Fund managers and defaultable debt*

Very preliminary, please do not circulate

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Abstract

We propose a simple, equilibrium model where investors hire fund managers to invest their capital either in a risky bond or in a riskless asset. The risky bonds are issued by a large number of borrowers who run risky investment and can decide to default ex-post. There is only a small fraction of talanted fund managers who have information on the fundamentals of the risky project. This generates career concerns that distort the investment decision of uninformed fund managers. When the probability of default is sufficiently high, they prefer to invest in safe bonds even at a lower expected return to reduce the probability of being fired. This is what we define "reputational premium". As the economic and financial conditions change, the reputational premium can switch sign. This generates an overreaction of the market leading to excess volatility of spreads, capital flows and economic activity.

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1 Introduction

In the last few years, before the subprime turmoil in August 2007, market observers seemed to be concerned about a growing "overenthusiasm" for risky investments, including mortgage-backed assets, emerging market bonds and high-yield corporate bonds. It was particularly visible on emerging-market spreads. As one observer puts it as early as 2005:

> "Bonds issued by Ecuador, which is politically very unstable, are among the riskiest bets in the emerging markets. It is hard to predict what will happen there next month, let alone in 10 years time. Yet buyers appear to be ready and willing to line up for a sale by the government of up to Dollars 750m in 10-years bonds, the first international bond offer since the country defaulted in 1999. The issue, [...] is the latest example that the prolonged love affair with emerging market debt is far from over." (December 9, 2005, Financial Times).

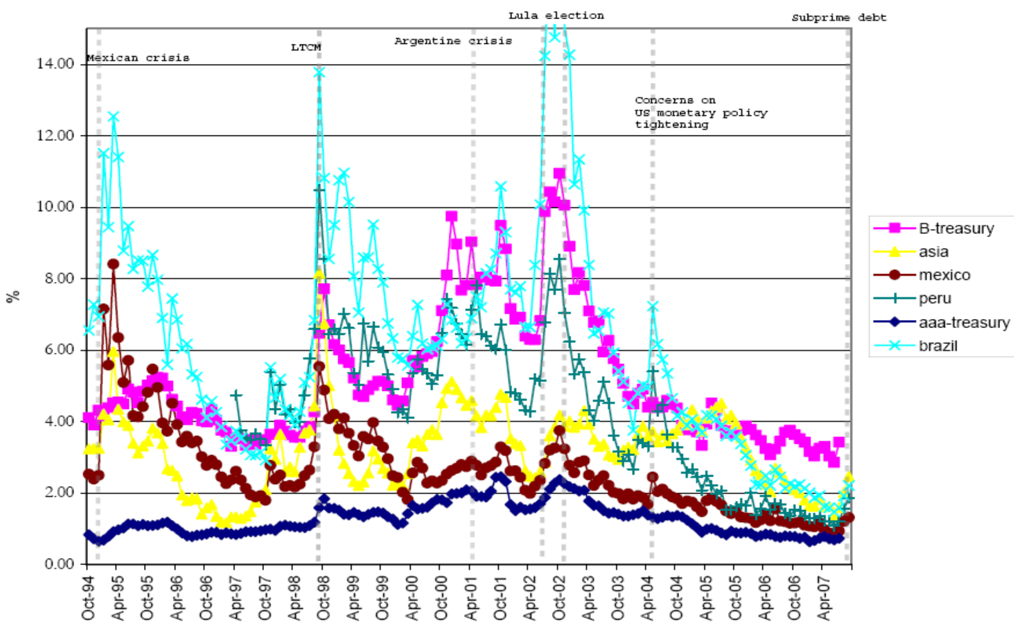

Figure 1: The JPMorgan EMBI+ spread for Asia, Brazil, Mexico, Peru, the yield spread of AAA corporate bonds and B-graded corporate bonds between 1994 and August 2007. Source: Datastream, St. Louis Fed.

Figure 1 shows that both emerging market spreads and high-yield corporate bond spreads had a pick in 2002 after which they started to decline and kept on declining even further after 2005. By April 2007, the spreads of all emerging countries represented in Figure 1 were close to the level of the investment grade corporate spread.[1] Many argue that in 1996-97, there was a similar overenthusiasm for East-Asian and Russia bonds, right before the emergence of crises in these areas (e.g. Kamin and von Kleist, 1999, IMF, 1999b, Duffie et al., 2003). These episodes are in sharp contrast to crises episodes, when virtually all high-risk bond spreads jump up and capital tends to flow out from these markets; a phenomenon frequently dubbed as flight-to-liquidity or flight-to-quality.

We propose a stylized dynamic general equilibrium model where investors rationally allocate their capital to fund managers, who can invest in risk-less bonds or finance defaultable risky projects. There is only a small fraction of talanted fund managers who have information on the fundamentals of the risky project. We argue that fund managers' career concerns lead to overinvestment in good times and underinvestment in bad times, generating excess volatility of prices, capital flows and economic activity.

Our economy is populated by three types of agents: investors, fund managers, and borrowers. Investors delegate their portfolio decision to risk-neutral fund managers. Fund managers can invest either in riskless assets or in risky bonds issued by a large number of borrowers. Borrowers invest in risky projects and can default on them after observing the realized project's productivity. As shown in Figure 2, the model is structured on two sets of interactions: investors/managers and managers/borrowers. On the one hand, the interaction between investors and managers shapes the managers career concers. There is a small portion of talented fund managers who have private information about the productivity of the risky project. Using this information, they can formulate a more precise estimate of the default probability of the risky bond than the untalented managers. At the end of each period, based on the manager's performance, each investor update his belief and decides whether to keep his manager or to fire him and hire a new one. The firing decision of the investors distorts the investment decision of untalented managers who would like to be perceived as talented managers.

[1] As a columnist of the Wall Street Journal observes, the 5-year credit default swap spread for Brazil, Peru, Columbia were at the record-tight levels of 0.70, 0.65 and 0.80 percentage point at the time when, for example, the Boston Scientific Corp, an investment grade company traded at 0.78 percentage point. (April 24, 2007, Tight spreads are emerging, WSJ).

On the other hand, the interaction between managers and borrowers determines the price of the risky bond, the probability of default and the level of economic activity in the economy. The investment choice of the fund managers determines the required rate of return on the bond for a given probability of default. The representative borrower issues bonds to cover her consumption and the fixed cost of the risky project. At the end of the period, she observes the productivity of the project and decides whether to pay-back the outstanding debt or to default and suffer a cost. For a given price, her default rule determines the ex-ante probability of default on the bond. Hence, the equilibrium bond price and default probability are jointly determined by the conditions of both the financial market and the fundamentals of the risky projects. Even though borrowers are homogenous once they start the risky project, they are ex-ante heterogenous in their outside option. The measure of borrowers who choose to start the project for a given bond price determines the level of economic activity.

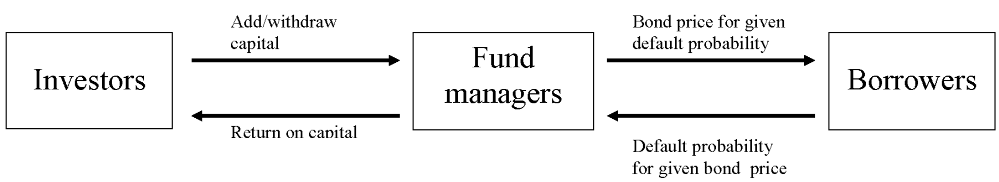

Figure 2: The structure of the model

The focus of our paper is to study the effect of the agency problem between investors and managers, the outcome of the first interaction, on the equilibrium bond price, default frequency and economic activity, that is, the outcome of the second interaction.

Our main result is that managers' career concerns amplify the effect of fundamental shocks on the bond price, the probability of default and the level of economic activity. This amplification effect arises in general equilibrium as outcome of two reenforcing mechanisms. First, on the real side, when borrowing is more expensive, the value of the same debt amount is lower. Hence, borrowers need to issue more bonds, and the probability of default increases. Second, on the financial side, career concerns impose a reputational premium on the price of risky bonds, that depends on the default probability. Untalented fund managers try to time the market in order to behave as if they were talented, and know in advance if there will be default or not. Default will hurt the reputation of managers who invest in the risky bond, and no default will hurt the reputation of managers who invest in the risk-less one. Thus, when the probability of default is high, the reputational premium is positive to compensate for the foregone reputation. Vice-versa, when the default probability is low the risky bond will trade with a reputational discount, due to the reputational gain. The real side of the model mechanism implies that a larger return on bond leads to a larger probability of default. The financial side of the model implies that a larger probability of default leads to a larger return on bond, because of a larger reputational premium. These two mechanisms reinforce each other in equilibrium and generate excess volatility in bond prices: bond spreads are particularly low in good times and high in bad times. As in our model economic activity is lower when borrowing is more expensive, the excess volatility in prices generates excess volatility in output.

We also explore an extension of the model where we introduce a second risky bond issued by a different group of borrowers. We show that career concerns introduce a common component in the required premium of the two bonds even if the underlying fundamentals are independent across the groups of borrowers. This result is in line with the large comovement of bond spreads shown in Figure 1. The presented channel of contagion is also distinct from the portfolio channel exposed in the literature (Calvo, 1999, Pavlova and Rigobon, 2007), as in our model none of the fund managers hold both type of risky bonds.

A natural application of our model would be to think of the borrowers as firms in an emerging economy. In this context, our results are in line with the empirical evidence that business cycle fluctuations in emerging economies are much more volatile than those in developed countries, and that such an excess volatility is partly driven by the volatility in bond spreads (Neumeyer and Perri, 2005, Uribe and Yue 2006). However, our result more generally apply to any type of bonds when the fundamentals of the underlying risky activity fluctuate substantially. The borrowers' need for funds can be due to a fixed cost of investment as in our example, or debt overhang which requires refinancing. This ensures that when the spreads rise, borrowers cannot decrease the dollar amount of their outstanding debt by cutting back their borrowing. Thus, other applications might include low-rated corporate debt and mortgaged-back assets of poor credit quality.

To our knowledge, this is the first paper to address the interaction between financial intermediation and defaultable debt. In particular, the application to emerging markets connects two distinct areas of economics and finance. On one hand, there is a vast literature on sovreign debt, reversal of capital flows and financial crisis in emerging economies, such as Atkenson, 1991, Cole and Kehoe, 2000, Aguiar and Gopinath, 2006, Caballero and Krishnamurthy, 2003, Calvo and Mendoza, 2000, Benczur and Ilut, 2005, Arellano, 2006, Uribe and Yue, 2006. On the other hand, there is a growing literature which analyzes the effect of delegated portfolio management on traders' decisions and asset prices in general, such as Dow and Gorton, 1997, Shleifer and Vishny, 1997, Allen and Gorton, 1993, Cuoco and Kaniel, 2007, Vayanos, 2003, Gümbel, 2005, Dasgupta and Prat, 2005, 2006, Kondor, 2007. However, the first group abstracts away from the effects of intermediation in financial markets, while the second group is silent on the real effects of these frictions.[2]

In the next section, we present the model. In Section 3, we define and characterize an equilibrium. In Section 4, we discuss the extension of the model with two risky bonds. Finally, Section 5 concludes. The appendix includes all the proofs that are not in the text.

2 The Model

The model is structured in three parts. First, the economy is populated by a large number of borrowers who need financing to undertake a risky project. They choose how much to borrow and under what circumstances to default, taking as given the cost of borrowing. Second, the international investors hire fund managers who decide whether to finance the risky project, or to invest their money in a risk-free bond. In any period, each investor decides whether to keep his manager, or to fire him and hire a new one, conditional on the realized returns. Third, the fund managers make their investment decisions, taking as given the probability of default of the risky project and the firing rule of the investors. We start by analyzing these three decision problems separately, taking as given the rest of the economy and, then, we merge them together to define the equilibrium concept.

