Competition, Innovation and Growth with Limited Commitment*

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Abstract

We study how limited enforcement of contracts and barriers to business start-up affect the investment in knowledge capital and the adoption of new technologies. We show that barriers to business start-up (limited competition) is the most important obstacle to growth. Limited enforceability of contracts is detrimental to growth only if there are barriers to business start-up. Our results are consistent with cross-country evidence.

1 Introduction

It is widely recognized that sustained economic growth—especially, in advanced societies—requires investment in R&D, adoption of advanced technologies and innovation. A distinguished feature of modern technologies such as information and communication technologies, biotechnologies and nanotechnologies, is the importance of *knowledge capital*, which is highly

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complementary to skilled human capital. The ability of a society to innovate and grow is then dependent on how knowledge capital is accumulated, organized and the returns shared among the parties who participate in the innovation process. This is especially important when the parties who finance investment differ from those who acquire the innovation skills. For instance, whether a certain innovation project is funded depends on the ability of the investors to recover at least some of the returns of the project. Similarly, workers and managers must have the right incentives to invest in knowledge and enhance their innovation skills. Of course, this depends on the contractual arrangements that are feasible and enforceable. The goal of this paper is to study how the accumulation of knowledge capital—which is distinct (although complementary) from physical capital—affects the rate of innovation and growth when contracts are not fully enforceable.

An important feature of knowledge capital is that it can hardly be collateralized. This implies that the contractual arrangements between entrepreneurs and investors are not trivial. The investors know that without the contribution of the innovator the project can seldom have an adequate return. For example, investment returns may be lost if the entrepreneur moves his attention and effort to a different—possibly, more rewarding—project. It follows that the 'degree of contractual enforcement' becomes central to the investment process. In the context of knowledge capital, however, limited enforcement is not caused by the ability of the entrepreneur to 'grab the money and run'. Rather, it is the ability to engage in different innovation projects relative to the ones preferred by the investor.

The limited enforceability of contracts is not only one-sided. If on the one hand the entrepreneur can engage in innovation activities that are not optimal for the investor, on the other, the investor can replace the entrepreneur and renege promises of payments. While the limited commitment of the entrepreneur may hold-up the investor from investing in physical capital, the limited commitment of the investor may hold-up the entrepreneur from investing in knowledge capital. A major finding of this paper is that the severity of the hold-up problem depends crucially on the presence of barriers to business start-up or more generally barriers to alternative uses of knowledge capital.

Without barriers, the entrepreneur can always quit the firm and start a new business. This implies that the entrepreneur can rely on the outside value of his or her knowledge capital as a threat against the investor's attempt to renegotiate. In this case, the entrepreneur may even over-accumulate knowledge in order to keep the threat value high. In contrast, when there are substantial barriers to business entry, knowledge capital does not have an independent value outside the firm. Under these conditions, the limited commitment of the investor implies that the entrepreneur will not be remunerated for the knowledge investment. As a result, there will be no accumulation of knowledge. Therefore, barriers to entry or lack of competition are detrimental to growth.

Our result differs from other models with hold-up problems. In some of these models the firm has full control over the accumulation of human capital. For example, in Acemoglu & Shimer (1999), is the employer that decides the amount of training. Under this assumption, greater mobility worsens the hold-up problem because it increases the workers' ability to capture the firm's rents. In other models, such as the one studied in Acemoglu (1997), workers do control the accumulation of skills, but the main conclusion does not change: greater mobility worsens the hold-up problem because workers are less likely to benefit from their accumulation of skills. In contrast to these studies, we show that mobility and competition increase human capital (knowledge) investment. The key factor leading to this result is the limited enforceability of long-term contracts from investors.

In our framework, investors prefer a lower rate of innovation than entrepreneurs because of the creative destruction of physical capital. In this context, without business entry or competition, it is the firm that holds up the entrepreneur. This conflict could be resolved if long-term contracts were enforceable. In this case the investor would retain the entrepreneur and adopt a slower pace of growth by promising higher payments. However, once the accumulation of innovation skills has been made, the investor would renege these payments. Consequently, the only way to retain the entrepreneur is by financing higher innovations. But for this to be the outcome, it is crucial that the entrepreneur has the option to quit and start a new firm: it is the threat of quitting that induces the investor to accept a faster rate of innovation. Because lower barriers generate greater potential *mobility* and greater *competition*, we will refer to an environment with no artificial barriers as a 'competitive economy'.

Whether competition enhances innovation has been a major topic of research and debate since Schumpeter's claim that, while product market competition could be detrimental to innovation, competition in the innovation sector enhances innovations (see, for example Aghion & Howitt (1999)). Most of the following literature has focused on market structure and product market competition. In particular, on the ability to appropriate the returns to R&D and to gain market shares by introducing new products. For example, Aghion, Blundell, Griffith, Howitt, & Prantl (2002) show that there is an inverted U relationship between product market competition and innovation. More closely related to our work is Aghion, Blundell, Griffith, Howitt, & Prantl (2004). They show—both, theoretically and empirically—that 'firm entry' spurs innovation in technological advanced sectors, while it may discourage innovation in lagged sectors. In the advance sector, the threat of competition spurs innovation as firms try to 'escape competition'. In contrast, we focus on the less studied dimension of 'knowledge capital competition'. There is also an 'escape competition' effect in our model; but of a very different nature. Competition for knowledge capital can spur innovation when investors try to retain their innovative entrepreneurs. When (and only when) investors are not able to commit to future payments, investing in innovations is a commitment device that prevents entrepreneurs from developing their innovations elsewhere. In practice, both 'escape competition' effects are likely to be complementary as innovative firms try to retain their knowledge capital and stay ahead of their competitors in the product market.

Our results are consistent with the technological advances of the U.S. economy. For example, Bresnahan & Malerba (2002) argue that the US' lead in the computer industry was possible thanks to a highly competitive environment, more prompt to stimulate innovations. For example, they claim that "The most important U.S. National institutions and policies supporting the emergence at this time [of the PC industry] were entirely non-directive: the existence of a large body of technical expertise in universities and the generally supportive environment for new firm formation in the United States", (p.69). In the next section we also provide cross-country evidence that growth is positively associated with the degree of contract enforcement and negatively associated with the cost of business start-up.

The paper relates to five strands of literature. First, the labor literature that studies the 'hold-up problem' (e.g., Acemoglu (1997), Acemoglu & Pischke (1999), Acemoglu & Shimer (1999)). Second, the literature that studies the linkages between 'competition and innovation' (e.g. again Aghion et al. (2002) and Aghion et al. (2004)). Third, the endogenous growth literature that studies the 'economics of ideas' and its impact on growth (starting from the pioneer work of Romer (1990, 1993)). Fourth, the recent growth literature that, building on the work of economic historians (e.g., Mokyr (1990)), emphasizes the role of 'barriers to riches' in slowing growth (Parente & Prescott

(1990)). Fifth, and foremost, the recent literature on dynamic contracts with enforcement constraints such as Marcet & Marimon (1992). Most of the models with limited enforcement ignore the issue of technology adoption and innovation. One exception, closely related to our work, is Kocherlachota (2001) who shows that limited enforcement may result in a lower rate of technological adoption when the division of the social surplus is sufficiently unequal, since in such context agents may not be induced to bear the adoption costs that they will bear in an economy with full commitment. Another exception is Cooley, Marimon, & Quadrini (2004). In that paper, however, the arrival of new technologies is exogenous and limited enforcement affects only the propagation of new technologies, not the rate of innovation. In the current paper, instead, we study how limited enforcement affects the development of new technologies and how this impacts on the long-run growth. Our paper also departs from the existing literature on dynamic contract enforcement that unambiguously predicts that more 'stringent enforceability'—in particular, severe punishments—enhance 'efficiency' (and 'technology adoption' in Kocherlachota (2001)). In contrast, in our model, limited enforceability coupled with competition can enhance efficiency, since—when investors can not commit—'severe punishments' are detrimental to innovation.

The plan of the paper is as follows. In Section 2 we provide cross-country evidence about contract enforcement, barriers to business start-up and macroeconomic performance. Section 3 describes the model. To facilitate the intuition about the theoretical results, Section 4 studies a simplified version of the model with only two periods. Sections 5 and 6 will generalize the results to the infinite horizon model. Section 7 concludes.

