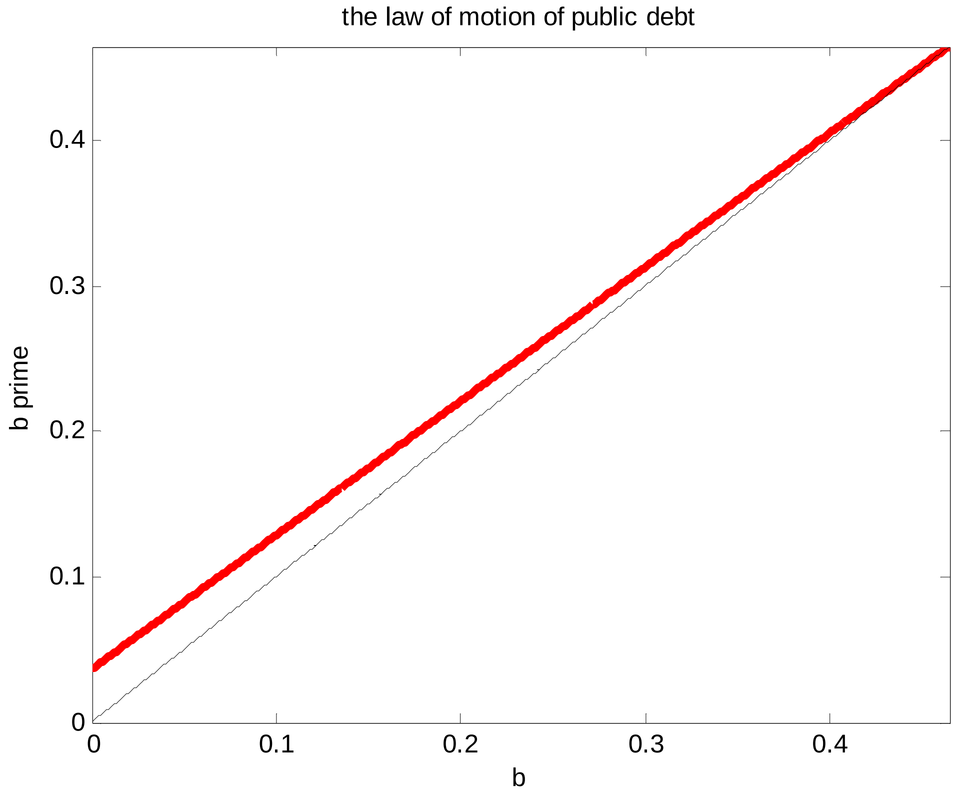

the law of motion of public debt
b prime
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
b

Figure 2: equilibrium policy rules when ξ=1

Figure 2-1

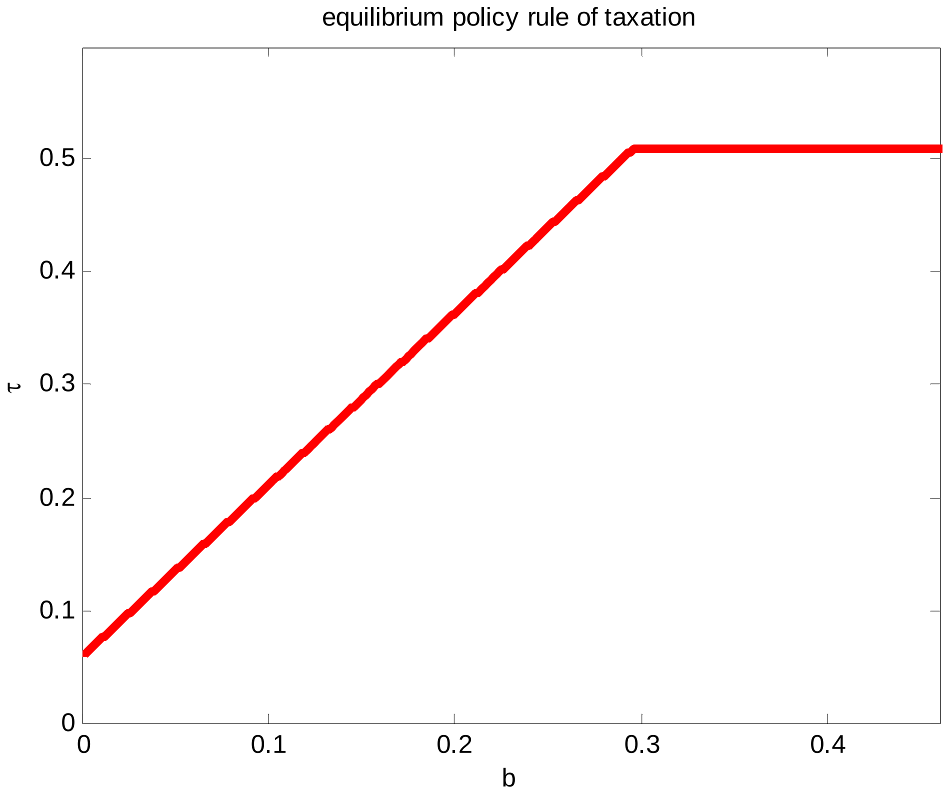


Figure 2-2

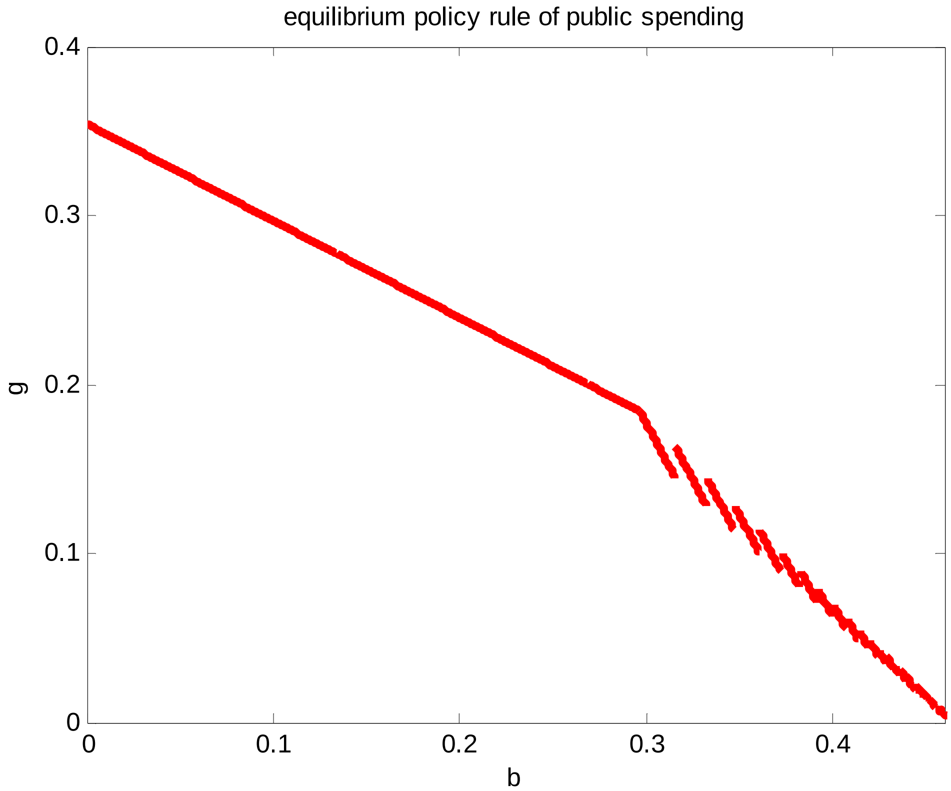
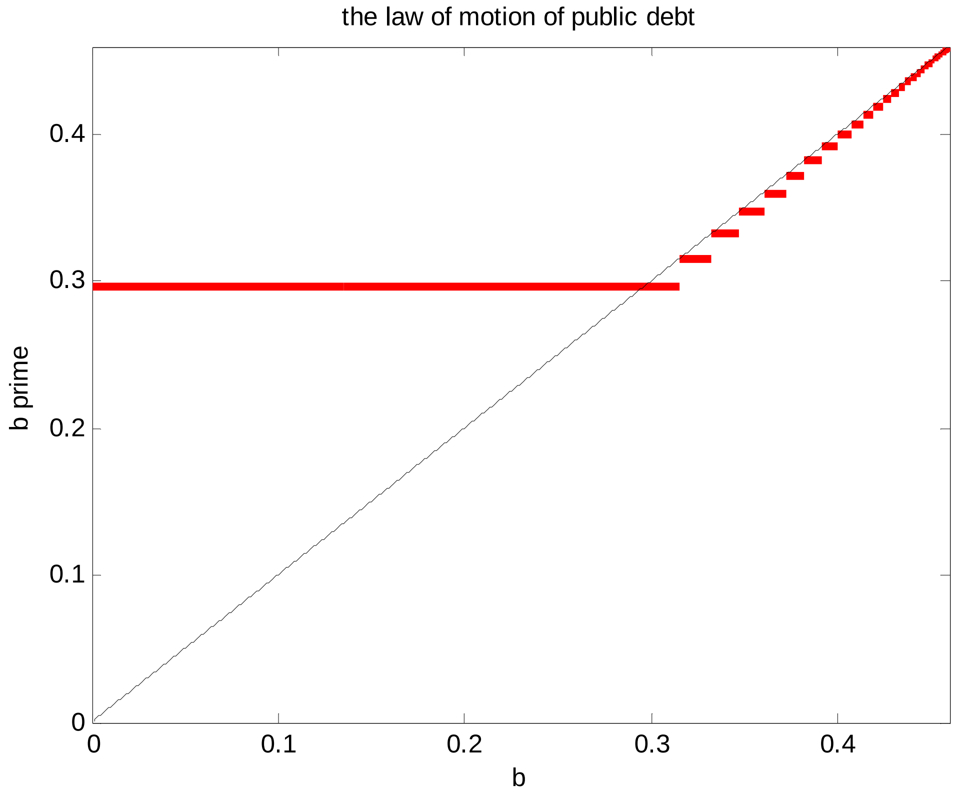

equilibrium policy rule of public spending
0.4
0.3
0.2
0.1
0
g
0
0.1
0.2
0.3
0.4
b

