Leverage, Incomplete Markets and Pareto Improving Regulation

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Abstract

In this paper, we give an example of an economy with production, collateral and default where regulating the margin requirement can lead to a Pareto-improvement.

Following Geanakoplos and Zame (2002), we assume that collateral is scarce and that therefore most contracts are not traded in equilibrium and margin requirements arise endogenously. Abstracting from price effects, these endogenous margins are constrained optimal. However, in our model, there is a mismatch between short term borrowing and long term investments in the presence of default and liquidation costs. A firm does not take into account the externality that by leveraging its own debt more it raises the probability that other firms in its industry will default. Reducing equilibrium default by exogenously regulating margin requirements might therefore be Pareto-improving.

We identify circumstances under which tougher requirements make everybody better off as well as situations in which weakening the requirements are improving.

VERY PRELIMINARY AND INCOMPLETE

1 Introduction

The vast majority of debt, especially if it extends over a long period of time, is guaranteed by tangible assets called collateral. For example, residential homes serve as collateral for short- and long-term loans to households, equipment and plants are often used as collateral for corporate bonds, and investors can borrow money to establish a position in stocks, using these as collateral. The margin requirement dictates how much collateral one has to hold in order to borrow one dollar. Market forces will generally play an important role in establishing these collateral requirements. In this paper we ask if these market forces, i.e. competition among financial intermediaries who set the margin requirements or among borrowers and lenders, lead to excessively low standards and an inefficient allocation of risk ? Can these low standards lead to liquidity crises which might threaten the stability of the whole financial system and can a regulation of the margin requirements avoid the crises and make everybody in the economy better off? We give a formal argument for regulation. We present an example where a regulation of margin requirements leads to a Pareto improvement. Without regulation, a financial crisis results in a dramatic loss of output and welfare, while regulation leads to more careful investment.

When markets are incomplete there will almost always be a mismatch between firm output across states of nature and asset promises. Default mechanisms are crucial institutions in allowing trade to go forward. Defaulters are forced to pay for as much of their debt as they can out of the money and goods they have on hand, and they are not punished. In this paper, we deal exclusively with collateral requirements, assuming that that there is no penalty, legal or reputational, to defaulting.

When default mechanisms are very sophisticated in that there is no social loss through defaults, they can go a long way toward compensating for the missing markets (see e.g. Zame (1993) or Dubey et al (2005) for examples). However, we suppose here that the default mechanism is less sophisticated. In our model, there is a mismatch between short term borrowing and long term investments. Production takes three periods and firms need to roll over their debt in the middle period. If the owner of the firm finds himself in a position where he cannot cover the additional margin requirement out of his own funds he must default in the middle period. No provision is made for goods in process that might be worth more later if the firm were not required immediately to sell all its assets but instead were permitted to continue to produce even after defaulting. We thus assume that default incurs liquidation costs and might be socially suboptimal.

How much default occurs in equilibrium will be governed by the margin requirements on the available loans. We build on Geanakoplos and Zame (2002) and Geanakoplos (2003) to model endogenous margin requirements, i.e. endogenous quantity constraints on the sale of promises. Scarce collateral rations the volume of trade since there will always be a gap between utility of buying and disutility of selling an asset. The rationing does not reduce volume of trade proportionally but chokes off all trade in most contracts. The investment/borrowing mismatch creates the potential for social losses deriving from the forced liquidation of goods in process. We show that when collateral levels are endogenous and debt is denominated in dollars, this mismatch leads to inefficiently high leverage (low collateral) and inefficiently many defaults.

If a firm is a debtor and finds that it must default in some state of the world because its productivity is unusually low, these liquidation costs will magnify the productivity shock. A national crisis, however, involves the simultaneous default of many firms. And this simultaneity gives rational firms and lenders a big incentive to avoid them, thus tending to lower their probability. If a firm anticipates that in some state of the world many of its competitors will default and go out of business, then it will anticipate that its output will sell for a much higher price in that state. The firm would thus try hard to adjust its production plan to remain in business in that state, and national crises will be curtailed. However, if by defaulting a firm makes it more difficult for other firms to roll over their debt (for example by lowering the market price of the other firms output in that period), a typical firm may thus find, by contrast to our last thought, that remaining in business in the state is more expensive, not more profitable. Thus the firm will not try to avoid the kinds of loans that lead to default in that state. In short, a borrowing firm does not take into account the externality that leveraging its debt makes it more likely that other firms in the same industry will default, provided that their output in that period is worth less. Since lenders and borrowers will then rationally anticipate higher defaults even for high collateral loans, they will be led to agree to loans with lower collateral (higher leverage).

