Credit Traps and Credit Cycles

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Abstract

We explore some of the implications of the heterogeneity of the investment projects in the dynamics of the aggregate investment and borrower net worth. The projects differ in productivity, in the investment size, and in the severity of the agency problems behind the borrowing constraints. With a variety of the investment projects with different characteristics competing with each other in the credit market for funding, a movement in borrower net worth can affect the composition of the credit. The model is simple enough to be tractable and yet it is rich enough that these composition effects cause a variety of nonlinear phenomena, such as traps and cycles, in the dynamics.

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1. Introduction

Following the seminal work of Bernanke and Gertler (1989), many recent studies on macroeconomics of credit market imperfections emphasized the credit multiplier (or financial accelerator) mechanism, which introduces persistence in the dynamics of the aggregate investment and borrower net worth. As the argument goes, a rise (a fall) in borrower net worth eases (aggravates) the borrowing constraint, thereby stimulating (discouraging) investment, which leads to further rise (fall) in borrower net worth. These studies typically consider the case where the investment projects facing the borrowing constraint are homogeneous. The alternatives available to the lenders are normally restricted to either consumption or the simple storage technology free of any borrowing constraint.

In the real world, of course, there are many different types of the investment projects. With different investment projects competing with each other in the credit market for funding, a change in the composition of the credit and investment projects can play additional roles in the dynamics.

In this paper, we propose a simple model of the dynamics of the aggregate investment and borrower net worth, which enables us to explore some of the implications of the heterogeneity of the investment projects. In the proposed model, the projects differ in productivity, in the investment requirement (i.e., the setup cost), and in the severity of agency problems behind the borrowing constraints, while the other sources of heterogeneity are ruled out in order to keep the analysis simple.[2] Furthermore, the model is set up in such a way that, without heterogeneity, the investment dynamics would be identical to those in the standard neoclassical growth model. In spite of all these restrictions, the model is rich enough to generate a variety of nonlinear phenomena, such as traps and cycles, in the dynamics of the aggregate investment and borrower net worth.

[2]More specifically, the model in this paper assumes that all the projects produce the same capital stock (but in different quantity) and that all the agents are identical. It turns out that introducing the heterogeneity along these dimensions in a nontrivial way makes the analysis of the dynamics considerably more demanding. Nevertheless, we have made some progress for a few cases, as reported in related papers. For example, Matsuyama (2004a) considers the cases where some projects produce the consumption good, while others produce the capital good. The world economy model of Matsuyama (2004b) may be viewed as an example of the cases, where different agents run different projects that produce different capital goods (the agents and the capital goods differ in their locations.) We have not been successful in integrating these different models within the unified framework.

Section 2 introduces the model and derives the system of equations that governs the equilibrium dynamics. Section 3 considers the case with the homogeneous investment projects and shows the dynamics are identical with the neoclassical growth model in spite of the presence of the credit market imperfections. This case helps to offer the benchmark against which we can identify the role of heterogeneity and changing composition of the credit. Sections 4 and 5 discuss the cases of heterogeneous investment projects. Section 4 demonstrates the possibility of credit traps and credit collapses. Section 5 demonstrates the possibility of (endogenous) credit cycles and growth miracles. Section 6 concludes.

2. The Model.

The basic framework used in the Diamond overlapping generations model with two period lives. The economy produces a single final good, using the CRS technology, $Y_t = F(K_t,L_t)$, where K_t is physical capital, and L_t is labor. The final good produced in period t may be consumed in period t or may be allocated to investment projects. Let $y_t \equiv Y_t/L_t = F(K_t/L_t,1) \equiv f(k_t)$, where $k_t \equiv K_t/L_t$ and f(k) satisfies $f'(k) > 0 > f''(k)$. The markets are competitive, and the factor rewards for physical capital and for labor are equal to $\rho_t = f'(k_t)$ and $w_t = f(k_t)-k_tf'(k_t) \equiv W(k_t) > 0$, which are both paid in the final good. For simplicity, physical capital is assumed to depreciate fully in one period.

In each period, a new generation of potential entrepreneurs, a unit measure of homogeneous agents, arrives with one unit of the endowment, called labor. They stay active for two periods. In the first period, they sell the endowment and earn $w_t = W(k_t)$. They consume only in the second period. Thus, they save all of the earning, w_t, and allocate it to maximize their second period consumption. They may become lenders or entrepreneurs. If they become lenders, they can earn the gross return equal to r_{t+1} per unit in the competitive credit market and consume $r_{t+1}w_t$ in the second period. Alternatively, they may become entrepreneurs by using their earning, w_t, to partially finance an investment project. They can choose from J-types of projects. All projects come in discrete, indivisible units and each entrepreneur can run only one project. A type-j (j = 1,2,…,J) project transforms m_j units of the final good in period t into m_jR_j units of physical capital in period t+1. Because of the fixed investment size, m_j, an entrepreneur needs to borrow by $m_j - w_t$ at the rate equal to r_{t+1}. (If $w_t > m_j$, they can entirely self-finance the project and lend $w_t - m_j$.)