Figure 2-3



the law of motion of public debt
0.4
0.3
0.2
0.1
0
b prime
0
0.1
0.2
0.3
0.4
b

Figure 3: equilibrium policy rules when ξ (0,1)

(solid line and dotted line stand for ξ=0.90 and ξ=0.50, respectively)

Figure 3-1

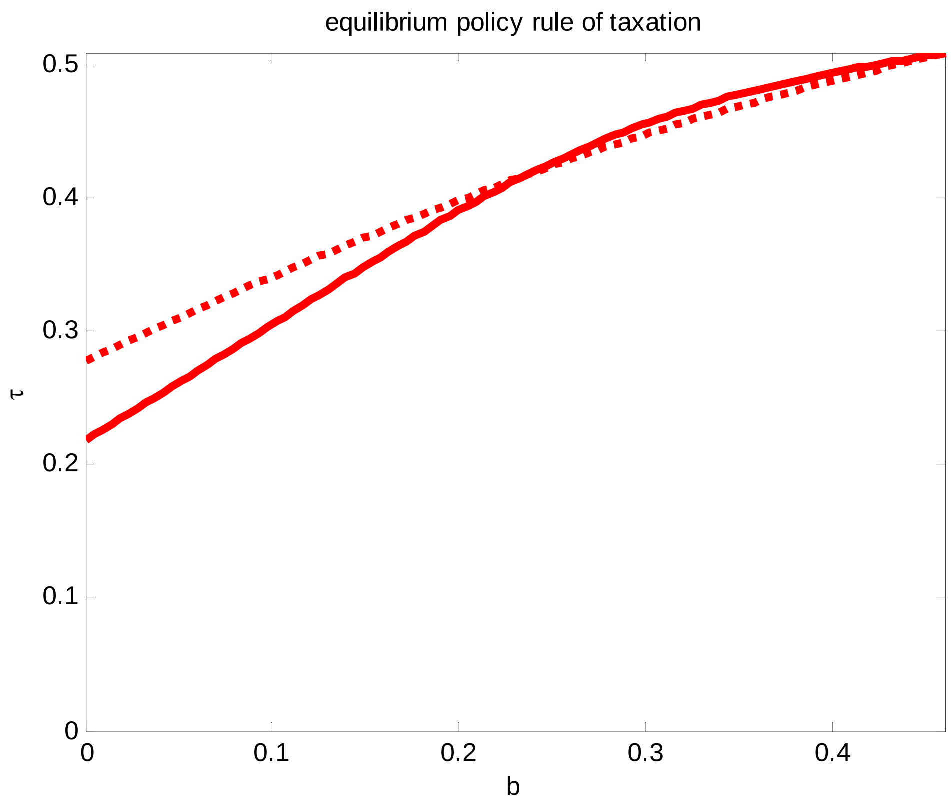
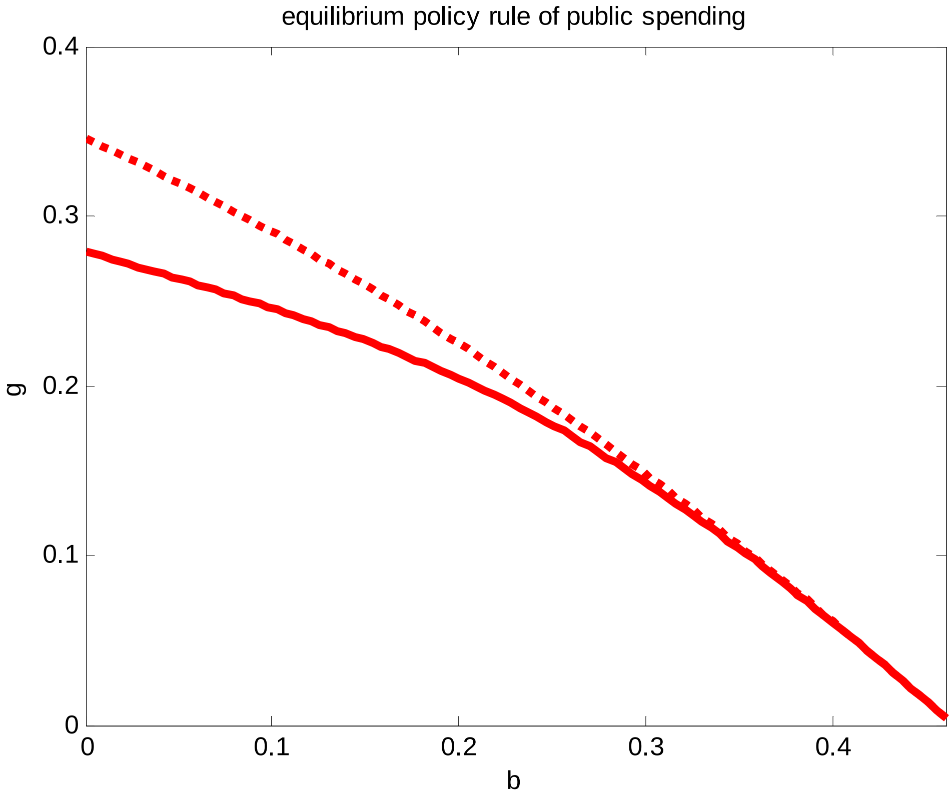


Figure 3-2



equilibrium policy rule of public spending
0.4
0.3
0.2
0.1
0
g
0
0.1
0.2
0.3
0.4
b

Figure 3-3

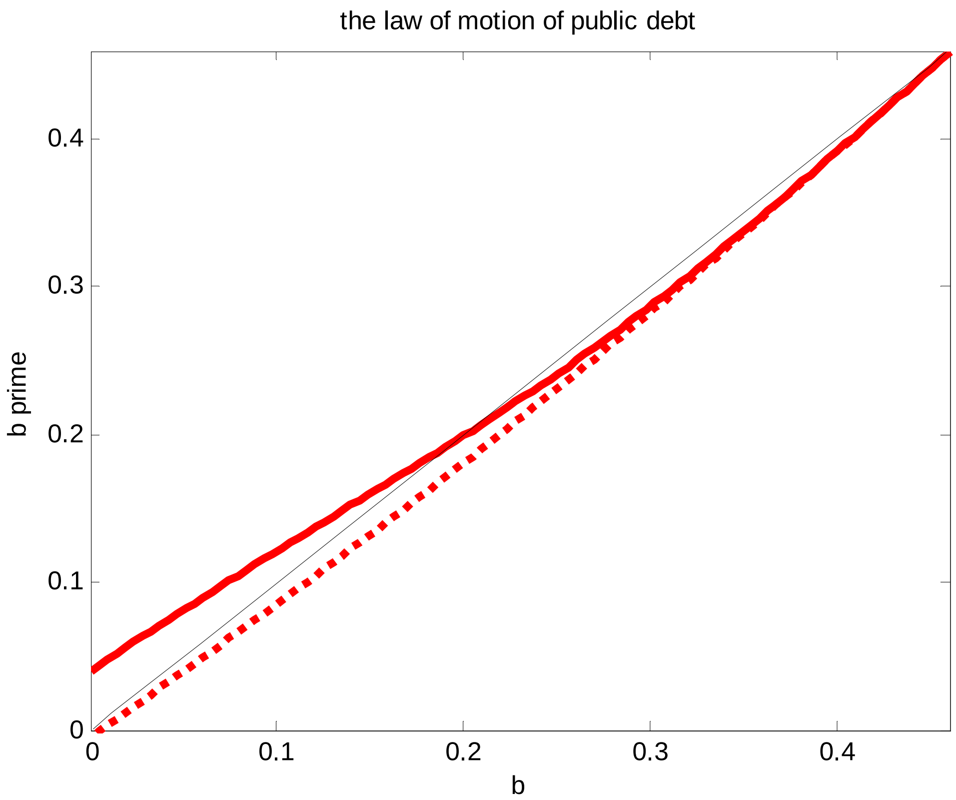


Figure 4: impulse response function for a unanticipated war

(solid line and dotted line stand for ξ=0.90 and ξ=1, respectively)

