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Unmeasured Investment and the Puzzling U.S. Boom in the 1990s*

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ABSTRACT

The basic neoclassical growth model accounts well for the postwar cyclical behavior of the U.S. economy prior to the 1990s, provided that variations in population growth, depreciation rates, total factor productivity, and taxes are incorporated. For the 1990s, the model predicts a depressed economy, when in fact the U.S. economy boomed. We extend the base model by introducing intangible investment and non-neutral technology change with respect to producing intangible investment goods and find that the 1990s are not puzzling in light of this new theory. There is compelling micro and macro evidence for our extension, and the predictions of the theory are in conformity with U.S. national products, incomes, and capital gains. We use the theory to compare current accounting measures for labor productivity and investment with the corresponding measures for the model economy with intangible investment. Our findings show that standard accounting measures greatly understate the boom in productivity and investment.

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The basic neoclassical growth model accounts well for the postwar cyclical behavior of the U.S. economy prior to the 1990s, provided that variations in population growth, depreciation rates, total factor productivity (TFP), and taxes are incorporated.[1] The behavior of the 1990s, however, is strikingly at variance with this model, particular in comparison with the boom in hours, but also in comparison with the behavior of most aggregate series that business cycle theorists study. To put it succinctly, the model predicts a depressed 1990s economy, when in fact it boomed.

In this paper, we extend the base model by introducing intangible investment and non-neutral technology change with respect to producing intangible investment goods and find that, in light of this new theory, the 1990s are not puzzling. Intangible investment is excluded from GDP because it is difficult to measure. Examples include research and development (R&D), advertising, and investments in building organizations. Some intangible investment is financed by owners of capital and is *expensed* rather than capitalized. Some is investment in *sweat equity* financed by worker-owners who allocate effort and time to their business and receive compensation at less than their market rate. These investments are made with the expectation of realizing future profits or capital gains when the business goes public or is sold.

We have found compelling evidence that both of these types of unmeasured investment were abnormally high in the 1990s. The National Science Foundation's report of R&D investment shows that R&D relative to GDP grew by 30 percent between 1994 and 2000.[2] The Current Population Surveys of the U.S. Department of Labor show a shift of labor into information technology–related and managerial occupations, with greater opportunities for business owners to make capital gains on expensed and sweat investment. If we look at patterns of national incomes, we also find evidence of abnormally high investment at the macroeconomic level Corporate profits were falling as output was rising, suggesting that investment in R&D and other intangible capital was abnormally high. Similarly,

[1] See, for example, Chen, İmrohoroğlu, and İmrohoroğlu (2007) and our technical appendix (McGrattan and Prescott 2007).

[2] Corrado, Hulten, and Sichel (2005, 2006) include other categories of expenditures and conclude that over the 1990s, total intangible investment was large and was increasing as a share of business output.

compensation per hour was low during the 1990s, suggesting that sweat investment was likewise abnormally high.

The fact that *measured* factor incomes were low when output and hours were booming is consistent with a theory that differentiates economic income and measured income, which need not move together and indeed did not move together in the 1990s. To uncover what actually happened during the 1990s, we use our extended theory and U.S. national income and product account (NIPA) data. Specifically, we incorporate intangible investment into an otherwise basic neoclassical growth model. Two technologies are available in the business sector: one for producing final goods and services and one for producing intangible capital. We use the extended model to determine the path for intangible investment, and show why including this type of investment is critical for understanding the boom in the U.S. economy in the 1990s.

We allow the rates of technological change to differ across these two technologies, thus allowing for a technology boom in the sector producing intangible capital—a boom like the one that occurred in the United States. During the 1990s, rapid technological advancements were being made in industries that are relatively intensive in producing intangible capital, such as those related to information technology. Two notable pieces of evidence (Doms 2004) are what happened to Intel processor speeds, which increased 4.6 percent per month in the period 1997–2000, and fiber-optic throughput, which rose from 2.5 gigabits to 400 gigabits per second between 1995 and 2000. Given that households equate wages and rental rates across production activities, we have a way to identify the TFP paths and to estimate the magnitude of intangible investment. We estimate that net intangible investment in the business sector was about 3 percent of GDP prior to 1990, rose to over 8 percent of GDP in the 1990s, and then returned to the level of the early 1990s in 2001.

We could have modified the basic model by introducing large and variable shocks to preferences for leisure, which is a common practice in business cycle research.[3] However,

[3] See, for example, Hall (1997), Chang and Schorfheide (2003), Galí (2005), Comin and Gertler (2006), Galí, Gertler, and López-Salido (2007), Ireland and Schuh (2007), Kahn and Rich (2007), and Smets and Wouters (2007) who point out that these shocks proxy for variations in tax rates and other labor

doing so would have violated two criteria that we require to successfully resolve the puzzling 1990s boom—or any other puzzle, for that matter. The first is the *input justification criterion*. By this, we mean that we require our exogenous inputs to be consistent with micro and macro empirical evidence. A large rise in factor productivity in the sector producing intangible capital is consistent with the U.S. technology boom and the shifts in employment into high-technology occupations and managerial positions. A large rise in preferences for leisure, on the other hand, cannot be justified by any observations on tax rates, tax credits, or welfare benefits.

The second of our two criteria for successfully resolving of the puzzling 1990s boom is the *prediction criterion*. At a minimum, to satisfy this criterion, a theory's predictions must not be counterfactual. A stronger requirement—one that is satisfied by our theory—is to make correct predictions for data that were not used to set parameters or exogenous inputs. Thus, we do not follow the widely-used practice in the business cycle literature of including the same number of exogenous inputs as observed time series, which is done to ensure a perfect fit between data and theory.

Here, there are two sequences of TFP parameters that are free in the analysis and many time series that must be in conformity with the theory. We find that the equilibrium paths of our extended theory are in close conformity with time series of both NIPA products and incomes and, most importantly, with the increase in capital gains that occurred in the second half of the 1990s. This increase was large, with the average real gains going from 6 percent of GDP in the period 1953–1994 to 12 percent of GDP in the period 1995–2003. Data on factor incomes and capital gains are not used to identify the TFP parameters. In contrast, a theory based on a large shift in preferences for leisure during the 1990s does not account for the observed changes in factor incomes and capital gains.[4]

After demonstrating that the model's predictions are in conformity with U.S. time series, we use the model to compare current accounting measures for investment and labor

market distortions, which are especially important in accounting for changes in hours. In McGrattan and Prescott (2007), we show that labor tax rates do account for much of the cyclical variation in hours prior to the 1990s but not in the 1990s.

[4] See McGrattan and Prescott (2007) for details.

productivity with corresponding measures that include expensed and sweat investment. Solow's (1987) remark that "you can see the computer age everywhere but in the productivity statistics" is pertinent for our findings. If expensed and sweat investments are included, the model predicts an earlier and larger boom in productivity than current accounting measures would indicate. Based on this prediction, we conclude that ignoring these two types of intangible investment distorts the true picture of the U.S. economy in the 1990s.

Our findings show that standard productivity measures greatly understate the actual rise in labor productivity whether we consider the overall economy or the business sector. Our analysis is not subject to the criticism of Brynjolfsson and Hitt (2000) who point out that intangible investment is "not well captured by traditional macroeconomic measurement approaches." Here, we explicitly model the intangible investment. Accounting for intangible investment, we find that the boom in business productivity began earlier and was bigger than standard statistics show. Over the period 1993–2000, the difference in labor productivity growth due to the inclusion of intangible investment is 0.8 percent per year.

The paper is organized as follows. In Section 1, we establish that basic theory—abstracting from unmeasured investment—generates strongly counterfactual predictions. In particular, the basic growth model does not generate a boom in the 1990s. If we extend the model to have two sectors, a private business sector and a non-business sector, then there is some improvement in the model's predictions given that business TFP boomed in the late 1990s. However, the rise in TFP is too small and occurs too late to account for the boom in hours that began in 1992. In Section 2, we summarize the evidence of increased intangible investment which motivates our extension of the basic theory. The extended theory includes expensed and sweat investment, and in Section 3, we assess its predictions. In Section 4, we reevaluate the performance of the U.S. economy in the 1990s through the lens of the extended theory. Conclusions are found in Section 5.

1. Predictions of the Basic Theory without Intangible Investment

Our starting point is the basic growth model used in the study of business cycles. This model abstracts from intangible investment. We treat TFP, tax rates, and population exogenously. We then use U.S. data for the 1990s to establish that there are large deviations from the basic model, indicating that this model must be abstracting from something important. We show that the nature of the deviations points to unmeasured investments.

1.1. The Basic Growth Model

In the standard one-sector growth model, given initial capital stock k_0, the problem for the stand-in household is to choose consumption c, investment x, and hours h to maximize

$$E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t \tag{1.1}$$

subject to the constraints

$$\begin{aligned} c_t + x_t = r_t k_t + w_t h_t - \tau_{ct} c_t - \tau_{ht} w_t h_t - \tau_{kt} k_t - \tau_{pt}(r_t k_t - \delta k_t - \tau_{kt} k_t) \\ - \tau_{dt}\{r_t k_t - x_t - \tau_{kt} k_t - \tau_{pt}(r_t k_t - \delta k_t - \tau_{kt} k_t)\} \end{aligned} \tag{1.2}$$

$$k_{t+1} = [(1-\delta)k_t + x_t]/(1+\eta), \tag{1.3}$$

where variables are written in per capita terms and $N_t = N_0(1+\eta)^t$ is the population in t. Capital is paid rent r_t, and labor is paid wage w_t. Households discount future utility at rate β, and capital depreciates at rate δ. Taxes are levied on consumption at rate τ_c, labor income at rate τ_h, tangible capital (that is, property) at rate τ_k, profits at rate τ_p, and capital distributions at rate τ_d. Note that taxable income for the tax on profits is net of depreciation and property tax, and taxable income for the tax on distributions is net of property tax and profits tax.

The aggregate production function is

$$Y_t = A_t F(K_t, H_t), \tag{1.4}$$

where capital letters denote aggregates. The parameter A_t is TFP that varies over time. The firm rents capital and labor. If profits are maximized, then the rental rates are equal to the marginal products. The goods market clears so $N_t(c_t + x_t) = Y_t$. Here, c includes both private and public consumption, and x includes both private and public investment.

We now show that this basic growth model generates grossly counterfactual predictions during the 1990s. To do so, we compute the model's equilibrium path with households having perfect foresight of future changes in tax rates, TFP, and populations. In Appendix A, we discuss our U.S. data sources and the adjustments we make to construct the empirical counterparts of the model variables. In Appendix B, we describe and motivate the parameterization we use for the model. The parameter values used to compute the equilibrium path here are summarized in Table B.1 under "One-Sector Model, No Intangible Investment." The paths for TFP and tax rates are reported in Table B.2.[5]

Our estimate for TFP is U.S. GDP divided by $F(K, H)$, with K equal to the total stock of U.S. tangible capital and H equal to total hours. Here, $F(K, H) = K^\theta H^{1-\theta}$. The tax rate changes we consider are variations in the labor tax rates τ_{ht} and consumption tax rates τ_{ct}, as constructed in Prescott (2004) with data from U.S. national accounts. During the 1990s, there was little change in legislation affecting capital taxation, and therefore we simply fix the rates τ_{kt}, τ_{pt}, and τ_{dt}.

The utility flow function is

$$U(c, h) = \log(c) + \psi \log(1 - h),$$

which is standard in the business cycle literature. We choose the level of capital tax rates, the depreciation rate δ, and the utility parameter ψ so that the model's consumption share, investment share, factor inputs, and tax revenues are consistent with U.S. levels in 1990. (See appendices A and B for details.)

In Figure 1, we plot the model's predicted per capita hours of work along with the U.S. actual per capita hours, indexed so that 1990 equals 100. The difference between the series is striking. Actual per capita hours rose 8 percent between 1992 and 1999, whereas the predicted series falls significantly during the same period.

[5] In McGrattan and Prescott (2007), we demonstrate that the perfect foresight assumption is innocuous by comparing the results to stochastic simulations. We also demonstrate the robustness of the results by varying parameters.

In Figure 2, we plot the model's predicted output along with U.S. real GDP. Both series are adjusted for population and a secular trend of 1.02^t. Although the boom in output was not quite as large as the boom in hours, the model predicts that the economy should have been depressed. This counterfactual prediction arises from the fact that the tax rates on labor rose during the 1990s and economy-wide TFP was below trend during most of the decade.

The basic model has neutral TFP change with respect to the business and non-business sectors. In fact, TFP change was non-neutral for these sectors. A question that arises is whether modeling this non-neutrality of TFP change could significantly narrow the large deviation from theory. In the two-sector version of the model, households solve the same problem, although they now have to allocate capital and labor to two technologies, "business" and "non-business." (See McGrattan and Prescott (2007) for model details.)

In Appendix A we describe how we categorize business and non-business activity. In Appendix B, we describe and motivate the parameterization for this version of the model. Parameter values used to compute the equilibrium path here are summarized in Table B.1 under "Two-Sector Model, No Intangible Investment." As before, the paths for TFP and tax rates are exogenous. These are reported in Table B.2. We assume that households in the model make exactly the same choices for non-business activity as U.S. households. We simply set the values of non-business hours, investment, and value added exogenously to U.S. levels. We treat this sector exogenously because it is quite small compared to the business sector. Furthermore, if there are any deviations between the model's predictions and U.S. data, we can attribute these deviations to our model of the business sector, which did have a boom in TFP.

We find that modeling the non-neutrality of TFP in business and non-business activity does not resolve the puzzling 1990s boom. Model predictions are still far from observations. In particular, output is still predicted to be below trend throughout the decade, and per capita hours are still predicted to be low. (Figures of these series and others are in McGrattan and Prescott 2007.) The boom in business-sector TFP is too small and too late. Clearly, something else gave rise to the puzzling behavior of the U.S. economy in the

1990s.

1.2. Investigating the Deviation

Why are the model's predictions so far off for the basic neoclassical growth model? The main reason is the behavior of TFP and tax rates. Given the behavior of these inputs, the model predicts an after-tax real wage, $(1-\tau_h)w$, below its secular trend. Not surprisingly, then, the model predicts that hours are low and output is below trend.

We turn next to evidence that using the wrong measure of output and understating labor productivity and, therefore, the after-tax real wage accounts for the large deviation between theory and data. The mismeasurement stems from abnormally large unmeasured intangible investment during this period. Standard measures of output growth are distorted when the importance of intangible investments grows.

2. Evidence of Increased Intangible Investment

We present two types of evidence that unmeasured intangible investment was abnormally large during the 1990s. One type of evidence is related to the behavior of NIPA compensation and profits, the other type to the technology boom going on during the period. Because intangible investments are expensed in the NIPA, measurements of factor incomes are understated to a greater extent in periods when these investments are high. We show that that was true for the 1990s. We then present evidence that, in fact, during the technology boom, the level of investment was indeed high and led to large capital gains that are missed by the NIPA's income measure.

2.1. Low Compensation and Corporate Profits

If all incomes were included in the national accounts, we would expect to see compensation per hour and profits to be high during a boom. But an examination of the U.S. national accounts reveals that compensation per hour and profits were low during the boom period, suggesting that unmeasured expensed and sweat investment was abnormally high.

In Figure 3, we plot average weekly hours of work for the noninstitutional population, age 16 to 64, which is the same series plotted in Figures 1 and 2. We also plot the wage rate corresponding to these hours, which is computed as follows. We take NIPA compensation and deflate it by the GDP deflator. We then correct for population growth by dividing real compensation by this population. Finally, because there is technological growth, we divide the wage rate by the factor 1.02^t, where t indexes time.[6] For all of the 1990s, NIPA real compensation per hour detrended in this way is below the 1990 level, despite the boom in hours.[7]

In Figure 4, we compare NIPA GDP and corporate profits, both deflated by the GDP deflator and adjusted for population and a secular trend of 1.02^t. We see that profits fall in the late 1990s when GDP, R&D, and capital gains are high.

2.2. The Technology Boom

We have found additional direct evidence that unmeasured investment was abnormally high in the 1990s. The 1990s was a period of rapid technological advances. Companies were increasing R&D, and the payoffs were evident in increased initial public offerings (IPOs) and mergers and acquisitions. Further, the rise in hours was particularly large for the more educated in occupations in which people make large sweat investments on average.

One indicator of increased intangible investment is increased funding of R&D, which is expensed by corporations. The National Science Foundation (NSF) (1953–2003) reports that industry R&D increased 68 percent between 1994 and 2000, whereas GDP rose only 39 percent. A significant fraction of the company-funded R&D was done by companies in information technology industries. Using data from the NSF, Doms (2004) estimates that the information technology share of company-funded R&D averaged 27 percent in the period 1997–2001.

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[6] The particular choice of 1.02 for the secular trend does not affect any results, but makes it easier to see the patterns in this and later figures.

[7] In earlier work (McGrattan and Prescott 2005b), we abstract from sweat equity investment and treat NIPA compensation as true labor income. Doing so reduces the estimate of intangible investment.

Another indicator of abnormally large intangible investment is the dramatic increase in IPOs and mergers and acquisitions. According to data from the Thomson Financial Securities Data Corporation (SDC) database (also available in Ritter and Welch 2002, Table 1), gross proceeds from IPOs were significantly higher in the 1990s than in the 1980s. Gross proceeds of IPOs averaged \$8.2 billion over the period 1980–89 and \$30.9 billion over the period 1990–99. Large increases in the value of existing equity, and therefore large capital gains, are typically associated with IPOs. Because these gains are not included in NIPA, NIPA incomes understate true income. Other related evidence available from the SDC database is the volume of announced mergers and acquisitions. The volume rose from \$0.6 trillion in 1994 to \$3.4 trillion by 2000. As in the case of IPOs, the accrued capital gains are not included in NIPA measures of income.

We have presented evidence that business owners made abnormally large unmeasured investments and accrued abnormally large capital gains during the 1990s boom. We now present evidence that the hours of certain categories of workers that tend to have a disproportionate number of sweat investors increased disproportionately in the 1992–2000 period. These categories include those who have had at least a year of college education and who work as managers, proprietors, computer analysts, and in certain financial occupations. We use Current Population Survey (CPS) data to determine hours worked in these categories.[8] These categories of workers accounted for 50 percent of the increase in hours between 1992 and 2000, even though they accounted for only 10 percent of hours in 1992.

The fall in hours in 2000 was coincident with a fall in information technology investment, which occurred partly because of regulatory factors. According to Couper, Hejkal, and Wolman (2003) and Doms (2004), the telecommunications sector contributed significantly to the *technology bust*. Part of the problem was that the demand for long-haul fiber capacity was not as great as anticipated. Another problem was what Federal Communications Commission chairman Michael Powell called "the problem of legal instability in the

[8] Here, we are referring to data compiled from the March supplement of the CPS survey. We split workers into two groups: those with variable EDUCREC greater than or equal to 8 and variable OCC in the set {4, 7, 9, 13, 14, 15, 18, 21, 22, 34, 37, 64, 65, 229, 23, 24, 25, 225} and the remainder. See Ruggles et al. (2004) for more details.

court system" (Powell 2002). The Telecommunications Act of 1996 brought competition to local telephone service, but it generated many legal battles as well. According to Couper, Hejkal, and Wolman (2003), the regulatory uncertainty has discouraged telecommunications companies from undertaking the large-scale investments needed for fiber-optic service to individual residences.

In summary, we find compelling evidence that intangible investment was abnormally high during the 1990s. This motivates an extension of the basic model with both tangible and intangible investment and technological change that gives rise to an increase in the importance of intangible investment.

3. Predictions of the Extended Theory with Intangible Investment

We now extend the basic growth model to include intangible investment and non-neutral technological change with respect to production of final goods and services and production of new intangible investment goods. Intangible investments are made by businesses, so the extended model distinguishes business and non-business activity. We start by describing the technologies available to businesses, the optimal business size, and the aggregate production technology. The household problem remains the same as in Section 2 except for an additional investment choice. We examine the extended model's predictions and show that these predictions are in conformity with U.S. observations during the 1990s.

3.1. Extensions

The aggregate production technology is characterized by the two aggregate production relations:

$$y_{bt} = A_t^1 (k_{Tt}^1)^{\theta_1} (k_{It})^{\phi_1} (h_t^1)^{1-\theta_1-\phi_1} \tag{3.1}$$

$$x_{It} = A_t^2 (k_{Tt}^2)^{\theta_2} (k_{It})^{\phi_2} (h_t^2)^{1-\theta_2-\phi_2}. \tag{3.2}$$

Firms produce business output y_b using their intangible capital k_I, tangible capital k_T^1, and labor h^1. Firms produce intangible capital x_I—such as new brands, new products R&D, patents, etc.—using intangible capital k_I, tangible capital k_T^2, and labor h^2.

Note that k_I is an input to both business sectors; it is not split between them as is the case for tangible capital and labor. A brand name is used both to sell final goods and services and to develop new brands. Patents are used by the producers and the researchers. The aggregation theory underlying this technology is developed in Appendix C.

Given (k_{T0}, k_{I0}), the stand-in household maximizes

$$E \sum_{t=0}^{\infty} \beta^t [\log c_t + \psi \log(1-h_t)] N_t$$

subject to

$$\begin{aligned}
c_t + x_{Tt} + q_t x_{It} = {} & r_{Tt} k_{Tt} + r_{It} k_{It} + w_t h_t + \zeta_t \\
& - \tau_{ct} c_t - \tau_{ht}(w_t h_t - (1-\chi) q_t x_{It}) - \tau_{kt} k_{Tt} \\
& - \tau_{pt}\{r_{Tt} k_{Tt} + r_{It} k_{It} - \delta_T k_{Tt} - \chi q_t x_{It} - \tau_{kt} k_{Tt}\} \\
& - \tau_{dt}\{r_{Tt} k_{Tt} + r_{It} k_{It} - x_{Tt} - \chi q_t x_{It} - \tau_{kt} k_{Tt} \\
& \quad - \tau_{pt}(r_{Tt} k_{Tt} + r_{It} k_{It} - \delta_T k_{Tt} - \chi q_t x_{It} - \tau_{kt} k_{Tt})\}
\end{aligned}$$

$$k_{T,t+1} = [(1-\delta_T) k_{Tt} + x_{Tt}]/(1+\eta) \tag{3.3}$$

$$k_{I,t+1} = [(1-\delta_I) k_{It} + x_{It}]/(1+\eta). \tag{3.4}$$

As before, all variables are in per capita units and there is growth in population at rate η. Consumption c includes both private and public consumption, and tangible investment x_T includes both private and public investment. The relative price of intangible investment and consumption is q.[9] The rental rates for business tangible and intangible capital are denoted by r_T and r_I, respectively, and the wage rate for labor is denoted by w. Inputs are paid their marginal products. The tax system is the same as in the standard model. Other income is denoted by ζ and is exogenous in the household's decision problem. Other income includes government transfers and non-business capital income net of taxes and investment. Non-business labor income is included in wh.

As before, we treat hours, investment, and output in the non-business sector exogenously because this sector is not important for the issues being addressed. To be precise,

[9] For some purposes, modeling the changes in technology that induce changes in the relative price of c_t and x_{Tt} is important. See, for example, Hornstein and Krusell (1996) and Greenwood, Hercowitz, and Krusell (1997). For our purposes, it is reasonable to abstract from this change in technology.

we set the levels of non-business hours $\bar{h}_n$, investment $\bar{x}_n$, and output $\bar{y}_n$ in the model's non-business sector equal to U.S. levels. Measured output, which corresponds to GDP, is the sum of y_b and $\bar{y}_n$. Measured tangible investment is the sum of business tangible investment x_T and non-business tangible investment $\bar{x}_n$. Measured hours h is the sum of business hours $h^1 + h^2$ and non-business hours $\bar{h}_n$.

Let χ denote the fraction of intangible investment financed by capital owners. The amount $\chi q x_I$ is *expensed investment*, financed by the capital owners who have lower accounting profits the greater this type of investment. The amount $(1-\chi)qx_I$ is *sweat investment*, financed by workers who have lower compensation the greater this type of investment.[10]

Gross domestic product in the economy is the sum of total consumption (public plus private) and tangible investment (public plus private) for business and non-business; in per capita terms GDP is $c + x_T + \bar{x}_n$. Gross domestic income (GDI) is the sum of all labor income less sweat investment $wh - (1-\chi)qx_I$, business capital income less expensed investment, $r_T k_T + r_I k_I - \chi q x_I$, and non-business capital income (which is found residually as the difference between GDP and the other components of GDI).

3.2. A Resolution of Seemingly Low Wages

We showed earlier that there is a large deviation between predictions of the basic one-sector and two-sector growth models and U.S. data. The models predict that after-tax real wages in the 1990s should have been below trend, leading to low per-capita hours and output below its secular trend. With our extended model, the measure of the real wage is different and is consistent with the behavior of output and hours.

The standard model measure of the business real wage is

$$\hat{w}_t = (1-\theta)y_{bt}/(h_t^1 + h_t^2), \tag{3.5}$$

where θ is the capital share, y_{bt} is measured business value added, and $h_t^1 + h_t^2$ is total

[10] In the absence of informational or financial constraints, the choice depends in a knife-edge way on the tax treatment of the expensed and sweat investment. This is analogous to the result of Miller (1977) for the debt-equity choice.

business hours. The problem with the measure of labor productivity on the right side of equation (3.5) is that some hours are used to accumulate intangible capital. The hours used to produce y_{bt} are h_t^1 and, therefore, the real wage measure is

$$w_t = (1-\theta)y_{bt}/h_t^1, \tag{3.6}$$

where $\theta = \theta_1+\phi_1$ and y_{bt}/h_t^1 is labor productivity in production of final goods and services. The labor input h_t^2 is used to produce output $q_t x_{It}$ and is not part of the labor input in producing y_{bt}. If the relative size of h_t^2 relative to h_t increases, then $w_t/\hat{w}_t$ increases and the percentage understatement of true wages becomes more severe.

The evidence presented earlier suggests that advances in technology were particularly large in activities related to intangible production. This would imply an increase in A_t^2/A_t^1. Our hypothesis is that A_t^2/A_t^1 did indeed increase significantly and that such an increase leads to an increase in the relative hours allocated to producing intangible investments, namely h_t^2/h_t.

3.3. Identifying Total Factor Productivities

In order to identify total factor productivities, the magnitude of the inputs and the outputs to the production functions must be determined. This requires determining the split of hours and tangible capital between two production activities in the business sector and the magnitude of intangible investment and capital.

To determine how much labor is allocated to the two production activities, we use the fact that the after-tax real wage rate is equal to the marginal rate of substitution between leisure and consumption, $\psi(1+\tau_{ct})c_t/(1-h_t)$. We have observations on consumption c, total hours h, business value added y_b, and tax rates. We use these observations to determine hours in production of final goods and services as follows:

$$h_t^1 = \left(\frac{1-\theta_1-\phi_1}{\psi}\right)\left(\frac{1-\tau_{ht}}{1+\tau_{ct}}\right)\left(\frac{y_{bt}}{c_t}\right)(1-h_t). \tag{3.7}$$

Hours in the accumulation of intangible capital is determined residually, $h_t^2 = h_t - h_t^1 - \bar{h}_{nt}$. Equation (3.7) is simply a rewriting of the household's intratemporal condition relating the marginal rate of substitution between leisure and consumption to $(1-\tau_h)w$ using w in (3.6).

Equating the marginal products of labor in the two activities yields an equation that can be solved for $q_t x_{It}$ as a function of the hours h^1 and h^2 determined above and business value added y_b which is observed,

$$q_t x_{It} = \left(\frac{1-\theta_1-\phi_1}{1-\theta_2-\phi_2}\right)\frac{y_{bt}}{h_t^1}\,h_t^2. \tag{3.8}$$

The allocation of tangible capital across the two activities in the business sector is determined in a similar way. The initial stock $k_{T,1990}$ and the sequence of business tangible investments imply the sequence of stocks $\{k_{Tt}\}$ from (3.3). Equating the marginal products of tangible capital across the two business activities implies

$$k_{Tt}^1 = \left(\frac{\theta_1 y_{bt}}{\theta_1 y_{bt} + \theta_2 q_t x_{It}}\right) k_{Tt},$$

where $k_{Tt}^2 = k_{Tt} - k_{Tt}^1$ is residually determined.

If we have a sequence for the price q_t of intangible investment, we could use the already computed sequence of outputs $q_t x_{It}$ to determine a sequence for intangible investment x_{It}, and, with an initial condition for the stock $k_{I,1990}$, we could use (3.4) to determine the sequence of stocks.[11] But we do not have q_t and, therefore, we use the intertemporal condition

$$1 = \beta\left(\frac{1+\tau_{ct}}{1+\tau_{c,t+1}}\right)\left(\frac{U_c(c_{t+1},h_{t+1})}{U_c(c_t,h_t)}\right) R_{t,t+1}, \tag{3.9}$$

where $R_{t,t+1}$ is the after-tax return realized by the household investing in intangible capital,

$$\begin{aligned} R_{t,t+1} = \Big\{ & q_{t+1}(1-\chi)(1-\tau_{h,t+1})(1-\delta_I) + (1-\tau_{p,t+1})(1-\tau_{d,t+1})[q_{t+1}\chi(1-\delta_I) \\ & + (\phi_1 y_{b,t+1} + \phi_2 q_{t+1} x_{I,t+1})/k_{I,t+1}]\Big\} / \\ & \Big\{ q_t[(1-\chi)(1-\tau_{ht}) + \chi(1-\tau_{pt})(1-\tau_{dt})]\Big\}, \end{aligned} \tag{3.10}$$

along with the capital accumulation equation and the already identified sequence for $q_t x_{It}$. This relation implicitly identifies q_{t+1} as a function of current variables and q_t. As a

[11] We use the steady-state stock to initialize intangible capital.

terminal condition, we assume that the price in the year following the end of our sample is equal to the price in the last year of our sample.

