# Pegs and Pain[*]

Stephanie Schmitt-Grohé[†]        Martín Uribe[‡]

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**Abstract**

This paper quantifies the costs of adhering to a fixed exchange rate arrangement, such as a currency union, for emerging economies. To this end it develops a dynamic stochastic disequilibrium model of a small open economy with downward nominal wage rigidity. In the model, a negative external shock causes persistent unemployment because the fixed exchange rate and downward wage rigidity stand in the way of real depreciation. In these circumstances, optimal exchange rate policy calls for large devaluations. In a calibrated version of the model, a large contraction, defined as a two-standard-deviation decline in tradable output, causes the unemployment rate to rise by more than 20 percentage points under a peg. The required devaluation under the optimal exchange rate policy is more than 50 percent. The median welfare cost of a currency peg is shown to be large, between 4 and 10 percent of lifetime consumption. Fixed exchange rate arrangements are found to be more costly when initial fundamentals are characterized by high past wages, large external debt, high country premia, or unfavorable terms of trade.

**JEL Classifications:** F41, E31.
**Keywords:** Currency pegs, currency unions, downward wage rigidity, unemployment, devaluation.

# 1   Introduction

Fixed exchange rate arrangements are easy to adopt but difficult to maintain. Countries can find themselves confined to a currency peg in a number of ways. For instance, a country could have adopted a currency peg as a way to stop high or hyperinflation in a fast and nontraumatic way. A classical example is the Argentine Convertibility Law of April 1991, which, by mandating a one-to-one exchange rate between the Argentine peso and the U.S. dollar, painlessly eliminated hyperinflation in less than a year. Another route by which countries arrive at a currency peg is the joining of a monetary union. Recent examples include emerging countries in the periphery of the European Union, such as Ireland, Portugal, Greece, and a number of small eastern European countries that joined the Eurozone. Most of these countries experienced an initial transition into the Euro characterized by low inflation, low interest rates, and economic expansion.

The Achilles' heel of currency pegs is, however, that they hinder the efficient adjustment of the economy to negative external shocks, such as drops in the terms of trade or hikes in the country interest-rate premium. The reason is that such shocks produce a contraction in aggregate demand that requires a decrease in the relative price of nontradables, that is, a real depreciation of the domestic currency, in order to bring about an expenditure switch away from tradables and toward nontradables. In turn, the required real depreciation may come about via a nominal devaluation of the domestic currency or via a fall in nominal prices or both. The currency peg rules out a devaluation. Thus, the only way the necessary real depreciation can occur is through a decline in the nominal price of nontradables. However, if nominal prices, especially factor prices, are downwardly rigid, the real depreciation will take place only slowly, causing recession and unemployment along the way.

This paper investigates how costly it is to maintain a currency peg, in terms of unemployment and welfare, for an emerging economy facing large external shocks. To this end, the paper embeds downward nominal wage rigidity into a dynamic, stochastic, disequilibrium (DSD) model of a small open economy with traded and nontraded goods. The key feature that distinguishes our theoretical framework from existing models of nominal wage rigidity is that in our formulation wage rigidity gives rise to involuntary unemployment. Specifically, downward wage rigidity introduces occasionally binding constraints that may prevent the real wage from falling to the level associated with full employment. A second distinguishing feature of our model of downward wage rigidity is that it does not rely on the assumption of imperfect competition in product or factor markets.

We show that in our DSD model, the optimal exchange rate policy takes the form of a time-varying rate of devaluation that allows the real wage to equal the full-employment

real wage at all times. Under the optimal exchange rate policy, the central bank devalues the nominal currency in periods in which the full-employment real wage rate experiences a sizable decline. These are typically periods in which the economy suffers negative external shocks.

The model is driven by stochastic disturbances to the traded endowment (which can be interpreted as variations in the terms of trade) and by disturbances to the country interest-rate premium. We estimate the joint law of motion of these two exogenous variables using Argentine data and feed it into a calibrated version of the model. We simulate the dynamics triggered by a large negative external shock like the one suffered by Argentina over the period 1998-2001, or peripherical Europe a decade later. Based on the Argentine experience, we define a large shock as a situation in which tradable output falls below trend by at least two standard deviations over a period of two and a half years. We compare the dynamics predicted by our model economy during the crisis under a currency peg and under the optimal exchange rate policy. Under the peg, the rate of unemployment rises by more than 20 percentage points. Even though tradable goods are scarce and there is open unemployment in the nontradable sector, the relative price of nontradables (the real exchange rate) fails to fall because of the combination of downward nominal wage rigidity and a fixed exchange rate. By contrast, under the optimal exchange rate policy the government devalues the currency by more than 50 percent. As a consequence, the real wage falls significantly and so does the relative price of nontradables. The adjustment of the real wage induced by the devaluation enables the economy to operate at full employment.

We measure the cost of maintaining a currency peg as the difference between the level of welfare enjoyed by households under a currency peg and the level of welfare associated with the optimal exchange rate policy. We find that the median welfare cost of living under a currency peg is enormous, between 4 and 10 percent of lifetime consumption. And weak fundamentals can bring this figure even higher. In particular, high levels of past wages, a large initial stock of external debt, a high country premium, and unfavorable terms of trade all exacerbate the welfare loss of living under a peg.

The remainder of the paper is organized in seven sections. Section 2 develops a dynamic disequilibrium model of a small open economy with unemployment due to downward nominal wage rigidity. This section also characterizes aggregate macroeconomic dynamics under a currency peg and the optimal exchange rate policy. Section 3 presents some empirical evidence on downward nominal wage rigidity and uses it as the basis to calibrate the model's structural parameter governing this source of nominal friction. Section 4 presents a quantitative analysis of external crises under currency pegs and the optimal exchange rate policy. Section 5 analyzes how exchange rate policy affects the long-run distribution of external

debt. Section 6 calculates the welfare benefits of switching from a currency peg to the optimal exchange rate policy. Section 7 studies the robustness of our findings to variations in the values taken by the calibrated parameters and preference specifications featuring endogenous labor supply. Finally, section 8 presents a fiscal policy that implements the efficient allocation when the exchange rate is fixed and then concludes.

# 2 A Dynamic Stochastic Disequilibrium Model

In this section, we develop a model of a small open economy in which nominal wages are downwardly rigid. In the model, the labor market is perfectly competitive. As a result, even though all participants understand that wages are nominally rigid, they do not act strategically in their pricing behavior. Instead, workers and firms take factor prices as given. The model features a traded and a nontraded sector and aggregate fluctuations are driven by stochastic movements in the value of tradable output and in the country-specific interest rate.

## 2.1 Households

The economy is populated by a large number of infinitely-lived households with preferences described by the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{1}$$

where $c_t$ denotes consumption, $U$ is a utility index assumed to be increasing and strictly concave, $\beta \in (0,1)$ is a subjective discount factor, and $\mathbb{E}_t$ is the expectations operator conditional on information available in period $t$. Consumption is assumed to be a composite good made of tradable and nontradable goods via the aggregation technology

$$c_t = A(c_t^T, c_t^N), \tag{2}$$

where $c_t^T$ denotes consumption of tradables, $c_t^N$ denotes consumption of nontradables, and $A$, defined over positive values of both of its arguments, denotes an aggregator function assumed to be homogeneous of degree one, increasing, concave, and to satisfy the Inada conditions. These assumptions imply that tradables and nontradables are normal goods.

Households are assumed to be endowed with an exogenous and stochastic amount of tradable goods, $y_t^T$, and a constant number of hours, $\bar{h}$, which they supply inelastically to the labor market. The assumption of an inelastic labor supply is motivated in part by microeconometric evidence (e.g., Blundell and MaCurdy, 1999) and macroeconometric

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evidence from models with nominal rigidities (e.g., Justiniano, Primiceri, and Tambalotti, 2010 ; and Smets and Wouters, 2007) suggesting that the labor supply elasticity is near zero. A second reason for assuming an inelastic labor supply is that it makes the workings of our two-sector model more transparent. In section 7, we relax this assumption by endogenizing the labor-supply decision.

Because of the presence of nominal wage rigidities in the labor market, households will in general only be able to work $h_t \leq \bar{h}$ hours each period. Households take $h_t$ as exogenously determined. Households also receive profits from the ownership of firms, denoted $\phi_t$, and expressed in terms of tradables. Households have access to an internationally traded risk-free pure discount bond that pays the country-specific interest rate, $r_t$, in terms of tradable goods when held between periods $t$ and $t+1$. The household's sequential budget constraint in period $t$ is then given by

$$c_t^T + p_t c_t^N + d_t \leq y_t^T + w_t h_t + \frac{d_{t+1}}{1+r_t} + \phi_t, \tag{3}$$

where $p_t$ denotes the relative price of nontradables in terms of tradables in period $t$, $d_{t+1}$ denotes the amount of debt assumed in period $t$ and maturing in period $t+1$, and $w_t$ denotes the real wage rate in terms of tradables in period $t$. Households are subject to a no-Ponzi game constraint of the form

$$d_{t+1} \leq \bar{d}, \quad \text{for } t \geq 0, \tag{4}$$

where $\bar{d} > 0$ is the natural debt limit.

The optimization problem of the household consists in choosing contingent plans $c_t$, $c_t^T$, $c_t^N$, and $d_{t+1}$ to maximize (1) subject to the aggregation technology (2), the sequential budget constraint (3), and the borrowing limit (4). Letting $\lambda_t$ and $\mu_t$ denote the Lagrange multipliers associated with (3) and (4), respectively, the optimality conditions associated with this dynamic maximization problem are (2), (3) holding with equality, (4), and

$$\frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = p_t, \tag{5}$$

$$\lambda_t = U'(c_t) A_1(c_t^T, c_t^N), \tag{6}$$

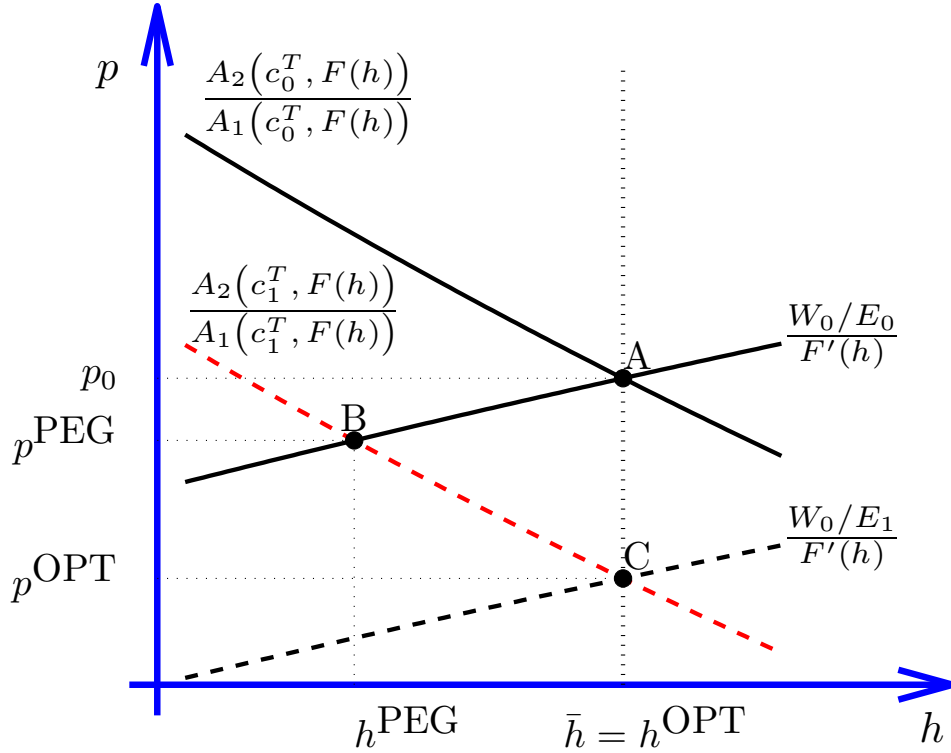$$\frac{\lambda_t}{1+r_t} = \beta \mathbb{E}_t \lambda_{t+1} + \mu_t, \tag{7}$$

$$\mu_t \geq 0, \tag{8}$$

$$\mu_t (d_{t+1} - \bar{d}) = 0, \tag{9}$$

where $A_i$ denotes the partial derivative of $A$ with respect to its $i$-th argument.

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Figure 1: General Disequilibrium

Optimality condition (5) can be interpreted as a demand function for nontradables. Intuitively, it states that an increase in the relative price of nontradables induces households to engage in expenditure switching by consuming relatively less nontradables. Our maintained assumptions regarding the form of the aggregator function $A$ guarantee that, for a given level of $c_t^T$, the left-hand side of this expression is a decreasing function of $c_t^N$. Figure 1 displays this downward sloping relationship with a solid line. The figure is drawn in the space $(h_t, p_t)$ by taking into account market clearing in the nontraded sector, which will be introduced later on. The level of $c_t^T$ acts as a shifter of the demand schedule for nontradables. A decline in $c_t^T$ causes the demand schedule to shift downward and to the left (see the broken downward sloping line in figure 1).

