

Government Deficits and Interest Rates: a No-Arbitrage Structural VAR Approach

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Abstract

What is the effect of government deficits on interest rates? This fundamental question has not been convincingly answered. We propose a no-arbitrage structural VAR method that allows us to incorporate the cross-sectional information in bond yields into a structural macroeconomic framework. We find that the government deficit is an important factor behind the yield curve: A one percentage point increase in the deficit increases the 10 year rate by 41 basis points.

Introduction

Empirical macroeconomic research has not been able to establish whether government deficits affect interest rates (Elmendorf and Mankiw (1999)). Yet, this issue is of crucial importance for policy making and for academic research. One reason for this lack of success is that the workhorse for empirical research in macroeconomics, the structural vector autoregression SVAR, has not been properly extended to account for no arbitrage restrictions in assets prices. As a result, macro-economists have to rely on descriptive evidences (see Barth, Iden, Russek, and mark Wohar (1991), Gale and Orszag (2003) and Engen and Hubbard (2004) for surveys of the existing literature). Recently, Canzoneri, Cumby, and Diba (2002) and Laubach (2003) present clever regressions that suggests that deficits matter, but their single-equation regression approach is open to criticism, and it leaves many questions unanswered.

On the other hand, recent theoretical and empirical research in finance has led to a better understanding of the dynamic properties of the term structure of interest rates: The models are parsimonious, financially coherent, and are able to capture some important stylized facts. (see Dai and Singleton (2003) for a recent survey of this literature). Most existing models, however, are based on unobserved or latent risk factors, which are not easy to interpret. The next step, currently under way, (see, e.g., Piazzesi (2003), Ang and Piazzesi (2003), Rudebusch and Wu (2003), and Hördahl, Tristani, and Vestin (2003)) is to draw explicit connections between latent risk factors that drive the term structure dynamics and observed macro-economic variables characterizing the state of the economy.

In this paper, we develop a model of the term structure that emphasizes the role of fiscal policy. Our empirical model is a structural VAR where we impose *no-arbitrage restrictions* on the joint dynamics of the state vector and the term structure of yields. The state space includes four factors: The government deficit to GDP ratio, inflation, the help wanted index and a latent factor that captures a persistent monetary shock. As far as we are aware, this is the first paper that introduces a fiscal policy variable into a dynamic term structure model. Also, following the macroeconomic literature (see, e.g., Christiano, Eichenbaum, and Evans (1999) and Blanchard and Perotti

(2002)), we allow full feedback among the four factors, and we impose structural restrictions to identify the structural shocks.

We find that government deficits matter for interest rates, especially at long horizons: A 1% increase in the deficit today increases the 10 year rate by 41 basis points. This result should be of interest for macro-economists and financial economists alike. We view this work as a step towards a fully structural model of the yield curve where all factors are observable, have a precise economic interpretation and follow dynamics consistent with standard macroeconomic theory.

1 Theoretical Framework

1.1 The Basic Setup

We assume that the central bank sets the short rate, r_t , according to the policy rule:

$$r_t = \delta_0 + \delta'_Y Y_t, \tag{1}$$

The state vector Y_t consists of a latent factor q_t , the deficit/GDP ratio d_t , the expected¹ growth rate of the GDP deflator π_t , and the help wanted index h_t , i.e.,

$$Y_t = \left(q_t \quad d_t \quad \pi_t \quad h_t \right)'. \tag{2}$$

The macro-economic dynamics is captured by a first-order VAR specification for the state vector:²

$$Y_t = \mu + \Phi Y_{t-1} + \Sigma \epsilon_t, \tag{3}$$

where ϵ_t is a vector of i.i.d. structural shocks with zero mean and an identity covariance matrix.

In order to examine the effect of fiscal policy on both short-term and long-term bond yields, we need to develop a no-arbitrage bond pricing model. This is achieved by positing that there exists a pricing kernel of the form:

$$M_{t+1} = \exp \left(-r_t - \frac{\Lambda'_t \Lambda_t}{2} - \Lambda'_t \epsilon_{t+1} \right), \tag{4}$$

¹We construct the expected inflation series using the survey of professional forecasters.

²If we write $\Phi = I - \kappa \Delta t$, where Δt is the observation interval, then κ is interpreted as the mean-reversion matrix. Let θ be the vector of long-run means for Y_t , then $\mu = \kappa \theta \Delta$. We omit the observation interval Δt in the theoretical formulation so long as no confusion arises.

where Λ_t is the vector of market prices of risk for the structural shocks. Following Dai and Singleton (2002) and Duffee (2002), we assume that the market prices of risk are affine in the state vector:

$$\Lambda_t = \Lambda_0 + \Lambda_Y Y_t. \quad (5)$$

To obtain an analytical bond pricing formula, we complete our specification by insisting that the structural shocks ϵ_{t+1} are conditionally jointly normal.

The defining property of the pricing kernel implies that the price, P_t^n , of a zero-coupon bond with n periods to maturity is given by

$$P_t^n = E_t [M_{t+1} \times M_{t+2} \times \dots \times M_{t+n}]. \quad (6)$$

Appendix A shows that under our maintained assumptions, zero-coupon bond prices are exponential affine in Y_t , and the zero-coupon bond yields, defined by $r_t^n \equiv -\frac{\log P_t^n}{n}$, are affine in Y_t , that is,

$$r_t^n = a_n + b_n' Y_t, \quad (7)$$

where the loadings a_n and b_n can be obtained through a recursion with initial conditions $a_1 = \delta_0$ and $b_1 = \delta_Y$.

To achieve structural identification, we follow Christiano, Eichenbaum, and Evans (1999) by imposing the following zero restrictions on the matrix Σ (a dot represents a free parameter):

$$\Sigma : \begin{array}{c} q \\ d \\ \pi \\ h \end{array} \begin{array}{c|ccc} & q & d & \pi & h \\ \hline & \cdot & 0 & \cdot & \cdot \\ & 0 & \cdot & 0 & \cdot \\ & 0 & 0 & \cdot & \cdot \\ & 0 & \cdot & \cdot & \cdot \end{array} \quad (8)$$

These restrictions imply that (i) monetary shocks cannot have any contemporaneous effect on other variables; (ii) unanticipated shocks to the deficit do not change money and inflation within the quarter; and (iii) inflation shocks do not change the deficit within the quarter.

1.2 Relation to Existing Work on the Term Structure

Equations (1), (3), (4), and (5) constitute a full-fledged term structure model, which belongs to the class of affine term structure models (see, e.g., Duffie and Kan (1996),

Dai and Singleton (2000), Dai and Singleton (2002), Duffee (2002), and Piazzesi (2003)). Unlike most previous term structure models in which the entire state vector is assumed to be latent, we use four macro-economic variables that proxy for monetary policy, fiscal policy, inflation, and the business cycle. This allows us to impose the restrictions (8) for identifying the structural shocks.