The final step in identifying the TFP parameters is to use the production functions (3.1) and (3.2) to determine the $\{A_t^1,\ A_t^2\}$. The resulting sequences for these two TFPs are plotted in Figure 5 along with a more standard measure of business-sector TFP that abstracts from intangible capital: business value added divided by $k_{Tt}^{0.33}(h_t - \bar{h}_{nt})^{0.67}$, where k_{Tt} is tangible capital in the business sector and $h_t - \bar{h}_{nt}$ is total business hours. The latter measure is labeled "U.S. Business Sector." All series are real and divided by 1.02^t.

The standard measure of business-sector TFP shows some acceleration beginning in 1996. The implied TFPs for the model with intangible investment show larger increases that begin earlier. For example, in the sector producing final goods and services, predicted growth in TFP between 1993 and 2000 is 0.7 percent greater per year than is found by constructing the standard measure. In the sector producing intangible capital, the implied growth in TFP between 1993 and 2000 is 2.7 percent greater per year than that found with the standard measure. All three measures show some decline after 2000, which could well have been due to regulatory and legal factors impinging negatively on efficiency.

The patterns of the TFP sequences are consistent with the micro and macro evidence we cite in Section 2. For this reason, we view this theory as one that satisfies our exogenous input justification criterion. We next show that the theory satisfies our prediction criterion.

3.4. Model Predictions

Treating the TFP sequences as exogenous inputs, we compute the equilibrium path of all of the variables and compare them to U.S. data. All of the parameters used in computing the equilibrium are described and motivated in Appendix B and summarized in Table B.1, under the heading "Extended Model, with Intangible Investment," and in Table B.2.[12]

In Figure 6, we display the implied intangible investment as a share of total output,

[12] Income shares listed in Table B.1 are the same across activities. We do sensitivity checks on these shares and other parameters of the model to ensure that the main quantitative results are robust. See McGrattan and Prescott (2007).

by which we mean GDP plus intangible investment. This figure displays the bottom line of our study: the value of this investment is large and increased dramatically in the 1990s. That is precisely what we see in Figure 6. Our estimate of *net* intangible investment—expensed plus sweat—in the business sector is a little over 3 percent of total output—GDP plus intangible investment—in 1990 and rises to nearly 8 percent of total output before returning to the level of the early 1990s.[13]

We now assess the conformity of two sets of predictions. The first are predictions of variables used to identify the sequences of TFP in Figure 5. These series are hours and components of GDP. The second are predictions of variables that are not used to identify the sequences of TFP. These series are NIPA factor incomes and capital gains reported in the U.S. Flow of Funds (1945–2005); the latter is especially important given the central role that capital gains play in our extended theory. We find that the model is in conformity with both sets of predictions.

Internal Conformity

We start with a comparison to total hours and to components of GDP. Note that although we used equilibrium conditions to identify the TFP parameters, the predicted and actual series may differ because we used *only one of the two* intertemporal conditions when determining the TFP paths. Condition (3.9) relates the marginal rate of substitution in consumption between period t and $t+1$ to the after-tax return on investing in intangible capital. The second intertemporal condition, which was not used, relates the marginal rate of substitution in consumption to the after-tax return on investing in tangible capital. If the latter condition is not satisfied by the data, the predicted and actual paths will differ.

Figure 7 shows the results for per capita total hours worked. Unlike the comparable figure with the standard model's predictions (Figure 1), here, the predictions and the actual series track each other closely. The extended model predicts a fall in hours used to produce final goods and services during the 1990s. However, because hours spent building intangible capital rise significantly, the model predicts the large overall increase in hours

[13] The estimate of net intangible investment here exceeds earlier estimates in McGrattan and Prescott (2005a). In earlier work we did not include sweat investment or noncorporate business activity.

worked h.

Similarly, the model and data paths for GDP are close. We plot these paths in Figure 8. The model's prediction for business value added is also close because, in the model, GDP is the sum of business value added y_b and non-business value added $\bar{y}_n$, where $\bar{y}_n$ is preset to be the same as in the United States. In the appendix, we display time series for consumption, tangible investment, and business labor productivity, which are all close in comparison to their U.S. analogues.

External Conformity

Now we consider a more demanding test of the theory: comparing model predictions to observations not used to determine the TFP paths. In particular, we compare predictions for business wage compensation as measured in the NIPA and for business capital gains as measured in the Flow of Funds accounts. We find that the model predicts these series remarkably well.

To compare the model's prediction for NIPA wage compensation in the business sector, we need to construct wages as a national accountant would. Such an accountant, placed in the model economy, would report wage compensation in the business sector as $w_t(h_t^1 + h_t^2) - (1-\chi)q_t x_{It}$, in effect, not including the value of sweat equity investment. In Figure 9, we plot this predicted series along with the actual U.S. series. Both are real series, detrended by 2 percent annually and set equal to 100 in 1990. The two are close. Relative to the 1990 trend level, both the model prediction and the actual wages are up nearly 8 percent in 2000. We note that our choice of $\chi = 0.5$ is relevant for this prediction. The value of χ determines the level of taxation on expensed versus sweat equity, which affects the equilibrium measured compensation. Higher values of χ increase the predicted value of compensation. We selected $\chi = 1/2$ given that we do not have independent evidence of the financing of expensed and sweat equity. In McGrattan and Prescott (2007), we show that our results are not sensitive to the choice of χ unless χ is far from 1/2.

Next, we compare the model's predictions for estimates of the increase in capital gains from expensed and sweat equity to U.S. household holding gains reported in the Flow of

Funds accounts. Those gains are the change in the value of assets outstanding (taken from Table L.100) less the net purchases during the period (taken from Table F.100). If Flow of Funds accountants recorded holding gains for our model households, they would compute differences in the total value of businesses (for which the household is the residual claimant). The value of all businesses in t, V_t, is composed of two parts:

$$V_t = (1-\tau_{dt})K_{T,t+1} + [\chi(1-\tau_{dt})(1-\tau_{pt}) + (1-\chi)(1-\tau_{ht})]q_t K_{I,t+1}, \tag{3.11}$$

where capital letters denote aggregates. On the right side of (3.11), the first term is the value of tangible capital and the second is the value of intangible capital. Notice that the price of intangible capital depends on χ, since income to capital and income to labor are taxed differently.

The change in the value V_t of businesses does not exactly reflect the additional income in the model economy. The additional income is $q_t X_{It}$ (in units of the final goods and services). However, during periods with large investments of intangible capital, the increase in holding gains, as defined in the Flow of Funds accounts, is a good approximation to the increase in intangible investment.

So that our estimates are comparable with the U.S. Flow of Funds, we make an adjustment to our model estimate to add foreign gains because our model includes only domestic sectors. Since many domestic corporations have foreign subsidiaries, the value of U.S. corporations includes equity from foreign capital, and the holding gains include gains from this foreign capital. We estimate these gains by assuming that the ratio of after-tax foreign corporate profits (excluding gains) to after-tax domestic corporate profits (excluding gains) is equal to the ratio of foreign to domestic holding gains. With this assumption, our estimate of foreign gains relative to total gains is approximately 23 percent on average for the period 1990–2003.

A significant break in U.S. real holding gains (relative to GDP) occurred in 1995. Before that year, the series averages around 6 percent of GDP. In 1995 and thereafter, the average is 12 percent. A difference of 6 percent of GDP is economically large. To determine whether the difference is statistically significant, we ran the following statistical test. We ran the regression

$$g_i = \alpha + \beta d_i,$$

where g_i is the real gain in decade i and d_i is a dummy variable, which is 1 for the 1995–2004 decade and 0 for the four decades preceding it (which is when data are available). Our estimate for β is 6.2 percent with a standard error of 2.9 percent, indicating that the change in the mean is statistically significant.

In Figure 10, we plot average real holding gains relative to GDP for the United States along with the extended model's prediction for them. Both curves rise significantly in the late 1990s. The rise is coincident with the dramatic rise in hours.

These results lead us to the conclusion that the model satisfies our prediction criterion.

3.5. Is Success Guaranteed?

A crucial element of the theory is intangible investment, which is directly measurable only in part. We treat the total as unmeasurable and, therefore, unobserved. Does this imply that intangible investment is simply making up for whatever is missing in the standard theory? In this section (and more fully in McGrattan and Prescott 2007), we demonstrate that intangible capital per se does not resolve the puzzling 1990s. If we extend the basic neoclassical model to include intangible capital but assume that technological change is *neutral* then the theory does not satisfy the two criteria we require to successfully resolve the puzzling 1990s boom.

In the alternative version of the model, we introduce a sequence of labor wedges $\{L_{wt}\}$ that are chosen so that the household's intratemporal first-order condition is satisfied,

$$\frac{\psi(1+\tau_{ct}c_t}{1-h_t} = L_{wt}(1-\tau_{ht})w_t,$$

and assume that the TFPs in (3.1) and (3.2) vary proportionally. The wedges are proxies for labor distortions other than government taxes on labor. If the income shares are common in the two activities, as assumed above, the relative price of intangible investment q is constant. We normalize it to one. As above, there are two "free" parameter sequences. In this case, they are $\{A_t^1, L_{wt}\}$.

We find the implications of the alternative model are grossly at odds with what is reasonable behavior for labor distortions and intangible investments. The resulting sequences of $\{L_{wt}\}$ and $\{x_{It}\}$ oscillate wildly. For example, the series for the labor wedge oscillates between 0.8 and 1.4 and displays little persistence. Given what we know about labor markets, this pattern is unreasonable. Furthermore, an hours boom is generated only if TFP and capital tax rates are also oscillatory and offsetting, which is unreasonable in the case of TFP and counterfactual in the case of capital tax rates.

In summary, it is not the inclusion of intangible capital per se that resolves the puzzling U.S. boom in the 1990s. We find that including both intangible capital and non-neutral technological change resolves the puzzle. We turn now to using our theory as a tool for uncovering what actually did happen in the 1990s.

4. Reevaluation of the U.S. Economy in the 1990s

What does all of this mean for U.S. labor productivity and investment? If some output is unmeasured relative to inputs, then GDP and productivity estimates are biased downward. If the mismeasurement is intangible investment, then the investment estimates are also biased downward. Our extended model's predictions for variables with and without intangible investment demonstrate how distorted standard data and models are for assessing the 1990s.

In Figure 11, we compare two predictions for business labor productivity, both computed from the extended model. One is the model's prediction for business value added—without intangible investment included—divided by total business hours. This is what a national accountant would construct. The other includes intangible investment as part of business value added. Both series are detrended by 2 percent annually and set equal to 100 in 1990. Notice how different the predictions are. Measured labor productivity, which is what national accountants would record, shows a significant fall relative to trend up to 1997 and then a sharp increase through 2000. But true productivity, including intangible investment, fell only until 1993 and then, starting in 1994, grew very quickly. Over the period 1993–2000, the difference in growth rates for these two series is 0.8 percent per year.

In Figure 12, we compare the model's two measures of total investment: tangible investment and tangible plus intangible investment. Again, both are detrended by 2 percent annually and normalized to 100 in 1990. And again, the predictions—with and without intangible investment—are very different. Between 1991 and 1999, tangible investment rose almost 20 percent. Total investment, however, rose more than 30 percent.

In summary, our results show that standard accounting measures and predictions of models without intangible investment do not accurately reflect what was going on in the U.S. economy during the 1990s.

5. Conclusion

We find that non-neutral technological change in the production of intangible investment goods was what gave rise to the puzzling behavior of the U.S. economy in the 1990s. This change resulted in a boom in intangible investment, which is not included in the national accounts measure of output. Once this feature of reality is introduced into the basic neoclassical growth model to obtain what we call the extended growth model, we see that the U.S. economy in the 1990s is in close conformity with theory for both the income and product sides of the national accounts and for the jump in average accrued capital gains. Furthermore, microeconomic observations strongly support our theory of non-neutral technological change.

The important implication of our analysis is that our extended growth model with intangible investments should be used in aggregate economic analyses. Indeed, we see it as a significant improvement on the basic neoclassical growth model.

Appendix A.

Data Appendix

The three main sources for our data are the Bureau of Economic Analysis (BEA), which publishes the national accounts and fixed asset tables; the Federal Reserve Board, which publishes the Flow of Funds tables; and the Bureau of Labor Statistics, which publishes data on hours and population. In this appendix we provide details on the specific data we use and the necessary revisions we make so that the data are consistent with growth theory.

A.1. National Accounts and Fixed Assets

A.1.1. Overview and Sources

Table A contains a summary of the revised national accounts along with averages for the period 1990–2003. The table numbers and sources of the raw data are listed in parentheses. The sources are tables from the BEA's national income and product accounts (NIPA) and fixed asset (FA) tables, and the Federal Reserve's Flow of Funds accounts (FOF). For example, NIPA 7.5 is Table 7.5 from the BEA NIPA tables. The averages for the period we study are relative to GDP and are included to help the reader with the magnitudes of each of the categories.

We have organized Table A as follows. Tables A1 and A2 are the income side of our revised accounts. In Table A1, we display the components of our measure of domestic business value added. This measure is close to, but not exactly the same as, the sum of the value added of corporate business, sole proprietorships and partnerships, and other private business as defined in the NIPA tables. In Table A2, we display the components of our measure of domestic non-business value added. This measure is close to, but not exactly the same as, the sum of value added of the household business sector, nonprofits, general government, and government enterprises. Table A3 provides details of the product side of the accounts along with totals for the income side (for comparison). We have categorized tangible investment into business and non-business as in the case of incomes. That is, investments of corporations and noncorporate business are included with business investment, and investments of household business, nonprofits, and government are included with non-business investment.

Data on capital stocks are used to impute some services of capital when we revise the accounts. They are also used to set certain model parameters and to initialize stocks when computing model equilibria. We use BEA reproducible stocks (FA Table 1.1 for totals and FA Table 6.1 by owner). To that we add land values based on Federal Reserve market values of real estate from balance sheets of households (FOF B100), nonfarm nonfinancial corporations (FOF B102), and nonfarm noncorporate (FOF B103). For farmland, we follow Hansen and Prescott (2002) and assume it is roughly 0.08 times GDP.

A.1.2. Revisions

We now describe how we revise the national accounts to make them consistent with our model. Three adjustments are necessary in the models with or without intangible investment. A fourth adjustment is necessary when we include intangible investment.

Consumption Taxes. Unlike the NIPA, our model output does not include consumption taxes as part of consumption and as part of value added. We thus subtract sales and excise taxes from the NIPA data on taxes on production and imports (line 26, Table A1 and line 24, Table A2) and from personal consumption expenditures (line 10, Table A3) since these taxes primarily affect consumption expenditures. As a result of this adjustment, we use producer prices rather than a mixture of producer and consumer prices.

Financial Services. We treat some of the NIPA's financial services as intermediate rather than as final and, therefore, need to subtract them from GDP and from consumption services. Specifically, we subtract personal business expenses for handling life insurance and pension plans from net interest (line 21, Table A1) and from personal consumption expenditures (line 9, Table A3).

Fixed Asset Expenditures. We treat expenditures on all fixed assets as investment. Thus, spending on consumer durables is treated as an investment rather than as a consumption expenditure and moved from private consumption (line 8, Table A3) to non-business tangible investment (line 22, Table A3). We introduce a consumer durables services sector in much the same way as the NIPA introduces owner-occupied housing services. Households rent the consumer durables to themselves. Specifically, we add depreciation of consumer durables to consumption of fixed capital of households (line 5, Table A2) and to private consumption (line 11, Table A3). We add imputed additional capital services for consumer durables to capital income (line 26, Table A2) and to private consumption (line 12, Table A3). We assume a rate of return equal to 4.1 percent, which is an estimate of the return on other types of capital. A related adjustment is made for government capital. Specifically, we add imputed additional capital services for government capital to capital income (line 27, Table A3) and to public consumption (line 15, Table A3).

Intangible Investment. We introduce intangible investment in the growth model in Section 3. Our output measure includes intangible investment. Thus, total product in the model is the sum of intangible investment and gross domestic product (which we define to be NIPA gross domestic product *after* adjustments are made for consumption taxes, intermediate financial services, and consumer durables). On the income side of our extended model accounts, we add capital gains $q_t x_{It}$. Fraction χ of these gains is "sweat investment," which is allocated to labor income (line 12, Table A1). Fraction $1 - \chi$ of these gains is "expensed investment," which is allocated to capital income (line 27, Table A1). On the product side, we add "business intangible investment" (line 27, Table A3). In Section 3, we describe our calculations.

A.1.3. Tax Rates on Consumption and Labor

We use data from the U.S. national accounts to construct estimates for the tax rates on consumption and labor.

The tax rate on consumption is found by taking the ratio of sales taxes in NIPA to consumption expenditures in NIPA (which include sales taxes). In our measure of sales taxes, we include federal excise taxes and customs, state and local sales taxes, and other non-property licenses and fees. Our measure of NIPA consumption expenditures includes adjustments for consumer durables. Denoting sales tax by $\tau_c c$ and NIPA consumption expenditures by $c + \tau_c c$, the ratio yields $\tau_c/(1 + \tau_c)$. It is easy to determine τ_c from this ratio.

For the marginal tax rate on labor, we apply essentially the same methodology as in Prescott (2004). Specifically, we take the effective labor tax to be the sum of a marginal income tax rate and a marginal tax rate for social security. The income tax rate is computed as follows. Take personal current taxes in NIPA (which are direct taxes paid by households) and divide them by GDP plus net gain from sale of assets less depreciation and taxes on production and imports. We include gains from asset sales because personal current taxes include taxes on these gains. Prescott (2004) multiplies the income tax rate by 1.6 in order to get estimates of the *marginal* rate comparable to Feenberg and Coutts (1993). The marginal tax rate on social security is computed as follows. Take contributions for government social insurance in NIPA and divide them by labor income. For labor income, we sum compensation and 70 percent of proprietors' income.

A.2. Hours and Population

The primary source of our hours and population data is the U.S. Department of Labor, Bureau of Labor Statistics, *Employment and Earnings*. They are based on the Current Population Survey (CPS). We briefly describe these data here. Full details are given in Prescott, Ueberfeldt, and Cociuba (2005).

The population covered by our series is the total noninstitutional population, ages 16 to 64, for the United States. Prior to 1982, military hours are estimated and added to civilian hours from the CPS. After 1982, they are included in the CPS estimate of total hours.

For versions of the growth model with business and non-business sectors, we also categorize CPS hours as business and non-business. Using the March supplement (through www.ipums.org), we construct business hours as the sum of hours for the self-employed—both incorporated and unincorporated—and hours for private wage and salary workers less hours for employees in nonprofits. Because private wage and salary workers include employees at nonprofits, we use BEA data on compensation in nonprofits, and assuming an average wage rate equal to the economy-wide average, we can infer hours for nonprofits. Hours in the non-business sector are found by subtracting business hours from the total. We use the hours from the March supplement sample to compute the fractions of hours in business and non-business. For our final series, we multiply these fractions by total hours in the monthly CPS sample.

Table A. Revised National Accounts, Averages Relative to GDP, 1990–2003

A1. Domestic Business Value Added

1	Domestic Business Value Added	.748
2	Consumption of fixed capital	.082
3	Corporate business (NIPA 7.5)	.067
4	Sole proprietorships and partnerships (NIPA 7.5)	.013
5	Other private business (NIPA 7.5)	.003
6	Labor income	.483
7	Compensation of employees	.421
8	Corporate business (NIPA 1.13)	.382
9	Sole proprietorships and partnerships (NIPA 1.13)	.036
10	Other private business (NIPA 1.13)	.002
11	70% proprietors' income with IVA and CCadj (NIPA 1.13)	.049
12	Sweat investment (authors' calculations)	.024
13	Capital income	.163
14	Corporate profits with IVA and CCadj (NIPA 1.13)	.073
15	30% proprietors' income with IVA and CCadj (NIPA 1.13)	.021
16	Rental income of persons with CCadj (NIPA 1.13)	.006
17	Net interest and miscellaneous payments	.022
18	Corporate business (NIPA 1.13)	.014
19	Sole proprietorships and partnerships (NIPA 1.13)	.012
20	Other private business (NIPA 1.13)	.005
21	*Less*: Intermediate financial services[a] (NIPA 2.5.5)	.009
22	Taxes on production and imports[b]	.026
23	Corporate business (NIPA 1.13)	.056
24	Sole proprietorships and partnerships (NIPA 1.13)	.008
25	Other private business (NIPA 1.13)	.002
26	*Less*: Sales tax (NIPA 3.5)	.040
27	Expensed investment (authors' calculations)	.024

See footnotes at the end of the table.

Table A. Revised National Accounts (Cont.)

A2. Domestic Non-business Value Added

1	DOMESTIC NON-BUSINESS VALUE ADDED	.337
2	Consumption of fixed capital	.099
3	Households	.084
4	Excluding consumer durables (NIPA 7.5)	.012
5	Consumer durable depreciation (FOF F10)	.062
6	Nonprofits (NIPA 7.5)	.004
7	General government (NIPA 7.5)	.018
8	Government enterprises (NIPA 7.5)	.003
9	Labor income	.154
10	Compensation of employees	.154
11	Households (NIPA 1.13)	.001
12	Nonprofits (NIPA 1.13)	.042
13	General government (NIPA 1.13)	.099
14	Government enterprises (NIPA 1.13)	.012
15	Capital income	.083
16	Current surplus of government enterprises (NIPA 1.13)	.001
17	Rental income of persons with CCadj (NIPA 1.13)	.008
18	Net interest and miscellaneous payments	.033
19	Households (NIPA 1.13)	.031
20	Nonprofits (NIPA 1.13)	.002
21	Taxes on production and imports[b]	.004
22	Households (NIPA 1.13)	.011
23	Nonprofits (NIPA 1.13)	.001
24	*Less*: Sales tax (NIPA 3.5)	.007
25	Imputed additional capital services[c]	.038
26	Household, consumer durables	.013
27	Government capital	.025

See footnotes at the end of the table.

Table A. Revised National Accounts (Cont.)

A3. Domestic Value Added and Product

1	TOTAL ADJUSTED DOMESTIC INCOME	1.091
2	DOMESTIC BUSINESS VALUE ADDED	.748
3	DOMESTIC NON-BUSINESS VALUE ADDED	.337
4	STATISTICAL DISCREPANCY	.006
5	TOTAL ADJUSTED DOMESTIC PRODUCT	1.091
6	Private consumption	.618
7	Personal consumption expenditures (NIPA 1.1.5)	.678
8	*Less*: Consumer durables (NIPA 1.1.5)	.083
9	*Less*: Intermediate financial services[a] (NIPA 2.5.5)	.009
10	*Less*: Sales tax, nondurables and services (NIPA 3.5)	.042
11	Consumer durable depreciation (FOF F10)	.062
12	Imputed additional capital services[c]	.013
13	Public consumption (NIPA 3.1)	.179
14	Government consumption expenditures (NIPA 3.1)	.154
15	Imputed additional capital services[c]	.025
16	Business tangible investment[d]	.112
17	Corporate gross private domestic investment (FOF F6)	.092
18	Noncorporate gross private domestic investment (FOF F6)	.020
19	Non-business tangible investment	.134
20	Household	.114
21	Excluding consumer durables (FOF F6)	.036
22	Consumer durables (NIPA 1.1.5)	.083
23	*Less:* Sales tax, durables (NIPA 3.5)	.005
24	Nonprofits (FOF F6)	.007
25	Government investment (NIPA 3.1)	.033
26	Net exports of goods and services (NIPA 1.1.5)	−.021
27	Business intangible investment (authors' calculations)	.048

Note: IVA, inventory valuation adjustment; CCadj, capital consumption adjustment.
[a] Expense is for handling life insurance and pension plans.
[b] This category includes business transfers and excludes subsidies.
[c] Imputed additional capital services are equal to 4.1 percent times the current-cost net stock of government fixed assets and consumer durables goods (FA 1.1).
[d] 10 percent of farm business is in corporate, with the remainder in noncorporate.

Appendix B.

Parameters

In this appendix, we report and motivate the parameters and exogenous technology and tax series used as inputs for the three models described in the main text. The parameter values are summarized in Table B.1. The exogenous technology and tax rate series are summarized in Table B.2. In a separate technical appendix (McGrattan and Prescott 2007), we also demonstrate, by doing sensitivity analysis, that our main results are robust.

For interest and growth rates, we use estimates based on U.S. trends. In particular, we set the interest rate at 4.1 percent (as in McGrattan and Prescott 2005a) and the annual growth in population η at 1 percent. We also assume that per capita GDP and its components grow at 2 percent annually ($\gamma = .02$). These choices imply that $\beta = .98$. These parameters are used in all versions of our model and ensure that the marginal rate of substitution is the same across experiments.

In all three models, we use constant tax rates on capital. In both versions of the growth model in which we distinguish business and non-business activity, we set the profits tax rate to $\tau_p = 0.35$. because most of the taxes on profits are corporate income taxes. In both cases, we set the distribution tax $\tau_d = 0.15$, which is slightly less than our estimate in earlier work (McGrattan and Prescott 2005a) for corporate distributions; this is appropriate because noncorporate taxes are not taxed twice. In the one-sector version of the growth model, we set the rates lower because most of the capital is not in the business sector and not affected by τ_p or τ_d. In fact we set these rates equal to 0.42 times the rates used in the business sector, since business tangible capital is 42 percent of total tangible capital. For property tax rates, we use NIPA property tax revenues (in "taxes on imports and production") to infer values for τ_k in the one-sector and two-sector versions of the model. Details of our labor and consumption tax rates are provided in Appendix A.

For the remaining parameters, we use 1990 levels of U.S. data to obtain estimates. (See Appendix A for a full description of the data.) Specifically, we use the consumption share of GDP, the tangible investment share of GDP, the total and business tangible capital stocks (including land) as shares of GDP, and hours as a fraction of discretionary time. We use 52 weeks times 100 hours per week as an estimate of discretionary time.

The ratio of tangible investment to the stock is used to infer the rate of depreciation of tangible capital in total or, if there are two sectors, in business. In Table B.1, we report estimates of depreciation for each of the three models. These rates are slightly lower than is typical in the literature because we include land and inventories in our estimates of the capital stock. If we do not include them, the estimates for annual depreciation are on the order of 5 or 6 percent. There is no way to determine δ_I. For this rate, we chose 0 and experimented with other values to make sure our main results did not change.

Given an interest rate, tax rates on capital, and the ratio of tangible capital to output, we can infer the share of tangible capital in producing final goods and services in the models without intangible investment.[14] For the one-sector version of the model, we find a capital share of 0.34. For the two-sector version, the capital share in business is 0.28. This is lower because the non-business sector is more capital intensive.

In the extended model, we need more information because we do not know the split of tangible capital in final goods production and intangible capital production. Here we use *reported* NIPA compensation for 1990, which, in theory, is equal to total compensation less sweat investment. For our baseline results, the input elasticities for producing both final goods and intangible capital are assumed to be the same (that is, $\theta_1 = \theta_2$ and $\phi_1 = \phi_2$). This restriction along with information on NIPA compensation allows us to determine all capital shares. In Table B.1, we report that the shares for tangible capital in production are 0.26, slightly lower than that for the two-sector model with no intangible investment. The intangible shares are 0.076.

The household's intratemporal condition, along with U.S. observables and estimates of the capital shares and tax rates, implies a value for the utility parameter ψ. In Table B.1, we report the values for each of the models. They are in the range of estimates used in the business cycle literature.

The final parameter to be set is χ, the fraction of intangible investment that is financed by capital owners. This parameter is used only in our extended growth model with intangible investment. As noted earlier, the only real ramification of this choice is for tax payments. But the evidence in Figures 2 and 3 indicates that some investment is being done by both shareholders and workers. We chose $\chi = 0.5$ and then experimented with other values. The main effect of varying χ is a change in the effective tax rates on labor and capital.

[14] An alternative procedure uses information on factor incomes to infer cost shares. However, this procedure is invalid when there are intangible investments because the NIPA accounts do not report the total compensation or profits.

TABLE B.1. MODEL PARAMETERS

PARAMETER	EXPRESSION	VALUE
COMMON PARAMETERS		
Growth in population	η	0.01
Growth in technology	γ	0.02
Discount factor	β	0.98
ONE-SECTOR MODEL, NO INTANGIBLE INVESTMENT		
Utility parameter	ψ	1.48
Depreciation rate	δ	0.031
Capital share	θ	0.34
Tax rate on property	τ_k	0.0073
Tax rate on profits	τ_p	0.15
Tax rate on distributions	τ_d	0.064
TWO-SECTOR MODEL, NO INTANGIBLE INVESTMENT		
Utility parameter	ψ	1.38
Depreciation rate, business	δ	0.033
Capital share, business	θ	0.28
Tax rate on business property	τ_k	0.014
Tax rate on business profits	τ_p	0.35
Tax rate on business distributions	τ_d	0.15
EXTENDED MODEL, WITH INTANGIBLE INVESTMENT		
Utility parameter	ψ	1.32
Tangible depreciation rate, business	δ_T	0.033
Intangible depreciation rate, business	δ_I	0
Tangible capital share, final goods & services, business	θ_1	0.26
Tangible capital share, intangible investment, business	θ_2	0.26
Intangible capital share, final goods & services, business	ϕ_1	0.076
Intangible capital share, intangible investment, business	ϕ_2	0.076
Tax rate on business property	τ_k	0.014
Tax rate on business profits	τ_p	0.35
Tax rate on business distributions	τ_d	0.15
Fraction of intangible financed by workers	χ	0.5

TABLE B.2. Time Series for Tax Rates and Technology

Year	Tax Rates		Technology Parameters			
			One-Sector	Two-Sector	Extended Model	
t	τ_{ct}	τ_{ht}	Model	Model	A_t^1	A_t^2
1990	6.6	31.1	1.49	1.75	1.66	1.53
1991	6.8	30.7	1.47	1.71	1.60	1.48
1992	6.8	30.3	1.48	1.73	1.60	1.44
1993	6.8	30.3	1.46	1.71	1.60	1.49
1994	7.0	30.7	1.45	1.71	1.63	1.59
1995	6.9	31.2	1.44	1.71	1.64	1.63
1996	6.7	31.9	1.44	1.71	1.67	1.69
1997	6.7	32.5	1.44	1.73	1.69	1.74
1998	6.7	33.3	1.45	1.76	1.73	1.77
1999	6.6	33.4	1.46	1.78	1.76	1.79
2000	6.5	34.3	1.46	1.80	1.76	1.79
2001	6.3	34.7	1.45	1.76	1.73	1.78
2002	6.2	30.8	1.45	1.73	1.60	1.59
2003	6.2	28.9	1.43	1.71	1.56	1.53

Appendix C.

