# Sovereign Debt, Default and Renegotiation

# with Endogenous Financial Contracting

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**Abstract**

This paper reconsiders the theory of sovereign debt, default and renegotiation in a setting that motivates conventional bond finance. A tax-smoothing model of government borrowing on international financial markets is used to analyze self-enforcing equilibrium in a private information economy. The main contribution of the model is to show how private debtor information can be used to motivate the implementation of borrowing through the issuance of conventional bonds that are renegotiated only in adverse states after debt has reached a sufficiently high level. The qualitative features of this equilibrium correspond favorably to the empirical experience of sovereign borrowing and default. The paper also considers the role of nominal public debt issues for achieving contingent repayments with unanticipated inflation. It shows that nominal public debt does not lead to unanticipated inflation outside adverse states with already high outstanding public debt.

## 1. Introduction

This paper revisits theoretical analyses of sovereign borrowing with renegotiation in a model of public finance that motivates the use of conventional bond contracts. The theoretical literature on international debt emphasizes the importance of a sovereign's willingness to repay for determining its ability to borrow following Eaton and Gersovitz (1981). Thus, the choice of whether to repay or default is based on an optimization calculation by the government. This notion applies to all debt issued by a sovereign whether it is held by foreign or domestic creditors in the approach of this paper. Outside exceptional cases, sovereign default results in debt renegotiation or is technically avoided by a debt restructuring. Theoretical models of debt renegotiation that take an optimal contracting approach interpret debt renegotiation as implementation of an implicit state-contingent contract. Such models imply that default is a frequent event. Recent research that reconsiders the experience of sovereign default in the wake of the Argentine default either returns to the original models with no renegotiation (for example, Arellano (2004)) or impose exogenous costs for default and debt restructurings that reduce the frequency of default (for example, Yue (2005)).

A different approach based on optimal contracting with private information and sovereign immunity is used in this paper to develop a model in which conventional bonds that are infrequently renegotiated implement equilibrium. The structure of incomplete financial market is endogenous. This approach yields several empirically observed phenomena. The first of these is that default and renegotiation are not frequent events that occur at all debt levels but, instead, sovereigns only default when highly in adverse states of nature. The second is that sovereign liabilities are structured as conventional bond contracts paying noncontingent interest when the sovereign's willingness to pay is not binding. A third implication is that bonds are only risky after a threshold level of indebtedness is reached, so that yield spreads on sovereign debt only rise with indebtedness for already highly indebted countries. Equilibrium renegotiations in the model also involve a maximum loss for creditors of the accumulated net opportunity interest. All of these correspond to empirical findings in the recent or historical record.

The absence of an international sovereign authority to define and enforce contractual obligations across borders constrains national fiscal policies in an integrated financial market. Both international and domestic public borrowing are possible to the extent that sovereign issuers willingly honor their liabilities aware of the consequences of default. While governments can waive sovereign immunity with regard to debtor assets abroad, they cannot credibly forgo immunity with respect to domestic policy, legislation, and enforcement. Sovereign immunity protects a debtor government's power to tax sources of income and wealth within national borders. The enforcement of private contracts between parties subject to different national jurisdictions also requires the enlightened self-interest of sovereign governments.

Sovereign immunity serves as a fundamental assumption in this paper to derive intertemporal budget constraints for sovereign nations and their governments. The capacity to finance current aggregate consumption, investment, and government expenditures from global savings is determined endogenously given that a national government acts only in the interest of its constituents while recognizing the consequences of its actions for future transactions and international cooperation. Limits on public debt and deficits are derived in a tax-smoothing model by finding an efficient global equilibrium with international financial flows constrained by borrower sovereign immunity and the willing participation of private creditors. The securities issued by sovereigns are endogenously determined; that is, the completeness or incompleteness of securities markets is determined in self-enforcing equilibria with and without restrictions on the information available to potential creditors.

The tax-smoothing model is first used to show how contingent securities are required in international financial market equilibrium when all information about the government is public. The next step is to introduce asymmetric information about shocks to the government's objective. These are only known to the government and cannot be verified by creditors even at a cost. The incentive compatible equilibria for tax smoothing is characterized and shown to implemented by contracts that still have state-contingent repayments for all debt levels. An essential feature of sovereign debt markets is that riskier borrowers have short maturity debt and roll over outstanding obligations by negotiating new loans or issuing new bonds. Relationships between a sovereign and its creditors is open to sequential entry by creditors. Thus, the long-term relationship modeled by repeated agency is not a very accurate portrayal of sovereign borrowing outside default. Sequential entry introduces unmonitored borrowing which rules out state-contingent contracts under private information about the government's objective and is shown to lead to an incentive compatible implemented by conventional loan contracts. With the additional constraints implied by willingness to pay, a constrained efficient equilibrium requires contingent repayment only when the outstanding government debt exceeds a threshold. This implies that entry must be detered after a high-debt threshold is reached so that state-contingent repayments can be implemented. Self-enforcement can allow this once the debt limit is reached if conventional bonds are renegotiated each period. Doing so allows the repayments to be contingent on the actions of other potential creditors.

The information assumptions used here to derive a constrained optimum implemented by conventional bonds contrasts with costly state verification. Verification serves to communicate private information to third parties that enforce the contractual relationship between the principal and the agent. In the sovereign borrowing setting, third-party enforcement, if it matters, arises endogenously as an action taken in self interest by other creditors or other sovereigns. Self-enforcement depends on the capacity of parties to the contractual relationship to observe payoff relevant information or to agree to contracts that elicit voluntary revelation of private information. Costly state verification provides information on which contractual arrangement exogenously enforced can be conditioned. The focus on sequential lender entry in debt roll overs is motivated by the problem that potential creditors act in their own self interest taking account of borrower past actions only as these affect their private returns. In the paper, entry is only used to motivate debt contracting without discussing the relationship between entry, debt dilution and the short maturity of sovereign liabilities noted elsewhere.[1]

The model is used to reconsider the role of nominal public debt. Efficiency requires that public debt repayments be state contingent in a stochastic environment. A line of research beginning with Lucas and Stokey (1983) studies the role of nominal public debt for achieving state-contingent payments with endogenous or exogenous monetary policy.[2] While several papers (Lucas and Stokey (1983) and Persson and Svensson (1984) in particular) emphasize how denominating public debt in domestic currency creates an inflationary bias for monetary policy, other papers (notably, Bohn (1988, 1990) and Barro (1999, 2003)) emphasize how nominal public debt can increase welfare when the government is restricted to issuing non-contingent (conventional) bonds.[3] With stochastic expenditures in a simple tax-smoothing model, payments to government liabilities need to be contingent on expenditures in every period in the Lucas and Stokey, and Bohn and Barro models.

In the complete information models of Bohn and Barro, unanticipated inflation is an everyday phenomenon just as debt renegotiation is with real-indexed debt. The private information model with endogenous incomplete markets is used to show that unanticipated inflation only arises in a constrained optimum once outstanding public debt is sufficiently high. Unanticipated inflation is associated with high public debt and adverse shocks in the model economy. In the optimum, the maximum creditor losses due to unanticipated inflation are limited to the net real opportunity interest on outstanding debt. The usefulness of domestic currency denominated debt for easing renegotiation in self-enforcing equilibria arises if renegotiation of foreign currency or real indexed debt is costly. The model, therefore, provides a rationale for issuing sovereign liabilities in noncontingent bonds denominated in debtor country currency which is absent from complete information models.

Section 2 sets up the model of public finance in an open economy under sovereign immunity and summarizes the properties of the constrained optimum with complete information as a benchmark. In section 3, the model is extended to allow incomplete information between the government and creditors that motivates conventional non-contingent bonds. The next section uses the analysis to discuss the stylized facts of sovereign debt and default exemplified by the survey of experience by Reinhart, Rogoff and

Savastano (2003) and others. The model is applied to nominal public debt in section 5, and the conclusion follows.

## 2. Sovereignty and International Debt with Complete Information

A simple model of a fiscal authority seeking to smooth distortionary taxes over time against random shocks to domestic demand, following Barro (1979), is used to motivate government debt. The shocks in the model are stochastic exogenous government expenditures for expositional convenience. An alternative approach is to introduce preference shocks reflected in the objective function of the sovereign policymaker. The two approaches will yield the same qualitative conclusions. In the model, fiscal authorities raise tax revenues to make transfers from the domestic private sector to creditors, domestic and foreign. More generally, taxation represents the capacity and willingness of the sovereign to ensure private and public debt repayments. Sovereign borrowing should be interpreted liberally to mean the issuing of any debt securities that can be held by foreign creditors by any debtor potentially protected by the sovereign.