The potential for social losses caused by an investment/borrowing mismatch also plays a crucial role in the large literature on bank-runs (see Diamond and Dybvig (1983) for the original model). The crucial difference between this literature and our model is that there the crises occurs through a lack of coordination and there typically exists another equilibrium where a crises can be avoided without any government intervention. In our model, the crises cannot be avoided simply by changing agents' expectations. Everybody who lends to the 'bad firm' knows that in the middle period the firm will (sub-optimally) liquidate and the receipts will be distributed among the lenders.

Aiyagari and Gertler (1999), Kiyotaki and Moore (1997) and Kocherlakota (2000) model the idea that borrowing on collateral might give rise to cyclical fluctuations in real activity. But they do not allow for endogenous collateral levels. These papers make no connection between endogenous collateral levels and promises. In our examples, a crises can be avoided with the 'right' margin requirements. Geanakoplos (1997), Geanakoplos and Zame (2002) and Dubey, Geanakoplos and Shubik (2005) were the first to model endogenous collateral levels in general equilibrium. The contribution of this paper is to show that the endogenously determined collateral levels can be precisely the ones that lead to a financial crisis. There is a substantial literature on the regulation of future exchanges. Although it is generally accepted that the market-mechanism results (at least in theory) in efficient allocations, it is often argued that in order to attract market volume, competing future exchanges settle for excessively low contractual guarantees. This "race to the bottom "argument is then used to advocate a regulation of minimal margin requirements for these exchanges. Academic economists often claim that competition among exchanges is necessary and, if quality is observable, also sufficient for an optimal amount of contractual guarantees. (see e.g. Santos and Scheinkman (2002)).

The paper is organized as follows. In Section 2 we give an example of a two period model without liquidation costs. We show under which conditions a unique collateral requirement arises in this model and argue that absent of price effects, equilibria are constrained efficient. In Section 3 we introduce a three period model with liquidation costs in the middle period. Section 4 provides examples of Pareto-improving margin regulations.

2 A two period model with endogenous leverage

To demonstrate the main ingredients of the three period model which we will present in the next section, such as the idea of endogenous collateral, we first consider a two period model with S states of the world in the second period. There are L perishable commodities in each period. We assume that there are two agents, $h = A, B$, a producer, agent B who needs to borrow to finance production and a lender who does not have access to the production technology directly. The agents have individual endowments $e^h(s) \in \mathbb{R}^L_+$ at all states s and von Neumann-Morgenstern utility with identical beliefs

$$U^h(c) = v_0^h(x_1(0)) + E\beta v_t^h(\tilde{x}).$$

Agent B has access to a linear technology, transforming goods $y_0 \in \mathbb{R}^L_+$, at $t = 0$, into goods at $t = 1$. The technology yields $y_s \in \mathbb{R}^L_+$ in each state s. Agent B can set up several firms to produce. We assume that each firm $f \in \mathcal{F}$ can borrow on collateralized loan-contracts, and has the option to default with limited liability to its owner. A firm that defaults must immediately sell off all the produced goods at $t = 1$ and turn the receipts over to its creditors. The simplest notation for depicting this is to suppose that for each loan type each borrower must set up a separate entity (i.e. firm) f that borrows exclusively in this loan. Thus without loss of generality we can use the index f to depict loan types as well as firms[1]. An owner who wishes to take out two kinds of loans simply operates two different firms. A loan f is characterized by its collateral κ_f and and promises to pay one unit of gold in the next period. The collateral κ_f represents additional money the firm owner must invest, along with the borrowed funds, into the operation of the firm. To simplify the model, we assume that there are finitely many possible collateral levels $\kappa_1, ..., \kappa_F$ available to finance a firm. However, F should be interpreted as an arbitrarily large finite number so that essentially any possible collateral level could be chosen. Investors feel more secure the higher is the collateral for two reasons. First, the tangible assets of the firms are higher