2.1 The borrowers

The economy is populated by a large number of borrowers running the same risky project. They can borrow from financial markets by issuing one-period discount bonds and can ex-post decide to default. We can think of borrowers as firms in an emerging economy, or, more

[2] Our mechanism is related to literature of reputational herding (Scharfstein and Stein (1990), Zweibel (1995), Ottaviani and Sorensen (2006)) to the extent that there is a group of managers who follow an action with inferior monetary payoff to increase their reputation. However, there are at least two significant differences. First, that literature concentrates on the incentives of informed agents to misreport their information, while in our model the informed agents always follow their signal. Only some uninformed agents change their actions because of career concerns. Second, all these models consider fixed, exogenously given prices, while we focus on the price effect of our mechanism.

generally, firms with the same risk characteristics, or even property owners whose loans are behind the same mortgage backed asset.

Time is discrete and there are overlapping generations of borrowers who live for two periods. In each period a new generation is born, which is represented by a continuum of measure 1 of agents, indexed by i, with logarithmic utility. Consider agent i of the generation born at time t. When she is young, she has the choice to invest in a risky project with return a_{t+1}, distributed according to the cumulative distribution function $F(a_{t+1})$, or to enjoy an outside option $\bar{u}_t^i$. We assume that agents, within a generation, differ in their outside option $\bar{u}_t^i$, which is distributed according to the cumulative distribution function $G(\cdot)$ with real support and $i.i.d$ across time. However, they have all access to the same risky project, so that all the agents who become active borrowers face the same problem.

Let us start to analyze the behavior of an agent who has decided to become an active borrower at time t. To simplify notation we drop the superscript i whenever this does not cause any confusion. At time t, the agent chooses how much to borrow and how much to consume, taking as given the price of borrowing p_t. As she does not have any income when she is young, she has to cover both the fixed cost of the investment, I, and her consumption by borrowing, i.e., her budget constraint when young is

$$p_t b_{t+1} \geq c_t + I, \tag{1}$$

where b_{t+1} represents the bonds issued at time t, p_t represents the price that the agent has to pay per dollar borrowed at time t, and c_t represents consumption at time t. We assume that there is a maximum amount of bonds that can be issued at each time, that is, $b_{t+1} \leq \bar{b}$ for each t. We will make sure that $\bar{b}$ is sufficiently large so that it is not binding in equilibrium. Moreover, notice that logarithmic utility implies that consumption must be non-negative, so $b_{t+1} \geq I/p_t$.

When the agent is old, she collects the project pay-off a_t and has the option to default on her debt b_{t+1} at a cost $D(b_{t+1})$ in terms of utility.[3] The function $D(\cdot) : [I/p_t, \bar{b}] \to [0, \infty)$

[3] We do not take a stand on the exact source of the cost of default. This is a particularly debated issue in the case of sovereign debt of an emerging country. Since the seminal paper of Eaton and Greskovitz (1981), it is recognized that there must be some cost of default on sovereign debt to enforce repayment. The theoretical literature on sovereign default has explored alternative possible punishments, such as partial or full exclusion from financial markets, or other economic or political sanctions (Eaton and Greskovitz, 1981, Bulow and Rogoff, 1989), loss of reputation (Grossman et al., 1988, Atkeson, 1991, Cole and Kehoe, 1996), or worse future terms of borrowing (Chang and Sundaresan, 2001, Kovrijnykh and Szentes, 2006). In this paper we abstract from the specific form of punishment and simply assume that default is costly enough to support an equilibrium where

satisfies mild conditions: *(i)* it is twice differentiable with $D'(b) > 0$ and $D''(b) > 0$ for all $b \in [I/p_t, \bar{b}]$; *(ii)* it satisfies $D(b) > \log(1 + bD'(b))$ for all $b \in [I/p_t, \bar{b}]$. Notice, that this last condition is not particularly strong, given that it is easily satisfied when $D(b) = D_1 + D_2(b)$ and the constant part D_1 is large enough. If the borrower chooses not to default she consumes her income after she repays her debt. If, instead, she decides to default, she can consume her entire income. Her budget constraint when old is

$$a_{t+1} - \left(1 - \chi_{t+1}(a_{t+1})\right) b_{t+1} \geq c_{t+1}, \tag{2}$$

where $\chi_{t+1} : \mathbb{R}_+ \mapsto \{0, 1\}$ denotes the default decision that the agent is making at time $t+1$, after observing the realization of a_{t+1}. However, if she decides to default she has a utility loss $D(b_t)$, so her objective function is

$$\log c_t + \beta \mathbb{E}\left[\log c_{t+1} - \chi_{t+1}(a_{t+1}) D(b_{t+1})\right]. \tag{3}$$

It is useful to define the ex-ante probability of default q_t as

$$q_t \equiv \int_0^\infty \chi_{t+1}(a_{t+1})\, dF(a_{t+1}).$$

The problem for the representative active borrower is to maximize (3) subject to (1) and (2), taking p_t as given. The problem can be written as

$$\begin{aligned} \max_{I/p_t \leq b_{t+1} \leq \bar{b}, \chi_{t+1}, c_t} \quad & \log c_t + \int_0^\infty \log\left[a_{t+1} - \left(1 - \chi_{t+1}(a_{t+1})\right) b_{t+1}\right] dF(a_{t+1}) - \\ & - \int_0^\infty \chi_{t+1}(a_{t+1}) D(b_{t+1})\, dF(a_{t+1}) \\ & s.t.\ p_t b_{t+1} = c_t + I. \end{aligned} \tag{4}$$

With a slight abuse of notation, let $V(p_t)$ represents the value of investing in the risky project when the bond price is p_t. Recall that the agents of generation t differ for their outside option $\bar{u}_t^i$ and, hence, for their choice of becoming or not an active borrower. A young agent decides to become an active borrower if and only if the value of investing in the risky project is bigger than her outside option, that is, $V(p_t) \geq \bar{u}_t^i$. Define $B(p_t)$ the aggregate supply of bonds when the bond price is p_t. It follows that

$$B(p_t) = G(V(p_t))\, b(p_t).$$

it is not always optimal to default.

2.2 Investors and fund managers

The financial market is populated by a mass Γ of risk-neutral investors, indexed by j, who can invest one unit of capital at each time t. They can invest their capital only through fund managers. At the beginning of each period there is a mass 2Γ of potential, risk-neutral fund managers. They do not have any capital, and become active fund managers only when they are hired by some investor. An investor can hire only one fund manager and a fund manager can be hired only by a single investor, so that in each period there is a mass Γ of active managers. For simplicity, we fix the contract between investors and fund managers: fund managers keep a share γ of the revenues and leave the rest to the investors. Both investors and managers fully consume their net revenues in each period.

A fund manager can invest either in a riskless bond with gross return R, or in the risky bond described in the previous section, with price p_t and aggregate supply $B(p_t)$. As we have described above, the borrowers can endogenously decide to default on that bond. The return on the bond will be 0 if the borrowers default, or $1/p_t$ if they do not. The managers take as given the probability of default q_t.

There are two types of fund managers: talented and untalented. Only a small fraction $\bar{\varepsilon}$ of all potential managers are talented. At the end of any period, each fund manager has a probability $(1-\delta)$ to die, and $(1-\delta)\,2\Gamma$ newly born managers, $\bar{\varepsilon}$ of which are talented, join the pool of unemployed, keeping the mass of managers constant.

Assumption 1 *Assume that $\bar{\varepsilon}$ is small enough and Γ big enough such that*

$$\begin{aligned} \bar{\varepsilon}2\Gamma &< \frac{I}{\bar{b}}B\left(\frac{I}{\bar{b}}\right) \\ \frac{1}{R}B\left(\frac{1}{R}\right) &< (1-2\bar{\varepsilon})\,\Gamma. \end{aligned}$$

This assumption will ensure that for any price, some untalented managers have to invest in the risky bond and some have to invest in the riskless one, otherwise the market of the risky bonds cannot clear.

If a fund manager is hired at time t by investor j, he gets a signal s_t^j about the productivity of the risky project, a_{t+1}. If the manager hired by investor j is talented, then he gets a signal that perfectly reveals whether borrowers will default, $s_t^j = d$, if $a_{t+1} \leq \hat{a}_{t+1}$,or will not default, $s_t^j = n$, otherwise. If he is untalented, then he gets an uninformative signal, that is, with abuse of notation, $s_t^j = 0$. Hence, any manager who is hired get a signal $s_t = \{n, d, 0\}$ before making his investment decision. From now on, manager j stays for "the manager hired from investor j".

Let us define $\mu_t(s_t^j, \varepsilon_t)$ the investment strategy of manager j, that is, the probability that a manager with signal s_t^j invests in the risky bond. Moreover, let us define $\tilde{\mu}_t^j$ the effective investment decision, that is $\tilde{\mu}_t^j = 1$ with probability $\mu_t(s_t^j, \varepsilon_t)$ and $\tilde{\mu}_t^j = 0$ with probability $1 - \mu_t(s_t^j, \varepsilon_t)$. Let define η_t^j the investors' belief at the beginning of time t that manager j is talented. At the end of the period, each investor observes the realized profit of his manager, and hence his investmetn decision $\tilde{\mu}_t^j$, and the state a_{t+1}. Hence, the investor can update his belief using the Bayes Rule, that is, $\eta_{t+1}^j = \zeta(\eta_t^j, \tilde{\mu}_t^j, a_{t+1})$.

At the beginning of time t, each investor j has a manager working for him, that he believes is talented with probability η_t^j. Let us define $U(\eta_t^j, \varepsilon_t)$ his expected utility at that stage, where ε_t is the probability that an unemployed manager is talented at the end period $t-1$. At the end of time t, after updating his belief, he chooses the firing rule $\phi(\eta_{t+1}^j, \varepsilon_{t+1})$ in order to maximize his expected utility from $t+1$ on, taking as given the risky bond price p_t, the strategy of the fund managers $\mu_t(s_t^j, \varepsilon_t)$, and the default probability q_t. The problem they solve can be written as

$$\begin{aligned} U(\eta_t^j, \varepsilon_t) &= (1-\gamma)\, \pi_t(\eta_t^j) + \\ &+ \delta E\left[\max_{\phi} \left\{(1-\phi)\, U(\eta_{t+1}^j, \varepsilon_{t+1}) + \phi U(\varepsilon_{t+1}, \varepsilon_{t+1})\right\} |\eta_t, \varepsilon_t\right], \end{aligned} \tag{5}$$

where the ex-ante investment pay-off is

$$\begin{aligned} \pi_t(\eta_t) &= \eta_t (1-q_t)\left[\mu_t(n)\frac{1}{p_t} + (1-\mu_t(n))\, R\right] + \eta_t q_t \left[(1-\mu_t(d))\, R\right] + \\ &+ (1-\eta_t)\left[\mu_t(0)(1-q_t)\frac{1}{p_t} + (1-\mu_t(0))\, R\right]. \end{aligned}$$

The probability ε_{t+1} of an unemployed manager to be informed at the end of period t is persistend and follows the low of motion $\varepsilon_{t+1} = \Psi(\varepsilon_t, a_{t+1})$,[4] that is public information. The investors' expected utility depends on ε_t because the investors use it to determine the probability that a new hire is informed and make their firing decision.

A fund manager with signal s_t^j chooses his investment strategy $\mu_t(s_t^j, \varepsilon_t)$ to maximize his expected utility, taking as given the risky bond price p_t, the firing rule adopted by the investors,

[4] See the appendix for the explicit derivation of the low of motion $\Psi(\varepsilon_t, a_{t+1})$.