Aggregation in the Extended Model

In this appendix, we develop the aggregation theory underlying the technology of our extended growth model with intangible investment. (See Section 3.)

A business is characterized by the stock of its (unmeasured) intangible capital, K_I. This capital can be used for two activities. One activity produces the composite output of the business Y_b, and the other produces intangible investment goods X_I.

Inputs of (measured) tangible capital K_T^i and hours H^i along with K_I produce an intermediate good Z^i via a standard constant returns to scale neoclassical production function f^i for $i \in \{1, 2\}$. In particular, the production functions are

$$Z^i = (K_T^i)^{\theta_i} K_I^{\phi_i} (H^i)^{1-\theta_i-\phi_i}, \quad i \in \{1, 2\}.$$

The quantity of Y_b produced is $g^1(Z^1)$, and the quantity of X_I produced is $g^2(Z^2)$. The functions g^i are increasing, initially strictly convex, then strictly concave, and they satisfy $g^i(0) = 0$. The slope of the maximal tangent ray from the origin is A^i. The point of tangency is $\hat{Z}^i$. The margin of adjustment is the number of units operated, which is variable. The capital stock K_I can be split over businesses through mergers, acquisitions, and spin-offs. All production units that are operated will have the same K_I. This K_I will depend upon the relative prices of the three inputs. Production units of type i will be operated at level $\hat{Z}^i$ and produce $g^i(\hat{Z}^i)$.

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Figure 1

U.S. and Basic Model Per Capita Hours Worked

Annual, 1990=100, 1990–2003

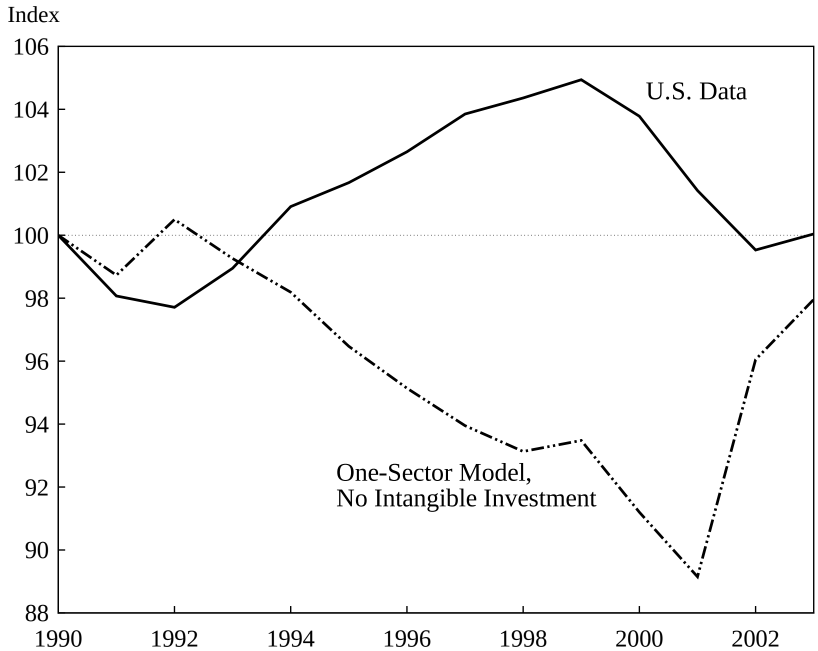


Figure 2

U.S. and Basic Model Per Capita Real GDP

Annual, Series Divided by 1.02^t, 1990=100, 1990–2003

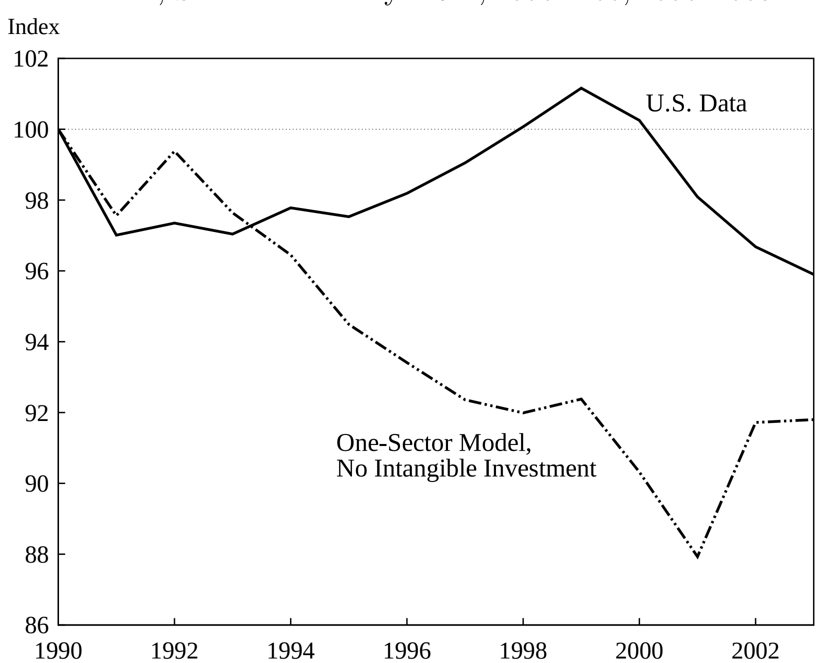


FIGURE 3

AVERAGE WEEKLY HOURS WORKED BY NONINSTITUTIONAL POPULATION, AGE 16–64, AND NIPA REAL COMPENSATION PER HOUR

Annual, Compensation Divided by 1.02^t, 1990=100, 1990–2003

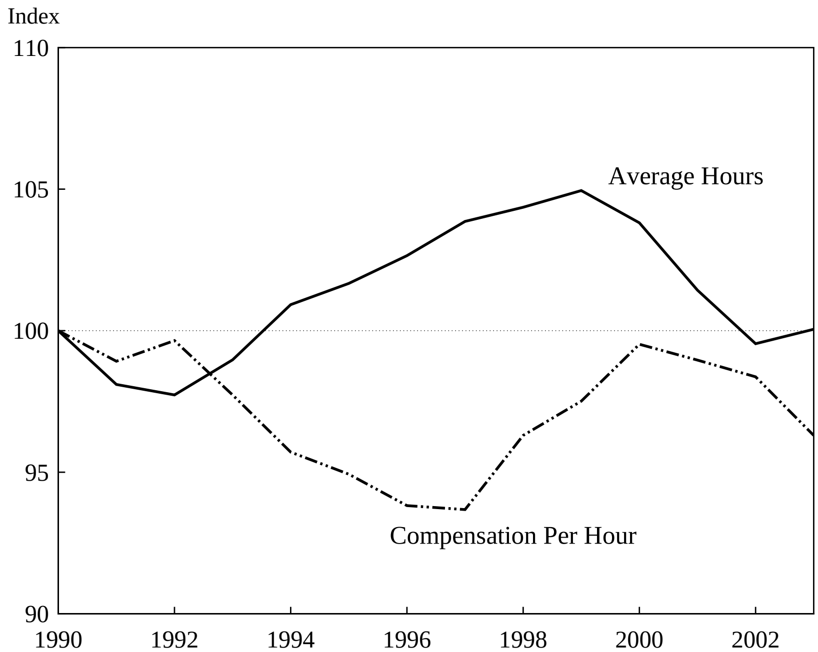


FIGURE 4

NIPA REAL PER CAPITA GDP AND CORPORATE PROFITS

Annual, Series Divided by 1.02^t, 1990–2003

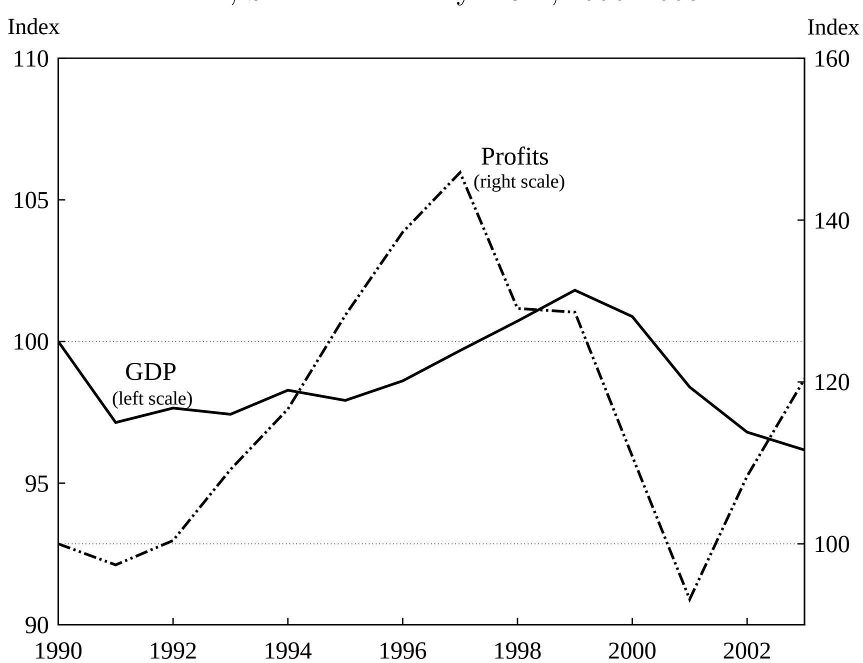


Figure 5

U.S. and Extended Model Real Total Factor Productivity

Annual, Series Divided by 1.02^t, 1990=100, 1990–2003

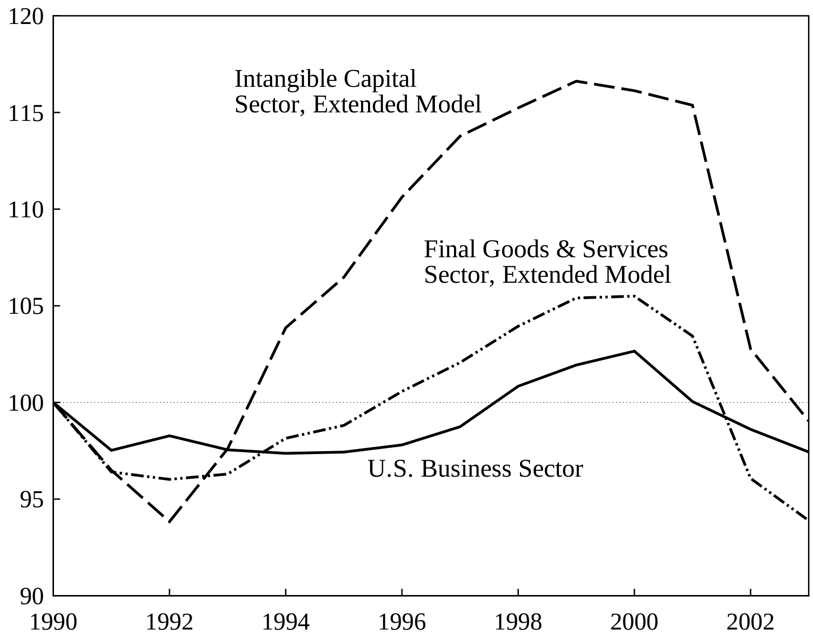


Figure 6

Extended Model Intangible Share of Total Output

Annual, 1990–2003

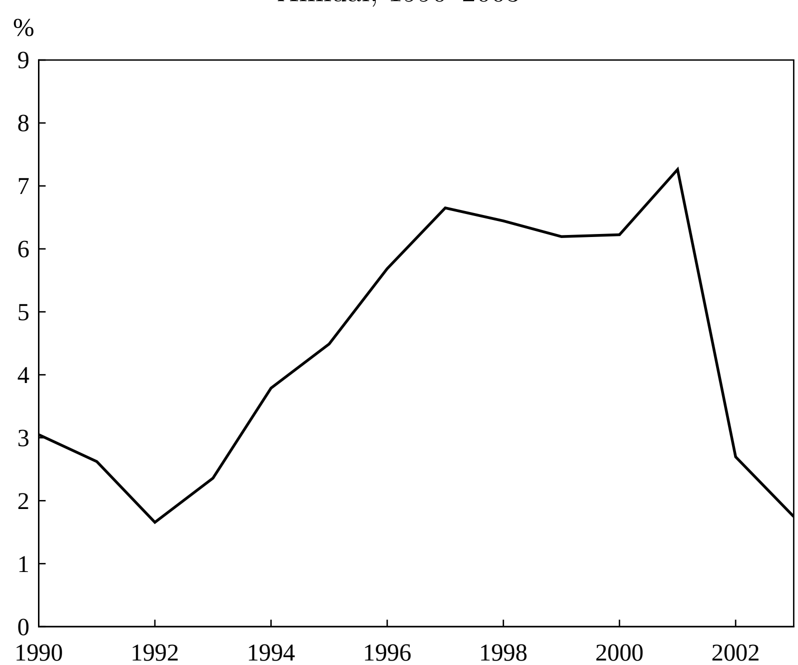


Figure 7

U.S. and Extended Model Per Capita Hours Worked

Annual, 1990=100, 1990–2003

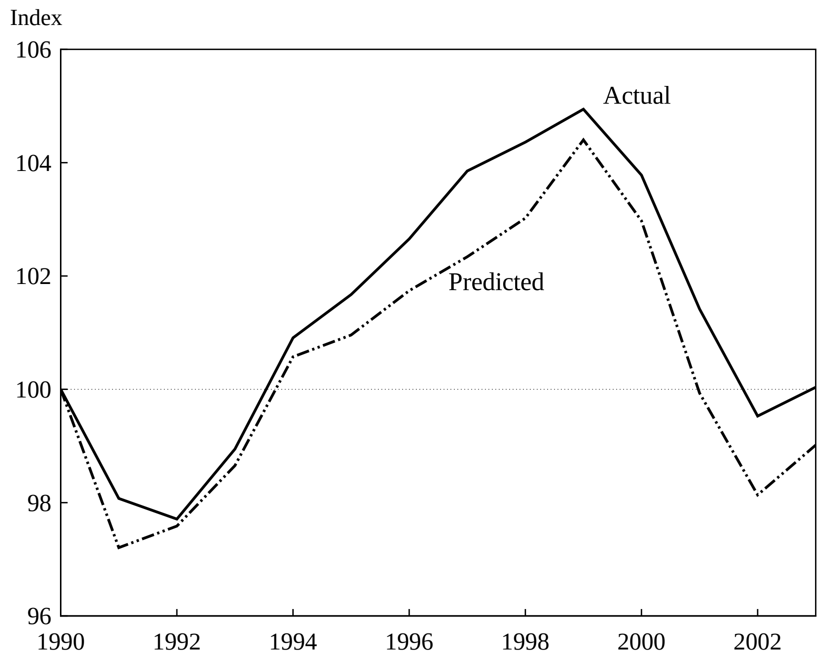


Figure 8

U.S. and Extended Model Per Capita Real GDP

Annual, Series Divided by 1.02^t, 1990=100, 1990–2003

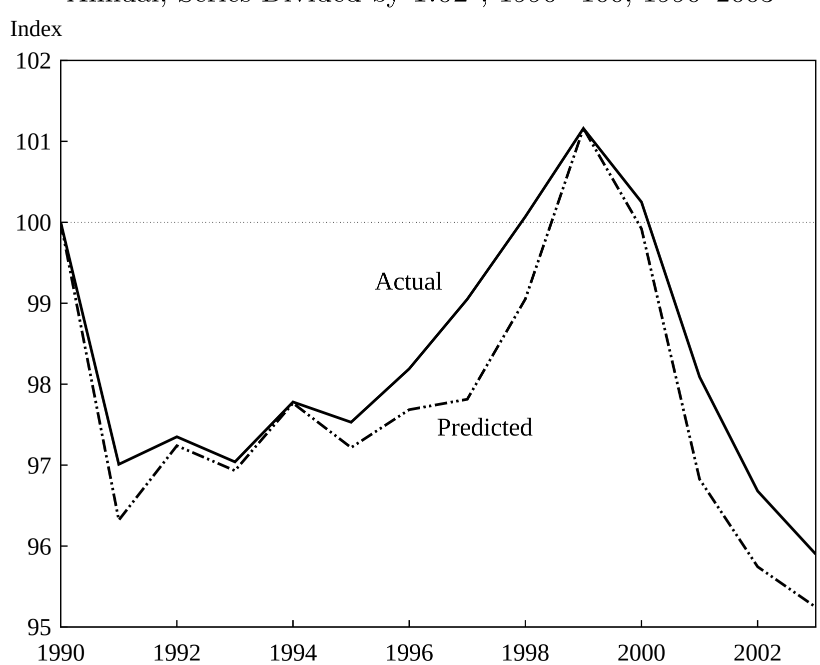


FIGURE 9

U.S. AND EXTENDED MODEL REAL BUSINESS COMPENSATION LESS SWEAT

Annual, Series Divided by 1.02^t, 1990=100, 1990–2003

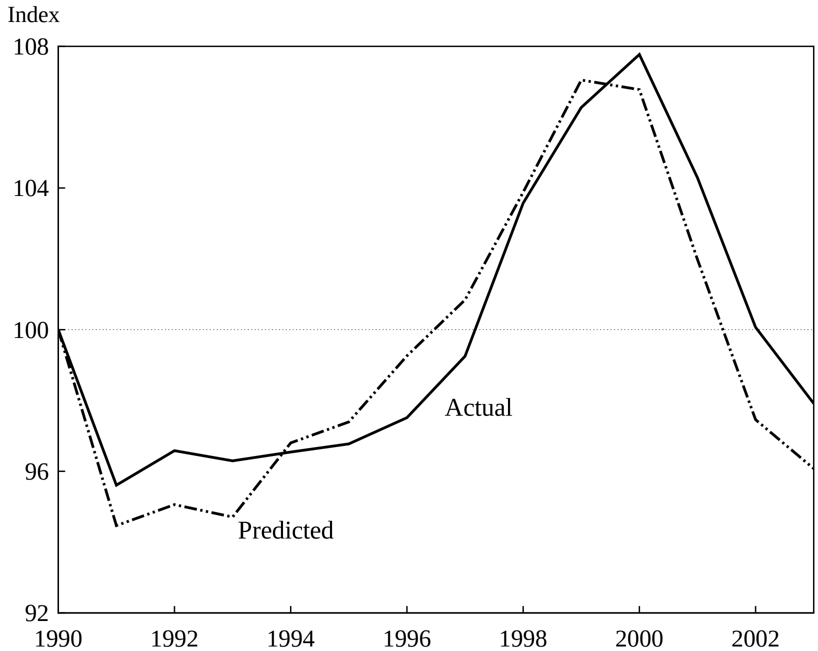


FIGURE 10

U.S. AND EXTENDED MODEL REAL HOLDING GAINS

Annual, % of Real GDP, Excluding Real Estate, 1990–2003

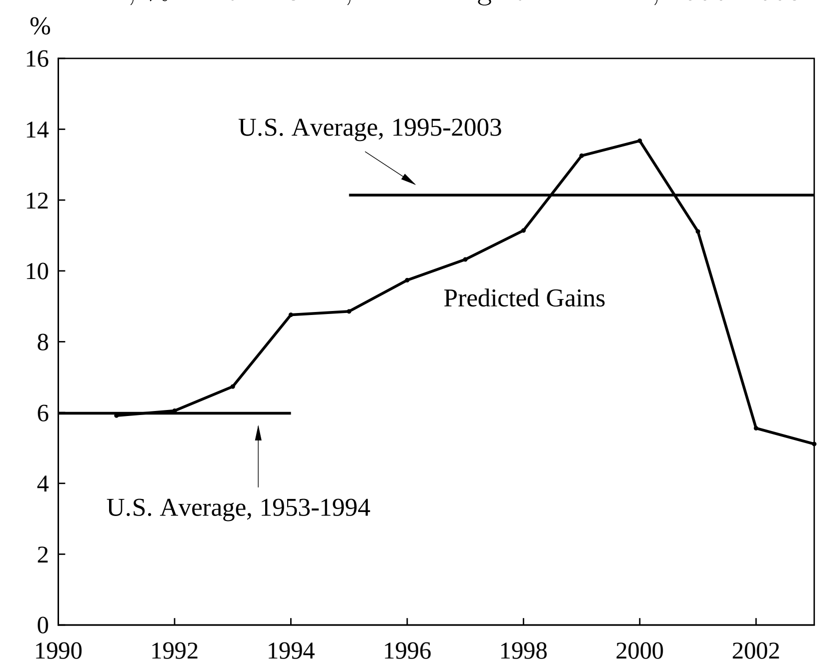


Figure 11

Extended Model Real Business Productivity

Annual, Series Divided by 1.02^t, 1990=100, 1990–2003

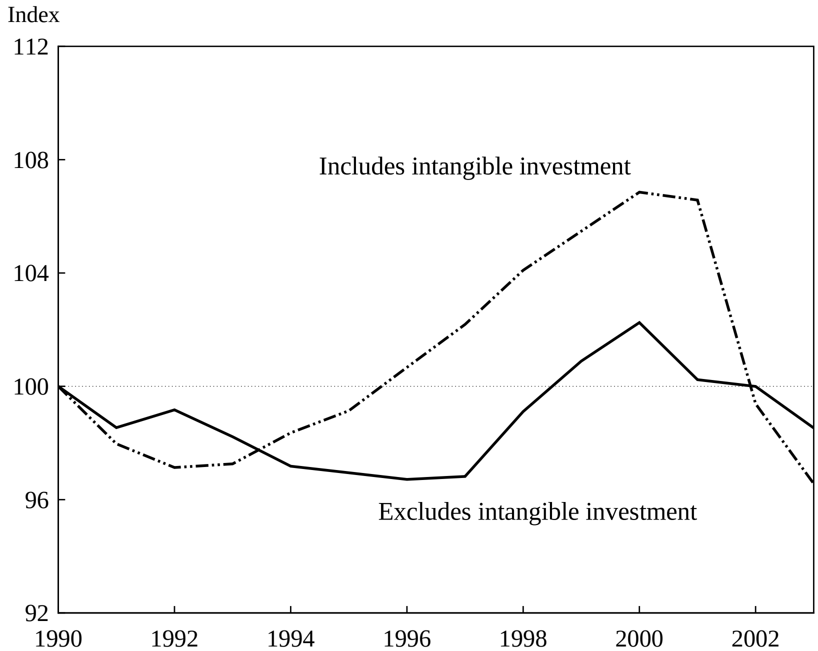
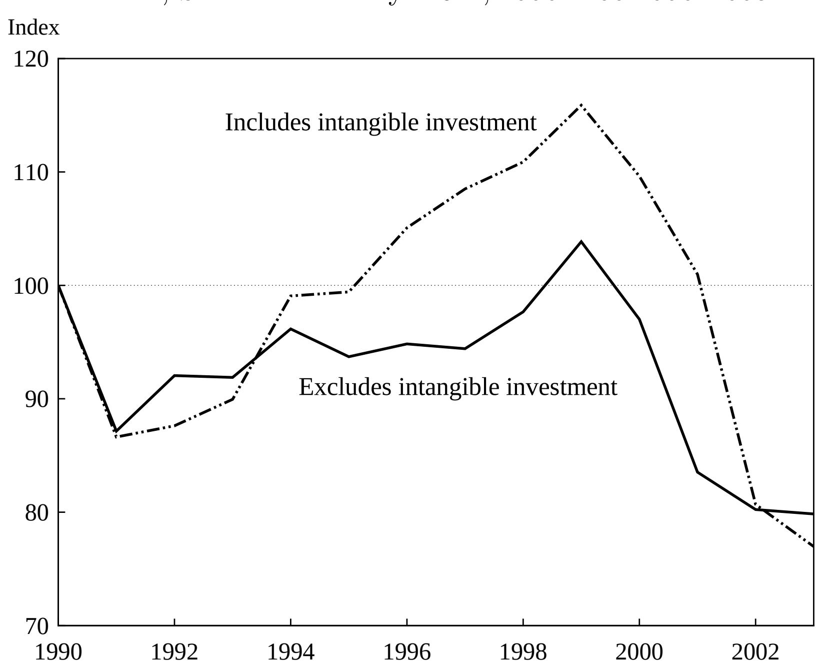


Figure 12

Extended Model Real Per Capita Investment

Annual, Series Divided by 1.02^t, 1990=100 1990–2003



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Technical Appendix: Unmeasured Investment and the Puzzling U.S. Boom in the 1990s*

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* The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Contents

Chapter 1.

Introduction

In this Appendix, we provide supporting evidence for claims made in our paper "Unmeasured Investment and the Puzzling U.S. Boom in the 1990s" and address issues raised in seminars and by referees. Here, we summarize our findings as responses to four common myths (which arise in most discussions of the paper).

The first common myth is that intangible investment is simply a free parameter that "makes up for whatever is missing to make standard theory work." (Here, we are quoting one of our referees.) To dispel this myth, we apply our methodology to three theories of the U.S. boom in the 1990s. *All three theories generate paths of GDP, consumption, investment, and hours that match U.S. data perfectly.* Despite the perfect fit for all theories, only one of the three theories satisfies our criteria for a successful theory.[1] One of the unsuccessful theories does include intangible capital and does generate a boom in the 1990s, but does not satisfy our criteria.

We also demonstrate that we would get a very different result if the data-generating mechanism were in fact inconsistent with our theory of intangible investment and non-neutral technological change. This is a slightly different way of making the point that intangible investment is not a free parameter that makes up for whatever is missing. We set up the following experiment. First, we generate artificial data from a model with *no* intangible investment that has hours fluctuating only because there have been changes in

[1] Our *input justification criterion* requires the exogenous inputs to be consistent with micro and macro evidence. Our *prediction criterion* requires conformity of theory and observed time series that were not used to set parameters or exogenous inputs.

labor market distortions (which are not due to changes in government labor tax rates). Second, with these data, we assess the theory *with* intangible investment and non-neutral technology. In this case, we find that our theory would satisfy neither our input justification criterion nor our prediction criterion.

The second common myth is that the neoclassical growth model does poorly over the *entire postwar period*—especially with regard to movements in hours of work—and not just in the 1990s as we claimed. For example, Stephanie Schmitt-Grohe and Martin Uribe (2005) motivated their work in an interview with the *NBER Reporter* by noting that "by the late 1990s empirical research using macroeconomic data from industrialized countries had cast compelling doubts on the ability of the neoclassical growth model to provide a satisfactory account of aggregate fluctuations" (p. 19). In dispelling the myth that neoclassical theory is doomed, we draw heavily on the work of Uhlig (2003) and Chen, İmrohoroğlu, and İmrohoroğlu (2007).[2]

A third common myth is that the class of new Keynesian models that Schmitt-Grohe, Uribe and others have adopted provides a better understanding of business cycles. With a new Keynesian model developed and used by Smets and Wouters (2007) to study U.S. business cycles, we show that this current-generation model, *which is designed to perfectly fit seven U.S. time series—GDP, consumption, investment, business hours, business wages, the federal funds rate, and inflation—fails to satisfy our criteria for a successful theory.*

The fourth and final myth is that conclusions based on perfect-foresight analyses are not robust. We use models with stochastic variation in the key exogenous variables to

[2] We thank Harold Uhlig, Kaiji Chen, and Rafael Wouters for providing us with data and codes used in their papers that we discuss in this Appendix. This made it relatively easy to reproduce their research and to make direct comparisons between our work and theirs.

show that the specific realizations of these exogenous stochastic variables, not the choice of household expectations, are what is important. We also do sensitivity analysis with respect to choices of parameters of preferences and technologies to show that our findings and conclusions are robust.

Chapter 2.

Assessing Three Theories

In this chapter we consider three theories. All three generate equilibrium paths for GDP, consumption, investment, and hours *that exactly match the U.S. time series during the 1990s*. The fact that all three theories can generate the boom of the 1990s—the phenomenon that is central to our paper—does not mean, however, that we view all of them as "successful." In fact, we will demonstrate later on that two of these theories are actually unsuccessful *in our sense of the word* because they do not satisfy the input justification criterion or the prediction criterion described in the paper. We deem them unsuccessful *even though they can generate the U.S. boom of the 1990s*. We finish the chapter by addressing the question, Could our preferred theory with intangible investment and non-neutral technological change ever fail to satisfy the criteria for a successful theory that we propose, or are we simply setting the bar too low for ourselves?

To satisfy the input justification criterion, the exogenous inputs of the theoretical model must be consistent with micro and macro empirical evidence. This criterion requires a theory for the exogenous inputs. To satisfy the prediction criterion, the model must not make counterfactual predictions. This is a minimum requirement. A stronger requirement is that the theory must predict time series that were not used to set parameters or exogenous inputs. For example, we can use the theories to make predictions for incomes and capital gains—data that were not used in setting any of the exogenous variables.

