Methods versus Substance: Measuring the Effects of Technology Shocks on Hours

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Abstract

Different empirical methodologies have coexisted in macroeconomics over the past decades: calibrated dynamic stochastic general equilibrium (DSGE) models, econometrically estimated DSGE models, and structural vector autoregressions. Using these methodologies we re-visit a long-standing question in business cycle research: what fraction of variation in hours worked is due to technology shocks. We analyze to what extent and why the methodologies generate different quantitative answers to our substantive question.

JEL CLASSIFICATION: C32, C52, E32

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1 Introduction

Different empirical methodologies have coexisted in macroeconomics over the past decades. Following Kydland and Prescott (1982) many researchers calibrate dynamic stochastic general equilibrium (DSGE) models to address quantitative questions. Other researchers use more formal econometric methods such as likelihood-based inference or generalized method of moments (GMM) to parameterize DSGE models and study their quantitative implications. Last but not least, a significant fraction of empirical work in macroeconomics is based on structural vector autoregressions (VARs) pioneered by Sims (1980).

Controversies over the virtues and perils of these three methods have persisted and are centered around the following two issues. First, how much theoretical structure should one impose on the empirical model that is used for the quantitative analysis? The appeal of structural VARs is that they are typically based on a minimal set of assumptions, just sufficient to answer the question of interest. DSGE models, on the other hand, are explicitly derived from specific assumptions on the tastes and technologies of economic agents and impose strong restrictions on the comovements of macroeconomic aggregates. While DSGE models can be used to address a wider class of questions, quantitative results are potentially distorted if the theory-based restrictions are at odds with the data.

The second controversy relates to the role of formal econometric methods in the quantitative work with DSGE models. While DSGE models deliver a complete multivariate stochastic process representation for the data they aim to explain, simple models are in many cases rejected against less restrictive specifications such as VARs. Apparent model misspecifications have been used as an argument in favor of a calibration approach, which eschews the notion that stylized DSGE models provide more or less realistic probability distributions for the data from which one could derive meaningful statistical measures of uncertainty about quantitative model implications. Econometricians, in turn, have developed statistical frameworks that formalize aspects of the calibration approach by taking model misspecification explicitly into account. Much of the debate about the role of econometrics is summarized in papers by Hansen and Heckman (1996), Kydland and Prescott (1996), and Sims (1996). Examples of econometric approaches that account for DSGE model misspecification can, for instance, be found in Watson (1993), Canova (1994), DeJong, Ingram, and Whiteman (1996), Geweke (1999), Schorfheide (2000), Del Negro and Schorfheide (2004), and Dridi, Guay, and Renault (2007).

Rather than re-visiting the philosophical underpinnings of the three empirical methodologies described above, the goal of our paper is to show how these methods can be applied to a substantive question and to what extent and why they generate different quantitative answers. More specifically, we revisit a long-standing question in business cycle research: what fraction of the variation in hours worked is due to technology shocks. Starting point for the analysis is a neoclassical growth model with two types of technology shocks. A neutral technology shock shifts total factor productivity and an investment-specific technology moves the slope of a linear transformation curve between consumption and investment goods. First, a quantitative answer to our question can be obtained by calibrating the parameters of the DSGE model and comparing the variability of model-generated data on hours worked to the variability of actual hours worked in the U.S. Second, we employ Bayesian estimation techniques and compute a posterior distribution for the model parameters and the fraction of the variation in hours worked due to technology shock. Third, we only use the theoretical model loosely to justify identifying restrictions for technology shocks in a structural VAR. We then assess the role of technology shocks for labor market fluctuations based on the estimated VAR.

Throughout the paper we will use a fairly simple, frictionless version of the neoclassical growth model for the following reasons. First, quantitative differences between the calibrated and the estimated DSGE model are mostly due to a single parameter, namely, the choice of the labor supply elasticity in view of the observed data. This simplifies our task of examining how the choice of methodology affects the substantive conclusion. Second, our setup maintains an important feature of most applications: the theoretical model is rather stylized and tends to fit worse, in terms of being able to track and forecast the observed time series, than a more densely parameterized model such as a VAR. In the presence of DSGE model misspecification, parameter estimates and hence quantitative model implications tend to be fairly sensitive to the criterion function that is used to map data into parameter values. To the extent that calibration and estimation methods potentially differ in terms of the mapping from data to parameters, a comparison of methodologies in terms of an obviously misspecified model is more interesting than a comparison based on a model that has been deliberately altered to fit the data as well as possible. Finally, while many of the above cited econometrics papers develop calibration-like methods to assess the fit of a DSGE model, the evaluation of the stochastic growth model itself is not the primary focus this paper. Our primary goal is to compare methods to determine the importance of technology shocks for fluctuations in hours worked. The empirical fit of the DSGE model is important to the extent that it affects the degree of confidence or uncertainty that we associate with its quantitative implications.

The literature on the importance of technology shocks for fluctuations of hours worked is very extensive and we will limit our discussion to a few references. The studies based on calibrated DSGE models mostly differ in regard to the model specification. A stylized version of the neoclassical growth model calibrated to U.S. data can generate around 20% of the observed variation in hours worked (see Cooley and Prescott (1995)). In Hansen (1985)'s indivisible labor model the volatility of hours reaches about 66% of the actual volatility. Other studies allow for variations of labor on both the intensive and extensive margin, variable capital utilization, home production, imperfect competition, incomplete markets, and labor search frictions. Overall, the fraction of variation of hours worked explained by technology shocks ranges from 10% to 80%, with a median of about 30%.

Altug (1989) estimated Kydland and Prescott (1982) one-shock time-to-build model using maximum likelihood techniques. She introduced measurement errors to account for the fluctuations of hours worked (and other variables used in the estimation procedure) that are not driven by technology shocks and obtains an estimate of 12%. McGrattan (1994) estimates a stochastic growth model with distortionary labor and capital taxes and finds that 20% of the fluctuations in hours worked are due to technology shocks. Chang and Schorfheide (2003) consider a home production model, which is estimated using Bayesian techniques based on data of aggregate output, hours worked, and consumption of durable goods. According to their analysis, technology shocks account for 50% of the variation in hours worked. Galí and Rabanal (2004) fit a New Keynesian DSGE model to observations on output, inflation, interest rates, and hours worked and find that technology shocks have virtually no effect on hours over the business cycle. Studies based on estimated DSGE models differ with respect to the model specification, the time series that enter the estimation objective functions, and the additional stochastic shocks that are included in the DSGE model to ensure that the model is able to reproduce the observed variation in the data. According to our reading of the DSGE model estimation literature, the median value obtained is about 20%.

Empirical work with structural VARs crucially depends on the variables included in the VAR and the assumptions that are used to identify technology shocks. Technology shocks are often identified using so-called long-run restrictions. They are the only shocks that can have a permanent effect on labor productivity. Shapiro and Watson (1988) estimate VARs using data on aggregate output, hours worked, the aggregate price level, and interest rates. They report that 32% of the variation in hours is due to technology shocks if it is assumed that hours have a stochastic trend. Under the assumption that hours is trend stationary the fraction rises to 40%. In general, the estimated effects are quite sensitive to the assumptions about the persistence and stationarity inducing transformations in hours worked. Following the work of Greenwood, Hercowitz, and Krusell (1997) the more recent literature distinguishes between neutral and investment-specific technology shocks. Galí and Rabanal (2004) find that neutral technology shocks explain very little of the variation in hours. Investment-specific technology shocks, on the other hand, can explain about 60%. According to Fisher (2006) neutral technology shocks account for 20%, whereas investment-specific technology shocks account for up to 47% of the fluctuations in hours worked.

The remainder of this paper is organized as follows. We present the theoretical model in Section 2. This model is calibrated to U.S. data in Section 3, estimated using Bayesian methods in Section 4, and used to motivate identifying restrictions for a structural VAR in Section 5. In each of the empirical sections, we parameterize the empirical model, then repeatedly simulate artificial data with the model using the two technology shocks as driving forces, and compute the ratio of the expected sample variance of simulated data to the variance of post-war U.S. data. To facilitate comparisons between the three empirical approaches, the steps taken to specify prior distributions for some of the DSGE model parameters resembles the calibration approach. To tie the VAR analysis to the estimation of the DSGE model, we use the framework of Del Negro and Schorfheide (2004) and create a family of structural VARs by systematically relaxing the restrictions that the DSGE model imposes on the structural VAR. Finally, Section 6 concludes. The appendix provides detailed information on the data set as well as the implementation of the empirical analysis.

2 The Theoretical Framework

We consider a stochastic growth model with two types of technology shocks, one of which – the neutral productivity shock – affects total factor productivity. The second shock is investment-specific and shifts the slope of the transformation curve between consumption and capital goods. Our model is a simplified version of the one studied by Greenwood, Hercowitz, and Krusell (2000) in that we only have one capital good and the degree of capital utilization is fixed. Our model is very similar to the one used by Fisher (2006).

The model economy is populated with a continuum of households solving the following problem:

$$\max_{\{C_t, X_t, H_t, K_{t+1}\}} \quad \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \xi \, \frac{H_t^{1+1/\nu}}{1+1/\nu} \right) \right] \tag{1}$$

$$\text{s.t.} \quad C_t + P_t X_t = W_t H_t + R_t P_t K_t \tag{2}$$

$$K_{t+1} = (1-\delta) K_t + X_t. \tag{3}$$

Here C_t denotes consumption, H_t is hours worked, X_t is investment measured in physical units, P_t is the price of a unit of the investment good using the consumption good as numeraire, W_t is the wage, and R_t the rental rate of capital. The households choose the level of consumption, investment, and hours worked subject to the budget constraint (2). Capital accumulates according to (3). The paramters β, δ, and ν denote the discount factor, the capital depreciation rate, and the Frisch labor supply elasticity, respectively. ξ affects the marginal rate of substitution between consumption and leisure and determines steady state hours. The static Euler equation associated with the households' optimization problem defines the labor supply schedule and can be expressed as

$$H_t = \left(\frac{1}{\xi} \frac{W_t}{C_t} \right)^{\nu}.$$

The dynamic Euler equation is of the form

$$1 = \beta \mathbb{E}_t \left[\frac{P_{t+1}/C_{t+1}}{P_t/C_t} \Big((1-\delta) + R_{t+1} \Big) \right].$$

In every period firms rent capital and hire labor from the households and produce consumption and investment goods according to the following technology

$$C_t + \frac{1}{V_t} X_t = A_t K_t^{\alpha} H_t^{1-\alpha}. \tag{4}$$

The left-hand-side of (4) can be interpreted as a linear transformation curve between consumption and investment goods. The slope of this curve is shifted by the investment-specific technology disturbance V_t. The right-hand-side takes the standard Cobb-Douglas form and A_t is an exogenous total factor productivity (or neutral) technology process. The firms choose outputs and inputs to maximize profits

$$\Pi_t = C_t + P_t X_t - W_t H_t - R_t P_t K_t$$

subject to (4). The optimal choice of capital and labor inputs by a representative firm implies that

$$W_t = (1-\alpha) A_t K_t^{\alpha} H_t^{-\alpha}, \quad \text{and} \quad R_t P_t = \alpha A_t K_t^{\alpha-1} H_t^{1-\alpha}.$$

Moreover, for both consumption and investment goods to be produced in equilibrium it has to be the case that the relative price of investment goods equals the reciprocal of the investment-specific technology process:

$$P_t = 1/V_t. \tag{5}$$

Equation (5) will be important for the empirical analysis as it implies that the investment-specific technology disturbance can be measured in the data as a relative price.

