

# RECURSIVE BARGAINING WITH ENDOGENOUS THREATS

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**ABSTRACT.** This paper studies the dynamic properties of partnerships characterized by bilateral monopoly. Contracts, assigning residual rents, are subject to bargaining. Participation is voluntary and competitive outside offers incite renegotiation of past commitments. Opportunistic behavior arise as partners direct effort towards unproductive activities in order to influence their value with outside competitors. We study how this form of strategic behavior affect the properties of bargaining and efficiency in a general framework of bilateral cooperation with one-sided limited enforcement. Several results emerge. We show that a strategic partner's valuation of the cooperation can be summarized by a surprisingly simple Bellman equation. Whenever contracts are renegotiated, multiplicity of continuation plans arise, and we turn attention to subgame perfect, or time-consistent, strategies. A partnership suffers from suboptimal levels of investment at early stages of cooperation, but converge towards a Pareto-optimal equilibrium as contracts are renegotiated. The proposed framework provides a novel perspective on, and solution to, the problem of holdups.

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## 1. INTRODUCTION

Partnerships are formed on a voluntarily basis if the surplus of cooperation exceeds that of isolation. When there are costs to mobility – e.g., search frictions – cooperation exhibits a long-term nature, and surplus is characterized by multilateral monopoly rents, subjected to bargaining. In most partnerships, however, participation is not enforceable, and past agreements may be renegotiated as competing outside offers arise.<sup>1</sup> As a consequence, participants have incentives to direct efforts towards activities influencing their value with outside competitors, in order to gain a bargaining advantage over their current partner. This paper studies how this type of opportunistic behavior affect the properties of bargaining and efficiency in a general framework of long-term bilateral cooperation with one-sided limited commitment.

We show that a strategically-, or opportunistically-, acting agent’s valuation of the partnership can be described by a surprisingly simple Bellman equation. The contraction property reveals that an equilibrium to a complicated strategic game with bargaining exists. Optimal policies, however, display a high degree of multiplicity, and we therefore turn attention to time-consistent, or subgame perfect, plans. At the early stage of a partnership, inefficiencies arise as the strategic agent over-engage in unproductive activities, in order to renegotiate the current contract at more favorable terms. At longer horizons, however, the partnership converges to a cooperative equilibrium, in which the strategic agent directs all her efforts to maximize common resources. Convergence towards cooperation, we show, occurs in finite time. We interpret and compare our results to the incomplete contracting literature, and conclude that our framework provides a novel solution to the “holdup problem”, in the case of one-sided investments.

At time zero, two individuals meet and form a partnership. Participation is voluntary, and each individual may freely walk away from any pre-commitments at will. One of the individuals, referred to as the *strategic agent*, is recurrently endowed with one unit of the single consumption good. The agent decides on what fraction of the endowment to invest in outside activities, and on what fraction to leave as surplus within her current partnership. Outside activities increase the value of potential future outside offers, while a larger surplus leaves more resources to be shared with the current partner. A competitive outside offer incite renegotiation of past agreements, leaving the strategic agent a larger share of surplus in the present and in the future. In each period, the strategic agent therefore faces the trade-off between increasing available surplus given existing bargaining

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<sup>1</sup>In most partnerships – such as employer/employee, investor/lending institution, and marriages – participation is not enforceable, but violation of contract is associated with a cost. This cost further strengthens the limitations to mobility and therefore magnify the monopoly rents available through bargaining.

power, or increasing her future bargaining power at the expense of current surplus. To keep the framework simple, the outside option of the *nonstrategic partner* is, by assumption, inoperative.

At the onset of the partnership, the agents bargain over current and future rents. Bargaining is conducted through a dynamic Pareto-problem that maximizes the joint welfare of the two individuals, treating the investment choices of the strategic agent as given. Due to the voluntary nature of partnerships, however, the Pareto-problem must respect constraints to participation. Using a constrained Pareto-problem as the outcome of bargaining has some strong intuitive appeal. Each agent's initial bargaining power is the product of some, unmodeled, ex-ante competition over potential partners. Whenever a partnership finally is formed, ex-post renegotiations are consistently refused unless an agent can present evidence of an outside offer with better contractual terms. On the occasions at which renegotiations do occur, the outside option of the allegedly defecting partner is exactly matched, leaving her a larger share of surplus in the current partnership, both in the present and in the future.

The strategic agent observes the bargaining process and decides on a (possibly stochastic) investment sequence to maximize her own private utility. As the bargaining solution will respect her sequence of outside options, optimality will generally entail an excessive devotion to outside activities. Providing the Bellman equation corresponding to the strategic problem above is a central contribution of this paper.

The constraint-set associated with the strategic problem is, itself, another constrained optimization problem – the Pareto-problem. Even when abstracting from the additional layer of complexity induced by the strategic agent's choices, the Pareto-problem itself does not straightforwardly admit a recursive representation. The presence of forward-looking participation constraints induce a dynamic tension between past and present choices, and the Markov-perfect solution is generally inefficient (Thomas and Worrall, 1988). Drawing heavily on Marcet and Marimon (2009), however, we show that the Pareto-problem observes a *partially* recursive representation at an augmented state-vector. In particular, *given* a process of strategic investment choices, the Pareto-problem satisfies Bellman's Principle of Optimality at an additional, non-Markovian, state variable: a recursively updated Pareto-weight, straightforwardly interpreted as the strategic agent's share of surplus.

Exploiting this result, we derive the strategic agent's Bellman equation as a binomial choice between exercising an outside option – and incite renegotiation – or proceed under current contractual terms, with repeated strategic choices of investment. Clearly, the

optimal choice is given by the alternative yielding the largest present value utility, and renegotiations occur at sufficiently competitive outside offers.

Whenever contracts are renegotiated, the new contract implicitly promises a continuation value of remaining in the partnership, exactly matching that of leaving. The level of the continuation value depends entirely on the outside alternative available to the agent, and is therefore independent of future investment choices. As a consequence, *all* feasible continuation strategies attain the *same* continuation value, and multiplicity arise. A continuation strategy, or plan, in which joint surplus is *maximized* is, for instance, attainable – but so is also one in which surplus is *minimized*. Clearly, each possible continuation plan implies a differently negotiated contract, such that the value of staying remains the same.

Are *all* continuation plans perfect equilibria in the continuation game? The answer is, generally, no. Each continuation plan is associated with a contract, assigning rents as shares of surplus. Depending on the perceived plan of investments, shares are contracted to exactly match outside offers. As a consequence, there lies a temptation in “promising” certain future actions at the time of renegotiation, only to act differently once that future arrives. Most continuation plans are *time-inconsistent*.

Are *some* continuation plans perfect equilibria in the continuation game? Yes, some are. We show that subgame perfect, or time-consistent plans, do exist. More precisely, there exist a renegotiated contract such that, at an opportunistic continuation game, the strategic agent attains an inside value that exactly match her outside offer at the time of renegotiation. The argument relies on continuity of the strategic agent’s valuation of the partnership with respect to contracts; at an opportunistic continuation game, there must exist some contract such that inside and outside values coincide. Our ensuing analysis then focus on the qualitative properties of these time-consistent continuation plans.

After a history at which renegotiation occurs, the strategic agent is offered a new contract yielding a larger share of surplus in the present and in the future. Subgame perfection, however, entails that the strategic agent, again, may engage in opportunistic behavior of soliciting outside offers, and the process repeats itself. Are partnerships in the present setting condemned to perpetually rent-seeking equilibria with low efficiency? We show that, under some simplifying assumptions, they are not. Opportunistic actions and renegotiation will occur finitely many times, and the partnership will eventually converge to a cooperative state in which all resources are devoted to maximize surplus. Even in models with opportunistic behavior and excessive engagements in outside activities, as here, limiting contracts are *self-enforcing* (cf. Thomas and Worrall (1988)). We consider this a central result in our analysis, which provides a novel perspective and solution to the problem of *holdups*.

A holdup arises in models of multilateral trade in which there are relation-specific investments and incomplete contracting (see Williamson (1975) and (1985)). In particular, in partnerships which require ex-ante investments, and in which rents are subject to ex-post bargaining, investments are typically suboptimally low (Grout, 1984; Hart and Moore, 1988). The reason being that while one party bears the full marginal cost of investment, all parties are claimants on residual rents, and the partnership therefore suffers from under-investments. The problem of holdups have received widespread attention in economics, and is considered a main candidate theory of the firm (Williamson, 1985; Klein, Crawford and Alchian, 1978), vertical integration (Grossman and Hart, 1986), and asset-ownership (Hart and Moore, 1988).

It is easy to see how the framework developed and analyzed in this paper fits the general idea of a holdup. A strategic agent decides on how much resources to invest in a partnership. Investments are relation-specific and sunk absent her current partner. Partners bargain over rents, and the strategic agent is therefore only a partial claimant on contemporaneous surplus. Associated with each investment choice is, of course, a cost. In the present setting, and in contrast with the main literature on holdups, the cost associated with investment is the foregone, current or future, outside opportunity. The intuition follows the idea that an agent's income is *solely* derived from the surplus shared within a partnership, and not from any alternative parallel source. An agent, for instance, engaged in on-the-job search forgoes presumptive job offers when devoting time and effort to her current employer. But her *source* of income remains unchanged (her current employer). A missed opportunity of physical investments leaves funds to be ventured in alternative partnerships, but does not yield any alternative income in the current relation. Lastly, an employee's engagement in *firm-specific*, as opposed to *general-*, training does not reduce her income, but may erode her appeal to alternative employers. The cost to investment, in all three examples, is borne, entirely, through forgone outside options.<sup>2</sup>

As the cost to investment is given by foregone outside offers, bounded offers implies a bounded cost to investment. At a sufficiently large share of surplus, the marginal cost to investment is therefore zero, and efficiency follows as an immediate consequence. In contrast, at suboptimally low shares of surplus, inefficiencies do arise, and renegotiations repeatedly occur. In the long-run, however, our qualitative results suggest that the partnership eventually reaches an efficient equilibrium, and that convergence occurs in finite time.

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<sup>2</sup>See Section 4.3 for an elaboration.