Existing works that are most closely related to our model are Ang and Piazzesi (2003), Rudebusch and Wu (2003), and Hördahl, Tristani, and Vestin (2003). Ang and Piazzesi (2003) use a no-arbitrage VAR where the maintained assumption is that latent factors (which include monetary policy) do not affect output or inflation. Therefore, the effects of monetary and fiscal policies on the term structure can not be identified in their model.

In contrast, Rudebusch and Wu (2003) and Hördahl, Tristani, and Vestin (2003) start from a simple textbook model of the macro-economy, with a price setting equation for firms, and a linearized Euler equation for consumption and output. We do not follow this strategy, for two reasons. First, while the price setting equation that governs the inflation process in the textbook model appears to be quite reasonable (see, e.g., Gali and Gertler (1999)), the Euler equation that governs the aggregate output process suffers from known failures (the most well-known being the equity premium puzzle or the risk-free puzzle). Indeed, for the purpose of pricing bonds, Rudebusch and Wu (2003) and Hördahl, Tristani, and Vestin (2003) posit a reduced-form pricing kernel on top of the marginal rate of substitution underlying the Euler equation. Second, to examine the effects of fiscal policy shocks on the term structure of interest rates, it is necessary to introduce the fiscal variables into either the pricing equation or the aggregate demand equation or both. There is hardly any consensus in the macro literature on how this can be achieved.

The above discussion provides the rationale for our approach, namely, a **No-Arbitrage Structural VAR** (henceforth NASVAR): (i) the bond pricing model (1), (3), (4), and (5) imposes "no-arbitrage" restrictions for the cross-section of bond yields; (ii) the identification of the state vector (2) in terms of observed macro variables allows the macro variables and bond yields to be described jointly as a VAR with full feedback effects; and (iii) the restrictions (8) allows us to identify the structural

shocks.

1.3 Fiscal Policy and Interest Rates: Descriptive Evidence

Before presenting the NASVAR results, we present some descriptive evidence using simple univariate regressions. Elmendorf and Mankiw (1999) offer the following summary in their review of existing evidences regarding the effect of budget deficit on interest rates: “our view is that this literature, like the literature regarding the effect of fiscal policy on consumption, is ultimately not very informative.” Some recent studies, however, have reached a more nuanced conclusion. In particular, Canzoneri, Cumby, and Diba (2002) show that when CBO forecasts of future deficit, rather than realized deficits, are used to forecast future interest rates, there is a much stronger case for the existence of a positive relationship.

In essence, one would like to know the effect of deficit d_t on interest rates r_t^n at some maturity n , while controlling for other factors that are likely to explain interest rates and deficits. The most obvious candidates are inflation, business cycle and monetary policy. We control for the business cycle using the help wanted index³ h_t , for inflation using the growth rate of the GDP deflator π_t and for monetary policy shocks using the Romer-Romer measure ρ_t (see Romer and Romer (2003)). Table 1 reports the slope coefficients on the deficit from the following regressions, with the dependent variable being nominal yields with various maturities or the 10-year/3-month term spread:

$$r_t^n = \alpha^n + \beta^n d_{t-1} + \sum_{i=1}^4 \gamma_i^n h_{t-i} + \sum_{i=1}^4 \phi_i^n \pi_{t-i} + \sum_{i=0}^4 \varphi_i^n \rho_{t-i} + \epsilon_t^n \quad (9)$$

We allow the error terms to be auto-correlated and we estimate equation (9) as a MA(1). Note that $\beta^n > 0$ and increases with n . The coefficients become significantly different from 0 for long enough maturities and for long-term spreads. So deficits seem to matter for the term structure, and the more so for longer maturities. Gale and Orszag (2003) have used CBO forecasts and have obtained similar results.

³It is the best proxy for the Taylor rule. Detrended output (using a backward looking moving average) or output growth perform very poorly. In fact, for both of these proxies, the coefficients $\{\gamma_i\}$ are all insignificant, but the estimated $\hat{\beta}$ does not change significantly.

Table 1: OLS Regressions of Yields on Deficits

n	3	6	12	24	36	48	60	120	10y-3m
β^n	.245	.386	.418	.477	.515	.534	.553	.509	.363
STD	.127	.147	.146	.148	.146	.147	.137	0.066	.0728

β^n reported in the second row is the slope coefficient associated with the lagged deficit variable d_{t-1} in the following regression:

$$r_t^n = \alpha^n + \beta^n d_{t-1} + \sum_{i=1}^4 \gamma_i^n h_{t-i} + \sum_{i=1}^4 \phi_i^n \pi_{t-i} + \sum_{i=0}^4 \varphi_i^n \rho_{t-i} + \epsilon_t^n,$$

for the sample period: 1965Q1 – 2003Q4. Controls are four lags of help wanted index, four lags of inflation, and current and four lags of Romer-Romer shocks.

However, there are several issues with this approach. First, it is clearly inefficient. There is information in all the yields, so one would like to use all of them at once. Second, we cannot learn about the channels through which deficits affect long-term rates: expected inflation, output, or risk premia. Third, deficits feed back into the economy. Suppose that high deficits cause high inflation. Then we would underestimate the importance of deficits by attributing too much causal role to inflation. Also, premia are endogenous so the error term will be correlated with the variables of interest.

So one would like to use a more structural approach. The standard way to do so in the empirical macroeconomic literature is to use a structural VAR. We argue that this is indeed a good idea, but that it needs to be complimented by a reasonable model of the term structure in order to achieve structural identification in the presence of long-term yields.

1.4 Relation to the Structural VAR Approach

To extend the OLS regression (9) to a structural VAR, the state vector should include $Y_t = (r_t^{3m} \ d_t \ \pi_t \ h_t)'$ and at least one long rate r_t^n . We write:

$$Y_t^n = \begin{pmatrix} Y_t \\ r_t^n \end{pmatrix},$$

A naive approach to a structural VAR would be to specify Y_t^n as a first-order VAR (first-order just for simplicity):

$$Y_t^n = \begin{bmatrix} \Phi & \Phi^{Yn} \\ \Phi^{nY} & \Phi^{nn} \end{bmatrix} \times Y_{t-1}^n + \begin{bmatrix} \Sigma & \Sigma^{Yn} \\ \Sigma^{nY} & \Sigma^{nn} \end{bmatrix} \times \begin{bmatrix} \epsilon_t \\ \epsilon_t^n \end{bmatrix}, \quad (10)$$

where the ϵ_t are the structural shocks to Y and ϵ_t^n is the structural shock to the long-term yield. Of course with OLS one can only estimate a reduced-form VAR:

$$Y_t^n = \begin{bmatrix} \Phi & \Phi^{Yn} \\ \Phi^{nY} & \Phi^{nn} \end{bmatrix} \times Y_{t-1}^n + \begin{bmatrix} u_t \\ u_t^n \end{bmatrix}, \quad (11)$$

where $u_t = \Sigma \epsilon_t + \Sigma^{Yn} \epsilon_t^n$ and $u_t^n = \Sigma^{nY} \epsilon_t + \Sigma^{nn} \epsilon_t^n$ are reduced-form shocks to Y and r_t^n , respectively, and the covariance matrix of $(u_t, u_t^n)'$ can be estimated.