[1] In our example, we assume constant-returns-to-scale production, so there is no interpretive difficulty in imagining one owner running many different firms.

which brings a higher liquidation value if the creditors must put the firm into default. Secondly the higher the collateral, the more capital the owner has at stake in the firm, and therefore the greater his incentives to keep the firm solvent, in particular by paying off all its debt. In order to promise one unit of the numéraire commodity for the next period at $t = 0$ firm f has to invest $\kappa_f \in \mathbb{R}$ units in its technology. At each subsequent node the firm either has to deliver on every loan or it has to sell all its assets and distribute the revenue among the lenders pro rata. In the last period, promises cannot be enforced, except through collateral. The payoff of asset j in the second period state s is then $d_s^j = \min(1, \kappa_j y_s \cdot p(s))$.

Since the firms are all controlled by the same agent, we do not need to model each firms decision separately but can combine them into a single period 0 budget constraint for agent B. There are F potential debt contracts, differing only in their collateral requirements κ_f. The producer can sell these contracts short, under the collateral constraint.

$$i - \sum_{f=1}^{F} \kappa_f \theta^f \geq 0,$$

where i denotes the total level (over all firms) at which the technology is operated.

We denoting lending by ϕ, and the price of bond f by q_f. The budget constraints are then

$$\begin{aligned}
p_0 \cdot c_0 &= p_0 \cdot e_0^h + \sum_{f=1}^{F} q_f \theta_f - \sum_f q_f \phi_f - i p_0 \cdot y_0 \\
i - \sum_f \kappa_f \theta_f &\geq 0 \\
p_s \cdot c_s &= p_s \cdot e_s^h + \sum_{f=1}^{F} d_s^f \phi_f - \sum_f \nu_s^f + i p_s \cdot y_s, \quad s = 1, ..., S \\
\nu_s^f &\geq \theta_f \min(1, \kappa_f p_s \cdot y_s), \quad s = 1, ..., S
\end{aligned}$$

Agents rationally anticipate delivery rates for assets traded

$$d_s^j = \frac{\nu_s^j}{\theta_j^B}.$$

In this two period model, if asset j is not traded, its delivery is still uniquely determined by $\min(1, \kappa_j p_s \cdot y_s)$. The situation will be a bit more complicated in the next section where the decision to default on an asset A GEI equilibrium with endogenous collateral can therefore be defined as usual by agents' optimality and market clearing.

We say that a collateral requirement κ is chosen endogenously in equilibrium if there is a firm f financed by asset f with collateral κ_f in equilibrium, i.e. if $\theta_f > 0$.

The margin requirement is then defined as

$$\text{margin} = \frac{\kappa p_0 \cdot y_0 - q_f}{\kappa p_0 \cdot y_0}$$

We also sometimes refer to 'leverage' which is defined as the inverse of the margin requirement, i.e.

$$\text{leverage} = \frac{\kappa \cdot \text{price of input}}{\kappa \cdot \text{price of input} - \text{price of bond}}.$$

As an example, suppose that there is one good per state, there are 2 states and $y_1 = 2$, $y_2 = 1$. It is easy to see that possible equilibrium collateral levels are $\kappa \in [1/2, 1]$. If collateral constraints are binding, no firm will ever choose a collateral higher than 1, such a contract delivers the same as a contract with a collateral requirement of 1, but the reservation price of the borrower is higher since he has to put up more collateral. All collateral levels lower than 1/2 are equivalent in that in equilibrium they yield same margin as a level of 1/2. For $\kappa \in [1/2, 1]$, reservation prices are

$$\begin{aligned} q^A(\kappa) &= \frac{1}{v_A'(c_0^A)}\left(\pi_1 v_A'(c_1^A) + \pi_2 \kappa v_A'(c_2^A)\right) \\ q^B(\kappa) &= \frac{1}{v_B'(c_0^B)}\left(\pi_1 v_B'(c_1^B) + \pi_2 \kappa v_B'(c_2^B) + \kappa\lambda\right), \end{aligned}$$