Figure 4-1

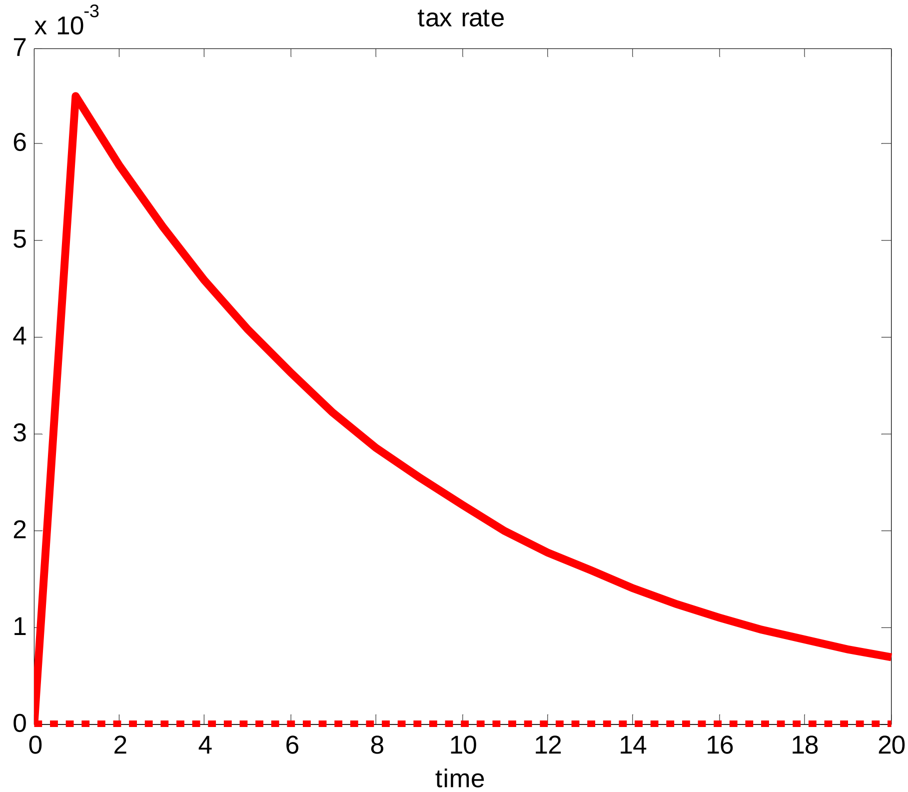


Figure 4-2

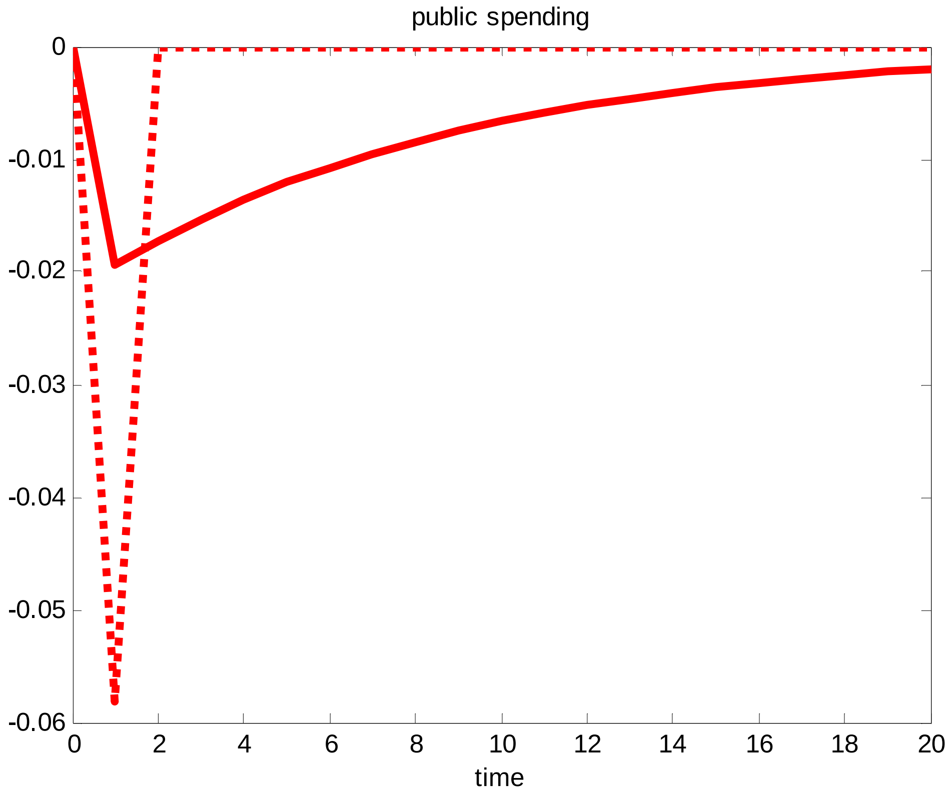

public spending
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
0
2
4
6
8
10
12
14
16
18
20
time

Figure 4-3

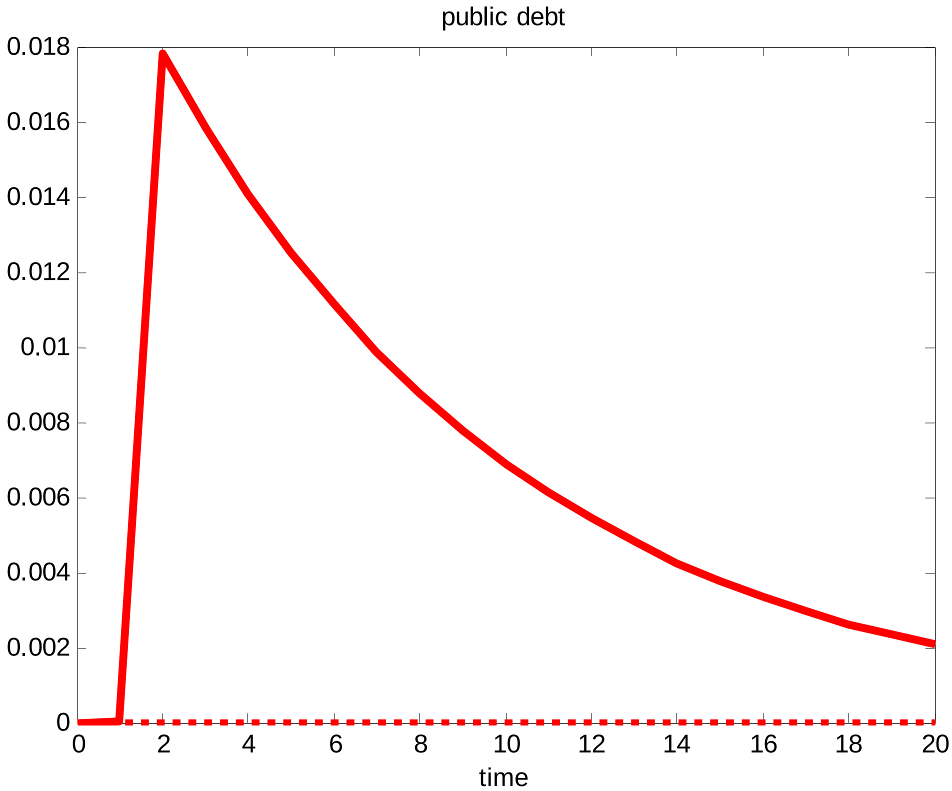

public debt
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0 2 4 6 8 10 12 14 16 18 20
time

Figure 5: equilibrium policy rules with war

(dotted line stands for peace and solid line stands for war)