$\phi(\zeta(\eta_t^j, \tilde{\mu}_t^j, a_{t+1}), \varepsilon_{t+1})$, and the default rule followed by the borrowers. The problem for a fund manager with signal $s_t = (d, n, 0)$ can be written as

$$\begin{aligned} W(d, \varepsilon_t) &= \max_{\mu} \gamma (1-\mu) R + \delta E\left[[1-\phi(\zeta(\eta_t^j, \tilde{\mu}_t^j, a_{t+1}), \varepsilon_{t+1})] W(s_{t+1}, \varepsilon_{t+1}) | d, \varepsilon_t, \mu\right] \quad (6) \\ W(n, \varepsilon_t) &= \max_{\mu} \gamma \left[\mu \frac{1}{p_t} + (1-\mu) R\right] + \delta E\left[[1-\phi(\zeta(\eta_t^j, \tilde{\mu}_t^j, a_{t+1}), \varepsilon_{t+1})] W(s_{t+1}, \varepsilon_{t+1}) | n, \varepsilon_t, \mu\right] \\ W(0, \varepsilon_t) &= \max_{\mu} \gamma \left[(1-q_t) \mu \frac{1}{p_t} + (1-\mu) R\right] + \delta E\left[[1-\phi(\zeta(\eta_t^j, \tilde{\mu}_t^j, a_{t+1}), \varepsilon_{t+1}) W(0, \varepsilon_{t+1}) | \varepsilon_t, \mu\right] \end{aligned}$$

The key feature of this problem is that the fund managers know that their investment decision affects the investors' firing decision by changing the belief's update. This generates career concerns affecting investment decision that are at the core of our model.

2.3 Definition of equilibrium

Let us summarize the timing of the model. At the beginning of period t the productivity shock a_t is realized, and, hence, old borrowers make their default decision according to the rule $\chi_t(a_t)$. The return of managers is also realized for each manager and it is shared between investors and managers. Then, each manager dies with probability $(1-\delta)$ and a measure of $(1-\delta) 2\Gamma$ newly born managers join the unemployed pool. Based on the return distribution of hired managers, investors with an alive manager decide whether to keep him or to fire him and hire a new one. Investors with a dead manager necessarily hire a new one. Next, hired managers receive the signal s_t and decide how to invest the investors' capital. At the same time, young agents decide whether to become active borrowers, how much to borrow and under what circumstances they will repay their loans. The bonds market clears.

We restrict attention to stationary equilibria where the investment strategies, the firing rule, the bond holdings of borrowers, the default's probability do not depend on the state ε_t. Thus, the only equilibrium objects that do vary with the state ε_t are the investors' beliefs. Moreover all the equilibrium object do not depend on the distribution of beliefs, given that the level of η_t^j does not matter for the equilibrium, as long as $\eta_t^j \in (0,1)$, as we show in the appendix. This allows us not to keep track of the distribution of the beliefs in the population and to simplify the analysis. In a stationary equilibrium, the amount of bond holdings, b, the probability of default, q, and the price, p, are time independent.[5]

[5] Similarly to a standard competitive equilibrium, the defined equilibrium can be implemented as a Perfect Bayesian Equilibrium of an augmented game where managers submit demand curves for the risky bond and a Walrasian auctioneer sets the price which clears the bond market.

Definition 1 *A stationary equilibrium is a set of strategies* $\{\mu(s_t^j, \varepsilon_t), \phi(\eta_{t+1}^j, \varepsilon_{t+1}), b, q\}$*, a belief function* $\eta_{t+1}^j = \zeta(\eta_t^j, \tilde{\mu}_t^j, a_{t+1})$*, where* $\tilde{\mu}_t^j = 1$ *with probability* $\mu_t(s_t^j, , \varepsilon_t)$ *and* $\tilde{\mu}_t^j = 0$ *otherwise, a price p, and a low of motion* $\varepsilon_{t+1} = \Psi(\varepsilon_t, a_{t+1})$ *such that*

1. *investors maximize their expected utility, taking as given the price p, and the strategies of fund managers and borrowers;*
2. *fund managers maximize their expected utility, taking as given the price p, and the strategies of international investors and borrowers;*
3. *borrowers maximize their expected utility, taking as given the price p;*
4. *the bonds market clears, that is*

$$E\left[\int_0^\Gamma \mu_t(s_t^j)dj\right] = pb\int_{V(p)\geq \bar{u}^i} di;$$

5. *investors' beliefs are consistent with the Bayes rule.*

3 Stationary Equilibrium

In this section we characterize the equilibrium. We proceed in three steps. First, we characterize the borrowers' *repayment rule*, that is the optimal default rule conditional on the bond price. Then we characterize the financial market *pricing rule,* that is, the equilibrium price for a given default probability. Finally, we show that these rules define a fixed point problem that determines the equilibrium bond price.

3.1 The repayment rule

Borrowers choose their default rule and how much to borrow and to consume in order to solve problem (4). Let us first consider the default decision of an old borrower. For any given pair, a_t and a_{t+1}, she will default if and only if

$$\log a_{t+1} - D(b_t) - \log(a_{t+1} - b_t) > 0.$$

Note that the left hand side of the condition above is decreasing in a_{t+1}, thus there will be a threshold $\hat{a}_{t+1}$ such that the agent will repay if the shock $a_{t+1} \geq \hat{a}_{t+1}$, and will not repay otherwise. Hence, $q_t = F(\hat{a}_{t+1})$. This result is summarized in the following lemma.

Lemma 1 *For a given cost function, $D(\cdot)$, there exists a threshold $\hat{a}_{t+1}$ such that $\chi_{t+1}(a_{t+1}) = 1$ if $a_{t+1} \le \hat{a}_{t+1}$ and $\chi_{t+1}(a_{t+1}) = 0$, otherwise, with*

$$\hat{a}_{t+1} = \frac{\exp\{D(b_{t+1})\}}{\exp\{D(b_{t+1})\} - 1} b_{t+1}. \tag{7}$$

If we substitute back the budget constraint and the default decision $\chi_{t+1}(a_{t+1})$ into problem (4), it becomes a maximization problem over the borrowing decision only. Hence, the optimal policy b_{t+1} must satisfy the first order condition

$$\frac{p_t}{(p_t b_{t+1} - I)} - \int_{\hat{a}_{t+1}}^{\infty} \frac{1}{(a_{t+1} - b_{t+1})} dF(a_{t+1}) - F(\hat{a}_{t+1}) \frac{dD(b_{t+1})}{db_{t+1}} = 0, \tag{8}$$

where $\hat{a}_{t+1}$ solves equation (7). Then, using equation (8) together with (7), we can solve for the optimal amount of bonds supplied b_{t+1}, for a given price p_t. Next, we can plug b_{t+1} back into equation (7) and solve for the equilibrium default probability, $F(\hat{a}_{t+1})$, for a given price p_t. We will refer to this condition as the borrowers' optimal repayment rule.

Notice that for a given productivity distribution, not all the cost functions $D(\cdot)$ support an equilibrium with a non-trivial default decision. Intuitively, if the marginal cost of default is not large enough compared to the advantage of additional borrowing, the agent would always like to borrow more and default more often. In this case, the solution for problem (4) would not be a finite b_{t+1}. To be more precise, let $h(b_{t+1})$ represent the total marginal cost of borrowing for a given level of debt b_{t+1}, that is,

$$h(b_{t+1}) \equiv \int_{\hat{a}_{t+1}}^{\infty} \frac{1}{(a_{t+1} - b_{t+1})} dF(a_{t+1}) + F(\hat{a}_{t+1}) \frac{dD(b_{t+1})}{db_{t+1}}, \tag{9}$$

where $\hat{a}_{t+1}$ is defined by (7). The first term is the cost of an additional unit of borrowing in terms of the foregone consumption of an old agent who pays back the debt, while the second term is the expected marginal increase of the cost of default. If the cost function $D(\cdot)$ is such that

$$h'(b_{t+1}) > -h(b_{t+1})^2 \text{ for any } b_{t+1} \in (I/p_t, \bar{b}), \tag{10}$$

the second order condition of the representative agent problem is negative, and the problem has a unique solution, as stated in the following lemma.

Proposition 1 *For given p_t, if $D(\cdot)$ satisfies condition (10), problem (4) has a unique solution.*

Finally, the next lemma establishes two important properties of the equilibrium. First, we show that as borrowing becomes more expensive, the representative agent will default with a higher probability, that is, he repayment rule is downward sloping in the (p_t, q_t) space. Second, we show that, under condition (10), when borrowing becomes less expensive, the nominal value of the outstanding debt for each agent increases.

Lemma 2 *Suppose that (10) holds. Then as the price of borrowing p_t decreases, (i) the probability of default $F(\hat{a}_{t+1})$ increases, and (ii) the value of capital borrowed $p_t b_{t+1}$ decreases.*

The intuition behind these results is straightforward. From one side, when borrowing becomes more expensive, the income effect makes the agent poorer. From the other side, the substitution effect makes consumption in the young age more expensive and induce the agent to substitute away from it towards consumption in the old age. Given that there is a fixed financing requirement of the investment project, the amount borrowed does not affect productivity but only consumption, and, in particular, a reduction in borrowing increases consumption in the old age. It follows that consumption in the young age decreases for both effects and, given that $p_t b_{t+1} = c_t + I$, the value $p_t b_{t+1}$ decreases, that is, as spreads increase, capital flows out from the country. Moreover, we show that the income effect dominates consumption in the old age and makes b_{t+1} increase and the default probability with it. As intuition suggests, the default probability increases when borrowing is more expensive.

Next, let us consider what happens to the aggregate value of capital inflows $p_t B(p_t)$ and to aggregate output. Define average aggregate output as $Y(p_t) \equiv G(V(p_t)) E(a_{t+1})$. The following proposition shows that both aggregate output and aggregate capital inflows are increasing with p_t.

Lemma 3 *Suppose that (10) holds. Then, as the price of borrowing p_t decreases, (i) the aggregate output Y_t decreases, and (ii) the aggregate value of capital borrowed $p_t B_{t+1}$ decreases.*

Proof. Agent j will decide to become an entrepreneur as long as his outside option is smaller than the value of being an active entrepreneur, that is, $\bar{u}^j \leq V_t(p_t)$. From the envelope theorem

$$V'(p_t) = \frac{b_{t+1}}{p_t b_{t+1} - I} > 0,$$

and hence,

$$\frac{\partial G(V(p_t))}{\partial p_t} = g(V(p_t)) \frac{b_{t+1}}{p_t b_{t+1} - I} > 0.$$

This also implies that aggregate output $Y(p_t)$ is increasing in p_t. Together with the second part of Lemma 2, this also implies that $p_t B(p_t)$ is also increasing in p_t, completing the proof. ■

The intuition is as follows. A young agent decides to become active whenever his outside option is smaller than the value of being an active entrepreneur. As borrowing becomes more expensive, this value decreases and there will be a bigger mass of agents who will prefer their outside option. This increases aggregate output and the aggregate value of capital inflows.

3.2 The pricing rule

Let us now turn to the characterization of the financial market. Both investors and fund managers take as given the bond price p_t and the borrowers' strategy $\left\{b_t, \chi_{t+1}(a_{t+1})\right\}$, and, hence, q_t.

First consider a benchmark model with $\bar{\varepsilon} = 0$. In this case, all managers are untalented, so investors will be indifferent among them and keep the one they start with. Managers will maximize their period by period profit. Thus, the bond price is determined by the standard no-arbitrage condition

$$(1 - q_t) \frac{1}{p_t} = R, \tag{11}$$

that is, the expected return on the bond must be equal to the return on the riskless asset, R.