One of the most natural interpretation of the framework proposed in this paper is within the labor market. In particular, the model largely resembles a situation in which an opportunistic employee faces the tradeoff between engaging in productive activities with her current employer – and consequently earning a higher income – or to devote a large share of time searching for competing offers – and therefore increasing her future bargaining power. In (sub-) section 4.3.2 we show how the concept of “outside activities” in the present model easily can be interpreted as on-the-job search, improving the probability of receiving offers from competing firms. This form of on-the-job search is widely perceived an important aspect of the labor market, capable of explaining several empirical regularities commonly found in the data (see, for instance, Shimer (2006) and Cahuc, Postel-Vinay and Robin (2006)). The literature considering the theoretical implications of on-the-job search is, however, small. Besides the aforementioned literature, Burdett and Mortensen (1998) makes a notable theoretical contribution, but treat on-the-job search as something exogenous, and do not consider the strategic aspects which is the focus of this paper. Christensen, Lentz, Mortensen, Neumann and Werwatz (2005) do consider a somewhat strategic aspect of on-the-job search, but ignore the possibility of counter-offers from the current employer, and therefore also the subgame perfect strategies analyzed here. Lastly, Postel-Vinay and Robin (2004), both endogenize on-the-job search *and* consider counter-offers, but they also assume the marginal cost to search being zero, and, therefore, abstract from the strategic considerations explored in this paper.

In a paper, at least conceptually, related to our strategic model, Lundberg and Pollak (2003) consider a model of bargaining within marriages. As here, they stress the importance of the strategic aspects of endogenous threatpoints, and the trade-off between current surplus and future bargaining power. Lundberg and Pollak (2003) show that inefficiencies are likely to arise due to the same strategic mechanism explored in this paper. However, their analysis is confined to a much less general framework, in which most of the endogenous interaction explored in this paper, is treated exogenous. In particular, their model is static, and shares of surplus are given as exogenously specified functions of outside investments. In contrast we consider repeated strategic interaction with repeated renegotiations, and derive dynamic results related to efficiency.

## 2. ECONOMIC ENVIRONMENT

At time zero, two individuals meet with the prospect of forming a partnership. The planning horizon is infinite and time is denoted  $t = 0, 1, \dots$ . The partnership is formed on a voluntary basis, and each individual may freely walk away from any pre-commitments at zero consequence. At any period,  $t$ , participation is, thus, not contractible.

A partnership is productive. In each period, the partnership is endowed with one unit of a single consumption good. The disposal of the endowment, however, is subject to the discretion of one of the involved individuals, referred to as the *strategic agent*. The strategic agent allocates a fraction of the endowment,  $e_t$ , to some outside activity, and leaves the remainder,  $1 - e_t$ , to be shared within the partnership.

The surplus of the partnership is subject to bargaining. The outcome of bargaining is given by an *implicit contract* (Azariadis, 1975; Thomas and Worrall, 1988). More precisely, the distribution of rents solves a dynamic Pareto-problem that maximizes the joint welfare of the two agents.<sup>3</sup> The voluntary nature of the partnership, however, subjects the Pareto-problem to observe constraints on participation. The value of participating in the partnership must exceed the value of some available outside alternative. The Pareto-problem treats the stochastic process of surplus and outside options as *given*. As a consequence, the implicit contract does not condition on the strategic agent's investment choice,  $1 - e_t$ , and is therefore *incomplete*.<sup>4</sup>

An individual's share of current surplus is, thus, given by her initial Pareto-weight, and her past and current outside options.<sup>5</sup> The *nonstrategic agent's* outside option is, however, assumed to be nil, and her bargaining power is exclusively summarized by her initial Pareto-weight. In contrast, the outside option of the strategic agent is governed by her current and past investments in outside activities, and possibly on some random component. In each period, the strategic agent therefore faces the trade-off between increasing available surplus given existing bargaining power, or increasing her future bargaining power at the expense of current surplus.

Let  $z$  be a random variable taking on values in the set  $Z = \{\omega_1, \dots, \omega_N\}$ . Let  $z_t$  be a particular realization of  $z$  in period  $t$ . The variable  $z_t$  will be the only source of "truly" exogenous variation in the model, and will affect the strategic agent's outside options. Conditional on  $z_t$ , the probability of  $z_{t+1}$  occurring is then given by the Markovian transition function  $f(z_{t+1}, z_t) = P(z_{t+1} = \omega_n | z_t = \omega_m)$ . Let  $Z^{t+1} = Z \times \dots \times Z$  define the set of all possible histories of  $z$  up to period  $t$ . An arbitrary element in  $Z^{t+1}$  is called a *history* and is denoted  $z^t$ . The probability of history  $z^t$  occurring is then given by  $\lambda(z^{t+1}) = f(z_{t+1}, z_t) \times \lambda(z^t)$ , with  $\lambda(z^0) = 1$  for  $z^0 = z_0$ , and zero elsewhere.

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<sup>3</sup>"Joint welfare" equals the sum of individual welfare, weighted by their respective, initial, Pareto-weight.

<sup>4</sup>We will be more specific of the exact nature of incompleteness in Section 4.3.

<sup>5</sup>An individual's initial Pareto weight represents her exogenous bargaining power, which is related to alternative and competing partnerships offered at time zero. Competing partnerships may for instance reflect alternative employers in firm/worker arrangements, or competing suitors in the marriage market.

Similarly, conditional on history  $z^t$ , the probability of history  $z^{t+s}$  occurring is given by  $\lambda(z^{t+s}, z^t) = f(z_{t+s}, z_{t+s-1}) \times \lambda(z^{t+s-1}, z^t)$ , all  $s > 0$ . The Markovian property implies that the conditional measure  $\lambda(z^{t+1}, z^t)$  equals  $f(z_{t+1}, z_t)$ , which, for notational consistency, will also be denoted  $\lambda(z_{t+1}, z_t)$ .

Associated with each history,  $z^t$ , is a random variable  $e_t : Z^{t+1} \rightarrow [0, 1]$ . The random variable  $e_t$  maps histories to outcomes and defines the *stochastic process*  $\tilde{e} = \{e_t(z^t)\}_{t=0}^\infty$ . Associated with each stochastic process,  $\tilde{e}$ , is a *plan*,  $\tilde{c} = \{c_t(z^t; \tilde{e})\}_{t=0}^\infty$ .<sup>6</sup> Thus, for each process,  $\tilde{e}$ , each element in the plan maps histories to *consumption levels*,  $c_t : Z^{t+1} \rightarrow_{\tilde{e}} [0, 1 - e_t]$ .<sup>7</sup> It is important to note the distinction in language here; a plan is an endogenous object, while a stochastic process is considered as exogenous. The stochastic process  $\tilde{e}$ , however, will fulfill the dual role of both a process (from the perspective of bargaining) and a plan (from the perspective of the strategic agent). As will become clear, the exogenous treatment of the process  $\tilde{e}$ , from the perspective of bargaining, renders the contract incomplete.

The strategic agent's bargaining power is related to her contemporaneous and past outside options. Her outside options, in turn, relate to her past investments in outside activities, denoted  $e_t(z^t)$ . Let  $h_t(z^t)$  represent the non-depreciated stock of all investments following history  $z^t$ , and let  $\varphi \in [0, 1]$  denote the depreciation rate. The law of motion for  $h_t(z^t)$  is then given by,

$$h_{t+1}(z^t) = (1 - \varphi)h_t(z^{t-1}) + e_t(z^t), \quad \text{all } z^t \in Z^{t+1}, t = 0, 1, \dots \quad (1)$$

which, itself, is a stochastic process.

The strategic agent ranks contemporaneous consumption allocations according to the utility function  $u(c_t)$ . The corresponding function for the non-strategic agent is given by  $v(1 - e_t - c_t)$ , where  $1 - e_t$  denotes the partnership's current surplus. Functions  $u(\cdot)$  and  $v(\cdot)$  are assumed to be increasing, concave, bounded, and once continuously differentiable. To guarantee interiority,  $\lim_{c \downarrow 0} u'(c) = \infty$  and  $\lim_{c \downarrow 0} v'(c) = \infty$ . The strategic agent's outside option is given by  $\hat{V}(h_t(z^{t-1}), z_t)$ , which may, thus, depend on the stochastic component  $z_t$ . The function  $\hat{V}(\cdot, \cdot)$  is assumed to be bounded, at bound  $B \leq \frac{u(1)}{1-\beta}$ . Although not necessary, it facilitates notation to assume that  $\hat{V}$  is non-decreasing in both  $h$  and  $z$ .

The agents repeatedly bargain over contemporaneous surplus. Bargaining is, as previously stated, conducted through a dynamic Pareto-problem that maximize the weighted

<sup>6</sup>Notice that notation is slightly imprecise here. Any plan,  $\tilde{c}$ , is of course contingent on process,  $\tilde{e}$ , underlying it. However, the relevant underlying process should be obvious from the context, and excessive notation is suppressed.

<sup>7</sup>The notation " $\rightarrow_{\tilde{e}}$ " captures the idea that the mapping depends on the process  $\tilde{e}$ . Alternative, we may define  $\tilde{E}$  as the space of all possible processes  $\tilde{e}$ , and  $c_t : Z^{t+1} \times \tilde{E} \rightarrow [0, 1 - e_t]$ .

welfare of the partnership. The strategic agent's initial Pareto-weight is given by  $\mu_0$ , while the corresponding weight for the non-strategic agent is normalized to one. Participation, however, is voluntary, and the strategic agent may receive outside offers that dominate the value of the current partnership, at current bargaining power. The Pareto-problem is therefore subject to constraints that ensure participation, and the strategic agent's share of surplus endogenously responds to match competing alternatives.