Optimality condition (7) equates the marginal benefits and the marginal costs of borrowing $1/(1 + r_t)$ units of tradables in period $t$. The left-hand side of this expression indicates the marginal utility of $1/(1 + r_t)$ units of tradables consumed in period $t$. The right-hand side presents the marginal cost of borrowing $1/(1 + r_t)$ units of tradables in period $t$. This cost is the sum of two components. One is $\beta \mathbb{E}_t \lambda_{t+1}$, which indicates the decline in utility caused by the reduction in tradable consumption in period $t+1$ necessary to repay the extra debt assumed in period $t$. The second component of the marginal cost is $\mu_t$, which measures

the shadow cost of tightening the borrowing limit. This second component is nil in periods in which $d_{t+1} < \bar{d}$, that is, in periods in which the borrowing limit does not bind.

## 2.2 Firms

The nontraded good is produced using labor as the sole factor input by means of an increasing and concave production function, $F(h_t)$. The firm operates in competitive product and labor markets. Profits, $\phi_t$, are given by

$$\phi_t = p_t F(h_t) - w_t h_t.$$

The firm chooses $h_t$ to maximize profits taking the relative price, $p_t$, and the real wage rate, $w_t$, as given. The optimality condition associated with this problem is

$$p_t F'(h_t) = w_t. \tag{10}$$

This first-order condition implicitly defines the firm's demand for labor. Alternatively, writing this expression as $p_t = w_t / F'(h_t)$, it can be interpreted as the supply schedule of non-tradables. Given the wage rate, an increase in $p_t$ induces firms to hire more hours and hence supply more nontradable goods. This supply schedule is shown with a solid upward sloping line in figure 1. Given the nominal wage rate, $W_t$, a devaluation acts as a shifter of the supply schedule. Specifically, given $W_t$, an increase in $E_t$ lowers the real labor cost, inducing firms to expand employment. This shift in the supply schedule is shown with an upward sloping dashed line in figure 1.

In the present model, firms are always on their labor demand curve. Put differently, firms never display unfilled vacancies nor are forced to employ more workers than desired. As we will see shortly, this will not be the case for workers, who will in general be off their labor supply schedule and will experience involuntary unemployment.

## 2.3 Unemployment and Wages

Let $W_t$ denote the nominal wage rate and $E_t$ the nominal exchange rate, defined as the domestic-currency price of one unit of foreign currency. Assume that the law-of-one-price holds for traded goods and that the foreign-currency price of traded goods is constant and normalized to unity. Then, the real wage in terms of tradables is given by

$$w_t \equiv \frac{W_t}{E_t}.$$

An assumption that distinguishes the present setup from the existing related literature is that nominal wages are assumed to be downwardly rigid. Specifically, we impose that

$$W_t \geq \gamma W_{t-1}, \quad \gamma > 0.$$

This setup nests the cases of absolute downward rigidity, when $\gamma \geq 1$, and full wage flexibility, when $\gamma = 0$. The presence of nominal wage rigidity implies the following restriction on the dynamics of real wages:

$$w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t}, \tag{11}$$

where

$$\epsilon_t \equiv \frac{E_t}{E_{t-1}}$$

denotes the gross nominal depreciation rate of the domestic currency.

The presence of downwardly rigid nominal wages implies that the labor market will in general not clear at the inelastically supplied level of hours $\bar{h}$. Instead, involuntary unemployment, given by $\bar{h} - h_t$, will be a regular feature of this economy. Actual employment must satisfy

$$h_t \leq \bar{h} \tag{12}$$

at all times. Finally, at any point in time, real wages and employment must satisfy the slackness condition

$$(\bar{h} - h_t) \left( w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) = 0. \tag{13}$$

This condition states that periods of unemployment must be accompanied by a binding wage constraint. It also states that when the wage constraint is not binding, the economy must be in full employment.

## 2.4   General Disequilibrium

Market clearing in the nontraded sector requires that

$$c_t^N = F(h_t). \tag{14}$$

Combining this market clearing condition, the definition of firm profits, and the household's sequential budget constraint, equation (3), yields

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t}. \tag{15}$$

We can then define the general disequilibrium of the economy as follows:

**Definition 1 (General Disequilibrium Dynamics)** *The general disequilibrium dynamics are given by stochastic processes $c_t$, $c_t^T$, $c_t^N$, $h_t$, $p_t$, $w_t$, $d_{t+1}$, $\lambda_t$, and $\mu_t$ satisfying (2) and (4)-(15) given the exogenous stochastic processes $y_t^T$ and $r_t$, an exchange rate policy, and initial conditions $w_{-1}$ and $d_0$.*

We consider two polar cases for the yet unspecified exchange rate policy: the optimal exchange rate policy and a currency peg.

## 2.5   Currency Pegs

Under a currency peg the government commits to keeping the nominal exchange rate constant over time, $E_t = E_{t-1}$ for $t \geq 0$. As a result, the gross rate of devaluation equals unity at all times:

$$\epsilon_t = 1, \tag{16}$$

for $t \geq 0$. Aggregate disequilibrium dynamics under a currency peg are then given by the exchange rate policy (16) and the conditions listed in Definition 1.

Under a currency peg, the economy is subject to two nominal rigidities. One is policy induced: The nominal exchange rate, $E_t$, is kept fixed by the monetary authority. The second is structural and is given by the downward rigidity of the nominal wage $W_t$. The combination of these two nominal rigidities results in a real rigidity. Specifically, under a currency peg, the real wage expressed in terms of tradables, $w_t$, is downwardly rigid, and adjusts only sluggishly, at the rate $(1 - \gamma)$, to negative demand shocks. The labor market is, therefore, in general, in disequilibrium and features involuntary unemployment. The magnitude of the labor market disequilibrium is a function of the amount by which the past real wage exceeds the current full-employment real wage. It follows that under a currency peg, the past real wage, $w_{t-1}$, becomes a relevant state variable for the economy.

Figure 1 depicts qualitatively the general disequilibrium for a given value of the aggregate domestic absorption of tradable goods, $c^T$. Assume that the initial desired level of tradable absorption is $c_0^T$. Market clearing in the nontraded sector occurs at point A, where the demand function, $A_2(c_0^T, F(h))/A_1(c_0^T, F(h))$, intersects the supply schedule, $(W_0/E_0)/F'(h)$. At point A, the economy enjoys full employment ($h = \bar{h}$) and the relative price of nontradables is equal to $p_0$. Suppose now that a negative demand shock, such as a deterioration in the terms of trade or a rise in the country premium, causes a decline in the desired aggregate absorption of tradables from $c_0^T$ to $c_1^T < c_0^T$. This adverse shock causes the demand function to shift down and to the left. This is because, at each level of nontraded consumption,

households are willing to consume less traded goods only if nontraded goods become relatively cheaper. At the same time the supply schedule does not shift.[1] This is because the combination of a currency peg and nominal downward rigidity of wages prevents the real wage from adjusting downward. The new intersection of the demand and supply schedules occurs at point B. At this point, employment is equal to $h^{PEG} < \bar{h}$ and the economy suffers from unemployment at the rate $\bar{h} - h^{PEG}$, where $h^{PEG}$ denotes the level of employment that obtains under the currency peg.

Alternatively, consider an exchange rate policy that maintains full employment in spite of the negative external shock. Specifically, suppose that in response to the negative external shock, the central bank were to devalue the domestic currency so as to deflate the purchasing power of nominal wages to a point consistent with full employment. That is, suppose that the central bank sets the exchange rate at the optimal level $E_1 > E_0$ satisfying $(W_0/E_1)/F'(\bar{h}) = A_2(c_1^T, F(\bar{h}))/A_1(c_1^T, F(\bar{h}))$. In this case the supply schedule would shift down and to the right intersecting the new demand schedule at point C, where unemployment is nil ($h = \bar{h}$). So, unlike the currency peg, the optimal exchange rate policy deflates the real value of wages, and in this way avoids involuntary unemployment. A further difference in the macroeconomic adjustment under a currency peg and the optimal exchange rate policy is that under a currency peg the relative price of non-tradables falls by less than under the optimal exchange rate policy. Specifically, in figure 1 the relative price of nontradables falls to $p^{PEG}$, whereas under the optimal policy, the relative price falls to $p^{OPT} < p^{PEG}$. This insufficient decline in the relative price of nontradables stands in the way of households switching expenditures away from tradables and toward nontradables, which is the root of the unemployment problem associated with currency pegs. Next, we analyze optimal exchange rate policy more formally.

## 2.6   Optimal Exchange Rate Policy

Consider an exchange rate arrangement in which the central bank always sets the devaluation rate to ensure full employment in the labor market, that is, to ensure that

$$h_t = \bar{h},$$

for all $t \geq 0$. We refer to this monetary arrangement as the full-employment exchange rate policy. This policy amounts to setting the devaluation rate to ensure that the real wage equals the full-employment real wage rate at all times. Formally, the optimal policy ensures

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[1]For expositional convenience, we are assuming here that $\gamma = 1$. A value of $\gamma$ close to but less than unity would imply a small displacement of the supply schedule down and to the right.

that

$$w_t = \omega(c_t^T),$$

where $\omega(c_t^T)$ denotes the full-employment real wage rate and is given by

$$\omega(c_t^T) \equiv \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h}). \tag{17}$$

The assumed properties of the aggregator function $A$ ensure that the function $\omega(\cdot)$ is strictly increasing in the domestic absorption of tradables, $c_t^T$,

$$\omega'(c_t^T) > 0.$$

The optimal exchange rate policy stipulates that should the nominal value of the full-employment real wage evaluated at last period's nominal exchange rate, $\omega(c_t^T)E_{t-1}$, fall below the lower bound $\gamma W_{t-1}$, then the central bank devalues the domestic currency to ensure that $\omega(c_t^T)E_t = \gamma W_{t-1}$. That is, the devaluation rate makes the nominal wage, $W_t$, equal to its lower bound, $\gamma W_{t-1}$, and at the same time guarantees that the real wage, $w_t$, equal the full-employment real wage $\omega(c_t^T)$. If, in this case, the central bank chose not to devalue, the economy would experience unemployment, because downward wage rigidities would prevent the real wage from falling to the full-employment level $\omega(c_t^T)$.

In general, any exchange rate policy satisfying $\epsilon_t \geq \gamma w_{t-1}/\omega(c_t^T)$ will ensure full employment at all times. All exchange rate policies pertaining to this family deliver the same real allocation and are therefore equivalent from a welfare point of view. From this class of policies we select the one that minimizes movements in the nominal exchange rate. Formally, the optimal exchange rate policy we consider is given by

$$\epsilon_t = \max\left\{1, \gamma\frac{w_{t-1}}{\omega(c_t^T)}\right\}. \tag{18}$$

This policy is of interest because it will inform us about the minimum devaluation required to maintain full employment during an external crisis.

The complete set of equilibrium conditions under the optimal exchange rate regime is then given by Definition 1 and the exchange rate policy given in equation (18).

Because under the optimal exchange rate policy the real wage rate is always equal to the full-employment real wage, equation (18) implies that for all $t > 0$ the devaluation rate is given by

$$\epsilon_t = \max\left\{1, \gamma\frac{\omega(c_{t-1}^T)}{\omega(c_t^T)}\right\}; \quad t > 0.$$

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Recalling that $\omega(\cdot)$ is a strictly increasing function of tradable absorption, this expression states that the central bank will devalue the domestic currency only when tradable expenditure falls. In light of this result, non-microfounded statistical analysis would conclude that devaluations are contractionary. However, the role of devaluations under the optimal exchange rate policy is precisely the opposite, namely, to prevent the contraction in the tradable sector to spill over into the nontraded sector. It follows that under the optimal exchange rate policy, devaluations are indeed expansionary in the sense that should they not take place, aggregate contractions would be even larger. Thus, under the optimal exchange rate regime, our model with downward nominal-wage rigidities turns the view that 'devaluations are contractionary' on its head and instead predicts that 'contractions are *devaluatory.*'

The full-employment exchange rate policy completely eliminates all real effects stemming from nominal wage rigidities. Indeed, one can show that the equilibrium under the optimal exchange rate policy is identical to the competitive equilibrium of an economy with full wage flexibility. Since wage rigidity is the only source of distortion in the present model, it follows that the equilibrium under the full-employment exchange rate policy is Pareto optimal. The equilibrium dynamics under the optimal exchange rate policy can therefore be characterized as the solution to the following value function problem:

$$v^{OPT}(y_t^T, r_t, d_t) = \max_{\{d_{t+1}, c_t^T\}} \left\{ U(A(c_t^T, F(\bar{h}))) + \beta \mathbb{E}_t v^{OPT}(y_{t+1}^T, r_{t+1}, d_{t+1}) \right\} \tag{19}$$

subject to (4) and (15), where the function $v^{OPT}(y_t^T, r_t, d_t)$ represents the welfare level of the representative agent under the full-employment exchange rate policy in state $(y_t^T, r_t, d_t)$. The equilibrium processes of all other endogenous variables of the model can be readily obtained from (18) and the conditions listed in Definition 1. The following proposition provides a formal statement of these results:

**Proposition 1** *The exchange rate policy given in equation (18) delivers a real allocation that exhibits full employment ($h_t = \bar{h}$) at all dates and states and, furthermore, is Pareto optimal.*

**Proof:** See appendix A.

The fact that the aggregate dynamics under optimal exchange rate policy can be described as the solution to a Bellman equation greatly facilitates the quantitative characterization of the model's predictions.