As is well-known, we must impose additional restrictions to achieve identification of Σ , Σ^{Yn} , Σ^{nY} , and Σ^{nn} , because, in our simple example above, the symmetric covariance matrix of the reduced-form shocks in (11) contains $\frac{5 \times (5+1)}{2} = 15$ free parameters, and the loading matrix in the structural model (10) contains $5 \times 5 = 25$ free parameters.

Where do the restrictions come from? The macro-economic literature has proposed enough restrictions to identify the four variables VAR (with only Y_t) which does not include the long rate r_t^n , but these restrictions are not enough to identify the VAR with r_t^n (or a fortiori a VAR with more than one long rate).

The solution is to use a no-arbitrage pricing model to determine Σ^{Yn} , Σ^{nY} , and Σ^{nn} in terms of Σ . To put it simply, in a no-arbitrage model, r_t^n is “priced” by Y_t , and adding a yield does not increase the number of unknown parameters. Therefore, we can use exactly the same structural identifying restrictions that one would use to estimate a structural VAR for the vector of pricing factors Y_t alone.

2 Empirical Results

We estimate the model using quarterly time-series observations of the deficit/GDP ratio, the one-quarter ahead forecast of the growth rate of the GDP deflator, the help wanted index, and zero-coupon bond yields with maturities two quarters, five years, and ten years.⁴ The sample period is from the first quarter of 1970 to the third quarter of 2003 and the summary statistics are reported in Table 2. Following standard practices (see, e.g., Chen and Scott (1993), Duffie and Singleton (1997), and Dai and Singleton (2000)), we assume that one of the bond yields (the two-quarter yield) is measured without error, so that the latent factor is implicitly observed given model parameters. The five-year and ten-year yields are measured with error with zero means. As a baseline case, we assume that the measurement errors are i.i.d. and uncorrelated cross-sectionally.

Before describing the empirical results in detail, let us summarize the key findings.

1. Our model recovers a reasonable Taylor rule, under which the short-term rate (1-quarter yield) increases with the deficit/GDP ratio, the growth rate of the GDP deflator, and the help wanted index, as well as the latent factor. The first difference of the latent factor is strongly correlated (with correlation coefficient 0.663) with the monetary shocks constructed by Romer and Romer (2003), despite the fact that we never used the Romer-Romer shocks directly or indirectly in our estimation.⁵
2. The estimated dynamics for the state vector appears to be sensible and broadly consistent with estimates from a traditional structural VAR.
3. Most importantly, we find strong effects of fiscal policy (as captured by the deficit/GDP ratio) on interest rates. The loading of nominal bond yields on the

⁴The macro time-series, including 3-month T-bill rates, are obtained from Federal Reserve Economic Database (FRED) and the bond yields (from maturities from 6 months to 10 years) are provided by Mark Fisher of the Federal Reserve Board. The inflation forecasts are from the survey of professional forecasters. Our results for effects of deficits are robust to using realized inflation, but the fit of the model for the early 1980's deteriorates, as one would expect.

⁵Romer and Romer construct their monetary shocks by taking out anticipated policy moves by the Fed based on published forecasts of future economic activities. In a very strong sense, the unanticipated shocks are true surprises to bond market participants.

deficit/GDP ratio increases with maturity up to 10 years. All else equal, the 10-year yield would increase by 41 basis points if the deficit/GDP ratio increases by 1%. A structural shock (one standard deviation) to the deficit/GDP ratio raises 1-quarter yield by 38 basis points and 10-year yield by 61 basis points.

2.1 Maximum Likelihood Estimates

Aside from the restrictions on Σ for identifying the structural shocks, we impose two normalizations that (i) the long-run mean of the latent factor is 0 (since the constant term δ_0 in the Taylor rule is a free parameter) and (ii) the loading of the Taylor rule on the latent factor is 1. Thus, the latent factor is literally the residual in the monetary policy instrument. The resulting model has 55 free parameters (4 for the Taylor rule, 3 for the long-run means of the observed macro variables, 16 for the mean reversion matrix κ , 10 for the covariance matrix, 20 for the market prices of risk, and 2 for the volatility of the measurement errors).

In addition, we impose 9 over-identifying restrictions. First, the loading of the deficit variable in the Taylor rule is set to 0, because it is well known that the U.S. central bank does not respond to budget deficit immediately (at least not within the quarter). Second, the second and third columns of Λ_Y are identically zero, which means that deficit and inflation do not affect the market prices of risk. The likelihood does not change appreciably even if we relax these restrictions.

Maximum likelihood estimates for the remaining 46 parameters are presented in Table 4. The t-ratios for most parameters are close to 0, which indicate clearly that the model is still over-parameterized. It is not clear, however, which parameters should be constrained to zero as such restrictions can potentially turn off economically important feedback effects. To give the model the best chance of recovering widely documented dynamic properties of the macro variables and the Taylor rule, we choose not to impose any additional restrictions.

To get a sense of how sharply model parameters are actually identified, we compute the standard errors for parameters related to the Taylor rule, (δ_0 and δ_Y), the state dynamics (κ , θ , and Σ), and the volatility of the measurement errors (Ω), while keeping the market prices of risk fixed at their estimated parameter values. These

standard errors are reported in the top half of Table 5, and are broadly consistent with those obtained from a standard VAR on the observed 1-quarter yield, the deficit/GDP ratio, the growth rate of the GDP deflator, and the help wanted index.⁶ In the bottom half, we report the standard errors for the market prices of risk, while keeping other parameters fixed. Although the t-ratios are very low, the relative magnitudes of the t-ratios indicate that market prices of risk are driven primarily by the latent and help wanted variables (i.e., monetary policy and business cycle variables).

2.2 Yield Loadings

The factor loadings a_n and b_n , which appear in equation (7), are plotted in the top panel of Figure 1.

One of the most important findings is the fact that the yield loading b_n^d for the deficit/GDP ratio increases monotonically with n from 0 at 1 quarter to about 41 basis points at 10 years (the loading peaks out at about 15 years and reverts back to 0 gradually). The effect of deficit on long-term rates is fairly robust: it is qualitatively the same under several alternative specifications of our model. For instance, the shape of the loading does not change qualitatively when we use realized inflation instead of inflation forecast or when we use the GDP growth rate or its forecast in lieu of the help wanted index.⁷

2.3 Time-series and Cross-sectional Fit for the Term Structure

The bottom panel of Figure 1 examines the fit for the average level of the observed yields and for the volatility of yield changes. The model fits the average shape of the yield curve fairly closely, but over-states the term structure of volatility by about 3.6% on average.