With  $\tilde{e}$  taken as given, the Pareto-problem solves,

$$W(h_0, \mu_0, z_0; \tilde{e}) = \max_{\tilde{c}} \sum_{t=0}^{\infty} \sum_{z^t \in Z^{t+1}} \beta^t \{ \mu_0 u(c_t(z^t; \tilde{e})) + v(1 - e_t(z^t) - c_t(z^t; \tilde{e})) \} \lambda(z^t) \quad (2)$$

$$\text{s.t.} \quad \sum_{s=0}^{\infty} \sum_{z^{t+s} \in Z^{t+s+1}} \beta^s u(c_{t+s}(z^{t+s}; \tilde{e})) \lambda(z^{t+s}, z^t) \geq \hat{V}(h_t(z^{t-1}), z_t) \quad (3)$$

for all  $z^t \in Z^{t+1}, t = 0, 1, \dots$

Considering a constrained Pareto-problem as the outcome of bargaining has some strong intuitive appeal. Each agent's initial bargaining power,  $\mu_0$ , is the product of some, unmodeled, ex-ante competition over potential partners. Whenever a partnership finally is formed, ex-post renegotiations are consistently refused *unless* an agent can present evidence of an outside offer with better contractual terms, at which the outside option of the (allegedly) defecting partner is precisely matched. Renegotiation therefore only occurs by mutual consent. The solution to the above problem is an implicit contract mapping histories to outcomes. As  $\tilde{e}$  is treated exogenous, the contract does not condition on the strategic agent's investment choices. The implicit contract is therefore incomplete. In Section 4.3, we will provide an explicit contract which mimics the implicit version above, and be more precise about the exact nature of incompleteness.

It is not immediate that a solution to (2)-(3) exist. In particular, for some process,  $\tilde{e}$ , and some histories,  $z^t$ , the constraint-set (3) may be empty.<sup>8</sup> For each such history, the partnership is terminated and the strategic agent receives her outside option. The continuation value of the partnership is then zero, effectively truncating the Pareto-problem into a finite horizon problem at relevant histories. Thus, with bounded return functions, the resulting truncated problem has a non-empty, compact constraint-set, and a solution exists.

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<sup>8</sup>This would, for instance, be the case if  $e_t(z^t) = 0$ , for all  $z^t \in Z^{t+1}, t = 0, 1, \dots$

The stochastic process,  $\tilde{e}$ , is a *given* in (2)-(3). In equilibrium, however,  $\tilde{e}$  is itself a *plan*, endogenously chosen by the strategic agent to maximize her own private utility.<sup>9</sup> With the notation developed above, it is straightforward to formulate the strategic agent's problem

$$V(h_0, \mu_0, z_0) = \max_{\tilde{e}} \sum_{t=0}^{\infty} \sum_{z^t \in Z^{t+1}} \beta^t u(c_t(z^t; \tilde{e})) \lambda(z^t) \quad (4)$$

where, again,  $\{c_t(z^t; \tilde{e})\}_{t=0}^{\infty}$  solves the bargaining problem (2)-(3), at process  $\tilde{e}$ . The strategic agent's problem in (4) is our main focus of interest. The plan which solves (4) is denoted  $\tilde{e}^* = \{e_t^*(z^t)\}_{t=0}^{\infty}$ .

The strategic problem above captures the ideas outlined in the introduction. A strategic agent decides on what fraction of the current endowment to share within her existing partnership, and on what fraction to invest in outside activities. Allocating a large share of resources to the partnership increases available surplus, of which the agent is a partial claimant. Investing a large share, instead, in outside activities is likely to increase her future bargaining power, but will also reduce current period surplus. The agent optimally trade-off these conflicting forces and chooses shares to maximize her present value expected utility. Stated somewhat differently, the strategic agent is the *Stackelberg leader* and the implicit contract the *Stackelberg follower*.

*Remarks.* There are many ways in which the above formulation could be extended, with all the main propositions in this paper remaining valid. One may, for instance, assume that the "endowment" is produced using physical capital,  $\omega = f(k)$ . In this case, the fraction  $e$  is still invested in outside activities,  $\hat{e}$  is left as surplus in the partnership, and the remainder,  $f(k) - e - \hat{e}$ , is invested in physical capital. Capital itself may accumulate according to the standard law of motion. While potentially interesting, all extensions come with additional notational complexity. To keep the paper simple, we focus on the model described above, which we consider sufficiently general to prove our point.

There are two assumptions, however, that are indispensable for the analysis. Firstly, only one of the agents is strategic. Secondly, the non-strategic agents outside option is inoperative.

As a final remark, it should be noted that we proceed under the hypothesis that a solution to the strategic problem exists. The purpose is to constructively derive a recursive formulation associated with (4). However, once we have a candidate Bellman equation, it is straightforward to exploit the contraction mapping theorem, together with a standard

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<sup>9</sup>Notice, again, that  $\tilde{e}$  is, indeed, is both a plan and a process. From the perspective of the bargaining problem,  $\tilde{e}$  is an exogenous stochastic process. From the perspective of the strategic agent, however,  $\tilde{e}$  is an endogenously chosen plan.

“verification theorem” (see for instance Theorem 4.3 in Stokey, Lucas and Prescott (1989)), to show that a solution to (4) indeed exists.

### 3. A RECURSIVE FORMULATION

One of the main goals of this paper is to derive a Bellman equation corresponding to the strategic agent’s problem, (4). This is a nontrivial task. The constraint-set associated with the strategic problem is itself another optimization problem – the bargaining problem in (2) – which is, itself, subject to a collection of infinite dimensional constraints, (3).

Even when abstracting from the additional layer of complexity induced by the strategic agent, the bargaining problem does not easily lend itself to a recursive representation. In particular, the presence of the forward-looking constraints entail that participation depends on future allocations. As a recursive formulation implies that future allocations – at some later stage – will indeed become current allocations, the agents generally lack incentives to fulfill previous commitments.<sup>10</sup>

Nevertheless, there has been substantial methodological progress in addressing problems of this kind. The general idea is to augment a payoff-relevant state vector with some additional (payoff-irrelevant) variable that effectively summarizes promises made in the past. A literature initiated by Spear and Srivastava (1987), Abreu, Pearce and Stacchetti (1990), and Thomas and Worrall (1988), show that a *promised value*, delivered if and only if an agent keeps participating in the continuation game, fulfills these requirements.

Although an elegant solution to the aforementioned complications, extending the idea to our strategic setting is not advisable. By engaging in outside activities, the strategic agent manipulates precisely those promised continuation values, which must therefore depend on the agent’s intensity of outside engagements. The appropriate state variable, a promised value, turns into a promised value *function* – an infinite dimensional object.

In contrast to these studies, Marcet and Marimon (2009) propose an alternative approach to deal with forward-looking constraints. In particular, Marcet and Marimon attach Lagrange multipliers to the constraints in (3), and show that a suitably scaled sum of past multipliers yields a sufficient state variable to honor past commitments. As we will see, the Lagrange multipliers has an interesting interpretation in the current context of dynamic bargaining: The sum of past multipliers act as an updated Pareto-weight on the strategic agent’s utility. This Pareto-weight can naturally be interpreted as the agent’s bargaining power in the continuation game. Whenever the strategic agent face outside alternatives that exceed the value of the current partnership, the agent’s bargaining power

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<sup>10</sup>Assuming a state vector of payoff relevant variables only.

permanently increases, yielding her a larger share of surplus both in the present and in the future.

As will become apparent in Section 3.1, Marcet and Marimon's (2009) approach avoids the infinite dimensional state vector implied by the "promised value" approach. As a consequence, their methodology will prove itself useful as a first step of deriving the Bellman equation associated with the strategic problem, (4).

Following the ideas of Marcet and Marimon (2009), let  $\tilde{\gamma} = \{\gamma_t(z^t; \tilde{e})\}_{t=0}^{\infty}$ , with  $\gamma_t : Z^{t+1} \rightarrow_{\tilde{e}} \mathbb{R}_+$ , denote an arbitrary, non-negative and bounded stochastic process. Let  $\tilde{c}$  denote an *arbitrary* consumption plan, feasible under the (arbitrary) process  $\tilde{e}$ . Then define the following object,

$$\begin{aligned} W(\tilde{c}, \tilde{\gamma}, \tilde{e}) &= \sum_{t=0}^{\infty} \sum_{z^t \in Z^{t+1}} \beta^t \{ \mu_0 u(c_t(z^t; \tilde{e})) + v(1 - e_t(z^t) - c_t(z^t; \tilde{e})) \} \lambda(z^t) \\ &+ \sum_{t=0}^{\infty} \gamma_t(z^t; \tilde{e}) \beta^t \lambda(z^t) \sum_{s=0}^{\infty} \sum_{z^{t+s} \in Z^{t+s+1}} \beta^s u(c_{t+s}(z^{t+s}; \tilde{e})) \lambda(z^{t+s}, z^t) - \hat{V}(h_{t+s}(z^{t+s-1}), z_{t+s}) \end{aligned}$$

in which dependence on initial  $h_0, \mu_0$  and  $z_0$  is suppressed.

A saddle-point to  $W(\tilde{c}, \tilde{\gamma}, \tilde{e})$  is, for any  $\tilde{e}$ , given by a  $\tilde{\gamma}^*$ , and a feasible  $\tilde{c}^*$ , such that,

$$\max_{\tilde{c}} W(\tilde{c}, \tilde{\gamma}^*, \tilde{e}) = W(\tilde{c}^*, \tilde{\gamma}^*, \tilde{e}) = \min_{\tilde{\gamma}} W(\tilde{c}^*, \tilde{\gamma}, \tilde{e}) \quad (5)$$

Notice that the problem associated with the first equality above is largely similar to the bargaining problem, (2)-(3). In contrast, however, the above formulation is an *unconstrained* optimization problem in which violations to the participation constraints are "penalized" at force  $\gamma_t^*(z^t; \tilde{e}) \beta^t \lambda(z^t)$ . Notice that for *any* process  $\tilde{\gamma}$ , the value associated with the above problem,  $\max_{\tilde{c}} W(\tilde{c}, \tilde{\gamma}, \tilde{e})$ , must weakly exceed that of the bargaining problem,  $W(h_0, \mu_0, z_0; \tilde{e})$ .

Using the "partial summation formula of Abel" (see Marcet and Marimon (2009)) we have

$$\begin{aligned} \max_{\tilde{c}} W(\tilde{c}, \tilde{\gamma}^*, \tilde{e}) &= \max_{\tilde{c}} \sum_{t=0}^{\infty} \sum_{z^t \in Z^{t+1}} \beta^t \{ (\mu_t(z^{t-1}; \tilde{e}) + \gamma_t^*(z^t; \tilde{e})) u(c_t(z^t; \tilde{e})) \\ &+ v(1 - e_t(z^t) - c_t(z^t; \tilde{e})) \} \lambda(z^t) \quad (6) \end{aligned}$$

where  $\mu_t(z^{t-1}; \tilde{e}) = \mu_0 + \gamma_0^*(z^0; \tilde{e}) + \gamma_1^*(z^1; \tilde{e}) + \dots + \gamma_{t-1}^*(z^{t-1}; \tilde{e})$ .

