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FIRMS AND THE NEUTRALITY OF MONEY**

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RATIONAL INATTENTION, MULTI - PRODUCT FIRMS AND THE NEUTRALITY OF MONEY

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Abstract

I augment the rational inattention model of price-setting (in which firms have a limited capacity to process information) to allow firms to produce multiple goods. My main contribution is to highlight the economies of scale in the use of information that arise in this context, which firms exploit by acquiring information of aggregate variables: Aggregate information is useful for pricing all goods; idiosyncratic information is only useful for pricing goods it is concerned with. The model quantitatively predicts average price changes consistent with the data for the U.S., low costs for firms due to the friction, and comovement of prices inside firms. Importantly, the economies of scale in the use of information cut by four the capacity of the model to deliver money non-neutrality when firms produce two goods instead of one. Money becomes almost neutral when firms produce five goods or more.

Resumen

Este trabajo extiende el modelo de fijación de precios cuando las firmas tienen una capacidad limitada de procesar información (inatención racional) a un contexto en el que las firmas producen múltiples bienes. El principal resultado es que existen economías de escala en el uso de información que las firmas explotan adquiriendo información sobre variables agregadas. Esto es así porque la información agregada es útil para fijar los precios de todos los bienes que la firma produce; la información idiosincrásica sólo es útil para fijar los precios de los bienes para los cuales dicha información es contingente. Se muestra cuantitativamente que el modelo predice cambios de precio coherentes con los datos para Estados Unidos, bajos costos de la fricción para las firmas, y correlación positiva de precios fijados por una misma firma. Cabe destacar que las economías de escala en el uso de información implican que la capacidad del modelo de generar efectos reales del dinero cae a la cuarta parte cuando las firmas producen dos bienes en vez de uno. El modelo predice neutralidad casi perfecta cuando las firmas producen cinco bienes o más.

I thank comments by Roland Benabou, Christian Hellwig, Francesco Lippi, Jean Tirole and seminar participants at the Central Bank of Chile, Central European University, Ente Einaudi, PUC-Chile, Toulouse and UChile-Econ. I also thank editorial advice by Consuelo Edwards. The views expressed herein are those of the author and do not necessarily reflect the position of the Central Bank of Chile. Errors and omissions are my own. E mail: epasten@bcentral.cl

1 Introduction

A standard assumption in macroeconomics is that pricing decisions are taken by single-product firms, but this assumption is clearly a simplification without empirical support. The empirical counterpart of firms in the model may stir some debate, be they productive firms or stores; in either case evidence against this assumption is strong. For productive firms, 98.5% of goods listed in the Production Price Index (PPI) in the U.S. are produced by multi-product firms with an average of about 5 goods per firm (Bhattarai and Schoenle, 2011).¹ For stores, the Food Marketing Institute reports that its members' stores sold an average of almost 40,000 different products in 2010.²

Relaxing the single-product assumption serves two main purposes. First, to check the robustness of aggregate implications of price rigidities. Second, to explore the capacity of price rigidities to deliver synchronization of price changes inside firms—an empirical fact documented by Lach and Tsiddon (1996), Fisher and Konieczny (2000) and Bhattarai and Schoenle (2011).

In this spirit, this paper introduces multi-product firms into the rational inattention model. Postulated by Sims (1998, 2003), this theory suggests that firms facing a limited capacity to process information may choose to allocate their "attention" away from monetary conditions when setting their prices. As a result, rational inattention is a source of money non-neutrality alternative to menu costs. Mackowiak and Wiederholt (2009) show quantitatively that this theory predicts average price changes consistent with the data together with small losses for firms due to the friction and large real effects of money in the aggregate.³

In this paper I redo Mackowiak and Wiederholt's exercise after augmenting their model to multi-product firms. My main contribution is to highlight the economies of scale in the use of information arising when firms have a limited information capacity. I mainly focus on the aggregate implications of these economies of scale by showing that the ability of the rational inattention model to quantitatively deliver money non-neutrality is severely undermined when firms produce multiple goods. I also show that these economies of scale imply that the model is qualitatively consistent with the synchronization of price changes inside firms. I use this feature to justify the key assumption behind my aggregate result.

To start, think on a multi-product, rationally inattentive firm. The firm must acquire aggregate and idiosyncratic information to set its prices, e.g. about nominal aggregate demand and specific

¹These are conservative estimates since not all goods firms produce are listed in the PPI.

²http://www.fmi.org/facts_figs/?fuseaction=superfact. The FMI's members are 1,500 food retailers and wholesalers in the U.S. accounting for 26,000 stores, 14,000 pharmacies and three-quarters of all retail food sales (<http://www.fmi.org/about/>).

³Small losses is an important result. It means that firms have little incentive to invest on increasing their information capacity. Thus, the friction remains active over time.

demand and costs for each good it produces. Economies of scale in the use of information arise because aggregate information is useful for the pricing decisions of all goods the firm produces while idiosyncratic information is only useful for the good it is concerned with. Besides, the cost of acquiring information in terms of information capacity is invariant to the number of decisions for which this information is used. Hence, the more goods the firm produces, the more incentives the firm has to acquire aggregate information. The overall effect of this force must be balanced with other two forces. The more goods the firm produces, the more sources of idiosyncratic information the firm must pay attention to. In addition, the information capacity of the firm may also depend on the size of the firm as measured by the number of goods it produces.

Using a special case in which all underlying variables are white noise, I show analytically that firms' attention to aggregate information is increasing in the number of goods they produce relative to the attention paid to idiosyncratic information. The more attention to the aggregate, the lesser the real effects of a money shock. Strategic complementarity in pricing decisions amplifies this effect. Moreover, since aggregate shocks are likely less volatile than good-specific shocks, shifting attention from idiosyncratic to aggregate information increases firms' expected losses per good due to the mispricing induced by the friction. Increasing firms' information capacity reduces such losses by allowing firms to pay more attention to idiosyncratic information. But firms also pay more attention to aggregate information, further reducing the money non-neutrality in the model.

When I numerically solve the model for a general specification of underlying variables, the model still predicts average price changes consistent with the data (9.7% excluding sales, according to Klenow and Kyvtsov, 2008) and small losses per good due to the friction. However, the real effects of money are cut by four and last for a third of the time length when firms produce two goods instead of one. Money becomes almost neutral when firms produce five goods or more.⁴ Alternative calibrations do not change this result. More weight to idiosyncratic variables in firms' profits increases money non-neutrality, but also increases average price changes and firms' losses due to the friction. The same occurs when idiosyncratic variables are assumed more volatile.

A key assumption in the analysis is that idiosyncratic variables are good-specific, not firm-specific.⁵ Here is where the prediction of the model regarding the synchronization of price changes plays an important role. Although the model predicts comovement instead of synchronization,⁶ if only aggregate and firm-specific information are relevant for pricing decisions, prices would perfectly comove inside firms for any number of goods they produce, which is counterfactual. I extend the analysis to allow for aggregate, firm-specific and good-specific information. Now even

⁴These are conservative estimates. If the calibration would target on keeping total firms' losses due to the friction in the same order of magnitude as those of single-product firms, my results were strongly reinforced.

⁵If they are firm-specific, the allocation of attention is invariant to the number of goods firms produce.

⁶Prices in the data are staggered while in the model they change every period, so the analysis remains qualitative.

more attention is taken away from good-specific information as the number of goods firms produce increases. Extra information capacity avoids the increase in firms' losses due to the friction, but some of this capacity is allocated to aggregate information. As a result, money non-neutrality is still sharply decreasing in the number of goods firms produce.

Literature review. The economies of scale in the use of information highlighted in this paper have not been stressed before in the fast-growing literature linking rational inattention and money non-neutrality, e.g. Sims (2006), Woodford (2009), Mackowiak and Wiederholt (2010), Matejka (2011) and Piacello and Wiederholt (2011). It is also unexplored in other applications of rational inattention, such as asset pricing (Peng and Xiong, 2006), portfolio choice (Huang and Liu, 2007; Mondria, 2010), rare disasters (Mackowiak and Wiederholt, 2011), consumption dynamics (Luo, 2008), home bias (Mondria and Wu, 2010), the current account (Luo, Nie and Young, 2010) and foundations for Logit models (Matejka and McKay, 2011). Besides, my work is complementary to the study of multi-production in the menu cost model as in Sheshinski and Weiss (1992), Alvarez and Lippi (2011), Bhattarai and Schoenle (2011) and Midrigan (2011). Despite being silent in an answer, this paper poses the question of the effects of multi-production on the observation cost model, e.g. Reis (2006). Finally, this paper stresses the difficulty of using rational inattention to motivate exogenous dispersed information among agents regarding aggregate variables as in Angeletos and La'O (2009), Lorenzoni (2009) and Albagli, Hellwig and Tsyvinski (2011).

Layout. Section 2 presents the multi-product, rational inattention model. Section 3 shows the frictionless case. Section 4 analytically solves the model when underlying variables are white noise. Section 5 illustrates the implications of multi-production. Section 6 introduces a special case of persistent underlying variables. Section 7 numerically solves the model for a general specification of underlying variables. Section 8 introduces firm-specific shocks to the white noise case of Section 4. Section 9 concludes. An appendix contains derivations absent in the main text.

2 A model of multi-product, rationally inattentive firms

This section introduces an augmented version of the rational inattention model of Mackowiak and Wiederholt (2009) that allows for multi-product firms.

Basic ingredients. An economy has a continuum of goods of measure one indexed by $j \in [0, 1]$ and a continuum of firms of measure $\frac{1}{N}$ indexed by $i \in [0, \frac{1}{N}]$. Each firm i produces an exogenous number N of goods randomly drawn from the pool of goods. Each good is produced by only one firm. Denote \aleph_i the countable set of N goods produced by firm i .

Each good j contributes to the profits of its producing firm according to

$$\pi (P_{jt}, P_t, Y_t, Z_{jt}), \quad (1)$$

where P_{jt} is the (fully flexible) price of good j , P_t is the aggregate price, Y_t is real aggregate demand, and Z_{jt} is an idiosyncratic variable, all at time t . The function $\pi(\cdot)$ is assumed independent of which and how many goods the firm produces, twice continuously differentiable and homogenous of degree zero in the first two arguments.