Let X_{jt} denote the measure of type-j projects initiated in period t. Then, the aggregate investment, the amount of the final good allocated to all the projects, is $I_t = \sum_j(m_j X_{jt})$. Since the aggregate saving is $S_t = W(k_t)$, the credit market equilibrium requires that

(1) $$W(k_t) = \sum_j(m_j X_{jt}).$$

The capital stock adjusts according to

(2) $$k_{t+1} = \sum_j(m_j R_j X_{jt}).$$

Let us now turn to the investment decisions. To invest in a project, the entrepreneurs must be both willing and able to borrow. By becoming the lenders, they can consume $r_{t+1}w_t$. By running type-j projects, they can consume $m_j R_j \rho_{t+1} - r_{t+1}(m_j - w_t)$. Thus, the agents are *willing* to borrow and to run a type-j project if and only if $m_j R_j \rho_{t+1} - r_{t+1}(m_j - w_t) \geq r_{t+1}w_t$, which can be simplified to

(PC-j) $$R_j f'(k_{t+1}) \geq r_{t+1},$$

where PC stands for the *profitability constraint*.

Even when (PC-j) holds, the agents may not be able to invest in type-j projects, due to the borrowing constraint. The borrowing limit exists because borrowers can pledge only up to a fraction of the project revenue for the repayment, $\lambda_j m_j R_j \rho_{t+1}$, where $0 \leq \lambda_j \leq 1$. Knowing this, the lender would lend only up to $\lambda_j m_j R_j \rho_{t+1}/r_{t+1}$. The agent can borrow to run a type-j project iff

(BC-j) $$\lambda_j m_j R_j f'(k_{t+1}) \geq r_{t+1}(m_j - W(k_t)),$$

where BC stands for the *borrowing constraint*.[3]

Suppose that $R_jf'(k_{t+1}) > r_{t+1}\max\{1, [1 - W(k_t)/m_j]/\lambda_j\}$, so that both (PC-j) and (BC-j) are satisfied with strict inequalities. Then, any agent would be able to borrow and run a type-j project and would be better off by doing so than by lending. This means that no agent would become a lender. Hence, in equilibrium, $R_jf'(k_{t+1}) \leq r_{t+1}\max\{1, [1 - W(k_t)/m_j]/\lambda_j\}$.[4] If this inequality holds strictly for some j, then at least one of (PC-j) and (BC-j) is violated, so that $X_{jt} = 0$. Since (1) requires that $X_{jt} > 0$ for some j, we have

$$(3) \qquad r_{t+1} = \max_{i=1,\dots J}\left\{\frac{R_i f'(k_{t+1})}{\max\{1,[1-W(k_t)/m_i]/\lambda_i\}}\right\} \geq \frac{R_j f'(k_{t+1})}{\max\{1,[1-W(k_t)/m_j]/\lambda_j\}},$$

where $X_{jt} > 0$ (j = 1, 2,…J) only if the inequality in (3) holds with the equality.

Equation (3) plays a central role in the following analysis. Hence, it is worth thinking of the intuitive meaning behind it. The RHS of the inequality in (3) is the rate of return that the agents could offer willingly and credibly to the lenders by running type-j projects. If this falls short of the equilibrium rate of return, type-j will not be run, because one of the two constraints is violated for j. In other words, the saving flows only to the projects for which the RHS of the inequality in (3) is the highest among all the projects. What matters in the following analysis is that the ranking of the projects, based on the RHS of the inequality in (3), determines the allocation of the credit, and that the ranking depends on the borrower net worth, $W(k_t)$. Note

[3]We have used this specification of the credit market imperfections elsewhere, e.g., Matsuyama (2000, 2004a, 2004b, 2005). It is possible to give any number of agency stories to justify the assumption that borrowers can pledge only up to a fraction of the project revenue. The simplest story would be that they strategically default, whenever the repayment obligation exceeds the default cost, which is proportional to the project revenue. Alternatively, each project is specific to the borrower, and requires his services to produce R_j units of physical capital. Without his services, it produces only λ_jR_j units. Then, the borrower, by threatening to withdraw his services, can renegotiate the repayment obligation down to $\lambda_jR_j\rho_{t+1}$. See Hart and Moore (1994) and Kiyotaki and Moore (1997). It is also possible to use the costly-state-verification approach used by Bernanke and Gertler (1989), or the moral hazard approach used by Tirole (2004). Nevertheless, the reader should interpret this formulation simply as a black box, a convenient way of introducing the credit market imperfection in a dynamic macroeconomic model, without worrying about the underlying causes of imperfections.