The simple model of tax smoothing assumes that distortionary taxes are required to raise government revenue to pay for public expenditures and make debt service payments. Tax distortions are captured by assuming that output is a decreasing and concave function of tax revenues given by $Y(T)$. This function satisfies $Y(T) > 0$, $Y'(T) \leq 0$, $Y'(0) = 0$ and $Y''(T) \leq 0$ for $0 \leq T \leq \overline{T}$, where $\overline{T} > 0$ is an upper bound on tax revenues. Authorities seek to maximize the utility of a representative agent given by

$$U_t = u(c_t) + E_t \sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s), \tag{1}$$

with respect to the consumption plan, where $c_t = y(T_t) = Y(T_t) - T_t$, subject to the need to finance an exogenous stream of government spending, $g_t$. The function $Y(T_t)$ is taken as given, leaving the optimal choice of individual tax instruments implicit.

Government finance in this simple model should be interpreted as follows. Domestic residents seek to smooth their consumption over time, but households do not have access to internationally integrated financial markets. A portion of domestic consumption cannot be smoothed by the private sector, so the government smooths this consumption by issuing debt in the integrated financial market. For simplicity, government spending will be treated as government expenditures on goods and services. However, the variable, $g_t$, may also be interpreted as the stochastic share of aggregate domestic demand for tradable goods that cannot be completely smoothed on international financial markets by households and firms. Government expenditures, $g_t$, are taken to be independently and identically distributed over a finite support for expositional simplicity.[4] Government expenditures can take on any of $N$ values ordered as

$$0 < g^1 < ... < g^N,$$

where a higher $g$ is a more adverse state.

Substitution into the representative household's utility function allows the government's objective to be rewritten as

$$\widetilde{U}_t = v\left(T_t\right) + E_t \sum_{s=t+1}^{\infty} \beta^{s-t} v\left(T_s\right),$$

where $v\left(T\right) = u\left(Y\left(T\right) - T\right)$ displays negative and decreasing marginal utility in taxes.[5] The trade balance is given by

$$\tau_t = Y\left(T_t\right) - c_t - g_t = T_t - g_t.$$

The government issues securities on a global financial market to finance the primary deficit, $g_t - T_t$. The present value of all financial claims against the government is given by

$$w_t = T_t - g_t + E_t \sum_{s=t+1}^{\infty} \beta^{s-t} \left(T_s - g_s\right). \tag{2}$$

The government can issue new securities to finance current primary deficits or the repayment of retiring debt. Debt holders can trade existing securities on the international financial market. The value of securities issued by the government can be rewritten in the form

$$w_t = -\left(g_t - T_t\right) + \beta E_t w_{t+1}, \tag{3}$$

where $w_{t+1}$ is the market value of outstanding debt conditional on the state in date $t+1$.

Any individual creditor purchases government securities willingly in any period. This assumption is expressed by the participation constraints,

$$w_t \geq 0 \quad \text{and} \quad w_{t+1} \geq 0,$$

for all states and dates. That is, tradable securities can only have nonnegative market values. This rules out securities for which the expected present value can be negative in some future event, such as pure insurance contracts. The conditional expected present value of current and future primary surpluses of the government is restricted to be nonnegative at all dates. The government can only force involuntary payments to it by taxing economic activities within its sovereign domain. It can raise revenue on an integrated financial market by issuing securities that always have nonnegative market values.

Another set of participation constraints is introduced to represent national sovereignty. A sovereign authority can elect to refuse to honor government-issued debt. The capacity of the government to choose to repay only if doing so is in the national interest at the time payments are due is expressed by a participation constraint. At any time, the sovereign issuer can switch to financing public expenditures on a pay-as-you-go basis by defaulting on its current debt and never attempting to issue debt again. The sovereignty constraint is expressed for all dates and states as

$$v\left(T_t\right) + E_t \sum_{s=t+1}^{\infty} \beta^{s-t} v\left(T_s\right) \geq v\left(g_t\right) + E_t \sum_{s=t+1}^{\infty} \beta^{s-t} v\left(g_s\right). \tag{4}$$

An equilibrium with symmetric information between potential creditors and the authorities of the issuing government is characterized first. In equilibrium, securities will have state-contingent payments. In the case of complete state-contingent markets, only the single constraint,

$$w_0 = T_0 - g_0 + E_0 \sum_{t=1}^{\infty} \beta^t \left(T_t - g_t\right) \geq 0, \tag{5}$$

would need to be imposed. With self-enforcing contracts, the constraint, $w_t \geq 0$, is imposed for all dates and states.

The equilibrium is found by maximizing

$$V_t = v(T_t) - v(g_t) + E_t \sum_{s=t+1}^{\infty} \beta^{s-t} \left( v(T_s) - v(g_s) \right) \tag{6}$$
$$= v(T_t) - v(g_t) + \beta E_t V_{t+1},$$

with respect to the current tax revenue, $T_t$, and future repayments, $w_{t+1}$, for each state of nature, subject to the constraints,

$$T_t - g_t + \beta E_t w_{t+1} \geq w_t, \tag{7}$$

$$w_{t+1} \geq 0 \quad \text{and} \quad V_{t+1} \geq 0 \quad \text{for each state.} \tag{8}$$

The surplus for the government, $V_t$, is a function of the outstanding value of its debt, $V_t(w_t)$.[6]

The solution for a self-enforcing equilibrium is familiar from Thomas and Worrall (1988), Kocherlakota (1996), Kletzer and Wright (2000) and Kehoe and Perri (2002). For this tax-smoothing model, the first-order condition for tax revenues can be derived and is given by

$$v'(T_t) = v'(T_{t+1}) \quad \text{if} \quad w_{t+1} > 0 \quad \text{and} \quad V_{t+1}(w_{t+1}) > 0, \tag{9}$$

$$v'(T_t) < v'(T_{t+1}) < 0 \quad \text{if} \quad w_{t+1} = 0 \tag{10}$$

and

$$v'(T_t) > v'(T_{t+1}) < 0 \quad \text{if} \quad V_{t+1}(w_{t+1}) = 0. \tag{11a}$$

The solution for $T_{t+1}$ depends on $g_{t+1}$ and $w_t$ (as does $w_{t+1}$) and can be written as

$$T_{t+1} = \theta(g_t, T_t),$$

since $T_t$ conveys all information about $w_t$.

In equilibrium, taxes are completely smoothed between dates if the participation constraints for neither the government nor its creditors are binding (as in equation (9)). Taxes rise with $g_{t+1}$ if creditors' participation constraints bind (as in inequality (10)), and fall as $g_{t+1}$ declines if the government's participation constraints bind as shown in equation (11a). For independently and identically distributed $g_t$, taxes and the primary surplus are non-decreasing with government expenditures. These are also increasing with the value of outstanding government obligations, $w_t$.

The value of financial claims against the government is state contingent in this equilibrium. As shown by Kletzer and Wright (2000) in the consumption-smoothing interpretation, this equilibrium can be implemented using single-period debt contracts with state-contingent, nonnegative repayments. The amount borrowed by the government at any date $t$ is

$$\ell_t = w_t + g_t - T_t,$$

the sum of outstanding public-sector liabilities, $w_t$, at the beginning of period $t$ and primary deficit for period $t$. Contingent repayments in period $t+1$ are given by setting

$$R_{t+1} = w_{t+1}.$$

An interpretation of the model is that equilibrium can be implemented through continual renegotiation of standard debt contracts with fixed contractual repayments equal to $\max\{w_{t+1}\}$. Renegotiation yields actual (*ex post*) repayments equal to $w_{t+1}$.[7]

The constraint on the government in the state-contingent economy differs from the conventional solvency constraint. By allowing for sovereign immunity, in the broad sense that foreign creditors can only indirectly influence the behavior of national executive, legislative, and judicial authorities by not purchasing assets issued by the country, the country's intertemporal budget constraint is given by

$$w_t = -(g_t - T_t) + \beta E_t w_{t+1} \geq 0 \quad \text{and} \quad w_{t+1} \geq 0, \tag{12}$$

and not by the conventional solvency constraint given by

$$\lim_{s\to\infty} E_t\beta^{s-t}b_{s+1} \leq 0 \quad \text{for all } t\text{, where} \quad \beta b_{s+1} = b_s + (g_s - T_s)\,. \tag{13}$$

The difference is that the conventional solvency constraint requires that the expectation of the present value of the primary surplus be at least as great as the current outstanding debt at all dates,

$$w_t = T_t - g_t + E_t \sum_{s=t+1}^{\infty} \beta^{s-t}(T_s - g_s) \geq b_t, \tag{14}$$

while sovereign immunity imposes the weaker constraint on borrowing that $w_t$ be greater or equal to zero at every date.