where λ denotes the Lagrange-multiplier associated with the collateral constraint. If collateral constraint are binding and if

$$\pi_2 \frac{v_A'(c_2^A)}{v_A'(c_0^A)} \neq \frac{\pi_2 v_B'(c_2^B) + \lambda}{v_B'(c_0^B)},$$

there is no equilibrium with $\kappa \in (1/2, 1)$. If the reservation price of borrowers and lenders for some bond with $1/2 < \kappa < 1$ would coincide in equilibrium one would have $q^A(\kappa) = q^B(\kappa)$. But then one would either obtain $q^A(1) > q^B(1)$ or $q^A(1/2) > q^B(1/2)$. Note that if

$$\frac{v_A'(e_2^A)}{v_A'(e_0^A)} \leq \frac{v_B'(e_2^B)}{v_B'(e_0^B)}$$

unique margin requirement in equilibrium is $\kappa = 1/2$, i.e. there is only one firm in equilibrium and this firm defaults in state 2 and is indifferent between delivering and defaulting in state 1.

Note that in this example, there are robust specifications of endowments and preferences for which in equilibrium

$$\pi_2 \frac{v_A'(c_2^A)}{v_A'(c_0^A)} = \frac{\pi_2 v_B'(c_2^B) + \lambda}{v_B'(c_0^B)}.$$

In this case, all margin requirements are possible. There is one equilibrium where exactly two bonds are traded with margin requirements 1/2 and 1 respectively. Changing the margin requirements then has no real effects. This will no longer be true in the three-period model below where there are liquidation costs of default.

2.1 Constrained optimality

In the absence of price effects, equilibrium is constrained efficient (see Geanakoplos and Zame (2002)). So in particular, in the two period model, if there is only one good, equilibrium is always constrained efficient. In the above example, it can be easily seen that if the uniquely traded collateral requirement is $\kappa = 1/2$, the lender is relatively richer in state 2 than in state 1. Therefore, the borrower defaulting in state 1 (but not in state 2) provides (constrained) optimal risk-sharing.

If there are several goods, this is no longer true because the prices of the commodities will generally change if one regulates margins. However, in general it is unlikely that just by the regulation of margins one can obtain Pareto-improvements if there are no liquidation costs. In particular, there is no intrinsic suboptimality built-in to the fact that margin requirements are endogenous.

3 A model with production and liquidation costs

We now consider a three period model of a production economy. To simplify the notation, we assume right away that there are two possible shocks in each period $t = 2, 3$. The nodes of the resulting event tree are identified by histories of shocks. We write (b) and (g) for the two nodes in the middle period and $(bb), (bg), (gb)$ and (gg) for the last period nodes.

There are two (perishable) commodities available for consumption at each node of the tree. 'Wine' can be used as the input to production but also for consumption, while 'gold', the numéraire commodity can only be consumed.

There is a stochastic linear technology which produces wine at $t = 3$ from wine in $t = 1$. The technology can also be used to produce wine at $t = 2$ but is highly unproductive for this. One unit of wine from $t = 1$ produces b unit of wine in the bad productivity states (gb) and (bb) at $t = 3$ and $g > b$ units in the good productivity state (gg) and (bg) . When used to produce wine at $t = 2$, it only produces γ units. The decision whether to produce wine at $t = 2$ or at $t = 3$ can be taken at $t = 2$ but the specifications of endowments and preferences will ensure that producing in the middle period is never optimal. This corresponds to liquidation costs. The event tree is shown in Figure 1.