⁶It is well known that risk premiums on financial assets are poorly identified because of limited sample size. By fixing the market prices of risk to their estimated values, the rest of the parameters should be reasonably sharply identified.

⁷It would also be useful to check whether it makes any difference if the CBO forecasts are used in lieu of the actual deficit levels. Unfortunately, the CBO forecasts are not available at quarterly frequency. We could, however, use our model to generate deficit forecasts and compare them with available CBO deficit forecasts.

The top panel of Figure 2 provides a time-series view of the fitted yields vs. their observed counterparts. The discrepancies are small across the entire term structure. For an example, the average pricing errors for 5-year and 10-year yields are 1.5 and -1.7 basis points, respectively, and the volatility of the pricing errors are 60 and 70 basis points, respectively

The remarkable fit for the yields is in part achieved by pushing any systematic discrepancy between model implied yields and observed yields into the latent factor. The bottom panel of Figure 2 shows the latent shocks are closely related to the Romer-Romer shocks. The high correlation between the latent shocks and the Romer-Romer shocks provides the justification for interpreting the latent shocks as monetary shocks.

2.4 Impulse Response Functions

Figure 3 plots the response functions of the four state variables.⁸ Focusing first on plots (a) – (d), which are impulse responses to one standard deviation of structural shocks:

(a) Impulse Responses to Monetary Tightening One standard deviation of monetary shock (2.09%) elicits delayed responses in the deficit/GDP ratio, the inflation rate, and the help wanted index: the deficit/GDP ratio increases gradually to a maximum response of 0.97% in about 11 quarters and declines gradually afterward; the expected inflation increases slightly to a maximum response of 0.32% in about 6 quarters – thus there is a small pricing puzzle;⁹ the help wanted index declines sharply, reaching a maximum impact of -0.68 in about 7 quarters. The output recovers to the pre-shock level in about 6 years, and overshoots to positive territory before declining to 0 eventually. The effect of

⁸The annualized sample standard deviations of the first differences of the latent factor, the deficit/GDP ratio, the growth rate of the GDP deflator, and the help wanted index are given by 2.27%, 1.37%, 1.08%, 0.0853, respectively (these are twice the values reported in the second column in the bottom panel of Table 2), which set the scales for the impulse response functions.

⁹The price puzzle is well-known in the structural VAR literature (see, e.g., ...) To confirm this, we ran a structural VAR on the 1-quarter rate, the deficit/GDP ratio, the inflation, and the help wanted index. With one-lag, we recover a small pricing puzzle similar to what we find in NASVAR. However, the pricing puzzle disappears when additional lags are added to our structural VAR. This suggests that NASVAR should also do better along this dimension by having a better forecast model.

the monetary shock is therefore most directly felt in real economic activity.

- (b) Impulse Responses to Deficit Increase** One standard deviation of structural shock to deficit (about 0.88% of GDP) elicits no response in monetary policy, a very slow delayed response in inflation (peaking in about 10 years), and an immediate and large response (0.0284 or about one third of its standard deviation) in the help wanted index. After the initial positive response, the real activity continues to grow, peaking out in about 12 quarters.
- (c) Impulse Responses to Inflation Shock** A standard deviation of the structural shock to inflation (about 1%) is accompanied by an easing of monetary policy (about 50 bps) and a small increase in the help wanted index. The monetary easing quickly reverses itself, but the real activity continues to grow, peaking out in about 11 quarters. Thus, like a positive deficit shock, a positive inflation shock also helps spur real acitivity.
- (d) Impulse Responses to Output Shock** A standard deviation of the structural shock to the help wanted index (which can be interpreted as a shock to real demand or output) induces practically no change in monetary policy or inflation. It leads to an immediate reduction in deficit (to the tune of 0.89% of GDP). The output shock reverses itself in about three years and overshoots to negative real growth before returning to 0.

2.5 Impulse Responses of Yields

The impulse responses of 1-quarter, 10-year, 5-year yields and the 10-year/1-quarter slope to the four structural shocks are plotted in the bottom four graphs, (i)–(iv), in Figure 3.

- (i) Monetary Shock** The effect of monetary shock to the term structure is relatively short-lived. A standard deviation of monetary shock (a tightening of 2.09%) leads to a rise of 1.40% in the 10-year yield and flattening of the yield curve by about 69 basis points. The yield curve will return to the pre-shock level relatively quickly (in about 3 years).

- (ii) **Deficit Shock** A standard deviation of the structural shock to the deficit/GDP ratio (about 0.88%) raises 10-year yield by 61 basis points, and 1-quarter yield by 38 basis points. Furthermore, the short-term rate is expected to continue to rise and the long-term rates decline very slowly. After 10 years, the 10-year yield is still 33 basis points above the pre-shock level, and the 1-quarter yield is about 58 basis points above the pre-shock level.
- (iii) **Inflation Shock** A standard deviation of inflation shock (about 1%) immediately raises 1-quarter yield by 40 basis points and 10-year yield by 28 basis points. All interest rates continue to rise, peaking out in about 11 quarters. In 10 years, the 10-year yield stays above the pre-shock level by 28 basis and the 1-quarter yield is 44 basis points above its pre-shock level.
- (iv) **Output Shock** A standard deviation of output shock leads to immediate increases in both short-term and long-term rates, with short-term rates increasing more. The yield curve flattens as a result. The effect of the shock all but disappears after about five years.

Of particular interest to us is the effect of deficit to interest rates. Our results suggest that, while an increase in deficit results a short-term gain (for about 3 years) in employment demand and real output growth, it has adverse long-term consequences: if deficit increases by 1% of GDP, the average level of interest rates will rise and stay high in the next 10 years by about 50 to 65 basis points.

3 Extensions and Robustness (incomplete)

We now present various extensions to our basic setup.

3.1 Sub-sample stability

It is widely believed that monetary policy has changed over time. We re-estimate our model before and after the Volcker disinflation. The results for the effects of deficits appear robust.

3.2 Effect of the stock of debt

In our basic setup, all variables are stationary. Consider the following thought experiment: Starting from no debt and no deficit, the government runs a deficit of 1% for 10 years, at which point the deficit returns to 0 forever. By construction, our model implies that after 10 years, all the yields will go back to their initial values, irrespective of the new stock of debt. Most macro-economists would be suspicious of such a result. However, incorporating the stock of debt is not so straightforward ...