Next, let us go back to our model with career concerns, where $\bar{\varepsilon} > 0$. The next proposition characterizes the equilibrium strategies of investors and managers and the equilibrium bond price for a given probability of default and bond holding strategy.

Proposition 2 *Suppose that the probability of default q_t is a constant $q > (1+2\delta-\sqrt{1+4\delta})/2\delta$, and that there is a fixed supply of bonds B with $B \in \left[B\left(I\bar{b}\right), B(1/R)\right]$. Let the bond price be*

$$p = \frac{(1 - \delta q)(1 - q)}{R\left[1 - \delta(1 - q)\right]}. \tag{12}$$

Then, the following strategies of investors and managers are optimal taking as given the strategies of the other players, under market clearing and a set of beliefs which are consistent with Bayes' rule:

1. *investors' firing rule*

$$\phi(\tilde{\mu}_t^j, a_{t+1}) = \begin{cases} 0 & if\ \tilde{\mu}_t^j = x_t(a_{t+1}) \\ 1 & otherwise \end{cases}, \tag{13}$$

where

$$x_t\left(a_{t+1}\right)=\left\{\begin{array}{ll}0 & if\ a_{t+1}\leq \hat{a}_{t+1}\\ 1 & if\ a_{t+1}>\hat{a}_{t+1}\end{array}\right. ;$$

2. *managers' strategies*

$$\mu_t\left(d,\varepsilon_t\right)=0,\ \ \mu_t\left(n,\varepsilon_t\right)=1,\ \ \mu_t\left(0,\varepsilon_t\right)\in(0,1) \tag{14}$$

where $\mu_t\left(0,\varepsilon_t\right)$ *is defined by*

$$\int_0^{\Gamma}\mu_t(s_t^j,\varepsilon_t)dj=pB.$$

This proposition shows that the optimal firing rule for the investors is to keep only the managers that invest in risk-less bonds when there is default, and in the risky ones bond when there is not. Then, the talented managers will follow their signal to avoid to be fired, and hence $\mu_t^j\left(d\right)=0$ and $\mu_t^j\left(n\right)=1$. Assumption 1 ensures that the market can clear if and only if a positive measure of untalented managers invest in each of the two types of bonds. The proportion of untalented managers who end up investing in the risky bond, $\mu_t\left(0\right)$, has to be such that the market clears for the equilibrium price. From the managers' problem (6), the untalented managers are indifferent if and only if

$$\left(1-q_t\right)\left(\frac{1}{p_t}+\delta W\left(0,\varepsilon_{t+1}\right)\right)=R+q_t\delta W\left(0,\varepsilon_{t+1}\right), \tag{15}$$

where

$$W\left(0,\varepsilon_{t+1}\right)=\frac{\gamma R}{1-\delta q}. \tag{16}$$

The left-hand side of equation (15) represents the expected payoff of a manager who invests in the risky bond. With probability $\left(1-q_t\right)$ borrowers do not default, that is $a_{t+1}\geq\hat{a}_{t+1}$. In this case, the manager succeeded to pool with the talented managers, he is not fired, and gets continuation utility $W\left(0,\varepsilon_{t+1}\right)$. If instead the manager invests in the bond and $a_{t+1}<\hat{a}_{t+1}$, that is, the borrowers default, with probability q_t, he gets zero return. Moreover, the investor learns that the manager was not talented and fires him, so that he gets no continuation utility. Similarly, the right-hand side of equation (15) represents the expected payoff of a manager who invests in the risk-free bond. He gets a return R with certainty. However, he is not fired, and gets continuation utility $W\left(0,\varepsilon_{t+1}\right)$, only if $a_{t+1}<\hat{a}_{t+1}$ and the borrowers do default. Otherwise, the investor learns that he was not talented and fires him. Equation (16) gives the continuation value of being an untalented manager who keeps the job. This condition is obtained by noticing that if a manager is indifferent in each point in time between investing in the risk-free asset or in the risky bond, his value function must be given by the value of always investing in the risk-free asset as long as with that strategy he is not fired. From combining equations (15) and (16) we immediately obtain the pricing condition. The lower bound on q_t implies that the return on the risky bond in the event of no default, $\frac{1}{p_t}$, is larger than R, i.e., the realized spread is non-negative.

Let us define the reputational premium Π_t, that is, the difference between the expected repayment and the risk free rate R

$$\Pi_t \equiv \frac{1-q_t}{p_t} - R. \tag{17}$$

This premium characterizes the price distortion generated by the career concerns of the untalented fund managers. In the benchmark model with no career concerns, equation (11) immediately implies that this premium is equal to zero. In the case with a positive measure of talented managers, the reputational premium can be negative or positive. Typically, it is positive when q_t is sufficiently large and negative when q_t is sufficiently small. Betting on large probability events is especially attractive for an untalented fund manager with career concerns, because it increases the chance that he will not make an unsuccessful decision and will not be fired. In contrast, even if the return compensates for the risk of default, holding a bond which pays off with small probability is especially unattractive for the untalented fund manager, as it increases the chance of being fired. In equilibrium, this preference for large probability events is priced. Fund managers are willing to give up a part of their expected return for a large probability of not being fired. Thus, in expectation untalented investors lose on large probability bets and gain on small probability bets in each period when they are hired.

3.3 Characterization of the equilibrium

In this section, we characterize the equilibrium of our model, which is jointly determined by the conditions of the financial market and the fundamentals of the borrowers.

In section 3.1, we have derived, for given price p_t, the endogenous probability of default of a representative borrower $q_t = F(\hat{a}_t)$. In section 3.2, we have derived, for a given default probability q_t, the price p_t determined by the financial market. The next corollary define a fixed point problem combining the repayment rule and the pricing rule, and shows that the equilibrium is characterized by a stationary default rule and bond price $\{\hat{a}^*, p^*\}$.

Proposition 3 *An equilibrium is characterized by a default rule and price $\{\hat{a}^*, p^*\}$ that solve the fixed point defined as follows:*

Corollary 1 *1. given $\hat{a}$, p^* solves the pricing rule, that is,*

$$p^* = \frac{(1-\delta F(\hat{a}))(1-F(\hat{a}))}{R[1-\delta(1-F(\hat{a}))]}; \tag{18}$$

2. given p, $\hat{a}^*$ solves the repayment rule, that is,

$$\hat{a}^* = \frac{\exp\{D(b^*)\}}{\exp\{D(b^*)\}-1} b^*, \tag{19}$$

where b^* satisfies

$$\frac{p}{pb^* - I} - \beta \int_{\frac{\exp\{D(b^*)\}}{\exp\{D(b^*)\}-1} b^*}^{\infty} \frac{1}{(a-b^*)} dF(a) - \beta F\left(\frac{\exp\{D(b^*)\}}{\exp\{D(b^*)\}-1} b^*\right) D'(b^*) = 0. \tag{20}$$

Next, as a point of comparison, we describe the equilibrium in the benchmark case, when $\bar{\varepsilon} = 0$. This is also a fixed point $\{\hat{a}^b, p^b\}$, but it has to satisfy the no-arbitrage condition (11) and the repayment rule, $F(\hat{a}_{t+1})$.

Proposition 4 *An equilibrium of the benchmark economy $\{\hat{a}^b, p^b\}$ solves the fixed point defined as follows:*

Corollary 2 *1. given $\hat{a}$, p^b solves the pricing rule, that is, $p^b = \left(1 - F\left(\hat{a}^b\right)\right)/R$,*

2. given p, $\hat{a}^b$ solves the repayment rule, that is, (19), where b^* satisfies (20).

Figure 3 represents graphically the equilibrium both of the economy with career concerns (E) and of the benchmark economy (B). The equilibrium prices p^* and p^b correspond to the intersections of the repayment rule and the corresponding pricing rule, graphed in the space (p_t, q_t).

In the baseline numerical exercise we have assumed that a is distributed as a lognormal random variable. In Figure 3 the parameters of the lognormal are such that $p^* > p^b$, that is, such that the reputational premium is positive. By reducing the mean of a we can easily obtain the symmetric figure where $p^* < p^b$ and the reputational premium is negative.

Proposition 1 shows that for any p, there is a unique pair $(\hat{a}^*, b^*)$ defined by (19) and (20). The next proposition proves the existence of the equilibrium, under some mild parameter restrictions.

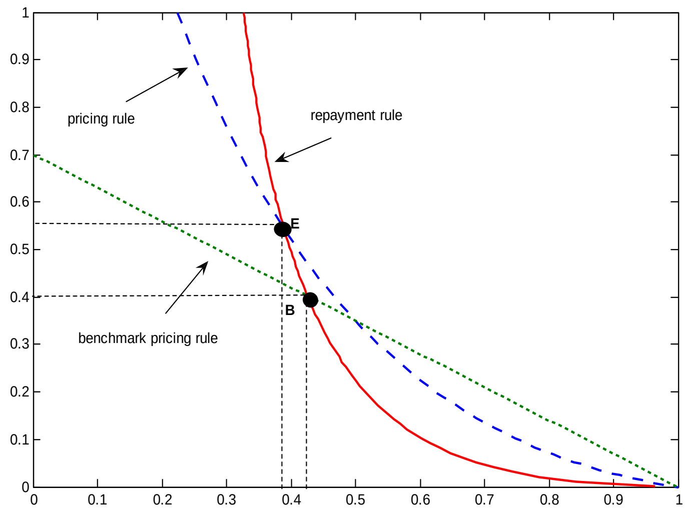

Figure 3: The solid line represents the repayment rule and the dashed curve and the dotted curve represent the pricing rule in the economy with career concerns and in the benchmark economy, respectively. Points E and B denote the equilibrium in the economy with career concerns and in the benchmark economy, respectively. Productivity is distributed according to a lognormal distribution with parameters 1.5 and 3.

Proposition 5 *Suppose that* $F(\hat{a}(1/R)) > (1 + 2\delta - \sqrt{1+4\delta})/2\delta$*. Then there exists an* $\hat{I} > 0$ *such that for any* $I \leq \hat{I}$*, there exists an equilibrium.*

Proof. See appendix. ■

3.4 Comparative statics

We now explore the properties of the equilibrium and analyze some comparative statics. In particular, we are interested in the reaction of the equilibrium both to shocks to financial markets and to shocks to the fundamentals of the borrowers. The first type of shocks affect the pricing rule and we refer to them as demand-side shocks; the second type affect the repayment rule and we label them supply-side shocks.

Notice that, depending on the parameters, two regimes are possible: the reputational premium might be negative or positive. The regime is determined jointly by the fundamentals of the risky project and the state of the financial market. For given fundamentals, the financial market can be such that the equilibrium is in any of the regimes. Similarly, for a given state of the financial market, the fundamentals can be such that the equilibrium is in any of the regimes. From Proposition 3, it is clear that when the reputational premium is positive, the demand for risky bonds is reduced compared to the benchmark case with no career concerns. The opposite is true when reputational premium is negative. The following proposition formalizes the conditions that determine the equilibrium regime.[6]

Proposition 6 *In equilibrium, one of the two following regimes arise: (i) if* $F(\hat{a}(1/2R)) < 1/2$*, the reputational premium is negative; (ii) if* $F(\hat{a}(1/2R)) > 1/2$*, the reputational premium is positive.*

This result is consistent with the empirical evidence that shows that emerging market bond prices fluctuate more than what is accounted for by changes in probability of default. On the one hand, Broner, Lorenzoni and Schmukler (2007) argue that the premium over the expected repayment on emerging market bonds is especially high during crises times. On the other hand, Duffie et al. (2003) document that the implied short spread of Russian bonds was very low during the first 10 months of 1997. Moreover, their estimation shows that in one short interval in 1997, bond prices were so high that the implied default adjusted short spread was negative. Although this observation is model specific, it is still interesting to point out that this is inconsistent with any risk-aversion based explanation, but consistent with our model. Our result is also consistent with the observation that business cycles are very volatile in emerging countries and spreads are countercyclical (e.g. Neumeyer and Perri, 2005, Uribe and Yue, 2006).