2 Cross-country evidence

A recent publication from the World Bank[1] provides data on the quality of the business environment for a cross-section of countries. Especially important for our paper is the *Cost of Starting a Business* and the *Degree of Contract Enforcement.*

Figure 1 plots the level of per-capita income against the cost of starting a business. This is the 'average pecuniary cost' needed to set-up a corporation in the country, expressed in percentage of the country per-capita income. As can be seen from the figure, there is a strong negative correlation between

[1] *Doing Business in 2005: Removing Obstacles to Growth.* World Bank, Washington.

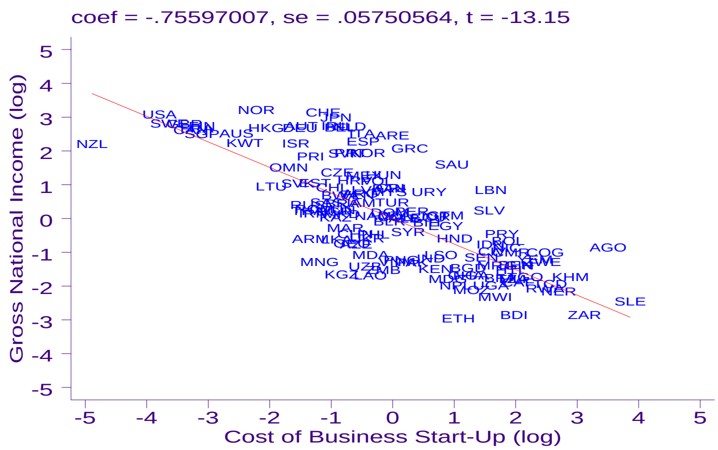

Figure 1: Cost of starting a business and level of development.

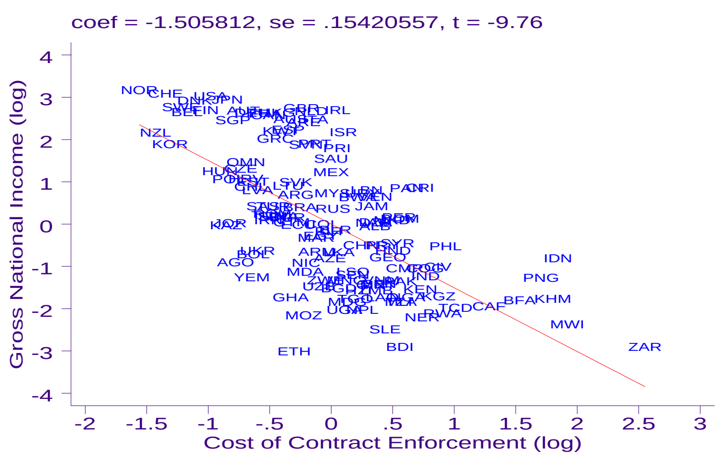

Figure 2: Cost of contract enforcement and level of development.

the cost of starting a business and the development of the country. A similar figure would result if we use alternative measures of the cost of business start-up, such as the 'number of bureaucratic procedures' or the 'number of days' needed to start a new business. These alternative measures are also reported by the World Bank. Therefore, the general pattern is that the set-up of a new business is more costly and cumbersome in poor countries.

Figure 2 plots the per-capita income against the cost of contract enforcement. This is the average pecuniary cost sustained to resolve a legal dispute in the country. The cost is expressed in percentage of the country per-capita income. Also in this case the figure shows a strong negative correlation between the cost of contract enforcement and the level of development. A similar pattern is obtained if we use alternatives measure of the cost of contract enforcement, such as the 'number of bureaucratic procedures' that need to be filed or the 'number of days' elapsed before a legal dispute is resolved. Hence, the enforcement of contracts in poorer countries is in general more problematic than in rich countries.

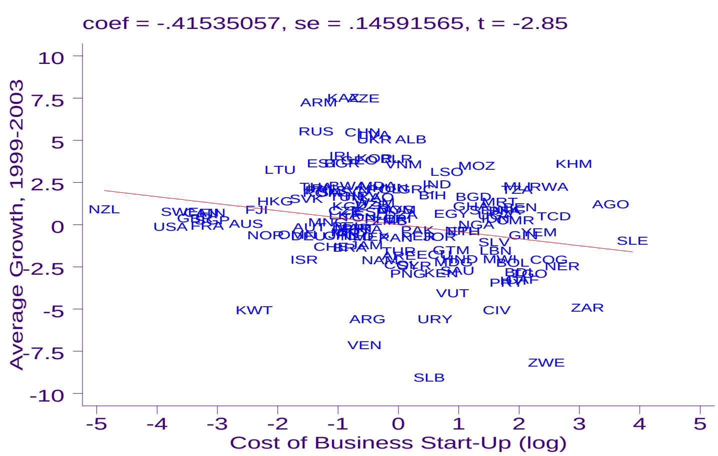

Figure 3: Cost of starting a business and economic growth.

The costs of business start-up and contract enforcement are also negatively associate to economic growth. Figure 3 correlates the average growth rate in per-capita GDP in the last five years of available data (1999-2003) to the cost of starting a business. The correlation is negative and statistically significant, showing that countries with lower barriers to entry tend to experience faster growth. This finding is robust to the choice of alternative years

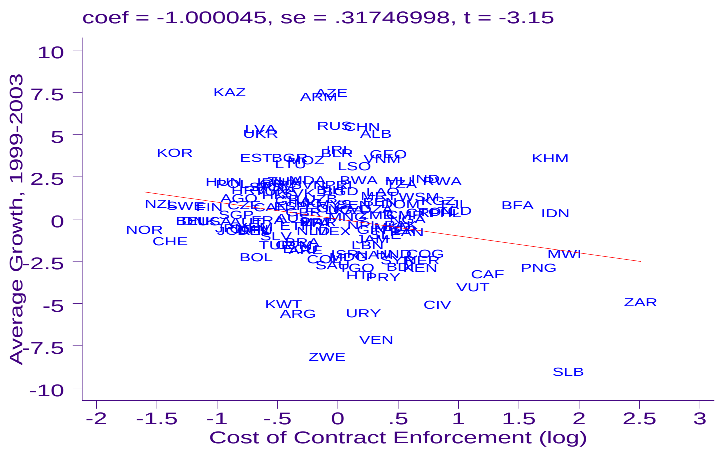

Figure 4: Cost of contract enforcement and economic growth.

to compute the average growth.

Figure 4 shows that the correlation is also negative and statistically significant between the average growth and the cost of contract enforcement. Therefore, countries with better enforcement of contracts tend to experience faster growth.

Because the cost of starting a business and the cost of contract enforcement are positively correlated, it is possible that the correlation with economic growth is not independent of each other. To investigate this possibility we regress the five years average growth in per-capita GDP on the cost of business start-up and the cost of contract enforcement. We also include the 1998 per-capita GDP to control for the initial level of development. The estimation results, with t-statistics in parenthesis, are reported in Table 1.

From the regression results we can see that both the cost of business start-up and the cost of contract enforcement are negatively correlated with the five-year average growth rate. Furthermore, the statistical significance of these correlations is not affected by the inclusion of the initial per-capita income, that controls for the development stage of the country. The results are robust to the choice of alternative periods to compute the average growth rate in per-capita GDP.

To summarize, the general picture portrayed by this section shows that the economic development and growth of a country is negatively associated with the cost of starting a business and the cost of enforcing contracts. In

Table 1: Business environment variables and growth.

	Constant	*Initial Per-Capita GDP*	*Cost of Business Start-Up*	*Cost of Contract Enforcement*
Coefficients	18.25	-1.21	-0.75	-1.25
t-Statistics	*(5.11)*	*(-3.86)*	*(-3.58)*	*(-3.26)*
R-square	0.177			
N. of countries	136			

NOTES: Dependent variable is the average annual growth rate in per-capita GDP for the five year period 1999-2003. Initial Per-Capita GDP is the log of per-capita GDP in 1998. The costs of business start-up and business enforcement are measured as a fraction of the per-capital Gross National Income as reported in *Doing Business in 2004*. These two variables enter the regression in logs.

the following sections we present a model that rationalizes these findings.

3 The model

There are two types of agents in the economy: 'investors' and 'entrepreneurs'. Investors maximize the lifetime utility from consumption $\sum_{t=0}^{\infty} \beta^t c_t$. Entrepreneurs' lifetime utility is $\sum_{t=0}^{\infty} \beta^t (c_t - e_t)$ where e_t is the effort to accumulate knowledge capital as specified below.