National income and product accounting for our model economy will be conducted in terms of the consumption good. We define $I_t = X_t P_t$ and hence the aggregate resource constraint can be written as

$$Y_t = C_t + I_t = A_t K_t^{\alpha} H_t^{1-\alpha}.$$

At this point our model has two exogenous disturbances, namely a neutral and a investment-specific technology process. To examine the effect of technology fluctuations on hours worked, we assume that

$$\begin{aligned}
(\ln A_t - \ln A_0 - \gamma_a t) &= \rho_{a,1}(\ln A_{t-1} - \ln A_0 - \gamma_a(t-1)) && (6)\\
&\quad +\rho_{a,2}(\ln A_{t-2} - \ln A_0 - \gamma_a(t-2)) + \sigma_a \epsilon_{a,t} \\
(\ln V_t - \ln V_0 - \gamma_v t) &= \rho_{v,1}(\ln V_{t-1} - \ln V_0 - \gamma_v(t-1)) && (7)\\
&\quad +\rho_{v,2}(\ln V_{t-2} - \ln V_0 - \gamma_v(t-2)) + \sigma_v \epsilon_{v,t}.
\end{aligned}$$

Thus, the log technologies fluctuate around a linear deterministic trend path, given by $\ln A_0 + \gamma_a t$ and $\ln V_0 + \gamma_v t$, respectively. If the autoregressive coefficients sum to one, the fluctuations are non-stationary and the technology processes can be rewritten as AR(1) processes in terms of growth rates. The most widely used specifications for the neutral technology process can be easily obtained as special case of (6). If $0 \leq \rho_{a,1} < 1$ and $\rho_{a,2} = 0$ then technology follows a stationary AR(1) process. If $\rho_{a,1} + \rho_{a,2} = 1$ then technology has a unit root and the serial correlation of its growth rates is $-\rho_{a,2}$, which is often assumed to be zero. In order to restrict the autoregressive processes in (6) and (7) to trend stationarity, it is convenient to re-parameterize them in terms of partial autocorrelations ψ_1 and ψ_2. Omitting the a and v subscripts, we let[1]

$$\rho_1 = \psi_1(1 - \psi_2), \quad \rho_2 = \psi_2.$$

Most of the analysis in this paper is conducted under the assumption that the two innovations $\epsilon_{a,t}$ and $\epsilon_{v,t}$ are normally distributed with mean zero and variances σ_a^2 and σ_v^2.

[1]In case of a unit root $\psi_1 = 1$.

Moreover, we assume that they are uncorrelated at all leads and lags. We will discuss some robustness analysis in which we relax these restrictions.

If both technology shocks have a stochastic trend component (unit root) then one can use the following transformations to induce stationarity:

$$\frac{Y_t}{Q_t}, \quad \frac{C_t}{Q_t}, \quad \frac{I_t}{Q_t}, \quad , \quad \frac{X_t}{Q_t V_t}, \quad \frac{K_{t+1}}{Q_t V_t}, \quad \frac{W_t}{Q_t}, \quad \text{where} \quad Q_t = A_t^{\frac{1}{1-\alpha}} V_t^{\frac{\alpha}{1-\alpha}}.$$

Similarly, if both technology shocks are trend stationary, then we can detrend the model variables as follows:

$$\frac{Y_t}{q^t}, \quad \frac{C_t}{q^t}, \quad \frac{I_t}{q^t}, \quad \frac{X_t}{q^t e^{\gamma_v t}}, \quad \frac{K_{t+1}}{q^t e^{\gamma_v t}}, \frac{W_t}{q^t}, \quad \text{where} \quad q = e^{\frac{1}{1-\alpha}\gamma_a + \frac{\alpha}{1-\alpha}\gamma_v}.$$

To approximate the model dynamics we rewrite the equilibrium conditions in terms of the appropriately detrended variables, derive a non-stochastic steady state, log-linearize the equilibrium condition around the steady state, and use a standard procedure to solve the resulting linear rational expectations system. We group the parameters of the DSGE model into three categories (Table 1): parameters that affect the steady states of the model, parameters that only affect the internal propagation mechanism but cannot be identified from the steady state relationships, e.g. the Frisch labor supply elasticity ν, and parameters that affect the law of motion of the technology disturbances.

We will subsequently examine the role of shifts in investment-specific technology and total factor productivity for fluctuations of hours worked. The model economy serves as theoretical framework for the quantitative analysis. In Sections 3 and 4 we will use the model directly. In particular, we use calibration and estimation techniques to determine numerical values for its parameters, simulate data from the model based on random draws of the technology processes, and compare the variance of the model-generated data to the sample variance of post-war U.S. data. In Section 5 we will use the model more loosely, to construct a VAR that relaxes some of the restrictions that the theory imposes on the time series dynamics, but contains sufficiently many restrictions to identify technology shocks.

3 Calibration

The first step in the calibration analysis is to choose values for those parameters that affect the model's steady state based on long-run averages in post-war U.S. data. Second, we proceed by constructing observations for the two types of technology shocks that appear in the theoretical model. The relative price of investment serves as our observation of the investment-specific technology shock. We construct quality-adjusted measures of investment and capital, that we can use to construct a time series for the neutral technology shock. Finally, we fit autoregressive models to the two technology series. Third, we discuss the calibration of the labor supply elasticity. Once all model parameters are determined, we simulate hours worked data from the calibrated DSGE model to assess the quantitative importance for technology shocks. Unless otherwise noted, our analysis is based on data from 1955:Q1 to 2004:Q4. Precise data definitions are provided in Appendix A.

3.1 Exploiting Steady State Relationships

We now describe in more detail how we choose the parameters of the stochastic growth model in view of U.S. data. We begin with the parameters α, β, δ, and ξ_*, which affect steady states and hence the long-run behavior of the model presented in Section 2. According to our specification, factor markets are competitive and the aggregate production function has a Cobb-Douglas form. Hence, the implied labor share $W_t H_t / Y_t$ is equal to $1 - \alpha$. Since the observed labor share is time-varying (see, for instance, Ríos-Rull and Santaeulàlia-Llopis (2007), we choose α to match the average labor share in the data. We used quality-adjusted depreciation and capital stock data (see Appendix) to determine the depreciation rate δ. The parameters β affects the steady state real interest rate and the investment-output ratio. Numerical values are chosen to reproduce the historical averages in U.S. data, which leads to

$$\alpha = 0.340, \quad \beta = 0.990, \quad \delta = 0.014.$$

The parameter ξ_* affects the steady state levels of output and hours, but not the interest rate or the so-called great ratios. Since ξ_* has no affect on the first-order dynamics of our model we set it equal to one without loss of generality.

3.2 Obtaining Measures for the Exogenous Disturbances

Our model has the rather strong implication that the investment-specific technology shock corresponds to the relative price of investment goods (in terms of consumption goods). We construct this relative price by combining a price index for investment in structures with a price index of quality-adjusted equipment investment. As a price index for investment in structures we use $PCONS_t$ (defined in the Appendix) which is a Tornquist aggregate that weights growth rates of price indexes for non-durable consumption goods and services by their nominal expenditure shares.

With regard to the equipment investment price index, Gordon (1989), Greenwood, Hercowitz, and Krusell (1997) and Cummins and Violante (2002), reveal substantial evidence of biases in the trend of official price indexes due to the lack of quality adjustment. We build on the annual series of Cummins and Violante (2002) to construct our quarterly series of quality-adjusted equipment investment.[2] Quarterly movements are imputed based on the official index reported by the Bureau of Economic Analysis (BEA) in the Fixed Asset Tables (FAT-BEA). The two investment price indices are combined with a Tornquist aggregator to obtain a quality-adjusted price index for total investment, $QAPI_t$. We then define

$$P_t = QAPI_t/PCONS_t \quad \text{and} \quad V_t = 1/P_t, \tag{8}$$

normalizing the index such that $P_0 = 1$ in 1947.

A series for the neutral technology process A_t is typically computed using measures of per capita real output Y_t, capital K_t, labor input H_t, and an estimate of the capital input share α, that is:

$$A_t = Y_t/K_t^{\alpha} H_t^{1-\alpha}. \tag{9}$$

The non-standard aspect of our analysis is that we have to construct a quality-adjusted capital stock. To do so, we begin by generating a quarterly series for investment in efficiency units:

$$X_t = (InvEQ_t + InvST_t)/QAPI_t,$$

where $InvEQ_t$ and $InvST_t$ are total nominal investment in equipment and structures, respectively, and $QAPI_t$ is the quality-adjusted price index that appears in (8). The quality-adjusted capital stock is obtained by the perpetual inventory method:

$$K_{t+1} = (1-\delta)K_t + X_t,$$

where δ's correspond to the average of Cummins and Violante (2002)'s physical depreciation rates for total capital. The initial capital stock K_0 is calibrated using the observed level of output and the capital-output ratio in 1947.