# 3  Some Evidence On Downward Nominal Wage Rigidity

A central parameter in our model is $\gamma$, which measures the degree of downward inflexibility in nominal wages. There is empirical evidence on the observed frequency of nominal wage cuts that strongly supports modeling nominal wage rigidity as one-sided, rather than as two-sided. Gottschalk (2005), for example, uses SIPP panel data to estimate the probability of wage declines, increases, and no changes for male and female hourly workers working for the same employer over the period 1986-1993 in the United States. He finds that for males the probabilities of wage increases, wage constancy, and wage declines are, respectively, 41.2, 53.7, and 5.1 percent per year. The corresponding probabilities for females are 46.5, 49.2, and 4.3 percent. This finding suggest that over the course of one year a very small fraction of workers experiences a decline in nominal wages, while about half of workers experience no change. For the purpose of our argument, it is important to note that the sample period used by Gottschalk comprises the 1991 U.S. recession, for it implies that the observed scarcity of nominal wage cuts took place in the context of elevated unemployment. Barattieri, Basu, and Gottschalk (2010) report similar findings using data from the 1996-2000 SIPP panel. Interestingly, this study is not restricted to workers working for the same employer. This evidence clearly favors modeling nominal wage rigidity as one-sided.

Evidence on the size of $\gamma$ can be found by examining the behavior of nominal wages during episodes in which an economy undergoing a severe recession keeps the nominal exchange rate fixed. A case in point is Argentina during the second half of the Convertibility Plan (1996-2001). Figure 2 displays nominal (peso) wages, real (dollar) wages, the nominal exchange rate, and subemployment (defined as the sum of unemployment and underemployment) for Argentina during the period 1996-2006. The subperiod 1998-2001 is of particular interest because the Argentine central bank was holding on to the currency peg in spite of the fact that the economy was undergoing a severe contraction and both unemployment and underemployment were in a steep ascent. In the context of a flexible-wage model, one would expect that the rise in unemployment would be associated with falling real wages. However, during this period, the average nominal wage never fell. Indeed, because the Argentine peso was pegged to the dollar, the dollar wage rose throughout the 1998-2001 contraction. We interpret this behavior of nominal wages as suggesting a value of $\gamma$ greater than or equal to unity. Additionally, the fact that real wages fell significantly and persistently with the devaluation of 2002 suggests that the 1998-2001 period was one of censored wage deflation, which further strengthens the view that nominal wages suffer from downward inflexibility.

We note that during the 1998-2001 Argentine contraction, consumer prices, unlike nom-

Figure 2: Nominal Wages and Unemployment in Argentina, 1996-2006
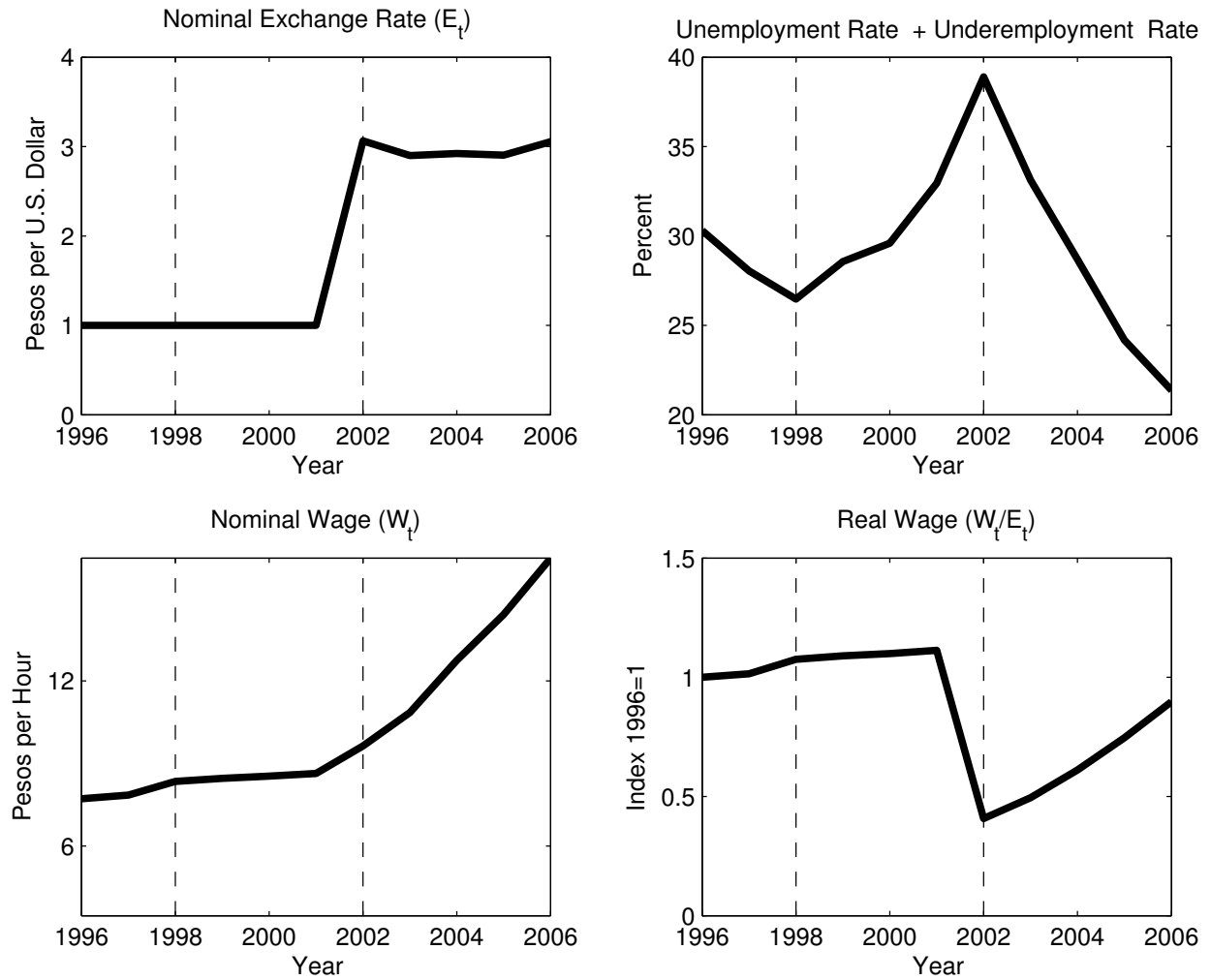
Source. Nominal exchange rate and nominal wage, BLS. Subemployment, INDEC.

inal wages, did fall significantly. The CPI rate of inflation was on average -0.86 percent per year over the period 1998-2001. It follows that real wages rose not only in dollar terms but also in terms of CPI units.[2]

Additional episodes that are relevant for inferring information about the value of $\gamma$ are provided by the behavior of unemployment and nominal wages in the periphery of the euro-zone in the aftermath of the great recession of 2008. Table 1 presents a rough estimate of $\gamma$ for ten European economies that are either on the euro or pegging to the euro. Our model predicts that nominal wages should fall at the rate $\gamma$ whenever the unemployment rate is above trend. The table shows the unemployment rate in 2008:Q1 and 2011:Q2. The starting point of this period corresponds to the beginning of the great recession in Europe according to the CEPR Euro Area Business Cycle Dating Committee, and the end point corresponds to the most recent data available at the time of the writing of this paper. The 2008 crisis caused unemployment rates to rise sharply across all ten countries. The table also displays the total growth of nominal hourly labor cost in manufacturing, construction and services over the thirteen-quarter period 2008:Q1-2011:Q2. Despite the large surge in unemployment, nominal wages grew in most countries and in those in which it fell, the decline was modest. The implied value of $\gamma$, shown in the last column of table 1, is given by the average growth rate of nominal wages over the period considered (that is, $\gamma = (W_{2011:Q2}/W_{2008:Q1})^{1/13}$). The estimated value of $\gamma$ ranges from 0.996 for Lithuania to 1.028 for Bulgaria. Again, this evidence suggests a value of $\gamma$ close to or even larger than unity.

In light of the above pieces of empirical evidence, we choose as the baseline value of $\gamma$ 0.99. This value means that nominal wages can fall frictionlessly by up to 4 percent per year. We regard this choice as conservative in the sense that it allows for more downward wage flexibility than what is suggested by the evidence presented above. Consider, for instance, the evidence presented in table 1. The largest fall in wages over the 13-quarter period since the start of the 2008 crisis is 5.1 percent and corresponds to Lithuania. Under our calibration, however, wages would have been allowed to fall by more than twice this amount, namely 13 percent. We also consider even more conservative values of $\gamma$ in the interval 0.96 to 0.99. The lower end of this range allows for a 52 percent decline in nominal wages over a 13-quarter period, a figure that vastly exceeds the observed declines in nominal wages in the eurozone since the onset of the great recession.

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[2]In addition, this evidence provides some support for our assumption that downward nominal rigidities are less stringent for product prices than for factor prices.

Table 1: <u>Unemployment, Nominal Wages, and $\gamma$: Evidence from the Eurozone</u>

| Country | Unemployment Rate 2008Q1 (in percent) | Unemployment Rate 2011Q2 (in percent) | Wage Growth $\frac{W_{2011Q2}}{W_{2008Q1}}$ (in percent) | Implied Value of $\gamma$ |
|---|---|---|---|---|
| Bulgaria | 6.1 | 11.3 | 43.3 | 1.028 |
| Cyprus | 3.8 | 6.9 | 10.7 | 1.008 |
| Estonia | 4.1 | 12.8 | 2.5 | 1.002 |
| Greece | 7.8 | 16.7 | -2.3 | 0.9982 |
| Lithuania | 4.1 | 15.6 | -5.1 | 0.996 |
| Latvia | 6.1 | 16.2 | -0.6 | 0.9995 |
| Portugal | 8.3 | 12.5 | 1.91 | 1.001 |
| Spain | 9.2 | 20.8 | 8.0 | 1.006 |
| Slovenia | 4.7 | 7.9 | 12.5 | 1.009 |
| Slovakia | 10.2 | 13.3 | 13.4 | 1.010 |

Note. $W$ is an index of nominal average hourly labor cost in manufacturing, construction, and services. Unemployment is the economy-wide unemployment rate. Source: EuroStat.

# 4    A Quantitative Analysis of External Crises

We wish to quantitatively characterize the response of our disequilibrium model to a large negative external shock. We have in mind extraordinary contractions like the 1989 or 2001 crises in Argentina, or the 2008 great recession in peripherical Europe. To this end, we estimate the joint stochastic process of our assumed exogenous driving forces (traded output and the country interest rate) using Argentine data. We calibrate the remaining structural parameters of the model to match salient aspects of the Argentine economy. Finally, we compute the economy's response to a large negative shock to traded output under the optimal exchange rate policy and under a currency peg.

## 4.1    Estimation of the Driving Process

The law of motion of tradable output and the country-specific interest rate is assumed to be given by the following autoregressive process:

$$\begin{bmatrix} \ln y_t^T \\ \ln \frac{1+r_t}{1+r} \end{bmatrix} = A \begin{bmatrix} \ln y_{t-1}^T \\ \ln \frac{1+r_{t-1}}{1+r} \end{bmatrix} + \epsilon_t, \tag{20}$$

where $\epsilon_t$ is a white noise of order 2 by 1 distributed $N(\emptyset, \Sigma_\epsilon)$. The parameter $r$ denotes the deterministic steady-state value of $r_t$. We estimate this system using Argentine data over

the period 1983:Q1 to 2001:Q4. We exclude the period post 2001 because Argentina was in default between 2002 and 2005 and excluded from international capital markets. The default was reflected in excessively high country premia (see figure 3(b)). Excluding this period is in order because interest rates were not allocative, which is at odds with our maintained assumption that the country never loses access to international financial markets. This is a conservative choice, for inclusion of the default period would imply a more volatile driving force accentuating the real effects of currency pegs on unemployment.

Our empirical measure of $y_t^T$ is the cyclical component of Argentine GDP in agriculture, forestry, fishing, mining, and manufacturing.[3] We obtain the cyclical component by removing a log-quadratic time trend. Figure 3(a) displays the resulting time series. We measure the country-specific real interest rate as the sum of the EMBI+ spread for Argentina and the 90-day Treasury-Bill rate, deflated using a measure of expected dollar inflation.[4]

Our OLS estimates of the matrices $A$ and $\Sigma_\epsilon$ and of the scalar $r$ are

$$A = \begin{bmatrix} 0.79 & -1.36 \\ -0.01 & 0.86 \end{bmatrix}; \qquad \Sigma_\epsilon = \begin{bmatrix} 0.00123 & -0.00008 \\ -0.00008 & 0.00004 \end{bmatrix}; \qquad r = 0.0316.$$

According to these estimates, both $\ln y_t^T$ and $r_t$ are highly volatile, with unconditional standard deviations of 12.2 percent and 1.7 percent per quarter (6.8 percent per year), respectively. Also, the unconditional contemporaneous correlation between $\ln y_t^T$ and $r_t$ is high and negative at -0.86. This means that periods of relatively low traded output are associated with high interest rates and vice versa. The estimated joint autoregressive process implies that both traded output and the real interest rate are highly persistent, with first-order autocorrelations of 0.95 and 0.93, respectively. Finally, we estimate a steady-state real interest rate of 3.16 percent per quarter, or 12.6 percent per year. This high average value reflects the fact that our sample covers a period in which Argentina underwent a great deal of economic turbulence.