[4]It is implicitly assumed here that the agents cannot entirely self-finance the projects, so that some agents must become lenders in equilibrium. This condition is satisfied unless the production is too productive. That is to say, if we let $f(k) = Ag(k)$ with $g'(k) > 0 > g''(k)$, it suffices to assume that A is not too big. (Alternatively, we can make R_j proportionately smaller, or m_j proportionately larger, which is isomorphic to choosing a smaller A.)

that, as $W(k_t) \rightarrow 0$, the RHS of the inequality in (3) converges to $\lambda_j R_j f'(k_{t+1})$, which means that, with low net worth, the credit goes to the project with the highest $\lambda_j R_j$. On the other hand, for a sufficiently high $W(k_t)$, the RHS of the inequality in (3) becomes $R_j f'(k_{t+1})$, which means the credit goes to the project with the highest R_j.

For any initial value, $k_0 > 0$, the sequence of k_t that solves (1), (2), and (3) is the equilibrium trajectory of the economy.[5]

Remark 1: The careful reader must have undoubtedly noticed that we deliberately avoid the use of the terminologies such as "debt capacity," "interest rate," and "loan market," and instead use "borrowing limit," "rate of return," and "credit market" in describing the credit market imperfections. This is because the present paper is concerned with dynamic general equilibrium implications of credit market imperfections, arising from the difficulty of external finance in general. Note that the borrowing constraint arises due to the inability of the borrowers to pledge the project revenue fully, not due to any restriction on the menus of the financial claims that they can issue. The main issues addressed here are general enough that they are independent of the financial structure. Indeed, we view it one of the advantages of the model that it is too abstract to make a meaningful distinction between the equity, the debt or any other forms of financial claims.[6]

3. Homogeneous Projects: The Neoclassical Convergence

To offer a benchmark, let us first consider the case, J = 1. Then, (1)-(3) become

(4) $X_t = W(k_t)/m$,

(5) $k_{t+1} = RW(k_t)$,

[5] Strictly speaking, eqs. (1)-(3) do not fully describe the equilibrium. It is also necessary to add the condition stating that, when (3) holds with equality for two or more types of projects, entrepreneurs would choose the one that would give them the highest second period consumption. However, this situation occurs only for a finite number of k_t, which means that the equilibrium trajectory would not encounter such a situation for almost all initial values of k_0. Hence, we omit the discussion of this condition for the ease of exposition.

[6] See Tirole (2004, pp.163-164), who also argues for the benefits of separating the general issues of credit market imperfections and the questions of the financial structure.

(6) $$r_{t+1} = \frac{Rf'(k_{t+1})}{\max\{1,[1-W(k_t)/m]/\lambda\}},$$

where subscript j = 1 has been omitted to simplify the notation.

Note that the equilibrium trajectory of k_t is determined entirely by eq. (5). Figure 1 depicts the dynamics defined by eq. (5) under the following assumption, which will be maintained for the rest of the paper.

(A) $W(k)/k$ is strictly decreasing in k, with $\lim_{k\to+0} W(k)/k = \infty$ and $\lim_{k\to+\infty} W(k)/k = 0$.

This assumption holds for many standard production functions, including a Cobb-Douglas, $f(k) = Ak^{\alpha}$ with $0 < \alpha < 1$. It provides a sufficient condition under which eq. (5) has a unique positive steady state, k^*, given by $k^* \equiv RW(k^*)$, for any $R > 0$, as seen in Figure 1.

Note also that eq. (5) is independent of λ and m. This is because, with all the investment projects being the same, the fact that an entrepreneur cannot fully pledge their project revenue does not affect the allocation of the aggregate saving across the investment projects. The dynamics is driven entirely by the credit supply, which is inelastic, as in the textbook Solow model.[7] A change in borrower net worth would be entirely offset by a change in the equilibrium rate of return, which adjusts to equate the saving and investment. The indivisibility of each project does not affect the dynamics, either, because the measure of the projects initiated (and the measure of the agents who become entrepreneurs) adjusts endogenously to equalize the investment and the saving in the aggregate. In equilibrium, the fraction of the agents equal to $X_t = W(k_t)/m < 1$ becomes entrepreneurs, while the rest, the fraction of the agents equal to $1 - X_t$, becomes the lenders.[8] In other words, in spite of the nonconvexity of each investment project, the aggregate investment technology is linear, as in the standard neoclassical growth model.[9]

[7]It is also worth noting that eq. (5) is isomorphic to the dynamics of the standard Diamond overlapping generations model for the case where each agent consumes only in the second period.
[8]That $W(k_t)/m < 1$ is ensured by the same procedure described in footnote 4.
[9]In the neoclassical growth model, the productivity of this linear technology, R, is commonly normalized to be one, which can be done without any loss of generality because of the homogeneity of the investment technologies. Here,