Based on the measure of A_t and V_t it is straightforward to estimate coefficients for the autoregressive models (6) and (7). We use a sample from 1955:Q1 to 2006:Q4. A Bayesian model selection criterion suggests to restrict the sum of the autoregressive coefficients to

[2]Gordon (1989) measures quality-adjusted prices for different types of equipment for the period 1947-1983. Cummins and Violante (2002) extrapolate for 1984 to 2000 the quality bias in the official NIPA price indexes for 1947-1983 implicitly estimated when using the Gordon's quality-adjusted price indexes for that period.

unity for both technology disturbances. Moreover, the criterion favors a specification in which the growth rates of the neutral technology disturbance are serially uncorrelated and have mean zero. Imposing the unit roots, as well as $\psi_{a,2} = 0$ and $\gamma_a = 0$ on the estimation leads to the following least squares estimates:

$$\begin{aligned} \Delta \ln A_t &= \Delta \ln A_{t-1} + 0.007\widetilde{\epsilon}_{A,t} \\ \Delta \ln V_t &= (1 - 0.799) \cdot 0.007 + 0.799 \Delta \ln V_{t-1} + 0.003\widetilde{\epsilon}_{V,t}. \end{aligned}$$

The estimated growth rates of the investment-specific technology process are positive on average and strongly serially correlated. It is interesting to note that the deterministic component of technology growth is solely due to $\ln V_t$, which implies that it is embodied in the physical capital stock.

3.3 Calibrating the Labor Supply Elasticity

At this point we have chosen numerical values for all model parameters except the Frisch labor supply elasticity ν. The implied steady state of the model is consistent with long-run averages in post-war U.S. data and the laws of motion for the technology processes capture the time series properties of our empirical measures of $\ln A_t$ and $\ln V_t$.

A variety of approaches have been pursued in the literature to parameterize ν. First, we could change the model such that the short-run labor supply elasticity is tied to the steady state of hours worked. Consider an instantaneous utility function of the form

$$U_t = \ln C_t + \ln(1 - H_t).$$

Here we set $\xi = 1$ and normalized the total endowment of time to 1 unit per period. The Frisch elasticity associated with this specification is given by $\nu = (1 - H_*)/H_*$, where H_* is the steady state value of hours worked. Under the assumption that households work about 1/3 of their time, we obtain a Frisch elasticity of 2. Second, one could use estimates obtained from micro-level data, e.g. Becker and Ghez (1975), MaCurdy (1981), Altonji (1986), Abowd and Card (1989), which tend to be small. In his survey paper, Pencavel (1986) reports that most estimates for men are between 0 and 0.45. While these studies try to measure the labor supply elasticity along the intensive margin, it is well documented that a large fraction of hours fluctuations is accounted for by movements in and out of employment. This extensive margin is emphasized by Hansen (1985) indivisible labor model, which generates an infinite Frisch labor supply elasticity.

Finally, one could use second moment properties of the aggregate hours data to determine the labor supply elasticity. However, the use of second moments presents a challenge, because the model presented in Section 2 is incompletely specified. It does not claim to explain all the observed fluctuations in the data. Recall that the goal of our analysis is to answer the question: what fraction of the variation in hours worked is due to technology shocks? If we would choose the value for the labor supply elasticity such that the model-implied volatility of hours matches the sample variance in our data set, then we would impose on our analysis that the answer has to be 100%. We will revisit this issue in Section 4. It turns out that the likelihood-based estimation of the DSGE model based on output, hours, and investment price data amounts to identifying the labor supply elasticity based on the relative response of hours and output to an investment-specific technology shock. Using this estimate of ν one can then obtain the response of hours to a neutral technology shock and determine the overall fraction of the variation in hours explained by technology shocks.

In the remainder of this section we will consider three values for the labor supply elasticity. First, $\nu = 2$ represents a value that is chosen based on strong functional form assumptions and steady state considerations. Second, we consider $\nu = 100$ which essentially captures the quasi-linear preferences in Hansen (1985)'s indivisible labor model. Third, $\nu = 0.2$ represents micro-level estimates based on labor adjustments of males along the intensive margin.

3.4 Quantitative Results

Although rarely done in practice, we can easily introduce parameter uncertainty by replacing the numerical values obtained so far by probability distributions. We do so for the parameters that govern the law of motion of the exogenous processes, since these parameters are estimated with econometric techniques that produce a statistical measure of uncertainty. Throughout the paper we adopt a Bayesian approach to econometric analysis and will use *a priori* and *a posteriori* distributions to represent parameter uncertainty.

The parameter choices and quantitative findings are summarized in Table 2. We fix α, β, δ, and ν and impose the following restrictions on the autoregressive technology processes in (6) and (7): $\psi_{a,1} = 1$, $\psi_{a,2} = 0$, $\gamma_a = 0$, and $\psi_{v,1} = 1$. Since $\ln A_0$ and $\ln V_0$ are irrelevant for our analysis, we only report moments of distributions for σ_a, γ_v, $\psi_{v,2}$, and σ_v in the table. We treat all parameters as independent, generate draws, and simulate the linearized DSGE model for 200 periods using (i) only the neutral technology process A_t, (ii) only investment-specific technology process V_t, (iii) both technology disturbances. Finally we compute the ratio of the variance of hours based on actual and model generated data for each simulated trajectory. Means and 90% intervals are reported in the last three rows of Table 2.

The quantitative results are very sensitive to the choice of labor supply elasticity. For $\nu = 2$ the neutral technology shock explains about 5% and the investment-specific technology shock explains about 23% of the variance of hours worked. The role of neutral technology shocks is smaller than in Cooley and Prescott (1995) in part because we are using a model in which technology shocks have a permanent effect. While the marginal product of labor rises in response to a neutral technology shock, the marginal utility of consumption falls because the households' wealth increases permanently. Thus, the overall effect on labor supply is small for $\nu = 2$. Figure 1 depicts impulse responses of labor productivity, hours, and the relative price of investment to technology shifts for $\nu = 0.2$ and $\nu = 100$. In response to a neutral technology shock that raises labor productivity by approximately 1% in the long-run, hours worked increase by 50 basis points in the short-run for $\nu = 100$, but essentially do not respond if $\nu = 0.2$. Overall, both technology shocks only explain about 1% of the variance of hours if $\nu = 0.2$ and 97% of its variance if $\nu = 100$.

Short run dynamics of labor productivity and hours in response to an investment specific technology shock are richer due to the serially correlated growth rates of this shock. In particular, if $\nu = 100$, current labor productivity increases by 50 basis points. It decreases for eight periods and then it increases reaching a new steady state. Under the same scenario hours first decrease by approximately 1%, increase for about 12 periods reaching its maximum at 50 basis points, and slowly die out. The serially correlated technology growth rate implies that the price of investment goods is expected to fall, which lowers the expected return on capital. Hence, households increase consumption sharply and, despite a higher wage, lower their labor supply. After the initial impact, the responses of labor productivity and hours mirror each other. When $\nu = 0.2$ the investment shock raises labor productivity in the long run by 0.5%. The response of hours, however, is negligible.

3.5 Sensitivity Analysis

To assess the robustness of our findings we repeat the above analysis under the assumption that the technology processes are trend stationary or follow a vector autoregressive process.

We use a nonlinear model solution technique, and we pass the actual and simulated hours worked data through the HP filter before computing the variance ratios.

We begin by studying the model under the assumption that technology is trend stationary. We maintain the restriction that $\gamma_A = 0$, set the first-order partial autocorrelations for both technology processes to 0.98, and re-estimate the coefficients of the autoregressive processes for $\ln A_t$ and $\ln V_t$ using least squares. The results are reported in Table 3. In case of $\nu = 2$ neutral technology shocks now explain almost 10% of the variance in hours worked. The less persistent the technology shock, the more attractive it is for the household to increase hours and enjoy the higher wages. For $\nu = 100$ the model generated hours series tend to be too volatile on average. Under the $\nu = 0.2$ calibration, technology shocks essentially play no role for the movement of hours over the business cycle. Neither the use of HP-filtered data in the computation of the variance ratio, nor the use of a second-order perturbation method to solve the model changed our findings in a substantive way.

4 Bayesian Estimation of the DSGE Model

There is a growing literature on the estimation of DSGE models with formal econometric methods. A detailed review of Bayesian approaches, which we will focus on in this paper, is provided in An and Schorfheide (2007). Bayesian estimates are obtained by combining a prior distribution for the parameters of the DSGE model, stacked in the vector $\theta \in \Theta$, with a likelihood function

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)}, \quad p(Y) = \int_{\theta \in \Theta} p(Y|\theta)p(\theta)d\theta.$$

Here $p(\theta)$ denotes the prior density, $p(Y|\theta)$ is the likelihood function, that is, the joint probability density function of the data Y given θ, $p(\theta|Y)$ is the posterior density, and $p(Y)$ is called the marginal data density or marginal likelihood that normalizes the posterior density so that it integrates to one.

A crucial step in the estimation is the choice of observables Y that enter the likelihood function $p(Y|\theta)$. Since we are interested in explaining the volatility of hours worked it is important to include hours data in our analysis. The analysis in Section 3 highlighted that the quantitative results are very sensitive to the labor supply elasticity. To obtain a reliable estimate of this elasticity, our set of observables should at a minimum span a measure of prices and quantities in the labor market. According to our model, wages are equal to the marginal product of labor, which in turn is proportional to average labor productivity.

Hence, we will also include labor productivity (and hence implicitly aggregate output) in Y. Finally, we are trying to disentangle the effects of neutral from investment-specific technology shocks. Since our model implies that the relative price of investment is a direct observation of the investment-specific technology process, we will also include the price series described in Section 3 in the estimation.

In principle, we could expand the list of observables even further. A natural candidate would be the quality-adjusted investment series, which in conjunction with the capital accumulation equation, the aggregate resource constraint, and data on output and hours, essentially identifies the neutral technology process. For now, however, we will omit investment data from Y and discuss at the end of this section how this choice affects our estimates.

If we use observations of output, hours worked, and the relative price of investment then the likelihood function associated with the model presented in Section 2 suffers from the well-known singularity problem: according to the model there exists a linear combination of the three series that can be predicted without error conditional on past observations. To overcome the singularity, researchers either introduce measurement errors, e.g. Altug (1989), or include additional shocks, e.g. Leeper and Sims (1994). We will follow the latter approach. Up to this point, the specification of the stochastic growth model has been incomplete in the following sense: we are allowing for the possibility that there exist other shocks that affect hours worked over the business cycle (hence the question: how much of the variation is explained by technology shocks). The calibration strategy pursued in Section 3 did not require us to be specific about these other shocks. The likelihood-based estimation pursued subsequently does require a probability model that is specified to capture all the fluctuations observed in the data.