Both demand-side and supply-side shocks can move the economy from one regime to the other. A typical demand-side shock can be represented by a change in the risk-free rate, R. From equations (11) and (18), it is easy to see that both pricing rules get flatter as R increases. Suppose the economy starts in a regime with negative reputational premium. Then, an increase in R can make the economy shift to a regime with positive premium. This shift amplifies the reduction of bond prices, capital flows, and production. We summarize this observation in the following Proposition.

Proposition 7 *If* $F(\hat{a}(1/2R) < (>)1/2$ *and a sufficiently large unexpected shock increases (decreases)* R*, then the price* p*, the value of capital flows* $pB(p)$*, and the aggregate level of production* Y*, change more with carrier concerns than in the benchmark model.*

[6] The proof is straightforward from Figure 3, and hence it is omitted.

The flight-to-quality phenomenon described by Proposition 7 is observationally equivalent to an increase in the "risk-appetite" of fund managers. However, in our story, all fund managers are risk-neutral and the mechanism is based only on career concerns.

The result that demand-side shocks can be important determinants of bond prices is broadly consistent with the empirical evidence that a large proportion of the variation in prices of both corporate bonds and emerging market bonds cannot be explained by the variation of fundamentals. Moreover, a large part of this unexplained component is common across bonds (see Collin-Dufresne at al. (2000), Gruber et al. (2001), Westphalen (2001)) and any demand-side shock will affect all bond prices simultaneously.

Alternatively, the economy can move from one regime to another because of a supply-side shock. When the fundamentals of the borrowers deteriorate, the default probability may increase for any given price, and the repayment rule may shift to the right. For example, think of a shock to the technology of the risky project. The next proposition shows that if the fixed cost of investment becomes higher, the default probability increases for a given bond price.

Proposition 8 *If the fixed investment cost increases, the default probability increases for any given bond price. If $F(\hat{a}(1/2R) < (>)1/2$ and the fixed investment cost increases (decreases) enough, then the price p, the value of capital flows $pB(p)$, and the aggregate level of production Y, change more with carrier concerns than in the benchmark model.*

Figure 3 illustrates that an increase in the default probability for any bond price can move the country from a negative premium regime to a positive premium regime. As in the case of the demand-side shock, this shift will generate a reduction in bond prices, capital flows and output, with the difference that this regime shift will be country-specific.

4 Persistent Productivity Shock

In this section, we introduce persistency in the productivity process. In particular, assume that a_{t+1} is distributed according to a first-order Markov process with cumulative density function $F(a_{t+1}|a_t)$. The environment is a natural generalization of the one with *i.i.d.* shock, where a_t represents an additional state variable. We look for Markovian equilibria.

4.1 Equilibrium characterization

The optimization problems for borrowers, investors and managers are natural generalizations of problems (4), (5), and (6), where a_t is added as a new state variable.

Hence, an equilibrium is an investment function $\mu(s_t, \varepsilon_t, a_t)$, a firing rule $\phi(\eta_{t+1}, \varepsilon_{t+1}, a_t)$, a borrowing decision $b(a_t)$, a default probability $q(a_t)$, a price $p(a_t)$ and a belief updating rule $\zeta(\eta_t, \mu_t, a_{t+1}, a_t)$ such that investors, fund managers, and borrowers maximize their expected utility taking prices and others' strategies as given, believes are consistent with the Bayes' rule and markets clear.

We propose a Markovian equilibrium with very similar properties to the *i.i.d.* case as it is described in the next proposition.

Proposition 9 *Suppose that there are default and pricing functions* $\{\hat{a}^*(\cdot), p^*(\cdot)\}$ *which solve the fixed point defined as follows:*

1. *given* $\hat{a}(\cdot)$, $p^*(\cdot)$ *solves the pricing rule, that is,*

$$p(a_t) = \frac{\gamma [1 - F(\hat{a}(a_t) | a_t)]}{W(0, a_t) - \delta \int_{\hat{a}(a_t)}^{\infty} W(0, a_{t+1}) \, dF(a_{t+1} | a_t)}, \tag{21}$$

where $W(0, \cdot)$ *satisfies*

$$W(0, a_t) = \gamma R + \delta \int_0^{\hat{a}(a_t)} W(0, a_{t+1}) \, dF(a_{t+1} | a_t). \tag{22}$$

2. *given* $p(\cdot)$, $\hat{a}^*(\cdot)$ *solves the repayment rule, that is,*

$$\hat{a}(a_t) = \frac{\exp\{D(b(a_t))\}}{\exp\{D(b(a_t))\} - 1} b(a_t), \tag{23}$$

where $b(\cdot)$ *satisfies*

$$\begin{aligned} & \frac{p(a_t)}{p(a_t) b(a_t) - I} - \beta \int_{\frac{\exp\{D(b(a_t))\}}{\exp\{D(b(a_t))\}-1} b(a_t)}^{\infty} \frac{1}{(a_{t+1} - b(a_t))} dF(a_{t+1} | a_t) \\ & -\beta F\left(\frac{\exp\{D(b(a_t))\}}{\exp\{D(b(a_t))\} - 1} b(a_t) \right) D'(b(a_t)) \\ = \quad & 0. \end{aligned} \tag{24}$$

then $\{\hat{a}^*(\cdot), p^*(\cdot)\}$ *is a Markov-equilibrium with investors' firing rule and managers' strategies analogous to (13)and (14).*

Proof. The proof is in the Appendix. ■

In our equilibrium investors and managers follow the same strategies as in the *i.i.d* case independently from the past realizations of the productivity shock, a_t. Borrowers repayment rule, $\hat{a}(a_{t+1})$, and the amount of the bonds issued, $b(a_t)$, are implicitly defined by equations (23) and (24) which are virtually identical to the analogous equations of (19) and (20) with the only exception that all decision variables must be conditional on the past realization of a_t as the distribution of the productivity shock is persistent. The pricing rule, (21) and the recursive formula for the value function, (22), is also analogous to equations (15) and (16) implied by the observation that untalented fund managers have to be indifferent whether to invest in the riskless asset or the risky bond.

Next, as a point of comparison, we propose the equilibrium for the benchmark economy with no career concerns. We omit the proof as it is identical to the i.i.d case.

Proposition 10 *An equilibrium of the benchmark economy* $\left\{\hat{a}^b(\cdot), p^b(\cdot)\right\}$ *solves the fixed point defined as follows:*

1. *given* $\hat{a}(\cdot)$, $p^b(\cdot)$ *solves the pricing rule, that is,*

$$p^b = \frac{1 - F\left(\hat{a}^b | a_t\right)}{R}, \tag{25}$$

2. *given* $p(\cdot)$, $\hat{a}^b(\cdot)$ *solves the repayment rule, that is, (19), where* $b^d(\cdot)$ *satisfies (20).*

4.2 Numerical example

In this section, we present some numerical examples that show that persistency in productivity can magnify the spread's volatility.

First, we show how the default probability, the bond price, and the reputational premium vary with the realization of the productivity shock. Let us see first the equilibrium behavior in the benchmark economy. As a bad shock hits, the financial market will realize that, even for a given default rule, the probability of default will be higher and will require a lower bond price. As borrowing becomes more expensive, borrowers will then reduce their default cut-off, magnifying the reduction in the bond price. Hence, for low realizations of productivity, the default cut-off will be higher and the bond price lower. Now, consider the economy with career concerns. Suppose the default probability is high enough that the reputational premium is positive. In this case, the financial market will require a bond price even lower than the benchmark economy because of the reputational premium. Given that productivity is persistent, a bad realization of the shock will further increase the probability of default, increasing the fear of the fund managers of being fired and pushing the bond price further down. This implies that the reputational premium itself is higher after bad shocks. Figures (4) and (5) show how the reputational premium, the bond price, and the default probability vary in equilibrium with the different realizations of the productivity shocks.

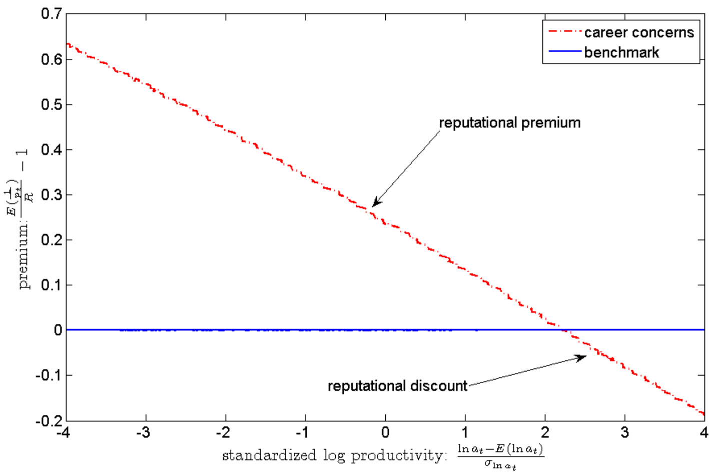

Figure 4: The figure shows the reputational premium as a function of the realization of the productivity shock. The dashed line is the premium with career concerns and the solid line shows the premium in the benchmark case.

Now, consider an economy that at time zero is hit by a shock. Figure 6 shows how the equilibrium prices react in expected terms to a bad and to a good shock, both with and without career concerns. Notice that the economy with career concerns reacts much more to the shocks than the benchmark economy. Moreover, notice that in the economy considered, the reputational premium would be positive in expected terms and a good shock can actually make the economy shift regime.

5 Two Groups of Borrowers

In this section, we allow fund managers to lend to two different groups of borrowers. We show that even if the fundamentals of the two groups are independent, prices and default

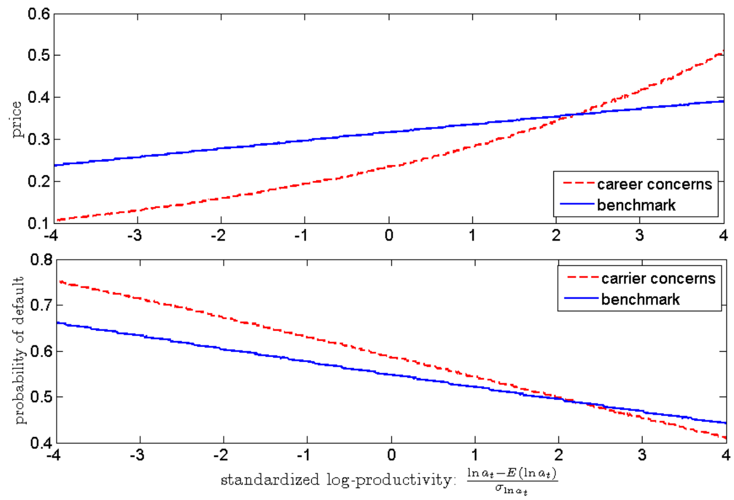

Figure 5: The upper panel shows the equilibrium price as a function of the productivity shock, while the lower panel shows the equilibrium probability of default as a function of the productivity shock. In both panels, the solid line represents the benchmark case and the dashed line represents the case with career concerns.

probabilities will be correlated if fund managers have career concerns.