This does not mean that credit market frictions play no role in the case considered here. From (5), eq. (6) becomes, for $W(k_t) > (1-\lambda)m$, $r_{t+1} = Rf'(RW(k_t))$. In this case, the agents do not face a binding borrowing constraint, and the equilibrium rate of return adjusts to ensure that they are indifferent between being the entrepreneurs and being the lenders, i.e., that (PC) holds with equality. The lender hence receives the rate of return equal to the marginal productivity of the investment project. For $W(k_t) < (1-\lambda)m$, eq. (6) becomes $r_{t+1} = \lambda Rf'(RW(k_t))/(1-W(k_t)/m) < Rf'(RW(k_t))$. That is, (BC) is binding, while (PC) holds with strict inequality. In this case, the rate of return for the lender is strictly less than the marginal productivity of the project, and hence the agents strictly prefer borrowing to become entrepreneurs to lending. Pinned down by (BC), r_{t+1} cannot adjust to make them indifferent. This means that the equilibrium allocation necessarily involves credit rationing, where some random mechanism allocates the credit to the fraction, X_t, of the agents, while the rest of the agents is denied the credit. These unlucky agents have no choice but to become the lenders; they would not be able to entice the potential lenders by promising a higher return, because that would violate (BC).

Remark 2: The two of the results above, (i) the dynamics converge to the unique positive steady state and (ii) the dynamics is independent of λ, are not robust features of the model when J = 1. The first result is ensured by Assumption (A). Without this assumption, the dynamics may have multiple steady states, or it may have no steady state for a sufficiently large R, or its unique steady state may be zero for a sufficiently small R. The second result depends on the assumption that the aggregate saving is inelastic.[10] The point is not to show that these results are inherent features of the homogeneous project case, because they are not. The point is to offer a benchmark, against which we can identify the role of the heterogeneity and the changing compositions of the credit and investments.

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we do not use the normalization, as we consider the cases where the investment technologies differ in the productivity.

[10]For example, suppose that, in their first periods, the agents can store the final good at the gross rate of return, ρ. When this storage technology is used (i.e., $k_{t+1} < RW(k_t)$), the credit supply becomes perfectly elastic at $r_{t+1} = \rho$. For $W(k_t) < (1-\lambda)m$, the dynamics is given by $f'(k_{t+1}) = (\rho/R\lambda)(1-W(k_t)/m)$, which shows the credit multiplier (financial accelerator) effect. Indeed, this case is effectively a reproduction of the Bernanke-Gertler (1989) model in its essentials.

Remark 3: Although the equilibrium credit rationing is important to understand the working of this model, one should not make too much out of it, because it is an artifact of the assumption that the agents are homogeneous. The homogeneity means that, whenever some agents face the borrowing constraint, all the agents face the borrowing constraint, so that coin tosses or some random devices must be evoked to determine the allocation of the credit. It is possible to extend the model to eliminate the equilibrium credit rationing without changing the essential feature of the model. For example, suppose that the endowment of the agents is given by a cumulative distribution, G(z), with the mean equal to one and with no mass point. Then, the allocation of the credit is determined by a critical value of the endowment, z_t, i.e., the agents, whose endowments are greater than or equal to z_t, become entrepreneurs and those whose endowments are less than z_t become the lenders. What is essential for the present analysis is whether the borrowing constraint is binding or not for the marginal agent, not whether there is an equilibrium credit rationing, which is merely an artifact of the assumption made solely to minimize the notation and to simplify the analysis.[11]

4. Heterogeneous Projects: Credit Traps and Credit Collapses

Let us consider the case, where $R_1 < R_2 < \ldots < R_J$ and $\lambda_1 R_1 > \lambda_2 R_2 > \ldots > \lambda_J R_J$. In words, there are trade-offs between productivity and pledgeability. Higher-indexed projects are more productive, hence appealing to the borrowers (and the next generations of the agents), while lower-indexed projects offer more pledgeable revenues per unit of investment, which make them potentially better alternatives for the lenders. For much of the discussion in this section, we focus on the case where J = 2, because it is straightforward (but cumbersome) to extend the analysis for the cases where J > 2.

Figures 2a and 2b show the graphs of

[11] While some authors use the term, "credit-rationing," whenever some borrowing limits exist, here it is used to describe the situation that the aggregate supply of credit falls short of the aggregate demand at the equilibrium rate of return, so that some borrowers cannot borrow up to their borrowing limit. In other words, there is no credit rationing if every borrower can borrow up to its limit. In such a situation, their borrowing may be constrained by their net worth, which affects the borrowing limit, but not because they are credit-rationed. This definition is in the same spirit with the following definition of credit rationing by Freixas and Rochet (1997, Ch.5), who attributed it to