We discretize the AR(1) process given in equation (20) using 21 equally spaced points for $\ln y_t^T$ and 11 equally spaced points for $\ln(1 + r_t)/(1 + r)$. We construct the transition probability matrix of the state $(\ln y_t^T, \ln((1+r_t)/(1+r)))$ by simulating a time series of length

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[3]The data were downloaded from www.indec.mecon.ar.

[4]Specifically, we construct the time series for the quarterly real Argentine interest rate, $r_t$, as $1+r_t = (1+i_t)E_t\frac{1}{1+\pi_{t+1}}$, where $i_t$ denotes the dollar interest rate charged to Argentina in international financial markets and $\pi_t$ is U.S. CPI inflation. For the period 1983:Q1 to 1997:Q4, we take $i_t$ to be the Argentine interest rate series constructed by Neumeyer and Perri (2005) and posted at www.fperri.net/data/neuperri.xls. For the period 1998:Q1 to 2001:Q4, we measure $i_t$ as the sum of the EMBI+ spread and the 90-day Treasury bill rate, which is in line with the definition used in Neumeyer and Perri. We measure $E_t\frac{1}{1+\pi_{t+1}}$ by the fitted component of a regression of $\frac{1}{1+\pi_{t+1}}$ onto a constant and two lags. This regression uses quarterly data on the growth rate of the U.S. CPI index from 1947:Q1 to 2010:Q2.

Figure 3: Traded Output and Interest Rate in Argentina, 1983:Q1-2008:Q3
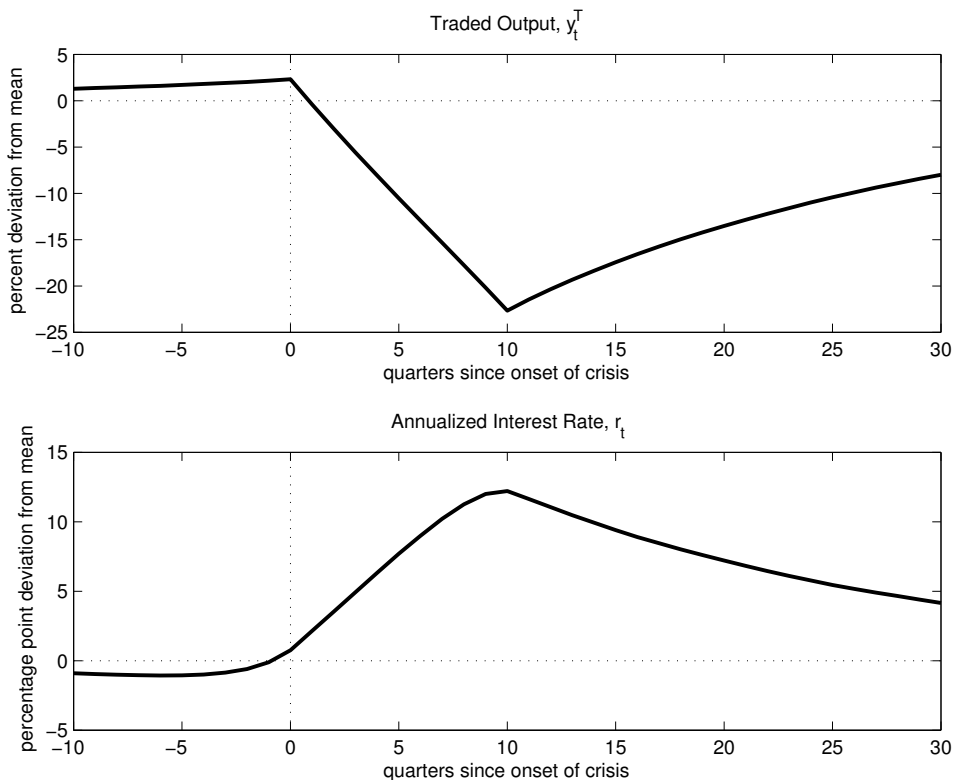
(a) Traded Output



(b) Interest Rate



Note. Traded output is expressed in log-deviations from trend. Source: See the main text.

Figure 4: The Source of a Crisis



1,000,000 drawn from the system (20). We associate each observation in the time series with one of the 231 possible discrete states by distance minimization. The resulting discrete-valued time series is used to compute the probability of transitioning from a particular discrete state in one period to a particular discrete state in the next period. The resulting transition probability matrix captures well the covariance matrices of order 0 and 1.

During the great Argentine crises of 1989 and 2001 traded output fell by about two standard deviations within a period of two and a half years. Accordingly, we define an external crisis in our theoretical model as a situation in which tradable output is at or above trend in period $t$ and at least two standard deviations below trend in period $t + 10$. To characterize the economy's behavior during such episodes, we simulate the model for 20 million quarters and identify episodes in which movements in traded output conform to our definition of an external crisis. We then average the responses of all variables of interest across the crisis episodes and subtract their respective means taken over the entire sample of 20 million quarters. The beginning of a crisis is normalized at $t = 0$.

Figure 4 displays the predicted average behavior of the two exogenous variables, traded output and the country interest rate, during a crisis. The downturn in traded output can

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Table 2: Calibration

| Parameter | Value | Description |
|---|---|---|
| $\gamma$ | 0.99-0.95 | Degree of downward nominal wage rigidity |
| $\sigma$ | 5 | Inverse of intertemporal elasticity of consumption |
| $y^T$ | 1 | Steady-state tradable output |
| $\bar{h}$ | 1 | Labor endowment |
| $a$ | 0.26 | Share of tradables |
| $\xi$ | 0.44 | Elasticity of substitution between tradables and nontradables |
| $\alpha$ | 0.75 | Labor share in nontraded sector |
| $\beta$ | 0.9375 | Quarterly subjective discount factor |

be interpreted either as a drastic fall in the quantity of tradables produced by the economy or as a collapse in the country's terms of trade. The figure shows that at the trough of the crisis (period 10), tradable output is 23 percent below trend. The contraction in tradable output is accompanied by a sharp increase in the interest rate that international financial markets charge to the emerging economy. The country interest rate peaks in quarter 10 at about 12 percentage points per annum above its average value. This behavior of the interest rate is dictated by the estimated negative correlation between tradable output and country interest rates.

## 4.2    Calibration

We adopt a CRRA form for the period utility function, a CES form for the aggregator function, and an isoelastic form for the production function of nontradables:

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma},$$

$$A(c^T, c^N) = \left[ a(c^T)^{1-\frac{1}{\xi}} + (1-a)(c^N)^{1-\frac{1}{\xi}} \right]^{\frac{\xi}{\xi-1}},$$

and

$$F(h) = h^{\alpha}.$$

We calibrate the model at a quarterly frequency using data from Argentina as shown in table 2. Reinhart and Végh (1995) estimate the intertemporal elasticity of substitution to be 0.21 using Argentine quarterly data. We therefore set $\sigma$ equal to 5. We normalize the steady-state levels of output of tradables and hours at unity. Then, if the steady-state trade-balance-to-output ratio is small, the parameter $a$ is approximately equal to the share of traded output in total output. We set this parameter at 0.26, which is the share of traded

output (as defined above) observed in Argentine data over the period 1980:Q1-2010:Q1. González Rozada et al. (2004) estimate the elasticity of substitution between traded and nontraded consumption, $\xi$, using time series data for Argentina for the period 1993Q1-2001Q3. A value of 0.44 is also consistent with the cross-country estimates of Stockman and Tesar (1995). These authors include in their estimation both developed and developing countries. Restricting the sample to include only developing countries yields a value of $\xi$ of 0.43 (see Akinci, 2011). Following Uribe's (1997) evidence on the size of the labor share in the nontraded sector in Argentina, we set $\alpha$ equal to 0.75.

The final parameter we calibrate is the subjective discount factor $\beta$. We set this parameter so as to match a foreign-debt-to-output ratio of 26 percent per year, a value in line with that reported for Argentina over our calibration period by Lane and Milesi-Ferretti (2007). In the context of our model, the task of calibrating $\beta$ is complicated by the fact that the debt-to-output ratio is highly sensitive to the assumed monetary regime. This is problematic because in emerging countries in general and in Argentina in particular, monetary regimes tend to change frequently and widely. A compromise is therefore in order. In calibrating $\beta$, we assume that the underlying monetary regime takes the form of a currency peg. This strategy results in a value of $\beta$ of 0.9375. In section 7 we explore the sensitivity of our results to changes in $\beta$.

## 4.3   Crisis Dynamics Under the Optimal Exchange Rate Policy

We numerically approximate the equilibrium dynamics under the optimal exchange rate policy by applying the method of value function iteration over a discretized state space. Under optimal exchange rate policy, the state of the economy in period $t \geq 0$ is the triplet $\{y_t^T, r_t, d_t\}$. In subsection 4.1, we explain the method employed for discretizing the exogenous state space $\{y_t^T, r_t\}$. In the discretization of the endogenous state $d_t$, we use 1001 equally spaced points. We set $\bar{d}$ at the natural debt limit, which we define as the level of external debt that can be supported with zero tradable consumption when the household perpetually receives the lowest possible realization of tradable endowment, $y^{T\min}$, and faces the highest possible realization of the interest rate, $r^{\max}$. Formally, $\bar{d} \equiv y^{T\min}(1 + r^{\max})/r^{\max}$. Given our discretized estimate of the exogenous driving process, $\bar{d}$ equals 8.34. We fix the upper bound of the debt grid at 8. The reason why we set this upper bound slightly below the natural debt limit is that our numerical algorithm requires evaluating the aggregator function for consumption at all points in the discretized state space. If we allowed the state to take the value $(y^{T\min}, r^{\max}, \bar{d})$, traded consumption would take zero or negative values for all possible choices of next-period debt in the grid. As a result, the aggregate level of

consumption would not be defined at this particular state. Experimenting with values for the upper bound of the debt grid closer to the natural debt limit did not affect our results.
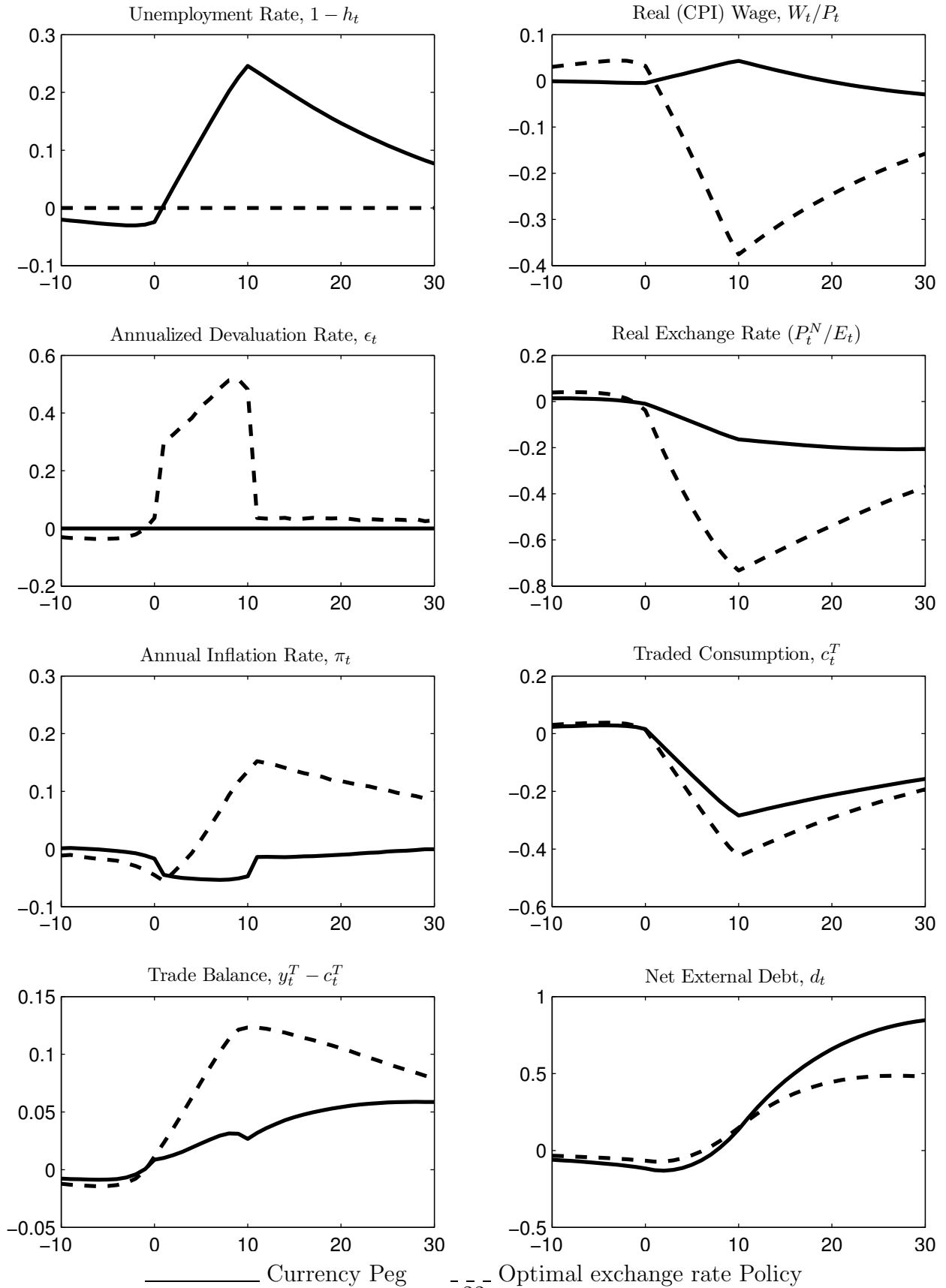
Figure 5 displays with broken lines the average response of the economy under the optimal exchange rate policy to the large negative external shocks displayed in figure 4. The collapse in tradable output and the interest-rate hike both exert substantial downward pressure on domestic absorption. To maintain full employment in the nontraded sector, the government engineers a significant expenditure switch away from tradables and toward nontradables. The instrument the government uses to accomplish this expenditure switch is a series of large devaluations of the domestic currency of about 40 percent per year during the contractionary phase of the crisis. The main purpose of these devaluations is to lower labor costs in the nontraded sector. In turn, the decline in labor costs allows firms to lower the relative price of nontradable goods in terms of tradable goods, which results in a depreciation of the real exchange rate. The purchasing power of wages in terms of the composite consumption good, $W_t/P_t$, falls by about 40 percent and the real exchange rate, $p_t \equiv P_t^N/E_t$, depreciates by about 70 percent, where $P_t$ denotes the nominal price of one unit of the composite consumption good in period $t$, and $P_t^N$ denotes the nominal price of one unit of nontradable consumption in period $t$.[5]