In equilibrium, the willingness of bondholders to purchase outstanding debt and accept new debt issues constrains the capacity of the government to smooth the distortionary cost of taxation. Incomplete smoothing arises because the participation constraints for the sovereign borrower can, and will, bind for some histories of realizations of the shock, $g_t$. Using this model to interpret debt renegotiation leads to the basic conclusion that renegotiation implements an implicit state-contingent contract guided by a simple explicit contract that ties a debtor to its creditors. Unfortunately, in the complete information economy, it implies that debt renegotiation is an everyday event, occurring in all but the lowest expenditure state of nature. Allowing asymmetric information about the government's willingness to repay debt can lead to incomplete state-contingent markets and motivate the use of conventional bond contracts without continuous renegotiation.

## 3. Sovereign Borrowing with Private Information

The equilibrium with complete information implies that sovereign borrowing should be characterized by either a rich set of state-contingent securities or very frequent international debt renegotiation. A standard approach for deriving incomplete state-contingent securities is to introduce asymmetric information between debtors and creditors. In the model, a simple way to add incomplete information is to let $g_t$ be observed by national authorities but never by creditors. For example, the variable, $g_t$, can be interpreted as an unobserved taste shock to the government's objective. That interpretation can be motivated because national authorities should be more informed about the government's capacity to transfer resources from the domestic private sector to creditors than are creditors themselves. Those who have achieved political power in a country are likely to know more about the willingness of residents to pay taxes for debt repayment than are individual bondholders or foreign authorities.

The assumption that shocks to expenditures are only observed by the borrowing government requires that equilibrium payments be incentive compatible for the government.[8] In an incentive-compatible equilibrium, the government will reveal the correct realization of $g_t$ to creditors by its current choice of the primary deficit, $g_t - T_t$, which is observed by creditors. The separate components, $g_t$ and $T_t$, are private information. The gains from tax smoothing suggest that higher reported expenditures should be associated with higher primary deficits. In the complete information case, current repayments and new borrowing can be conditioned separately on the publicly observable state, $g_t$. With incomplete information, incentive compatibility requires that the government cannot pay less when $g_t$ is high without repaying more in the future. Otherwise, the government could falsely report high expenditures to lower the present value of its net repayments. In an incentive-compatible equilibrium, a favorable shock (lower $g_t$) should lead to a larger current net repayment (a larger primary surplus) and lower future repayments.

Under private information, an equilibrium is found by maximizing the expected value of government liabilities, $W_t = E_{t-1}w_t$, given a constraint set that includes the incentive-compatibility condition for the government. The expected future surplus for the government, $\mathcal{V}_{t+1} \equiv E_t V_{t+1}$, can be written as a function of the reported current state, denoted $\widetilde{g}_t$, and the value of future repayments to creditors, $W_{t+1}$. Current taxes should also be a function of the reported state. The incentive-compatibility condition requires authorities to be at least as well off reporting the actual state, $g_t$, as reporting any other state. It is written as

$$V_t\left(g_t, g_t\right) \geq V_t\left(g_t, \widetilde{g}_t\right), \tag{15}$$

for each state $g_t$ and all possible $\widetilde{g}_t$, where

$$V_t(g_t, \widehat{g}_t) \equiv v(T_t(\widetilde{g}_t)) - v(g_t) + \beta \mathcal{V}_{t+1}(\widetilde{g}_t).^{9} \tag{16}$$

The properties of incentive-compatible equilibrium with commitment on one or both sides of the market are well known (the one-sided commitment case is analyzed by Thomas and Worrall (1990) and Atkeson and Lucas (1992)). It is helpful to use the first-order conditions for the incentive-compatible equilibrium with full commitment to write the appropriate version of an Euler condition for this model. A standard result is that the incentive compatibility condition will only bind in any state for misreporting the next worst state. That is, the only incentive-compatibility conditions that bind in equilibrium are

$$v\left(T_t\left(g^i\right)\right) - v\left(g^i\right) + \beta \mathcal{V}_{t+1}\left(g^i\right) \geq v\left(T_t\left(g^{i-1}\right)\right) - v\left(g^i\right) + \beta \mathcal{V}_{t+1}\left(g^{i-1}\right),$$

for each state $i = 2, ..., N$. This constraint does not bind in the most favorable state, $g^1$. Another standard property for the repeated-agency problem is that the solution is efficient in the most favorable state. In this tax-smoothing model, the first-order condition for the lowest expenditure state, $g^1$, is given by

$$\frac{1}{v'(T_t)} = W'(\mathcal{V}_{t+1}),$$

where $W(\mathcal{V}_{t+1})$ is the constrained optimal value of outstanding public debt carried forward into period $t+1$. Creditor surplus, $W$ is decreasing in the government's surplus.

The constrained effiicient equilibrium is found by solving for the function $W'(\mathcal{V}_t)$ given the incentive compatibility and participation constraints. The details are given in the appendix which shows the derivation and simplification of the necessary conditions for a constrained optimum. The first-order conditions for the equilibrium for the model yield a pair of difference equations that are solved to derive the Euler conditions for this model. The solution for the derivative of the value function for creditors with respect to the government's surplus is given by

$$W'(\mathcal{V}_{t+1}) = E_t\left[\frac{\theta_{t+1}}{v'(T_{t+1})}\right],$$

where $\theta_{t+1}$ in a state $i$ is a normalized product of the cost of each incentive compatibility constraint,

$$\frac{v'\left(T_{t+1}\left(g^{i+1}\right)\right)}{v'\left(T_{t+1}\left(g^{i}\right)\right)},$$

for all expenditure states below $i$. The exact solution for the first-order condition governing the dynamics of government liabilities is given by

$$\frac{1}{v'\left(T_t\right)}+\gamma_t=E_t\left[\frac{\theta_{t+1}}{v'\left(T_{t+1}\right)}\right], \tag{17}$$

where $\gamma_t$ in state $i$ is a linear function of the $\theta_t$ for states $i$ and $i+1$. Adding self-enforcement constraints converts this equation of motion for tax revenues to an inequality. The inequality is

$$\frac{1}{v'\left(T_t\right)}+\gamma_t\leq E_t\left[\frac{\theta_{t+1}}{v'\left(T_{t+1}\right)}\right]$$

for the case in which the government's constraint binds (it is indifferent between tax smoothing and pay-as-you-go financing) and the opposite when the constraint for its creditors binds (that is, the value of the liabilities is zero).

This condition is similar to the inverse Euler condition,

$$\frac{1}{v'\left(T_t\right)}=E_t\left(\frac{1}{v'\left(T_{t+1}\right)}\right),$$

derived for optimal taxation with observable consumption by Kocherlakota (2005), Golosov and Tsyvinski (2006) and Golosov, Kocherlakota and Tsyvinski (2003). The modification to the inverse Euler condition (17) is due to the assumption that government expenditures are unobservable. Allowing for unobservable government expenditures in the tax-smoothing model implies that the inverse Euler condition is written as an inequality,

$$\frac{1}{v'\left(T_t\right)}\leq E_t\left(\frac{1}{v'\left(T_{t+1}\right)}\right).$$

The important qualitative feature of repeated borrowing with private information is due to Jensen's inequality,

$$E_t\left(\frac{1}{v'(T_{t+1})}\right) < \frac{1}{E_t v'(T_{t+1})},$$

for the negative random variable, $v'(T_{t+1})$, which implies that

$$v'(T_t) > E_t v'(T_{t+1}).$$

This Euler inequality implies that the government borrows more than it would under an equality. In the constrained private information optimum, the government borrows at the margin at a rate of interest less than the common discount rate.