Entrepreneurs do not save, and therefore, they are unable to acquire the capital (and the control) of the firm. This assumption should be interpreted as an approximation to the case in which entrepreneurs discount more heavily than workers and/or there is continuous entrance of new entrepreneurs with zero initial wealth. The risk neutrality of investors implies that the equilibrium interest rate is equal to their intertemporal discount rate, that is, $r = 1/\beta - 1$.

The output produced by a firm at time t is:

$$y_t = z_t k_t^{\alpha}$$

where z_t is the level of technology and k_t is the input of capital financed by the investor at time $t - 1$.

The variable z_t changes over time as the firm adopts new technologies. The key assumption is that more advanced technologies (characterized by higher z_t) require higher knowledge capital embodied in the skills of the entrepreneur. Let h_t be the knowledge capital. This enables the entrepreneur to adopt technologies with productivity:

$$z_t = Ah_t^{1-\alpha}$$

This assumption formalizes the idea that innovations are complementary to knowledge capital. Therefore, even though the investor has the control of the firm, innovations are ultimately controlled by the entrepreneur.

The accumulation of knowledge capital requires effort: Higher is the accumulation of knowledge, $h_{t+1} - h_t$, and higher is the effort cost. This cost also depends on the economy-wide level of knowledge, H_t, due to leakage or spillover effects. We formalize this by denoting the effort cost as $e_t = \varphi(H_t; h_t, h_{t+1})$. This function is homogeneous of degree 1, strictly decreasing in H_t and h_t, strictly increasing in h_{t+1} and satisfies $\varphi(H_t; h_t, h_t) = 0$.

Physical capital is technology-specific. Therefore, when the firm replaces the current technology, only part of the physical capital can be used with the new technology. Furthermore, the obsolescence of the existing capital increases with the degree of innovation. This is formalized by assuming that physical capital depreciates at rate $\delta(z_{t+1}/z_t)$, where the function δ is strictly increasing, strictly convex and satisfies $\delta(1) = 0$. Because of capital obsolescence, *incumbent* firms have a lower incentive to innovate than *new* firms, still uncommitted to any previous investment.

The final assumption is that the entrepreneur has a reservation value equal to $R(\mathbf{s}_t)$, where $\mathbf{s}_t$ denotes the aggregate state variables that will be define later. This imposes a lower bound to the value that the entrepreneur receives from innovating and managing a firm.[2]

In characterizing the optimal contract problem, we distinguish the case in which the entrepreneur can leave the firm and start a new business from the case in which this is not feasible or allowed. We refer to the first case as the *competitive* economy and to the second as the *non-competitive* economy. For each environment we will separately consider the case in which the investor commits to the long-term contract from the case of limited commitment.

[2]Although we assume that the reservation value is exogenous, we could make it endogenous by assuming that labor is also an input of production. In this case $R(\mathbf{s}_t)$ would be the value of being a worker.

4 Equilibrium in a two-period model

To gauge some intuitions about the key properties of the model, it would be convenient to consider first a simplified version with only two periods. We will identify these two periods as period zero and period one. The state variables of the firm at the beginning of period zero are h_0 and k_0. After making the investment decisions h_1 and k_1, the firm generates output $y_1 = z_1 k_1^\alpha$ in the second period. Because $z_1 = A h_1^{1-\alpha}$, the output can also be written as $y_1 = A h_1^{1-\alpha} k_1^\alpha$. After production, knowledge and physical capital fully depreciate. The entrepreneur receives a payment from the firm (compensation) at the end of the first period, after the choice of h_1. Allowing for additional payments in the first period before the choice of h_1 and/or in the second period does not change the results as we explain below. To simplify the analysis, we also assume that the effort cost does not depend on the economy-wide knowledge H and there is no discounting.

The timing of the model can be summarized as follows: The firm starts period zero with initial states h_0 and k_0. At this stage the entrepreneur decides whether to stay or quit the firm. If he quits and the economy is competitive, he will start a new business funded by a new investor. The repudiation value is then given by the part of the surplus generated by the new firm appropriated by the entrepreneur. If the economy is not competitive, the repudiation value is the reservation utility R. We are assuming that R is sufficiently small that the repudiation value from starting a new business is bigger than R. If the entrepreneur decides to stay, he will choose the new level of knowledge capital h_1 and the investor provides the funds to accumulate the new physical capital k_1. After the investment decision has been made, the investor pays the entrepreneur d_0. At this stage the entrepreneur can still quit, but he cannot change the knowledge investment h_1. The investor is the residual claimant of the firm in the second period.

4.1 Competitive economy with investor commitment

When the investor commits to the long-term contract, all variables are chosen at the beginning of the first period to maximize the surplus of the contract. Because the economy is competitive, the repudiation value for the entrepreneur is the value of starting a new business. Let $D(h_0)$ be the repudiation value before choosing h_1 and $\widehat{D}(h_1)$ the repudiation value after the choice of h_1. The participation of the entrepreneur requires that the value of staying with the firm is greater than the repudiation values before and after the knowledge investment, that is,

$$\begin{aligned} d_0 - \varphi(h_0, h_1) &\geq D(h_0) \\ d_0 &\geq \widehat{D}(h_1) \end{aligned}$$

The first is the participation constraint before choosing h_1 and the second is the participation constraint after choosing h_1. For the moment we assume that the repudiation values are known. We will derive them after writing the optimization problem. At that point we will also show that, if the first participation constraint is satisfied, the second is also satisfied. Using this result, the optimization problem when the investor commits to the contract can be written as:

$$\max_{h_1,k_1,d_0} \left\{ -\varphi(h_0, h_1) - k_1 + \left[1 - \delta\left(\frac{h_1^{1-\alpha}}{h_0^{1-\alpha}}\right)\right] k_0 + Ah_1^{1-\alpha}k_1^{\alpha} \right\} \tag{1}$$

$$\textbf{s.t.} \quad d_0 - \varphi(h_0, h_1) \geq D(h_0)$$

where we have substituted $z_1 = Ah_1^{1-\alpha}$ in the production and depreciation function.

From the optimization problem it is clear that the choice of the entrepreneur's payment d_0 is independent of the investment choices. This follows from the fact that d_0 does not enter the objective function. The only constraint is that this payment, net of the effort cost, is not smaller than the value that the entrepreneur would get from quitting. To determine the value of quitting before the choice of h_1, we have to solve for the optimal investment when the entrepreneur starts a new firm. Also in this case the optimal contract maximizes the total surplus, that is:

$$S(h_0) = \max_{h_1,k_1,d_0} \left\{ -\varphi(h_0, h_1) - k_1 + Ah_1^{1-\alpha}k_1^{\alpha} \right\} \tag{2}$$

$$\textbf{s.t.} \quad d_0 - \varphi(h_0, h_1) \geq D(h_0)$$

Notice that this problem differs from the previous problem only because a new firm does not have any physical capital to start with. But it is still the case that the choice of h_1 and k_1 is independent of d_0.

The part of the surplus going to the entrepreneur depends on the bargaining powers between the entrepreneur and the investor. Without loss of generality we can assume that the entrepreneur gets the whole surplus, which is the outcome if financial markets are competitive. This implies that $D(h_0) = S(h_0) = d_0 - \varphi(h_0, h_1)$, where h_1 solves problem (2). The alternative assumption that the entrepreneur gets only a fraction of the surplus does not change the main conclusion.