During the crisis, the constraint on nominal wage cuts is typically binding. Therefore, the median quarterly proportional decline in nominal wages is $1 - \gamma$, or 1 percent per quarter, implying a cumulative decline of about 10 percent over the course of the crisis. The nominal price of nontradables mimics the behavior of nominal wages. To see this, note that under the optimal exchange rate policy $P_t^N$ must satisfy $P_t^N F'(\bar{h}) = W_t$, which implies that the nominal price of nontradables falls at the same rate as nominal wages. It follows that the nominal price of nontradables remains relatively flat as the large nominal devaluations occur. This prediction of our model is remarkable because nominal prices of nontradables are assumed to be fully flexible. The predicted sluggish adjustment of the nominal price of nontradables is in line with the empirical findings of Burstein, Eichenbaum, and Rebelo (2005) who report that the primary force behind the observed large drop in the real exchange rate that occurred after the large devaluations in Argentina (2002), Brazil (1999), Korea (1997), Mexico (1994), and Thailand (1997) was the slow adjustment in the nominal prices of nontradable goods.

In the tradable sector, the expenditure switch is reflected in a drastic fall in tradable consumption of about 40 percent. In fact the fall in the domestic absorption of tradable goods is larger than the contraction in the supply of tradables. This results in a significant improvement in the trade balance from slightly below trend to about 10 percent of tradable

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[5]Formally, $P_t \equiv \left[ a^\xi E_t^{1-\xi} + (1-a)^\xi P_t^{N\,1-\xi} \right]^{\frac{1}{1-\xi}}$.

Figure 5: The Dynamics of a Crisis: Response to a Two-Standard-Deviation Fall in $y_t^T$



Unemployment Rate, $1 - h_t$

Real (CPI) Wage, $W_t/P_t$

Annualized Devaluation Rate, $\epsilon_t$

Real Exchange Rate $(P_t^N/E_t)$

Annual Inflation Rate, $\pi_t$

Traded Consumption, $c_t^T$

Trade Balance, $y_t^T - c_t^T$

Net External Debt, $d_t$

——— Currency Peg        - - - Optimal exchange rate Policy

GDP above trend. However, because of the elevated debt-service cost stemming from the interest-rate hike, the large improvement in the trade balance is not sufficient to prevent external debt from growing during the crisis.

The large optimal devaluations that take place during the crises are prima facie an indication that rigidly adhering to a currency peg during times of duress might carry nontrivial real effects in terms of unemployment, output, and welfare. We turn to this issue next.

## 4.4   Crisis Dynamics Under A Currency Peg

At center stage in our analysis is the characterization of the costs of maintaining a currency peg. A currency peg is meant to capture, for example, the monetary policy in place in Argentina between April 1991 and December 2001 or the monetary restrictions faced by the small emerging economies that are members of the Eurozone, such as Greece, Portugal, and Ireland.

Approximating the dynamics of the model under a currency peg is computationally more demanding than doing so under optimal exchange rate policy due to the emergence of a fourth state variable, $w_{t-1}$. In addition, the disequilibrium dynamics cannot be cast in terms of a Bellman equation because of the distortions introduced by nominal rigidities. We therefore approximate the solution by Euler equation iteration over a discretized version of the state space $(y_t^T, r_t, d_t, w_{t-1})$. The discretization of the exogenous states $y_t^T$ and $r_t$ is as described in section 4.2. For the discretization of the endogenous states, we use 501 equally spaced points for external debt, $d_t$, and 500 equally spaced points for the logarithm of $w_{t-1}$. The calibration of the model is the same as the one discussed in section 4.2.

Figure 5 depicts with solid lines the dynamics induced by a large crisis when the monetary policy takes the form of an exchange rate peg. As in section 4.3, a crisis is defined as the average response of the economy to a situation in which in period 0 tradable output is at or above trend and in period 10 it is at least two standard deviations below trend. By construction, the behavior of the exogenous variables, traded output and the country interest rate, are identical to that shown for the case of optimal exchange rate policy (see figure 4).

The central difference in the response to an external crisis between the currency-peg economy and the optimal exchange rate economy is that under a currency peg the contraction in the traded sector spills over to the nontraded sector. This is because the monetary authority fails to devalue the domestic currency preventing real wages and thus also marginal costs in the nontraded sector from adjusting downward. With real wages too high, the labor market is in disequilibrium and unemployment emerges. At the same time, because marginal costs are high, firms are unwilling to cut the relative price of nontradables. Consequently,

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there is no expenditure switching away from tradables toward nontradables. Put differently, the combination of downward nominal wage rigidity and a currency peg hinders the ability of the price system to signal to firms and consumers the relative aggregate scarcity of tradable goods and the relative aggregate abundance of nontradable goods.

The collapse in the demand for nontradables pushes the unemployment rate up by more than 20 percentage points. Moreover, unemployment is highly persistent. Even five years after the trough of the crisis, the unemployment rate remains more than 7 percentage points above average. This sluggishness in unemployment is due to the fact that real wages adjust slowly downward at the rate $\gamma$. The predicted unemployment is involuntary, in the sense that every unemployed individual would be willing to work at the going wage rate.

The predictions of our disequilibrium model suggest that a hands-off policy approach whereby the necessary real depreciation is let to occur via deflation, while maintaining a currency peg, is highly costly in terms of unemployment and forgone consumption of non-tradables. Although the currency-peg economy displays a mild deflation during the crisis, the fall in nominal product prices is too small and too slow to bring about the efficient real depreciation required to avoid unemployment. It is interesting to point out that nominal product prices fail to decline to the level consistent with full employment in spite of the fact that they are fully flexible. The reason for the seemingly sticky behavior of product price inflation is that, because nominal wages are downwardly rigid, marginal costs remain high and therefore firms are unable to cut prices without making losses.

A further distortion introduced by the combination of downward wage rigidity and a currency peg is an excessive absorption of tradable goods during the crisis. The excess consumption of tradables (relative to what is optimal) is driven by the fact that the exchange rate does not depreciate sufficiently in real terms. Under a currency peg the improvement in the trade balance during the crisis is more modest than under the optimal exchange rate policy, spurring a more rapid rise in external debt driven by the higher interest rates. Indeed, the suboptimal behavior of the external debt affects the country's long-run ability to accumulate external financial obligations. Next we explore the relationship between the exchange rate regime and the level and volatility of external debt.

## 5   Debt Dynamics

An important finding of the present study is that the distribution of external debt depends significantly on the exchange rate regime in place. Figure 6 documents this finding. It displays the unconditional distribution of the external debt in the stochastic steady state of the currency-peg and optimal exchange rate economies. The debt distribution associated with

Figure 6: The Distribution of External Debt
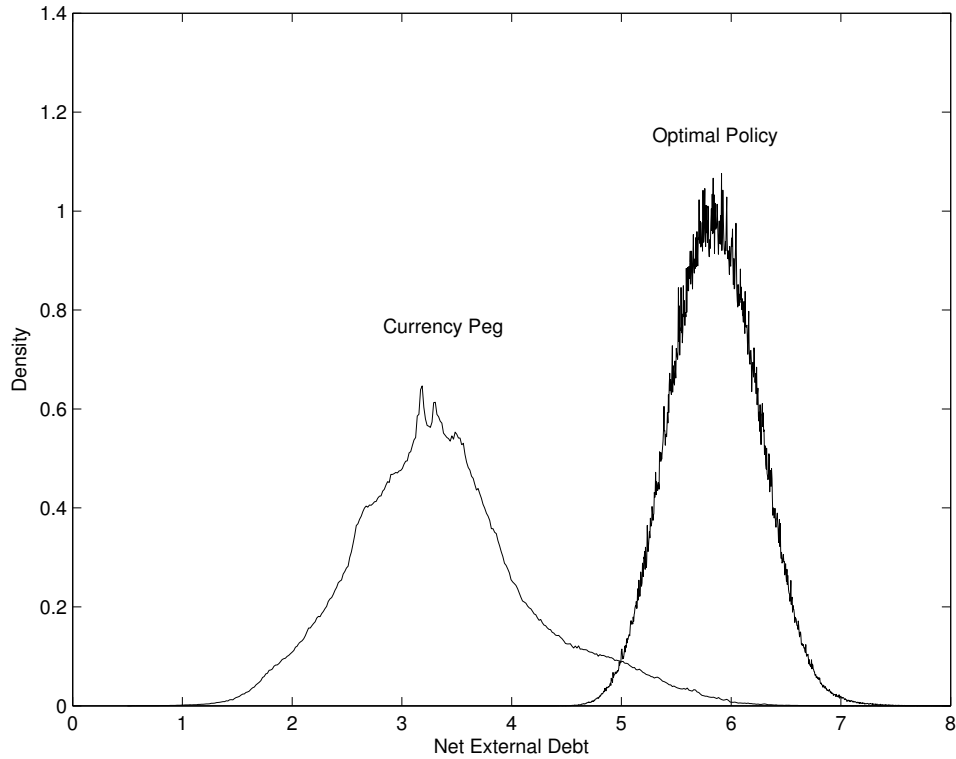


the optimal exchange rate economy features a higher mean and is more concentrated about the mean than the debt distribution associated with the currency-peg economy. Specifically, the mean debt under the optimal exchange rate policy is 5.8 (or 146 percent of the annual steady-state endowment of traded goods) and the standard deviation is 0.4 (or 10 percent of annual traded endowment). By contrast, in the currency-peg economy the mean external debt is 3.4 (or 84 percent of annual steady-state tradable output) with a standard deviation of 0.8 (or 20 percent of annual tradable output).

These differences in the distribution of the level of external debt are due to the fact that in the currency-peg economy the price mechanism that induces households to drastically cut their absorption of tradable goods during periods in which the traded endowment is low and the interest rate is high malfunctions due to the real rigidity imposed by the combination of downward nominal wage rigidity and a currency peg. During crises, the currency-peg economy takes on too much external debt. The resulting path of debt is excessively volatile. The only way such debt dynamics can be supported in the long-run is by lowering the average level of external debt. In summary, inefficient short-run volatility acts as a borrowing constraint and has long-run effects on the debt distribution.

The prediction that the average level of external debt is higher under the optimal exchange

Figure 7: The Distribution of External Debt when $\xi < \frac{1}{\sigma}$
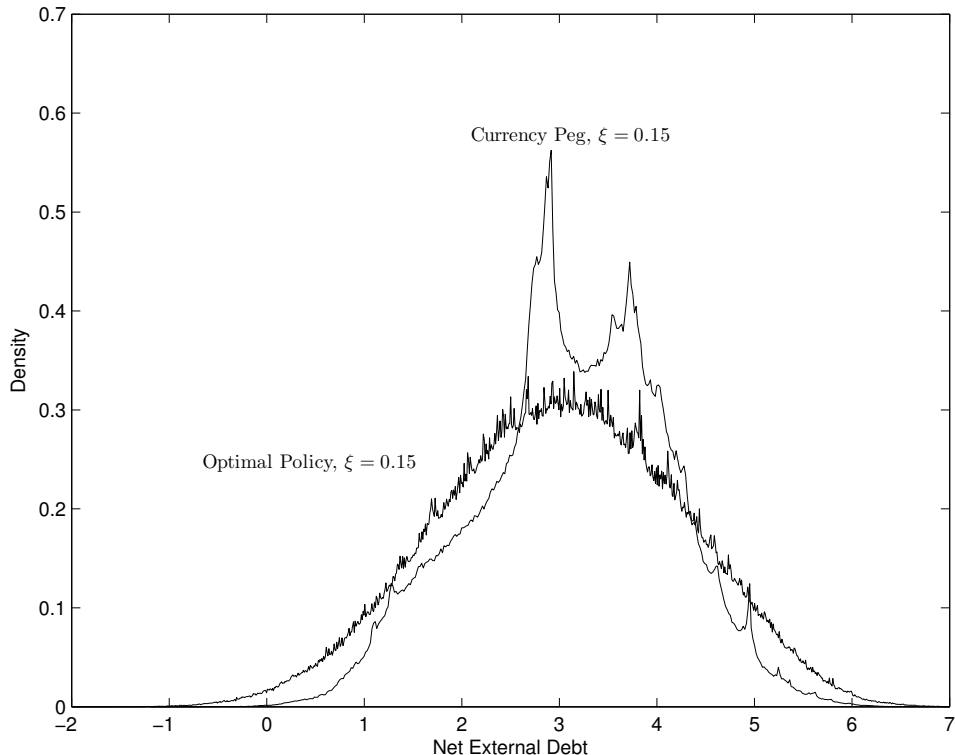


rate policy than under a currency peg depends, however, on the relative magnitudes of the intra- and intertemporal elasticities of substitution, $\xi$ and $1/\sigma$, respectively. Our baseline calibration assumes that the intratemporal elasticity exceeds the intertemporal elasticity, 0.44 versus 0.2. If the intertemporal elasticity of substitution is set at a higher value than the intratemporal elasticity, it becomes possible that the average level of external debt is higher in the currency-peg economy than in the optimal exchange rate economy. Figure 7 illustrates this point. It displays the distribution of external debt under the optimal exchange rate policy and under the currency peg for the case that the intratemporal elasticity of substitution is smaller than the intertemporal elasticity of substitution, $\xi < 1/\sigma$. Specifically, in producing the figure we set $\xi$ at 0.15 and maintain all other parameters at their baseline values, including $1/\sigma = 0.2$. Under this calibration, the mean external debt is larger under a peg than under the optimal exchange rate policy.