For each theory that we investigate, we generate an exact match between predicted and actual paths for GDP, consumption, investment, and hours by introducing either a *labor wedge* or an *investment wedge* or both. The labor wedge is an exogenous input that results in an exact fit for the household's intratemporal first-order condition relating the marginal rate of substitution between leisure and consumption and the marginal product of labor.[3] The investment wedge is an exogenous input that results in an exact fit for the household's intertemporal first-order condition relating the intertemporal marginal rate of substitution and the intertemporal marginal rate of transformation.

For a theory to satisfy the input justification criterion, we require either (*i*) the variation in U.S. time series attributed to the wedges is small or (*ii*) some empirical justification for these wedges. If no empirical support is available, then theory is, for all practical purposes, vacuous. For a theory to successfully resolve the puzzling U.S. boom of the 1990s, it must then satisfy the more demanding prediction criterion.

The first theory we analyze in this section is the standard *theory without intangible capital*, commonly referred to as neoclassical growth theory. We consider a specific model with fluctuations driven by TFP, tax rates on hours and consumption, a labor wedge, and an investment wedge. We consider a version of the model with one sector that combines business and non-business activity and another version that distinguishes between them. We do both because the behavior of economy-wide TFP and of business-sector TFP was quite different during the 1990s. The economy-wide TFP was a little below trend throughout the 1990s, and the business-sector TFP started below trend and rose rapidly in the late 1990s. To give the simple theory the best chance of success, we want to allow for a rapid increase in business TFP.

Neither model for the standard theory satisfies our criteria for a successful theory.

[3] Unlike Chari, Kehoe, and McGrattan (2007), we separately include tax rates on labor. The labor wedge would have to be a proxy for labor distortions other than taxes.

The labor wedge has to be huge and is inconsistent with all measures of effective tax rates and all measures of worker benefits available (e.g., tax credits and welfare). Furthermore, the model's predictions of factor incomes and capital gains are not consistent with U.S. observations.

The second and third theories include intangible capital and also distinguish between business and non-business activity. The second theory assumes that technological change is neutral with respect to two activities: production of final goods and services and production of new intangible investment goods. We refer to this as the *theory with intangible capital and neutral technology.* The specific model we analyze has fluctuations driven by the same exogenous variables used for the standard model: TFP, tax rates on hours and consumption, a labor wedge, and an investment wedge. Like the standard model without intangible capital, this extension with intangible capital *does not* satisfy our criteria for a successful theory. The labor wedge and the implied intangible investment are wildly oscillatory and inconsistent with all micro evidence on labor distortions and all direct measures of intangible investments. Furthermore, the model's predictions are also grossly inconsistent with the U.S. data. This theory shows that intangible capital is not "making up" for whatever is missing in standard theory. In fact, the theory of intangible capital with neutral technology does considerably worse than the standard theory.

The third theory assumes that technological change is non-neutral. The non-neutrality is at the heart of the theoretical contribution. We consider two different activities within the business sector and refer to the theory as the *theory with intangible capital and non-neutral technology.* The specific model that we analyze has fluctuations driven by TFP in the final goods and services sector, TFP in the intangible-investment sector, tax rates on hours and consumption, and an investment wedge. The effect of the investment wedge—

which is simply an addition to get a perfect fit—is small and well within the range of estimates of capital tax rate changes. The other inputs are consistent with micro evidence of this period in which a technology boom occurred. Specifically, they are consistent with micro evidence on R&D, which is an important component of intangible investment, and they are consistent with the shift in employment to occupations in which sweat equity is important. We compare the model's predictions for factor incomes and capital gains, which were not used in the determination of sectoral TFPs. We show that the model does well here too.

These findings lead us to conclude that there is now one theory of the 1990s boom, whereas before there was none.

2.1. Theory without Intangible Capital

In this section we describe a particular growth model without intangible capital. Fluctuations in the model are driven by changes in TFP, tax rates on hours and consumption, a labor wedge, and an investment wedge. We use the model to demonstrate that this theory fails to satisfy the criteria we propose for a successful theory.

2.1.1. A Specific Model

Given an initial capital stock k_0, the stand-in household chooses sequences of consumption $\{c_t\}$, investment $\{x_t\}$, and hours $\{h_t\}$ to maximize

$$\max E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t$$

subject to

$$c_t + x_t = r_t k_t + w_t h_t - \tau_{ct} c_t - \tau_{ht} w_t h_t - \tau_{kt} k_t - \tau_{pt}(r_t - \delta - \tau_{kt}) k_t - \tau_{xt} x_t$$

$$- \tau_{dt}\{r_t k_t - x_t - \tau_{kt} k_t - \tau_{pt}(r_t - \delta - \tau_{kt}) k_t - \tau_{xt} x_t\} + Tr_t \tag{2.1.1}$$

$$k_{t+1} = [(1-\delta)k_t + x_t]/(1+\eta), \tag{2.1.2}$$

where variables are written in per capita terms, N_t is population at t which grows at rate η, and r_t and w_t are rental and wage rates. Taxes are assessed on consumption (τ_c), investment (τ_x), property (τ_k), profits (τ_p), dividends (τ_d), and labor income (τ_h). Transfers are given by Tr_t.

In equilibrium factors are paid their marginal products. Per capita output is given by

$$y_t = k_t^{\theta} (Z_t h_t)^{1-\theta},$$

and, therefore, $r_t = \theta y_t / k_t$ and $w_t = (1-\theta) y_t / h_t$ are the rental rate and wage rate, respectively, in equilibrium. The parameter Z_t is labor-augmenting technology change which grows at rate γ, that is, $Z_t = z_t (1+\gamma)^t$ with z_t stationary.

Suppose $U(c,h) = \log c + \psi \log(1-h)$. In this case, the household's first-order conditions—after substituting for r_t and w_t—are given by

$$\frac{\psi(1+\tau_{ct})\hat{c}_t}{1-h_t} = (1-\tau_{ht}) \frac{(1-\theta)\hat{y}_t}{h_t} \tag{2.1.3}$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} \left[R_{t,t+1} + (1-\delta)\xi_{t,t+1} \right], \tag{2.1.4}$$

where $\hat{\beta} = \beta/(1+\gamma)$, $\mu_t = 1/[(1+\tau_{ct})\hat{c}_t]$, and

$$R_{t,t+1} = \frac{1-\tau_{d,t+1}}{(1-\tau_{dt})(1+\tau_{xt})} \left[(1-\tau_{p,t+1}) \left(\theta \frac{\hat{y}_{t+1}}{\hat{k}_{t+1}} - \tau_{k,t+1} \right) + \delta \tau_{p,t+1} \right] \tag{2.1.5}$$

$$\xi_{t,t+1} = \frac{1-\tau_{d,t+1}}{1-\tau_{dt}} \cdot \frac{1-\tau_{p,t+1}}{1-\tau_{pt}}. \tag{2.1.6}$$

The hat on a variable indicates that it has been detrended by $(1+\gamma)^t$, e.g., $\hat{c}_t = c_t/(1+\gamma)^t$. To close the model, we add the resource constraint,

$$\hat{c}_t + \hat{x}_t + \hat{g}_t = \hat{y}_t.$$

When setting parameters for our numerical experiments, we use 1990 estimates from U.S. data for $\hat{y}$, $\hat{c}$, $\hat{g}$, h, and $\hat{k}$ along with estimates for the growth rates γ, η, the tax rate on labor τ_h, the tax rate on consumption τ_c, tax rates on capital τ_p, τ_d, τ_x, τ_k, and an interest rate i. We can use these estimates to evaluate the following expressions for β, δ, θ, ψ, and z:

$$\beta = \frac{1+\gamma}{1+i} \tag{2.1.7}$$

$$\delta = \hat{x}/\hat{k} + 1 - (1+\eta)(1+\gamma) \tag{2.1.8}$$

$$\theta = \frac{(1-\hat{\beta}(1-\delta))(1+\tau_x) - \hat{\beta}\delta\tau_p + \hat{\beta}(1-\tau_p)\tau_k}{\hat{\beta}(1-\tau_p)} \frac{\hat{k}}{\hat{y}} \tag{2.1.9}$$

$$\psi = \frac{(1-\tau_h)(1-\theta)(1-h)\hat{y}}{(1+\tau_c)\hat{c}h} \tag{2.1.10}$$

$$z = \left(\hat{k}/y\right)^{\theta/(\theta-1)} \frac{\hat{y}}{h} \tag{2.1.11}$$

with $\hat{x} = \hat{y} - \hat{c} - \hat{g}$.

The U.S. levels of (detrended) variables in 1990 that we use when parameterizing the model are as follows: $\hat{y} = 1$ (which is a normalization), $\hat{c} = .7626$, $\hat{x} = .2377$, $\hat{g} = 0$, $h = .2751$, and $\hat{k} = 3.91$.[4] The growth rates are set equal to $\eta = 1\%$ and $\gamma = 2\%$ and the interest rate to $i = 4.1\%$. Tax rates on labor and consumption are $\tau_h = .3109$ and $\tau_c = .0657$, respectively. When we compute equilibrium paths for the 1990s, we assume

[4] All estimates are based on the data described in the paper. Note that we included public consumption in $\hat{c}$ and public investment in $\hat{x}$. See the data appendix for further details.

that these tax rates affecting the intratemporal margin (2.1.3) are varying. We assumed that capital tax rates were roughly constant throughout the 1990s, since there was little change in corporate tax policy and capital tax rates have little effect on hours. The constant rates we use are as follows: $\tau_k = .0073$, $\tau_x = 0$, $\tau_p = 0.1487$, and $\tau_d = 0.0637$. The tax on profits τ_p and distributions τ_d are computed by multiplying effective corporate tax rates times the ratio of business capital to total capital. Substituting these values in the expressions (2.1.7)–(2.1.11) implies $\beta = .98$, $\delta = 0.0306$, $\theta = .3358$, $\psi = 1.4841$, and $z = 1.8243$.

2.1.2. Business Cycle Accounting in the 1990s

We observe sequences for $\hat{y}_t$, $\hat{c}_t$, $\hat{x}_t$, $\hat{g}_t$, h_t, τ_{ct}, and τ_{ht}, and an initial capital stock for capital $\hat{k}_0$. The initial capital stock plus sequence of investments imply a sequence of capital stocks if we apply the capital accumulation equation in (2.1.2). Given inputs for capital and labor, we have a measure of (detrended) total factor productivity, $A_t = \hat{y}_t/(\hat{k}_t^{\theta} h_t^{1-\theta})$, which is also equal to $z_t^{1-\theta}$.

If we compute a perfect-foresight equilibrium path for this model, assuming households take as given time paths for TFP and tax rates on hours and consumption, we cannot get a perfect match between the model predictions and the data.[5] For example, if we substitute U.S. data for $\hat{c}_t$, h_t, $\hat{y}_t$, τ_{ht}, and τ_{ct} into (2.1.3), the relation does not hold exactly.

We could get a perfect match if we introduce a labor wedge that forces (2.1.3) to hold and an investment wedge that forces (2.1.4) to hold. Specifically, we define the labor wedge

[5] Later, we relax that assumption to determine if our results are sensitive to this specification of expectations.

L_{wt} and investment wedge X_{wt} as follows:

$$L_{wt} = \frac{\psi(1+\tau_{ct})\hat{c}_t}{1-h_t} \cdot \frac{h_t}{(1-\theta)\hat{y}_t} \cdot \frac{1}{1-\tau_{ht}} \tag{2.1.12}$$

$$X_{w,t+1} = \frac{\hat{\beta}(1-\delta)\mu_{t+1}X_{wt}}{\mu_t - \hat{\beta}R_{t,t+1}\mu_{t+1}X_{wt}}\xi_{t,t+1} \tag{2.1.13}$$

with equation (2.1.13) solved recursively starting with $X_{w0} = 1$. Then we replace $1-\tau_{ht}$ in (2.1.3) with $(1-\tau_{ht})L_{wt}$ and $1/(1+\tau_{xt})$ in (2.1.4) with $X_{wt}/(1+\tau_{xt})$. If there is some mismeasurement in the effective rates on labor and capital, these wedges will pick it up. Ideally, they should be quantitatively insignificant.

Increases in the wedges have a positive effect on output and hours. The labor wedge has the same effect as a tax on labor (in the form $1-\tau_{ht}$), and the investment wedge has the same effect as a tax on investment (in the form $1/(1+\tau_{xt})$). We distinguish movements in the wedges from movements in these tax rates because we want to set τ_{ht} and τ_{xt} equal to effective rates set by the government. Without further interpretation, these time-varying inputs are just wedges that force first-order conditions to hold. Thus, it is desirable, unless we have some theory of these wedges, that their effect be quantitatively insignificant. They should be interpreted as small measurement errors in constructing national accounts and tax data.

In Table 1, we report the values of the implied exogenous variables; when all are fed into the model, the model exactly reproduces the U.S. sequences for detrended output $\hat{y}_t$, detrended consumption $\hat{c}_t$, detrended investment $\hat{x}_t$, and hours of work h_t. This is true by construction.

Figure 1 is a comparison of U.S. per capita hours and the model's prediction of per capita hours in the case that only TFP and tax rates on labor and consumption are varying

Year (t)	A_t	τ_{ht}	τ_{ct}	L_{wt}	X_{wt}
1990	1.4909	0.3109	0.0657	1.0000	1.0000
1991	1.4651	0.3070	0.0675	0.9838	1.0132
1992	1.4760	0.3028	0.0678	0.9716	1.0101
1993	1.4609	0.3034	0.0678	0.9859	1.0164
1994	1.4544	0.3068	0.0702	1.0076	1.0196
1995	1.4435	0.3116	0.0686	1.0228	1.0255
1996	1.4441	0.3190	0.0674	1.0403	1.0274
1997	1.4448	0.3254	0.0674	1.0561	1.0316
1998	1.4530	0.3327	0.0670	1.0730	1.0252
1999	1.4612	0.3335	0.0662	1.0870	1.0119
2000	1.4568	0.3424	0.0649	1.0960	1.0112
2001	1.4469	0.3472	0.0625	1.0874	1.0156
2002	1.4459	0.3076	0.0617	1.0119	1.0159
2003	1.4328	0.2885	0.0621	0.9990	1.0153

Table 1. Exogenous Variables for Model without Intangible Capital

(i.e., $L_{wt} = X_{wt} = 1$ for all t). By construction, if the wedges were varying, then the model would fit exactly and the predicted and actual series would lie on top of each other. The difference in the actual and predicted series is therefore attributed to the wedges. Clearly, this difference is large.

Figure 2 compares U.S. per capita real GDP and the model's prediction for per capita real GDP. We divide both series by 1.02^t, since our technological growth rate is chosen to be $\gamma = .02$. The model predicts a depressed economy (relative to a 2 percent trend), but the U.S. economy boomed. Figure 3 shows GDP per hour relative to the 2 percent trend for both the data and the model. This figure shows that the deviations in Figures 1 and 2 are not offsetting and, therefore, the prediction for labor productivity is also inconsistent with observations.

Figures 4 and 5 show the model predictions for per capita real investment and consumption along with U.S. data. Neither match up well.

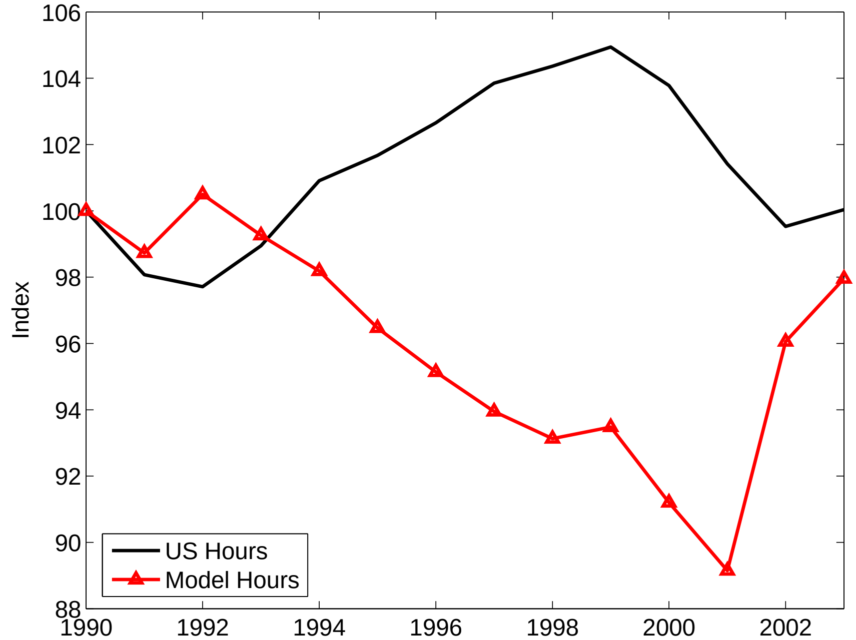

FIGURE 1. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITHOUT INTANGIBLE CAPITAL

(Labor and investment wedges constant)

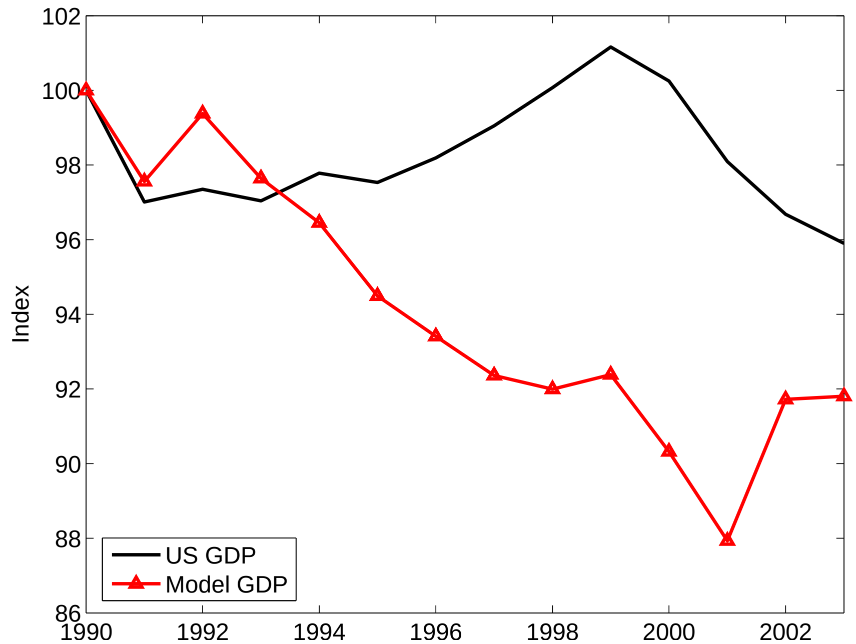

FIGURE 2. U.S. PER CAPITA REAL GDP AND PREDICTION OF MODEL WITHOUT INTANGIBLE CAPITAL, SERIES DIVIDED BY 1.02^t

(Labor and investment wedges constant)

In Figures 6 and 7, we examine the model's predictions for per capita hours without TFP or tax rates varying. In Figure 6, we plot U.S. per capita hours along with the model's prediction for hours in the case that only L_{wt} is varying. Figure 7 is the prediction when only X_{wt} is varying.[6] These figures show that the labor wedge is key to getting the hours boom. To generate an hours boom of 7 percent between 1992 and 1999, the labor wedge has to rise nearly 10 percent.

Summary

The main problem with the standard theory driven by the labor wedge is the interpretation of the wedge. It certainly cannot be interpreted as mismeasurement of effective labor tax rates, which were rising—not falling—for all estimates we have seen. Could it be that other policies such as the Earned Income Tax Credit (EITC) or welfare benefits were affecting how much people work?[7] The answer, given the aggregate spending and coverage of these programs, is most surely no. For example, in 1990, the EITC total amount of credit was 7.5 billion, or roughly 0.13 times GDP. (See the U.S. House of Representatives Green Book, Table 13-14.) That figure rose over the 1990s to 0.34 times GDP and then flattened. It is not clear whether it had a positive or negative effect on hours, but the upper bound of the effect on tax rates is tiny. Furthermore, the EITC credits did not decline after 1999, but hours did. There are other tax credits and income-tested benefit programs that affect hours but are much smaller than the EITC.[8]

Without some other empirical motivation for this wedge, the theory does not sat-

[6] When summed, the predicted series in Figures 1, 6, and 7 are approximately but not exactly equal to the U.S. series. It is not exact because there are endogenous movements in the capital stock.

[7] These policies appear in transfers to persons and do not come into the calculation of τ_{ht}.

[8] Examples include the Work Opportunity Tax Credit, the Welfare-to-Work Tax Credit, the Welfare-to-Work Grant Program, and work-related Temporary Assistance for Needy Families. For further details, see Appendix K of the the U.S. House of Representatives Green Book.

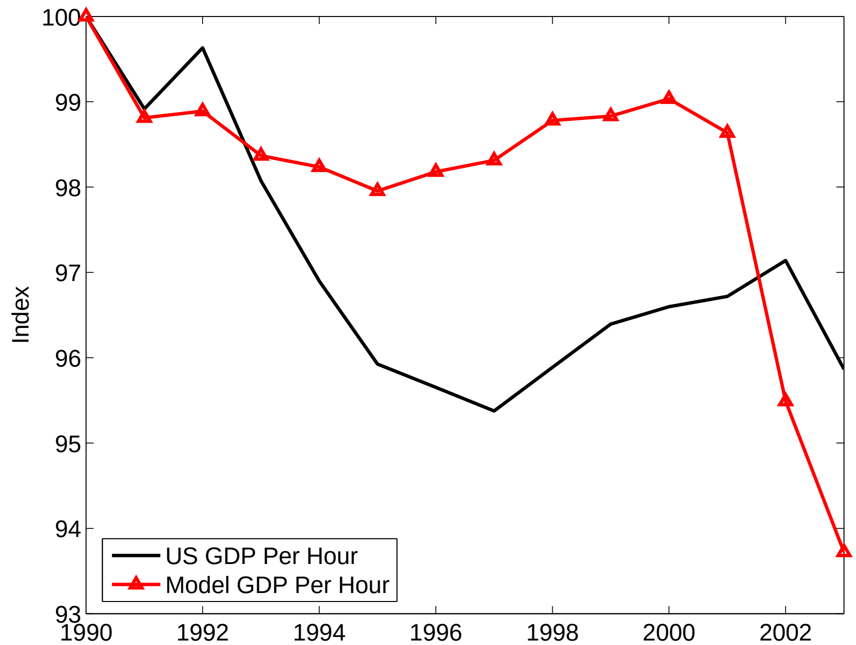

Figure 3. U.S. Real GDP Per Hour and Prediction of Model without Intangible Capital, Series Divided by 1.02^t

(Labor and investment wedges constant)

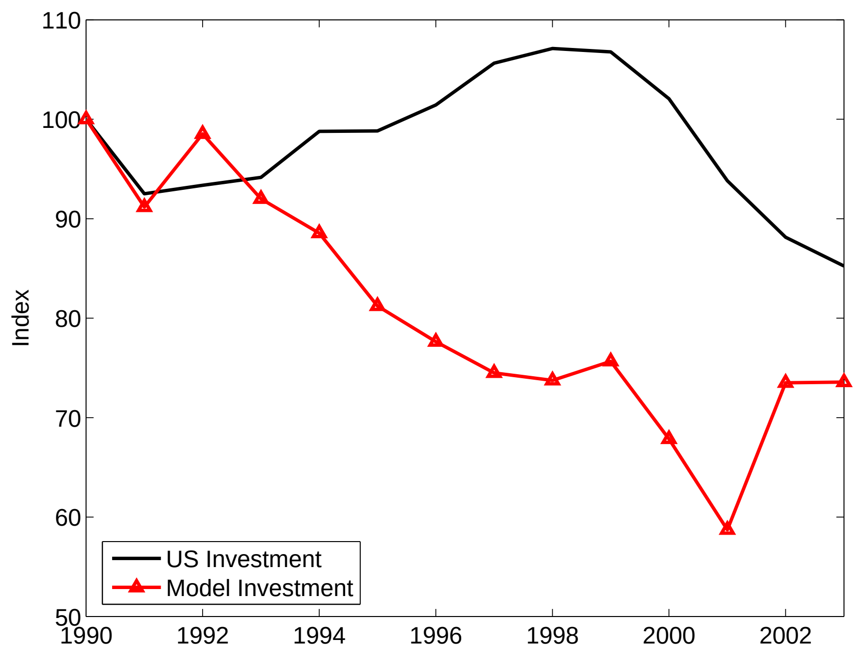

Figure 4. U.S. Per Capita Real Investment and Prediction of Model without Intangible Capital, Series Divided by 1.02^t

(Labor and investment wedges constant)

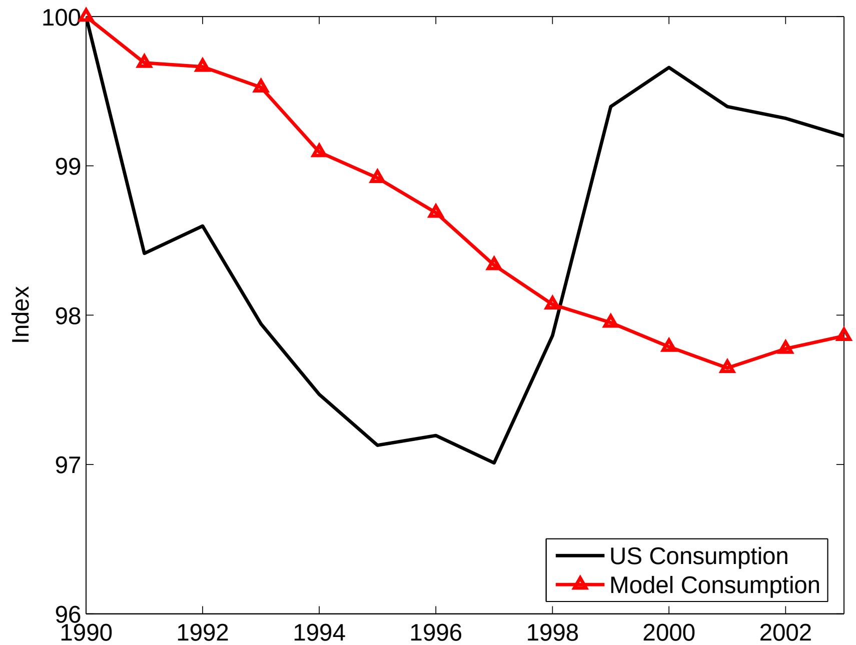

Figure 5. U.S. Per Capita Real Consumption and Prediction of Model without Intangible Capital, Series Divided by 1.02^t

(Labor and investment wedges constant)

isfy the input justification criterion and does not provide a plausible answer the question, Why did hours boom in the 1990s? It also does not satisfy the prediction criterion. The model's predictions for factor incomes and capital gains are inconsistent with U.S. observations. The model's estimate of compensation is $(1-\theta)y_t$. U.S. wages rose by more than U.S. output. The model's estimate for the market value of capital is $(1-\tau_d)k_t$. Changes in the reproducible stock of tangible capital are much too small to rationalize the large U.S. capital gains in the late 1990s.

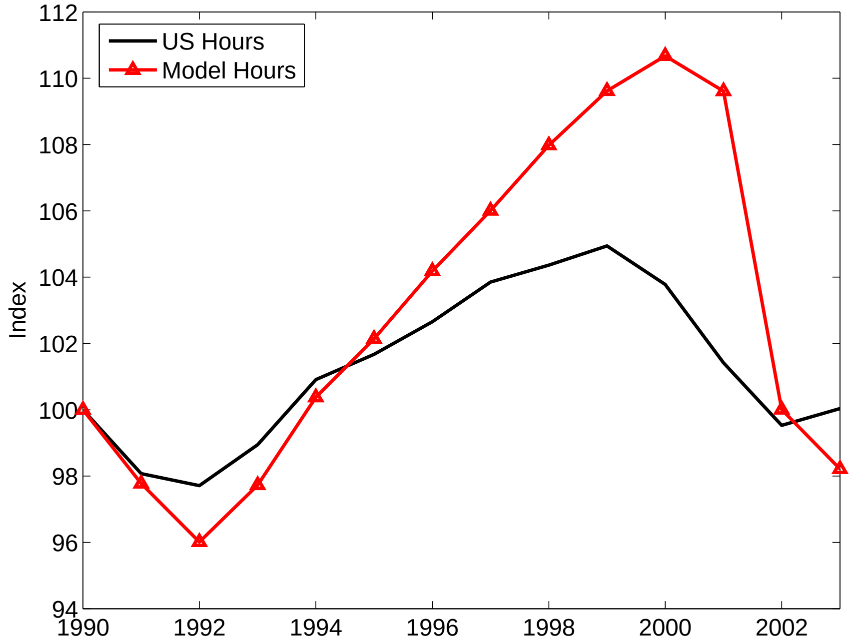

FIGURE 6. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITHOUT INTANGIBLE CAPITAL

(Labor wedge only)

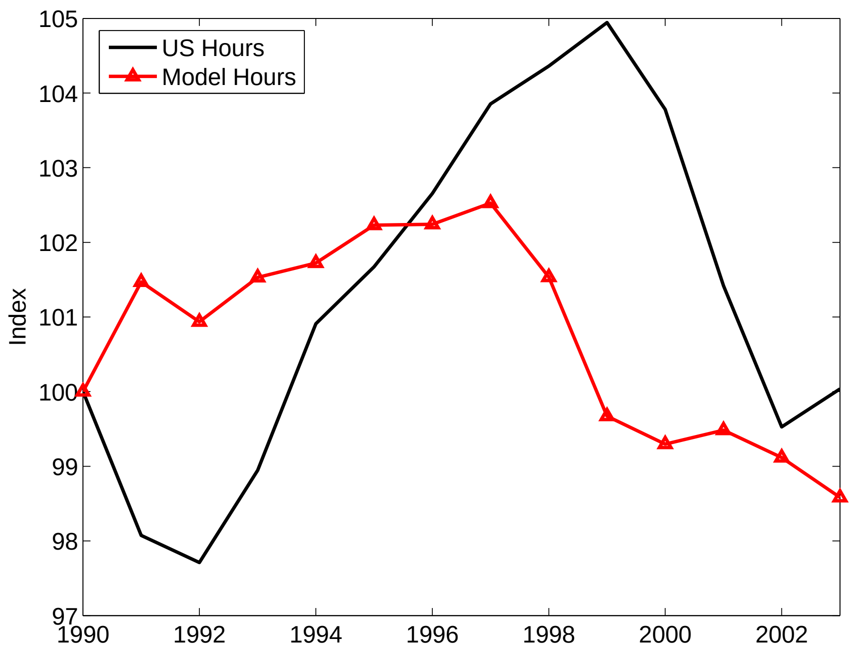

FIGURE 7. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITHOUT INTANGIBLE CAPITAL

(Investment wedge only)

2.1.3. A Version with a Business Sector

We extend the standard model slightly to include both a business and non-business sector, where the latter includes households, government, and nonprofits. As we saw from Table 1, the TFP of the aggregate economy is falling relative to trend. Business TFP, on the other hand, rose rapidly at the end of the 1990s. Here, we investigate whether focusing on the business sector helps the standard theory satisfy our criteria for a successful theory.