Idiosyncratic variables Z_{jt} are such that

$$\int_0^1 z_{jt} dj = 0, \quad (2)$$

where small case generically denotes log-deviations from steady-state levels.

Nominal aggregate demand Q_t is assumed exogenous and stochastic but satisfies

$$Q_t = P_t Y_t, \quad (3)$$

where

$$p_t = \int_0^1 p_{jt} dj. \quad (4)$$

The problem of the firm. The momentary profit function of firm i is

$$\sum_{n \in \mathbb{N}_i} \pi (P_{nt}, P_t, Y_t, Z_{nt}).$$

Denote s_{it} the vector of all signals that firm i receives. As in Sims (2003), firms face a capacity constraint in the "flow of information" that they can process at every period t ,

$$I(\{Q_t, Z_{it}\}; \{s_{it}\}) \leq \kappa(N).$$

Without loss of generality, I allow the firm's information capacity $\kappa(N)$ to depend on the number N of goods the firm produces. The function $I(\{T_t\}, \{O_t\})$ measures the information contained in a vector of observable variables O_t , called "signals", regarding a vector of target variables T_t . For instance, if T_t and O_t are Gaussian i.i.d. processes, then

$$I(\{T_t\}, \{O_t\}) = \frac{1}{2} \log_2 \left(\frac{1}{1 - \rho_{T,O}^2} \right).$$

In words, the higher the correlation between T_t and O_t , the higher the information flow. The vector of signals s_{it} may be partitioned into $N + 1$ subvectors

$$\{s_{ait}, \{s_{nt}\}_{n \in \mathbb{N}_i}\};$$

$\{q_t, s_{ait}\}, \{z_{nt}, s_{nt}\}_{n \in \mathbb{N}_i}$ are Gaussian and independent of each other, not necessarily i.i.d. This notation implicitly assumes that either the firm i only receives idiosyncratic information about the goods it produces or simply the firm discards useless information. In addition, assume

$$s_i^1 = \{s_{i-\infty}, \dots, s_{i1}\};$$

there exists an initial infinite history of signals such that all prices are stationary processes.

Therefore, the problem of the firm i may be represented as

$$\max_{\{s_{it}\} \in \Gamma} \mathbb{E}_{i0} \left[\sum_t \beta^t \left\{ \sum_{n \in \mathbb{N}_i} \pi(P_{nt}^*, P_t, Y_t, Z_{nt}) \right\} \right] \quad (5)$$

where

$$P_{nt}^* = \arg \max_{P_{nt}} \mathbb{E} [\pi(P_{nt}, P_t, Y_t, Z_{nt}) \mid s_{it}] \quad (6)$$

subject to

$$\begin{aligned} I(\{P_t, Y_t\}, \{s_{ait}\}) + \sum_{n \in \mathbb{N}_i} I(\{Z_{n\tau}\}, \{s_{nt}\}) &\leq \kappa(N) \\ \Leftrightarrow \kappa_a + \sum_{n \in \mathbb{N}_i} \kappa_n &\leq \kappa(N). \end{aligned} \quad (7)$$

The firm's pricing problem in (6) is static since prices are fully flexible. The firm, however, must consider its whole discounted expected stream of profits to decide how to allocate its information flow capacity, its "attention", among a set Γ of admissible signals. A signal must not contain information about future realizations of its target variable and must be Gaussian, jointly stationary and independent. If a firm chooses more precise signals about, for instance, $\{P_t, Y_t\}$, the information flow κ_a increases, reducing the information capacity to be allocated to other signals. A target $\{P_t, Y_t\}$ is equivalent to $\{Q_t\}$ since Q_t is the only source of aggregate uncertainty.

The pricing problem in (6) is independent of the number N of goods the firm produces. However, N enters the attention problem through three channels. First, the momentary objective in (5) sums up the contribution to profits of the N goods the firm produces. This is the source of the economies of scale in the use of information stressed in this paper. Second, the number of variables that the firm must pay attention to increases with N in the left-hand side of (7). Finally,

the capacity constraint $\kappa(N)$ in the right-hand side of (7) may or may not depend on N .

Equilibrium. An equilibrium is a collection of signals $\{s_{it}\}$, prices $\{P_{jt}\}$, the price level $\{P_t\}$ and real aggregate demand $\{Y_t\}$ such that:

1. Given $\{P_t\}$, $\{Y_t\}$ and $\{Z_{jt}\}_{j \in [0,1]}$, all firms $i \in [0, \frac{1}{N}]$ choose the stochastic process of signals $\{s_{it}\}$ at $t = 0$ and the price of goods they produce, $\{p_{nt}\}_{n \in \mathbb{N}_i}$ for $t \geq 1$.
2. $\{P_t\}$ and $\{Y_t\}$ are consistent with (3) and (4) for $t \geq 1$.

3 The frictionless case

Assume that $\kappa(N) \rightarrow \infty$, so the firm is able to choose infinitely precise signals regarding all target variables. Hence, I only need to solve for (6).

To do so, I denote as \bar{Q} and $\bar{Z}_j = \bar{Z} \forall j$ the non-stochastic steady state level of these variables. The properties of $\pi(\cdot)$ imply

$$\pi_1(1, 1, Y_t, \bar{Z}) = 0,$$

which follows from the optimality of prices. This equation solves for the steady-state level of real aggregate demand \bar{Y} , and (3) for the steady-state aggregate price level $\bar{P} = \bar{Q}/\bar{Y}$.

A second-order approximation of the problem of firm i around the steady-state is

$$\max_{\{p_{nt}\}_{n \in \mathbb{N}_i}} \sum_{n \in \mathbb{N}_i} \left\{ \hat{\pi}_1 p_{nt} + \frac{\hat{\pi}_{11}}{2} p_{nt}^2 + \hat{\pi}_{12} p_{nt} p_t + \hat{\pi}_{13} p_{nt} y_t + \hat{\pi}_{14} p_{nt} z_{nt} \right\} + \text{terms independent of } p_{nt}.$$

with $\hat{\pi}_1 = 0$, $\hat{\pi}_{11} < 0$, $\hat{\pi}_{12} = -\hat{\pi}_{11}$ and all parameters identical for all goods and all firms. Hence, the optimal pricing rule for each good $j \in [0, 1]$ is

$$p_{jt}^\diamond = p_t + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} y_t + \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} z_{jt} \equiv \Delta_t + \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} z_{jt}. \quad (8)$$

Using equations (2), (4) and $y_t = q_t - p_t$, this rule implies neutrality of money:

$$p_t^\diamond = q_t.$$

Importantly, the multi-production assumption is so far innocuous.

4 The white noise case

Assume now that $\kappa(N)$ is finite. Further, assume that q_t and z_{jt} —the log deviation from the steady-state of nominal aggregate demand and the idiosyncratic component of each good’s profits—follow white noise processes, respectively with variances σ_q^2 and $\sigma_j^2 = \sigma_z^2$ for $j \in [0, 1]$. This case has full analytic solution which I use in section 5 to illustrate the implications of multi-production.

From (8), the optimal price p_{jt}^* that solves (6) is

$$p_{jt}^* = \mathbb{E}[\Delta_t | s_{ait}] + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} \mathbb{E}[z_{jt} | s_{jt}], \quad (9)$$

where i is an arbitrary firm that produces good j , i.e., $j \in \aleph_i$.

The second-order approximation for the expected loss in profits of good j due to the friction is

$$\widetilde{\pi}(p_{jt}^\diamond, p_t, y_t, z_{jt}) - \widetilde{\pi}(p_{jt}^*, p_t, y_t, z_{jt}) = \frac{|\widehat{\pi}_{11}|}{2} (p_{jt}^\diamond - p_{jt}^*)^2. \quad (10)$$

To solve the allocation of attention problem, let $p_t = \alpha q_t$, so

$$\Delta_t = \left[\alpha + \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} (1 - \alpha) \right] q_t. \quad (11)$$

In addition, signals chosen by firm $i \in [0, \frac{1}{N}]$ are restricted to have the structure

$$\begin{aligned} s_{ait} &= \Delta_t + \varepsilon_{it}, \\ s_{nt} &= z_{nt} + \psi_{nt}, \end{aligned}$$

where $n \in \aleph_i$, and σ_ε^2 and $\sigma_{\psi_n}^2$ are the variance of ε_{it} and ψ_{nt} .⁷ Note that, since I am now characterizing the problem of the firm, I change the good subindex from j to n .

Given that q_t and z_{jt} for $j \in [0, 1]$ are white noise, the constraint in (7) becomes

$$\begin{aligned} \frac{1}{2} \log_2 \left(\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2} + 1 \right) + \frac{1}{2} \sum_{n \in \aleph_i} \log_2 \left(\frac{\sigma_z^2}{\sigma_{\psi_n}^2} + 1 \right) &\leq \kappa(N) \\ \kappa_a + \sum_{n \in \aleph_i} \kappa_n &\leq \kappa(N) \end{aligned} \quad (12)$$

⁷Mackowiak and Wiederholt (2009) show that this structure of signals is optimal. This result is not affected by the multi-production assumption.