The "reorganized problem" in (6) has an intuitive interpretation in the current setting. According to the original formulation in (2)-(3), the strategic agent receives an ex-ante sub-optimally large share of net present value resources ensuing a binding outside option. Following (6), this turns isomorphic to a permanent increase in the strategic agent's

Pareto-weight. If we, for the sake of argument, interpret the process of Pareto-weights,  $\{\mu\}$ , as a given, the allocation  $\tilde{c}$  *is indeed* Pareto-optimal – as opposed to *constrained* Pareto-optimal – since the participation constraint is never binding. The strategic agent then aims to influence the process of Pareto-weights by eliciting outside options through her choice of  $\tilde{e}$ .

Following Theorem 2 in Marcet and Marimon (2009) we have the following, quite standard, result,

$$W(h_0, \mu_0, z_0; \tilde{e}) = W(\tilde{c}^*, \tilde{\gamma}^*, \tilde{e}) \quad (7)$$

With a slight abuse of notation, we will henceforth refer to any saddle-point,  $\tilde{c}^*$  and  $\tilde{\gamma}^*$  simply as  $\tilde{c}$  and  $\tilde{\gamma}$ . As a consequence,  $\tilde{c}$  is a solution to the bargaining problem (2)-(3), and given  $\tilde{c}$ ,  $\tilde{\gamma}$  solves  $\min_{\tilde{\gamma}} W(\tilde{c}, \tilde{\gamma}, \tilde{e})$ . Notice, however, that  $\tilde{c}$  and  $\tilde{\gamma}$  are only optimal in association with a certain process  $\tilde{e}$ . The relevant  $\tilde{e}$  is given by the context and will not be explicitly stated unless there is risk for confusion.

*Remarks.* The existence of a saddle point is an immediate corollary of the existence of Lagrange multipliers (see, for instance, Luenberger (1969)). In addition to the previously stated assumption, Lagrange multipliers exist if the return functions are bounded, and if, for each  $\tilde{e}$ , there exist a  $\tilde{c}'$  such that

$$\sum_{s=0}^{\infty} \sum_{z^{t+s} \in Z^{t+s+1}} \beta^s u(c'_{t+s}(z^{t+s}; \tilde{e})) \lambda(z^{t+s}, z^t) > \hat{V}(h_t(z^{t-1}), z_t)$$

for all  $z^t \in Z^{t+1}, t = 0, 1, \dots$  (“Slater’s condition”) Clearly, no such  $\tilde{c}'$  exist *for all* processes  $\tilde{e}$ . The constraint-set may indeed be empty for sufficiently small values of  $e_t(z^t)$ . Following the discussion in the preceding section, however, we may consider a truncated problem in which the partnership is terminated at empty constraint-sets. More precisely, for any history  $z^t$ , such that no plan fulfills Slater’s condition, the continuation value of the partnership is zero.<sup>11</sup> For the remaining truncated problem, Lagrange multipliers do exist, and  $W(h_0, \mu_0, z_0; \tilde{e}) = W(\tilde{c}, \tilde{\gamma}, \tilde{e})$ .

**3.1. A Bellman equation.** The goal of this section is to present a Bellman equation associated with the strategic agent’s problem in (4). To this end, we proceed in three separate steps. First we derive a partially recursive formulation to (7). The formulation is only *partially* recursive as the infinite dimensional process  $\tilde{e}$  appears as an argument in the associated value function. The result relies heavily on the arguments presented in Marcet and Marimon (2009), and is included for completeness. Exploiting this result, we show, as a second step, that the strategic agent’s problem fulfills Bellman’s principle of

<sup>11</sup>See the following section for a sharper definition of a continuation value.

optimality when the participation constraint is *non-binding*. In the third and final step, we combine results to attain a fully recursive Bellman equation.

Given plans  $\tilde{e}$  and  $\tilde{c}$ , define  $\vec{V}(z^t; \tilde{e})$  as the strategic agent's *continuation value* at node  $z^t$ .<sup>12</sup> That is,

$$\vec{V}(z^t; \tilde{e}) = \sum_{s=0}^{\infty} \sum_{z^{t+s} \in Z^{t+s+1}} \beta^s u(c_{t+s}(z^{t+s}; \tilde{e})) \lambda(z^{t+s}, z^t) \quad (8)$$

and note that

$$\vec{V}(z^0; \tilde{e}^*) = u(c_0(z^0; \tilde{e}^*)) + \beta \sum_{z_1 \in Z} \vec{V}(z^1; \tilde{e}^*) \lambda(z_1, z_0) = V(h_0, \mu_0, z_0) \quad (9)$$

Using similar ideas we define the continuation value associated with the bargaining problem (2)-(3),

$$\begin{aligned} \vec{W}(z^t; \tilde{e}) = \sum_{s=0}^{\infty} \sum_{z^{t+s} \in Z^{t+s+1}} \beta^s \{ \mu_0 u(c_{t+s}(z^{t+s}; \tilde{e})) \\ + v(1 - e_{t+s}(z^{t+s}) - c_{t+s}(z^{t+s}; \tilde{e})) \} \lambda(z^{t+s}, z^t) \end{aligned} \quad (10)$$

And again note that

$$\begin{aligned} \vec{W}(z^0; \tilde{e}) = \mu_0 u(c_0(z^0; \tilde{e})) + v(1 - e_0(z^0) - c_0(z^0; \tilde{e})) \\ + \beta \sum_{z_1 \in Z} \vec{W}(z^1, \tilde{e}) \lambda(z_1, z_0) = W(h_0, \mu_0, z_0; \tilde{e}) \end{aligned} \quad (11)$$

The following lemma is essentially equivalent to Theorem 3 in Marcat and Marimon (2009), and the proof is included for completeness.

**Lemma 1.** *Let  $h_t(z^t)$  be generated by  $\tilde{e}$ , and  $\mu_t(z^{t-1}; \tilde{e})$  by  $\tilde{\gamma}$ . Then  $\vec{W}(z^t; \tilde{e})$  equals  $W(h_t, \mu_t, z_t; \tilde{e})$  for all  $z^t \in Z^{t+1}$ .*

*Proof.* Pick  $t = 1$ . Then  $W(h_1, \mu_1, z_1; \tilde{\gamma}, \tilde{e}) \geq \vec{W}(z^1; \tilde{e})$ . Suppose the inequality is strict for some  $z^1 \in Z^2$ . Then there exist an alternative  $\tilde{c}'$  with  $c'_0(z^0) = c_0(z^0)$  such that

$$\begin{aligned} (\mu_0 + \gamma_0(z^0; \tilde{e})) u(c_0(z^0; \tilde{e})) + (1 - \mu_0) v(e_0(z^0) - c_0(z^0; \tilde{e})) \\ + \beta \sum_{z_1 \in Z} W(h_1, \mu_1, z_1; \tilde{\gamma}, \tilde{e}) > W(h_0, \mu_0, z_0; \tilde{\gamma}, \tilde{e}) \end{aligned}$$

which contradicts that  $\tilde{c}$  attains the maximum in (6). Hence,  $W(h_1, \mu_1, z_1; \tilde{\gamma}, \tilde{e}) = \vec{W}(z^1, \tilde{e})$ .

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<sup>12</sup>Recall that  $\tilde{c}$  is always assumed to be optimal. That is,  $\tilde{c}$  solves (2)-(3).

Using the previous result we know that,  $\min_{\tilde{\gamma}} W(\tilde{c}, \tilde{\gamma}, \tilde{e}) = W(h_1, \mu_1, z_1; \tilde{e}) \leq \vec{W}(z^1, \tilde{e})$ . Suppose the inequality is strict for some  $z^1 \in Z^2$ . Then there exist an alternative  $\tilde{\gamma}'$  with  $\gamma'_0(z^0) = \gamma_0(z^0; \tilde{e})$  such that

$$\begin{aligned} (\mu_0 + \gamma_0(z^0; \tilde{e}))u(c_0(z^0; \tilde{e})) + (1 - \mu_0)v(e_0(z^0) - c_0(z^0; \tilde{e})) \\ + \beta \sum_{z_1 \in Z} W(h_1, \mu_1, z_1; \tilde{e}) < W(h_0, \mu_0, z_0; \tilde{e}) \end{aligned}$$

contradicting that  $\tilde{\gamma}$  indeed was the minimizer to (6). As a consequence,  $\vec{W}(z^1; \tilde{e}) = W(h_1, \mu_1, z_1; \tilde{e})$ , for all  $z^1 \in Z^2$ . Repeating the argument yields the equivalent result for any arbitrary period  $t$ .  $\square$

As previously noted, forward-looking constraints induce an tension between past promises and current outcomes. In the present setting, the strategic agent remains within the partnership as long as the value of staying exceed that of leaving. The value of staying depends, of course, on current rent, but also on commitments regarding the future. Following equation (6), an allegedly defecting agent receives rents related to her current Lagrange multiplier,  $\gamma_t(z^t; \tilde{e})$ . Future commitments are made through a permanent increase in her Pareto-weight,  $\mu_{t+1}(z^t; \tilde{e})$ . As the Pareto-weight effectively summarizes *all* past promises, it provides sufficient information to align the non-strategic agent's incentives with all her previous commitments. Using the Pareto-weight as a state variable is sufficient to guarantee a time-consistent solution.

The proof of the lemma is also quite intuitive, and is essentially based on a repeated application of Bellman's Principle of Optimality. Given minimizers  $\tilde{\gamma}$ , re-optimizing in period one can only improve on the current allocation. However, the improvement cannot be strict as this would violate the optimality of the initial choice of  $\tilde{c}$ . Conversely, given a consumption plan,  $\tilde{c}$ , re-minimizing with respect to  $\tilde{\gamma}$  can only reduce the value of the current allocation. If the reduction was strict,  $\tilde{\gamma}$  cannot be the optimal solution to the minimization problem at given  $\tilde{c}$ .

Lemma 1 provides sufficient theoretical foundation to derive a *partially* recursive formulation to the bargaining problem. The formulation, however, is only partially recursive as it conditions on the infinite dimensional object  $\tilde{e}$ , which is - thus far - not Markovian in structure.