Assume now that measured investment includes business sector investment x_{bt} plus non-business investment $\bar{x}_{nt}$. Throughout, we will use b as a subscript for business and n for non-business. The problem for the household given the initial capital stock k_{b0} is to maximize

$$\max E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t$$

subject to

$$\begin{aligned} c_t + x_{bt} = {} & r_t k_{bt} + w_t h_{bt} - \tau_{ct} c_t - \tau_{ht} w_t h_{bt} - \tau_{kt} k_{bt} \\ & - \tau_{pt}(r_t - \delta - \tau_{kt}) k_{bt} - \tau_{xt} x_{bt} \\ & - \tau_{dt}\{r_t k_{bt} - x_{bt} - \tau_{kt} k_{bt} - \tau_{pt}(r_t - \delta - \tau_{kt}) k_{bt} - \tau_{xt} x_{bt}\} \\ & + [\bar{y}_{nt} - \bar{x}_{nt} - \bar{\tau}_{nt}] + Tr_t \end{aligned} \tag{2.1.14}$$

$$k_{b,t+1} = [(1-\delta) k_{bt} + x_{bt}]/(1+\eta) \tag{2.1.15}$$

$$h_t = h_{bt} + \bar{h}_{nt} \tag{2.1.16}$$

where $\bar{y}_{nt}$, $\bar{x}_{nt}$, $\bar{h}_{nt}$, and $\bar{\tau}_{nt}$ are value added, investment, hours, and taxes paid, respectively, in the non-business sector.

Here and later, we assume that sequences for non-business investment, hours, and output are taken as given by the households. Essentially we are assuming that prices in the non-business sector are such that households optimally chose the U.S. levels. Treating the non-business sector this way simplifies the modeling and allows us to directly compare the model national accounts and U.S. national accounts. Furthermore, our interest is U.S. boom in the 1990s, which occurred in the business sector.

The resource constraint is now

$$c_t + x_{bt} + \bar{x}_{nt} + g_t = y_{bt} + \bar{y}_{nt} = y_{mt},$$

where model GDP is the measured output y_{mt}. Value added in the business sector is

$$y_{bt} = k_{bt}^{\theta}(Z_{bt}h_{bt})^{1-\theta},$$

where $Z_{bt} = z_{bt}(1+\gamma)^t$. A NIPA accountant in this economy would measure the following product and income:

NIPA product $= c + x_b + \bar{x}_n + g$

Private consumption $= c$

Public consumption $= g$

Investment $= x_b + \bar{x}_n$

NIPA income $= y_b + \bar{y}_n$

Business profits $= (r - \tau_k - \delta)k_b$

Business wages $= wh_b$

Business depreciation $= \delta k_b$

Business production tax $= \tau_k k_b$

Nonbusiness income $= \bar{y}_n$.

The first-order conditions for the household's problem (assuming factors are paid their marginal product) in the case of log utility are as follows:

$$\frac{\psi(1+\tau_{ct})\hat{c}_t}{1-h_t} = (1-\tau_{ht})\frac{(1-\theta)\hat{y}_{bt}}{h_{bt}} \tag{2.1.17}$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} \left[R^b_{t,t+1} + (1-\delta)\xi_{t,t+1} \right], \tag{2.1.18}$$

where $\hat{\beta} = \beta/(1+\gamma)$ and $\mu_t = 1/[(1+\tau_{ct})\hat{c}_t]$ as before,

$$R^b_{t,t+1} = \frac{1-\tau_{d,t+1}}{(1-\tau_{dt})(1+\tau_{xt})} \left[(1-\tau_{p,t+1}) \left(\theta \frac{\hat{y}_{b,t+1}}{\hat{k}_{b,t+1}} - \tau_{k,t+1} \right) + \delta \tau_{p,t+1} \right] \tag{2.1.19}$$

and $\xi_{t,t+1}$ is defined in (2.1.6). Notice that the only difference between the first-order conditions (2.1.3)–(2.1.4) and (2.1.17)–(2.1.18) are the marginal products of labor and capital, which in the first case is economy-wide and in the second case is for the business sector.

We'll again assume that we have values for some of the endogenous variables in 1990 and use them to set some of the parameters. Suppose we observe $\hat{y}_b$, $\hat{y}_n$, $\hat{c}$, $\hat{g}$, h, h_b, and $\hat{k}_b$ along with estimates for the growth rates γ, η, the tax on labor τ_h, the tax on consumption τ_c, tax rates on capital, τ_p, τ_d, τ_x, τ_k, and an interest rate i. We can use these estimates to evaluate the following expressions for δ, θ, ψ, and z_b:

$$\delta = \hat{x}_b/\hat{k}_b + 1 - (1+\eta)(1+\gamma) \tag{2.1.20}$$

$$\theta = \frac{(1-\hat{\beta}(1-\delta))(1+\tau_x) - \hat{\beta}\delta\tau_p + \hat{\beta}(1-\tau_p)\tau_k}{\hat{\beta}(1-\tau_p)} \frac{\hat{k}_b}{\hat{y}_b} \tag{2.1.21}$$

$$\psi = \frac{(1-\tau_h)(1-\theta)(1-h)\hat{y}_b}{(1+\tau_c)\hat{c}h_b} \tag{2.1.22}$$

$$z_b = \left(\hat{k}_b/y_b\right)^{\theta/(\theta-1)} \frac{\hat{y}_b}{h_b}, \tag{2.1.23}$$

where $\hat{x}_b = \hat{y}_b + \bar{y}_n - \bar{x}_n - \hat{g} - \hat{c}$ and $\beta = (1+\gamma)/(1+i)$ as before.

The U.S. levels of (detrended) variables in 1990 that we use when parameterizing the model are as follows: $\hat{y}_m = 1$ (which is a normalization), $\hat{y}_b = .6621$, $\bar{y}_n = .3379$, $\hat{c} = .7626$, $\hat{x} = .2377$, $\hat{g} = 0$, $h = .2751$, and $\hat{k}_b = 1.66$. Growth rates, the interest rate, the discount

Year (t)	A_t	τ_{ht}	τ_{ct}	L_{wt}	X_{wt}
1990	1.7544	0.3109	0.0657	1.0000	1.0000
1991	1.7136	0.3070	0.0675	0.9882	1.0122
1992	1.7267	0.3028	0.0678	0.9761	1.0081
1993	1.7127	0.3034	0.0678	0.9866	1.0135
1994	1.7066	0.3068	0.0702	1.0088	1.0162
1995	1.7069	0.3116	0.0686	1.0163	1.0224
1996	1.7118	0.3190	0.0674	1.0327	1.0253
1997	1.7269	0.3254	0.0674	1.0403	1.0316
1998	1.7639	0.3327	0.0670	1.0394	1.0284
1999	1.7837	0.3335	0.0662	1.0449	1.0185
2000	1.7985	0.3424	0.0649	1.0402	1.0218
2001	1.7559	0.3472	0.0625	1.0488	1.0285
2002	1.7330	0.3076	0.0617	0.9864	1.0298
2003	1.7118	0.2885	0.0621	0.9746	1.0297

Table 2. Exogenous Variables for Model with Business but No Intangible Capital

factor, and taxes on labor and consumption are as before. The capital tax rates are now $\tau_k = .0144$, $\tau_x = 0$, $\tau_p = 0.35$, and $\tau_d = 0.15$, which are the effective rates for the business sector. Substituting these values in the expressions (2.1.20)–(2.1.23) implies $\delta = 0.0331$, $\theta = .277$, $\psi = 1.375$, and $z_b = 2.176$.

2.1.4. Business Cycle Accounting in the 1990s

We observe sequences for $\hat{y}_{bt}$, $\bar{y}_{nt}$, $\hat{c}_t$, $\bar{x}_{bt}$, $\bar{x}_{nt}$, $\hat{g}_t$, h_t, h_{bt}, $\bar{h}_{nt}$, τ_{ht}, and τ_{ct}, and an initial business capital stock $\hat{k}_{b0}$. Given $\hat{k}_{b0}$ and the sequence for $\hat{x}_{bt}$, we can use the law of motion for business capital (2.1.15) to derive the sequence of stocks $\{k_{bt}\}$. Then, we have business TFP as follows: $A_t = y_{bt}/[k_{bt}^{\theta} h_{bt}^{1-\theta}]$.

As in the one-sector version of the model, we can define the labor wedge as follows:

$$L_{wt} = \frac{\psi(1+\tau_{ct})\hat{c}_t}{1-h_t} \cdot \frac{\hat{h}_{bt}}{(1-\theta)\hat{y}_{bt}} \cdot \frac{1}{1-\tau_{ht}} \tag{2.1.24}$$

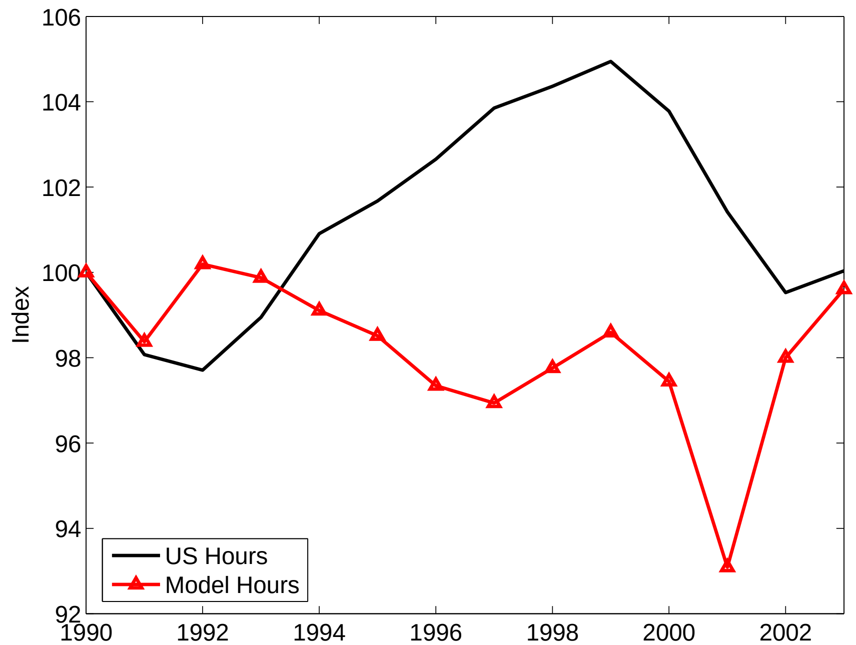

Figure 8. U.S. Per Capita Hours and Prediction of Model with Business but No Intangible Capital
(Labor and investment wedges constant)

and the investment wedge is as in (2.1.13), except that the capital return appearing in this expression is now the return on business capital $R^b_{t,t+1}$ instead of $R_{t,t+1}$.

We conduct the same experiment as before of computing equilibrium paths for per capita hours. In Table 2, we report the values of the implied exogenous variables.

Figures 8–14 show the results in the case that only TFP (A_t) and tax rates (τ_{ht}, τ_{ct}) vary. Figure 8 is a comparison of per capita hours for the United States and for the model. As in the one-sector version of the model, there is a large deviation. Figures 9 and 10 show per capita real GDP and value added in the business sector. As before, the model predicts that the U.S. economy should have been depressed (relative to trend). Because TFP in the business sector rises at the end of the 1990s, there is a rise in business value added. However, the growth is too modest relative to what we observed in the actual economy.

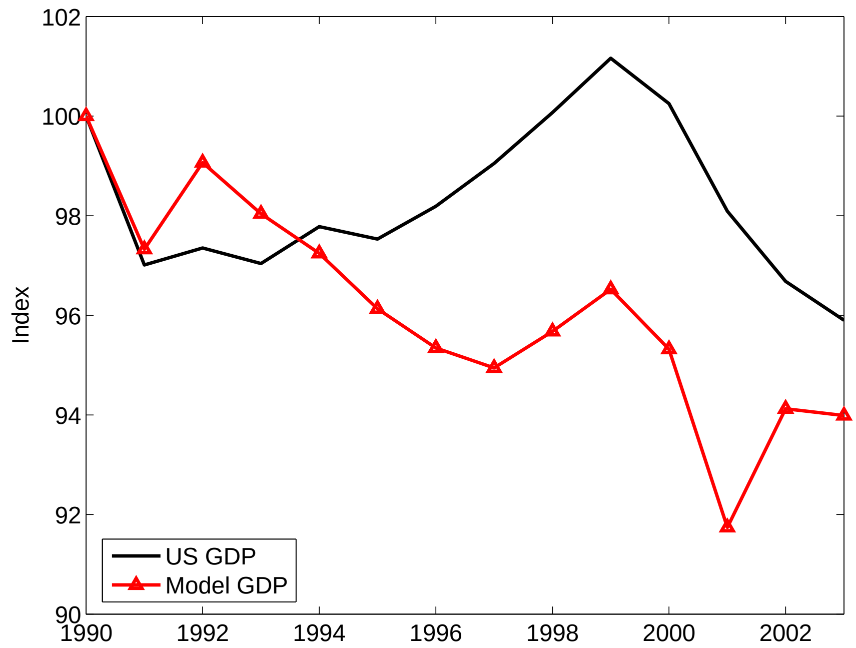

FIGURE 9. U.S. PER CAPITA REAL GDP AND PREDICTION OF MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL, SERIES DIVIDED BY 1.02^t

(Labor and investment wedges constant)

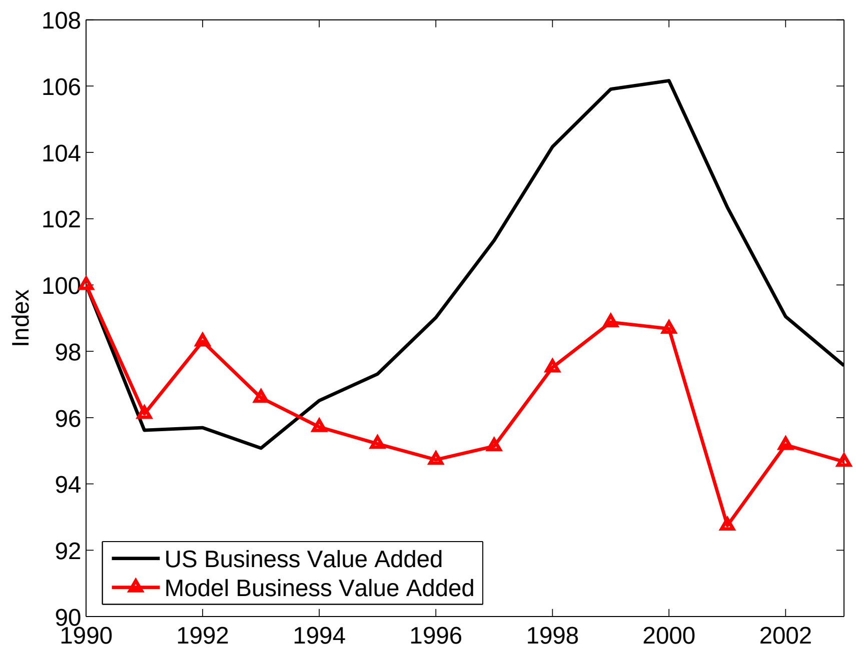

FIGURE 10. U.S. PER CAPITA REAL BUSINESS VALUE ADDED AND PREDICTION OF MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL, SERIES DIVIDED BY 1.02^t

(Labor and investment wedges constant)

Figures 11–12 show labor productivity for the aggregate economy and for the business sector. The prediction of business labor productivity is reasonable, but the prediction of overall labor productivity is no better than in the one-sector version of the growth model.

For completeness, we also include figures for the components of GDP. These are shown in Figures 13 and 14. In this case, there is little improvement in the model's predictions relative to the one-sector model.

In Figures 15 and 16, we show the predictions for per capita hours when we allow only the labor wedge or only the investment wedge to vary. These results are comparable to those in Figures 6 and 7 for the one-sector version of the model. They can also be compared to Figure 8, which assumes no variation in either the labor wedge or the investment wedge. These results make it clear that to generate the hours boom, we again need an implausible boom in the labor wedge. However, as before, we have no empirical evidence supporting any theory of this labor wedge boom.

Summary

To account for the boom in the U.S. economy using the standard growth model, whether we use the one-sector model or the two-sector model, we must rely on implausible movements in the labor and investment wedges.

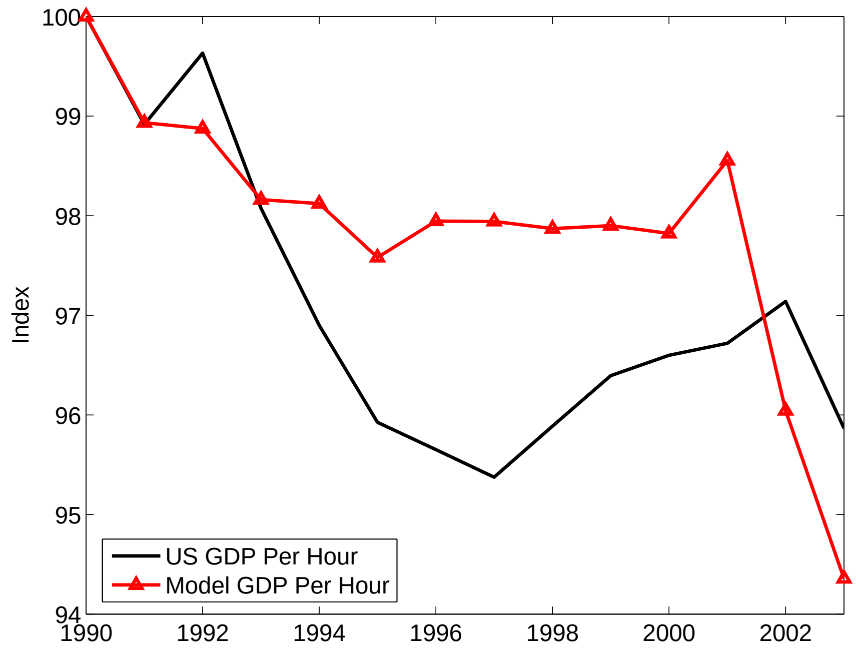

Figure 11. U.S. Real GDP Per Hour and Prediction of Model with Business but No Intangible Capital, Series Divided by 1.02^t

(Labor and investment wedges constant)

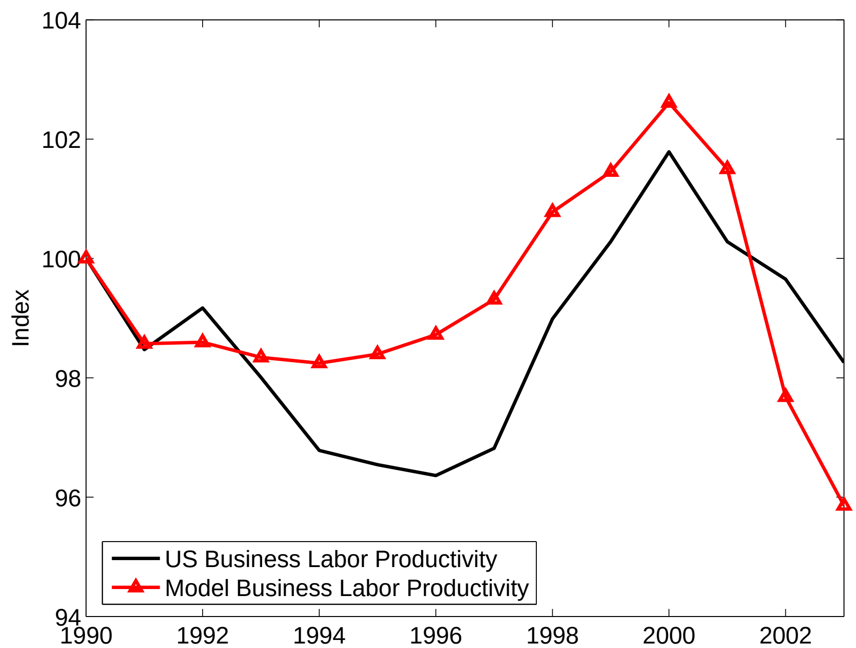

Figure 12. U.S. Real Business Value Added Per Hour and Prediction of Model with Business but No Intangible Capital, Series Divided by 1.02^t

(Labor and investment wedges constant)

2.2. Theory with Intangible Capital and Neutral Technology

We now extend the basic theory described above by incorporating intangible capital. We have used this theory before to study the U.S. and U.K. stock markets. We show, however, that to generate a boom like that observed in the United States during the 1990s, we require wildly implausible exogenous wedges as before, thus demonstrating that intangible capital per se cannot make up for whatever is missing in standard theory.

2.2.1. A Specific Model

The problem for the household given initial stocks of tangible capital k_b and intangible capital k_u (which is *unmeasured* in NIPA) is to maximize

$$\max E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t$$

subject to

$$\begin{aligned}
c_t + x_{Tt} + x_{It} = {} & r_{Tt} k_{Tt} + r_{It} k_{It} + w_t h_{bt} \\
& - \tau_{ct} c_t - \tau_{ht} w_t h_{bt} - \tau_{kt} k_{Tt} \\
& - \tau_{pt}\{r_{Tt} k_{Tt} + r_{It} k_{It} - \delta_T k_{Tt} - \tau_{kt} k_{Tt} - x_{It}\} - \tau_{xt} x_{Tt} \\
& - \tau_{dt}\{r_{Tt} k_{Tt} + r_{It} k_{It} - x_{Tt} - \tau_{kt} k_{Tt} - x_{It} \\
& \qquad - \tau_{pt}(r_{Tt} k_{Tt} + r_{It} k_{It} - \delta_T k_{Tt} - \tau_{kt} k_{Tt} - x_{It}) - \tau_{xt} x_{Tt}\} \\
& + [\bar{y}_{nt} - \bar{x}_{nt} - \bar{\tau}_{nt}] + Tr_t
\end{aligned} \tag{2.2.1}$$

$$k_{T,t+1} = [(1-\delta_T) k_{Tt} + x_{Tt}]/(1+\eta) \tag{2.2.2}$$

$$k_{I,t+1} = [(1-\delta_I) k_{It} + x_{It}]/(1+\eta) \tag{2.2.3}$$

$$h_t = h_{bt} + \bar{h}_{nt}. \tag{2.2.4}$$

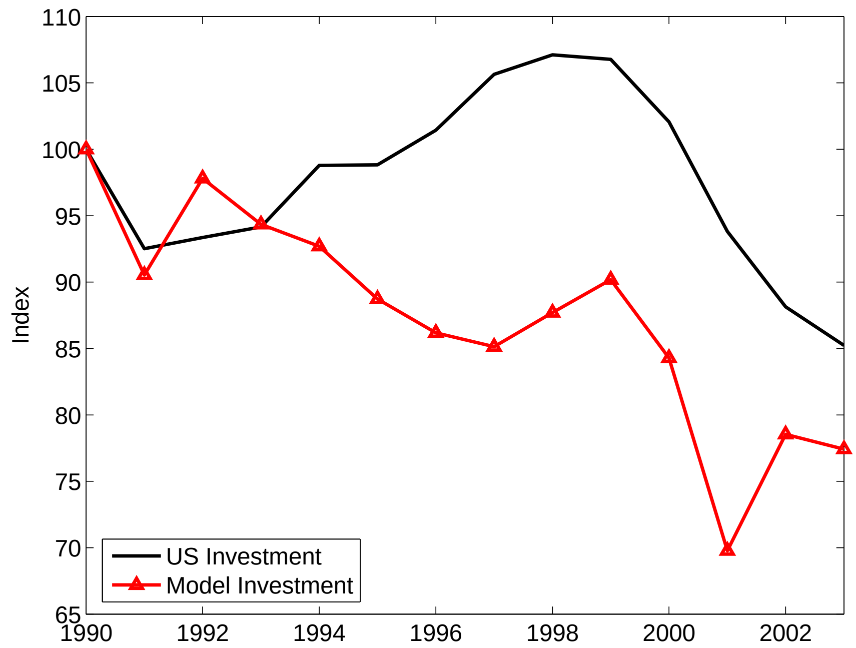

FIGURE 13. U.S. PER CAPITA REAL INVESTMENT AND PREDICTION OF MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL, SERIES DIVIDED BY 1.02^t

(Labor and investment wedges constant)

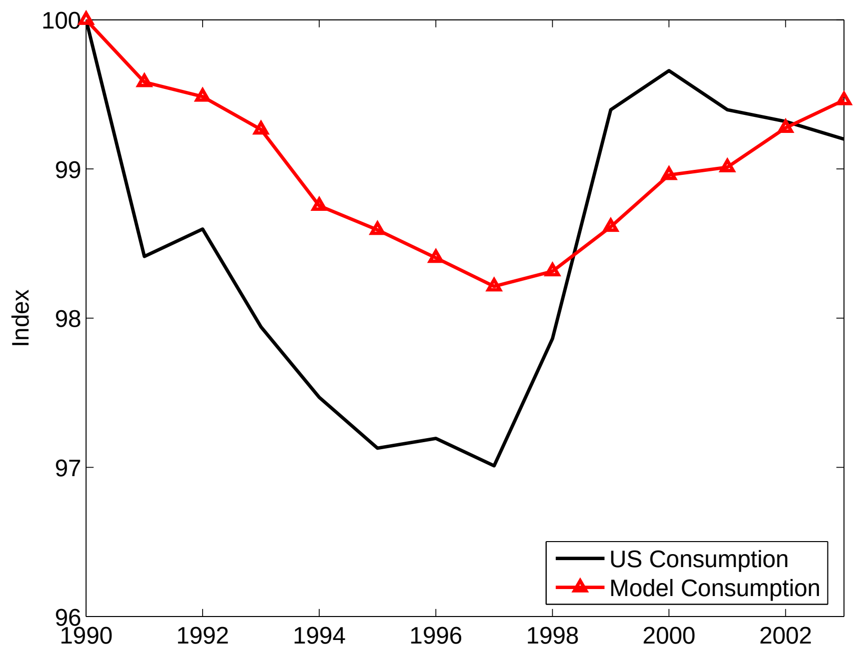

FIGURE 14. U.S. PER CAPITA REAL CONSUMPTION AND PREDICTION OF MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL, SERIES DIVIDED BY 1.02^t

(Labor and investment wedges constant)

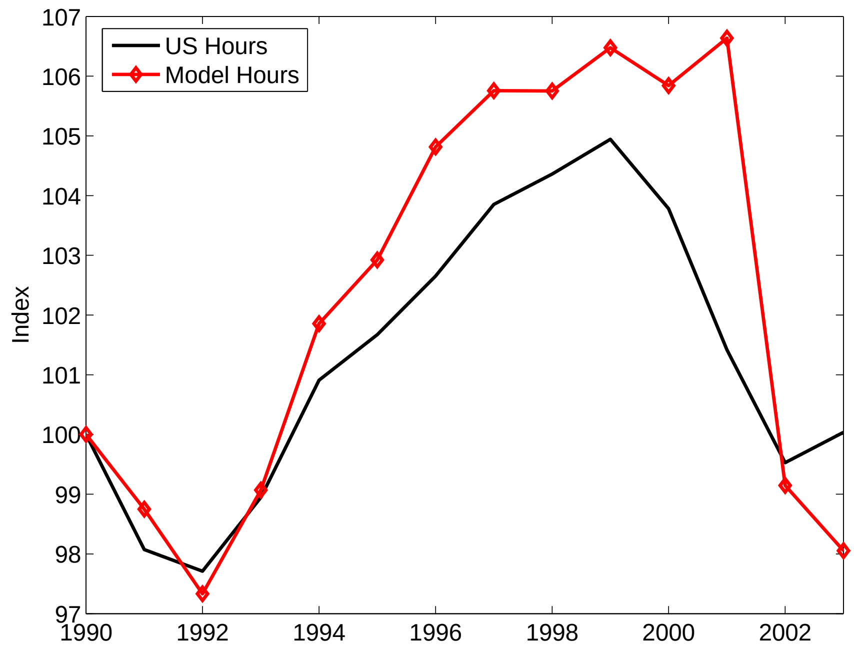

FIGURE 15. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL

(Labor wedge only)

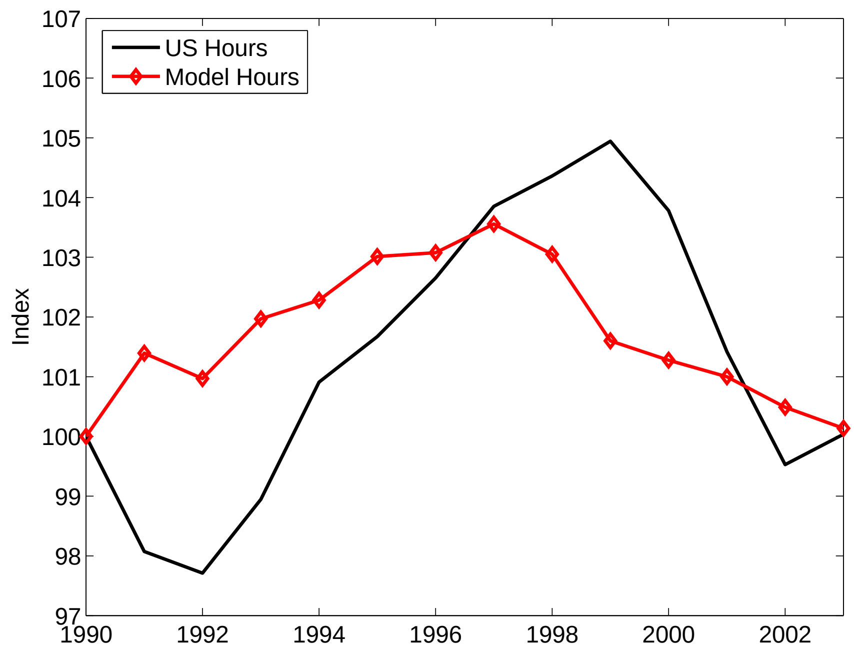

FIGURE 16. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL

(Investment wedge only)

All variables are as defined in Section 2.1.3 except that we have added the stock k_{It} and investment x_{It} of intangibles. There are no additional exogenous variables. Notice that x_{It} is expensed and thus subtracted from taxable profits in (2.2.1).