The optimal pricing rule in (9) for a good $n \in \mathfrak{N}_i$ may be rewritten as

$$\begin{aligned} p_{nt}^* &= \frac{\sigma_\Delta^2}{\sigma_\Delta^2 + \sigma_\varepsilon^2} s_{ait} + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} \frac{\sigma_z^2}{\sigma_z^2 + \sigma_{\psi n}^2} s_{nt} \\ &= (1 - 2^{-2\kappa_a}) (\Delta_t + \varepsilon_{it}) + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} (1 - 2^{-2\kappa_n}) (z_{nt} + \psi_{nt}). \end{aligned} \quad (13)$$

Therefore, the problem of a firm i may be reduced to

$$\begin{aligned} &\min_{\kappa_a, \{\kappa_n\}_{n \in \mathfrak{N}_i}} \sum_{n \in \mathfrak{N}_i} \left\{ \sum_{t=1}^{\infty} \beta^t \frac{|\widehat{\pi}_{11}|}{2} \mathbb{E} \left[(p_{nt}^\diamond - p_{nt}^*)^2 \right] \right\} \\ \rightarrow &\min_{\kappa_a, \{\kappa_n\}_{n \in \mathfrak{N}_i}} \frac{\beta}{1 - \beta} \frac{|\widehat{\pi}_{11}|}{2} \left[2^{-2\kappa_a} \sigma_\Delta^2 N + \left(\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} \right)^2 \sum_{n \in \mathfrak{N}_i} 2^{-2\kappa_n} \sigma_z^2 \right] \end{aligned}$$

subject to

$$\kappa_a + \sum_{n \in \mathfrak{N}_i} \kappa_n \leq \kappa(N).$$

which delivers the optimality condition

$$\kappa_a^* = \kappa_n^* + \log_2 \left(x \sqrt{N} \right), \quad \forall n \in \mathfrak{N}_i \quad (14)$$

for $x \equiv \frac{|\widehat{\pi}_{11}| \sigma_\Delta}{\widehat{\pi}_{14} \sigma_z}$. This condition implies that all κ_n are identical, $\kappa_n^* = \kappa_z^*$ for $n \in \mathfrak{N}_i, \forall i$. In words, all firms choose the same variance for the noise of idiosyncratic signals they are concerned with, $\sigma_{\psi n} = \sigma_\psi$ for $n \in \mathfrak{N}_i, \forall i$. This condition in (14) along the information constraint implies

$$\kappa_a^* = \begin{cases} \kappa(N) & \text{if } x \geq \frac{2^{\kappa(N)}}{\sqrt{N}}, \\ \frac{1}{N+1} \kappa(N) + \frac{N}{(N+1)} \log_2 \left(x \sqrt{N} \right) & \text{if } x \in \left(\frac{2^{-\kappa(N)/N}}{\sqrt{N}}, \frac{2^{\kappa(N)}}{\sqrt{N}} \right), \\ 0 & \text{if } x \leq \frac{2^{-\kappa(N)/N}}{\sqrt{N}}. \end{cases} \quad (15)$$

All firms are identical, so the price of any good $j \in [0, 1]$ follows

$$p_{jt}^* = (1 - 2^{-2\kappa_a^*}) (\Delta_t + \varepsilon_{it}) + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} (1 - 2^{-2\kappa_z^*}) (z_{jt} + \psi_{jt}) \quad (16)$$

where ε_{it} is the realization of the noise of the aggregate signal for the firm i that produces the good j . Aggregating among all goods and all firms,

$$p_t^* = (1 - 2^{-2\kappa_a^*}) \Delta_t = (1 - 2^{-2\kappa_a^*}) \left[\alpha + \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} (1 - \alpha) \right] q_t$$

which confirms the guess $p_t^* = \alpha q_t$,

$$\alpha = \frac{(2^{2\kappa_a^*} - 1) \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}}{1 + (2^{2\kappa_a^*} - 1) \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}}. \quad (17)$$

Summing up, a binding information constraint implies, in contrast to the frictionless case, that an innovation q_t in nominal aggregate demand—for instance, a monetary shock—has real effects. This result holds for any finite $\kappa(N)$; in fact, $\alpha \rightarrow 1$ when $\kappa(N) \rightarrow \infty$. Moreover, the more attention the firm pays to idiosyncratic information, the lower is κ_a^* , so real effects of money are stronger. Similarly, a higher complementarity in pricing decisions, a smaller $\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} > 0$, also leads to stronger money non-neutrality. Equation (16) collapses to that found by Mackowiak and Wiederholt (2009) when $N = 1$. The next section explores the implications of allowing for $N > 1$.

5 Multi-product firms and rational inattention

I study now the white noise case solved above to illustrate the implications of multi-product firms on the predictions of the rational inattention model.

5.1 Comparative statics with respect to N

Allocation of attention. Let for now focus on the optimality condition in (14) and treat x as an exogenous parameter. I start the analysis by assuming that $\kappa(N) = \kappa$. Equation (14) implies that the difference between the allocation of attention to aggregate information κ_a^* with respect to idiosyncratic information κ_z^* is increasing in the number N of goods the firm produces,

$$\kappa_a^* - \kappa_z^* = \log_2 \left(x\sqrt{N} \right).$$

This result is a direct consequence of the increasing importance of aggregate variables in firms' total profits as N increases. Given an interior solution in (15), this force implies that κ_a^* increases with N unless κ_z^* decreases by larger magnitude. This is exactly what happens when N is below a given threshold. To see why, note that, after imposing the symmetry of attention to idiosyncratic signals, the information flow constraint in (12) becomes

$$\kappa_a^* + N\kappa_z^* = \kappa,$$

which captures the second key force that the multi-product firms' assumption introduces into the

attention problem: The more goods the firm produces, the more sources of information the firm must pay attention to. From the interior solution of (15) I obtain

$$\frac{\partial \kappa_a^*}{\partial N} = \frac{\log(x\sqrt{N}) - \kappa \log 2 + \frac{1}{2}(N+1)}{(N+1)^2 \log 2},$$

such that $\frac{\partial \kappa_a^*}{\partial N} < 0$ for $N < \hat{N}$, where \hat{N} solves $\log \hat{N} + \hat{N} = 2\kappa \log 2 - 2 \log x - 1$.

$N \in \mathbb{N}$ and $\log 2 < 0$, so $\hat{N} > 0$ only if $x > 0$ is small enough. Since $x \equiv \frac{|\hat{\pi}_{11}| \sigma_\Delta}{\hat{\pi}_{14} \sigma_z}$, a multi-product firm pays less attention to the aggregate than a single-product firm when $|\hat{\pi}_{11}| \sigma_\Delta$ is sufficiently small with respect to $\hat{\pi}_{14} \sigma_z$. This happens when the frictionless price in (8) is highly sensitive to idiosyncratic variables ($\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|}$ is high) and/or when such variables are highly volatile relative to the compound aggregate variable Δ_t ($\frac{\sigma_\Delta}{\sigma_z}$ is small). Otherwise, the prices of multi-product firms absorb a larger extent of an aggregate shock than those of single-product firms. In particular, a corner solution $\kappa_a^* = \kappa$ is reached when $\sqrt{N} \geq \frac{2^\kappa}{x}$.

In either case, with $\kappa(N) = \kappa$, the allocation of attention to idiosyncratic information κ_z^* is unambiguously decreasing in N since

$$\kappa_z^* = \frac{1}{N+1} \kappa - \frac{1}{(N+1)} \log_2(x\sqrt{N}).$$

Allowing $\kappa(N)$ to vary with N —specifically, to increase with N —does not affect qualitatively these results. The gap between κ_a^* and κ_z^* is still increasing in N according to (15). The marginal effect of an increase in N on κ_a^* is now

$$\frac{\partial \kappa_a^*}{\partial N} = \frac{\log(x\sqrt{N}) - \kappa \log 2 + \frac{1}{2}(N+1)}{(N+1)^2 \log 2} + \frac{\kappa'(N)}{N+1}, \quad (18)$$

which makes more restrictive the conditions in x such that $\frac{\partial \kappa_a^*}{\partial N} < 0$ for a given N . A corner solution $\kappa_a^* = \kappa$ is reached now when $\sqrt{N} \geq \frac{2^{\kappa(N)}}{x}$. Regarding κ_z^* , $\frac{\partial \kappa_z^*}{\partial N}$ becomes ambiguous, but, if positive, it is smaller than $\frac{\partial \kappa_a^*}{\partial N}$.

Fixed point and money neutrality. Consider now the endogeneity of x . Given that $\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} > 0$, α in (17) is increasing in κ_a^* . Hence, the reaction of aggregate prices to an innovation in q is increasing in κ_a^* , according to (9). In addition, from (11),

$$\sigma_\Delta^2 = \left[\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} + \left(1 - \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} \right) \alpha \right]^2 \sigma_q^2. \quad (19)$$

σ_{Δ}^2 is increasing in α if $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} < 1$, i.e., if firms' pricing decisions are strategic complements. As a result, σ_{Δ}^2 is increasing in κ_a^* . Taking into account this feedback effect, the equilibrium allocation of attention jointly solves equations (15) and (17).

Figure 1 draws these two equations in the space (α, κ_a) . The interior solution of (15) is drawn in red, while (17) is drawn in blue. In addition, $\kappa_a \in [0, \kappa(N)]$ and $\alpha \in [0, 1]$; dashed lines represent $\alpha = 1$ and $\kappa_a = \kappa(N)$. Equilibrium α is denoted as α_1^* .

Suppose now that N increases. Equation (17) is invariant in N but the intercept of (15) may decrease or increase while its slope is decreasing with N . The green line in Figure 1 depicts the case of a higher intercept of (15), so equilibrium α is now α_2^* . Importantly, both functions (15) and (17) are increasing in α , hence an interior solution is highly sensitive to changes in parameters, for instance in N . As a result, the effect of N on κ_a^* studied when x is treated as exogenous is amplified by an indirect effect of κ_a^* on σ_{Δ}^2 in the same direction.

Volatility of prices. Most macroeconomic studies modeling firms' pricing decisions focus on the average size of price changes for numerical exercises.⁸ In fact, I do the same in section 7 when I solve numerically the general model. However, for illustrative purposes, I focus now on the variance of individual prices—for which there is a closed form solution.

κ_a^* and κ_z^* pin down the variance of aggregate and idiosyncratic signals, σ_{ε}^2 and σ_{ψ}^2 , according to (12). Denoting σ_p^2 as the time variance of the individual prices, (16) implies

$$\sigma_p^2 = (1 - 2^{-2\kappa_a^*}) \sigma_{\Delta}^2 + \left(\frac{\widehat{\pi}_{14}}{\widehat{\pi}_{11}} \right)^2 (1 - 2^{-2\kappa_z^*}) \sigma_z^2. \quad (20)$$

which is increasing in κ_a^* , κ_z^* but also in σ_{Δ}^2 and σ_z^2 . Increasing N affects the volatility of prices since it affects κ_a^* , κ_z^* and σ_{Δ}^2 .