The following proposition appears deceptively simple, but will prove crucial for the derivation of the Bellman equation associated with the strategic problem in (4). The proposition states that *if* the participation constraint in (3) is *non-binding* in period zero, then the strategic problem fulfills Bellman's principle of optimality in period one.

**Proposition 1.** *Given an optimal plan  $\tilde{e}^*$ , if  $\gamma_0(z^0; \tilde{e}) = 0$ , then  $\vec{V}(z^1; \tilde{e}^*) = V(h_1, \mu_0, z_1)$  for all  $z_1 \in Z$ .*

*Proof.* A necessary and sufficient condition for optimality of  $c_0$  with respect to the bargaining problem is given by<sup>13</sup>

$$\mu_0 u'(c_0) = v'(e_0^*(z^0) - c_0) \quad (12)$$

Under the hypothesis that constraint (3) is non-binding, the optimal choice of  $c_0$  is therefore independent of any future variables. By Lemma 1,  $\tilde{c}$  is such that  $\vec{W}(z^1, \tilde{e}^*) = W(h_1, \mu_0, z_1; \tilde{e}^*)$ . As a consequence  $\vec{V}(z^1; \tilde{e}^*) \leq V(h_1, \mu_0, z_1)$ . Suppose the inequality is strict for some  $z_1 \in Z$ . Then there exist an  $\tilde{e}'$ , and an associated  $\tilde{c}'$  – with  $e'_0 = e_0^*$  and  $c'_0 = c_0$  – such that  $\tilde{c}'$  solves (2)-(3) given  $\tilde{e}'$ , and  $\tilde{e}'$  attains

$$u(c_0) + \beta \sum_{z_1 \in Z} V(h_1, \mu_0, z_1) \lambda(z_1, z_0) > V(h_0, \mu_0, z_0) \quad (13)$$

Since the last inequality contradicts that  $V(h_0, z_0)$  attains the maximum in (4),  $\vec{V}(z^1; \tilde{e}^*)$  must equal  $V(h_1, \mu_0, z_1)$ .  $\square$

The proposition is an adaptation of Bellman's Principle of Optimality to the current setting. Following the usual logic, the Principle relies on a certain time-consistency of the constraint-set; re-optimizing at any future date should neither expand nor contract the set of feasible choices. In the strategic problem, however, the set of feasible choices is dictated by dynamic bargaining. As a consequence, the proposition above relies on Lemma 1, which provides a time-consistent formulation of bargaining.

In a standard dynamic problem, Proposition 1 would provide sufficient information to derive a Bellman equation associated with the original formulation. The proposition, however, is valid only under the hypothesis of a non-binding participation constraint, which limits its immediate applicability. Instead, let  $J(h_0, \mu_0, z_0)$  denote the strategic agent's valuation of the current partnership, conditional on participation. Then, using well-known arguments (Stokey et al., 1989), we have

$$J(h_0, \mu_0, z_0) = \max_{c_0, e_0} \{u(c_0) + \beta \sum_{z_1 \in Z} V(h_1, \mu_0, z_1) \lambda(z_1, z_0)\} \quad (14)$$

$$\text{s.t.} \quad \mu u'(c_0) = v'(1 - e_0 - c_0) \quad (15)$$

$$h_1 = (1 - \varphi)h_0 + e_0 \quad (16)$$

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<sup>13</sup>The first order condition is, itself, not necessary to attain the result. It is important to note that the choice of  $c_0$  is independent of future variables. The first order condition provides a concise illustration of this.

Albeit a functional equation, the formulation above is incomplete and does not identify one unique solution; there is one equation and two unknowns. However, whenever the participation constraint is binding, the strategic agent's valuation of the partnership equals  $V(h_0, \mu_0, z_0) = \hat{V}(h_0, z_0) > J(h_0, \mu_0, z_0)$ . And whenever the participation constraint is slack,  $V(h_0, \mu_0, z_0) = J(h_0, \mu_0, z_0)$ . As a consequence,  $V(h_0, \mu_0, z_0) = \max\{J(h_0, \mu_0, z_0), \hat{V}(h_0, z_0)\}$ , and the Bellman equation associated with the strategic problem, (4), is therefore given by

$$V(h, \mu, z) = \max\{\max_{c,e}\{u(c) + \beta \sum_{z' \in Z} V(h', \mu, z')\lambda(z', z)\}, \hat{V}(h, z)\} \quad (17)$$

$$\text{s.t.} \quad \mu u'(c) = v'(1 - e - c) \quad (18)$$

$$h' = (1 - \varphi)h + e \quad (19)$$

in which time subscripts are superfluous and therefore dropped. The Bellman equation (17)-(19) captures the main tension facing the strategic agent. Devoting a large share of resources to be shared within the partnership, increases current rents, but at the expense of reduced possibilities of soliciting outside offers.

It is important to note that, under the stipulated assumptions, the Bellman equation above is bounded and satisfies Blackwell's sufficient condition. The right hand side defines a contraction mapping in the space of bounded continuous functions, of which  $V(h, \mu, z)$  is the unique fixed point. As a consequence, problem (17)-(19) has a unique solution,  $V(h, \mu, z)$ , which is bounded, continuous, nondecreasing in  $h$ , and strictly increasing in  $\mu$ . Evoking a standard verification theorem, e.g. Theorem 4.3 or Theorem 9.2 in Stokey et al. (1989), reveals that a solution to (4) exists.

#### 4. ANALYSIS

The Bellman equation in (17)-(19) provides a recursive framework for studying the strategic agent's valuation of the partnership, (4). In particular, the equation defines a contraction mapping that permits us to find the value associated with a complicated strategic game with dynamic bargaining. The contraction property further allows us to conclude that a solution to (4) exists, and that the associated value function carries some important qualitative properties with respect to  $h$  and  $\mu$ . Yet the formulation is surprisingly simple.

With respect to *plans*, however, problem (17) is moot. Whenever the agent finds it optimal to first exercise her outside option, continuation plans are, in fact, undetermined.<sup>14</sup>

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<sup>14</sup>A continuation plan is the plan following the history  $z^t$  that led up to state  $h_t, \mu_t$  and  $z_t$ .

To see this more clearly, notice that the strategic agent's problem at a *binding state* is given by,

$$\begin{aligned} V(h, \mu, z) &= \max_{c, e} \hat{V}(h, z) \\ \text{s.t.} \quad \mu u'(c) &= v'(1 - e - c) \\ h' &= (1 - \varphi)h + e \end{aligned}$$

in which, clearly, the choice of  $e$  – and therefore  $c$  – is undetermined. As a consequence, the optimal policy is not a function mapping states to choices, but rather a compact- and convex-valued correspondence.

The indeterminacy with respect to plans is not, however, due to some incompleteness of formulation (17)-(19). Rather, it is a consequence of a more general indeterminacy inherent in bargaining problems with occasionally binding endogenous threats. To appreciate this, let  $z^\infty \in Z^\infty$  denote an arbitrary infinite history and let  $n$  denote the period in which the outside option was binding *for the first time*. If  $z^j$  represent a predecessor to  $z^n$ , we define  $z^n$ , if it exists, as  $\vec{V}(z^n; \tilde{e}) = \hat{V}(h_n(z^n), z_n)$  and  $\vec{V}(z^j; \tilde{e}) > \hat{V}(h_j(z^j), z_j)$ , for all  $j < n$ . Notice that  $z^j \subset z^n \subset z^\infty$ . We then have the following proposition.

**Proposition 2.** *Let  $\tilde{e}^*$  denote an optimal plan. Define  $\tilde{e}'$  as  $e'_j(z^j) = e_j^*(z^j)$ , but  $e'_{n+s}(z^{n+s}) \neq e_{n+s}^*(z^{n+s})$ , for  $s \geq 0$ . The strategic agent is then indifferent between plans  $\tilde{e}^*$  and  $\tilde{e}'$ .*

*Proof.* Since  $e'_j(z^j) = e_j^*(z^j)$ , we know that  $h_n(z^{n-1}; \tilde{e}') = h_n(z^{n-1}; \tilde{e}^*)$ . As a consequence,  $\hat{V}(h_n(z^{n-1}; \tilde{e}'), z_n) = \hat{V}(h_n(z^{n-1}; \tilde{e}^*), z_n)$ , and

$$\begin{aligned} \sum_{s=0}^{\infty} \sum_{z^s \in Z^{n+s}} \beta^s u(c_{n+s}(z^{n+s}; \tilde{e}')) \lambda(z^{n+s}, z^n) &= \vec{V}(z^n; \tilde{e}') \\ &= \vec{V}(z^n; \tilde{e}^*) = \sum_{s=0}^{\infty} \sum_{z^s \in Z^{n+s}} \beta^s u(c_{n+s}(z^{n+s}; \tilde{e}^*)) \lambda(z^{n+s}, z^n) \quad (20) \end{aligned}$$

By the first order condition in (12), consumption at any predecessor node,  $z^j$ , is independent of any future allocations, and therefore  $c_j(z^j; \tilde{e}') = c_j(z^j; \tilde{e}^*)$ . Combining results completes the proof.  $\square$

The proposition above is intuitive. The strategic agent's outside value at history  $z^n$  is given by  $\hat{V}(h_n(z^n), z_n)$ . Since both this outside value, and  $h_n(z^n)$ , are independent of any continuation plan, all continuation plans must deliver the same continuation value. In addition, by the first order condition in (12), all consumption allocations preceding  $z^n$  are independent of all continuation plans. Together, unaltered consumption allocations and

continuation values imply unaltered initial values, and indifference with respect to plans continuing history  $z^n$  follows.

Following a binding history,  $z^n$ , Proposition 2 implies that *any* continuation plan is a feasible solution. In particular, full cooperation is attainable, and no long-run distortions remain.<sup>15</sup> The “continuation process” of Lagrange multipliers, and therefore Pareto-weights, can straightforwardly be imputed from the relevant continuation plan, together with the associated future outside options.

Although full cooperation is possible, it relies on an important assumption; the strategic agent can credibly commit to future actions,  $e$ , and will keep his promise even though there may exist a temptation in violating them. In the case of full cooperation in the continuation game, for instance, the strategic agent would be tempted to, yet again, act according to (17)-(19), at the newly gained bargaining weight,  $\mu + \gamma$ , in order to attain an even higher value.