Figure 5-1

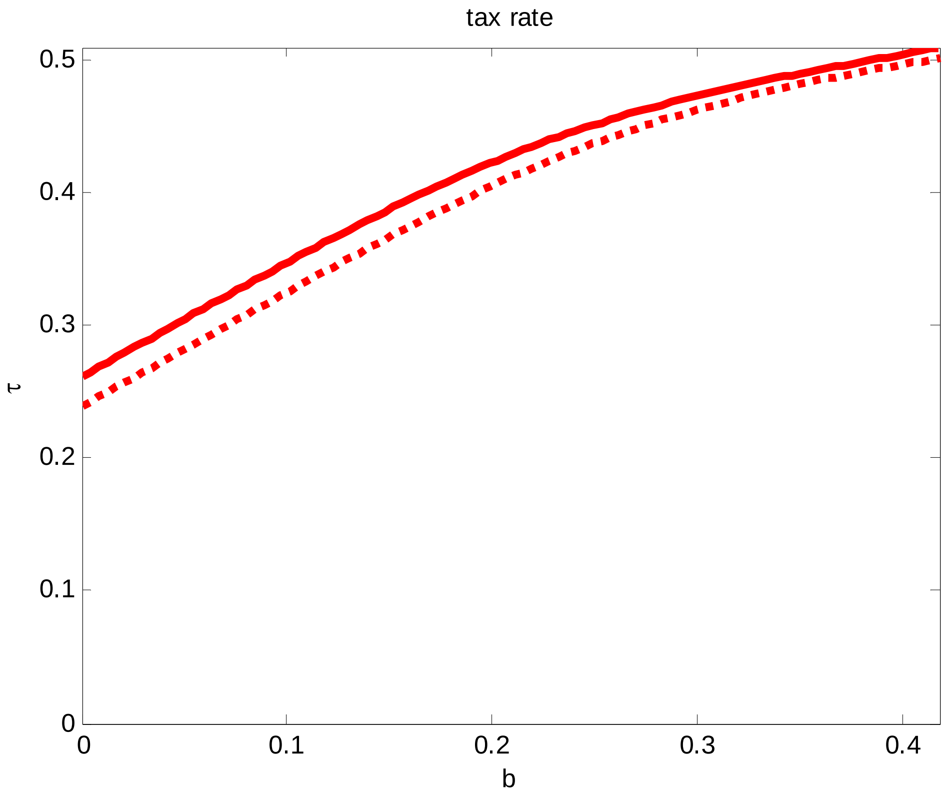


Figure 5-2

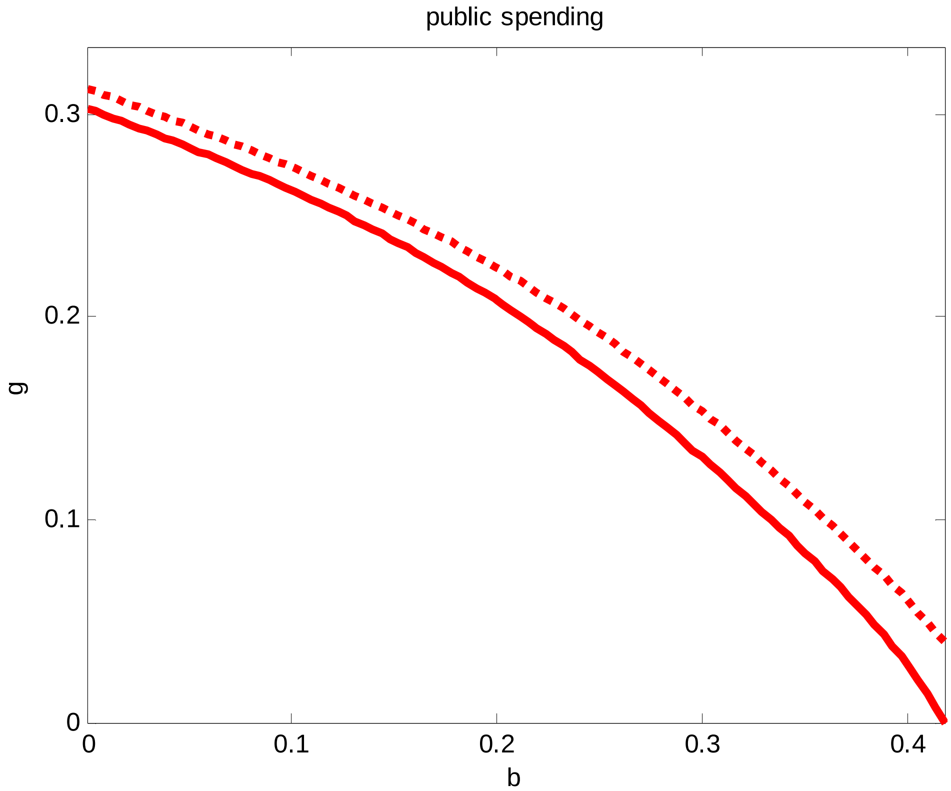
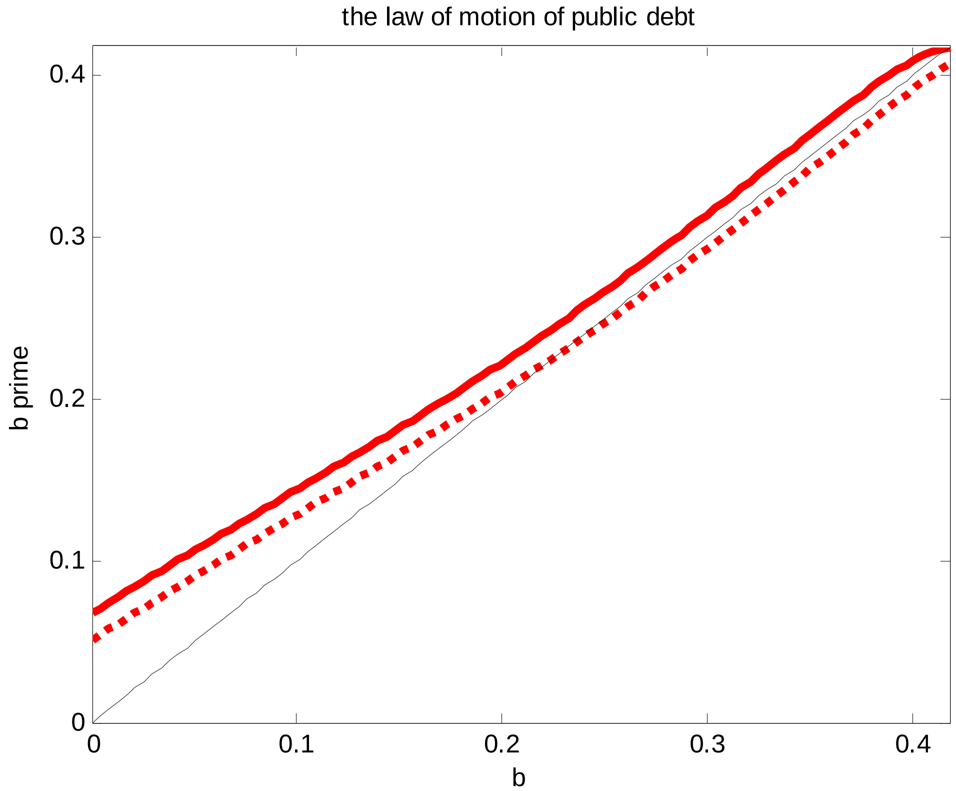
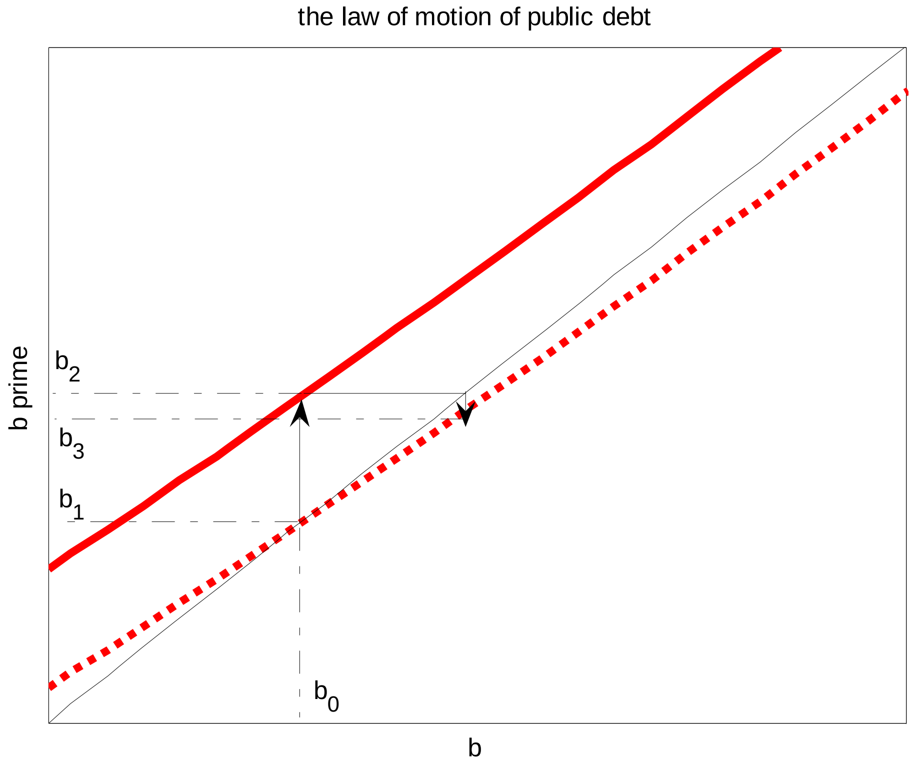