Therefore, if the entrepreneur stays with the incumbent firm, the payment d_0, net of the effort cost $\varphi(h_0, h_1)$, must be at least as large as $S(h_0)$. Formally,

$$d_0 \geq S(h_0) + \varphi(h_0, h_1) \tag{3}$$

As anticipated above, if the participation constraint is satisfied before investing in knowledge, it will also be satisfied after the choice of h_1. To show this result, we have to consider the problem solved by a new firm started after choosing h_1. This can be written as:

$$\widehat{S}(h_1) \;=\; \max_{k_1, d_0} \Big\{ -k_1 + A h_1^{1-\alpha} k_1^{\alpha} \Big\} \tag{4}$$

$$\textbf{s.t.} \quad d_0 \geq \widehat{D}(h_1)$$

Assuming that the entrepreneur gets the whole surplus, we have that $\widehat{D}(h_1) = \widehat{S}(h_1) = d_0$. Therefore, the participation constraint, after the investment in knowledge can be written as:

$$d_0 \geq \widehat{S}(h_1) \tag{5}$$

It is now easy to show that, if constraint (3) is satisfied, then constraint (5) is also satisfied. This follows from the fact that $S(h_0) + \varphi(h_0, h_1) \geq \widehat{S}(h_1)$, as can be verified from problems (2) and (4).[3]

Problems (1) and (2) show the different incentive to invest for an incumbent firm versus a new firm. A new firm does not have any physical capital and innovations do not generate capital obsolescence. Therefore, new firms have a greater incentive to innovate than incumbent firms. This is clearly shown from the first order conditions with respect to h_1, in problems (1) and (2) respectively. They are:

$$(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} \;=\; \varphi_{h_1} + \delta_{h_1} \cdot k_0 \tag{6}$$

[3]Given problems (2) and (4), we can write $S(h_0) = \max_h \{-\varphi(h_0, h) + \widehat{S}(h)\}$, which implies that $S(h_0) + \varphi(h_0, h_1) \geq \widehat{S}(h_1)$.

$$(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} \quad = \quad \varphi_{h_1} \tag{7}$$

where the terms φ_{h_1} and δ_{h_1} are the derivatives of the effort cost and the depreciation functions respectively.

The first condition is for problem (1), that is, for an incumbent firm, while the second is for problem (2), that is, the problem solved by a new firm. Because the term δ_{h_1} is positive, the marginal productivity of knowledge capital (the left-hand-side) must be bigger for incumbent firms. This implies that they choose a lower value of h_1. This result will be used below to study the equilibrium when the investor does not commit to the contract.

4.2 Competitive economy without investor commitment

Does the limited commitment from the investor lead to lower accumulation of knowledge? We show that, as long as there is competition (free entry), the opposite holds true.

We show first that, after the accumulation of knowledge, the investor has an incentive to renegotiate the optimal contract. We have shown above that $S(h_0)+\varphi(h_0,h_1) \geq \widehat{S}(h_1)$. But, as long as the depreciation of existing physical capital increases with h_1, this inequality is strict. In fact, from problems (2) and (4) we can write:

$$S(h_0) = \max_h \Big\{ -\varphi(h_0,h) + \widehat{S}(h) \Big\} \geq -\varphi(h_0,h_1) + \widehat{S}(h_1)$$

where h_1 is the knowledge investment chosen by an incumbent firm. Let h_{max} be the knowledge investment that solves the maximization problem. This is the investment that a new firm chooses. Obviously, the inequality above will be strict if $h_{max} \neq h_1$, which is the case as shown in the previous section (see conditions (6) and (7)). Therefore,

$$S(h_0) + \varphi(h_0, h_{max}) > \widehat{S}(h_1)$$

But this implies that the participation constraint after the choice of knowledge investment is not binding, that is,

$$d_0 > \widehat{S}(h_1)$$

and the investor has an incentive to renegotiate down the payment promised to the entrepreneur. The ability to renegotiate can be justified by assuming that the investor can replace the current entrepreneur by poaching other entrepreneurs with the same knowledge capital.

The entrepreneur anticipates that the promised payments will be renegotiated after the knowledge investment. Therefore, he will quit the firm unless the investor agrees to the same knowledge investment chosen by a new firm, that is, $h_1 = h_{max}$, which is higher than the one preferred by the investor. This implies that $S(h_0) + \varphi(h_0, h_1) = \widehat{S}(h_1)$ and $d_0 = \widehat{S}(h_1)$. In this way the entrepreneur keeps the repudiation value high and prevents the investor from renegotiating.

Therefore, we conclude that the lack of commitment from the investor does not lead to lower investment in knowledge. It is important to emphasize, however, that this is true only if there is competition. As we will see below, in absence of competition, limited commitment does lead to lower investment in knowledge.

4.3 Non-competitive economy with investor commitment

When the investor commits, the optimal contract chooses h_1 and k_1 to maximize the total surplus as in (1) and (2). The only difference is that now the repudiation values $D(h_0)$ and $\widehat{D}(h_1)$ are equal to the reservation utility R. But we have seen that, when the investor commits, the investment in knowledge does not depend on the repudiation values. They only affect the entrepreneur's payment d_0. Therefore, we conclude that the lack of competition is not harmful for innovations as long as there is commitment from the investor.

4.4 Non-competitive economy without investor commitment

In this case, the payment to the entrepreneur must satisfy the following two constraints:

$$\begin{aligned} d_0 &\geq \varphi(h_0, h_1) + R \\ d_0 &\geq R \end{aligned}$$

The first is the participation constraint at the beginning of the period, before investing in knowledge. The second is the participation constraint after the investment in knowledge.

Because the investor does not commit to the optimal contract, after the investment in knowledge, he will renegotiate down any payment that exceeds the reservation utility. This implies that the second participation constraint will be satisfied with equality, that is, $d_0 = R$. But then the first constraint will not be satisfied unless $h_1 = h_0$. This implies that there will be no investment in knowledge and the economy stagnates.

Intuitively, the entrepreneur anticipates that any promise of payments above R will be renegotiated and he will not be rewarded for the effort in accumulating knowledge. This follows from the fact that without competition knowledge does not have any value outside the firm. It is important to point out that making the payment before the investment in knowledge does not solve the problem. This is because the entrepreneur could quit after receiving the payment. The limited commitment is for both, the investor and the entrepreneur.

4.5 Summary results

We summarize the properties of the two-period version of the model in Table 2. We denote with $g^* = h_1/h_0 - 1 > 0$ the growth rate of knowledge capital in the competitive economy with one-side commitment. This economy acts as a reference of comparison. The key finding is that limited enforcement of contracts is not a cause of stagnation as long as there is competition. On the contrary, limited enforcement may even enhance growth if there is competition. At the same time, the lack of competition is not a cause of stagnation if there is commitment from the investor. What is harmful for growth is the lack of both commitment and competition.

Table 2: Summary results

	Competitive Economy	*Non-competitive Economy*
Commitment	Growth=g^*	Growth=g^*
No commitment	Growth>g^*	Growth=0

In the next section we study the general model with an infinite number of periods and with knowledge spillovers. The analysis is more complex but the general results are similar to the ones summarized in Table 2.

5 The infinite horizon model

In this section we study the general model with infinitely lived agents. We first characterize the equilibrium for the competitive economy with investor's commitment.

For the analysis that follows, it will be convenient to define the end-of-period resource function for the firm as follows:

$$\pi(h_t, k_t, h_{t+1}, k_{t+1}) = z_t k_t^\alpha + \left[1 - \delta\left(\frac{z_{t+1}}{z_t}\right)\right] k_t - k_{t+1}$$

Given the current level of knowledge h_t and physical capital k_t, the firm produces output $z_t k_t^\alpha$, to which it subtracts the gross investment in physical capital. In writing $\pi(h_t, k_t, h_{t+1}, k_{t+1})$, we take into account that $z_t = A h_t^{1-\alpha}$. Therefore, once we know h_t and h_{t+1}, we also know z_t and z_{t+1}.

In the competitive economy the entrepreneur has the option to quit the current firm and start a new business funded by new investors. The decision to start a new business can be made before or after investing in knowledge. We start with the optimization problem solved by a new firm created at the beginning of period t by an entrepreneur with knowledge capital h_t. This is before the current investment in knowledge.

The optimal contract can be characterized by maximizing either the value for the entrepreneur or the value for the investor, subject to the enforceability and participation constraints. A third strategy, used in the analysis of the two-period model, is to maximize the whole surplus. Of course, the three approaches are equivalent and will give the same results. For analytical convenience we choose to maximize the entrepreneur's value.