$$\frac{R_j}{\max\{1,[1-W(k_t)/m_j]/\lambda_j\}} \qquad (j = 1, 2)$$

as functions of $W(k_t)$. These graphs, when multiplied by $f'(k_{t+1})$, show the RHS of the inequality in (3), i.e., the rate of return that each project type can offer willingly and credibly to the lender. As shown, each graph is increasing in $W(k_t)$, for $W(k_t) < (1-\lambda_j)m_j$, i.e., when (BC-j) is the relevant constraint. The reason is that an increase in $W(k_t)$ eases the borrowing constraint, as the entrepreneurs need to borrow less. This makes it possible for them to promise a higher return to the lenders. The graphs are flat for $W(k_t) > (1-\lambda_j)m_j$, i.e., when (PC-j) is the relevant constraint. With $R_2 > R_1 > \lambda_1R_1 > \lambda_2R_2$, the two graphs intersect once at k_c. At this intersection, (BC-2) is always binding. For type-1 projects, (PC-1) is binding, if $m_2/m_1 > (1-\lambda_1)/(1-\lambda_2R_2/R_1)$, as shown in Figure 2a; (BC-1) is binding if $m_2/m_1 < (1-\lambda_1)/(1-\lambda_2R_2/R_1) < 1$, as shown in Figure 2b. In either case, for $k_t < k_c$, type-1 projects can offer a higher return to the lender than type-2 projects, and hence, all the saving flows into type-1 projects; $X_{1t} = W(k_t)/m_1$ and $X_{2t} = 0$. Therefore, from (3), $k_{t+1} = R_1W(k_t)$. Likewise, for $k_t > k_c$, $k_{t+1} = R_2W(k_t)$. To summarize,

$$(7) \qquad k_{t+1} = \begin{cases} R_1W(k_t) & \text{if} \quad k_t < k_c, \\ R_2W(k_t) & \text{if} \quad k_t > k_c. \end{cases}$$

The intuition behind eq. (7) should be clear. When the entrepreneurs have low net worth, they have to rely heavily on borrowing. Thus the saving flows into type-1 projects, which generate the higher rate of pledgeable return. When the net worth improves, the borrowers need to borrow less, which enables the entrepreneurs to offer the higher return to the lender with type-2 projects, despite they generate the lower pledgeable return per unit of investment.

Since $R_1 < R_2$, the map defined in eq. (7) jumps up as k_t passes k_c, which means that there are three generic cases, depending on whether $k_c < k^*$ (Figure 3a), or $k^* < k_c < k^{**}$ (Figure 3b),

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Baltensperger: “some borrower’s demand for credit is turned down, even if this borrower is willing to pay all the price and nonprice elements of the loan contract.”

or $k^* < k^{**} < k_c$ (Figure 3c), where k^* and k^{**} ($> k^*$) are defined by $k^* \equiv R_1W(k^*)$ and $k^{**} \equiv R_2W(k^{**})$, respectively. One can easily verify that all three cases are feasible.[12]

In Figure 3b, both k^* and k^{**} are stable steady states. The lower steady state, k^*, may be interpreted as a *credit trap*. In this steady state, the borrower net worth is low, so that the saving flows into the projects that generate the higher pledgeable return per unit of investment, although they produce less physical capital. The resulting lower supply of physical capital leads to a lower price of the endowment held by the next generation of the agents, hence, a low borrower net worth. Even when a credit trap does not exist, a low net worth can contribute to a slow growth of the economy, as illustrated by Figure 3a. In this case, if the economy starts with a low value of k_0, the saving will fail to flow into more productive projects for long time, thereby slowing down an expansion of the economy. In Figure 3c, the higher steady state fails to exist, and the saving will eventually stop flowing into more productive type-2 projects, even if the economy starts with a high value of k_0. This case may be called a *credit collapse*.

It should be obvious to the reader how the above analysis can be extended to the case with $J > 2$. With J types of the projects, there can be at many as J stable steady states and $J - 1$ credit traps. Furthermore, it is also possible that credit traps and credit collapses may exist at any level of k_t. While this may seem trivial, it helps to clarify some widespread misunderstandings on the implications of models with multiple stable steady states. For example, it is often argued that models with stable multiple steady states offer an explanation for variations of economic performances across the countries. When a graph similar to Figure 3b is used to make this point, the lower (higher) steady state is interpreted as representing the location of poorer (richer) countries. One should not conclude from this, however, that the argument suggests that the poor "developing" countries are in the trap, while the rich "developed" countries are out of the trap. It is also false to say that the argument suggests that the distribution is bimodal. The logic of the argument does not require that there are only two stable steady states. Models with multiple stable steady states mean that there are many states towards which a country may gravitate. If the countries are scattered across an arbitrary number of stable steady states, there is no reason to believe that the argument suggests a bimodal distribution. Furthermore, it may well be the case

[12]To see this, note that k^* and k^{**} are independent of the parameters, λ_1, λ_2, m_1, and m_2, and that k_c can take any positive value by changing these parameters without violating the assumption, $R_2 > R_1 > \lambda_1R_1 > \lambda_2R_2$.