Agents have endowments $e^h(\sigma) \in \mathbb{R}^2_+$ at all nodes $\sigma \in \{(0), (b), (g), (bb), (bg), (gb), (gg)\}$ and von Neumann-Morgenstern utility with identical beliefs

$$U^h(c) = v_0^h(x_1(0)) + E \sum_{t=2}^{3} v_t^h(x_{t1}, x_{t2})$$

As before, at $t = 0$ there are F contracts which differ only in their margin requirement. Each contract is a short term bond which promises one unit of gold in the middle period. In the middle period, agent B can decide which firms default, sell off their goods in process (i.e. produce in the middle period) and then shut down and which firms continue operating. If a firm continues operating, its debt has to be paid back in full and the new debt has to be backed again by collateral. In the middle period, goods in process serve as collateral. In order to promise one unit in the last period on loan $f(b)$, a firm must have κ_{fb} units of

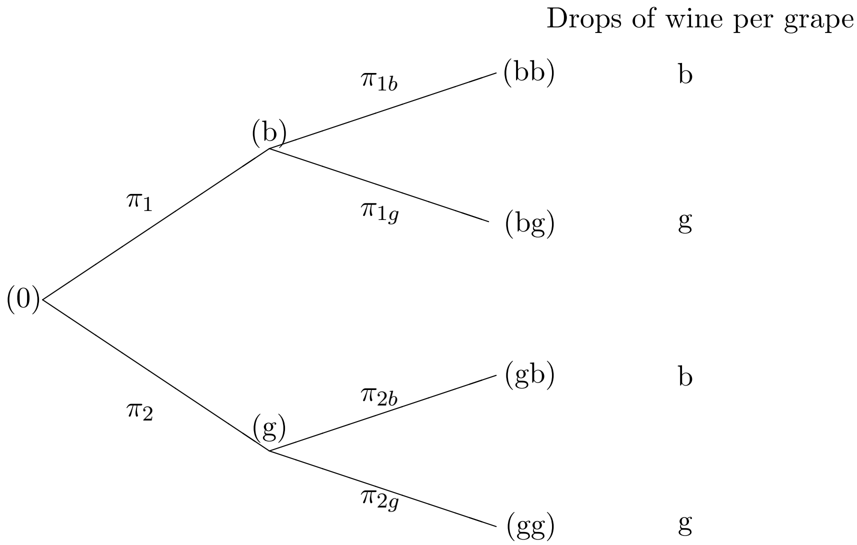

Figure 1. Event tree.

goods still invested. In order to roll over loans, agent B can also use his personal wealth in the middle period.

The budget constraints of the borrower in the first period are

$$p_0 \cdot c_0 = p_0 \cdot e_0^B + \sum_{f=1}^{F} q_f(0)\theta_f(0) - \sum_{f \in \mathcal{F}} i_f p_0 \cdot y_0, \quad i_f - \kappa_f \theta_f \geq 0 \text{ for all } f$$

In the middle period, at node $\sigma = (b), (g)$ the borrower faces the following constraints.

$$\begin{aligned} p(\sigma) \cdot c(\sigma) &= p(\sigma) \cdot e^B(\sigma) - \sum_f \nu^f(\sigma) + \sum_f \theta_f(\sigma) q_f(\sigma) \\ \nu^f(\sigma) &\geq \theta_f(0) \min(1, \kappa_f p(\sigma) \cdot y(\sigma)) \\ \rho_f(\sigma) &= i_f - (1 - \nu^f(\sigma))\kappa_f \text{ for all } f \\ \rho_f(\sigma) - \theta_f(\sigma) &\geq 0 \text{ for all } f \end{aligned}$$

Finally, in the last period,

$$p(\sigma) \cdot c(\sigma) = p(\sigma) \cdot e^B(\sigma) - \sum_f \nu^f(\sigma) + \sum_f \rho_f p(\sigma) \cdot y(\sigma), \quad \sigma = (bb), (bg), (gg), (gb)$$

with

$$\nu^f(\sigma) = \theta_f(\sigma_-) \min(1, \kappa_f p(\sigma) \cdot y(\sigma))$$

3.1 Endogenous margin at $t = 0$

The crucial difference to 2-period case is that now borrower stops defaulting before the market value of goods in process equals to value of promises.

First order conditions are

$$\eta_0^A q(\kappa) = \eta_1^A \pi_1 + \eta_2^A \pi_2 d_s^j(\kappa)$$

$$\eta_0^B q(\kappa) = \eta_1^B \pi_1 + \eta_2^B \pi_2 d_s^j(\kappa) + \lambda\kappa$$

There is a cutoff value κ^* at which borrower is indifferent between defaulting and not defaulting in state b. For all $\kappa < \kappa^*$ full default will occur in that state and $d_s^j = p\kappa$. Inefficiency of default implies that $p\kappa^* < 1$. For $\kappa \geq \kappa^*$ there is no default.