Comovement of prices inside firms. According to (16),

$$\text{corr}(p_{nt}^*, p_{-nt}^*) = (1 - 2^{-2\kappa_a^*}) \sigma_{\Delta}^2 / \sigma_p^2$$

for $n, -n \in \aleph_i$. Prices inside a firm comove because aggregate information is a common input for the pricing rule of all goods the firm produces. An important result for the subsequent analysis is that this comovement is increasing in κ_a^* , so it is also increasing in N if $\frac{\partial \kappa_a^*}{\partial N} > 0$.

Firms' expected losses. Another important dimension in the analysis is the per-good expected loss in profits each period that firms bear because of their limited information capacity. After imposing

⁸For instance, Golosov and Lucas (2007), Mackowiak and Wiederholt (2009) and Midrigan (2011).

$\kappa_n^* = \kappa_z^* \forall n \in \mathbb{N}_i$, the firms' per-good expected momentary loss due to the friction is

$$\frac{|\widehat{\pi}_{11}|}{2} \left[2^{-2\kappa_a^*} \sigma_\Delta^2 + \left(\frac{\widehat{\pi}_{14}}{\widehat{\pi}_{11}} \right)^2 2^{-2\kappa_z^*} \sigma_z^2 \right],$$

which is quickly decreasing in κ_a^* and κ_z^* .

5.2 A numerical illustration

I now conduct three exercises for the white noise case solved so far to further illustrate the effect of multi-production on the predictions of the rational inattention model. In all exercises I solve for the fixed point between equations (15) and (17).

Exercise 1: Baseline calibration. For numerical exercises, Mackowiak and Wiederholt (2009) choose

$$\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} = 1; \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} = .15; \kappa = 3; \sigma_q = 2.68\%.$$

$\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} = 1$ means that aggregate and idiosyncratic variables q_t and $\{z_{jt}\}_{j \in [0,1]}$ have the same weight in profits; $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} = .15$ implies strong strategic complementarity in pricing decisions; $\kappa = 3$ means that, if all attention is devoted to one target variable, the variance of the noise of the signal is $\frac{1}{63}$ of the variance of the target variable; and $\sigma_q = 2.68\%$ is obtained from U.S. quarterly data.

Solving the model for a general specification of q_t and $\{z_{jt}\}_{j \in [0,1]}$, Mackowiak and Wiederholt (2009) find that calibrating $\sigma_z = 11.8\sigma_q$ matches the average price observed in microlevel data. In this calibration, the attention to aggregate information is $\kappa_a^* = .19$, which implies that aggregate prices absorb on impact only 2.8% of an innovation in nominal aggregate demand q_t . They also find that firms suffer small expected momentary losses in profits per-good due to the friction.

I find, for the white noise case, that calibrating $\sigma_z = 11.8\sigma_q$ yields a corner solution $\kappa_a^* = 0$ and microlevel moments cannot be matched. I thus focus on matching the aggregate predictions of Mackowiak and Wiederholt (2009), which I obtain by calibrating $\sigma_z = 1.166\sigma_q$. The implied volatility of prices σ_p is 3.1% and firms' expected per-good loss due to the friction is 3.6×10^{-5} per period. I use these numbers as fictional targets for the exercises in this section.

Table 1 summarizes results when N is increased with $\kappa(N) = 3$. In particular, it reports results for

$$N = \{1, 2, 5, 10, 100, 1000\}.$$

This choice responds to the interpretation of N . If one thinks that productive firms are the relevant decision makers for price setting, one may be interested in a low N . Bhattacharai and Schoenle

(2011) report that most firms produce between 2 and 10 goods listed in the PPI with an average of about 5. This is a conservative estimate since firms may produce goods that are not listed in the PPI. In contrast, if one has in mind stores, a high N is informative. The FMI reports that, in average, its members' stores sold about 40,000 goods in 2010.⁹ Thus, $N = 100$ or 1000 are also conservative estimates, which may be interpreted as the number of clusters of goods with highly correlated idiosyncratic shocks within clusters but not so much between clusters.¹⁰

Table 1 shows that α increases five times when N goes from 1 to 2, from 2.8% for $N = 1$ to $\alpha = 15.5\%$ for $N = 2$. A corner solution with $\kappa_a^* = 3$ and $\alpha = 90.4\%$ is reached for $N \geq 100$. In words, money non-neutrality is greatly undermined for $N > 1$. In addition, the attention to idiosyncratic information falls quickly as N increases. The volatility of prices σ_p decreases with N , from 3.1% for $N = 1$ to 2.4% for $N \geq 100$. Consistently, firms' average momentary loss due to the friction quickly increases as the attention shifts with N to aggregate information.

Exercise 2: Using σ_z as free variable. As N increases, I keep all parameters identical to the exercise 1 but I use idiosyncratic volatility σ_z to match price volatility $\sigma_p = 3.1\%$ obtained in the exercise 1 for $N = 1$. Table 2 summarizes the results.

For $N = 2$, σ_p falls below 3.1% in the exercise 1; σ_z must increase to $1.26\sigma_q$ to restore $\sigma_p = 3.1\%$. The increase in σ_z shifts attention to idiosyncratic information, so κ_a^* and κ_z^* are respectively smaller and higher than in the exercise 1 for $N = 2$. As a result, real effects of money increase since α decreases to 10.9% with respect to the exercise 1 for $N = 2$, but it is still four times higher than $\alpha = 2.8\%$ when $N = 1$. Firms' momentary loss due to the friction remains similar to that of exercise 1 for $N = 2$, which is five times higher than when $N = 1$ (3.6×10^{-5}).

As N increases, the assumed volatility of idiosyncratic variables σ_z must increase quickly to keep $\sigma_p = 3.1\%$, reaching 19.9 times σ_q when $N = 1000$. This high volatility allows to keep reasonably high real effects of a money shock, with α about 20%. However, the little attention to idiosyncratic information and the high volatility of idiosyncratic variables deliver momentary losses quickly increasing in N . The average loss per good due to noise reaches implausible levels up to five orders of magnitude higher when $N = 1000$ than when $N = 1$.

Exercise 3: Using $\kappa(N)$ as free variable. I keep the target on $\sigma_p = 3.1\%$ for calibration as N increases, but I now use the information capacity constraint $\kappa(N)$ as free variable. I keep σ_z and all other parameters as in the exercise 1. Table 3 summarizes the results.

When $N = 2$, $\kappa(N) = 4$ matches $\sigma_p = 3.1\%$, so $\kappa_a^* = 1.1$ and $\alpha = 36.7\%$. Firms' loss due to the friction is similar to that obtained in exercise 1 for $N = 2$. However, prices now absorb a

⁹See footnote 1.

¹⁰I introduce aggregate, firm-specific and good-specific shocks in Section 8.

proportion twelve times higher of a nominal aggregate demand shock than when $N = 1$.

Similarly, firms' expected per-good losses stay in the same range as N increases. But since the attention to idiosyncratic information κ_z^* decreases quickly with N , $\kappa(N)$ must increase also quickly to match $\sigma_p = 3.1\%$. As a result, prices absorb 59% of a money shock for $N = 5$; 71% for $N = 10$; 94% for $N = 100$; and 99% for $N = 1000$.

Notice that firms' expected momentary losses per good due to the friction are still higher than when $N = 1$. If the calibration would target on losses, either σ_z should decrease or $\kappa(N)$ should increase even more with N . Both cases imply higher attention to aggregate information, so such exercise would enforce results obtained here.

Discussion. Exercise 1 has another two parameters where calibration may be arguable. One is $\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} = 15$. This choice implies high strategic complementarity, which ensures a high level of unresponsiveness of aggregate prices to an aggregate demand shock. There is no much room to decrease $\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}$; e.g., Woodford (2003) suggests that $\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} \in [.1, .15]$.

The other is $\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} = 1$ —aggregate and idiosyncratic variables have the same weight in the contribution of one good to firms' profits. One of the key forces that multi-production introduces to the attention problem is that aggregate information becomes more important in firms' total profits. Thus, one may think that increasing $\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|}$ should solve the calibration problem.

In the joint solution of κ_a^* and α^* , equation (15) depends on $\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|}$ through $x \equiv \frac{|\hat{\pi}_{11}|\sigma_\Delta}{\hat{\pi}_{14}\sigma_z}$ while equation (17) is independent of $\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|}$. Since σ_Δ is governed by σ_q and α , increasing $\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|}$ has the same aggregate implications as increasing σ_z . In terms of σ_p , an increase in $\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|}$ is also equivalent to an increase in σ_z , according to (20). Exercise 2 shows that increasing σ_z delivers high per-good losses due to the friction when $N > 1$.

Tables 4A and 4B report results after redoing exercises 2 and 3 for $\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} = 2$. Table 4A shows that the required increase in σ_z to match $\sigma_p = 3.1\%$ as N increases is less steep, but firms' per-good losses still quickly increase with N . Table 4B shows that the required increase of $\kappa(N)$ is also less steep, but still prices absorb a high proportion of a shock in q_t for $N \geq 100$. Real effects of money are high ($\alpha = 0$) for $N \leq 5$, but information capacity $\kappa(N)$ must be very low to match $\sigma_p = 3.1\%$, so per-good losses due to the friction are high relative to the exercise 1.

5.3 Summary

This section presents in a simplified framework the main messages of this paper, so it deserves a detailed summary. Section 5.1 shows analytically that firms' attention to aggregate information is increasing in the number of goods firms produce unless under very restrictive conditions about the

volatility and importance in profits of idiosyncratic variables relative to aggregate variables. Firms' attention to idiosyncratic information decreases in the number of goods firms produce unless information capacity increases fast enough, in which case the attention to aggregate information increases more. Strategic complementarity in pricing decisions amplifies these effects. Besides, the volatility of prices is increasing in the attention paid to either type of information; the expected cost of the friction per good produced is decreasing in firms' overall attention; and the correlation of price changes inside firms increases in firms' attention to aggregate information.