To make matters worse, the strategic agent has further incentives to *lie* about her future actions, only to act differently once those future dates arrive. To see this more clearly, consider two arbitrary continuation plans that follows the “binding history”  $z^n$ . The first continuation plan is *stingy* and delivers, say,  $e_{n+s}(z^{n+s}) = 0.1$  for all  $s \geq 0$ . The second is *lavish* and delivers, say,  $e_{n+s}(z^{n+s}) = 0.9$ . By proposition 2, both continuation plans attain the same continuation value, but brings different intertemporal “temptations”. In particular, as both plans generate the same continuation value, the strategic agent’s bargaining power – represented by her Pareto-weight  $\mu_{n+1}(z^n)$  – must differ considerably across plans. A stingy plan necessitates a relatively large share of (scarce) future surplus in order to deliver the same continuation value as the lavish plan. Since the agent’s current bargaining power is related to her promised continuation plan, there lies a temptation in “promising” stingy plans, only to act lavishly once the higher bargaining share is attained. Put simply, most continuation plans are *time-inconsistent*.

Therefore, to gain further insights, and to provide a sharper characterization of the strategic problem, we turn attention to *time consistent plans*.

**4.1. Time consistent plans.** As noted above, various continuation plans give rise to different temptations of renegeing on contracts. Time consistent plans are those in which no such temptations remain. More precisely, a time-consistent plan is such that all continuation plans are optimal with respect to (17)-(19), at *each* respective future date. In addition, each “continuation process” of Pareto-weights,  $\mu_{n+s}(z^{n+s-1})$ , are consistent with each continuation plan.

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<sup>15</sup>“Full cooperation” implies  $e_{n+s}(z^{n+s}) = 0$ , all  $s \geq 0$ .

Following equation (14), let  $J(h, \mu, z)$  denote the strategic agent's valuation of the partnership, conditional on participation. We then have the following definition of a time-consistent plan.

**Definition 1.** *The plan  $\tilde{e}$  is said to be time-consistent if for all histories  $z^t \in Z^{t+1}$ ,  $t = 0, 1, \dots$ , there exist a  $\gamma_t(z^t) \geq 0$  such that*

$$\vec{V}(z^t; \tilde{e}) = J(h_t(z^t), \mu_{t+1}(z^t), z_t) \quad (21)$$

and  $\tilde{e}$  attains  $J(h_t(z^t), \mu_{t+1}(z^t), z_t)$ .

That is, a time-consistent plan is comprised by a process of Lagrange multipliers  $\tilde{\gamma}$ , and a plan  $\tilde{e}$ , such that  $\tilde{e}$  attains  $J$ , and  $J$  obeys the participation constraints at multipliers  $\tilde{\gamma}$ .

A time-consistent plan can be thought of as a subgame perfect equilibrium. Each expected continuation plan conveys a continuation process of multipliers. And each continuation plan is optimal under the implied continuation process, at each respective date. Expectations are therefore aligned with outcomes, and are consequently rational. A natural question follows, whether a subgame perfect equilibrium exists. The following proposition reveals that it does.

**Proposition 3.** *There exist a time-consistent plan that attains  $V(h_0, \mu_0, z_0)$ .*

*Proof.* Suppose not. Let  $\tilde{e}$  represent some other plan that attains  $V(h_0, \mu_0, z_0)$ , and let  $\mu_t(z^{t-1}; \tilde{e}) > \mu_{t+1}(z^t; \tilde{e})$  be the associated multiplier, at some *binding* history  $z^t$ . Lending ideas from Proposition 1 we note that

$$V(h_t(z^t), \mu_{t+1}(z^t; \tilde{e}), z_t) > \vec{V}(z^t; \tilde{e}) \quad (22)$$

for some  $z^t$ . By definition  $\vec{V}(z^t; \tilde{e}) \geq \hat{V}(h_t(z^t), z_t)$ , and therefore  $J(h_t(z^t), \mu_{t+1}(z^t; \tilde{e}), z_t) = V(h_t(z^t), \mu_{t+1}(z^t; \tilde{e}), z_t)$ . Since  $J$  is non-decreasing and continuous in  $\mu$ , there either exist a  $\mu' \geq \mu_t(z^{t-1}; \tilde{e})$  with

$$J(h_t(z^t), \mu', z_t) = \vec{V}(z^t; \tilde{e}) \quad (23)$$

or

$$J(h_t(z^t), \mu_t(z^{t-1}; \tilde{e}), z_t) > \vec{V}(z^t; \tilde{e}) \quad (24)$$

In the former case, the plan that attains  $J(h_t(z^t), \mu', z_t)$  fulfills the requirements of time consistency, while the latter inequality reveals, by Proposition 1, that  $\tilde{e}$  is sub-optimal. Since, by construction,  $\tilde{e}$  is optimal, there exist a time-consistent plans, with  $\mu_{t+1}(z^t) = \mu'$ .<sup>16</sup>  $\square$

<sup>16</sup>Note that the implied multiplier  $\gamma_t = \mu' - \mu_t$  fulfills the complementary slackness condition.

The proposition relies on two distinct ideas. Starting at period zero, an optimal plan is time-consistent as long as the participation constraint is non-binding. In the alternative case, continuation plans are generally undetermined, and not all are time-consistent. However, by continuity of the value function, there exists a Pareto-weight such that the participation constraint is satisfied *and* the associated time-consistent plan attains a value equal to the outside option. This plan can then be employed as a time-consistent continuation plan.<sup>17</sup>

The proof to the proposition above is useful, not only to show that time-consistent plans exist, but it also shows how to recover the optimal time-consistent plans and associated Lagrange multipliers. For a given value of  $h$ ,  $\mu$ , and  $z$ , the optimal choice of  $e$  solves the “unconstrained” strategic problem (14)-(16). If the participation constraint is binding, however, the Lagrange multiplier,  $\gamma$ , solves  $\hat{V}(h, z) = J(h, \mu + \gamma, z)$ . Again, the optimal choice of  $e$  is given by the solution to (14)-(16), but now at Pareto-weight  $\mu' = \mu + \gamma$ .

**4.2. Qualitative results.** By inspecting the Bellman equation (17)-(19), it is immediate that some substantial inefficiencies can arise. In particular, the strategic agent may devote large amounts of resources to outside activities, and therefore reduce surplus below the socially optimal level of one. The partnership suffers from a “holdup”.<sup>18</sup> However, whether or not inefficiencies will arise depends on the value of present and future, potential, outside options, vis-à-vis the partnership’s inside value. The inside value, in turn, depends on the strategic agent’s bargaining share. At a time-consistent solution, the bargaining share awarded to the strategic agent relates to her past and present outside alternative. Thus, as time progresses, the strategic agent’s bargaining power improves, and the desirability of soliciting outside options diminishes. Will the partnership eventually converge to a cooperative equilibrium, in which the strategic agent maximizes the joint surplus, and engagement in outside activities is zero? Proposition 4 provides an affirmative answer. Under some simplifying assumptions, the partnership will eventually reach a cooperative state, and convergence to which will occur in finite time.

We will make two simplifying assumptions: First,  $h$  depreciates fully in each period. That is,  $\varphi = 0$ . Second, the outside option,  $\hat{V}(h, z)$ , is independent of  $z$  – the problem is deterministic. We analyze the following Bellman equation

$$V(h, \mu) = \max\{J(\mu), \hat{V}(h)\} \tag{25}$$

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<sup>17</sup>Of course, the relevant continuation plan is only time-consistent up to the point at which the participation constraint is again binding. At that stage, we simply repeat the above logic to construct yet another time-consistent continuation plan, and so on.

<sup>18</sup>See elaboration in Section 4.3.

with,

$$J(\mu) = \max_{c,e} \{u(c) + \beta V(h', \mu)\} \quad (26)$$

$$\text{s.t.} \quad \mu u'(c) = v'(1 - h' - c) \quad (27)$$

The optimal, time-consistent, policy is derived using (26)(27) whenever participation is of no concern. When the participation constraint is binding, we find  $\mu' > \mu$  as the solution to  $J(\mu') = \hat{V}(h)$ , and, again, recover the policy follows from (26), but now at Pareto-weight  $\mu'$ . Let  $g(h, \mu)$  denote the function mapping current values of  $h$  and  $\mu$ , to future Pareto-weights,  $\mu'$ . That is,  $\mu' = g(h, \mu)$ . Notice that at any optimal partnership, the implied sequence  $\mu_{t+1} = g(h_t, \mu_t)$ , is *monotonically increasing*.

The strategic agent's bargaining power in period  $t$  is given by  $\mu_{t+1}$ , and generally not by  $\mu_t$ . To see this, notice that if the participation constraint is binding, the strategic agent's Pareto-weight is given by  $\mu' = \mu_t + \gamma_t$ , such that  $J(\mu') = \hat{V}(h)$ . By definition,  $\mu_{t+1} = \mu_t + \gamma_t$ . If the participation constraint is non-binding, however,  $\gamma_t = 0$ , and  $\mu_{t+1} = \mu_t$ . Therefore, to simplify notation, we will consider the lagged sequence  $\{\hat{\mu}\}$ , simply defined as  $\hat{\mu}_t = \mu_{t+1}$ . By construction, the optimal policy in period  $t$  solves (26)-(27) at  $\mu = \hat{\mu}_t$ , and the strategic agent's bargaining weight in period  $t$  is given by  $\hat{\mu}_t$ .

The following lemma shows that the sequence  $\{\hat{\mu}_t\}_{t=0}^{\infty}$  converges to some *finite* value  $\hat{\mu}$ .

**Lemma 2.** *The sequence  $\{\hat{\mu}_t\}$  converges to limit  $\hat{\mu}^* < \infty$ .*

*Proof.* For any monotonically increasing sequence,  $\{\mu_n\}_{n=0}^{\infty}$ , converging to infinity, the sequence  $\{c_n\}_{n=0}^{\infty}$ , such that  $\mu_n u'(c_n) = v'(1 - c_n)$ , converges to one. To see this, notice that  $\{c_n\}$  is a monotonically increasing sequence bounded by one. The sequence must therefore converge. Suppose that  $c_n \rightarrow \bar{c} < 1$ . Then there exist some  $N < \infty$  such that  $\mu_N u'(\bar{c}) = v'(1 - \bar{c})$ , and  $c_n > \bar{c}$  for all  $n > N$ . This is a contradiction, and therefore  $c_n \rightarrow 1$ .