5.1 Equilibrium characterization

Let us suppose that there are two groups of borrowers in the economy. The two groups are identical, except that group $s = A, B$ faces the productivity shock a_t^s. The stochastic Markov processes a_t^A and a_t^B are described by the joint conditional cumulative density function, $F\left(a_{t+1}^A, a_{t+1}^B | a_t^A, a_t^B\right)$. The informational environment is the same as before, but talented fund managers get a signal about only one of the bonds. To keep the analysis symmetric, let us suppose that conditional on getting any signal, the fund manager gets a signal about bond A with probability $\frac{1}{2}$. We also change Assumption 1 as follows.

Assumption 2 *Assume that $\bar{\varepsilon}$ and Γ are such that*

$$\begin{aligned} \bar{\varepsilon}\Gamma &< \frac{I}{\overline{b}} B^s \left(\frac{I}{\overline{b}}, a_t^s\right) \\ \frac{1}{R} B^s \left(\frac{1}{R}, a_t^s\right) &< \frac{1}{2}\left(1 - 2\bar{\varepsilon}\right)\Gamma \end{aligned}$$

for $s = A, B$ and for every a_t^A and a_t^B in the support of $F\left(a_{t+1}^A, a_{t+1}^B | a_t^A, a_t^B\right)$.

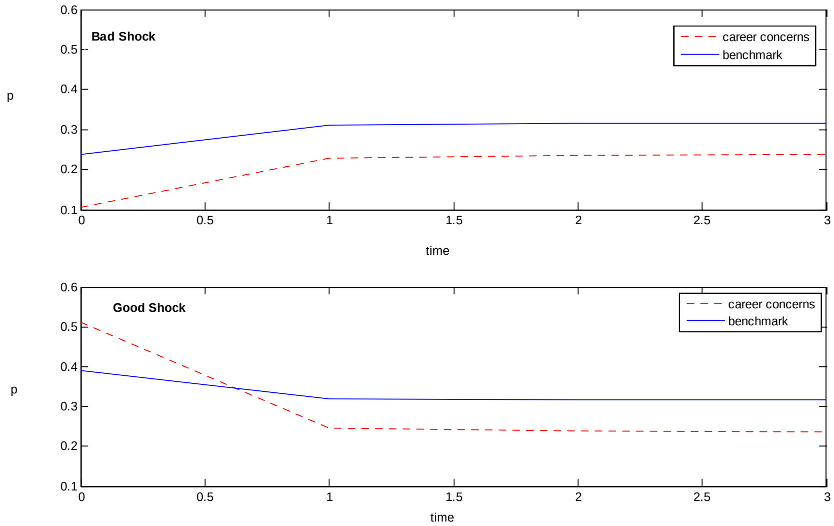

Figure 6: The two panels show the reaction of the equilibrium prices to a bad and a good shock, respectively. The solid line represents prices in the benchmark economy, and the dashed line prices in the economy with career concerns. At time zero productivity drops to the lowest possible realization in the first case and rises to the highest possible one in the second case.

In the following proposition, we propose an equilibrium very similar to the baseline equilibrium described in Proposition 9. The only major difference is the pricing rule described by (26) and (27). Although prices of the two bonds are still determined by indifference conditions of untalented managers regarding all available strategies, now this implies that the price of each risky bond will depend on both shocks.

Proposition 11 *Suppose that* $\{\hat{a}^{s*}(\cdot), p^{s*}(\cdot)\}$ $s = A, B$ *solves the fixed point defined as follows:*

1. *given* $\hat{a}^A(\cdot)$ $\hat{a}^B(\cdot)$, $p^{s*}(\cdot)$ *solves the pricing rule, that is,*

$$p^{s*}(a_t) = \frac{1 - \int_0^{\hat{a}^s(a_t^s)} \int_0^{\infty} dF\left(a_{t+1}^A, a_{t+1}^B | a_t^A, a_t^B\right)}{W\left(a_t^A, a_t^B\right) - \delta \int_{\hat{a}^s(a_t^s)}^{\infty} \int_0^{\infty} W\left(a_{t+1}^A, a_{t+1}^B\right) dF\left(a_{t+1}^A, a_{t+1}^B | a_t^A, a_t^B\right)}, \tag{26}$$

where $W(\cdot)$ *satisfies*

$$\begin{aligned} W\left(a_t^A, a_t^B\right) &= R + . \int_0^{\hat{a}^A\left(a_t^A\right)} \int_0^{\infty} W\left(a_{t+1}^A, a_{t+1}^B\right) dF\left(a_{t+1}^A, a_{t+1}^B | a_t^A, a_t^B\right) \\ &\quad + \int_0^{\hat{a}^B\left(a_t^B\right)} \int_{\hat{a}^A\left(a_t^A\right)}^{\infty} W\left(a_{t+1}^A, a_{t+1}^B\right) dF\left(a_{t+1}^A, a_{t+1}^B | a_t^A, a_t^B\right) \end{aligned} \tag{27}$$

2. given $p^s(\cdot)$, $\hat{a}^{s*}(\cdot)$ solves the repayment rule, that is,

$$\hat{a}^s\left(a_t^s\right)=\frac{\exp\left\{D^s\left(b^s\left(a_t^s\right)\right)\right\}}{\exp\left\{D^s\left(b^s\left(a_t^s\right)\right)\right\}-1}b^s\left(a_t^s\right), \tag{28}$$

where $b^s(\cdot)$ satisfies

$$\begin{aligned}&\frac{p^s\left(a_t^s\right)}{\left(p^s\left(a_t^s\right)b\left(a_t^s\right)-1\right)}-\beta\int_{\frac{\exp\left\{D^s\left(b^s\left(a_t^s\right)\right)\right\}}{\exp\left\{D^s\left(b^s\left(a_t^s\right)\right)\right\}-1}b^s\left(a_t^s\right)}^{\infty}\int_0^{\infty}\frac{1}{\left(a_{t+1}^s-b^s\left(a_t^s\right)\right)}dF\left(a_{t+1}^A,a_{t+1}^B|a_t^A,a_t^B\right) \\ &-\beta F\left(\frac{\exp\left\{D^s\left(b^s\left(a_t^s\right)\right)\right\}}{\exp\left\{D^s\left(b^s\left(a_t^s\right)\right)\right\}-1}b^s\left(a_t^s\right)\right)D^{s\prime}\left(b^s\left(a_t^s\right)\right) \\ =\ &0;\end{aligned} \tag{29}$$

then there exists a $\bar{\delta}<1$ and a $\bar{t}>0$ such that if $\delta>\bar{\delta}$ and the second risky bond is introduced at time $t>\bar{t}$, $\{\hat{a}^{s*}(\cdot),p^{s*}(\cdot)\}$ for $s=A,B$ is a Markov-equilibrium with the investors' firing rule and managers' strategies analogous to (13) and (14).

Proof. See the Appendix. ■

Observe that the prices of the two risky bonds are related through the common terms in the denominator in (26), even if the two group-specific shocks are independent.

In the rest of this section, first, we derive analytical results for the special case of i.i.d. shocks to highlight the intuition behind the reputational link between prices of different risky bonds. Second, we show numerical results that illustrate the reputational link in the general case.

5.2 The stationary equilibrium

Let us suppose that the shock which affects the two groups of borrowers are independent across time:

$$F\left(a_{t+1}^A,a_{t+1}^B|a_t^A,a_t^B\right)=F\left(a_{t+1}^A,a_{t+1}^B\right).$$

Similarly to the baseline model, this assumption results in a stationary equilibrium. Let us use the notation

$$\begin{aligned}q^{AB}&=F\left(\hat{a}^A,\hat{a}^B\right)\\ q^s&=\int_0^{\hat{a}^s}\int_0^{\infty}f\left(a^A,a^B\right)da^Ada^B\text{ for }s=A,B\end{aligned}$$

where $\hat{a}^s$ is the equilibrium cut-off to default for group s, q^s is the probability that group s alone defaults, and q^{AB} the probability that both groups default.

As in the baseline case, untalented managers have to be indifferent to invest in any bond. The indifference conditions take the form

$$(1-q^s)\left(\frac{1}{p^s}+\delta W\right)=R+\left(q^A+q^B-q^{AB}\right)\delta W \text{ for } s=A,B \tag{30}$$

Using these indifference conditions, the value function (27) reduces to

$$W=\frac{R}{1-\delta\left(-q^{AB}+q^A+q^B\right)}.$$

Thus, the pricing rule becomes

$$p^s=\frac{(1-q^s)}{R}\frac{1-\delta\left(q^A+q^B-cov_{AB}-q^Aq^B\right)}{1-\delta\left(1-q^s\right)}$$

for $s=A,B$, where the covariance between the payoffs of the bonds A and B is $cov_{AB}=q^{AB}-q^Aq^B$.

Observe that the price of any of the two risky bonds is decreasing in the probability of default of the other bond, regardless of the covariance between the payoffs of the bonds, since

$$\frac{\partial p^s}{\partial q^{s'}}=1-\delta\frac{\left(1-q^s\right)^2}{R\left(1-\delta+q^s\delta\right)}<0$$

for $s\neq s'$. The return of a bond increases in the probability of default of any of the two bonds. The intuition is immediate from the indifference conditions (30). The reputational cost of investing in the riskless asset depends on the probability that at least one of the bonds defaults. If none of the bonds defaults, the manager who invested in the risk-less bond is perceived to be untalented and loses his job. Thus, if the probability of default of any of the risky bonds decreases, the riskless asset will be less attractive, so the prices of both bonds have to go up to keep managers indifferent between different strategies.

The equilibrium price of the bond s, p^s, is determined by the intersection of the pricing rule of bond s and the repayment rule of group s given by $F^s\left(\hat{a}^s\left(b^s\left(p^s\right)\right)\right)$ where $\hat{a}^s(\cdot)$ and $b^s(\cdot)$ are determined as in the 1-asset case.

5.3 A numerical example

We consider two symmetric groups of borrowers with the same fundamentals as in our numerical example of the baseline case. The productivity processes of the two countries follow the same Markov-process. Assume that there is no fundamental link between the two bonds, that is, the two productivity processes are independent:

$$F\left(a^A_{t+1},a^B_{t+1}|a^A_t,a^B_t\right)=F\left(a^A_{t+1}|a^A_t\right)F\left(a^B_{t+1}|a^B_t\right).$$

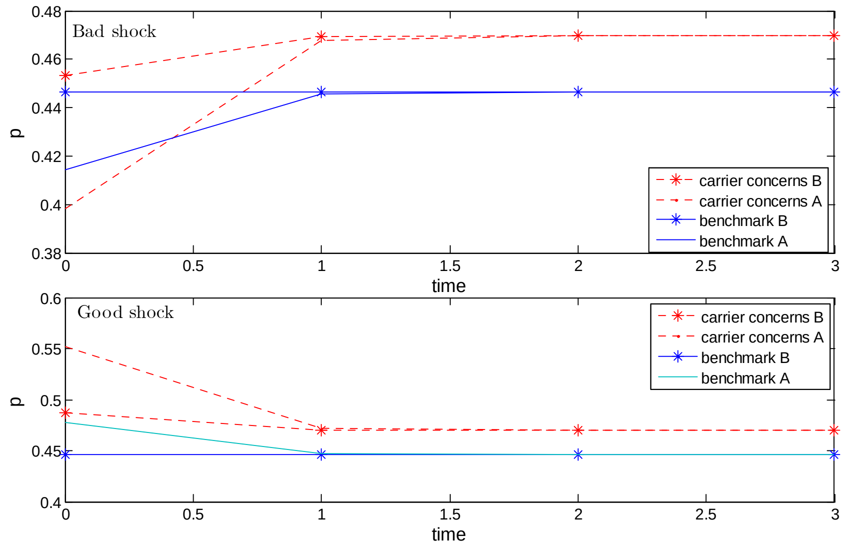

Figure 7: The two panels show the reaction of the equilibrium prices to a bad and a good shock, respectively. Dashed lines show the price response with career concerns.Solid lines show the price response in the benchmark case. Starred lines show the price response of the bond of the group with unaffected productivity process. At time zero, productivity of one group drops to the lowest possible realization in the first case and rises to the highest possible one in the second case. The productivity of the other group is unaffected.