Section 5.2 numerically show that taking the process of nominal aggregate demand from the data and targeting on moments regarding price changes (in this case, σ_p) and expected costs of the friction for each good similar to those of single-product firms impose enough discipline for calibrations. Only setting information capacity increasing in N yields satisfactory results, which implies low money non-neutrality. Natural candidates to solve the calibration problem, such as increasing volatility or importance in profits of idiosyncratic relative to aggregate variables, do not yield satisfactory results since they imply too high price volatility and expected cost of the friction.

6 The auto-regressive case

Assume now that the process of q_t is such that Δ_t is $AR(1)$ with persistency ρ_Δ . Idiosyncratic variables $\{z_{jt}\}_{j \in [0,1]}$ are also $AR(1)$ with persistency $\rho_j = \rho_z$. This case allows to introduce persistency keeping at least partial analytical solution. In a nutshell, all results qualitatively remain.

The starting guess is now

$$p_t = \sum_{l=0}^{\infty} \alpha_l v_{t-l}, \quad (21)$$

where $v^t \equiv \{v_{t-l}\}_{l=0}^{\infty}$ is the history of nominal aggregate demand innovations.

Given the structure of signals, optimal prices under rational inattention follow

$$p_{jt}^* = \mathbb{E} [\Delta_t | s_{ai}^t] + \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} \mathbb{E} [z_{jt} | s_j^t] = \hat{\Delta}_{it} + \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} \hat{z}_{jt}$$

for any $j \in [0, 1]$. This rule is equivalent to (9) with the only difference that the producing firm i uses the whole history $\{s_{ai}^t, s_j^t\}$ of its observed aggregate and idiosyncratic signals.

The loss of profits for deviations from the frictionless prices due to the friction is still governed

by (10). The firm's problem may be cast in two stages. In the first stage, the firm chooses

$$\begin{aligned} & \min_{\hat{\Delta}_{it}, \{\hat{z}_{nt}\}_{n \in \mathbb{N}_i}} \sum_{n \in \mathbb{N}_i} \left\{ \sum_{t=1}^{\infty} \beta^t \frac{|\hat{\pi}_{11}|}{2} \mathbb{E} \left[(p_{nt}^{\diamond} - p_{nt}^*)^2 \right] \right\} \\ \rightarrow & \min_{\hat{\Delta}_{it}, \{\hat{z}_{nt}\}_{n \in \mathbb{N}_i}} \frac{\beta}{1-\beta} \frac{|\hat{\pi}_{11}|}{2} \left\{ \mathbb{E} \left(\Delta_t - \hat{\Delta}_{it} \right)^2 N + \left(\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} \right)^2 \sum_{n \in \mathbb{N}_i} \mathbb{E} (z_{nt} - \hat{z}_{nt})^2 \right\} \end{aligned}$$

subject to

$$\begin{aligned} I \left(\left\{ \Delta_t, \hat{\Delta}_{it} \right\} \right) & \leq \kappa_a, \\ I \left(\left\{ z_{nt}, \hat{z}_{nt} \right\} \right) & \leq \kappa_n, \text{ for } n \in \mathbb{N}_i \\ \kappa_a + \sum_{n \in \mathbb{N}_i} \kappa_n & \leq \kappa(N) \end{aligned}$$

after assuming, for simplicity, that there is an univariate signal for each target variable.

For the second stage, the firm must choose the signals structure that delivers $\hat{\Delta}_{it}^*, \{\hat{z}_{nt}^*\}_{n \in \mathbb{N}_i}$. Proposition 4 in Mackowiak and Wiederholt (2009) shows that many information structures meet this condition, including "target plus noise" signals which volatilities are pinned down by κ_a and $\{\kappa_n\}_{n \in \mathbb{N}_i}$. These results are not affected by the multi-production assumption.

Multi-production affects the allocation of attention by the same three forces studied above. First, aggregate information becomes more important for firms as the number N of goods they produce increases. This force is captured in the first term in the objective. Second, the more goods the firm produces, the more idiosyncratic signals the firm must pay attention to. This force is captured here in the number of constraints and in the left-hand side of the last constraint. Third, the information capacity in the right-hand side of the last constraint may depend on N .

This problem may be further manipulated. Following Sims (2003),¹¹ the solution of

$$\min_{b,c} \mathbb{E} (T_t - O_t)^2$$

where T_t is a target variable and O_t is an observable variable, subject to

$$\begin{aligned} T_t &= \rho T_{t-1} + a u_t, \\ O_t &= \sum_{l=0}^{\infty} b_l u_{t-l} + \sum_{l=0}^{\infty} c_l \varepsilon_{t-l}, \\ \kappa &\geq I(\{T_t, O_t\}) \end{aligned}$$

¹¹Also in Proposition 3 of Mackowiak and Wiederholt (2009).

implies the value of the objective

$$\mathbb{E} (T_t - O_t^*)^2 = \sigma_T^2 \frac{1 - \rho^2}{2^{2\kappa} - \rho^2}.$$

Using this result, the firms' problem in the first stage may be represented as

$$\min_{\kappa_a, \{\kappa_n\}_{n \in \mathfrak{N}_i}} \frac{\beta}{1 - \beta} \frac{|\hat{\pi}_{11}|}{2} \left[\sigma_\Delta^2 \frac{1 - \rho_\Delta^2}{2^{2\kappa_a} - \rho_\Delta^2} N + \left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}} \right)^2 \sum_{n \in \mathfrak{N}_i} \frac{1 - \rho_z^2}{2^{2\kappa_z} - \rho_z^2} \sigma_z^2 \right]$$

subject to

$$\kappa_a + \sum_{n \in \mathfrak{N}_i} \kappa_n \leq \kappa(N)$$

which is identical to the firms' problem in the white noise case if $\rho_\Delta = \rho_z = 0$.

Its optimality condition is

$$\kappa_a^* + f(\rho_\Delta, \kappa_a^*) = \kappa_z^* + f(\rho_z, \kappa_z^*) + \log_2 x \sqrt{N} \quad (22)$$

with $x \equiv \frac{|\hat{\pi}_{11}| \sigma_\Delta}{\hat{\pi}_{14} \sigma_z}$ and $f(\rho, \kappa) = \log_2(1 - \rho^2 2^{-2\kappa})$.

Notice that $f(0, \kappa) = 0$. Hence, (22) collapses to (14) when q_t and $\{z_{jt}\}_{j \in [0,1]}$ are white noise. In addition, $f(\rho, \kappa)$ is weakly negative, increasing in κ and decreasing in ρ . Therefore, keeping x and N constant, more attention is given to aggregate signals relative to idiosyncratic signals, i.e., $\kappa_a^* - \kappa_z^*$ increases, when the compound aggregate variable Δ_t is less persistent relative to the idiosyncratic variable, i.e. when $|\rho_\Delta| - |\rho_z|$ is lower.

The solution of κ_a^* given x may be obtained combining equation (22) and the constraint

$$\kappa_a^* + N \kappa_z^* = \kappa(N)$$

which remains invariant to the introduction of persistency into the attention problem.

Allocation of attention. The result in the white noise case that $\kappa_a^* - \kappa_z^*$ is increasing in N for a constant x remains here. Note that

$$f(\rho_\Delta, \kappa_a^*) - f(\rho_z, \kappa_z^*) = \log_2 \left(\frac{1 - \rho_\Delta^2 2^{-2\kappa_a^*}}{1 - \rho_z^2 2^{-2\kappa_z^*}} \right)$$

is increasing in $\kappa_a^* - \kappa_z^*$. Therefore, $\kappa_a^* - \kappa_z^*$ is increasing in N in (22) but to a lesser extent than in the white noise case in (14). The higher is either $|\rho_\Delta|$ or $|\rho_z|$, the weaker is the effect of N on $\kappa_a^* - \kappa_z^*$. Since the constraint is invariant to the introduction of persistency, the allocation of

attention has the same properties than in the white noise case.

Fixed point and money neutrality. Now not only x is endogenous, but also ρ_Δ . In addition, prices are allowed to respond to the whole history of innovations in nominal aggregate demand. Therefore, there is no closed form solution for parameters in the guess in (21) given the allocation of attention; i.e, there is no explicit counterpart to equation (17).

However, this relationship remains independent of N . Thus, despite persistence in q_t and $\{z_{jt}\}_{j \in [0,1]}$ may add non-linearity in the allocation of attention and persistence in the response of aggregate prices to nominal shocks, the ways in which N affects the predictions of the rational inattention model is still captured by the optimality condition (22).

The rest of the statistics, such as the magnitude of price changes and losses due to the friction, are better studied in the quantitative exercise that follows.

7 The general case

I now solve numerically the model studied in this paper for a general specification of the log-deviation of nominal aggregate demand q_t and idiosyncratic variables at the good level $\{z_{jt}\}_{j \in [0,1]}$.¹²

As in section 5.2, I use the parameters used by Mackowiak and Wiederholt (2009):

$$\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} = 1; \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} = .15; \kappa(N = 1) = 3; \sigma_q = 2.68\%.$$

I add now their assumed process for q_t : $AR(1)$ with persistency $\rho_q = .95$ and standard deviation $\sigma_v = 1\%$ for its innovation v_t .¹³ This process is approximated by a $MA(20)$:

$$q_t = \sum_{k=0}^{20} \rho_q \left(1 - \frac{k}{20}\right) v_{t-k} \quad (23)$$

The process for $\{z_{jt}\}_{j \in [0,1]}$ is assumed to follow the same $MA(20)$ structure than q_t with an innovation 11.8 times more volatile than v_t . Each period is meant to be a quarter.

For $N = 1$, I replicate exactly the results of Mackowiak and Wiederholt (2009). The attention allocation is $\kappa_a^* = .08$ and $\kappa_z^* = 2.92$. The average size of price changes is 9.7% in line with the findings of Klenow and Kryvtsov (2008). Firms' expected losses due to the friction are small,

¹²The appendix displays the analytic problem and describes the solution algorithm.

¹³These estimates are obtained from GNP quarterly data spanning 1959:1–2004:1.