For any value of  $\mu$ ,  $J(\mu) \geq \frac{u(c(\mu))}{1-\beta}$ , where  $c(\mu)$  solves  $\mu u'(c) = v'(1 - c)$ . By construction,  $J(\mu)$  is bounded and by continuity,  $J(\mu_n) \rightarrow \bar{J} \geq \frac{u(1)}{1-\beta}$ . As a consequence, there exist some  $\bar{\mu} < \infty$  such that  $J(\bar{\mu}) = \max_{h'} \hat{V}'(h) < B = \frac{u(1)}{1-\beta}$ . The sequence  $\{\mu_n\}$  therefore has upper bound  $\bar{\mu}$ . Since  $\{\mu_n\}$  is a monotonically increasing bounded sequence, it converges to limit  $\hat{\mu}^* < \bar{\mu} < \infty$ .  $\square$

In lieu of outside options, the strategic agent's bargaining share is constant across time. At periods in which participation is binding, the outside option is precisely matched through a permanent increase in the agent's Pareto-weight. As a consequence, the sequence of weights must be monotonically increasing. At some monotone sequence of

weights converging to infinity, the strategic agent's valuation of the partnership must weakly exceed that of consuming the maximum surplus for perpetuity. As the outside options are bounded at  $B < \frac{u(1)}{1-\beta}$ , equilibrium Pareto-weights cannot converge to infinity. Since every monotonically bounded real sequence has a limit, the result follows.

The following proposition shows that the partnership will reach a cooperative equilibrium in finite time.

**Proposition 4.** *For some  $N < \infty$ , total surplus equals one, for all  $t = N, N + 1, \dots$*

*Proof.* The proof is in three parts. First it will be shown that the strategic agent's consumption sequence must converge. Second it will be shown that consumption must converge to its cooperative value. And lastly it will be shown that convergence must occur in finite time.

By the construction of  $\{\hat{\mu}_t\}$  notice that

$$J(\hat{\mu}_t) = u(c_t) + \beta J(\hat{\mu}_{t+1})$$

By Lemma 2,  $\{\hat{\mu}_t\}$  converges to  $\hat{\mu}^* < \infty$ . Since  $J(\cdot)$  is a continuous function,  $\lim J(\hat{\mu}_t) = J(\hat{\mu}^*)$ . As a consequence,  $c_t$  converges to limit  $c^*$ , implicitly defined by

$$u(c^*) = J(\hat{\mu}^*) - \beta J(\hat{\mu}^*)$$

Now, define  $\bar{c}$  as  $\hat{\mu}^* u'(\bar{c}) = v'(1 - \bar{c})$ . That is  $\bar{c}$  is the *cooperative level* of consumption at Pareto-weight  $\hat{\mu}^*$ . Notice that  $c_t \leq \bar{c}$  and therefore that  $c^* \leq \bar{c}$ . Suppose that  $c^* < \bar{c}$ . Then

$$J(\hat{\mu}^*) \geq u(\bar{c}) + \beta J(\hat{\mu}^*) > u(c^*) + \beta J(\hat{\mu}^*)$$

which is a contradiction by the definition of  $c^*$ . Therefore,  $c^* = \bar{c}$ , and consumption converges to its cooperative level.

It remains to be shown that convergence occurs in finite time. Notice that either  $h_{t+1} > h_t$  or  $h_{t+1} = 0$ .<sup>19</sup> If the latter is true for some  $t$ , then

$$J(\hat{\mu}_t) = u(c_t) + \beta J(\hat{\mu}_t)$$

and convergence has occurred. Thus, consider the alternative in which  $h_{t+1} > h_t$  for all  $t$ . Define the auxiliary sequence  $\{\hat{c}_t\}$  as  $\hat{\mu}^* u'(\hat{c}_t) = v'(1 - \hat{c}_t - h_{t+1})$ , and notice that  $\bar{c} > \hat{c}_t \geq c_t$  for all  $t$ . In addition,  $\hat{c}_{t+1} > \hat{c}_t > \dots$  so  $\{\hat{c}_t\}$  is a non-increasing, bounded sequence with  $\bar{c} > \lim \hat{c}_t \geq \lim c_t = c^*$ . Since this contradicts that  $c^* = \bar{c}$ ,  $h_{t+1} = 0$  for some  $t < \infty$ , and surplus equals one.  $\square$

<sup>19</sup>Any other choice of  $h_{t+1}$  can trivially be declared suboptimal.

The intuition underlying the proof of Proposition 4 is similar to the intuition underlying many transversality conditions. The strategic agent may choose to engage in outside activities in order to gain future bargaining power. A higher bargaining power yields larger claims on future surplus, but at the expense of current consumption. If the agent would indefinitely invest in outside activities, the partnership's surplus would steadily decline, and the agent would be void of reaping the benefits of the higher bargaining share. Postponing consumption indefinitely is clearly suboptimal.

In the long-run equilibrium, the strategic agent donates the entire endowment to the partnership and she does not engage in any outside activities. Once the agent has attained some optimal level of bargaining power, she may use this, together with a maximized surplus, to get as much as possible out of the partnership. As a consequence, there may be short-run inefficiencies, but there are no long-run distortions of endogenous threats.

**4.3. Interpretations.** The Pareto-, or bargaining-, problem in (2)-(3) defines an *implicit contract*. As previously noted, the contract does not distribute rents contingent upon the strategic agent's investment choices. As a consequence, the contract is incomplete. Incompleteness of contracts is commonly justified on grounds of their simplicity. In particular, it may be difficult to foresee, describe and/or verify that certain states of the world actually have occurred (see for instance Hart and Moore (1988)). Investments can often take intangible forms of informal actions, such as learning, the nurturing of relationships, or R&D. Noisy outcomes and unverifiable actions limits enforcement and therefore also the richness of contracts. While it is clear that the implicit contract of study does not assign rents based on investment choices, it is not immediate to what extent additional information is excluded. Put differently, how incomplete is our incomplete contract?

Consider again the saddle-function in (7). The first order condition with respect to  $c_t$  observes,

$$\mu_{t+1}u'(c_t) = v'(1 - e_t - c_t) \quad (28)$$

in which reliance on  $z^t$  and  $\tilde{e}$  is left implicit. An alternative formulation of the contract is therefore given by,

$$c_t = f(1 - e_t, \mu_{t+1}) \quad (29)$$

The strategic agent's income in period  $t$ , is a time-invariant function of *current surplus*, and her *current Pareto-weight*,  $\mu_{t+1}$ . Her current Pareto-weight differs from her past, if and only if a more favorable contract is currently offered to her.

The explicit form of the contract is therefore immediate, and can be described as follows: In period zero, the agents meet and sign a legal document. The contract assigns rents as constant shares of surplus. Renegotiation occurs only upon mutual consent. More

precisely, whenever the strategic agent receives a more favorable outside option, the agents renegotiate the terms of the contract. The nonstrategic agent extends a “take it or leave it” offer to the strategic agent, exactly matching her outside opportunity. The new contractual terms include a permanently larger share of surplus to the strategic agent.

Would it be favorable for the nonstrategic agent to suggest a slightly more complicated contract? If, for instance, the random component,  $z_t$ , displays some persistence, a contract specifying shares correlated with histories, may efficiently deter the strategic agent from soliciting outside offers. Clearly, increased sophistication of contracts may increase efficiency of partnerships. Nevertheless, it is not obvious that a court can verify a certain realization of shocks. The stochastic element,  $z$ , affect the strategic agent’s valuation of (or, *value at*) alternative partnerships, and may well assume a subjective or unverifiable nature. As a consequence, while deterring the strategic agent of engaging in outside activities, a more sophisticated contract provides insufficient incentives to honor past commitments.

4.3.1. *Holdup*. A holdup occurs in bilateral trade if there are ex-ante relationship specific investments, and contracting is incomplete. Consider, for instance, the scenario in which one of two parties may venture some sunk investment in period zero. The investment increases joint surplus available in period one, but the cost is borne entirely by the investing party. A *complete* contract would entail a mapping from investments to rents, and the resulting allocation would be Pareto-optimal. At an *incomplete* contract, however, ex-ante assigned rents are non-enforceable and, instead, the outcome of ex-post bargaining.<sup>20</sup> As bargaining permits the investing agent to recover only a fraction of the societal benefits of investments, incomplete contracts renders investments below the Pareto-optimum (Grout, 1984; Williamson, 1985; Hart and Moore, 1988).

The holdup problem has received considerable attention in economics. Klein et al. (1978) and Williamson (1985) consider the holdup problem a major reason why relationships involving specific investment are rarely observed within the market context, but instead organized within institutions (e.g. firms), thus providing a theory of the extent and structure of firm organization. Grossman and Hart (1986) consider holdups a basis of the theory of vertical integration, and Hart and Moore (1990) similarly develop an influential theory of asset ownership.

It is straightforward to see how the analysis in this paper fits the general idea of a holdup. At any period  $t$ , investments, denoted  $1 - e_t$ , are relation-specific and carries no value outside the partnership. Recalling the bargaining problem in (2)-(3), investments are

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<sup>20</sup>The holdup literature usually describe this as investments are non-verifiable by the court.

considered given, and are therefore not contractible. Rents are negotiated independently of investment choices, and the contract is incomplete. The long-term nature of bargaining further strengthens the interpretation as a study of an incomplete, but *simple*, contract. In the absence of a competing outside option, renegotiation is consistently refused. When the participation constraint is binding, however, renegotiation occurs through a permanent increase in the strategic agent's bargaining share (cf. the permanent change in price of the traded good in MacLeod and Malcomson (1993)). In slight deviation from the previous literature, the strategic agent is recurrently endowed with one unit of resources. Relation-specific investments are then given by the fraction of endowments left to be shared within the partnership,  $1 - e_t$ . The associated cost of investment is the forgone, current or future, outside opportunity, intimately related to  $e_t$ .