Total output in the business sector is equal to

$$y_t = k_{Tt}^{\theta} k_{It}^{\phi} (Z_{bt} h_{bt})^{1-\theta-\phi},$$

where $Z_{bt} = z_{bt}(1+\gamma)^t$, and the economy's resource constraint is

$$c_t + x_{Tt} + x_{It} + \bar{x}_{nt} + g_t = y_t + \bar{y}_{nt}.$$

Measured value added in the business sector is $y_{bt} = y_t - x_{It}$ and aggregate GDP is $y_{bt} + \bar{y}_{nt}$. A NIPA accountant in this economy would measure the following product and income:

NIPA product $= c + x_T + \bar{x}_n + g$

Private consumption $= c$

Public consumption $= g$

Investment $= x_T + \bar{x}_n$

NIPA income $= y_b + \bar{y}_n$

Business profits $= (r_T - \tau_k - \delta_T)k_T + r_I k_I - x_I$

Business wages $= w h_b$

Business depreciation $= \delta_T k_T$

Business production tax $= \tau_k k_T$

Nonbusiness income $= \bar{y}_n$,

which differs from the earlier model only in the category of business profits. Business profits now include a dividend to intangible, $r_I k_I - x_I$.

Assuming log utility, we can derive the first-order conditions, which are given by

$$\frac{\psi(1+\tau_{ct})\hat{c}_t}{1-h_t} = (1-\tau_{ht})\frac{(1-\theta-\phi)\hat{y}_t}{h_{bt}} \tag{2.2.5}$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} \left[R^T_{t,t+1} + (1-\delta_T)\xi_{t,t+1} \right] \tag{2.2.6}$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} \left[R^I_{t,t+1} + (1-\delta_I)\zeta_{t,t+1} \right], \tag{2.2.7}$$

where

$$R^T_{t,t+1} = \frac{1-\tau_{d,t+1}}{(1-\tau_{dt})(1+\tau_{xt})} \left[(1-\tau_{p,t+1}) \left(\theta \frac{\hat{y}_{t+1}}{\hat{k}_{T,t+1}} - \tau_{k,t+1} \right) - \delta_T \tau_{p,t+1} \right] \tag{2.2.8}$$

$$R^I_{t,t+1} = \phi \frac{\hat{y}_{t+1}}{\hat{k}_{I,t+1}} \tag{2.2.9}$$

$$\zeta_{t,t+1} = \frac{1-\tau_{d,t+1}}{1-\tau_{dt}} \cdot \frac{1-\tau_{p,t+1}}{1-\tau_{pt}} \tag{2.2.10}$$

and $\xi_{t,t+1}$ is given by (2.1.6). Relative to the standard theory without intangible capital, we are adding one dynamic equation (2.2.7).

Assigning parameters for the model is done as above except we need to also assign δ_I and θ_I. We assume the intangible capital is long-lived (e.g., organizations) and set $\delta_I = 0$. This choice will not matter for our results. To set θ_I we use one piece of additional information from the national accounts for 1990, namely business compensation. Then, given estimates for the rental rate r_T, the intangible capital stock $\hat{k}_I$, and the intangible investment $\hat{x}_I$, we have θ and ϕ:

$$r_T = \frac{(1-\hat{\beta}(1-\delta_T))(1+\tau_x) - \delta_T \hat{\beta} \tau_p + \hat{\beta}(1-\tau_p)\tau_k}{\hat{\beta}(1-\tau_p)} \tag{2.2.11}$$

$$\hat{k}_I = \frac{\hat{y}_b - r_b \hat{k}_b - \text{1990 NIPA business compensation}}{1+i-(1+\gamma)(1+\eta)} \tag{2.2.12}$$

$$\hat{x}_I = ((1+\gamma)(1+\eta) - 1 + \delta_I)\hat{k}_I \tag{2.2.13}$$

$$\phi = \frac{(i+\delta_u)\hat{k}_u}{\hat{y}_b + \hat{x}_u} \tag{2.2.14}$$

$$\theta = \frac{r_T \hat{k}_T}{\hat{y}_T + \hat{x}_I} \tag{2.2.15}$$

where the values for $\hat{y}_b$, $\hat{k}_T$, i, γ, and η are as before.

The U.S. levels of (detrended) variables in 1990 that we use when parameterizing the model are exactly the same as those used above. The only additional information is NIPA business compensation, which is equal to 0.443 times GDP in 1990. Growth rates, the interest rate, the discount factor, tax rates, and the depreciation rate on tangible capital are also exactly the same as in Section 2.1.3. Because total output includes intangible investment, some parameters are changed slightly. The new parameters are $\theta = 0.240$, $\phi = 0.180$, $\psi = 1.273$, and $z = 1.635$.

2.2.2. Business Cycle Accounting for the 1990s

Suppose that we have observations on $\hat{y}_{bt}$, $\bar{y}_{nt}$, $\hat{c}_t$, $\hat{x}_{Tt}$, $\bar{x}_{nt}$, $\hat{g}_t$, h_t, $\bar{h}_{nt}$, τ_{ht}, and τ_{ct}. Then, given $\hat{x}_{Tt}$, we can use the law of motion for capital to get

$$\hat{k}_{T,t+1} = [(1-\delta_T)\hat{k}_{Tt} + \hat{x}_{Tt}]/[(1+\gamma)(1+\eta)]$$

given an initial condition $\hat{k}_{T0}$.

We can infer the magnitude of intangible capital and investment using (2.2.3) and (2.2.7) along with data on consumption, business output, and tax rates.[9] Given sequences for intangible investments and stocks, we can compute TFP,

$$A_t = \frac{\hat{y}_{bt} + \hat{x}_{It}}{\hat{k}_{Tt}^{\theta}\hat{k}_{It}^{\phi}h_{bt}^{1-\theta-\phi}}.$$

To get a perfect match to U.S. data, we again rely on a labor wedge and an investment

[9] We use the steady-state level of $\hat{k}_I$ (given 1990 values of other variables) to initialize the stock. We assume the growth in the per capita stock in the last period is γ.

Year (t)	A_t	τ_{ht}	τ_{ct}	L_{wt}	X_{wt}
1990	1.6586	0.3109	0.0657	0.8020	1.0000
1991	0.9014	0.3070	0.0675	1.4051	0.9911
1992	1.4223	0.3028	0.0678	0.8955	0.9919
1993	1.1352	0.3034	0.0678	1.1216	0.9875
1994	1.2306	0.3068	0.0702	1.0634	0.9846
1995	1.1315	0.3116	0.0686	1.1695	0.9788
1996	1.2365	0.3190	0.0674	1.0995	0.9740
1997	1.1647	0.3254	0.0674	1.1924	0.9668
1998	1.4442	0.3327	0.0670	0.9874	0.9627
1999	1.6581	0.3335	0.0662	0.8722	0.9625
2000	1.3061	0.3424	0.0649	1.0993	0.9575
2001	1.1055	0.3472	0.0625	1.2776	0.9471
2002	1.1913	0.3076	0.0617	1.1043	0.9358
2003	1.1942	0.2885	0.0621	1.0793	0.9235

Table 3. Exogenous Variables for Model with Intangible Capital and Neutral Technology

wedge,

$$L_{wt} = \frac{\psi(1+\tau_{ct})\hat{c}_t}{1-h_t} \cdot \frac{h_{bt}}{(1-\theta)\hat{y}_t} \cdot \frac{1}{1-\tau_{ht}} \tag{2.2.16}$$

$$X_{w,t+1} = \frac{\hat{\beta}(1-\delta)\mu_{t+1}X_{wt}}{\mu_t - \hat{\beta}R^T_{t,t+1}\mu_{t+1}X_{wt}}\xi_{t,t+1} \tag{2.2.17}$$

with equation (2.1.13) solved recursively starting with $X_{w0} = 1$. As before, we would replace $1-\tau_{ht}$ with $(1-\tau_{ht})L_{wt}$ and $1/(1+\tau_{xt})$ with $X_{wt}/(1+\tau_{xt})$.

In Figures 17–19, we repeat the exercise of plotting actual hours and predicted hours for versions of the model with wedges off and wedges on. Table 3 has the values of the implied exogenous variables. Figure 17 shows the predicted hours for the case that $L_{wt} = X_{wt} = 1$ in all periods. Clearly, this is not an improvement on standard theory because the hours prediction is wildly oscillatory.[10] The same strange behavior is evident

[10] An alternative strategy is to use (2.2.5) to infer the sequence of intangible investments. In this case,

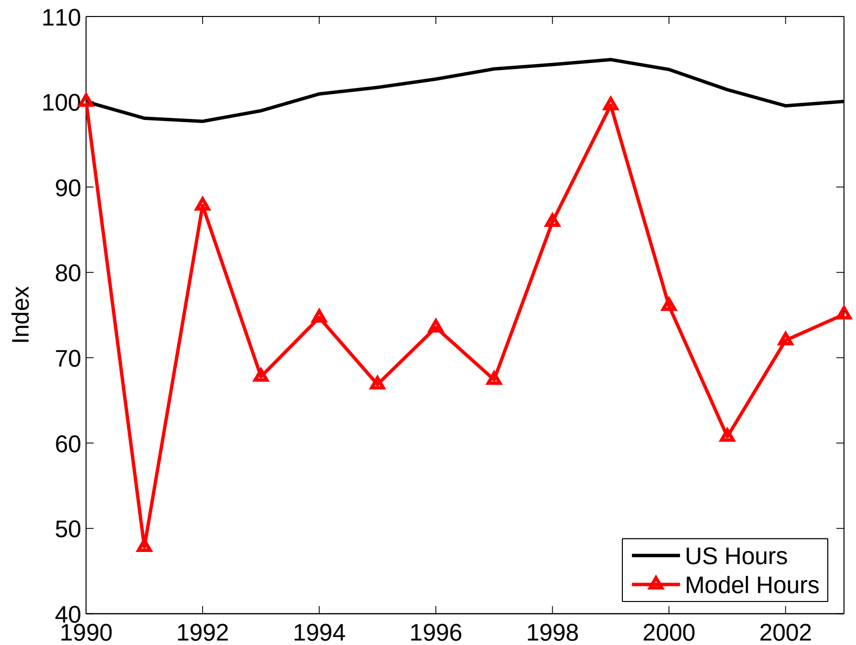

Figure 17. U.S. Per Capita Hours and Prediction for Model with Intangible Capital and Neutral Technology

(Labor and investment wedges constant)

in the labor-wedge-only case shown in Figure 18. Essentially, the labor wedge is canceling out the other exogenous variables in such a way that the prediction of Figure 19, with the investment-wedge-only case, is relatively smooth. Interestingly, if we input all of these exogenous variables, *we get a perfect fit.*

Why do the results look so strange? The logic that intangible capital makes up for whatever is missing is faulty. When we added a labor wedge to the theory without intangible capital, we simply added an exogenous term to one of the existing first-order conditions and let the wedge be whatever it had to be. *In the model with intangible capital, we are adding two more endogenous variables, and, therefore, we are adding two more equations and more restrictions in the dynamical system.* The new larger system

a "tangible investment wedge" and an "intangible investment wedge" are needed for (2.2.6) and (2.2.7) to hold, given U.S. observations. We applied this strategy and found again that the model's predictions were grossly at odds with the data when the wedges were off.

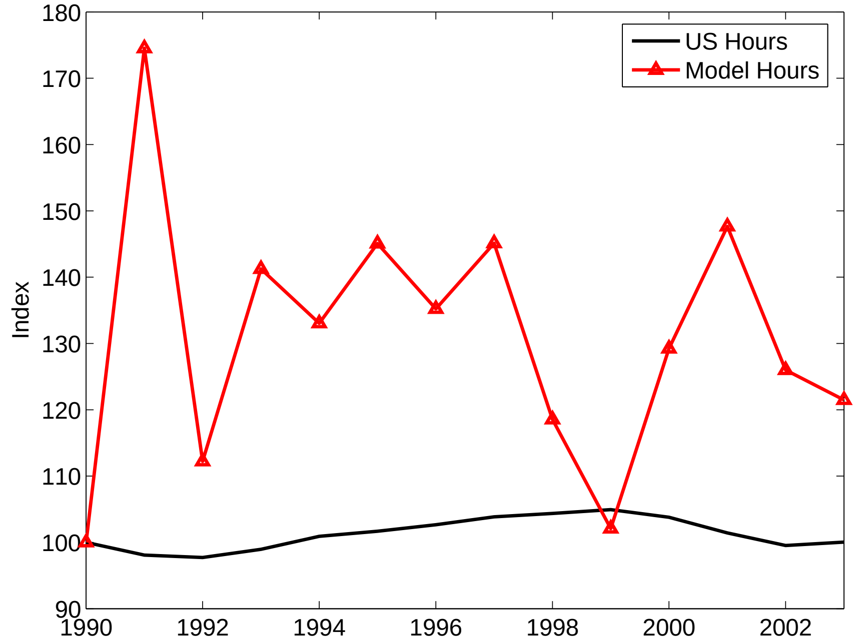

Figure 18. U.S. Per Capita Hours and Prediction for Model with Intangible Capital and Neutral Technology

(Labor wedge only)

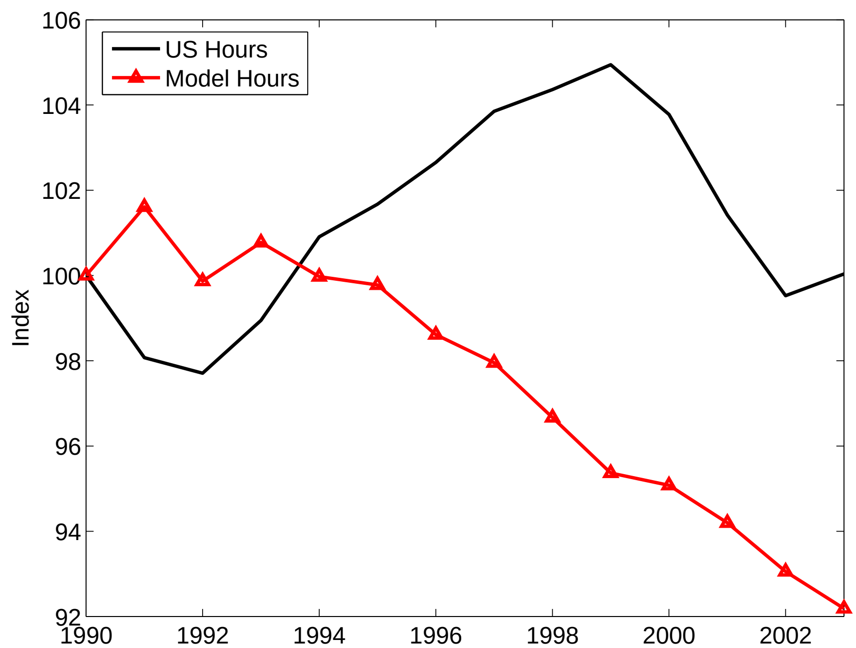

Figure 19. U.S. Per Capita Hours and Prediction for Model with Intangible Capital and Neutral Technology

(Investment wedge only)

of first-order conditions are not in block form. Thus, the intangible variables enter the original first-order conditions, and the observed variables enter the additional conditions. There is no mathematical basis for thinking that adding the intangible capital will improve the predictive power of the model with the wedges turned off. Indeed, it is worse in this case.

Summary

We showed that intangible capital is not a free parameter that makes up for whatever is missing to make standard theory work. Next, we show that central to understanding the boom of the 1990s is non-neutral technological change.

2.3. Theory with Intangible Capital and Non-neutral Technology

We turn now to the "extended model" developed in our paper that has *both* intangible capital and non-neutral technological change. This theory satisfies our input justification criterion, and we show here that it also satisfies our prediction criterion. We also demonstrate that we would get a very different result if the data-generating mechanism were inconsistent with the theory proposed, implying that the bar we set is not too low.

2.3.1. A Specific Model

There are two additions relative to the model of Section 2.2. First, we allow for *non-neutral technology*. The idea is that we are modeling a technology boom that is concentrated in intangible activity. Second, we allow for *sweat equity*. Some intangible investment is financed by shareholders and some is financed by worker-owners of businesses. We were motivated to include this, since the pattern of incomes suggests that both types of financing are done.

The problem for the household given initial stocks of business tangible capital k_{T0} and business intangible capital k_{I0} is to maximize

$$\max E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t$$

subject to

$$\begin{aligned}
c_t + x_{Tt} + q_t x_{It} = r_{Tt} k_{Tt} + r_{It} k_{It} + w_t h_{bt} \\
&- \tau_{ct} c_t - \tau_{ht}(w_t h_{bt} - (1-\chi) q_t x_{It}) - \tau_{kt} k_{Tt} \\
&- \tau_{pt}\{r_{Tt} k_{Tt} + r_{It} k_{It} - \delta_T k_{Tt} - \tau_{kt} k_{Tt} - \chi q_t x_{It}\} - \tau_{xt} x_{Tt} \\
&- \tau_{dt}\{r_{Tt} k_{Tt} + r_{It} k_{It} - x_{Tt} - \tau_{kt} k_{Tt} - \chi q_t x_{It} \\
&\qquad - \tau_{pt}(r_{Tt} k_{Tt} + r_{It} k_{It} - \delta_T k_{Tt} - \tau_{kt} k_{Tt} - \chi q_t x_{It}) \\
&\qquad - \tau_{xt} x_{Tt}\} - \tau_{gt} q_t x_{It} \\
&+ [\bar{y}_{nt} - \bar{x}_{nt} - \bar{\tau}_{nt}] + Tr_t \qquad (2.3.1)
\end{aligned}$$

$$k_{T,t+1} = [(1-\delta_T) k_{Tt} + x_{Tt}]/(1+\eta) \qquad (2.3.2)$$

$$k_{I,t+1} = [(1-\delta_I) k_{It} + x_{It}]/(1+\eta) \qquad (2.3.3)$$

$$h_t = h_{bt}^1 + h_{bt}^2 + \bar{h}_{nt} \qquad (2.3.4)$$

$$k_{Tt} = k_{Tt}^1 + k_{Tt}^2, \qquad (2.3.5)$$

where q_t is the relative price of intangible investment goods and final output, and χ is the fraction of intangible investment financed by shareholders. The remaining $1-\chi$ of intangible investment is financed by workers who own their businesses and put in sweat equity, which is uncompensated labor. The effective compensation is through capital gains when the business is sold.

The total produced in the business sector is $y_t = y_{bt} + q_t x_{It}$, where

$$y_{bt} = (k_{Tt}^1)^{\theta_1} k_{It}^{\phi_1} (Z_t^1 h_{bt}^1)^{1-\theta_1-\phi_1}$$

$$x_{It} = (k_{Tt}^2)^{\theta_2} k_{It}^{\phi_2} (Z_t^2 h_{bt}^2)^{1-\theta_2-\phi_2}$$

and $Z_t^1 = z_t^1(1+\gamma)^t$, $Z_t^2 = z_t^2(1+\gamma)^t$. If $\chi = 1$ and technologies are neutral (so that $q_t = 1$ in equilibrium), then we are back to the model of Section 2.2.1.

The economy's resource constraint is

$$c_t + x_{Tt} + q_t x_{It} + \bar{x}_{nt} + g_t = y_t + \bar{y}_{nt}$$

and aggregate GDP is $y_{bt} + \bar{y}_{nt}$. A NIPA accountant in this economy would measure the following product and income:

NIPA product $= c + x_T + \bar{x}_n + g$

Private consumption $= c$

Public consumption $= g$

Investment $= x_T + \bar{x}_n$

NIPA income $= y_b + \bar{y}_n$

Business profits $= (r_T - \tau_k - \delta_T)k_T + r_I k_I - \chi q x_I$

Business wages $= wh_b - (1-\chi)qx_I$

Business depreciation $= \delta_T k_T$

Business production tax $= \tau_k k_T$

Nonbusiness income $= \bar{y}_n$

Simplifying the first-order conditions for the log utility case, we get

$$\frac{\psi(1+\tau_{ct})\hat{c}_t}{1-h_t} = (1-\tau_{ht})\frac{(1-\theta_1-\phi_1)\hat{y}_{bt}}{h_{bt}^1} \qquad (2.3.6)$$

$$(1-\theta_1-\phi_1)\frac{\hat{y}_{bt}}{h_{bt}^1} = (1-\theta_2-\phi_2)\frac{q_t\hat{x}_{It}}{h_{bt}^2} \qquad (2.3.7)$$

$$\theta_1 \frac{\hat{y}_{bt}}{\hat{k}^1_{Tt}} = \theta_2 \frac{q_t \hat{x}_{It}}{\hat{k}^2_{Tt}} \tag{2.3.8}$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} \left[R^T_{t,t+1} + (1-\delta_T)\xi_{t,t+1} \right] \tag{2.3.9}$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} \left[R^I_{t,t+1} + (1-\delta_I) q_{t+1} \zeta_{t,t+1}/q_t \right], \tag{2.3.10}$$

where

$$R^T_{t,t+1} = \frac{1-\tau_{d,t+1}}{(1-\tau_{dt})(1+\tau_{xt})} \left[(1-\tau_{p,t+1}) \left(\theta_1 \frac{\hat{y}_{b,t+1}}{\hat{k}^1_{b,t+1}} - \tau_{k,t+1} \right) - \delta_b \tau_{p,t+1} \right]$$

$$R^I_{t,t+1} = \frac{\phi_1 \hat{y}_{b,t+1} + \phi_2 q_{t+1} \hat{x}_{I,t+1}}{q_t \hat{k}_{I,t+1}} \left(\frac{(1-\tau_{d,t+1})(1-\tau_{p,t+1})}{\chi(1-\tau_{dt})(1-\tau_{pt}) + (1-\chi)(1-\tau_{ht})} \right)$$

$$\zeta_{t,t+1} = \frac{\chi(1-\tau_{d,t+1})(1-\tau_{p,t+1}) + (1-\chi)(1-\tau_{h,t+1})}{\chi(1-\tau_{dt})(1-\tau_{pt}) + (1-\chi)(1-\tau_{ht})} \tag{2.3.11}$$

and $\xi_{t,t+1}$ is given by (2.1.6). Assigning parameters for the model is done as above except that we have three additional parameters, χ, θ_2, and ϕ_2 (and we normalize q to 1). In our benchmark experiments, we set $\chi = 1/2$ and equated capital shares, $\theta_2 = \theta_1$ and $\phi_2 = \phi_1$. In this case, the shares θ_1 and ϕ_1 are constructed as follows:

$$r_I = \frac{q(1-\hat{\beta}(1-\delta_I))[(1-\chi)(1-\tau_h) + \chi(1-\tau_d)(1-\tau_p)]}{\hat{\beta}(1-\tau_d)(1-\tau_p)}$$

$$\hat{k}_I = \frac{\hat{y}_b - r_T \hat{k}_T - 1990 \text{ NIPA business compensation}}{r_I - \chi q[(1+\gamma)(1+\eta) - 1 + \delta_I]}$$

$$\hat{x}_I = ((1+\gamma)(1+\eta) - 1 + \delta_I)\hat{k}_I$$

$$\phi_1 = \frac{r_I \hat{k}_I}{\hat{y}_b + q\hat{x}_I}$$

$$\theta_1 = \frac{r_T \hat{k}_T}{\hat{y}_b + q\hat{x}_I},$$

where r_T is given by (2.2.11). In Chapter 5, we will check to see if our results are sensitive to our choices of χ, α_b, and α_u.

The U.S. levels of (detrended) variables in 1990 that we use when parameterizing the model are exactly the same as those used in Section 2.2.1. Because $\chi < 1$ and technology is non-neutral, some parameters are changed. These are changed to $\theta_1 = 0.263$, $\phi_1 = 0.076$, $\psi = 1.323$, $z^1 = 2.161$, and $z^2 = 1.539$.

2.3.2. Business Cycle Accounting for the 1990s

We have observations on $\hat{y}_{bt}$, $\bar{y}_{nt}$, $\hat{c}_t$, $\hat{x}_{Tt}$, $\bar{x}_{nt}$, $\hat{g}_t$, h_t, h_{bt}, $\bar{h}_{nt}$, τ_{ht}, and τ_{ct}. Given $\hat{x}_{Tt}$, we can use the law of motion for capital as before to get the sequence of capital stocks $\{\hat{k}_{Tt}\}$ given an initial condition $\hat{k}_{T0}$.

We can infer how many hours are used in final production in the business sector by rearranging the intratemporal condition as follows:

$$\hat{h}^1_{bt} = \frac{(1-\theta_1-\phi_1)(1-\tau_{ht})\hat{y}_{bt}(1-h_t)}{\psi(1+\tau_{ct})\hat{c}_t}$$

in terms of observables. Since we observe total business hours, we know the hours spent accumulating intangible, $h^2_{bt} = h_{bt} - h^1_{bt}$. The fact that we had to put in a large L_{wt} in the standard theory works in favor of the technology boom hypothesis. If a rise in h^2_{bt} is the source of the hours boom, then we will get a boom in $\hat{y}_{bt}/\hat{h}^1_{bt}$. This is our story for the earlier labor wedge.

Total output is $\hat{y}_{bt} + \bar{y}_{nt} + q_t\hat{x}_{It}$ where

$$q_t\hat{x}_{It} = \left(\frac{1-\theta_1-\phi_1}{1-\theta_2-\phi_2}\right)\frac{\hat{y}_t}{h^1_t}h^2_t. \tag{2.3.12}$$

The relation follows from the fact that households equate wages in the two business-sector activities. Similarly, households equate returns to tangible business capital in the two business-sector activities and, therefore, it must be that case that

$$\hat{k}^1_{Tt} = \frac{\theta_1 \hat{y}_{bt}}{\theta_1 \hat{y}_{bt} + \theta_2 q_t \hat{x}_{It}} \hat{k}_{Tt}$$

and $\hat{k}^2_{Tt} = \hat{k}_{Tt} - \hat{k}^1_{Tt}$.

To infer sequences for intangible capital $\hat{k}_{It}$, we guess a path for the price q_t and use (2.3.10) and the capital accumulation equation (2.3.3) to derive sequences for intangible flows and stocks. It must be the case that x_{It} multiplied by the guess q_t is equal to the left-hand side of (2.3.12).

Technology parameters are given as follows:

$$A^1_t = \hat{y}_{bt}/[(\hat{k}^1_{Tt})^{\theta_1}(\hat{k}_{It})^{\phi_1}(h^1_t)^{1-\theta_1-\phi_1}]$$

$$A^2_t = \hat{x}_{It}/[(k^2_{Tt})^{\theta_2}(k_{It})^{\phi_2}(h^2_t)^{1-\theta_2-\phi_2}]$$

We also include an investment wedge in (2.3.9) to account for any mismeasurement in capital tax rates (which we assumed to be constant over this period). The expression is exactly as in the three earlier examples with $R^T_{t,t+1}$ given by (2.2.8). We need the effect of the investment wedge to be quantitatively small if our theory is to satisfy the input justification criterion. Otherwise, we need to rethink our assumption of constant tax policy.[11]

We turn now to the numerical experiments. Table 4 has the values of the implied exogenous variables. In Figure 20, we plot U.S. per capita hours along with the model's

[11] In our sensitivity analysis, we consider feeding in some estimates of capital tax rates for the 1990s.