According to the log-linearized DSGE model the labor supply function is of the form

$$\widehat{h}_t = \nu(\widehat{w}_t - \widehat{c}_t), \tag{10}$$

where $\widehat{x}_t$ denotes a percentage deviation of a variable x its long-run growth path. It is well-known, e.g. Hall (1997), that (10) does not hold in the data. To capture the wedge we introduce a preference or labor supply shock, ξ_t that enters the instantaneous utility function as follows:

$$U_t = \ln C_t - \xi_t \frac{H_t^{1+1/\nu}}{1+1/\nu}, \quad \text{where} \quad \ln(\xi_t/\xi) = \rho_\xi \ln(\xi_{t-1}/\xi) + \frac{\sigma_\xi}{\nu}\epsilon_{\xi,t}.$$

4.1 From Priors to Posteriors

Bayesian analysis requires the specification of a prior distribution, which is summarized in Table 4. Using the same arguments as in Section 3 we fix the discount factor and the depreciation rate: $\beta = 0.99$ and $\delta = 0.013$. Moreover, we choose a prior for α that is centered at 0.36. While data on wages, real interest rate, capital, and investment do not enter our likelihood function $p(Y|\theta)$ directly, we use them to elicit a rather tight prior distribution for these three parameters. Our prior for the Frisch labor supply elasticity is centered at 2, but with a standard deviation of one. Hence, a 90% *a priori* credible interval encompasses values found in studies that use micro-level data for employed males, as well as the values necessary to be able to explain most of the observed volatility in hours worked in a stochastic growth model driven by technology shocks.

We estimate a version of the model in which the technology processes have a unit root, that is, $\psi_1 = \rho_1 + \rho_2 = 1$ in (6) and (7), as well as a version in which technology disturbances are trend stationary. For both specifications we assume that the second order partial autocorrelations are negative, which means that for first-order partial autocorrelations near unity the growth rates are positively correlated. For the trend stationary specification our prior implies that both technology processes are highly persistent. Our priors are fairly agnostic with respect to the average growth rate of the technology processes. The prior distribution for the autocorrelation of the preference shock is centered at 0.8 and has a standard deviation of 0.1. To generate predictions about the level of output, hours, and the relative price of investment we include $\ln Y_0$, $\ln H_*$, and $\ln V_0$ into the parameter vector θ.[3]

Two versions of the DSGE model are estimated based on observations from 1955:QI to 2004:Q4: in one version both technology processes have stochastic trends whereas they are trend-stationary in the second version. We use the Markov-Chain Monte-Carlo methods reviewed in An and Schorfheide (2007) to obtain draws from the posterior distribution of the DSGE model parameters. Posterior means and 90% credible intervals for the parameters and the variance ratios for model generated and actual hours worked data are summarized in Table 5. The most important finding is that the estimated labor supply elasticities are 0.67 (deterministic trend) and 0.30 (stochastic trend), respectively. Hence, overall the effect of technology shocks on hours worked is small. According to the deterministic trend specification, 10% of the fluctuations are due to technology shock. The stochastic trend version implies that the explained fraction is only 2%. Parameter uncertainty matters to

[3]For estimation purposes it is convenient to parameterize the model in terms of $\ln Y_0$ and $\ln H_*$ rather than $\ln A_0$ and ξ. The latter can be inferred from the steady state relationships.

some extent: the upper bounds for the 90% credible intervals associated with the two models are 18% and 5%, respectively.

In addition to the parameter estimates, the Bayesian analysis also delivers a measure of relative fit of the two estimate DSGE model specifications. According to Bayes Theorem, prior model odds are updated according to

$$\frac{\pi(\mathcal{M}_{ST}|Y)}{\pi(\mathcal{M}_{DT}|Y)} = \frac{\pi(\mathcal{M}_{ST})}{\pi(\mathcal{M}_{DT})} \times \frac{p(Y|\mathcal{M}_{ST})}{p(Y|\mathcal{M}_{DT})}.$$

In our application, the Bayes factor

$$\frac{p(Y|\mathcal{M}_{ST})}{p(Y|\mathcal{M}_{DT})} = \exp[2.86]$$

favors the stochastic trend specification by a small margin, pointing toward a negligible role for technology shocks with respect to hours movements.

The Kalman smoother can be used to back out the neutral technology process and the capital stock, both of which have been treated as latent variables in the estimation process. Figure 2 overlays time series plot for the smoothed $\ln A_t$ series and the measure of $\ln A_t$ constructed in Section 3. The two series are strongly correlated, which is consistent with the similarity between the Bayesian estimates and the calibrated values for the parameters that determine the law of motion of the neutral technology process. The second panel of Figure 2 depicts the changes in the capital stock. The measured series is based on data on quality-adjusted investment data, whereas the smoothed series is constructed only from observations on output, hours, and the relative price of investment goods. While the smoothed series misses some of the high frequency variation in the measured series, there is some strong correlation between the two.

4.2 Identification of the Labor Supply Elasticity

The most important difference between the calibration and the estimation of our simple DSGE model is the identification of the labor supply elasticity. In the context of calibration we considered three approaches: (i) use steady state considerations; (ii) emphasize that most variation of hours occurs along the extensive margin and use an indivisible labor model with an infinite Frisch elasticity; (iii) use a micro-level estimate of the elasticity along the intensive margin. A likelihood-based estimator, on the other hand, generates a parameter value for which the model implied autocovariance function of the observables matches the sample autocovariance function as closely as possible in terms of a statistical metric. Unfortunately, it is often difficult to disentangle what pattern in the data leads to a particular estimate.

Our DSGE model has hard-wired the restriction that the relative price of investment goods is exogenous and can be interpreted as investment-specific technology shock. This restriction is sufficient to identify the dynamic response of an autoregressive system to an innovation $\epsilon_{v,t}$. Movements of hours and labor productivity in response to the investment-specific technology shock are informative about the labor supply elasticity. According to the linearized DSGE model, we have the following labor market equations (in terms of percentage deviations from the stochastic trend path):

$$\begin{aligned} \text{Demand} &: \widehat{h}_t = -\widehat{z}_t + \widehat{k}_t - \frac{1}{\alpha}\widehat{w}_t \\ \text{Supply} &: \widehat{h}_t = \nu\widehat{w}_t - \nu\widehat{c}_t \end{aligned}$$

Here $\widehat{z}_t$ represents a technology shock and wages are given by $\widehat{w}_t = \widehat{y}_t - \widehat{h}_t$. Shifts of the labor market equilibrium in response to an investment-specific technology shock are depicted in Figure 3. Panels (1,1), (2,1), and (2,2) show the responses of the investment-specific technology process, labor productivity, and hours worked to an innovation $\epsilon_{v,t}$ for $\nu = 2$ versus $\nu = 0.2$. Panel (1,2) depicts the wage-hours locus along the response, starting from the south-east corner. Here wages are in terms of deviations from the stochastic trend and eventually revert back to zero. While the response of labor productivity and hence wages is very similar for the two values of ν, hours respond more forcefully if the labor supply elasticity is large. This mechanism leads to the identification of the labor supply elasticity in the estimation. Meanwhile the volatility of the preference shock is determined to match the total volatility of hours and its correlation with aggregate output.

We also estimated the DSGE model based on observations of labor productivity, quality adjusted investment, and hours worked. By changing the data set, we are changing the identification of the labor supply elasticity. Based on the investment data, the model restrictions are sufficient to generate a path for the capital stock, which in combination with data on output and hours and a tight prior on the capital share parameter identifies the neutral technology process. The exogeneity of the technology process generates an exclusion restriction that is sufficient to identify the response of labor productivity and wages to a neutral technology shock. Along this response, we can infer the labor supply elasticity. It turns out that using this different set of observables, we obtain slightly larger estimates of the Frisch elasticity: $\hat{\nu} = 0.78$ for the deterministic trend specification and $\hat{\nu} = 0.60$ for the stochastic trend specification.

5 VAR Analysis

At last, we will use a structural VAR to assess the importance of technology shocks for cyclical fluctuations of hours worked. The advantage of the VAR analysis is that we can avoid the use of potentially misspecified over-identifying restrictions generated by the DSGE model. Let y_t be a $n \times 1$ vector, composed of the growth rates of the investment goods price and labor productivity, and the log level of hours worked. We assume that the law of motion of y_t can be described by a p'th order structural VAR:

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + \ldots + \Phi_p y_{t-p} + \Phi_\epsilon \epsilon_t. \tag{11}$$

We adopt the interpretation that the 3×1 vector of structural shocks ϵ_t is composed of the two technology process innovations as well as the innovation to a third shock. This third shock could potentially capture exogenous labor supply shifts, but unlike in the estimation of the DSGE model, we do not have to make specific assumptions about the role of the third shock. We can define the reduced-form innovations $u_t = \Phi_\epsilon \epsilon_t$ and denote the covariance matrix of u_t by Σ. The VAR can be conveniently expressed in matrix form as a linear regression model:

$$Y = X\Phi + U,$$

where Y is the $T \times n$ matrix with columns y_t', and X is the $T \times k$ matrix with columns $x_t' = [1, y_{t-1}', \ldots, y_{t-p}']$.

5.1 Connecting the VAR with the DSGE Model

Employing the framework of Del Negro and Schorfheide (2004), we link the DSGE model of Section 2 and the VAR by assuming that we estimate a VAR based on infinitely many observations generated from the DSGE model, conditional on the structural parameter vector θ. Let $I\!E_\theta^D[\cdot]$ be the expectation under the DSGE model and define the autocovariance matrices

$$\Gamma_{XX}(\theta) = I\!E_\theta^D[x_t x_t'], \quad \Gamma_{XY}(\theta) = I\!E_\theta^D[x_t y_t'].$$

A VAR approximation of the DSGE model can then be obtained by population least squares:

$$\Phi_*(\theta) = \Gamma_{XX}^{-1}(\theta)\Gamma_{XY}(\theta), \quad \Sigma_*(\theta) = \Gamma_{YY}(\theta) - \Gamma_{YX}(\theta)\Gamma_{XX}^{-1}(\theta)\Gamma_{XY}(\theta). \tag{12}$$

An important concern when estimating a DSGE model is that we are imposing invalid cross-coefficient restrictions on the data. VARs are in general less restrictive and, loosely speaking, try to let the data speak. To relax the cross-coefficient restrictions, we can use a prior distribution of the VAR parameters Φ and Σ that has a lot of mass near the DSGE model restrictions but does not dogmatically impose them:

$$\begin{aligned} \Sigma|\theta &\sim \mathcal{IW}\Big(\lambda T \Sigma_*(\theta), \lambda T - k, n\Big) \\ \Phi|\Sigma, \theta &\sim \mathcal{N}\left(\Phi_*(\theta), \frac{1}{\lambda T}\Big[\Sigma^{-1} \otimes \Gamma_{XX}(\theta)\Big]^{-1}\right). \end{aligned} \tag{13}$$

The larger the hyperparameter λ, the more tightly the prior contours are concentrated.