.0021 \bar{Y} per quarter,¹⁴ so $\kappa(1) = 3$ allows firms to track well optimal frictionless prices and thus firms have little incentives to increase their information capacity if such decision were endogenous. Importantly, real effects of money are strong and long lasting: Prices respond weakly and slowly to an innovation in q_t . Figure 2 draws this response. The black line represents the response of frictionless prices after a shock of magnitude .01 in q_t —according to (23), this response decreases linearly to die after 20 quarters. The blue line represents the response of prices under rational inattention for $N = 1$. Prices absorb only 2.8% of the shock on impact with a maximum of 17.6% of the shock 10 quarters after the impact (at this point, 55% of the shock remains in the response of frictionless prices) to die after 20 quarters. The accumulated response of prices of rationally inattentive firms for $N = 1$ is only 22% of the accumulated response of frictionless prices.

For $N \geq 2$, the information capacity $\kappa(N)$ is calibrated to be

$$\kappa(N) = \kappa_a^*(N = 1) + N \times \kappa_z^*(N = 1). \quad (24)$$

In words, the optimal allocation of attention when $N = 1$ is feasible for the firm for any N . This calibration ensures similar firms' expected losses per good due to the friction for any N .

Figure 2 draws in red the response of prices of rationally inattentive firms for $N = 2$. Information capacity is $\kappa(2) = 5.9$; its allocation is $\kappa_a^* = .3$ and $\kappa_z^* = 2.8$ for information about each good the firm produces. The average size of price changes is still 9.7%. Firms' expected losses per good and quarter due to the friction are .0023 \bar{Y} . Prices absorb on impact 13% of the shock with a maximum of 60% after 7 quarters; the response of prices is almost identical to the response of frictionless prices thereafter. The accumulated response of prices is 70.2% of the accumulated response of frictionless prices. In few words, non-neutrality is largely undermined, in both magnitude and duration: The response of output is cut by four and its duration by three.

Similarly, Figure 2 draws in green the response of prices of rationally inattentive firms for $N = 5$. Information capacity is $\kappa(5) = 14.6$, so $\kappa_a^* = .6$ and $\kappa_z^* = 2.8$ for each good. Firms' expected losses per good and quarter are .0023 \bar{Y} . Prices absorb 30% of the shock on impact with a maximum of 70% only 3 quarters after the shock; prices response is almost identical to the response of frictionless prices thereafter. The accumulated response of prices is 87% of the accumulated response of frictionless prices. Money non-neutrality almost disappears.

Figure 3 draws responses of prices for $N = 10, 100$ and 1000 (in red, green and magenta) and the response of frictionless prices and prices of rationally inattentive firms for $N = 1$ (in black and blue). Average price changes are 9.7% and firms' losses per good and quarter are .0023 \bar{Y} . For $N = 10$, prices absorb 53% of the shock on impact and their response is almost identical to the

¹⁴This result comes from equation (10) after assuming $|\hat{\pi}_{11}| = 15\bar{Y}$, which is consistent with the calibration.

response of frictionless prices after 2 quarters. Money is fully neutral for $N = 100, 1000$.

Discussion. As in section 5, some parameters may be manipulated to try to improve these results. Such attempts are unsuccessful; the same happens here. Setting $\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} = 2$ for $N = 2$, real effects of money are large ($\kappa_a^* = .06$), but the implied average size of price changes doubles (19.4%) and firms' losses quadruples ($.0077\bar{Y}$) those when $\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} = 1$. If in addition the volatility of idiosyncratic variables is cut by half, all results are restored, including the low real effects of money.

When the volatility of idiosyncratic variables is doubled for $N = 2$, results are identical to doubling $\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|}$. As in section 5, increasing σ_z is isomorphic to increasing $\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|}$. When I cut information capacity for $N = 2$ from $\kappa(2) = 5.9$ to 4, I obtain $\kappa_a^* = .05$, average size of price changes equals 9.6%, but firms' losses are large, $.0075\bar{Y}$.

These exercises target on low firms' expected per-good losses due to the friction. The argument to keep losses low is to keep incentives low for firms to increase their information capacity if such decision were endogenous. This is a conservative criterion; the total cost of the friction increases with the number N of decisions that firms must take, in this case, pricing of goods produced. The cost of acquiring information is independent of N ; it is likely that investment in information capacity is also independent of N . Targeting on total losses would strongly reinforce my results.

8 Firm-specific and good-specific information

Relevant idiosyncratic variables in firms' profits are so far good-specific. This means that multi-product firms can only exploit the economies of scale in the use of information, which are in the core of this paper, by acquiring aggregate information. This section introduces firm-specific variables to make two points. First, good-specific variables are necessary to generate increasing comovement of prices inside firms in the number of goods they produce. Second, if good-specific variables are relevant for pricing decisions, all results remain.

Comovement and synchronization of price changes. Lach and Tsiddon (1996), Fisher and Konieczny (2000) and Bhattarai and Schoenle (2010) document the synchronization of price changes inside firms. For instance, Bhattarai and Schoenle (2010) report that prices of a two-product firm have 30% probability to change in the same direction at the same time. When firms produce 3 goods, this probability increases to 37%. One difficulty of the rational inattention model is that it predicts prices changing every period while in reality prices change only infrequently. Hence, synchronization is not a well defined concept inside the model.

Abstracting from this issue and interpreting synchronization as comovement, two desired features the model should generate are prices imperfectly comoving inside firms and increasing co-

movement in the number of goods firms produce. The model is able to generate these features if good-specific shocks are relevant for pricing decisions. To show this point, I first replace good-specific by firm-specific information in the model. Then I allow aggregate, good-specific and firm-specific information to interact.

Aggregate and firm-specific information. Assume that a firm $i \in [0, \frac{1}{N}]$ observes signals about q_t and firm-specific shocks $\{f_{it}\}_{i \in [0, \frac{1}{N}]}$ in profits instead of good-specific shocks $\{z_{jt}\}_{j \in [0, 1]}$ in the setup of section 4 and 5—when all variables are white noise and univariate signals have the structure "fundamental plus noise". The firm's objective now is

$$\min_{\kappa_a, \kappa_f} \frac{\beta}{1 - \beta} \frac{|\widehat{\pi}_{11}|}{2} \left[2^{-2\kappa_a} \sigma_{\Delta}^2 N + \left(\frac{\widehat{\pi}_{14}}{\widehat{\pi}_{11}} \right)^2 2^{-2\kappa_f} \sigma_f^2 N \right]$$

where κ_f is the attention given to the signal about f_{it} and σ_f is the volatility of f_{it} for all i . The information capacity constraint now is

$$\kappa_a + \kappa_f \leq \kappa(N).$$

From the three forces introduced by multi-production into the attention allocation problem—on the objective, on the left-hand side of information capacity constraint and on $\kappa(N)$ in the right-hand side of this constraint—only the effect on $\kappa(N)$ remains active. Calibrating $\kappa(N)$ consistently with (24) yields $\kappa_a^*(N) = \kappa_a^*(1)$; the allocation of attention is invariant in N .

Individual prices follow a process

$$p_{nt}^* = (1 - 2^{-2\kappa_a^*}) (\Delta_t + \varepsilon_{it}) + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} (1 - 2^{-2\kappa_f^*}) (f_{it} + \vartheta_{it})$$

for $n \in \mathbb{N}_i$ and $i \in [0, \frac{1}{N}]$, where ϑ_{it} is the noise of the signal for f_{it} . Thus, prices inside firms perfectly comove:

$$\text{corr}(p_{nt}^*, p_{-nt}^*) = 1 \text{ for all } n, -n \in \mathbb{N}_i.$$

In contrast, if in the setup of Section 5—when idiosyncratic shocks are good-specific—I calibrate $\kappa(N)$ consistently with (24), the optimality condition in (14) implies

$$\kappa_a^*(N) = \frac{1}{2} \left[\kappa(1) + \log_2 \left(x\sqrt{N} \right) \right]. \quad (25)$$

Individual prices follow (16). Prices' comovement now is

$$\text{corr}(p_{nt}^*, p_{-nt}^*) = (1 - 2^{-2\kappa_a^*(N)}) \sigma_\Delta^2 / \sigma_p^2 < 1.$$

which is increasing in N if $\kappa_a^*(N)$ is increasing in N .

As a result, an interior solution for the optimal allocation of attention to good-specific information is necessary for the rational inattention model to generate imperfect comovement of prices inside firms together with increasing comovement as firms produce more goods.

Aggregate, firm-specific and good-specific information. As a robustness check, assume now that the setup of section 4 is augmented to include firm-specific variables $\{f_{it}\}_{i \in [1, \frac{1}{N}]}$ and good-specific variables $\{z_{jt}\}_{j \in [0, 1]}$. The contribution of profits of good $n \in \mathbb{N}_i$ for its producing firm i is

$$\pi(P_{nt}, P_t, Y_t, Z_{nt}, F_{it});$$

its approximation up to a second-order around zero log deviations from steady state is

$$\widehat{\pi}_1 p_{nt} + \frac{\widehat{\pi}_{11}}{2} p_{nt}^2 + \widehat{\pi}_{12} p_{nt} p_t + \widehat{\pi}_{13} p_{nt} y_t + \widehat{\pi}_{14} p_{nt} z_{jt} + \widehat{\pi}_{15} p_{nt} f_{it} + \text{terms independent of } p_{nt}.$$

The firms' problem is now

$$\min_{\kappa_a, \{\kappa_n\}_{n \in \mathbb{N}_i}, \kappa_f} \frac{\beta}{1 - \beta} \frac{|\widehat{\pi}_{11}|}{2} \left[2^{-2\kappa_a} \sigma_\Delta^2 N + \left(\frac{\widehat{\pi}_{14}}{\widehat{\pi}_{11}} \right)^2 \sum_{n \in \mathbb{N}_i} 2^{-2\kappa_n} \sigma_z^2 + \left(\frac{\widehat{\pi}_{15}}{\widehat{\pi}_{11}} \right)^2 2^{-2\kappa_f} \sigma_f^2 N \right]$$

$$s.t. \quad \kappa_a + \sum_{n \in \mathbb{N}_i} \kappa_n + \kappa_f \leq \kappa(N).$$

Following the spirit of (24), I calibrate $\kappa(N)$ to be

$$\kappa(N) = \kappa_a^*(N=1) + N \times \kappa_z^*(N=1) + \kappa_f^*(N=1), \quad (26)$$

so the optimal allocation to aggregate information is

$$\kappa_a^* = \frac{1}{3} \left[\kappa(1) + \log_2 \left(x_1 \sqrt{N} \right) + \log_2(x_2) \right]$$

where $x_1 \equiv \frac{|\widehat{\pi}_{11}| \sigma_\Delta}{\widehat{\pi}_{14} \sigma_z}$ and $x_2 \equiv \frac{|\widehat{\pi}_{11}| \sigma_\Delta}{\widehat{\pi}_{15} \sigma_f}$. Comparing with (25), firm-specific shocks to profits affect the attention paid by firms to the aggregate. But the implications of multi-production remain strong: $\frac{2}{3}$ of the already shown large effects on mitigating money non-neutrality obtained when idiosyncratic shocks are good-specific. Importantly, the volatility of f_{it} plays no role. These

results depend on an interior solution for the optimal allocation of attention to the good-specific signal, which in turn is necessary to get the desired properties of prices' comovement inside firms.