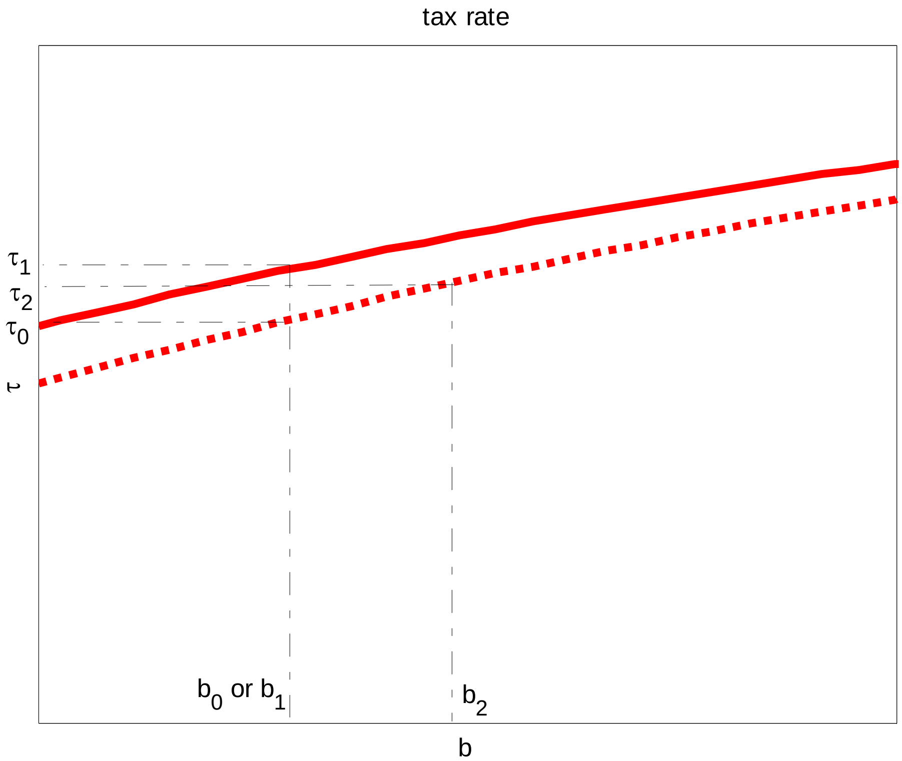
Figure 5-3


the law of motion of public debt
b prime
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
b

Panel 5-4



Panel 5-5



Panel 5-6

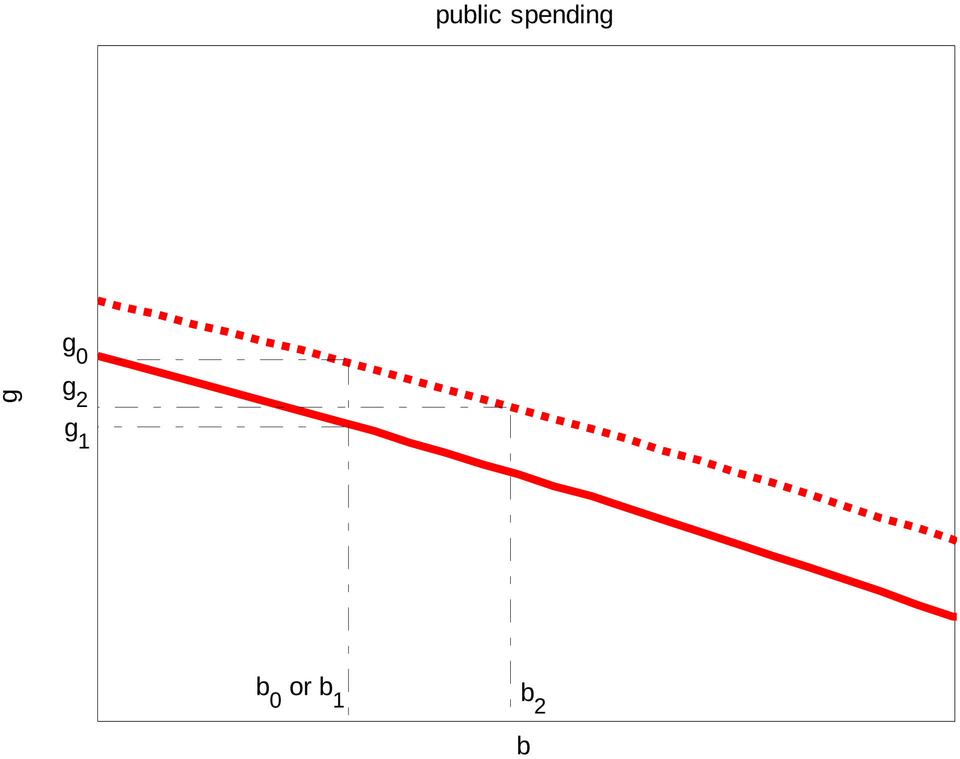


Figure 6: Ramsey policy rules with war after the initial period

(dotted line stands for peace and solid line stands for war)