To address our substantive question about the role of technology shocks it is crucial that we parameterize the VAR in terms of Φ_ϵ instead of Σ. Let Σ_{tr} be the Cholesky factor of Σ and let Ω be an orthonormal matrix. Then, without loss of generality, we can write

$$\Phi_\epsilon = \Sigma_{tr}\Omega$$

The above prior distribution for Σ induces a prior for Σ_{tr} and we only have to specify a distribution for the orthonormal matrix Ω. Following the approach in Del Negro and Schorfheide (2004), we link Ω to the DSGE model parameters as follows. According to the DSGE model, upon impact the effect of the structural shocks ϵ_t on the endogenous variables y_t is given by

$$\left(\frac{\partial y_t}{\partial \epsilon_t'}\right)_{DSGE} = A(\theta),$$

say. We now use a QR factorization of $A(\theta)$ to decompose $A(\theta)$ uniquely into a lower triangular matrix and an orthonormal matrix $\Omega_*(\theta)$. To link the reduced form and the structural innovations in the VAR we use:

$$\Phi_\epsilon = \Sigma_{tr}\Omega_*(\theta)$$

Hence, along the restriction function $\Phi_*(\theta), \Sigma_*(\theta)$ the VAR impulse responses to the structural shocks will closely resemble the DSGE model impulse responses, at least at short horizons.

Overall, we obtain a hierarchical model composed of a prior distribution for the DSGE model parameters, $p(\theta)$, a prior for the VAR parameters, $p(\Phi, \Sigma, \Omega|\theta, \lambda)$, and the VAR likelihood function, $p(Y|\Phi, \Sigma)$. We refer to this model as DSGE-VAR. Conditional on λ we can write the joint density of data and parameters as

$$p(Y|\Phi, \Sigma)p(\Phi, \Sigma|\theta, \lambda)p(\Omega|\theta)p(\theta).$$

We use the MCMC methods developed in Del Negro and Schorfheide (2004) and reviewed in An and Schorfheide (2007) to generate draws from the joint posterior distribution of θ, Φ, Σ, and Ω.

As discussed in detail in Del Negro, Schorfheide, Smets, and Wouters (2007), we can study the fit of the DSGE model and determine by how much the cross-coefficient restrictions need to be relaxed by examining the marginal likelihood function of the hyperparameter λ:

$$p(Y|\lambda) = \int p(Y|\Phi, \Sigma)p(\Phi, \Sigma, \Omega, \theta|\lambda)d(\theta, \Phi, \Sigma, \Omega). \tag{14}$$

The marginal likelihood penalizes the in-sample-fit of the estimated VAR by a measure of complexity. The larger λ, the more restricted the prior, the smaller the model complexity, and the smaller the penalty.

5.2 Estimation Results

We report estimates for the DSGE model parameters obtained from the DSGE-VAR analysis in Table 6. We consider two choices of the hyperparameter: $\lambda = \infty$ corresponds to the VAR(4) approximation of the DSGE model; $\lambda = 1$ implies that we are allowing for substantial deviations of the VAR coefficients from the restrictions implied by the VAR approximation of the DSGE model – loosely speaking we are estimating the VAR based on a sample that consists to equal parts of actual and DSGE-model-generated observations.

The results in Table 6 can be summarized as follows. First, the log marginal data densities indicate that the fit of the vector autoregression improves drastically if we relax the restrictions generated by the theoretical model. Second, the estimated labor supply elasticity rises from 0.23 ($\lambda = \infty$) to 0.48 ($\lambda = 1$). The estimates of θ obtained for $\lambda = 1$ can be interpreted as a Bayesian version of minimum distance estimates[4]: we create an estimate of the VAR coefficients, albeit by tilting it toward the theoretical model, and then look for a parameterization of the DSGE model that minimizes the discrepancy between the estimated VAR coefficients and the restriction function $\Phi_*(\theta), \Sigma_*(\theta)$. Third, after relaxing the DSGE model restrictions we find a larger role of technology shocks for fluctuations in hours worked: according to the 90% posterior credible intervals, the two technology shocks explain 0 to 3% of the variation in hours if $\lambda = \infty$ and between 0 and 30% if $\lambda = 1$.

To shed some light on the findings obtained from the VAR estimation, it is useful to examine impulse response functions. In Figure 4 we compare responses obtained from the

[4] A rigorous statement is provided in Del Negro and Schorfheide (2004).

state-space representation of the DSGE model to responses calculated based on the VAR(4) approximation of the theoretical model, using the same values for θ. While the VAR(4) approximation is not exact, it reproduces the responses of the DSGE model for hours and the investment-specific technology at all horizons, and the responses of labor productivity at least over the first 12 quarters accurately. Thus, by and large, the DSGE-VAR($\lambda = \infty$) specification has similar dynamics as the DSGE model studied in Sections 3 and 4.

In Figure 5 we overlay DSGE-VAR responses obtained for $\lambda = 1$ and $\lambda = \infty$. Most important for our understanding of the variance ratios is the response of hours to the two technology shocks. First note that the responses of labor productivity to an innovation $\epsilon_{a,t}$ and the responses of the relative price of investment to $\epsilon_{v,t}$ are quantitatively very similar for $\lambda = 1$ and $\lambda = \infty$. Moreover, the response of hours worked to an investment-specific technology shock does not change significantly as we relax the restrictions implied by our theoretical model. What changes drastically, is the magnitude of the hours response to a neutral technology shock. For $\lambda = 1$ hours rise sharply in response to the neutral technology shock, and hence this shock can account for about 13% of the variance of hours.

If the technology processes are difference stationary and we denote the growth rates of A_t and V_t by $\widehat{a}_t$ and $\widehat{v}_t$, we can express hours (in percentage deviations from steady state) as

$$\widehat{h}_t = \frac{1}{(\nu+1)/\nu - (1-\alpha)} \left[\alpha \widehat{k}_t - \widehat{c}_t - \frac{\alpha}{1-\alpha}(\widehat{a}_t + \widehat{v}_t) \right].$$

Moreover, it is straightforward to verify that the two technology growth rates affect the law of motion of $\widehat{k}_t$ and $\widehat{c}_t$ in the same manner. Thus, the only difference between the propagation of the innovations $\epsilon_{a,t}$ and $\epsilon_{v,t}$ is that $\widehat{a}_t$ is essentially serially uncorrelated, whereas $\widehat{v}_t$ has an autocorrelation of about 0.8. To the extent that the likelihood-based estimation procedure of the DSGE model determines the labor supply elasticity ν from the response of hours and wages to an investment-specific technology shock, this estimate also determines the response of hours to a neutral shock. A VAR that relaxes the cross-coefficient restrictions of the DSGE model can break this link. According to our DSGE-VAR($\lambda = 1$) estimates, neutral technology shocks have a much larger effect on hours work than implied by the fairly low estimate of the Frisch elasticity.

6 Conclusion

The analysis in the previous sections suggests that all the methods considered in this paper have their advantages and disadvantages. Conditional on imposing the restrictions of the dynamic equilibrium on the empirical analysis the main difference between the calibration analysis and the Bayesian estimation was the identification inherent in the likelihood function. Carefully implemented calibration studies try to create a tight link between model parameters and informative empirical observations, drawn from a wide variety of sources. In poorly implemented calibration studies this link is often lost.

Econometric approaches impose a lot of discipline on the specification of estimation objective functions and, as reward, produce statistical measures of uncertainty or reliability of parameter estimates and quantitative model implications. Much of the state-of-the-art empirical work is based on likelihood functions and parameters are determined by matching the model implied autocovariances to sample autocovariances. The potential downside of the mechanical application of likelihood functions is that it becomes much harder to link patters in the data to particular parameter estimates and to provide a compelling story about identification. In our application a key parameter turned out to be the labor supply elasticity and we provided some insights into how this parameter is identified from the observed autocovariances. Bayesian techniques can be used to build a bridge between calibration and estimation approaches, in that they allow us to incorporate observations that are not captured by the likelihood function through the prior distribution.

Potential model misspecification may cast some doubts on the reliability of quantitative answers obtained from calibrated or estimated DSGE models and it is advisable to study the sensitivity of the quantitative results to reasonable perturbations of the theoretical structure. Vector autoregressions provide a convenient way for doing so. We gradually relaxed the cross-coefficient restrictions using the DSGE-VAR framework. We maintained sufficiently many restrictions to be able to identify the two technology shocks, while allowing for more general patterns of propagation. In doing so, we found a greater effect of technology shocks on hours fluctuations.

Perhaps of most interest for all is that the methodologies themselves did not really change the answers, but the implicit identification of the labor supply elasticity did. We should probably talk more about that, instead of arguing for or against methodologies with reasons that are not too different from the ones that we use to justify the choice of soccer team we root for.