We conduct an experiment very similar to the one which leaded to Figure 6. At time 0, the economy of group A is hit by a large negative or positive shock. We check how the prices of both bonds react to these shocks with and without career concerns. The results are shown in Figure 7. Dashed lines show the price responses with career concerns. Solid lines show the price responses in the benchmark case. Starred lines show the price responses of the bond of the group with unaffected productivity process. Naturally, with no career concerns the price of the bond with unaffected fundamentals remains constant. However, with career concerns both prices respond to the shock. There is a reputational link which leads to comovement in bond prices, even if the underlying fundamentals are independent.

6 Conclusion

As the economic and financial conditions change, markets can overreact generating excess volatility of spreads, capital flows and economic activity. Is it possible in a rational model that fund managers buy risky bonds with expected returns smaller than the riskfree rate?

In this paper we have introduced an equilibrium model of delegated portfolio management with endogenous default. In our model, investors hire fund managers to invest their capital either in a defaultable bond or in a riskless one. Looking at the past performance, investors update their beliefs on the information of their fund managers. This leads to career concerns that affect the funds' investment decisions, generating a "reputational premium". When the probability of default is sufficiently high, fund managers prefer to invest in safe bonds even at a lower expected return to reduce the probability of being fired. The reputational premium can switch sign in response to shocks, both to the financial market and to the fundamentals of the borrowers (for example, to the economic conditions of an emerging economy). This can generate an overreaction of the market leading to excess volatility of spreads, capital flows, and output. In an extension, we also show that the presented reputational mechanism can lead to contagion between assets with no fundamental links.

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Appendix

A.1 Proof of Proposition 1

Using expression (9), we can rewrite the first order condition of the problem (8) as

$$\frac{p_t}{p_t b_{t+1} - I} - h\left(b_{t+1}\right) = 0. \tag{31}$$

By deriving with respect to b_{t+1}, we obtain the second order condition

$$-\left(\frac{p_t}{p_t b_{t+1} - I}\right)^2 - \frac{\partial h\left(b_{t+1}\right)}{\partial b_{t+1}} < 0.$$

When $D\left(\cdot\right)$ satisfies (10), the second order condition is immediately satisfied, completing the proof.

A.2 Proof of Lemma 2

1. First, notice that for a given a_t

$$\frac{dF\left(\hat{a}_{t+1}\right)}{dp_t} = \frac{dF\left(\hat{a}_{t+1}\right)}{\hat{a}_{t+1}} \frac{d\hat{a}_{t+1}}{b_{t+1}} \frac{db_{t+1}}{dp_t}. \tag{32}$$

For given a_t, let us define the function

$$\Phi\left(p_t, b_{t+1}\right) \equiv \frac{p_t}{p_t b_{t+1} - I} - h\left(b_{t+1}\right).$$

The first order condition (31) implies $\Phi\left(p_t, b_{t+1}\right) = 0$. Applying the implicit function theorem, we obtain

$$\frac{db_{t+1}}{dp_t} = -\frac{\frac{\partial \Phi\left(p_t, b_{t+1}\right)}{\partial p_t}}{\frac{\partial \Phi\left(p_t, b_{t+1}\right)}{\partial b_{t+1}}}, \tag{33}$$

where

$$\frac{\partial \Phi\left(p_t, b_{t+1}\right)}{\partial p_t} = -\frac{p_t b_{t+1}}{\left(p_t b_{t+1} - 1\right)^2},$$

and $\partial \Phi\left(p_t, b_{t+1}\right)/\partial b_{t+1} < 0$ because it coincides with the second order condition of problem 4, which is satisfied by lemma 1. It follows that $db_{t+1}/dp_t < 0$. Moreover, by differentiating (7), we get

$$\frac{d\hat{a}_{t+1}}{db_{t+1}} = \frac{1}{1 - \exp\left\{-D\left(b_{t+1}\right)\right\}} \left[1 - \frac{D'\left(b_{t+1}\right) b_{t+1}}{\exp\left\{D\left(b_{t+1}\right)\right\} - 1}\right]$$

and the assumption that $D\left(b_{t+1}\right) > \log\left(1 + b_{t+1} D'\left(b_{t+1}\right)\right)$ implies immediately that $d\hat{a}_{t+1}/db_{t+1} > 0$. Hence, from (32), it follows that $dF\left(\hat{a}_{t+1}\right)/dp_{t+1} < 0$, competing the proof of the first claim of the proposition.

2. Notice that

$$\frac{dp_t b_{t+1}}{dp_t} = b_{t+1} + p_t \frac{db_{t+1}}{dp_t},$$

where from (33)

$$\frac{db_{t+1}}{dp_t} = \frac{p_t b_{t+1}}{(p_t b_{t+1} - I)^2} \left[h\left(b_{t+1}\right)^2 + \frac{\partial h\left(b_{t+1}\right)}{\partial b_{t+1}} \right]^{-1}.$$

After some algebra, we obtain

$$\frac{dp_t b_{t+1}}{dp_t} = b_{t+1} \left[1 + h\left(b_{t+1}\right)^2 \left[h\left(b_{t+1}\right)^2 + \frac{\partial h\left(b_{t+1}\right)}{\partial b_{t+1}} \right]^{-1} \right]$$

Using condition (10), we see that $dp_t b_{t+1}/dp_t > 0$, completing the proof.

A.3 Proof of Proposition 2

Let us define ρ_t as the proportion of talented managers that are not hired at the beginning of period t. After a mass $(1-\delta)\,2\Gamma$ of unemployed managers die and a new mass $(1-\delta)\,2\Gamma$ is born, there are going to be $[\delta\rho_t + (1-\delta)\,2\bar{\varepsilon}]\,\Gamma$ of untalented unemployed and a total mass of $(2-\delta)\,\Gamma$ unemployed mangers. Hence, the probability that an unemployed manager is talented is $\varepsilon_{t+1} \equiv [\delta\rho_t + (1-\delta)\,2\bar{\varepsilon}] \,/\, (2-\delta)$. The talented unemployed managers at time $t+1$ are going to be the ones that were untalented at time t and were not fired, minus the proportion of talented in the ones that were newly hired. Define μ_t^c the proportion of untalented managers who make the same investment decisions of the talented guys and hence are not fired, that is,

$$\mu_t^c = (1-\mu(0))\,q_t + \mu(0)\,(1-q_t)\,.$$

It follows that $\rho_{t+1} = F\left(\rho_t, a_{t+1}\right)$ where

$$F\left(\rho_t, a_{t+1}\right) = [\delta\rho_t + (1-\delta)\,2\bar{\varepsilon}] - \{1 - \delta\,[(2\bar{\varepsilon} - \rho_t) - (1 - (2\bar{\varepsilon} - \rho_t))\,\mu_t^c]\}\,\varepsilon_{t+1} \tag{34}$$

Lemma 4 *The following series of inequalities hold for any* $t \geq 0$:

$$\bar{\varepsilon} > \rho_{t+1} > 0.$$

Proof. The proof proceeds by induction. We know that $\rho_0 = \bar{\varepsilon}$. Then

$$\rho_1\left(a_{t+1}, \bar{\varepsilon}\right) = \bar{\varepsilon}\left(1 - \left(\delta\left(1-\bar{\varepsilon}\right)\left(1 - \mu_t^U\left(a_{t+1}\right)\right)\right)\right)$$

and $\bar{\varepsilon} > \rho_1 > 0$, so the statement is true for ρ_1. Now let us suppose that it is true for ρ_t. Observe that

$$\frac{2\bar{\varepsilon}(1-\delta)+\delta\rho_t}{2-\delta} > 0$$

so (34) is increasing in μ_t^c. So for any fixed ρ_t, (34) is maximal when $\mu_t^c = 1$ and minimal when $\mu_t^c = 0$. first we show that for any $\bar{\varepsilon} > \rho_t > 0$, $\bar{\varepsilon} > \rho_{t+1}$. It is true for all μ_t^c if it is true for $\mu_t^c = 1$. But

$$\rho_{t+1}(1,\rho_t) = \frac{\delta}{2-\delta}\rho_t + 2\bar{\varepsilon}\frac{1-\delta}{2-\delta}$$

which is increasing in ρ_t and

$$\rho_{t+1}(1,\bar{\varepsilon}) = \bar{\varepsilon}$$

so

$$\rho_{t+1}(\mu_t^c,\rho_t) < \bar{\varepsilon}$$

for any $\mu_t^c \in (0,1) .\rho_t < \bar{\varepsilon}$. Now observe that

$$\rho_{t+1}(0,\rho_t) = -\frac{\delta^2}{2-\delta}\rho_t^2 + \delta\frac{1-\delta-2\bar{\varepsilon}+4\delta\bar{\varepsilon}}{2-\delta}\rho_t + 2\bar{\varepsilon}(1-\delta)\frac{1-\delta+2\delta\bar{\varepsilon}}{2-\delta}$$

is quadratic and concave and

$$\rho_{t+1}(0,0) = 2\bar{\varepsilon}(1-\delta)\frac{1-\delta+2\delta\bar{\varepsilon}}{2-\delta} > 0$$

and

$$\rho_{t+1}(0,\bar{\varepsilon}) = \bar{\varepsilon}(1-(1-\bar{\varepsilon})\delta) > 0$$

This implies that

$$\rho_{t+1}(\mu_t^c,\rho_t) > \rho_{t+1}(0,\rho_t) > 0$$

for all $\rho_t \in (0,\bar{\varepsilon})$. ■

The proof Proposition 2 proceeds in 3 steps: first, we show that given the equilibrium firing rule, the investment strategies are optimal, second we show that given the equilibrium investment strategies, the firing rule is optimal, third we show that given that the optimal investment strategy for the untalented managers is mixed, the equilibrium price is derived.

Step 1. Suppose that managers follow the investment strategies (14), that is, follow their signal when there are talented and randomize when they are not. Then, given that $1/p_t > R$, it follows that the managers will always prefer to hire talented managers who never make "mistakes". Suppose investor j has hired at the beginning of time t a manager. At the end of the period the investor observes the investment realization $\tilde{\mu}_t^j$ and the productivity realization a_{t+1}. Then if $a_{t+1} \leq \hat{a}_{t+1}$ and $\tilde{\mu}_t^j = 1$, or $a_{t+1} > \hat{a}_{t+1}$ and $\tilde{\mu}_t^j = 0$, he realizes that the his manager is not talented, that is, $\eta_{t+1}^j = 0$, and fires him, given that there are no cost of firing, while there is always a positive probability that a new manager is talented, that is, $\rho_{t+1} > 0$, from the previous lemma. On the other hand, if the manager does not make a mistake, that is, if $a_{t+1} \leq \hat{a}_{t+1}$ and $\tilde{\mu}_t^j = 0$, or $a_{t+1} > \hat{a}_{t+1}$ and $\tilde{\mu}_t^j = 1$, then he does not fire him if and only if the updated belief on the manager η_{t+1}^j is higher than the probability that a new hire is talented, that is, $\eta_{t+1}^j \geq \varepsilon_{t+1}$. In this case,

$$\begin{aligned}
\eta_{t+1}^j &= \zeta(\eta_t^j, \tilde{\mu}_t^j, a_{t+1}) = \frac{\Pr(\tilde{\mu}_t^j = \mu_t^I\left(a_{t+1}\right)|s_t \neq 0)\eta_t^j}{\Pr(\tilde{\mu}_t^j = \mu_t^I\left(a_{t+1}\right)|s_t \neq 0)\eta_t^j + \Pr(\tilde{\mu}_t^j = \mu_t^I\left(a_{t+1}\right)|s_t = 0)(1-\eta_t^j)} \\
&= \frac{\eta_t^j}{\eta_t^j + \mu_t^c(1-\eta_t^j)}
\end{aligned}$$

Notice that if the manager is a new hire, then $\eta_t^j = 2\bar{\varepsilon} - \rho_{t+1}$. In this case

$$\eta_{t+1}^j = \frac{2\bar{\varepsilon} - \rho_{t+1}}{2\bar{\varepsilon} - \rho_{t+1} + \mu_t^c\left(1 - \left(2\bar{\varepsilon} - \rho_{t+1}\right)\right)}.$$

Given that $\mu_t^c \in [0,1]$, then

$$\eta_{t+1}^j \geq 2\bar{\varepsilon} - \rho_{t+1} \geq \frac{2\bar{\varepsilon}\left(1-\delta\right) + \delta\rho_{t+1}}{2-\delta},$$

and hence $\eta_{t+1}^j \geq \varepsilon_{t+1}$ given that from previous lemma $\rho_{t+1} < \bar{\varepsilon}$. Moreover, notice that the investors' believes about any other manager who is working but was hired before time t must be higher than the one that has been hired at time t, given that if he was not fired he never made any mistake. Hence, a fortiori, $\eta_{t+1}^j \geq \varepsilon_{t+1}$, completing the proof.