Figure 6-1

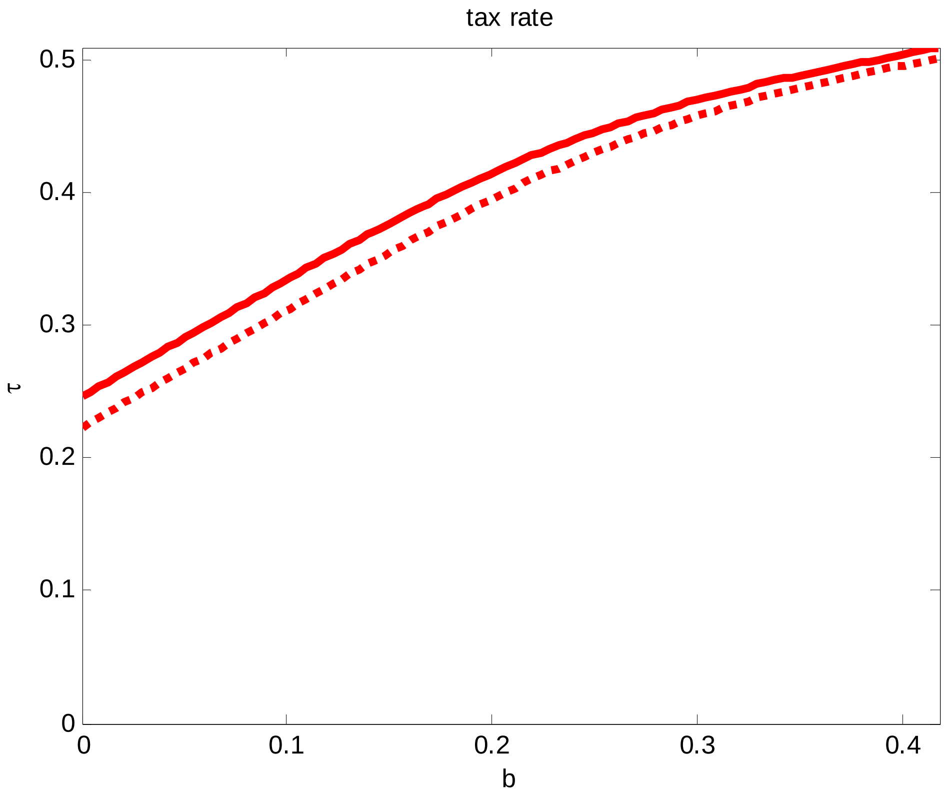


Figure 6-2

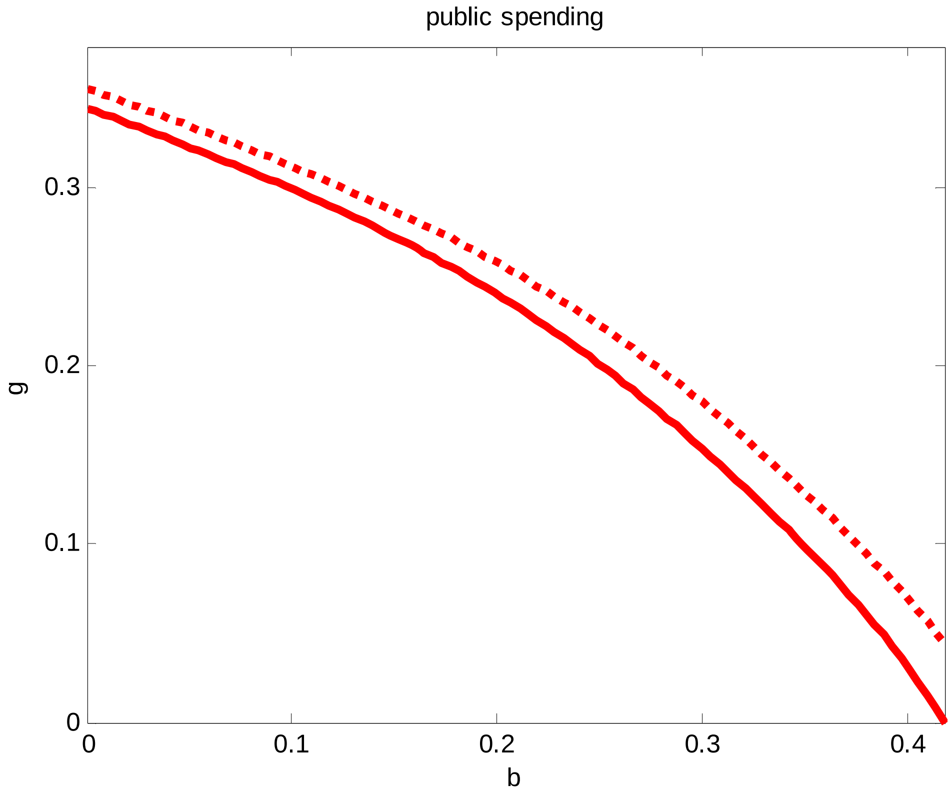


Figure 6-3

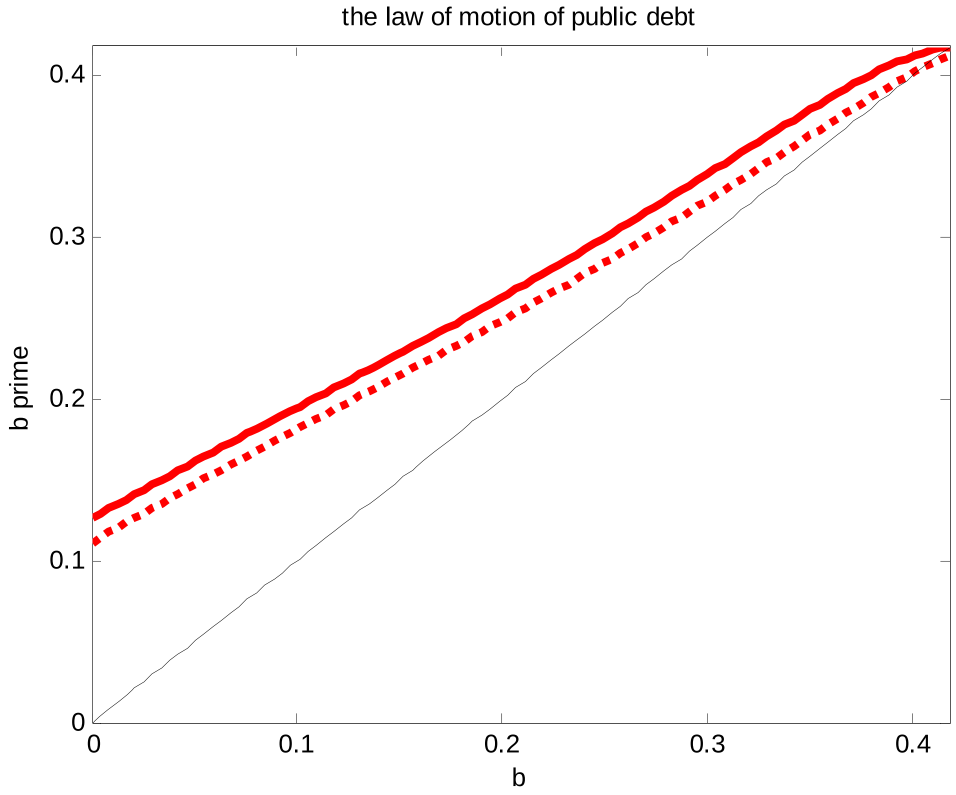
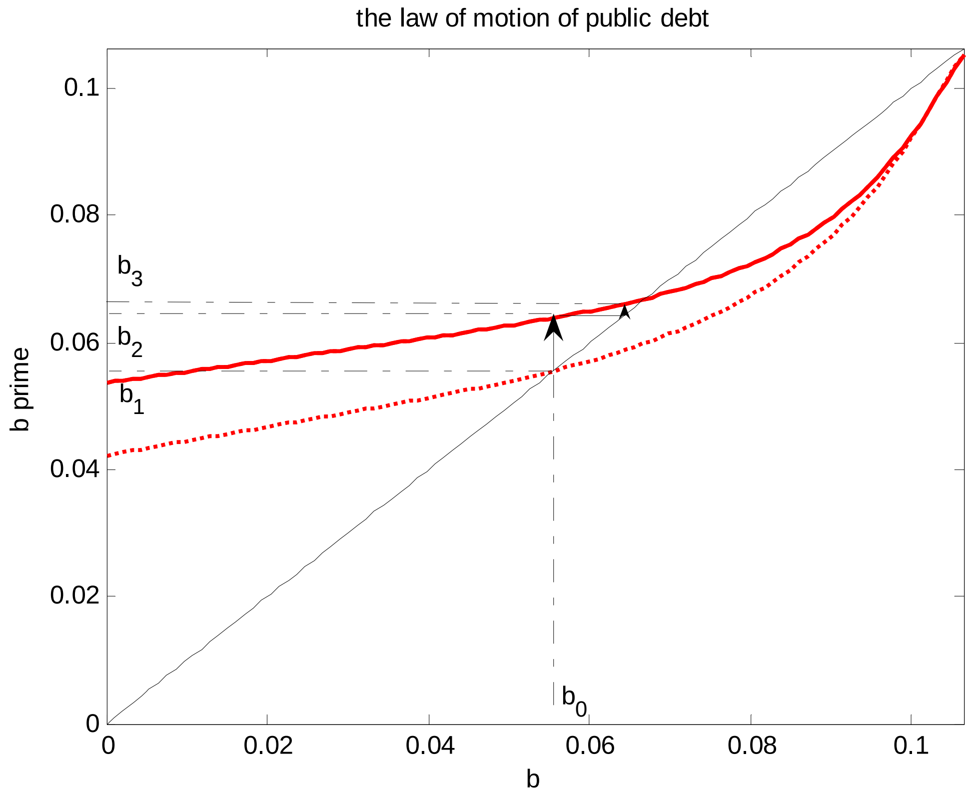

the law of motion of public debt
b prime
b
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4

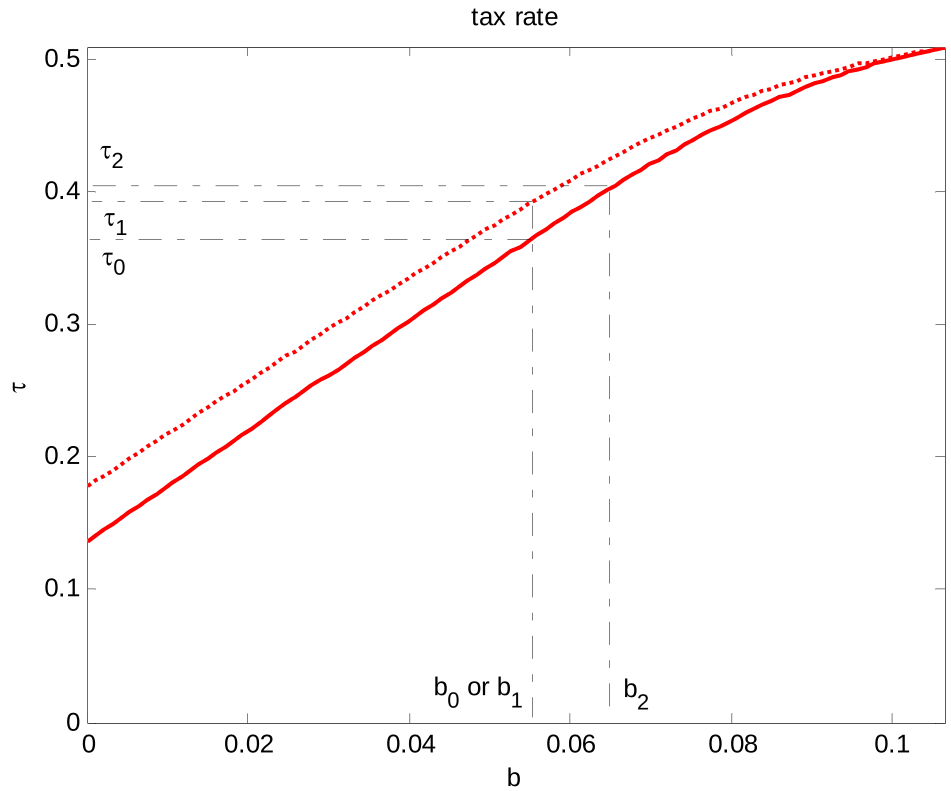
Figure 7: Political Regime Shifts

(dotted lines for the left-wing regime, and solid lines for the right-wing)