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Table 1: Parameter Categories

Steady States:	α, β, δ, ξ_*
Internal Propagation Only	ν
Exogenous Disturbances	$\ln A_0$, $\psi_{a,1}$, $\psi_{a,2}$, σ_a,
	$\ln V_0$, $\psi_{v,1}$, $\psi_{v,2}$, σ_v

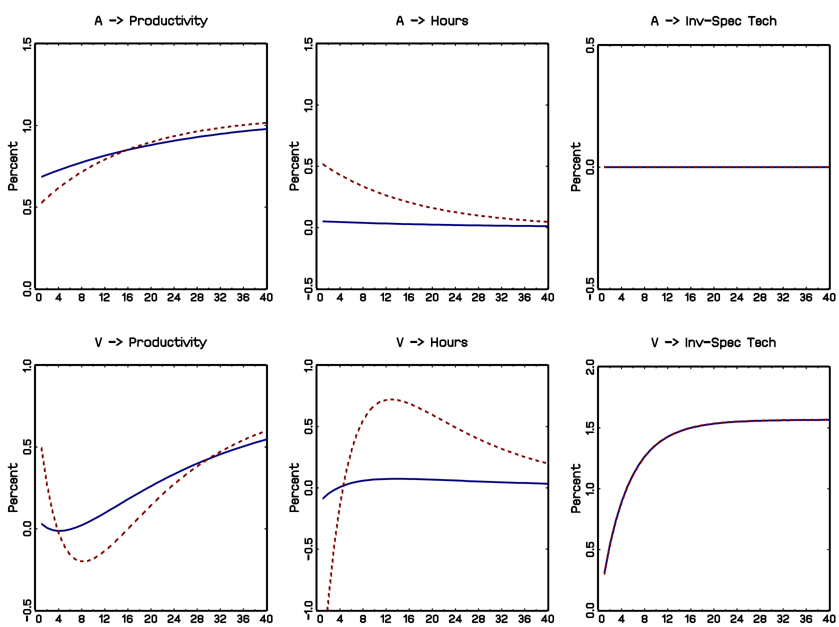
Notes: (to be added)

Table 2: Calibration - Stochastic Trend Version

	Calibration 1		Calibration 2		Calibration 3	
	$\nu = 2.0$		$\nu = 100$		$\nu = 0.2$	
	Mean	90% Cred. Intv	Mean	90% Cred Intv	Mean	90% Cred Intv
Parameter Values						
α	0.340		0.340		0.340	
β	0.990		0.990		0.990	
δ	0.013		0.013		0.013	
γ_a	0.000		0.000		0.000	
$\psi_{1,a}$	1.000		1.000		1.000	
$-\psi_{2,a}$	0.000		0.000		0.000	
σ_a	0.007	[0.006, 0.008]	0.007	[0.006, 0.008]	0.007	[0.006, 0.008]
γ_v	0.007	[0.005, 0.009]	0.007	[0.005, 0.009]	0.007	[0.005, 0.009]
$\psi_{1,v}$	1.000		1.000		1.000	
$-\psi_{2,v}$	0.799	[0.737, 0.868]	0.800	[0.737, 0.865]	0.800	[0.733, 0.865]
σ_v	0.003	[0.003, 0.003]	0.003	[0.003, 0.003]	0.003	[0.003, 0.003]
Variance Ratios for Hours: Model / Data						
A	0.050	[0.020, 0.070]	0.140	[0.070, 0.210]	0.002	[0.001, 0.003]
V	0.230	[0.060, 0.420]	0.830	[0.250, 1.450]	0.010	[0.002, 0.017]
A, V	0.280	[0.090, 0.470]	0.970	[0.330, 1.610]	0.012	[0.003, 0.020]

Notes: (to be added)

Figure 1: Impulse Response Functions for Calibrated Model



Notes: The figures depict responses to one-standard-deviation shocks for $\nu = 0.2$ (solid) and $\nu = 100$ (dashed).

Table 3: Calibration - Deterministic Trend Version

	Calibration 1		Calibration 2		Calibration 3	
	$\nu = 2.0$		$\nu = 100$		$\nu = 0.2$	
	Mean	90% Cred. Intv	Mean	90% Cred Intv	Mean	90% Cred Intv
			Parameter Values			
α	0.340		0.340		0.340	
β	0.990		0.990		0.990	
δ	0.013		0.013		0.013	
γ_a	0.000		0.000		0.000	
$\psi_{1,a}$	0.980		0.980		0.980	
$-\psi_{2,a}$	0.050	[-0.067, 0.161]	0.050	[-0.067, 0.161]	0.050	[-0.067, 0.161]
σ_a	0.007	[0.006, 0.008]	0.007	[0.006, 0.008]	0.007	[0.006, 0.008]
γ_v	0.007	[0.005, 0.009]	0.007	[0.005, 0.009]	0.007	[0.005, 0.009]
$\psi_{1,v}$	0.980		0.980		0.980	
$-\psi_{2,v}$	0.770	[0.701, 0.832]	0.770	[0.701, 0.832]	0.770	[0.701, 0.832]
σ_v	0.003	[0.003, 0.003]	0.003	[0.003, 0.003]	0.003	[0.003, 0.003]
			Variance Ratios for Hours: Model / Data			
A	0.090	[0.040, 0.130]	0.310	[0.140, 0.450]	0.003	[0.001, 0.005]
V	0.220	[0.090, 0.340]	0.980	[0.340, 1.540]	0.007	[0.003, 0.010]
A, V	0.310	[0.150, 0.460]	1.290	[0.560, 1.910]	0.010	[0.005, 0.015]

Notes: (to be added)

Table 4: Prior Distribution for DSGE Model Parameters

Name	Domain	Density	Para (1)	Para (2)
α	$[0, 1)$	Beta	0.36	0.20
β		fixed	0.99	
δ		fixed	.013	
ν	$I\!R^+$	Gamma	2.00	1.00
γ_a	$I\!R$	Normal	0.00	0.10
$\psi_{1,a}$	$I\!R^+$	Beta	0.95	0.02
$-\psi_{2,a}$	$I\!R^+$	Beta	0.20	0.10
σ_a	$I\!R^+$	InvGamma	0.01	4.00
γ_b	$I\!R$	Normal	0.00	0.10
$\psi_{1,v}$	$I\!R^+$	Beta	0.95	0.02
$-\psi_{2,v}$	$I\!R^+$	Beta	0.50	0.20
σ_v	$I\!R^+$	InvGamma	0.01	4.00
ρ_ξ	$I\!R^+$	Beta	0.80	0.1
σ_ξ	$I\!R^+$	InvGamma	0.01	4.00
$\ln H_*$	$I\!R$	Normal	0.00	10.0
$\ln Y_0$	$I\!R$	Normal	0.00	100
$\ln V_0$	$I\!R$	Normal	0.00	100

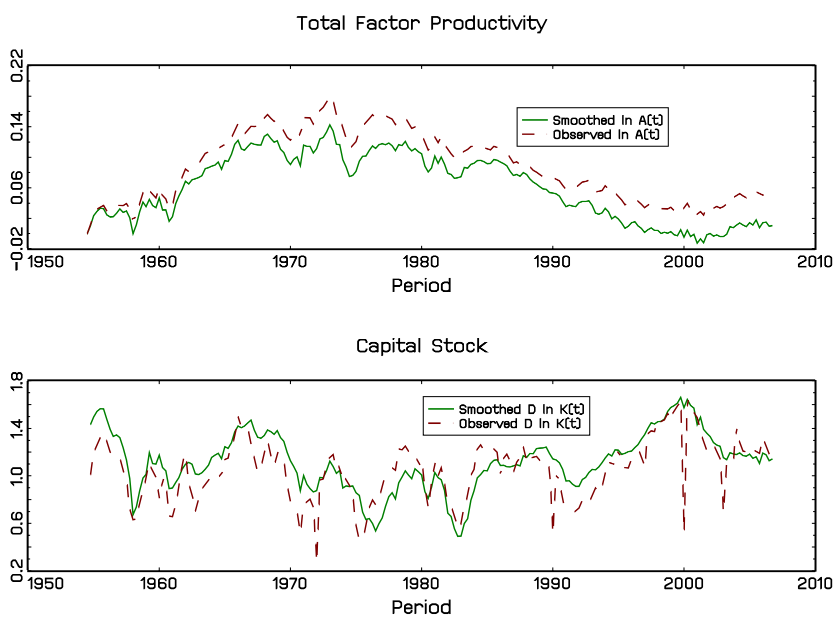
Notes: Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; the upper and lower bound of the support for the Uniform distribution; s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. To estimate the stochastic growth version of the model we set $\psi_{1,a} = \psi_{1,v} = 1$.

Table 5: POSTERIOR ESTIMATES FOR DSGE MODEL

	Deterministic Trend		Stochastic Trend	
	Mean	90% Cred. Intv	Mean	90% Cred Intv
	Parameter Values			
α	0.375	[0.342, 0.407]	0.354	[0.322, 0.387]
ν	0.670	[0.296, 1.038]	0.302	[0.050, 0.533]
γ_a	-0.001	[-0.002, -0.001]	-0.001	[-0.002, 0.001]
$\psi_{1,a}$	0.975	[0.962, 0.990]	1.000	
$-\psi_{2,a}$	0.087	[-0.041, 0.202]	0.121	[0.038, 0.207]
σ_a	0.007	[0.007, 0.008]	0.007	[0.007, 0.008]
γ_v	0.007	[0.007, 0.008]	0.007	[0.005, 0.008]
$\psi_{1,v}$	0.990	[0.988, 0.994]	1.000	
$-\psi_{2,v}$	0.728	[0.646, 0.807]	0.714	[0.636, 0.794]
σ_v	0.003	[0.003, 0.004]	0.003	[0.003, 0.004]
ρ_ξ	0.970	[0.952, 0.990]	0.972	[0.955, 0.993]
σ_ξ	0.011	[0.010, 0.013]	0.010	[0.009, 0.011]
	Variance Ratios for Hours: Model / Data			
A	0.030	[0.010, 0.060]	0.010	[0.000, 0.020]
V	0.060	[0.010, 0.100]	0.010	[0.000, 0.030]
A, V	0.100	[0.020, 0.180]	0.020	[0.000, 0.050]
$\ln p(Y)$	2264.74		2267.60	

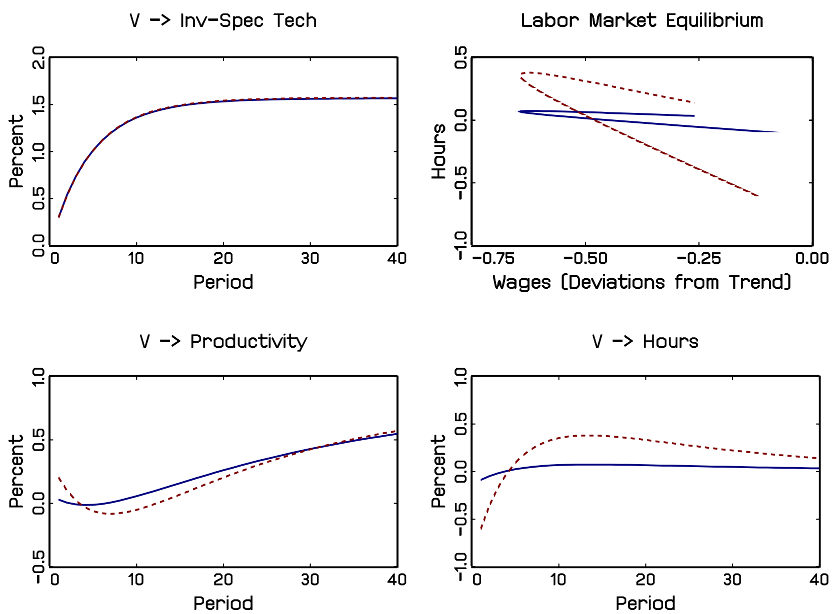
Notes: The following parameters were fixed during the estimation: $\beta = 0.99$, $\delta = 0.013$.