The reason why the difference in mean debt under the peg and the optimal policy depends on the difference between the intertemporal and intratemporal elasticities of substitution is that this difference governs the relative response of tradable consumption to disturbances in the marginal utility of wealth. In appendix B, we show that the elasticity of traded

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consumption with respect to the marginal utility of wealth satisfies the following expression:

$$
\frac{d\ln(c_t^T)}{d\ln(\lambda_t)} = \begin{cases} \dfrac{-1}{\eta\sigma+(1-\eta)\frac{1}{\xi}} & \text{Under the optimal exchange rate policy} \\ & \text{or when wage rigidity does not bind in a peg} \\ \\ \dfrac{-1}{\eta\sigma+(1-\eta)\frac{1}{\xi}+\left(\sigma-\frac{1}{\xi}\right)\frac{1-\eta}{1+\xi\left(\frac{1-\alpha}{\alpha}\right)}} & \text{when wage rigidity does bind in a peg} \end{cases},
$$

where $\eta \in (0,1)$ is a measure of the share of tradable consumption in total consumption. This expression shows that in the special case in which the intratemporal and intertemporal elasticities of substitution are equal to each other ($\xi = 1/\sigma$), the wealth elasticity of traded consumption is the same under both exchange rate regimes. That is, a given rise in the marginal utility of wealth, due, for instance to a fall in tradable endowment or a rise in the world interest rate when households are net external debtors, causes the same proportional decline in the consumption of tradable goods under a peg and under the optimal exchange rate policy. However, when the intratemporal elasticity of substitution exceeds the intertemporal elasticity ($\xi > 1/\sigma$), as is the case in our baseline calibration, then the contraction in tradable consumption triggered by a given proportional increase in the marginal utility of wealth is smaller under a peg than under the optimal exchange rate policy. It follows that in this case, during a crisis, the economy experiences a smaller improvement in the trade balance and larger accumulation of external debt under the currency peg than it does under the optimal exchange rate policy. The reverse is true when the intratemporal elasticity is smaller than the intertemporal one.

The intuition behind this finding is as follows. One can show that tradable and nontradable goods are gross substitutes in consumption if the intratemporal elasticity of substitution, $\xi$, is larger than the intertemporal elasticity of substitution, $1/\sigma$. That is, if $\xi > 1/\sigma$, then traded consumption falls when the relative price of nontradables declines. During a crisis, the relative price of nontradables falls by much more under the optimal exchange rate regime than under a currency peg. Therefore, under gross substitutability, traded consumption falls by more under the optimal exchange rate regime than under the currency peg. This implies, in turn, that the economy displays a smaller improvement in the trade balance and a larger deterioration in the external debt position during a crisis under a currency peg than it does under the optimal exchange rate policy. The reverse is the case if tradables and nontradables are gross complements ($\xi < 1/\sigma$).

In the special case in which the intra- and intertemporal elasticities of substitution are equal to each other ($\xi = 1/\sigma$), one can show analytically that the stochastic processes of external debt, $d_t$ and consumption of tradables, $c_t^T$, are independent of the exchange rate

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regime and, in particular, identical under a currency peg and the optimal exchange-rate policy. In other words, in this special case, the dynamics of $d_t$ and $c_t^T$ are independent of the presence of downward wage rigidity. We collect this result in the following proposition.

**Proposition 2** *Suppose $\xi = 1/\sigma$. Then the dynamics of $d_t$ and $c_t^T$ are independent of the exchange-rate regime.*

**Proof:** Note that when $\xi = 1/\sigma$, we have that condition (6) becomes $\lambda_t = a(c_t^T)^{-1/\xi}$. Now, consider the following subset of conditions describing aggregate dynamics (4), (6)-(9), and (15). These conditions can be solved for $c_t^T$, $d_t$, $\lambda_t$, and $\mu_t$ independently of the exchange-rate regime.■

It is worth noting that this proposition does not imply the absence of welfare costs associated with currency pegs. For downward wage rigidity in combination with a fixed exchange rate will continue to cause unemployment and hence lower production and consumption of nontradable goods when the economy is hit by negative external shocks. The proposition does say, however, that unemployment in the nontraded sector will not disrupt the allocation of expenditures in tradable goods.

# 6　The Welfare Costs of Currency Pegs

In our model, the most important source of welfare loss associated with a currency peg is that a currency-peg economy is prone to persistent unemployment spells (see figure 5). Under our calibration the average unemployment rate under a currency peg is 14 percent. As a consequence the average supply of nontraded goods is lower than in the optimal exchange rate economy, in which unemployment is zero at all times.

We quantify the welfare cost of living in an economy in which the central bank pegs the currency by computing the percent increase in the consumption stream of the representative household living in the currency-peg economy that would make him as happy as living in the optimal exchange rate economy. This computation must take explicitly into account the transitional dynamics induced by the switch from a peg to the optimal policy. Specifically, one can express the value function associated with the currency-peg economy as

$$v^{PEG}(y_t^T, r_t, d_t, w_{t-1}) = E_t \sum_{s=0}^{\infty} \beta^s \frac{\left(c_{t+s}^{PEG}\right)^{1-\sigma} - 1}{1 - \sigma},$$

where $c_t^{PEG}$ denotes the stochastic process of consumption of the composite good in the currency-peg economy. Then, define the proportional compensation rate $\lambda(y_t^T, r_t, d_t, w_{t-1})$

implicitly as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\left[c_{t+s}^{PEG}(1 + \lambda(y_t^T, r_t, d_t, w_{t-1}))\right]^{1-\sigma} - 1}{1 - \sigma} = v^{OPT}(y_t^T, r_t, d_t),$$

where $v^{OPT}(y_t^T, r_t, d_t)$ denotes the value function associated with the optimal exchange-rate policy, defined in equation (19). Solving for the welfare cost $\lambda(y_t^T, r_t, d_t, w_{t-1})$, we obtain

$$\lambda(y_t^T, r_t, d_t, w_{t-1}) = \left[\frac{v^{OPT}(y_t^T, r_t, d_t)(1 - \sigma) + (1 - \beta)^{-1}}{v^{PEG}(y_t^T, r_t, d_t, w_{t-1})(1 - \sigma) + (1 - \beta)^{-1}}\right]^{1/(1-\sigma)} - 1.$$

This expression makes it clear that the compensation $\lambda(y_t^T, r_t, d_t, w_{t-1})$ is state dependent. Specifically, the distribution of $\lambda(y_t^T, r_t, d_t, w_{t-1})$ depends upon the distribution of the state $(y_t^T, r_t, d_t, w_{t-1})$.

The ergodic distributions of debt under the currency peg and under the optimal policy do not have the same support. Therefore, in order to compute $\lambda(y_t^T, r_t, d_t, w_{t-1})$ one must evaluate the welfare function $v^{OPT}(y_t^T, r_t, d_t)$ at levels of debt outside of the support of its ergodic distribution under the optimal policy. Having computed $\lambda(y_t^T, r_t, d_t, w_{t-1})$ for all values of the state in its ergodic distribution under the currency peg, we proceed to compute the probability density function of $\lambda(y_t^T, r_t, d_t, w_{t-1})$ by sampling from the ergodic distribution of the state under the currency peg.

Figure 8 displays with a solid line the unconditional probability density function of $\lambda(y_t^T, r_t, d_t, w_{t-1})$ expressed in percentage points. The probability density is highly skewed to the right, implying that the probability of very high welfare costs is nonnegligible. The median of the welfare cost of a currency peg is 10.4 percent of the consumption stream. That is, households living in a currency peg economy require 10.4 percent more consumption in every date and state in order to be indifferent between staying in the currency-peg regime and switching to the optimal exchange rate regime. This is an enormous number as welfare costs go in monetary business-cycle theory. Even under the most favorable initial conditions, the welfare cost of a currency peg is large, 5.6 percent of consumption each period. (This figure corresponds to the lower bound of the support of the probability density of $\lambda(y_t^T, r_t, d_t, w_{t-1})$.)

The welfare consequences of unemployment under a currency peg are reflected in the implied average levels of consumption. Eventhough the currency-peg economy enjoys 8.5 percent higher average consumption of tradables, due to lower average debt holdings (see figure 6), aggregate consumption under the currency peg is 5 percent lower than under the optimal exchange rate policy. This is because, due to unemployment in the nontraded sector,

Figure 8: Probability Density Function of the Welfare Cost of Currency Pegs
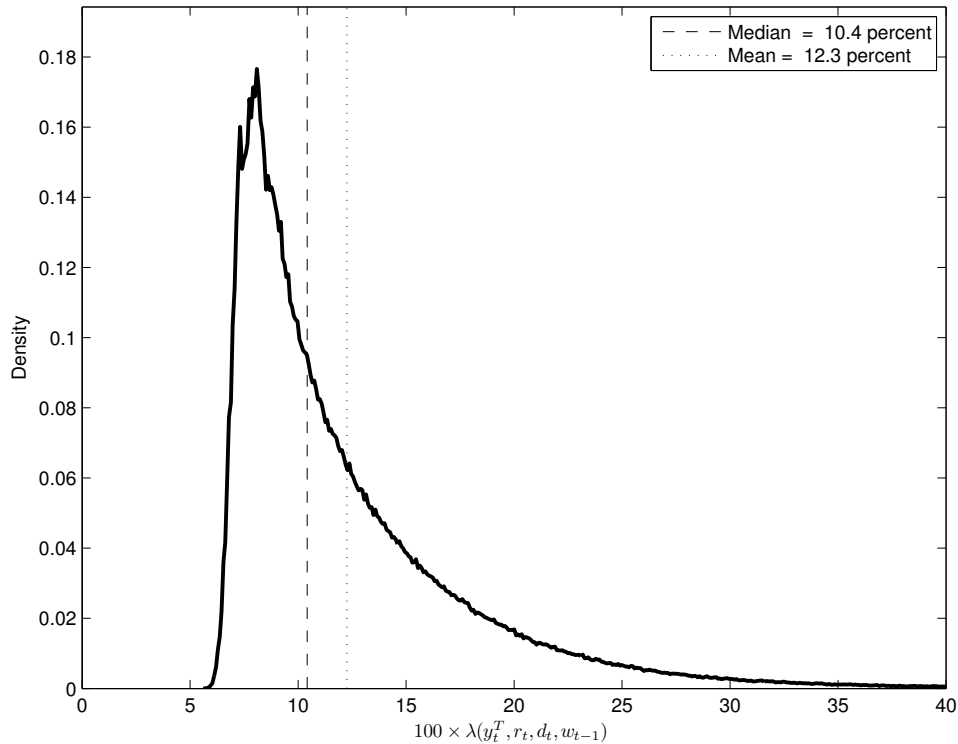


consumption of nontradables under the currency peg is on average 11 percent lower than under the optimal exchange rate regime. As mentioned earlier, this is the main reason why the welfare costs of currency pegs are so high in our model.

Our finding of large welfare costs of currency pegs stands in stark contrast to a large body of work, pioneered by Lucas (1987), suggesting that the costs of business cycles (not just of suboptimal monetary policy) are minor. Lucas' approach to computing the welfare costs of business cycles consists in first removing a trend from a consumption time series and then evaluating a second-order approximation of welfare using observed deviations of consumption from trend. Implicit in this methodology is the assumption that the trend is unaffected by policy. In our model, however, suboptimal monetary policy, through its effect on the average level of unemployment, has a significant effect on the average level of consumption. And indeed lower average consumption is the main reason currency pegs are so costly in our model. It follows that applying Lucas' methodology to data stemming from our model would overlook the effects of policy on trend consumption and therefore would result in spuriously low welfare costs.