3.3 Spending and taxes

The deficit can increase because government spending increases for given taxes or because taxes decrease for given expenditures. However, while almost all theories predict that government spending should increase interest rates, there is no consensus about the role of taxes and deficits for given expenditures: This is the debate about Ricardian equivalence. The conventional view is that debt financing of government expenditures increases aggregate demand and therefore output in the short run, while reducing national saving and national wealth in the long run. The Ricardian theory is that government financing is irrelevant, because the households' lifetime wealth depends only on the present value of government spending. As a result, a \$1 decrease in public saving is compensated by a \$1 increase in private saving, leaving output, investment and interest rates unchanged. By definition, the Ricardian equivalence can fail because taxes distort the allocation of resources, because government financing decisions redistribute resources across generations, or because households do not have the same discount factor as the government (which, in turn, may be due to myopia or capital market imperfections).

We therefore follow Blanchard and Perotti (2002) and model separately government spending and taxes ...

3.4 Policy Inertia vs. Serially Correlated Monetary Shocks

It would also be interesting to examine whether the effect of deficit on the term structure is robust to different specifications of the monetary policy rule.

To achieve this, we will extend our basic setup to a more general specification in which the observable state vector $Y_t = (r_t \ d_t \ \pi_t \ h_t \ \text{“spending” or “tax”} \ \dots)'$ follows a vector ARMA process:

$$Y_{t+1} = \mu + \Phi(L)Y_t + \sigma\Psi(L)\epsilon_{t+1}, \quad (12)$$

where $\Phi(L)$ and $\Psi(L)$ are distributed lag operators and L is the lag operator. This framework captures both the notion of interest rate smoothing or policy inertia (as in ?) and serial autocorrelation in the monetary shocks (as in Rudebusch and Wu (2003)).

This framework also allows us to address an apparent model mis-specification: Figure 4, which plots the structural shocks implied by the model, and Table 3, which tabulates the correlation structure of these shocks, indicate that a more appropriate specification for the help wanted index is AR(2) rather than AR(1) (the first-order serial correlation of the shocks to the help wanted index is 0.58).

4 Conclusion

We have found that government deficits increase interest rates, especially long ones.

To reach this conclusion, we have developed a No-Arbitrage Structural Vector Auto-Regression, which we believe could be useful for exploring a number of intricate issues in macro-economics and finance. We have shown that it is necessary to impose the no-arbitrage restrictions in order to maintain the structural identification of the system, while exploiting the richness of the information contained in the cross-section of bond prices.

A number of important extensions are still missing from this draft. First, we did not distinguish between the two sources of changes in the deficit: Spending and taxes need not have symmetric effects on interest rates. Second, we did not take into account the impact of the stock of government debt. Finally, it will be interesting to extend the dynamic specification beyond one lag, and to explore systematically the interactions between the financial variables and the real variables.

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Appendices

A Bond Pricing

Our no-arbitrage term structure model is specified by the following pricing kernel

$$\begin{aligned} M_{t+1} &= \exp\left(-r_t - \frac{\Lambda'_t \Lambda_t}{2} - \Lambda'_t \epsilon_{t+1}\right), \\ r_t &= \delta_0 + \delta'_Y Y_t, \quad \Lambda_t = \Lambda_0 + \Lambda_Y Y_t, \quad Y_t = \mu + \Phi Y_{t-1} + \Sigma \epsilon_t. \end{aligned}$$

where the structural shocks ϵ are assumed to be conditionally independent and jointly normally distributed.

By definition, the price of a zero-coupon bond with n periods to maturity satisfies

$$P_t^n = E_t [M_{t+1} \times M_{t+2} \times \dots \times M_{t+n} \times 1] = E_t [M_{t+1} \times P_{t+1}^{n-1}], \quad (13)$$

where the last equality follows from the law of iterated expectations. This pricing equation is solved by conjecturing that

$$P_t^n = \exp(A_n + B'_n Y_t),$$

where the "price loadings" A_n and B_n satisfy the following recursion: for $n \geq 2$,

$$\begin{aligned} A_n &= -\delta_0 + A_{n-1} + B'_{n-1}(\mu - \Sigma \Lambda_0) + \frac{1}{2} B'_{n-1} \Sigma \Sigma' B_{n-1}, \\ B_n &= -\delta_Y + (\Phi - \Sigma \Lambda_Y)' B_{n-1}, \end{aligned}$$

with initial conditions $A_1 = -\delta_0$ and $B_1 = -\delta_Y$.

It follows that the zero-coupon bond yields are given by equation (7), with $a_n = -\frac{A_n}{n}$ and $b_n = -\frac{B_n}{n}$, for $\forall n \geq 1$.

B Likelihood Function

Let β be the vector of free parameters in our model. To estimate β by maximum likelihood method, we need to construct the likelihood function. To this end, let

$$X_t = (r_t^{6m} \quad d_t \quad \pi_t \quad h_t) ', \quad \text{and} \quad Z_t = (r_t^{5y} \quad r_t^{10y}) ',$$

be, respectively, the vector of observed variables measured without error and the vector of observed yields measured with error.

The log likelihood function is given by

$$\log \mathcal{L}(\beta \mid X_t, Z_t : 1 \leq t \leq T) = \frac{1}{T-1} \sum_{t=1}^{T-1} f(X_{t+1}, Z_{t+1} \mid I_t, \beta),$$

where $I_t \equiv \{X_{t-i}, Z_{t-i}\}_{i \geq 0}$ is the information set at t , and

$$f(X_{t+1}, Z_{t+1} | I_t, \beta) = f(X_{t+1} | Z_{t+1}, I_t, \beta) \times f(Z_{t+1} | I_t, \beta),$$

the joint conditional density of X and Z . Under our Markovian structure, and the assumption that the measurement errors are uninformative about the state of the economy, we have

$$f(X_{t+1} | Z_{t+1}, I_t, \beta) = f(X_{t+1} | X_t, \beta) = \left| \frac{\partial X_{t+1}}{\partial \hat{Y}_{t+1}} \right|^{-1} \times f(\hat{Y}_{t+1} | \hat{Y}_t, \beta),$$

where \hat{Y}_t is the vector of *implied state variables*, given by

$$\hat{Y}_t = \begin{bmatrix} b'_{6m} \\ 0_{3 \times 0} & I_{3 \times 3} \end{bmatrix}^{-1} \times (X_t - a_{6m}).$$

Furthermore,

$$f(Z_{t+1} | I_t, \beta) = f(Z_{t+1} | Z_t, X_t, \beta) = f(\eta_{t+1} | \eta_t, \beta),$$

where $\eta_t \equiv Z_t - \hat{Z}_t$ is the vector of measurement errors, and $\hat{Z}_t = a_Z + b'_Z \hat{Y}_t$ is the vector of model prices.