that no country has succeeded in reaching the highest stable steady state. If so, all the countries are in the traps, and in this sense, they are all "developing."[13]

Before moving to the next section, let us briefly consider the implications of an increase in the pledgeability. In the above analysis, one reason why the saving may fail to flow into the more productive projects is that the borrowers cannot fully pledge their project revenues. So, one might think that a better corporate governance or contractual enforcement technology, which helps to improve the pledgeability would always cause the saving to flow into the more productive investment projects. That is certainly the case, if the improvement means a higher λ_J, i.e., a higher pledgeability of the most productive projects. How about a higher pledgeability of the other projects? To answer this question, let us go back to the case where J = 2. In particular, look at the case illustrated in Figure 2b. Note that a higher λ_1 leads to a higher k_c. Since k^* and k^{**} are independent of λ_1, this means that the dynamics could change from Figure 3a to Figure 3b, in which case the credit trap is created as a result of "an improvement" in the credit market. Or the dynamics could change from Figure 3b to Figure 3c, in which case the credit collapse occurs as a result of "an improvement in the credit market. More generally, a higher pledgeability of the projects, except those most productive, could end up causing credit traps and credit collapses.[14]

5. Heterogeneous Projects: Credit Cycles and Growth Miracles

In the previous section, we considered the cases where there are trade-offs between the productivity and the pledgeability, so the interests of the borrowers and the lenders are diametrically opposed when it comes to the choice of the project to be funded. This does not mean that the heterogeneity plays no role when there is no such conflict of interest. To see this, consider the case where J = 2 with $R_1 > R_2 > \lambda_1 R_1 > \lambda_2 R_2$, and $m_2/m_1 < (1-\lambda_1 R_1/R_2)/(1-\lambda_2) < 1$. Thus, type-1 projects produce more physical capital *and* generate more pledgeable revenue than

[13] Indeed, one could allow for $J = \infty$, and an infinite number of stable steady states, in which case it is impossible for any country to reach the highest stable steady state, because there is no highest stable steady state.

[14] Recall that Figure 2b is applied when $R_2 > R_1 > \lambda_1 R_1 > \lambda_2 R_2$, and $m_2/m_1 < (1-\lambda_1)/(1-\lambda_2 R_2/R_1) < 1$. Suppose that type-1 projects use well-known but less productive technologies, which comes with relatively minor agency problems. Type-2 projects are new, using highly productive technologies, run by small venture capital, which come with huge agency problems. If an attempt to improve corporate governance is effective only for the well-established

type-2 projects, but the investment size is much smaller for type-2 projects. Figure 4 shows the two graphs, $R_j/\max\{1, [1 - W(k_t)/m_j]/\lambda_j\}$ (j = 1, 2), as functions of $W(k_t)$ for this case. This time, the two graphs intersect twice, at k_c and k_{cc}. For an intermediate range, $k_c < k_t < k_{cc}$, type-2 projects offer a higher return to the lenders than type-1 projects, and hence all the saving flows into type-2 projects and $k_{t+1} = R_2W(k_t)$. Otherwise, $k_{t+1} = R_1W(k_t)$. To summarize,

$$(8) \qquad k_{t+1} = \begin{cases} R_1W(k_t) & \text{if} \quad k_t < k_c \text{ or } k_t > k_{cc} \\ R_2W(k_t) & \text{if} \quad k_c < k_t < k_{cc}. \end{cases}$$

Since $R_1 > R_2$, the map defined in eq. (8) jumps down as k_t passes k_c and jumps up as k_t passes k_c. The intuition should be clear. When the net worth is very low, the entrepreneurs must rely almost entirely on external finance, so that the saving flows into type-1 projects that generate more pledgeable return per unit of investment. As the net worth rises, the entrepreneurs can offer more attractive return with type-2 projects than with type-1 projects, because they do not need to borrow much for type-2 projects. Hence, a rise in the net worth leads to a shift of the credit toward less productive projects. If the net worth rises even further, then the borrowing need becomes small enough for type-1 projects that the saving shifts back to type-1 projects.

Figure 5a and Figure 5b depict some possibilities generated by eq. (8). In Figure 5a, where $k^* < k_c < k^{**} < k_{cc}$, the equilibrium path fluctuates forever for all k_0.[15] Along these credit cycles, an improvement in the current net worth causes a shift in the credit towards the less productive projects that help less to create the future net worth. The resulting decline in the net worth causes the credit to shift back towards the projects that help more to build the net worth in the following period. In Figure 5b, where $k^* < k_c < k_{cc} < k^{**}$, these endogenous fluctuations co-exist with the steady state, k^{**}. If $RW(k_c) < k_{cc}$, the economy fluctuates indefinitely for $k_0 < k_{cc}$, while it converges to k^{**} for $k_0 > k_{cc}$. Thus, this is the case, where the credit trap takes the form of credit cycles, instead of the lower steady state, k^*. The situation is far more complicated if

industries, whose nature of the agency problems are relatively understood, it would end up preventing the saving from flowing into new, but more productive technologies.