Similarly there is a cutoff $\underline{\kappa}$ at which borrower is on the verge of defaulting in state g.

If

$$\frac{\pi_2\eta_2^A}{\eta_0^A} > \frac{\pi_2\eta_2^B + \lambda}{\eta_0^B},$$

no asset with $\kappa \in (\underline{\kappa}, \kappa^*)$ will be traded

If

$$\frac{\pi_2\eta_2^A}{\eta_0^A} \leq \frac{\pi_2\eta_2^B + \lambda}{\eta_0^B},$$

only asset with collateral κ^* is traded Note that for first case, there might be two assets being traded with collateral levels κ^* and $\underline{\kappa}$ On-the-verge-refinement implies that agents who are indifferent do not default!

4 A simple example

Suppose there is no uncertainty after the middle period, i.e. (bb) follows (b) with probability 1 and (gg) follows (g) with probability 1.

Both agents have identical Cobb-Douglas (log) utilities.

$$v^h(x_1, x_2) = \log(x_1) + \log(x_2), \quad h = A, B$$

Individual endowments and (unconditional) probabilities are as follow

State	Lender A's end.	Borrower B's end.	probability
0	(1,2)	(0,0.25)	1
b	(1,2)	(0.5,0)	0.5
g	(1,2)	(0.5,0)	0.5
bb	(1,1)	(0.25,1.75)	0.5
gg	(1,1)	(1.75,0.25)	0.5

The technology is given by

$$g = 2, \quad b = 2, \quad \gamma = 1.$$

This example is simply meant to illustrate that endogenous margin requirement might be suboptimal, obviously it is not 'realistically calibrated' in any way. In particular, it will turn out to be crucial for the improvement that the borrower's endowments are 'twisted' in the last period. The output of the technology is the same in both state, but at (bb) aggregate endowments in good 2 are much higher, so the output of technology is much less valuable.

We consider two cases: In case 1, collateral levels are endogenous. In the first period 2 bonds (called asset 1 and asset 2 in the table) are traded, in the middle period there is a unique bond (asset (b) in state (b), asset (g) in state (g)). In Case 2, margin in first period is regulated to a level that ensures that there is no default in the middle period. Only one bond is traded in the first period (asset 2). In the middle periods it is endogenous, but regulating it would not change anything since there is no uncertainty after this point. As explained above, in the unregulated equilibrium, the low margin requirement will ensure that the borrower is indifferent between delivering and defaulting in the good state. In this particular example, this implies that the margin requirement is actually zero. This will play an important role in the interpretation later on.

The following table shows some of the crucial differences between the regulated and the unregulated equilibrium.

	Case 1	Case 2
Asset price (01)	1.11	NA
Asset price (02)	1.72	1.77
Asset price b	1.69	1.64
Asset price g	0.74	0.71
Total investment at (0)	0.187	0.136
Investment left at (b)	0.086	0.136
Margin for asset (01)	0	NA
Margin for asset (02)	0.0864	0.0842
Promises in asset (01)	0.34	NA
Default in asset (01)	total	NA
Promises in asset (02)	0.074	0.184
Default in asset (02)	0	0
Debt in gold at (b)	0.074	0.184
Debt at (g) in gold	0.466	0.347
Utils agent A	0.0798	0.0811
Utils agent B	-4.033	-4.012

In this simple example, since there is no uncertainty after the middle period, the producer can borrow fully up to the value of the output in the last period. In (b), this is in fact what happens. The good firm manages to roll over its entire debt without the owner having to put in any of his own capital. The bad firm defaults in full. The crucial reason why margin regulation is Pareto improving in this example is that the bad firm competing for inputs at period zero drives up the price and makes it more difficult for the good firm to be profitable.