Year (t)	A_t^1	A_t^2	τ_{ht}	τ_{ct}	X_{wt}
1990	1.6534	1.3111	0.3109	0.0657	1.0000
1991	1.5956	1.2357	0.3070	0.0675	1.0116
1992	1.5910	1.1608	0.3028	0.0678	1.0064
1993	1.5940	1.2445	0.3034	0.0678	1.0110
1994	1.6213	1.3818	0.3068	0.0702	1.0140
1995	1.6312	1.4321	0.3116	0.0686	1.0208
1996	1.6579	1.5176	0.3190	0.0674	1.0252
1997	1.6814	1.5740	0.3254	0.0674	1.0336
1998	1.7125	1.5948	0.3327	0.0670	1.0327
1999	1.7358	1.6245	0.3335	0.0662	1.0254
2000	1.7380	1.6132	0.3424	0.0649	1.0314
2001	1.7027	1.6180	0.3472	0.0625	1.0416
2002	1.5901	1.3214	0.3076	0.0617	1.0431
2003	1.5556	1.2302	0.2885	0.0621	1.0428

Table 4. Exogenous Variables for Model with Intangible Capital and Non-neutral Technology

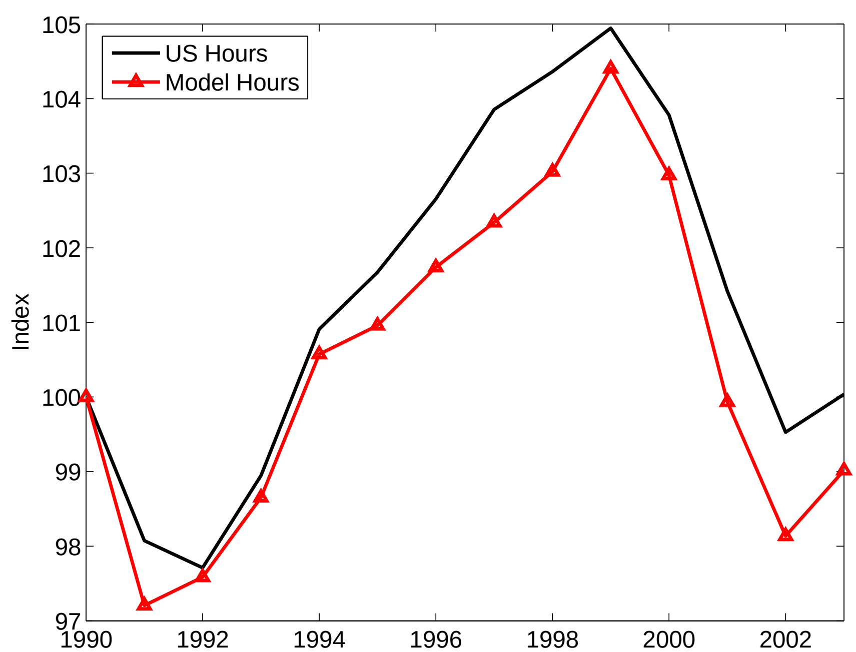

Figure 20. U.S. Per Capita Hours and Prediction of Model with Intangible Capital and Non-neutral Technology
(Investment wedge constant)

prediction for the case with only the TFPs and tax rates on labor and consumption varying

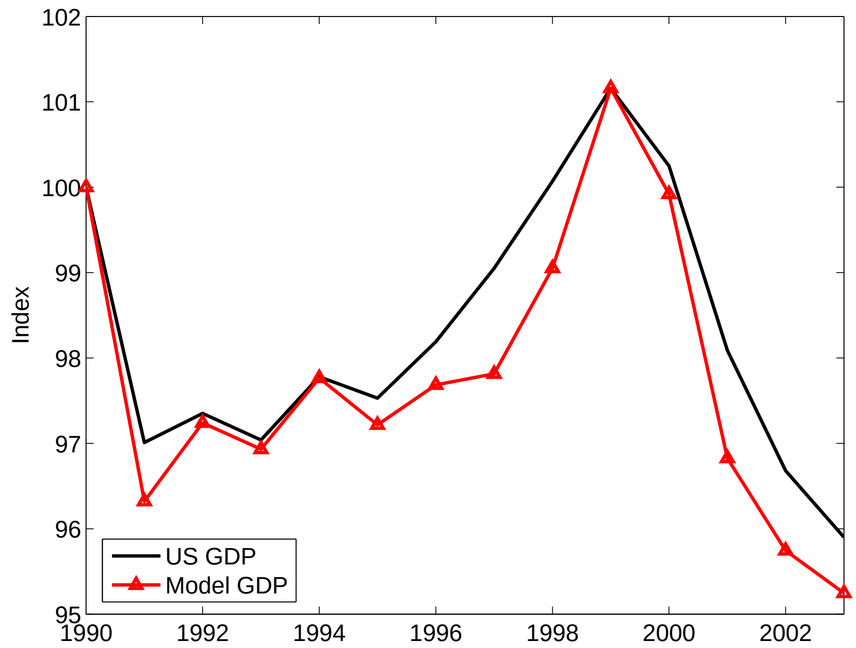

Figure 21. U.S. Per Capita Real GDP and Prediction of Model with Intangible Capital and Non-neutral Technology, Series Divided by 1.02^t

(Investment wedge constant)

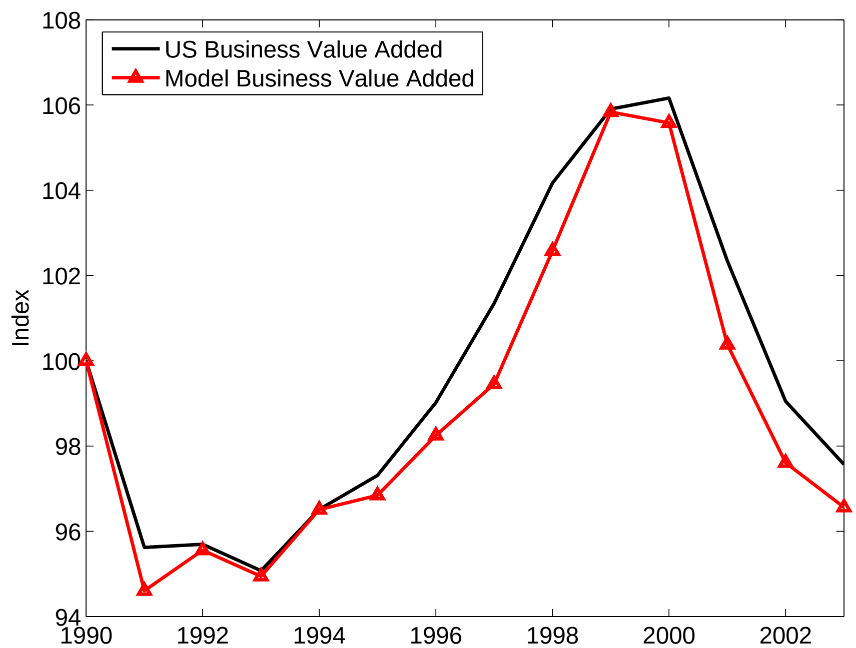

Figure 22. U.S. Per Capita Real Business VA and Prediction of Model with Intangible Capital and Non-neutral Technology, Series Divided by 1.02^t

(Investment wedge constant)

(i.e., $X_{wt} = 1$ for all t). By construction, if the investment wedge were varying, then the model would fit exactly and the predicted and actual series would lie on top of each other. The difference in the actual and predicted series is therefore attributed to the wedges. Clearly, this difference is small.

Figures 21 and 22 show output for the aggregate economy and the business sector. In this case, the match is extremely close. As a contrast, compare these figures to Figures 9 and 10.

Figures 23 and 24 show labor productivity for the aggregate economy and the business sector. Given the good agreement between theory and data for both outputs and hours, it is not a surprise that predicted and actual series are close. The significant improvement in fit can be seen by comparing these figures to Figures 11 and 12.

Figure 25 displays tangible investment for both the model and the data. The deviation from theory seems larger than it really is. For example, if we set 1991=100, the series line up nicely. The same is true for consumption displayed in Figure 26.

As a check on the model's predictions, we compared the model's prediction for factor incomes and capital gains to U.S. measures in, respectively, the NIPA and the U.S. Flow of Funds accounts. This is shown in Figures 27 and 28. In Figure 28, the data have been averaged. The line at the beginning of the 1990s is the average for the period 1953–1994, and the line at the end of the 1990s is the average for the period 1995–2003. Neither the income data nor the capital gains data are used to infer our measures of sectoral TFPs. We find that the model's predictions of both incomes and capital gains are in conformity with U.S. observations.

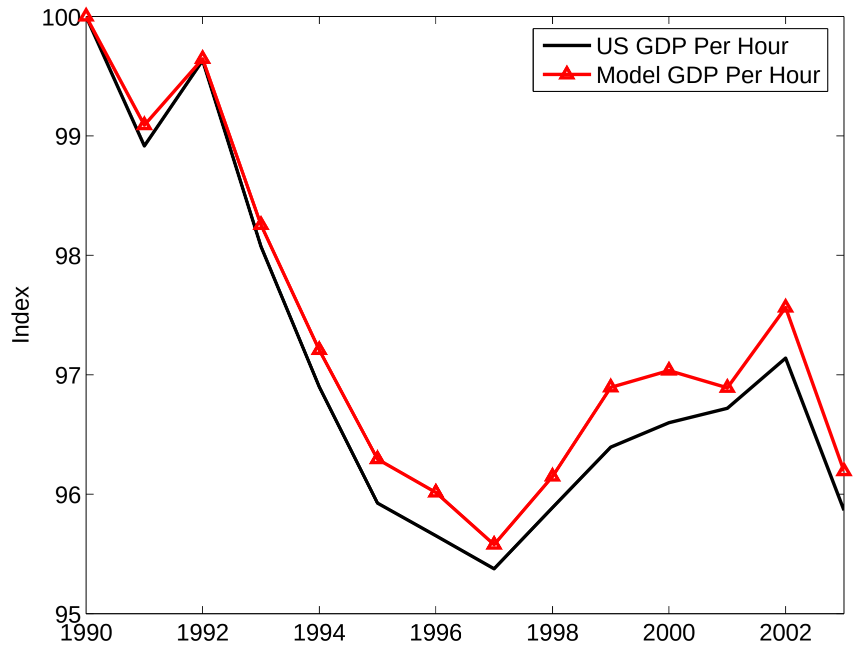

FIGURE 23. U.S. REAL GDP PER HOUR AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY 1.02^t

(Investment wedge constant)

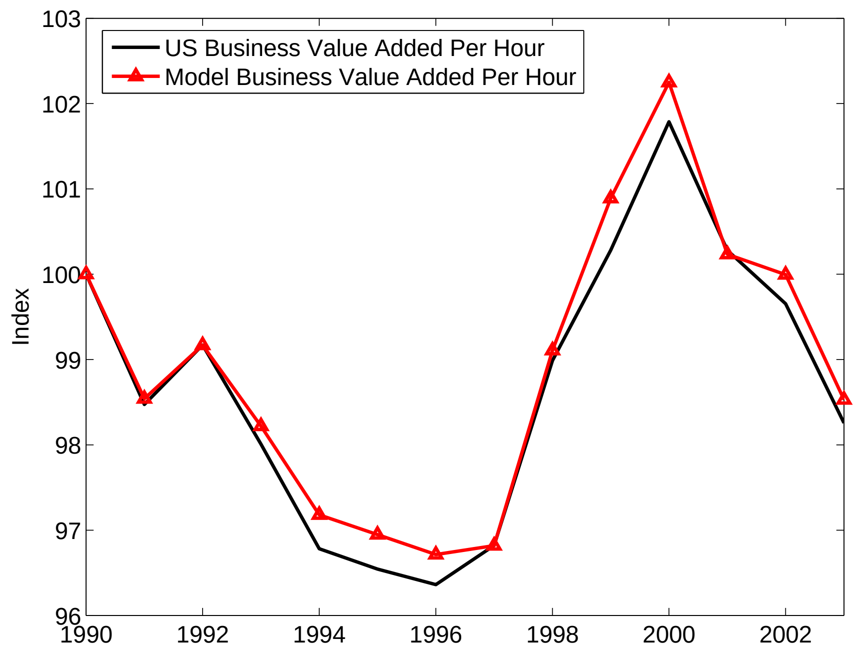

FIGURE 24. U.S. REAL BUSINESS VALUE ADDED PER HOUR AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY 1.02^t

(Investment wedge constant)

2.3.3. Can Our Theory Be Unsuccessful?

An issue is whether our theory can ever be unsuccessful given that the path of intangible capital is inferred from first-order conditions of the theory. To address this issue, we carry out the following experiment. We simulate data for the model with no intangible capital, assuming that TFP is on trend, no change in capital tax rates, and a large decline in labor market distortions other than τ_{ht} or τ_{ct}. In other words, we simulate a large rise in the labor wedge that proxies for labor market distortions other than government taxes. (See Section 2.1.1.) We treat these simulated data as the true economic data.

With these data, we ask, If we analyzed these data using our preferred theory with intangible capital and non-neutral technology, would we say that the theory satisfies our two criteria for a successful theory?

The short answer is no: the model with intangible capital and non-neutral technology would not satisfy either criterion. The theory would not satisfy the input justification criterion because it predicts a huge boom in R&D and other expensed investments ($q_t x_{It}$) when there was not a shred of micro evidence for it—in fact, the true economy being analyzed has *no intangible capital at all*. Even if there was a sizable intangible capital stock, we would face the same problem if it was not in fact abnormally high during the period of interest. With our model, we would infer an abnormally high intangible stock, but would find no micro evidence for the rise. We would also predict a big temporary shift in employment in certain occupations (h_{2t} high relative to h_{1t}) but would find no evidence of this given that the changes in the true economy are neutral with respect to employment sectors. The model would not satisfy the prediction criterion because the capital gains in the true economy barely changed, whereas we would be predicting a huge boom.

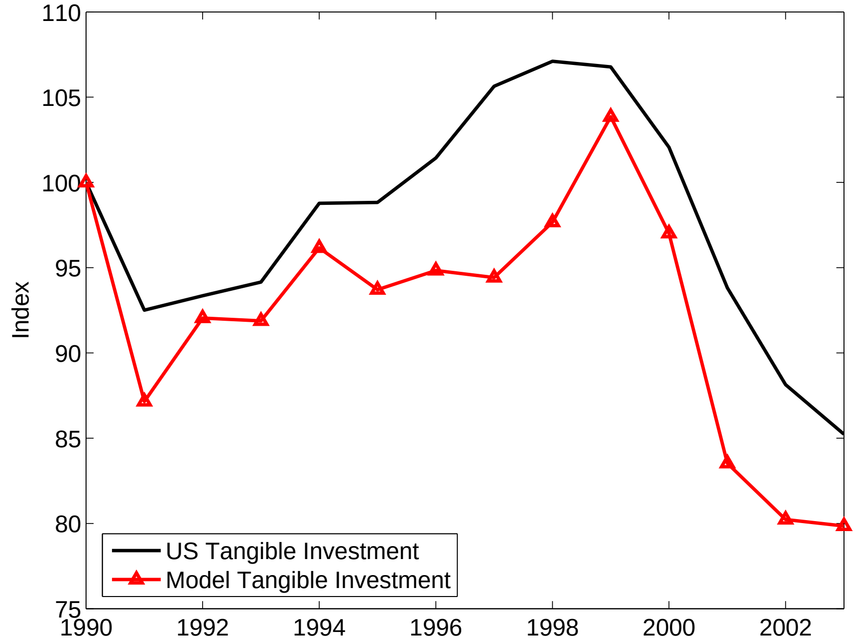

Figure 25. U.S. Per Capita Real Investment and Prediction of Model with Intangible Capital and Non-neutral Technology, Series Divided by 1.02^t

(Investment wedge constant)

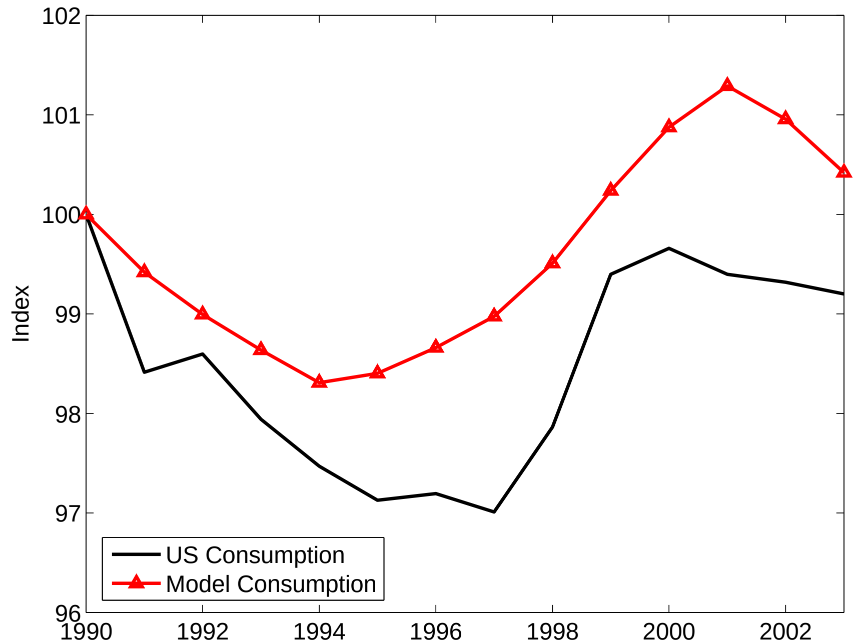

Figure 26. U.S. Per Capita Real Consumption and Prediction of Model with Intangible Capital and Non-neutral Technology, Series Divided by 1.02^t

(Investment wedge constant)

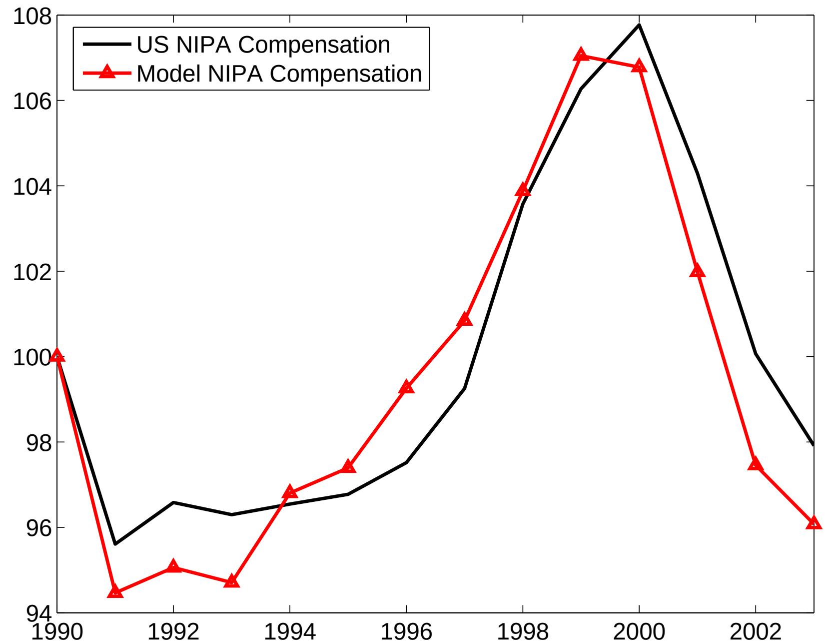

FIGURE 27. U.S. PER CAPITA REAL BUSINESS COMPENSATION AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY 1.02^t

(Investment wedge constant)

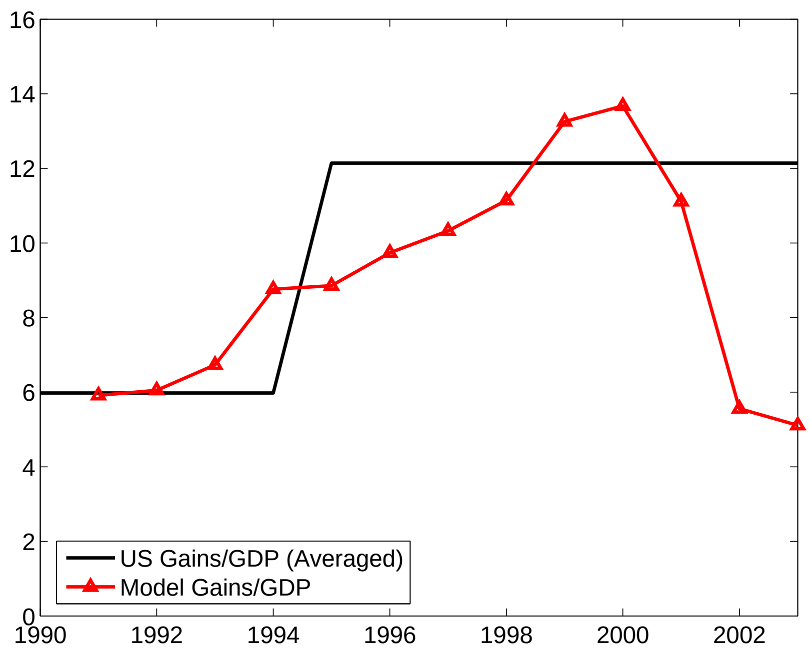

FIGURE 28. U.S. REAL HOLDING GAINS AS % OF GDP AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY

(Investment wedge constant)

Summary

In this chapter, we assessed three theories designed to generate equilibrium paths for GDP, consumption, investment, and hours that matched the U.S. time series exactly. Despite the perfect fit, only one theory satisfied our criteria for a successful theory. We also showed that this was not a foregone conclusion just because intangible investment was included in the model.

Chapter 3.

Accounting for Business Cycles During 1960–1989

In our paper we focus on the 1990s as a period during which a huge unexplained deviation from theory occurred. Many people have noted "that the baseline model from which you [McGrattan and Prescott] start has difficulties in replicating the unfiltered and TFP-detrended macroeconomic time series not only in the 90s, but generally." (Here, we are quoting the editor handling the paper.) In this chapter, we demonstrate that the basic neoclassical growth model accounts well for the postwar cyclical behavior of the U.S. economy prior to the 1990s, provided that variations in population growth, depreciation rates, total factor productivity, and taxes are incorporated.

The view that the "basic" neoclassical growth model does poorly in general is largely due to key missing factors in the basic model. For example, to account for movements in hours and labor productivity, one must account for key distortions to the labor market, in particular tax rates on labor. If we compare the predictions of Uhlig's (2003) real business cycle model and Chen et al.'s (2007) real business cycle model, we reach a different conclusion about the performance of the "basic" neoclassical growth model driven by real factors.

Figure 29 shows Uhlig's (2003) prediction for the log deviation in business along with the U.S. measure. In Uhlig's model, fluctuations are driven by changes in TFP, government spending, and population growth. Periods in his model are quarters. The figure is reproduced from Uhlig's Table 6, which has the caption "uncomfortably big gaps are visible." In terms of hours, the most uncomfortable gaps are during the recessions of

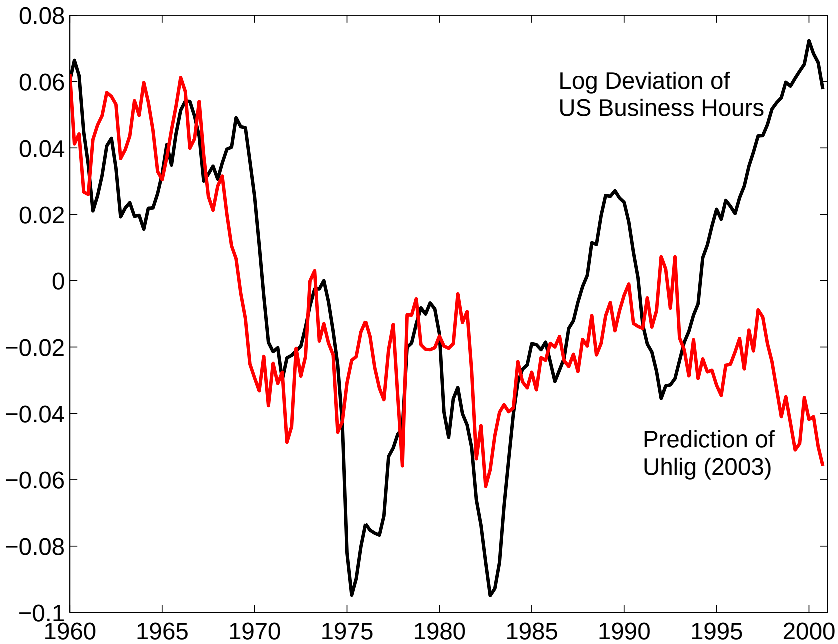

Figure 29. Deviation of Logarithm of U.S. Per Capita Business Hours and Prediction of Uhlig (2003, Table 6) Model, 1960:1–2000:4

the 1970s and 1980s and especially during the boom of the 1990s. The model gets 1/3 of the 1970s decline in hours, 1/2 of the 1980s decline, and predicts a depression in the 1990s when in fact there was a boom.

Figure 30 shows Chen et al.'s prediction of total hours along with the U.S. measure (based on the establishment survey). This figure is reproduced from their paper (page 15). In Chen et al.'s model, fluctuations are driven by changes in TFP, labor tax rates, capital tax rates, depreciation rates, government spending, and population growth. Periods in their model are years. Interestingly, if we focus on the cyclical movements of hours prior to the 1990s, the model does quite well. To see this better, we apply a Hodrick-Prescott (1997) filter (with the smoothing parameter set equal to 100). Figure 31 shows the filtered series for 1960–1989. The cyclical predictions are extremely good until the late 1980s. In contrast to Uhlig's model, this model accounts for most of the decline in the 1970s and all

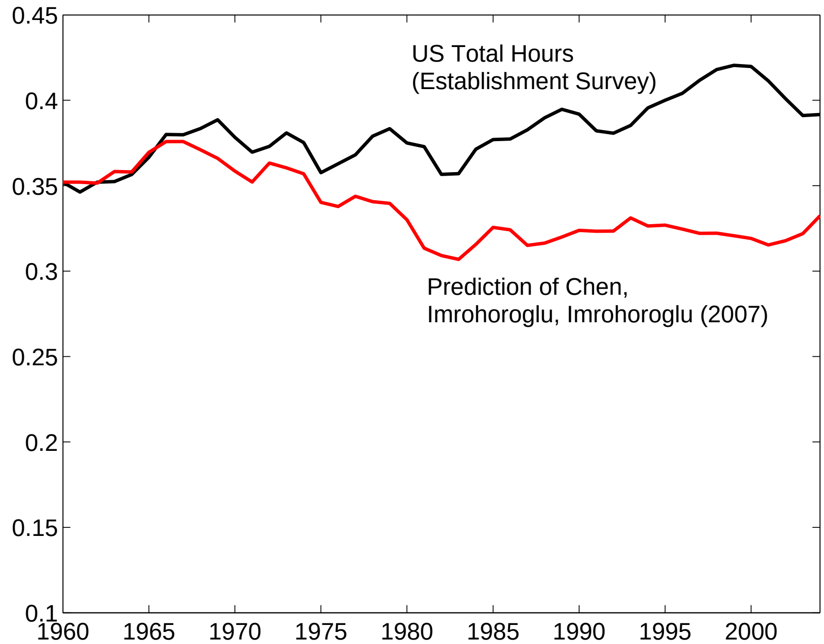

Figure 30. U.S. Per Capita Total Hours and Prediction of Chen et al. (2007, p.15) Model, 1960–2004

of the decline in the 1980s. An important difference between Uhlig (2003) and Chen et al. (2007)—especially in the case of hours worked—is that Chen et al. include variations in taxes, whereas Uhlig does not.

However, during the 1990s, both the Uhlig model and the Chen et al. model do poorly. Both predict depressions when in fact there was a boom. Figure 32—which displays the filtered series of Chen et al. for 1990–2004—is a dramatic contrast to Figure 31 for the earlier period. Clearly, there is something missing. This was our starting point. This is what puzzled us for many years.

An open question remains about the basic model's (of Chen et al.) predictive ability for secular trends. Some of the deviation between theory and data in Figure 30 may be due to measurement. For example, the aggregate CPS-based hours series that we use does not rise as much as the aggregate establishment-based hours series that Chen et al. use.

Some of the deviation between theory and data may be due to treatment of households as uni-sex. Work is beginning on this important topic. In our opinion, the deviation in the 1990s was a big and important puzzle, so we started there.

One final note concerns the role of intangible capital in the pre-1990 period. If technological change is neutral, the predictions of our model *with intangible capital* is the same as Chen et al.'s model *without intangible capital* for the relevant set of (measured) variables. In particular, we too would generate results like that in Figure 32 as long as we allow for variations in TFP, tax rates, government spending, population, and depreciation as in Chen et al. (2007).

Summary

In this chapter, we displayed the results of Chen et al. (2007) to motivate our claim that the basic neoclassical growth model accounts well for the postwar cyclical behavior of the U.S. economy prior to the 1990s, provided that variations in population growth, depreciation rates, total factor productivity, and taxes are incorporated. We contrasted these results with the model in Uhlig (2003), which does not include variation in tax rates and, therefore, does poorly in accounting for movements in hours and productivity.