Define $D(\mathbf{s}_t; h_t)$ the entrepreneur's value from quitting the firm at the beginning of the period, before investing in knowledge. This is the value that the entrepreneur with knowledge h_t would get from starting a new firm. Furthermore, define $\widehat{D}(\mathbf{s}_t; h_{t+1})$ the value of quitting after choosing the knowledge investment, and therefore, after exerting the investment effort. At this point the stock of knowledge is h_{t+1}. For the moment we take these two functions as given. The optimization problem when the investor commits to the long-term contract is:

$$V(\mathbf{s}_t; h_t) = \max_{\{d_\tau, k_{\tau+1}, h_{\tau+1}\}_{\tau=t}^{\infty}} \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Big[d_\tau - \varphi(\mathbf{s}_\tau; h_\tau, h_{\tau+1}) \Big] \right\} \tag{8}$$

subject to

$$\sum_{j=\tau}^{\infty} \beta^{j-\tau}\Big[d_j - \varphi(\mathbf{s}_j; h_j, h_{j+1})\Big] \geq D(\mathbf{s}_\tau; h_\tau), \qquad \text{for } \tau \geq t \tag{9}$$

$$d_\tau + \sum_{j=\tau+1}^{\infty} \beta^{j-\tau}\Big[d_j - \varphi(\mathbf{s}_j; h_j, h_{j+1})\Big] \geq \widehat{D}(\mathbf{s}_\tau; h_{\tau+1}), \quad \text{for } \tau \geq t \tag{10}$$

$$-d_t - k_{t+1} + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}\Big[\pi(h_\tau, k_\tau, h_{\tau+1}, k_{\tau+1}) - d_\tau\Big] \geq 0 \tag{11}$$

The objective is the discounted flow of utilities for the entrepreneur. In each period, the entrepreneur receives the payment (consumption) d_τ and faces the disutility from effort $\varphi(\mathbf{s}_\tau; h_\tau, h_{\tau+1})$. Even though we did not impose it explicitly, the entrepreneur's payments are subject to a non-negativity constraint.

Constraints (9) and (10) are the enforcement conditions. Starting at time $t+1$, the entrepreneur could quit at the beginning of the period, before choosing the investment in knowledge. In this case the repudiation value is $D(\mathbf{s}_\tau; h_\tau)$. After choosing the knowledge investment, the value of quitting becomes $\widehat{D}(\mathbf{s}_\tau; h_{\tau+1})$. The last constraint is the participation constraint for the investor or break-even condition. This simply says that the value of the contract for the investor cannot be negative.

For an entrepreneur who starts a new firm *after* investing in knowledge, the value of the new contract is:

$$\widehat{V}(\mathbf{s}_t; h_{t+1}) = \max_{\{d_\tau, k_{\tau+1}, h_{\tau+2}\}_{\tau=t}^{\infty}} \left\{ d_t + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}\Big[d_\tau - \varphi(\mathbf{s}_\tau; h_\tau, h_{\tau+1})\Big] \right\} \tag{12}$$

subject to (9), (10) and (11)

The key difference respect to problem (8) is that the current effort cost has already being sustained by the entrepreneur and h_{t+1} is given. Therefore, the current return for the entrepreneur is only d_t. This also explains why the choice of knowledge starts at $t+2$. We are using the *hat* sign to denote all functions that are defined *after* the investment in knowledge.

Given the above definitions of $V(\mathbf{s}_t; h_t)$ and $\widehat{V}(\mathbf{s}_t; h_{t+1})$, it is easy to see that these two functions are related as follows:

$$V(\mathbf{s}_t; h_t) = \max_{h_{t+1}} \Big\{ \varphi(H_t; h_t, h_{t+1}) + \widehat{V}(\mathbf{s}_t; h_{t+1}) \Big\} \tag{13}$$

The optimization problems above assume that we know the repudiation functions $D(\mathbf{s}_t; h_t)$ and $\widehat{D}(\mathbf{s}_t; h_{t+1})$. But these functions are unknown because they dependent on the value functions $V(\mathbf{s}_t; h_t)$ and $\widehat{V}(\mathbf{s}_t; h_{t+1})$. More specifically, they are defined as:

$$D(\mathbf{s}_\tau; h_\tau) \;=\; \max \Big\{ R(\mathbf{s}_\tau)\,,\, V(\mathbf{s}_\tau; h_\tau) \Big\} \tag{14}$$

$$\widehat{D}(\mathbf{s}_\tau; h_{\tau+1}) \;=\; \max \Big\{ R(\mathbf{s}_\tau)\,,\, \widehat{V}(\mathbf{s}_\tau; h_{\tau+1}) \Big\} \tag{15}$$

Because the entrepreneur has always the option to the reservation utility $R(\mathbf{s}_\tau)$, the repudiation value is the maximum between $R(\mathbf{s}_\tau)$ and the value of starting a new business. However, in the analysis that follows we assume that the reservation utility $R(\mathbf{s}_\tau)$ is sufficiently small that managing a firm is always preferable. Therefore, the repudiation value is simply equal to the value of starting a new business.

From equations (14) and (15) it is clear that the derivation of the value functions requires the solution of a non-trivial fixed point problem: given the repudiation functions D and $\widehat{D}$, we can solve for the value functions V and $\widehat{V}$, which in turn allows us to solve for the repudiation values.

Before proceeding we prove a property that simplifies the optimization problem. In particular, we show that constraint (10) is always satisfied if constraint (9) is satisfied. This is formally established in the following lemma:

Lemma 1 *Constraint (10) is always satisfied if constraint (9) is satisfied.*

Proof 1 *See Appendix A.*

Hence, in characterizing the solution to the problem with commitment we can ignore the enforcement constraint (10). This constraint becomes relevant when the investor does not commit, as we will see later.

5.1 Recursive formulation and equilibrium

Because the economy can generate persistent growth, the optimization problem is not stationary. It is then convenient to adopt a normalization of variables that will make the problem stationary.

Define the variables $g_t = H_{t+1}/H_t$, $\tilde{h}_t = h_t/H_t$, $\tilde{k}_t = k_t/H_t$ and $\tilde{d}_t = d_t/H_t$. Because the effort cost and the end-of-period resource functions are homogeneous of degree 1, we can rewrite these two functions as follows:

$$\begin{aligned} \varphi(H_t; h_t, h_{t+1}) &= \varphi(\tilde{h}_t, g_t\tilde{h}_{t+1}) \cdot H_t \\ \pi(h_\tau, k_\tau, h_{\tau+1}, k_{\tau+1}) &= \pi(\tilde{h}_\tau, \tilde{k}_\tau, g_t\tilde{h}_{\tau+1}, g_t\tilde{k}_{\tau+1}) \cdot H_t \end{aligned}$$

We then have the following lemma:

Lemma 2 *The functions $V(H_t; h_t)$, $D(H_t; h_t))$, $\widehat{D}(H_t; h_{t+1})$ are linear in H_t, that is,*

$$\begin{aligned} V(H_t; h_t) &= V(\tilde{h}_t) \cdot H_t \\ D(H_t; h_t) &= D(\tilde{h}_t) \cdot H_t \\ \widehat{D}(H_t; h_{t+1}) &= \widehat{D}(g_t\tilde{h}_{t+1}) \cdot H_t \end{aligned}$$

Proof 2 *See Appendix B.*

Given this lemma, we can use the aggregate stock of knowledge H_t—which grows over time—as a scaling factor for all growing variables. The optimization problem can then be rewritten as:

$$V(g_t\tilde{h}_{t+1}) = \max_{\{\tilde{d}_\tau, \tilde{k}_{\tau+1}, \tilde{h}_{\tau+1}\}_{\tau=t}^{\infty}} \left\{ \sum_{\tau=t}^{\infty} \tilde{\beta}_{t,\tau} \Big[\tilde{d}_\tau - \varphi(\tilde{h}_\tau, g_\tau\tilde{h}_{\tau+1}) \Big] \right\} \qquad (16)$$

subject to

$$\sum_{j=\tau}^{\infty} \tilde{\beta}_{\tau,j} \Big[\tilde{d}_j - \varphi(\tilde{h}_j, g_j\tilde{h}_{j+1}) \Big] \geq D(\tilde{h}_\tau), \qquad \text{for } \tau \geq t \qquad (17)$$

$$-\tilde{d}_t - g_t\tilde{k}_{t+1} + \sum_{\tau=t+1}^{\infty} \tilde{\beta}_{t,\tau} \Big[\pi(\tilde{h}_\tau, \tilde{k}_\tau, g_\tau\tilde{h}_{\tau+1}, g_\tau\tilde{k}_{\tau+1}) - \tilde{d}_\tau \Big] \geq 0 \qquad (18)$$

where $\tilde{\beta}_{t,\tau} = \left(\Pi_{j=t}^{\tau-1}\beta g_t\right)$. In the analysis that follows we concentrate on the balanced growth path equilibrium where the stock of aggregate knowledge grows at the constant rate g and the discount factor becomes $\tilde{\beta}_{t,\tau} = (g\beta)^{\tau-t}$.