The long-run effects of the exchange rate regime on asset accumulation also play a role in determining the welfare cost of currency pegs. Under a peg, the inability of relative prices

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Figure 9: Welfare Cost of Currency Pegs as a Function of the State Variables
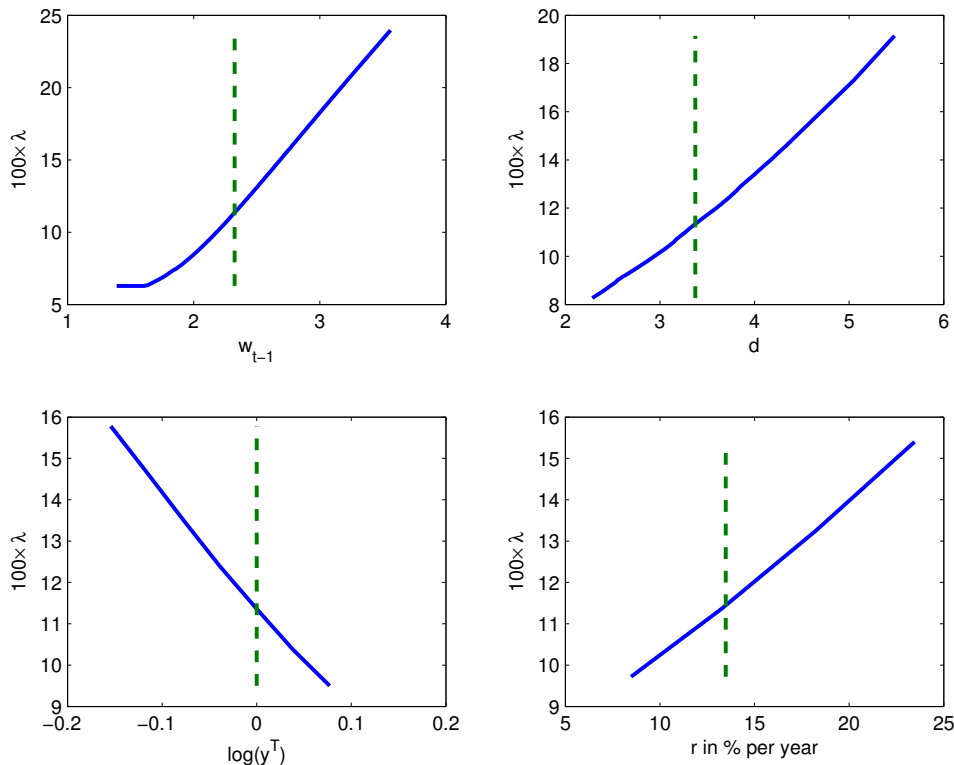


Note. In each plot, all states except the one shown on the horizontal axis are fixed at their unconditional mean values. The dashed vertical lines indicate the unconditional mean of the state displayed on the horizontal axis (under a currency peg if the state is endogenous).

to correctly signal the relative scarcity of traded goods implies large swings in the country's external debt position (see figure 6). As a consequence of the high volatility in the net debt position, risk-averse agents overaccumulate foreign assets. This excessive asset accumulation is costly. A switch from a currency peg to the optimal exchange rate regime allows agents to reduce their average asset holdings, which results in temporarily high levels of consumption of tradable goods.

The marked skewness of the unconditional distribution of the welfare cost of currency pegs suggests that there are initial states at which an economy undergoing a currency peg is particularly vulnerable. Figure 9 sheds light on what these states are. It displays the welfare cost of currency pegs as a function of the four state variables. In each panel only one state variable is allowed to vary (along the horizontal axis) and the remaining three state variables are fixed at their respective unconditional means (under a currency peg, in the case of endogenous states). The figure shows that currency pegs are more painful when the country is initially more indebted, when it inherits higher past real wages, when the tradable

sector is undergoing a contraction (due, for example, to unfavorable terms of trade), or when the country interest-rate premium is high. Viewing the recent debt crisis among eurozone emerging countries through the lens of our model, it is not difficult to understand why doubts about the optimality of European monetary union are the strongest for member countries like Greece, Portugal, and Spain: These are countries with highly inflexible labor markets that before the 2008 crisis experienced large increases in wages and sizable current account deficits.

# 7    Sensitivity Analysis

In this section, we study how the welfare costs of currency pegs are affected by alternative parameterizations of the model. We consider lower degrees of downward wage rigidity, endogenous labor supply, a higher intertemporal elasticity of substitution, a higher intratemporal elasticity of substitution, and preferences featuring more patient households.

## 7.1    Lower Downward Nominal Wage Rigidity

The fact that unemployment is the main source of welfare losses associated with currency pegs suggests that a key parameter determining the magnitude of these welfare losses should be $\gamma$, which governs the degree of downward nominal wage rigidity. Our baseline calibration ($\gamma = 0.99$) implies that nominal wages can fall frictionlessly up to four percent per year. As argued in section 3, this is a conservative value in the sense that it allows for falls in nominal wages during crises that are much larger than those observed either in the 2001 Argentine crisis or the ongoing crisis in peripherical Europe. We now consider alternative values that allow for nominal wage declines of up to 16 percent per year. Taking into account that the largest wage decline observed in Argentina in 2001 or in the periphery of Europe since the onset of the great recession was 1.6 percent per year (Lithuania, see table 1), it follows that we are considering degrees of wage rigidity substantially lower than those implied by observed wage movements during recent large contractions. Table 3 shows that the median welfare cost of a currency peg is 4.3 percent for the lowest value of $\gamma$ considered, 0.96. This welfare cost, although smaller than the one obtained under our baseline calibration of $\gamma$, is still a large figure compared to existing results in monetary economics. The intuition why currency pegs are less painful when wages are more downwardly flexible is straightforward. A negative aggregate demand shock reduces the demand for nontradables which requires a fall in the real wage rate to avoid unemployment. Under a currency peg this downward adjustment must be brought about exclusively by a fall in nominal wages. The less downwardly rigid

are nominal wages, the faster is the downward adjustment in both the nominal and the real wage and therefore the smaller is the resulting level of unemployment.

## 7.2  Endogenous Labor Supply

We now relax the assumption of an inelastic labor supply schedule. Specifically, we consider a period-utility specification of the form

$$U(c_t, h_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma} + \varphi \frac{(\bar{h} - h_t)^{1-\theta} - 1}{1 - \theta},$$

where $\varphi$, $\theta$, and $\bar{h}$, are positive parameters, and $c_t$ and $h_t$ denote, respectively, consumption of the composite good and hours worked. Under this specification, the household's optimization problem features a new efficiency condition of the form

$$\varphi(\bar{h} - h_t^s)^{-\theta} = w_t \lambda_t,$$

where $h_t^s$ denotes the number of hours supplied to the market, which may exceed the number of hours actually worked, $h_t$. In addition, conditions (12) and (13) of the baseline economy are replaced, respectively, by

$$h_t^s \geq h_t,$$

and

$$(h_t^s - h_t) \left( w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) = 0.$$

All other conditions describing aggregate dynamics are as before. In the present economy, the variable $h_t^s - h_t$ measures the level of involuntary unemployment. We set $\varphi = 4.4$, $\bar{h} = 3$, and $\theta = 1.001$ or 6. The values of $\varphi$ and $\bar{h}$ are chosen so that the average level of employment is about unity, which is the level of full-employment in the baseline economy with inelastic labor supply. (It is somewhat above unity for $\theta = 6$ and somewhat below unity for $\theta = 1.001$.) The two values of $\theta$ are meant to capture an environment with a relatively large labor supply elasticity ($\theta = 1.001$, or near logarithmic specification) and one with relatively low labor supply elasticity ($\theta = 6$). Specifically, business-cycle studies measure the Frisch labor supply elasticity for preference specifications of the type used here as $(\bar{h} - h_t)/h_t/\theta$ evaluated at some average level of employment. In our model, under a peg this measure of the labor supply elasticity equals 0.3 on average when $\theta$ equals 6 and 4 when $\theta$ equals 1.001. (The corresponding values under the optimal policy are 0.2 and 3.) The median of the welfare cost of a currency peg implied by this model specification is 4.5 when $\theta$ equals 1.001 and 6.8 when $\theta$ equals 6. These values continue to be large compared to

results in the related literature. The intuition behind this result is that with a higher labor supply elasticity wages rise by less during booms, which reduces the required fall during contractions, making the lower bound constraint on nominal wages less stringent.

It is worth noting that the welfare costs of currency pegs obtained under endogenous labor supply are likely to be biasded downward. The reason is that the preference specificaiton considered in this section treats voluntary leisure and involuntary leisure (i.e., involuntary unemployment) as perfect substitutes. There exists an extensive empirical literature on subjective well being when unemployed suggesting that the nonpecuniary costs of unemployment are substantial (see, Winkelmann and Winkelmann, 1998, and the references cited therein).

## 7.3   Patience

We now consider the case that households are more patient. A reason for considering this case is that in principle there exist two equivalent strategies to calibrate the subjective discount factor $\beta$. One strategy is to match the mean net external debt to output ratio. The other is to match the mean trade-balance-to-output ratio. When working with actual data, however, these two strategies are not equivalent because of the necessity to use finite samples and because of the presence of measurement error. In our baseline calibration we adopt the strategy of matching the average debt-to-output ratio in Argentina, which as mentioned earlier is 26 percent over the calibration period. This approach delivers a trade-balance-to-output ratio of 3.7 percent which is higher than the value of 2.2 percent observed over our calibration period. We now calibrate $\beta$ so as to match this value of the trade-balance-to-output ratio. This requires raising $\beta$ from its baseline value of 0.9375 to 0.945. The raising of the discount factor results in a decline in the average debt-to-output ratio in the currency peg economy from 26 to 15 percent of annual output. Table 3 shows that under this alternative calibration the welfare costs of a currency peg relative to the optimal exchange rate policy has a median of 8 percent and a mean of 9.2 percent of the consumption stream. These figures are smaller than those corresponding to the baseline calibration but still extremely large as costs of business cycles go. As discussed earlier, in our economy with downwardly rigid wages a currency peg acts like a borrowing constraint. This is because a currency peg induces a more dispersed debt distribution, which requires a lower mean debt (or higher precautionary savings) to ensure long-run solvency. More patient agents are in general less negatively affected by a given borrowing limit than are less patient agents. This explains why the welfare cost of a currency peg under the current calibration is smaller than under the baseline calibration.

Table 3: Sensitivity of the Welfare Costs of Currency Pegs

| Parameterization | Welfare Cost | |
| --- | --- | --- |
| | Median | Mean |
| Baseline | 10.4 | 12.3 |
| Less downward wage rigidity: $\gamma = 0.98$ | 6.7 | 8.1 |
| Less downward wage rigidity: $\gamma = 0.97$ | 5.1 | 6.2 |
| Less downward wage rigidity: $\gamma = 0.96$ | 4.3 | 5.2 |
| Endogenous labor supply: $\theta = 1$ | 4.5 | 6.2 |
| Endogenous labor supply: $\theta = 6$ | 6.8 | 8.6 |
| Higher patience: $\beta = 0.945$ | 8.0 | 9.2 |
| Higher elasticity of substitution: $\xi = 0.88$ | 8.6 | 10.8 |
| Higher intertemporal elast. of sub: $\sigma = 2$ | 9.9 | 10.8 |

Note. The welfare cost of a currency peg is given by the percent increase in consumption that would make an individual living in the currency-peg economy as well off as an individual living in the optimal-exchange-rate economy for a given state of the economy $(y_t^T, r_t, d_t, w_{t-1})$.

## 7.4   Intratemporal Elasticity of Substitution

An important parameter in our model is $\xi$ measuring the intratemporal elasticity of substitution between traded and non-traded goods. A reason why this parameter is important in our model is that the higher is the elasticity of substitution between tradables and nontradables, the smaller is the real depreciation required to bring about the necessary expenditure switch away from tradables and towards nontradables in response to negative aggregate demand shocks. It follows that the higher is $\xi$ the less disruptive is the presence of nominal downward rigidities in wages for macroeconomic adjustment. We therefore consider a value of $\xi$ that is twice as large as our baseline value. This value is in line with the one obtained by restricting the cross-country sample of Stockman and Tesar (1995) to include only developed countries, (see Akinci 2011). This higher value of $\xi$ is also consistent with the estimates reported in Ostry and Reinhart (1992) for a panel of Latin American countries. Consistent with our intuition, we find that the average unemployment rate falls from 14 to 11 percent as $\xi$ increases from its baseline value of 0.44 to 0.88. And the median welfare cost of a currency peg relative to the optimal exchange rate policy falls from 10.4 to 8.6 percent of the consumption stream. These figures show that even in an environment in which consumers can substantially more easily substitute tradables for nontradables, currency pegs continue to be extremely painful.