The idea that costs to investment are borne through forgone outside values, constitutes a substantial departure from the aforementioned literature, and is key to understanding Proposition 4. The intuition follows the idea that an agent's income is solely derived from the surplus shared within a partnership, and not from any alternative parallel source. An agent engaged in on-the-job search, for instance, forgoes presumptive job offers when devoting time and effort to her current employer. But her *source* of income remains unchanged (her current employer). A missed opportunity of physical investments leaves funds to be ventured in alternative partnerships, but does not yield any alternative income in the current relation. Lastly, an employee's engagement in *firm-specific*, as opposed to *general-*, training does not reduce her income, but may erode her appeal to alternative employers. The cost to investment, in all three examples, is borne through forgone outside options.

If the potential value of outside options is *bounded*, the cost to investment is also bounded. Following Lemma 2, at some bargaining share, or price, the inside value exceed *any* outside value, and the cost to investments is zero. The resulting allocation is consequently Pareto-optimal. Proposition 4 strengthens this result and reveals that even at sub-optimal contracts, yielding short-run inefficiencies, the dynamic nature of bargaining eventually leads the partnership to a cooperative state. The holdup, in this setting, is a short-run problem in a long-term partnerships.

The idea that outside options, or market competition, can provide a solution the holdup problem is not new. MacLeod and Malcomson (1993) consider in many aspects a similar framework, but in which investment costs predicate a direct reduction in current income. They show that an enforceable fixed price of trade, renegotiated whenever participation constraints bind, can yield an efficient outcome. However, on occasions at which investments have adverse effect on outside options, as in this paper, their analysis is very limited.

Our study therefore complements MacLeod and Malcomson’s (1993) Proposition 6 to incorporate occasionally binding participation constraints. In two related studies, Cole, Mailath and Postlewaite (2001) and de Meza and Lockwood (2007), conclude that outside competition may not only solve the holdup problem, but also lead to *over-investments*. Their results, however, hinge upon a *general* nature of investments, in which capital can readily be transferred between competing partnerships. This general nature of investment, however, departs quite substantially from the question addressed in this paper, and from the holdup literature in general. Lastly, in an influential article, Che and Sákovics (2004) study the holdup problem under some dynamic considerations. In particular, they permit repeated investments leading up to the *endogenous* trade date, at which the relationship is terminated. Che and Sákovics (2004) show that in this setting, there are Markov-perfect equilibria at which holdup is of no concern. Their mechanism, however, is fundamentally different from ours. Che and Sákovics (2004) show how the expectation of “sub-optimal” actions from a future self, can be self-confirming and generate sub-optimal actions already today.<sup>21</sup> In contrast, we allow for repeated trade, repeated investment, and long-term bargaining, at a unique time-consistent equilibrium.

4.3.2. *On-the-job search.* One of the most natural interpretations of the present framework is within the labor market. The strategic agent’s investment choices particularly resembles those of strategic *on-the-job* search. A strategically acting employee divides her endowed time between working and searching for alternative employers. Searching improves the likelihood of lucrative outside offers, while working yields somewhat higher income. The employee strategically tradeoff these conflicting forces in order to maximize her present value utility.

At time  $t = -1$ , an entrepreneur and a potential employee meet and form a firm. Assume for simplicity that both the employee and the entrepreneur share the same utility function, such that  $u'^{-1}(v'(x)) = x$ . At time zero, the partners agree on a wage rate  $w_0 = \frac{1}{1+u'^{-1}(\mu_0)}$ , and the employee earns income  $w_t \times (1 - e_t)$ . Here,  $1 - e_t$  denotes the time devoted to work, and  $e_t$  the time devoted to on-the-job search. The employees outside option,  $\hat{V}(h_t, z_t)$ , denotes a job-offer, carrying value  $\hat{V}$ . Let  $\underline{i}(h_t, V)$  denote the *smallest* index  $i$  such that  $\hat{V}(h_t, \omega_i) \geq V$  for all  $i \geq \underline{i}(h_t, V)$ . That is,

$$\underline{i}(h_t, V) = \inf\{i \in \{1, \dots, N\} : \hat{V}(h_t, \omega_i) \geq V\}$$

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<sup>21</sup>Of course, there also exist alternative equilibria at which expectations of optimal future actions induce optimal actions already today.

and notice that  $\underline{i}(h_t, V)$  is *non-increasing* in  $h$ . The probability of receiving a job-offer greater or equal to  $V$  is given by

$$P(\hat{V} \geq V | h_t, z_{t-1}) = \sum_{i=\underline{i}(h_t, \bar{V})}^N \lambda(\omega_i, z_{t-1})$$

which is *non-decreasing* in  $h$ . As a consequence,  $e_t$  can be interpreted as the strategic agent's on-the-job search in period  $t$ , which implicitly affects the probability of receiving a competitive outside offer, through the stock of past searches,  $h_t$ .

The Bellman equation in (17)-(19) can therefore be thought of as an employee's problem of dividing her time between working, or strategically engaging in on-the-job search. As in the holdup problem, the long-term nature of the partnership has a straightforward interpretation also in the labor market. In any period  $t$  the employee earns  $w_t(1 - e_t)$ . Renegotiation occurs by mutual consent, in which the contracted wage is permanently increases to exactly match a competing outside offer.

This model provides a number of interesting and empirically relevant predictions (cf. Burdett and Mortensen (1998)): Wages are downward sticky, but upward mobile; turnover is likely to be higher for younger workers than for older; higher paid employees search less actively; and turnover decrease with wages.<sup>22</sup>

Admittedly, this is not the first study considering on-the-job search, although the literature is very small. Burdett and Mortensen (1998) has a strong focus on job-to-job transitions, but treats on-the-job search as something exogenous. Christensen et al. (2005) endogenize search effort, but ignore the aspect of counteroffers from the current employers: Outside offers terminate the ongoing match. Postel-Vinay and Robin (2004) endogenize on-the-job search *and* consider counter-offers. However, Postel-Vinay and Robin (2004) assume the marginal cost to search is zero, and each agent either search the maximal amount, or nothing at all. Job-search is therefore "endogenously exogenous".

**4.3.3. Human capital.** Human capital accumulation, or training, on the job is commonly thought of as a *firm-sponsored* event. Firms pay for workers' training activities, which may be of both general or specific character.<sup>23</sup> However, as noted by Prendergast (1993), workers can largely affect the acquisition of skills outside formal training arrangements, and these skills are rarely observable. On a daily basis, workers are routinely engaged in activities that may increase their stock of human capital. The nature of learning however, can be directed towards firm-specific skills, but also skills of more general character. A sales representative can, for instance, spend a larger fraction of time nurturing his personal ties

<sup>22</sup>With "turnover" we refer to the likelihood of receiving a competitive outside offer.

<sup>23</sup>See Acemoglu and Pischke (1999) for a survey.

with potential customers, rather than establishing a more solid firm-customer relationship. The opportunity of such choices are likely to generate a tension between the interest of the firm and the interest of the employee. The present framework is well suited for studying such tensions.

Suppose an employee can direct part of her working day to training activities. She may engage in both firm specific training and training of a more general character. Let  $g_t$  and  $s_t$  denote the *fraction* of total training time devoted to general and firm-specific activities, respectively. As there is no other form of training,  $s_t = 1 - g_t$ . Total training time is then given by  $\tilde{f}(g_t, 1 - g_t)$ , which is simply denoted  $f(g_t)$ . The remaining time of the day is spent working, producing  $1 - f(g_t)$  units of output. Assume that the function  $f$  is increasing in  $g$ : General training activities are more time consuming than specific.<sup>24</sup> Thus, as  $e_t = f(g_t)$ , all previous results follow without modification. Proposition 4, again, reveals that the employee eventually will direct all her time towards specific training, and all short-term distortions vanish.

There are, of course, alternative solutions to the tensions arising from the conflicting interests of the firm and the worker. Malcomson (1997) casually states that many solutions to the holdup problem indeed solves the above inefficiency of we reverse the roles of the firm and the employee. At some fixed level of profits,  $\pi$ , the worker is the residual claimant to additional profits, and their incentives are align. In a more elaborate setting, Prendergast (1993) suggests that promotion schemes may provide appropriate incentives.

## 5. CONCLUSIONS

This paper studied of model of partnerships in which rents are characterized by a bilateral monopoly. Contracts, assigning residual rents, are subjected to bargaining. Due to the voluntary nature of any partnership, sufficiently lucrative outside offers incite renegotiation of past agreements. As a consequence, an opportunistic agent has incentives to engage in strategic behavior, devoting resources to an inefficient activity of soliciting outside offers. We explored how strategic behavior of this sort affect the properties of bargaining and efficiency in a general framework of bilateral cooperation with one-sided limited commitment.

Several results emerged. We showed that there exist a surprisingly simple Bellman equation associated with the strategic game representing the choices facing the opportunistic agent. The solution to this Bellman equation, the optimal plan, displayed a large degree of multiplicity, and we put focus on subgame perfect equilibria, which we showed

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<sup>24</sup>Alternatively, the total effect of firm-specific training on output is increasing, *net of the time-cost*, but decreasing in general training.

exist. Even though repeated renegotiations and repeated opportunistic behavior arise in equilibrium, the associated inefficiencies are short-lived. In particular, we concluded that the equilibrium converges to a cooperative, Pareto-optimal state, and that convergence occurs in finite time.

Our study may be extended along several interesting dimensions. The nonstrategic agent treats the opportunistic actions of her partner as given. Contracts are therefore not only incomplete, but also quite simplistic. One could easily imagine a contract assigning rents based on histories of shocks, and not only histories of competing offers. If appropriately designed, such a contract may well deter the strategic agent from engaging in unproductive activities. However, the possibility of writing such contracts clearly depend on the observability, and verifiability, of shocks, which may vary substantially across professions. For certain professions with a high degree of unobservability of productivity (such as, for instance, academics), the current framework may be an accurate description. For other, more simple jobs, it may not be.

Lastly, it should be noted that we consider the case of one-sided limited commitment.<sup>25</sup> Some of the results derived in this paper crucially rely on this. In particular, in a two-sided case, it is not obvious that the partnership will converge to a cooperative equilibrium. One could imagine repeated rent-seeking in an oscillating equilibrium in which both parties perpetually engage in opportunistic behavior.

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<sup>25</sup>Notice that one-sided limited commitment implies one-sided investments.

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