Appendix C shows that, after writing the Lagrangian and re-arranging terms, the transformed problem can be reformulated as follows:

$$\min_{\lambda,\tilde{\mu}_{t+1}} \max_{\tilde{d}_t\geq 0,\tilde{h}_{t+1},\tilde{k}_{t+1}} \lambda\Big\{ -\tilde{d}_t - g\tilde{k}_{t+1} + \tilde{\mu}_{t+1}\Big[\tilde{d}_t - \varphi(\tilde{h}_t, g_t\tilde{h}_{t+1})\Big] \\ -(\tilde{\mu}_{t+1} - 1/\lambda)D(\tilde{h}_t) + g\beta W(\tilde{\mu}_{t+1}, \tilde{h}_{t+1}, \tilde{k}_{t+1})\Big\} \tag{19}$$

where λ is the Lagrange multiplier associated with the participation constraint (18) and the problem is subject to the constraint $\tilde{\mu}_{t+1} \geq 1/\lambda$. The function W is defined recursively as:

$$W(\tilde{\mu}, \tilde{h}, \tilde{k}) = \min_{\tilde{\mu}'} \max_{\tilde{d}\geq 0,\tilde{h}',\tilde{k}'} \Big\{\pi(\tilde{h}, \tilde{k}, \tilde{h}', \tilde{k}') - \tilde{d} + \tilde{\mu}'\Big[\tilde{d} - \varphi(\tilde{h}, g\tilde{h}')\Big] \\ -(\tilde{\mu}' - \tilde{\mu})D(\tilde{h}) + g\beta W(\tilde{\mu}', \tilde{h}', \tilde{k}')\Big\} \tag{20}$$

Problem (19) is the problem solved by a new firm started at time t by an entrepreneur with (normalized) knowledge capital $\tilde{h}_t$. After starting the firm and choosing the first period investment, the problem becomes recursive as written in (20). Therefore, (20) is the problem solved by an incumbent firm. The variable $\tilde{\mu}$ can be interpreted as the weight that an hypothetical planner gives to the entrepreneur. The weight given to the investor is 1. Over time the planner increases $\tilde{\mu}$ to make sure that the entrepreneur does not quit the firm, until $\tilde{\mu} = 1$. See Marcet & Marimon (1997) for details about the use of the saddle-point formulation to write the problem recursively.

The solution to the above two problems can be characterized by deriving the first order conditions. For problem (19), the first order conditions are:

$$D(\tilde{h}_t) = \tilde{d}_t - \varphi(\tilde{h}_t, g\tilde{h}_{t+1}) + g\beta D(\tilde{h}_{t+1}) \tag{21}$$

$$\tilde{\mu}_{t+1} \leq 1 \tag{22}$$

$$\tilde{\mu}_{t+1}\varphi_2(\tilde{h}_t, g\tilde{h}_{t+1}) = \beta W_2(\tilde{\mu}_{t+1}, \tilde{h}_{t+1}, \tilde{k}_{t+1}) \tag{23}$$

$$\beta\pi_2(\tilde{h}_{t+1}, \tilde{k}_{t+1}, g_{t+1}\tilde{h}_{t+2}, g_{t+1}\tilde{k}_{t+2}) = 1 \tag{24}$$

where subscripts denote the derivatives with respect to the particular argument in the function. The first order conditions for problem (20) are:

$$D(\tilde{h}) = \tilde{d} - \varphi(\tilde{h}, g\tilde{h}') + g\beta D(\tilde{h}') \tag{25}$$

$$\begin{aligned}
\tilde{\mu}' &\leq 1 & (26)\\
-\pi_3(\tilde{h},\tilde{k},g\tilde{h}',g\tilde{k}') + \tilde{\mu}'\varphi_2(\tilde{h},g\tilde{h}') &= \beta W_2(\tilde{\mu}',\tilde{h}',\tilde{k}') & (27)\\
\beta\pi_2(\tilde{h}',\tilde{k}',g'\tilde{h}'',g'\tilde{k}'') &= 1 & (28)
\end{aligned}$$

In conditions (22) and (26) the inequality constraint is strict if the entrepreneur's payment is zero, that is, $\tilde{d} = 0$. Finally, we have the envelope condition:

$$W_2 = \pi_1(\tilde{h},\tilde{k},g\tilde{h}',g\tilde{k}') - \tilde{\mu}'\varphi_1(\tilde{h},g\tilde{h}') - (\tilde{\mu}' - \tilde{\mu})D_1(\tilde{h})$$

The first order conditions along with the envelope conditions characterize the solution to the firm's problem. Because in equilibrium there is no entry, all firms have been in operation for a long period of time. This implies that $\tilde{\mu} = \tilde{\mu}' = 1$. Furthermore, in the balanced growth path all firms are alike, which implies $\tilde{h} = 1$. Therefore, we can rewrite the first order conditions for an incumbent firm as:

$$\tilde{d} - \varphi(1,g) = (1 - g\beta)D(1) \tag{29}$$

$$\pi_3(1,\tilde{k},g,g\tilde{k}) - \varphi_2(1,g) + \beta\Big[\pi_1(1,\tilde{k},g,g\tilde{k}) - \varphi_1(1,g)\Big] = 0 \tag{30}$$

$$\beta\pi_2(1,\tilde{k},g,g\tilde{k}) = 1 \tag{31}$$

Conditions (30) and (31) can be used to solve for g and $\tilde{k}$. Condition (29) determines the payment to the entrepreneur $\tilde{d}$ given the repudiation value $D(1)$. In order to find $D(1)$, we need to solve for the whole transition in the event of repudiation as characterized by the first order conditions (21)-(28). We state this formally:

Proposition 1 *In the balance growth path, the steady state values of $\tilde{d}$, g and $\tilde{k}$ solve equations (29)-(31), given $D(1)$. The value of $D(1)$ is found by solving for the transitional dynamics of a new firm as characterized by conditions (21)-(28).*

Proof 1 *It follows from the discussion above.*

6 The other economies

After characterizing the equilibrium of the competitive economy with one-side commitment, we can now characterize the equilibrium of the other economic environments.

6.1 Competitive economy without investor's commitment

In this case the investor can renegotiate the payments promised with the long-term contract. The renegotiation of these payments is made credible by the ability of the investor to replace the current entrepreneur by poaching other entrepreneurs (currently running other firms). In particular, the investor will renegotiate payments whose present value exceeds the repudiation value of the entrepreneur.

We have shown above that in the long-term contract with one-side commitment, constraint (10) is never binding. This implies that, after the entrepreneur has chosen the knowledge investment, the investor may have an incentive to renegotiate down the payments up to the point in which the value of staying is just equal to the value of quitting. This implies that constraint (10) must be satisfied with equality. The optimization problem can still be written as in (8), but with the second enforcement constraint satisfied with equality. We then have the following proposition:

Proposition 2 *Without investor's commitment, the knowledge investment $h_{\tau+1}$ chosen by an incumbent firm is equal to the knowledge investment chosen by a newly created firm.*

Proof 2 *See Appendix D.*

This result has a simple intuition. Because the investor can renegotiate the promised payments after the knowledge investment, the entrepreneur would not stay with the firm unless the investor agrees on the same knowledge investment chosen by a new firm. In this way, the entrepreneur keeps his outside value high, avoiding the risk of renegotiation.

The next step is to show that, because of capital obsolescence for incumbent firms, the knowledge investment chosen by a new firm is higher than the value preferred by an incumbent firm and this leads to faster growth.

Because incumbent firms innovate at the same rate as new firms, it becomes important to study the optimality conditions for newly created firms. This can be done by comparing the first order conditions of problem (19) with those of problem (20). In particular, the first order conditions with respect to $\tilde{h}'$ for these two problems can be written as:

$$\varphi_2(1, g\tilde{h}') = \beta W_3(\lambda, 1, \tilde{h}', \tilde{k}') \tag{32}$$

$$-\pi_3(1, \tilde{k}^*, g, g\tilde{k}^*) + \varphi_2(1, g) = \beta W_3(\lambda, \lambda, 1, \tilde{k}^*) \left(\frac{1}{\lambda}\right) \tag{33}$$

where $\tilde{k}^*$ is the steady state capital-knowledge ratio.