The marginal rate of interest should not be confused with the average rate of interest determined by the change in the value of outstanding government liabilities each period. In the constrained optimum, the risk of the tax burden associated with stochastic public expenditures is shared between the government (that is, tax payers) and its creditors. In the complete information economy with non-binding self-enforcement constraints, taxes are completely smoothed so that taxpayer utility is no longer risky. All risk is borne by creditors whenever the self-enforcement constraints do not bind in the next period for any state. However, with private information tax smoothing is constrained so that both the government's surplus and the value of its liabilities decrease with government expenditures. An adverse shock in period $t$ leads to an increase in taxes in period $t$ and an increase in the present value of government liabilities. That is, the government cannot fully shed its risk but can still partially smooth an adverse expenditure shock forward. Given the expected present value of the government's liabilities, $W_t$, the *ex post* value $w_t$ is higher than $W_t$ for the lowest expenditure state, $g^1$, and lower than $W_t$ for the highest expenditure state, $g^N$. In a low expenditure state, tax revenues are below their *ex ante* expectation so that the utility from current consumption, $v(T_t)$, is higher than expected. The continuation welfare for the government, $\mathcal{V}_{t+1} =$, is also higher as the benefits of a favorable outcome are smoothed forward. The future value of government liabilities, $W_{t+1}$, also falls, but because $w_t$ increases as $g_t$ decreases the effective rate of interest for repayments is higher than the discount rate in a low expenditure state. The opposite holds for high expenditure states: government liabilities, $W_t$, rise but $w_t$ falls below its expected value. The interest rate applied to debt increases is less than the discount rate for creditors.

## 4. Sequential Lending and Bond Contracting under Private Information

The constrained efficient equilibrium with private information requires the government's expected surplus in any period to depend on its surplus and primary deficit in the previous period. This is an intermediate case implemented by complicated contracts that lies between the complete information of tax smoothing and the self-insurance models of tax smoothing when financial instruments are restricted to noncontingent bonds. The value of government liabilities, $w_t$, is not stochastic in the standard self-insurance model with an exogenously imposed solvency constraint. An incentive compatible equilibrium can generate the conventional bond contract if the government can avail itself of a hidden saving or borrowing opportunity. Access to unmonitored borrowing or savings rules out the constrained optimum (Rogerson (1985) as shown by Allen (1985) and Cole and Kocherlakota (2001)).

Unmonitored borrowing arises in the sovereign debt model if sequential creditor entry is allowed. The constrained optimum with only private information refers to a long-term relationship between a single principal (for example, a delegated monitor) and an agent. This long-term relationship can be implemented through a series of short-term state-contingent contracts, but the permanence of the relationship is essential. For sovereign borrowers, short maturity contracts do not necessarily guide long-term relationships between a consolidated lender and the borrower. Rather, sovereigns roll over short-term liabilities by selling new debt to any potential creditor including new ones. The entry of new creditors accepting new short-maturity contracts means that the government faces a sequence of principals whose objectives do not include the wealth of previous lenders. Therefore, new creditors may not have an incentive to enforce the conditionality of the government's future surplus on payments to current creditors required by the efficient private information equilibrium.

The possibility of sequential lender entry adds an additional constraint for incentive compatible equilibrium with private information under the assumption that expiring contract payments are made by issuing new liabilities that cannot be monitored by exiting creditors. This restriction on equilibrium contracts is seen in a failure of incentive compatibility for equilibria in which creditor surplus varies with the government's reported state. In the equilibrium of the previous section, creditor surplus decreases with the government's report of $g_t$ even when the self-enforcement constraint for the borrower does not bind. With sequential lender entry, the total surplus of new lenders at date $t$ is written as $\tau_t + \beta W_{t+1}$ where $\tau_t$ is the total amount paid for an expected gross return equal to $W_{t+1}$. The ex post surplus for holders of expiring short-term contracts is

$$w_t = T_t - g_t - \tau_t.$$

With free entry by lenders, any contract that earns non-negative present value in expectation can be offered. Under the incentive compatible contract for a long-term relationship, the surplus at date $t$ for the lender is a function, $w_t(\widetilde{g}_t)$, decreasing in the report $\widetilde{g}_t$. With sequential free entry by lenders, suppose the government reports $\widetilde{g}_t > g_t$ when the true state is $g_t$. With short-term contracts and entry, the government pays its state-contingent liability for date $t$ given this misrepresentation, $w_t(\widetilde{g}_t) < w_t(g_t)$, and accepts a new contract offer. In the same period, it can borrow an amount $\widetilde{\tau}_t$ that just finances the equilibrium primary deficit, $g_t - T_t$, for the constrained optimum given by

$$w_t(\widetilde{g}_t) = T_t - g_t - \widetilde{\tau}_t < w_t(g_t).$$

For this choice of the report and level of borrowing, the present value of government liabilities carried into the next period (date $t+1$) is

$$\beta \widetilde{W}_{t+1} = -\widetilde{\tau}_t = w_t(\widetilde{g}_t) + g_t - T_t < \beta W_{t+1}.$$

The government increases its surplus by misrepresenting its true state as the worst state, paying its minimum liability and choosing a new contract with lower future liabilities than under the long-term agency contract. Therefore, the equilibrium for the repeated financial relationship between the government and a given principal lender is not incentive compatible for a sequence of lenders.

The argument that incentive compatibility fails for the equilibrium allocation with a single long-term lender reveals the incentive compatibility condition for short-maturity government liabilities with creditor entry. This is the requirement that lender surplus cannot vary with the reported private information of the borrowing government. That is, the ex post payment to lenders at date $t$ given by

$$w_t = T_t - g_t + \beta W_{t+1}$$

depends on the prior surplus of the government (implying the value of its outstanding liabilities, $W_t$) but not on $g_t$. The expected present value of government liabilities must equal the actual present value ($w_t\left(g_t\right) = W_t$ for all $g_t$). Therefore, a sequence of conventional noncontingent debt contracts implements the incentive compatible equilibrium with private government information and free lender entry over time. The discounted level of the future debt, $\beta W_{t+1}$, then varies one for one with the primary surplus, and the government has no incentive to misrepresent its current state.

Using this model, we can observe that unmonitored government borrowing from an entering creditor is equivalent to hidden government savings. The unmonitored new liabilities of the government, $\beta W_{t+1}$, are the negative of the government's financial wealth at the end of period $t$. In their model of the role of hidden savings, Cole and Kocherlakota allow access to a private storage technology in a model of household consumption smoothing with hidden income. They constrain accumulated savings to be non-negative. As proposed here, sequential lending works similarly except that savings is not accumulated in the same way. With state contingent liabilities, savings in period $t$ is only hidden temporarily since this is all that matters for the holder of a maturing government liability. In the presence of state contingent contracts (that is, out of equilibrium), gross savings in each successive period do not need to evolve according to conventional debt dynamics. With the equilibrium noncontingent contracts, unmonitored borrowing has no effect on the returns to holders of maturing liabilities and the government has an incentive to correctly reveal its state,

$g_t$, to new creditors through its net repayment.

Tax smoothing by the government could also be motivated by introducing hidden storage directly. It may well be governments are better informed of their internal assets and liabilities than are either foreign or domestic bondholders. This could suggest a model with costly monitoring rather than strictly unobservable savings. An example of unobservable savings may be available from the optimal taxation model. Basseto and Kocherlakota (2004) show that first-order debt neutrality can hold in a model with distortionary taxation by adopting the Diamond-Mirrlees approach in which the government's objective function is based on household welfare. Neutrality arises if postponed tax collections financed by issuing debt depend only on current information. This works because optimal taxes are non-linear and have an intercept that works as a lump-sum portion of the tax revenue. An interpretation of their result is that the government levies taxes in period $t$ but collects these in period $t' > t$. If the government has private information about its optimal tax problem, then it can create hidden savings by reducing this portion of Diamond-Mirrlees taxes and imposing it on the holders of its maturing contingent liabilities.[10]

The introduction of sequential lenders offering short-maturity contracts in this model of sovereign borrowing is an example of a common agency problem. In a common agency problem, a single agent contracts with a number of principals who may act non-cooperatively monitoring the agent's actions. The private information problem posed here could be analyzed with non-cooperative contemporaneous creditors. However, sequential entry of new principals captures an essential feature of sovereign borrowing that interacts with enforcement of repayments. Non-cooperative behavior between contemporaneous creditors is an important issue in the literature on the resolution of sovereign defaults and is not analyzed here.[11]

A self-enforcing equilibrium for the sovereign borrowing model requires the addition of the participation constraints for the government and its creditors. With sequential lender entry, lender surplus is government debt, and the solution for the constrained tax smoothing equilibrium under this assumption maximizes government surplus subject to the conventional debt identity,

$$W_t = T_t - g_t + \beta W_{t+1}$$

where $W$ is not stochastic. When the participation constraints for the government and lenders are added, the dynamics for the conventional tax-smoothing problem imply that these will bind in some feasible borrowing histories. Setting the government's surplus equal to zero leads to an upper bound for government debt. Once such a bound is reached, net repayments cannot be positive in every state for the government's surplus to remain non-negative and so they must be state contingent. This issue is addressed in two parts. First, we consider the optimal tax-smoothing with the restricted incentive compatibility constraints and participation constraints. Then, the problem of enforcement with common agency and how debt markets may resolve it is discussed.