Collecting all the intermediate results, we obtain the joint conditional density of the observed yields:

$$f(X_{t+1}, Z_{t+1} | I_t, \beta) = [b^q_{6m}]^{-1} \times \mathcal{N}(\hat{\epsilon}_{t+1}) \times f(\eta_{t+1} | \eta_t, \beta),$$

where $\mathcal{N}(\cdot)$ is the density function for a 4×1 vector of independent normal variates, and $\hat{\epsilon}_{t+1}$ is the 4×1 vector of structural shocks implied by the model:

$$\hat{\epsilon}_{t+1} \equiv \Sigma^{-1} \left(\hat{Y}_{t+1} - \mu - \Phi \hat{Y}_t \right).$$

Table 2: Summary Statistics. Sample Period: 1971:1 to 2003:3

	Mean	Std. Dev.	Min	Max
	Levels			
Government Deficit	0.0188	0.0193	-0.0279	0.0633
Realized Inflation	0.0408	0.0260	0.0065	0.1180
Forecasted Inflation	0.0409	0.0207	0.0130	0.0971
Help Wanted Index	0.7635	0.1722	0.3633	1.0367
3 Month Treasury Bill Rate	0.0598	0.0260	0.0089	0.1340
6 Month Zero-coupon Rate	0.0672	0.0294	0.0100	0.1594
5 Year Zero-coupon Rate	0.0752	0.0247	0.0250	0.1531
10 Year Zero-coupon Rate	0.0777	0.0224	0.0376	0.1486
	First Differences			
Government Deficit	0.0004	0.0068	-0.0282	0.0337
Realized Inflation	-0.0003	0.0128	-0.0389	0.0377
Forecasted Inflation	-0.0001	0.0053	-0.0225	0.0132
Help Wanted Index	-0.0029	0.0429	-0.1633	0.0833
3 Month Treasury Bill Rate	-0.0005	0.0104	-0.0573	0.0325
6 Month Zero-coupon Rate	-0.0005	0.0123	-0.0740	0.0415
5 Year Zero-coupon Rate	-0.0004	0.0081	-0.0309	0.0251
10 Year Zero-coupon Rate	-0.0003	0.0065	-0.0211	0.0181

Notes: Realized inflation is log growth rate of GDP deflator. Forecasted inflation is from the Survey of Professional Forecasters. Index of Help Wanted Advertising in Newspapers is from the Conference Board (normalized to 1 in 1987). 3 month T-Bill is secondary market rate from FRED. Zero-coupon rates were provided by Mark Fisher from the Federal Reserve Board.

Table 3: Correlation Matrix for Quarterly Changes

Observed Variables

Government Deficit	1							
Realized Inflation	-0.2679	1						
Forecasted Inflation	-0.1512	0.115	1					
Help Wanted Index	-0.4716	0.1117	-0.0228	1				
3 Month Treasury Bill Rate	-0.2429	-0.0132	-0.0006	0.435	1			
6 Month Zero-coupon Rate	-0.2649	-0.0133	-0.0032	0.4512	0.9285	1		
5 Year Zero-coupon Rate	-0.2324	0.0182	0.0221	0.3425	0.703	0.8434	1	
10 Year Zero-coupon Rate	-0.2336	0.0509	0.0576	0.3055	0.5805	0.7165	0.9434	1

Model-Implied Structural Shocks

Structural Shocks	Correlation Matrix				First-order Autocorrelation
Monetary	1				-0.2199
Deficit	-0.0772	1			-0.2878
Inflation	-0.0025	-0.0034	1		-0.0644
Help Wanted	0.0013	0.0025	0.0024	1	0.5814

Table 4: MLE Estimates

19.8341 (46)	Estimates				t-ratio			
δ_0	-0.07068				-0.00063			
δ_Y	1	-	0.84601	0.13212	0.20631 0.00125			
κ	0.84182	-0.00412	-0.01067	0.00004	0.00178	-0.00002	-0.00009	-
	0.11383	0.93427	-0.00139	-0.00356	0.00065	1.34343	-0.00177	-0.00008
	0.06187	-0.00501	0.95739	0.00571	0.00065	-0.01345	2.26773	0.00022
	-1.14649	0.52097	0.24096	0.95171	-0.00065	0.08296	0.03533	0.00201
θ	-	0.02418	0.04040	0.77499	1.05521 0.93714 3.92029			
Σ	0.02092	-	-0.00525	-0.00026	0.00261		-0.00216	-0.00005
	-	0.00879	-	-0.00891	0.00030		-	-0.00031
	-	-	0.01042	-0.00126	-		0.02110	-0.00031
	-	0.02839	0.00325	0.06707	0.00013		0.00009	0.00073
Λ_0	0.71630	-1.03040	1.42290	-6.27080	-	-	-	-0.00003
Λ_Y	32.44582	-	-	0.37318	0.00003		-	0.00008
	-104.89489	-	-	-2.02691	-0.00005		-	-0.00014
	-14.85669	-	-	-0.17093	-0.00002		-	-0.00003
	-165.65233	-	-	3.15244	-0.00010		-	0.00021
Ω	60.20218				4.24781			
	70.47455				4.56967			

Table 5: Standard Errors

Fixed Market Prices of Risk									
19.7659 (34)	Estimates				t-ratio				
δ_0	-0.07068				-1.02875				
δ_Y	1	-	0.84601	0.13212	2.22959 1.73504				
κ	0.84182	-0.00412	-0.01067	0.00004	1.93652	-0.01910	-0.08027	0.00220	
	0.11383	0.93427	-0.00139	-0.00356	0.41239	5.20055	-0.00575	-0.07756	
	0.06187	-0.00501	0.95739	0.00571	0.47675	-0.03233	5.59452	0.22774	
	-1.14649	0.52097	0.24096	0.95171	-0.93870	0.54966	0.17758	2.94617	
θ	-	0.02418	0.04040	0.77499	1.30114 1.27795 4.95125				
Σ	0.02092	-	-0.00525	-0.00026	11.41584		-1.16668 -0.04696		
	-	0.00879	-	-0.00891	2.47086 -2.56070				
	-	-	0.01042	-0.00126	16.09178 -0.81281				
	-	0.02839	0.00325	0.06707	0.99902	0.30206	5.32977		
Ω	60.42349	(5-year)			5.25594				
	70.51659	(10-year)			5.32875				
Fixed Dynamics									
19.7659 (10)	Estimates				t-ratio				
Λ_0	0.71630	-1.03040	1.42290	-6.27080	0.00038	-0.00250	0.00018	-0.00625	
Λ_Y	32.44582	-	-	0.37318	0.00805 0.00400				
	-104.89489	-	-	-2.02691	-0.02995 -0.02677				
	-14.85669	-	-	-0.17093	-0.00142 -0.00064				
	-165.65233	-	-	3.15244	-0.04217 0.03412				