These two conditions illustrate the different incentives of new and incumbents firms in the accumulation of knowledge capital, and therefore, in the adoption of new technologies. The terms on the left-hand-side measure the marginal cost of innovation while the terms on the right-hand-side capture the marginal benefit. The term $-\pi_3(1, \tilde{k}^*, g, g\tilde{k}^*)$, which is positive, quantifies the obsolescence in physical capital generated by innovations. This term is only present in the optimality condition of incumbent firms. New firms are not committed to any physical capital and this term is zero. It is then clear that the marginal cost of innovating is greater for incumbent firms. This leads to greater innovation and growth when the investor does not commit to the long-term contract as stated in the next proposition.

Proposition 3 *In a competitive economy, the lack of commitment from the investor leads to faster growth (higher g).*

Proof 3 *See Appendix E.*

6.2 Non-competitive economy with investor's commitment

In this case the entrepreneur cannot start a new business. Therefore, the repudiation value becomes the reservation utility $R(\mathbf{s}_\tau)$.

The optimization problem can be written as in (8). However, the repudiation values $D(\mathbf{s}_\tau, h_\tau)$ and $\widehat{D}(\mathbf{s}_\tau, h_{\tau+1})$ are now equal to the reservation utility $R(\mathbf{s}_\tau)$. The enforcement constraints become:

$$\sum_{j=\tau}^{\infty} \beta^{j-\tau}\Big[d_j - \varphi(\mathbf{s}_j; h_j, h_{j+1})\Big] \geq R(\mathbf{s}_\tau), \qquad \text{for } \tau \geq t+1 \tag{34}$$

$$d_\tau + \sum_{j=\tau+1}^{\infty} \beta^{j-\tau}\Big[d_j - \varphi(\mathbf{s}_j; h_j, h_{j+1})\Big] \geq R(\mathbf{s}_\tau), \quad \text{for } \tau \geq t+1 \tag{35}$$

As in the competitive economy, the second constraint is always satisfied once we impose the first. Therefore, in characterizing the solution we can ignore constraint (35).

The growth rate g and the capital-knowledge ratio $\tilde{k}$ are still determined by the first order conditions (30) and (31). Therefore, we have the same solution as in the competitive economy with commitment. What changes is the entrepreneur's payment $\tilde{d}$. The payment is still determined by condition (31). However, $D(1)$ is now exogenous. Assuming that the reservation utility can be written as $R(\mathbf{s}_\tau) = R \cdot H_\tau$, we have that $D(1) = R$. Because the repudiation value is smaller than in the case of competition, the entrepreneur will receive smaller payments. We summarize this in the following proposition.

Proposition 4 *If the investor commits to the long-term contract, the growth rate of the economy is not affected by competition. Competition only affects the distribution of the rents.*

Proof 1 *See Appendix F.*

6.3 Non-competitive economy without investor's commitment

The optimization problem is as in the economy with investor's commitment but constraint (35) must be satisfied with equality. Substituting this constraint in (34) and re-arranging we get:

$$-\varphi(\mathbf{s}_\tau; h_\tau, h_{\tau+1}) + R(\mathbf{s}_\tau) \geq R(\mathbf{s}_\tau)$$

which is satisfied only if $h_{\tau+1} = h_\tau$. Therefore,

Proposition 5 *Without competition and investor's commitment, there is no investment in knowledge and the economy stagnates (g is zero).*

Proof 4 *It trivially follows from the above condition.*

Also this result has a simple intuition. Without commitment from the investor, the entrepreneur is unable to get rewarded for the effort to accumulate knowledge. With competition, the entrepreneur is still willing to invest in knowledge because of its value outside the firm. But in absence of competition, the entrepreneur's knowledge has a value only inside the firm, which is fully controlled by the investor. Therefore, we reach the conclusion that, in an economy without investor's commitment and without competition, there will be no investment in knowledge capital and the economy stagnates.

6.4 Summary results

The results obtained in this section can be summarized as we did in Section 4 using Table 2. The key finding is that limited enforcement of contracts is not a cause of stagnation as long as there is competition. On the contrary, limited enforcement may even enhance growth if there is competition. At the same time, the lack of competition is not a cause of stagnation if there is commitment from the investor. What is harmful for growth is the lack of both commitment and competition.

7 Conclusion

Modern technologies are highly complementary to skilled labor. This implies that the adoption of these technologies requires the accumulation of innovation skills or knowledge from workers/managers. In absence of a commitment devise or enforcement for the investors, under-accumulation of skills may result. We have shown that limited enforcement alone is not sufficient to impair the accumulation of knowledge capital and the long-term growth. It is the simultaneous lack of enforcement and competition—that is, the ability of workers/managers to use their skills to start new businesses—that is detrimental to growth.

Our paper provides a theoretical foundation for the empirical finding that the cost of starting a business and the cost of contract enforcement are negatively associated to the level of development and growth of a country.

A Proof of Lemma 1

Conditions (9) and (10) can be rewritten as:

$$\begin{aligned}\sum_{j=\tau}^{\infty}\beta^{j-\tau}\Big[d_j-\varphi(\mathbf{s}_j;h_j,h_{j+1})\Big] &\geq D(\mathbf{s}_\tau;k_\tau)\\ \sum_{j=\tau}^{\infty}\beta^{j-\tau}\Big[d_j-\varphi(\mathbf{s}_j;h_j,h_{j+1})\Big] &\geq -\varphi(\mathbf{s}_\tau;h_\tau,h_{\tau+1})+\widehat{D}(\mathbf{s}_\tau;h_{\tau+1})\end{aligned}$$

Therefore, to show that the second constraint is satisfied when the first constraint is satisfied, it is enough to show that $D(\mathbf{s}_\tau;h_\tau)\geq-\varphi(\mathbf{s}_\tau;h_\tau,h_{\tau+1})+\widehat{D}(\mathbf{s}_\tau;h_{\tau+1})$ for any value of h_{t+1}. From the definition of the repudiation values—equations (14) and (15)—we have that $D(\mathbf{s}_\tau;h_\tau)=\max_h\{-\varphi(\mathbf{s}_\tau;h_\tau,h)+\widehat{D}(\mathbf{s}_\tau;h)\}$. This is at least as big as $-\varphi(\mathbf{s}_\tau;h_\tau,h_{\tau+1})+\widehat{D}(\mathbf{s}_\tau;h_{\tau+1})$. *Q.E.D.*

B Proof of Lemma 2

We use a guess and verify procedure. Suppose that $\widehat{D}(H_t;h_{t+1})=\widehat{D}(g_t\tilde{h}_{t+1})\cdot H_t$. Obviously $D(H_t;h_t)$ takes a similar form:

$$\begin{aligned}D(H_t;h_t) &= \max_{g_t}\Big\{\varphi(\tilde{h}_t,g_t\tilde{h}_{t+1})\cdot H_t+\widehat{D}(g_t\tilde{h}_{t+1})\cdot H_t\Big\}\\ &= \max_{g_t}\Big\{\varphi(\tilde{h}_t,g_t\tilde{h}_{t+1})+\widehat{D}(g_t\tilde{h}_{t+1})\Big\}\cdot H_t\\ &= D(\tilde{h}_t)\cdot H_t\end{aligned}$$

We want to show next that $V(H_t;h_t)=V(\tilde{h}_t)\cdot H_t$. This can be easily proved by normalizing all variables by H_t in problem (8). After normalizing, the optimization is over $\{\tilde{d}_\tau,\tilde{k}_{\tau+1},\tilde{h}_{\tau+1}\}_{\tau=t}^{\infty}$. Simple inspection of the normalized problem proves our claim. *Q.E.D.*

C Saddle-point formulation

Consider problem (16). Given γ_τ the Lagrange multiplier associated with the enforcement constraint (17) and λ the Lagrange multiplier associated with the enforcement constraint (18), the Lagrangian can be written as:

$$\begin{aligned}\mathcal{L} &= \sum_{\tau=t}^{\infty}\tilde{\beta}_{t,\tau}\Big[\tilde{d}_\tau-\varphi(\tilde{h}_j,g_j\tilde{h}_{j+1})\Big]\\ &+ \sum_{\tau=t}^{\infty}\tilde{\beta}_{t,\tau}\gamma_\tau\left\{\sum_{j=\tau}^{\infty}\tilde{\beta}_{\tau,j}\Big[\tilde{d}_j-\varphi(\tilde{h}_j,g_j\tilde{h}_{j+1})\Big]-D(\tilde{h}_\tau)\right\}\end{aligned}$$

$$+ \quad \lambda \left\{ -\tilde{d}_t - g_t \tilde{k}_{t+1} + \sum_{\tau=t+1}^{\infty} \tilde{\beta}_{t,\tau} \left[\pi(\tilde{h}_\tau, \tilde{k}_\tau, g_\tau \tilde{h}_{\tau+1}, g_\tau \tilde{k}_{\tau+1}) - \tilde{d}_\tau \right] \right\}$$