Figure 2: Neutral Technology and Capital in the Estimated DSGE Model



Notes:

Figure 3: Labor Market Equilibrium: $\nu = 0.2$ (solid) versus $\nu = 2$ (dashed)



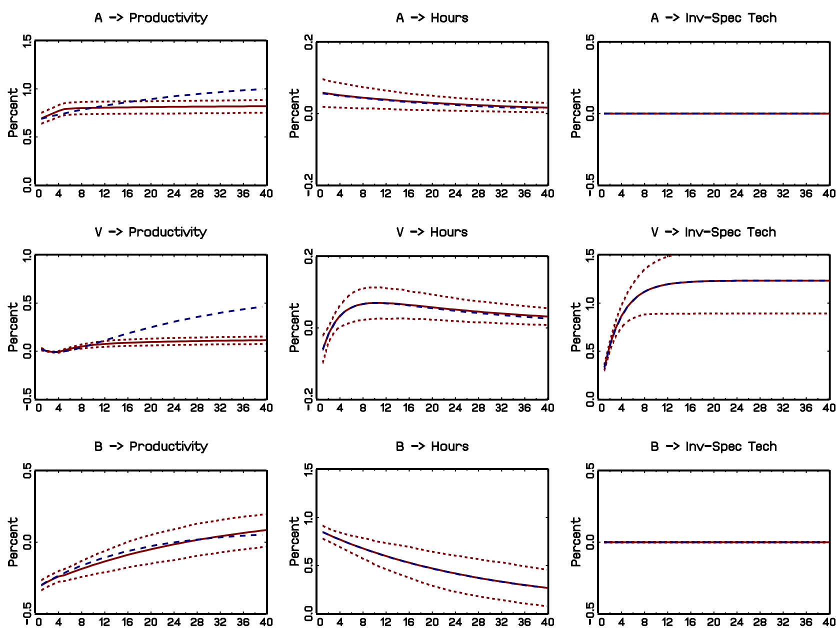
Notes:

Table 6: Posterior Estimates for the DSGE-VAR

	DSGE-VAR($\lambda = \infty$)		DSGE-VAR($\lambda = 1$)	
	Mean	90% Cred. Intv	Mean	90% Cred Intv
Parameter Values				
α	0.353	[0.322, 0.386]	0.360	[0.327, 0.395]
ν	0.229	[0.056, 0.395]	0.484	[0.151, 0.815]
γ_A	0.000	[-0.001, 0.001]	0.000	[-0.001, 0.001]
$\psi_{1,a}$	1.000		1.000	
-$\psi_{2,a}$	0.000		0.000	
σ_a	0.007	[0.007, 0.008]	0.007	[0.006, 0.007]
γ_v	0.007	[0.005, 0.008]	0.007	[0.005, 0.009]
$\psi_{1,v}$	1.000		1.000	
-$\psi_{2,v}$	0.727	[0.652, 0.800]	0.615	[0.506, 0.725]
σ_v	0.003	[0.003, 0.004]	0.003	[0.003, 0.003]
ρ_ξ	0.970	[0.952, 0.989]	0.958	[0.931, 0.985]
σ_ξ	0.010	[0.008, 0.011]	0.008	[0.006, 0.009]
Variance Ratios for Hours: Model / Data				
A	0.004	[0.000, 0.009]	0.128	[0.004, 0.249]
V	0.010	[0.000, 0.021]	0.030	[0.001, 0.070]
A, V	0.014	[0.000, 0.029]	0.158	[0.001, 0.298]
$\ln p(Y)$	2278.14		2322.83	

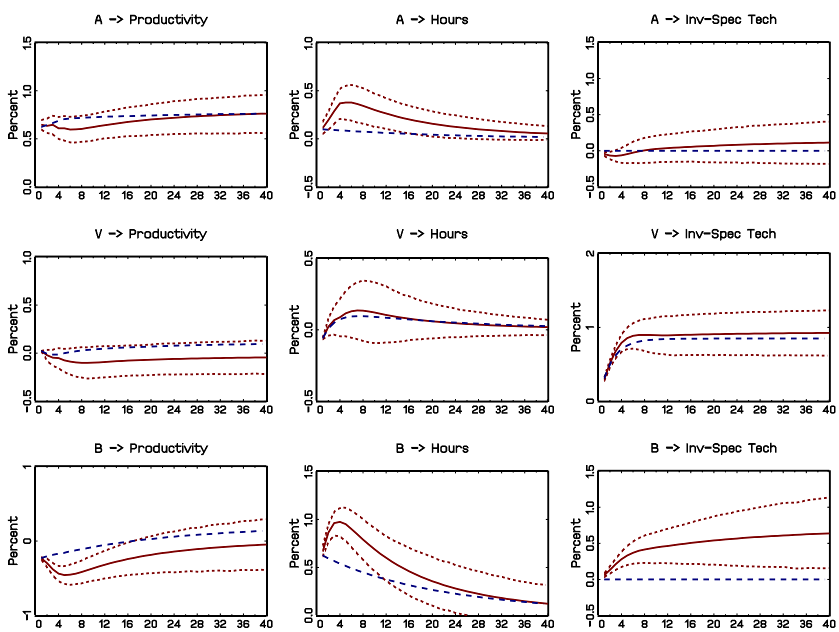
Notes: The following parameters were fixed during the estimation: $\beta = 0.99$, $\delta = 0.013$.

Figure 4: Impulse Responses: DSGE (dashed) versus DSGE-VAR($\lambda = \infty$) (solid)



Notes: (to be added)

Figure 5: Impulse Responses: DSGE-VAR($\lambda = 1$) (solid) versus DSGE-VAR($\lambda = \infty$) (dashed)



Notes: (to be added)

A Data Construction

We use U.S. 1948.I-2006.IV data from NIPA-BEA, FAT-BEA, BLS and Cummins and Violante (2002) to construct quarterly series of investment-specific technological change and neutral technological change. First, we construct the real price of investment that, as Greenwood, Hercowitz, and Krusell (1997), Cummins and Violante (2002) and Fisher (2006), identifies investment-specific technological change. Second, we use the aggregate resource constraint of the model economy to recover a series of neutral technological change that incorporates capital quality improvement.

A.1 Raw Data Series

All raw data series retrieved from the the Bureau of Economic Analysis (BEA; www.bea.gov) and the Bureau of Labor Statistics (BLS; www.bls.gov) for the period 1948.I-2006.IV were current as of April 19, 2007

National Income and Product Accounts (NIPA-BEA)

1. Table 1.1.5: Consumption of Durable Goods (CD_t), Change in Inventories (ChInv_t)
2. Table 1.7.5: Gross National Product (GNP_t).
3. Table 2.3.3 and 2.3.5: Quantity Index (QCONS_t^i) and Nominal (CONS_t^i) Nondurables Consumption (excluding Energy) and Services (excluding Housing)[5]
4. Table 3.9.5: Government Investment in Equipment (GovIEQ_t), Government Investment in Structures (GovIST_t)
5. Table 5.3.5: Private Fixed Investment in Equipment ($\mathrm{PrivIEQ}_t$), Private Fixed Investment in Structures ($\mathrm{PrivIST}_t$)

Fixed Asset Tables (FAT-BEA)

1. Tables 5.3.4: Official Price Index for Investment in Equipment (OPIEQ_t)

[5]Goods i correspond to nondurables consumption in food, clothing and shoes, and others, and services in households operations, transportation, medical care, recreation and others.

Bureau of Labor Statistics(BLS)

1. Aggregate Hours Index (H_t), BLS ID PRS85006033
2. Civilian Noninstitutional Population +16 ($Pop16_t$), BLS ID LNU00000000

Cummins and Violante (2002), 1947-2000

1. Annual Quality-Adjusted Price Index for Investment in Equipment ($\text{QAPIEQ}^{CV}_{year}$)
2. Annual Quality-Adjusted Depreciation Rates for Total Capital (δ^{CV}_{year})

A.2 The Relative Price of Quality-Adjusted Investment

We construct the relative price of quality-adjusted investment (in terms of the consumption good) as a Tornquist aggregate of the price index of structures investment and the price index of quality-adjusted equipment investment. We use the Tornquist procedure to aggregate price indexes of nondurables consumption (excluding energy) and services (excluding housing) into a structures investment price index.

Quarterly Price Index for Investment in Structures, PCONS$_t$. We use a Tornquist price index aggregate that weights growth rates of price indexes for nondurables consumption (food, clothing and shoes, and others) and services (households operations, transportation, medical care, recreation and others) by their nominal shares. That is, the quarterly price index for investment in structures is a consumption price index for nondurables consumption and services. Let PCONS^i_t be the price index for nondurable consumption/service good i in quarter t computed as the ratio between nominal consumption of good i, CONS^i_t, and the quantity index of good i, QCONS^i_t. Let s^i_t be the corresponding current and last period nominal share of good i. Then, the growth rate [6] of the price index for investment in structures is

$$\lambda\left(\text{PCONS}_t\right) = \sum_i \lambda\left(\text{PCONS}^i_t\right)\left(\frac{s^i_t + s^i_{t-1}}{2}\right)$$

The level of the consumption price index is recovered recursively,

$$\text{PCONS}_t = \text{PCONS}_{t-1}\ \left(1 + \lambda\left(\text{PCONS}_t\right)\right)$$

where we set PCONS_0 such that the initial relative price of investment is equal to one, see below.