The supporting (Arrow-Debreu) prices of the commodities are as follows

State	gold unreg.	wine unreg.	gold reg.	wine reg.
0	1	2.77	1	2.60
g	0.87	0.65	0.91	0.69
b	0.85	0.60	0.86	0.64
gg	0.64	1.08	0.65	1.17
bb	1.44	0.63	1.40	0.58

The main externality of default is obvious now. The price of the input at state 0 decreases making it easier for agent B to set up firms which do not default.

The allocations are

State	Agent A Case 1	Agent B Case 1	Agent A Case 2	Agent B Case 2
0	(2.13,0.77)	(0.12,0.04)	(2.14,0.82)	(0.11,0.04)
b	(1.25,1.78)	(0.25,0.36)	(1.25,1.67)	(0.25,0.33)
g	(1.22,1.63)	(0.28,0.37)	(1.17, 1.56)	(0.33,0.44)
bb	(0.74,1.69)	(0.51,1.16)	(0.76, 1.84)	(0.49,1.18)
gg	(1.66,0.98)	(1.09 ,0.64)	(1.65, 0.91)	(1.10,0.61)

The main effect of the regulation of the margin requirement is that aggregate consumption increases in state (bb), this helps both agents. However, it is crucial most of the increase goes to the lender since he will lose at (g). For this, several endowments and technology have to satisfy several crucial conditions. The following example illustrates that while it is easy to avoid crises with a regulation of margin requirements one does not always get Pareto-improvements.

The allocations show what happens. Without regulation, the price of the input at $t = 0$ is determined by the lender's marginal utilities. Both with and without regulation, the good firms make positive profits under the lender's marginal utilities. The point is, however, that the borrower's marginal utilities determine production. In the unregulated equilibrium only very few firms produce, since the margin requirement is so high that the borrower is constraint to produce on a small level. Although the price of the input falls through regulation, both borrower and lender agree that the production of good firms better than the bad production.

4.1 A crises that is not Pareto-inferior

Suppose now that in the above example the probability of a bad state is 0.4 instead of 0.6. This will imply that there is much more investment and much more highly leveraged borrowing in the first period. At (b), there is now a real crises. However, a margin regulation hurts the lender, because of the price effect on the interest rate in the first period. The extra consumption he gets at (bb) does not compensate him for the loss of consumption at (g) and (gg)

The equilibria without and with regulation are now as follows.

	Case 1	Case 2
Asset price (01)	1.21	NA
Asset price (02)	1.71	1.89
Total investment at (0)	0.40	0.32
Investment at (b)	0.15	0.32
Margin for asset (01)	0	NA
Margin for asset (02)	0.12	0.10
Promises in asset (01)	0.68	NA
Default in asset (01)	total	NA
Promises in asset (02)	0.28	0.47
Default in asset (02)	0	0
Debt in gold at (b)	0.28	0.47
Debt at (g) in gold	0.64	0.57
Utils agent A	9.47	9.24
Utils agent B	-19.77	-19.13

4.2 Sensitivity analysis

We return to the example where a Pareto-improvement was possible and ask what role the endowments play. It turns out that increasing endowments for the borrower are crucial to obtain default in the middle period at all. The twisted endowments are crucial to obtain an improvement for the lender. It is fairly easy to find specifications where the borrower gains from a margin regulation. However, it is difficult to identify circumstances under which both borrower and lender gain.

4.3 Improving regulation in the middle period

We now assume that there is uncertainty after node (b) - only the conditional probability of a bad shock is higher at (b) than at (g) (where it is 0). Utility functions are as above.

The endowments and (unconditional) probabilities are as follow

State	Lender A's end.	Borrower B's end.	probability
0	(1.9,1)	(0.1,0)	1
b	(1,1.5)	(0,2)	0.7
g	(2.5,2.5)	(0.2,0.2)	0.3
bb	(5,5)	(10,10)	0.2
bg	(5,5)	(0.1,0.1)	0.5
gg	(1,0.1)	(10,0.1)	0.3

The technology is

$$g = 5, \quad b = 0.5, \quad \gamma = 0.75$$

We consider first consider two cases: In case 1, collateral levels are endogenous. In the first period 2 bonds (called asset 1 and asset 2 in the table) are traded, in the middle period there is a unique bond (asset 3 in state (b), not reported for state (g)). In Case 2, collateral is chosen to maximize equilibrium utilities in all periods ! Only one bond is traded in the first period (asset 2). γ denotes the amount of wine produced by one unit in the middle period.