Figure 1: Goodness-of-fit for the Term Structure

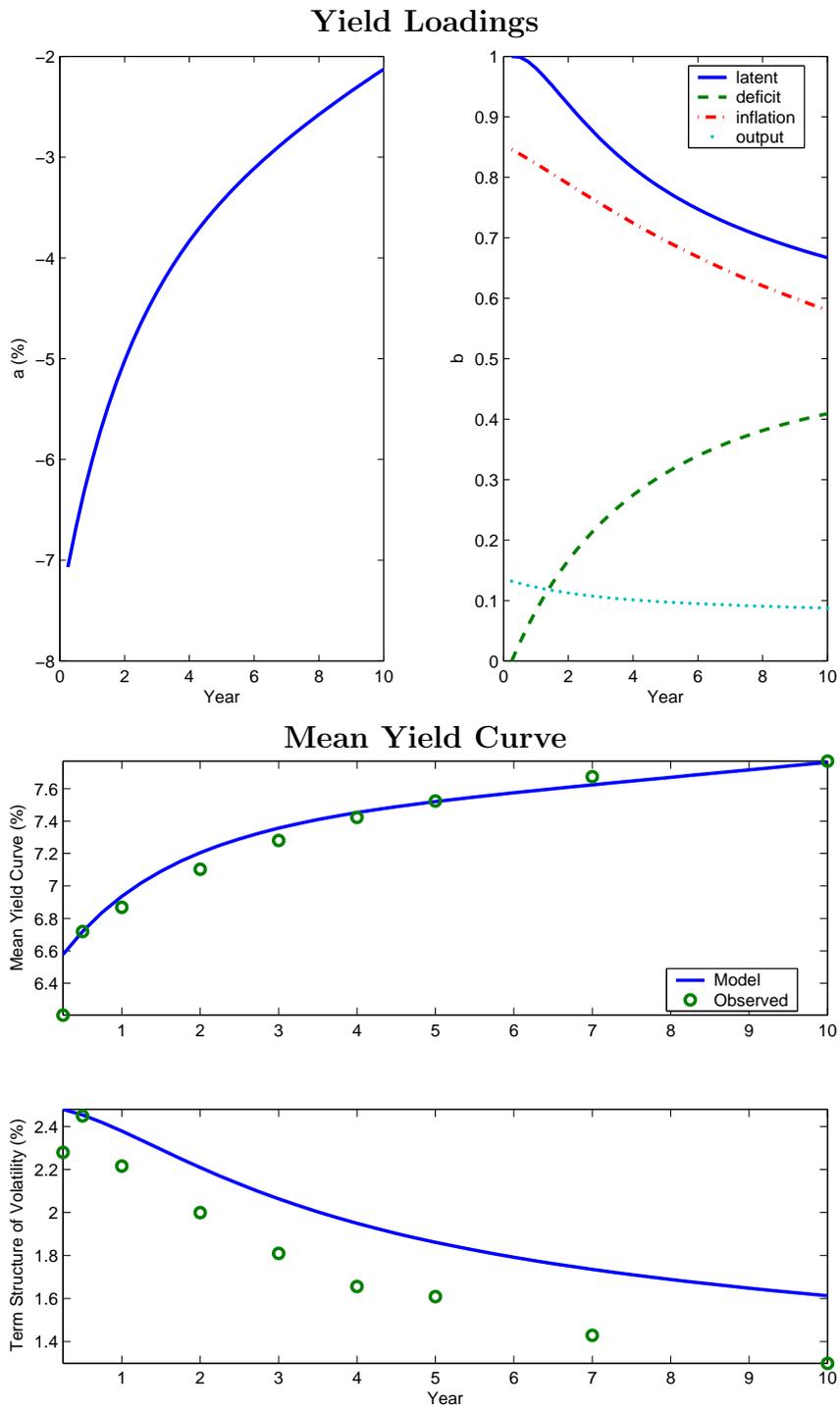
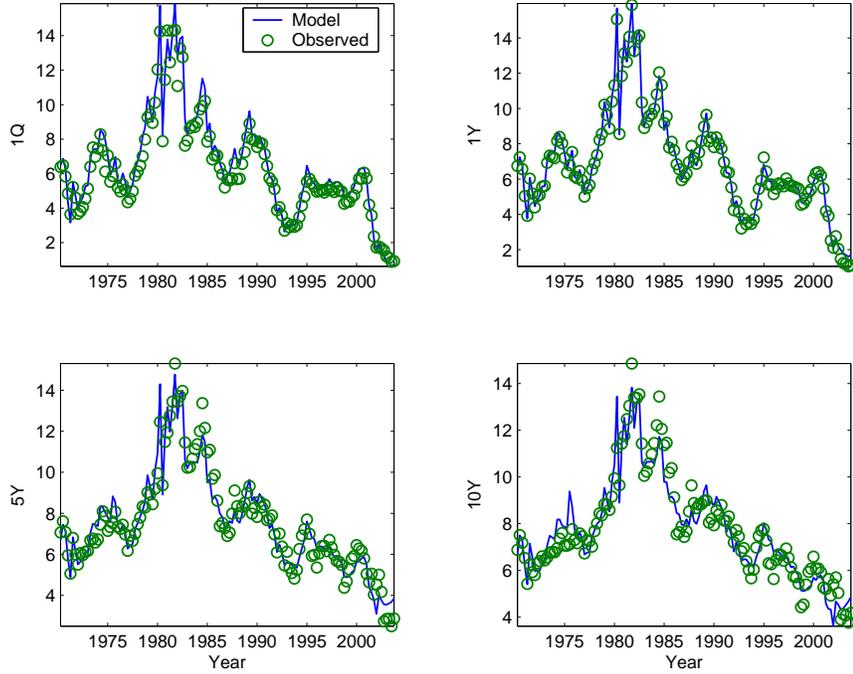