The first-order condition for tax smoothing with noncontingent borrowing if the participation constraints do not bind in any state in period $t+1$ is given by

$$v'\left(T_t\right) = E_t v'\left(T_{t+1}\right), \tag{18}$$

the standard Euler condition for self-insurance.[12] The participation constraints for bondholders and for the sovereign remain

$$w_{t+1} \geq 0 \quad \text{and} \quad V_{t+1} \geq 0,$$

respectively. If neither of these constraints binds in any possible state, $g_{t+1}$, then the first-order condition (18) holds. If the government's participation constraint binds for some states, then the first-order condition becomes

$$v'\left(T_t\right) \leq E_t v'\left(T_{t+1}\right) < 0. \tag{19}$$

In this case, the government's primary surplus is constrained at time $t$ in state $g_t$ by its willingness to raise taxes to make repayments in some states at time $t+1$. When the participation constraints hold for the government's creditors, the first-order condition is given by

$$v'(T_t) \geq E_t v'(T_{t+1}). \tag{20}$$

In this case, the debt limit for the government is reached with positive probability.

The equilibrium when the sovereignty constraint for the government binds helps us to understand the renegotiation of conventional debt contracts issued by sovereigns. When participation constraints are not binding, equilibrium borrowing and repayment are implemented by conventional, noncontingent, single-period debt contracts. When the sovereignty constraint is binding with positive probability in the next period, the continuation surplus for the government, $E_t V_{t+1}$, equals its lowest possible value, which is zero. Therefore, in any state $g_t$ that

$$v'(T_t) < E_t v'(T_{t+1}), \tag{21}$$

the government's continuation surplus must be zero. Otherwise, taxes could be smoothed more by reducing current taxes, $T_t$, and increasing the debt carried into the next period (that is, lowering the continuation surplus, $E_t V_{t+1}$). The present value of outstanding debt is maximized when the government's constraint binds. The debt limit equals the expected surplus for creditors, $W_{t+1}$, when the expected surplus for the government $E_t V_{t+1}$ equals zero. The debt limit is denoted $\overline{W}$.[13] This debt limit is the maximum willingness to pay in Eaton and Gersovitz (1981) applied to a model allowing debt renegotiation. Incentive compatibility requires that the primary deficit be the same for any state such that inequality (21) holds. If this were not true, then the government could increase its primary deficit without increasing its debt. Whenever the inequality (21) holds in a period, the value of outstanding debt is $\overline{W}$ in the next period.

At the debt limit, taxes are a function of the current state and, naturally, at their highest level in equilibrium. Let $\overline{T}(g_t)$ be the maximum tax revenue collected in state $g_t$ in equilibrium. At the debt limit, expected marginal utility, $E_{t-1} v'\left(\overline{T}(g_t)\right)$, is a constant if shocks are independently and identically distributed. In this case, there must be a pivotal state, $\overline{g}$, such that inequality (21) holds for expenditure shocks greater than $\overline{g}$. In these states, the primary surplus will equal zero because the debt cannot rise above the limit so that taxes equal expenditures, $\overline{T}(g_t) = g_t$, for $g_t \geq \overline{g}$. The pivotal state is determined by

$$v'(\overline{g}) = Ev'\left(\overline{T}(g_t)\right) \tag{22}$$

for $g^1 < \overline{g} < g^N$. For expenditure states below $\overline{g}$, the government runs a primary surplus and its debt falls (so that it has an incentive to run a primary surplus).

As the government's debt rises to the debt limit, it follows the standard identity,

$$\beta W_{t+1} = W_t + (g_t - T_t), \tag{23}$$

until

$$v'\left(g^N\right) < Ev'\left(\overline{T}(g_t)\right)$$

(recall that $g^N$ is the largest shock to expenditures). For shocks such that the inequality (21) holds, the primary deficit can be positive but must satisfy

$$\beta\overline{W} = W_t + (g_t - T_t).$$

In these circumstances, the primary deficit is independent of expenditures and current taxes are given by

$$T_t = W_t - \beta\overline{W} + g_t.$$

Therefore, debt rises from $W_t < \overline{W}$ to $\overline{W}$ in period $t+1$ if the shock at date $t$ is equal to or greater than $\widehat{g}(W_t)$, which is determined by the condition

$$v'\left(W_t - \beta\overline{W} + \widehat{g}(W_t)\right) = Ev'\left(\overline{T}(g_t)\right). \tag{24}$$

Another critical debt level, $\widehat{W}$, can be defined using the first-order condition as

$$v'\left(\widehat{W} - \beta\overline{W} + g^N\right) = Ev'\left(\overline{T}(g_t)\right). \tag{25}$$

If the current debt, $W_t$, is less than $\widehat{W}$, then debt the next period, $W_{t+1}$, remains below the debt limit with certainty. But if the current debt is above the critical level, $\widehat{W}$, then the debt limit will be reached in one period with the probability that $g_t \geq \widehat{g}(W_t)$. This probability rises with current outstanding debt in the interval, $\widehat{W} \leq W_t \leq \overline{W}$, as $\widehat{g}(W_t)$ decreases from $g^N$ to $\overline{g}$ as $W_t$ increases. For $g_t \geq \widehat{g}(W_t)$, the primary deficit equals $\beta\overline{W} - W_t$. For $g_t < \widehat{g}(W_t)$, the primary deficit is decreasing in the expenditure shock and must be in surplus for low expenditure shocks.

If the debt is below the critical level, $\widehat{W}$, public debt follows conventional debt dynamics and repayments are not state contingent. The ex post value of the debt, $w_t$, is the same as the ex ante expected value, $W_t$. The real rate of interest on single-period bonds is equal to the riskless rate equal to the discount rate, $\rho = (1-\beta)/\beta$. When the debt level exceeds the critical level, the present value of outstanding bonds varies with the expenditure shock. The return to single-period bonds is risky. Past the critical level, the present value of public debt satisfies

$$W_t = E_{t-1}\left[(T_t - g_t) + \beta W_{t+1}\right], \tag{26}$$

rather than

$$W_t = (T_t - g_t) + \beta W_{t+1}$$

because the *ex post* value of debt claims, $w_t = (T_t - g_t) + \beta W_{t+1}$, is state dependent. The single-period budget identity can be rewritten as

$$W_{t+1} = (1 + r_{t+1})\left[W_t + (g_t - T_t)\right], \tag{27}$$

where $r_{t+1}$ is the *ex post* yield, which is stochastic if debt exceeds the critical level and equal to $\rho$ if debt is below the critical level. For example, if the debt limit has already been reached, the rate of return is zero for adverse shocks because

$$\overline{W} = (1 + r_{t+1})\left[\overline{W} + (g_t - T_t)\right] = (1 + r_{t+1})\,\overline{W} \quad \text{for } g_t > \overline{g}. \tag{28}$$

For $g_t$ less than $\overline{g}$, the yield is positive and falls with higher expenditures. Combining equations (26) and

(27) leads to

$$\rho\left(W_t + E_{t-1}\left(g_t - T_t\right)\right) = W_t E_{t-1} r_t + E_{t-1}\left[r_{t+1}\left(g_t - T_t\right)\right], \tag{29}$$

implying that the actual return exceeds the discount rate for low expenditure shocks.