[6] We denote by $\lambda(x_t)$ the growth rate of variable x_t, that is, $\lambda(x_t) = \frac{x_t - x_{t-1}}{x_t}$

Quarterly Quality-Adjusted Price Index for Investment in Equipment, $\mathbf{QAPIEQ}_t$. We use the U.S. 1947-2000 annual series provided by Cummins and Violante (2002) for the price index of equipment investment, $\text{QAPIEQ}^{CV}_{year}$, and impute the quarterly movements of the official FAT-BEA price index of equipment investment, OPIEQ_t.[7] For the years after 2000, we use the official price index.[8] This way, we consider that the hedonic methods used to compute the official price index correctly quality-adjust most of types of equipment investment after 2000.

Quarterly Quality-Adjusted Price Index for Total Investment, $\mathbf{QAPI}_t$. We use a Tornquist price index aggregate that weights growth rates of the price index of investment in structures and the price index of investment in equipment by their nominal shares, respectively s_t^{IST} and s_t^{IEQ} and where nominal structures investment is the sum of private structures investment (PrivIST_t) and government structures investment (GovIST_t), and nominal equipment investment is the sum of private equipment investment (PrivIEQ_t), government equipment investment (GovIEQ_t), changes in inventories (ChInv_t) and consumer durables (CD_t). The growth rate of the quarterly quality-adjusted price index for total investment is

$$\lambda(\text{QAPI}_t) = \lambda(\text{PCONS}_t)\left(\frac{s_t^{IST} + s_{t-1}^{IST}}{2}\right) + \lambda(\text{QAPIEQ}_t)\left(\frac{s_t^{IEQ} + s_{t-1}^{IEQ}}{2}\right)$$

The level of quarterly quality-adjusted price index for total investment is recovered recursively,

$$\text{QAPI}_t = \text{QAPI}_{t-1}\ \left(1 + \lambda\left(\text{QAPI}_t\right)\right)$$

where we use the initial value QAPI_0 suggested in Cummins and Violante (2002). The price index QAPI_t quality adjusts nominal investment, that is, it transforms nominal investment into investment in efficiency units.

Quarterly Relative Price of Investment, P_t. The relative price of investment goods (using the consumption good as numeraire) is defined as the ratio between the quality-adjusted investment price index, QAPI_t (investment deflator), and the consumption price index, PCONS_t (consumption deflator),

$$P_t = \frac{\text{QAPI}_t}{\text{PCONS}_t}$$

[7] As Fisher (2006), in order to impute the quarterly official price index fluctuations to the quality-adjusted annual series in Cummins and Violante (2002) we use the proportional Denton method of interpolation. The Denton method is a standard subroutine in most econometric software packages.

[8] We re-scale the official series such that it equates the value in Cummins and Violante (2002) in year 2000.

Its inverse, $V_t = \frac{1}{P_t}$, is investment-specific technological change. We set $V_0 = \frac{1}{P_0} = 1$, that is, we assume real capital is equal to capital in efficiency units in 1947.

A.3 Neutral Technological Change

The series of neutral technological change is standardly computed using measures of real output Y_t,[9] real capital K_t and labor input H_t,[10] together with an estimate of the input shares of production. Here, we explicitly consider capital quality improvement represented by the the historical fall in the real price of investment. To do so we build quarterly series for investment in efficiency units and physical depreciation rates that we use to construct series of quality-adjusted capital stock. Quality adjustments change substantially the series of capital - real capital falls below capital in efficiency units, and affect the trend of neutral technological change.

Quarterly Quality-Adjusted Investment, X_t. Total investment in efficiency units is defined as total de-annualized nominal investment deflated by the quality-adjusted price of investment

$$X_t = \frac{\text{InvEQ}_t + \text{InvST}_t}{\text{QAPI}_t}$$

Quarterly Quality-Adjusted Depreciation Rates, δ_t. We build on the time-varying annual physical depreciation rates for total capital provided in Cummins and Violante (2002) for the period 1947-2000, δ^{CV}_{year}.[11] They argue that physical (rather than economic) depreciation must be used to construct the quality-adjusted productive capital stock when investment is measured in efficiency units. We define δ as the average depreciation rate over the period 1955:Q1 to 2004:Q4: $\delta = 0.137$.

Quarterly Quality-Adjusted Capital Stock, K_t. We have created quarterly quality-adjusted investment series, X_t, and quarterly series for the quality-adjusted depreciation rate, δ_t. Then we can construct the series of capital in efficiency units recursively using the perpetual inventory method,

$$K_{t+1} = (1 - \delta)\, K_t + X_t$$

where the initial capital stock in efficiency units, K_0, is calibrated using the steady-state investment equation

$$\frac{K_0}{Y_0} = \frac{V_0\, I_0}{Y_0} \left(1 - (1 - \delta) \exp\left(-\lambda_K\right)\right)^{-1}.$$

[9] Real output Y_t is computed as the nominal gross national product, GNP_t, deflated with PCONS_t

[10] In addition, we convert these series in per capita terms dividing by civilian noninstitutional population $Pop16_t$.

[11] For the years after 2000 we assume a constant depreciation rate equal to that in year 2000.

We obtain the unconditional mean of the investment-output ratio is 0.284, and the quarterly capital per capita growth rate averages 1.08%.[12] This yields an initial quarterly capital-output ratio of 11.6 (or 2.92 annually) which together with the initial value of real output pins down an initial efficient capital stock.

Neutral Technological Change, A_t. The series of neutral technological change is computed as

$$A_t = \frac{Y_t}{K_t^{\alpha}\ H_t^{1-\alpha}},$$

where $\alpha = \sum_t \frac{\alpha_t}{T}$ is the average capital share augmented to incorporate capital income from government capital and durables.

B The Model

B.1 Stochastic Trend

Let us define $\hat{x}_t = ln(\tilde{X}_t/X^*)$, the log-linearized equilibrium conditions are given by:

$$\hat{y}_t = \hat{c}_t\frac{C^*}{Y^*} + \hat{i}_t\frac{I^*}{Y^*} \tag{15}$$

$$\hat{y}_t = -\alpha(\hat{q}_t + \hat{v}_t) + \alpha\hat{k}_t + (1-\alpha)\hat{h}_t \tag{16}$$

$$\hat{k}_{t+1} = (1-\delta)\left(\frac{1}{q^*v^*}\right)\left[\hat{k}_t - (\hat{q}_t + \hat{v}_t)\right] + \hat{i}_t\frac{I^*}{K^*} \tag{17}$$

$$0 = E_t\left[\hat{c}_t - \hat{c}_{t+1} - (\hat{q}_{t+1} + \hat{v}_{t+1}) + \hat{r}^k_{t+1}\right] \tag{18}$$

$$\hat{r^k_t} = \left(\frac{R^*}{1-\delta+R^*}\right)\hat{r}_t \tag{19}$$

$$\hat{h}_t = \nu(\hat{w}_t - \hat{c}_t - \hat{\xi}_t) \tag{20}$$

$$\hat{\xi}_t = \rho_\xi\hat{\xi}_{t-1} + \sigma_\xi\epsilon_{\xi,t} \tag{21}$$

$$\hat{r}_t = \hat{y}_t - \hat{k}_t + \hat{q}_t + \hat{v}_t \tag{22}$$

$$\hat{w}_t = \hat{y}_t - \hat{h}_t \tag{23}$$

$$\hat{q}_t = \frac{1}{1-\alpha}\hat{a}_t + \frac{\alpha}{1-\alpha}\hat{v}_t \tag{24}$$

Here $\hat{a}_t$ and $\hat{v}_t$ are the demeaned growth rates of the neutral and investment-specific technology processes.

[12]Note that in order to compute the growth rate of capital we do not need an initial value for the series of capital because the growth rate is a unit independent statistic.

The steady state conditions are given by

$$q^* = e^{\frac{1}{1-\alpha}\gamma_a + \frac{\alpha}{1-\alpha}\gamma_v} \tag{25}$$

$$v^* = e^{\gamma_v} \tag{26}$$

$$R^* = \frac{q^* v^*}{\beta} - (1-\delta) \tag{27}$$

$$\frac{X^*}{K^*} = 1 - (1-\delta)\frac{1}{q^* v^*} \tag{28}$$

$$I^* = X^* \tag{29}$$

$$\frac{K^*}{Y^*} = \frac{1}{\alpha q^* v^*} R^* \tag{30}$$

$$\frac{C^*}{Y^*} = \left(\frac{1-\delta}{q^* v^*} - 1\right)\frac{K^*}{Y^*} + 1 \tag{31}$$

$$\frac{I^*}{Y^*} = 1 - \frac{C^*}{Y^*} \tag{32}$$

$$H^* = \left[(1-\alpha)\frac{Y^*}{C^*}\right]^{\frac{\nu}{1+\nu}} \tag{33}$$

B.2 Deterministic Trend

The log-linearized system of equilibrium conditions is given by:

$$\hat{y}_t = \frac{C^*}{Y^*}\hat{c}_t + \frac{I^*}{Y^*}\hat{i}_t \tag{34}$$

$$\hat{y}_t = \alpha \hat{k}_t + (1-\alpha)\hat{h}_t + \hat{A}_t \tag{35}$$

$$q_* v_* \hat{k}_{t+1} = (1-\delta)\hat{k}_t + (q_* v_* - (1-\delta))(\hat{i}_t + \hat{V}_t) \tag{36}$$

$$0 = E_t\left[\hat{c}_t - \hat{c}_{t+1} - \hat{V}_{t+1} + \left(\frac{R^*}{1-\delta+R^*}\right)\hat{r}_{t+1}\right] \tag{37}$$

$$\hat{h}_t = \nu(\hat{w}_t - \hat{c}_t - \hat{\xi}_t) \tag{38}$$

$$\hat{\xi}_t = \rho_\xi \hat{\xi}_{t-1} + \sigma_\xi \epsilon_{\xi,t} \tag{39}$$

$$\hat{r}_t = \hat{y}_t - \hat{k}_t + \hat{V}_t \tag{40}$$

$$\hat{w}_t = \hat{y}_t - \hat{h}_t \tag{41}$$

Here $\hat{A}_t$ and $\hat{V}_t$ denote percentage deviations of the level of technology from the deterministic trend path. The steady state are described by (25)-(33), except that $I^* = X^*/V_0$ and $K^*/Y^* = R^*/(\alpha V_0)$.