$RW(k_c) > k_{cc}$. In this case, starting from $k_0 < k_{cc}$, the economy may fluctuate forever around k_c, or, depending on the value of k_0, it may escape and succeed in reaching k**, possibly after long periods of fluctuating around k_c. Thus, this case suggests the possibility of *growth miracles*, where some countries succeed in escaping the trap, and which countries succeed and which countries fail may depend on subtle differences in the initial conditions.

Again, the above analysis can be extended to the case with J > 2. In particular, it is possible that the map jumps down and up for many times, creating fluctuations around different levels of k_t. Therefore, one should *not* conclude by looking at Figure 5b that only the poor countries are subject to credit cycles.[16]

6. Concluding Remarks

The recent literature on macroeconomics of credit market imperfections emphasizes the importance of borrower net worth in the aggregate investment dynamics. When a variety of investment projects with different characteristics compete with each other in the credit market for funding, a movement of borrower net worth can have additional roles in the aggregate investment dynamics through its effect on the composition of the credit. In this paper, we proposed a model with heterogeneous investment projects, which is simple enough to be tractable and yet rich enough to capture some of the implications of the heterogeneity in the dynamics of the aggregate investment and borrower net worth. The purpose of this paper is not to challenge the results obtained in the existing models with homogeneous investment projects. Rather, it is to identify the mechanisms through the changing composition of the credit, which are complementary to those identified in the models with homogeneous investment projects. The importance of the composition effects may, of course, depend on the applications. For example, it might be reasonable to ignore them on a first approximation, when applied to the high frequency dynamics to deal with the issues such as the short-run analysis of the monetary policy (see, e.g., Bernanke

[15] Although these figures depict period-2 cycles, the fluctuations can take a more complicated form. Providing a full characterization of the dynamics could easily double the length of this paper, without adding much economic insight.

[16] Empirically, it may be the case that poor countries are more volatile. However, this is not a robust implication of the model presented here.

and Gertler 1995, 1999).[17] The composition effects may be more important in the low frequency dynamics, which is one reason why some of the analysis above was discussed using the language of economic growth and development.

Keep in mind that this paper offers only a glimpse of what might happen in the investment dynamics in the presence of credit market frictions, when we allow for the composition of the credit to change. The model presented here does not take into account all the potential sources of the heterogeneity across the investment projects. They are assumed to be different only in productivity, pledgeability, and the investment size. Among other things, it is assumed that all the investment projects produce the same capital good, and that all the agents are identical. These restrictions are responsible for certain unrealistic features of the equilibrium. For example, the model has the property that, in any period, all the credit goes to only one type of the projects. When a change in borrower net worth causes the credit to switch from one type to another, the switch occurs quite abruptly. And this abrupt switch causes the discontinuity of the dynamical systems studied here. One could remove these features of the models by relaxing the above restrictions. For example, one could assume instead that some projects produce the consumption good, while others produce the capital good, as in Matsuyama (2004a). Introducing such an additional element of heterogeneity could make the dynamical system continuous and prevent any abrupt change in the composition of the credit from occurring along the equilibrium path. It would also enable us to address certain issues that cannot be addressed in the above model.[18] While that would be certainly more appealing, it would make the model more difficult to analyze. Indeed, the analysis in Matsuyama (2004a) requires the use of fairly sophisticated techniques from the nonlinear dynamical system theory, which are not among the standard tools

[17] It is only as a first approximation, because the existing studies in this area assume exogenous productivity shocks to study the role of borrower net worth in the propagation mechanism. Arguably, some of these productivity shocks may be due to the composition effects that cause the credit to switch between the projects with different productivity levels.

[18] In the present model, all the projects produce the same capital good, which means that the interest of the agents as the borrower/entrepreneur is completely aligned with the interest of the next generation of the agents. In the model of Matsuyama (2004a), the borrower/entrepreneur may invest in the projects that produce the consumption good, although such projects do not improve the net worth of the next generation. This feature of the model makes it easier to generate endogenous credit cycles under less stringent conditions. Furthermore, this mechanism can be combined with the credit multiplier mechanism of Bernanke and Gertler (1989) to generate asymmetric fluctuations, where the economy experiences a long and slow process of recovery from a recession, followed by a rapid expansion, and

in economics. One advantage of the model presented above is that it is simple enough that one could analyze it by relatively simple graphic techniques familiar to many economists. The message here is that *even* such a simple model can generate a wide range of nonlinear phenomena, such as traps and cycles, in the dynamics of the aggregate investment and borrower net worth. What has been uncovered in this paper is only the tip of the iceberg.

possibly after a period of high volatility, plunges into a recession. Such an asymmetry does not occur in the present model.