The debt limit derived for the tax-smoothing model under sovereign immunity is not the same as the conventional debt limit imposed in most models of government borrowing. In the model with private information, government securities have noncontingent repayments unless the sovereignty constraints can bind with positive probability before maturity. The natural interpretation is that equilibrium borrowing and lending follow an implicit contract guided by standard, noncontingent debt instruments. Contingent repayments are made through renegotiation of the net interest on conventional bonds issued at interest rates that include a positive risk premium when debt outstanding exceeds the critical level. Alternatively, an explicit state-contingent contract would specify contingent repayments only when the debt is above the same threshold. That is, the contract would depend on the state of nature, $g_t$, and the state variable, $W_t$.

Imposing a conventional solvency constraint on bond borrowing does not allow constrained efficient equilibrium tax smoothing in the presence of sovereign immunity. The conventional approach, for example as followed by Aiyagari (1994) and others, will set an upper bound on outstanding public debt, $b_t$, for this economy given by

$$\left(\beta^{-1} - 1\right) b_t \leq \overline{T}\left(g^N\right) - g^N, \tag{30}$$

where the right-hand side of this inequality is the largest primary surplus sustainable in equilibrium in all states. This ensures that noncontingent debt can be repaid in all events. An equilibrium for the tax-smoothing model could be derived after adding the restriction that only noncontingent securities can be issued by the borrower.[14] Restricting contracts to be noncontingent in all events reduces the set of securities that can be used in equilibrium for the tax-smoothing model under private information, since it eliminates securities that were needed to implement an efficient solution. Doing so must lead to a lower debt limit than the limit, $\overline{W}$, derived above and to less smoothing of taxes. In the model of this paper, sovereign immunity is taken as a fundamental and the set of securities is endogenous while in the conventional model the nature of securities and contract enforcement are taken as exogenous.

The question of how state-contingent repayments can be made in the presence of sequential creditor entry has not been addressed. In particular, the constrained efficient equilibrium with the conventional flow identity for a borrower replacing the incentive compatibility condition for long-term debtor-creditor relationship requires contingent repayments whenever government debt exceeds the critical level, $\widehat{W}$. State-contingent contracts are not incentive compatible when subsequent borrowing cannot be monitored. A solution may be found in another aspect of lender coordination. For example, when the outstanding debt in period $t$ equals debt limit, the minimum present value of the debt is $\beta\overline{W}$. Under a state-contingent contract, the government should report $g_t > \overline{g}$ and pay just $\beta\overline{W}$ by issuing new bonds in this amount. If, instead, the debt obligation remains equal to $\overline{W}$, a new creditor will not lend more than $\beta\overline{W}$ requiring the borrower to generate a primary surplus equal to the difference or rollover the obligations with the old creditor. With the upper bound on the present value of all debt obligations, total repayments cannot increase so that new creditors can only lend a positive amount if they gain seniority over old debt. If existing creditors can ensure that they will share in eventual repayments, then they can deter entry by new lenders by rolling over the debt when the upper bound on the present value of payments is reached with positive probability.

The solution to this problem arises from free ridership associated with common agency in which new loans cannot be made because existing creditors do not relinguish their claims to future payments. This coordination problem has been identified frequently as a barrier to resolving sovereign debt difficulties. The monitoring problem can be overcome by writing noncontingent bond contracts and renegotiating repayments ex post when borrowing from other sources is observable with a lag. In this case, creditors should not write down accumulated debt until the incentive compatibility constraint ceases to bind in the constained optimum with sequential entry. This occurs when the equilibrium creditor surplus becomes equal or less than the critical level, $\widehat{W}$.

This model adopts the self-enforcement constraints for both lenders and the sovereign borrower that allow the demonstration of coalition-proof equilibrium in Kletzer and Wright (2000). These allow the existence of self-enforcing equilibria with tax smoothing without exogenous contract enforcement. One way for existing creditors to deter entry with bond contracts is to adapt the punishments proposed by Kletzer and Wright for the complete information model to this bond lending economy.

## 5. Relationship to Stylized Facts of Sovereign Debt

Introducing private information into a self-enforcing equilibrium model of sovereign debt significantly changes the implications of theoretical models of sovereign debt with endogenous renegotiation. With complete information, repayments are also state contingent so that deviations from a conventional noncontingent bond contract occur in each date in all states of nature except the most favorable. This implies frequent default and renegotiation of simple one-period debt contracts independently of the debt level. Actual default is an infrequent event, even for serial defaulters. Defaults are associated with high debt-to-GDP ratios and poor growth performance.[15] Tax smoothing with complete information also implies that the gross return to creditors is zero in the worst state when tax smoothing is incomplete. That is, the full value of the debt can be erased with positive probability in equilibrium.

In contrast, equilibrium under incomplete information can be implemented by one-period contracts that are only state contingent when the present value of outstanding government obligations is above a threshold. Renegotiation will only take place with positive probability after the debt level exceeds the critical level. The probability of default rises above zero only after debt has reached a sufficiently high level, and default only occurs in adverse states. Therefore, the interest premium on short-maturity public debt rises above the discount rate once outstanding debt passes the threshold. In the model, creditors are risk neutral or, equivalently, renegotiation risk is uncorrelated with global market risk. Extending the model to allow for risk-averse creditors would yield a risk premium on short-maturity debt that only turns positive when the critical level is attained. In the economy with private information, the lowest single-period *ex post* net yield on bonds is zero. At the debt limit, bondholders lose net interest with an adverse shock but not bond principal.

Each of these implications is found in the data. The probability of default and debt restructuring is strongly associated with the debt-to-GDP ratio. The yield spread on sovereign bonds rises non-linearly with outstanding debt. For example, Eichengreen, Kletzer and Mody (2005; Table 4) find that the debt-to-GDP ratio has a significant (and positive) effect on emerging market bond yields for countries with debt levels in excess of 60% of GDP but not for countries with debt-to-GDP ratios below 60%. In historical data, Eichengreen and Portes (1995) study a sample of bonds issued in the 1920s in dollars and in Sterling of which approximately half went into default in the 1930s. Bonds in default eventually yielded small positive internal rates of return substantially less than the average rates of return to benchmark US and UK treasury debt. Bonds issued by Brazil in the sample all entered default and yielded an eventual rates of return between 1.1 and 2.3 percent compared with 4.1 percent return to benchmark US treasuries.

The incomplete information economy without unobservable wealth, but with self-enforcement constraints, yields a similar debt limit but implies that marginal rates of interest differ from average rates of interest. In particular, the inverse Euler condition combined with the monotonicity of the value functions in government expenditures implies that the yield spread on loans should decrease with the amount of new borrowing conditional on current indebtedness. It implies that the spread on new loans decreases in the amount lent for a given initial indebtedness. In the large sample of emerging market bonds and bank loans analyzed by Eichengreen, Kletzer and Mody (2005; Table 4), the amount borrowed does not have a significant effect on yield spread for bonds but does for bank loans for all ranges of country indebtedness. The theoretical model may be consistent with their interpretation that bank lenders are better informed monitors of borrowers than are bondholders.

## 6. Nominal Debt, Inflation and Renegotiation

The suggestion by Bohn (1988), Barro (2003), and others that issuing public debt in domestic currency units allows state-contingent real repayments on standard bonds was made in the context of complete information models. In those models, noncontingent bond contracts do not implement the constrained optimum, but noncontingent nominal bonds and unanticipated inflation can generate the required state-contingent real payments to bondholders.[16] In the model of tax smoothing with incomplete information, nominal debt and unanticipated inflation can serve the same purpose of allowing state-contingent real payments with standard, noncontingent securities. An important difference is that state-contingent repayments are only needed to implement a constrained optimum at high levels of outstanding public debt. In complete information models, such as Bohn (1988), unanticipated changes in inflation must be an everyday event to achieve state contingent real returns to nominal bonds. In those models, unanticipated increases and decreases in the inflation rate can occur with any level of outstanding debt issued in nominal terms. In the model with unobservable expenditures and assets, real yields on bonds is not stochastic for all states when the outstanding level of debt is below the threshold. Inflationary surprises should only occur after public debt has reached its threshold level in adverse states. These are the same events in which the debtor's participation constraint binds.