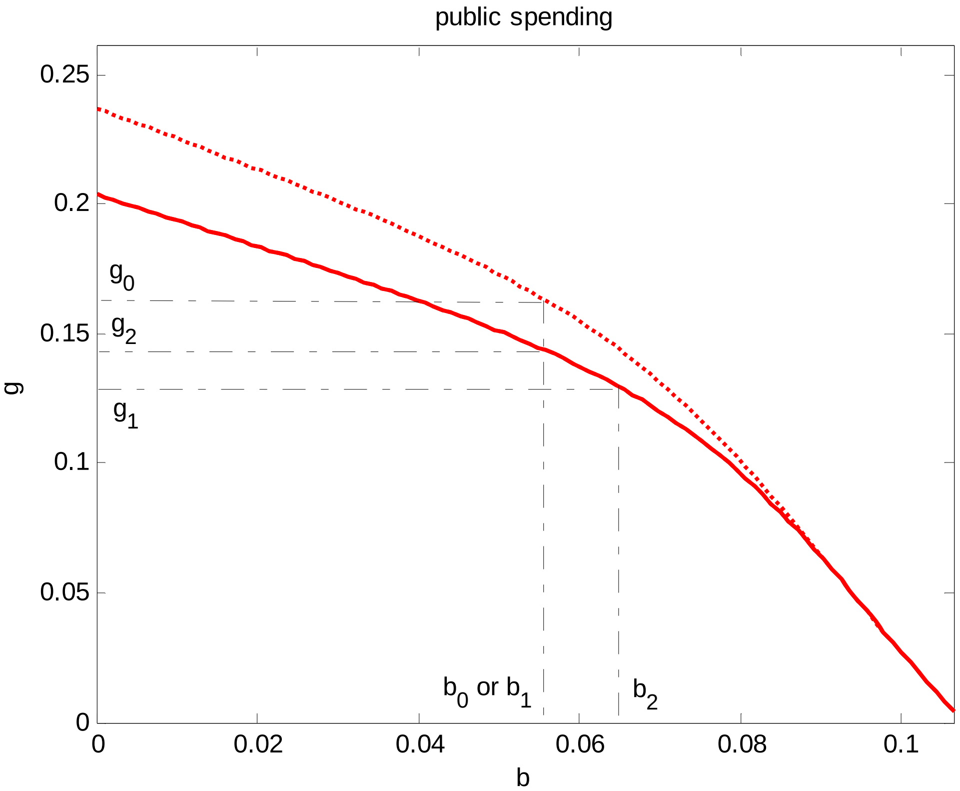
Panel 7-1



Panel 7-2


tax rate
0.5
0.4
0.3
0.2
0.1
0
τ
τ_2
τ_1
τ_0
b_0 or b_1
b_2
0
0.02
0.04
0.06
0.08
0.1
b

Panel 7-3

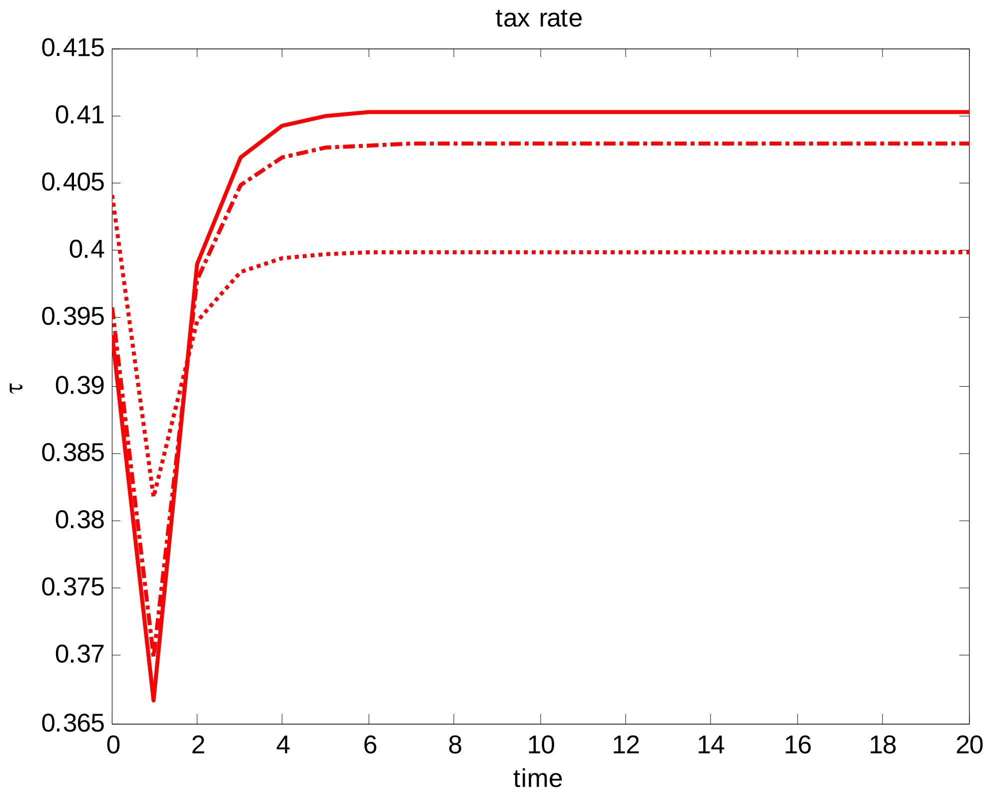


public spending
0.25
0.2
0.15
0.1
0.05
0
g
g_0
g_2
g_1
0
0.02
0.04
0.06
0.08
0.1
b
b_0 or b_1
b_2

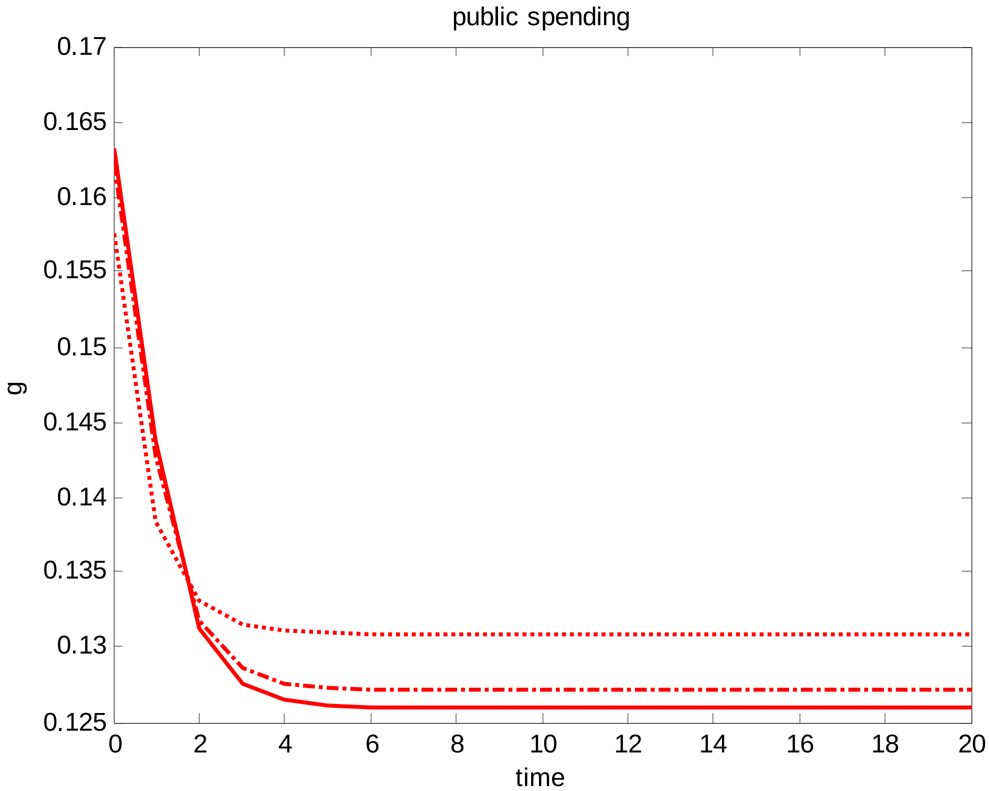
Figure 8: Time-Series for Political Regime Switches

(solid lines for p=1, dashed lines for p=0.9 and dotted lines for p=0.5)

Panel 8-1



Panel 8-2


public spending
g
time
0.17
0.165
0.16
0.155
0.15
0.145
0.14
0.135
0.13
0.125
0
2
4
6
8
10
12
14
16
18
20