Define μ_τ recursively as follows: $\mu_{\tau+1} = \mu_\tau + \gamma_\tau$, with $\mu_t = 0$. Using this variable and rearranging terms, the Lagrangian can be written as:

$$\begin{aligned} \mathcal{L} &= \sum_{\tau=t}^{\infty} \tilde{\beta}_{t,\tau} \left\{ (1+\mu_{\tau+1}) \left[\tilde{d}_\tau - \varphi(\tilde{h}_\tau, g_\tau \tilde{h}_{\tau+1}) \right] - (\mu_{\tau+1} - \mu_\tau) D(\tilde{h}_\tau) \right\} \\ &+ \lambda \left\{ -\tilde{d}_t - g_t \tilde{k}_{t+1} + \sum_{\tau=t+1}^{\infty} \tilde{\beta}_{t,\tau} \left[\pi(\tilde{h}_\tau, \tilde{k}_\tau, g_\tau \tilde{h}_{\tau+1}, g_\tau \tilde{k}_{\tau+1}) - \tilde{d}_\tau \right] \right\} \end{aligned}$$

Define $\tilde{\mu}_\tau = (1+\mu_\tau)/\lambda$ for $\tau \geq t$ with $\tilde{\mu}_t = 1/\lambda$. After substituting we get:

$$\begin{aligned} \mathcal{L} &= -\lambda(\tilde{d}_t + g_t \tilde{k}_{t+1}) + \lambda \tilde{\mu}_{t+1} \left[\tilde{d}_t - \varphi(\tilde{h}_t, g_t \tilde{h}_{t+1}) \right] - (\lambda \tilde{\mu}_{t+1} - 1) D(\tilde{h}_t) \\ &+ \lambda \sum_{\tau=t+1}^{\infty} \beta_{t,\tau} \left\{ \pi(\tilde{h}_\tau, \tilde{k}_\tau, g_\tau \tilde{h}_{\tau+1}, g_\tau \tilde{k}_{\tau+1}) - \tilde{d}_\tau \right. \\ &\qquad \left. + \tilde{\mu}_{\tau+1} \left[\tilde{d}_\tau - \varphi(\tilde{h}_\tau, g_\tau \tilde{h}_{\tau+1}) \right] - (\tilde{\mu}_{\tau+1} - \tilde{\mu}_\tau) D(\tilde{h}_\tau) \right\} \end{aligned}$$

Looking at the special case in which the stock of aggregate knowledge grows at the constant rate g, the problem can be rewritten as:

$$\begin{aligned} \mathcal{L} &= \min_{\lambda, \tilde{\mu}_{t+1}} \max_{\tilde{d}_t \geq 0, \tilde{h}_{t+1}, \tilde{k}_{t+1}} \left\{ \lambda \left[-\tilde{d}_t - g \tilde{k}_{t+1} + \tilde{\mu}_{t+1} \left[\tilde{d}_t - \varphi(\tilde{h}_t, g_t \tilde{h}_{t+1}) \right] \right. \right. \\ &\qquad \left. \left. - (\tilde{\mu}_{t+1} - 1/\lambda) D(\tilde{h}_t) + g\beta W(\tilde{\mu}_{t+1}, \tilde{h}_{t+1}, \tilde{k}_{t+1}) \right] \right\} \end{aligned}$$

which is subject to the constraint $\tilde{\mu}_{t+1} \geq 1/\lambda$ and the function W is defined recursively as follows:

$$\begin{aligned} W(\tilde{\mu}, \tilde{h}, \tilde{k}) &= \min_{\tilde{\mu}' \geq \tilde{\mu}} \max_{\tilde{d} \geq 0, \tilde{h}', \tilde{k}'} \left\{ \pi(\tilde{h}, \tilde{k}, \tilde{h}', \tilde{k}') - \tilde{d} + \tilde{\mu}' \left[\tilde{d} - \varphi(\tilde{h}, g\tilde{h}') \right] \right. \\ &\qquad \left. - (\tilde{\mu}' - \tilde{\mu}) D(\tilde{h}) + \beta W(\tilde{\mu}', \tilde{h}', \tilde{k}') \right\} \end{aligned}$$

D Proof of Proposition 2

After imposing the equality sign in (10), we can substitute this constraint in (9). The resulting expression is:

$$D(\mathbf{s}_\tau; h_\tau) \;\leq\; -\varphi(H_\tau; h_\tau, h_{\tau+1}) + \widehat{D}(\mathbf{s}_\tau; h_{\tau+1})$$

From the previous analysis of the competitive environment with one-side commitment we know that $D(\mathbf{s}_\tau; h_\tau) = V(\mathbf{s}_\tau; h_\tau)$ and $\widehat{D}(\mathbf{s}_\tau; h_{\tau+1}) = \widehat{V}(\mathbf{s}_\tau; h_{\tau+1})$. Therefore, the above condition can also be written as:

$$V(\mathbf{s}_\tau; h_\tau) \;\leq\; -\varphi(H_\tau; h_\tau, h_{\tau+1}) + \widehat{V}(\mathbf{s}_\tau; h_{\tau+1})$$

However, from (13) we also know that

$$V(\mathbf{s}_\tau; h_\tau) \;\geq\; -\varphi(H_\tau; h_\tau, h_{\tau+1}) + \widehat{V}(\mathbf{s}_\tau; h_{\tau+1})$$

Therefore, it must be that this condition is satisfied with the equality sign. This requires that the value of $h_{\tau+1}$ chosen by an incumbent firm is equal to the knowledge capital chosen by a new firm. *Q.E.D.*

E Proof of Proposition 3

Two cases are possible: The case in which $\lambda = 1$ and the case in which $\lambda > 1$. Let's consider first $\lambda = 1$. In this the entrepreneur gets a positive payment at time t and $\tilde{\mu} = \lambda$ from the beginning. Because $\pi_3(1, \tilde{k}^*, g, g\tilde{k}^*)$ is negative, $\tilde{h}' = 1$ cannot be a solution to (32). In particular, because $\varphi_2(1, g\tilde{h}')$ increases with $\tilde{h}'$ and $W_3(1, 1, \tilde{h}', \tilde{k}')$ decreases with $\tilde{h}'$, the solution must be $\tilde{h}' > 1$.

The solution is characterized by the conditions (25)-(28) after imposing the steady state conditions, that is,

$$\begin{aligned}
&\tilde{d} - \varphi(1, g) = (1 - g\beta)D(1) \\
&-\varphi_2(1, g) + \beta\Big[\pi_1(1, \tilde{k}, g, g\tilde{k}) - \varphi_1(1, g)\Big] = 0 \\
&\beta\pi_2(1, \tilde{k}, g, g\tilde{k}) = 1
\end{aligned}$$

Let's consider now the case $\lambda > 1$. This implies that the entrepreneur gets zero initial payments. The solution for the choice of the initial knowledge investment cannot be $\tilde{h}' < 1$. If this was the case, we would have that the entrepreneur gets an initial flow of utility, which is negative, and a smaller continuation utility $D(\tilde{h}')$. By staying with the firm, the entrepreneur enjoys a positive flow of utility in the current period plus a higher continuation utility $D(1)$. Therefore, it must be that $\tilde{h}' > 1$. *Q.E.D.*

F Proof of Proposition 4

The optimization problem can still be written as in (16) after replacing $D(\tilde{h}_\tau)$ with R. We can then derive the recursive formulation as in (19) and (20), where now $D(\tilde{h}_\tau) = R$.

The first order conditions determining the values of g and $\tilde{k}$ in the balanced growth path are still (30) and (31). Therefore, the equilibrium values of these two variables do not change. The entrepreneur's payment $\tilde{d}$ is still determined by condition (29) but with $D(1) = R$. This implies that the entrepreneur's payments are different from the case of competition. *Q.E.D.*

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