Introducing nominal debt into the model requires adding a motive for holding money by households. Formally, real balances can be introduced as an argument in the utility function for domestic residents in an additively separable way. This gives the standard relationship between current consumption, the nominal stock of money, the nominal interest rate, and the price level. For example, for logarithmic utility in real balances, real balance demand will be given by

$$\frac{M_t}{p_t} = u'(c_t)\,\kappa \frac{1+i_{t+1}}{i_{t+1}}, \tag{31}$$

for a constant $\kappa$ and nominal interest rate $i_{t+1}$. Outstanding nominal debt is given by $B_t = p_t W_t$, so that nominal debt evolves according to

$$B_{t+1} = (1 + i_{t+1}) [B_t + p_t (g_t - T_t)], \tag{32}$$

and real debt follows

$$W_{t+1} = \frac{p_t}{p_{t+1}} (1 + i_{t+1}) [W_t + (g_t - T_t)]. \tag{33}$$

If inflation is stochastic, then the change in the real value of the debt is also stochastic. Replication of the equilibrium with sovereign bonds requires that the real return to nominal bonds equals the equilibrium yield for real bonds,

$$\frac{p_t}{p_{t+1}} (1 + i_{t+1}) = 1 + r_{t+1}. \tag{34}$$

When the real value of the debt is less than the critical level, the Fisher interest parity condition,

$$(1 + i_{t+1}) = \frac{p^e_{t+1}}{p_t} \beta,$$

holds, where actual inflation must equal expected inflation to support the constrained optimum with noncontingent, nominal interest rate bonds. When the debt is above the critical level, then actual inflation should deviate from expected inflation to satisfy equation (34).

The motivation for issuing public debt in nominal terms is exactly the same as in Bohn (1988). State-contingent fluctuations in the *ex post* real yield on bonds can be achieved through unanticipated inflationary increases in the money supply. The difference is that the model implies that nominal debt issues should not lead to frequent inflationary surprises for low levels of outstanding public debt. It also predicts that unanticipated inflation taxes are associated with high levels of debt and adverse shocks.

Issuing debt denominated in domestic currency can allow the implementation of the constrained efficient equilibrium without explicit state-contingent government securities or formal debt default and renegotiation at high levels of outstanding debt. Bonds indexed in commodity units or denominated in units of foreign currency must be renegotiated and restructured explicitly when the debt limit is reached.

The ease of reducing the outstanding stock of debt through unanticipated inflation together with the possibility of issuing debt on other terms suggests a credibility problem. The equilibrium of the incomplete information model of sovereign debt can be supported by renegotiation-proof punishment strategies following Kletzer and Wright (2000). The possibility that the government can issue debt in terms of home or foreign currency offers another strategy for the debtor. This is the opportunity to rapidly inflate away the value of outstanding domestic currency debt while issuing new bonds in foreign currency units to achieve the required rollover. The intuitive reputational equilibrium in which once inflation deviates from the constrained optimum, the government cannot reissue bonds in its own currency could be constructed for the self-enforcing equilibrium model with asymmetric information.

Two approaches to punishment for excessive inflation with nominal debt are useful to consider. Suppose the government's outstanding debt is denominated in domestic currency and the money supply increase exceeds the amount given by equation (34). Enforcement of the original equilibrium requires the debtor's continuation surplus, $E_t V_{t+1}$, be reduced enough to offset the increase in the government's present gain from reducing the real value of its current debt stock. This can be accomplished by either issuing new domestic currency debt or issuing foreign currency or real-indexed debt. In the first case, the existing debt may be refinanced by issuing new bonds denominated in the debtor's currency at nominal interest rates exceeding the required equilibrium real rate of return by an inflation rate that is a credible upper bound for the sovereign's currency. The excessive money supply growth at time $t-1$ results in inflation, $\frac{\widehat{p}_t}{p_{t-1}}$, reducing the value of nominal debt claims to $W_t'$ from what was expected, $W_t$,

$$W_t > W_t' = \frac{p_{t-1}}{p_t'} (1 + i_t) \left[ W_{t-1} + (g_{t-1} - T_{t-1}) \right] . \tag{35}$$

The terms of the new issues, including the primary surplus at time $t$, will need to satisfy

$$W_{t+1} \leq \frac{p_t'}{p_{t+1}'} \left( 1 + i_{t+1}'' \right) \left[ W_t' - s_t'' \right] , \tag{36}$$

where primes denote the deviation from equilibrium and double primes the terms of new loans. These terms ensure no gain from deviation. This only works if there is a credible upper bound on the inflation rate. If the government otherwise cares about inflation, then continuing to inflate away the value of the debt at ever higher rates might be too costly. In this case, nominal debt could be sustainable, but if the government is insufficiently inflation averse, then reversion to foreign currency bonds can be a punishment strategy. In this case, new foreign currency bonds achieve the appropriate punishment if the interest terms are chosen so that condition (36) is satisfied as

$$W_{t+1} \leq \left(1 + r''_{t+1}\right) \left[W'_t - s''_t\right],$$

where the real interest rate and primary surplus leave the borrower with no gain for devaluing the outstanding debt the previous period.[17] This second punishment strategy may not suffice to sustain nominal debt issues simply because the holders of the new foreign currency bonds face no consequences for accepting new bonds at the equilibrium rate of interest consistent with the post-inflation real value of the government's outstanding debt. With free bondholder entry, the model implies that a credible equilibrium with nominal debt requires either unassociated inflation costs or costs to debt renegotiation in default. The equilibrium is derived assuming costless renegotiation. Inflationary surprises are a low cost means of renegotiation, whereas default and subsequent debt restructuring are costly. The ease of unilateral renegotiation through inflationary monetary expansions and the large immediate costs of default favor issuing debt in domestic currency. However, a reputational model of “original sin” is consistent with the framework of this model.

## 7. Conclusion

A model of sovereign borrowing to finance public spending in the presence of exogenous shocks and distortionary taxation is used to motivate sovereign debt renegotiation. The debt limit for the government is determined endogenously given information asymmetries that motivate conventional bond contracts. The model shows that the interpretation of debt renegotiation as the playing out of an implicit contract with state contingent repayments that is guided by noncontingent bond contracts implies continuous renegotiation under complete information. With private information, renegotiation only occurs when the debt level exceeds a sufficiently high threshold and domestic shocks are adverse. As a consequence, sovereign risk premiums should only rise above the opportunity cost for foreign creditors after the debt threshold is reached for single-period maturity bonds. This conclusion is consistent with the nonlinear effect of debt on bond spreads found empirically (for example, by Eichengreen, Kletzer and Mody (2004)). Further, the model shows that creditor losses in default are bounded by the flow of net interest accruing during periods of distress, consistent with historical empirical studies.

The usefulness of nominal public debt for achieving contingent repayments with a tax-smoothing motive under uncertainty is reconsidered by comparing equilibrium with and without debtor private information. In the complete information case (corresponding to previous models, for example, Bohn (1988) and Barro (2003), with conventional solvency constraints), optimal inflation is stochastic each period. The incomplete information model implies that debt reduction through unanticipated inflation should not be an everyday contingency. Debt reduction achieved by inflationary surprises is optimal only when outstanding public debt exceeds a critical threshold and limited to the extent of holding real outstanding debt equal to the sovereign debt limit. The convenience of nominal debt is that renegotiation can be achieved unilaterally subject to maintaining credibility in the reputational equilibrium sustaining public debt and repayment in the model. This is an advantage if bilateral or multilateral renegotiations of foreign currency denominated debt are costly as experience suggests. It also raises interesting issues for sustaining nominal debt in a reputational equilibrium in the presence of the alternative of issuing debt in foreign currency. Inflation indexation of nominal debt may offer an intermediate step, allowing unilateral renegotiation, but restricting real reduction to explicit default on the indexation clause.