Panel 8-3

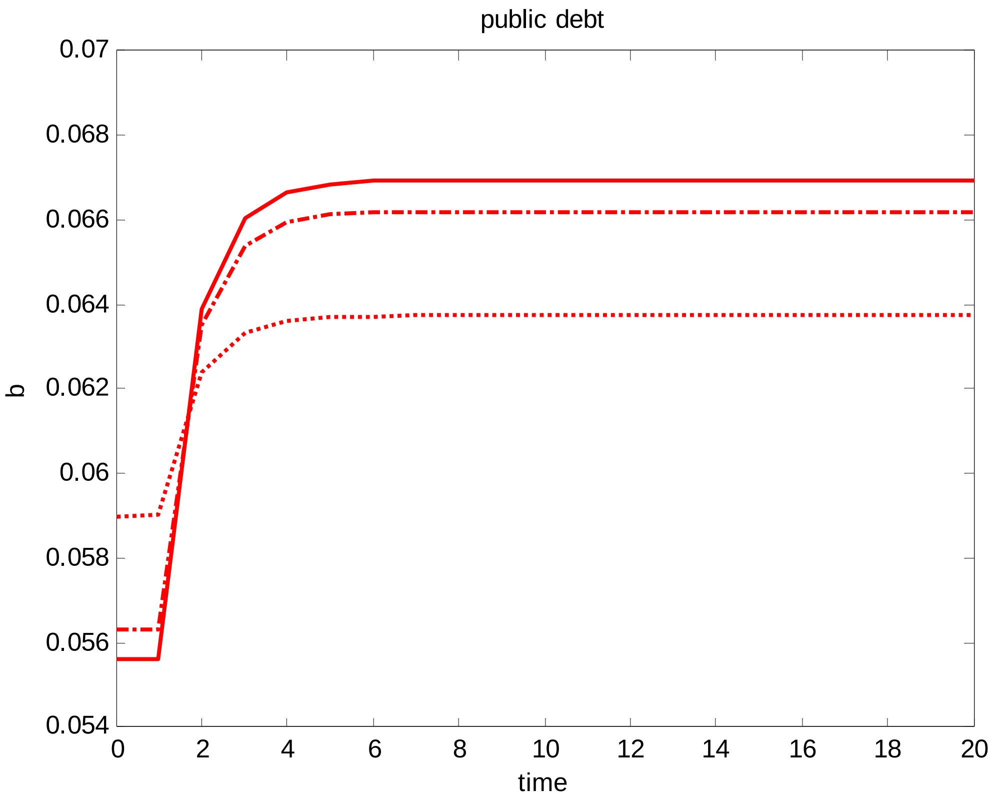
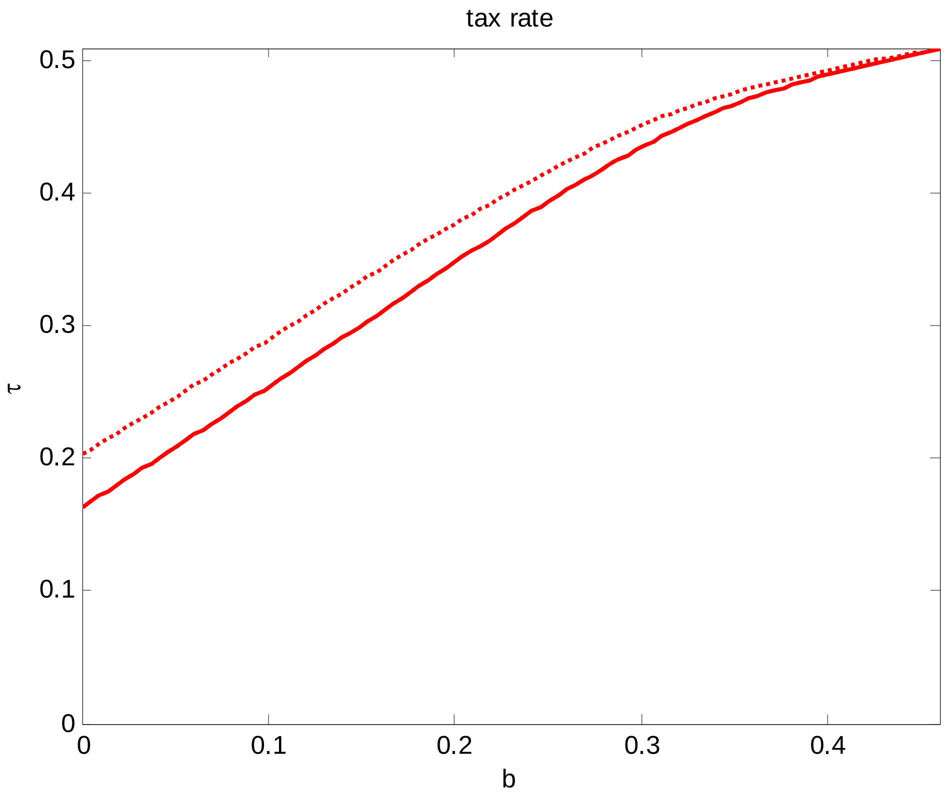

public debt
0.07
0.068
0.066
0.064
0.062
0.06
0.058
0.056
0.054
b
0
2
4
6
8
10
12
14
16
18
20
time

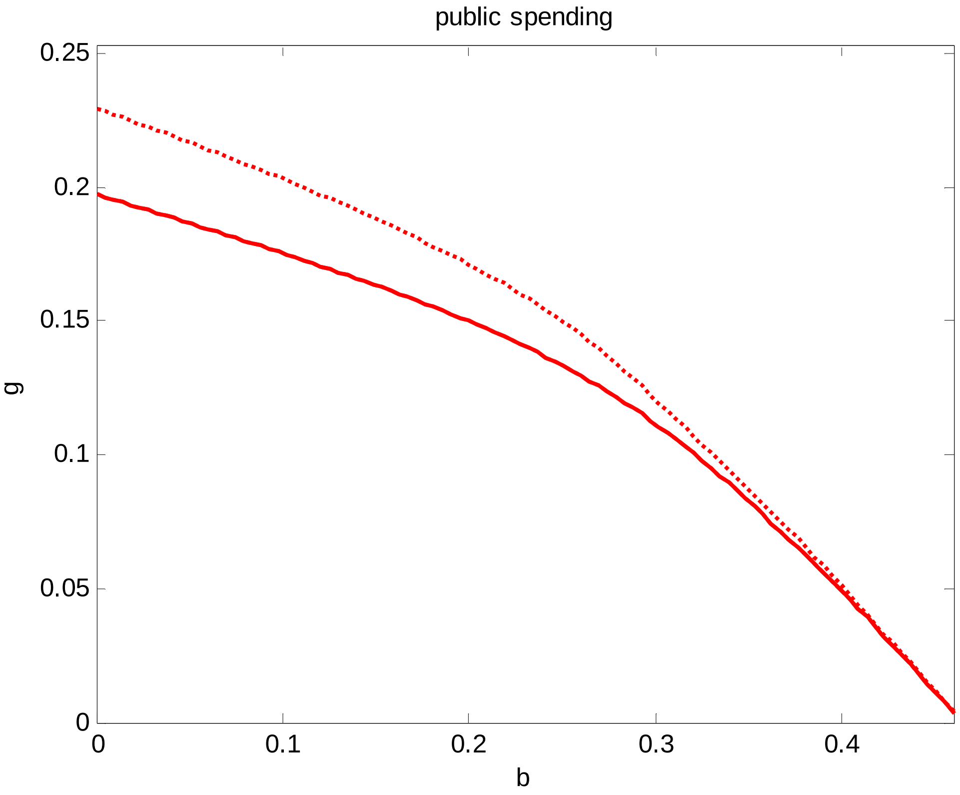
Figure 9: Political Uncertainty with Altruism

(dotted lines for the left-wing regime and solid lines for the right-wing)

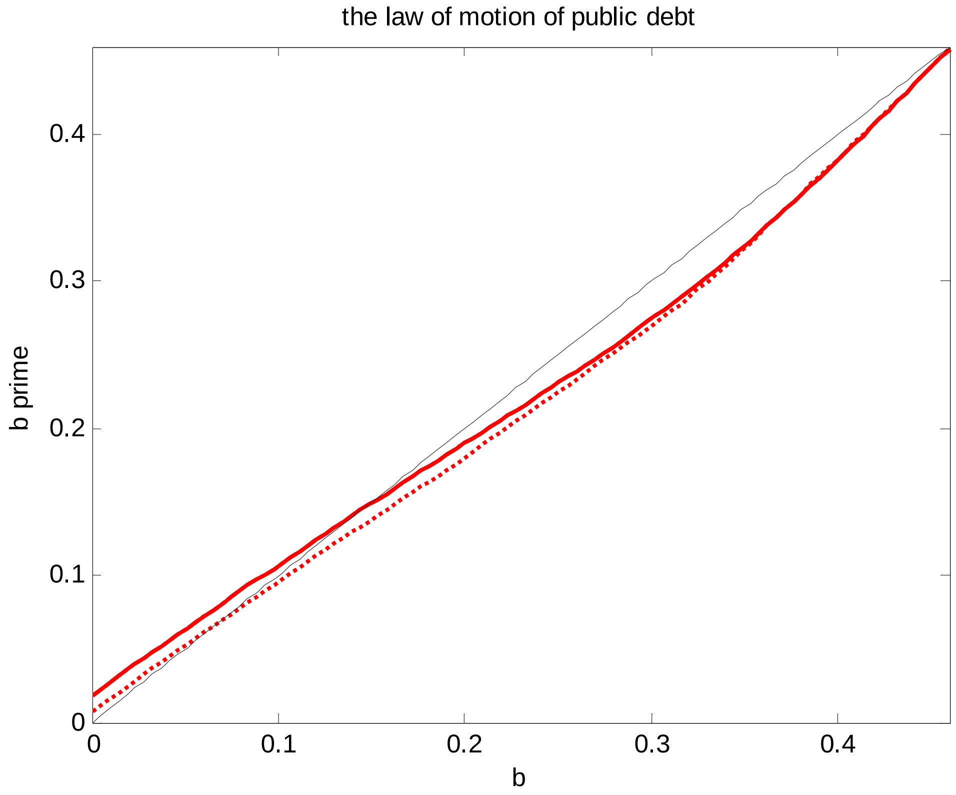
Panel 9-1



Panel 9-2



Panel 9-3


the law of motion of public debt
b prime
b
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4