Intuitively, the story is not very different from above. Aggregate and firm-specific information becomes more important for firms relative to good-specific information as N increases. If information capacity is invariant to N , even more attention than above is taken away from good-specific signals with N . Thus, firms' per-good losses due to the friction rapidly increase with N . Setting $\kappa(N)$ as in (26) prevents losses to spike up by giving more information capacity to the firm as N increases. The firm chooses to allocate some of this extra capacity to aggregate information (as well as to firm-specific information). As a result, multi-product firms' prices respond quicker than single-product firms to an innovation in nominal aggregate demand q_t .

9 Conclusions

This paper highlights the importance of economies of scale in the use of information arising in the rational inattention model. In this model, agents must optimally allocate their limited capacity to process information. I explore the effects of these economies of scale on the implications of the model regarding money neutrality by using the work of Mackowiak and Wiederholt (2009) as benchmark. Multi-product firms exploit these economies of scale by acquiring aggregate information since this type of information is useful for firms to take all their pricing decisions, as opposed to good-specific information which is only useful for the pricing of the good for which the information is concerned with. Hence, firms' capacity allocated to process aggregate information increases in the number of goods they produce. I quantitatively show that this effect reduces significantly the ability of model to deliver large real effects of money when firms are allowed to produce multiple goods. The introduction of firm-specific information does not change this result as long as good-specific information remains relevant for pricing decisions. Additionally, these economies of scale allow the model to predict increasing comovement of prices inside firms in the number of goods they produce—which is a feature qualitatively consistent with the data.

The economies of scale in the use of information find application in the wide variety of contexts in which the rational inattention model has been used. I leave these applications for future research.

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Appendix: General problem and its solution

This appendix displays the analytical representation of firms' problem in the setup of section 7 and explains the numerical algorithm applied. Both, the analytical representation and the numerical algorithm, are extensions of Mackowiak and Wiederholt (2009).

The MA representations of q_t and $\{z_{jt}\}$ are

$$q_t = \sum_{l=0}^{20} a_l v_{t-l}, \quad (27)$$

$$z_{jt} = \sum_{l=0}^{20} b_l \zeta_{t-l}, \quad (28)$$

for $j \in [0, 1]$, where $\{v_t\}$ and $\{\zeta_{jt}\}$ are innovations following Gaussian independent processes.

Given the definition of Δ_t in (8), $y_t = q_t - p_t$ and the guess

$$p_t = \sum_{l=0}^{20} \alpha_l v_{t-l}$$

yield

$$\Delta_t = \left(1 - \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}\right) \sum_{l=0}^{20} \alpha_l v_{t-l} + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} \sum_{l=0}^{20} a_l v_{t-l}. \quad (29)$$

The problem of firm $i \in [0, \frac{1}{N}]$ has two stages. In the first stage, the firm must choose conditional expectations for Δ_t and $\{z_{nt}\}_{n \in \mathcal{N}_i}$ to minimize the deviation of prices with respect to frictionless optimal prices

$$\min_{\hat{\Delta}_{it}, \{\hat{z}_{nt}\}_{n \in \mathcal{N}_i}} \sum_{n \in \mathcal{N}_i} \left\{ \sum_{t=1}^{\infty} \beta^t \frac{|\hat{\pi}_{11}|}{2} \mathbb{E} \left[(p_{nt}^\diamond - p_{nt}^*)^2 \right] \right\}$$

subject to the information capacity constraint. This problem is equivalent to

$$\min_{\hat{\Delta}_{it}, \{\hat{z}_{nt}\}_{n \in \mathcal{N}_i}} \left\{ \mathbb{E} \left[(\Delta_t - \hat{\Delta}_{it})^2 \right] N + \left(\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} \right)^2 \sum_{n \in \mathcal{N}_i} \mathbb{E} \left[(z_{nt} - \hat{z}_{nt})^2 \right] \right\}$$

subject to (27), (28), (29), and the information capacity constraint, which takes the form

$$-\frac{1}{4\pi} \int_{-\pi}^{\pi} \log_2 \left[1 - C_{\Delta, \hat{\Delta}_i}(\omega) \right] d\omega - \frac{1}{4\pi} \sum_{n \in \mathcal{N}_i} \int_{-\pi}^{\pi} \log_2 \left[1 - C_{z_n, \hat{z}_n}(\omega) \right] d\omega \leq \kappa(N)$$

where the functions inside the square brackets are called coherence,

$$C_{\Delta, \hat{\Delta}_i}(\omega) = \frac{\frac{G(e^{-i\omega})G(e^{i\omega})}{H(e^{-i\omega})H(e^{i\omega})}}{\frac{G(e^{-i\omega})G(e^{i\omega})}{H(e^{-i\omega})H(e^{i\omega})} + 1}},$$

where $G(e^{i\omega}) = g_0 + g_1e^{i\omega} + g_2e^{i2\omega} + \dots$ and $H(e^{i\omega}) = h_0 + h_1e^{i\omega} + h_2e^{i2\omega} + \dots$

$$C_{z_n, \hat{z}_n}(\omega) = \frac{\frac{R_n(e^{-i\omega})R_n(e^{i\omega})}{S_n(e^{-i\omega})S_n(e^{i\omega})}}{\frac{R_n(e^{-i\omega})R_n(e^{i\omega})}{S_n(e^{-i\omega})S_n(e^{i\omega})} + 1}},$$

where $R(\cdot)$ and $S(\cdot)$ are defined similar to $G(\cdot)$ and $H(\cdot)$.

The solution must have the form

$$\begin{aligned} \hat{\Delta}_{it}^* &= \sum_{l=0}^{20} g_l^* v_{t-l} + \sum_{l=0}^{20} h_l^* \varepsilon_{t-l}, \\ \hat{z}_{nt}^* &= \sum_{l=0}^{20} r_{nl}^* v_{t-l} + \sum_{l=0}^{20} s_{nl}^* \varepsilon_{nt-l} \text{ for } n \in \aleph_i. \end{aligned}$$

To find these coefficients, the problem is partitioned in two. First, solve

$$\begin{aligned} \min_{\hat{\Delta}_{it}} \mathbb{E} \left[\left(\Delta_t - \hat{\Delta}_{it} \right)^2 \right] N &= \min_{g, h} \left[\sum_{l=0}^{20} (a_l - g_l)^2 + \sum_{l=0}^{20} h_l^2 \right] N \\ \text{s.t. } -\frac{1}{4\pi} \int_{-\pi}^{\pi} \log_2 \left[1 - C_{\Delta, \hat{\Delta}_i}(\omega) \right] d\omega &= \kappa_a. \end{aligned}$$

The optimality conditions are

$$\begin{aligned} g_l &: 2(a_l - g_l) N = -\frac{\mu_a}{4\pi \log(2)} \int_{-\pi}^{\pi} \frac{\partial \log \left[1 - C_{\Delta, \hat{\Delta}_i}(\omega) \right]}{\partial g_l} d\omega, \\ h_l &: 2h_l N = \frac{\mu_a}{4\pi \log(2)} \int_{-\pi}^{\pi} \frac{\partial \log \left[1 - C_{\Delta, \hat{\Delta}_i}(\omega) \right]}{\partial h_l} d\omega \end{aligned}$$

where μ_a is the Lagrangian multiplier. This system plus the constraint yield the coefficients $\{g^*(\kappa_a, N)\}$ and $\{h^*(\kappa_a, N)\}$ and thus $\hat{\Delta}_{it}(\kappa_a, N)$. Similarly, coefficients $\{r_n^*(\kappa_n, N)\}$ and

$\{s_n^*(\kappa_n, N)\}$ and thus $\widehat{z}_{nt}(\kappa_n, N)$ for $n \in \aleph_i$ solve the problem

$$\begin{aligned} \min_{\widehat{\Delta}_{it}} \mathbb{E} [(z_{nt} - \widehat{z}_{nt})^2] &= \min_{g,h} \left[\sum_{l=0}^{20} (b_l - r_l)^2 + \sum_{l=0}^{20} s_l^2 \right] \\ \text{s.t.} \quad & -\frac{1}{4\pi} \int_{-\pi}^{\pi} \log_2 [1 - C_{z_n, \widehat{z}_n}(\omega)] d\omega = \kappa_n. \end{aligned}$$

Finally, the coefficients of g^*, h^*, r^*, s^* , are obtained solving

$$\begin{aligned} \min_{\kappa_a, \{\kappa_n\}_{n \in \aleph_i}} \left\{ \mathbb{E} \left[\left(\Delta_t - \widehat{\Delta}_{it}(\kappa_a, N) \right)^2 \right] N + \left(\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} \right)^2 \sum_{n \in \aleph_i} \mathbb{E} [(z_{nt} - \widehat{z}_{nt}(\kappa_n, N))^2] \right\} \\ \text{s.t.} \quad \kappa_a + \sum_{n \in \aleph_i} \kappa_n \leq \kappa(N). \end{aligned}$$

which gives $\kappa_a^*(N)$ and $\kappa_n^*(N) = \kappa_z^*(N)$ since the problem is symmetric for all $n \in \aleph_i$.