Figure 2: Interpretation of the Latent Factor

Time-Series of Fitted Yields



Latent Shocks vs. Romer-Romer Shocks

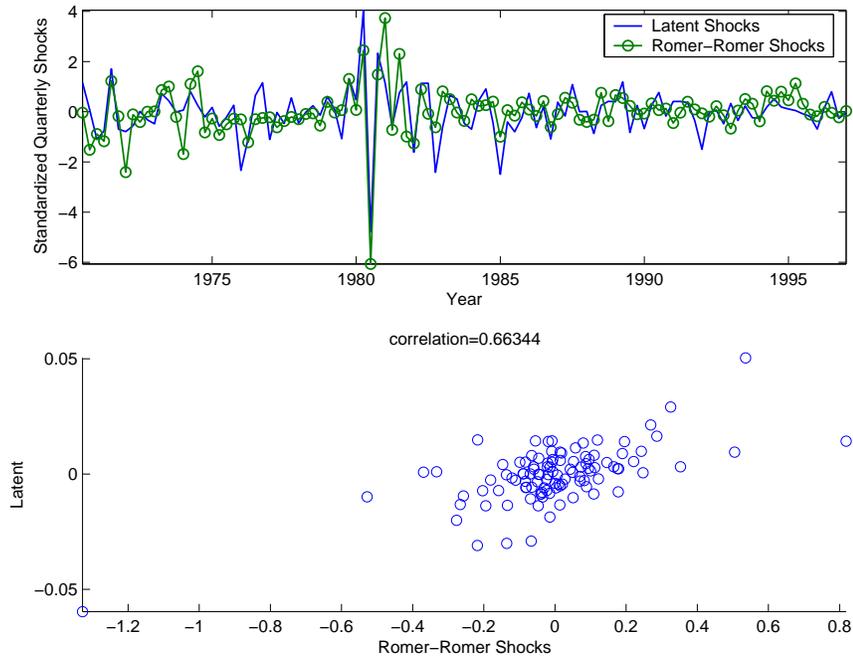
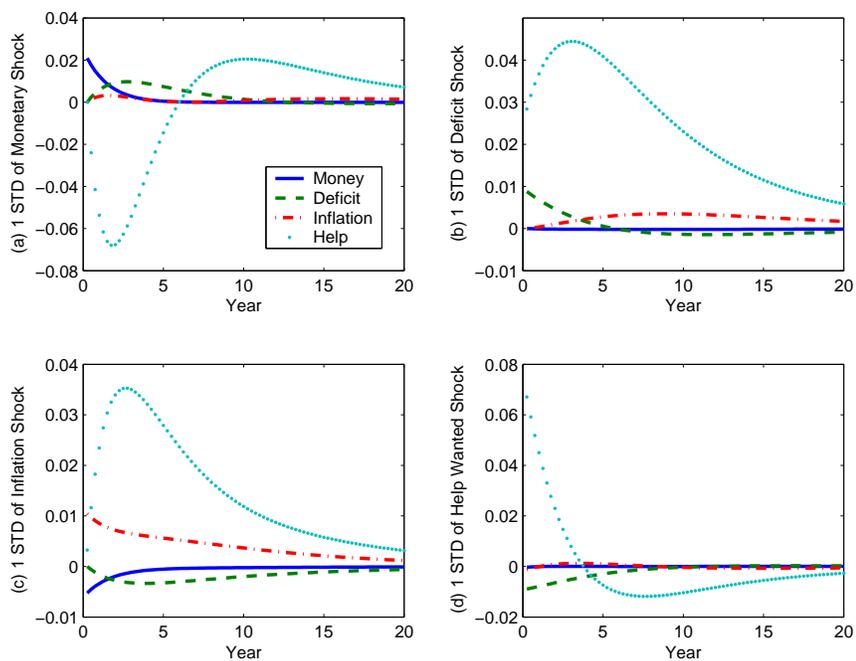


Figure 3: Impulse Response Functions to Structural Shocks

Impulse Response of Factors



Impulse Response of Yields

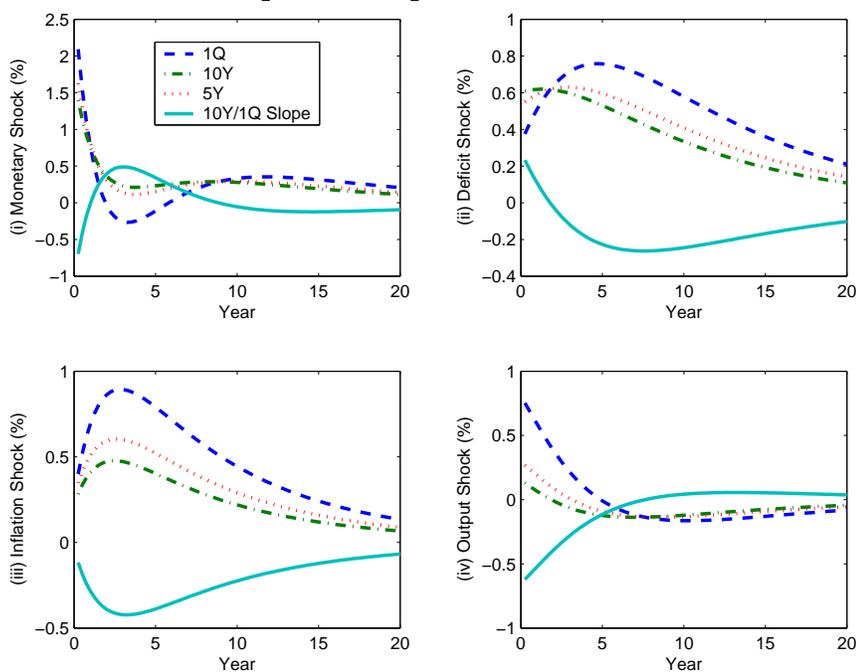
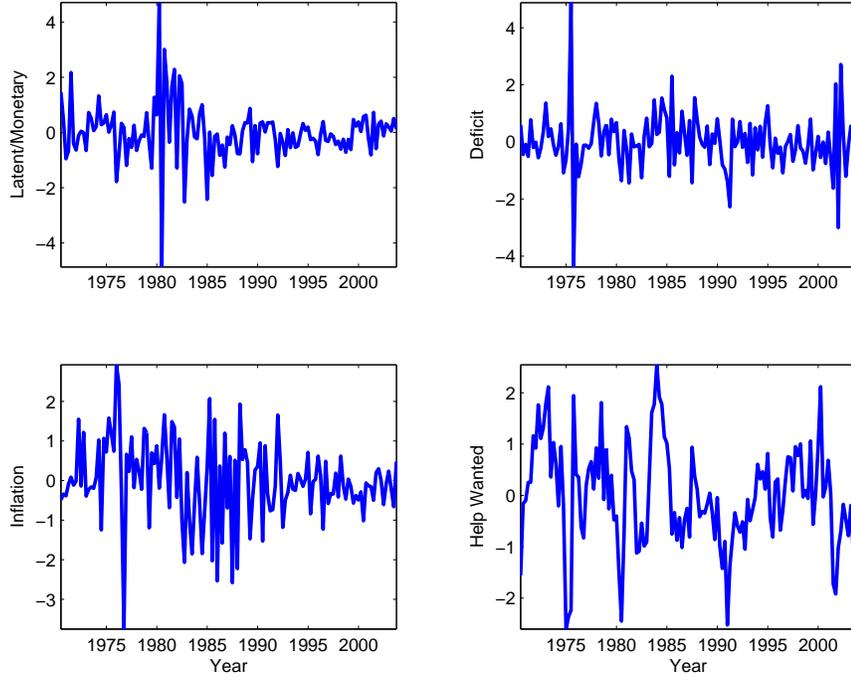


Figure 4: Model-Implied Structural Shocks



The model-implied structural shocks are given by

$$\hat{\epsilon}_{t+1} = \Sigma^{-1} \left[\hat{Y}_{t+1} - \mu - \Phi \hat{Y}_t \right], \quad \hat{Y}_t = \left(\hat{q}_t \quad d_t \quad \pi_t \quad h_t \right)',$$

$$\hat{q}_t = \left(r_t^{6m} - a_{6m} - b_{6m}^d d_t - b_{6m}^\pi \pi_t - b_{6m}^h h_t \right) / b_{6m}^q.$$