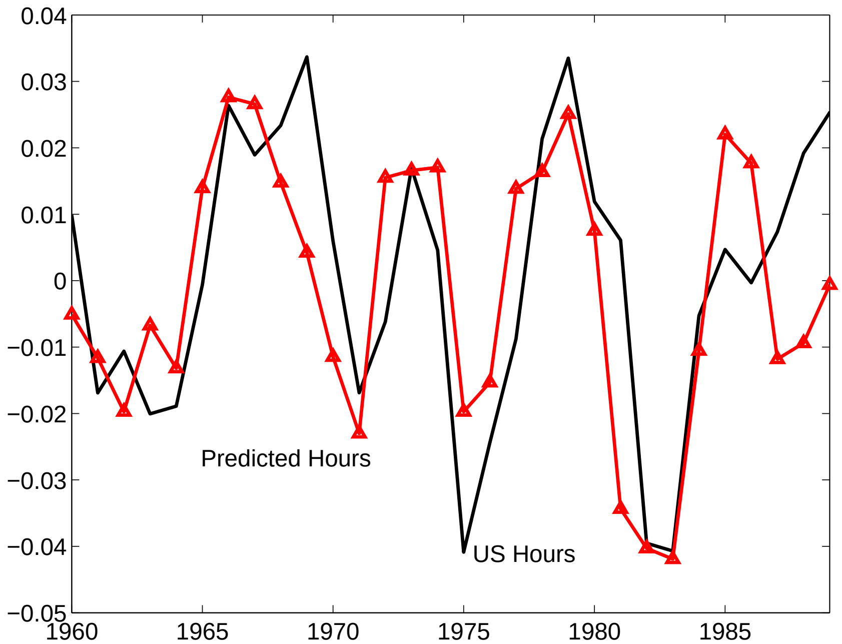

FIGURE 31. FILTERED U.S. PER CAPITA HOURS AND PREDICTION OF CHEN ET AL. (2007) MODEL, 1960–1989

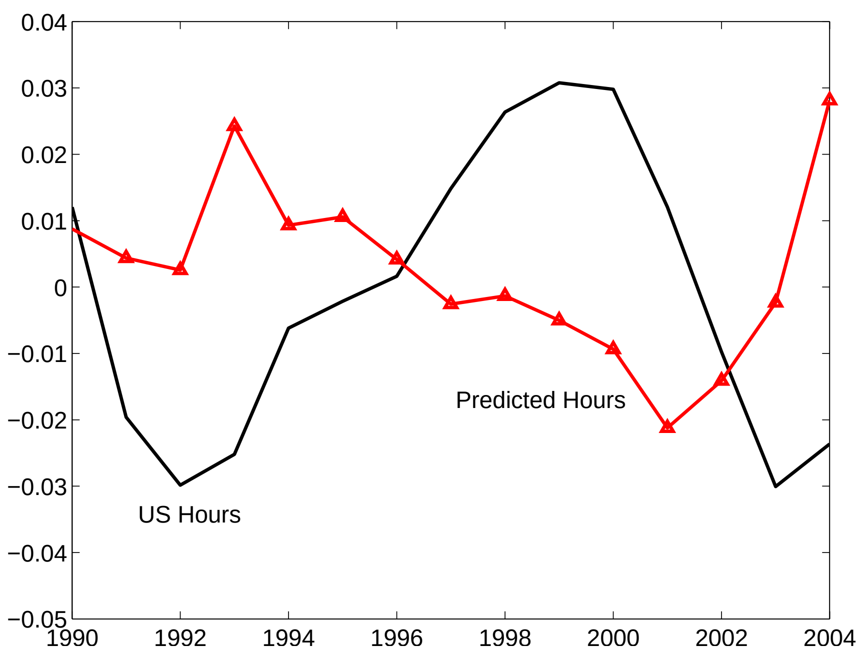

FIGURE 32. FILTERED U.S. PER CAPITA HOURS AND PREDICTION OF CHEN ET AL. (2007) MODEL, 1990–2004

Chapter 4.

Assessing New Keynesian Business Cycle Theory

One reasonable reaction to predictions such as those shown in Figures 1–16 is to abandon the standard neoclassical theory and replace it with something else. That has happened to some extent in the macro literature as many researchers have switched to analyzing and using new Keynesian theories. In this chapter, we briefly describe a typical model in this class that Smets and Wouters (2007) analyzed. We then ask, in the context of their model, Why did hours boom in the 1990s? The answer is the same one given in Section 2.1: because a labor wedge boom occurred. The Smets-Wouters model says that there was a shock to wage markups and point out that "alternatively, we could interpret this disturbance as a labour supply disturbance coming from changes in preferences for leisure" (p. 15). Here, we report on some key predictions of the Smets-Wouters model and compare them to the same predictions of the model of Section 2.1.

The new Keynesian models are more complicated than the models worked out in Chapter 2 because they typically have many nominal and real "rigidities" intended to help propagate shocks. Here, we will report results based on the Smets-Wouters (2007) model, which has sticky wages and prices, habit formation in consumption, investment adjustment costs, variable capital utilization, and fixed costs in production. Fluctuations are driven by seven exogenous variables (to account perfectly for seven observables). Five of the exogenous stochastic variables are modeled as AR(1) processes, and two are modeled as ARMA(1,1) processes. None of the exogenous variables are taken from outside data sources or estimated outside the model (e.g., tax rates or government spending). (See Smets and Wouters 2007 for details.)

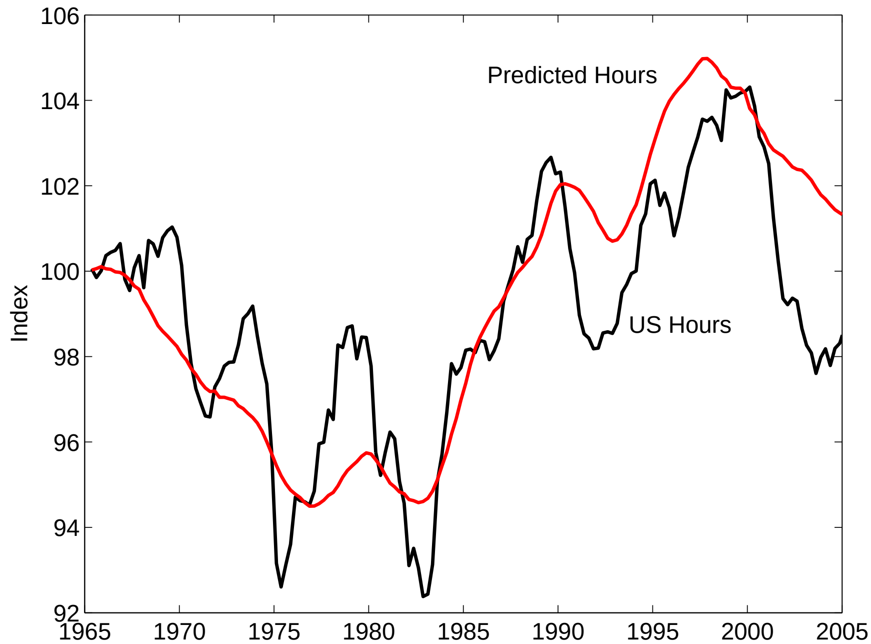

Figure 33. U.S. Per Capita Business Hours and Smoothed Prediction for Smets-Wouters (2007) Model

(Preference shock to leisure only, 1965:1–2004:4)

Smets and Wouters (2007) estimate the parameters of their model and use the estimates to compute a variance decomposition for the observed variables. For business hours, they find that 67 percent of variation in the 1965–2005 period is due to shocks to preferences for leisure—what we call the labor wedge. In Figure 33, we plot the series for business hours that they use (which is based on establishment data) along with the model's prediction of hours. It is not a perfect fit because we shut off all shocks except the shock to preferences for leisure. (Note that if we turn on all shocks, the model's prediction will be the same as the data by construction.) The figure shows why most of the variation in hours is being attributed to the labor wedge.

If we zoom in on the figure in the 1990s, we get Figure 34. We can compare this Smets-Wouters prediction for the path of business hours due to the labor wedge with the prediction of standard theory (of Section 2.1) for business hours that is shown in Figure

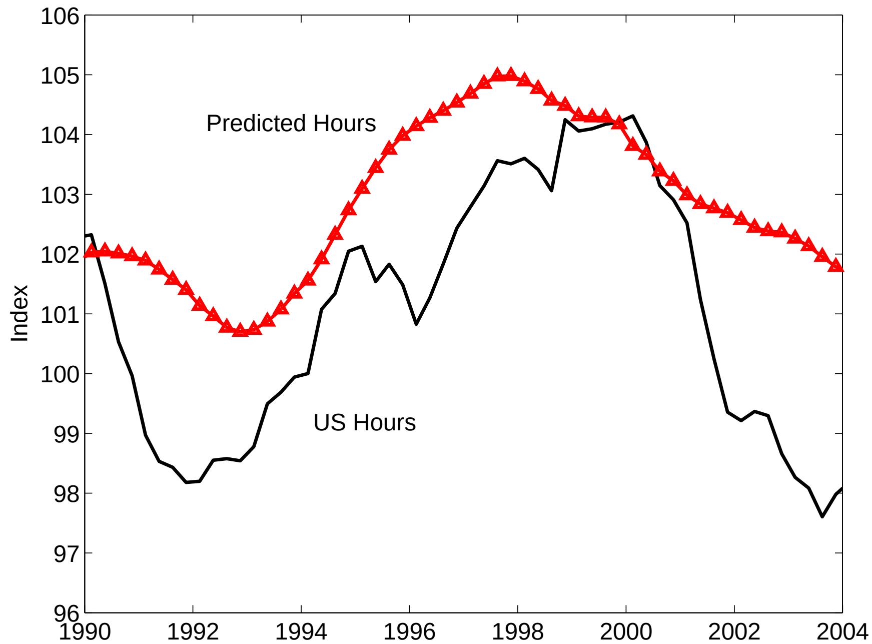

Figure 34. U.S. Per Capita Business Hours and Smoothed Prediction for Smets-Wouters (2007) Model

(Preference shock to leisure only, 1990:1–2003:4)

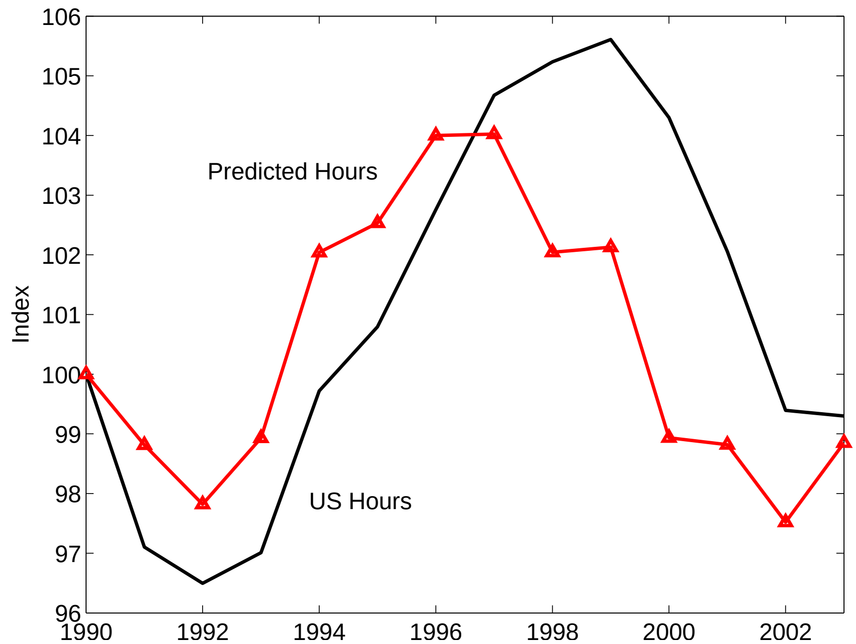

Figure 35. U.S. Per Capita Business Hours and Prediction for Model without Intangible Capital

(TFP and investment wedge constant)

35. We find them similar. Both models attribute the rise in hours to this wedge. In neither model is the boom due to TFP or to monetary shocks. So, what is the implication of this new line of research? It is that the hours boom of the 1990s is primarily due to a preference shock for leisure, which is of the same order of magnitude as the neoclassical model requires to generate the observed hours boom. In building in monetary factors necessary to evaluate the effects of monetary policy, a key puzzle has not been resolved.

Summary

Without evidence or theory for labor wedges, new Keynesian theory does not provide any better understanding of the 1990s than the neoclassical model without intangible capital and non-neutral technological change.

Chapter 5.

Checking the Sensitivity of Our Results

In this section, we report on our sensitivity analysis for the model with intangible capital and non-neutral technology. In Section 5.1, we examine the sensitivity of our findings to the values of the parameters and find that the results are robust to their specification. Consequently, the parameters are not being selected to fit the episode. In Section 5.2, we establish that the expectation assumption with regard to the total factor productivity parameters has almost no consequence for the realized path of the economy.

5.1. Varying Parameters of the Model

Here, we describe how our results are affected as we vary factor shares (θ_i, ϕ_i), the share of intangible investment financed by shareholders (χ), the depreciation rate on intangible capital (δ_I), and tax rates on labor and profits (τ_{ht}, τ_{pt}). In all cases, we follow the same procedure as described in Section 2.3 to determine the parameters that are not pre-set.

In the main paper, we imposed symmetry in technologies. In particular, we set $\theta_1 = \theta_2$ and $\phi_1 = \phi_2$. Here, we show results for a plausible alternative with an intangible sector that is intangible-capital intensive. Specifically, we set $\phi_2 = .2$ and $\theta_2 = .1$. (We could go to the extreme point of $\theta_2 = 0$, but it seems implausible that no structures or computers are being used in the sector producing intangible capital.) With these new parameters, we determine a new sequence of TFP parameters consistent with our theory. These parameters, along with the tax rates of Table 4, are input in the model.

In Figure 36, we plot hours of work in the baseline economy and the alternative

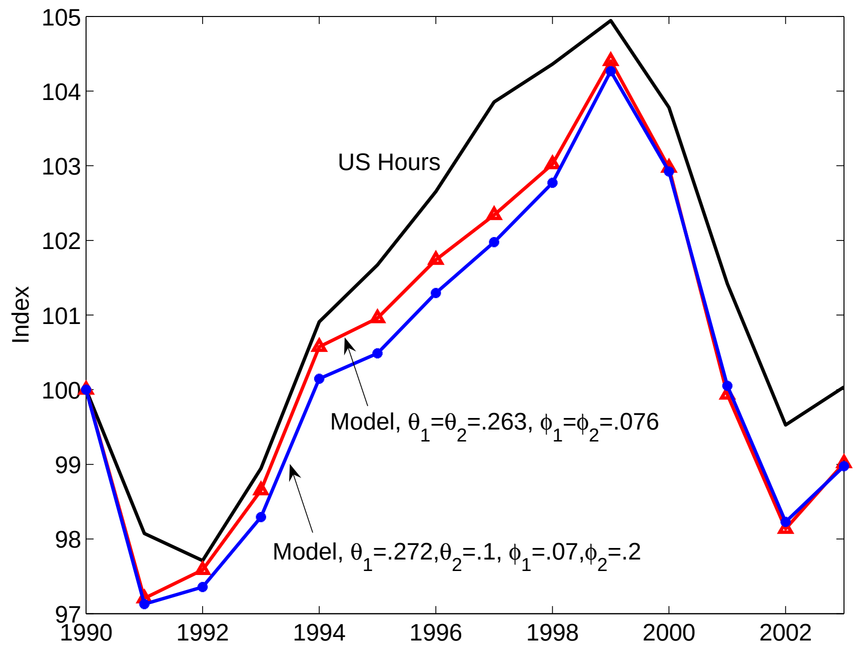

Figure 36. U.S. Per Capita Hours and Predictions of Model Intangible Capital and Non-neutral Technology

(Investment wedge constant)

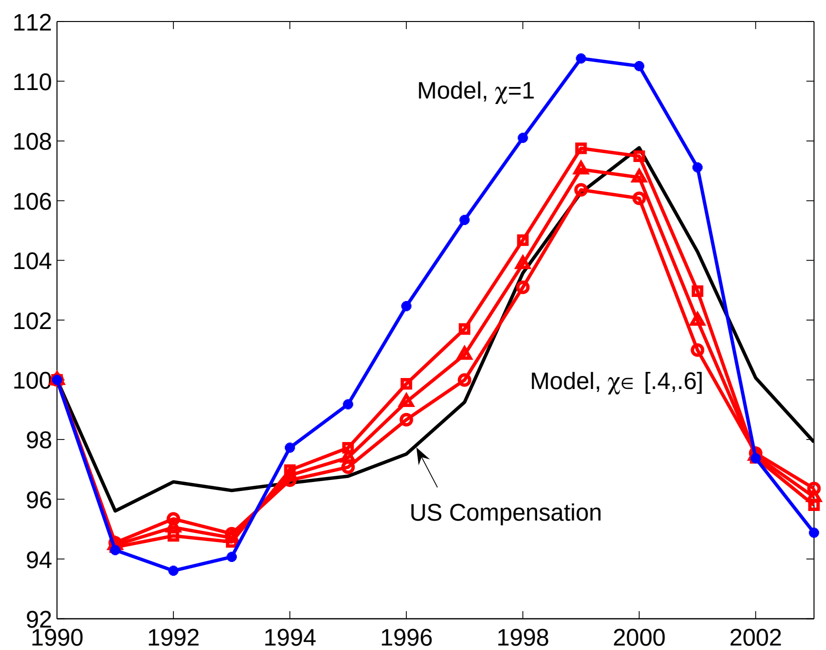

Figure 37. U.S. Per Capita NIPA Compensation and Predictions of Model with Intangible Capital and Non-neutral Technology, Series Divided by 1.02^t

(Investment wedge constant)

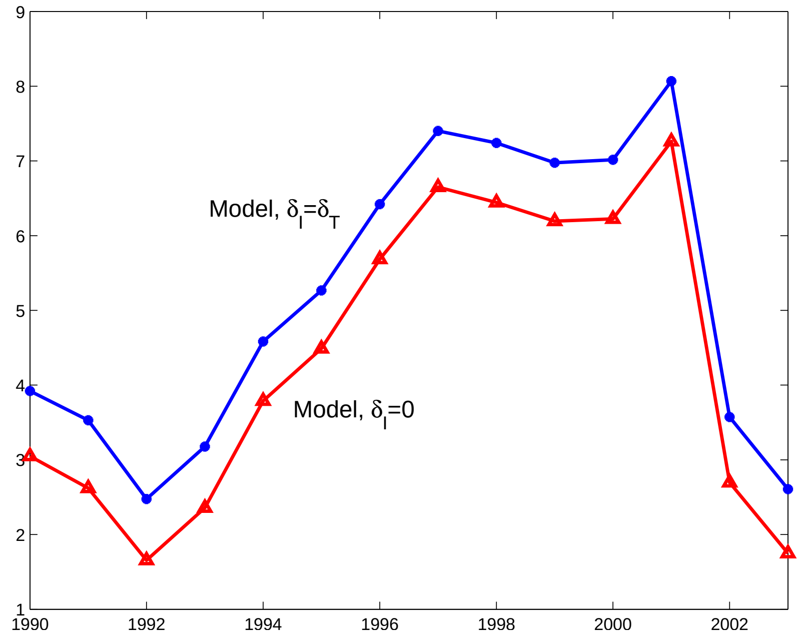

Figure 38. Predicted Intangible Share of Output in Model with Intangible Capital and Non-neutral Technology, Series Divided by 1.02^t

(Investment wedge constant)

economy. As the figure shows, there is only a small difference in the predictions. The same is true for the other series displayed earlier. Other variations of the technologies did not produce large deviations.

For the next experiment, we varied the parameter χ, which determines the amount of intangible capital expensed from shareholder profits versus the amount expensed from compensation of business owners. At this point, we do not have any direct estimates of this parameter. As we noted in the main paper, values for χ in a range near the baseline value of 1/2 yield similar results. This is shown in Figure 37, where we display measured (NIPA concept) compensation for $\chi = 0.4$, 0.5, and 0.6 (all in red). We then set $\chi = 1$— which is the case where all intangible capital is financed by shareholder profits. This is the assumption we made in earlier work before we realized the importance of sweat equity.

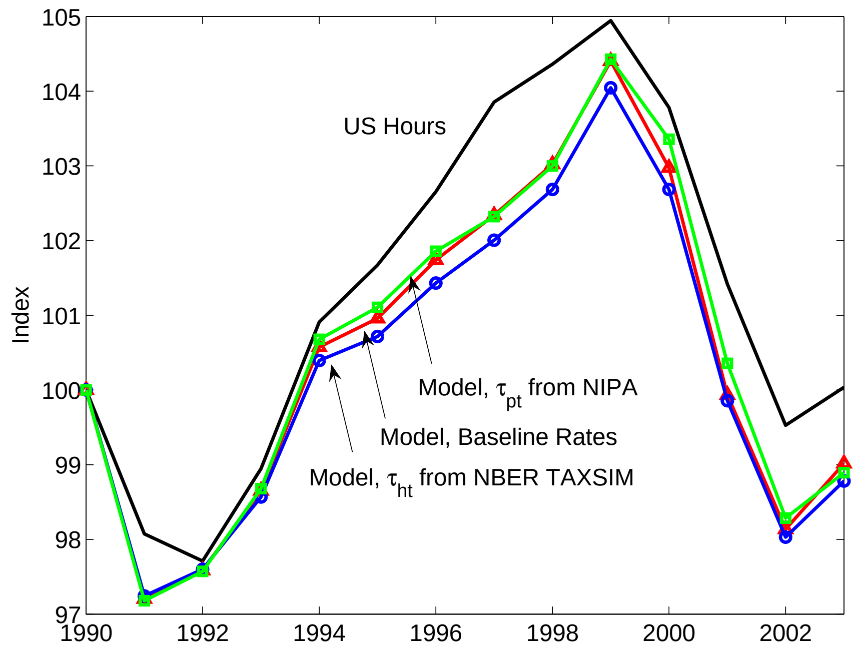

FIGURE 39. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY

(Investment wedge constant)

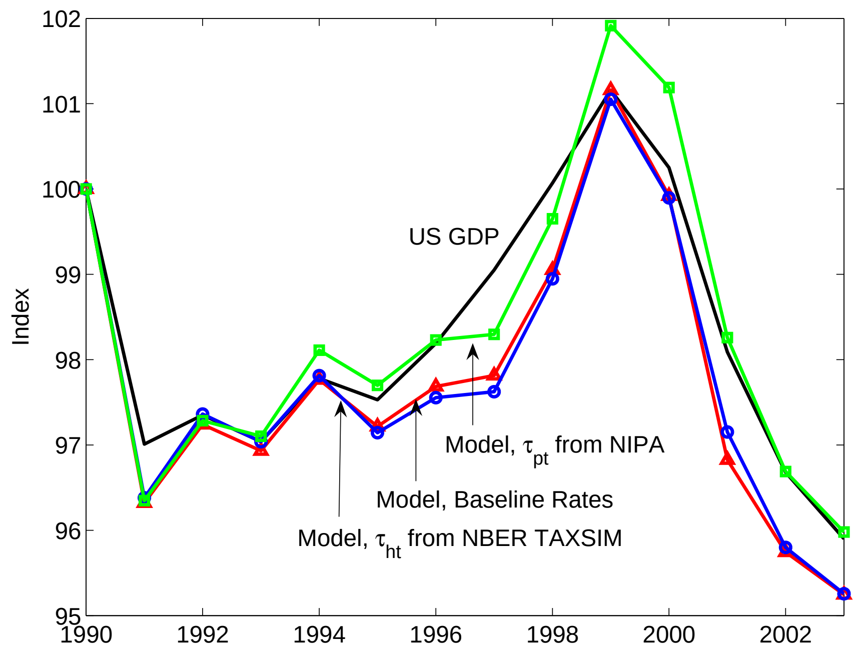

FIGURE 40. U.S. PER CAPITA REAL GDP AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY 1.02^t

(Investment wedge constant)

In this case, we do see that predicted (measured) compensation is quite a bit higher than compensation reported in NIPA.

Another parameter for which there is no direct estimate is δ_I. In our baseline simulations, this parameter was set to 0, in part because intangible investments include investments in organizations that are long-lived. To see how important this assumption is, we considered an alternative economy with $\delta_I = \delta_T$. With the exception of the level of the share of intangible capital in output, the figures hardly change. In Figure 38, we show the share of intangible investment in output. Notice that the curve shifts up roughly the same amount at each date. The growth in the share is unaffected.

We turn next to the tax rates. Given the fact that most current models rely on large preference shocks to leisure—which are isomorphic to large declines in the tax on labor—we thought it important to try other measures of the labor tax. In Figures 39 and 40, we display the predictions for per capita hours and real GDP for the baseline rate τ_{ht} in Table 4 and the estimates of the NBER TAXSIM model available at www.nber.org. (See Feenberg and Coutts 1993 for details.) It is hard to distinguish the predictions for these alternative measures in the figures.

We also examined how our results change if we relax the assumption of a constant profit tax rate. We use the following measure for τ_{pt}: NIPA corporate tax liabilities divided by corporate income (with Federal Reserve profits subtracted from both the numerator and the denominator). The difference in the hours predictions for this case and the baseline case is hard to see in Figure 39. There is some noticeable difference in the predictions for GDP because the constructed τ_{pt} series has some effect on our prediction for tangible capital. One could reasonably argue that adding variable profits taxes improves the fit of the model.

5.2. Varying Expectations

In this section, we demonstrate that the perfect-foresight assumption is innocuous. We do this by comparing results for two versions of our model that differ only in the assumption of expectations.

To simplify our analysis, we set all tax rates and non-business variables equal to their 1990 levels and ignore the investment wedge. Another simplification involves the TFP processes. We want to generate a non-neutrality in A_t^2 relative to A_t^1. We do this by modeling the two TFPs as functions of the same AR(1) process. Thus, the problem boils down to one with a one-dimensional shock process.

More specifically, we use Tauchen's (1986) discrete approximation for AR processes to construct a Markov chain for an AR(1) process $s_{t+1} = \rho s_t + \epsilon_t$ with $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. We set A_t^1 equal to its steady-state value plus s_t. We set A_t^2 equal to its steady-state value plus 1.5 times s_t. The transition matrix is the matrix computed using Tauchen's method for a specific value of ρ.

Given the processes for the TFP parameters, we find an equilibrium stochastic process for the economy. We pick a particular realization of the exogenous stochastic TFPs, making sure that our choice implies increases in hours and output that are of similar magnitude as their observed counterparts. We then find the equilibrium realization of the endogenous variables for the realization of exogenous stochastic elements of the economy. Finally, we compare this to the realization if households had perfect foresight about the paths of the TFP parameters.

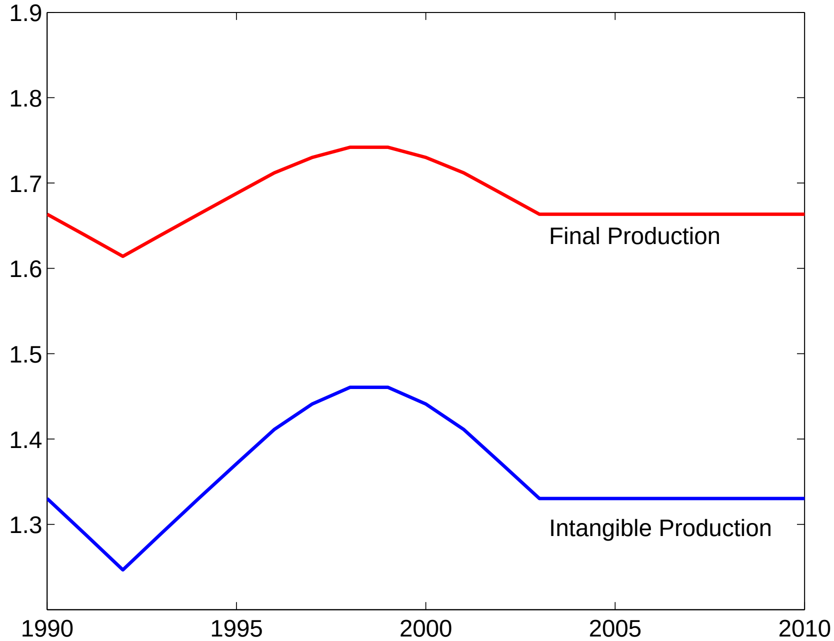

Figure 41. Realizations of TFP Inputs in Stochastic Model with Intangible Capital and Non-neutral Technology

(All other exogenous inputs constant)

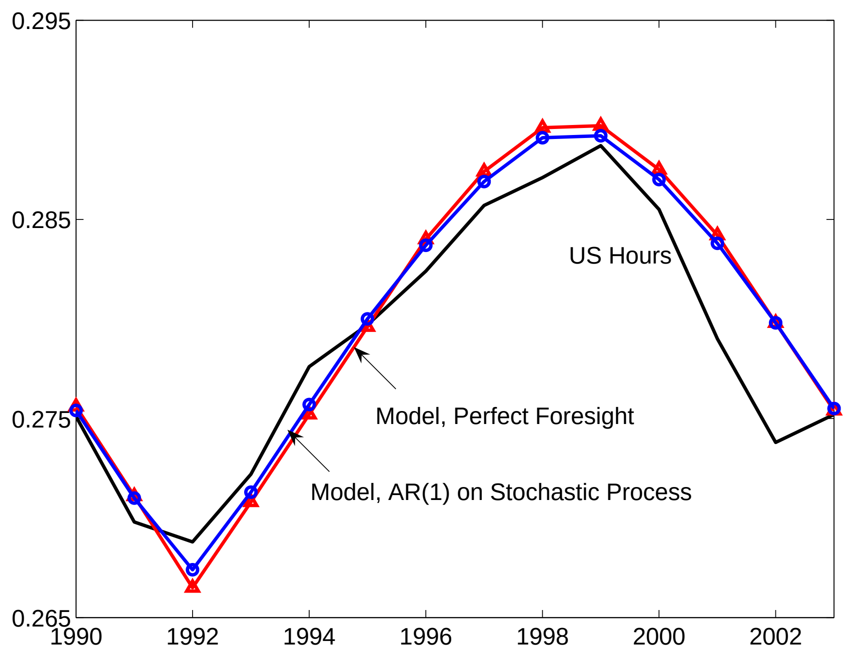

Figure 42. U.S. Per Capita Hours and Predictions of Stochastic Model with Intangible Capital and Non-neutral Technology

(All other exogenous inputs constant)

In Figure 41, we display the realizations of the paths of TFP in final production A_t^1 and TFP in intangible production A_t^2 that we use in our experiment. These are not the same paths used in generating the results of the paper because of the restrictions on the exogenous inputs. However, these values of TFPs do imply a technology boom of the same magnitude as we saw in the United States during the 1990s.

In Figure 42, we display the household's realized hours in the case that TFPs are governed by an AR(1) process and in the perfect-foresight case. The difference in predicted paths for hours is small. Similarly, the difference in terms of other endogenous variables is small. The reason is that the realizations of A_t^1 and A_t^2 are what is important, not the choice of household expectations. For this experiment, we use $\rho = .95$, but varying ρ does not change the result. Also plotted are U.S. hours. As we noted earlier, this model does generate a boom of the right magnitude.

Summary

We find that our conclusions are robust to plausible variations in parameters and expectations.

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