The second stage of the problem is to obtain optimal signals structures that deliver $\widehat{\Delta}_{it}^* = \widehat{\Delta}_{it}(\kappa_a^*(N), N)$ and $\widehat{z}_{nt}^* = \widehat{z}_{nt}(\kappa_n^*(N), N)$. Since I am interested in the aggregate implications of the model, I do not solve this part. However, Mackowiak and Wiederholt (2009) show that there always exist multiple signal structures that yield these results (Proposition 4).

Numerically, I first start from a guess for $\{\alpha_l^{(0)}\}_{l=0}^{20}$. Then I obtain $\kappa_a^*(N)$ and $\kappa_z(N)$ by solving the non-linear system of first-order conditions using the Levenberg-Marquardt algorithm. Then I compute optimal prices and get a new sequence $\{\alpha_l^{(1)}\}_{l=0}^{20}$. I iterate upon convergence.

Tables and Figures

N	1	2	5	10	100	1000
κ_a^*	.13	.58	1.08	1.51	3	3
α	2.8%	15.5%	34.5%	51.7%	90.4%	90.4%
κ_z^*	2.87	1.21	.38	.15	0	0
σ_p	3.1%	2.9%	2.3%	2%	2.4%	2.4%
$loss$	3.6×10^{-5}	2.1×10^{-4}	6.1×10^{-4}	8.3×10^{-4}	9.6×10^{-4}	9.6×10^{-4}

Table 1 – Exercise 1, white noise

N	1	2	5	10	100	1000
κ_a^*	.13	.43	.58	.63	.66	.67
α	2.8%	10.9%	15.5%	17.1%	18.5%	18.6%
κ_z^*	2.87	1.29	.48	.24	.02	.002
σ_z/σ_q	1.17	1.26	1.63	2.14	6.34	19.9
$loss$	3.6×10^{-5}	2.2×10^{-4}	1.0×10^{-3}	2.4×10^{-3}	2.8×10^{-2}	2.8×10^{-1}

Table 2 – Exercise 2, white noise

N	1	2	5	10	100	1000
κ_a^*	.13	1.1	1.69	2.05	3.38	4.98
α	2.8%	36.7%	58.7%	70.7%	94.2%	99.3%
κ_z^*	2.87	1.42	.88	.65	.29	.22
$\kappa(N)$	3	4.0	6.1	8.6	31.9	202.7
$loss$	3.6×10^{-5}	1.7×10^{-4}	3.2×10^{-4}	4.2×10^{-4}	6.6×10^{-4}	7.2×10^{-4}

Table 3 – Exercise 3, white noise

N	1	2	5	10	100	1000
κ_a^*	.13	.43	.58	.63	.66	.67
α	2.8%	10.9%	15.5%	17.1%	18.5%	18.6%
κ_z^*	2.87	1.29	.48	.24	.02	.002
σ_z/σ_q	.58	.63	.81	1.07	3.17	9.95
$loss$	3.6×10^{-5}	2.2×10^{-4}	1.0×10^{-3}	2.4×10^{-3}	2.8×10^{-2}	2.8×10^{-1}

Table 4A – Exercise 2 for $\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} = 2$, white noise

N	1	2	5	10	100	1000
κ_a^*	0	0	0	.44	2.2	3.82
α	0%	0%	0%	11.1%	75.2%	96.8%
κ_z^*	.21	.21	.21	.20	.11	.05
$\kappa(N)$.21	.41	1.0	2.43	12.8	58.7
$loss$	3.0×10^{-3}	3.0×10^{-3}	3.0×10^{-3}	3.0×10^{-3}	3.4×10^{-3}	3.6×10^{-3}

Table 4B – Exercise 3 for $\frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} = 2$, white noise

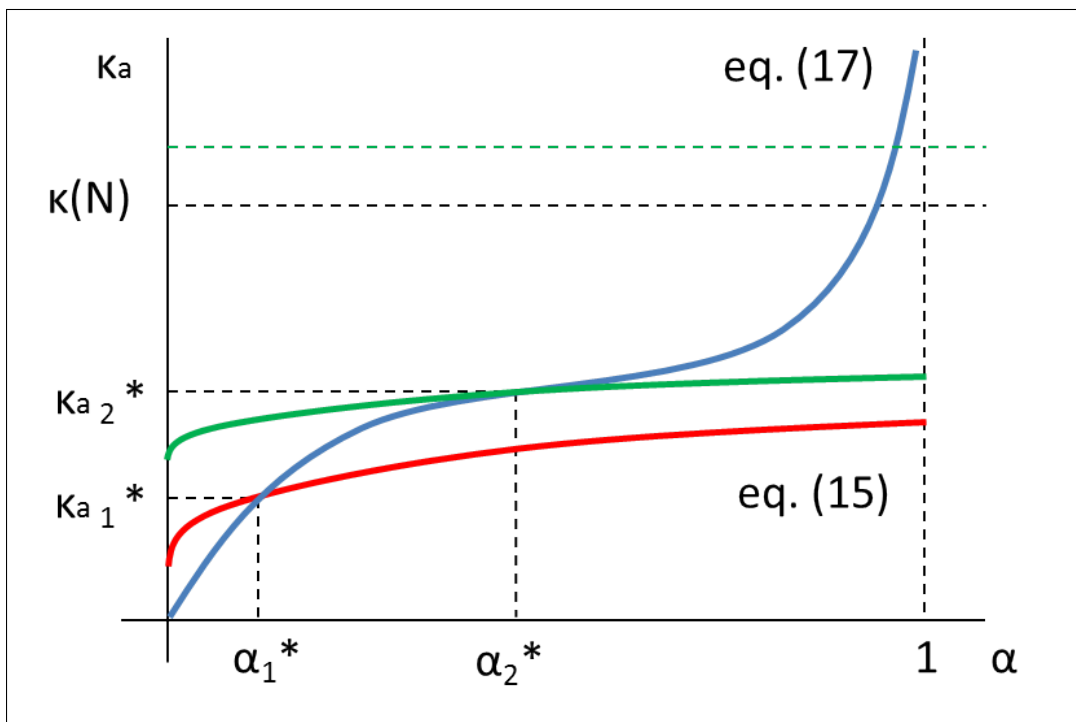


Figure 1 – Equations (15) and (17) in the space (α, κ_a)

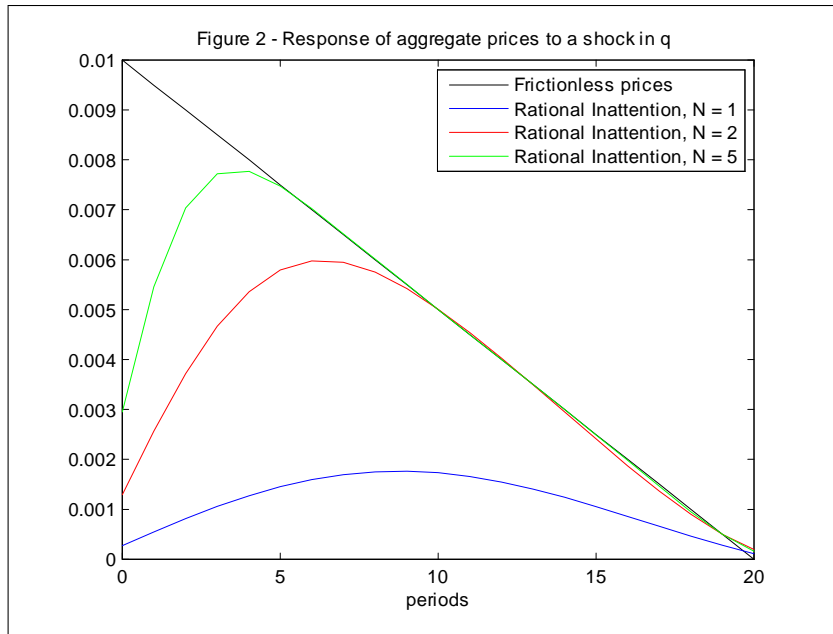


Figure 2 – Response of aggregate prices to a shock in q_t

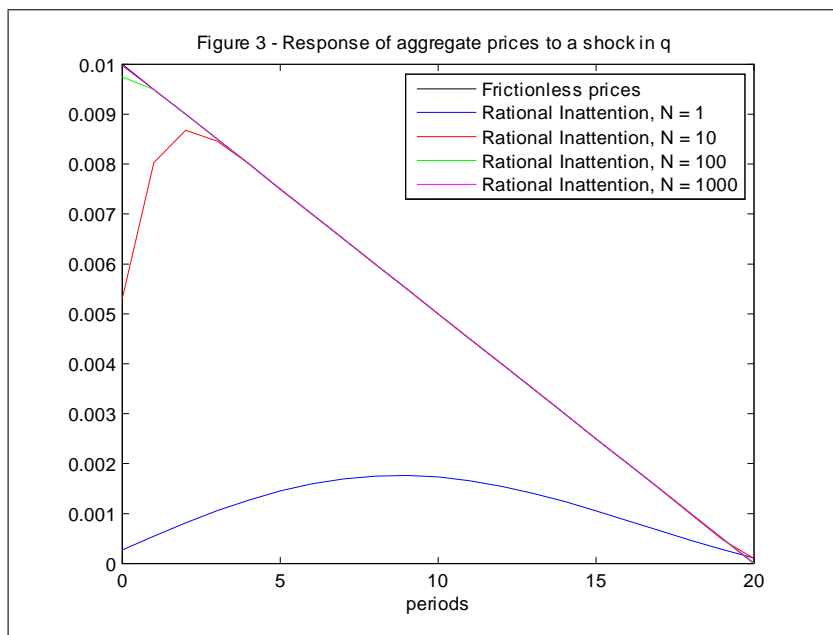


Figure 3 – Response of aggregate prices to a shock in q_t

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