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Figure 1

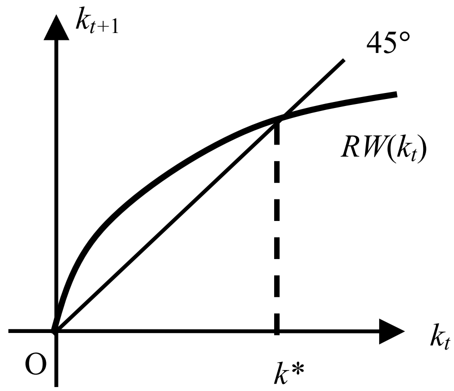


Figure 2a

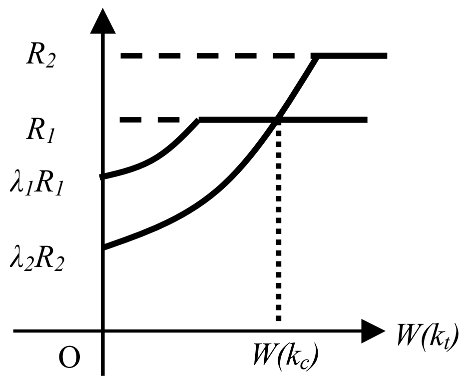
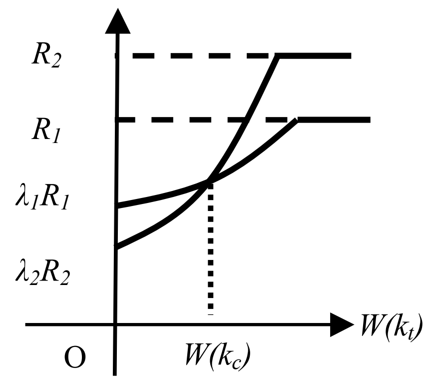


Figure 2b



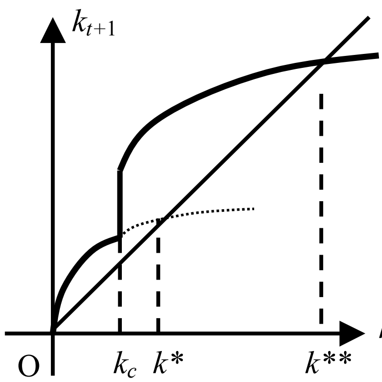


Figure 3a

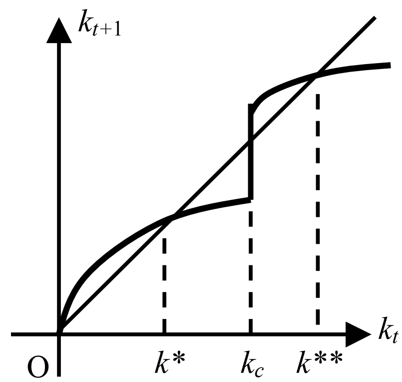


Figure 3b

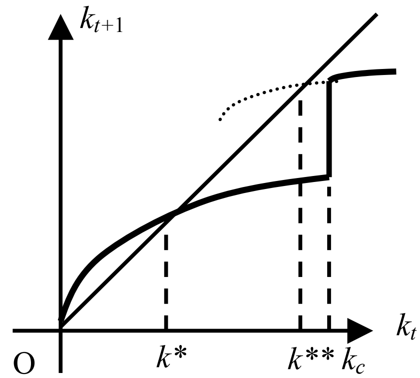


Figure 3c

Figure 4

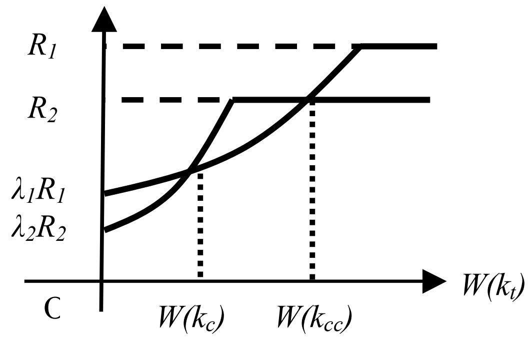


Figure 5a

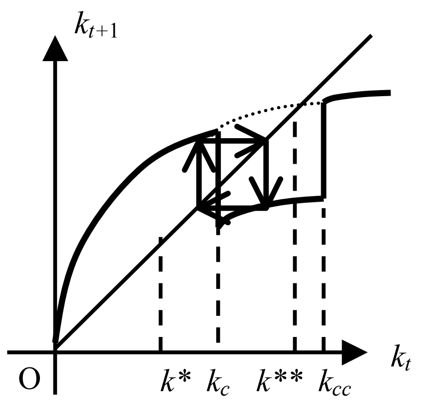


Figure 5b