Step 2. Suppose now that the investors follow the strategy (13).

1. We show that talented managers must follow their signal, that is, $\mu\left(d\right) = 0$ and $\mu\left(n\right) = 1$. Let us suppose that this is not the case. There are two alternatives. First, notice that $\mu\left(d\right) = 1$ and $\mu\left(n\right) = 0$, that is, talented managers always act against their signal, cannot be optimal for them. The second possibility is that talented managers are indifferent between investing in risky bonds or risk-free ones. Let assume that $a_{t+1} \leq \hat{a}_{t+1}$ and $s_t^j = d$. If j is indifferent at time t, given that we are looking at stationary equilibria, he

must be indifferent for any t. Hence, we can evaluate his expected utility as if he follows the strategy $\mu(n) = \mu(d) = 1$. Given that he knows that $a_{t+1} \leq \hat{a}_{t+1}$, that is, that $q_t = 1$, then he knows his expected utility is zero, while if he had chosen $\mu(d) = \mu(n) = 0$, then his expected utility would have been $W = \gamma R/(1-\delta) > 0$, yielding a contradiction. Similarly, one can prove that if $a_{t+1} > \hat{a}_{t+1}$, the talented manager cannot be indifferent. It follows that the talented managers must follow their signals.

2. We show that untalented managers must adopt a mixed strategy, that is, $\mu(0) \in (0,1)$. The market clearing condition for the risky bond market can be written as

$$[(2\bar{\varepsilon} - \rho_t) + (1 - (2\bar{\varepsilon} - \rho_t))\,\mu_t(0)]\,\Gamma = pb(p)\,G(V(p)).$$

Notice that assumption (1) implies that there must always be some untalented managers that invest in the risky bonds and always some that do not. If no untalented manager was investing in risky bonds, then there would be excess supply, since

$$(2\bar{\varepsilon} - \rho_t)\,\Gamma < \frac{I}{\bar{\bar{b}}} B\left(\frac{I}{\bar{\bar{b}}}\right).$$

If instead all the untalented managers were investing in the risky bonds, even if no untalented was doing the same, there would be excess demand, since

$$(1 - (2\bar{\varepsilon} - \rho_t))\,\Gamma > \frac{1}{R} B\left(\frac{1}{R}\right).$$

This implies that untalented managers must be indifferent, completing the proof of step 2.

Step 3. Given steps 1 and 2, the equilibrium price condition (12) comes directly from the indifference condition for the untalented guys (15) that is obtained by combining the managers' optimization problem (6) and the firing rule (13), completing the proof.

A.4 Proof of Proposition 5

We prove the existence of an equilibrium by showing that there exists a fixed point of the pricing rule and the repayment rule defined in definition 3 for the special case of i.i.d. productivity shock. The proof proceeds in two steps: first, we show that for a default probability low enough the price demanded by the financial side of the market is lower than the price offered by the emerging country; second, we show that for a default probability big enough the opposite is true. This implies that there must be a fix point.

Step 1. First, suppose the default probability is equal to $F(\overline{a})$, where

$$\overline{a} = \frac{\overline{b}}{1 - \exp\left\{-D(\overline{b})\right\}}.$$

Then, according to the pricing rule, the price required by the financial market would be

$$\overline{p}^D = \frac{(1 - F(\overline{a}))(1 - \delta F(\overline{a}))}{R(1 - \delta(1 - F(\overline{a})))},$$

while, according to the repayment rule, the price that would implement that default decision by the borrowers would be $\overline{p}^S$ such that

$$\frac{1}{\overline{b} - I/\overline{p}^S} - \int_{\overline{a}}^{\infty} \frac{1}{(a - \overline{b})} dF(a) + F(\overline{a}) D'\left(\overline{b}\right) = 0.$$

This expression pins down only the ratio $I/\overline{p}^S \equiv \alpha$. Hence, we can ensure that $\overline{p}^S < \overline{p}^D$, by choosing $I \leq \hat{I}$, where

$$\hat{I} = \alpha \frac{(1 - F(\overline{a}))(1 - \delta F(\overline{a}))}{R(1 - \delta(1 - F(\overline{a})))},$$

completing this step of the proof.

Step 2. Now, consider $p = 1/R$. From the pricing rule, one can show that this price is required by the financial market, when the default probability $F(\hat{a})$ satisfies

$$F(\hat{a}) = \frac{1}{2\delta}\left(1 + 2\delta - \sqrt{1 + 4\delta}\right)$$

On the other side, given $p = 1/R$, the borrowers will decide to default with probability $F(\hat{a}(1/R))$, where

$$\hat{a}(1/R) = \frac{\underline{b}}{1 - \exp\left\{-D(\underline{b})\right\}},$$

where $\underline{b}$ is such that

$$\frac{1}{\underline{b} - R * I} - \int_{\frac{\underline{b}}{1 - \exp\{-D(\underline{b})\}}}^{\infty} \frac{1}{(a - \underline{b})} dF(a) + F\left(\frac{\underline{b}}{1 - \exp\left\{-D(\underline{b})\right\}}\right) D'(\underline{b}) = 0.$$

The assumption that $F(\hat{a}(1/R)) > (1 + 2\delta - \sqrt{1 + 4\delta})/2\delta$, together with the fact that both the repayment rule and the pricing rule are monotonically decreasing, ensures that when the default probability is $F(\hat{a}(1/R))$, the price required by the financial market $\underline{p}^D$ is smaller than $1/R$, completing the proof.

A.5 Proof of Proposition 8

The equilibrium condition is

$$f\left(b', I\right) = \frac{p}{\left(pb' - I\right)} - \int_{\hat{a}(b')}^{\infty} \frac{1}{\left(a' - b'\right)} dF\left(a'\right) - F\left(\hat{a}\left(b'\right)\right) \frac{dD\left(b'\right)}{db'} = 0.$$

Using the implicit function theorem it is straightforward to show that

$$\frac{db'}{dI} = -\frac{\frac{\partial f(b',I)}{\partial I}}{\frac{\partial f(b',I)}{\partial b'}} > 0$$

given that

$$\frac{df}{dI} = \frac{p}{\left(pb' - I\right)^2} > 0$$

and $\partial f\left(b', I\right)/\partial b' < 0$ by Lemma 1. Hence, from equation (7) we get that $d\hat{a}\left(b'\right)/dI > 0$ and hence $dF\left(\hat{a}\left(b'\right)\right) > 0$, completing the proof.

A.6 Proof of Proposition 9

If $(p^*\left(a_t\right), \hat{a}^*\left(a_t\right))$ is the fixed point of the system described in the Proposition, then untalented managers are indifferent whether to invest in the risky bond or the riskless asset and each borrower optimize her value function for the given price. For the proposed firing rule and $(p^*\left(a_t\right), \hat{a}^*\left(a_t\right))$, talented managers prefer to follow their signals, as in each period this provides both a larger monetary gain and a larger probability of being rehired. Given the strategies of managers, investors' firing rule is optimal as it is shown in Proposition (2).

A.7 Proof of Proposition 11

Observe first that if $\delta \to 1$, the fixed point defined as the unique solution, $\varepsilon^* \in [0, \bar{\varepsilon}]$

$$\varepsilon_{t+1}\left(\mu_t^c, \varepsilon^*\right) = \varepsilon^*$$

converges to zero for any μ_t^c as

$$\lim_{\delta \to 1} \left(\varepsilon_{t+1}\left(\mu_t^c, \varepsilon_t\right) - \varepsilon_t\right) = \varepsilon_t\left(1 - \mu_t^c\right)\left(2\varepsilon - \varepsilon_t - 1\right).$$

Thus, if $\delta = 0$, as $t \to \infty$, $\varepsilon_t \to 0$, regardless of the changes in μ_t^c. Consequently, for any small positive number υ, there must be a critical threshold $\bar{\delta}$ and $\bar{t}$ that if $\delta > \bar{\delta}$, there is a $\bar{t}$, that for any $t > \bar{t}$, $\|\varepsilon_t\| < \upsilon$.

Then, observe that if managers follow the described optimal strategy, it is optimal for investors to follow the described strategy if and only if the following two conditions hold. First, the probability that a manager is talented if he invested in the riskless asset and there is no default in any of the markets must be larger than the probability that an unhired manager is talented, that is,

$$\Pr\left(s_t = d^s | \mu_t = 0, a^A\left(a_{t+1}\right) < \hat{a}\left(a_{t+1}\right), a^B\left(a_{t+1}\right) < \hat{a}\left(a_{t+1}\right)\right) =$$
$$= \frac{\left(2\bar{\varepsilon} - \varepsilon_t\right)}{\left(2\bar{\varepsilon} - \varepsilon_t\right) + \left(1 - \mu_t^A - \mu_t^B\right)\left(1 - \left(2\bar{\varepsilon} - \varepsilon_t\right)\right)} \geq \frac{2\bar{\varepsilon}\left(1 - \delta\right) + \delta\varepsilon_t}{2 - \delta}.$$

Just as in the 1-asset case, this condition always holds as $(2\bar{\varepsilon} - \varepsilon_t) > \varepsilon_t$. Second, the probability that a manager is talented if he invested in one of the risky bonds and that bond did not default must be higher than the probability that an unhired manager is talented.

$$\Pr\left(s_t = n^s | \mu_t^s = 1, a^A\left(a_{t+1}\right) > \hat{a}\left(a_{t+1}\right)\right) =$$
$$= \frac{\frac{1}{2}\left(2\bar{\varepsilon} - \varepsilon_t\right)}{\frac{1}{2}\left(2\bar{\varepsilon} - \varepsilon_t\right) + \mu_t^s\left(1 - \left(2\bar{\varepsilon} - \varepsilon_t\right)\right)} \geq \frac{2\bar{\varepsilon}\left(1 - \delta\right) + \delta\varepsilon_t}{2 - \delta}.$$

Observe that as $\delta \to \infty$, and, consequently, $\varepsilon_t \to 0$, this condition must hold.

Given the optimal firing rule of investors, managers' strategy is optimal if untalented managers are indifferent between investing in the risky bond or in the riskless asset. This is the case if (26) holds. Given the price, borrowers' optimal strategy is given by the first order condition (29).