	Case 1	Case 2
Asset price (01)	0.269	NA
Asset price (02)	0.92	0.97
Asset price b	0.43	0.34
Asset price g	0.67	0.55
Total investment at (0)	1	1
Investment at (b)	0.37	1
κ for asset (01)	0.21	NA
'Leverage for asset (01)'	944.8	NA
κ for asset (02)	0.756	0.832
'Leverage for asset (02)'	20.1	42.7
κ for asset b	2.01	0.517
κ for asset g	0.138	0.138
Promises of asset (01)	3.00	NA
Debt in asset (01) (gold,wine)	(0.81,0.63)	NA
Default in asset (01)	total	NA
Promises in asset (02)	0.49	1.20
Debt in asset (02) (gold,wine)	(0.45,0.35)	(1.17,0.98)
Default in asset (02)	0	0
Promises in asset b	0.18	1.93
Debt in asset b in gold	0.08	0.66
Promises in asset g	7.27	7.27
Debt in asset g in gold	4.85	4.01
Utils agent A	34.76	34.79
Utils agent B	6.90	8.19
Subtree-utils agent A	22.68	23.46
Subtree-utils agent B	12.79	13.16

These are the allocations

State	Agent A Case 1	Agent B Case 1	Agent A Case 2	Agent B Case 2
0	1.924	0.076	1.928	0.072
b	(0.86,2.82)	(0.14,1.16)	(0.84,2.43)	(0.16,1.07)
g	(2.21,1.84)	(0.49,0.86)	(1.52, 1.43)	(1.18,1.27)
bb	(5.06,5.12)	(9.94,10.06)	(5.16, 5.33)	(9.84,10.17)
bg	(4.37,5.95)	(0.73,0.99)	(4.36, 8.62)	(0.75, 1.47)
gg	(4.99,2.36)	(6.01,2.84)	(4.99, 2.36)	(6.01,2.84)

Prices of wine (gold is the numeraire)

State	Case 1	Case 2
0	1.28	1.20
b	0.65	0.75
g	1.61	1.72
bb	0.99	0.98
bg	0.86	0.71
gg	1.45	1.45

4.4 Improving regulations at (b)

Now we take the same example as above but assume that margin requirements are only regulated at (b), i.e. at (0) there are (endogenously) 2 assets traded, while at (b) agents are limited to trade in one asset, whose collateral requirement is set to a level much lower than what the market would pick.

Note that the lender is better off than in **all** other cases !

	Case 3
Asset price (01)	0.28
Asset price (02)	0.93
Asset price (b)	0.35
Asset price g	0.65
Total investment at (0)	1
Investment at (b)	0.47
κ for asset (01)	0.216
'Leverage for asset (01)'	1118.0
κ for asset (02))	0.75
'Leverage for asset (02)'	23.9
κ for asset (b)	0.7
κ for asset g	0.138
Promises in asset (01)	2.45
Default in asset (01)	total
Debt in asset (01) in (gold,wine)	(0.69,0.53)
Promises in asset (02)	0.63
Debt in asset (02) in (gold,wine)	(0.58,0.45)
Default in asset (02)	none
Promises in asset (b)	0.63
Debt in asset (b) in gold	0.22
Delivery of asset (b) (across states)	(0.35,1)
Promises in asset g	7.27
Debt in asset g in gold	4.71
Utils agent A	34.81
Utils agent B	7.05
Subtree-utils agent A	22.86
Subtree-utils agent B	12.80

Allocations and Prices

State	Agent A	Agent B	Price of wine
0	1.926	0.074	1.30
b	(0.85,2.73)	(0.15,1.17)	0.67
g	(2.10,1.76)	(0.60,0.94)	1.65
bb	(5.07,5.15)	(9.93,10.09)	0.99
bg	(4.42,6.44)	(0.68,0.99)	0.83
gg	(4.99,2.36)	(6.01,2.84)	1.45

This example shows that it might well be Pareto-improving to ease the margin.

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