## 7.5 Intertemporal Elasticity of Substitution

We also investigate the sensitivity of our findings to increasing the intertemporal elasticity of substitution, $1/\sigma$. Specifically, we lower $\sigma$ from its baseline value of 5 to a value of 2. This latter value is widely used in emerging-country business-cycle analysis, see Uribe (2011) and the references cited therein. Raising the intertemporal elasticity of substitution makes households less risk averse and as a result more willing to assume external debt. Holding all parameters other than $\sigma$ constant at their baseline values, the lowering of $\sigma$ results in debt distributions (under both the currency peg regime and the optimal exchange rate regime) that pile up to the left of the natural debt limit. The implied debt-to-output ratios are many times larger than those observed over our calibration period. For this reason, we adjust the value of $\beta$ from its baseline value of 0.9375 to 0.964 to ensure that together with a value of $\sigma = 2$, the currency-peg economy delivers an external debt share in line with that observed over the calibration period (about 26 percent of annual output). In contrast to the baseline calibration, under the present calibration the intertemporal elasticity of substitution, $1/\sigma$, exceeds the intratemporal elasticity, $\xi$. As a consequence and as suggested in section 5, the average debt to output ratio is higher in the currency peg economy than in the optimal exchange rate economy (0.24 versus 0.14, respectively), which represents a reversal of the predictions obtained under the baseline calibration. Table 3 shows that under this alternative calibration the welfare costs of currency pegs continue to be extremely high with a median of 9.9 percent of consumption per period. This figure is slightly smaller than its baseline counterpart. This is expected because less risk averse agents are more tolerant to economic fluctuations, and because consumption is more volatile in the currency peg economy than in the optimal exchange rate policy economy. However, in our general disequilibrium model, the bulk of the welfare losses associated with a currency peg stems not from this second-order source but from the average unemployment induced by this type of policy, which is a first-order effect. And the high rate of unemployment induced by a currency peg appears to be robust to changes in $\sigma$. Specifically, when $\sigma$ takes the value of 2, the average rate of unemployment continues to be high, above ten percent. Such a high rate of unemployment implies a permanent loss of nontradable output of about 12 percent per period, which given the weight of 0.74 of nontradable consumption in the aggregator function implies a permanent loss of total consumption of about 9 percent per period. This loss of consumption is entirely avoided under the optimal exchange rate policy because in that case the economy is always operating at full employment.

# 8 Discussion and Conclusion

In this paper we show that currency pegs can be extremely costly to sustain when nominal wages are downwardly rigid. The reason is that the presence of two nominal rigidities, namely a fixed exchange rate and a downwardly sticky nominal wage, results in a real rigidity. In the context of our model, this real rigidity takes the form of downward sluggish adjustment of the real wage expressed in terms of tradables. In turn, the sluggish downward adjustment in the real wage makes the labor market vulnerable to large unemployment spills when the economy suffers adverse external shocks such as increases in the country interest rate premium or deteriorations in its terms of trade.

We develop a dynamic stochastic disequilibrium model of a small open economy that captures well the real affects of the aforementioned dual nominal rigidity. We then quantify the real effects and welfare consequences of a currency peg in the context of our disequilibrium model. To this end, we estimate the driving forces of the model using data from a small emerging economy. We then feed these processes into our disequilibrium model, which allows us to characterize the macroeconomic dynamics implied by a currency peg. We find that a large external shock, defined as a two-standard-deviation collapse in the value of tradable output, causes massive unemployment of about 20 percent of the labor force. This figure is consistent with the unemployment observed in the aftermath of recent large contractions in emerging market economies that followed a fixed exchange rate regime, including Argentina 1998-2001 and the periphery of the European Union (e.g., Latvia, Greece, Portugal, Spain, and Ireland) post 2008. Furthermore, we find that under a currency peg unemployment is highly persistent. Our model predicts that even five years after the trough of the crisis the unemployment rate remains above 8 percentage points above average.

We contrast the behavior of the economy under a currency peg to that under the optimal exchange rate policy. We show that the optimal exchange rate policy completely eliminates the real effects of nominal downward wage rigidities. In particular, the optimal policy is able to bring about full employment of the labor force at all times. An important feature of the optimal exchange rate policy is that during an external crisis of the type described above the central bank engages in a series of large devaluations of the domestic currency. These devaluations are aimed at lowering the real value of the downwardly rigid nominal wage to a level consistent with full employment. Interestingly, these devaluations occur in the context of a severe contraction of economic activity and domestic spending driven by adverse shocks in the traded sector. As a result, under optimal exchange rate policy, the model predicts a positive correlations between the rate of devaluation and economic crisis. This pattern of correlation could lead non-structural econometric analysis to wrongly

conclude that devaluations are contractionary. The reason that such a conclusion would be misplaced is that under optimal exchange rate policy devaluations prevent economic crises originating in the traded sector from spreading to the non-traded sector, that is, absent a devaluation the recession caused by the negative external shock would be more severe. We therefore conclude that in the context of our model devaluations are not contractionary but rather contractions are 'devaluatory.'

Our quantitative model allows us to address the question of how costly currency pegs are in terms of welfare. We find that these costs are enormous. At the median of the distribution of welfare costs, households living under a currency peg require a ten percent increase in consumption every period to be as well off as households living in an economy in which the central bank implements the optimal exchange rate policy. The source of these large welfare costs is primarily that currency pegs entail high levels of unemployment which affects negatively the supply and ultimately the domestic absorption of nontradable goods. We find that the welfare costs of currency pegs are larger than its median of ten percent when the initial state of the economy is characterized by weak fundamentals such as high external debt, high past real wages, high country premia, or weak terms of trade. These findings shed light on why pressures to abandon the currency peg emerged with force in Argentina in 2001 and across the emerging-market members of the European Union in the wake of the great contraction of 2008. Besides being on a fixed exchange rate, these countries had in common high country premia, high levels of external debt, weak terms of trade, and a highly unionized labor force that all but prevented nominal wage cuts.

Many observers have suggested the use of fiscal policy to ease the pains of currency pegs currently felt in the periphery of the European Union. However, advocates of active fiscal policy do not speak with a single voice. Some argue that the right medicine for emerging country members of the European Union is fiscal restraint via tax increases and cuts in public expenditures. Others hold diametrically opposed views and argue that only widespread increases in government spending and tax cuts can offer pain relief. Our model suggests that both of these extreme views are misguided. Instead, the model suggests that the way to ease the pain of a currency peg by means of fiscal policy is more sophisticated in nature. Specifically, optimal fiscal policy in the context of a currency peg consists in a time-varying labor income subsidy that targets industries with high degrees of downward wage rigidity. It can be shown that in our currency-peg economy the full-employment equilibrium can be reached by implementing a proportional wage subsidy at the rate $\tau_t$, where

$$\tau_t = \max\left\{0, 1 - \frac{\omega(c_t^T)}{\gamma w_{t-1}}\right\}, \tag{21}$$

where $\omega(c_t^T)$ denotes the full-employment real wage and $\gamma w_{t-1}$ denotes the real wage that prevails when the wage rigidity is binding. If the combination of the currency peg and the downward nominal wage rigidity prevent the real wage from falling to the full-employment real wage, the subsidy is positive. Otherwise, the subsidy is zero. This subsidy scheme can be financed in a revenue neutral fashion, by an appropriate proportional tax on any source of income (labor income, $w_t h_t$, tradable income, $y_t^T$, profits, $\phi_t$, or any combination thereof). We note that these financing schemes work even when the labor supply is elastic. The reason is that the subsidy is positive only in states of the world in which, in the absence of the subsidy, households are off their labor supply schedule, or involuntarily unemployed.[6] It is clear from equation (21) that the optimal subsidy inherits the stochastic properties of the optimal devaluation rate studied in previous sections (see equation (18)). Because the optimal devaluation rate is found to be highly volatile at business-cycle frequency, it follows that the fiscal alternative presented here may indeed introduce an impractically high level of volatility in the tax/subsidy regime. This conclusion gives credence to the dictum that nominal rigidities are best addressed by monetary policy.

In closing this discussion, we wish to stress that our analysis focuses on the welfare costs of currency pegs stemming exclusively from nominal wage rigidity. In particular, it does not consider other sources of welfare loss, nor any benefits of adhering to a currency peg, such as those created by the presence of liability dollarization, the elimination of exchange rate risk, or the amelioration of time inconsistency problems in monetary policy. We therefore interpret the results of this paper as providing indirect evidence that for long lasting currency pegs or monetary unions, the sum of the pecuniary and non-pecuniary benefits of a single currency must outweigh the large welfare costs identified in our analysis.

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[6]In a more recent contribution, Farhi et al. (2011) expand this idea to other economic environments.

# Appendix A: Proof of Proposition 1

Consider a set of stochastic processes $\{c_t, c_t^T, c_t^N, h_t, p_t, w_t, d_{t+1}, \lambda_t, \mu_t, \epsilon_t\}_{t=0}^{\infty}$ satisfying the conditions given in Definition 1 and the exchange rate policy (18). We first show that $h_t$ must equal $\bar{h}$ at all times. This part of the proof is by contradiction. Suppose $h_t < \bar{h}$. Then, by (13) we have that

$$w_t = \frac{\gamma w_{t-1}}{\epsilon_t}. \tag{22}$$

If $\omega(c_t^T) \geq \gamma w_{t-1}$, then, by (18) , $\epsilon_t = 1$, which, together with (22) implies that $w_t \leq \omega(c_t^T)$. Using (5), (10), and (17) we can write this inequality as

$$\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} F'(h_t) \leq \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h}).$$

Because the left-hand side of this expression is strictly decreasing in $h_t$, we have that $h_t$ must equal $\bar{h}$, which is a contradiction.

Alternatively, if $\omega(c_t^T) < \gamma w_{t-1}$, then, by (18), we have that $\epsilon_t = \gamma w_{t-1}/\omega(c_t^T)$. Then, combining this expression with (22), we obtain $w_t = \omega(c_t^T)$. Using again (5), (10), and (17), we can write this equality as

$$\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} F'(h_t) = \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h}).$$

Because the left-hand side of this expression is strictly decreasing in $h_t$, we have that the only solution is $h_t = \bar{h}$, which, again, is a contradiction.

We have therefore shown that under the exchange rate policy given in (18), unemployment is nil at all dates and states. Evaluating (2), (4), (6)-(9), (14) and (15) at $h_t = \bar{h}$, we obtain

$$d_{t+1} \geq \bar{d}$$

$$\lambda_t = U'(A(c_t^T, \bar{h})) A_1(c_t^T, \bar{h})$$

$$\frac{\lambda_t}{1 + r_t} = \beta \mathbb{E}_t \lambda_{t+1} + \mu_t$$

$$\mu_t \geq 0,$$

$$\mu_t(d_{t+1} - \bar{d}) = 0,$$

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t},$$

which are precisely the first-order necessary and sufficient conditions associated with the

social planner's problem consisting in maximizing (19) subject to (4) and (15). The fact that the first-order conditions of the social planner's problem are necessary and sufficient follows directly from the strict concavity of the planner's objective and the convexity of the planner's constraint set.

# Appendix B: The Wealth Elasticity of Traded Consumption

Using the assumed functional forms for preferences and technology, we can write expressions (2), (5), and (6) , respectively, as

$$c = \left[a(c^T)^{1-1/\xi} + (1-a)(c^N)^{1-1/\xi}\right]^{1/(1-1/\xi)},$$

$$p = \left(\frac{1-a}{a}\right)\left(\frac{c^T}{c^N}\right)^{1/\xi},$$

and

$$ac^{\sigma-1/\xi}c^{T1/\xi} = \lambda^{-1}.$$

Throughout this analysis we drop time subscripts unless otherwise necessary. In the labor market, the situation depends on the monetary policy in place. Under the optimal policy, we have full employment at all times, which implies that

$$c^N = 1.$$

This expression will also hold under a currency peg in states in which the downward-wage-rigidity constraint is not binding. By contrast, when this constraint does bind under a currency peg, the labor market will be in disequilibrium. In this case, we have that $pF'(h) = \gamma w_{-1}$ and $c^N = F(h)$, where $w_{-1}$ denotes the previous period's real wage rate. Using the assumed functional forms, we can combine these two expressions to obtain

$$\alpha p c^{N(\alpha-1)/\alpha} = \gamma w_{-1}$$

We now proceed to log linearize all of the above expressions around some point $\tilde{c}^T$ for tradable consumption and unity or nontraded consumption. We denote with a circumflex accent log deviations from this reference point. The log-linearized version of the above expressions is

$$\hat{c} = \eta\hat{c}^T + (1-\eta)\hat{c}^N,$$

$$\hat{p} = \frac{1}{\xi} \left( \hat{c}^T - \hat{c}^N \right)$$

$$\left( \sigma - \frac{1}{\xi} \right) \hat{c} + \frac{1}{\xi} \hat{c}^T = -\hat{\lambda},$$

and

$$\hat{c}^N = \begin{cases} 0 & \text{Under the optimal exchange rate policy} \\ & \text{or when wage rigidity does not bind in a peg} \\ \\ \frac{\alpha}{1-\alpha} \hat{p} & \text{when wage rigidity does bind in a peg} \end{cases}$$

where $\eta \equiv \frac{a(\tilde{c}^T)^{1-1/\xi}}{a(\tilde{c}^T)^{1-1/\xi}+(1-a)}$. Solving for $\hat{c}^T$ as a function of $\hat{\lambda}$, we obtain

$$\hat{c}^T = \begin{cases} \dfrac{-\hat{\lambda}}{\eta\sigma+(1-\eta)\frac{1}{\xi}} & \text{Under the optimal exchange rate policy} \\ & \text{or when wage rigidity does not bind in a peg} \\ \\ \dfrac{-\hat{\lambda}}{\eta\sigma+(1-\eta)\frac{1}{\xi}+\left(\sigma-\frac{1}{\xi}\right)\frac{1-\eta}{1+\xi\left(\frac{1-\alpha}{\alpha}\right)}} & \text{when wage rigidity does bind in a peg} \end{cases} .$$

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