**Appendix**

The private information constrained optimum is found by maximizing the expectation of creditor surplus,

$$W_t = E_{t-1}\left(T_t - g_t + \beta W_{t+1}\right)$$

with respect to the primary surplus, $\tau_t = T_t - g_t$ and the expected government surplus $\mathcal{V}_{t+1}(g_t) = E_t V_{t+1}(g_t)$, subject to the government's surplus promise for period $t$,

$$E_{t-1}\left(v\left(\tau_t + g_t\right) - v\left(g_t\right) + \beta \mathcal{V}_{t+1}\left(g_t\right)\right) \geq \mathcal{V}_t,$$

the incentive compatibility constraints

$$v\left(\tau_t^i + g^i\right) - v\left(g^i\right) + \beta E_t V_{t+1}\left(g^i\right) \geq v\left(\tau_t^i + g^{i-1}\right) - v\left(g^i\right) + \beta E_t V_{t+1}\left(g^{i-1}\right),$$

for each $i = 2, ..., N$, and the participation constraints,

$$\mathcal{V}_{t+1}\left(g_t\right) \geq 0, \qquad \text{for } g_t = g^i \text{ for each } i = 1, ..., N$$

and

$$W_{t+1}\left(\mathcal{V}_{t+1}\left(g_t\right)\right) \geq 0, \qquad \text{for } g_t = g^i \text{ for each } i = 1, ..., N.$$

The notation abuse in the use of time subscripts on the surpluses for the government and creditor indicate dependence on the state variable, $\mathcal{V}_t$, is used to ease reading. Sufficient conditions will be satisfied under the concavity assumptions made in the text (see Thomas and Worrall (1990)).

The first-order conditions include

$$\pi_j\left(\frac{1}{v'\left(\tau_t^j + g^j\right)} + \lambda + \mu_j\right) - \pi_{j+1}\mu_{j+1}\frac{v'\left(\tau_t^j + g^{j+1}\right)}{v'\left(\tau_t^j + g^j\right)} = 0$$

and

$$\pi_j\left(\left(1 + \psi_{t+1}^j\right) W'_{t+1}\left(\mathcal{V}_{t+1}\left(g^j\right)\right) + \lambda + \mu_j + \varphi_{t+1}^j\right) - \pi_{j+1}\mu_{j+1} = 0,$$

where the multiplier for the government's surplus for period $t$ is $\lambda$, the multiplier for the incentive compatibility constraint for state $j$ is $\mu_j$, the multipliers for the participation constraints for the government and the creditor are $\varphi_{t+1}^j$ and $\psi_{t+1}^j$, respectively, and $\pi_j$ is the probability of state $g^j$.

Taken together, these are two difference equations in the multipliers $\mu_j$. For simplicity, these are solved leaving the multipliers $\varphi_{t+1}^j$ and $\psi_{t+1}^j$ equal to zero. Solving these forward over $j = 1, ..., N$ by setting

$\mu_1 = 0$, the necessary conditions (including the envelope condition) lead to

$$W'_{t+1}\left(\mathcal{V}_{t+1}\left(g^j\right)\right) = -\lambda_{t+1} = E_t\left[\frac{\theta_{t+1}}{v'\left(\tau_{t+1}+g_{t+1}\right)}\right]$$

and

$$\begin{aligned} & W'_{t+1}\left(\mathcal{V}_{t+1}\left(g^j\right)\right) - \frac{1}{v'\left(\tau_t^j + g^j\right)} \\ = & \left(\frac{v'\left(\tau_t^j+g^j\right)}{v'\left(\tau_t^j+g^{j+1}\right)} - 1\right)\left[E_{t-1}\left(\frac{\theta_t}{v'\left(\tau_t+g_t\right)} \mid g_t \leq g^j\right) + \lambda_t\right]\frac{\sum_{i=1}^j \pi_i\theta^i}{\pi_j\theta^j} \end{aligned}$$

where

$$-\lambda_t = E_{t-1}\left(\frac{\theta_t}{v'\left(\tau_t+g_t\right)}\right),$$

and

$$\theta^j \equiv \frac{R_j}{\sum_{j=1}^N \pi_j R_j}$$

for

$$R_j = \prod_{i=1}^{j-1}\frac{v'\left(\tau_t^i + g^{i+1}\right)}{v'\left(\tau_t^i+g^i\right)} \quad \text{for } j = 2, ..., N \quad \text{and } R_1 = 1.$$

The expression $\frac{\sum_{i=1}^j \pi_i\theta^i}{\pi_j\theta^j}$ is the hazard rate familiar to principal-agent problems for the distribution $\pi_j\theta^j$ for the case in which the best state is $1$ and the worst state is $N$.

These necessary conditions modify the inverse Euler condition and are written in the text as

$$\frac{1}{v'\left(T_t\right)} + \gamma_t = E_t\left[\frac{\theta_{t+1}}{v'\left(T_{t+1}\right)}\right],$$

where $\gamma_t$ is positive. If the participation for the government binds for period $t+1$, the inverse inequality is

$$\frac{1}{v'\left(T_t\right)} + \gamma_t \leq E_t\left[\frac{\theta_{t+1}}{v'\left(T_{t+1}\right)}\right],$$

with the opposite inequality if the participation constraint for the creditor binds.

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**Endnotes**

[1]The time inconsistency of long-maturity debt has been shown by several authors, for example, by Bolton and Jeanne (2006). The point that asymmetric information about the action of other lenders can also lead to debt dilution appears to be first pointed out by Kletzer (1984) for sovereign debt.

[2]For example, Persson and Svensson (1984), Bohn (1988, 1990), Calvo and Guidotti (1990), Giavazzi and Pagano (1990), Chari, Christiano and Kehoe (1994), Missale and Blanchard (1994), Aiyagari, Marcet, Sargent and Seppala (2002) and Barro (1999, 2003).

[3]Bohn (1988, 1990) considers why nominal public debt is predominant in light of the conclusions of Lucas and Stokey (1983) and shows that nominal debt indexation is constrained optimal in a model following Barro (1979). Barro (1999, 2003) argues that nominal indexation allows potentially welfare improving contingent repayment with exogenous stochastic inflation.

[4]All of the analytics can be extended to the case in which $g_t$ follows a Markov chain.

[5]The replacement of a tax-smoothing objective for a consumption-smoothing objective for the representative household is demonstrated in Zhu (1992).

[6]For the stochastic $g_t$ following a Markov chain (including independently and identically distributed $g$), this surplus can be written as $V(w_t, g_t)$. The model extends immediately to the case in which $g_t$ follows a Markov chain.

[7]This interpretation of debt renegotiation as the implementation of an implicit contract was suggested by Grossman and van Huyck (1988).

[8]Atkeson (1991) introduces asymmetric information in a repeated moral hazard model of sovereign debt with one-sided commitment. Kletzer (2005) discusses bond contracts with no observability of debtor income in a model with two-sided noncommitment.

[9]The time subscripts on the functions reflects dependence on ex ante surplus, $E_{t-1}V_t$.

[10]In the case of state-contingent contracts, eliminating tax collections reduces debt so that this policy shift is not neutral. Debt neutrality does hold in an incentive compatible equilibrium that accounts for hidden savings absent sovereign default risk.

[11]Eichengreen, Kletzer and Mody (2005) discuss delegated monitoring and creditor coordination problems. They also provide a large number of references to the literature on creditor coordination failures in crisis resolution.

[12]The model implicitly assumes no global constraints, and this first-order condition implies convergence toward a steady state in which taxes are zero and the interest on government credit covers the upper bound for expenditures. With risk-averse counterparts, the steady-state gross interest rate, $R$, would be smaller than $\beta^{-1}$ and the first-order condition would become $v'(T_t) = R\beta E_t v'(T_{t+1})$.

[13]The debt limit $\overline{W}$ is determined endogenously by the condition,

$$v\left(\widehat{T}(g_t)\right) - v(g_t) + \beta E_t V_{t+1} = 0,$$

and the inequality (21), which holds with equality for $E_t V_{t+1} > 0$.

[14]Two papers that endogenize the debt limit but place exogenous constraints on contracts are Aiyagari (1995) and Aiyagari and others (2002).

[15]Reinhart, Rogoff and Savastano (2003) survey the experience of repeated default and generate these stylized facts, among others.

[16]Barro (2003) does not model optimal fiscal and monetary policy with nominal debt, but instead argues that nominal debt with independently determined inflation allows state contingency that may be welfare-improving. Bohn (1988, 1990) considers optimal policy without endogenizing debt limits.

[17]As in deviations from equilibrium payment with contingent contracts and sovereign risk, holders of debt at time $t-1$ do suffer capital losses. The borrower cannot gain by deviating from the implicit contract in perfect equilibrium (see Kletzer and Wright, 2000).